MULTIPLE CRITERIA
DECISION MAKING '10-11

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CONTENTS

PREFACE

This book includes theoretical and application papers from the field of multicriteria decision making. The authors are faculty members of the University of Economics in Katowice, Department of Operations Research, and researchers from Poland and abroad, collaborating with the Department.

In the paper *Improving Teaching Efficiency: An Application of QFD and AHP* A. Anis and R. Islam identify the requirements of students in the Faculty of Business to improve the efficiency of the teaching and learning system and show how to derive the priorities of the requirements.

In the paper *An ANP-based framework for revenue management* P. Fiala presents network revenue management models to maximize revenue when customers buy bundles of multiple resources.

In the paper *An interactive approach determining the indifference thresholds in Promethee II* H. Moalla Frikha, H. Chabchoub and J.M. Martel consider usual and quasi criteria, exploit the information provided by the decision-maker and by using mathematical programming.

In the paper *Sustainability in mining: An application of Prométhée II* L. Gomes, M. Macedo and L. Rangel show how applied approach can be put into practice, providing a more global and transparent result.

In the paper *On the choice of method in multi-criteria decision aiding process concerning European projects* D. Górecka presents the main strengths and weaknesses of selected decision aiding tools applicable to the problem considered as well as of chosen procedures aiming at facilitating the process of selecting an appropriate one. Moreover, an extension of EXPROM II by stochastic dominance rules is proposed.

In the paper *R&D rivalry and cooperation in duopoly: Firm organization, welfare and policy implications* J. Hanna argues that the most efficient industry organization occurs when firms cooperate and fully share R&D results but remain competitive in the final goods market.

In the paper *Application of DEA model with bootstrap to evaluation of SMEs efficiency in the Spanish textile industry* M. Kapelko shows that firms in the sample are on average relatively highly efficient in their productive process and the bias-corrected efficiency score slightly fluctuates during the period analyzed.

In the paper *Manager preferences modelling for stochastic aggregate planning* M. Mezghani, T. Loukil and B. Aouni aims to develop a goal programming model where the goals and the right-hand sides of constraints are random and normally distributed.

In the paper *Multi-criteria decision aiding in project planning using decision trees and simulation* B. Nowak and M. Nowak present a simple, yet comprehensive, methodology that permits the consideration of both multiple criteria and risk.

In the paper *An approach to modeling altruistic equilibrium in games* D. Podkopaev proves that any Pareto optimal strategy profile can be an equilibrium if the level of player altruism is high enough.

In the paper *Multi-criteria decision making models by applying the Topsis method to crisp and interval data* E. Roszkowska systematises the knowledge about the method and solves simple numerical examples that show practical applications of different aspects of the method, especially in the negotiation process

In the paper *Compromise Hypersphere for Stochastic Dominance Model* S. Sitarz presents a method of ranking a finite set of discrete random variables.

In the paper *Application of TOPSIS Methodology to the Scoring of Negotiation Issues Measured on the Ordinal Scale* T. Wachowicz focuses on the pre--negotiation preparation and the process of negotiation template evaluation, which results in building a scoring system for the negotiation offers

In the paper *Analytic Network Process in ERP system selection* P. Wieszała, T. Trzaskalik and K. Targiel present a framework for selecting a suitable ERP system for a small manufacturing enterprise.

In the paper *The dominance-based rough set approach* (*DRSA) applied to bankrupcy prediction modeling for small and medium businesses* K. Zaras proposes a hybrid DRSA model, in which the discrimination analysis results are used to explain the decision rules obtained from regional experts.

The volume editors would like to thank the authorities of the University of Economics in Katowice for support in editing the current volume in the series Multiple Criteria Decision Making.

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Azilah Anis

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IMPROVING TEACHING EFFICIENCY: AN APPLICATION OF QFD AND AHP

Abstract

HSBL University College is one of the privately run institutions of higher learning in Malaysia which offers, among others, various programs in Economics, Accounting and Business Administration. A recent survey conducted by the Faculty of Business (FB) of HSBL found that the students were not fully satisfied with the teaching and learning system of the college. The present work has been carried out to identify the FB students' requirements to improve the efficacy of the teaching and learning system. Having identified the requirements, a number of lecturers were contacted to extract the design requirements that would address the students' needs. The novelty of the paper is that Quality Function Deployment and Analytic Hierarchy Process both have been applied to derive the priorities of the design requirements. The results obtained by the above two methods have shown close resemblance.

Keywords

Quality in higher education, ranking, Quality Function Deployment, Analytic Hierarchy Process.

Introduction

Quality in education is important to ensure an adequate supply of qualified, highly skilled and well trained manpower [Doherty, 2008]. Quality in higher education has attracted greater interest and wider discussion as society has come to realize the crucial importance of trained manpower to its socio--economic development and well being. Higher education plays an important role in providing quality, trained manpower, which is crucial to an economy in creating and maintaining a competitive edge over its competitors [Hwarng and Teo, 2000]. Quality education means adding value to students and ultimately to the society, so that students are enriched not only in their knowledge, skills and techniques but also in intellectual growth and development [Hwarng and Teo, 2001].

In Malaysia, a number of initiatives have been taken to ensure quality in education. In 1996, the Ministry of Education launched a customer charter, formalizing the inception of Total Quality Management (TQM) in the Malaysian education sector. The ministry formed a policy and quality section to monitor the implementation of the country's education policy at all levels, based on the TQM principles, with a vision that all schools and universities will eventually adopt the TQM principles. To control the standards of public higher education institutions, the National Higher Education Council was formed in 1996. A grading system was put in place to assess the effectiveness of each department and faculty. In 1997, the ministry launched the National Accreditation Board (LAN) to assess the quality of higher education institutions. On 1 November 2007, the Malaysian Qualification Agency (MQA) was established that enforced its own Act (Malaysian Qualification Agency Act 2007). The MQA is responsible for monitoring and overseeing the quality assurance practices and accreditation of national higher education. The establishment of MQA saw LAN dissolved and its personnel absorbed into the MQA.

Currently the ISO 9001 has been widely implemented by most of the universities and colleges to assure good performance and the customers of higher education are being well served (Sohail et al., 2003). Currently, there are a number of universities and university colleges in Malaysia that have already been certified with the ISO 9001 and one university college which is on the list is the HSBL University College.

Even though the HSBL University College has already obtained the ISO 9001, there are certain issues that are worth mentioning.

– Most of the students enrolled in HSBL University College have minimum entry requirements. The lecturers encounter difficulties to deliver their lectures due to the low ability of the students to understand, apply and conceptualize the theory and practical issues that are being taught. The lecturers keep on complaining on this matter, but there is no formal action plan to find solutions to improve the situation. As such, opinions and comments from both parties (lecturers and students) are important in developing effective strategies for the teaching and learning process.
– In order to improve the quality of teaching, a teaching evaluation is conducted at the end of every semester. The evaluation provides comments on lecturers' effectiveness for every module that has been taught. However, the existing evaluation system does not have a good impact on the lecturers since the feedback that is given by the respective department is not precise and actionable enough.
– Faculty of Business (FB) in HSBL University College lacks core programs to offer to the market, thus the number of students in FB was decreasing from year to year. In June 2004, drastic changes were made and various programs were offered. This effort resulted in increasing the number

of students and academic staff, but not as much as the FB was expecting. Besides the weak marketing strategies, one of the factors is the negative word of mouth among students about the poor teaching methodologies.

It shows that from the issues raised above, the main area seems to be the teaching effectiveness of the lecturers. Thus, based on the previous discussions, this paper will focus on: *How the students' voices play its role in contributing feedbacks to improve the teaching effectiveness among lecturers, particularly in the Faculty of Business, HSBL University College.*

The identification of the problem statement above has led to constructing the main objective of this paper which is: *To develop policies to be implemented by HSBL University College in order to improve the students' satisfaction by improving the lecturers' teaching effectiveness*.

1. Quality Function Deployment

Quality Function Deployment (QFD) is a quality assurance tool that helps ensure that the voice of the customer is heard and followed in the development of a product or service [Pitman et al., 1996]. Ermer [1995] emphasized that QFD is a design tool that matches customers' requirements with the necessary system design elements. This structured approach gives increased focus to understanding customers' requirements. According to Hwarng and Teo [2001], QFD is a methodology for the development or deployment of features, attributes, or function that give a product or service high quality. QFD can be very useful in answering the question on how to deliver quality products and services based on the needs of customers. It is simply a planning tool that begins with market research that identifies what the customers like, which is called the Voice of Customers (VOC). It is through the QFD process that the VOC is translated into system and part requirements.

QFD found its first use at Mitsubishi's Kobe shipyard site in 1972. Today QFD is used successfully by manufacturers of electronics appliances, clothing and construction equipment firms such as General Motors, Ford, Mazda, Motorola, Xerox, Kodak, IBM, Procter and Gamble, Hewlett-Packard, AT&T, etc. [Evans and Lindsay, 2005, p. 387]. QFD is also proved to be an effective tool in improving quality in higher education. One of the earliest uses of QFD in education was by Emer at the Mechanical Engineering Department of the University of Wisconsin in 1991 where the department chairman used it to assess and respond to the needs of his department. Other applications of QFD in higher education are reported by Jaraiedi and Ritz [1994], Pitman et al. [1996], Hilmer et al. [1995], Sandvik and Hakun [1996], Mohamad and Aspinwall [1998], Lam and Zhao [1998], Hwarng and Teo [2001], Peters et al. [2005], Bier and Cornesky [2001] and Salih et al. [2003].

According to Pitman et al. [1996], the fundamental tasks of the QFD are:

- To identify the customers.
  In identifying the customers, the organization must objectively determine the group that best describe its current and/or desired customer base. After the customer base has been identified, the *wants* of the customers are determined.
- To identify what the customers' wants.
  These wants are commonly referred to as the *whats*, and can be derived by using a wide variety of methods. When collecting these *whats*, it is critical for the organization to use the terms, phrases and languages of the customers. After collecting the *whats*, the QFD team works with the customers to determine priorities of the *whats*.
- To identify the design requirements that can fulfill the customers' requirements.
  Once the *whats* are identified, the QFD team determines the mechanism that would satisfy the *whats*. These mechanisms are commonly referred as the *hows*. The *whats* are expressed in customers terms, whereas the *hows* are expressed in technical, corporate terms.

With the *whats* and *hows* in place, the QFD team establishes relationship between them. Evans and Lindsay [2005, p. 573] noted that the purpose of the relationship is to show whether the final technical requirements (*hows*) have adequately addressed customers requirements. In indicating the relationship between the *whats* and the *hows*, the QFD team assigns a strength value of none, weak, medium or strong to each relationship.

After the relationship matrix has been developed, there is a need to place a priority on each issue that was considered in the design process [Peters et al., 2005]. By using the value of 9 (high), 3 (medium) and 1 (low) and 0 (none) as weights, a design issue's importance weighting measure can be calculated by taking the weighted sum of its relationship i.e., ∑[(value of relationship strength) x (customer importance rating)]. Thus, the value of the weighting measure will indicate the rank of the design issue. The highest weighting measure will indicate the importance of the design issue in fulfilling the voice of customers and vice versa.

The translation process uses a series of matrices, commonly known as the House of Quality (HoQ) as shown in Figure 1. Normally, a HoQ diagram consists of the following information:

- What's? (Voice of Customers),
- How's? (Design Requirements),
- Relationship Matrix,
- Correlation Matrix,
- Customers' Assessment,
- Technical Assessment.

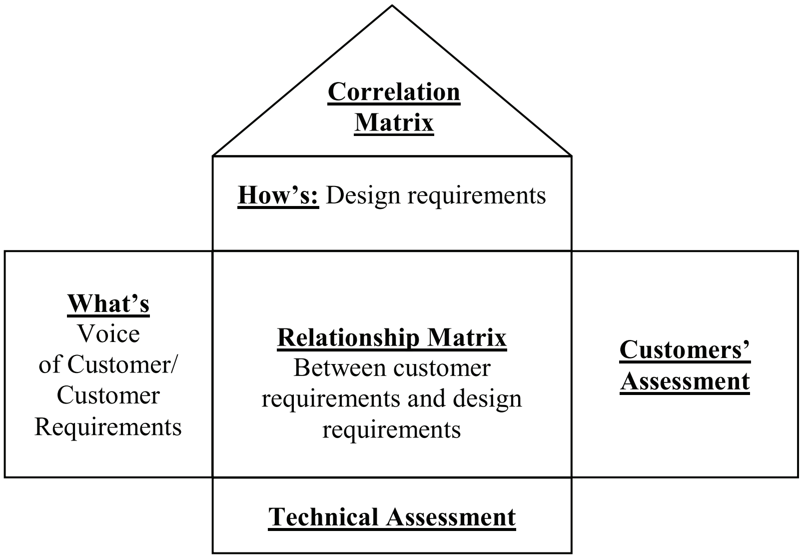

Figure 1. Framework of House of Quality

2. Analytic Hierarchy Process (AHP)

According to Lam and Zhao [1998], AHP is a decision making method for prioritizing and selecting decision alternatives when multiple decision--making criteria are considered. The AHP offers a methodology to rank alternative courses of actions based on decision maker's judgments concerning the importance of the criteria and the extent to which they are met by each alternative. The Analytic Hierarchy Process (AHP) was introduced by Saaty in 1977 and solves a multiple criteria decision making problem using three steps:
(1) Find out the overall goal, criteria, subcriteria, and alternatives and form a linear hierarchy involving all of them in several levels,
(2) Form pairwise comparison matrices for all the criteria, subcriteria and alternatives and compute their weights by using a suitable weight determination technique,
(3) Synthesize all the local sets of weights to obtain a set of overall or global weights for the alternatives. A pairwise comparison matrix in Step 2 has the form:

$$A = \begin{array}{c|cccc} & F_1 & F_2 & \dots & F_n \\ \hline F_1 & a_{11} & a_{12} & \dots & a_{1n} \\ F_2 & a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ F_n & a_{n1} & a_{n2} & \dots & a_{nn} \end{array}$$

where F_i's are the factors (meaning either criteria or alternatives whose weights are to be determined), $a_{ij} = w_i / w_j$, for all i, j, and $w = (w_1, w_2, \ldots, w_n)^T$ is the underlying weight vector for the n factors. Each entry a_{ij} of A is the answer to a typical question, "Given two factors F_i and F_j, which is more dominant (or preferable or important) and what is the degree of this dominance?" The answers are usually given verbally, for instance: F_1 is weakly (or strongly) more dominant over F_j. Later, these verbal qualitative phrases (weakly or strongly more) are quantified by means of the (1/9-9) ratio-scale. For example, if F_1 is strongly more dominant over F_2, then $a_{12} = 5$. The interpretation of all the numerical judgments of the (1/9-9) scale is given in the Table 1.

Table 1

AHP verbal Scale

Verbal Judgment of Preference	Numerical Rating
Equal importance	1
Equal to moderate importance	2
Moderate importance	3
Moderate to strong importance	4
Strong importance	5
Strong to very strong importance	6
Very strong importance	7
Very strong to extreme importance	8
Extreme importance	9

Note: If any factor F_i has importance strength over F_j as any of the above nonzero numbers, then F_j has the reciprocal importance strength with F_i, i.e., $a_{ji} = 1 / a_{ij}$.

From the foregoing discussion, it is intuitively clear that if F_1 is 5 times more important than F_2, then F_2 is 1/5 times more important than F_1. It has been stated that each a_{ij} is the ratio of the two weights w_i and w_j. Now, if we multiply A by the weight vector w from the right, we get

$$Aw = nw \tag{1}$$

where n is the order of the matrix, i.e., the number of factors compared. So, we can recover the weight vector w from (1), provided $(A-nI)w = 0$ has non-trivial solution, i.e., $|A-nI| = 0$, i.e., n is the eigenvalue of A. We also note that $a_{ij} = (w_i / w_k)(w_k / w_j) = a_{ik}a_{kj}$, which is known as *cardinalconsistency relation.* If all the elements of A satisfy this relation, then we say that the matrix is consistent, otherwise that it is inconsistent. In reality, especially within the framework of the AHP, the matrix A is hardly consistent. In the inconsistent case, Equation (1) becomes

$$A'w' = \lambda_{\max} w' \tag{2}$$

where λ_{max} is the largest eigenvalue of A'. Here the matrix A has been perturbed to A' and the consistency relation is violated at least once. For simplicity, the primes are omitted in the following notations and expression. To find out the weights, first we determine the largest eigenvalue λ_{max} of A. Then the weights w_i's are determined by solving the following system of linear simultaneous equations:

$$w_i = \frac{1}{\lambda_{\max}} \sum_{j=1}^{n} a_{ij} w_j, \quad i = 1, 2, \ldots, n \tag{3}$$

For uniqueness, we normalize the set of weights so that $\sum_{i=1}^{n} w_i = 1$. In practice, Expert Choice software is used to compute the weights from the pairwise comparison matrices.

3. Teaching Effectiveness

Lecturers are directly entrusted with providing education to the students. Therefore, quality in education is substantially dependent on lecturers' commitment. Quality lecturers produce quality students. Excellent lecturers will be able to provide more satisfaction, exceeding the expectation of the students [Wan Jaafar, 1996].

The definition of teaching effectiveness varies from researcher to researcher. Abrami [1989] recognized that the nature of effective teaching could vary across instructors, courses, and students. But since this paper pertains to the ways through which the lecturers are going to fulfill their students' needs, therefore, the area that will be covered for teaching effectiveness is only on the lecturers' side and the categories considered are the following [Rosenshine and Furst, 1973; Swan et al., 2003].

a) *Clarity*: It involves cognitive clearness of a lecturer's presentation. A lecturer with greater clarity presents points that are clear and easy to understand, explains concepts clearly and answers questions with clear and good organization [Swan et al., 2003]. It describes the ability of the learners to clearly see, hear and understand what is being said. Clarity was found to be the number one factor leading to improved learning [Rosenshine and Furst, 1973].
b) *Variability*: A lecturer's ability to use a variety of materials. High variability lecturers use a variety of instructional materials, teaching devices, types of tests and different level of learners' task [Swan et al., 2003]. Some students learn better by listening, some by seeing and some by doing. Regardless of the best mode of learning, it helps students if the lecturer covers the material in a variety of ways [Rosenshine and Furst, 1973].
c) *Enthusiasm*: This refers to a lecturer's enthusiasm. Highly enthusiastic lecturers use movement, gesture, voice inflections and questioning of interpretation of test [Swan et al., 2003]. The enthusiasm of a lecturer is contagious. If the lecturer shows interest in a topic, students are more likely to be interested. If the lecturer apologizes for how boring a topic is, do not expect the students to stay awake and listen to the lecturer [Rosenshine and Furst, 1973].
d) *Task orientation*: This relates to a lecturer's degree of task orientation, achievement-orientedness. People tend to learn better when they are engaged in a task. Lecturers who keep guiding their students back to the topic have a better chance of achieving their objectives. Checklist, procedure sheets and other aids may help students stay on the task [Rosenshine and Furst, 1973].
e) *Opportunities for students to learn additional material*: The degree of opportunities of a lecturer provides the students with opportunities to practice what is being taught. A positive relationship exists between the material learned in the course and student achievement on a certain test [Swan et al., 2003]. Students should be given the opportunity to engage with the materials. This could mean that the lecturer remains silent at times

to allow the students to digest what they have learnt. Or, perhaps, there is an activity where the student writes something or discusses with the person next to them.

4. Data Collection

According to the requirements of QFD, data have been collected from the following respondents:

- Students – to gather the voices of customers (stated as students' needs).
- Lecturers – to obtain the design requirements (mentioned as lecturers' designs).

4.1. Voices of Customers (Students' Needs)

The voices of customers were obtained through focus group interviews and discussion with 18 students from semester 4, 5 and 6. Most of them are diploma holders with years of experience in learning process. Thus, they are more determined to get their needs and expectations fulfilled. The students articulated their requirements for effective teaching. The voices of customers were then synthesized to identify genuine needs, as opposed to unnecessary wants of the students. All the items of students' needs were grouped based on similar characteristics to finalize FB students' needs. The students' needs were then classified into five categories: clarity, variability, enthusiasm, task orientation, and opportunities to learn. The details of their needs are presented in Table 2.

Table 2

Students' needs towards improving lecturers' teaching effectiveness

Clarity – Detailed explanation – Detailed notes – Clear and understandable English – Examples	*Variability* – Variety of materials – Case studies
Enthusiasm – Energetic – Efforts	*Task Orientation* – Questioning – Handouts
Opportunities to learn – Questions and answer session – Discussions	

The following is the description of the students' needs towards lecturers' teaching effectiveness:

1. Clarity:

– Detailed explanation – ability of the lecturers to explain the content of the subject systematically and precisely. The explanation should consist of introduction, e.g. definition, main body, e.g. methods, steps, theories, concepts, examples and conclusion, e.g. summary of the content.
– Detailed notes – the notes given to the students should be properly structured in guiding them to understand the content easily. It will help the students to find the exact answer for their quizzes, tests, assignments and final examination questions.
– Clear and understandable English – lecturers use simple English in delivering the content.
– Examples – ability of the lecturers to relate the theories and concepts to the real world environment or any recent local and international scenarios.

2. Variability:

– Variety of materials – lecturers are able to hand in various forms of materials that are related to the subject, e.g. diagram, figure, graph, pictures, articles and others for the students to understand the subject better.
– Case studies – any scenarios, business situation or relevant data/numbers that are required for students to discuss and think critically in accordance with the case given.

3. Enthusiasm:

– Energetic – lecturers' passion, movement and voice refexion in transferring the knowledge and making their students understand the content of the subject.
– Efforts – passion and action that are shown by the lecturers in terms of time spent to prepare and deliver the content to the students.

4. Task orientation:

– Questioning – lecturers ask questions frequently to get the students' attention and at the same time, to assess the students' comprehension on the content delivered.
– Handouts – extra information in form of a diagram, figures, graph to capture the students' attention and to help them comprehend the subject.

5. Opportunities to learn:

– Q & A session – lecturers allocate several minutes for the students to ask questions regarding the material delivered.

– Discussion – lecturers allocate time and allow the students to express their opinion, ideas and experiences related to the subject. Lecturers will act as moderators in controlling the flow and scope of the discussion.

A questionnaire survey was then conducted and distributed to 140 students to obtain importance ratings of various students' requirements (presented in the QFD: Figure 2). For the AHP approach, the importance of rank has been identified by means of pairwise comparison of students' needs with teaching effectiveness as a factor to be evaluated. Two students with excellent academic performance had been selected to make the comparison.

4.2. Technical Design (Lecturers' Design)

To gain information on lecturers' designs, interviews were conducted with the lecturers. Lecturers' designs need to be defined in fulfilling the requirements of the students. To complete this task, three senior lecturers, one associate professor and one professor with more than ten years of teaching experience were contacted. The selected lecturers were asked to answer the following question: *How each of the students' needs towards teaching effectiveness best be fulfilled?* The lecturers provided the answers in reference to the students' needs according to the teaching effectiveness categories and they are shown in Table 3.

Table 3

Lecturers' designs to satisfy the students' needs towards teaching effectiveness

1. Preparation	2. Assignments
3. References	4. Presentation
5. Resourcefulness	6. Subject Matter Expert

Below is the description of the lecturers' design in fulfilling the students' needs towards lecturer teaching effectiveness at the FB.

1. Preparation. The process of preparing the material, obtaining the knowledge and information to be delivered confidently by the lecturers during lecture hours. Thus, the delivery process can be smoothly implemented and students are able to understand the content easily.

2. Assignments. They can be categorized into individual and group assignments, consisting of simple to complex questions, conducted during or after the class hour.
3. Presentation. The skill of transferring the knowledge and information to the students during the class hour. It consists of lecturers' voice, eye contact, body language, movement, activities conducted in the class. The creativity of the lecturers in structuring the content to the form related diagram or mechanism can be considered as presentation, too.
4. References. Text book, articles, newspaper clippings, magazines that can be used by the lecturers in delivering lecture and information related to the subject.
5. Resourcefulness. The degree of knowledge and understanding possessed by the lecturers on the subject. It depends considerably on the references used and preparation done by the lecturers.
6. Subject Matter Expert. The ability of the lecturers in mastering the subject assigned to them. It has a strong correlation with the educational background and the level of education of the lecturers, number of years teaching the subject, prior industrial experiences related to the subject and books or references used by the lecturers.

The students' needs and lecturers' design are presented in the House of Quality, as shown in Figure 2.

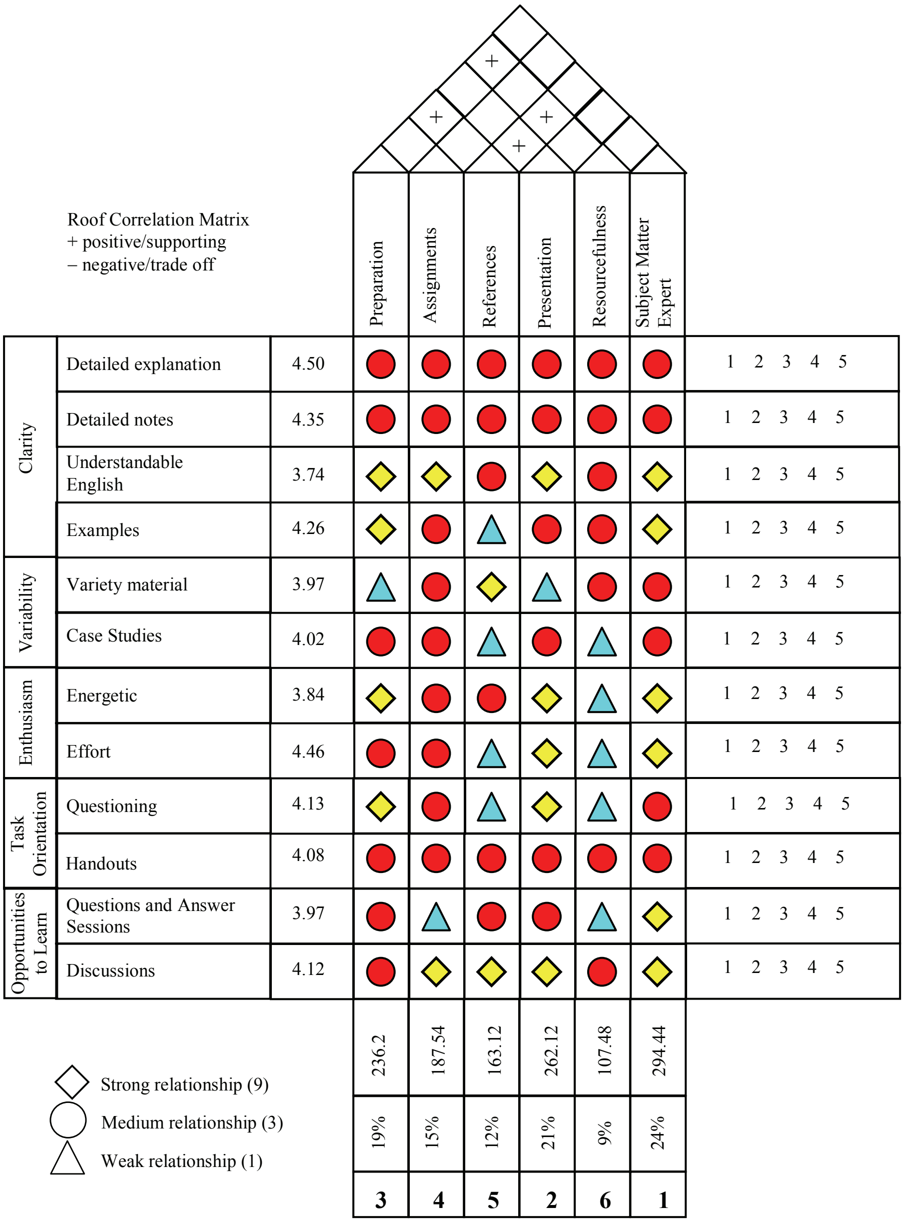

Figure 2. Complete HoQ

5. Data Analysis

The details of the data analysis are provided in Table 4.

Table 4

Analysis of the data by QFD and AHP

Type of data	Analysis
What's? (Voice of Customers) – to identify and prioritize the customer requirements.	*QFD Approach:* A five-point Likert-Scale was used and the mean scores for each students' need were calculated in order to obtain their importance rating. A questionnaire survey was conducted and distributed to 140 students to obtain importance ratings of various students' requirements (presented in the QFD: Figure 2). *AHP Approach:* To calculate the priorities of students' requirements, pairwise comparisons of students' needs with teaching effectiveness as a factor to be evaluated were conducted. Two students with excellent academic performance had been selected to make the comparison. The priorities of the *whats* obtained by AHP are shown in Table 4
Relationship Matrix – to identify the relationship between students' needs and lecturers' design	*QFD Approach:* It is based on the judgments obtained from selected lecturers as required by the QFD. In this case, the lecturers involved have to indicate the relationship of how the lecturers' design is able to fulfill students' requirements by using the symbols of '◊' (with the value of 9) for the strong relationship, while medium relationship was indicated by 'O' (with the value of 3) and the weak relationship, by the symbol of 'Δ' (with the value of 1) to determine the relationship between students' needs and lecturers' design. The relationship is exhibited in Figure 2. *AHP approach:* AHP was used to determine the relationship between students' needs and lecturers' designs in the following manner: 1. Define the problem – to improve teaching effectiveness. 2. Structure the hierarchy of teaching effectiveness, which is presented in Figure 3. The top level consists of the goal which is improving teaching effectiveness, the second level represents the criteria of teaching effectiveness, namely: clarity, variability, enthusiasm, task orientation and opportunities to learn. The next level consists

Table 4 contd.

Type of data	Analysis
	of the sub-criteria and the lowest level of the hierarchy shows the alternatives which are the lecturers' design requirements. 3. Construct the pairwise comparison matrix for the lecturers' designs based on the students' needs, as presented in Appendix 1. 4. Obtain judgments required to develop the set of matrices in step 3. In indicating this particular relationship, selected senior lecturers and professor had been involved. Pairwise comparison of lecturers' design was constructed with students' need as the factor to be evaluated (refer to Appendix 1). 5. Construct pairwise comparison matrix, calculate the priority values and consistency ratio for each students' need. The outcome is presented as in Appendix 1, too. 6. Perform steps 3, 4 and 5 for all the students' needs
Technical Assessment – to identify and prioritize lecturers' designs according to the students' needs	*QFD Approach:* Peters et al. [1996] demonstrated the calculation in a formula which is as follows: Lecturers' design importance = $\sum$ (customer importance rating) × (strength of relationship) Customer importance rating was identified by using the mean score for each students' need, while the strength of the relationship was determined by selected lecturers. *AHP Approach:* The lecturers' design priorities were calculated by multiplying the importance rating for the students' needs with the priority value of the individual design requirement and summing across each of the lecturers' design. The highest score of the lecturers' designs indicate the most important one in fulfilling the students' needs towards improving teaching effectiveness [Lam and Zhao, 1998].
Correlation Matrix – to identify the relationship between each lecturer's design	The purpose is to complete the roof of the HoQ diagram by examining the relationship between each pair of design issues [Peters et al., 2005]. A positive correlation between two designs indicates that the two designs are likely to reinforce each other. A negative correlation indicates that two designs are likely to negatively affect each other, while empty cells represent the fact that no correlation exists between the pairs [Hwarng and Teo, 2001]

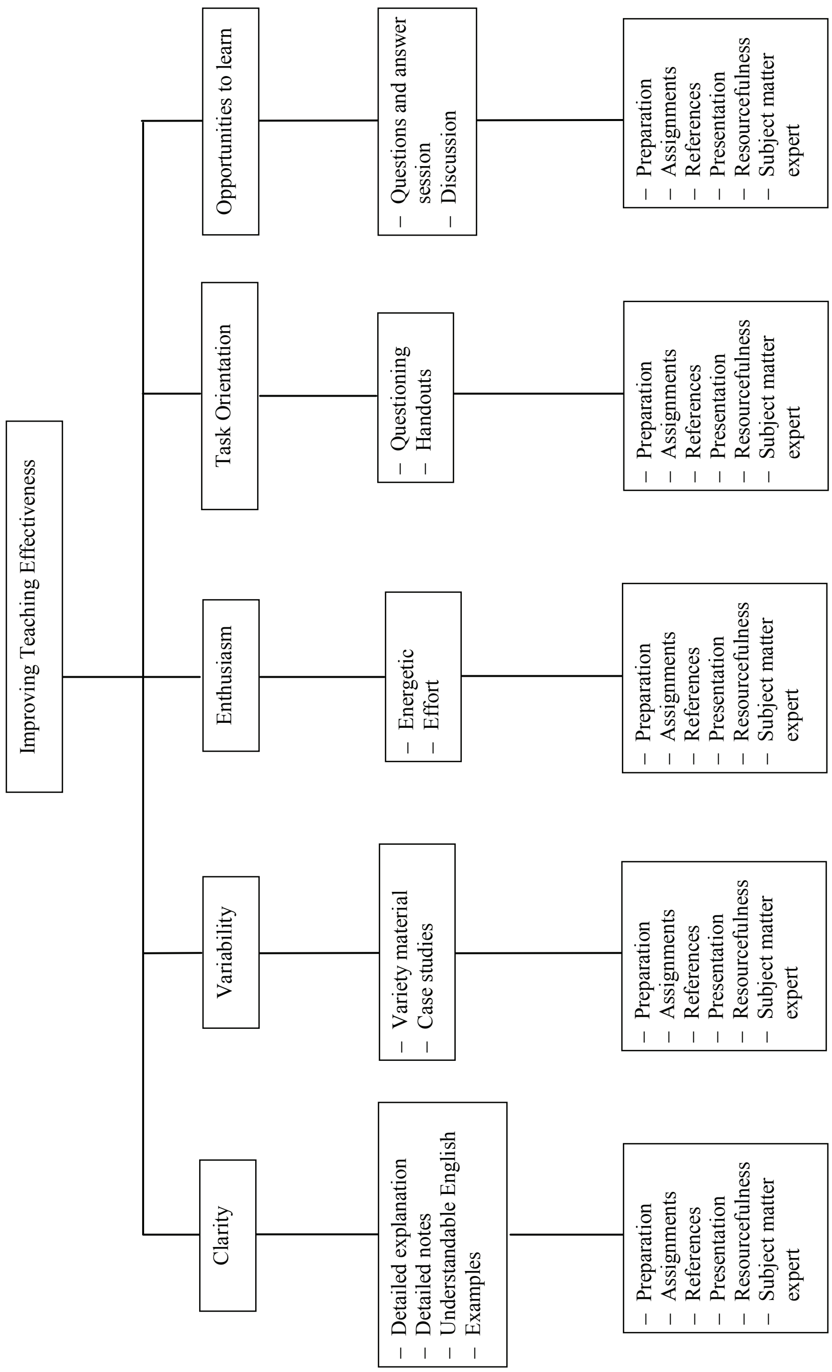

Figure 3. The AHP hierarchy representing the criteria and alternatives for improving teaching effectiveness

5.1. Identifying and Prioritizing Students' Needs towards Lecturers' Teaching Effectiveness

QFD Approach:
By referring to the information in the complete HoQ which is presented in Figure 2, it was found that *Explain in detail* was most important need with the mean value of 4.50, followed by *Making efforts to ensure that students understand well* with the mean 4.46 and *Provide detailed notes* holds the third rank. The least importance was placed on*Use easy and understandable English* with the mean value of 3.74.

AHP Approach:
The priorities of the students' requirements determined by the AHP are shown in Table 5.

Table 5

The priorities of the students requirements determined by AHP

Attribute of effective teaching	Priority value
Detailed explanation	0.1424
Detailed notes	0.1600
Understandable English	0.0344
Examples	0.0766
Varied material	0.0474
Case study	0.0621
Energetic	0.0409
Effort	0.1407
Questioning	0.1260
Handouts	0.0658
Q & A session	0.0308
Discussion	0.0730

The consistency ratio = 0.0964.

Since the consistency ratio was 0.0964 which is less than 0.1, therefore it can be concluded that the priorities of the students requirements calculated are acceptable. The results show that *Detailed explanation* was ranked as the most important of students need followed by *Detailed notes* and *Understandable English* with the priority value of 0.1424, 0.1600 and 0.0344 respectively.

5.2. Relationship between Students' Needs and Lecturers' Designs

QFD Approach:
The information presented in Figure 2 shows the relationship between lecturers' designs and students' needs based on the judgments of selected lecturers. As mentioned before, the symbol '◊' (with the value of 9) denotes strong relationship, medium relationship was indicated by 'O' (with the value of 3) and weak relationship was indicated by the symbol 'Δ' (with the value of 1).

AHP approach:
First and foremost, a four-level decision hierarchy for teaching effectiveness was constructed. The top level consists of the goal of the problem, which is improving teaching effectiveness. The second level describes the criteria to be considered for teaching effectiveness, namely: clarity, variability, enthusiasm, task orientation and opportunities to learn. The next level consists of the students' needs which can be considered as the sub-criteria according to the categories of teaching effectiveness. The lowest level of the decision hierarchy consists of the alternatives. The hierarchy of lecturers' teaching effectiveness is presented in Figure 3.

By using pairwise comparisons and a (1-9) scale, pairwise comparison matrix for each students' need was constructed, followed by determining the priority value and consistency ratio for each of the students' needs. The consistency ratio (CR) was computed to measure the consistency of the decision maker's responses. In general, if the CR is less than 0.1, then the decision maker's answers are considered acceptable. The summary of priority values and consistency ratio (CR) for the lecturers' design requirements with respect to each of the students' needs are provided in Table 5. As it is shown in the table, the values of CR for each of the students' needs was less than 0.1, therefore it can be said that the evaluation for the 12 students' needs for FB lecturers was acceptable.

By using the AHP synthesis rule, the global priorities of the Lecturers' design requirements were determined, as shown in Table 6.

Table 6

The AHP priority value of the design requirements and CR
for each students' need

Students' needs	Preparation	Assign-ments	References	Presen-tation	Resource-fulness	Subject Matter Expert	CR
Detailed Explanation	0.2960	0.1105	0.0642	0.2905	0.0601	0.1787	0.08317
Detailed notes	0.2639	0.1017	0.0880	0.1934	0.0756	0.2867	0.08439
English	0.2151	0.0909	0.1245	0.1539	0.1134	0.3022	0.08009
Examples	0.2242	0.0751	0.1165	0.0493	0.1617	0.3732	0.08967
Variety material	0.1698	0.0892	0.2781	0.0601	0.2781	0.1247	0.09281
Case studies	0.1559	0.1369	0.1241	0.0595	0.1753	0.3482	0.08163
Energetic	0.1716	0.0584	0.0989	0.3382	0.0989	0.2341	0.09027
Effort	0.2351	0.1242	0.0596	0.2906	0.0888	0.2017	0.08023
Questioning	0.1704	0.1252	0.0672	0.3371	0.0672	0.2329	0.09101
Handouts	0.2906	0.1242	0.0888	0.0596	0.2017	0.2351	0.08023
Q & A sessions	0.3448	0.1920	0.0591	0.1237	0.0883	0.1920	0.09249
Discussions	0.0861	0.1314	0.0694	0.1979	0.1701	0.3451	0.09192

5.3. Priorities of the design requirements

QFD approach:
By referring to Figure 2, the importance of rank for the lecturers' designs had been identified and prioritized. It shows that *Subject Matter Expertise* was ranked at the top of the lecturers' designs (294.44) followed by *Presentation* (262.12) and *Preparation* (236.2). *Resourcefulness* was ranked at the bottom (107.48) of the lecturers' designs list.

AHP approach:
From Table 7 we find that the element *'Subject Matter Expertise'* (0.2517) was the most important lecturers' design in fulfilling the students' needs. *Preparation* was at the second position (0.2300) followed by *Presentation* (0.2073). *References* was the last lecturers' design requirement (0.0909) in fulfilling the FB students' needs.

Table 7

Global priorities of the design requirements

Students' needs	Importance rating	Lecturers' Design requirements					
		Preparation	Assign-ment	References	Presen-tation	Resource-fulness.	Subject Matter Expert
Detailed Explanation	0.1424	0.2960	0.1105	0.0642	0.2905	0.0601	0.1787
Detailed notes	0.1600	0.2639	0.1017	0.0880	0.1934	0.0756	0.2867
English	0.0344	0.2151	0.0909	0.1245	0.1539	0.1134	0.3022
Examples	0.0766	0.2242	0.0751	0.1165	0.0493	0.1617	0.3732
Variety material	0.0474	0.1698	0.0892	0.2781	0.0601	0.2781	0.1247
Case studies	0.0621	0.1559	0.1369	0.1241	0.0595	0.1753	0.3482
Energetic	0.0409	0.1716	0.0584	0.0989	0.3382	0.0989	0.2341
Effort	0.1407	0.2351	0.1242	0.0596	0.2906	0.0888	0.2017
Questioning	0.1260	0.1704	0.1252	0.0672	0.3371	0.0672	0.2329
Handouts	0.0658	0.2906	0.1242	0.0888	0.0596	0.2017	0.2351
Q & A sessions	0.0308	0.3448	0.1920	0.0591	0.1237	0.0883	0.1920
Discussions	0.0730	0.0861	0.1314	0.0694	0.1979	0.1701	0.3451
Global weights →		0.2300	0.1129	0.0909	0.2073	0.1144	0.2517
Rank →		**2**	**4**	**6**	**3**	**5**	**1**

5.4. Comparison of Lecturers' Design requirements for the QFD and AHP Approaches

The ranks obtained by both approaches are shown in Table 8. The Table shows a slight difference in the ranks of lecturers' designs for both approaches in fulfilling students' needs towards lecturers teaching effectiveness. For both approaches, *Subject Matter Expertise* ranks at the top of the lecturers' designs list, which proves the value and importance of this element for teaching effectiveness.

Table 8

Comparison of ranks of design requirements obtained by QFD and AHP Approaches

Lecturers' Design Requirements	**QFD Rank**	**AHP Rank**
Subject Matter Expert	1	1
Presentation	2	3
Preparation	3	2
Assignments	4	4
References	5	6
Resourcefulness	6	5

Conclusions

This study demonstrates the application of QFD and AHP in fulfilling students' needs towards teaching effectiveness at the Faculty of Business, HSBL University College. A survey was conducted among the FB students in an attempt to determine their requirements/expectations from the lecturers. All the attributes have been placed at the left side of the House of Quality.

Interviews with lecturers were conducted for the purpose of satisfying the students' needs. The survey was then carried out to obtain the relationship between each lecturer's design. Once the students' needs and lecturers' designs were established and properly placed, the next step was to build the relationship between these two types of requirements by using the judgment of selective lecturers as required by the principle of QFD and the analysis of AHP. By using both approaches, the most effective lecturers' designs have been identified. Although the findings for both approaches are slightly different due to the students' rank of importance, however *Subject Matter Expertise* emerges as the most important lecturers' design in fulfilling the students' needs in FB.

As for extension of the study, one can consider applying ANP to address the inner dependency among the elements at various levels of the AHP hierarchy.

Appendix

Sample pairwise comparison matrices and the priority values of the lecturers design requirements for each students' need.

Detailed explanation	**Prep.**	**Assign.**	**Referen.**	**Present.**	**Resource.**	**SME**	**Priority value**
Preparation	1	5	3	1	3	3	0.2960
Assignment	0.2	1	3	0.3333	3	0.3333	0.1105
References	0.3333	0.3333	1	0.3333	1	0.2	0.0642
Presentation	1	3	3	1	5	3	0.2905
Resourcefulness	0.3333	0.3333	1	0.2	1	0.3333	0.0601
Subject Matter Expertise	0.3333	3	5	0.3333	3	1	0.1787

CR = 0.08317

Detailed notes	**Prep.**	**Assign.**	**Referen.**	**Present.**	**Resource.**	**SME**	**Priority value**
Preparation	1	5	3	1	3	1	0.2639
Assignment	0.2	1	3	0.3333	1	0.3333	0.1017
References	0.3333	0.3333	1	1	1	0.3333	0.0880
Presentation	1	3	1	1	3	0.3333	0.1934
Resourcefulness	0.3333	1	1	0.3333	1	0.3333	0.0756
Subject Matter Expertise	1	3	3	3	3	1	0.2867

CR = 0.0843

Understandable English	**Prep.**	**Assign.**	**Referen.**	**Present.**	**Resource.**	**SME**	**Priority value**
Preparation	1	3	1	1	3	1	0.2151
Assignment	0.3333	1	1	1	0.3333	0.3333	0.0909
References	1	1	1	1	1	0.3333	0.1245
Presentation	1	1	1	1	3	0.3333	0.1539
Resourcefulness	0.3333	3	1	0.3333	1	0.3333	0.1134
Subject Matter Expertise	1	3	3	3	3	1	0.3022

CR = 0.0800

Examples	**Prep.**	**Assign.**	**Referen.**	**Present.**	**Resource.**	**SME**	**Priority value**
Preparation	1	3	3	3	3	0.3333	0.2242
Assignment	0.3333	1	0.3333	3	0.3333	0.2	0.0751
References	0.3333	3	1	3	0.3333	0.3333	0.1165
Presentation	0.3333	0.3333	0.3333	1	0.3333	0.2	0.0493
Resourcefulness	0.3333	3	3	3	1	0.3333	0.1697
Subject Matter Expertise	3	5	5	5	3	1	0.3732

CR = 0.0896

Material Variety	**Prep.**	**Assign.**	**Referen.**	**Present.**	**Resource.**	**SME**	**Priority Value**
Preparation	1	3	0.3333	3	0.3333	3	0.1698
Assignment	0.3333	1	0.3333	3	0.3333	0.3333	0.0892
References	3	3	1	3	1	3	0.2781
Presentation	0.3333	0.3333	0.3333	1	0.3333	0.3333	0.0601
Resourcefulness	3	3	1	3	1	3	0.2781
Subject Matter Expertise	0.3333	3	0.3333	3	0.3333	1	0.1247

CR = 0.0928

Case Studies	**Prep.**	**Assign.**	**Referen.**	**Present.**	**Resource.**	**SME**	**Priority value**
Preparation	1	1	3	3	0.3333	0.3333	0.1559
Assignment	1	1	1	3	1	0.3333	0.1369
References	0.3333	1	1	3	1	0.3333	0.1241
Presentation	0.3333	0.3333	0.3333	1	0.3333	0.3333	0.0595
Resourcefulness	3	1	1	3	1	0.3333	0.1753
Subject Matter Expertise	3	3	3	3	3	1	0.3482

CR = 0.0816

Energetic	Prep.	Assign.	Referen.	Present.	Resource.	SME	Priority value
Preparation	1	3	3	0.3333	3	0.3333	0.1716
Assignment	0.3333	1	0.3333	0.3333	0.3333	0.3333	0.0584
References	0.3333	3	1	0.3333	1	0.3333	0.0989
Presentation	3	3	3	1	3	3	0.3382
Resourcefulness	0.3333	3	1	0.3333	1	0.3333	0.0989
Subject Matter Expertise	3	3	3	0.3333	3	1	0.2341

CR = 0.0927

Effort	Prep.	Assign.	Referen.	Present.	Resource.	SME	Priority value
Preparation	1	3	3	1	3	1	0.2351
Assignment	0.3333	1	3	0.3333	3	0.3333	0.1242
References	0.3333	0.3333	1	0.3333	0.3333	0.3333	0.0596
Presentation	1	3	3	1	3	3	0.2906
Resourcefulness	0.3333	0.3333	3	0.3333	1	0.3333	0.0888
Subject Matter Expertise	1	1	3	0.3333	3	1	0.2017

CR = 0.8023

Questioning	Prep.	Assign.	Referen.	Present.	Resource.	SME	Priority value
Preparation	1	3	3	0.3333	3	0.3333	0.1704
Assignment	0.3333	1	3	0.3333	3	0.3333	0.1252
References	0.3333	0.3333	1	0.3333	1	0.3333	0.0672
Presentation	3	3	3	1	3	3	0.3371
Resourcefulness	0.3333	0.3333	1	0.3333	1	0.3333	0.0672
Subject Matter Expertise	3	3	3	0.3333	3	1	0.2329

CR = 0.0910

Handouts	**Prep.**	**Assign.**	**Referen.**	**Present.**	**Resource.**	**SME**	**Priority value**
Preparation	1	3	3	3	3	1	0.2906
Assignment	0.3333	1	3	3	0.3333	0.3333	0.1242
References	0.3333	0.3333	1	3	0.3333	0.3333	0.0888
Presentation	0.3333	0.3333	0.3333	1	0.3333	0.3333	0.0596
Resourcefulness	0.3333	3	3	3	1	1	0.2017
Subject Matter Expertise	1	3	3	3	1	1	02351

CR = 0.080231

Q & A session	**Prep.**	**Assign.**	**Referen.**	**Present.**	**Resource.**	**SME**	**Priority value**
Preparation	1	3	3	3	3	3	0.3448
Assignment	0.3333	1	3	3	3	1	0.1920
References	0.3333	0.3333	1	0.3333	0.3333	0.3333	0.0591
Presentation	0.3333	0.3333	3	1	3	0.3333	0.1237
Resourcefulness	0.3333	0.3333	3	0.3333	1	0.3333	0.0883
Subject Matter Expertise	0.3333	1	3	3	3	1	0.1920

CR = 0.092486

Discussion	**Prep.**	**Assign.**	**Referen.**	**Present.**	**Resource.**	**SME**	**Priority value**
Preparation	1	0.3333	1	0.3333	1	0.3333	0.0861
Assignment	3	1	3	0.3333	0.3333	0.3333	0.1314
References	1	0.3333	1	0.3333	0.3333	0.3333	0.0694
Presentation	3	3	3	1	1	0.3333	0.1979
Resourcefulness	1	3	3	1	1	0.3333	0.1701
Subject Matter Expertise	3	3	3	3	3	1	0.3541

CR = 0.09192

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Petr Fiala

AN ANP-BASED FRAMEWORK FOR REVENUE MANAGEMENT

Abstract

Revenue management (RM) is the process of understanding, anticipating and influencing consumer behavior in order to maximize revenue. The challenge is to sell the right resources to the right customer at the right time for the right price through the right channel. Network revenue management models attempt to maximize revenue when customers buy bundles of multiple resources. An Analytic Network Process (ANP)-based framework for RM problems structuring and combining specific methods is presented. RM addresses three basic categories of demand-management decisions: price, quantity, and structural decisions. Specific models are used to model and to solve basic RM decisions. Combinations of the solutions are given by sub-networks in an ANP model.

Keywords

Revenue management, multicriteria decisions, price decisions, quantity decisions, structural decisions, Analytic network process, Dynamic Network Process.

Introduction

Revenue management (RM) is the process of understanding, anticipating and influencing consumer behavior in order to maximize revenue or profits from fixed, perishable resources. The RM area encompasses all work related to operational pricing and demand management. This includes traditional problems in the field, such as capacity allocation, overbooking and dynamic pricing, as well as newer areas, such as oligopoly models, negotiated pricing and auctions. Recent years have seen great successes of revenue management, notably in the airline, hotel, and car rental business. Currently, an increasing number of industries is exploring possibilities of adopting similar concepts [see Talluri, van Ryzin, 2004]. What is new about RM is not the demand--management decisions themselves but rather how these decisions are made. The true innovation of RM lies in the method of decision making.

Revenue Management is to sell the right product, to the right customer at the right time, for the right price through the right channel by maximizing revenue. RM is the art and science of predicting real-time customer demand and optimizing the price and availability of products according to the demand. RM addresses three basic categories of demand-management decisions:

- structural,
- price, and
- quantity decisions.

Network revenue management models attempt to maximize revenue when customers buy bundles of multiple resources. The dependence among the resources in such cases is created by customer demand.

For the basic specific problems many appropriate methods have been proposed [see Talluri, van Ryzin, 2004]. An Analytic Network Process (ANP)-based framework for RM problems structuring and combining specific methods is presented in this paper. Combinations of the solutions are given by subnetworks in an ANP model. RM problems are complex dynamic problems. The DNP (Dynamic Network Process) method was used for dynamic extensions.

1. ANP structure of the problem

The Analytic Hierarchy Process (AHP) is the method for setting priorities [Saaty, 1996]. A priority scale based on reference is the AHP way to standardize non-unique scales in order to combine multiple performance measures. The AHP derives ratio scale priorities by making paired comparisons of elements on a common hierarchy level by using a 1 to 9 scale of absolute numbers. The absolute number from the scale is an approximation to the ratio w_j / w_k and then it is possible to derive values of w_j and w_k as weights, i.e. measures of relative importance. The AHP method uses the general model for synthesis of the performance measures in the hierarchical structure:

$$u_i = \sum_{j=1}^{n} v_j w_{ji} ,$$

where u_i are global weights of alternatives, v_j are weights of criteria, and w_{ji} are weights of alternatives by individual criteria.

The Analytic Network Process (ANP) is the method [Saaty, 2001] that makes it possible to deal systematically with all kinds of dependence and feedback in the performance system. The well-known AHP theory is a special case of the Analytic Network Process that can be very useful for incorporating linkages in the system.

The structure of the ANP model is described by clusters of elements connected by their dependence on one another. A cluster groups elements that share a set of attributes. At least one element in each of these clusters is connected to some element in another cluster. These connections indicate the flow of influence between the elements.

The challenge in RM is to sell:

– the right resources,
– to the right customer,
– at the right time,
– for the right price,
– through the right channel.

There are two possibilities for a decision: to accept or to reject a request for a product. The clusters in an RM problem can consist of resources, customers, time, prices, channels, and decisions (see Figure 1).

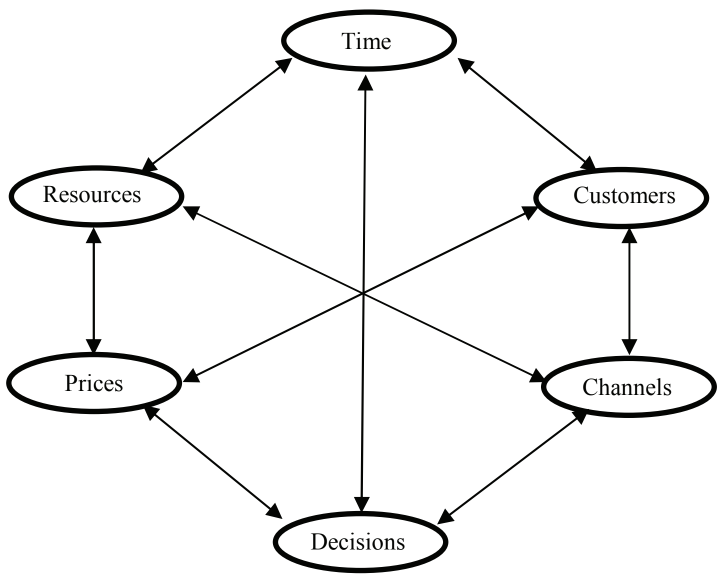

Figure 1. Clusters and connections in an RM problem

Pairwise comparisons are inputs for preference elicitation in revenue problems. A supermatrix is a matrix of all elements by all elements. The weights from the pairwise comparisons are placed in the appropriate column of the supermatrix. The sum of each column corresponds to the number of comparison sets. The weights in the column corresponding to the cluster are multiplied by the weight of the cluster. Each column of the weighted supermatrix sums to one and the matrix is column stochastic. Its powers can stabilize after some iterations to a limited supermatrix. The columns of each block of the matrix are identical in many cases, though not always, and we can read off them the global priority of units.

2. Sub-networks

The basic ANP model is completed by specific sub-networks. The sub--networks are used to model important features of the RM problems. The most important features in our ANP-based framework for revenue management are captured in sub-networks:
– time dependent resources,
– products,
– network revenue management,
– price-quantity-structure network.

Time dependent resources

A specific sub-network models time-dependent amounts of resources. The time-dependent amount of resources is given by previous decisions. The sub-network connects clusters: time, resources and decisions.

Products

A product is a sub-collection of available resources. An (m, n) matrix $A = [a_{ij}]$ is defined such that a_{ij} represents the amount of resource i used to produce one unit of product j. Every column j of A represents a different product and the collection $M = \{A_{.1}, \ldots , A_{.n}\}$ is the menu of products offered by the seller.

Network revenue management

The quantity-based revenue management of multiple resources is referred to as network revenue management. This class of problems arises for example in airline, hotel, and railway management. In the airline case, the problem consists in managing capacities of a set of connecting flights across a network, a so called hub-and-spoke network (see Figure 2). In the hotel case, the problem is managing room capacity on consecutive days when customers stay multiple nights.

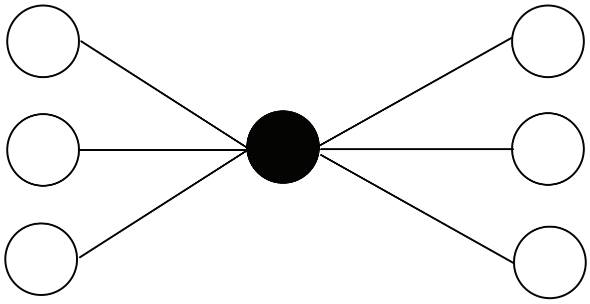

Figure 2. Hub-and-spoke network

Network revenue management models attempt to maximize some reward function when customers buy bundles of multiple resources. The Inter-dependence of resources, commonly referred to as network effects, creates difficulties in solving the problem. The classical technique of approaching this problem has been to use a deterministic LP solution to derive policies for the network capacity problem. Initial success with this method has triggered considerable research on possible reformulations and extensions, and this method has become widely used in many industrial applications. A significant limitation of the applicability of these classical models is the assumption of independent demand. In response to this, interest has arisen in recent years to incorporate customer choice into these models, further increasing their complexity. This development drives current efforts to design powerful and practical heuristics that still can manage problems of practical scope.

The basic model of the network revenue management problem can be formulated as follows [see Talluri, van Ryzin, 2004]: The network has m resources which can be used to provide n products. We define the incidence matrix $A = [a_{ij}]$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, where

$a_{ij} = 1$, if resource i is used by product j, and
$a_{ij} = 0$, otherwise.

The j-th column of A, denoted a_j, is the incidence vector for product j. The notation $i \quad a_j$ indicates that resource i is used by product j.

The state of the network is described by a vector $x = (x_1, x_2, \dots, x_m)$ of resource capacities. If product j is sold, the state of the network changes to $x - a_j$.

Time is discrete, there are T periods and the index t represents the current time, $t = 1, 2, \dots, T$. We assume that within each time period t at most one request for a product can arrive.

Price-quantity-structure network

RM addresses three basic categories of demand-management decisions:

1. Price decisions:
   – How to set posted prices.
   – How to price across product categories.
   – How to price over time.
   – How to markdown over the product lifetime.
2. Quantity decisions:
   – Whether to accept or reject an offer to buy.
   – How to allocate output or capacity to different segments, products and channels.
   – When to withhold a product from market and sale it at later points in time.
3. Structural decisions:
   – How to bundle products.
   – Which selling format to use.
   – Which segmentation or differentiation mechanisms to use.
   – Which terms of trade to offer.

The price-quantity-structure network is given by interdependences of the three very important factors. The solutions of three basic categories of demand--management decisions are solved by basic methods described in next paragraphs. Interdependencies are modeled and analyzed in the ANP sub--network.

3. Price decisions

The basic pricing model of the network revenue management problem is formulated as a stochastic dynamic programming problem whose exact solution is computationally intractable. Most approximation methods are based on one of two basic approaches: to use a simplified network model or to decompose the network problem into a collection of single-resource problems.

The Deterministic Linear Programming (DLP) method is a popular one in practice. The DLP method is based on a wrong assumption that demand is deterministic and static. Approximation methods based on extensions of the basic approaches are proposed.

The revenue management general model [Bitran, Caldentey, 2003] provides a global view of the different elements and their interrelations:

- Supply.
- Product.
- Information.
- Demand.

A seller has a fixed amount of initial capacity that is used to satisfy a price-sensitive demand during a certain selling period $[0, T]$. This initial capacity is modeled by an m-dimensional vector of m resources. Capacity can be interpreted for example as rooms in a hotel, available seats for a specific origin-destination flight on a given day, etc. Capacity is essentially given and the seller is committed exclusively to finding the best way to sell it. From a pricing perspective, two important attributes of the available capacity are its degree of flexibility and its perishability. Flexibility measures the ability to produce and offer different products using the initial capacity C_0. Perishability relates to the lack of ability to preserve capacity over time. As time progresses and resources are consumed, capacity decreases. The available capacity at time t is denoted by $C_t = (c_1(t), \dots , c_m(t))$.

The knowledge of the system and its evolution over time is crucial to any dynamic pricing policy. Given an initial capacity C_0, a product menu M, and a demand and price processes, the observed history H_t of the selling process is defined as the set of all relevant information available up to t. This history should include at least the observed demand process and available capacity, and it can also include some additional information such as demand forecasts.

The set of potential customers is divided into different segments, each one having its own set of attributes. A d-dimensional stochastic process is defined as $N(t, H_t) = (N_1(t, H_t), \dots , N_d(t, H_t))$ where $N_j(t, H_t)$ is the cumulative potential demand up to time t from segment j given the available information H_t. An (n, d) matrix $B(P) = [b_{ij}]$ is defined where b_{ij} represents the units of product $i \in M$ requested by a customer in segment $j = 1, \dots , d$. The demand depends on the pricing policy $P = \{p_t, t \in [0, T]\}$ where $p_t(i, H_t)$ is the price of product $i \in M$ at time t given a current history H_t. The effective cumulative demand process in $[0, t]$ at the product level is defined as the n-dimensional vector $D(t, P, H) = B(P)N(t, H_t)$. The set of all admissible pricing policies, those that satisfy all the relevant constraints, is denoted by Π. The seller has the ability to partially serve demand if it is profitable to do so. An n-dimensional vector $S(t)$ that represents the cumulative sales up to time t is defined.

The general revenue management problem is to find the solution to the following optimal control problem:

$$\sup_{P,S} E_N\left[\int_0^T p_t dS(t)\right]$$

subject to

$$C_t = C_0 - AS(t) \geq 0 \text{ for all } t \in [0; T],$$
$$0 \leq S(t) \leq D(t, P, H_t) \text{ for all } t \in [0; T],$$
$$P \in \Pi, \text{ and } S(t) \in H_t.$$

Deterministic models assume that the seller has perfect information about the demand process. They are easy to analyze and provide a good approximation for the more realistic yet complicated stochastic models. Deterministic solutions are in some cases asymptotically optimal for the stochastic demand problem [Cooper, 2002].

The simplest deterministic model considers the case of a monopolist selling a single product to a price sensitive demand during a period $[0, T]$. The initial inventory is C_0, demand is deterministic with time dependent and price sensitive intensity $\mu(p, t)$. The instantaneous revenue function $r(p, t) = p\,\mu(p, t)$ is assumed to be concave as in most real situations. The general revenue management problem can be written in this case as follows:

$$\max_P \int_0^T p_t \mu(p_t, t)dt$$

subject to

$$\int_0^T \mu(p_t, t)dt \leq C_0 .$$

This is a standard problem in calculus of variations. The optimality condition is given by

$$p_t^* = \lambda - \frac{\mu(p_t^*, t)}{\mu_p(p_t^*, t)},$$

where λ is the Lagrangian multiplier for the constraint, μ_p is the partial derivative of μ with respect to the price.

