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Marcin Anholcer

ALGORITHM FOR DERIVING PRIORITIES FROM INCONSISTENT PAIRWISE COMPARISON MATRICES

Abstract

In several multiobjective decision problems Pairwise Comparison Matrices (PCM) are applied to evaluate the decision variants. The problem that arises very often is inconsistence of given PCM. In such a situation it is important to approximate the PCM with a consistent one. The most common way is to minimize the Euclidean distance between the matrices. In the paper we consider minimization of the maximum distance.

Keywords

Heuristics, nonlinear programming, decision making, pairwise comparison.

Introduction

One of the popular tools of multiobjective decision making is the Analytic Hierarchy Process, introduced by Saaty [see e.g. Saaty 1980; Erkut and Tarimcilar 1991] and studied by numerous authors. During the process, the Decision Maker compares pairwise n given decision variants. Usually the comparisons are represented by the *pairwise comparison matrix* $A = [a_{ij}]$, where the number a_{ij} says how many times the variant i is preferred to the variant j.

The values of a_{ij}, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, n$ should fulfill the following conditions:

$$a_{ji} = \frac{1}{a_{ij}} \text{ for } i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n, \tag{1}$$

$$a_{ij}a_{jk} = a_{ik} \text{ for } i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, n, k = 1, 2, \ldots, n. \tag{2}$$

If the above conditions are satisfied, the pairwise comparison matrix A is called *consistent*. The condition (1) is rather easy to fulfill in practice (the decision maker may e.g. fill only the elements of A above the diagonal and then the remaining ones are easily calculated). The condition (2) is much more difficult to satisfy and is the main source of the inconsistency.

It is easy to prove that the matrix A is consistent if and only if there exist positive weights $w_1, w_2, \ldots, w_n$ (forming the vector w) such that

$$a_{ij} = \frac{w_i}{w_j}, i = 1,2, \ldots, n, j = 1, 2, \ldots, n \tag{3}$$

The elements of w are interpreted as the explicit values representing the priorities of the decision variants. Finding their values is thus essential. Note that if some vector w defines the matrix A then also the vector λw for every $\lambda > 0$.

1. Problem formulation

As in real-life problems the matrix A is very often not consistent, it is impossible to find the vector w (in fact, it does not exist). In such a situation the goal is to find the vector w that defines the matrix B which is as close as possible to the original pairwise comparison matrix A.

The distance between matrices A and B may be calculated in various ways. One of the methods is to calculate Saaty's inconsistency index using the eigenvalues of the (relative) estimation error matrix, which can be approximated by the row-wise geometric means [see e.g. Saaty 1980; Mogi and Shinohara 2009]. Estimation errors are calculated as the quotients or differences of the respective elements of A and B. Another approach, based on the additive PCM (a formulation equivalent to the one discussed in this paper), may be found e.g. in Fedrizzi, Giove [2007].

Another approach is to calculate some kind of average of errors. The most popular measure is the square mean calculated according to the formula

$$G_2(A, v) = \left(\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(a_{ij} - \frac{v_i}{v_j}\right)^2\right)^{\frac{1}{2}}. \tag{4}$$

This method of the inconsistency measurement (called least square method, LSM) was introduced in this context by Chu et al. [1979] and used e.g. by Anholcer et al. [2011], Bozóki [2008], Fülöp, Koczkodaj and Szarek [2010], Fülöp [2008], Bozóki and Rapcsàk [2008], Mogi and Shinohara [2009].

In the last two papers other inconsistency measures were also considered. Mogi and Shinohara analyzed the general mean which can be defined as

$$G_p(A,v) = \left(\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|a_{ij} - \frac{v_i}{v_j}\right|^p\right)^{\frac{1}{p}}. \quad (5)$$

If $p = 2$, we obtain the LSM. Other special cases, also considered in the paper, are $p = -\infty$ (minimum), $p = -1$ (harmonic mean), $p = 0$ (geometric mean), $p = 1$ (arithmetic mean) and $p = \infty$ (maximum). In the remainder of this paper we will be interested in the last measure. To be more precise, we want to solve the following problem:

$$\min\left\{G_\infty(A,v) = \max_{1\le i,j\le n}\left\{\left|a_{ij} - \frac{v_i}{v_j}\right|\right\}\right\}, \quad (6)$$

s.t.

$$v_1 = 1, \quad (7)$$

$$v_j > 0,\ j = 1, 2, \dots, n. \quad (8)$$

The condition $v_1 = 1$ has been introduced to normalize the vector v (if some vector v is the solution to the above problem, then also every vector λv for every $\lambda > 0$). Of course other normalizing conditions can be used [compare e.g. Anholcer et al. 2011; Bozóki 2008; Fülöp 2008].