4. Quantity decisions

Demand in time period t is modeled as the realization of a single random vector $r(t) = (r_1(t), r_2(t), \dots , r_n(t))$. If $r_j(t) = r_j > 0$, this indicates that a request for product j occurred and that its associated revenue is r_j. If $r_j(t) = 0$, this indicates that no request for product j occurred. A realization $r(t) = 0$ (all components equal to zero) indicates that no request from any product occurred at time t. The assumption that at most one arrival occurs in each time period means that at most one component of $r(t)$ can be positive. The sequence $r(t)$, $t = 1, 2, \dots , T$, is assumed to be independent with known joint distributions in each time period t. When revenues associated with product j are fixed, we will denote these by r_j and the revenue vector $r = (r_1, r_2, \dots , r_n)$.

Given the current time t, the current remaining capacity x and the current request $r(t)$, the decision is to accept or not to accept the current request. We define the decision vector $u(\mathrm{t}) = (u_1(t), u_2(t), \dots , u_n(t))$ where

$\mathrm{u}_j(t) = 1$, if a request for product j in time period t is accepted, and
$\mathrm{u}_j(t) = 0$, otherwise.

The components of the decision vector $u(\mathrm{t})$ are functions of the remaining capacity components of vector x and the components of the revenue vector r, $u(\mathrm{t}) = u(t, x, r)$. The decision vector $u(t)$ is restricted to the set

$$U(x) = \{u \in \{0, 1\}^n,\ Au \le x \}.$$

The maximum expected revenue, given remaining capacity x in time period t, is denoted by $\mathrm{V}_t(x)$. Then $\mathrm{V}_t(x)$ must satisfy the Bellman equation:

$$V_t(x) = E\left[\max_{u \in U(x)} \{r(t)^T u(t,x,r) + V_{t+1}(x - Au)\}\right] \tag{1}$$

with the boundary condition

$$V_{T+1}(x) = 0, \forall x.$$

A decision u^* is optimal if and only if it satisfies:

$u_j\,(t, x, r_j) = 1$, if $r_j \ge V_{t+1}(x) - V_{t+1}(x - a_j),\ a_j \le x$,

$u_j\,(t, x, r_j) = 0$, otherwise.

This reflects the intuitive notion that revenue r_j for product j is accepted only when it exceeds the opportunity cost of the reduction in resource capacities required to satisfy the request.

Basic approximation approach

The equation (1) cannot be solved exactly for most networks of realistic size. Solutions are based on approximations of various types. There are two important criteria when judging network approximation methods: accuracy and speed. Among the most useful information provided by an approximation method are estimates of bid prices [see Talluri, van Ryzin, 2004].

Given an approximation method M that yields an estimate of the value function $V_t^M(x)$ we can approximate the displacement cost of accepting product j by gradient of the value function approximation. The bid prices are then defined by:

$$\pi_i^M(t,x) = \frac{\partial}{\partial x_i} V_t^M(x) .$$

We introduce Deterministic Linear Programming (DLP) method. The approach is to use a simplified network model, for example formulating the problem as a static mathematical program.

The DLP method uses the approximation:

$$V_t^{LP}(x) = \max r^T y \qquad (2)$$

subject to

$Ay \leq x$

$0 \leq y \leq E[D]$

where $D = (D_1, D_2, \dots, D_n)$ is the vector of demand over the periods t, $t+1, \dots, T$, for product $j, j = 1, 2, \dots, n$, and $r = (r_1, r_2, \dots, r_n)$ is the vector of revenues associated with the n products. The decision vector $y = (y_1, y_2, \dots, y_n)$ represents partitioned allocation of capacity for each of the n products. The approximation effectively treats demand as if it were deterministic and equal to its mean $E[D]$.

The optimal dual variables, π^{LP}, associated with the constraints $Ay \leq x$, are used as bid prices. The DLP was among the first models analyzed for network RM [see Talluri, van Ryzin, 2004]. The main advantage of the DLP model is that it is computationally very efficient to solve. Due to its simplicity and speed, it is a popular one in practice. The weakness of the DLP approximation is that it considers the mean demand only and ignores all other distributional information. The performance of the DLP method depends on the type of network, the order in which fare products arrive and the frequency of re-optimization.

5. Structural decisions

One of structural decisions is how to bundle products. We will show on this example how to use models of combinatorial auctions. Auctions are important market mechanisms for the allocation of goods and services. Combinatorial auctions are those auctions in which bidders can place bids on combinations of items, so called bundles. The advantage of combinatorial auctions is that the bidder can more adequately express his preferences. This is particularly important when items are complements. The auction designer also derives value from combinatorial auctions. Allowing bidders more adequately to express preferences often leads to improved economic efficiency and greater auction revenues. However, alongside their advantages, combinatorial auctions raise a host of questions and challenges [see Cramton et al. (eds.), 2006; de Vries and Vohra, 2003].

The problem, called the winner determination problem, has received considerable attention in the literature. The problem is formulated as follows: Given a set of bids in a combinatorial auction, find an allocation of items to bidders that maximizes the seller's revenue. It introduced many important ideas, such as the mathematical programming formulation of the winner determination problem, the connection between the winner determination problem and the set-packing problem as well as the issue of complexity.

Winner determination problem

Many types of combinatorial auctions can be formulated as mathematical programming problems. From among different types of combinatorial auctions we present an auction of indivisible items with one seller and several buyers. Let us suppose that one seller offers a set G of m items, $j = 1, 2, \dots, m$, to n potential buyers. Items are available in single units. A bid made by buyer i, $i = 1, 2, \dots, n$, is defined as:

$$B_i = \{S, v_i(S)\},$$

where

$S \subseteq M$, is a combination of items,
$v_i(S)$, is the valuation or offered price by buyer i for the combination of items S.

The objective is to maximize the revenue of the seller given the bids made by buyers. The constraints ensure that no single item is allocated to more than one buyer and that no buyer obtains more than one combination.

Problem formulation

Bivalent variables are introduced for model formulation:
$x_i(S)$ is a bivalent variable specifying if the combination S is assigned to buyer i ($x_i(S) = 1$).

The winner determination problem can be formulated as follows:

$$\sum_{i=1}^{n} \sum_{S \subseteq M} v_i(S)\, x_i(S) \rightarrow \max$$

subject to:

$$\sum_{S \subseteq M} x_i(S) \leqq 1, \ \forall i, \ i = 1, 2, \dots, n,$$

$$\sum_{i=1}^{n} \sum_{S \subseteq M} x_i(S) \leqq 1, \ \forall j \in \mathrm{M},$$

$$x_i(S) \in \{0, 1\}, \ \forall \ S \subseteq M, \ \forall i, \ i = 1, 2, \dots, n.$$

The objective function expresses the revenue. The first constraint ensures that no bidder receives more than one combination of items. The second constraint ensures that overlapping sets of items are never assigned.

6. Dynamic Network Process

RM problems are complex dynamic problems. The ANP is static but for the RM problem, time dependent decision making is very important. The DNP (Dynamic Network Process) method was introduced [Saaty, 2003]. There are two ways to study dynamic decisions: structural, by including scenarios, and functional, by explicitly involving time in the judgment process. For the functional dynamics there are analytic or numerical solutions. The basic idea of the numerical approach is to obtain the time dependent principal eigenvector by simulation.

The Dynamic Network Process seems to be an appropriate instrument for analyzing dynamic networks [Fiala, 2006]. The method is appropriate also for the specific features of RM problems. The method computes time dependent weights for decisions and combinations of decisions.

We used the ANP software package Super Decisions developed by Creative Decisions Foundation (CDF) for some experiments in testing the possibilities of the expression and evaluation of the dynamic RM models [see Figure 3].

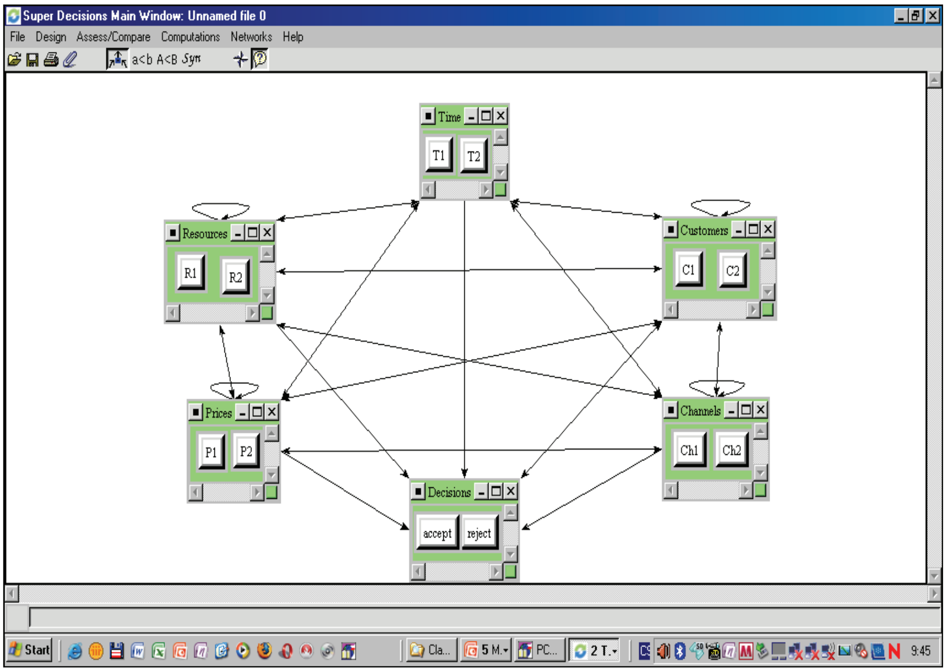

Figure 3. Super Decisions – RM model

Conclusions

RM problems are important subjects of intensive economic research. A possible flexible ANP/DNP framework is presented. Analytic Network Process methodology is useful for structuring the RM problem and for combining specific models. Sub-networks are used for sophisticated analyses of RM processes. Specific models are used to model and to solve basic RM decisions (price, quantity, structure). Approximations, heuristics, or iterative approaches are used for solving the specific models. Dynamic Network Process is an appropriate approach for explicitly involving time in the RM processes. The combination of such approaches can give more complex views on RM problem.

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AN INTERACTIVE APPROACH DETERMINING THE INDIFFERENCE THRESHOLDS IN PROMETHEE II

Abstract

The implementation of most multi-criteria decision aid methods requires fixing of certain parameters in order to model the decision-maker's preferences. The fixing of these parameter values must be naturally done with the decision-maker's collaboration. The parameter determination constitutes an important task, which is generally quite delicate and difficult to accomplish, for the decision-maker. In fact, the information provided at this level is inevitably subjective and partial. In this paper, we intend to determine the values of the indifference thresholds associated to usual and quasi criterion in PROMETHEE, by exploiting the information provided by the decision-maker and by using mathematical programming.

Keywords

Multi-criteria analysis, Integer programming, Preference disaggregation, PROMETHEE, Indifference thresholds.

Introduction

At the time of a multi-criteria decision aid activity, the basic preoccupation concerns the manner in which the decision will be taken in a given context. However, it can also be pertinent to pose the problem inversely: supposing that a decision has been taken, is it possible to find the rational bases allowing to explain or to justify the decision taken? Or is it possible to explain the decision-maker's preference model which leads precisely to the same decision or at least to a very "similar" decision? The philosophy of the preference disaggregation approach in the framework of a multi-criteria analysis is to determine preference modelling elements from preferential structures provided by the decision-maker, taking into account the method used for multi--criteria aggregation.

The implementation of PROMETHEE method requires fixing relative importance coefficients of the criteria, preference thresholds and indifference thresholds. In fact, we proposed an approach inferring relative importance coefficients of the criteria from preference relations provided by the decision maker [Frikha et al., 2010].

In this paper, we consider the problem of the preference disaggregation, inferring, from preference relations provided by the decision-maker, the indifference threshold values. We will focus our interest only on usual criteria and quasi-criteria. In a subsequent work, we intend to extend our results to the general case, with the other criteria's functions, requiring preference and indifference thresholds simultaneously. The organization of the paper is as follows: a brief presentation of the disaggregation approaches constitutes section 2. Section 3 is dedicated to the description of the PROMETHEE method. In section 4, we will describe the model proposed to determine the indifference threshold values. This model, including an interactive aspect, is based on mathematical programming of the goal-programming type. A fictitious numerical example is the object of section 5 and finally Section 6 contains a brief conclusion.

1. Preference disaggregation methods

Several disaggregation approaches have been developed to infer the ELECTRE method's parameters. Indeed, a first trial of ELECTRE III parameter determination from a given ranking has been presented by Richard [1981] without eventually leading to satisfactory results. Then Kiss et al. [1994] developed an interactive system called ELECCALC that determines indirectly the ELECTRE II method's parameters from decision-makers' answers to questions concerning their global preferences. In the same context of preference disaggregation methods allowing to determine certain of ELECTRE parameters' values on the basis of information provided by the decision-maker, Mousseau contributed to the development of several works. Indeed, Mousseau and Slowinski [1998] proposed a global inference approach that deduces ELECTRE TRI's parameters simultaneously from assignment examples. Continuing the same idea, Mousseau et al. [2001] proposed a partial inference approach that consists in inferring only the criteria's relative importance coefficients and the cut levels in order to deduce some trivial relations from valued ranking relations. Ngo The and Mousseau [2002] presented also an inference procedure that determines the limits of categories of ELECTRE TRI method from assignment examples provided by the decision-maker. Finally, Dias and Mousseau [2006] proposed a mathematical program to deduce the veto threshold values of ELECTRE III from ranking examples.

In the same context of preference disaggregation, Jacquet-Lagreze [1979] had proposed an approach to construct an additive value model that consists in assessing indirectly the model's parameters on the basis of preference holistic information. This approach is mathematically integrated in the UTA method by Jacquet-Lagrèze and Siskos [1982] through a disaggregation model of ordinal regression type, based on the formulation of linear programming. The preference disaggregation methods also appear in other versions of UTA. Indeed, the UTADIS method [Utilité Additive Discriminante] of Devaud et al. [1980] is an ordinal regression method based on the preference disaggregation approach. Given a predefined action ranking in classes, the objective of UTADIS is to estimate the additive utility function and utility thresholds that assign actions in their original classes with a minimum of ranking errors. The method UTA II, developed by Siskos [1980] is another version of the UTA method. This preference disaggregation approach is useful to assess the additive utility model. Greco et al. [2008] developed the UTAGMS method, which allows the determination of all additive value functions compatible with the preference information provided by the decision maker [a set of pairwise comparisons on a subset of alternatives, called reference alternatives]. Besides, Figueira et al. [2009] developed the UTAGRIP allowing constructing a set of additive value functions compatible with preference information composed of comparisons of reference action pairs. Moreover, Bous et al. [2010] proposed a new method called ACUTA based on the computation of the analytic centre of a polyhedron, for the selection of additive value functions that are compatible with holistic assessments of preferences provided by the decision maker. In the same context of determining additive value functions, Köksalan and Özpeynirci [2009] developed an approach that estimates an additive utility function. In fact, the decision maker is invited to assign some reference alternatives into categories during the interactive process. Else, Greco et al. (2010) proposed a model that aims at assigning actions evaluated on multiple criteria to *p* pre-defined and ordered classes. In this work, the decision maker supplies a set of assignment examples on a subset of actions, called reference actions. This information is used to determine a set of general additive value functions compatible with these assignment examples.

In the framework of multi-criteria decision aid under uncertainty, Siskos [1983] developed a stochastic ordinal regression method from UTA (stochastic UTA).

The disaggregation approaches are also applicable in a specific multi--objective optimization field, mainly in the field of the linear programming with multiple objective functions. For example, in the classic methods of Geoffrion et al. [1972] and Zionts and Wallenius [1976], the weights of the objective linear combinations are inferred locally from the judgments provided by the decision-maker at each iteration of these methods. Stewart [1987] proposed a procedure of action pruning using the UTA method, whereas Jacquet-Lagrèze et al. [1987] developed a method similar to UTA to estimate the utility function of multi-objective systems for the linear programming systems. Siskos and Despotis [1989] proposed an interactive method called ADELAIS that uses UTA in an iterative way in order to optimize an additive value function in the feasible region defined on the basis of satisfaction levels determined during each iteration. Tangian [2001] proposed a disaggregation technique to calculate quadratic multi-objective functions.

2. The PROMETHEE method

The PROMETHEE method (Preference Ranking Organization METHod for Enrichment Evaluation) [Brans and Vincke, 1985] is based on the principle of pairwise action comparison according to each criterion. It consists in defining a preference function P^k_{ij}, allowing the modeling of the decision-maker's preferences with respect to each criterion k.

When the decision-maker compares two alternatives x_i and x_j, P^k_{ij} represents the degree of preference for x_i, considering only the criterion k. The preference function's value varies between 0 and 1 and is defined separately for every criterion by:

$$P^k_{ij} = \begin{cases} 0 & \text{if} \quad d^k_{ij} \le 0 \\ P^k_{ij} & \text{if} \quad d^k_{ij} > 0 \end{cases} \tag{1}$$

For the usual-criterion:

$$P^k_{ij} = \begin{cases} 0 & \text{if} \quad d^k_{ij} \le 0 \quad \text{(Indifference)} \\ 1 & \text{if} \quad d^k_{ij} > 0 \quad \text{(Strict preference)} \end{cases}$$

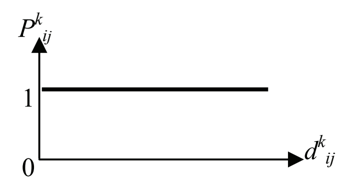

Figure 1. The preference function for the usual-criterion

In this case, there is an indifference between x_i and x_j only if $g_k(x_i) = g_k(x_j)$. As soon as these values are different, there is a strict preference for one of the alternatives. There is no parameter to determine.

For the quasi-criterion:

$$P^k_{ij} = \begin{cases} 0 & \text{if} \quad d^k_{ij} \leq q_k \quad \text{(Indifference)} \\ 1 & \text{if} \quad d^k_{ij} > q_k \quad \text{(Strict preference)} \end{cases}$$

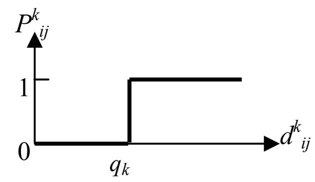

Figure 2. The preference function for the quasi-criterion

In this case, there is an indifference between x_i and x_j as a long as the slack between $g_k(x_i)$ and $g_k(x_j)$ does not exceed the indifference threshold q_k. Beyond this, the preference becomes strict. We find the usual-criterion if q_k=0.

To define the criteria preference function, it is necessary to determine the indifference threshold values (q). The indifference threshold (q) corresponds to the maximum value of d^k_{ij} below which the decision-maker is indifferent between the two alternatives x_i and x_j according to the considered criterion.

The preference modelling, at the time of the decision process, requires for each alternative x_i, the use of the preference indexes C_{ij}, the outgoing flow ϕ_i^+, the incoming flow ϕ_i^- and the net flow ϕ_i.

Therefore, it is necessary to calculate for every alternative x_i:

– The preference index C_{ij} which represents the degree of preference for x_i with regard to x_j over all the criteria simultaneously.

$$C_{ij} = \sum_{k=1}^{n} w_k P^k_{ij} \tag{2}$$

Where w_k is the relative importance coefficient (RIC) given to each criterion k with $w_k \geq 0$ and Σw_k= 1, the greater the RIC, the more important the criterion.

– The outgoing flow ϕ_i^+ which represents the dominance of a with regard to other alternatives.

$$\phi_i^+ = \sum_{j=1,\, i \neq j}^{r} C_{ij} \tag{3}$$

– The incoming flow ϕ_i^- which represents the weakness of a with regard to other alternatives.

$$\phi_i^- = \sum_{i=1,\, j\neq i}^{r} C_{ji} \tag{4}$$

– The net flow ϕ_i which is the difference between the outgoing and the incoming flows.

$$\phi_i = \phi_i^+ - \phi_i^- \tag{5}$$

3. Determination of PROMETHEE's indifference thresholds

Within the framework of the PROMETHEE method, the decision-maker is invited to provide directly the analyser with information concerning alternatives, criteria, the assessment of each alternative according to each criterion, the nature of each criterion function, as well as all parameters' values required for the implementation of the method. The quantitative information that he must provide (relative importance coefficients, preference and indifference thresholds) is not always easy to put in evidence. Besides, many other factors such as the order in which criteria are presented to the decision-maker, the moment at which he is interrogated or the type of the alternative assessed, can lead to considerable variation of parameter values. Consequently, the parameters' values provided directly by the decision maker are subjective and not very reliable. In what follows, we propose to deduce some of these parameters from global information given by the decision-maker.

We suppose that the criteria relative importance coefficients (r.i.c) w_k are known. Criterion functions can only take the form of the usual-criterion or the quasi-criterion. The decision matrix (which is composed of alternatives, criteria as well as the assessment of alternatives according to each criterion) is known and the decision-maker is invited to provide us with p preference relations on some alternatives; relations of the type: alternative x_i is preferred ($\succ$) to alternative x_j. Our objective is to determine the indifference thresholds q_k associated to each criterion k through the resolution of the first mixed integer linear program. When q_k takes the value zero, the k^{th} criterion is a usual one.

Program 1:

$$Minimize\ Z = \sum_{m=1}^{p} S_m^- \tag{6}$$

Subject to

$$\left[\sum_{\substack{i=1\\i\neq l}}^{r}\sum_{k=1}^{n} w_k P_{il}^k - \sum_{\substack{l=1\\l\neq i}}^{r}\sum_{k=1}^{n} w_k P_{li}^k\right] - \left[\sum_{\substack{j=1\\j\neq l}}^{r}\sum_{k=1}^{n} w_k P_{jl}^k - \sum_{\substack{l=1\\l\neq j}}^{r}\sum_{k=1}^{n} w_k P_{lj}^k\right] + S_m^- - S_m^+ = 0 \quad \forall\ m = 1,\dots,p \tag{7}$$

$$P_{ij}^k = 0 \qquad \forall\ d_{ij}^k \leq 0 \quad \forall\ i \neq j,\ i = 1,\dots,r \text{ and } j = 1,\dots,r,\ \forall\ k = 1,\dots,n \tag{8}$$

$$P_{ij}^k \leq P_{hl}^k \qquad \forall\ d_{ij}^k \leq d_{hl}^k \quad \forall\ i \neq j,\ i = 1,\dots,r \text{ and } j = 1,\dots,r\ ;\ \forall\ k = 1,\dots,n\ ;\ \forall\ h \neq l,\ h = 1,\dots,r \text{ and } l = 1,\dots,r \tag{9}$$

$$P_{ij}^k + P_{ji}^k \leq 1 \qquad \forall\ i \neq j,\ i = 1,\dots,r \text{ and } j = 1,\dots,r,\ \forall\ k = 1,\dots,n \tag{10}$$

$$P_{ij}^k \in \{0,1\} \qquad \forall\ i \neq j,\ i = 1,\dots,r \text{ and } j = 1,\dots,r,\ \forall\ k = 1,\dots,n \tag{11}$$

$$S_m^+ \geq \alpha_m \qquad \forall\ m = 1,\dots,p \tag{12}$$

$$S_m^-, S_m^+ \geq 0 \qquad \forall\ m = 1,\dots,p \tag{13}$$

where:

P^k_{ij} is the preference index of x_i on x_j according to the criterion k,
d^k_{ij} is the difference between of assessment of x_i and x_j according to the criterion k,

where

$d_{ij}^k = g_k(x_i) - g_k(x_j)$ if the criterion k is to be maximised,
$d_{ij}^k = g_k(x_j) - g_k(x_i)$ if the criterion k is to be minimised,

p is the preference relation number provided by the decision-maker,
n is the criteria number,
r is the alternative number.

The objective function (6) consists in minimizing the sum of the negative deviations. In this paper, we regard the p preference relations expressed by the decision-maker ($x_i \succ x_j$) as goals to be achieved. In fact, in PROMETHEE method, the "goal" of having x_i preferred to x_j ($x_i \succ x_j$) means that $\phi_i > \phi_j$

then $\phi_i - \phi_j > 0$. We can transform these inequalities into equalities by introducing two slack variables which represent the deviations between the achievements and the decision-maker's preferences (goals).

Let's note by S^+_m the positive deviation in case of objective exceeding and by S^-_m the negative deviation in the opposite case.

Therefore, $(x_i \phi x_j)$ means that $\phi_i - \phi_j + S_m^- - S_m^+ = 0$ with $S_m^+ \geq 0$ and $S_m^- \geq 0$.

In order to reach the goal $(x_i \phi x_j)$, it is necessary that $\phi_i > \phi_j$. We can transform this inequality into equality, by subtracting a positive deviation S^+_m then $\phi_i^+ - \phi_j^- - S_m^+ = 0$ with $S_m^+ > 0$ and $S_m^- = 0$.

In order to satisfy all the preferences expressed by the decision-maker, we must minimize all the negative deviations, which must be ideally null. The objective function will be, therefore, the minimization of the negative deviation sum (6), then the risk encountered is that at the optimality, all the positive and negative deviations are null. In this case, $\phi_i - \phi_j + S_m^- - S_m^+ = 0$ becomes $\phi_i - \phi_j = 0$ and the decision-maker will be indifferent between the alternatives i and j, which is in contradiction with the preference relations provided.

However, $(x_i \phi x_j)$ means that $\phi_i - \phi_j > 0$. We must therefore have at least a small difference between ϕ_i and ϕ_j. In order to have $\phi_i > \phi_j$, and to satisfy the equality $\phi_i - \phi_j + S_m^- - S_m^+ = 0$ with S_m^- taking its minimal value (we prefer that $S_m^- = 0$), we must have $S_m^+ > 0$. For this reason, we introduce in the program constraints of the type $S_m^+ \geq \alpha_m \quad \forall\ m = 1, \ldots, p$ (12) fixing a minimum threshold α_m to each positive deviation S^+_m in order to prevent it from being null. Now, the question is how to choose these thresholds?

We start with fixing an arbitrary threshold α_m to each S^+_m and we solve the mathematical program. At this level, we are interested in positive deviation values S^+_m only.

If the positive deviation values found are much larger than threshold values fixed in the constraints ($S_m^+ > \alpha_m$), then thresholds are well fixed.

However, if the positive deviation values found are equal to the threshold values fixed in the constraints ($S_m^+ = \alpha_m$), then there exists a risk that S^+_m would have another value smaller than α_m, but this cannot happen because of the constraint $S_m^+ \geq \alpha_m$. Hence, it took the minimum, which is equal to α_m. In this case, we decrease the threshold's value α_m and we solve the program again. We verify if the positive deviation values found are much greater than threshold values, and so forth… If the mathematical program does not have a solution, we must reduce the α_m's and we solve it again. In fact, when many constraints are satisfied, the program may have no solution. When α_m (the alternatives preference degree) are reduced, the program may have solution(s).

Concerning the other program constraints, the constraint (7) is related to preference relations provided by the decision-maker. In fact, the relation ($x_i \phi x_j$), expressed by the equality $\phi_i - \phi_j + S_m^- - S_m^+ = 0$, leads us to write the 7th constraint.

Every modification in the preference relations' information provided by the decision-maker will induce modifications at the level of preference functions' values, variables of the program. The modifications of the P^k_{ij}'s values can result either in changes in the indifference threshold values, or in its maintenance at its initial value (because different preference function matrices can give the same value of q_k).

As for the constraint (8) of the program, it expresses the cases where the preference function is null. The preference function P^k_{ij}, whose assessment differs according to the criterion nature, is defined separately for every criterion in (1). Hence, the constraint: If $d_{ij}^k \leq 0$ then $P_{ij}^k = 0$.

The constraint (9) expresses the comparison between preference functions' values, while basing on the comparison between alternative assessment differences. Three cases are presented:

– $\boldsymbol{d^k_{hl} \leq q_k}$ hence $P^k_{hl} = 0$, and since $d^k_{ij} \leq d^k_{hl}$ then $P^k_{ij} = 0$ (therefore $P^k_{ij} = P^k_{hl} = 0$).

– $\boldsymbol{d^k_{ij} > q_k}$ hence $P^k_{ij} = 1$, and since $d^k_{ij} \leq d^k_{hl}$ then $P^k_{hl} = 1$ (therefore $P^k_{ij} = P^k_{hl} = 1$).

– $\boldsymbol{d^k_{ij} \leq q_k < d^k_{hl}}$ hence $P^k_{ij} = 0$ and $P^k_{hl} = 1$ (therefore $P^k_{ij} < P^k_{hl}$).

From these three cases, we conclude that $P_{ij}^k \leq P_{hl}^k$ if $d_{ij}^k \leq d_{hl}^k$

The constraint (10) requires that the sum of the symmetrical preference functions' values not exceed 1. Indeed, the assessment difference matrix is symmetrical with regard to the diagonal, where $i = j$. It means that if $d^k_{ij} = a$, then $d^k_{ji} = -a$ ($a \in \Upsilon$). In fact, when $d^k_{ij} \leq 0$ then $P^k_{ij} = 0$ and when $d^k_{ji} > 0$ then $P^k_{ji} = 0$ or $P^k_{ji} = 1$, all depends on the indifference threshold value q_k. Therefore, P^k_{ij} and P^k_{ji} cannot, both of them, take the value 1. Either one is null and the other is equal to 1 or each of them is null. Hence, $P_{ij}^k + P_{ji}^k \leq 1$.

Besides, the constraint (11) indicates that the preference function in the cases of usual-criterion and quasi-criterion is a binary variable that can only take the values 0 and 1. Indeed,

$$P_{ij}^{k} = \begin{cases} 0 & \text{if} \quad d_{ij}^{k} \leq q_{k} \\ 1 & \text{if} \quad d_{ij}^{k} > q_{k} \end{cases} \tag{14}$$

The last constraint represented in the program of indifference threshold determination is (13): the constraint of no negativity ($S_m^-, S_m^+ \geq 0$) which requires that slack variables not be negative.

The solution of this program provides us with the values of the variables $P^k{}_{ij}$, $S^+{}_m$ and $S^-{}_m$.

From the values of the $P^k{}_{ij}$, $d^k{}_{ij}$ and the relation (14), and by taking into account that q_k is the indifference threshold that corresponds to the smallest assessment difference leading to conclude the strict preference, we deduce the indifference threshold values q_k.

The program can have multiple solutions. In this case, we determine all the program solutions (possible values of the $P^k{}_{ij}$), and we deduce indifference thresholds q_k associated with each solution. We assume that all indifference threshold values are integer. All these threshold values found permit to respect the preference relations provided by the decision-maker.
In addition, the threshold values found permit us to find out the nature of the criteria. Indeed, if $q_k = 0$, the criterion is usual, and if q_k is strictly positive, we have the quasi-criterion.

After having found out all the possible solutions of the thresholds q_k, and in the framework of an interactive approach, we ask the decision-maker to provide information concerning intervals for the indifference thresholds q_k. Among solutions, we look for the one or ones that belong to the intervals.

– If none of the solutions belong to the interval, we ask the decision-maker to change the q_k's intervals.
– If a solution is found, we communicate it to the decision-maker.
– If more than one solution are found in an interval, we ask the decision-maker to reduce the k^{th} interval, or we give him solutions (whose number is reduced), and ask him to choose one of them.

After having deduced the indifference threshold values associated with each criterion, we apply the PROMETHEE method in order to get the total alternative ranking. We present them, together with the preference functions, to the decision-maker. He can then modify the alternative ranking (change the starting information on his preferences or add another preference relation in contradiction with the final alternative ranking).

In this case, the modified information will be modelled in the mixed integer linear program in order to determine the new indifference threshold values that will be presented to the decision-maker. This interactive approach of the indifference threshold determination is summarized in the following chart (Figure 3).

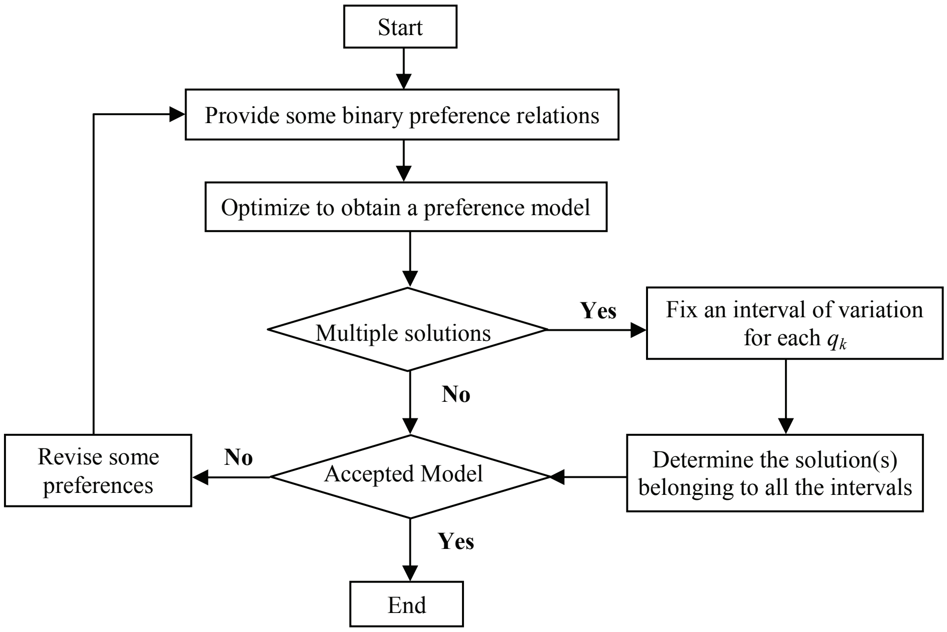

Figure 3. General diagram of the interactive approach

4. Illustrative example

We suppose a decision problem with three criteria C_1, C_2 and C_3 and six alternatives *A*, *B*, *C*, *D*, *E* and *F* is given. The criteria's r.i.c w_k are given, the indifference threshold q_k as well as the type of each criterion function (usual-criterion or quasi-criterion) are to be determined. The decision-maker provides the following decision matrix (Table 1).

Table 1

Decision matrix

Alternatives	Criteria		
	C_1 (Max)	C_2 (Max)	C_3 (Min)
A	6	5	28
B	4	2	25
C	5	7	35
D	6	1	27
E	6	8	30
F	5	6	26
Normalized r.i.c	0,3	0,5	0,2

The decision-maker provides the following information concerning some binary preference relations: $E\phi F$, $A\phi D$, $D\phi B$, $F\phi C$, $E\phi B$.

The assessment difference matrices d^k_{ij}, as well as the preference function matrices P^k_{ij} are represented in Table 2, using the following formula: $P^k_{ij} = 0$ if $d^k_{ij} \leq 0$.

Table 2

Assessment difference matrices and preference function matrices

For $k = 1$

d^1_{ij}	**A**	**B**	**C**	**D**	**E**	**F**
A		2	1	0	0	1
B	–2		–1	–2	–2	–1
C	–1	1		–1	–1	0
D	0	2	1		0	1
E	0	2	1	0		1
F	–1	1	0	–1	–1	

P^1_{ij}	**A**	**B**	**C**	**D**	**E**	**F**
A		P^1_{12}	P^1_{13}	0	0	P^1_{16}
B	0		0	0	0	0
C	0	P^1_{32}		0	0	0
D	0	P^1_{42}	P^1_{43}		0	P^1_{46}
E	0	P^1_{52}	P^1_{53}	0		P^1_{56}
F	0	P^1_{62}	0	0	0	

For $k = 2$

d^2_{ij}	**A**	**B**	**C**	**D**	**E**	**F**
A		3	–2	4	–3	–1
B	–3		–5	1	–6	–4
C	2	5		6	–1	1
D	–4	–1	–6		–7	–5
E	3	6	1	7		2
F	1	4	–1	5	–2	

P^2_{ij}	**A**	**B**	**C**	**D**	**E**	**F**
A		P^2_{12}	0	P^2_{14}	0	0
B	0		0	P^2_{24}	0	0
C	P^2_{31}	P^2_{32}		P^2_{34}	0	P^2_{36}
D	0	0	0		0	0
E	P^2_{51}	P^2_{52}	P^2_{53}	P^2_{54}		P^2_{56}
F	P^2_{61}	P^2_{62}	0	P^2_{64}	0	

For $k = 3$

d^3_{ij}	**A**	**B**	**C**	**D**	**E**	**F**
A		–3	7	–1	2	–2
B	3		10	2	5	1
C	–7	–10		–8	–5	–9
D	1	–2	8		3	–1
E	–2	–5	5	–3		–4
F	2	–1	9	1	4	

P^3_{ij}	**A**	**B**	**C**	**D**	**E**	**F**
A		0	P^3_{13}	0	P^3_{15}	0
B	P^3_{21}		P^3_{23}	P^3_{24}	P^3_{25}	P^3_{26}
C	0	0		0	0	0
D	P^3_{41}	0	P^3_{43}		P^3_{45}	0
E	0	0	P^3_{53}	0		0
F	P^3_{61}	0	P^3_{63}	P^3_{64}	P^3_{65}	

We deduce the preference index matrix C_{ij}, which is given in Table 3. From this matrix, we calculate the incoming flows, the outgoing flows as well as the net flows.

In order to determine the indifference threshold values q_k, we model the information provided by the decision-maker in a mathematical program 2.

Table 3

Preference index matrix

C_{ij}	**A**	**B**	**C**	**D**	**E**	**F**
A		$0.3P^1_{12}+0.5P^2_{12}$	$0.3P^1_{13}+0.2P^3_{13}$	$0.5P^2_{14}$	$0.2P^3_{15}$	$0.3P^1_{16}$
B	$0.2P^3_{21}$		$0.2P^3_{23}$	$0.5P^2_{24}+0.2P^3_{24}$	$0.2P^3_{25}$	$0.2P^3_{26}$
C	$0.5P^2_{31}$	$0.3P^1_{32}+0.5P^2_{32}$		$0.5P^2_{34}$	0	$0.5P^2_{36}$
D	$0.2P^3_{41}$	$0.3P^1_{42}$	$0.3P^1_{43}+0.2P^3_{43}$		$0.2P^3_{45}$	$0.3P^1_{46}$
E	$0.5P^2_{51}$	$0.3P^1_{52}+0.5P^2_{52}$	$0.3P^1_{53}+0.5P^2_{53}+0.2P^3_{53}$	$0.5P^2_{54}$		$0.3P^1_{56}+0.5P^2_{56}$
F	$0.5P^2_{61}+0.2P^3_{61}$	$0.3P^1_{62}+0.5P^2_{62}$	$0.2P^3_{63}$	$0.5P^2_{64}+0.2P^3_{64}$	$0.2P^3_{65}$	

Program 2:

$$Min\ S_1^- + S_2^- + S_3^- + S_4^- + S_5^-$$

Subject to

$$E \succ F:\ 0.3P^1_{52} + 0.3P^1_{53} + 0.3P^1_{56} - 0.3P^1_{62} + 0.3P^1_{16} + 0.3P^1_{46} + 0.3P^1_{56} + 0.5P^2_{51} + 0.5P^2_{52} + 0.5P^2_{53} + 0.5P^2_{54} + 0.5P^2_{56} - 0.5P^2_{61} - 0.5P^2_{62}$$
$$-0.5P^2_{64} + 0.5P^2_{36} + 0.5P^2_{56} + 0.2P^3_{53} - 0.2P^3_{15} - 0.2P^3_{25} - 0.2P^3_{45} - 0.2P^3_{65} - 0.2P^3_{61} - 0.2P^3_{63} - 0.2P^3_{64} - 0.2P^3_{65} + 0.2P^3_{26} + S^-_1 - S^+_1 = 0;$$

$$A \succ D:\ 0.3P^1_{12} + 0.3P^1_{13} + 0.3P^1_{16} - 0.3P^1_{42} - 0.3P^1_{43} - 0.3P^1_{46} + 0.5P^2_{12} + 0.5P^2_{14} - 0.5P^2_{31} - 0.5P^2_{51} - 0.5P^2_{61} + 0.5P^2_{14} + 0.5P^2_{24} + 0.5P^2_{34}$$
$$+0.5P^2_{54} + 0.5P^2_{64} + 0.2P^3_{13} + 0.2P^3_{15} - 0.2P^3_{21} - 0.2P^3_{41} - 0.2P^3_{61} - 0.2P^3_{41} - 0.2P^3_{43} - 0.2P^3_{45} + 0.2P^3_{24} + 0.2P^3_{64} + S^-_2 - S^+_2 = 0;$$

$$D \succ B:\ 0.3P^1_{42} + 0.3P^1_{43} + 0.3P^1_{46} + 0.3P^1_{12} + 0.3P^1_{32} + 0.3P^1_{42} + 0.3P^1_{52} + 0.3P^1_{62} - 0.5P^2_{14} - 0.5P^2_{24} - 0.5P^2_{34} - 0.5P^2_{54} - 0.5P^2_{64} - 0.5P^2_{24}$$
$$+0.5P^2_{12} + 0.5P^2_{32} + 0.5P^2_{52} + 0.5P^2_{62} + 0.2P^3_{41} + 0.2P^3_{43} + 0.2P^3_{45} - 0.2P^3_{24} - 0.2P^3_{64} - 0.2P^3_{21} - 0.2P^3_{23} - 0.2P^3_{24} - 0.2P^3_{25} - 0.2P^3_{26} + S^-_3 - S^+_3 = 0;$$

$$F \succ C:\ 0.3P^1_{62} - 0.3P^1_{16} - 0.3P^1_{46} - 0.3P^1_{56} - 0.3P^1_{32} + 0.3P^1_{13} + 0.3P^1_{43} + 0.3P^1_{53} + 0.5P^2_{61} + 0.5P^2_{62} + 0.5P^2_{64} - 0.5P^2_{36} - 0.5P^2_{56} - 0.5P^2_{31}$$
$$-0.5P^2_{32} - 0.5P^2_{34} - 0.2P^2_{36} + 0.5P^2_{53} + 0.2P^3_{61} + 0.2P^3_{63} + 0.2P^3_{64} + 0.2P^3_{65} - 0.2P^3_{26} + 0.2P^3_{13} + 0.2P^3_{23} + 0.2P^3_{43} + 0.2P^3_{53} + 0.2P^3_{63} + S^-_4 - S^+_4 = 0;$$

$$E \succ B:\ 0.3P^1_{52} + 0.3P^1_{53} + 0.3P^1_{56} + 0.3P^1_{12} + 0.3P^1_{32} + 0.3P^1_{42} + 0.3P^1_{52} + 0.3P^1_{62} + 0.5P^2_{51} + 0.5P^2_{52} + 0.5P^2_{53} + 0.5P^2_{54} + 0.5P^2_{56} - 0.5P^2_{24}$$
$$+0.5P^2_{12} + 0.5P^2_{32} + 0.5P^2_{52} + 0.5P^2_{62} + 0.2P^3_{53} - 0.2P^3_{15} - 0.2P^3_{25} - 0.2P^3_{45} - 0.2P^3_{65} - 0.2P^3_{21} - 0.2P^3_{23} - 0.2P^3_{24} - 0.2P^3_{25} - 0.2P^3_{26} + S^-_5 - S^+_5 = 0;$$

If $d^k_{ij} \le d^k_{hl}$ then $P^k_{ij} \le P^k_{hl}$ $\quad \forall\ i \ne j,\ i = 1,\ldots,6$ and $j = 1,\ldots,6$, $\forall\, k = 1,\ldots,3$

$\forall\ h \ne l,\ h = 1,\ldots,6$ and $l = 1,\ldots,6$

$P^k_{ij} \in \{0,1\}$ $\quad \forall\ i \ne j,\ i = 1,\ldots,6$ and $j = 1,\ldots,6,\ \forall\, k = 1,\ldots,3$

$S^+_m \ge 0{,}01$ $\quad \forall\, m = 1,\ldots,5$

$S^-_m, S^+_m \ge 0$ $\quad \forall\, m = 1,\ldots,5$

By solving the second program, we notice that it has multiple solutions respecting all preference relations provided by the decision-maker.

From the P^k_{ij}'s matrices, the d^k_{ij}'s matrices and the relation (14), we determine the indifference threshold values q_k and we deduce the type associated with each criterion. The results are given in Table 4.

Table 4

The multiple solutions of the mathematical program

Solutions	***1st criterion***		***2nd criterion***		***3rd criterion***	
	q_1	***Criterion type***	q_2	***Criterion type***	q_3	***Criterion type***
1st solution	0	Usual-criterion	6	Quasi-criterion	8	Quasi-criterion
2nd solution	0	Usual-criterion	2	Quasi-criterion	4	Quasi-criterion
3rd solution	0	Usual-criterion	0	Usual-criterion	4	Quasi-criterion
4th solution	0	Usual-criterion	1	Quasi-criterion	3	Quasi-criterion
5th solution	0	Usual-criterion	2	Quasi-criterion	2	Quasi-criterion
6th solution	0	Usual-criterion	1	Quasi-criterion	1	Quasi-criterion
7th solution	0	Usual-criterion	2	Quasi-criterion	1	Quasi-criterion
8th solution	0	Usual-criterion	2	Quasi-criterion	0	Usual-criterion
9th solution	0	Usual-criterion	0	Usual-criterion	2	Quasi-criterion
10th solution	0	Usual-criterion	2	Quasi-criterion	3	Quasi-criterion
11th solution	0	Usual-criterion	0	Usual-criterion	0	Usual-criterion
12th solution	0	Usual-criterion	0	Usual-criterion	1	Quasi-criterion
13th solution	0	Usual-criterion	1	Quasi-criterion	0	Usual-criterion
14th solution	0	Usual-criterion	1	Quasi-criterion	2	Quasi-criterion
15th solution	0	Usual-criterion	6	Quasi-criterion	4	Quasi-criterion
16th solution	0	Usual-criterion	5	Quasi-criterion	1	Quasi-criterion
17th solution	0	Usual-criterion	4	Quasi-criterion	1	Quasi-criterion
18th solution	0	Usual-criterion	4	Quasi-criterion	0	Usual-criterion
19th solution	0	Usual-criterion	6	Quasi-criterion	2	Quasi-criterion
20th solution	0	Usual-criterion	5	Quasi-criterion	0	Usual-criterion
21th solution	0	Usual-criterion	6	Quasi-criterion	3	Quasi-criterion

All the indifference threshold values q_k of each solution permit to respect preference relations provided by the decision-maker.

The decision-maker communicates to us the following information concerning intervals of indifference threshold values: $q_1 \in [0, 2]$, $q_2 \in [0, 4]$, $q_3 \in [2, 6]$. Among the solutions found, and taking into account the assumption that the indifference thresholds must have integer values, seven solutions belong to the fixed intervals (the 2nd, the 3rd, the 4th, the 5th, the 9th, the 10th and the 14th). In this case, we ask the decision-maker to reduce the intervals already fixed. Then, he presents to us the following new intervals: $q_1 \in [0, 2]$, $q_2 \in [2, 4]$,

$q_3 \in [4, 6]$. The 2nd solution belongs to the given intervals, therefore $\boldsymbol{q_1 = 0}$, $\boldsymbol{q_2 = 2}$, $\boldsymbol{q_3 = 4}$. The first criterion is then a true-criterion, whereas the second and the third are quasi-criteria.

While applying the **PROMETHEE** method with the indifference thresholds found, we get the following alternative ranking: *E*, *A*, *F*, *C*, *D*, *B* which satisfy the decision-maker.

Conclusions

In this paper, we clarified and illustrated an approach which permits to determine the indifference threshold values associated with each criterion in the framework of the PROMETHEE II method. This approach of indifference threshold determination presents the advantage of modelling with the unavoidable subjectivity and uncertainty at the level of the alternative assessment, as well as the direct intervention of the decision-maker in the decision process. In addition, it offers us the possibility to start from partial information concerning the preference relations on some pairs of alternatives in order to reach a total ranking, and this is in the context of PROMETHEE II method.

The extension of the methodology for the simultaneous determination of indifference and preference threshold values associated with the criteria function of type criterion with linear preference, level criterion, criterion with linear preferences and indifference area is a direction of research that we pursue presently, the preference threshold (p) corresponds to the minimum value of d^k_{ij} above which we consider that the alternative x_i is strictly preferred to x_j.

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SUSTAINABILITY IN MINING: AN APPLICATION OF PROMÉTHÉE II

Abstract

This article deals with the use of a Multicriteria Decision Aiding method in the evaluation of environmental sustainability. Environmental indicators related to sustainable development require interconnected systems to provide a progress evaluation within a development context for a country, a region, a community or an industrial sector of the economy. The focus of this article is the evaluation process that involves the main players in the mining and metallurgy industries, based on the principles of the Global Report Initiative (GRI). The use of multicriteria analysis aims to offer stakeholders, in particular risk agencies and investment funds, a structured approach to the environmental performance of the mining sector. In this article it is shown how that approach can be put into practice by using the PROMÉTHÉE II method of Multi-Criteria Decision Aiding, providing a more global and transparent result. The selection of some specific indicators led to capturing potential problems in a clear and concise way. The multi-criteria evaluation study presented in this article can be complemented in the future by considering the other environmental indicators of the GRI, even those of a qualitative nature as described by specific actions of environmental management.

Keywords

Mining industry, sustainability, outranking methods, environmental management.

Introduction

This paper deals with the application of a Multicriteria Decision Aiding method in the evaluation of environmental sustainability. The evaluation involves the main players in the mining and metallurgy industries, based on the principles of the Global Report Initiative (GRI). The use of the multicriteria analysis aims to offer stakeholders, in particular risk agencies and investment funds, a structured approach to the environmental performance of the mining sector. This is accomplished by providing an evaluation of the risks associated with environmental sustainability in the sector, and, in this way, orientate investors on the application of funds in organizations whose environmental risks are to be found within the accepted boundaries defined by these entities. In order to do this, specific environmental indicators are considered and, in this fashion, greater transparency is provided to environmental management in this economic sector [Villas Boas and Beinhoff (eds.), 2002]. The decision analysis carried out through this approach is indeed a decision aiding tool within the decision making process, as it permits a relatively large problem to be broken down into a set of situations of less complexity.

The concept of sustainable development has arisen from a relatively long historical process of critical re-evaluation of the relationship of society and its natural environment. As it deals with a continuous and complex problem, even today a variety of approaches can be observed which seek to explain the concept of sustainability. The term sustainable development was first discussed by the World Conservation Union, according to which, for development to be sustainable, it must consider aspects related to social and ecological dimensions, as well as economic factors, living and non-living resources and the short and long term advantages of alternative actions. The focus of the concept is environmental integrity and, only from the definition of the Brundtland Report does the emphasis shift to the human element, creating a balance between the economic, environmental and social dimensions [WWF-Brazil, 2009]. The GRI, in turn, is a broad network of multiple stakeholders composed of thousands of specialists in dozens of countries around the world. The guidelines of the GRI are a set of indicators and recommendations which define a global standard of distribution of information on economic, environmental and social performance [GRI, 2009]. Environmental indicators related to sustainable development require interconnected systems to provide a progress evaluation within a development context for a country, a region, a community or an industrial sector of the economy.

For the process of evaluation and decision making, particularly in the presence of multiple criteria – which are often conflicting – the main role of the analysis is to make clear to those involved in the process the understanding of the problem in question, including here all the variables and actors involved [Belton and Stewart, 2002]. Recent references on multi-criteria sustainability evaluation in mining are scarce in the literature [Esteves, 2008; Slowinski, Greco and Matarazzo, 2002].

Each evaluation or decision criterion, in particular, is a tool which permits the comparison of alternatives according to a particular point of view. The success of the decision aiding process is strongly dependent on the way in which the family of criteria is created. In this way, under the multicriteria focus, there is a need to construct several criteria representing different points of view, allowing the evaluators to express their preferences, which should be seen as partial, as they are restricted to the aspects which each particular criterion covers [Bouyssou, 1990].

The major financial institutions, either national or international, as well as risk credit agencies have professionals dedicated to socio-environmental risk analyses of companies. Those analyses provide bases for credit concession processes as well as investment information to the stock market [City Group, 2011; Goldman Sachs, 2011; Standard & Poors, 2011]. Setting an investment strategy in the stock market relies on an evaluation on socio-environmental grounds. This evaluation aims to keep investment risks within a tolerable margin and at the same time to provide adequate long run returns. Evaluating the way mining companies manage environmental issues by coping with legal requirements and corporate obligations is a fundamental procedure for checking how such companies differ in their governance models and responses to risk exposures.

At present risk agencies such as Goldman Sachs, Citigroup and Standard & Poors perform evaluations of the environmental sustainability of large companies (i.e. companies with market values above 3 billion dollars). This is normally accomplished by taking into account environmental requirements in an isolated fashion. In other words, criteria such as: emission of greenhouse effect gases, consumption of new water, area affected by mining, generation of wastes, etc. are not considered jointly within a broad framework. A visit to the sites of the main risk agencies and financial institutions can verify that reports on environmental sustainability are based on analyzing each indicator separately, without relying on a holistic approach. Through this paper we show how that approach can be put into practice by using the PROMÉTHÉE II method of Multicriteria Decision Aiding, providing a more global and transparent result.

1. Problem definition

1.1. The GRI

The GRI is a broad multi-stakeholder network composed of specialists in dozens of countries around the world. They participate in the GRI work-groups and governance bodies, use its guidelines in their reports, access information in reports based on it and contribute to the development of its structure of reports in other ways, both formally and informally [Gallopin, 1996]. The GRI guidelines are a set of indicators and recommendations which define a global standard of disclosure of information on economic, environ-mental and social performance [GRI, 2009].

1.2. Environmental performance

The environmental dimension of sustainability relates to the impact of the organization on natural living and non-living systems, including biotic and physical environments (soil, air, water). The environmental indicators cover the performance related to raw materials (such as materials, energy, water) and generation (air emissions, wastes water, solid wastes). In addition to this, they consider performance in relation to biodiversity, to environmental legal conformity and other important information such as environment expenses and the impacts of products and services.

1.3. Management

The report must supply a concise description of the environmental management approach, with references to the following environmental aspects: materials, water, biodiversity, air emissions, wastewater and solid wastes, products and services, legal conformity, transport, and general aspects [GRI, 2009].

1.4. Indicators of environmental performance

The aspects contained in the environmental indicators are structured so as to reflect the raw materials, outputs and types of impact that the organization generates in the environment. Energy, water and materials represent three basic types of raw materials used by the majority of the organizations. These raw materials result in relevant outputs from the environmental point of view and are described in the environmental aspects related to air emissions, wastewater and solid wastes. Biodiversity is also related to the concept of raw materials, in the sense that it can be considered a natural resource. However, biodiversity also suffers the direct impact of outputs such as pollutants.

Aspects related to transport, products and services represent areas in which an organization can also have a negative impact on the environment. Generally, this occurs through third parties, such as clients or logistic service providers. Legal conformity and general aspects, in turn, are specific actions that the organizations, according to the GRI, adopt in the management of their environmental performance, such as, for example: ensuring that the industrial wastewater is correctly treated before being released into water courses or implementing and maintaining water sprinklers through internal mineshafts in order to avoid the emission of particulate material.

1.5. Energy

The energy indicators cover the five most important areas of energy consumption by organizations and include both direct and indirect energy. The consumption of direct energy is the amount consumed by the organization to obtain products and to provide services. The consumption of indirect energy, in turn, is all that consumed by others which serve the organization.

The five different areas of energy consumption to which the indicators are associated are described as follows:

- The EN3 indicator relates to the consumption of direct energy by the organization, produced on site.
- The EN4 indicator supplies information related to the consumption of energy necessary for the production of energy purchased externally.
- The EN5 indicator supplies information on energy economized due to improvements in conservation and efficiency.
- The EN6 indicator covers the development of products and services with low energy consumption.
- The EN7 indicator covers the consumption of indirect energy by the activities of the organization.

1.6. Emissions

The aspect related to air emissions, wastewater and solid wastes deals with indicators which measure standard emissions in the environment and which are considered pollutants. These indicators include various types of pollutants which are typically considered in regulatory structures (EN20 to EN23 and EN24). In addition to this, there are indicators for two types of emissions which are the subject of international conventions: greenhouse effect gases (EN16 and EN17) and substances which destroy the ozone layer (EN19). Indicator EN18 covers, in a qualitative way, reductions in emissions achieved and initiatives to reduce these emissions.

2. Case study

2.1. Choice of analytical method

The difficulty in decision making when classifying companies with respect to their environmental performances, by means of the GRI indicators, naturally imposes the use of multicriteria analysis, in the sense that different subjective attributes and aspects are considered, such as: initiatives to supply products and services with low energy consumption, a description of the significant impact of the activities on biodiversity, products and services in protected areas, strategies, measures in operation and future plans for the management of impact on biodiversity, among others. The choice of the multicriteria method to be used, however, depends on the type of problem under analysis, the context studied, the actors involved, the structure and preferences and the type of response which is sought; in other words, the reference problem [Figueira, Greco and Ehrgott, 2005]. The problem approached in this case study, in particular, has as its objective a classification and ranking of alternatives, considering the principal players in the global mining industry, subject to the influence of various environmental performance indicators, according to the GRI standard. The group selected is composed of companies which have an estimated market value of more than USD 10 billion and which published their sustainability reports in 2006. In this way, the selected companies were: BHP Billiton, Vale, Anglo American, Rio Tinto and Xstrata, all open companies with stocks negotiated on the stock exchanges of the United States or the United Kingdom. Each of the environmental indicators will be considered as an evaluation criterion and, therefore, will require inter-criteria information which corresponds to its relative importance in the context of environmental sustainability. For these cases, a special use of the French School methods is recommended, using an approach based on the aforementioned concept of the relation of outranking [Roy and Bouyssou, 1993].

From among the methods based on relations of outranking developed to select, rank and classify the environmental performance indicators of the main players in the mining industry, considering the premises of the GRI, the PROMÉTHÉE family of multicriteria methods was selected as the problem requires a ranking of the alternatives (companies) taking into account the indicators of environmental sustainability. Within that family the PROMÉTHÉE II method was chosen due to its advantage of requiring very clear additional information, which can easily be obtained and managed both by the decision agent and the analyst. This additional information is introduced through the aforementioned generalized criterion, to capture the range of the differences among the evaluations of each of the criteria, enriching the preference structure. Furthermore, PROMÉTHÉE II is a flexible multicriteria method, offering two degrees of freedom to the decision agent: the first relates to the selection of the type of preference function and the second one, to the selection of defining thresholds [Brans and Mareschal, 2002].

It can be observed that the choice of PROMÉTHÉE II is based on the fact that the method, like other methods of the French School, requires intense interaction between the decision agent and the analyst to ensure that the parameters used are clearly defined. In addition to this, the parameters of the model must represent the unanimous consensus of the group or at least the position of a significant majority [Leyva-López and Fernández-González, 2003]. The PROMÉTHÉE II method provides a definition of degrees of preference represented by a real number, which varies from 0 (indifference) to 1 (strong preference). In the case study, this means: a) a comparison of the environmental performance indicators of the main global mining companies, considering the advantages of one over another, without neglecting the common characteristics among them, b) that the criteria for the definition of the environmental performance indicators and the alternatives for each of them are not clearly defined and c) that the criteria and the alternatives are connected, in such a way that one determined indicator can partially reflect another one.

PROMÈTHÈE II was the chosen multicriteria method, although a number of other methods could be used. The highlights of the method were explained to the experts and they felt comfortable with the kind of information they were supposed to provide for its use. They also seemed to understand the notion of generalized criteria, a notion that would serve for capturing the strength of differences between evaluations according to various criteria. This last aspect of PROMÈTHÈE II is regarded as a way to enrich the structure of preference. Coupled with its relative understandability and used by participants in the evaluation process it led to the decision to use PROMÉTHÉE II for tackling the problem. Another important aspect that favored the choice of PROMÈTHÈE II was the intense interaction required among participants and analyst to search for a group consensus on the values of the parameters of that method [Leyva-Lopez and Fernandez-Gonzalez, 2003]. The participants were experts with an average of 20 years of professional experience in different aspects of the mining industry. None of them had a previous experience with the use of methods of Multicriteria Decision Aiding. A number of meetings with these professionals took place to obtain the evaluations needed by the analytical method.

2.2. Computations by PROMÉTHÉE II

This phase included the processing of the data from the sustainability reports of the mining companies using the Decision Lab software [Visual Decision, 2009], with the aim of obtaining the results of the calculations according to the PROMÉTHÉE II method. In this phase a sensitivity analysis was then carried out in relation to the weights used. For the purposes of the research, the environmental performance indicators chosen were those most representative from the environmental sustainability point of view, in relation to the mining industry. The indicators selected are presented in Table 1.

Table 1

Indicators of environmental performance chosen

Type of management	Indicator	Description
Materials	EN1	Materials used by weight and volume
Energy	EN3	Consumption of direct energy discriminated by primary source of energy
	EN4	Consumption of indirect energy discriminated by primary source
Water	EN8	Total water removed by source
	EN10	% and total volume of water recycled or reused
Biodiversity	EN11	Location and size of area owned, leased or administered within protected areas, or next to them, and areas with a high level of biodiversity outside the protected areas
	EN13	Habitats protected or restored
Emissions, effluents and residues	EN16	Total direct and indirect emissions of greenhouse effect causing gases
	EN20	NOx, SOx and other significant emissions by type and weight
	EN21	Total water disposal, by quantity and destination
	EN22	Total weights of residues, by type and method of disposition
Conformity	EN28	Monetary values of significant and total number of non--monetary sanctions resulting from non-conformity to environmental laws and regulations

Table 2 presents the data of the companies researched, obtained from the 2006 sustainability report with the respective weights. The environmental indicators associated with water consumption (EN8), recirculation of water (EN10) and the size of the areas impacted (EN11) received the greatest weights on the grounds that they are the most significant in environmental terms for the mining industry.

Table 2

Values and weights of the selected environmental performance indicators

Indicator	Anglo Gold	BHP	Vale	Rio Tinto	Xstrata	Weight
EN1 – t	12061000	4186100	0	0	0	2.5
EN3 – peta joules	300	304	0	258	25.5	5.0
EN4 – peta joules	0	0	0	0	37.7	5.0

Table 2 contd.

Indicator	Anglo Gold	BHP	Vale	Rio Tinto	Xstrata	Weight
EN8 – m^3	582000	204250000	140000000	391000	85600000	20.0
EN10 – m^3	0	170000000	114800000	0	101300000	20.0
EN11 – ha	8000	0	0	350	8829	15.0
EN13 – ha	0	2400	400	401	992	15.0
EN16 – t	36447000	51000000	0	28300000	0	5.0
EN20 – t	136000	259850	0	0	252888	2.5
EN21 – m^3	208328000	88180000	0	0	7258000	2.5
EN22 – t	278913	202530	0	2192000	957600000	2.5
EN28 – USD	0	141526	0	56800	8100	5.0

Environmental problems caused by mining activities are of different types. One of them concerns disturbing the land surface through mining and it is foremost present in open-pit mines. Mining activities can also contribute to polluting surface and groundwater by mining materials, concentration of chemical products used in the processing stage, lixiviation and flow of sediments to hydric bodies. Based on those major environmental impacts, higher values for weights were therefore assigned taking into consideration impacts associated with water impounding, generation of wastes, biodiversity and emission of greenhouse gases. Lower weights were assigned to other impacts. This rationale is not only aligned with the environmental perspective in terms of impacts, but it also meets the 2000 Millenium Goals concerning loss of biodiversity, access to potable water and rehabilitation of degraded areas [United Nations, 2000].