The problem under consideration is a difficult optimization problem, as the objective function is neither convex nor concave and thus no local search algorithm may be applied to find the global optimum.

The LSM problem (with G_2 instead of G_∞) was studied e.g. in Anholcer et al. [2011] – heuristic approach, Bozóki [2008] – systems of nonlinear equations and Fülöp [2008] – branch and bound algorithm. The statistical approach was used by Hovanov, Kolari and Sokolov [2008], while Mogi and Shinohara [2009] used simulation. Our goal is to give an effective method to derive the weights minimizing the value of function G_∞ as the inconsistency measure.

2. New algorithm

The problem (6)-(8) may be reformulated as follows. Let us introduce additional variable $z = G_\infty(A,v)$. Then we can rewrite the problem as

$$\min\{z\}, \quad (9)$$

s.t.

$$\left|a_{ij} - \frac{v_i}{v_j}\right| \leq z, \qquad i,j = 1,2,\dots,\mathrm{n} \tag{10}$$

$$v_1 = 1, \tag{11}$$

$$v_j > 0, j = 1,2,\dots,n. \tag{12}$$

Note that the problems are not identical (the sets of feasible solutions are distinct), but they are equivalent (the optimal solutions to both problems are the same and they always exist). The problem (9)-(12) is a difficult mathematical programming problem – the constraints (10) are nonlinear, neither convex nor concave. Moreover, the set of feasible solutions is not closed and thus not compact (although the optimum exists). In order to find its approximate solution we are going to treat z as a parameter.

If we assume that the value of z is given, the problem (9)-(12) reduces to the following system of linear equations and inequalities:

$$(a_{ij} - z)v_j \leq v_i \leq (a_{ij} + z)v_j \qquad i,j = 1,2,\dots,\mathrm{n} \tag{13}$$

$$v_1 = 1, \tag{14}$$

$$v_j \geq 0, j = 1,2,\dots,n. \tag{15}$$

Note that the constraint (12) may be replaced with (15) as none of v_j can be equal to 0 – otherwise all of them would be equal to 0 according to the constraints (13). That would in turn contradict the constraint (14).

The number of inequalities (13) may be reduced. First, for every i, the inequalities $(a_{ii} - z)v_i \leq v_i \leq (a_{ii} + z)v_i$ are always satisfied as $a_{ii} = 1$ and $z \geq 0$.

Another operation lets us remove half of the remaining inequalities. Let us consider the two inequalities in which the variables v_i and v_j occur for some $i \neq j$. They can be rewritten in the following form:

$$-v_i + (a_{ij} - z)v_j \leq 0, \tag{16}$$

$$-v_i + \frac{1}{(a_{ji} + z)}v_j \leq 0, \tag{17}$$

$$\frac{1}{(a_{ij} + z)}v_i - v_j \leq 0, \tag{18}$$

$$\left(a_{ji}-z\right)v_i - v_j \leq 0. \tag{19}$$

Exactly one of the inequalities (16) and (17) implies the other one, so one of them can be removed. More precisely, we leave the inequality

$$-v_i + \left(\max\left\{\left(a_{ij}-z\right), \frac{1}{\left(a_{ji}+z\right)}\right\}\right) v_j \leq 0. \tag{20}$$

Analogously, we can eliminate one of the inequalities (18) and (19), by choosing the following one

$$\left(\max\left\{\left(a_{ji}-z\right), \frac{1}{\left(a_{ij}+z\right)}\right\}\right) v_i - v_j \leq 0. \tag{21}$$

Note that in both cases the chosen maxima have positive values. To solve the resulting system of linear inequalities and equations, we formulate the following auxiliary linear programming problem.

$$\min\{z_0\} \tag{22}$$

s.t.