The value 0 shown in Table 2 is used to represent information not available in sustainability reports analyzed. As a matter of fact, the GRI allows different levels of reporting and in 2006 not all companies surveyed disclosed or had information on all environmental sustainability indicators. The weight values ranging from 2.5 to 20, also shown in Table 2, were defined through meetings with the experts. Those were then asked to associate the degree of importance of criteria by weights. Therefore, weights in Table 2 add to 100. Using sensitivity analyses, the analyst deals with cases where information was incomplete.

2.3. Results

2.3.1. Outputs from Decision Lab

Using the command *View*, option *Rankings*, the total classification shown in Figure 1 is obtained.

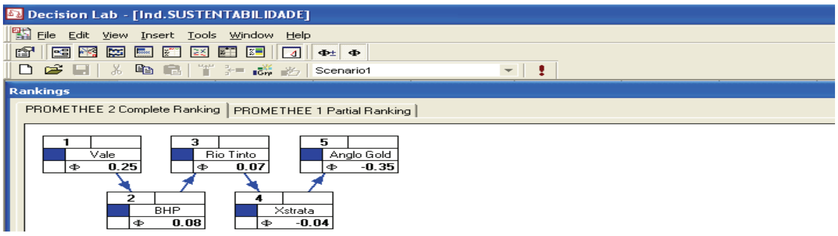

Figure 1. Classification of the companies

2.3.2. Sensitivity analysis of PROMÉTHÉE II

The sensitivity analysis of the results of PROMÉTHÉE II was carried out with respect to the variation of weights, with the purpose of evaluating alterations in the results due to fluctuations in their values. With respect to the criteria weights, five additional options to the scenario were chosen in which results were obtained, namely:

– Uniformity: considering all the weights distributed equally.
– Reduction: maintaining the importance of the greatest criterion and reducing the values of the others.
– Inversion 1: inverting the order of the scenario considered, giving greater importance to the second criterion.
– Inversion 2: inverting the order of the scenario, giving greater importance to the second criterion and reducing the rest.

From the results processed by Decision Lab the following conclusions could be reached:

– In all the scenarios tested, the net flows are not practically altered. this shows a tendency for small alterations in the order of the alternatives and values of the flows.
– The cut-off line between positive and negative flows remained constant between the options Xstrata, Rio Tinto and Anglo Gold.
– An inversion of the order was discovered between the alternatives Vale and BHP, when the option Inversion 2 was applied.
– The variations of the values of the flows were not very sensitive to changes in the weights.

In this way, it was concluded that the results obtained with the weights selected behave in a consistent way when evaluated in relation to other scenarios deemed probable.

2.3.3. Discussion of results

Applying PROMÉTHÉE II led to identifying Vale as the company that corresponds to the best result as regards the 12 criteria used in the analysis covering the areas of materials, energy, water, biodiversity, emissions, effluents and residues, and conformity. The net flow provided by that method can be visualized in Figure 1. Nevertheless, that net flow does not indicate that Vale is about 3 times better than BHP, nor that Vale is 7 times better than Xstrata. In order to confirm the rank obtained by PROMÉTHÉE II some changes have been introduced in the data. New computations were then performed as part of the sensitivity analysis.

The generalized criterion of type I (usual) was used in this application of PROMÉTHÉE II [Brans and Mareschal, 2002]. The experts agreed that the Type I preference function should be used in the evaluation of environmental impact, since a situation of indifference would only be identified between the performances of alternatives if their values were equal. As long as a difference exists there is a strict preference for the alternative with a higher performance. This contributes to minimizing the values of indicators thus characterizing a lower environmental impact in the region of the mineral venture. No corporate parameter is then set because reducing the environmental footprint is always sought in mining operations. It is desirable that that footprint be kept as low as possible.

Conclusions

Indicators of environmental sustainability need to be aggregated without the loss of precious information. That aggregation provides an effective evaluation of environmental performance. The multicriteria analysis performed through the use of PROMÉTHÉE II allowed to identify the level of environmental sustainability of the major players in the mining industry. The selection of some specific indicators led to capturing potential problems in a clear and concise way. A higher degree of transparency associated with that level of environmental sustainability was thus provided.

The experts in different aspects of the mining industry that participated in the evaluation concluded that the application of PROMÉTHÉE II was useful in the context analyzed. It helped to aid the decisions involved in the case studied, because it combined a form of classifying alternatives – the main mineral companies – by market value, with results of environmental performance based on internationally recognized methodology accepted by the risk classification agencies of the market. Among the best results obtained by the implementation of the method the following can be cited: the construction of an organized way to think about the environmental performance of the main global players in the mining sector. and the possibility of disclosure with greater transparency for Vale shareholders, risk classification agencies and other stakeholders, by means of a structured methodology, in this way avoiding loss of precious time without a meaningful practical result. From the initial construction of the table of alternatives, criteria and weights the alternative solutions could be shared and easily understood, with their validation obtained in a very practical manner. The results were simulated by variations of weights, based on the importance attributed to the environmental performance criteria.

With respect to the practical questions related to the application of the method, chiefly in terms of the results processed, it was possible to conclude its applicability, through the result of the net flows of PROMÉTHÉE II tested in sensitivity analyses. This permits a clear view of the fluctuations as regards the modifications of the values associated with the weights. In addition to this, data processing through the Decision Lab software permitted a simple approach to the problem, based on sensitivity analyses, which led to objective and easily understood results. In this particular case study, one observed limitation was lack of information related to the environmental performance indicators of the companies, meaning that the method assumed null values, not due to operational excellence in a determined topic. With respect to the absence of information on some environmental indicators, the perception of the risk agencies as regards the environmental performance of Vale improved, confirming the need to provide the interested parties in the company, quantitative information and a minimum of conjectures not adequately founded on its environmental sustainability. The conclusion was reached that the application of the PROMÉTHÉE II method managed to fulfill its objective completely in the sense of organizing a complex decision making process, which presupposes interactivity and simulations arriving at a result which provided transparency to the effectiveness of the environmental management of the main players in the global mining industry.

The multicriteria evaluation study presented in this paper can be complemented in the future by considering the other environmental indicators of the GRI, even those of a qualitative nature (described by specific actions of environmental management), associating perhaps the Verbal Decision Analysis approach [Larichev and Moshkovich, 1997. Gomes, Moshkovich and Torres, 2010] with PROMETHÉE II. This work would provide a wider ranging evaluation of the environmental management of the mining companies and thus make an evaluation in terms of sustainability more representative.

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ON THE CHOICE OF METHOD IN MULTI-CRITERIA DECISION AIDING PROCESS CONCERNING EUROPEAN PROJECTS

Abstract

In this paper the problem of selecting the most appropriate multi-criteria decision aiding method for a particular application is considered. It is illustrated by a real-life example concerning applications for project co-financing by the European Union.

Making a proper decision on which method to choose is difficult because of the great diversity of MCDA techniques proposed so far within the literature. Thus, the systematic analysis of their assumptions and properties is required.

The paper presents the main strengths and weaknesses of particular decision aiding tools applicable to the problem of ordering European projects as well as chosen procedures aiming at facilitating the process of selecting an appropriate one.

Moreover, an extension of EXPROM II by stochastic dominance rules is proposed.

Keywords

Multi-criteria decision aiding methods, model choice algorithm, model selection process, EXPROM II with stochastic dominances.

Introduction

European regional policy is nowadays one of the most vital factors in strengthening the socio-economic development of Poland and other European Union countries, especially those that entered the EU in 2004 and 2007, whose economies have lagged far behind the economies of the old Member States of EU-15 and whose needs in the areas of environment, infrastructure, research and innovation, industry, services and SMEs are truly significant.

Regional policy helps to reduce disparities between countries, increase the regions' competitiveness and attractiveness, improve the employment prospects and support cross-border co-operation through financing specific projects for regions, towns and their inhabitants. In the previous programming period 2000-2006 over 233 billion EUR was earmarked for all regional instruments for the 15 old Member States. Moreover, around 24 billion EUR was allocated for the 10 new Member States for the years 2004-2006, not to mention 22 billion EUR granted for pre-accession aid. In the present programming period 2007-2013 cohesion policy will benefit from total allocation of about 347 billion EUR, which represents nearly 36% of the entire Union's budget.

Because of the enormous amount of money devoted to the structural aid it is crucially important to allocate the means in the most effective way possible. And that depends among other things on the proper choice of projects that are going to be co-financed. In order to help the decision-makers in this challenging and difficult task, multi-criteria decision aiding techniques, which refers to making decision in the presence of multiple, usually conflicting criteria, should be applied as evaluation of the European projects requires taking into account many diverse aspects: economic, financial, ecological, technical, technological, social and legal.

Since many different MCDA methods are available and there are specific advantages, disadvantages and limitations of each of them, a detailed analysis must be carried out in order to choose an appropriate technique for a particular decision-making problem. Otherwise the solution may be misleading or unsatisfactory, useful methods may be rejected incurring losses in valuable time, energy and money and, last but not least, the potential users may be discouraged from applying MCDA methods to real-world problems [Gilliams et al., 2005].

The main aim of this paper is to examine and compare the applicability of various MCDA methods to the problem of ordering projects applying for co-financing from the EU. Decision concerning the usefulness of the selected methods will be taken on the basis of the analysis of the decision-making problem and the decision-making process as well as on the basis of the examination of the information constraints and the profile of the decision-makers.

Selecting the correct MCDA method is in itself an MCDA problem as there is a wide variety of criteria upon which the choice should be based. Therefore a number of procedures to assist both the analyst and the decision-maker to choose a suitable method for a specific decision problem has been presented in literature [Al-Shemmeri et al., 1997]. These procedures are helpful inasmuch as they can confirm or deny the results of the qualitative analysis mentioned above. While the confirmation of the outcome of the qualitative analysis indicates that the process of method selection reaches the end as the choice that has been made is appropriate, the denial implies rethinking of the problem.

In this paper the model choice algorithm of Gershon (see [Gershon, 1981]) and the model selection process of Tecle (see [Tecle, 1988]) will be applied in order to support the qualitative analysis.

1. Comparative analysis of selected MCDA methods

The choice of a correct tool to solve the decision-making problem depends on the type of the problem as well as on the goals of the decision-makers and the desired properties of the solution obtained. Sometimes 'the simpler the method, the better', but complicated decision problems may also require complex methods.

In the case of the European projects the approach based on the multiattribute utility theory (see [Keeney, Raiffa, 1976]) may be implemented. Methods falling into this category assume that there exist global utility function to represent the decision-maker's preferences and it can be built through aggregating variants' partial utilities (according to each criterion). But the reduction of a multidimensional evaluation to a one-dimensional one via the formulation of global utility function is possible only when certain rigorous conditions* are met. Besides, it may lead to the complete compensation between criteria – the situation in which the variant evaluated low against one or even more criteria is ranked high because it has achieved high grades against the remaining criteria. In this approach a not very realistic assumption is accepted that the decision-maker's preferences are given and fixed, i.e. they are expressed clearly and result in good ordering alternatives against criteria – the decision-maker is able to indicate, without any hesitation, even the smallest differences in utilities and confidently, consequently and precisely assign the scores to variants considered. In addition, determining an analytical form of the global utility function is usually very difficult and sometimes even infeasible – it happens frequently that the decision-maker is not able to provide information essential to build this function [Trzaskalik et al., 1998].

An interesting alternative is the approach based on the outranking relation and on the fundamental partial comparability axiom (see [Roy, 1990]) in which incomparability plays a key role [Martel, 1998]. The basic idea of this approach is as follows: alternative A outranks B if on a great part of the criteria A performs at least as good as B (concordance condition), while its worse performance is still acceptable on the other criteria (non-discordance condition). Indifference thresholds and preference thresholds are introduced in order to build outranking relations that represent decision-makers' preferences and constitute partial relations of the global preferences. In this kind of approach there is place for incomparability, explained e.g. by the lack of sufficient

* For instance, the necessary and sufficient condition of applying an additive form of the utility function in the situation when the evaluations are deterministic is mutual preferential independence of the criteria. If the evaluations have the form of probability distributions the above mentioned condition is not sufficient – in that case the utility independence condition must be satisfied [Trzaskalik et al., 1998].

information to define preferential situation [Trzaskalik et al., 1998]. The procedures exploited according to this approach – among which the ELECTRE and PROMETHEE methods stand out – are usually less demanding for their users at the informational level and result in more balanced recommendations than those belonging to the first approach of a single criterion synthesis [Martel, 1998]. Since their assumptions correspond to reality they can definitely improve the procedure of appraising and selecting projects applying for co-financing from the European Union.

Although expected utility models and outranking relation models used to be often treated as competitors, it is possible to benefit from both approaches in the situation when the performances of various alternatives are evaluated in a probabilistic way (as it is in the case of the European projects because the number of experts participating in evaluation is greater than 1). Namely, stochastic dominance rules can be employed to establish preferences with respect to each criterion and the criteria aggregation method based on the outranking relation procedure can be used to obtain global preference [Martel, Zaraś, 1995]. Moreover, the concept of pseudo-criteria can be employed to distinguish situations of strict preference, weak preferences and indifference [Nowak, 2004]. As a matter of fact, applying this combined approach seems to be an appropriate solution in the case of appraisal of European projects.

In Table 1 the main advantages and drawbacks of various MCDA techniques in the context of the European projects selection are presented.

Table 1

Strengths and weaknesses of selected MCDA methods

No.	Method	Characteristics
1	Arithmetic mean of weighted sums of the scores of all experts participating in the Panel (see [Ministerstwo Gospodarki i Pracy, 2004])	The possibility of ranking the European projects with help of arithmetic mean of the weighted scores given by the members of the Panel of Experts – the procedure that was used in Poland in the period 2004-2006 – seems somewhat illusory in view of uncertainty, inaccuracy, instability and indefiniteness concerning data, evaluations and preferences characteristic for decision-making problems. Especially in the situation such as discussed here, when a large number of different stakeholders (e.g. various decision-makers, experts in the appropriate fields, consulting companies responsible for preparation of the projects, political parties, civic organisations, inhabitants and interest groups affected by the decision) with conflicting preferences are involved and costs and benefits of the alternatives are difficult to assess. Furthermore, this method – as others based on multi-attribute utility theory – allows for complete compensation between criteria, what can be even dangerous in the case of investment projects as poor performance on one criterion (e.g. technical feasibility) can be easily counterbalanced with a good one on another one. Moreover, it is

Table 1 contd.

No.	Method	Characteristics
		also imperfect as far as group decision-making is concerned since it does not take into account the distributions of the evaluations. On the other hand, this method is completely comprehensible for the potential users and because of that it is easy to implement. Besides, it requires neither a skilled analyst to operate the system nor specialized software
2	ELECTRE III with stochastic dominances (see [Nowak, 2004])	This method requires from its users determination of indifference, preference and veto threshold as well as the weights of criteria. The thresholds values are easily interpretable and allows for better reflection of the decision-maker's preferences but their determination is time-consuming. Thanks to the veto thresholds the technique is partially compensatory (really bad score on one criterion cannot be compensated by a good score on another). Besides, it takes into consideration distributions of the evaluations thanks to applying the stochastic dominance rules. The final rankings are not transitive and the results are partial orders. The technique is complex and mathematically complicated – hence an analyst as well as specialized software are required. Because not every step of this method is understandable for the decision-makers it may be difficult to persuade them to apply it. Another drawbacks of this method are as follows: – on one level a few projects can be classified, so in some cases we are not sure which projects should be co-financed, – the form of the final solution – final ranking without any points may be unconvincing for the potential users, – the possibility of incomparability occurrence – no potential beneficiary will accept the explanation that his/her application was rejected because it was incomparable with others
3	PROMETHEE II with stochastic dominances (see [Nowak, 2005])	This method allows to discard the last three shortcomings of the ELECTRE III method with stochastic dominances as a complete pre-order of the projects is proposed to the decision-maker and points are assigned to the alternatives. The idea of calculation of the net flow for each project connected with this method is much preferable to the idea of distillation procedure connected with ELECTRE III from the point of view of participants of the decision-making process. They consider it as more user-friendly: easier to understand and to implement
4	Modified BIPOLAR* with stochastic dominances (see [Górecka, 2008])	Rankings obtained with help of both ELECTRE III and PROMETHEE II methods do not allow for stating whether highly ranked projects are really good or just the best of the weak ones. This problem can be solved by applying BIPOLAR method with modifications introduced by the author of this paper which enable ranking and sorting projects as well as determining their quality by taking into account what is good and undesirable from the decision-maker's point of view in the decision-making problem. At the same time, the problem of the projects' incomparability is eliminated

Table 1 contd.

No.	Method	Characteristics
5	EXPROM II with stochastic dominances (see Appendix)	This method has similar strengths and weaknesses as PROMETHEE II with stochastic dominances but is based on the notion of ideal and anti-ideal solutions and enables the decision-maker to rank variants on a cardinal scale
6	SMART (see [Edwards, 1977; Edwards, Barron, 1994])	This method is a simple way to implement the multi-attribute utility theory by using the weighted linear averages. Its advantages and disadvantages are similar to those of arithmetic mean of weighted sums
7	TOPSIS (see [Hwang, Yoon, 1981])	According to this method the most preferred alternative should have a profile which is nearest to the ideal solution and farthest from the anti-ideal solution. It is slightly more complicated than, for example, SMART and potential users may not be completely aware of its consequences
8	AHP (see [Saaty, 1980])	Applying this method to European projects' selection is impossible as it requires from its users making comparisons between all the alternatives, and it happens that there are over 100 projects in the competition – in such situations pair-wise comparisons are infeasible

* The original version of BIPOLAR method was proposed by Konarzewska-Gubała. Detailed description of this technique is presented in [Konarzewska-Gubała, 1991].

To sum up, the following characteristics of the decision-making problem analysed and the following expectations of the decision-makers involved in the realisation of the EU regional policy should be taken into consideration in the process of selecting the most appropriate multi-criteria decision aiding method for the problem of choosing project applying for co-financing by the EU:

– the decision-making problem should be formulated as a problem of ordering a finite number of alternatives – it is indispensable to each beneficiary to be classified in the ranking and to know its own result,
– the problem is a group decision-making problem – experts engaged in the projects' appraisal individually and independently evaluate a finite number of competing projects and it is required to incorporate diverse individual views into a blended final decision,
– there should be a possibility to employ both quantitative and qualitative criteria,
– decision-makers are able to present the information about their preferences but they do not have much time for the interaction and cooperation with the analyst,
– participants of the decision-making process have very diverse educational background and their knowledge about multi-criteria decision aiding methods is usually limited,

- the decision aiding technique should not be too complicated so as to enable decision-makers to explain to the applicants how it works and elucidate the reasons of their projects rejection,
- the decision-making method should not be too simple so as to limit the possibilities of manipulating the results,
- it should be taken into account that experts appraising many projects during several days may not be consistent in their evaluations, especially in view of uncertainty and inaccuracy characteristic for the decision-making problem discussed,
- the possibility of a complete compensation occurrence should be removed – in the case of some criteria it may be hazardous and in the case of others, projects should fulfil the so-called "minimal quality",
- there is no room for the incomparability of the alternatives – ranking should be complete as the explanation that the project has not been selected for co-financing because of the incomparability with the others will not be accepted by the applicants,
- the possibility that a few projects will be classified on the same place in the ranking should be limited as it may create problems with dividing the funds,
- the final solution should take the form in which the scoring points occur, otherwise it may be unconvincing for the applicants,
- it is desired that the decision aiding method enables to determine whether the highly ranked projects are really good or just better than the weak ones.

Taking into account all the above-mentioned information on the properties of the decision-making problem analysed, its participants and the selected MCDA techniques, the most suitable method to aid the decision-making process seem to be one based on the outranking relation combined with stochastic dominance rules, namely PROMETHEE II, EXPROM II or modified BIPOLAR technique. On the one hand, PROMETHEE II is the simplest and the most user-friendly of these three but, on the other hand, it allows only to determine the relative quality of the projects. This drawback does not occur in the modified BIPOLAR method from which the problem of projects incomparability has been also removed but this method is unfortunately more complicated than PROMETHEE II and it may be hard to explain to people without mathematical background. EXPROM II, in turn, enables to create cardinal rankings of the projects and it is only a bit more complex than PROMETHEE II – accordingly it seems that this is the method that should be recommended.

2. Model choice algorithm of Gershon

Gershon's model contains 27 criteria as a basis of comparison between different MCDA methods. They are divided into 4 groups:

– criteria 1-5: compulsory binary criteria which remove candidate techniques from further consideration if they are not fulfilled, rated as 1 (if selected) or 0 (if not selected),
– criteria 6-12: non-obligatory binary criteria, rated as 1 (if satisfied) or 0 (if not satisfied),
– criteria 13-19: technique dependent criteria rated on a 0-10 subjective scale,
– criteria 20-27: application dependent criteria rated on a 0-10 subjective scale [Al-Shemmeri et al., 1997].

The procedure involves the selection of a subset of the criteria that are relevant to the problem, assignment of weights to the criteria in the subset and appraisal of the candidate methods with respect to predetermined criteria.

The decision situation was examined in the context of the problem of choosing European projects. Ten criteria were found to be irrelevant to the situation. Criteria 3 and 4 (continuous sets and dynamic problems) are not applicable, as the problem involves an explicit list of predefined alternatives and the input data is not changing. In turn, criterion 5 was eliminated because treating the problem as a stochastic one is neither required nor indispensable (although desired). Criteria 12, 18 and 20-23 were also eliminated because either they are meaningless for ordering the European projects or refer to conditions not encountered in this problem.

Compromise programming was applied[*] to rank the 7 methods listed in Table 2 and select the one that is closest to the ideal solution determined as follows: [1,1,1,1,1,10,10,10,10,10,10,10,10,10,10]. The distance metric to minimize was defined in the following way:

$$L_1 = \sum_{k=1}^{n} w_k \cdot \frac{f_k^* - f_k(a_i)}{f_k^* - f_k^{\min}},$$

where w_k is the weight, f_k^* is the optimal value of the criterion *k*, $f_k^{\min}$ is the worst value attainable for criterion *k* and $f_k(a_i)$ is the evaluation of the *i*th technique with respect to the *i*th criterion.

As a result of detailed analysis of the selected MCDA techniques properties against chosen criteria and applying the model choice algorithm[**] it turned out that the method closest to the ideal solution is EXPROM II combined with stochastic dominances with a distance value of 9,8 as it is shown in Table 2. On the opposite end of the ranking ELECTRE III method with stochastic dominance rules was placed.

[*] It is possible to use other multi-criteria methods, e.g. ones based on the outranking relation such as PROMETHEE.

[**] The opinions expressed herein are those of the author and do not necessarily represent those of the decision-makers involved in the implementation of the EU regional policy.

Table 2

Model choice algorithm and evaluations of chosen multi-criteria methods

No.	Criterion	Eliminated	Weight	Weighted sum	ELECTRE III	PROMETHEE II	EXPROM II	Modified BIPOLAR	SMART	TOPSIS
1	Handle qualitative criteria		x	1	1	1	1	1	1	1
2	Handle discrete sets		x	1	1	1	1	1	1	1
3	Handle continuous sets	x	x	x	x	x	x	x	x	x
4	Handle dynamic problems	x	x	x	x	x	x	x	x	x
5	Handle stochastic problems	x	x	x	x	x	x	x	x	x
6	Comparison to goal point		1	0	0	0	1	1	0	1
7	Comparison to aspiration level		1	1	0	0	0	0	1	0
8	Direct comparison		1	0	1	1	1	0	0	0
9	Strongly efficient solution	x	x	x	x	x	x	x	x	x
10	Complete ranking		7	1	0	1	1	1	1	1
11	Cardinal ranking		5	1	0	0	1	1	1	1
12	Ability to handle integer variables	x	x	x	x	x	x	x	x	x
13	Computer time required		1	10	8	9	9	9	10	10
14	Implementation time required		1	10	2	4	4	4	9	9

Table 2 contd.

No.	Criterion	Eliminated	Weight	Weighted sum	ELECTRE III	PROMETHEE II	EXPROM II	Modified BIPOLAR	SMART	TOPSIS
15	Interaction time required		3	10	6	6	6	2	10	10
16	Decision maker's awareness		2	10	2	3	3	3	8	6
17	Consistency of results		4	10	8	10	10	10	10	10
18	Robustness of results	x	x	x	x	x	x	x	x	x
19	Handle group decision maker		4	4	10	10	10	10	4	4
20	Number of objectives	x	x	x	x	x	x	x	x	x
21	Number of systems	x	x	x	x	x	x	x	x	x
22	Number of constraints	x	x	x	x	x	x	x	x	x
23	Number of variables	x	x	x	x	x	x	x	x	x
24	Decision maker's level of knowledge		3	10	6	6	6	2	10	10
25	Time available for interaction		2	10	8	8	8	6	10	10
26	Desire for interaction		3	10	7	7	7	5	10	10
27	Confidence in preference structure		7	6	9	10	10	10	5	5
Distance metric				**11.6**	**29.2**	**15.8**	**9.8**	**16**	**13.625**	**14.125**

Source: Own calculation.

3. Model selection process of Tecle

This method consists of identifying a set of feasible MCDA techniques and evaluating them with respect to 49 criteria which are divided into 4 sets:

- criteria 1-13: problem related criteria,
- criteria 14-20: decision-maker or analyst related criteria,
- criteria 21-40: technique related criteria,
- criteria 41-49: solution related criteria.

They are presented in Table 3.

The main steps of the procedure are as follows:

- the desired objectives to be satisfied by the MCDA techniques are determined,
- the evaluation criteria relating methods' capabilities to the objectives are chosen,
- the MCDA techniques available for achieving the aims defined in the first step are selected,
- the methods' capabilities or the levels of performances of the techniques with respect to the successive evaluation criteria are determined according to the opinions and beliefs of the user,
- an evaluation matrix is constructed, whose elements represent the capabilities of competing techniques in terms of the selected criteria,
- the performances of the alternative MCDA methods specified in the third step are analysed [Al-Shemmeri et al., 1997].

Seven MCDA methods (the same as in the previous part of the paper) have been examined and appraised* for their performance in solving a multi--criteria European projects problem and only 23 out of the 49 criteria have been utilized in order to do that. The subset of the criteria and techniques which were selected for considered problem are presented in Table 3.

As far as the weighting coefficients are considered every criterion in each group was assigned a weight relative to its importance in that group as perceived by the user. It was done with help of 'resistance to change' grid proposed by Hinkle (see [Rogers, Bruen, 1998]). The calculated weights are presented in Table 3.

* The opinions expressed herein are those of the author and do not necessarily represent those of the decision-makers involved in the implementation of the EU regional policy.

Table 3

Model selection process and evaluations of chosen multi-criteria methods

No.	Criterion	Selected	Weight	Weighted sum	ELECTRE III	PROMETHEE II	EXPROM II	Modified BIPOLAR	SMART	TOPSIS
Characteristics Describing the Problem										
1	Ability to choose among a finite number of alternatives	x	50	1	1	1	1	1	1	1
2	Ability to choose among an infinite number of alternatives									
3	Ability to deal with problems having nonnumerical criteria	x	50	1	1	1	1	1	1	1
4	Ability to handle stochastic multi-objective problems									
5	Ability to solve dynamic MO problems									
6	Size of problem (small scale or large scale)									
7	Number of objectives									
8	Number of alternatives									
9	Number and type of constraints									
10	Number of variables									
11	Ability to handle problems with fuzzy information									
12	Ability to handle MO integer of mixed integer problems									
13	Ability to handle MO nonlinear problems									

Table 3 contd.

No.	Criterion	Selected	Weight	Weighted sum	ELECTRE III	PROMETHEE II	EXPROM II	Modified BIPOLAR	SMART	TOPSIS
Characteristics Describing the DM or Analyst										
1	Individual or group decision making	x	28	4	10	10	10	10	4	4
2	Time available of DM for interaction	x	6	10	8	8	8	6	10	10
3	The DM's desire for interaction	x	28	10	7	7	7	5	10	10
4	Assumption of DM's preference information									
5	DM's actual level of knowledge	x	16	10	1	2	2	2	8	8
6	DM's level of knowledge required	x	11	10	6	6	6	2	10	10
7	Need for skilled analyst to operate and interpret system	x	11	10	4	5	5	5	9	9
Characteristics Describing the Techniques										
1	Ability of comparison to goal point									
2	Ability of comparison to aspiration level									
3	Ability of direct comparison									
4	Ability to predict best alternative	x	19	3	7	8	9	10	4	4
5	Ability to determine efficient points									

Table 3 contd.

No.	Criterion	Selected	Weight	Weighted sum	ELECTRE III	PROMETHEE II	EXPROM II	Modified BIPOLAR	SMART	TOPSIS
6	Applicability to real-world problems	x	19	5	7	10	10	10	4	4
7	Plausibility of algorithm									
8	Ease of use	x	14	10	1	3	2	2	9	8
9	Flexibility									
10	Instrumentality to achieve a solution									
11	Ease of coding	x	7	10	4	6	6	6	9	9
12	Computational burden	x	2	10	7	8	8	8	10	10
13	Interaction time required	x	14	10	6	6	6	2	10	10
14	CPU time required	x	2	10	8	9	9	9	10	10
15	Rapidity of convergence									
16	Parameters specifications required (like weights, trade--offs)	x	14	10	7	7	7	4	10	10
17	Amount of total information required									
18	Is algorithm an ad hoc procedure									
19	Is the technique interactive									
20	Need for specialized software	x	9	10	1	2	2	2	8	8

Table 3 contd.

No.	Criterion	Selected	Weight	Weighted sum	ELECTRE III	PROMETHEE II	EXPROM II	Modified BIPOLAR	SMART	TOPSIS
Characteristics Describing the Solution Obtained										
1	Type of nondominance of solution (rank or numerical)	x	15	10	4	7	10	10	10	10
2	Strength of efficient solution									
3	Number of solutions obtained at each iteration									
4	Complete ranking	x	15	10	5	10	10	10	10	10
5	Cardinal ranking	x	5	10	0	0	10	10	10	10
6	Consistency of result	x	15	10	8	10	10	10	10	10
7	Robustness of result									
8	Confidence in the solution obtained	x	25	6	9	10	10	10	5	5
9	Usefulness of result to DM	x	25	7	4	7	10	10	7	7
Distance metric				**95,33**	**197,94**	**119,01**	**92,85**	**123,83**	**110,90**	**112,45**

Source: Own calculation.

After establishing the weights for the criteria the evaluation process was continued:

– criteria describing the problem were appraised using "yes" or "no" response, rated as 1 (if satisfied) or 0 (if not satisfied),
– criteria describing the decision-maker or analyst as well as the technique related criteria and solution related criteria were evaluated using a 0-10 subjective scale [Al-Shemmeri et al., 1997].

The evaluation matrix was analysed with help of compromise programming resulting in construction of the ranking of the methods according to ascending order of the values of the distance metric. The compromise programming was utilized as in Gershon's algorithm instead of composite programming which was originally used by Tecle in his model.

The analysis carried out in this part of the paper confirmed that EXPROM II method together with stochastic dominance rules is the most suitable method for ordering projects applying for co-financing from the European Union. On the last place ELECTRE III method with stochastic dominances was classified, as it was in the former order.

Conclusions

Two algorithms were implemented to aid the process of selecting a suitable technique for ranking projects applying for co-financing from the European Union funds and in both cases EXPROM II with stochastic dominances was found to be the most preferred technique, which confirmed the results of the analysis carried out before applying these two algorithms and led to the conclusion that this method is appropriate for the considered decision-making problem. Consequently, reanalysis turned out to be unnecessary.

It is worth mentioning that on the second place the arithmetic mean of weighted sums was classified, mainly thanks to its simplicity.

In turn, the lowest ranked technique in both rankings was ELECTRE III with stochastic dominances. The properties of this method, especially its complexity and the form of solution obtained, make it practically useless when dealing with the problem of ordering European projects.

On the one hand, the huge diversity of MCDA methods is really helpful and may be seen as an advantage, on the other – it is rather a weakness as the selection of the right technique for a specific problem is becoming extremely difficult. The models described in the paper can be applied to any multi-criteria decision-making problem to support the process of selecting the appropriate MCDA technique.

The approach presented in the article, based on the qualitative analysis and fulfilling auxiliary function algorithms, could lead to the preparation of the catalogue of problem types and methods best suited to solve them, which could serve as a general guide for participants of the decision-making processes.

Appendix

APPLICATION OF THE EXPROM METHOD WITH STOCHASTIC DOMINANCES TO THE EUROPEAN PROJECTS' SELECTION

EXPROM is a modification and extension of the PROMETHEE method* proposed by Diakoulaki and Koumoutsos [1991]. It is based on the notion of ideal and anti-ideal solutions and enables the decision-maker to rank variants on a cardinal scale. Assuming that all criteria are to be maximized, the ideal and anti-ideal solutions' values are defined as follows:

– ideal variant: $f_k(a^*) = \max\limits_{a_i \in A} f_k(a_i)$,

– anti-ideal variant: $f_k(a_*) = \min\limits_{a_i \in A} f_k(a_i)$**,

where $A = \{a_1, a_2, \ldots, a_m\}$ is finite set of m variants and $F = \{f_1, f_2, \ldots, f_n\}$ is set of n criteria examined.

After introducing stochastic dominance rules to the EXPROM method the procedure of ordering projects consists of the following steps***:

1. Identification of stochastic dominances for all pairs of projects with respect to all criteria****. Because all criteria are measured on ordinal scale the ordinal stochastic dominance approach proposed in [Spector et al., 1996] is applied:

Definition 1: Ordinal First-Degree Stochastic Dominance (OFSD):

$$X_k^i \text{ OFSD } X_k^j \text{ if and only if } \sum_{l=1}^{s} p_{kl}^i \le \sum_{l=1}^{s} p_{kl}^j \text{ for all s = 1, ... , z,}$$

---

* The idea of the PROMETHEE methodology is presented in [Brans, Vincke, 1985] and a description of PROMETHEE techniques can be found in [Brans et al., 1986].

** The values can be also defined independently from the examined variants, representing – in the case of the ideal solution – some realistic goals and in the case of the anti-ideal solution – the situation that should be avoided.

*** The PROMETHEE method with stochastic dominances was proposed by Nowak. A detailed description of this method is presented in [Nowak, 2005].

**** According to the results of experiments presented in [Kahneman, Tversky, 1979] it is assumed that the decision-maker(s) is (are) risk-averse.

where:

X_k^i – distribution of the evaluations of project a_i with respect to criterion f_k,

p_{kl} – probability of obtaining given evaluation by the project in the case of criterion f_k.

Definition 2: Ordinal Second-Degree Stochastic Dominance (OSSD):

$$X_k^i \text{ OSSD } X_k^j \text{ if and only if } \sum_{r=1}^{s} \sum_{l=1}^{r} p_{kl}^i \leq \sum_{r=1}^{s} \sum_{l=1}^{r} p_{kl}^j \text{ for all s = 1, ... , z.}$$

For modeling preferences the ordinal almost stochastic dominances are also utilized*:

Definition 3: Ordinal Almost First-Degree Stochastic Dominance (OAFSD):

$$X_k^i \ \varepsilon_1^* - OAFSD \ X_k^j \text{, if for } 0 < \varepsilon_1^* < 0{,}5$$

$$\sum \left(\sum_{l=1}^{s_1} p_{kl}^i - \sum_{l=1}^{s_1} p_{kl}^j \right) \leq \varepsilon_1^* \left\| X_k^i - X_k^j \right\| \text{ for all } s_1 = 1, \dots, z,$$

where: $s_1 = \left\{ s : \sum_{l=1}^{s} p_{kl}^j < \sum_{l=1}^{s} p_{kl}^i \right\}$, $\left\| X_k^i - X_k^j \right\| = \sum \left(\left| \sum_{l=1}^{s} p_{kl}^i - \sum_{l=1}^{s} p_{kl}^j \right| \right)$,

ε_1^* – allowed degree of OFSD rule violation, which reflects the decision--makers preferences; $\varepsilon_1^* \geq \varepsilon_1$, where ε_1 – the actual degree of OFSD rule violation.

Definition 4: Ordinal Almost Second-Degree Stochastic Dominance (OASSD):

$$X_k^i \ \varepsilon_2^* - OASSD \ X_k^j \text{, if for } 0 < \varepsilon_2^* < 0{,}5$$

$$\sum \left(\sum_{l=1}^{s_2} p_{kl}^i - \sum_{l=1}^{s_2} p_{kl}^j \right) \leq \varepsilon_2^* \left\| X_k^i - X_k^j \right\| \text{ for all } s_2 = 1, \dots, z \text{ and } \mu_k^i \geq \mu_k^j,$$

* Almost stochastic dominances were proposed by Leshno and Levy in [Leshno, Levy, 2002].

$$\text{where } s_2 = \left\{ s_1 : \sum_{r=1}^{s_1} \sum_{l=1}^{r} p_{kl}^j < \sum_{r=1}^{s_1} \sum_{l=1}^{r} p_{kl}^i \right\},$$

$$\left\| X_k^i - X_k^j \right\| = \sum \left(\left| \sum_{l=1}^{s} p_{kl}^i - \sum_{l=1}^{s} p_{kl}^j \right| \right),$$

μ_k^i and μ_k^j – average performances (expected values of the evaluation distributions) of the projects a_i and a_j on the criterion f_k,

ε_2^* – allowed degree of OSSD rule violation, which reflects the decision--makers preferences; $\varepsilon_2^* \geq \varepsilon_2$, where ε_2 – the actual degree of OSSD rule violation.

2. Calculation of concordance indexes for each pair of projects (a_i, a_j):

$$c(a_i, a_j) = \sum_{k=1}^{n} w_k \varphi_k (a_i, a_j)$$

where: $\sum_{k=1}^{n} w_k = 1,$

$$\varphi_k (a_i, a_j) =$$
$$= \begin{cases} 1 \quad \mathit{if} \quad X_k^i SD\, X_k^j \quad \mathit{and} \quad \mu_k^i > \mu_k^j + p_k[\mu_k^i], \\ \dfrac{\mu_k^i - q_k[\mu_k^i] - \mu_k^j}{p_k[\mu_k^i] - q_k[\mu_k^i]} \quad \mathit{if} \quad X_k^i SD\, X_k^j \quad \mathit{and} \quad \mu_k^j + q_k[\mu_k^i] < \mu_k^i \leq \mu_k^j + p_k[\mu_k^i], \\ 0 \quad \mathit{otherwise} \quad , \end{cases}$$

w_k – coefficient of importance for criterion f_k,

$q_k[\mu_k^i]$, $p_k[\mu_k^i]$ – indifference and preference threshold for criterion f_k, respectively.

3. Calculation of discordance indexes for each pair of projects and for each criterion:

$$d_k(a_i,a_j)=$$

$$=\begin{cases} 1 \quad \textit{if}\, X_k^j SD\, X_k^i \quad \textit{and} \quad \mu_k^j > \mu_k^i + v_k[\mu_k^i], \\ \dfrac{\mu_k^j - p_k[\mu_k^i] - \mu_k^i}{v_k[\mu_k^i] - p_k[\mu_k^i]} \quad \textit{if} \quad X_k^j SD\, X_k^i \quad \textit{and} \quad \mu_k^i + p_k[\mu_k^i] < \mu_k^j \le \mu_k^i + v_k[\mu_k^i], \\ 0 \quad \textit{otherwise} \end{cases}$$

where $v_k[\mu_k^i]$ – veto threshold for criterion f_k.

4. Calculation of credibility indexes for each pair of projects (a_i,a_j):

$$\sigma(a_i,a_j) = c(a_i,a_j) \prod_{k \in D(a_i,a_j)} \frac{1-d_k(a_i,a_j)}{1-c(a_i,a_j)}$$

where: $D(a_i,a_j) = \{k : d_k(a_i,a_j) > c(a_i,a_j)\}$.

5. Determination of strict preference indexes for each pair of projects (a_i,a_j):

$$\pi(a_i,a_j) = \nu(a_i,a_j) \cdot \sum_{k=1}^{n} w_k \pi_k(a_i,a_j),$$

where:

$$\nu(a_i,a_j) = \begin{cases} 1, \quad \textit{if} \quad \forall k : d_k(a_i,a_j) \le c(a_i,a_j), \\ 0, \quad \textit{if} \quad \exists k : d_k(a_i,a_j) > c(a_i,a_j), \end{cases}$$

$$\pi_k(a_i,a_j) = \begin{cases} \dfrac{(\mu_k^i - \mu_k^j) - p_k[\mu_k^i]}{(\mu_k^* - \mu_{k*}) - p_k[\mu_k^i]} \quad \textit{if} \quad \varphi_k(a_i,a_j) = 1, \\ 0 \quad \textit{otherwise}, \end{cases}$$

$$\mu_k^* = \max_{a_i \in A} \mu_k^i \quad \text{and} \quad \mu_{k*} = \min_{a_i \in A} \mu_k^i .$$

The aim of the strict preference function $\pi_k(a_i,a_j)$ is to distinguish the state of the strict preference found to be valid for more than one pair of projects at a given criterion f_k. Their values belong to the interval [0, 1] and $\pi_k(a_i,a_j) = 0$ denotes weak preference or indifference between two projects.

6. Calculation of total preference index for each pair of projects (a_i, a_j):

$$\omega(a_i, a_j) = \min\{1;\ \sigma(a_i, a_j) + \pi(a_i, a_j)\}.$$

The total preference index gives an accurate measure of the intensity of preference of project a_i over a_j for all the criteria. It combines two aspects: subjective – expressed by the credibility index and referring only to the relation between two examined projects and objective – expressed by the strict preference index and representing the relation between two projects considered with regard to other projects examined.

7. Calculation of outgoing flow $\phi^+(a_i)$ and incoming flow $\phi^-(a_i)$ for each project:

$$\phi^+(a_i) = \frac{1}{m-1}\sum_{j=1}^{m}\omega(a_i, a_j)$$

$$\phi^-(a_i) = \frac{1}{m-1}\sum_{j=1}^{m}\omega(a_j, a_i)$$

In EXPROM I a final partial ranking is obtained as follows:

$$\begin{cases} a_i P a_j, \quad gdy \begin{cases} \phi^+(a_i) > \phi^+(a_j) \quad i \quad \phi^-(a_i) < \phi^-(a_j) \quad or \\ \phi^+(a_i) = \phi^+(a_j) \quad i \quad \phi^-(a_i) < \phi^-(a_j) \quad or \\ \phi^+(a_i) > \phi^+(a_j) \quad i \quad \phi^-(a_i) = \phi^-(a_j); \end{cases} \\ a_i I a_j, \quad gdy \quad \phi^+(a_i) = \phi^+(a_j) \quad i \quad \phi^-(a_i) = \phi^-(a_j); \\ a_i R a_j, \quad gdy \begin{cases} \phi^+(a_i) > \phi^+(a_j) \quad i \quad \phi^-(a_i) > \phi^-(a_j) \quad or \\ \phi^+(a_i) < \phi^+(a_j) \quad i \quad \phi^-(a_i) < \phi^-(a_j); \end{cases} \end{cases}$$

where P, I and R stands for preference, indifference and incomparability respectively.

In EXPROM II a final complete ranking is constructed according to the descending order of the net flows $\phi(a_i)$, where $\phi(a_i) = \phi^+(a_i) - \phi^-(a_i)$.

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Joseph Hanna

R&D RIVALRY AND COOPERATION IN DUOPOLY: FIRM ORGANIZATION, WELFARE AND POLICY IMPLICATIONS

Abstract

The objective of the paper is to reveal the optimal organization of industry when firms, facing externalities, compete or cooperate in R&D as well as in the final output market. The model hinges on a two-stage game setting. A ranking of solutions is established for alternative organizations. We focus on welfare issues and allow for public intervention. Subsidizing R&D is used to draw the industry to match the social welfare solution. The paper shows that targeting the optimal level of R&D leaves final output fall short of the welfare solution. Whereas targeting the final output leads to overinvestment in R&D. The ranking of policies reveals that the most efficient industry organization occurs when firms cooperate and fully share R&D results, but remain competitive in the final good market.

Keywords

R&D, subsidies, welfare, spillover, two stage-game, Cournot equilibrium.

Introduction

The literature dealing with R&D cooperation and policy regulations has focussed on the main private advantages and disadvantages of such agreements as well as the main public costs and benefits*. There are more difficulties encountered in setting up R&D cooperation, compared to other fields in economy, even though social welfare benefits are more likely to occur from such agreements. Cooperation in R&D appears as an alternative to pure market transactions on one hand and to full integration within a firm on the other hand. A cooperative research arrangement for instance, can reduce problems of asymmetric information as market transactions are liable to be affected

* The main papers related to the subject are those of A. Jacquemin [1988], M. Katz [1986] and M. Spence [1984].

by moral hazard and adverse selection. The opposite case of mergers can tend to create quite rigid structures curtailing attempts to switch research capacity and strategy, or more generally to respond quickly to innovation over time.

Despite their many private advantages, cooperative agreements in R&D are not very frequent to observe. When they occur, they are usually fragile constructions with various difficulties to overcome and are either dismantled or absorbed through merger operations. The main argument in favour of co-operation stems from market failure. Such a situation prevents the firm from appropriating completely the benefits of R&D activity. The amount of research produced and diffused by private firms may be socially inefficient, whatever the market structure is. We need to distinguish between two situations:

1. The one without externalities: that is when each firm's R&D affects only its own cost. Competition among firms will usually lead to wasteful duplication of research. Investment in R&D is greater than what is socially needed.
2. In the case of substantial R&D externalities or spillovers, the benefits of each firm's R&D effort flow freely to other firms. In such a situation there is underinvestment in R&D compared to what the social optimal level would require. Incentive to innovate will be reduced as the innovator is aware that competitors will strengthen, in a costless way, their competitive position through his R&D investment.

It can then be argued that cooperative R&D can improve both situations. According to M. Spence [1984], the incentive of a firm to invest in R&D needs a sufficient amount of appropriation of the benefits, therefore a limited diffusion of knowledge. At the same time tightening conditions, to create a nearly perfect appropriation, impedes spillovers of R&D to other firms and will thus prevent cost reduction to spread across the sector. Cooperative R&D is then viewed as the means through which these two objectives can be achieved simultaneously, that is:

– internalizing, into an appropriate organization, the externalities generated by a high level of R&D spillover, and
– providing a better sharing of information among the participant firms.

The incentive to invest in R&D is improved, at the same cooperation will avoid to devote resources to wasteful duplication.

Our objective is to address the question of the socially optimal organization of the industry when firms compete or cooperate in R&D as well as in the final good market. We also discuss the policy of subsidizing R&D activities (not final output) as an incentive to reach welfare objectives.

Our analysis shares a good deal of similarity with the pioneering approach introduced by C. d'Aspermont and A. Jacquemin [1988]. They consider a two-stage game in a duopolistic setting: in the first stage firms conduct research in order to reduce unit costs, and are Cournot competitors in the second stage; that is in the final output market. The focus of their analysis is on comparison of cost reducing achieved when firms conduct R&D cooperatively or as competitors, in the presence of spillover effects.

There are many extensions and related papers to the d'Aspermont--Jacquemin approach, but they do not explicitly address policies, such as subsidies, and their welfare issues related to the model*.

Having sketched out the economic background, some technical aspects of the model are now underlined:

1. Any one firm, while maximizing its objective function, or profit, decides on the level of R&D output (x) as well as on the final good production level (q). These two choice variables are technically determined by a sequence of operations: R&D in the first step and final output in the subsequent stage. Moreover the firm's own decision on output depends on the other firm's behaviour, thus reflecting the market structure. The underlying industry organization can range from full rivalry at both stages to a completely integrated monopoly.
2. The other feature is the spillover effect (β), as all output solutions depend on this parameter. It can also be interpreted as the proportion of R&D results, the firms are willing to share, either within a coalition or if they remain rivals. Alternative information sharing hypothesis and objective functions consistent with industry organization are summarized in the Appendix.
3. The main focus of our model is optimizing a social welfare objective. Together with profit maximization by firms, we allow for public intervention. Taking into account anti-trust regulations, subsidies are used to fund R&D activities in order to drive the industry organization to meet the welfare solution. The social planner controls one instrument, the subsidy, whereas there are two target variables: R&D as well as final good outputs. Some compromise has to be established, and welfare solutions will hinge on the spillover parameter β.

The paper is organized in the following way. In section 1, we introduce the two stage-game model. Two alternative cases are discussed, one when rivalry between firms occurs at both stages, the other when firms coordinate R&D activities aiming to maximize joint profits but remain competitive in the final product market. The analysis focuses on stability issues as well as the switch of the slope of the reaction functions as the spillover parameter increases. We use numerical simulations to plot the behaviour of some

* Contributions like those of M. Kamien et al. [1992], K. Suzumura [1992], De Bondt et al. [1992] are direct extensions of the effect of spillovers in R&D with many firms and many stages (mainly two) in R&D operations. Welfare issues are limited to R&D levels in Suzumura and are imbedded in a broader comparison of cooperative issues, in M. Kamien et al., namely joint ventures. This last point will be discussed further in this paper.

significant variables. We show that investment in R&D is a decreasing function of spillovers in the competitive case, whereas it is increasing in the cooperative case. For this latter case, R&D spending grows faster than the final good output beyond the switch point, magnifying the increase in profits. These results are indications of the performances of cooperation relations among firms; from simply coordinating R&D to the full sharing of information in order to eliminate duplication and free riding (discussed in section 3). Section 2 examines two alternative cases when industry is fully integrated. The first case is a private monopoly, while the other can be considered as a public monopoly that seeks to maximize total surplus. This last case unambiguously yields the highest levels of R&D spending as well as final product output. This is the standard welfare case in the d'Aspermont-Jacquemin model against which all other equilibriums are compared. We can then establish a ranking of solutions as to guide the implementation of economic policy. This point is taken up in section 3 where we introduce subsidies in order to highlight the cost of drawing the industrial organization to match the social welfare solution. We show that targeting the optimal R&D investment level would still leave final output fall short from the welfare solution. Whereas targeting the final output objective will overshoot the optimal level of spending on R&D. Subsidies are by no means substitutes to cooperation among firms. The analysis reveals the most adequate industrial organization liable to fulfil the welfare objective: this is cooperation and full sharing of information in R&D, while remaining competitive in the final good market. Subsidies are viewed as efficient incentives to stabilize such cooperative agreements in R&D activities. The final section gathers conclusions and considers some extensions of the analysis.

The Appendix contains tables summarizing the notations for alternative models discussed in the paper.

1. Competition and cooperation in R&D with spillovers

1.1. General assumptions

We consider an industry with two firms. They face the linear inverse demand function given by:

$$p = a - bQ \tag{1}$$

where p is the price and $Q = q_i + q_j$ is the total amount of a homogenous good produced by firms i and j, with $a, b > 0$ and $Q \leq \frac{a}{b}$.

Production cost is such as $C_i(q_i, x_i, x_j)$ is a function of its own final good output q_i, the amount of research x_i that it undertakes and the amount of the rival's firm research x_j:

$$C_i = (q_i, x_i, x_j) = (A - x_i - \beta x_j) q_i, \quad i = 1,2 \quad i \neq j. \tag{2}$$

With unit cost $c_i = (A - x_i - \beta x_j) \geq 0$, $0 < A < a$ and $0 \leq \beta \leq 1$ where β is the spillover parameter*. Moreover the cost of R&D is chosen to be quadratic as we may assume the production process to exhibit diminishing returns to scale; that is: $\frac{\gamma}{2} x_i^2$, $(i = 1,2)$.

The model and its variants feature a two-stage game with two firms. In this section, during the second stage, firms are assumed to engage in Cournot competition, while in the first stage they invest in R&D. For the first variant, there is R&D competition in which firms maximize their individual profits by deciding unilaterally on their R&D investments. In the second case firms coordinate R&D activities such as to maximize joint profits while maintaining competition in the final output production stage.

There are alternative organizations and different levels of cooperation in which firms can be involved while coordinating R&D activities. In one sub--case, we may consider that coordination does not necessarily mean total sharing of results between partners. One participant firm may be allowed (by an agreement) to carry out some propriety research; and hence duplication is not completely eliminated ($\beta < 1$). When results of R&D activities are fully shared, the spillover rate is at its maximal level, that is $\beta = 1$**.

* When $\beta = 0$, we have the Brander/Spencer [1983] two- stage duopoly in R&D game. When $\beta = 1$, in a cooperative duopoly in R&D, then means full sharing of information as in M. Kamien et al. [1992].

** These cases are used by M. Kamien et al. [1992] to distinguish between *research joint venture* $(\beta = 1)$ and R&D *cartelization* $\beta < 1$. Further cases are discussed, particularly the one with competition in both stages, but with fully sharing of R&D results. These "other" cases are imbedded in our analysis in this section and also while discussing policy implications in section 3. The authors do not introduce the two full cooperation cases of our section 2.

A firm's payoff consists of the second stage production profit less the first stage R&D cost. The cooperative or non-cooperative solutions to this first stage are then obtained by maximizing profits with respect to levels of R&D(x_i, x_j). We can then compare the corresponding sub--game perfect equilibrium.

1.2. The rivalry solution

Both firms act non-cooperatively at both stages of the game. Firm *i* maximizes its second stage profit, conditional on x_i and x_j, by choosing its output and assuming the output of the rival firm *j* is fixed: $\underset{q_i / q_j \, fixed}{Max} \ \pi_i = pq_i - C_i - \frac{\gamma}{2} x_i^2$.

The first order condition yields:

$$\frac{\partial \pi_i}{\partial q_i} = -bq_i + \left[a - b\left(q_i + q_j\right)\right] - \left[A - x_i - \beta x_j\right] = 0. \tag{3}$$

By collecting terms and using relation (2), the profit maximization condition (3) gives:

$$\left(p - c_i\right) = bq_i \text{, and the maximized profit is: } \pi_i^* = bq_i^2 - \frac{\gamma}{2} x_i^2 \tag{4}$$

where q_i and x_i are to be replaced by the optimal levels of output and R&D expenditure.

Solving for condition (3) also yields the reaction function for output: $q_i = -\frac{1}{2} q_j + \frac{\left(a - c_i\right)}{2b}$. Using relation (2) and arranging terms gives:

$$q_i = -\frac{1}{2} q_j + \frac{\left(a - A\right)}{2b} + \frac{x_i + \beta x_j}{2b}. \tag{5}$$

By the symmetry assumption there is a similar function for firm *j*. Solving the two reaction functions yields the second stage output:

$$q_i = \frac{\left(a - A\right) + \left(2 - \beta\right) x_i + \left(2\beta - 1\right) x_j}{3b}. \tag{6}$$

The maximized profit expression for firm *i* in (4) can be written as:

$$\pi_i^* = \frac{1}{9b} \left[\left(a - A\right) + \left(2 - \beta\right) x_i + \left(2\beta - 1\right) x_j\right]^2 - \frac{\gamma}{2} x_i^2 \quad i = 1,2 \quad i \neq j. \tag{7}$$

Expression (7) shows the influence of R&D levels on the profit through the output of the final good, the unit cost of production and the expenses devoted to R&D levels themselves. At the initial stage of the game, the non--cooperative level of R&D, x_i, is chosen to maximize the profit given in (7), assuming that the rival's investment in R&D x_j is fixed: $\underset{x_i / x_j \, fixed}{Max} \ \pi_i^*$.

The first order condition for profit maximization is given by:

$$\frac{\partial \pi_i}{\partial x_i} = \frac{1}{9b} 2(2-\beta)\left[(a-A)+(2-\beta)x_i + (2\beta - 1)x_j\right] - \gamma x_i = 0. \tag{8}$$

The second order condition for a maximum: $\frac{\partial^2 \pi_i}{\partial x_i} < 0$, requires: $\frac{9}{2} b\gamma > (2-\beta)^2$ *.

The reaction function for R&D levels associated with this initial stage of the game is found by solving (8):

$$x_i = \frac{2(2-\beta)(2\beta-1)x_j + 2(2-\beta)(a-A)}{9b\gamma - 2(2-\beta)^2} \qquad i = 1,2 \quad i \neq j. \tag{9}$$

Solving the reaction functions for R&D levels yields:

$$x_i^* = x_j^* = \frac{(a-A)(2-\beta)}{\frac{9}{2} b\gamma - (2-\beta)(1+\beta)}. \tag{10}$$

The optimal output for final goods is obtained using relation (6):

$$q_i^* = q_j^* = \frac{(a-A)}{3b}\left[\frac{\frac{9}{2} b\gamma}{\frac{9}{2} b\gamma - (2-\beta)(1+\beta)}\right]. \tag{11}$$

Total industry output is: $Q^* = q_i^* + q_j^*$. We can revert to relation (4) and compute the maximized profit for any one firm:

$$\pi_i^* = \frac{(a-A)^2 \gamma \left[9b\gamma - 2(2-\beta)^2\right]}{4\left[\frac{9}{2} b\gamma - (2-\beta)(1+\beta)\right]^2}. \tag{12}$$

---

* For $\gamma = b = 1$, even if β is at its maximum value, $\beta = 1$, the condition is always met. More constraining conditions appear when we discuss elaborate organization and policy issues as in section 3.

We can check that these results, contained in relations (10), (11) and (12) for the rivalry case in both stages are consistent when the following condition holds:

$$b\gamma > \frac{2}{9}(2-\beta)(1+\beta)^{*}.$$

1.3. The cooperative R&D solution

Let us consider that firms, while still being competitors in the product market, coordinate their R&D effort in order to maximize the joint profit. The second stage of the game is therefore unchanged and equation (6) and (7) still hold. The joint profit is given by $\Pi = \pi_i + \pi_j$. Considering the symmetric solution $x_i = x_j = x$, and using equation (7) yields the joint profit as a function of the R&D investment of any one firm:

$$\Pi = \frac{2}{9}\left[(a-A)+(2-\beta)x+(2\beta-1)x\right]^2 - \gamma x^2.$$

When arranging the terms in the brackets the joint profit can be written as:

$$\Pi = \frac{2}{9}\left[(a-A)+(1+\beta)x\right]^2 - \gamma x^2. \tag{13}$$

The first order condition $\frac{\partial \Pi}{\partial x} = 0$, yields the solution for the R&D level of expenditure**:

$$\bar{x} = \frac{(a-A)(1+\beta)}{\frac{9}{2}b\gamma - (1+\beta)^2}. \tag{14}$$

The corresponding output of the final good for any firm is:

$$\bar{q} = \frac{(a-A)}{3b}\left[\frac{\frac{9}{2}b\gamma}{\frac{9}{2}b\gamma - (1+\beta)^2}\right]. \tag{15}$$

* We can compare this result with the second order condition on the profit and the indication given in footnote on p. 110.

** The second order condition for profit maximization is given by: $b\gamma > \frac{2}{9}(1+\beta)^2$.

We can then easily compute total output as $\overline{Q} = 2\overline{q}$. The profit of any one firm is given by:

$$\overline{\pi} = \frac{(a-A)^2 \gamma \left[9b\gamma - 2(1+\beta)^2\right]}{4\left[\frac{9}{2}b\gamma - (1+\beta)^2\right]^2} \,. \tag{16}$$

Results given by (14), (15) and (16) depend on the degree of spillovers which does not necessarily reach its maximum value ($\beta = 1$) because of cooperation. The agreement set between firms does not totally eliminate duplication because the sharing of information is not complete among the participants. The spillover parameter β plays a crucial role in the analysis as the following comparisons of output and R&D expenditure show. We can easily check that $\overline{x} > x^*$ if $\beta > \frac{1}{2}$, that is R&D effort is greater when firms cooperate compared to the case when they are rivals, only if externalities are high. We can also establish that $\overline{Q} > Q^*$, total output level is also higher when firms cooperate under the same condition on the spillover parameter β.

1.4. The potential gains from cooperation

In the traditional quantity competition Cournot model, reaction functions are downward sloping, and stability of the solution can be examined by comparing these slopes in the quantity space. When plotted in the (x_i, x_j) space, the slope of the reaction functions of firm i and j, using (9), are given by the following expressions:

Slope for firm $j = \dfrac{2(2-\beta)(2\beta-1)}{9b\gamma - 2(2-\beta)^2}$, and by the symmetry assumption,

Slope for firm $i = \dfrac{9b\gamma - 2(2-\beta)^2}{2(2-\beta)(2\beta-1)}$. By the second order condition on profit maximization, $9b\gamma - 2(2-\beta)^2 > 0$, therefore expressions of the slopes are negative if: $2(2-\beta)(2\beta-1) < 0$, but as $\beta \leq 1$, the condition reduces to $\beta < \frac{1}{2}$. Reaction functions are downward sloping for weak values of the spillover parameter[*].

[*] The useful reference for discussing stability issues is I. Henriques [1990].

As Figure 1 shows, there is a stable equilibrium when the slope of the reaction function of firm i is greater, in absolute value, than the slope of firm j 's reaction function. Such a condition holds when:

$$\frac{2(2-\beta)^2 - 9b\gamma}{2(2-\beta)(2\beta-1)} > 1. \tag{17}$$

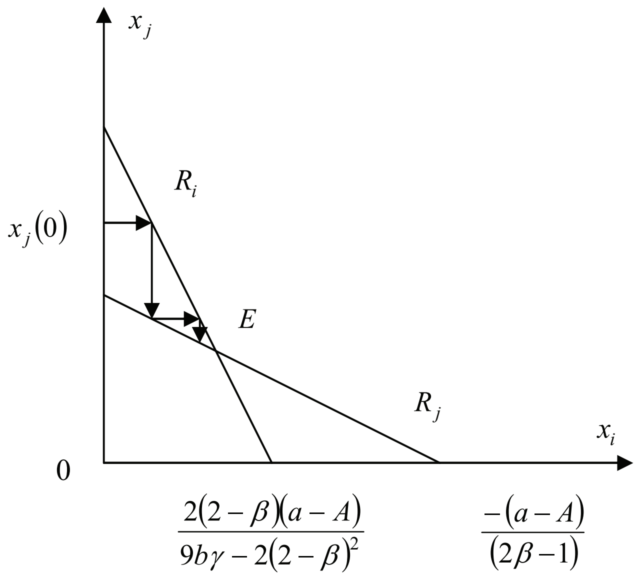

Figure 1. Adjustment path with downward sloping reaction functions

R_i and R_j are the reaction functions of firm i and firm j respectively. They are shown for $\beta < \frac{1}{2}$.

In order to maintain comparisons with other key variables, we set $b = \gamma = 1$. From equation (17), we get the equivalent condition*: $2\beta^2 - 6\beta + 1 > 0$. It can easily be checked that this inequality holds for $\beta > 0.177$. Therefore for low spillovers there is a stable solution only if: $0;177 < \beta < \frac{1}{2}$.

* The condition is equivalent to show that the intercept with the x_i axis of reaction function R_j is greater than the intercept of the R_i reaction function.

Beyond the critical value of $\beta = \frac{1}{2}$, the slope of the reaction functions is reversed and the level of R&D of any one firm is an increasing function of the rival's expenditure on R&D as depicted in Figure 2.

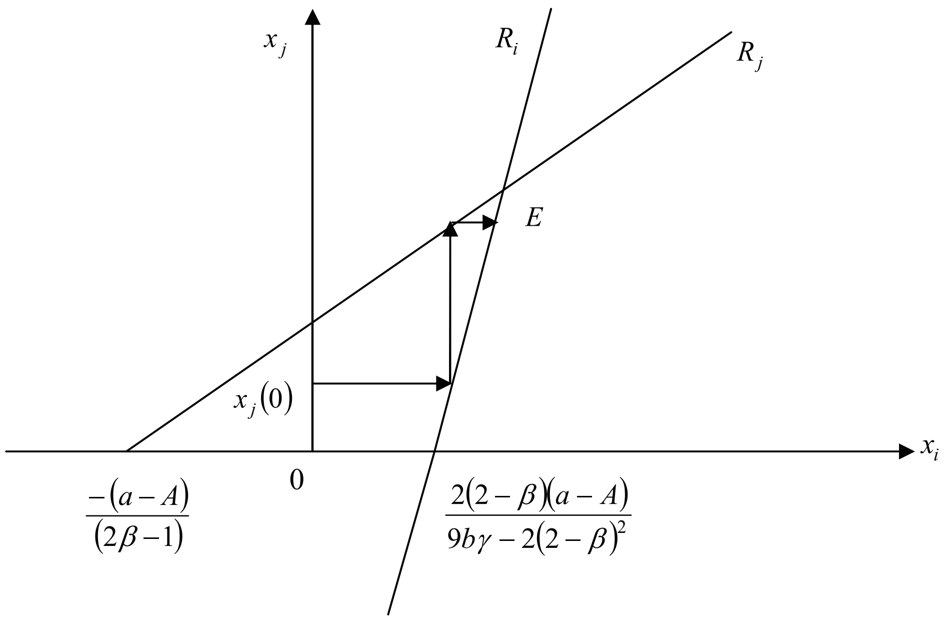

Figure 2. Adjustment path with upward sloping reaction functions

It can easily be shown that stability then holds for all values of $\beta > \frac{1}{2}$ *

What are the potential gains expected from cooperation when we allow for externalities? This issue is best addressed when we compare cooperation and rivalry key variables of the model for the whole range of values of the spillover parameter β . The significant variables chosen for both cases are output, R&D levels and also profits. The numerical values have been computed using equations (10), (11) and (12) for the rivalry case and equations (14), (15) and (16) for the cooperative case. The values for parameters b and γ are unchanged from the preceding discussion; that is $b = \gamma = 1$.

* We can check that the reaction function of firm i is steeper than the one of firm j , and stability occurs as the condition $2\beta^2 - 2\beta + 5 > 0$ is always satisfied for $\beta > 0.5$.

In equations (10) and (14), we set $m = (a - A) > 0$, and plot the behaviour of R&D levels from the values obtained by Table 1.

Table 1

β	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x^*	$0.77m$	$0.74m$	$0.70m$	$0.66m$	$0.6m$	$0.56m$	$0.51m$	$0.45m$	$0.4m$
$\bar{x}$	$0.39m$	$0.46m$	$0.55m$	$0.66m$	$0.82m$	$1.06m$	$1.42m$	$2.11m$	$4m$

These values are plotted in Figure 3.

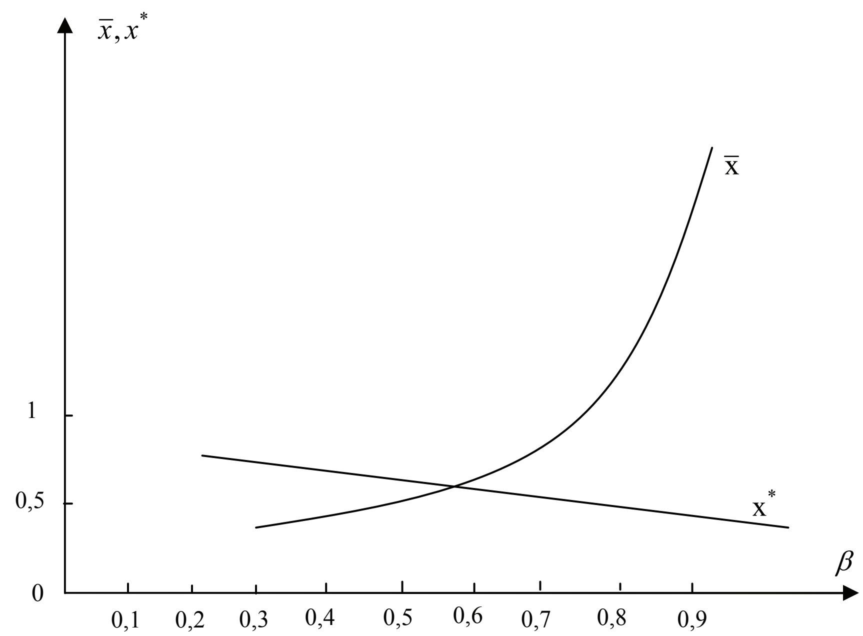

Figure 3. Comparative behaviour of R&D levels between competition and cooperation

The R&D level is decreasing in the non-cooperative situation for all ranges of β, whereas it is increasing in the cooperative case. R&D expenditures are greater, when rivalry prevails, compared to cooperation for weak values of the spillovers. The significant feature is the very rapid growth of R&D investment in the cooperation case when the curve crosses the switch point value $\beta = \frac{1}{2}$.

Final outputs behaviour, equations (11) and (15), reveals similar results as shown by Table 2.

Table 2

β	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q^*	$0.64m$	$0.65m$	$0.66m$	$0.66m$	$0.66m$	$0.65m$	$0.64m$	$0.62m$	$0.60m$
$\overline{q}$	$0.49m$	$0.53m$	$0.59m$	$0.66m$	$0.77m$	$0.93m$	$1.12m$	$1.68m$	$3m$

Final output rises, is stationary, and then declines very slowly in the non--cooperative case. It is an increasing function of β in the cooperative case, also showing a very quick growth after the switch point. Rather than plotting these results one against the other, it is significant for further interpretations to link the behaviour of R&D to output in each case as depicted in Figures 4 and 5.

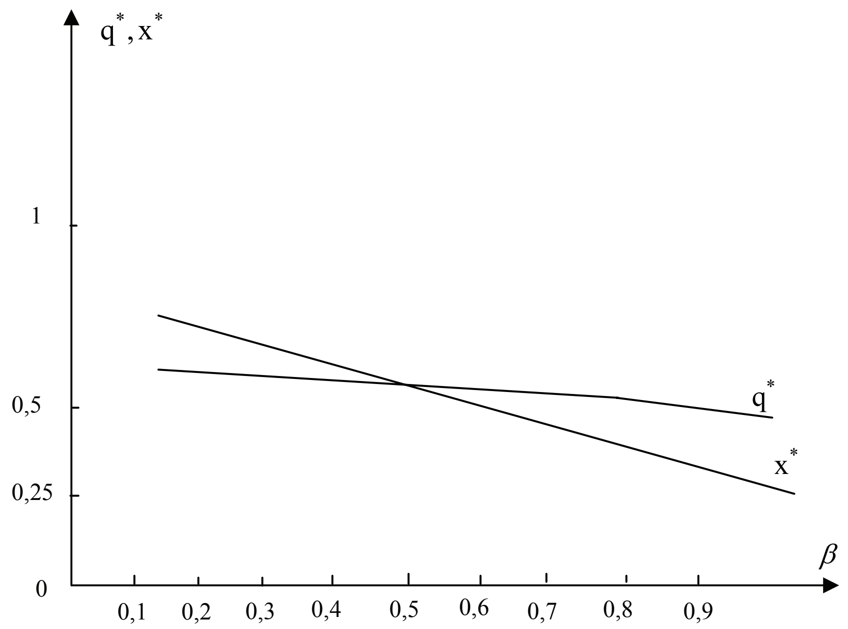

Figure 4. R&D and final output levels in the rivalry case

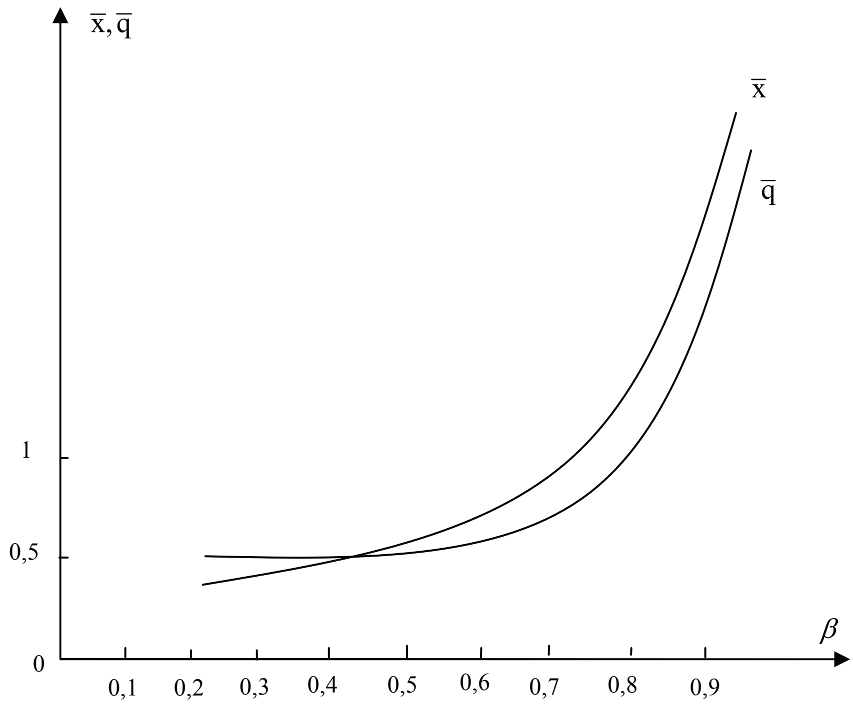

Figure 5. R&D and final output levels in the cooperative case

For high spillover rates, the reduction in cost is greater when firms coordinate R&D activities than when they remain rivals. There are two types of externalities generated by R&D activities when spillovers are meaningfully high:

- The first type is linked to a firm's competitiveness relative to its rivals. Any firm investing in R&D to reduce its unit cost, takes into account the fact that the spillover reduces to some extent the cost of the rival firm making it a tougher competitor.
- The other type, affects the performance of the industry as a whole. This second aspect is ignored under R&D competition. It is internalized in the process of choosing the level of R&D spending to maximize joint profits when firms cooperate within an adequate structure such as a cartel.

Another interesting way to look at the problem is to point out that cooperation acts to eliminate duplication. Moreover in the non-cooperative model, as the level of spillover rises firms tend to "free-ride" on the other firm's knowledge as we observe that R&D levels fall when β rises.

These results are also reflected by comparing the behaviour of profits in Table 3.

Table 3

β	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
π^*	$0.11\,m^2$	$0.15\,m^2$	$0.19\,m^2$	$0.22\,m^2$	$0.24\,m^2$	$0.26\,m^2$	$0.28\,m^2$	$0.28\,m^2$	$0.28\,m^2$
$\overline{\pi}$	$0.16\,m^2$	$0.18\,m^2$	$0.20\,m^2$	$0.22\,m^2$	$0.26\,m^2$	$0.30\,m^2$	$0.39\,m^2$	$0.56\,m^2$	m^2

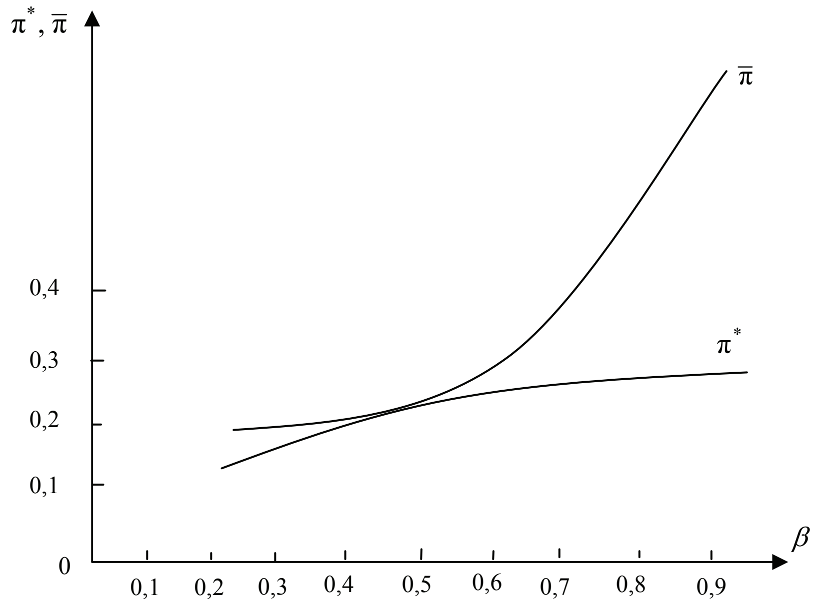

Figure 6. Comparative profit behaviour

In the rivalry case, the decline in both the R&D level and final output (the latter inhibits a sharp fall in price) drives the profit to a stationary value for high spillovers. Even if the firms form a *joint venture*, as discussed in M. Kamien et al. [1992], to share R&D results, it is not clear why there should be substantial gains from such an agreement. On the contrary when firms form a cartel and coordinate activities in order to maximize the joint profit, then the gains are magnified as R&D results are shared between participants (spillover reaches its maximal value).

2. The integrated industry

We assume in this section that firms cooperate in both stages of the game. In the first sub-case the fully integrated industry behaves like a private monopoly. In the other, which is the interesting alternative introduced by d'Aspermont and Jacquemin [1988], there is full cooperation in order to maximize total surplus. We look at this second situation as a public monopoly seeking to achieve a social welfare objective.

2.1. The private monopoly

As the industry is now fully integrated, the joint profit is given by:

$$\Pi = [a - bQ]Q - AQ + (x_i + \beta x_j)q_i + (x_j + \beta x_i)q_j - \frac{\gamma}{2}x_i^2 - \frac{\gamma}{2}x_j^2.$$

From the symmetry assumption, we can set the following equalities:

$$x_i = x_j = x^M \text{ and } q_i = q_j = q^M.$$

The expression for the joint profit is then given by:

$$\Pi = [a - bQ]Q - AQ + (1+\beta)xQ - \gamma x^2 \tag{18}$$

or by: $\Pi = pQ - [A - (1+\beta)x]Q - \gamma x^2$, which is also equivalent to $\Pi = (p - c)Q - \gamma x^2$
where $c = [A - (1+\beta)x]$ is the unit cost of producing the final output.

The first order condition for a maximum is given by:

$$\frac{d\Pi}{dQ} = -bQ + [A - bQ] - A + (1+\beta)x = 0.$$

Solving for Q leads to the monopoly output as a function of R&D expenditure:

$$Q^M = \frac{1}{2b}[(a - A) + (1+\beta)x]. \tag{19}$$

Substituting the value of monopoly output in (18) leads to:

$$\Pi \equiv \pi^M = \frac{1}{b}\left[\frac{(a - A) + (1+\beta)x}{2}\right]^2 - \gamma x^2.$$

The integrated firm now chooses the level of R&D to maximize the profit*, which yields:

$$x^M = \frac{(a-A)(1+\beta)}{\left[4b\gamma-(1+\beta)^2\right]}. \tag{20}$$

The second order condition on profit maximization is given by: $b\gamma > \frac{1}{4}(1+\beta)^2$. It is sufficient for a positive output level in R&D as well as for final output:

$$Q^M = \frac{1}{2b}\left[\frac{4b\gamma(a-A)}{4b\gamma-(1+\beta)^2}\right]. \tag{21}$$

2.2. The public monopoly

Let us look at the welfare objective of the monopoly. Total surplus is made up of consumer's surplus S_c and profits Π. We call this the welfare objective noted as:

$$W(Q) = S_c + \Pi \tag{22}$$

From the inverse demand function given in relation (1), we see that the maximum price, which drives output to zero is $p_{\max} = a$. The expression of consumer's surplus is given by:

$S_c = \frac{(a-p)Q}{2}$, using the inverse demand function given in relation (1) leads to:

$$S_c = \frac{bQ^2}{2}. \tag{23}$$

By using the expression of the joint profit in (18) and consumer's surplus given by (23), the welfare function can be expressed as:

$$W(Q) = -\frac{1}{2}bQ^2 + \left[(a-A)+(1+\beta)x\right]Q - \gamma x^2. \tag{24}$$

---

* It can be shown that $x^M > x^*$ if $\beta > 0.41$, and $x^M > \bar{x}$ always holds.

Maximizing with respect to Q leads to: $Q = \frac{(a-A)+(1+\beta)x}{b}$, and the maximized welfare by the choice of Q is given by:

$$W(Q) = \frac{1}{2}bQ^2 - \gamma x^2 \quad \text{or as:} \quad W(Q) = \frac{1}{2}b\left[\frac{(a-A)+(1+\beta)x}{b}\right]^2 - \gamma x^2.$$

The social planner now chooses the level of R&D to maximize total surplus. The second order condition is satisfied if: $b\gamma > \frac{1}{2}(1+\beta)^2$, and the solution is given by:

$$\overset{\#}{x} = \frac{(a-A)(1+\beta)}{2b\gamma - (1+\beta)^2}. \tag{25}$$

and the corresponding output is:

$$\overset{\#}{Q} = \frac{(a-A)}{b}\left[\frac{2b\gamma}{2b\gamma - (1+\beta)^2}\right]. \tag{26}$$

Solutions given by (25) and (26) are the social standard to classify the various results. By reverting to equations (10), (11), (14), (15), (20) and (21), we can establish the following rankings:

$$\overset{\#}{x} > x^M > \overline{x} > x^*$$

and

$$\overset{\#}{Q} > \overline{Q} > Q^* > Q^M. \tag{27}$$

Being fully integrated, the monopoly is more efficient in R&D activities. This effort is devoted to the sole objective of maximizing the profit when the organization is a private monopoly. Final output, and therefore consumer's surplus, then shows to be the least compared to all other situations. The case of the public monopoly maximizing social welfare is the main feature of the model as it shows that both, the R&D and final output, levels can be increased.

There are many reasons, such as anti-trust regulations, that prevent firms from cooperating in the final output market. The policy maker may revert to subsidies, in funding R&D activities to reach welfare objectives. This point is picked up in the following section.

3. Achieving welfare: subsidies and firm organization

Let "s" be the "marginal cost reducing subsidy" used to fund R&D activities. We shall henceforth refer to "s" as the "unit marginal subsidy". The quadratic cost function* for producing R&D is now changed to: $\frac{1}{2}(\gamma - s)x^2$. We also assume that $\gamma > s$, that is funding will only pay a share of total costs devoted to R&D expenditures.

3.1. Funding R&D of the private monopoly

From the ranking conditions given in (27), we notice that the private monopoly's R&D level is the nearest to the socially optimal level compared to the other cases discussed earlier. It is therefore tempting to ask what would be the consequence of subsidizing R&D activities to attain a welfare objective.

1. The optimal R&D objective.

The maximized profit by the choice of output is $\Pi = bQ^2 - (\gamma - s)x^2$, but monopoly output is still given by equation (19). Maximizing the profit when subsidies enter the R&D cost function yields:

$$x^{Ms} = \frac{(a-A)(1+\beta)}{4b(\gamma - s) - (1+\beta)^2}. \tag{28}$$

We can easily check that monopoly output given in equation (21), changes to:

$$Q^{Ms} = \frac{1}{2b}\left[\frac{4b(\gamma - s)(a-A)}{4b(\gamma - s) - (1+\beta)^2}\right]. \tag{29}$$

All solutions are fundamentally unaltered in their general structure; the term γ being replaced by $(\gamma - s)$. If the policy maker sets "s" to reach the socially optimal R&D investment level $\overset{\#}{x}$, we can then compute the adequate subsidy by setting: $x^{Ms} = \overset{\#}{x}$, that is:

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* Introduced in section 1's general assumptions.

$$\frac{(a-A)}{\left[4b(\gamma-s)-(1+\beta)^2\right]}=\frac{(a-A)(1+\beta)}{\left[2b\gamma-(1+\beta)^2\right]}.$$

The unit marginal subsidy, when the optimal level of R&D is targeted, is given as:

$$s^{MR}=\frac{1}{2}\gamma. \tag{30}$$

What is the impact of such funding on the output of the final good? Monopoly output when R&D is chosen as a target, Q^{MsR}, is obtained by substituting the value of the unit subsidy given by (30) in expression (29), which yields:

$$Q^{MsR}=\frac{1}{2b}\left[\frac{2b\gamma(a-A)}{2b\gamma-(1+\beta)^2}\right]. \tag{31}$$

When we compare this result with the socially optimal output of final goods, $\overset{\#}{Q}$ given in (26), we get:

$$Q^{MsR}=\frac{1}{2}\overset{\#}{Q}. \tag{32}$$

Even if output is increased*, the cost reduction due to subsidies, will be used to improve profits by charging consumers a relatively high price compared to the public monopoly. Nevertheless, with higher output for final goods, the subsidy will shift some of the rent captured by the private monopoly to consumers.

2. The optimal output objective.

If we seek to reach the welfare output using subsidies that reduce costs of producing R&D, we have to solve for "*s*" such that $Q^{Ms}=\overset{\#}{Q}$:

$$\frac{1}{2b}\left[\frac{4b(\gamma-s)(a-A)}{4b(\gamma-s)-(1+\beta)^2}\right]=\frac{1}{b}\left[\frac{2b\gamma(a-A)}{2b\gamma-(1+\beta)^2}\right].$$

* We can check that $Q^{MsR}>Q^{M}$.

Solving for "s" yields the subsidy needed to attain this objective:

$$s^{MQ} = \frac{2b\gamma^2}{\left[2b\gamma + (1+\beta)^2\right]}. \tag{33}$$

We can check that $s^{MQ} > s^{MR}$ if $2b\gamma > (1+\beta)^2$. This last inequality is the second order condition for maximizing the welfare objective. This result is predictable as it is more costly to "pull" monopoly output to the socially optimal value than to do the same with R&D levels when the ranking given in (27) holds. This higher value of unit marginal subsidy will mechanically increase the R&D level beyond its socially optimal value. Substituting (23) in equation (28), gives the subsidized R&D level, when final output is the target:

$$x^{MsQ} = \frac{(a-A)\left[2b\gamma + (1+\beta)^2\right]}{(1+\beta)\left[2b\gamma - (1+\beta)^2\right]}. \tag{34}$$

Comparing x^{MsQ} given by (34) with the value of $\overset{\#}{x}$ reveals that: $x^{MsQ} > \overset{\#}{x}$. There is excessive R&D compared to the socially necessary level when subsidies are used to attain the welfare maximizing output.

3.2. Subsidizing the R&D cartel

1. The optimal R&D objective.

We use the same device as previously to compute subsidies when firms remain competitive in the final good market but cooperate in the R&D stage. The subsidized R&D level is given using equation (14) as:

$$\bar{x}^{s} = \frac{(a-A)(1+\beta)}{\frac{9}{2}b(\gamma - s) - (1+\beta)^2}.$$

When the social planner targets the welfare solution for R&D, he (or she) solves "s" such that $\bar{x}^{s} = \overset{\#}{x}$, which yields: $\bar{s}^{R} = \frac{5}{9}\gamma$. We can immediately notice that:

$$\bar{s}^{R} > s^{MR} \tag{35}$$

The unit marginal subsidy needed to reach the welfare R&D level is slightly greater (0.55 compared to 0.50) than the one needed for the monopoly to hit the same objective. The impact of this first type of subsidy on final output can be deduced from equation (15)*:

$$\bar{Q}^{s}=\frac{2(a-A)}{3b}\left[\frac{\frac{9}{2}b(\gamma-s)}{\frac{9}{2}b(\gamma-s)-(1+\beta)^{2}}\right].$$

For $\bar{s}^{R}=\frac{5}{9}\gamma$, the corresponding output is:

$$\bar{Q}^{R}=\frac{2(a-A)}{3b}\left[\frac{2b\gamma}{2b\gamma-(1+\beta)^{2}}\right]. \tag{36}$$

Comparing with the welfare output given in (26), we get:

$$\bar{Q}^{sR}=\frac{2}{3}Q^{\#}. \tag{37}$$

The performance of subsidies in the cartel is greater than what it would achieve in the monopoly organization, when the level of R&D is the objective.

2. The optimal output objective,

In the case the welfare output is targeted, we solve for "*s*" equating the subsidy driven output of the cartel $\bar{Q}^{s}$ to its welfare counterpart $Q^{\#}$:

$$\frac{2(a-A)}{3b}\left[\frac{\frac{9}{2}b(\gamma-s)}{\frac{9}{2}b(\gamma-s)-(1+\beta)^{2}}\right]=\frac{(a-A)}{b}\left[\frac{2b\gamma}{2b\gamma-(1+\beta)^{2}}\right].$$

Solving for "*s*" we find:

$$\bar{s}^{Q}=\frac{\gamma}{3}\left[\frac{3b\gamma+(1+\beta)^{2}}{b\gamma+(1+\beta)^{2}}\right]. \tag{38}$$

* Recall that $\overline{Q}=2\overline{q}$.

For this situation also, we can show that: $\bar{s}^{Q} > \bar{s}^{R}$ A greater amount of subsidies is needed to reach the welfare output compared to the former case. Once again the level of R&D, when final output is targeted by subsidies exceeds the one obtained in the welfare solution:

$$\bar{x}^{sQ} = \frac{(a-A)\left[b\gamma + (1+\beta)^2\right]}{(1+\beta)\left[2b\gamma - (1+\beta)^2\right]} \tag{39}$$

and $\bar{x}^{sQ} > x^{\#}$.

It will be more efficient to subsidize the cartel in order to attain the welfare output solution compared to the private subsidized monopoly if:

$$\bar{s}^{Q} < s^{MQ}. \text{ This inequality holds if: } b\gamma > (1+\beta)^2. \tag{40}$$

The inequality may go either way as it is not supported by any second order condition on profit maximization. Therefore the ranking of subsidies may be reversed: $\bar{s}^{Q} > s^{MQ}$.

The reasons why monopoly may be more efficient in using subsidies are discussed below.

There are sets of values of b and γ for which the inequality in (40) should hold as the value of β increases. When there is strong cooperation in R&D, let it be in the monopoly organization or within the cartel with substantial result sharing, the spillover parameter is set to its maximal value: $\beta = 1$.

In such a case, $(1+\beta)^2 = 4$, we need to set $b\gamma > 2$ for the second order condition on profit maximization to hold in the welfare case[*]. Recall that satisfying the second order condition for monopoly was less restrictive. If we assume that the inequality $b\gamma > (1+\beta)^2$ holds for all values of β, then all other conditions are automatically satisfied and comparisons can be carried out for all cases.

Away from these technical considerations, it is important to ask under what circumstances and for what organization reasons this inequality may be reversed. Put differently, we may ask why is the monopoly more efficient, when R&D is subsidized, to achieve the welfare output compared to the situation where firms, in the R&D cartel, remain competitive in the final good market.

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[*] In section 2, the values $b = \gamma = 1$, while correct for comparisons with rivalry and cooperation in R&D, do not carry for the monopoly case as $4b\gamma > (1+\beta)^2$. $b\gamma$ will fall short of $(1+\beta)^2$ when β reaches its maximal value $\beta = 1$.

Expression $b\gamma$ puts together elements of cost of producing R&D, γ, and market power considerations for the final output through slope of the inverse demand function parameter b. A lower value for $b\gamma$ could simultaneously mean that the monopoly is more efficient if developing research can be undertaken by one firm and that the elasticity of demand is relatively high (as linked to the inverse of the slope given by b). The opposite case necessitates cooperation among a number of firms, to cut costs, combined with a high market power.

The economic reasons supporting a greater efficiency of the monopoly in using R&D cost reduction are more subtle. These were discussed in the introductory section. They are linked to the degree of appropriating benefits in the final good market. These benefits stem from cost improvements incurred by R&D realized in the first stage. The monopoly has a greater incentive to use efficiently the subsidies when it can capture profits in the second stage of the game.

We look at this question by setting $b = 2$ and $\gamma = 3$ so that the inequality in (40) holds. When targeting outputs, subsidies depend on the spillover parameter β. Either in monopoly or when cooperation in the cartel is based on full sharing of information, externalities reach their maximal value $\beta = 1$. Comparing subsidies needed to reach the welfare output reveals that:

$s^{MQ} = 1.5$, whereas $\overset{-Q}{s} = 1.46$. It will cost less to subsidize the cartel than the private monopoly when $\beta = 1$. On the contrary if cooperation among the participants is loose and the spillover parameter is set to equal $\beta = 0.6$ and $\overset{-Q}{s} = 1.6$. This value is greater than the amount of the subsidy devoted to the monopoly to reach the same objective.

Targeting output with many sellers dissipates the incentive to supply R&D at the first stage because the opportunity of capturing profits in the second stage decreases with competition. However comparing subsidies in the monopoly case and in the cooperative situation is meaningful only if $\beta = 1$. As long as the cost of developing and improving technology is high and the firm has a strong market power, which are the main features of modern industrial structures, cooperation in producing R&D, will prove to be an efficient organization only if results are fully shared.

The above discussion is intended to reveal the performance of industry organization that simultaneously internalizes externalities generated by a high level of R&D spillover and provides a better sharing of information among participant firms.

We may ask to what extent targeting welfare output with subsidies is a socially efficient policy. The output of R&D overshoots the optimal level given by the welfare solution. This outcome means that resources are wasted by excessive investment. A private firm may be willing to reach such high levels of R&D if the objective is to deter entry of potential competitors. The consequence of subsidizing excessively R&D in order to reach the socially optimal output is to shift potential producer's rent to increase consumer surplus. This situation inhibits the incentives of firms to use efficiently the subsidies to increase output. Economic policy may gain in efficiency if it was to be limited in funding R&D activities to reach the socially optimal level and allow competition in the final good market to increase output and consumer surplus.

3.3. Subsidizing R&D among competitors

It may still be interesting to look at the fully competitive case. We use the same device to compute the unit marginal subsidy that drives R&D expenditure to its welfare level, we find that:

$$s^{*R} = \gamma\left[1 - \frac{4}{9}\frac{(2-\beta)}{(1+\beta)}\right]. \tag{41}$$

The unit marginal subsidy now depends on the spillover parameter β, contrary to the other cases examined earlier. The subsidy will depend on the degree of information sharing within a coalition as discussed by M. Kamien et al. [1988]: firms are allowed to maintain competition in R&D but are willing to share all their results.

Once the socially optimal level of R&D is reached, the non-cooperative case will yield the same output of final goods as the cooperative solution, because in both cases firms are competitors in the second stage of the game. It can be readily shown that:

$$Q^{*sR} = \frac{2}{3}\overset{\#}{Q}. \tag{42}$$

This result is somewhat misleading if we are to carry out comparisons. The socially optimal output $\overset{\#}{Q}$ is computed as a function of β, as well as the subsidies for this case*. The spillover is actually fully internalized when the organization is a public monopoly that yields the value of $\overset{\#}{Q}$. The meaningful value of the externalities is to be set to $\beta = 1$, in order to allow comparisons.

* In the former cases, the subsidies were given as a fixed proportion of γ, and were therefore independent of the spillover parameter. Comparisons with the socially efficient output could be conducted directly.

Firms in the non-cooperative case do not coordinate their action to maximize joint profit. The agreement is built on result sharing among participants. This organization is assumed to induce the spillover parameter to reach its maximal value: $\beta = 1$. As R&D output is a decreasing function of β (as discussed in section II), it will pay a high cost to incite participants to cooperate in order to reach the socially optimal level. The corresponding unit marginal subsidy is given by:

$$s^{*R} = \frac{7}{9}\gamma \quad \text{for} \quad \beta = 1. \tag{43}$$

The policy maker may be only willing to subsidize coalitions that are significantly committed to cooperate*.

Conclusion

When we put together the results discussed in the precedent sections, economic policy will have to operate a compromise between conflicting goals while choosing the level and the destination of subsidies. In the case where the socially optimal level of R&D, is chosen as an objective, we can establish the following results:

Table 4

Firm organization, alternative policies and corresponding outcomes

	Unit marginal subsidy for β = 1	Corresponding output
Private monopoly	$s^{MR} = \frac{1}{2}\gamma$	$Q^{MR} = \frac{1}{2}Q^{\#}$
R&D cartel	$s^{-R} = \frac{5}{9}\gamma$	$Q^{-sR} = \frac{2}{3}Q^{\#}$
Duopoly competition	$s^{*R} = \frac{7}{9}\gamma$	$Q^{*sR} = \frac{2}{3}Q^{\#}$

The ranking of solutions is given by:

$$s^{MR} < s^{-R} < s^{*R}.$$

And the corresponding output by:

$$Q^{MR} < Q^{-sR} = Q^{*sR}.$$

* When firms are competitors the R&D output is higher, for weak spillovers, and there is no rationale for public policy to fund firms engaged in competition.

It will cost the policy planner a high subsidy, $\frac{7}{9}\gamma$, to fund a coalition in which firms are competitors in R&D but are willing to share completely their knowledge. The same result on final output will require a lesser amount of subsidy, $\bar{s}^{R} = \frac{5}{9}\gamma$, when firms coordinate the R&D activities in a cartel (and maximize joint profit) with full sharing of information.

In the monopoly case the organization is completely integrated and shows a higher degree of efficiency in using the subsidy to reach the R&D target. It will cost the policy maker a lesser amount, $s^{MR} = \frac{1}{2}\gamma$, compared to the cartel. There is however a social price to such a performance as output of final goods reaches a much weaker level than the one obtained by the cartel: $\frac{1}{2}\overset{\#}{Q}$ compared to $\frac{2}{3}\overset{\#}{Q}$. The cost/benefit ratio is unambiguously in favour of the cooperative cartel: $\dfrac{\frac{1}{2}\gamma}{\frac{1}{2}\overset{\#}{Q}} > \dfrac{\frac{5}{9}\gamma}{\frac{2}{3}\overset{\#}{Q}} \Leftrightarrow 1 > \frac{5}{6}$.

Cooperation in R&D activities among participants with full sharing of information (while keeping competitive in the final good market) proves to be the most efficient organization the policy maker in willing to fund.

We also explored the case of subsidizing R&D to reach the socially optimal level of final output. Such a policy may seem globally inefficient as improving consumer surplus by providing higher output is obtained at the expense of wasting resources, as subsidies induce excessive R&D levels. There may be an incentive to increase R&D output with the intention of using such a potential as a barrier to entry. However the discussion was intended to reveal industry performance when firms seek through their organization to simultaneously internalize externalities generated by high levels of R&D spillovers, and to provide a better sharing of information among participants. The fully integrated industry, as a monopoly, has an incentive to use efficiently the subsidies as it can capture a large share of profits in the final output stage of production. Competition among firms dissipates these potential gains. However for plausible assumptions such as high cost in developing research as well as for firms with large market power, the fully integrated industry is not necessarily more efficient. Comparing subsidies needed to reach the welfare output reveals that competition in the final good market and cooperation in producing R&D proves superior to full integration when these activities are coordinated and results completely shared.

Subsidizing integrated industries may be less acceptable by anti-trust regulations because the degree of capturing profits in the second stage is high. Policy makers may act indirectly to provide some protection to research consuming industries (so as to create barriers to entry) as well as improving consumer surplus. Encouraging coalitions with subsidies flowing to participants can be achieved through cooperation involving private or public organization should these be firms, research units or universities. One of the possible extensions of the analysis would be to define rules of cooperation within these structures. Distribution of subsidies between participants will have to provide mechanisms to cope with moral hazard and adverse selection.

Appendix

1. Monopoly.

	Decision structure	Objective	Spillover β	Outcome
Monopoly	Two stage cooperation: R&D and final output	Private monopoly: Max π	Full sharing of results: $\beta=1$	x^M, Q^M
	Two stage cooperation: R&D and final output	Public monopoly: Maximise social welfare: $S_c + \pi$	Full sharing of results: $\beta=1$	$x^\#, Q^\#$

2. R&D cartel.

	Decision rule	Objective	Spillover β	Outcome
R&D cartel	Coordination in R&D stage Competition in output stage	Max joint profit by coordinating R&D: Max: $\pi_1 + \pi_2$	Full sharing of results if $\beta = 1$ Or partial sharing of results if $\beta < 1$	$\bar{x}$,$\bar{Q}$ as functions of β

3. Competition.

	Decision structure	Objective	Spillover β	Outcome
Duopoly competition	Two stage competition: R&D and final output	Max: π_1 Max: π_2	Full sharing of results if β=1 Or partial sharing of results if $\beta < 1$	x^*, Q^* as functions of β

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APPLICATION OF DEA MODEL WITH BOOTSTRAP TO EVALUATION OF SMES EFFICIENCY IN THE SPANISH TEXTILE INDUSTRY

Abstract

The Spanish textile industry underwent an important transformation during the 1990s. To survive under new market conditions, firms had to refocus their competitive strategies towards an increase in productive efficiency or an investment in technological development. The purpose of this paper is to evaluate the technical efficiency in the sample of 66 micro-, small- and medium-sized textile companies that operated in the Spanish region of Catalonia during the 1996-2001 period. Based on the firm-level accounting data we derive efficiency estimates using Data Envelopment Analysis model with bootstrap. The general result of this study shows that firms in the sample are on average relatively highly efficient in their productive process. The bias-corrected efficiency score reaches the 0.817 level and it slightly fluctuates during the period analyzed.

Keywords

Textile industry, efficiency, DEA, SMEs

Introduction

Textile industry in Spain since the 1990s has undergone major restructuring and readjustment in order to improve the competitiveness of companies, which faced the increased competition from low-wage developing countries. As the answer to those competitive pressures, companies had to substantially reduce the mass production and refocus their competitive strategies towards an increase in productive efficiency or an investment in technological development.

Within this context, the aim of this paper is to evaluate the efficiency of Spanish micro-, small- and medium-sized firms (*SMEs*) in the textile industry. We analyze textile firms operating in Spanish region of Catalonia, where they have traditionally been mostly concentrated. In particular, we are interested to analyze if the competitive pressures have impacted the level of companies' efficiency. The empirical part is based on the firm-level data, which consists of accounting information covering the time period of six years: from 1996 to 2001. We use the Data Envelopment Analysis (*DEA*) model for assessing the efficiency of companies and we perform a bootstrap of efficiency scores to derive the confidence intervals and to measure the bias for those indices.

The focus of the study on micro-, small- and medium-sized firms is of particular importance. While firm-level performance and efficiency among larger companies was studied intensively, such research on *SMEs* is rather rare [Hill and Kalirajan, 1993]. In addition, Spanish textile sector is predominantly based on micro-, small- and medium-sized companies [Stengg, 2001].

The paper proceeds as follows. We first describe the evolution of the Spanish textile industry. Then we present the Data Envelopment Analysis model with bootstrap that permits to measure the technical efficiency of firms. The subsequent sections describe the data, the variables and discuss the results. Conclusions are presented in the final section.

1. The textile industry in Spain

The textile sector in the European Union (*EU*) in 2004 represented some 77 288 firms with production of 104 billion euros, while an average firm produced 1.4 million euros*. Its importance for social and economic cohesion is increased by the fact that it is dominated by a large number of micro, small and medium-sized companies, which are often concentrated in particular regions contributing greatly to their wealth and cultural heritage [Stengg, 2001]. Spanish textile industry is not an exception. It has traditionally been a major sector in the manufacturing industry. In 2003 its production exceeded 13 000 million euros manufactured in 7200 companies, giving the employment to almost 260 000 people. The Spanish textile industry occupies the fifth place among the *EU-15*** countries with the share of 8% of the *EU-15* total, and falls behind Italy (28%), the United Kingdom (14%), France (12%) and Germany (13%) [Stengg, 2001]. The risk factor for the Spanish textile sector is the fact that imports continue to grow dangerously (Table 1).

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* According to the European Commission survey *Study on the competitiveness, economic situation and location of production in the textiles and clothing, footwear, leather and furniture industries*.

** *EU*-15 refers to the *EU* countries prior to accession of 12 members in 2004 and 2007 that is comprising of Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden and the United Kingdom.

Table 1

Data for Spanish textiles

Data for Spanish textiles	2000	2001	2002	2003
Employment	278 200	277 900	268 200	257 500
Number of firms	7 615	7 590	7 470	7 200
Production (millions of euros)	14 827	14 800	13 912	13 258
Imports (millions of euros)	7 671	8 231	8 620	9 431
Exports (millions of euros)	5 475	5 991	6 143	6 437

Source: [Centre of Information about Textile and Clothing Industry – Centro de Información Textil y de la Confección *CITYC*].

The textile industry in Spain confronts the radical changes posed by the internationalisation, the advance in technology with a development of new fabrics, the rapid progress in information technology, and the increasing demand for variety [Owen, 2001]. In particular, the sector experiences a high competition from developing countries, especially from South-East Asia [Stengg, 2001]. Spanish micro-, small- and medium-sized firms are mostly affected by the changes in the environment due to the fact that the increased competition places obviously large companies in a privileged position. As the answer to the competitive pressures, Spanish textile companies improved their competitiveness by substantially reducing mass production and concentrating instead on a wider variety of products with a higher value added. The direct results of those transformations are considerable reductions in production and employment. As a final consequence, the companies are in the process of developing the specific competitive advantages based on innovation, design, quality, creativity and use of information technologies [Stengg, 2001].

Since the early nineteenth century, the Spanish textile industry has been strongly concentrated in Catalonia, especially the leading textile activity of weaving. In 2000 there were 98 210 people employed in the Catalan textile industry, representing 35% of the whole employment in the sector. The production equalled 6176 millions of euros, which stands for the 41% of the national textile production*.

* According to the *Textile-clothing industry 2004/Spain. Practical guide*.

2. Methodology

This section explains the foundations of Data Envelopment Analysis model and its recent development in the form of bootstrapping methods. *DEA* is a nonparametric technique for identifying efficient production frontiers and evaluating the relative efficiency of decision making units (*DMUs*), each of which is responsible for converting multiple inputs into multiple outputs. As such it considers multidimensional aspects of organizational performance, which is a characteristic not available in other models such as financial ratio analysis. In ratio models, as opposed to DEA, it is difficult to gain an overall image of performance as every ratio usually indicates a different level of performance. Only if we are able to combine well several financial ratios into a summary measure of performance, they conform better to *DEA* conclusions. Hence, both techniques are usually regarded as complementing each other [Thanassoulis et al., 1996].

DEA involves the application of the linear programming techniques to given inputs consumed and outputs produced by firms. Next *DEA* constructs an efficient production frontier based on the best practices. Each firm's efficiency is then measured relative to this frontier. In recent years there has been an extensive methodological growth of the *DEA*, giving rise to the development of many different models[*]. Concerning the technology, *DEA* specifications invoke different assumptions about returns to scale. Returns to scale measure the change in output levels due to the changes in input levels. Constant returns to scale (*CRS*) imply that an increase in input levels results in a proportional increase in output levels. On the other hand, variable returns to scale (*VRS*) imply that an increase in the input levels does not necessarily result in a proportional increase in output levels, that is, the output levels can increase (increasing returns to scale) or decrease (decreasing returns to scale) by a different proportion than the input increment. The original *DEA* model proposed by Charnes et al. [1978] assumes constant returns to scale. This premise is only appropriate when all companies are operating at an optimal scale, however in practice certain constraints might cause the optimal scale to be impossible to achieve. As the answer to those problems, Banker et al. [1984] developed the *VRS* model, which permits the firms to be compared with those of similar size.

[*] A detailed review of majority of existing models can be found in Cook and Seiford [2009].

From the efficiency measure point of view, input- and output-oriented models can be distinguished. The input-oriented model aims at minimizing inputs while maintaining outputs constant, while the output-oriented one focuses on maximization of outputs and still utilizing the input levels specified originally*. In both cases, efficiency can be considered from two perspectives: technical and scale [Dimara et al., 2003]. Technical efficiency is the distance from the point of the company current input-output combination to the production frontier under constant returns to scale. It is often referred to as global technical efficiency. On the other hand, scale efficiency shows if correct scale of inputs for the output level was chosen. We can talk also about pure technical efficiency, free of scale efficiency effects, that is calculated under variable returns to scale specification.

In this study, an input-oriented model is used due to the characteristics of the industry chosen. In order to survive, textile firms cannot assume to expand their market share in a significant way because of the increasing competition. Instead, companies change to the type of products based on intangible assets directed to niche markets, subcontract parts of the manufacturing process, reduce the size of factories as well as decrease the employment, which is a clear orientation towards the input reduction. In the input-oriented model, the efficiency score is bounded from above by 1, when the score of 1 means that the firm is efficient. The technology chosen is *VRS* because our dataset includes numeric values of various magnitudes. However, we calculate also the scores under *CRS* to be able to measure scale efficiency.

The mathematical formulation of the *VRS* input-oriented model goes as follows. Suppose we have *n DMUs* to be evaluated and each of them consumes varying amounts of *m* different inputs to produce *s* different outputs. DMU_k consumes the quantity $X_k = \{x_{ik}\}$ of inputs $i = \{1,2, \dots , m\}$ and produces the quantity $Y_k = \{y_{jk}\}$ of outputs $j = \{1,2, \dots , s\}$. The model evaluates the efficiency score of each *DMU* observed called DMU_o relative to other *DMUs*. The linear model can be described as below:

* There exists also a hyperbolic orientation which simultaneously focuses on increasing outputs and minimizing inputs.

$$
\begin{aligned}
& Min \quad \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{j=1}^{s} s_j^+\right) \\
& subject\ to \quad \sum_{k=1}^{n} x_{ik} z_k + s_i^- = \theta x_{io}, \qquad i = \{1,2,\ldots,m\} \\
& \qquad \sum_{k=1}^{n} y_{jk} z_k - s_j^+ = y_{jo}, \qquad j = \{1,2,\ldots,s\} \\
& \qquad \sum_{k=1}^{n} z_k = 1 \\
& \qquad z_k, s_i^-, s_j^+ \geq 0
\end{aligned} \tag{1}
$$

where:

θ is the efficiency coefficient,
ε is a very small – archimedean – positive number,
x_{ik} stands for quantity of input $i = \{1,2, \ldots, m\}$ consumed by DMU_k ($k = 1, \ldots, n$),
y_{jk} stands for quantity of output $j = \{1,2, \ldots, s\}$ produced by DMU_k,
x_{io} represents quantity of input i consumed by the observed unit under analysis DMU_o,
y_{jo} represents quantity of output j produced by the observed unit under analysis DMU_o,
z_k symbolises the activity levels associated with inputs and outputs of DMU_k,
s^- is the input slack,
s^+ is the output slack.

Note that the restriction $\sum_{k=1}^{n} z_k = 1$ corresponds to the *VRS* model. The computation of efficiency scores involves solving one linear program for each *DMU*. The firm is efficient when the slacks are equal to zero.

Such formulation of *DEA* is deterministic as it does not allow for random error. In other words, *DEA* assumes that the distance between the observation and the efficient boundary reflects only inefficiency. However, it reflects both inefficiency and noise as the input-output levels could be subject to a measurement error or some input-output variables might be omitted. Hence, it would be desirable to determine the statistical properties of estimated *DEA* scores as they are essential for their interpretations and inference. Recent *DEA* literature allows for this. In particular, Simar and Wilson [1998] proposed to use bootstrapping technique to correct *DEA* estimators for a bias and to estimate the confidence intervals for those indices. Bootstrapping could be defined as a repeated simulation of the data-generating process through resampling and applying the original estimator to each simulated sample so that resulting estimates imitate the original unknown sampling distribution of the estimators of interest. To introduce the bootstrap procedure mathematically, we denote $\chi = \{(\mathbf{x}_k, \mathbf{y}_k), k = 1, \ldots, n\}$ as an original sample of *n DMUs* for which bootstrap should be estimated. The algorithm can be summarized in the following steps [Simar and Wilson, 1998, 2000]:

1. The computation of the efficiency scores $\hat{\delta}_k$ for each DMUs $k = 1, \ldots, n$ by solving the linear programming model described by (1).
2. Using kernel density estimation and reflection method (smooth bootstrap*), the generation of the random sample of size j from $\{\hat{\delta}_k, k = 1, \ldots, n\}$, resulting in $\{\delta^*_{kb}, k = 1, \ldots, n\}$.
3. The generation of the pseudo sample $\chi^* = \{(\mathbf{x}^*_k, \mathbf{y}^*_k), k = 1, \ldots, j\}$ to form the reference bootstrap technology.
4. The computation of the bootstrap estimation of efficiency $\hat{\delta}^*_{kb}$ of $\hat{\delta}_k$ for each $k = 1, \ldots, n$.
5. The repetition of steps 2)-4) B times in order to obtain a set of estimates $\{\hat{\delta}^*_{kb}, b = 1, \ldots, B\}$.

Having the bootstrap values computed, we obtain the following measures:

a) bootstrap bias for the original estimator $\hat{\delta}_k$: $bia\hat{s}_B(\hat{\delta}_k) = B^{-1}\sum_{b=1}^{B} \hat{\delta}^*_{kb} - \hat{\delta}_k$,

b) bias-corrected estimator of δ: $\hat{\hat{\delta}}_k = \hat{\delta}_k - bia\hat{s}_B(\hat{\delta}_k) = 2\hat{\delta}_k - B^{-1}\sum_{b=1}^{B} \hat{\delta}^*_{kb}$,

c) confidence intervals for efficiency scores, which involves the following steps:
   - sort the values $(\hat{\delta}^*_{kb} - \hat{\delta}_k)$ for $b = 1, \ldots, B$ and delete $(\frac{\alpha}{2} \times 100)$ – percent of the elements at either end of this sorted array,
   - set $-\hat{b}^*_\alpha$ and $-\hat{a}^*_\alpha$ $(\hat{a}^*_\alpha \le \hat{b}^*_\alpha)$, equal to the endpoints of the resulting array, then the estimated $(1-\alpha)$ – percent confidence interval is formulated as: $\hat{\delta}_k + a^*_\alpha \le \delta_k \le \hat{\delta}_k + b^*_\alpha$.

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* For details of statistical rationale see: Simar and Wilson [1998].

3. Data

The data used in this study come from the *SABI* database. *SABI* (Sistema de Análisis de Balances Ibéricos) contains financial accounts of Spanish and Portuguese companies classified according to the *NACE Rev. 1.1 code**. To delimit the scope of our study we searched for micro, small and medium-sized textile companies operating in the Spanish region of Catalonia. Following the *EU* definition, the category of *SMEs* is made up of enterprises which employ fewer than 250 persons and which have an annual turnover not exceeding 50 million euros and/or an annual balance sheet total not exceeding 43 million euros**. Furthermore, we delimit the sample to the firms representing the leading textile activity – textile weaving***. After filtering out some firms that did not provide all the information necessary or with negative auditors' opinion on the data, the final sample consists of 66 firms that operated in Catalonia from 1996 to 2001.

We have some year-end variables from the balance sheet and the profit and loss account for individual firms. Although there are a number of potential problems with accounting data, the *DEA* literature using this type of information is very extensive. The studies apply a huge variety of different variables for inputs and outputs and there is no consensus which combinations are the most appropriate. Among outputs, the most frequently applied in the studies are sales revenues [Zheka, 2005; Zhang et al., 2001]. Sometimes this variable is used in conjunction with others, such as profit before tax [Worthington, 1998; Zhu, 1996]. However, it is believed that profits are not a good approximation of outputs, because they can be strongly influenced by the environmental conditions [Al-Shammari, 1999]. With regard to input variables, Hill and Kalirajan [1993] for example work with three inputs: cost of employees, material cost and value of investments, while Thore et al. [1994] use operating cost, fixed assets and number of employees. Basing on those studies and given the limitation of available data, in this paper we consider the production

* *NACE Rev. 1.1* is a classification of economic activities used by *EUROSTAT* and published in 2002. It is an extension of *ISIC Rev. 3* activity representation created by the United Nations. According to this classification number 17 represents Manufacture of textiles.

** Within this category, small firms are those that employ fewer than 50 persons and whose annual turnover or annual balance sheet total does not exceed 10 million euro, while micro companies employ less than 10 persons and have annual turnover or annual balance sheet total that does not exceed 2 million euro.

*** Textile weaving includes cotton-type, woollen-type and worsted-type weaving. It is represented in *NACE Rev. 1.1* classification by the number 172.

of woven textiles as the outcome of labour, fixed assets and variable inputs [materials]. The production is estimated via sales revenues. Hence, we apply the following variables*:

Inputs	***Output***
a. Number of employees	a. Sales revenues
b. Fixed assets	
c. Material expenses	

The basic descriptive statistics of these data are presented in Table 2.

Table 2

Input/output specification for *DEA* (descriptive statistics for 1996-2001)

Variable	Mean	Std. Dev.	Min	Max
Output				
Sales revenues	2778.705	4024.828	93	23502
Input				
Number of employees	23	19.458	1	114
Fixed assets	459.879	978.957	0	11822
Material expenses	1862.152	3313.954	1	19872

Note:
All variables, except for number of employees, are expressed in thousands of euros.

We can observe that the mean textile company in our sample has 23 employees and almost 3 million of sales revenues. Hence, it belongs to the category of small companies. In addition, our average sample firm represents well the European population as the mean textile company in Europe has 19 employees [Stengg, 2001].

4. Results

The bootstrap algorithm of Simar and Wilson [1998, 2000] described before in this paper was applied with *FEAR 1.1 package*** with $B = 2000$

* We checked the correlations among inputs and outputs and we did not find any high correlation between inputs neither a very low correlation between inputs and outputs, which reasonably validates our DEA model (no input could be excluded and all variables fit the model).

** *FEAR* is a software package for frontier efficiency analysis with *R*, which allows computing many different estimates of efficiency. The software is freely available from: http://www.clemson.edu/economics/faculty/wilson/Software/FEAR/fear.html.

bootstrap replications. Table 3 summarizes the means of the following measures: efficiency scores, bias-corrected efficiency scores and confidence intervals for true efficiency scores in *VRS* specification.

Table 3

Mean *DEA* efficiency scores under *VRS*

Years	Efficiency score	% of efficient firms	Bias-corrected efficiency score	Confidence interval	
				Lower bound	Upper bound
1996	0.876 (0.136)	34.848	0.807 (0.113)	0.732 (0.097)	0.871 (0.135)
1997	0.877 (0.130)	33.333	0.814 (0.109)	0.738 (0.095)	0.873 (0.130)
1998	0.880 (0.141)	40.909	0.810 (0.121)	0.730 (0.110)	0.875 (0.140)
1999	0.888 (0.128)	37.878	0.829 (0.109)	0.751 (0.103)	0.884 (0.128)
2000	0.896 (0.116)	36.364	0.837 (0.097)	0.757 (0.085)	0.892 (0.116)
2001	0.872 (0.128)	34.848	0.806 (0.105)	0.729 (0.090)	0.868 (0.128)
1996-2001	0.881 (0.130)	36.364	0.817 (0.109)	0.740 (0.097)	0.877 (0.129)

Note:
The values presented in the brackets show the standard deviations.

The first thing to note in Table 3 is that during the period under investigation textile firms in our sample, on average, have the relatively high levels of efficiency of 0.881 with standard deviation of 13%. The number of firms that are classified as relatively efficient is rather high in individual years (more than 30%). When taking bias-corrected estimates, mean efficiency in the sample decreases to 0.817 with standard deviation of 10.9%. This value indicates that there is still a room for efficiency improvement for firms by reducing the input. In particular, to be efficient the companies should be able to obtain the same sales revenues by reducing the consumption of production resources (employees, fixed assets and material costs) at least in 18.3%. Furthermore, Table 3 reveals a slight increase in the original efficiency scores up to the year 2001 when the efficiency dropped. To sum up: on average, the bias-corrected efficiency scores are lower than the original ones (indicating higher level of inefficiency) and the values of real efficiency scores are contained in the interval between 0.740 and 0.877 as indicated by confidence intervals.

We further applied *DEA* with bootstrap in *CRS* specification. If there is a difference in the *CRS* and *VRS* scores, it indicates that the company is suffering from scale inefficiency. The scores for scale efficiency can be computed by dividing the efficiency scores in *CRS* by the efficiency scores in *VRS*. Obviously, if scale efficiency score is equal to 1, it means that *CRS* efficiency is equal to *VRS* efficiency and the firm is said to be scale efficient. Otherwise, the firm is scale inefficient. The Table below summarizes the decomposition of bias-corrected efficiency scores computed under *CRS* (global technical efficiency) into *VRS* bias-corrected efficiency scores (pure technical efficiency) and scale bias-corrected efficiency scores during the period 1996-2001 (yearly average).

Table 4

Global bias-corrected technical efficiency decomposition

Years	*CRS* bias-corrected efficiency scores	*VRS* bias-corrected efficiency scores	Scale bias-corrected efficiency scores
1996	0.773 (0.123)	0.807 (0.113)	0.959 (0.080)
1997	0.754 (0.138)	0.814 (0.109)	0.927 (0.115)
1998	0.740 (0.147)	0.810 (0.121)	0.914 (0.116)
1999	0.801 (0.124)	0.829 (0.109)	0.966 (0.075)
2000	0.798 (0.115)	0.837 (0.097)	0.953 (0.074)
2001	0.762 (0.124)	0.806 (0.105)	0.945 (0.083)
1996-2001	**0.772 (0.130)**	**0.817 (0.109)**	**0.944 (0.094)**

Note:
The values presented in the brackets show the standard deviations.

Table 4 reveals a slight decrease in the level of global efficiency until 1998, which was largely due to a dramatic reduction in scale efficiency. Global efficiency decreased from a mean value of 0.773 in 1996 to 0.740 in 1998, which reflects an increase in the distances separating the best practices from the rest of textile firms in the sample. Scale efficiency decreased from a value of 0.959 in 1996 to 0.914 in 1998, hence the companies were positioned farther from the optimal scale. This effect was partially offset by the moderate improvement in pure efficiency in 1997. In 1999 overall efficiency increased due to the increment in pure efficiency and scale efficiency, while it continued to drop until 2001 as a result of a substantial drop in scale efficiency in spite of an increase in pure efficiency. In addition, it is worth observing that despite an almost continuous decrease, the mean scores of scale efficiency are still relatively high: ranging from 0.966 to 0.914.

Furthermore, we perform a test of stochastic dominance to evaluate if distributions of bias-corrected efficiency scores of micro companies are different to those of small and medium-sized. Stochastic dominance refers to the differences that may exist between two distributions, characterized by their cumulative distribution functions*. Formally, let us suppose that we have two distributions *A* and *B* with cumulative distribution functions *F* and *G*, respectively. First order stochastic dominance of *A* relative to *B* is defined by: $F(x)-G(x)\leq 0$ for any argument $x \in R$ [Delgado et al., 2002]. We need to test the following hypothesis: $H_0 : F(x) = G(x)$ *for all* $x \in R$ versus $H_1 : F(x) \neq G(x)$ *for at least one value of* x. To test this hypothesis the Kolmogorov-Smirnov two-sided test is used [Conover, 1971]. Because the application of this test requires independence of observations and as we possess data of six years, we calculate it separately for each time period. Table 5 presents the mean values of bias-corrected efficiency scores under *VRS* and the P-values of Kolmogorov-Smirnov test.

Table 5

Micro versus small and medium-sized firms

DMU	1996	1997	1998	1999	2000	2001	Mean 1996-2001
Micro	0.856	0.866	0.849	0.866	0.863	0.847	0.858
Small- and medium-sized	0.789	0.794	0.795	0.815	0.828	0.791	0.802
P-value	0.011	0.003	0.138	0.007	0.579	0.085	0.000

The results of the tests suggest that the null hypothesis of equality between distributions of micro firms and small and medium-sized can be rejected in all years analyzed, except for 1998 and 2000. As the mean values of bias-corrected efficiency are higher for micro firms, micro companies statistically dominate small- and medium-sized ones in all years, except for 1998 and 2000. In addition, Figure 1 reports the differences between the bias-corrected efficiency scores distributions for those types of companies. It can be seen on the graphs that the position of the distribution for micro companies with respect to small- and medium-sized ones indicates higher levels of efficiency for micro firms for all years. However, the distributions in 2000 lie very close one to another, which can further confirm the insignificance of P-value results for this year.

* Test of stochastic dominance is more general than the Wilcoxon test as it tests if the entire distribution is different.

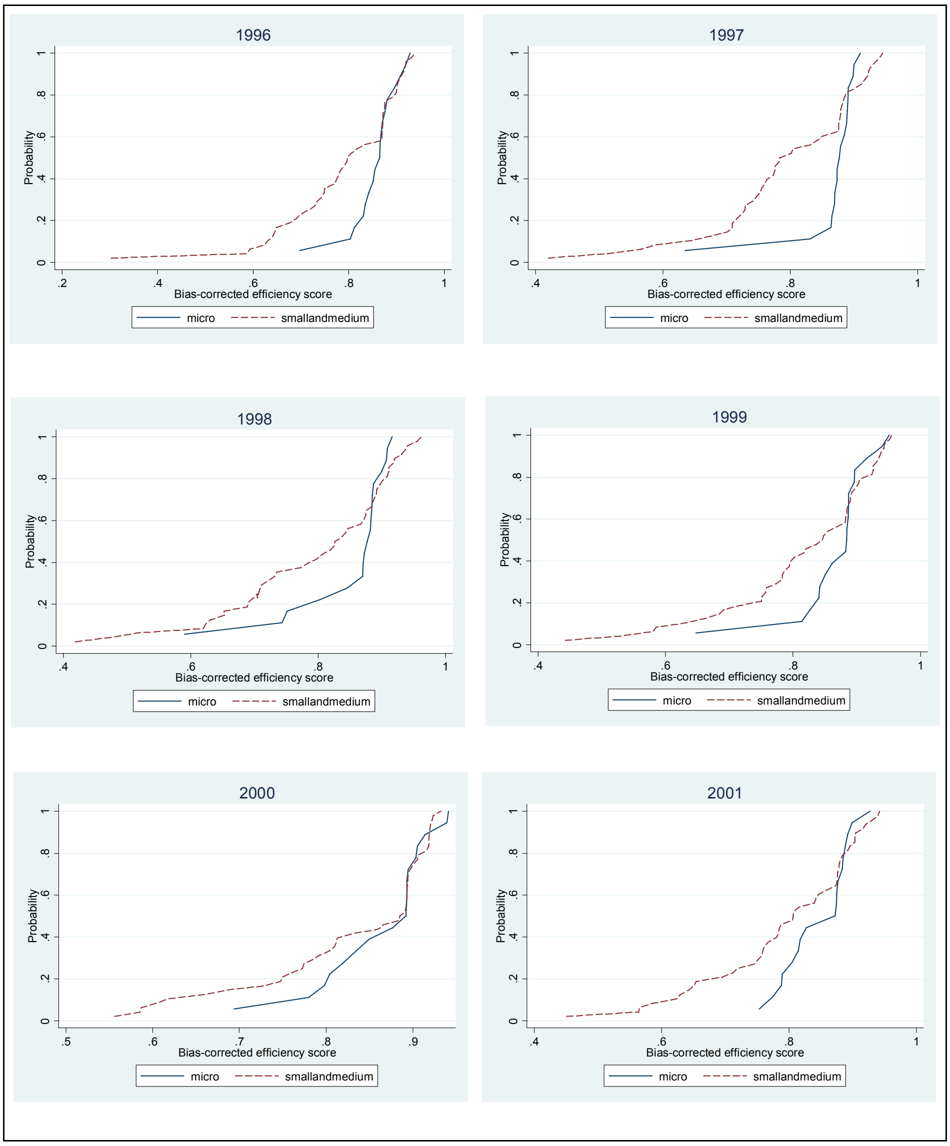

Figure 1. Differences in bias-corrected efficiency scores: micro- versus small- and medium-sized firms (smooth sample distribution function)

Conclusions

This paper aimed at assessing the efficiency of micro-, small- and medium-sized firms in the textile industry in Catalonia during the second half of the 1990s and the beginning of 2000. In the empirical analyses we applied an input-oriented *DEA* model and we used the bootstrap method to give the statistical significance to indices computed. The results have shown that textile firms in our sample are on average relatively highly efficient in their productive process as efficiency score reached the value of 0.881 or 0.817 when the bias-corrected score was taken into account. The efficiency indices fluctuated only slightly, hence the effect of increased competition in the sector cannot be observed by augmented efficiencies. Probably, firms focused mainly on the investment in technological development. In addition, when performing the test of stochastic dominance we found that micro companies are more efficient than small- and medium-sized ones in all years, except for 1998 and 2000. The conclusions from the inefficiencies observed in the sample are the following. First of all, there is room to decrease input for textile firms. In order to improve efficiency companies should be able to obtain the same output as reflected by the sales revenues by reducing the production resources of employees, fixed assets and material costs. Secondly, the firms in the sample experience some small problems with scale efficiency by choosing an incorrect scale of inputs for output level.

The empirical study presented here has some limitations, which open the areas for future research. In particular, efficiency indices computed do not separate the effect of firms moving towards the benchmark frontier from the effect of the movements of the frontier across time (technological development). To separate both moves, in the future research the Malmquist index must be used, also to analyze more precisely the impact of increased competition in the textile sector. Moreover, in the future research, the *DEA* approach could be compared with financial ratio analysis to find out, for example, if and to what extent the two models agree or disagree on the performance of firms.

Acknowledgements

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MANAGER PREFERENCES MODELLING FOR STOCHASTIC AGGREGATE PLANNING

Abstract

In the Aggregate Production Planning (APP) the manager considers simultaneously conflicting objectives such as total cost, inventories level, workforce fluctuation, and utilization level of the physical facility and equipment. The goals associated with these objectives may be uncertain in nature. The aim of this paper is to develop a Goal Programming (GP) model where the goals and the right-hand sides of constraints are random and normally distributed. The concept of satisfaction functions will be used for modeling the uncertainty as well as to explicitly integrate the manager preferences. The proposed model is applied to APP problem to generate the most satisfying aggregate plan.

Keywords

Aggregate Production Planning; Goal Programming; satisfaction functions.

Introduction

Aggregate Production Planning (APP) deals with matching capacity to forecasted demand. The APP aims to set overall production levels for each family of products to meet fluctuating or uncertain demand in the medium term to set decisions and policies concerning hiring, firing, overtime, backorders, subcontracting and inventory level, and thus determining the appropriate resources that will be used. The APP is one of the most important functions in production and operations management.

Traditionally, the objective of the APP is either to maximize profit or minimize costs and is formulated as a single objective function in linear programming [Hanssmann and Hess, 1960; Bowman, 1956]. Many researchers and practitioners are increasingly aware of the presence of multiple objectives in real life problems [Masud and Hwang, 1980; Baykasoglu, 2001; Leung and Chan, 2009]. The existing APP models assume that the information related to the decision making situation is precise and deterministic. Nevertheless, the demand level, the resources and the costs are not usually known in advance. In such situations, the models mentioned above are no longer realistic. The manager should take into account uncertainty while formulating his/her model. To incorporate uncertainty, some mathematical programming methods such as fuzzy programming and stochastic programming have been developed in the literature.

Wang and Fang [2001] developed a linear programming model to solve the APP where the parameters such as demand, machine time, machine capacity and relevant costs, are fuzzy, in which four objectives are optimized. The fuzzy parameters are represented by trapezoidal fuzzy numbers. Wang and Liang [2005] developed a novel interactive possibilistic linear programming approach for solving the multi-product APP decision problem where cost coefficients in the objective function, forecast demand and capacity are imprecise. This approach attempts to minimize the total cost which is the sum of the production costs and the costs of changes in labor levels over the planning horizon. In the last four decades, many studies have addressed the formulation of risk-averse decision making in the stochastic programming models. Leung and Wu [2004] developed a robust optimization model for stochastic APP by optimizing four objectives under different economic growth scenarios. Leung et al. [2007] proposed a robust optimization model to address a multi-site APP problem in an uncertain environment. Gfrerer and Zapfel [1995] present a multi-period hierarchical production planning model with two planning levels: aggregate and detailed, and with uncertain demand.

In this paper, we will present a stochastic goal programming formulation for APP problem where the goals and some parameters are regarded as random. This model will explicitly integrate the manager’s preferences through the concept of satisfaction function developed by Martel and Aouni [1990].

1. The Goal Programming model

The Goal Programming (GP) was originally developed by Charnes and Cooper [1961] and it became the most popular model in multi-objective programming. The standard formulation of the GP model is as follow:

Minimize $\sum_{i=1}^{P}\left(\delta_i^+ + \delta_i^-\right)$

Subject to $\sum_{j=1}^{n} c_{ij}x_j + \delta_i^- - \delta_i^+ = g_i \qquad (i = 1, \dots, p)$

$$x_j \in S = \left\{ x_j \in R^n / \sum_{j=1}^{n} a_{kj}x_j \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b_k, \;\; x_j \geq 0, \;\; b_k \in R^m \right\}$$

$$\delta_i^+, \; \delta_i^- \geq 0 \qquad (i = 1, \dots, p)$$

where g_i represent the aspiration level associated with the objective i, δ_i^+ and δ_i^- indicate the positive and negative deviations of the achievement level from the aspiration level, x_j are decision variables, c_{ij} and a_{kj} are technological coefficients associated with goals and constraints, respectively, and b_k are the limitations of resources. With this formulation, the goal values are considered precise and deterministic. Nevertheless, many uncertain aspiration levels may exist.

2. Stochastic Goal Programming model

The first formulation of Stochastic Goal Programming (SGP) was presented by Contini in 1968 [Contini, 1968]. He considered the goals as random variables with normal distribution. This model is based on the maximization of the probability that the decision belongs to a region encompassing the random goals. In other words, this model tries to obtain a solution which is as close as possible to the random goals. Stancu-Minasian [1984] and Stancu-Minasian and Giurgiutiu [1985] presented a synthesis of methodologies used in multiple objectives programming in a stochastic context. The various approaches proposed use the solution of a deterministic equivalent program. The Chance Constrained Programming (CCP) was introduced by Charnes and Cooper [1959, 1963] to obtain a deterministic program. The main idea of the CCP is to maximize the expected value of the objectives while assuring a certain probability of realization of the various constraints. Some approaches using or referring to SGP are proposed by Ben

Abdelaziz and Mejri [2001], Tozer and Stokes [2002], Bordley and Kirkwood [2004], Sahoo and Biswal [2005]. When time series of probability distributions are not explicitly known, they can be assumed to be defined by fuzzy logic [Ben Abdelaziz and Masri, 2005]. The SGP model formulation proposed by Aouni et al. [2005] explicitly integrates the decision maker's preferences in an uncertain environment. The goals specified by the decision maker $\tilde{g}_i$ are normally distributed with known mean μ_i and variance σ_i^2. This formulation is as follows:

Maximize $\quad Z = \sum_{i=1}^{p} \left(w_i^+ F_i^+(\delta_i^+) + w_i^- F_i^-(\delta_i^-) \right).$

Subject to: $\quad \sum_{j=1}^{n} c_{ij} x_j - \delta_i^+ + \delta_i^- = \mu_i; \qquad (i = 1, \dots, p),$

$x_j \in S,$

$\delta_i^+ \text{ and } \delta_i^- \le \alpha_{iv} \qquad (i = 1, \dots, p),$

$\delta_i^+, \delta_i^- \text{ and } x_j \ge 0 \qquad (i = 1, \dots, p), \ (j = 1, \dots, n).$

where $F_i(\delta_i)$ are the satisfaction functions associated with positive and negative deviations (δ_i^+, δ_i^-) as presented in Figure 1. The coefficients w_i^+ and w_i^- express the relative importance of the positive and negative deviations, respectively; α_{id} is the indifference threshold; α_{i0} is the null satisfaction threshold and α_{iv} is the veto threshold.

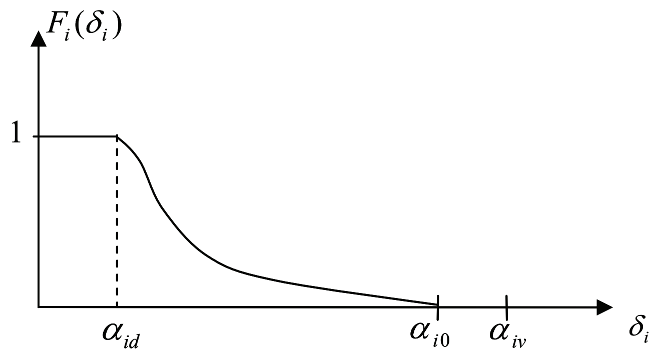

Figure 1. General form of the satisfaction function

In the following section, we extend the SGP model formulation proposed by Aouni et al. [2005] to take into account the randomness goals and the right-hand sides of the constraints.

3. The proposed model in an uncertain environment

In this section, we begin by introducing the following goal programming model with uncertain goals and the right-hand sides of the constraints.

$$\text{Optimize} \sum_{j=1}^{n} c_{ij}x_j \qquad (i = 1, \dots, p).$$

Subject to

$$\sum_{j=1}^{n} a_{kj}x_j \le \tilde{b}_k \qquad (k = 1, \dots, m_1),$$

$$\sum_{j=1}^{n} a_{kj}x_j \le b_k \qquad (k = m_1 + 1, \dots, m).$$

$$x_j \ge 0$$

Let $\tilde{g}_i$ be the uncertain aspiration levels for the i^{th} objective function $\sum_{j=1}^{n} c_{ij}x_j$. $\sum_{j=1}^{n} a_{kj}x_j \le (=, \ge)\ \tilde{b}_k$ indicates that the k^{th} uncertain right-hand side parameter is greater than or equal to (equal or less than or equal to) $\sum_{j=1}^{n} a_{kj}x_j$. The other variables are defined as in standard GP. We assume that the goals and the right-hand sides of the constraints are uncertain variables with a normal distribution (μ_i, μ_k, σ_i^2 and σ_k^2 are known): $\tilde{g}_i \in N(\mu_i, \sigma_i^2)$ and $\tilde{b}_k \in N(\mu_k, \sigma_k^2)$. Therefore, we have: $P\left(\tilde{g}_i = \sum_{j=1}^{n} c_{ij}x_j\right)$ are equivalent to

$$P\left[\left((\tilde{g}_i - \mu_i)/\sigma_i\right) = \left(\left(\sum_{j=1}^{n} c_{ij}x_j - \mu_i\right)/\sigma_i\right)\right], \text{ where } \left((\tilde{g}_i - \mu_i)/\sigma_i\right) \in N(0,1)$$

$(\mathrm{i}=1,\ldots,\mathrm{p})$ and $P\left(\tilde{b}_k \geq \sum_{j=1}^{n} a_{kj}x_j\right)$ are equivalent to

$$P\left[\left(\left(\tilde{b}_k-\mu_k\right)/\sigma_k\right) \geq \left(\left(\sum_{j=1}^{n} a_{kj}x_j-\mu_k\right)/\sigma_k\right)\right] \quad (k=1,\ldots,m_1), \text{ where } \left(\left(\tilde{b}_k-\mu_k\right)/\sigma_k\right) \in N(0,1).$$

By introducing the satisfaction functions, the goal programming model in stochastic environment can be formulated as follows:

$$Maximize \ \sum_{i=1}^{p} w_i\left(F_i^+\left(\delta_i^+\right)+F_i^-\left(\delta_i^-\right)\right)+\sum_{k=1}^{m_1} w_k F_k^+\left(\gamma_k^+\right).$$

Subject to:

Goal constraints

$$\sum_{j=1}^{n} c_{ij}x_j-\delta_i^+ +\delta_i^- = \mu_i \qquad (i=1,\ldots,p),$$

$$\sum_{j=1}^{n} a_{kj}x_j-\gamma_k^+ +\gamma_k^- = \mu_k \qquad (k=1,\ldots,m_1).$$

System constraints

$$\sum_{j=1}^{n} a_{kj}x_j \leq b_k \qquad (k=m_1+1,\ldots,m),$$

$$x_j \geq 0 \qquad (j=1,\ldots,n),$$

$$\delta_i^+ \text{ and } \delta_i^- \leq \alpha_{iv},\ \gamma_k^+ \text{ and } \gamma_k^- \leq \alpha_{kv},$$

$$\delta_i^+, \delta_i^-, \gamma_k^+, \gamma_k^- \geq 0.$$

where:

δ_i^+, γ_k^+ indicate the over achievement of the goals $\tilde{g}_i$ and $\tilde{b}_k$ and δ_i^-, γ_k^- indicates the under achievement of these goals. α_{iv} and α_{kv} represent the veto thresholds.

The formulation proposed seeks not only to determine a solution that the probabilities $\tilde{g}_i \in (\mu_i-\varepsilon_i, \mu_i+\varepsilon_i)$ and $\tilde{b}_k \in (\mu_k+\varepsilon_k)$ are maximized (where ε_i and ε_k are very small positive numbers), but also to take into account the manager's preferences regarding the deviations from the target values of each objective. The threshold values of the satisfaction functions depend on the manager's appreciation of the deviations σ_i^2 and σ_k^2 . The indifference thresholds for each goal are greater than or equal to ε_i and ε_k.

4. APP model formulation

We illustrate the stochastic goal programming model proposed for an aggregate production planning problem where the goals associated with the objectives and market demands for each period of the planning horizon are uncertain and normally distributed. The objective functions of this decision problem are to minimize the total production cost, the changes in workforce level and the total inventory and backorder cost.

In the following, the parameters and the variables for the model are defined. Mathematical formulation of the model proposed, including various goal constraints related to the respective goals, system constraints, and the achievement function are also described.

4.1. Notations

Parameters and constants

T: Planning horizon or number of periods;
CP_t: Production cost per unit of regular time in period t;
CO_t: Production cost per unit of overtime in period t;
CR_t: Labor cost in period t;
CI_t^+ : Inventory cost per unit in period t;
CI_t^-: Backorder cost per unit in period t;
CH_t: Cost to hire one worker in period t;
CF_t: Cost to lay off one worker in period t;
$\widetilde{D}_t$: Forecasted demand in period t;
i_t : Labor time in period t (man hour/unit);
a: Regular working hours per worker;
b_t: Fraction of working hours available for overtime production.

Decision variables

P_t: Regular time production in period t;
O_t: Overtime production in period t;
W_t: Workforce level in period t;
I_t^+: Inventory level at the beginning of period t;
I_t^-: Backorder level at the beginning of period t;
H_t: Number of workers hired in period t;
F_t: Number of workers laid off in period t.

4.2. Goal constraints and objective functions

Goal 1: *Total production cost goal.*
The total production cost goal constraint is illustrated below; it takes into account regular time production costs, overtime production costs, and the labor cost at regular time.

$$\sum_{t=1}^{T}\left(CP_t.P_t + CO_t.O_t + CR_t.W_t\right) - \delta_1^+ + \delta_1^- = \mu_{g_1}.$$

Parameter μ_{g_1} denotes the mean production cost. A positive deviational variable δ_1^+, represents the over achievement of the goal $\tilde{g}_1$ and a negative deviational variable δ_1^-, represents the under achievement of this goal. This gives $\delta_1^+.\delta_1^- = 0$.

Goal 2: *The changes in workforce level goal.*
This objective includes the hiring cost and the lay-off cost. The goal constraint is formulated below.

$$\sum_{t=1}^{T}\left(CH_t.H_t + CF_t.F_t\right) - \delta_2^+ + \delta_2^- = \mu_{g_2}.$$

Parameter μ_{g_2} denotes the mean change in workforce level. A positive deviational variable δ_2^+, represents the over achievement of the goal $\tilde{g}_2$ and a negative deviational variable δ_2^-, represents the under achievement of this goal. This gives $\delta_2^+.\delta_2^- = 0$.

Goal 3: *Total inventory and backorder cost goal.*

This goal includes two components: the inventory carrying cost and the backorder cost. The goal constraint is formulated below.

$$\sum_{t=1}^{T}\left(CI_t^+ . I_t^+ + CI_t^- . I_t^-\right) - \delta_3^+ + \delta_3^- = \mu_{g_3}.$$

Parameter μ_{g_3} denotes the mean cost of the total inventory and backorder. A positive deviational variable δ_3^+, represents the over achievement of the goal $\tilde{g}_3$ and a negative deviational variable δ_3^-, represents the under achievement of this goal. This gives $\delta_3^+ . \delta_3^- = 0$.

Goal 4: *Demand goal.*

The demand goal constraint is illustrated as follows: the sum of regular and overtime production, inventory level, and backorder level should equal approximately the market demand.

$$I_{t-1}^+ - I_{t-1}^- + P_t + O_t - I_t^+ + I_t^- - \rho_t^+ + \rho_t^- = \mu_{g4} \qquad (t = 1, \dots, T).$$

Parameter μ_{g_4} denotes the mean demand. A positive deviational variable ρ_t^+, represents the over achievement of the goal $\tilde{g}_4$ and a negative deviational variable ρ_t^-, represents the under achievement of this goal. This gives $\rho_t^+ . \rho_t^- = 0 \ \ (t = 1, \dots, T)$.

By introducing the satisfaction functions, the multi-objective aggregate production planning problem in stochastic environment is formulated as follows:

$$Maximize \quad Z = \sum_{i=1}^{3} w_i F_i^+\left(\delta_i^+\right) + \sum_{t=1}^{T}\left(F_t^+\left(\rho_t^+\right) + F_t^-\left(\rho_t^-\right)\right).$$

4.3. System constraints

$$\mathrm{W_t = W_{t-1} + H_t - F_t} \qquad (t = 1, \dots, T), \tag{1}$$

$$\mathrm{i_t P_t \le a W_t} \qquad (t = 1, \dots, T), \tag{2}$$

$$\mathrm{i_t O_t \le a\, b_t W_t} \qquad (t = 1, \dots, T), \tag{3}$$

$$\delta_i^+ \text{ and } \delta_i^- \leq \alpha_{iv} \qquad (i=1, \dots, 3), \tag{4}$$

$$\rho_t^+ \text{ and } \rho_t^- \leq \alpha_{tv} \qquad (t=1, \dots, T) \tag{5}$$

Constraints (1) ensure that the available workforce in any period equals workforce in the previous period plus the change of workforce in the current period (hiring minus firing). Constraints (2) ensure that the labor times for manufacturing the products during regular time should be limited to the available regular time workforce. Constraints (3) limit the fraction of workforce available for overtime production. Finally, the two kinds of deviations should not exceed the veto threshold (4) and (5).

5. Computational results

In this section, the same data set as presented by Gen et al. [1992] is used to illustrate the proposed stochastic goal programming model for the aggregate production planning problem. A six period's planning horizon with probabilistic demands is considered. The market demands for the five last years (N-1 to N-5) are presented in Table 1. Our objective is to generate a production plan for year N where the goals values are random and where the decision-maker's preferences are explicitly integrated. Table 2 shows the random goals. Table 3 shows the different costs (production, inventory, backorder, labor, hiring and firing). The number of labor hours needed for each unit of production is three and the regular work day is eight man-hour per day. The initial workforce is 100 workers (man-day). The initial inventory and backorder are nil (equal to zero). Overtime production is limited to no more than 14% of regular time production.

Table 1

Market demands per year

	1	**2**	**3**	**4**	**5**	**6**
N-1	190	173	250	200	255	*310*
N-2	250	156	288	240	300	*270*
N-3	196	232	310	280	210	*210*
N-4	240	168	344	190	284	*216*
N-5	220	220	309	240	350	*280*
Mean values	***220***	***190***	***300***	***230***	***280***	***257***

Table 2

Goals per year

	Total production cost	The changes in workforce level cost	*Total inventory and backorder cost*
N-1	57 715	40	*16*
N-2	49 400	70	*8*
N-3	60 000	140	*0*
N-4	61 885	110	*8*
N-5	56 000	30	*8*
Mean values	***57 000***	***78***	***8***

Table 3

Production costs

Production cost other than labor cost	\$16 per unit
Labor cost at regular time	\$60 per worker
Hiring cost	\$30 per worker
Firing cost	\$40 per worker
Inventory carrying cost	\$2 per unit
Backorder cost	\$10 per unit
Overtime production cost	\$49 per unit

For the above three objectives we have used the satisfaction function of type III where the target's mean and the thresholds are summarized in Table 4. For the market demands objective, we use a satisfaction function of type II and the thresholds for the positive and negative deviations are the same during the planning horizon $\left(\alpha_{td}^{+} = \alpha_{td}^{-} = 20 \text{ and } \alpha_{tv}^{+} = \alpha_{tv}^{-} = 30\right)$. The shape of these functions is as follows (Figure 2).

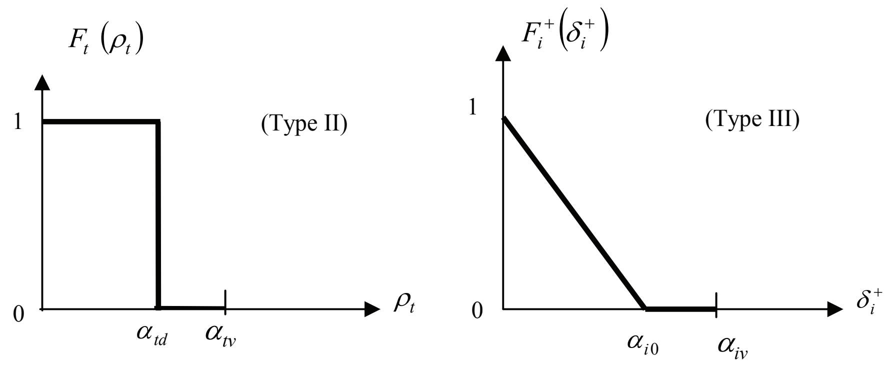

Figure 2. Shape of satisfaction function

The relative weights associated with the four goals are equal.

Table 4

Set of targets and satisfaction thresholds

Objectives	Target's mean μ_i	Nil-satisfaction threshold	Veto threshold
Total production cost	57 000	3000	4000
The changes in workforce level cost	78	30	40
Total inventory and backorder cost	8	16	20

The satisfaction functions are used to explicitly incorporate the manager's preferences in a stochastic environment. The equivalent representation of the various satisfaction functions requires the introduction of binary variables. The obtained model is non linear. The linearization procedure developed by Oral and Kettani [1992] and modified by Aouni [1996] is used to generate the linear equivalent formulation of the stochastic APP problem. The software package Lindo 6.1 is used to solve the mathematical programming problem.

Using the above data, the aggregate production plan is performed and the results are given in Table 5. The satisfaction level of the objective function is 98%. In fact, the achievement levels of the objectives are: total production cost is \$57 223, the change in workforce level cost is \$80, the total inventory and backorder cost is \$24 and finally the values of market demand for the planning horizon are respectively: 200, 170, 280, 210, 260 and 237 units. Therefore, the goal values of market demand reached are within the indifference region. The regular time production for period 2 exceeds the market demand by 12 units. The production level reaches the peak during the third period.

Table 5

Production plan

Period	1	2	3	4	5	6
Regular time production	200	182	261	210	260	237
Overtime production	0	0	7	0	0	0
Inventory level	0	12	0	0	0	0
Backorder level	0	0	0	0	0	0
Workforce level	98	98	98	98	98	98
Workers hired	0	0	0	0	0	0
Workers layoff	2	0	0	0	0	0

Conclusion

In this paper, we have proposed a stochastic goal programming model for solving an aggregate planning problem where the concept of satisfaction function was used to integrate explicitly the manager's preferences.

The model proposed has been illustrated through a hypothetical example of aggregate production planning problem. This model can be applied to large--scale production planning. Moreover, the model proposed can be easily based on information technology tools.

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MULTI-CRITERIA DECISION AIDING IN PROJECT PLANNING USING DECISION TREES AND SIMULATION

Abstract

A good plan is fundamental for a project's success. Inaccuracies in planning are reported to be among the main reasons of a project's fiasco. Planning means making a variety of decisions. As these decisions refer to the future, so when faced with them, the decision maker has also to face uncertainty. The selection of a new project or a group of projects, as well as decisions how to implement them, involve prediction and comparison of future outcomes. In real world, not every possible future outcome is known with certainty. Thus, decisions made during the project planning process are usually based on past experience, either rationally or intuitively with some degree of uncertainty, and thus are made under risk.

The aim of the paper is to present a simple, yet comprehensive, methodology for project planning that permits the consideration of both multiple criteria and risk. Our approach combines decision trees, simulation modelling and stochastic dominance rules. An example is presented to show the applicability of the procedure. It is based on the experiences of a company providing solutions for the railway industry.

Keywords

Project planning, decision tree, simulation, multiple criteria decision making.

Introduction

A good plan is fundamental for a project's success. Inaccuracies in planning are reported to be among the main reasons of a project's fiasco. The term "project planning" is not uniformly defined. Some authors suggest that planning is just scheduling – determining the dates for performing schedule activities and meeting schedule milestones. However, project planning is also more broadly understood, as a process that includes a number of phases and starts shortly after a business need, contract request, or request for proposal has been received [Nicholas and Steyn, 2008]. A Guide to the Project Management Body of Knowledge [PMBOK, Guide, 2004] defines a project management plan as follows: "A formal, approved document that defines how the project is executed, monitored and controlled". In this paper we focus on the initial phase of the planning process, when basic assumptions defining the project are made.

Planning means making a variety of decisions. As these decisions refer to the future, so when faced with them, the decision maker (DM) has also to face uncertainty. The selection of a new project or a group of projects, as well as decisions how to implement them, involve prediction and comparison of future outcomes. In real world, all possible future outcomes are not known with certainty. Project planning is usually based on past experience. Decisions within this phase of project life cycle are made either rationally or intuitively with some degree of uncertainty, and thus are made under risk.

Although financial analysis plays the key role in project planning, other criteria are also important. It is usually assumed that the purpose for project is to achieve an objective, that cannot be attained by standard operational work. However, the overall goal of the project is often expressed in general terms. A widely used statement says that the goal of a project is to hit a three-dimensional target: to complete the work in accordance with budget, schedule, and performance. As a result, project management problems can be considered as decision problems with multiple criteria. It should be also mentioned that projects are tools for achieving the organization's strategic plan. As profitability is not the only goal considered when the strategy is formulated, various criteria should be taken into account when various ways of project completion are compared.

The aim of this paper is to present a simple, yet comprehensive, methodology for project planning that permits the consideration of both multiple criteria and risk. Our approach combines decision trees, simulation modelling and stochastic dominance rules. The paper is organized as follows. Section 1 describes the project planning process and defines problems considered in this paper. Next section gives a literature overview. In section 3 new methodology for project planning decisions is introduced. Section 4 presents a numerical example. We finish with some conclusions and suggestions for future research in the last section.

1. Decision problems within planning phase of the project's life cycle

In this paper we focus on projects realized by manufacturing organizations by applying the so-called Project Management style of business management. In such companies, most business activity is focused on implementation projects with clearly defined goals and precisely specified due dates. A rough taxonomy of projects implemented by them involves: research and development, engineering, and service. The first group involves projects aimed at developing new products. The main feature of such projects is the lack of direct profit. Their implementation involves significant costs, resulting mainly from the salaries of engineers, designers and constructors.

The next group includes a wide rage of undertakings, from small modernization projects to large-scale ones with budgets of hundreds of millions of euro. The realization of such projects is often the main source of company's revenue and involves all the departments and divisions. Due to their complexity, projects of this type are often subdivided into three stages: the preliminary stage, a middle stage involving preparing and negotiating tenders, and a final implementation stage. Each phase, while significantly distinctive in scope from the others, is a part of the project as a whole. Engineering projects implementation is based on widely used project management methodologies and techniques.

The last group consists of service projects. As with engineering projects, their realization is based on classical project management approaches. They can be divided in two main groups:

- modernization projects involving replacement and upgrades of existing contractor's infrastructure,
- repair projects implemented mainly in manufacturing companies and involving the removal of defects in products delivered to customers.

Although repair projects are a small percentage of all projects, their implementation has a significant contribution to the overall company image. Time and resource availability play an important role in such projects. Since for the company a repair project represents only costs, shortening the completion time is crucial.

Going back to engineering projects we should point out that various teams are usually responsible for the implementation of various project phases. The initial phase of the project focuses on the analysis of business opportunities for planned activities from the perspective of the company itself and its business partners (investor, suppliers). This stage begins from the receipt of the first information about the customer plans of the future investment. At this stage the tender conditions are not yet known, so the analysis is based only on the experience with similar projects completed previously. The bid team has to decide whether the company will be able to accomplish the project, and to specify the project configuration optimal both from financial and scheduling point of view. Various solutions are considered, taking into account production capacity of sub-contractors and suppliers. Inspections in the area, where investment is to be implemented, are often necessary to propose a spectrum of alternatives to the person (or the team) responsible for deciding whether to continue the project.

The next phase of project planning starts with obtaining detailed information about the investment or purchasing tender conditions and continues until the final tender is submitted. At this stage the organization focuses on gatheri ng offers from suppliers and analyzing availability of resources: project manager and team members, equipment, financing, etc. Before preparing a final offer, a preliminary schedule should be prepared. Project planning is completed as soon as the contract is signed. At this stage a detailed project schedule is prepared, taking into account the availability of resources.

In this study we focus only on the initial phase of the project planning process. Highly skilled staff is required to complete it successfully. To make good decisions, both experience from the previous contracts and the knowledge of customers needs and local conditions must be exploited. A knowledge base containing all the experience gathered by the organization while executing previous projects could be an advantage. Such information can be used for estimating probabilities of various states of nature taken into account in the analysis. Otherwise, the decision process must be based on experience and intuition of the project team members only.

In general, the problems analyzed within the first stage can be divided into three categories: strategic, technical, and organizational. Strategic perspective requires, for example, the decision whether the project should be implemented and if so, whether the organization should play the role of a general contractor, realize it in cooperation or be only a supplier of products.

Taking into account technical issues, a preliminary analysis should be carried out to answer the question whether the organization provides products that meet customer expectations. A positive answer to this question implies the need for further analysis in order to determine whether the products offered will be capable of working with the customer's existing infrastructure, or will require additional adaptation. At this phase the organization's production capacity should also be analyzed. The team preparing the offer must make sure that the organization will be able to produce and adapt all products on time. Otherwise, other options, such as the use of products offered by subcontractors have to be considered.

The last group of issues includes organizational problems. This refers primarily to the availability of organization's own staff, as well as the possibility of hiring external cooperators, such as legal support, consulting, design offices, etc.

Since such analysis takes time, it is necessary to permanently "keep track of the market" in order to identify opportunities for future projects as soon as possible. The effort in this phase of the project planning process often determines the success of the entire project.

2. Related work

Although the nature of problems that the DM faces in the initial phase of project planning differs from what he/she has to do while selecting a project, both issues considered and methods used to solve them are largely similar. In fact, by making preliminary decisions on the way in which the project should be executed, the DM refines the project and thus, to some extent, selects the project to be implemented. However, if the project selection process is essentially static, the decision-making process during the project planning phase is dynamic. An initial decision determines the alternatives, that can be taken into account in subsequent phases of the process.

Various techniques have been proposed for project planning and estimation. Considering research and development projects Doctor et al. (2001) point out that two approaches have been particularly useful in practice: decision tree and Option Pricing Theory. While the former has been around for a long time, the latter has only become of interest in the last two decades.

Decision trees have found wide use both in literature and also in industry [Magee, 1964; Raiffa, 1968; Thomas, 1972]. Hespos and Strassmann [1965] proposed stochastic decision-tree concept that permits the use of continuous probability density functions instead of the usual discrete ones. Heidenberger [1996] uses decision trees in a mixed integer linear programming model for dynamic project selection and funding problems. He extends a classical approach by adding a new node type that allows for continuous control of discrete branching probability distributions. Examples for the use of decision trees for project selection and resource allocation were also presented by Chiu and Gear [1979], Gear and Lockett [1973], Granot and Zuckerman [1991], Hess [1993], Stonebraker and Kirkwood [1997], Thomas [1985].

Risk environments often require better understanding of the possible range of outcomes. Simulation models try to solve this problem. They allow representation of real-world systems in greater detail than optimization models, at the expense of answering only what-if questions per simulation run. Various simulation approaches are proposed for project selection, resource allocation and other project planning problems. Two main approaches are used in simulation modeling: Monte Carlo simulation and systems simulation. The former uses probability distributions of all stochastic elements to calculate probability distribution of objective values. Such approach is used by Martino [1995], Souder and Mandakovic [1986]. Systems simulation models analyze sequences of events that occur over time. Thus, it is possible, for example, to study results and reactions in certain markets after a new product has been launched [Milling, 1996]. Fox and Baker [1985] combine both approaches and propose a model consisting of three components: a net-present value profitability module, a project generation module and a project portfolio selection submodel based on zero-one programming.

A variety of multi-criteria approaches are also proposed for project selection and planning, including techniques based on the utility function, methods based on the outranking relation, goal programming approaches and algorithms using stochastic dominance relation. Multiattribute utility analysis is used, for example, by Moselhi and Deb [1993], who treat uncertainty in a similar way to that used in PERT technique. In this procedure the total expected utility is calculated as the product of three matrices: utility matrix, objective matrix and scaling matrix. Wong et al. [2000] incorporate fuzzy analysis into multi-attribute utility theory. Their procedure uses stochastic dominance rules for ordering projects.

Outranking relation is used by Martel and D'Avignon [1982]. They consider a case study, where each project is evaluated by experts according to a set of criteria. These evaluations lead to distributive evaluation, i.e. to the calculation of the distribution of the anticipated performance of each project with respect to each attribute. The problem is solved by establishing a confidence index, which is based on probabilities that one project is as good as another.

A goal programming approach is also successively employed in project selection. This technique attempts to find a solution that is as close as possible to the goals specified by the decision maker. A goal programming concept is used, for example, by Santhanam and Kyprasis [1995], Lee and Kim [2000], de Oliveira et al. [2003].

When faced with the project selection and planning problems the decision maker has also to face uncertainty. Stochastic dominance rules are an efficient and flexible tool for comparing alternative solutions under uncertainty. Multicriteria techniques based on this approach are proposed by Nowak [2005, 2006].

3. Methodology

The methodology proposed here combines the decision tree, simulation, lexicographic approach and stochastic dominance rules. Nearly all decisions made during the planning phase of the project's life cycle are made sequentially. The choices made at the initial phase of the process determine the set of alternatives that can be considered at subsequent steps. The decision tree is an efficient tool to analyze such problems, as it makes possible to decompose the whole process into separate stages and analyze them sequentially. However, some disadvantages of this technique are mentioned. It is usually supposed that a crisp value representing the profit, loss or score is assigned to each end node. In many complex decision problems we are not able to quantify evaluations in such a form, unless the decision tree is significantly enlarged. The usefulness of the decision tree lies in its simple form, which is lost if the tree is increased. Instead of enlarging the tree, one can try to decompose the problem into sub-problems analyzed separately. In such a case, a probability distribution is assigned to each end node in the "master" decision tree. Such a distribution can be obtained by a detailed model constructed for the scenario represented by a particular end node. In our procedure we employ such approach. We use a simulation model to analyse each scenario. The results obtained from simulation runs are used for constructing probability distributions, which are assigned to end nodes of the decision tree.

Once the decision tree is constructed and scores are assigned to each end node, it is possible to identify the optimal solution. Two main principles are usually used for comparing alternative solutions in the decision tree: expected value maximization and expected utility maximization. The former is easy to employ, but ignores risk. The latter takes risk into consideration, but is difficult to implement because of the problems with the utility function estimation. In our approach we propose to apply a combined approach that uses the expected utility maximization principle and employs simulation modelling for analysing the risk. Thus, simulation is used twice in our procedure: first, to evaluate scores in end nodes, second, to evaluate risk associated with the implementation of a particular solution.

As already mentioned, various criteria are usually considered during the project planning process. In this paper we analyze a two-criteria problem, taking into account profit margin and completion time. However, our procedure can also be used for more complex problems. We employ a lexicographic approach. First, the most important criterion (profit margin) is optimized. The best solutions with respect to this criterion are identified. In the second phase, the less important goal (completion time minimization) is taken into account.

The procedure consists of five main steps:

1. Defining the decision problem and constructing the decision tree.
2. Performing simulations to assess distributional evaluations of criteria assigned to decision tree end nodes.
3. Identifying decision strategies to be considered.
4. Performing simulations on the decision tree.
5. Solving the multi-criteria problem using lexicographic approach and stochastic dominance rules.

Details for each step of the procedure are provided below.

Step 1: Defining the decision problem and constructing the decision tree.

The initial phase of the procedure focuses on problem definition. In order to describe the DM's situation properly, we should specify decision points: the choices that should be made and the decision alternatives that can be selected. These decision points should be arranged in a logical sequence, as choices made in the initial phase of the decision process determine alternatives that can be considered at subsequent steps. The events that are not under the DM's control (states of nature) should also be identified. Finally, probabilities should be assigned to each state of nature.

The estimation of these probabilities is probably the most difficult part of the work. Usually two sources of data are suggested: historical and experts' assessments. Real-world organizations usually do not collect a sufficient amount of data required by formal probability estimation techniques. Moreover, the DM often has to solve a problem for which historical data are not available. As a result, subjective feelings have to be translated into quantitative estimates. The shortcomings of subjective assessments are often pointed out. People usually overestimate the probability of a rare event, while underestimating the probability of a frequent one [Fischhoff, De Bruin, 1999]. Nevertheless, Teale et al. [2003] argue that „[…] it is better to have imperfect information than perfect 'misinformation' because a fateful event with severe consequences is one in which we may be particularly reluctant to commit ourselves to a value". In order to assess the probability of a particular state of nature, it can be helpful to use a probability scale from 0 to 100. Obviously, if the problem is important enough the organization may try to assess additional information in order to gain more precise estimations of probabilities. In this paper, however, we do not consider this issue.

Step 2: Performing simulations to assess the distributional evaluations of criteria assigned to end nodes of the decision tree.

Once the decision tree is constructed, payoffs or losses should be assigned to end nodes. In classical approach results are represented by crisp values. It is assumed, that all risks are represented by state-of-nature nodes. Real problems, however, are usually much more complex. As a variety of risks has to be taken into account, it is not convenient to present all of them on the decision tree. Moreover, in the decision tree we are able to present only those risks for which a finite and relatively small number of possible states of nature are identified.

In this paper we assume that the decision tree represents only the general scheme of the problem. Each end node corresponds to one possible scenario, which should be analyzed in details. Here we assume that simulation modeling is used to analyze such scenarios. Another possibility is to construct additional decision subtrees for each end node.

To estimate distributional evaluations with respect to the criteria the following steps should be performed:

a) analyzing sources of risks,
b) identifying appropriate probability distributions for input data,
c) constructing simulation models,
d) performing simulation runs.

A spreadsheet model can be used for evaluating a particular scenario with respect to the criterion "profit margin". In such a case additional tools, like CrystalBall or @Risk, can be used to perform simulations. As with the construction of the decision tree, also in this case, estimating probability distributions is the most difficult step. Three types of data can be used for this task [Robinson, 2004]:

– category A: data that are available because they are known or they have been collected earlier,
– category B: data that need to be collected,
– category C: data that are not available and cannot be collected.

In the absence of data, approximate distributions not based on strong theoretical underpinnings are used. Among them the uniform distribution and triangle distributions are used most often.

Once the simulation model is built, verified, and validated, simulation runs can be performed. The results are used for constructing the distributional evaluation assigned to a particular end node.

Step 3: Identifying decision strategies to be considered.

In this step we focus on identifying the decision strategies that can be implemented by the DM. By a strategy we mean a rule that is followed by the decision maker, when he/she has to make a decision at any stage of the decision process. As in this study we analyze only small-scale problems with up to ten decision nodes, the identification of all possible strategies is quite easy. However, if the decision tree is large, it would not be feasible to analyze all of them. In such a case we suggest identifying the subset of the strategies that provide the best evaluations with respect to the most important criterion. They can be identified using the expected value optimization rule. If, for example, profit margin is considered to be the most important criterion, strategies with the highest expected profit should be identified. While finding the strategy that optimizes this value is quite easy, the identification of sub-optimal solutions is not trivial and requires a special procedure. However, we do not analyze that problem in this study.

Step 4: Performing simulations on the decision tree.

During the next phase of our procedure simulations are performed to analyze how risky the strategies identified at the previous step are. For each strategy a series of simulation runs is performed. In each run sampling methods are used to determine the path through the tree and to generate the values of the criteria at the end node taking into account distributions generated in step 2. The simulation procedure is presented in Figure 1.

As a result, for each strategy and for each criterion a series of observations is obtained. These data are used to generate probability distributions expressing how good the strategy with respect to each criterion is.

Step 5: Solving the multi-criteria problem.

In our approach stochastic dominance rules are used for comparing uncertain outcomes. This concept is based on the axioms of the utility theory, but does not require estimating the utility function. Instead, probability distributions are compared by pointwise comparison of some performance functions. In this study we assume that the DM is risk-averse. In such a case two types of stochastic dominance relations can be used for modeling DM's preferences: First Stochastic Dominance (FSD) and Second Stochastic Dominance (SSD).

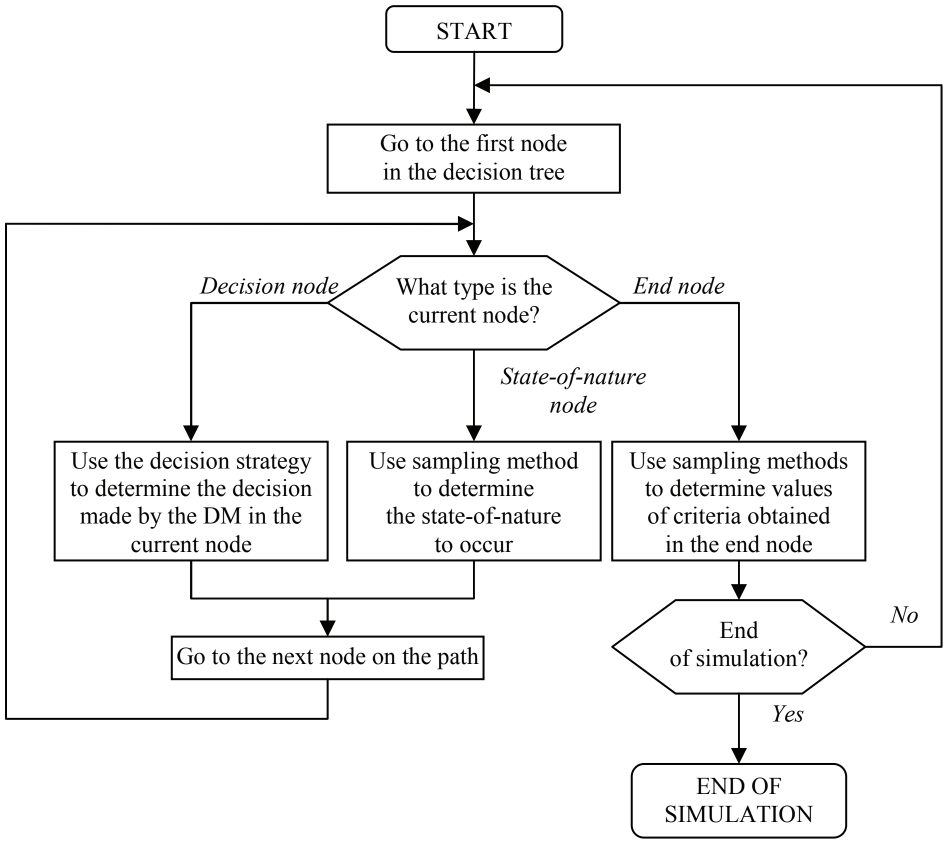

Figure 1. Simulation procedure

Let us assume the following notation:

$\mathbf{A} = \{a_1, a_2, \ldots, a_m\}$ – the set of strategies under consideration,

m – number of strategies,

n – number of criteria,

X_{ik} – evaluation of strategy a_i with respect to k-th criterion,

$\mathbf{A}^{(l)}$ – the set of strategies considered at the l-th step of the multi-criteria procedure.

We will assume that criteria are defined so that larger values are preferred to smaller ones. Let $F_{ik}(x)$ and $F_{jk}(x)$ be right-continuous cumulative distribution functions representing evaluations of ai and aj respectively over criterion X_k:

$$F_{ik}(x) = \Pr(X_{ik} \le x),$$
$$F_{jk}(x) = \Pr(X_{jk} \le x).$$

The definitions of the first and second degree stochastic dominance relations are as follows:

Definition 1. (FSD – First Degree Stochastic Dominance)

X_{ik} dominates X_{jk} by FSD rule ($X_{ik} \succ_{\text{FSD}} X_{jk}$) if and only if $F_{ik}(x) \neq F_{jk}(x)$ and $H_1(x) = F_{ik}(x) - F_{jk}(x) \le 0$ for $x \in R$.

Definition 2. (SSD – Second Degree Stochastic Dominance)

F_{ik} dominates F_{jk} by SSD rule (F_{ik} SSD F_{jk}) if and only if

$F_{ik}(x) \neq F_{jk}(x)$ and $H_2(x) = \int_{-\infty}^{x} H_1(y)dy \le 0$ for $x \in R$.

Hadar and Russel [1969] show that the FSD rule is equivalent to the expected utility maximization rule for all decision makers preferring larger outcomes, while the SSD rule is equivalent to the expected utility maximization rule for risk-averse decision makers preferring larger outcomes.

The multi-criteria procedure is based on lexicographic approach. First, the DM is asked to define a strict hierarchy of criteria according to their importance. Next, strategies are compared using stochastic dominance rules starting from the most important criterion. For each criterion, strategies dominated according to FSD/SSD rules are identified and removed. Finally, when all criteria have been considered (or there is only one strategy to be taken into account), the results are presented to the DM. He/she is asked to make a final choice. However, if the DM is not able to do this, some additional procedure must be employed. An interactive procedure for discrete multi--criteria decision making problems under risk proposed in Nowak [2006] can be used to complete the analysis.

Let us assume that the criteria are numbered according to their importance: the most important is criterion no. 1, while the least important in criterion no. n. The procedure operates as follows:

1. Assume: l := 1, $\mathbf{A}^{(l)} := \mathbf{A}$.
2. For each pair (a_i, a_j), such that $a_i, a_j \in \mathbf{A}^{(l)}$, $a_i \neq a_j$ identify FSD/SSD relation with respect to l-th criterion.

3. Identify the set of nondominated strategies with respect to l-th criterion:

$$\mathbf{A}^{(l+1)} = \mathbf{A}^{(l)} \setminus \{a_j : a_j \in \mathbf{A}^{(l)} \wedge \underset{i}{\exists}(X_{ik} \succ_{\text{FSD}} X_{jk} \vee X_{ik} \succ_{\text{SSD}} X_{jk})\}$$

4. If $l < n$, assume $l := l + 1$, go to 2, otherwise go to 5.
5. Present the results to the DM and ask him/her to make a final choice.

The procedure presented here differs from the ones that are usually used for project planning problems. In previous studies a decision tree was used mainly for single criterion problems. Simulation techniques were also popular. However, these approaches were usually used for comparing no more than two or three alternatives. In a multi-criteria framework goal programming was often employed. In such approaches, however, the risk was either ignored, or included in the model using some risk measures. In our approach we take into account both multiple criteria and risk. By using stochastic dominance rules we are able to take into account the DM's attitude to risk.

4. Illustrative example

The example presented in this section is based on the experience of the employees of the company providing solutions for the railway industry. The company is famous for the exceptional care it takes with regard to the safety of equipment and the range of services offered. Due to the specialised nature of its business, the execution of each project requires particular attention to detail and care both in preparation and implementation phase.

As a part of a global corporation, the company adopted standards according to which each project is divided into a number of crucial steps. This study focuses only on the first stage, i.e. project planning. Besides this stage, the company breaks down the project life cycle into 3 additional steps: tender preparation, project's implementation, and warranty coverage. The project life cycle is presented in Figure 2.

For each project project groups are formed responsible for preparing the documents required. At the end of each phase the documentation of project implementation strategy – white book, blue book, orange book, or red book – is presented to the management body responsible for deciding whether the project should be continued. Although the preparation of this data is time consuming and laborious, it makes possible to rationalize the decision process, as well as eliminate various weaknesses in the offer.

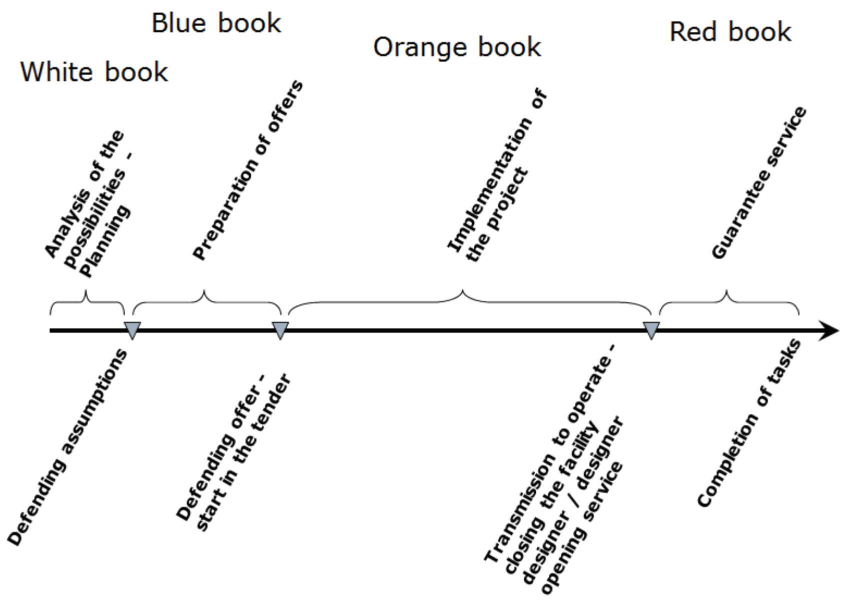

Figure 2. Project life cycle phases

In this study we consider both technical and organizational problems that have to be solved within the project planning phase. They consist mainly in assessing the potential use of company resources and experience to estimate the number of essential project elements. At this stage the team should verify that the company will be able to implement the project having won the tender. The most important factors determining the implementation of the planned tasks include: accessibility of the resources required for effective project management (project and construction managers, experienced contract engineers and contractors), production capacity adequate to produce the equipment required, the availability of the technology suitable for satisfying investor's needs. Knowledge of the local market and local circumstances is also very important. Combined, all these factors affect the decision regarding preparation and submitting a bid for the investor.

This example describes how the procedure proposed in the paper can be used when the decision on the tender preparation is made. The company considers entering a new market. It is possible to operate as a general contractor or to cooperate with a local company. Two criteria are considered: profit margin and project completion time. The final decision should specify whether the company should prepare and submit the tender or give up the contract.

Step 1: Defining the decision problem and constructing the decision tree.

The decision process involves several steps. Initially, the DM is faced with the choice between executing the project as a general contractor, or collaborating with a local company. The latter option leads to the necessity to search for a cooperator. Such a search, however, may be unsuccessful. In this case the company can either try to carry out the contract alone or to abandon it. On the other hand, if the cooperator is found, it can be employed as a supplier of some part of equipment, or hired for completing the installation work only. Figure 3 exemplifies this decision making process in the form of a decision tree.

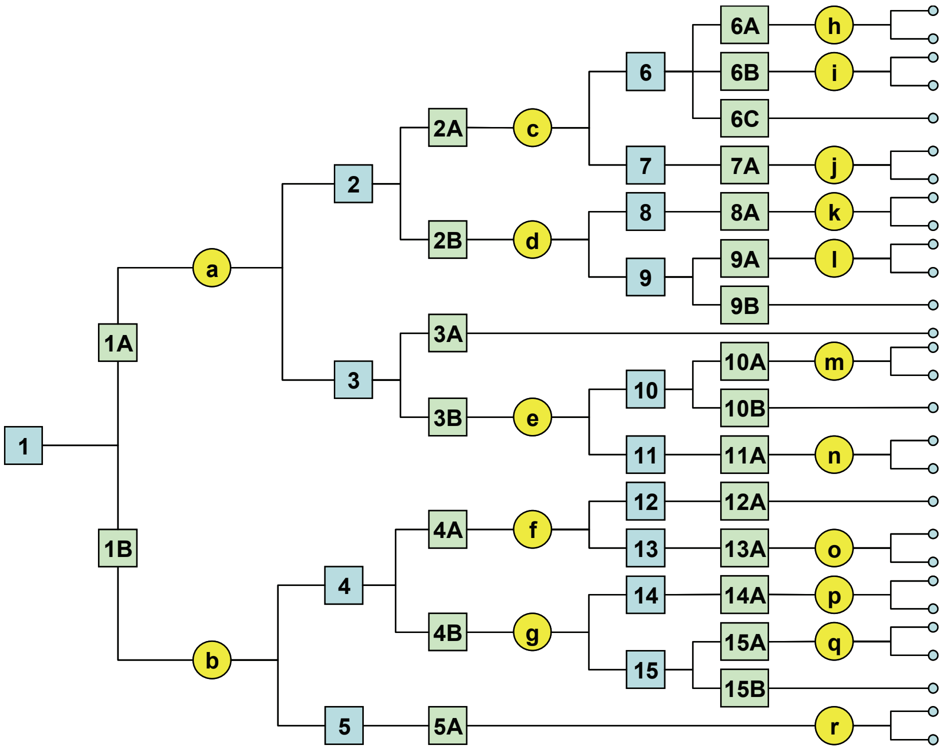

Figure 3. The decision tree describing the decision-making process

The details of the decision-making process under consideration are given below. At the first stage (decision node 1) the choice between two options must be made:

– implementation of the project in collaboration with a local company (decision 1A),
– implementation of the project as a general contractor (decision 1B).

The first option leads to the state-of-nature node a, in which two states of nature can arise:

– the company finds a local representative for cooperation (state a1),
– the company is not able to find a cooperator (state a2).

If a1 arises, the decision process proceeds to the decision node 2, otherwise it proceeds to the decision node 3. The decision 1B leads to the state-of-nature node b, in which the following states of nature are considered:

– the company is facing technical and organizational problems during the tender preparation (state b1),
– the company is able to prepare the tender without too much trouble (state b2).

While the occurrence of b1 moves the decision-making process to the decision node 4, the occurrence of b2 moves it to node 5.

The decisions considered in node 2 are as follows:

– the collaborating company is employed as the supplier of some part of equipment (decision 2A),
– the collaborating company is employed for completing a part of installation work only (decision 2B).

If decision 2A is made, the process proceeds to state-of-nature node c, otherwise it proceeds to the node d. The following states of nature are considered in node c:

– problems with adaptation of devices supplied by the local cooperator occurred (state c1),
– no problems with adaptation are identified (state c2).

The occurrence of these states moves the decision-making process to decision nodes 6 and 7, respectively. The states of nature taken into account in node d are as follows:

– an agreement concerning the distribution of responsibilities has been reached, no problems arise from the implementation of the assigned tasks (state d1),
– an agreement concerning the distribution of responsibilities has been reached, there are problems arising from implementation of the assigned tasks (state d2).

If d1 occurs, the decision-making process goes to the node 8, otherwise to the node 9.

In the decision node 3 the DM can choose between two alternatives:

- to give up tender submission (decision 3A),
- to turn back to the original concept – the completion of the task as general contractor (decision 3B).

If the first option is chosen, the decision-making process is finished, otherwise it goes to the state-of-nature e, where two states are considered:

- the company is facing problems with the organisation of the project (state e1),
- the company is not facing any problems with the organisation of the project (state e2).

The occurrence of these two states leads to nodes 10 and 11, respectively.

The decision node 4 represents the situation when the DM has to choose between two alternatives:

- to hire a consulting firm to support project implementation (decision 4A),
- to turn back to the original concept – to establish cooperation with a local company (decision 4B).

If the first option is chosen, the process goes to state-of-nature node f, otherwise it is moved to node g. The former represents the possibility of the occurrence of two states:

- problems with implementation are not solved (state f1),
- with the help of the consulting firm problems are solved (state f2).

The occurrence of these states moves the decision-making process to nodes 12 and 13, respectively. In node g two possibilities are considered:

- cooperation with a local company makes it possible to solve problems (state g1),
- problems identified during tender preparation are not solved (state g2).

State g1 leads to node 14, while state g2 to node 15.

The last decision node that has to be considered at the second stage of the process is node 5. It represents the situation in which the company is able to prepare the tender without too much trouble. In such a case the decision to submit the tender is made.

The decisions made at the third stage are as follows:

1. Decision node 6:
   - deciding to complete the contract by using only devices produced by the company itself and submitting the tender (decision 6A),
   - deciding to propose adaptation works and submitting the tender (decision 6B),
   - giving up tender submission (decision 6C).

2. Decision node 7:
   – tender submission (decision 7A).
3. Decision node 8:
   – tender submission (decision 8A).
4. Decision node 9:
   – organizing additional training for the employees of the cooperator and submitting the tender (decision 9A),
   – giving up tender submission (decision 9B).
5. Decision node 10:
   – hiring a consulting company and submitting the tender (decision 10A),
   – giving up tender submission (decision 10B).
6. Decision node 11:
   – tender submission (decision 11A).
7. Decision node 12:
   – giving up tender submisision (decision 12A).
8. Decision node 13:
   – tender submission (decision 13A).
9. Decision node 14:
   – tender submission (decision 14A).
10. Decision node 15:
   – organizing additional training for the employees of the cooperator and submitting the tender (decision 15A).
   giving up tender submission (decision 15B).

The decision to submit the tender in each case leads to a state-of-nature node in which two states are considered: the company's offer is accepted or rejected. Finally, probabilities are assigned to each state of nature (Table 1).

Table 1

Probabilities assigned to each state of nature

State of nature node	State of nature	Probability	State of nature node	State of nature	Probability
a	a1	0.7	e	e1	0.4
	a2	0.3		e2	0.6
b	b1	0.6	f	f1	0.6
	b2	0.4		f2	0.4
c	c1	0.6	g	g1	0.3
	c2	0.4		g2	0.7
d	d1	0.6	h-r	h1 … r1	0.6
	d2	0.4		h2 … r2	0.4

Step 2: Performing simulations to assess distributional evaluations of criteria assigned to end nodes of the decision tree.

For each end node a calculation sheet and project network describing tasks required for completing the project are developed. At this stage precise data are not available. However, the expertise of team members and data contained in company's knowledge base can be used to estimate probability distributions of uncertain variables. To analyze the profit margin made on contract, not only project implementation cost, but also tender preparation cost are taken into account.

Spreadsheet models are constructed for performing simulations. The simulation results – means of distributions obtained for criteria – are presented in Table 2. Each scenario is defined by a sequence of decisions and states of nature. The completion time equal to 0 means that either the company decides to give up submitting the tender, or the offer is not accepted.

Table 2

Results of simulations performed for scenarios represented by end nodes

Scenario	Values of criteria (means)		Scenario	Values of criteria (means)	
	profit margin (PLN)	completion time (days)		profit margin (PLN)	completion time (days)
1A-a1-2A-c1-6A-h1	634.733	80	1A-a2-3B-e1-10A-m2	–46.400	0
1A-a1-2A-c1-6A-h2	–46.233	0	1A-a2-3B-e1-10B	–34.333	0
1A-a1-2A-c1-6B-i1	800.867	80	1A-a2-3B-e2-11A-n1	744.667	65
1A-a1-2A-c1-6B-i2	–34.500	0	1A-a2-3B-e2-11A-n2	–46.750	0
1A-a1-2A-c1-6C	–27.867	0	1B-b1-4A-f1-12A	–39.220	0
1A-a1-2A-c2-7A-j1	870.333	75	1B-b1-4A-f2-13A-o1	694.340	70
1A-a1-2A-c2-7A-j2	–34.783	0	1B-b1-4A-f2-13A-o2	–46.700	0
1A-a1-2B-d1-8A-k1	819.467	75	1B-b1-4B-g1-14A-p1	750.467	75
1A-a1-2B-d1-8A-k2	–34.730	0	1B-b1-4B-g1-14A-p2	–46.733	0
1A-a1-2B-d2-9A-l1	760.567	85	1B-b1-4B-g2-15A-q1	710.833	85
1A-a1-2B-d2-9A-l2	–46.467	0	1B-b1-4B-g2-15A-q2	–46.033	0
1A-a1-2B-d2-9B	–39.333	0	1B-b1-4B-g2-15B	–39.167	0
1A-a2-3A	–27.200	0	1B-b2-5A-r1	756.000	65
1A-a2-3B-e1-10A-m1	694.167	70	1B-b2-5A-r2	–34.221	0

Step 3: Identifying decision strategies to be considered.

At this step decision strategies are identified. As the example considered here is not very large, it is quite easy to list all decision strategies (Table 3).

Table 3

Decision strategies under consideration

Decision strategy		Decision strategy	
a_1	1A – 2A – 3A – 6A – 7A	a_{10}	1A – 2B – 3A – 8A – 9A
a_2	1A – 2A – 3A – 6B – 7A	a_{11}	1A – 2B – 3A – 8A – 9B
a_3	1A – 2A – 3A – 6C – 7A	a_{12}	1A – 2B – 3B – 8A – 9A – 10A – 11A
a_4	1A – 2A – 3B – 6A – 7A – 10A – 11A	a_{13}	1A – 2B – 3B – 8A – 9A – 10B – 11A
a_5	1A – 2A – 3B – 6A – 7A – 10B – 11A	a_{14}	1A – 2B – 3B – 8A – 9B – 10A – 11A
a_6	1A – 2A – 3B – 6B – 7A – 10A – 11A	a_{15}	1A – 2B – 3B – 8A – 9B – 10B – 11A
a_7	1A – 2A – 3B – 6B – 7A – 10B – 11A	a_{16}	1B – 4A – 5A – 12A – 13A
a_8	1A – 2A – 3B – 6C – 7A – 10A – 11A	a_{17}	1B – 4B – 5A – 14A – 15A
a_9	1A – 2A – 3B – 6C – 7A – 10B – 11A	a_{18}	1B – 4B – 5A – 14A – 15B

Step 4: Performing simulations on the decision tree.

The next step of the procedure involves performing simulation runs for each strategy identified in the decision tree. The procedure used for analyzing the profit margin differs from the one used for analyzing the completion time. When the profit margin is analyzed, all potential states of nature and decisions are taken into account, including the ones which result either in giving up tender submission or having the offer rejected. However, if completion time is analyzed, such procedure does not make sense, as we cannot take into account the scenarios that do not result in project implementation (giving up tender submission or having the offer rejected). Thus, only scenarios resulting in offer acceptation are taken into account while analyzing completion time.

The results of simulation runs were used for generating distributional evaluations of each solution with respect to both criteria. The summary of the results is presented in Table 4.

Table 4

Results of simulations performed on the decision tree

Decision strategy	Means of probability distributions		Decision strategy	Means of probability distributions	
	profit margin (PLN)	completion time (days)		profit margin (PLN)	completion time (days)
a_1	244.158	57.8	a_{10}	265.446	37.2
a_2	203.071	52.4	a_{11}	138.666	42.0
a_3	78.588	35.0	a_{12}	380.129	55.2
a_4	331.328	52.0	a_{13}	310.040	50.4
a_5	268.953	53.7	a_{14}	266.929	44.2
a_6	358.841	51.4	a_{15}	218.128	42.6
a_7	317.754	49.9	a_{16}	240.371	37.0
a_8	193.271	39.3	a_{17}	387.623	53.2
a_9	144.471	37.7	a_{18}	214.224	33.4

Step 5: Solving the multi-criteria problem.

The last step includes multi-criteria analysis of the problem. First, probability distributions of profit margin are compared according to FSD/SSD rules (Table 5).

Table 5

Stochastic dominance relations with respect to criterion "profit margin"

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}
a_1			FSD					FSD	FSD		FSD				FSD	FSD		FSD
a_2	FSD		FSD					FSD	FSD	FSD	FSD			FSD	FSD	FSD		FSD
a_3																		
a_4	FSD	FSD	FSD		FSD			FSD	FSD	FSD	FSD		FSD	FSD	FSD	FSD		FSD
a_5	FSD	FSD	FSD					FSD	FSD	FSD	FSD			FSD	FSD	FSD		FSD
a_6	FSD	FSD	FSD		FSD			FSD	FSD	FSD	FSD		FSD	FSD	FSD	FSD		FSD
a_7	FSD	FSD	FSD		FSD			FSD	FSD	FSD	FSD		SSD	FSD	FSD	FSD		FSD
a_8			FSD						FSD		FSD							SSD
a_9			FSD								FSD							
a_{10}	FSD		FSD					FSD	FSD		FSD				FSD	FSD		FSD
a_{11}			FSD															
a_{12}	FSD	FSD	FSD		FSD			FSD	FSD	FSD	FSD		FSD	FSD	FSD	FSD		FSD
a_{13}	FSD	FSD	FSD		FSD			FSD	FSD	FSD	FSD			FSD	FSD	FSD		FSD
a_{14}	FSD		FSD					FSD	FSD		FSD				FSD	FSD		FSD
a_{15}			FSD					FSD	FSD		FSD					SSD		FSD
a_{16}			FSD						FSD		FSD							FSD
a_{17}	FSD	FSD	FSD		FSD			FSD	FSD	FSD	FSD		FSD	FSD	FSD	FSD		FSD
a_{18}			FSD						FSD		FSD							

Five strategies are nondominated according to FSD/SSD rules with respect to profit margin criterion: a_4, a_6, a_7, a_{12}, and a_{17}. Thus, to identify the final solution, relationships between these alternatives with respect to the second criterion "completion time" should be analyzed (Table 6).

Table 6

Stochastic dominance relations with respect to criterion "completion time"

Decision strategy	Decision strategy				
	a_4	a_6	a_7	a_{12}	a_{17}
a_4				FSD	FSD
a_6				FSD	FSD
a_7				FSD	FSD
a_{12}					
a_{17}					

Three strategies are nondominated according to stochastic dominance rules with respect to the criterion "completion time": a_4, a_6, and a_7. These solutions are presented to the DM. The simulation results can be used to provide additional information, such as the probability of making a profit not smaller than a specified value or the probability of meeting the due date. In our case, the probability of making a profit not smaller than 400 000 PLN is equal to 0.63 for a_4, 0.78 for a_6, and 0.56 for a_7, while the probability of meeting the due date (65 days) is equal to 0.81 for both a_4 and a_6, and 0.89 for a_7. Thus, it seems that the DM should choose a_6 as the final solution. According to this, the company should first of all try to find a local partner. If it is successful, it will employ it as a supplier of some part of equipment. Next, if any problems with adaptation of devices supplied by the local cooperator arise, the company will perform adaptation works. However, if the search for a local partner is not successful, the company should return to the original concept – the completion of the task as general contractor.

Conclusions

Project planning involves making a series of decisions. As these decisions are made under risk, the decision tree seems to be an efficient tool. However, the results that are obtained in end nodes often cannot be expressed as crisp values. In such situations, computer simulation can be employed for analyzing results of various strategies.

In the example presented in this paper, a two-criteria problem was analyzed. Obviously profit and completion time are usually taken into account when various project implementation strategies are considered. Nevertheless, other issues are also taken into account, such as resources usage. Our procedure can be successfully used in those cases as well.

The example presented in this paper is relatively simple. The number of end nodes in our tree is not very large. Thus, we were able to analyze all alternative strategies identified in the tree. Such approach is applicable for small problems. In real-life situations the size of the problem is usually much larger. However, some segments of the tree are replicated. Moreover, such fragments can occur in various projects. Thus, when faced with a new problem, the DM can adapt some parts of decision trees constructed previously. Our idea for future work is to construct a "library" or "database" of tree segments which can be used while constructing a decision tree for a new problem. For each problem a "master tree" describing only the main idea of the problem will be constructed, and a subtree will be assigned to each end node. As these subtrees will be considered separately, the problem will become simpler. Additionally, the procedure should approximate the knowledge about results that can be achieved for each scenario. These data can be used at the initial phase of the procedure, for selecting the most promising solutions. Next, simulation should be used for in-depth analysis of selected solutions.

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AN APPROACH TO MODELING ALTRUISTIC EQUILIBRIUM IN GAMES

Abstract

We present an approach to modeling equilibrium in non-cooperative non-zero sum games taking into account player altruism. The altruistic preferences concern the relations between changes of the given player's and the other players' pay-offs. The degree of altruism is represented by the altruistic coefficient for each pair of players. We prove that any Pareto optimal strategy profile can be an equilibrium if the level of player altruism is high enough.

Keywords

Game theory, altruistic equilibrium, altruistic trade-off.

Introduction

In recent years one can observe a tendency toward enhancing the game--theoretical apparatus integrated into the economic theory. The "classical" game theory models are based on the assumption that a player aims only at increasing his/her own pay-off. Such models are unable to explain why cooperation emerges in the wide variety of prisoner's dilemma-like economic affairs in real life. Let us refer to the journalistic article by Paul Krugman[*] [2009], where he criticizes the current state of economical science in the context of the world economy crisis. He points out, among other methodological defects, that the view of individual behavior of economic agents is primitively rational. The cognitive and behavioral approaches to economics, in contrast, are considered to be new directions of research for tackling the complex behavior of economic agents in the context of their personality. Thus, new models explaining player behavior are needed.

[*] The winner of the Nobel Memorial Prize in Economic Sciences in 2008.

Since the initial models of player behavior are based on the assumption of absolute egoism, the attempts to enhance them imply introducing non--egoistic features into the player behavior. The idea of modeling altruistic behavior can be traced back to Edgeworth [1881] (as described in Collard [1975]). Edgeworth proposed to increase an individual's utility by a value proportional to the utility of another person. The most popular models of altruistic player behavior in games have the form of utility functions which depend not only on the player's pay-off, but also on pay-offs of the other players. For example, the utility function by Fehr and Schmidt [1999] includes negative terms as penalties for distributional unfairness. The function by Bolton and Ockenfels [2000] depends on the relation between the player's own and the average pay-offs. Charness and Rabin [2002] built their function assuming that the player is interested in increasing the minimal and the average pay-offs of the other players.

Our approach differs from those mentioned above. We consider the situation where a player chooses his/her strategy while the other players' strategies are fixed. We formulate a condition when the player prefers not to change his/her strategy. Thus, the proposed preference model is bound directly to the notion of equilibrium, and the existence of a utility function characterizing the player preferences is not required.

1. The definition of altruistic equilibrium

Consider a p-person, $p > 1$, non-cooperative non-zero sum game $(\mathbf{S},\mathbf{a})$, where

$\mathbf{S}=S_1\times S_2\times \dots \times S_p$ is the set of *strategy profiles*, $S_k := \{1,2, \dots , m_k\}$, $m_k > 1$, is the strategy set of k-th player, $k\in \mathrm{N}_p := \{1,2, \dots , p\}$;

$\mathbf{a}=(a^1,a^2, \dots , a^p)$: $\mathbf{S} \to \mathbf{R}^p$ is the vector of pay-off functions, a^k: $\mathbf{S} \to \mathbf{R}$ is the *pay-off function* of k-th player yielding pay-off $a^k(I)$ for each strategy profile $I \in \mathbf{S}$.

For any strategy profile $I=(i_1,i_2, \dots , i_p)$ and any player k, define another strategy profile which differs from I only by strategy of player k:

$$I_{\langle k,j\rangle}=(i'_1, i'_2, \dots , i'_p), \text{ where } i'_l=i_l \text{ for any } l\neq k \text{ and } i'_k=j, j\in S_k, j\neq i_k.$$

Definition 1. *Strategy profile I is a* ***Nash equilibrium*** *in game* $(\mathbf{S},\mathbf{a})$, *if*

$$a^k(I) \geq a^k(I_{\langle k,j\rangle}) \text{ for any } k \in N_p \text{ and any } j \in S_k.$$

Definition 2. *Strategy profile I is* ***Pareto optimal*** *in game $(\boldsymbol{S},\boldsymbol{a})$, if there does not exist any other strategy profile I' such that*

$$\boldsymbol{a}(I') \geq \boldsymbol{a}(I), \quad \boldsymbol{a}(I') \neq \boldsymbol{a}(I).$$

We propose the following assumption about the altruistic behavior of players:

each player evaluating one strategy versus another,
prefers not to gain in his/her pay-off, if this leads to
disproportionately large loss in pay-offs of other players.

To quantify this assumption, for each pair of players we introduce the *altruistic coefficient*. Denote two players by k and l, $k \neq l$, and denote the altruistic coefficient of player k with respect to player l by α_{kl}, $\alpha_{kl} \geq 0$. This coefficient applies in the following situation. Let player k evaluate one of his/her strategies, say i, over another his/her strategy, say j, under the assumption that the strategies of the other players are known. Let strategy i give player k a greater pay-off in comparison to j, but if player k chooses i over j, then player l loses in his/her pay-off. In these terms, the above assumption is reformulated as follows:

player ***k*** *does not prefer strategy* ***i*** *to strategy* ***j****,*
if the pay-off loss of player ***l*** *multiplied by α_{kl}*
is greater or equal to the pay-off gain of player ***k****.*

We define the matrix of altruistic coefficients $A = (\alpha_{kl})_{p\times p} \in \mathbf{R}_{>}^{p\times p}$, where $\alpha_{kk} := 1$, $k \in N_p$, and $\mathbf{R}_{>}^{p\times p}$ is the set of non-negative matrices with ones on the main diagonal.

Definition 3. *Strategy profile I is called* ***altruistic equilibrium*** *or* ***A-equilibrium****, if for any player $k \in N_p$ and any his/her strategy $j \in S_k$, $j \neq i_k$, the following implication holds:*

if $a^k(I_{\langle k,j\rangle}) > a^k(I)$, then for some player $l \in N_p$, $l \neq k$, it follows

$$\alpha_{kl}(a^l(I) - a^l(I_{\langle k,j\rangle})) \geq a^k(I_{\langle k,j\rangle}) - a^k(I). \tag{1}$$

Literally, A-equilibrium is a strategy profile such that no player wants to change his/her strategy for the following reason: if the player can gain in pay--off by changing his/her strategy, then this leads to pay-off loss of another player such that the absolute value of the pay-off loss multiplied by the corresponding altruistic coefficient is greater than or equal to the pay-off gain of the first player.

A player's altruistic behavior restricts the domains in which players act exclusively in their own interests. The greater a player's altruistic coefficient is, the more severe this restriction. On the other hand, if $\alpha_{kl} = 0$ then player k does not feel any altruism with respect to player l and acts as in an "ordinary" game. Indeed, for $\alpha_{kl} = 0$ the implication in Definition 3 takes the form:

$$\text{if } a^k(I_{\langle k,j\rangle}) > a^k(I) \text{ then } a^k(I_{\langle k,j\rangle}) - a^k(I) \leq 0$$

which holds true if and only if $a^k(I_{\langle k,j\rangle}) \leq a^k(I)$. It follows that the definition of A-equilibrium is equivalent to the definition of Nash equilibrium, if all the altruistic coefficients are equal to zero.

Let us compare our concept of altruistic equilibrium to the concept implied by the Edgeworth's [1881] proposition on altruism (see Collard [1975]). Under the Edgeworth's assumption that the player's pay-off is increased by a value proportional to the pay-off of the other player, the equilibrium definition in a two player game takes the following form:

Strategy profile I is an Edgeworth A-equilibrium in game $(\boldsymbol{S},\boldsymbol{a})$
if and only if
$$a^1(I)-a^1(I_{\langle 1,j\rangle}) \geq \alpha^{12}(a^2(I_{\langle k,j\rangle})-a^2(I)) \text{ and}$$
$$a^2(I)-a^2(I_{\langle 1,j\rangle}) \geq \alpha^{21}(a^1(I_{\langle k,j\rangle})-a^1(I)) \text{ for any } j \in S_k.$$

Our Definition 3 has the following form in the case of two players:

Strategy profile I is an A-equilibrium in game $(\boldsymbol{S},\boldsymbol{a})$
if and only if
$$a^1(I) - a^1(I_{\langle 1,j\rangle}) \geq \min\{0,\ \alpha^{12}(a^2(I_{\langle k,j\rangle}) - a^2(I))\} \text{ and}$$
$$a^2(I) - a^2(I_{\langle 1,j\rangle}) \geq \min\{0,\ \alpha^{21}(a^1(I_{\langle k,j\rangle}) - a^1(I))\} \text{ for any } j \in S_k.$$

The main difference is that the approach based on the Edgeworth's proposition may lead to a situation where a player sacrifices a small part of his/her pay-off to the benefit of another player. Our approach does not imply such a possibility.

Note that in the general case, the Edgeworth's A-equilibrium in game (**S**,**a**) is equivalent to the Nash equilibrium in game (**S**,**a**′) with the linearly transformed pay-off function: $\mathbf{a}'(I) = A\,\mathbf{a}(I)$ for any $I \in S$.

2. Altruism and cooperation

One important consequence of altruistic behavior is that it may lead to cooperation among players. We will prove that if the players are altruistic enough, then there exists an altruistic equilibrium which is efficient (Pareto optimal).

Definition 4. We call strategy profile I **locally efficient**, if there does not exist $k \in p$, $j \in S_k \setminus \{i_k\}$ such that

$$a^k(I_{\langle k,j\rangle}) > a^k(I) \text{ and } a^l(I_{\langle k,j\rangle}) \geq a^l(I) \text{ for any } l \in N_p \setminus \{k\}.$$

In other words, I is locally efficient if it is not "dominated" by any "neighbor" strategy profile $I_{\langle k,j\rangle}$. Here "domination" differs from the Pareto domination relation by the requirement $a^k(I_{\langle k,j\rangle}) > a^k(I)$, and "neighborhood" of strategy profiles is understood as difference in only one player's strategy.

Theorem 1. *Let $I \in \mathbf{S}$. There exists $A = (\alpha_{kl}) \in \mathbf{R}_{>}^{p\times p}$ such that I is an A-equilibrium if and only if I is locally efficient.*

Proof. Sufficiency. Suppose that I is locally efficient. Then for any player k such that:

$$a^k(I_{\langle k,j\rangle}) > a^k(I) \text{ for some strategy } j \in S_k,\ j \neq i_k,$$

there exists another player l such that

$$a^l(I_{\langle k,j\rangle}) < a^l(I).$$

If α_{lk} satisfies

$$\alpha_{lk} \geq \frac{a^k\left(I_{\langle k,j\rangle}\right) - a^k(I)}{a^l(I) - a^l\left(I_{\langle k,j\rangle}\right)},$$

then we have (1). It follows that I is an A-equilibrium, if the altruistic coefficients are large enough.

Necessity. If I is not locally efficient, then for some $k \in p$ and some $j \in S_k \setminus \{i_k\}$ we have

$$a^k(I_{\langle k,j \rangle}) > a^k(I) \text{ and } a^l(I_{\langle k,j \rangle}) \geq a^l(I) \text{ for any } l \in N_p \setminus \{k\}.$$

It follows that there does not exist $l \in N_p$, $l \neq k$, and positive α_{lk} satisfying (1). Therefore I is not an A-equilibrium for any altruistic coefficients. □

It is evident that any Pareto optimal strategy profile is locally efficient. Therefore Theorem 1 implies:

Corollary 1. *For any Pareto optimal strategy profile $I \in \mathbf{S}$ in game ($\mathbf{S}$, $\mathbf{a}$), there exists $A \in \mathbf{R}_{>}^{p \times p}$ such that I is an A-equilibrium.*

Actually, Corollary 1 is a stronger proposition than the existence of an altruistic equilibrium being Pareto optimal. We have proved that **any** Pareto optimal strategy profile can be an altruistic equilibrium, if the altruistic coefficients of players are large enough. Observe that the existence of a Nash equilibrium in the game is not required.

Let us illustrate the altruistic equilibrium concept by the example of the prisoner's dilemma game. The classical interpretation of the game is that both players are suspected in a crime they committed together. They are separated from each other and interrogated simultaneously. Each of them have to decide either to betray the partner or to stay silent. The absolute values of pay-offs indicate how many years of imprisonment will a player get depending on both players' decisions.

Example 1. *Denote the players by Player A and Player B. The pay-off matrix is following:*

	Player A stays silent	***Player A betrays***
Player B stays silent	*(–0.5, –0.5)*	*(0, –10)*
Player B betrays	*(–10, 0)*	*(–2, –2)*

Here the two numbers in parentheses denote Player A's and Player B's pay-offs, respectively.

The paradox is that the cooperative solution (stay silent, stay silent) is not an equilibrium (in the sense of Nash), if the players behave rationally. On the contrary, the unique equilibrium is (betray, betray) which yields a non-efficient outcome.

Now suppose that both altruistic coefficients are equal to 0.1.

Consider the situation where Player A stays silent. If Player B had betrayed instead of staying silent, he/she would condemn Player A to additional 9.5 years of imprisonment while avoiding only 0.5 year imprisonment for him/herself. The pay-off loss of Player A multiplied by the altruistic coefficient is greater than the pay-off gain of Player B ($9.5 \cdot 0.1 > 0.5$). Then according to our assumption, Player B prefers to stay silent. Analogously, if Player B stays silent, then Player A prefers to stay silent too. Thus, (stay silent, stay silent) is an equilibrium in the sense that no player deviates from his/her strategy if the partner does not.

3. Characterization of strategy profiles in terms of altruistic equilibrium

According to Theorem 1, any (and only such) locally efficient strategy profile can be an altruistic equilibrium for sufficiently large altruistic coefficients. The following question arises: *for what values of altruistic coefficients a given strategy profile is an altruistic equilibrium?*. Answering this question means characterizing a locally efficient strategy profile I by a set of matrices $\Omega(I)$ such that I is an A-equilibrium if and only if $A \in \Omega(I)$. We can build such a characterization with the help of the trade-off concept.

Trade-off coefficients are widely used in multiple criteria decision making to characterize solutions in terms of partial preferences concerning relative importance of criteria (see Kaliszewski [2006]). We define the trade-off coefficient in a game as the ratio between the improvement of a player's pay-off and the worsening of another player's pay-off caused by the former player's strategy change.

Definition 5. *For any strategy profile I, any pair of players $k,\ l \in N_p$, $k \neq l$, and any k-th player's strategy $j \in S_k$ such that $a^k(I_{\langle k,j \rangle}) > a^k(I)$ and $a^l(I_{\langle k,j \rangle}) < a^l(I)$, the number*

$$T_{kl}(I,j) = \frac{a^k\left(I_{\langle k,j \rangle}\right) - a^k(I)}{a^l(I) - a^l\left(I_{\langle k,j \rangle}\right)}$$

is called ***altruistic trade-off coefficient*** *of player k with respect to player l for strategy profile I and strategy j.*

In the following obvious proposition, we reformulate the definition of A-equilibrium in terms of altruistic trade-off coefficients.

Proposition 1. *A locally efficient strategy profile I is an A-equilibrium if and only if for any player* $k \in N_p$ *and any strategy from his/her strategy set* $j \in S_k$, $j \neq i_k$, *the following implication holds:*

$$\text{if } a^k(I_{(k,j)}) > a^k(I), \text{ then there exists}$$
$$\text{another player } l \in N_p \text{ such that } a^l(I_{(k,j)}) < a^l(I) \text{ and } T_{kl}(I,j) \le \alpha_{kl}.$$

Let us apply this proposition to characterize the cooperative solution of the Prisoner's Dilemma game

Example 2. *Consider the Prisoner's Dilemma game described in Section 3, where the players and the strategies are numbered in the following way: Player A = 1, Player B = 2, "stay silent" = 1 and "betray" = 2.*

Consider the strategy profile $I := (1,1)$. *It is locally efficient. Let us calculate the altruistic trade-off coefficients for I:*

$$T_{12}(I,2) = T_{21}(I,2) = 0.5/9.5 = 1/19.$$

According to Proposition 1, strategy profile I is an A-equilibrium if and only if $\alpha_{12} \ge 1/19$ *and* $\alpha_{21} \ge 1/19$. *So it suffices that each player considers the other player's interests 19 times less important than his/her own interests, to make the cooperation possible.*

It is easy to characterize a strategy profile in a game with two players with the help of the following evident corollary from Proposition 1.

Corollary 2. *Let* $p = 2$. *A locally efficient strategy profile* $I = (i_1, i_2)$ *is an A-equilibrium if and only if* $\alpha_{12} \ge \tau_{12}$ *and* $\alpha_{21} \ge \tau_{21}$, *where*

$$\tau_{kl} = \max\Big\{T_{kl}(I,j) : j \in S_k,\ j \neq i_k,\ a^k(I_{(k,j)}) > a^k(I),\ a^l(I_{(k,j)}) < a^l(I)\},$$
$$(k,l) \in \{(1,2),(2,1)\}\Big\}$$

and the maximum over the empty set is assumed to be zero.

Unfortunately, in a game with more than two players it is impossible to characterize a strategy profile by lower bounds of altruistic coefficients. In other words, it is impossible to represent the characterization in the following form: *the strategy profile is A-equilibrium if and only if* $\alpha_{kl} \ge \tau_{kl}$ *for any* $k,l \in N_p$, $k \neq l$, *where* τ_{kl} *is the lower bound for altruistic coefficient*. This difficulty is illustrated by the following example.

Example 3. *Consider the game with 3 players each having 2 strategies and following pay-off functions:*

Player 1 pay-off function

	$i_2=1$		$i_2=2$	
	$i_3=1$	$i_3=2$	$i_3=1$	$i_3=2$
$i_1=1$	**4**	**3**	**0**	**1**
$i_1=2$	**5**	**1**	**1**	**3**

Player 2 pay-off function

	$i_1=1$		$i_1=2$	
	$i_3=1$	$i_3=2$	$i_3=1$	$i_3=2$
$i_2=1$	**3**	**3**	**5**	**1**
$i_2=2$	**4**	**1**	**1**	**4**

Player 3 pay-off function

	$i_1=1$		$i_1=2$	
	$i_2=1$	$i_2=2$	$i_2=1$	$i_2=2$
$i_3=1$	**10**	**2**	**0**	**1**
$i_3=2$	**8**	**1**	**1**	**6**

It is easy to check that the strategy profile $I := (1,1,1)$ is locally efficient. Let us characterize it with the help of altruistic trade-off coefficients.

Altruistic trade-off coefficients of Player 1: $T_{13}(I,2) = 0.1$; $T_{12}(I,2)$ *is undefined because when Player 1 changes his/her strategy from 1 to 2, Player's 2 pay-off is not decreased.*

Altruistic trade-off coefficients of Player 2: $T_{21}(I,2) = 0.5$; $T_{23}(I,2) = 0.25$.

Altruistic trade-off coefficients of Player 3 *are undefined because $a^3(1,1,2) < a^3(1,1,1)$, which means that Player 3 is not interested in changing his/her strategy. So the degree of this player's altruism does not influence the equilibrium.*

Applying Proposition 1, we obtain that I is an A-equilibrium if and only if

$$\alpha_{13} \geq 0.1 \text{ and } (\alpha_{21} \geq 0.5 \text{ or } \alpha_{23} \geq 0.25).$$

Thus, instead of a set of constraints on the altruistic coefficients, we have a logical expression which does not necessarily include all of them.

In general, the strategy profile characterization implied by Proposition 1 can be formulated as follows:

Locally efficient strategy profile I is A-equilibrium if and only if

$$\bigwedge_{k \in \hat{N}(I)} \bigwedge_{j \in \hat{S}_k(I)} \bigvee_{l \in \tilde{N}(I,k,j)} \left(\alpha_{kl} \geq T_{kl}(I,j)\right) \tag{2}$$

where

$\hat{N}(I) = \left\{k \in N_p : \hat{S}_k(I) \neq \emptyset\right\}$ is the subset of players who can improve their pay-offs by changing their strategies,

$\hat{S}_k(I) = \left\{j \in S_k : j \neq i_k, a^k\left(I_{\langle k,j \rangle}\right) > a^k(I)\right\}$ is the subset of player k's strategies, for which his/her pay-off is greater than the initial pay-off,

$\tilde{N}(I,k,j) = \left\{l \in N_p : a^l\left(I_{\langle k,j \rangle}\right) < a^l(I)\right\}$ is the set of players who suffer from player k changing his/her strategy to j.

Thus, the set of matrices characterizing a strategy profile may have a rather complicated structure in the case of a large number of players.

Conclusion

We presented an approach to modeling equilibrium in multi-player non--zero sum games taking into account the relative preferences of players, namely the relative importance of their own gains and other players' losses. We suppose that in addition to striving for their own profit, players are concerned with not harming the interests of other players disproportionately, which can be referred to as altruism. Such a deviation from the egoistic behavior in real life may be conditioned by moral and ethic concerns, fear, reputation concern, and many other motivation factors.

The intensity of mutual altruism of players is quantified by a non--negative altruistic coefficient. When the altruistic coefficients are zero, the altruistic equilibrium is reduced to the Nash equilibrium, so the concept proposed may be considered as a generalization of the Nash equilibrium concept.

Our approach does not require to characterize player preferences in terms of a utility function (in contrast to other approaches, see Fehr and Schmidt [1999], Bolton and Ockenfels [2000], Charness and Rabin [2002]). On the contrary, the proposed equilibrium condition is based on direct comparison of pay-off differences. It is worth to note that the proposed model is not the only possible formalization of the player altruism concept in terms of relative importance preferences. The model based on the Edgeworth's proposition (see the end of Section 2) describes a slightly different variant of the altruistic behavior.

The clear interpretation of the model proposed makes it useful for analyzing equilibrium situations in terms of relative preferences. For example, after estimating the degree of player altruism, one can describe a range of possible equilibria. And vice versa, analyzing the information about equilibria achieved and unachieved among locally efficient solutions, one can estimate the degree of player altruism in terms of bounds on altruistic coefficients. This can be done by characterizing strategy profiles in terms of altruistic trade-off coefficients (see Section 4).

Another possible application of the model proposed is in the field of repeatedly played games. This research area attempts to explain cooperative behavior through natural selection mechanisms. For example, Axelrod [1980, 1984] conducted game tournaments with two players and found out a long-term incentive for cooperation in their behavior. In the framework of evolutionary approach, Robson [1990] proposed a model where a prisoner's dilemma-like game is repeatedly played in a population of players and there are "mutants" who cooperate by playing with other "mutants" and betray by playing with the rest of individuals. An invasion of "mutants" displays the advantage of the cooperation strategy. Chlebuś et al. [2009] built a computer simulation of a society, where economic activity is modeled dynamically by repetitive playing of random prisoner's dilemma-like games. By varying parameters of players' behavior, one can analyze how the propensity to cooperate influences social welfare. Other examples of the evolutionary approach applied to the game theory can be found in Nowak and Sigmund [2004], Szabó and Fáth [2007]. Our model can be used to quantify the degree of player altruism in evolutionary simulations. It would be interesting to trace the dependence between the inclination to altruism and the survivability or welfare of the player population.

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MULTI-CRITERIA DECISION MAKING MODELS BY APPLYING THE TOPSIS METHOD TO CRISP AND INTERVAL DATA

Abstract

In this paper, one of the multi-criteria models in making decision, a Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS), is described. Some of the advantages of TOPSIS methods are: simplicity, rationality, comprehensibility, good computational efficiency and ability to measure the relative performance for each alternative in a simple mathematical form.

The paper has a review character. It systematises the knowledge within the scope of techniques of decision taking with the use of the TOPSIS method. Simple numerical examples that reference real situations show practical applications of different aspects of this method.

The paper is organized as follows. The Introduction presents a short overview of the decision making steps as well as MCDM techniques. Section 1 presents matrix representation of the MCDM problem. Section 2 describes the TOPSIS procedure for crisp data, and Section 3 for interval data. The TOPSIS algorithm in group decision environment in the case of crisp and interval data is also presented. In Section 4 the problem of qualitative data in TOPSIS model is discussed. The numerical examples showing applications of those techniques in the negotiation process are presented in Section 5. Finally, conclusions and some concluding remarks are made in last section.

Keywords

TOPSIS method, numerical data, interval data, positive ideal solution, negative ideal solution.

Introduction

Multi-criteria decision making (MCDM) refers to making choice of the best alternative from among a finite set of decision alternatives in terms of multiple, usually conflicting criteria. The main steps in multi-criteria decision making are the following [Hwang, Yoon, 1981; Jahanshahloo, Hosseinzadeh, Lofti, Izadikhah, 2006a]:

– establish system evaluation criteria that relate system capabilities to goals,
– develop alternative systems for attaining the goals (generating alternatives),
– evaluate alternatives in terms of criteria,
– apply one of the normative multiple criteria analysis methods,
– accept one alternative as "optimal" (preferred),
– if the final solution is not accepted, gather new information and go to the next iteration of multiple criteria optimization.

Multi-criteria decision making techniques are useful tools to help decision maker(s) to select options in the case of discrete problems. Especially, with the help of computers, those methods have become easier for the users, so they have found great acceptance in many areas of decision making processes in economy or management. Among many multi-criteria techniques, MAXMIN, MAXMAX, SAW, AHP, TOPSIS, SMART, ELECTRE are the most frequently used methods [Chen, Hwang, 1992]. The nature of the recommendations of one of those methods depends on the problem being addressed: choosing, ranking or sorting. The selection of models/techniques can be also based on such evaluation criteria as:
– internal consistency and logical soundness,
– transparency,
– ease of use,
– data requirements are consistent with the importance of the issue being considered,
– realistic time and manpower resource requirements for the analytical process,
– ability to provide an audit trail,
– software availability, where needed.

The classification methods can be categorized by the type of information from the decision maker (no information, information on attributes or information on alternatives), data type or by solution aimed at [Chen, Hwang, 1992, p.16-25]. The MAXMIN technique assumed that the overall performance of an alternative is determined by its weakest attribute, in the MAXMAX technique an alternative is selected by its best attribute value. The SAW (Simple Additive Weighting) method multiplies the normalized value of the criteria for the alternatives with the importance of the criteria and the alternative with the highest score is selected as the preferred one. The TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution) selects the alternative closest to the ideal solution and farthest from the negative ideal alternative. The classical TOPSIS method is based on information on attribute from decision maker, numerical data; the solution is aimed at evaluating, prioritizing and selecting and the only subjective inputs are weights. The AHP (The Analytical Hierarchy Process) uses a hierarchical structure and pairwise comparisons. An AHP hierarchy has at least three levels: the main objective of the problem at the top, multiple criteria that define alternatives in the middle and competing alternatives at the bottom. The major weaknesses of TOPSIS are that it does not provide for weight elicitation, and consistency checking for judgments; on the other hand, the use of AHP has been significantly restrained by the human capacity for the information process. From this point of view, TOPSIS alleviates the requirement of paired comparisons and the capacity limitation might not significantly dominate the process. Hence, it would be suitable for cases with a large number of criteria and alternatives, and especially where objective or quantitative data are given [Shih, Shyur, Lee, 2007]. SMART (The Simple Multi Attribute Rating Technique) is similar to AHP, a hierarchical structure is created to assist in defining a problem and in organizing criteria. However, there are some significant differences between those techniques: SMART uses a different terminology. For example, in SMART the lowest level of criteria in the value tree (or objective hierarchy) are called attributes rather than sub-criteria and the values of the standardized scores assigned to the attributes derived from value functions are called ratings. The difference between a value tree in SMART and a hierarchy in AHP is that the value tree has a true tree structure, allowing one attribute or sub-criterion to be connected to only one higher level criterion. SMART does not use a relative method for standardizing raw scores to a normalized scale. Instead, a value function explicitly defines how each value is transformed to the common model scale. The value function mathematically transforms ratings into a consistent internal scale with lower limit 0 and upper limit 1. The ELECTRE (Elimination and Choice Expressing Reality) method was to choose the best action(s) from a given set of actions, but it can also be applied to three main problems: choosing, ranking and sorting. There are two main parts to an ELECTRE application: first, the construction of one or several outranking relations, which aims at comparing in a comprehensive way each pair of actions; second, an exploitation procedure that elaborates on the recommendations obtained in the first phase.

This paper is focused on the TOPSIS method, which was presented by Hwang and Yoon [1981] and developed later by many authors [Jahanshahloo, Lofti, Izadikhah, 2006a; 2006b; Zavadskas,Turskis, Tamosaitiene, 2008; Hung, Chen, 2009]. The acronym TOPSIS stands for **T**echnique for **O**rder **P**reference by **S**imilarity to the **I**deal **S**olution. It is worth noting that the TOPSIS method corresponds to the Hellwig taxonomic method of ordering objects [Hellwig, 1968]. The main advantages of this method are the following [Hung, Cheng, 2009]:

– simple, rational, comprehensible concept,
– intuitive and clear logic that represent the rationale of human choice,
– ease of computation and good computational efficiency,

– a scalar value that accounts for both the best and worst alternatives ability to measure the relative performance for each alternative in a simple mathematical form,
– possibility for visualization.

In general, the process for the TOPSIS algorithm starts with forming the decision matrix representing the satisfaction value of each criterion with each alternative. Next, the matrix is normalized with a desired normalizing scheme, and the values are multiplied by the criteria weights. Subsequently, the positive-ideal and negative-ideal solutions are calculated, and the distance of each alternative to these solutions is calculated with a distance measure. Finally, the alternatives are ranked based on their relative closeness to the ideal solution. The TOPSIS technique is helpful for decision makers to structure the problems to be solved, conduct analyses, comparisons and ranking of the alternatives. The classical TOPSIS method solves problems in which all decision data are known and represented by crisp numbers. Most real-world problems, however, have a more complicated structure. Based on the original TOPSIS method, many other extensions have been proposed, providing support for interval or fuzzy criteria, interval or fuzzy weights to modeled imprecision, uncertainty, lack of information or vagueness.

In this paper, the classical TOPSIS algorithms for crisp, as well as interval data are described. Interval analysis is a simple and intuitive way to introduce data, uncertainty for complex decision problems, and can be used for many practical applications. An extension of the TOPSIS technique to a group decision environment is also investigated. The context of multi-criteria group decision making in both crisp and interval data are described. Finally, situations where criteria and their weight are subjectively expressed by linguistic variables are considered. The practical applications of the TOPSIS technique in estimating offers, for instance, in buyer-seller exchange are also proposed.

1. The matrix representation of the MCDM problem

The MCDM problems can be divided into two kinds. One is the classical MCDM set of problems among which the ratings and the weights of criteria are measured in crisp numbers. Another one is the multiple criteria decision-making set of problems where the ratings and the weights of criteria evaluated on incomplete information, imprecision, subjective judgment and vagueness are usually expressed by interval numbers, linguistic terms, fuzzy numbers or intuitive fuzzy numbers.

In the classical MCDM model, we assume exact data, objective and precise information, but this is often inadequate to model real life situations. Human judgments are often vague under many conditions. The socio-economic environment becomes more complex, the preference information provided by decision-makers is usually imprecise, and can create hesitation or uncertainty about preferences. A decision may have to be made under time pressure and lack of knowledge or data, or the decision-makers have limited attention and information processing capacities. Most input information is not known precisely, so that the values of many criteria are expressed in subjective or uncertain terms. The criteria, as well as their weight, could be subjectively expressed by linguistic variables. Thus, many researchers extended the MCDM approach for decision making problems with subjective criteria, interval data or fuzzy environment using grey system theory or fuzzy set theory.

The grey system theory, developed by Deng [1982, 1988] is based upon the concept that information is sometimes incomplete or unknown [Jadidi, Hong, Firouzi, Yusuff, 2008; Liu, Lin, 2006]. Exactly, the theory is based on the degree of information known which is modeled by intervals. If the system information is unknown, it is called a black system, if the information is fully known, it is called a white system. And a system with information known partially is called a grey system. The fuzzy set theory cannot handle incomplete data and information, but is adequate to deal with uncertain and imprecise data [Kahraman, 2008; Chen, Hwang, 1992]. The advantage of the grey theory over the fuzzy theory is that the grey theory takes into account the condition of the fuzziness; that is, the grey theory can deal flexibly with the fuzziness situation.

We can also consider single decision making and group decision making. Group decision making is more complex than single decision making because it involves many contradicting factors, such as: conflicting individual goals, inefficient knowledge, validity of information, individual motivation, personal opinion, power. In both multi-criteria decision making (MCDM) and group decision making (GDM), there are two steps: aggregation and exploitation. In MCDM, aggregation consists in combining satisfaction over different criteria while GDM problem consists in combining the experts' opinions into a group collective one. Group decision making can be approached from two points of view. In the first approach, individual multi-criteria models are developed based on individuals' preferences. Each decision maker formulates a multi--criteria problem defining the parameters according to these preferences and solves the problem getting an individual solution set. Next, the separate solutions are aggregated by aggregation of operations resulting in the group solution. In the second approach, each decision maker provides a set of parameters which are aggregated by appropriate operators, providing finally a set of group parameters. Upon this set the multi-criteria method is applied and the solution expresses group preference [Rigopoulos, Psarras, Askounis, 2008].

Solving of each multi-criteria problem (individual or group decision) begins with the construction of a decision making matrix (or matrices). In such matrixes, values of the criteria for alternatives may be real, intervals numbers, fuzzy numbers or qualitative labels.

Let us denote by $D = \{1,2,\dots,K\}$ a set of decision makers or experts. The multi-criteria problem can be expressed in $k-$ matrix format in the following way:

$$\begin{array}{c|cccc} & \mathrm{C}_1 & \mathrm{C}_2 & \cdots & \mathrm{C}_n \\ \hline \mathrm{A}_1 & x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ \mathrm{A}_2 & x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \dots & \dots & \dots & \dots & \dots \\ \mathrm{A}_m & x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{array}$$

where:

- $A_1, A_2, \dots, A_m$ are possible alternatives that decision makers have to choose from,
- $C_1, C_2, \dots, C_n$ are the criteria for which the alternative performance is measured,
- x_{ij}^k is the $k-$decision maker rating of alternative A_i with respect to the criterion C_j (x_{ij}^k is numerical, interval data or fuzzy number).

In this way for m alternatives and n criteria we have matrix $X^k = \left(x_{ij}^k\right)$ where x_{ij}^k is value of $i-$alternative with respect to $j-$criterion for $k-$ decision maker, $j = 1,2,\dots,n,\ k = 1,2,\dots,K$.

The relative importance of each criterion is given by a set of weights which are normalized to sum to one. Let us denote by $W^k = [w_1^k, w_2^k, \dots, w_n^k]$ a weight vector for $k-$decision maker, where $w_j^k \in \Re$ is the $k-$decision maker weight of criterion C_j and $w_1^k + w_2^k + \dots + w_n^k = 1.$

In the case of one decision maker we write x_{ij}, w_j, X, respectively.

Multi-criteria analysis focuses mainly on three types of decision problems: *choice* – select the most appropriate (best) alternative, *ranking* – draw a complete order of the alternatives from the best to the worst, and *sorting* – select the best k alternatives from the list.

2. The classical TOPSIS method

In the classical TOPSIS method we assume that the ratings of alternatives and weights are represented by numerical data and the problem is solved by a single decision maker. Complexity arises when there are more than one decision makers because the preferred solution must be agreed on by interest groups who usually have different goals. The classical TOPSIS algorithm for a single decision maker and for group decision making is systematically described in Section 2.1 and Section 2.2, respectively.

2.1. The classical TOPSIS method for a single decision maker

The idea of classical TOPSIS procedure can be expressed in a series of following steps [Chen, Hwang, 1992; Jahanshahloo, Lofti, Izadikhah, 2006a].

Step 1. Construct the decision matrix and determine the weight of criteria.
Let $X = (x_{ij})$ be a decision matrix and $W = [w_1, w_2, \ldots, w_n]$ a weight vector, where $x_{ij} \in \Re$, $w_j \in \Re$ and $w_1 + w_2 + \ldots + w_n = 1$.
Criteria of the functions can be: benefit functions (more is better) or cost functions (less is better).

Step 2. Calculate the normalized decision matrix.
This step transforms various attribute dimensions into non-dimensional attributes which allows comparisons across criteria. Because various criteria are usually measured in various units, the scores in the evaluation matrix X have to be transformed to a normalized scale. The normalization of values can be carried out by one of the several known standardized formulas. Some of the most frequently used methods of calculating the normalized value n_{ij} are the following:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \tag{2.1}$$

$$n_{ij} = \frac{x_{ij}}{\max_i x_{ij}}, \tag{2.1*}$$

$$n_{ij} = \begin{cases} \dfrac{x_{ij} - \min\limits_i x_{ij}}{\max\limits_i x_{ij} - \min\limits_i x_{ij}} & \text{if } C_i \text{ is a benefit criterion} \\ \dfrac{\max\limits_i x_{ij} - x_{ij}}{\max\limits_i x_{ij} - \min\limits_i x_{ij}} & \text{if } C_i \text{ is a cost criterion} \end{cases} \tag{2.1**}$$

for $i = 1, \dots, m$; $j = 1, \dots, n$.

Step 3. Calculate the weighted normalized decision matrix.

The weighted normalized value v_{ij} is calculated in the following way:

$$v_{ij} = w_j n_{ij} \text{ for } i = 1, \dots, m; j = 1, \dots, n. \tag{2.2}$$

where w_j is the weight of the j-th criterion, $\sum_{j=1}^{n} w_j = 1.$

Step 4. Determine the positive ideal and negative ideal solutions.

Identify the positive ideal alternative (extreme performance on each criterion) and identify the negative ideal alternative (reverse extreme performance on each criterion). The ideal positive solution is the solution that maximizes the benefit criteria and minimizes the cost criteria whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria.

Positive ideal solution A^+ has the form:

$$A^+ = \left(v_1^+, v_2^+, \dots, v_n^+\right) = \left(\left(\underset{i}{max} v_{ij} \mid j \in I\right), \left(\underset{i}{min} v_{ij} \mid j \in J\right)\right). \tag{2.3}$$

Negative ideal solution A^- has the form:

$$A^- = \left(v_1^-, v_2^-, \dots, v_n^-\right) = \left(\left(\underset{i}{min} v_{ij} \mid j \in I\right), \left(\underset{i}{max} v_{ij} \mid j \in J\right)\right) \tag{2.4}$$

where I is associated with benefit criteria and J with the cost criteria, $i = 1, \dots, m$; $j = 1, \dots, n$.

Step 5. Calculate the separation measures from the positive ideal solution and the negative ideal solution.

In the TOPSIS method a number of distance metrics can be applied*.
The separation of each alternative from the positive ideal solution is given as

$$d_i^+ = \left(\sum_{j=1}^{n} \left(v_{ij} - v_j^+ \right)^p \right)^{1/p}, \quad i = 1,2, \ldots , m. \tag{2.5}$$

The separation of each alternative from the negative ideal solution is given as

$$d_i^- = \left(\sum_{j=1}^{n} \left(v_{ij} - v_j^- \right)^p \right)^{1/p}, \quad i = 1,2, \ldots , m. \tag{2.6}$$

Where $p \geq 1$. For $p = 2$ we have the most used traditional n-dimensional Euclidean metric.

$$d_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+ \right)^2}, \quad i = 1,2, \ldots , m, \tag{2.5*}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^- \right)^2}, \quad i = 1,2, \ldots , m. \tag{2.6*}$$

Step 6. Calculate the relative closeness to the positive ideal solution.

The relative closeness of the *i*-th alternative A_j with respect to A^+ is defined as

$$R_i = \frac{d_i^-}{d_i^- + d_i^+}, \tag{2.7}$$

where $0 \leq R_i \leq 1$, $i = 1,2, \ldots , m$.

Step 7. Rank the preference order or select the alternative closest to 1.

A set of alternatives now can be ranked by the descending order of the value of R_i.

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* Possible metrics the first power metric (the least absolute value terms), Tchebychev metric or others [see Kahraman, Buyukozkan, Ates, 2007; Olson 2004].

2.2. The classical TOPSIS method for group decision making

In this part we explain the detailed TOPSIS procedure for group decision making based on the Shih, Shyur and Lee proposition [Shih, Shyur, Lee, 2007].

Step 1. Construct the decision matrixes and determine the weights of criteria for k-decision makers.

Let $X^k = \left(x_{ij}^k\right)$ be a decision matrix, $W^k = [w_1^k, w_2^k, \dots, w_n^k]$ weight vector for $k-$ decision maker or expert, where $x_{ij}^k \in \Re$, $w_j^k \in \Re$, $w_1^k + w_2^k + \dots + w_n^k = 1$ for $k = 1,2, \dots, K$.

Step 2. Calculate the normalized decision matrix for each decision maker.

In this step some of the earlier described methods of normalization can be used. Let us assume that we use

$$r_{ij}^k = \frac{x_{ij}^k}{\sqrt{\sum_{i=1}^{m}\left(x_{ij}^k\right)^2}}. \tag{2.8}$$

In this procedure weights are manipulated in the next step.

Step 3. Determine the positive ideal and negative ideal solutions for each decision maker.

The positive ideal solution A^{k+} for $k-$ decision maker has the form

$$A^{k+} = \left\{r_1^{k+}, r_2^{k+}, \dots, r_n^{k+}\right\} = \left\{\left(\underset{i}{max}(r_{ij}^k) \mid j \in I\right), \left(\underset{i}{min}(r_{ij}^k) \mid j \in J\right)\right\}. \tag{2.9}$$

The negative ideal solution A^{k-} for k- decision maker has the form:

$$A^{k-} = \left\{r_1^{k-}, r_2^{k-}, \dots, r_n^{k-}\right\} = \left\{\left(\min_i(r_{ij}^k) \mid j \in I\right), \left(\max_i(r_{ij}^k) \mid j \in J\right)\right\}, \tag{2.10}$$

where I is associated with the benefit criteria and J with the cost criteria.

Step 4. Calculate the separation measures from the positive ideal solution and the negative ideal solution.

Step 5.1. Calculate the separatation measure for individuals.

The separation of i-th alternative A_i from the positive ideal solution A^{k+} for each k – decision maker is given as

$$d_i^{k+} = \left(\sum_{j=1}^{m} w_j^k \left(r_{ij}^k - r_j^{k+} \right)^p \right)^{1/p}, \quad i = 1,2, \dots, m. \tag{2.11}$$

The separation of i-th alternative A_i from the negative ideal solution A^{k-} for each k – decision maker is given as

$$d_i^{k-} = \left(\sum_{j=1}^{m} w_j^k \left(r_{ij}^k - r_j^{k-} \right)^p \right)^{1/p}, \quad i = 1,2, \dots, m, \tag{2.12}$$

where $p \geq 1$. For $p = 2$ we have the Euclidean metric.

Step 5.2. Calculate the separation measure for the group.

The aggregation for measure for the group measures of the positive ideal d_i^{*+} and negative ideal solution d_i^{*-} for the i-th alternative A_i is given by one of the operators:

arithmetic mean:

$$d_i^{*+} = \frac{\sum_{k=1}^{K} d_i^{k+}}{K} \quad \text{and} \quad d_i^{*-} = \frac{\sum_{k=1}^{K} d_i^{k-}}{K} \tag{2.13}$$

or

geometric mean:

$$d_i^{*+} = \sqrt[K]{\prod_{k=1}^{K} d_i^{k+}} \quad \text{and} \quad d_i^{*-} = \sqrt[K]{\prod_{k=1}^{K} d_i^{k-}}\ . \tag{2.13*}$$

Step 6. Calculate the relative closeness to the positive ideal solution.

The relative closeness of the alternative A_i to the positive ideal solution is defined as

$$R_i^* = \frac{d_i^{*-}}{d_i^{*-} + d_i^{*+}} \quad \text{for} \quad i = 1,2, \dots, m \tag{2.14}$$

where $0 \le R_i^* \le 1$.

The larger the index value, the better the evaluation of the alternative.

Step 7. Rank the preference order or select the alternative closest to 1.

A set of alternatives can now be ranked by the descending order of the value of R_i^*.

3. The TOPSIS method with criteria values determined as interval

In some cases determining the exact value of criteria is difficult and decision makers are usually more comfortable providing intervals to specify model input parameters. An interval number data formulation is a simple and intuitive way to represent uncertainty, which is typical of real decision problems. Here, the TOPSIS method using interval as the basis for evaluating value alternatives is described. However, we can also consider an interval weights description [Jadidi, Hong, Firouzi, Yusuff, 2008].

3.1. The TOPSIS method with attributed values determined as interval for a single decision maker

An algorithmic method which extends TOPSIS for decision-making problems with interval data was proposed by Jahanshahloo, Lofti, Izadikhah. This procedure can be described in the following steps [Jahanshahloo, Lofti, Izadikhah, 2006a].

Step 1. Construct the decision matrix and determine the weight of criteria.

Let $X = (x_{ij})$ be a decision matrix and $W = [w_1, w_2, \dots, w_n]$ a weight vector, where $x_{ij} = [\underline{x}_{ij}, \overline{x}_{ij}]$, $\underline{x}_{ij}, \overline{x}_{ij} \in \Re$, $w_j \in \Re$ and $w_1 + w_2 + \dots + w_n = 1$.

Step 2. Calculate the normalized interval decision matrix.

The normalized values $\underline{n}_{ij}, \overline{n}_{ij}$ are calculated in the following way:

$$\underline{n}_{ij} = \frac{\underline{x}_{ij}}{\sqrt{\sum_{i=1}^{m}\left((\underline{x}_{ij})^2 + (\overline{x}_{ij})^2\right)}} \quad \text{for } i = 1, \dots, m;\ j = 1, \dots, n. \tag{3.1}$$

$$\overline{n}_{ij} = \frac{\overline{x}_{ij}}{\sqrt{\sum_{i=1}^{m}\left(\left(\underline{x}_{ij}\right)^2 + \left(\overline{x}_{ij}\right)^2\right)}} \text{ for } i = 1, \dots, m; \; j = 1, \dots, n. \tag{3.2}$$

The interval $\left[\underline{n}_{ij}, \overline{n}_{ij}\right]$ is normalized value of interval $\left[\underline{x}_{ij}, \overline{x}_{ij}\right]$.

Step 3. Calculate the weighted normalized interval decision matrix.

The weighted normalized values $\underline{v}_{ij}$ and $\overline{v}_{ij}$ are calculated in the following way:

$$\underline{v}_{ij} = w_j \underline{n}_{ij} \text{ for } i = 1, \dots, m; \; j = 1, \dots, n, \tag{3.3}$$

$$\overline{v}_{ij} = w_j \overline{n}_{ij} \text{ for } i = 1, \dots, m; \; j = 1, \dots, n, \tag{3.4}$$

where w_j is the weight of the j-th criterion, $\sum_{j=1}^{n} w_j = 1$.

Step 4. Determine the positive ideal and negative ideal solutions.

The positive ideal solution has the form A^+ :

$$A^+ = \left(v_1^+, v_2^+, \dots, v_n^+\right) = \left(\left(\max_i \overline{v}_{ij} \mid j \in I\right), \left(\min_i \underline{v}_{ij} \mid j \in J\right)\right). \tag{3.5}$$

The negative ideal solution has the form A^- :

$$A^- = \left(v_1^-, v_2^-, \dots, v_n^-\right) = \left(\left(\min_i \underline{v}_{ij} \mid j \in I\right), \left(\max_i \overline{v}_{ij} \mid j \in J\right)\right), \tag{3.6}$$

where I is associated with benefit criteria and J with cost criteria.

Step 5. Calculate the separation measures from the positive ideal solution and the negative ideal solution.

The separation of each alternative from the positive ideal solution is given as*:

$$d_i^+ = \sqrt{\sum_{j=1}^{n}\left(\underline{v}_{ij} - v_j^+\right)^2 + \sum_{j=1}^{n}\left(\overline{v}_{ij} - v_j^+\right)^2}, \; i = 1, 2, \dots, m. \tag{3.7}$$

---

* Traditional TOPSIS applied to Euclidean norm is presented here. However, we can also use other metrics.

The separation of each alternative from the negative ideal solution is given as:

$$d_i^- = \sqrt{\sum_{j=1}^{n} \left(\bar{v}_{ij} - v_j^-\right)^2 + \sum_{j=1}^{n} \left(\underline{v}_{ij} - v_j^-\right)^2}\ , \ i = 1,2, \dots, m. \tag{3.8}$$

Step 6. Calculate the relative closeness to the positive ideal solution.

The relative closeness of the alternative A_i with respect to A^+ is defined as:

$$R_i = \frac{d_i^-}{d_i^- + d_i^+} \ \text{ for } \ i = 1,2, \dots, m, \tag{3.9}$$

where $0 \le R_i \le 1$.

Step 7. Rank the preference order or select the alternative closest to 1.

The set of alternatives can now be ranked by the descending order of the value of R_i.

3.2. The TOPSIS method with attributed values determined as intervals for group decision making

We assume here that values are considered as intervals and we have group of k – decision makers [Zavadskas, Turskis, Tamosaitiene, 2008].

Step 1. Construct the decision matrixes and determine the weights of criteria for k – decision makers.

Let $X^k = \left(x_{ij}^k\right)$ be a decision matrix, $W^k = [w_1^k, w_2^k, \dots, w_n^k]$ is weight vector for k – decision maker, where $x_{ij}^k = \left[\underline{x}_{ij}^k, \bar{x}_{ij}^k\right]$, $\underline{x}_{ij}^k, \bar{x}_{ij}^k \in \Re$, $w_i \in \Re$, $w_1^k + w_2^k + \dots + w_n^k = 1$ for $k = 1,2, \dots, K$.

Step 2. Calculate the normalized grey decision matrixes.

This step transforms various attribute dimensions into non-dimensional attributes $r_{ij}^k = \left[\underline{r}_{ij}^k, \bar{r}_{ij}^k\right]$, which allows comparisons across criteria. The normalized values $\underline{r}_{ij}^k, \bar{r}_{ij}^k$ are calculated in the following way:

$$\underline{r}_{ij}^{k} = \frac{\underline{x}_{ij}^{k}}{\max_i(\overline{x}_{ij}^{k})} , \qquad \overline{r}_{ij}^{k} = \frac{\overline{x}_{ij}^{k}}{\max_i(\overline{x}_{ij}^{k})} \tag{3.10}$$

for $i = 1, \dots , m$; $j = 1, \dots , n$, $k = 1,2, \dots , K$.

Step 3. Determine the positive ideal and the negative ideal solutions for each decision maker.

The positive ideal solution A^{k+} for k-decision maker has the following form:

$$A^{k+} = \left\{r_1^{k+}, r_2^{k+}, \dots , r_n^{k+}\right\} = \left\{\left(\max_i(\overline{r}_{ij}^{k}) \mid j \in I \right), \left(\min_i(\underline{r}_{ij}^{k}) \mid j \in J \right)\right\}. \tag{3.11}$$

The negative ideal solution A^{k-} for k-decision maker has the form:

$$A^{k-} = \left\{r_1^{k-}, r_2^{k-}, \dots , r_n^{k-}\right\} = \left\{\left(\min_i(\underline{r}_{ij}^{k}) \mid j \in I \right), \left(\max_i(\overline{r}_{ij}^{k}) \mid j \in J \right)\right\}, \tag{3.12}$$

where I is associated with benefit criteria and J with cost criteria.

Step 4. Calculate the separation measures from the positive ideal solution and the negative ideal solution.

Step 5.1. Calculate the separation measure for individuals.

The separation of i-th alternative A_i from the positive ideal solution A^{k+} for each k – decision maker is given as

$$d_i^{k+} = \left(\frac{1}{2} \sum_{j=1}^{m} w_j^k \left(\left(r_j^{k+} - \underline{r}_{ij}^{k} \right)^p + \left(r_j^{k+} - \overline{r}_{ij}^{k} \right)^p \right) \right)^{1/p} , \quad i = 1,2, \dots , m. \tag{3.13}$$

The separation of i-th alternative A_i from the negative ideal solution A^{k-} for each k – decision maker is given as

$$d_i^{k-} = \left(\frac{1}{2} \sum_{j=1}^{m} w_j^k \left(\left(r_j^{k-} - \underline{r}_{ij}^{k} \right)^p + \left(r_j^{k-} - \overline{r}_{ij}^{k} \right)^p \right) \right)^{1/p} , \quad i = 1,2, \dots , m. \tag{3.14}$$

If p = 2, then the metric is a weighted grey number Euclidean distance function. Equations (3.13) and (3.14) will be as follows:

$$d_i^{k+} = \sqrt{\frac{1}{2}\sum_{j=1}^{m} w_j^k\left(\left(r_j^{k+} - \underline{r}_{ij}^{k}\right)^2 + \left(r_j^{k+} - \overline{r}_{ij}^{k}\right)^2\right)},\ i = 1,2,\ \dots,\ m. \qquad (3.13^*)$$

The separation of i-th alternative A_i from the negative ideal solution A^{k-} is given as

$$d_i^{k-} = \sqrt{\frac{1}{2}\sum_{j=1}^{m} w_j^k\left(\left(r_j^{k-} - \underline{r}_{ij}^{k}\right)^2 + \left(r_j^{k-} - \overline{r}_{ij}^{k}\right)^2\right)},\quad i = 1,2,\ \dots,\ m. \qquad (3.14^*)$$

Step 5.2. Calculate the separation measure for the group.

The aggregation of the measure for the group measures of the positive ideal d_i^{*+} and the negative ideal solution d_i^{*-} for the i-th alternative A_i is given by:

arithmetic mean:

$$d_i^{*+} = \frac{\sum_{k=1}^{K} d_i^{k+}}{K} \quad \text{and} \quad d_i^{*-} = \frac{\sum_{k=1}^{K} d_i^{k-}}{K}, \qquad (3.15)$$

or

geometric mean:

$$d_i^{*+} = \sqrt[K]{\prod_{k=1}^{K} d_i^{k+}} \quad \text{and} \quad d_i^{*-} = \sqrt[K]{\prod_{k=1}^{K} d_i^{k-}}. \qquad (3.15^*)$$

Step 6. Calculate the relative closeness to the positive ideal solution.

The relative closeness of the alternative A_i with respect to A^+ is defined as

$$R_i^* = \frac{d_i^{*-}}{d_i^{*-} + d_i^{*+}} \quad \text{for } i = 1,2,\ \dots,\ m, \qquad (3.16)$$

where $0 \le R_i^* \le 1$.

The larger the index value, the better the evaluation of alternative.

Step 7. Rank the preference order or select the alternative closest to 1.

The set of alternatives can now be ranked by the descending order of the value of R_i^*.

4. The quantitative and qualitative criteria in the TOPSIS method. Weights expressed by linguistic variable

In the TOPSIS algorithm the quantitative criteria are scaled using their own real numbers and for representation of the imprecision of spatial data, and human cognition over the criteria of the theory of linguistic variables is used. A linguistic variable is a variable where values are words or sentences in a natural or artificial language. Especially, since traditional quantification methods are difficult to describe situations that are overtly complex or hard to describe, the notion of a linguistic variable is necessary and useful. We can use this kind of expression for rating qualitative criteria as well as to compare two evaluation criteria.

The qualitative criterion can be described using linguistic variables; next the criteria ratings on the 1-9 number scale (Table 1) or on the 1-7 interval scale (Table 2) can be provided, respectively [Jadidi, Hong, Firouzi, Yusuff, Zulkifli, 2008].

Table 1

The scale of alternative ratings for qualitative criterion in the case of classical TOPSIS method

Scale	**Rating**
Poor (P)	1
Medium poor (MP)	3
Fair (F)	5
Medium good (MG)	7
Good (G)	9
Intermediate values between the two adjacent judgments	2,4,6,8

Table 2

The scale of alternative ratings in the case of interval TOPSIS method

Scale	**Rating**
Very Poor (VP)	[0,1]
Poor (P)	[1,3]
Medium poor (MP)	[3,4]
Fair (F)	[4,5]
Medium good (MG)	[5,6]
Good (G)	[6,9]
Very good (VG)	[9,10]

Each decision maker individually uses linguistic variables transformed for numerical scale (Table 1) or interval scale (Table 2) to identify the alternative rankings for the subjective criterion. Then the rating value for group decision makers can be calculated using the following formula

$$x_{ij} = \frac{1}{K}\left[x_{ij}^{1} + x_{ij}^{2} + \ldots + x_{ij}^{K}\right], \tag{4.1}$$

where:

$x_{ij}^{s}-$ is the rating value of alternative A_i with the respect to quantitative criterion C_j (crisp or interval) of $s-$decision maker ($i=1,2,\ldots,n;$ $j=1,2,\ldots,m;$ $s=1,2,\ldots,K$).

In this way for m alternatives and n criteria and $K-$decision makers we can obtain one aggregated matrix $X=(x_{ij})$ where x_{ij} is value of $i-$alternative with respect to $j-$criterion for $i=1,2,\ldots,m;$ $j=1,2,\ldots,n.$

The weights of the factors are subjective input, which directly influences the final evaluated result. In the evaluating system, the importance of every index is, in general, not equal, so they must be set different weight factors. Among many ways to set the weight factors are for instance the Delphi method or the AHP method [Olson, 2004]. The Delphi method is the most popular expert evaluating technique. The Delphi method is a forecasting and evaluating method both qualitative and quantitative which collects experts' ideas anonymously, exchanges and corrects this information many times to reach a consistent idea and gives the subject a final evaluation according to the experts' ideal. AHP (The Analytical Hierarchy Process) uses a hierarchical structure and pairwise comparisons. An AHP hierarchy has at least three levels: the main objective of the problem at the top, multiple criteria that define alternatives in the middle and competing alternatives at the bottom. The AHP method uses system analysis and continuously decomposes the evaluating indices according to the main evaluating indices of every level [Saaty, 1980]. The classical TOPSIS method does not consider a hierarchical structure consisting of main attributes and subattributes. This method evaluates the alternatives with respect to main attributes only with a single level. The common property of these methods is their ease of implementation, so this method is often used to obtain weight criteria.

In the case where the criterion weights are linguistic variables, the weights can be expressed by the 1-9 scale shown in Table 3 [Jadidi, Hong, Firouzi, Yusuff, Zulkifli, 2008].

Table 3

The scale of criterion weights

Scale	Weight
Very very low (VVL)	0,005
Very low (VL)	0,125
Low (L)	0,175
Medium low (ML)	0,225
Medium (M)	0,275
Medium hight (MH)	0,325
Hight (H)	0,375
Very Hight (VH)	0,425
Very Very Hight (VVH)	0,475

The vector of attribute weights must sum up to 1; if not, it is normalized. Each decision maker individually uses linguistic variables (Table 3) to identify the criterion weights. Then the criterion weights for all decision makers can be calculated using the following formula

$$w_r = \frac{1}{K}\left[w_r^1 + w_r^2 + \ldots + w_r^K\right], \tag{4.2}$$

where:

w_r^s – is the weight of r – criterion for s – decision makers ($r = 1,2, \ldots , n$; $s = 1,2, \ldots , K$).

5. Practical application

In this section, to demonstrate the calculation process of the approaches described, two examples are provided.

Example 1.

A firm intends to choose the best offer (or ranking of the offers) from the set of proposals submitted by potential contractors. Two experts evaluate five proposals using several criteria. In order to simplify the calculation, only four criteria are considered: deadline of payment after receipt the goods (in days), unitary price (in euro), conditions of warranty and contractor reputation, C_1, C_2, C_3, C_4, respectively. The criteria C_1, C_3, C_4 are benefit criteria, the greater values being better, and C_2 is the cost criterion, the smaller values are better. Criteria C_3, C_4 are subjectively evaluated by experts on basic available information and they are considered as linguistic variables, while the other criteria are scaled using their own real numbers, respectively. This is shown in Table 4. Based on Table 1, the decision matrixes for two decision makers are obtained (Table 5).

Table 4

Criteria rating values for two decision makers

		C_1	C_2	C_3	C_4
D_1					
	A_1	7	21	F&MP	MG
	A_2	7	24	MG&F	F&MP
	A_3	14	25	MP	G& MG
	A_4	14	26	G	MP
	A_5	21	35	MP &F	F& MP
D_2					
	A_1	7	21	F&MP	MG&F
	A_2	7	24	MG	F
	A_3	14	25	MP&P	G &MG
	A_4	14	26	MG	MP&P
	A_5	21	35	MP	MP

Table 5

Decision matrixes for two decision makers

		C_1	C_2	C_3	C_4
D_1					
	A_1	7	21	4	7
	A_2	7	24	6	4
	A_3	14	25	3	8
	A_4	14	26	9	3
	A_5	21	35	4	4
D_2					
	A_1	7	21	4	6
	A_2	7	24	7	5
	A_3	14	25	2	8
	A_4	14	26	7	3
	A_5	21	35	3	3

Based on linguistic variables, the evaluation values of attribute weights for the first and second decision makers can be obtained and the results are shown in Table 6. The normalized criteria weights for each decision maker obtained using Table 3 are shown in Table 7.

Table 6

Criteria weights for two decision makers

	C_1	C_2	C_3	C_4
D_1				
	L	VH	L	ML
D_2				
	ML	VVH	VL	L

Table 7

Normalized criteria weights for two decision makers

	C_1	C_2	C_3	C_4
D_1				
	0,175	0,425	0,175	0,225
D_2				
	0,225	0,475	0,125	0,175

CASE 1. Rank the preference order for individual decision makers

Using formulas 2.1-2.7 the calculation results on data from Table 5 and Table 7 and rank order for each decision maker are shown in Table 8 and Table 9, respectively.

Table 8

Calculation results for 1-decision maker

Alternatives	The separation of alternatives to positive ideal one	The separation of alternatives to negative ideal one	The relative closeness of alternatives to the positive ideal one	Rank
A_1	0.107805	0.124281	0.535497	**2**
A_2	0.117942	0.090785	0.434945	**4**
A_3	0.096981	0.122181	0.557491	**1**
A_4	0.105379	0.112779	0.516961	**3**
A_5	0.141766	0.083486	0.370633	**5**

Table 9

Calculation results for 2-decision maker

Alternatives	The separation of alternatives to positive ideal one	The separation of alternatives to negative ideal one	The relative closeness of alternatives to the positive ideal one	Rank
A_1	0.112485	0.128548	0.533321	**2**
A_2	0.115014	0.113042	0.495676	**3**
A_3	0.082214	0.130468	0.613440	**1**
A_4	0.110660	0.104396	0.485436	**4**
A_5	0.141415	0.104894	0.425865	**5**

CASE 2. Rank the preference order for group decision makers (1 method)

The decision matrix is calculated using formula (4.1) and attributes weights of the criteria using (4.2). The results are shown in Table 10 and Table 11, respectively.

Table 10

Decision matrix for group decision makers

	C_1	C_2	C_3	C_4
A_1	7	21	4	6,5
A_2	7	24	6,5	4,5
A_3	14	25	2,5	8
A_4	14	26	8	2,5
A_5	21	35	3,5	3,5

Table 11

Normalized criteria for group decision makers

C_1	C_2	C_3	C_4
0.20	0.45	0.15	0.20

Using formulas 2.1-2.7 the calculation results on data based on Table 10 and Table 11 and rank order for group decision making are shown in Table 12.

Table 12

Calculation results for group decision making (1 method)

Alternatives	The separation of alternatives to positive ideal one	The separation of alternatives to negative ideal one	The relative closeness of alternatives to the positive ideal one	Rank
A_1	0.107701	0.126396	0.539930	**2**
A_2	0.112581	0.102844	0.477401	**4**
A_3	0.088631	0.127144	0.589244	**1**
A_4	0.108991	0.107566	0.496711	**3**
A_5	0.141512	0.094110	0.399412	**5**

CASE 3. Rank the preference order for group decision makers (2 method)

Using formulas 2.8-2.14 in the case of the Euclidean metric ($p = 2$) and arithmetic mean (formula 2.13) the calculation results on data based on Table 5 and Table 7 and rank order for group decision making are shown in Table 13.

Table 13

Calculation results

Alternatives	The separation of alternatives to positive ideal one	The separation of alternatives to negative ideal one	The relative closeness of alternatives to the positive ideal one	Rank
A_1	0.250522	0.222860	0.470783	**3**
A_2	0.253325	0.199743	0.440868	**4**
A_3	0.210667	0.255295	0.547889	**1**
A_4	0.235766	0.229436	0.493196	**2**
A_5	0.273169	0.211055	0.435863	**5**

Remark 1. Let us observe that we obtain different rank order in Case 2 and Case 3.

Example 2.

A firm intends to choose the best offer (or ranking of the offers) from the set of three proposals submitted by potential contractors. As in example 1, two experts evaluate each proposal using the same four criteria: deadline of payment after receipt of the goods (in days), unitary price (in euro), conditions of warranty and contractor's reputation, C_1, C_2, C_3, C_4 respectively. The criteria C_1, C_3, C_4 are benefit criteria, greater values being better, and C_2 is the cost criterion, smaller values being better. Criteria C_3, C_4 are subjectively evaluated by the experts on basic available information and they are considered now as linguistic variables, and the other criteria are scaled using interval data, respectively. This is shown in Table 14.

Table 14

The interval decision matrix for two decision makers

	C_1		C_2		C_3	C_4
	$\underline{x}_{i1}^{1}$	$\overline{x}_{i1}^{1}$	$\underline{x}_{i2}^{1}$	$\overline{x}_{i2}^{1}$		
D_1						
A_1	0	7	20	22	P	P
A_2	7	14	22	24	G	MG
A_3	14	21	24	26	MP	F
	$\underline{x}_{i1}^{2}$	$\overline{x}_{i1}^{2}$	$\underline{x}_{i2}^{2}$	$\overline{x}_{i2}^{2}$		
D_2						
A_1	0	7	20	22	P	MP
A_2	7	14	22	24	MP	P
A_3	14	21	24	26	MP	MP

Based on Table 2, the decision matrixes of two decision makers are obtained (Table 15).

Table 15

The interval decision matrix for two decision makers

	C_1		C_2		C_3		C_4	
	$\underline{x}_{i1}^{1}$	$\overline{x}_{i1}^{1}$	$\underline{x}_{i2}^{1}$	$\overline{x}_{i2}^{1}$	$\underline{x}_{i3}^{1}$	$\overline{x}_{i3}^{1}$	$\underline{x}_{i4}^{1}$	$\overline{x}_{i4}^{1}$
D_1								
A_1	0	7	20	22	1	3	1	3
A_2	7	14	22	24	6	9	5	6
A_3	14	21	24	26	3	4	4	5

Table 15 contd.

	$\underline{x}_{i1}^{2}$	$\overline{x}_{i1}^{2}$	$\underline{x}_{i2}^{2}$	$\overline{x}_{i2}^{2}$	$\underline{x}_{i3}^{2}$	$\overline{x}_{i3}^{2}$	$\underline{x}_{i4}^{2}$	$\overline{x}_{i4}^{2}$
D_2								
A_1	0	7	20	22	1	3	3	4
A_2	7	14	22	24	3	4	1	3
A_3	14	21	24	26	3	4	3	4

Based on linguistic variables the evaluation values of attribute weight for each decision maker can be obtained and the results are shown in Table 16.

Table 16

Criteria weights for two decision makers

	C_1	**C_2**	**C_3**	**C_4**
D_1				
	ML	VVH	M	L
D_2				
	ML	VVH	VL	L

The normalized criteria weights for two decision makers are shown in Table 17.

Table 17

Normalized criterion weights for two decision makers

	C_1	**C_2**	**C_3**	**C_4**
D_1				
	0.196	0.413	0.239	0.152
D_2				
	0.225	0.475	0.125	0.175

CASE 1: Rank the preference order for individual decision makers

Using formulas 3.1-3.9 the calculation results on data from Table 15 and Table 17 and rank order for each decision maker are shown in Table 18 and Table 19, respectively.

Table 18

Calculation results for 1-decision maker

Alternatives	The separation of alternatives to positive ideal one	The separation of alternatives to negative ideal one	The relative closeness of alternatives to the positive ideal one	Rank
A_1	0.266630	0.084412	0.240462	**3**
A_2	0.121513	0.230392	0.654700	**1**
A_3	0.169564	0.191156	0.529928	**2**

Table 19

Calculation results for 2-decision maker

Alternatives	The separation of alternatives to positive ideal one	The separation of alternatives to negative ideal one	The relative closeness of alternatives to the positive ideal one	Rank
A_1	0.195026	0.118375	0.377710	**3**
A_2	0.141770	0.141991	0.500388	**2**
A_3	0.084299	0.211994	0.715488	**1**

CASE 2. Rank the preference order for group decision makers (1 method)

The decision matrix is calculated using formula (4.1) and attributes weights of the criteria using (4.2). The results are shown in the Table 20 and the Table 21, respectively.

Table 20

Decision table for group decision making

	C_1		C_2		C_3		C_4	
	$\underline{x}_{i1}^{1}$	x_{i1}^{-1}	$\underline{x}_{i2}^{1}$	x_{i2}^{-1}	$\underline{x}_{i3}^{1}$	x_{i3}^{-1}	$\underline{x}_{i4}^{1}$	x_{i4}^{-1}
A_1	0	7	20	22	1	3	2	3,5
A_2	7	14	22	24	4,5	6,5	3	4,5
A_3	14	21	24	26	3	4	3,5	4,5

Table 21

Normalized criteria weights for group decision makers

C_1	C_2	C_3	C_4
0.2105	0.4440	0.1820	0.1635

Using formulas 3.1-3.9 the calculation results on data based on Table 20 and Table 21 and rank order for group decision making are shown in Table 22.

Table 22

Calculation results for group decision making (1 method)

Alternatives	The separation of alternatives to positive ideal one	The separation of alternatives to negative ideal one	The relative closeness of alternatives to the positive ideal one	Rank
A_1	0.217917	0.087577	0.286673	**3**
A_2	0.122574	0.172678	0.584850	**2**
A_3	0.110306	0.194664	0.638305	**1**

CASE 3. Rank the preference order for group decision making (2 method)

Using formulas 3.10-3.16 in the case of the Euclidean metric (formula 3.13*-3.14*) and arithmetic mean (3.15) the calculations results on data based on Table 15 and Table 17 and rank order for group decision making are shown in Table 23.

Table 23

Calculation results for group decision making (2 method)

Alternatives	The separation of alternatives to positive ideal one	The separation of alternatives to negative ideal one	The relative closeness of alternatives to the positive ideal one	Rank
A_1	0.224004	0.125446	0.358981	**3**
A_2	0.125382	0.223736	0.640861	**2**
A_3	0.173915	0.322996	0.650008	**1**

Remark 2. Let us observe that we obtain the same rank order in Case 2 and Case 3.

Remark 3. The TOPSIS method presents a universal methodology and a simplified practical model for ordering and choosing offers in buyer-seller exchange. This indicator system and evaluation model can be used widely in the area of bargaining process which is usually complex and uncertain. Negotiators have to consider qualitative issues such as price, time of payments, as well as quantitative ones such as reputation, power of negotiation, relationships between sides and so on. Moreover, human thinking is imprecise, lack of information, imprecision and evaluations are always restricted by some objective factors. The concept of the TOPSIS method is clear, the calculation is simple and convenient and the methodology can be extended and adjusted to specific environments. According to the TOPSIS analysis results, a negotiator can choose the most effective alternative that is possible to implement. The decision maker's evaluation could be based on linguistic variables, crisp or interval data. The example of the practical application proves that this method is efficient and feasible.

Concluding remarks

There are a variety of multiple criteria techniques to aid selection in conditions of multiple-criteria problems. One of them is the TOPSIS method, where the ranking of alternatives is based on the relative similarity to the ideal solution, which avoids the situation of having the same similarity index to both positive ideal and negative ideal solutions.

The TOPSIS method is a practical and useful technique for ranking and selecting alternatives. In this paper we focused mainly on the concept of the TOPSIS algorithm for crisp and interval data. An extension of the TOPSIS technique to a group decision environment was also investigated.

The high flexibility of the TOPSIS concept is able to accommodate further extensions to make best choices in various situations. Practically, TOPSIS and its modifications are used to solve many theoretical and real-world problems. In addition, the preferences of more than one decision makers can be also aggregated into the TOPSIS procedure. The classical TOPSIS have been extended according to the requirements of different real-world decision making problems providing support for interval or fuzzy criteria, interval or fuzzy weights to modeled imprecision, uncertainty, lack of information or vagueness, such as TOPSIS with interval data, Fuzzy TOPSIS, Fuzzy AHP and TOPSIS and group TOPSIS.

In the TOPSIS model based on the theory of fuzzy sets the rating of each alternative is expressed in triangular or trapezoidal fuzzy numbers, the weight of each criterion is represented by fuzzy or crisp values, and different normalization (for instance Euclidean, linear or others) are used[*]. The normalized fuzzy numbers can be calculated by using the concept of α-*cuts* [Jahanshaholoo, Lofti, Izadikhah, 2006b]. The TOPSIS model based on the intuitionistic fuzzy set (IFS) allows also to measure the degree of satisfiability and the degree of non-satisfiability, respectively, of each alternative evaluated across a set of criteria [Hung, Chen, 2009; Saghafian, Hejazi, 2005]. The hierarchical TOPSIS method is developed to benefit both from the superiority of the hierarchical structure of AHP and ease of implementation of TOPSIS method [Kahraman, Buyukozkan, Ates 2007; Chiang, Cheng, 2009].

In Polish literature, among many applications, the TOPSIS method (to rank objects) and analytical hierarchy process (to calculate weight of criteria) was employed to assess the socioeconomic development of rural Wielkopolska seen as a collection of counties [Łuczak, Wysocki, 2006], the fuzzy TOPSIS method based on α-level sets was employed to assess the level of people life in chosen counties in Wielkopolska Province [Łuczak, Wysocki, 2008], TOPSIS methods for crisp and interval data were used for ordering offers in buyer-seller transactions [Roszkowska 2009].

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Sebastian Sitarz

COMPROMISE HYPERSPHERE FOR STOCHASTIC DOMINANCE MODEL

Abstract

The aim of the work is to present a method of ranking a finite set of discrete random variables. The proposed method is based on two approaches: the stochastic dominance model and the compromise hypersphere. Moreover, a numerical illustration of the method presented is given.

Keywords

Stochastic dominance, compromise programming, multiple criteria optimization.

Introduction

This paper presents a method of ranking a finite set of discrete random variables. The method is based on one of the multiple criteria methods: the compromise hypersphere, Gass and Roy [2003]. The source of the compromise hypersphere is the compromise programming, Charnes and Cooper [1957], Zeleny [1982]. Adaptations of the compromise hypersphere, in optimization with random variables, are based on stochastic dominance, Levy [1992]. The proposed method consists of the following steps:

Step 1. Establish feasible decisions and corresponding random variables.

Step 2. Compute nondominated random variables in the sense of stochastic dominance.

Step 3. Find the compromise hypersphere.

Step 4. Build a ranking of nondominated random variables using the compromise hypersphere.

Our paper consists of four sections: Section 1 presents a description and properties of the compromise hypersphere; in Section 2 a model of stochastic dominance is considered; Section 3 presents the four steps of the method in detail and the numerical illustration of the proposed algorithm is presented in section 4. The paper concludes with remarks and suggestions for further research.

1. Compromise hypersphere

The presented method originates in the work of Gass and Roy [2003]. The aim of this method is to rank the finite set of nondominated vectors $\mathbf{y}^1 \in R^n, \dots, \mathbf{y}^m \in R^n$. In detail, the method looks as follows:

1. Solve the program:

$$\min_{\mathbf{y}^0, r_0} \max_{i=1,\dots,m} \left| r_0 - d\left(\mathbf{y}^i, \mathbf{y}^0\right) \right|, \tag{1}$$

where

d: $R^n \times R^n \to R$ denotes the distance between two vectors.

We denote the optimal solution of (1) by $\overline{\mathbf{y}^0}$, $\overline{r^0}$ and the minimal value of the cost function as $\overline{\min(1)}$.

2. Find the ranking of the points $\mathbf{y}^1, \mathbf{y}^2, \dots, \mathbf{y}^m$ based on the distances:

$$| \overline{r^0} - d(\overline{\mathbf{y}^0}, \mathbf{y}^i)| \qquad i = 1, \dots, m. \tag{2}$$

In particular, we look for the point $\mathbf{y}^i$ closest to the hypersphere:

$$\min_{i=1,\dots,m} | \overline{r^0} - d(\overline{\mathbf{y}^0}, \mathbf{y}^i)| . \tag{3}$$

Remark 1

Problem (1) is to find a hypersphere with the centre $\mathbf{y}^0 \in R^n$ and the radius $r_0 \in R$ with a minimal distance from the set $\{\mathbf{y}^1, \mathbf{y}^2, \dots, \mathbf{y}^m\}$.

Remark 2

In problem (1) one can use the well known family of metrics $l^p : R^n \times R^n \to R$ as the function d with the parameter $p \in [1,\infty]$. The function $l^p : R^n \times R^n \to R$ is described as follows:

$$l^p\left(\mathbf{y},\mathbf{z}\right)=\begin{cases}\left(\sum_{j=1}^{n}\left|y_j-z_j\right|^p\right)^{\frac{1}{p}}, & p\in[1,\infty)\\ \max\limits_{j=1,\dots,n}\left|y_j-z_j\right|, & p=\infty\end{cases}$$

where $\mathbf{y}=(y_1,\dots,y_n)\in R^n$, $\mathbf{z}=(z_1,\dots,z_n)\in R^n$.

Remark 3

In general, problem (1) is a complicated optimization problem and we use genetic algorithms to solve it, Koza [1992, 1994].

Remark 4

Problem (3) is to find the point closest to the hypersphere found in step 1. Problems (2) and (3) are trivial; it is enough to compare n numbers, used in step 1.

2. Stochastic dominance

In this section, we use the first order stochastic dominance, Shaked and Shanthikumar [1993], Ogryczak and Ruszczynski [1999].

The relation of the first order stochastic (FSD) dominance is defined as follows:

$$\xi_1 \leq_{\mathrm{FSD}} \xi_2 \Leftrightarrow \forall_{x\in \mathrm{R}}\ F_{\xi_1}\left(x\right) \geq F_{\xi_2}\left(x\right),$$

where $F_\xi(x)=\mathbb{P}(\xi\leq x)$ is the right-continuous cumulative distribution function of the random variable ξ. We consider the family of discrete random variables $\{\xi_i : i=1,2,\dots,m\}$. Moreover, we assume that the following set:

$$\mathrm{X}=\{x\in R\colon \exists_{i\in\{1,2,\dots,m\}}\ \mathrm{P}(\xi_i=x)>0\}$$

is finite. It means that we are able to enumerate the elements of the set X in the following way:

$$\mathrm{X}=\{x_1,x_2,\dots,x_{\mathrm{n}}\},$$

Where $x_s<x_t$ for $s<t$.

We call ξ^* a nondominated random variable in set $\Omega=\{\xi_i\colon i=1,2,\dots,m\}$ in the sense of FSD if

$$\neg \exists_{\xi \in \Omega} \quad \xi^* \leq_{FSD} \xi \wedge F_\xi \neq F_{\xi^*}\ .$$

We build the vector $\mathbf{y}^i$ connected with discrete random variables ξ_i in the following way:

$$\mathbf{y}^i = [\, y_1^i,\ y_2^i\ , \ldots ,\ y_n^i\,] = [\, F_{\xi_i}(x_1), F_{\xi_i}(x_2), \ldots , F_{\xi_i}(x_n)\,].$$

In this case the FSD relation has the following form:

$$\xi_1 \leq_{FSD} \xi_2 \ \Leftrightarrow\ \mathbf{y}^1 \geq \mathbf{y}^2 \ \wedge\ \mathbf{y}^1 \neq \mathbf{y}^2.$$

Some additional aspects of FSD models one can find in papers by Ogryczak [2002] and Ogryczak and Romaszkiewicz [2001].

3. Method of ranking

The aim of the proposed procedure is to choose a decision from a finite set of decisions. The returns of decisions are described by means of random variables. The method is based on the stochastic order and the compromise hypersphere method. The procedure looks as follows:

Step 1. Establish feasible decisions with corresponding random variables $\{\xi_i : i = 1, 2, \ldots , m\}$ and the right-continuous cumulative distribution function.

We obtain:

$$\mathbf{y}^1, \mathbf{y}^2, \ldots , \mathbf{y}^m,$$

where $\mathbf{y}^i = [\, y_1^i,\ y_2^i , \ldots ,\ y_n^i\,] = [\, F_{\xi_i}(x_1), F_{\xi_i}(x_2), \ldots ,\ F_{\xi_i}(x_n)\,]$.

Step 2. Compute nondominated vectors in the set $\{\mathbf{y}^1, \mathbf{y}^2, \ldots , \mathbf{y}^m\}$ in the sense of minimalization, i.e. $\mathbf{y}^j$ is nondominated if

$$\neg \underset{i=1,\ldots,m}{\exists} \mathbf{y}^j \leq \mathbf{y}^i \ \wedge\ \mathbf{y}^j \neq \mathbf{y}^i \quad .$$

We obtain

$$\mathbf{y}^{i_1}, \mathbf{y}^{i_2}, \ldots , \mathbf{y}^{i_{kp}} \quad (p \leq m).$$

The above vectors are connected with the nondominated random variables in the sense of FSD.

Step 3. Solve problem (1) for $\mathbf{y}^{i_1}, \mathbf{y}^{i_2}, \ldots , \mathbf{y}^{i_p}$.

Step 4. Use values (2) to obtain the ranking of $\mathbf{y}^{i_1}, \mathbf{y}^{i_2}, \ldots , \mathbf{y}^{i_p}$ and corresponding nondominated random variables in the sense of FSD.

4. Example

Step 1. Let us consider a set of seven discrete random variables:

$$\xi_i, \quad i \in \{1, 2, \ldots, 10\}.$$

The probabilities characterizing these random variables are presented in table 1.

Table 1

Description of random variables

	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6	ξ_7	ξ_8	ξ_9	ξ_{10}
$P(\xi_i=0)$	0	0.3	0.4	0.1	0	0.2	0.2	0.1	0.4	0.1
$P(\xi_i=1)$	0	0.1	0	0.4	0.5	0.3	0.1	0.6	0	0.4
$P(\xi_i=2)$	1	0.1	0	0.3	0.4	0.1	0.4	0.1	0.4	0.4
$P(\xi_i=3)$	0	0.5	0.6	0.2	0.1	0.4	0.3	0.2	0.2	0.1

Vectors $\mathbf{y}^i$ built for the random variables considered are presented in table 2.

Table 2

Vectors $\mathbf{y}^i$ for considered random variables

$\mathbf{y}^1$	$\mathbf{y}^2$	$\mathbf{y}^3$	$\mathbf{y}^4$	$\mathbf{y}^5$	$\mathbf{y}^6$	$\mathbf{y}^7$	$\mathbf{y}^8$	$\mathbf{y}^9$	$\mathbf{y}^{10}$
0	0.3	0.4	0.1	0	0.2	0.2	0.1	0.4	0.1
0	0.4	0.4	0.5	0.5	0.5	0.3	0.7	0.4	0.5
1	0.5	0.4	0.8	0.9	0.6	0.7	0.8	0.8	0.9
1	1	1	1	1	1	1	1	1	1

Step 2. Compute the nondominated vectors in the set $\{\mathbf{y}^1, \mathbf{y}^2, \ldots, \mathbf{y}^{10}\}$. The nondominated vectors are as follows:

$$\{\mathbf{y}^1, \mathbf{y}^2, \mathbf{y}^3, \mathbf{y}^4, \mathbf{y}^5, \mathbf{y}^6, \mathbf{y}^7\}.$$

We denote the set of indices of the nondominated vectors by N, i.e.: $N = \{1, 2, 3, 4, 5, 6, 7\}$.

Step 3. By solving problem (1) with the set $\{\mathbf{y}^i\colon\ i \in N\}$ and $d = l^2$:

$$\min_{\mathbf{y}^0, r_0} \max_{i \in N} \left| r_0 - \sqrt{\sum_{j=1}^{4} \left(y_j^0 - y_j^i \right)^2} \right|,$$

we obtain the following optimal solution:

$$\overline{\mathbf{y}^0} = (-0.73808;\ 0.05522; -0.02151; -2.39575), \quad \overline{r^0} = 3.61347$$

and the minimal value of the cost function:

$$\overline{\min(1)} = 0.00908.$$

Step 4. By solving problem (3)

$$\min_{i \in N} \left| \overline{r^0} - \sqrt{\sum_{j=1}^{4} \left(\overline{y_j^0} - y_j^i \right)^2} \right|,$$

we obtain values (as distances between points and the hypersphere) shown in Table 3. Moreover, Table 3 presents the ranking based on these values.

Table 3

Ranking for $d = l^2$

	$\mathbf{y}^1$	$\mathbf{y}^2$	$\mathbf{y}^3$	$\mathbf{y}^4$	$\mathbf{y}^5$	$\mathbf{y}^6$	$\mathbf{y}^7$
$\left\| \overline{r^0} - \sqrt{\sum_{j=1}^{4} \left(\overline{y_j^0} - y_j^i \right)^2} \right\|$	0.00902	0.00798	0.00908	0.00678	0.00908	0.00858	0.00908
Ranking	4	2	5	1	5	3	5

Conclusions and further research

In this paper we have proposed a method of ranking discrete random variables. We have used two approaches: the stochastic dominance and the compromise hypersphere. In future, the following aspects of the presented method are worth studying: comparing with other methods of random variables ranking, the case of continuous random variables, an interactive version of the method, analysis of the method for different metrics *d*, applications to real life problems.

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Tomasz Wachowicz

APPLICATION OF TOPSIS METHODOLOGY TO THE SCORING OF NEGOTIATION ISSUES MEASURED ON THE ORDINAL SCALE

Abstract

The aim of this paper is to apply TOPSIS method for negotiation support. The support we focus on concerns the pre-negotiation preparation and the process of negotiation template evaluation, which results in building a scoring system for the negotiation offers. Since the negotiation template may contain different types of criteria (negotiation issues), both quantitative (price, time) and qualitative (verbal description of warranty), the mechanisms of measuring distances for different types of data need to be incorporated into TOPSIS scoring procedure. We will use GDM (generalized distance measure) for interval and ordinal data. For weakly structured negotiation templates an alternative approach is proposed, one that does not use pair-wise comparisons of the evaluated alternatives. To illustrate the performance of TOPSIS in negotiation support we present a numerical example of business negotiations.

Keywords

Negotiation support, negotiation template, preference elicitation, TOPSIS, interval scale, generalized distance measure.

Introduction

Many researchers and negotiation practitioners emphasize that the strategic element of negotiations, that influence the following process of exchanging offers and outcomes, is negotiation preparation that should be conducted within the pre-negotiation phase [Thompson 1998, Lewicki et al., 1999]. One of the key elements of the pre-negotiation phase is negotiation template building [Raiffa et al., 2002]. Negotiation template specifies the structure of the potential decision problem negotiators face. It contains the definition of the issues under consideration (equivalent to criteria defined in decision making problem) and options (potential resolution levels defined for each criterion). A well defined negotiation template helps negotiator to identify the negotiation space and support them in searching the compromise.

The negotiation template should be built jointly by negotiators during the pre--talks conducted in the pre-negotiation phase. However, some negotiation problems may be too complicated or the negotiators may wish not to reveal any of their position or goals, so the template cannot be well defined. No matter how well the template is defined, it should be scored, which will help negotiators to evaluate the offers proposed later in actual negotiation phase. The offer scoring process corresponds in fact to the negotiator's preference elicitation, therefore typical multiple attribute decision making procedures and algorithms are usually proposed to score the template. The additive scoring model [Keeney and Raiffa, 1976] is most often used. It has already been successfully applied in electronic negotiation support systems, such as Inspire [Kersten and Noronha, 1999], SmartSettle [Thiessen and Soberg, 2003], Negoisst [Schoop et al., 2003] or NegoCalc [Wachowicz 2008] and used for supporting real world problems, such as First Nations Negotiations in Canada [Thiessen and Shakun, 2009]. The additive scoring model is methodologically a simple tool, but it requires from decision makers (negotiators) the definition of their preferences for each single resolution level (issue option), that can be used for building the decision alternatives (offers). It is easy to conclude that for large decision problems the multitude of the score assignments may be tiresome and discouraging for decision maker. Therefore other methods for scoring a negotiation template are proposed. AHP [Saaty, 1980] is suggested frequently as an alternative to the additive scoring model [Mustajoki and Hamalainen, 2000; Wachowicz, 2008a]. In AHP the preference elicitation approach is different and is based on pair-wise comparisons of all atomic elements of the decision process and the subsequent preference aggregation. For large decision problems it may be, however, as tiresome as the additive scoring model and may result in ranking reversal if the negotiation space changes. Other methods and models have been also proposed for scoring templates, such as rule-based models [Chen et al., 2004], simulation [Matwin et al., 1989] or ELECTRE-TRI [Wachowicz, 2010], but all of them require either professional mathematical (or decision making) knowledge of negotiators or very complicated calculations that make the elicitation process not transparent to the decision maker.

In this paper we propose an alternative approach for elicitation of the negotiator's preferences that allows for scoring the negotiation template quickly and reduce the negotiator's workload and involvement in the scoring process. It is based on a straightforward statistical method and calculates the offers scores using their distances from the ideal and negative ideal solutions. The approach is based mostly on TOPSIS [Hwang and Yoon, 1981], however, the method needs to be modified to allow the ordinal variables (issues) to be taken into account. In this modification the notion of a generalized distance measure [Walesiak, 2002] and measuring distances for various types of variables [Bock and Diday, 2000] is mainly used. Two alternative procedures GDM-TOPSIS and TOPSIS-WDT are proposed for evaluating well and weakly defined templates respectively. In the following sections we give a brief review of TOPSIS (Section 2) and propose the TOPSIS modifications (Section 2). Then an algorithm for negotiator's preference elicitation is proposed (Section 3) and the examples of GDM-TOPSIS and TOPSIS-WDT algorithm are presented. We conclude with some final remarks and future work required.

1. Foundations of TOPSIS

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) was developed by Hwang and Yoon [1981] and later widely described with its modifications and adjustments [see Hwang et al., 1993; Lai et al., 1994]. It allows to build a ranking of alternatives described by a number of criteria. The underlying principle of TOPSIS is a bipolar comparison of each alternative under consideration with both the positive ideal (PIS) and the negative ideal (NIS) solutions. The distances to these two solutions are calculated for each alternative and then the aggregated criterion is built that combines these two factors and describes the quality of each alternative, assuming that the chosen alternative should have the shortest distance to the ideal solution and the farthest distance to the negative ideal one.

To conduct TOPSIS analysis we assume that the decision making problem is presented in the form of a matrix:

	C_1	C_2	…	C_n
A_1	x_{11}	x_{12}	…	x_{1n}
A_2	x_{21}	x_{22}	…	x_{2n}
…	…	…	…	…
A_m	x_{m1}	x_{m2}	…	x_{mn}

where A_j describes the alternative j under consideration ($j = 1, \dots , m$), C_k describes the criterion k for measuring the alternatives' performance ($k = 1, \dots , n$) and x_{jk} is the resolution level (performance) of alternative A_j with respect to criterion C_k. Furthermore, the criteria importance is specified in the form of a vector of weights $w = (w_1, w_2, \dots , w_n)$, where $\sum_{k=1}^{n} w_k = 1$. Let us assume, for each criterion C_k, without loss of generality, that a higher value of the alternative's performance is more preferred by the decision maker. In other words, we face the problem of vector maximization.

Having the decision making problem described as above, we can conduct the TOPSIS analysis for building the ranking of the alternatives. The TOPSIS algorithm consists of six subsequent steps:

1. Building the normalized decision matrix:

$$N = \begin{bmatrix} \widehat{x}_{11} & \widehat{x}_{12} & \cdots & \widehat{x}_{1n} \\ \widehat{x}_{21} & \widehat{x}_{22} & \cdots & \widehat{x}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \widehat{x}_{m1} & \widehat{x}_{m2} & \cdots & \widehat{x}_{mn} \end{bmatrix} \tag{1}$$

where:

$$\widehat{x}_{jk} = \frac{x_{jk}}{\sqrt{\sum_{j=1}^{m} x_{jk}^{2}}}, \tag{2}$$

for $j = 1, \ldots, m$ and $k = 1, \ldots, n$*.

2. Computing the weighted normalized decision matrix:

$$V = \begin{bmatrix} w_1\widehat{x}_{11} & w_2\widehat{x}_{12} & \cdots & w_n\widehat{x}_{1n} \\ w_1\widehat{x}_{21} & w_2\widehat{x}_{22} & \cdots & w_n\widehat{x}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ w_1\widehat{x}_{m1} & w_2\widehat{x}_{m2} & \cdots & w_n\widehat{x}_{mn} \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix}. \tag{3}$$

3. Determining the positive ideal (A^+) and negative ideal (A^-) solutions:

$$A^+ = (v_1^+, v_2^+, \ldots, v_n^+), \text{ where } v_k^+ = \max_j(x_{jk}), \text{ for } k = 1, 2, \ldots, n \tag{4}$$

$$A^- = (v_1^-, v_2^-, \ldots, v_n^-), \text{ where } v_k^- = \min_j(x_{jk}), \text{ for } k = 1, 2, \ldots, n \tag{5}$$

4. Calculating the separation measures (distance) for each alternative from PIS (d_j^+) and NIS (d_j^-) respectively:

$$d_j^+ = \sqrt[p]{\sum_{k=1}^{n} \left| v_{jk} - v_k^+ \right|^p}, \text{ for } j = 1, 2, \ldots, m \tag{6}$$

$$d_j^- = \sqrt[p]{\sum_{k=1}^{n} \left| v_{jk} - v_k^- \right|^p}, \text{ for } j = 1, 2, \ldots, m \tag{7}$$

---

* Apart from the above vector normalization procedure other normalization procedures are also proposed, such as different types of linear normalization or non-monotonic normalization and their effects on the final ranking results is studied [see Hwang and Yoon, 1981, Milani et al., 2005]. One of them will be proposed later in Section 2.2.

where p is the distance coefficient. Usually, the Euclidean distance is used in TOPSIS analysis, for which $p = 2$*.

5. Determining the relative closeness of each alternative to the ideal solution:

$$S_j = \frac{d_j^-}{d_j^+ + d_j^-}, \text{ for } j = 1, 2, \ldots, m. \tag{8}$$

where $0 \le S_j \le 1$. The closer the alternative A_j to PIS is, the larger the value of S_j.

6. Ranking the alternatives in descending order using S_j.

As can be derived from the above algorithm, to use the TOPSIS effectively the problem under consideration should be well structured and described with quantitative data. What is more, the criteria must use strong scales (such as ratio and interval ones), for which measuring distances according to the Minkowski formulas (6) and (7) may be applied only**. However, in the negotiation process some issues (criteria) may be described qualitatively or even verbally. For instance, in business negotiation the details of the warranty or returns may be such a complex issue that the full written returns policy (a few-pages-long text) is perceived as a resolution level. Negotiators are usually able to build a preorder for these resolution levels, indicating the best one (scored as 1), the second best (scored as 2), etc., but the distances between the numbers that reflect the order cannot be interpreted. Therefore another method for measuring distances for weak-scale data must be incorporated, if TOPSIS is going to be used for negotiation support.

2. TOPSIS and the problem of measuring distances for variables on ordinal scale

2.1. Generalized Distance Measure (GDM)

If the negotiation template was well discussed by negotiators in pre--negotiation phase and may be perceived as fixed and stable (no options are expected to be introduced later within the negotiation process) another approach for measuring distances between PIS and NIS may be applied. The notion of Generalized Distance Measure (GDM) may be used for calculating distances for different types of data. Generalized distance measure was proposed first

* Other metrics are also proposed such as the Manhattan or Tchebycheff ones or even the weighted L_p metrics [see Jones and Mardle, 2004].

** Since addition and subtraction are mathematical operations that cannot be applied to the ordinal or nominal scales.

by Walesiak [2002]* who based his idea on the conception presented in a research book by Bock and Diday [2000]. GDM is based mainly on the notion of generalized correlation coefficient, which derives from Pearson linear correlation coefficient and Kendall tau rank correlation coefficient. GDM is given by the formula

$$d_{yz}^{GDM} = \frac{1}{2} - \frac{\sum_{k=1}^{n} w_k a_{yzk} b_{zyk} + \sum_{k=1}^{n} \sum_{\substack{j=1 \\ j \neq x,y}}^{m} w_k a_{yjk} b_{zjk}}{2\left[\sum_{k=1}^{n}\sum_{j=1}^{m} w_k a_{yjk}{}^2 \cdot \sum_{k=1}^{n}\sum_{j=1}^{m} w_k b_{zjk}{}^2\right]^{\frac{1}{2}}}, \quad \text{for } j = 1, 2, \dots, m \tag{9}$$

where:

d_{yz}^{GDM} is a distance measure between objects (alternatives) A_y and A_z, $d_{yz}^{GDM} \in [0;1]$,

w_k is a weight of k-th variable (criterion): $w_k \in (0;m) \wedge \sum_{k=1}^{n} w_k = m$,

a_{yzk} and b_{zyk} are the distance indicators between objects (alternatives) A_y and A_z, for criterion k, and are calculated differently, depending on a type scale the criterion is measured with.

For ratio and interval variables the distance indicators are calculated intuitively using the following formulas

$$a_{y\alpha k} = x_{yk} - x_{\alpha k}, \quad \text{for } \alpha = z, j, \tag{10}$$

$$b_{z\beta k} = x_{zk} - x_{\beta k}, \quad \text{for } \beta = y, j. \tag{11}$$

For ordinal scale, for which the inequality statements for the objects compared (such as the state of being equal, grater or less than) may only be counted, Walesiak proposes to determine the distance indicator in the following way

$$a_{y\alpha k}(b_{z\beta k}) = \begin{cases} 1 \text{ for } x_{yk} > x_{\alpha k} \, (x_{zk} > x_{\beta k}) \\ 0 \text{ for } x_{yk} = x_{\alpha k} \, (x_{zk} = x_{\beta k}), \\ -1 \text{ for } x_{yk} < x_{\alpha k} \, (x_{zk} < x_{\beta k}) \end{cases} \tag{12}$$

for $\alpha = z, j$ and $\beta = y, j$.

* GDM was described first by Walesiak [2002] originally in Polish. The detailed analysis of GDM and its properties was published later in English in the research paper by [Jajuga et al., 2003].

Walesiak proposes also the formulas for determining the distance indicators for the nominal scale variables. However, since we assumed that our negotiator is always able to define his general preferences for the qualitative issues by building a preorder of the options we will not use the nominal issues in the negotiation template.

Applying GDM to TOPSIS analysis requires two small changes in the general algorithm (Section 1). In the first step of the algorithm, the normalized matrix N should be computed for metric data only. Secondly, we omit step 2, since the variables will be weighted while calculating the GDM distance (formula (9)). Finally, in the step 4 while calculating d_j^+ and d_j^- the equations (6) and (7) need to be replaced with the GDM formula (9). Such modified TOPSIS algorithm we will call GDM-TOPSIS.

It is easy to conclude, while analyzing the above approach, that GDM--TOPSIS may be applied for negotiation support only if the template does not change within the negotiation process. It is a strong assumption, however, lots of negotiation support systems work with pre-defined fixed templates (such as Inspire [Kersten and Noronha, 1999], SmartSettle [Thiessen and Soberg, 2003]. The TOPSIS-GDM-based scoring system (offers' ranking) is built based on the distance comparisons between all feasible resolution levels that can be distinguished within the template* (see the second component of the addition formula in the numerator and the whole denominator of the equation (9)) therefore any future change in the sets of feasible resolution levels will affect the previous calculations and consequently the final ranking itself. In other words, to keep the scoring system determined by means of TOPSIS and GDM legitimate, only the offers comprised of the predefined (salient) options may be proposed during the negotiation process.

2.2. Alternative approach for weakly defined negotiation templates

Let us assume that the pre-negotiation talks did not lead negotiators to the formulation of a fixed negotiation template. Negotiators were able**, however, to find the negotiation space by defining the maximal and minimal acceptable values for quantitative issues but not for qualitative ones (e.g. returns policy). Each negotiator may have a few pre-defined options for this issue, but the smallest modification within this pre-defined contracts creates in fact another option. While making trade-off within this issue negotiators may create hundreds of versions of such a contract within the actual negotiation phase.

* It is based in fact on the pair-wise comparison of the offers.
** And usually are.

Therefore a special procedure needs to be introduced in the process of scoring the template (option evaluation), that will be insensible to new options that may appear later during the actual negotiation phase and the process of exchanging offers.

In this paper we propose to apply a very simple solution based on the pre--defined and ordered categories (clusters) of options*. We suggest to the negotiators to build the categories of options for each qualitative issue in pre--negotiation phase that will reflect the general quality of all feasible options that may appear in the negotiation process for this issue (e.g. the category of excellent options, the category of very good options, etc.). The number of the categories depends on the expected precision of the scoring system but should not be too big to avoid problems with assigning options to the pre-defined categories. This assignment process will be conducted by the negotiator himself, therefore he should define the optimal number of categories he is able to handle comfortably later on. By applying this approach we move from the verbally defined options (the set of which is not known at the beginning of the negotiation) to the numerically defined ones, while the numbers assigned to the categories are of the ordinal scale.

Since the weakly defined negotiation template (as described above) does not allow to build the set of feasible alternatives, some modifications need to be introduced into TOPSIS algorithm to remove all mathematical operations that require any information about this set of alternatives. First, the whole GDM distance formula needs to be changed, to avoid a calculation of some distance indicators that refer to the set of alternatives (i.e. the multipliers in the denominator of the equation (9)). We will change the Walesiak's formula (9), but we will still keep the general notion he used to build it. Walesiak used the Bock and Diday [2000] approach for measuring distance for ratio, interval, ordinal and nominal variables describes by formula

$$d_{yz} = \frac{w_1 d_{yz}^N + w_2 d_{yz}^O + w_3 d_{yz}^I + w_4 d_{yz}^R}{w_1 + w_2 + w_3 + w_4}, \tag{13}$$

where:

$N(O,I,R)$ is a subset of the nominal (ordinal, interval, ratio) variables under consideration,

$d_{yz}^{N(O,I,R)}$ is a distance calculated for the nominal (ordinal, interval, ratio) variables describing alternatives A_y and A_z,

$w_1(w_2, w_3, w_4)$ is a weight assigned to the nominal (ordinal, interval, ratio) variables.

* Similar categories-based approach for scoring the complete packages of offers by means of calibrated ELECTRE-TRI was previously proposed by Wachowicz [2010].

We will use the formula (13) for calculating the separation measures between alternatives and the PIS and NIS in the fourth step of TOPSIS removing the first component of the addition formula in the numerator of equation (13)*. Since the weights are taken into consideration during the distance aggregation, Step 2 of the original TOPSIS may be omitted here.

For measuring distances we will use the following formula

$$d_{yz}^{O(I,R)} = \frac{|x_{yk} - x_{zk}|}{v_k^+ - v_k^-}, \quad \text{for} \quad k = 1, \dots, n \,. \tag{14}$$

where:

v_k^+, v_k^- are the maximal and maximal values defined by negotiators in weakly structured template for issue *k*.

Originally the measure (14) was proposed only for interval and ratio variables, but we will use the Kaufman and Rousseeuw [1990] rationale, according to which the formula (14) may be also used for ordinal variables. Some authors argue against Kaufman and Rousseeuw proposition, stating that the addition and subtraction are properties of interval and ratio scales only, but in our case – assuming that the negotiators build the option categories that differ by the same value of quality – the above formula may be applied. What is more, using the GDM for measuring the distances within the group of ordinal variables** will result in the same values of distances as the ones obtained with the formula (14). As we are using the formula (14) to calculate distances we do not need to normalize variables, therefore Step 1 of the classic TOPSIS procedure may be omitted.

Since all the above modifications were proposed for negotiation problem with weakly defined negotiation template we will call the whole modified TOPSIS procedure TOPSIS-WDT (TOPSIS-WeaklyDefinedTemplate).

3. Negotiation support for offers evaluation

Here we will summarize the notions presented in Section 3 and describe the procedure for negotiation support for the evaluation of negotiation offers. The procedure represents an asymmetric approach, i.e. it focuses on supporting only one party of the negotiation process. The structured algorithm of the supporting procedure is presented in Figure 1.

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* We assumed there are no ordinal variables in the negotiation template.

** It is legitimate since we previously assumed that negotiators pre-define the quality categories for these variables, so the set of options is known and fixed for this type of variables.

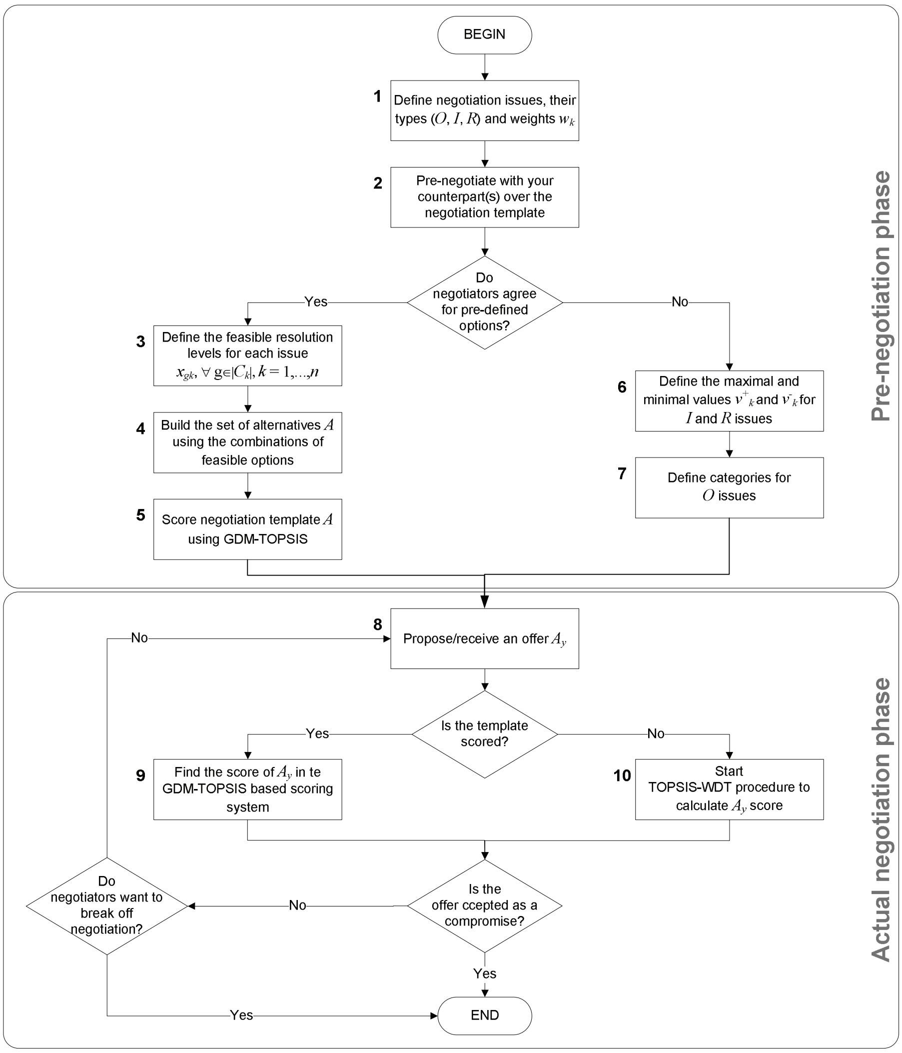

Figure 1. The algorithm for the evaluation of negotiation offers using TOPSIS

The template is defined in the pre-negotiation phase. Template definition begins with the identification of negotiation issues under consideration and the types of variables describing these issues (step 1). The individual, subjective importance of each issue should also be defined by the negotiator in this step in the form of weights. Next the pre-talks between negotiators are conducted to define the structure of the template (step 2). The aim of this step is to agree on the set of feasible resolution levels for each issue, which will remain stable during the negotiation process (no other options will be allowed). If negotiators agree on such pre-defined sets (which make the negotiation template fixed and the problem itself discrete), they specify these sets within step 3. When the sets of resolution level are agreed upon, the set of all feasible alternatives A is created* (step 4). The offers that comprise set A must take into consideration all possible combinations of feasible options defined within step 3. For the set A GDM-TOPSIS calculation procedure is run (step 5), which results in the construction of the negotiation offer scoring system, that may be used in pre-negotiation phase for simulation of the future negotiation process or later, in actual negotiation phase, to evaluate each offer proposed by the counterpart or to construct the negotiator's own proposal of agreement (step 9).

If the template was weakly defined (it is impossible to find the finite sets of feasible options for the issues), the negotiator defines the negotiation space only. In step 6 he sets the maximal and minimal acceptable values for each metric issue defining their feasible ranges. For ordinal issues he defines the categories and orders them from the most to the least preferred ones (step 7). The facilitator should assign numbers to the categories in descending order (i.e. the more preferred the category is the higher score it receives).

After the pre-negotiation actions, the actual negotiation support begins. It starts with the formulation of the offer by the negotiator or his counterparts (step 8). The negotiator now expects to have this offer evaluated. If he operates with fixed and well defined template, previously scored by the GDM-TOPSIS procedure he simply finds the offer proposed on the list of offers scored. He may compare it with the previous offers proposed within the negotiation process (or with his aspiration levels) and find other alternatives that will improve his score. If he operates with the weakly defined template he needs to start the TOPSIS-WDT procedure now and calculate the score of the offer proposed (step 10). The only reference points he has is the ideal offer PIS (of score 1) and the NIS (of score 0), so having the proposed offer scored he may analyze how close it is to PIS and NIS. If the offer is not satisfying he may try to build another one making an intuitive trade-off and score it running the TOPSIS--WDT procedure again. If he is satisfied with the score of the newly composed offer he may send it to his counterpart as an agreement suggestion. The steps 8, 9/10 are repeated until an agreement is set or negotiation is broken off.

The main difference between these two alternative paths in the algorithms is that for the weakly defined template (the right hand path of the algorithm) the offer evaluation process is conducted in the actual negotiation phase, just after the offer was proposed by negotiator. For well defined templates the scoring procedure is conducted in the pre-negotiation phase and later for

* It is a facilitator or negotiation support system task to prepare such a set for negotiators.

any offer proposed by the parties the scoring system is only browsed to find the score of this offer. Thus, when the template scored before the actual negotiation phase, the negotiator sees all the alternatives for agreement and at every stage (round) of negotiations he knows how far from the aspired level he is and what are the offers (complete packages) that may improve his current score. For the weakly defined template the negotiator may only score the current offer but he needs to construct a counteroffer by himself. What is more, this counteroffer will be scored after being constructed, so while building it he is not aware of the scale of concessions he is just making.

4. Example

4.1. GDM-TOPSIS application

Let us consider a simple business-to-business negotiation between a buyer (B) and a seller (S). They want to agree on the contract for new delivery of the components the buyer needs for production process. The negotiator S will be supported by the procedure proposed in the previous section of this paper*.

Step 1.

The negotiators want to agree on three different issues: price (Pr) per unit (in USD), time of delivery (TD) in days, and returns policy (RP). The first two issues are metric, while the last issue is ordinal. S has assigned the following weights to the issues: 0.6, 0.2, 0.2.

Step 2.

The negotiators agreed to prepare a fully defined negotiation template by defining no more than 6 salient options to for each negotiation issue.

Step 3.

The resolution levels defined by the negotiators for each issue are:
– for Pr: {3.60, 4.00, 4.20, 4.50},
– for TD: {30, 40, 60},
– for RP: {“any defects no penalty”, “3% defects no penalty”, “5% defects 2% penalty”, “7% defects 4% penalty”}.

Since we assumed that the negotiators are able to build a preorder on resolution levels of any issue, B must define his preferences over the options of RP. The order (from the most to the least preferred) of the options with the ordinal scores assigned to them by a facilitator is presented in Table 1.

* The case is based on the assignment implied in electronic negotiation support Inspire.

Table 1

Ordinal scores for verbally defined options

	Options		
Order by negotiator	“any defects no penalty”	“3% defects no penalty”	“5% defects 2% penalty”, “7% defects 4% penalty”.
Scores by facilitator	3	2	1

Step 4.

The alternatives are built in the form of complete packages consisting of different combinations of options pre-defined in step 3. In our negotiation problem there is $4 \times 3 \times 4 = 48$ feasible offers that comprise the set A. One of such packages may be specified as $A_1 = (4.50, 60,$ “any defects no penalty”) while another as $A_{20} = (4.20, 40,$ “7% defects 4% penalty”). The full list of the offers is presented in Appendix 1, Table 2.

Step 5.

The set A is scored by means of GDM-TOPSIS procedure:

– RP options are replaced with their numerical equivalents (see Table 1),
– Pr and TD options are normalized using formula (2),
– PIS and NIS are defined: $A^+ = (4.50, 60, 3)$, $A^- = (3.60, 30, 1)$,
– separation measures d_j^+ and d_j^+ are calculated using formula (9) and distance indicators (10) and (11) for Pr and TD issues; and (12) for RP (see Appendix, Table 2),
– relative closeness S_j is calculated for each alternative (see Appendix, Table 2) and the ranking is built (see Appendix, Table 3)*.

Step 8.

An offer is send by B, $A_{26}^B = (4.00, 60,$ “3% defects no penalty”).

Step 9.

Since the negotiation template was well defined, S may now find the score of the offer A_{26}^B. He looks into ranking (Appendix, Table 3) and finds that $S_{26} = 0.63$. Having the template scored S also knows that there are two other offers that satisfy his preferences at the same level of 0.63:

* We used R language (ver. 2.11.0) and `pattern.GDM1()` and `pattern.GDM2()` procedures for determining the distance matrix in Appendix 2.

A_{11} = (4.50, 30, "5% defects 2% penalty"),

A_{12} = (4.50, 30, "7% defects 4% penalty").

If he is satisfied with this score but wishes to obtain a higher price, he may send a counteroffer to B choosing one of the above alternatives. If he expects the compromise to satisfy his preferences at the level no lower than 0.75* he may choose one of the first nine offers from the scored template (Appendix, Table 3).

Analyzing Table 3 (Appendix) he has also the insight into the values of the potential trade-off he may do. Let us assume that his offer A_5^S = (4.50, 40, "any defects no penalty") with the score $S_6 = 0.89$ was rejected by his counterpart and he may consider making small concession. If he decides to give in on TD (moving from 40 to 30 → from A_5 to A_9) his score will fall to the level of 0.78. If he decides to give in on RP (moving from "any defects no penalty" to "5% defects 2% penalty"→ from A_5 to A_6) his score will fall to the level of 0.85 only. Despite the fact that both issues TD and RP have the same weights it is more profitable for S to make a concession on RP, since it "costs" him less than the concession made on TD.

A similar analysis can be conducted in the next rounds of the negotiation process.

4.2. TOPSIS-WDT application

Let us now consider the same negotiation problem as described in Section 4.1, but for a weakly defined template. The steps 1 and 2 of the algorithm remain the same. The procedure now reaches the step 6.

Step 6.

Negotiator S defines the negotiation space by defining the maximal and minimal values for each metric issue: He sets:

– $v_{\mathrm{Pr}}^{+} = 4.50$ and $v_{\mathrm{Pr}}^{-} = 3.60$,
– $v_{TD}^{+} = 60$ and $v_{TD}^{-} = 30$.

* Interpreted on the ratio scale as 75% of satisfaction or referring to offers being at least in 75% as good as the ideal one (PIS).

Step 7.

S defines categories for possible resolution levels of RP. Let us assume that he defines 3 categories of: good, average and weak options. The facilitator assigns the numerical equivalents to the categories: 3, 2, 1, respectively.

Step 8.

An offer is send by B, A^B = (4.00, 60, "3% defects no penalty").

Step 10.

TOPSIS-WDT calculation procedure is started by the facilitator or the negotiator himself. The score of A^B is equal to 0.56. S knows now that A^B is somewhere in the middle between the ideal and the negative ideal solutions. He does not have a scored template, so he can not find other solutions with the same score. If he would like to propose an offer giving him a score of 0.89 (as in previous case) he simply needs to try to improve the resolution levels of each issue intuitively and recalculate the score of the offer using the scoring system.

It is not a problem when NSS supports him and the calculations can be conducted automatically. Despite the fact that there is no well defined template for this negotiation NSS may find for S some equivalents of A^B lowering values of selected criteria and rising the values of others. NSS supports him similarly in making tradeoffs on the selected issues. If S's offer A^S = (4.50, 40, "any defects no penalty"), scored now with 0.87 points, is rejected, NSS may find another solution using a different combination of trade-off for declared concession level. Let us assume that S decided to make a concession of 0.05 scoring points. NSS finds for him such offers of 0.82 score*:

A_1 = (4.30; 53; "any defects no penalty"),

A_2 = (4.50; 48; "3% defects no penalty").

Conclusions

In this paper we have proposed two approaches for negotiation offer evaluation, both based on TOPSIS, as alternatives to the classic scoring systems widely used in negotiation support (such as additive scoring models or AHP-based scoring models). For a well structured template, where all feasible options are defined, GDM-TOPSIS procedure was proposed, whilst for a weakly

* These offers may be easily found by solving simple mathematic programming problem.

structure template, TOPSIS-WDT is suggested. Both procedures derive from the classic TOPSIS algorithm proposed by Hwang and Yoon [1981], but include some formal modifications that allow to analyze the negotiation problem for which ordinal issues were declared by negotiators.

The modified TOPSIS seems to be very effective in scoring a negotiation template. It does not require a tiresome interaction with negotiator to build a negotiation offer scoring system and releases him from an unintuitive assignment of scores to issues and options, but there are some drawbacks of the TOPSIS approach. Since it is based on distance measuring only, it does not take into account a nonlinearity of the negotiator's evaluation function. The negotiator may differently perceive the difference between the resolution levels of one issue, depending on how far these resolutions are from the ideal value of this issue. For instance, alternatives A and B may result in resolution levels 2000 and 1990 for issue x respectively (having the difference of 10 units) and the negotiator may perceive the difference between them as significant. Simultaneously, alternatives C and D may have the same difference of 10 points, but for the resolution levels 20 and 10, respectively. These two numbers may be perceived by the negotiator as equally bad, whilst TOPSIS will assign them different scores (distances) keeping the proportion of the differences for A, B and C, D at the same level. TOPSIS makes the differences between all options equally scored for any decision maker, as if the distance was the only objective measure of preferences. We are sure that there is a great number of scientists and researchers exploring the field of multiple attribute decision making that would not be willing to agree with this approach.

What should be noticed about the application of GDM in TOPSIS procedure, is the dependence of the distances between the ordinal options (their scores) on the number of these options. The distance is measured by the pair-wise comparisons between these options (see numerator in formula (9)). The greater number of options is worse than the hypothetical option o the greater "power" of option o is and the closer it is to the PIS. It is very important for scoring a well defined template, where the number of occurrences of an option for one particular ordinal issue depends on the number of options defined for other issues. The negotiator and facilitator should be aware of the potential problem that this may cause. For the same negotiation problem described by templates with different calibration of metric issues* different scorings may be obtained.

However, all the drawbacks presented above do not change the fact that scoring a negotiation template with TOPSIS is much quicker and less tiresome than using an additive scoring model or an AHP-based scoring model, since the only information we need from the negotiators are the weight coefficients

* E.g. in the first template the price issue will change of 10 cents (between 5 and 10 USD), while in the second it will change of 50 cents.

for the issues defined. Therefore the future work will focus on building a software tool for supporting negotiations according to GDM-TOPSIS and TOPSIS-WDT procedures, and comparing user satisfaction from using the classic scoring models and the TOPSIS-based ones.

Acknowledgements

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Appendix

Tabela 1

List of the feasible offers for B vs. S negotiations

Offer number	Issues		
	Pr	TD	RP
1	4.50	60	any defects no penalty
2	4.50	60	3% defects no penalty
3	4.50	60	5% defects 2% penalty
4	4.50	60	7% defects 4% penalty
5	4.50	40	any defects no penalty
6	4.50	40	3% defects no penalty
7	4.50	40	5% defects 2% penalty
8	4.50	40	7% defects 4% penalty
9	4.50	30	any defects no penalty
10	4.50	30	3% defects no penalty
11	4.50	30	5% defects 2% penalty
12	4.50	30	7% defects 4% penalty
13	4.20	60	any defects no penalty
14	4.20	60	3% defects no penalty
15	4.20	60	5% defects 2% penalty
16	4.20	60	7% defects 4% penalty
17	4.20	40	any defects no penalty
18	4.20	40	3% defects no penalty
19	4.20	40	5% defects 2% penalty
20	4.20	40	7% defects 4% penalty
21	4.20	30	any defects no penalty

Table 1 contd.

Offer number	Issues		
	Pr	TD	RP
22	4.20	30	3% defects no penalty
23	4.20	30	5% defects 2% penalty
24	4.20	30	7% defects 4% penalty
25	4.00	60	any defects no penalty
26	4.00	60	3% defects no penalty
27	4.00	60	5% defects 2% penalty
28	4.00	60	7% defects 4% penalty
29	4.00	40	any defects no penalty
30	4.00	40	3% defects no penalty
31	4.00	40	5% defects 2% penalty
32	4.00	40	7% defects 4% penalty
33	4.00	30	any defects no penalty
34	4.00	30	3% defects no penalty
35	4.00	30	5% defects 2% penalty
36	4.00	30	7% defects 4% penalty
37	3.60	60	any defects no penalty
38	3.60	60	3% defects no penalty
39	3.60	60	5% defects 2% penalty
40	3.60	60	7% defects 4% penalty
41	3.60	40	any defects no penalty
42	3.60	40	3% defects no penalty
43	3.60	40	5% defects 2% penalty
44	3.60	40	7% defects 4% penalty
45	3.60	30	any defects no penalty
46	3.60	30	3% defects no penalty
47	3.60	30	5% defects 2% penalty
48	3.60	30	7% defects 4% penalty

Tabela 2

Distance parameters for the offers

d_j^+	d_j^-	S_j
0.00	0.93	1.00
0.04	0.85	0.96
0.14	0.79	0.85
0.14	0.79	0.85
0.10	0.82	0.89
0.14	0.74	0.85
0.24	0.68	0.74
0.24	0.68	0.74
0.21	0.76	0.78
0.25	0.68	0.73
0.36	0.62	0.63
0.36	0.62	0.63
0.07	0.72	0.91
0.11	0.64	0.86
0.21	0.57	0.73
0.21	0.57	0.73
0.19	0.53	0.74
0.23	0.45	0.66
0.33	0.39	0.54
0.33	0.39	0.54
0.34	0.48	0.58
0.38	0.40	0.51
0.49	0.33	0.41
0.49	0.33	0.41
0.23	0.54	0.70
0.26	0.46	0.63
0.37	0.39	0.51
0.37	0.39	0.51
0.38	0.30	0.44
0.42	0.22	0.34
0.52	0.16	0.23
0.52	0.16	0.23
0.51	0.27	0.35
0.55	0.19	0.26
0.66	0.13	0.17
0.66	0.13	0.17

Table 2 contd.

d_j^+	d_j^-	S_j
0.58	0.33	0.37
0.61	0.25	0.29
0.72	0.19	0.21
0.72	0.19	0.21
0.72	0.17	0.19
0.76	0.09	0.10
0.87	0.02	0.03
0.87	0.02	0.03
0.79	0.14	0.15
0.83	0.06	0.07
0.93	0.00	0.00
0.93	0.00	0.00

Table 3

Negotiation offers' GDM-TOPSIS final ranking

Offer number	Issues			S_j
	Pr	TD	RP	
1	4.5	60	any defects no penalty	1.00
2	4.5	60	3% defects no penalty	0.96
13	4.2	60	any defects no penalty	0.91
5	4.5	40	any defects no penalty	0.89
14	4.2	60	3% defects no penalty	0.86
3	4.5	60	5% defects 2% penalty	0.85
4	4.5	60	7% defects 4% penalty	0.85
6	4.5	40	3% defects no penalty	0.85
9	4.5	30	any defects no penalty	0.78
7	4.5	40	5% defects 2% penalty	0.74
8	4.5	40	7% defects 4% penalty	0.74
17	4.2	40	any defects no penalty	0.74
10	4.5	30	3% defects no penalty	0.73
15	4.2	60	5% defects 2% penalty	0.73
16	4.2	60	7% defects 4% penalty	0.73
25	4	60	any defects no penalty	0.70
18	4.2	40	3% defects no penalty	0.66
11	4.5	30	5% defects 2% penalty	0.63
12	4.5	30	7% defects 4% penalty	0.63
26	4	60	3% defects no penalty	0.63
21	4.2	30	any defects no penalty	0.58
19	4.2	40	5% defects 2% penalty	0.54
20	4.2	40	7% defects 4% penalty	0.54

Table 3 contd.

Offer number	Issues			S_j
	Pr	TD	RP	
27	4	60	5% defects 2% penalty	0.51
28	4	60	7% defects 4% penalty	0.51
22	4.2	30	3% defects no penalty	0.51
29	4	40	any defects no penalty	0.44
23	4.2	30	5% defects 2% penalty	0.41
24	4.2	30	7% defects 4% penalty	0.41
37	3.6	60	any defects no penalty	0.37
33	4	30	any defects no penalty	0.35
30	4	40	3% defects no penalty	0.34
38	3.6	60	3% defects no penalty	0.29
34	4	30	3% defects no penalty	0.26
31	4	40	5% defects 2% penalty	0.23
32	4	40	7% defects 4% penalty	0.23
39	3.6	60	5% defects 2% penalty	0.21
40	3.6	60	7% defects 4% penalty	0.21
41	3.6	40	any defects no penalty	0.19
35	4	30	5% defects 2% penalty	0.17
36	4	30	7% defects 4% penalty	0.17
45	3.6	30	any defects no penalty	0.15
42	3.6	40	3% defects no penalty	0.10
46	3.6	30	3% defects no penalty	0.07
43	3.6	40	5% defects 2% penalty	0.03
44	3.6	40	7% defects 4% penalty	0.03
47	3.6	30	5% defects 2% penalty	0.00
48	3.6	30	7% defects 4% penalty	0.00

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ANALYTIC NETWORK PROCESS IN ERP SYSTEM SELECTION

Abstract

An Enterprise Resource Planning (ERP) system has a major impact on a company's performance; therefore it is a critical investment. This paper presents a framework for selecting a suitable ERP system using the Analytic Network Process (ANP) methodology. The proposed framework establishes a set of criteria with respect to the support of business goals and enterprise strategies. The method is explained on a numerical example based on the choice of an ERP for a small manufacturing enterprise.

Keywords

ERP system selection, Analytic Network Process (ANP).

Introduction

The first years of the 21st century show a highly dynamic market, fierce market competition, global call for an effective way of doing business. One of the main assets is an information system. Various methods and procedures are combined in many ways and into various subsystems to create what may be called an information system. Early business information systems were limited to the information processed by accounting systems, or, in a production enterprise, to inventory control systems. Today such systems must integrate information from all resources in the enterprise. They are known as Enterprise Resource Planning systems (ERP), which are complete information systems, that can support an enterprise by integrating all its data assets and automate some of its business processes.

From the systematic point of view, if an enterprise has problems with resource planning and wants to improve its processes, the way to change the current state into a desired one is to choose a new ERP system.

A successful project involves the selection of a vendor and a software application, as well as implementation and verification of the system selected. Because of the complexity of business environment, limitation of available financial resources and system availability, the selection of an ERP system is a very difficult, and at the same time an important element of a project; a wrong choice leads to implementation which could be very difficult, time consuming and very expensive [Wei, Chien, Mao Wang, 2005]. Most of the existing ERP systems are similar, but also have fundamental design differences. Different companies have different needs, business models and key business processes. Although the system must have the functionalities desired, not all systems are suitable for every company. Therefore companies must carefully organize the process of the selection of an ERP system.

There are many different quantitative techniques being used for the ERP system selection problem, such as: ranking scoring, mathematical optimization, Analytic Hierarchy Process (AHP), Quality Function Deployment (QFD), DEA, etc. However, many of these methods have limitations and don't include a wide spectrum of expert knowledge in selection criteria.

In this paper a new, easy and flexible proposition of ERP system selection is given. The proposition is based on the Analytic Network Process (ANP) methodology. ANP is an extension of AHP [Rao, 2000], a well known decision making method proposed by T. Saaty. The ANP method is a more general form of AHP, incorporating internal and external dependencies among the decision model's elements and alternatives [Percin, 2008]. The full description of the model can be found in [Saaty, 1999]. The main aim of this paper is to adopt ANP methodology to ERP system selection, with the proper choice of criteria.

This paper is organized as follows. After a short introduction to the ERP system selection problem, in the first chapter ERP systems are presented. In Chapter 2 a description of the proposed method for the ERP system selection, based on the ANP methodology is given. In Chapter 3 a case study is presented. A small enterprise intending to implement an ERP system is described. The aim of the ERP system implementation in that enterprise and the criteria applied are shown. In Chapter 4 a numerical example is given. Finally, overall conclusions are presented.

1. ERP systems

The acronym ERP was first employed in the early 1990s as an extension of the Material Requirement Planning (MRP) standard and later of the Manufacturing Resource Planning (MRP II) standard. Systems known today as ERP systems have no official standard, but generally such systems integrate internal and external management information across the entire enterprise, including manufacturing, finance and accounting, sales, service, human resource management etc. One of the most complete definitions is given by the American Production and Inventory Control Society:

An accounting-oriented information system for identifying and planning the enterprise-wide resources needed to take, make, ship and account for customer orders. An ERP system differs from the typical MRP II system in technical requirements such as graphical user interface, relational database, use of fourth generation language and computer assisted software engineering tools in the development of client/server architecture and open-system portability.

This definition points out that the main advantage of ERP is the ability to integrate most of the business functions. Owing to this, the company can easily and quickly analyze all business data from every organization area with respect to enterprise as a whole.

The current ERP development aims to utilize ERP to realize and sustain a competitive advantage. Complementary technologies are beginning to extend the functionality of enterprise application to include the Internet and telecommunication technologies to fulfill the needs of e-commerce [Wei, Chien, Mao Wang, 2005].

One of the most important characteristics of ERP systems is their modularity. Figure 1 presents the main modules of a typical ERP system, however, the number and names of modules may differ. A typical system integrates all those modules by allowing them to share and use all business data from one central database.

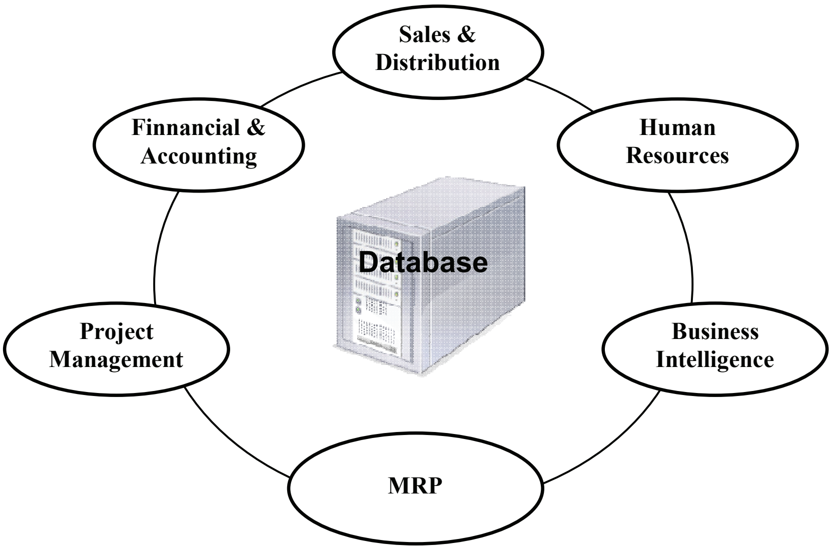

Figure 1. Main modules of an ERP system

2. Proposed Method for ERP system selection

The deployment of ERP system consists of two stages: selection and implementation. While most ERP packages have similarities, they also have design differences. Most papers about ERP explicitly focus on the critical success factors for the implementation process. The issue of the selection process for ERP software is for the most part ignored. Anyway, this issue is important, because, as the stage preceding the implementation process, it presents the opportunity for both researchers and experienced people to examine all the dimensions and implications (benefits, risk challenges, cost, etc.) of buying and implementing ERP software prior to the commitment of a formidable amount of money, time and resources. Hence a better understanding of critical factors could amount to substantial savings in terms of economics (actual cost), time and improved administrative procedures and could lessen the risk and uncertainty associated with the acquisition of these types of systems [Verville, Bernadas, 2005].

The proposed ANP model for the ERP system selection is given in Figure 2.

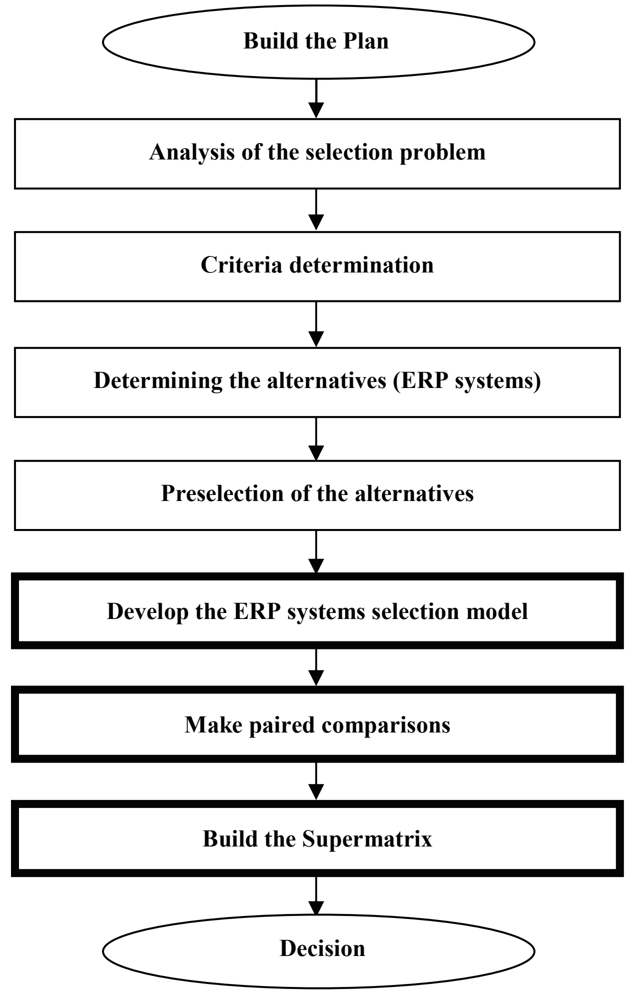

Figure 2. The proposed ANP model for ERP system selection

We explain each step of this model.

Step 1: The first step of the algorithm is the analysis of the selection problem. The main task at this stage is to form a project team, plan and collect all possible information related to the next stages. The plan should define the structure of the process and identify the general criteria of the ERP.

Step 2: In this step all criteria must be determined. In this paper we propose criteria divided into three main clusters as in Verville and Halingten [Verville, Halingten, 2002]:

Technical Features
Functionality Features
Vendor Factors

This is an extension of models proposed in the ERP selection problem (examples of these models can be found in [Percin, 2008; Wei, Chien, Mao Wang, 2005]). Most of them propose two main group of criteria: system factors and vendor factors, but the incorporation of technical features and redefinition of functionality features allows the team members to focus separately on the functionality and the technical aspects of ERP systems. However, those criteria are not the only possibilities, every project team should discuss the form of the ANP model in the context of the organization needs.

Step 3: During this step, a list of available vendors and technologies is created.

Step 4: If the list of possible alternatives is long, preselection is made.

Step 5: In this step an interaction network is created. The project team must identify all dependencies among the elements of the network.

Step 6: The alternatives from the short list are pairwise compared by expert judgments, according to the method proposed by Saaty [1999].

Step 7: During this step, the system's Supermatrix is constructed corresponding to the interactions in network created in step 5. The impact of a given set of elements in a component on another element in the system is represented by the ratio scale priority vector derived from paired comparisons in the same way as it was derived in the AHP method. Each priority vector is entered in the appropriate position as a column of the Supermatrix. The Supermatrix structure is shown in Figure 3.

$$W = \begin{array}{l} \text{Technical} \\ \text{Functionality} \\ \text{Vendor} \\ \text{Alternatives} \end{array} \begin{bmatrix} W_{11} & W_{12} & W_{13} & W_{14} \\ W_{21} & W_{22} & W_{23} & W_{24} \\ W_{31} & W_{32} & W_{33} & W_{34} \\ W_{41} & W_{42} & W_{43} & W_{44} \end{bmatrix}$$

Figure 3. The Initial Supermatrix

In the structure above, the terms W_{ij} represent the sub-matrix of priority vectors derived with respect to a given element. In the Supermatrix the values W_{21} mean that the cluster "*Functionality features*" depends on the cluster "*Technical features*". In this step the consistency of each comparison is checked and analyzed.

Step 8: The Initial Supermatrix derived in step 5 is often called unweighted, because it consist of several normalized eigenvectors (priority vectors [Saaty, 2004]), and hence the entire column of the matrix may sum to an integer greater than one. The Supermatrix has to be stochastic to allow the derivation of meaningful limiting priorities. Saaty proposes to multiply the cluster weights by the corresponding elements in the Supermatrix [Saaty, 1999]. To get the cluster weights the standard pairwise comparisons algorithm is used. As the result we receive the Weighted Supermatrix in which each column sums to one.

Step 9: In the last step we compute the Limit Supermatrix. The Weighted Supermatrix is multiplied by itself, until the Supermatrix row values converge to the same value for each column of the matrix. This matrix yields the long-run or limit priority of influence of each element on every other element. The most suitable ERP system is that which has the highest priority.

As a result of this method we receive a scale of priorities. It is read from the Limit Supermatrix and then normalized.

3. Case study

3.1. Description of the enterprise

The enterprise under analysis belongs to the manufacturing and installation of steel construction market. It has been created as a result of the merger of three steel industry companies, specializing in various stages of the production cycle. This merger made it possible to service the entire production cycle, starting with the purchase of materials, through manufacturing, and ending with the final installation and service.

The strategic goal of the enterprise is to strengthen its position in the sector of steel construction manufacturing and installation. In the long-term, a dynamic growth of demand for steel products is expected, which is related to investments planned in the energy and oil industries. The company plans extensive investments, raising its competitiveness and production capabilities. Thanks to the diversification of revenues into trade, manufacturing, and services, the company is able to achieve a high margin and to decrease the risk caused by market fluctuations.

The company's focus is on distribution , manufacturing, and construction and installation services. Its main customer is the Polish market, but a part of the products goes to the European Union countries, Asia and South America. The company offers industrial constructions and equipment, bridge constructions, buildings with steel supporting structure, narrow-gauge railway junctions, installations and equipment for environmental protection. Additionally, the company's contractors can use the services of its design office.

The merger of such differentiated enterprises in one company involves many organizational problems. The necessity to arrange the processes of norm adjustment and to establish a system of information flow became a significant challenge for the company managers. Another problem has been created by the location of the individual firms within the enterprise. Their location is very advantageous because of their activities, but it makes the control and information flow between its individual branches more difficult. The board of directors has decided to implement an integrated management system, since the solutions used in the individual companies comprising the enterprise did not fulfill their functions enterprise-wide.

When analyzing the situation of an enterprise, we can distinguish several factors in favor of the implementation of an ERP. The basic factor is the necessity to arrange and make uniform the individual processes within the entire enterprise, to ensure integration of reporting originating in the individual companies, and to provide access to the resources and data of the enterprise. The introduction of an ERP system should contribute to the increase of control over the individual projects, to stock reduction and to storing costs decrease.

3.2. Goals of the ERP system implementation

The analysis of the requirements of an enterprise is based on the premise that an ERP system is selected for at least 5-6 years and therefore the stated goals of the implementation of the system should take into account the development strategy of the enterprise. The strategic goals of the enterprise under discussion are presented in Figure 4. All the goals included in the pyramid are related to the improvement of the efficiency and profitability of the company and with the streamlining of the information flow among the individual divisions of the company. At the top of the pyramid, the main goal of the company, that is, the maximization of its value, is located. The lower the level, the smaller the importance of the individual goal for the realization of the strategy of the entire enterprise.

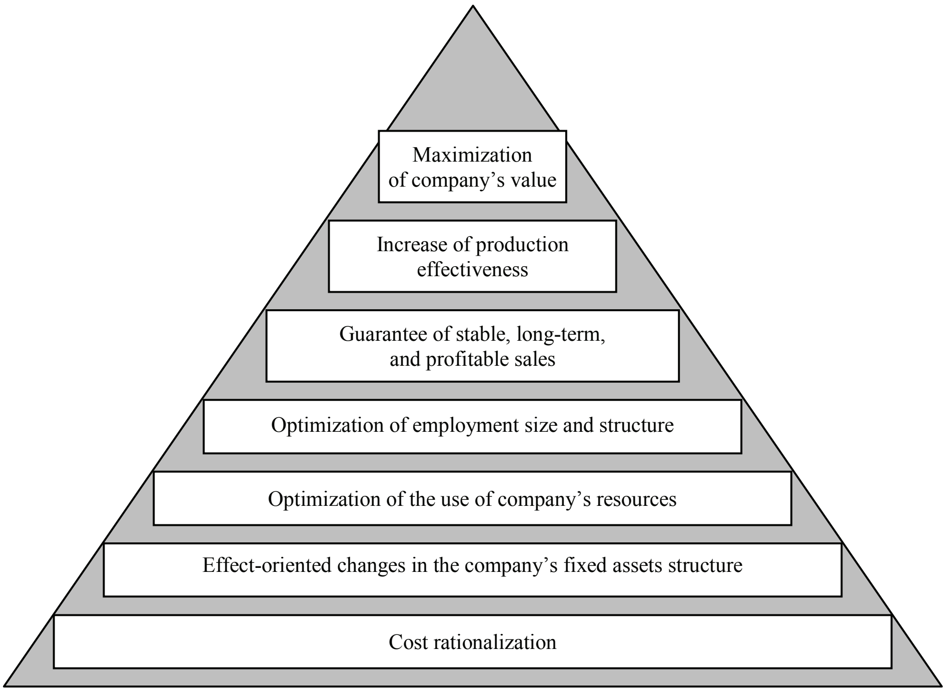

Figure 4. Pyramid of strategic goals

Source: Wieszała [2009].

As regards strategic goals, the enterprise has defined several detailed goals whose realization should be made easier by the ERP system. The main of them are: streamlining of information flow, increased control over the resources, uniformization of procedures, streamlining of key processes, enabling the introduction of project management methods, introduction of an integrated system for the management of human resources and skills, automation of order settlement, streamlining of the manufacturing logistics and transport management, computerization of the archives of certificates, attestations and manufacturing documentation, and the utilization of the e-commerce potential (in particular of B2B). Most of them result directly from the premises which caused the decision to implement ERP, as well as from the main problems related to the functioning of the enterprise.

The main goal of the system to be implemented is the streamlining of the information flow within the organization. The enterprise is the result of a merger and the companies comprising it belonged to various markets and have various experiences. It is necessary to create a platform bonding the companies together within the new organizational structure, as well as a flexible and extensive report system for the board of directors and the shareholders. The enterprise has divisions in three voivodeships, and for this reason the system has to ensure an adequate level of data integration and to enable information flow by means of an internet network.

The next group of goals is related to the issues of organization functioning. Ensuring control over the enterprise resources means streamlining of the management and planning processes and controlling all its resources, in particular: storing, supply, utilization of equipment and machinery as well as processes related to the finances of the enterprise. Streamlining of the basic processes of the enterprise means automation of some of time-consuming tasks related to administration, accounting, or human resources management. Procedures recorded in the system should service the largest possible scope of the activities of the enterprise, while being easy to monitor. The next goal, procedure standardization, is related to the main goal. Certain standard procedures have to be established when the new organization is being created, and adherence to them has to be based on the system. The chief asset of the enterprise analyzed is its ability to provide full project support, from planning, to manufacturing, to installation. This means that the system has to capable of supporting each project in such a way as to provide access to all the data related to the consecutive stages, by means of a central database.

The remaining goals are related to the expectations of the enterprise with respect to the system. Business-to-Business (B2B) is the totality of relationships between the enterprise and its partners , middlepersons, suppliers, distributors, points of sale and service shops. The use of this technique makes it possible to automate part of the communication with suppliers by means of the systems, and therefore to streamline the processes of the supply chain and to monitor the project realization. The next goal, partly related to the possibilities of B2B, is the management of manufacturing logistics and goods distribution, in particular, the management of transportation between the individual branches of the enterprise or between the enterprise and its customers. The management of human resources and skills is of particular importance when it comes to specialized tasks, such as welding or work at height, since they require special certifications and medical check-ups. The specific character of the enterprise requires gathering of all attestations and certificates for the individual construction elements. It is expected that it will be possible to automate and computerize the order repository management by means of the system.

3.3. Criteria for the selection of an ERP system

The main issue in the selection process is the establishment of the set of the estimation criteria for the individual variants. It should be large enough to encompass all relevant features of the variants analyzed, but not so large as to make the entire selection process difficult and to generate additional costs related to the longer duration of the analysis and a larger amount of information. Various sets of criteria for the ERP selection problem have been proposed in the literature; here we try to discuss only some of them to find the overall direction suggested by the authors.

Bernroider and Koch, in their paper on the selection of ERP systems for enterprises, pointed out the differences in the selection process depending on the size of the organization [Bernoider, Koch, 2001]. Everdingen, Hillegersberg and Waarts attach particular importance to such criteria as: adjustment to the business processes of the enterprise, flexibility, costs, ease of use, user-friendliness, implementation time, and functionality [Everdingen, Hillegersberd, Waarts, 2000]. According to the Epicor consultants, there are eleven key criteria for finding and selecting a solution satisfying the enterprise's expectations. They are: ability to support the enterprise in the future, solidity, expert knowledge as regards system replacement, elimination of implementation guesswork, good knowledge of the industry in question, utilizing the development of technology for the good of the enterprise, guaranteed scalability, high level of technical support and service, integrity and dedication, and guarantee of return on investment.

In the context of solutions for small and middle-size enterprises, Rao has suggested taking into account in the assessment process: cost analysis, market sector in which the vendor specializes, proximity of the vendor, as well as the development abilities of both the technology and the system [Rao, 2000]. Verville and Halingten have grouped the criteria into three groups: vendor assessment criteria, technical criteria of the system, and criteria for the system functionality assessment [Verville, Bernadas, 2005]. Similarly, Neves, Fenn and Sulcas also divided the criteria into three main groups. Within these groups they have defined 21 detailed criteria to be taken into account by the enterprise in the analysis. Among them are: the number of implementations done by the vendor on the local market, the assessment of his market position, adaptation to the functional requirements of the organization, and capacity for development [Das Neves, Fenn, Sulcas, 2004].

Most papers mentioned here suggest that the assessment of software vendor plays a great role in the system selection process. The most often repeated criteria, other than costs, are: flexibility, ability to adapt the system to the business process specific for the organization, user-friendliness, implementation time, and development perspectives. Vendor criteria encompass the assessment of his market position and quality of cooperation, and the assessment of the services offered by the vendor.

Using the papers cited, the selection criteria discussed in this paper have been divided into three groups: technical-technological criteria, system functionality criteria, and vendor assessment criteria. Additionally, detailed criteria of assessment have been established within each group.

The first group consists of functional criteria, defining the functions of the system which are directly perceived by the user. This group includes mostly criteria related to the functionality, flexibility, and ease of use of the system.

The scope of functionality determines the detailed abilities of the system and the range of activities of the enterprise which the system can support. It is assumed that the functions of the system overlap as much as possible with the business processes of the enterprise. This criterion estimates the extent to which the system satisfies the enterprise's requirements. In the context of the enterprise analyzed this is the support for individual production, integration of CAD-based project systems, and a project management module.

A separate criterion of strategic adaptability has been singled out from the functionality criterion. It estimates the ability to satisfy future needs of the enterprise, resulting from the realization of its strategy.

By system flexibility we mean its ability to adapt to the existing market situation. Also, the system should be scalable, that is, it should be possible to install only those components which will actually be used; on the other hand, it should allow for the development of the system as the organization grows. System flexibility is understood also as its capacity for introduction of structural changes, so as to be able to adapt the solutions to the enterprise's needs as fully as possible. The system should also ensure smooth integration with other applications, in particular with industry-specific solutions. An advantage of the system is also its self-dependence as regards both hardware and platform, thanks to which the enterprise can freely use computer-based solutions.

A user-friendly system is a system easy to use and not requiring a long learning process of each individual function. In this criterion, particular attention has been paid to the ergonomics of the interaction with the system, easy adaptation to the needs of the given user (for instance, through menu personalization or interface look and feel), adherence to generally adopted standards (for instance, with regard to document appearance), and intuitiveness.

Features such as a clear graphic interface, well-written user manual, on-line help with expert assistance, or interactive on-line courses encourage users to accept the system.

The last criterion in this group concerns data security. The data gathered constitute a valuable resource of the enterprise; their loss can cause a significant deterioration of its situation. Within this criterion, we will analyze, first of all, security levels, security functions, encrypting, and ability to manage permissions.

Another group consists of technical and technological criteria, which are not perceived directly by the average user, but translate into many features that determine such system abilities as flexibility, processing efficiency, openness, scalability.

The first criterion in this group is system architecture. This criterion assesses the technology used in building the system. Here are assessed, among other things, methods of data management, communication protocols used, supported device interfaces, and overall system architecture, including network capacities, built-in procedures for multi-division enterprise management, capacities for parameterization and for user influence on the functioning of the system.

The criterion of adaptation to the technological needs of the enterprise assesses the extent to which the technology used by the system will support the functioning of the enterprise. Only actual needs of the enterprise are analyzed, to avoid unnecessary involvement of too advanced or obsolete technology. This criterion reflects the criterion of the strategic adaptability (from the group of functional criteria). The technology chosen by the enterprise should be capable of supporting the enterprise at each stage of the realization of the strategy, both now and in the future.

The fundamental condition of the usability of the system is its stability; an ERP system integrates the entire information system of the enterprise, and thus any dysfunction can cause significant losses. It is easy to imagine a situation in which a shortage of components for manufacturing occurs or an invoice is incorrectly entered.

The innovativeness criterion should assess how the given system differs from others as regards the use of new solutions which increase its output, functionality or flexibility.

The last group consists of criteria assessing system vendors. The fundamental criterion in this group is the assessment of the market position of the vendor. It reflects the market strength of the vendor and indicates the popularity of the solutions proposed by him. The better the vendor's position, the higher the probability that his system will satisfy the highest requirements and that it will be capable of future development.

The cost criterion is still one of the most important factors influencing the decision to purchase a given system. It is essential to take into account the actual cost of the system, that is the costs of the license, technical infrastructure, consultants, training, as well as the costs of new modules, upgrades, and updates.

The next criterion – the organizational and financial abilities of the vendor – assesses the contractor's stability on the market. The implementation of an ERP system requires a significant organizational and financial involvement of both parties. That is why the financial situation of the contractor is a very important factor in the selection of an ERP system. It should be kept in mind that a system will be used at least five years, and therefore one should have contractor's support ensured for that period. It is also important to analyze the organizational capabilities of the vendor. A small vendor will not be able to provide adequate support for a large client and vice versa. When analyzing implementation in our enterprise, we should consider a large vendor with an adequate base of highly qualified consultants who will be able to implement the system efficiently and quickly.

An oft-touched upon issue is the ensuring of system integration, security, and stability. For that reason, attention should be paid to the support by the vendor. Availability of the consultants and assistance with problem solutions can be critical success factors of the entire implementation. Other services provided by vendors are also assessed within this criterion. Such services are, for instance, assistance with purchase of specialized equipment and its installation. If the enterprise decides to use outsourcing, the vendor should also help with finding trusted partners. Often, software vendors can help their customers in the search of financing for the system.

The criterion of implementation time and methodology allows to assess the implementation method offered by the vendor. Almost every larger company specializing in the implementation of integrated management systems has its own methods and schedules of implementation depending on the experience of its employees, the number of system modules, the scope of implementation, expenses for training, and infrastructure.

All the criteria described here are shown in Table 1 together with symbols used in later calculations.

4. A Numerical example

The presented method is explained on the example of choosing an ERP system for a hypothetical small enterprise whose main activity is manufacturing.

Step 1. After the formation of the project team and the collection of needed data, problem analysis based on method proposed in part 2, was conducted. The results of the analysis are important for the next steps.

Step 2. Based on the analysis performed in step 1, three clusters, with 17 criteria are proposed; they are presented in Table 1.

Table 1

Criteria of the proposed ERP selection model

Cluster	Name	Symbol
Functionality Features	Functionality	F
	Strategic Alignment	SA
	FleXibility	FX
	User Friendliness	UF
	Safety	S
Technical Features	System aRchitecture	SR
	Technical Alignment	TA
	Solution Innovation	SI
	Reliability	R
Vendor Factor	Market Share	MS
	Total Cost	TC
	Financial and Organizational Capabilities	FOC
	Service Support	SS
	Implementation Time	IT

Step 3: The few ERP systems available on the Polish market are presented in Table 2. The proposed method will be used to choose the optimal system.

Table 2

ERP systems available on the Polish market

No.	Name	Manufacturer/Vendor	Main Field of Usage	Usage References
1	BAAN IV c4	INFOR, USA/ BEELC Poland	Manufacturing (Aerospace, Automotive, Shipbuilding) Services (Financial Services, Health-care, Insurance, Telecommunications) Distribution (Transportation & Logistics, Electrical, Industrial)	More than 13 000 customers in over 90 countries, a few companies in Poland
2	IFS Applications	IFS / IFS Industrial and Financial Systems Poland Sp. z o.o.	Small & Medium Size Enterprises (Special solutions for Construction Companies)	Many in over 46 countries also many in Poland
3	IMPULS 5	BPSC SA, Poland	Small & Medium Size Enterprises (Production Companies, Automotive Industry, Wood and Furnish Industry, Food Industry, Cloth Industry, Public Utility Companies, Distribution Companies, Construction Companies, Research and Education Institution)	More than 300 customers, mainly in Poland
4	SAP	SAP, Waldorf Germany	All types of business: Financial and Public Services: Banking, Defense & Security, Healthcare, Higher Education & Research, Insurance, Public Sector Manufacturing & operations: Aerospace & Defense, Automotive, Chemicals, Consumer Products, Industrial Machinery & Components, Engineering, Construction & Operations	More than 92,000 customers in over 120 countries, many companies in Poland
5	VANTAGE	EPICOR, USA / Epicor Software Poland Sp. z o.o.	Mainly Medium & Large Size Production, Trade / Service Companies	A few companies in Poland

Step 4: Based on usage references on the Polish market and field of usage – small enterprises – three systems have been chosen for the short list: IFS Applications (IFS), IMPULS 5 (IM5) and SAP (SAP).

Step 5: A dependence matrix has been defined by the project team. Figure 5 shows the ANP interaction network for the selection of the most suitable ERP system software, created using the dependence matrix presented in Table 3.

Table 3

Dependence matrix

	F	FX	S	SA	UF	R	SI	SR	TA	FOC	IT	MS	SS	TC	IFS	IM5	SAP
F	0	1	1	1	0	0	1	0	1	0	1	0	0	1	1	1	1
FX	1	0	0	1	1	0	0	1	1	1	1	0	0	1	1	1	1
S	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1
SA	1	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1
UF	0	1	1	0	0	1	1	0	0	1	1	0	0	0	1	1	1
R	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1
SI	0	1	1	0	0	1	0	0	0	1	0	0	0	1	1	1	1
SR	1	1	1	1	0	1	1	0	1	0	0	0	0	1	1	1	1
TA	1	1	0	1	1	0	0	0	0	0	0	0	0	1	1	1	1
FOC	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1	1	1
IT	0	1	0	0	1	0	0	0	0	1	0	0	0	1	1	1	1
MS	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
SS	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1
TC	1	0	0	0	0	0	1	0	0	1	1	1	1	0	1	1	1
IFS	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
IM5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
SAP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

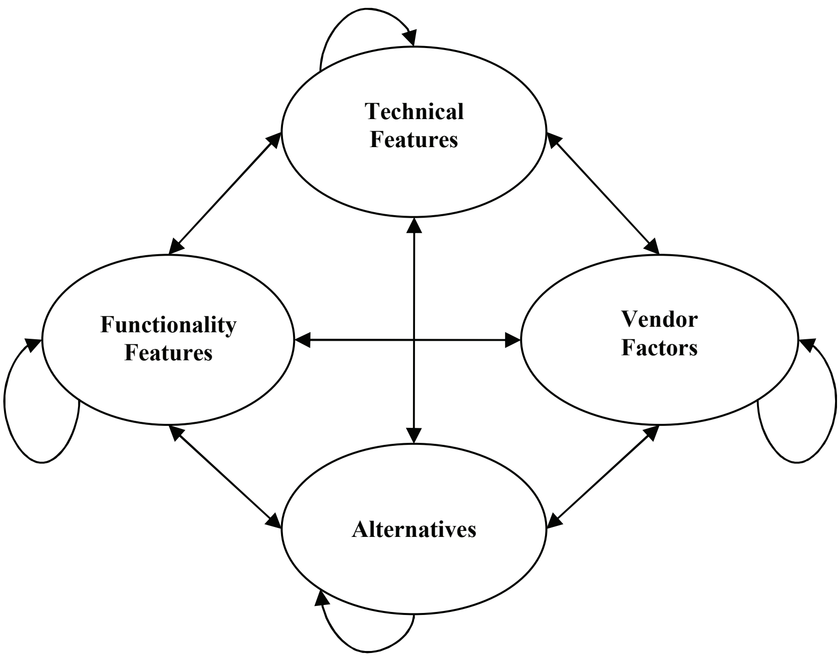

Figure 3. ANP model for the selection of a suitable ERP system

Step 6: In this step criteria and alternatives are compared with respect to the selected criteria. Due to limited space in this paper this process is shown for one criterion only, namely Functionality (F) (Tables 4 to 8).

First of all, based on expert judgments, Implementation Time (IT) are compared with Total Costs (TC) with respect to F. For example, one of the experts evaluates that TC is six times more important than IT, as shown in Table 4. Based on this information the parameter δ_j is computed and used for computing the normalized matrix *B* (Table 5) with respect to the relation:

$$\beta_{ij} = \frac{a_{ij}}{\delta_j}$$

Table 4

Computing δ_j

F	IT	TC
IT	1	0,1667
TC	6	1
$\delta_j = \sum_{i=1}^{n} a_{ij}$	7	1,1667

Table 5

Computing the normalized matrix $B = [\beta_{ij}]_{i,j=1\ldots n}$

F	IT	TC
IT	0,1429	0,1429
TC	0,8571	0,8571

Now the vector of priorities w_i can be computed from the equation:

$$w_i = \frac{1}{n}\left(\sum_{j=1}^{n} \beta_{ij}\right)$$

The results are presented in Table 6.

Table 6

Computing the vector of priorities w_i

F	$w_i = \frac{1}{n}\left(\sum_{j=1}^{n}\beta_{ij}\right)$
IT	0,1429
TC	0,8571

Next the alternatives are pairwise compared with respect to each criterion in our example in Table 7 with respect to Functionality. In this Table Consistency Index (*CI*) is also computed.

Table 7

Comparing F element in Alternative's cluster

F	IFS	IM 5	SAP	w_i
IFS	1	1	2	0,4000
IM 5	1	1	2	0,4000
SAP	0,5	0,5	1	0,2000
$\lambda_{\max} = \frac{1}{n}\left(\sum_{i=1}^{n}\frac{(Aw)_i}{w_i}\right)$ $CI = \frac{\lambda_{\max} - n}{n-1}$				0,0000

As F depends on Flexibility (FX), Safety (S) and Strategic Alignment (SA), it must be also compared in Feature cluster (Table 8).

Table 8

Comparing F element in Functionality Feature cluster

F	FX	S	SA	w_i
FX	1	5	4	0,6738
S	0,2	1	0,3333	0,1006
SA	0,25	3	1	0,2255
$\lambda_{\max} = \frac{1}{n}\left(\sum_{i=1}^{n}\frac{(Aw)_i}{w_i}\right)$ $CI = \frac{\lambda_{\max} - n}{n-1}$				0,0825

Step 7: Based on the priority vectors computed in step 6, the Initial Supermatrix is constructed. This Supermatrix is shown in Table 14. The values computed in this example are in the first column.

Step 8: The Weighted Supermatrix is computed. Since all clusters depend on each other, as shown in Figure 3, we must compare pairwise all clusters with respect to each other. The comparisons are shown in Tables 9 to 12.

Table 9

Comparing clusters with respect to Alternatives

Alternatives	Functionality Features	Technical Features	Vendor Factors	Alternatives	Priorities
Functionality Features	1,00	3,00	4,00	6,00	0,5609
Technical Features		1,00	1,00	3,00	0,1898
Vendor Factors			1,00	3,00	0,1783
Alternatives				1,00	0,0710
				CI	0,0172

Table 10

Comparing clusters with respect to Vendor Factors

Vendor Factors	Functionality Features	Technical Features	Vendor Factors	Alternatives	Priorities
Functionality Features	1,00	1,00	0,33	1,00	0,1728
Technical Features		1,00	0,33	0,33	0,1300
Vendor Factors			1,00	1,67	0,4331
Alternatives				1,00	0,2640
				CI	0,0432

Table 11

Comparing clusters with respect to Functionality Features

Functionality Features	Functionality Features	Technical Features	Vendor Factors	Alternatives	Priorities
Functionality Features	1,00	0,33	4,00	3,00	0,2542
Technical Features		1,00	6,00	6,00	0,5790
Vendor Factors			1,00	1,00	0,0808
Alternatives				1,00	0,0860
				CI	0,0172

Table 12

Comparing clusters with respect to Technical Features

Technical Features	Functionality Features	Technical Features	Vendor Factors	Alternatives	Priorities
Functionality Features	1,00	2,00	5,00	3,00	0,4539
Technical Features		1,00	4,00	5,00	0,3531
Vendor Factors			1,00	0,50	0,0752
Alternatives				1,00	0,1178
				CI	0,0609

The Weighted Supermatrix is shown in Table 15.

Step 9: The Limit Supermatrix G is computed from the formula

$$\frac{1}{n}\sum_{k=1}^{n} W^k = G$$

The result is shown in Table 16.
Finally we receive priorities shown in Table 13.

Table 13

Synthesis for the alternatives

Alternatives	Derived Priorities	Priorities (Normalized)	Rank
IFS	0,0448	0,3035	2
IM5	0,0651	0,4415	1
SAP	0,0376	0,2550	3

Summary and Conclusions

The selection of a suitable ERP system, in particular for a small enterprise, is a strategic decision which should be carefully prepared and organized. In this paper we propose the Analytic Network Process model for ERP system selection. The ANP model can provide a more accurate mechanism to better understand the nature of trade-offs between various criteria than standard selection methods, because it is capable of dealing with all kinds of feedback and dependence, when modeling a complex decision environment [Rashid Hossain, Patrick, 2002].

If standard selection models are applied, managers might base their decisions on a subset of important criteria only, without understanding their relative importance and interactions. The major advantage of this approach is that it assists them to approach the selection comprehensively. Furthermore, our model is flexible, easy to understand, and does not require an increase of IT costs.

Although the model proposed provides a comprehensive framework to guide the management of any company, the methods proposed have limitations. First, the model does not consider all possible clusters, elements and their interactions. Depending on the decision-making team, additional factors and interactions, within and between decision elements and alternatives could be added. However, the additional factors and their interactions require additional time and effort necessary for completion of such a model. In this case, the number of pairwise comparisons required would be quite high. Second, the model is very dependent on the weightings provided by decision makers. While this model effectively incorporates qualitative and quantitative measures into the evaluation process, its efficacy depends on the accuracy and the value of judgment provided by the decision making team.

In the example presented, the reduction of the list of alternatives plays an important role. With a longer list numerical problems have been observed. It is best to reduce the list of alternatives to three elements.

The ideas presented in this paper can be applied to real-life selection problems. The goal of future research is to improve the ANP model and to prove usefulness of this method by applying the ANP-based models to different companies operating in various industries. A comparison of the model proposed here with other tools and a different ANP base model should be investigated.

Acknowledgements

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Appendix

Table 1

Initial Supermatrix

	F	FX	S	SA	UF	R	SI	SR	TA	FOC	IT	MS	SS	TC	IFS	IM5	SAP
F	0,0000	0,3333	0,0000	1,0000	0,0000	0,0000	0,0000	0,2500	0,2756	0,0000	0,0000	0,2180	0,0000	1,0000	0,2031	0,1169	0,4804
FX	0,6738	0,0000	0,0000	0,0000	0,5000	0,0000	0,5000	0,2500	0,1412	0,0000	0,8333	0,5153	0,0000	0,0000	0,1276	0,2122	0,2402
S	0,1007	0,0000	0,0000	0,0000	0,5000	1,0000	0,5000	0,2500	0,0000	0,0000	0,0000	0,0603	0,0000	0,0000	0,0535	0,0587	0,1146
SA	0,2255	0,3333	0,0000	0,0000	0,0000	0,0000	0,0000	0,2500	0,4827	0,0000	0,0000	0,1199	0,0000	0,0000	0,5270	0,3697	0,1098
UF	0,0000	0,3333	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1006	0,0000	0,1667	0,0865	1,0000	0,0000	0,0889	0,2425	0,0549
R	0,0000	0,0000	1,0000	0,0000	1,0000	0,0000	1,0000	0,3333	0,0000	0,0000	0,0000	0,2581	0,0000	0,0000	0,1704	0,1851	0,0640
SI	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,3333	0,0000	1,0000	0,0000	0,1076	0,0000	1,0000	0,0724	0,1163	0,1005
SR	0,0000	0,5000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1896	0,0000	0,0000	0,3418	0,2404	0,5731
TA	1,0000	0,5000	0,0000	1,0000	0,0000	0,0000	0,0000	0,3333	0,0000	0,0000	0,0000	0,4448	0,0000	0,0000	0,4154	0,4582	0,2624
FOC	0,0000	0,3333	0,0000	0,0000	0,0000	0,0000	0,5000	0,0000	0,0000	0,0000	0,8571	0,2175	0,0000	0,2500	0,2000	0,1135	0,2728
IT	0,1429	0,3333	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1237	0,0000	0,2500	0,2000	0,2054	0,1327
MS	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,2500	0,2000	0,0868	0,4923
SS	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,2000	0,0000	0,0894	0,0000	0,2500	0,2000	0,2302	0,0650
TC	0,8571	0,3333	0,0000	0,0000	0,0000	1,0000	0,5000	1,0000	1,0000	0,8000	0,1429	0,5694	0,0000	0,0000	0,2000	0,3641	0,0372
IFS	0,4000	0,0936	0,3108	0,5469	0,2493	0,1958	0,1634	0,2403	0,4000	0,3108	0,1220	0,2403	0,1311	0,2583	0,0000	0,6667	0,8571
IM5	0,4000	0,6267	0,4934	0,3445	0,5936	0,4934	0,2970	0,2098	0,4000	0,1958	0,6483	0,2098	0,6608	0,6370	0,1250	0,0000	0,1429
SAP	0,2000	0,2797	0,1958	0,1085	0,1571	0,3108	0,5396	0,5499	0,2000	0,4934	0,2297	0,5499	0,2081	0,1047	0,8750	0,3333	0,0000

Table 2

Weighted Supermatrix

	F	FX	S	SA	UF	R	SI	SR	TA	FOC	IT	MS	SS	TC	IFS	IM5	SAP
F	0,0000	0,0847	0,0000	0,2765	0,0000	0,0000	0,0000	0,1135	0,1934	0,0000	0,0000	0,0377	0,0000	0,1728	0,1139	0,0656	0,2695
FX	0,1713	0,0000	0,0000	0,0000	0,1383	0,0000	0,2270	0,1135	0,0991	0,0000	0,1655	0,0891	0,0000	0,0000	0,0716	0,1190	0,1347
S	0,0256	0,0000	0,0000	0,0000	0,1383	0,7017	0,2270	0,1135	0,0000	0,0000	0,0000	0,0104	0,0000	0,0000	0,0300	0,0329	0,0643
SA	0,0573	0,0847	0,0000	0,0000	0,0000	0,0000	0,0000	0,1135	0,3387	0,0000	0,0000	0,0207	0,0000	0,0000	0,2956	0,2074	0,0616
UF	0,0000	0,0847	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0706	0,0000	0,0331	0,0149	0,3957	0,0000	0,0499	0,1360	0,0308
R	0,0000	0,0000	0,8706	0,0000	0,6299	0,0000	0,3531	0,1177	0,0000	0,0000	0,0000	0,0336	0,0000	0,0000	0,0323	0,0351	0,0122
SI	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1177	0,0000	0,1572	0,0000	0,0140	0,0000	0,1301	0,0138	0,0221	0,0191
SR	0,0000	0,2895	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0247	0,0000	0,0000	0,0649	0,0456	0,1088
TA	0,5790	0,2895	0,0000	0,6299	0,0000	0,0000	0,0000	0,1177	0,0000	0,0000	0,0000	0,0579	0,0000	0,0000	0,0789	0,0870	0,0498
FOC	0,0000	0,0269	0,0000	0,0000	0,0000	0,0000	0,0376	0,0000	0,0000	0,0000	0,4268	0,0942	0,0000	0,1083	0,0357	0,0202	0,0486
IT	0,0115	0,0269	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0536	0,0000	0,1083	0,0357	0,0366	0,0237
MS	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1083	0,0357	0,0155	0,0878
SS	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,1047	0,0000	0,0387	0,0000	0,1083	0,0357	0,0410	0,0116
TC	0,0693	0,0269	0,0000	0,0000	0,0000	0,1163	0,0376	0,0752	0,1163	0,4189	0,0711	0,2466	0,0000	0,0000	0,0357	0,0649	0,0066
IFS	0,0344	0,0081	0,0402	0,0512	0,0233	0,0356	0,0192	0,0283	0,0728	0,0992	0,0370	0,0634	0,0792	0,0682	0,0000	0,0473	0,0608
IM5	0,0344	0,0539	0,0638	0,0323	0,0556	0,0898	0,0350	0,0247	0,0728	0,0625	0,1967	0,0554	0,3993	0,1682	0,0089	0,0000	0,0101
SAP	0,0172	0,0241	0,0253	0,0102	0,0147	0,0566	0,0635	0,0648	0,0364	0,1575	0,0697	0,1452	0,1258	0,0277	0,0621	0,0237	0,0000

Table 3

Limit Supermatrix

	F	FX	S	SA	UF	R	SI	SR	TA	FOC	IT	MS	SS	TC	IFS	IM5	SAP
F	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928	0,0928
FX	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619
S	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167	0,1167
SA	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921	0,0921
UF	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341	0,0341
R	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373	0,1373
SI	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179	0,0179
SR	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282	0,0282
TA	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448	0,1448
FOC	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216	0,0216
IT	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151	0,0151
MS	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127	0,0127
SS	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143	0,0143
TC	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630	0,0630
IFS	0,0448	0,0448	0,0448	0,0448	0,0448	0,0448	0,0448	0,0448	0,0448	0,0448	0,0448	0,0448	0,0448	0,0448	0,0448	0,0448	**0,0448**
IM5	0,0651	0,0651	0,0651	0,0651	0,0651	0,0651	0,0651	0,0651	0,0651	0,0651	0,0651	0,0651	0,0651	0,0651	0,0651	0,0651	**0,0651**
SAP	0,0376	0,0376	0,0376	0,0376	0,0376	0,0376	0,0376	0,0376	0,0376	0,0376	0,0376	0,0376	0,0376	0,0376	0,0376	0,0376	**0,0376**

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Kazimierz Zaras

THE DOMINANCE-BASED ROUGH SET APPROACH (DRSA) APPLIED TO BANKRUPCY PREDICTION MODELING FOR SMALL AND MEDIUM BUSINESSES

Abstract

The preferential information given in the form of ranking or classification examples is more natural than those given in the form of functional parameters or the relational model of the preferences. Nevertheless, processing of these data cause certain difficulties related to a lack of coherence and contradictions in these examples. These contradictions often result from granularity of description language, inaccuracy or uncertainty of the information which makes the decision maker hesitate before the decision making. The model of the preferences will not correct or ignore these contradictions, but rather consider them to release a certain doubtful part of them. Then, exploitation of this model within the framework of decisional problems will lead to unquestionable and possible recommendations.

The Rough Set Theory takes into account this postulate making the contradiction analyze possible. This theory was introduced in the early 1980s by Polish researcher Z. Pawlak and developed by S. Greco, B. Matarazzo and R. Slowinski as the Data-based Rough Set Approach (DRSA).

In this proposal we will apply the DRSA to hybrid bankruptcy prediction modeling for small businesses. In this modeling the discrimination analysis results are used to explain the decision rules obtained from regional experts.

Keywords

Multi Criteria Decision Analysis (MCDA), Preference Modelling, Discriminate Analysis, Hybrid Model, Rough Set Theory, Dominance-Based Rough Set Approach (DRSA).

Introduction

There are many preference modelling methods where the model is adjusted to the decisional situation by determination of parameter values. In practice, the task of parameter values determination is not easy, because the DM does not understand the decisional situation in terms of parameters.

More realistic is the model construction from examples, called learning approach based on examples. Usually, referential activities are well known to the DM, and he is able to order them and express his preferences in this way. Simply, he shows us how he does his job.

However, processing of the information, coming from the DM creates certain difficulties because of the lack of consistency of examples and contradictions.

According, to Polish researcher Zdzislaw Pawlak the preference model should neither correct nor ignore these contradictions. It should rather consider them to induce certain and uncertain decision rules. The exploration of this model will allow us give to the DM two kinds of certain and possible recommendations.

The Rough Set Theory suggested by [Pawlak, 1982] respects the above principle. In 2001 Greco, Matarazzo and Slowinski have introduced Dominance-Based Rough Set Approach (DRSA) which is an extension of rough set theory for Multicriteria Decision Analysis (MCDA). The main change comparing to the classical rough sets is the substitution of the indiscernibility relation by a dominance relation, which permits to deal with inconsistencies typical for considerations of criteria and preferences. In DRSA, examples of decision making are presented in the form of a decision table.

1. Decision table

Formally, a decision table is the 4-tuplex T = (U,Q,V,f) where U is a finite set of objects, Q is a finite set of criteria, where Q is divided into non-empty condition criteria set C and the decision criterion d. Notice, that f(x,q) which belongs to V_q is an evaluation of the object x on criterion q which belongs to the set C, while f(x,d) is the class assignment (decision value) of the object.

Table 1

Example of decision table

***object* (*cand*)**	***q_1* *Piano***	***q_2* *Violin***	***q_3* *Trumpet***	***q_4* *Guitar***	***d* (*decision*)**
x_1	4	4	3	4	A
x_2	5	5	2	4	A
x_3	4	4	2	4	A
x_4	4	4	2	4	R
x_5	5	5	2	4	A
x_6	4	4	2	3	A
x_7	4	3	2	3	R

As an illustrative example, consider the problem selection of candidates to a high music school by the committee. The candidates are assigned to two disjunctive classes: accepted (A) or rejected (R) (see Table 1). The performance of each candidate is described by four criteria: level piano, violin, trumpet and guitar playing, each taking one of three possible values: 3;4;5 with respect to two first criteria and 2;3;4 with respect to the two second criteria. Criteria are ordered so that grater values are better.

The classical rough set approach allows us to obtain a partition of indiscernible classes of objects in the decision table. The objects are indiscernible if their performance is described by the same conjunction of the values with respect to the conditional criteria (C). The inconsistency of the first kind is identified by the classical rough set approach if two indiscernible objects correspond to two different disjunctive decision classes (d).

In our example, it is the case of $\{x_3,x_4\}$. The classical rough set approach doesn't allow us to identify the second kind of inconsistency where a principle of dominance is not respected. In our example, it is the case of the relation between candidates x_4 which dominates and x_6 which is dominated and the first one is rejected while the second one is accepted. This is why an extension of the classical rough set approach was suggested by [Greco, Matarazzo and Slowinski, 2001], called DRSA (Dominance-based Rough Set Approach).

2. Dominace-based Rough Set Approach (DRSA)

Let $\succsim_q$ be outranking relation such that:

$$x \succsim_q y \Leftrightarrow f(x,q) \geq f(y,q) \tag{1}$$

This relation is straightforward for gain-type (the more is better), for cost-type (the less, the better) can be satisfied by negating the values from V_q.

Dominance

We say that x **dominates** y with respect to $P \subseteq C$ denoted by xD_Py, if x is better than y on every criterion from P, $x \succsim_q y$, $\forall q \in P$.

Given $P \subseteq C$ and $x \in U$, let

$$\begin{aligned} D_P^+(x) &= \{y \in U : yD_Px\} \\ D_P^-(x) &= \{y \in U : xD_Py\} \end{aligned} \tag{2}$$

represent **P-dominating** and **P-dominated** sets for each $x \in U$, respectively.

Next, with respect to the decisional attribute we consider n disjoint classes CL=$\{CL_t, t \in T\}$, where $Cl_t = \{x \in U: f(x,d) = t\}$. Each object x$\in U$ is assigned to one and only one classCl_t. The classes are preference-ordered according to an increasing order of class indices. This is why for each class t cumulated decision classes are considered "at most" or "at least", defined respectively (3), as:

$$Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s \ et \ Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s \quad for\ each\ t \in T. \tag{3}$$

In DRSA, to identify inconsistency cases, we do **P-lower** and the **P-upper approximations** (4) of $Cl_t^{\geq}$ and $Cl_t^{\leq}$, t$\in T$ for each $P \subseteq C$ denoted as $\underline{P}(Cl_t^{\geq})$, $\overline{P}(Cl_t^{\geq})$ and $\underline{P}(Cl_t^{\leq})$, $\overline{P}(Cl_t^{\leq})$, respectively are defined:

$$\begin{aligned} &\underline{P}(Cl_t^{\geq}) = \{x \in U : D_P^+(x) \subseteq Cl_t^{\geq}\}, \\ &\overline{P}(Cl_t^{\geq}) = \bigcup_{x \in Cl_t^{\geq}} D_P^+(x), \\ &\underline{P}(Cl_t^{\leq}) = \{x \in U : D_P^-(x) \subseteq Cl_t^{\leq}\}, \\ &\overline{P}(Cl_t^{\leq}) = \bigcup_{x \in Cl_t^{\leq}} D_P^-(x). \end{aligned} \tag{4}$$

The **P-boundaries (**P-doubtful regions) (5) of $Cl_t^{\geq}$ and $Cl_t^{\geq}$ are defined as:

$$\begin{aligned} &BN_P(Cl_t^{\geq}) = \overline{P}(Cl_t^{\geq}) - \underline{P}(Cl_t^{\geq}), \\ &BN_P(Cl_t^{\leq}) = \overline{P}(Cl_t^{\leq}) - \underline{P}(Cl)_t^{\leq}. \end{aligned} \tag{5}$$

Coming back to our example we have only two classes, which will correspond to two cumulated classes: $Cl^{\geq} = \mathrm{Cl_A}$ and $\mathrm{Cl}^{\leq} = \mathrm{Cl_R}$. We identify all **P-dominating** and **P-dominated** sets for each candidate and we are doing **P-lower** and **P-upper** approximations for each class.

$$\begin{aligned} &\underline{P}(Cl_A) = \{x_1, x_2, x_5\}, \\ &\overline{P}(Cl_A) = \{x_1, x_2, x_3, x_4, x_5, x_6\} \\ &\underline{P}(Cl_R) = \{x_7\}, \\ &\overline{P}(Cl_R) = \{x_3, x_4, x_6, x_7\}, \\ &BN_C(Cl_A) = BN_C(Cl_R) = \{x_3, x_4, x_6\}. \end{aligned} \tag{6}$$

We can see that two P-doubtful regions allow us to identify three candidates where decision of committee was inconsistent. Two of them have been obtained the same evaluations, but x_3 was accepted and x_4 was rejected. The evaluation of sixth candidate is dominated by these of x_3 and x_4, but he was accepted. This example illustrates two kinds of inconsistencies which are identified by the DRSA.

3. Decision rules

On the basis of the approximations obtained by means of the dominance relations, it is possible to derive a generalized description of the preferential information contained in the decision table, in terms of decision rules.

In our example we have obtained the following rules:

Rule 1: If $q_1 \geq 4 \wedge q_3 = 3$ then $d = A$
Rule 2: If $q_1 = 5$ then $d = A$
Rule 3: If $q_2 = 3$ then $d = R$
Rule 4: If $q_1 = 4 \wedge q_2 = 4 \wedge q_3 = 2$ then $d = A \vee d = R$.

Forth rule covers three examples from the decision table where a decision of the committee was contradictory.

The reduced subset of the criteria, which give the same candidate classification as original set is composed of.

4. Application of DRSA to bankrupcy prediction modeling for small and medium businesses

Application of the DRSA in the prediction of the bankruptcy modelling of SMALL AND MEDIUM-SIZED BUSINESSES (SMB) in Abitibi--Témiscamingue was the object of the Master's essay of my student Mohamed Kaba in 2008. In this modelling we coupled results of the discriminate analysis [Altman, 1968] and of the DRSA analysis.

The discriminate analysis consists in calculation of the score Z according to the formula proposed by Altman:

$$Z = 1{,}2X_1 + 1{,}4X_2 + 3{,}3X_3 + 0{,}6X_4 + 0{,}999X_5 \tag{7}$$

where:

X_1 – is working capital / total assets,
X_2 – is retained earnings / total assets,
X_3 – is earning before interest and taxes / total assets,
X_4 – is market value of equity / total liabilities,
X_5 – is sales / total assets.

The value of Z was calculated for seven manufacture companies chosen among thirty in Abitibi-Témiscamingue, the region of Quebec, for a period of five years. According to Altman the discriminate value of Z for the prosperous companies is equal to at least 2.67. The value of Z for any SMB in manufacture sector of Abitibi-Témiscamingue which went bankrupt was negative. The manufacture SMB whose Z was between 0 and 2.67, were classified as on the edge of bankruptcy.

Then, in the analysis the DRSA we considered three decisional classes of SMB: prosperous-(P), bankrupt-(B) and on the edge of bankruptcy-(M). These seven SMB were evaluated by four experts in charge of regional development with respect to five criteria on the scale of seven levels (Tables 2 and 3).

Table 2

Scale of evaluation

Criterion level	Signification
1	Very strong(Very developed)
2	Strong (Developed)
3	Strong enough (Developed enough)
4	Weak (Little developed)
5	Weak enough(Very little developed)
6	Very weak (Not developed)
7	I have no idea

The criteria were five capacities of the successful promoters (in according to Filion [1991]): leadership, vision, network of contacts, management abilities and differentiation. The experts had the perfect knowledge of the promoters, but they did not know results of the discriminate analysis.

Table 3

Decision Table

Prom.	Leader.	Vision	Network	Manage.	Dif	d
1	1	2	1	1	2	P
2	2	2	4	4	3	B
3	4	2	4	6	2	M
4	4	4	4	4	2	M
5	4	2	4	2	2	M
6	3	2	1	2	4	P
7	4	5	6	6	4	B

The calculations were done with software package 4eMka2 developed by Laboratory of Intelligent Decision Support System (IDSS) in the Institute of Computing Science, Poznan University of Technology. We identified three reduced subsets of criteria:

1. Management abilities, Network of contacts.
2. Management abilities, Leadership and Vision.
3. Management abilities, Leadership and Differentiation.

In fact, since Management abilities are in each reduced subset of criteria, this criterion cannot be ignored in the analysis without influencing the quality of approximation. In this case, the quality of approximation was not very high and equal to 0.57 what can be explain by difficulties of the experts to distinguish between the class of the bankruptcy (B) and the class on the edge of bankruptcy (M) (Rule 5).

We have obtained five decision rules:

Rule 1: **If** Network of contacts very developed **then** SMB are prosperous;
Rule 2: **If** Management abilities at least strong **then** SMB are at least on the edge of bankruptcy
Rule3: **If** Network of contacts at most little developed **then** SMB are at most on the edge of bankruptcy;
Rule 4: **If** Vision at most very little developed **then** bankruptcy.
Rule 5: **If** Management abilities at most weak and vision at least little developed **then** bankruptcy or on the edge of bankruptcy.

From these rules we can see that the criteria **Network of contacts** and **Vision** also are very important. If **Network of contacts** is very developed than SMB are prosperous, if this criterion is evaluated on the level at most little developed, SMB are on the edge of bankruptcy or on the level of the bankruptcy. **Vision** is discriminatory in the negative terms. If it is at most very little developed then SMB are bankrupt.

Conclusions

In this paper, the Dominance-Based Rough Set Approach is proposed as an operational tool for aid to the regional developing and assistance of Small and Medium Businesses. This problem was treated by [Dimitras et al., 1999; Slowinski and Zopounidas, 1995], but based on classical approach of rough sets and used for the prediction of corporate businesses failure in the particular bank of Greece. They considered only financial criteria. For regional development problems, it is important to consider quantitative and qualitative criteria. For this reason our proposal consists of hybrid model composed of quantitative and qualitative data. The quantitative part (Z-score) is used to identify decision classes of the rough set model which is based on qualitative criteria. The prediction model has the form of decision rules which are particularly useful for evaluation of new promoters.

Z-score method was adapted to evaluate SMB and it was validated for regional population of manufacture businesses. We observed that the discriminate values in the case of SMB are lower than these suggested by Altman. In particular, the lower value of Z, suggested by Altman to be close to 1.8, is rather close to zero. This explains the difficulties which experts had in distinguishing between two classes: bankruptcy and on the edge of bankruptcy.

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