$$-v_i + \left(\max\left\{\left(a_{ij}-z\right), \frac{1}{\left(a_{ji}+z\right)}\right\}\right) v_j + z_{ij}^1 = 0, \;\; 1 \leq i < j \leq \text{n}, \tag{23}$$

$$\left(\max\left\{\left(a_{ji}-z\right), \frac{1}{\left(a_{ij}+z\right)}\right\}\right) v_i - v_j + z_{ij}^2 = 0, \;\; 1 \leq i < j \leq \text{n}, \tag{24}$$

$$v_1 + z_0 = 1, \tag{25}$$

$$v_j \geq 0, \;\; j = 1, 2, \dots, n, \tag{26}$$

$$z_0 \geq 0, \;\; z_{ij}^k \geq 0, \;\; 1 \leq i < j \leq n, \;\; k = 1,2. \tag{27}$$

We solve the above problem using the adapted version of the simplex method. The initial feasible base solution is formed by the variables included in constraint (27): $z_0 = 1$ and $z_{ij}^k = 0, \; 1 \leq i < j \leq n, \; k = 1,2$. The reduced costs are equal to the coefficients in the constraint (25). Also, we use additional stopping criterion: $z_0 = 0$. If this criterion is used, the initial system of inequalities has feasible solution where the values of v_j are equal to those in the optimal solution of the problem (22)-(27). On the other hand, if the standard optimality condition is in use, that means that $z_0 = 1$ and the problem (22)-(27) is inconsistent.

Note also that if the feasible solution exists for some value of $z = z^\star$, then it is also the solution for every value $z \geq z^\star$. That means also that if the system (13)-(15) is inconsistent for some value of $z = z^\star$, then it is also inconsistent for every $z \leq z^\star$. This leads us to the following algorithm, where the starting point is generated by the geometric means of rows of A.

Algorithm 1

1. Assume the accuracy level $\varepsilon > 0$. Let $v_i^\star = \left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}$ and $v_i = \frac{v_i^\star}{v_1^\star}$ for $i = 1, 2, \dots, n$. Let $z = z_{max} = G_\infty(A, v)$ and $z_{min} = 0$. Proceed to step 2.
2. If $z - z_{min} < \varepsilon$ then STOP. The vector v is the desired approximation of the weight vector w. Otherwise go to step 3.
3. Set $z := \frac{(z_{max} - z_{min})}{2}$. Solve the problem (22)-(27). If $z_0 = 0$, save the new value of v and set $z_{max} := z$. Otherwise do not change the value of v and set $z_{min} := z$, $z := z_{max}$. Go back to step 2.

In every step of the algorithm the value of $z_{max} - z_{min}$ decreases twice, so in the finite number of iterations we obtain the approximation of the optimal solution (more precisely, if $z_{max}^\star$ denotes the initial value of z_{max}, then the algorithm stops after $\left\lceil \log_2\left(\frac{z_{max}^\star}{\varepsilon}\right)\right\rceil$ steps).

3. Numerical example

Let us present a small illustrative example. Assume that

$$A = \begin{bmatrix} 1 & 2 & 1 & 5 & 2 \\ 0.5 & 1 & 0.8 & 2.5 & 0.4 \\ 1 & 1.25 & 1 & 2.5 & 1 \\ 0.2 & 0.4 & 0.4 & 1 & 0.8 \\ 0.5 & 2.5 & 1 & 1.25 & 1 \end{bmatrix}$$

and $\varepsilon = 0.1$.

Step 1. We derive the initial solution as the geometric means of the rows and divide all of them by v_1, so $v_1 = 1.000$, $v_2 = 0.457$, $v_3 = 0.690$, $v_4 = 0.264$, $v_5 = 0.821$. The matrix derived with the values v_j has the form

$$B = \begin{bmatrix} 1.000 & 2.187 & 1.450 & 3.789 & 1.665 \\ 0.457 & 1.000 & 0.663 & 1.733 & 0.761 \\ 0.690 & 1.509 & 1.000 & 2.614 & 1.149 \\ 0.264 & 0.577 & 0.383 & 1.000 & 0.439 \\ 0.601 & 1.313 & 0.871 & 2.276 & 1.000 \end{bmatrix}.$$

As one can easily check, the inconsistency measure equals 1.211. Thus $z_{min} = 0$ and $z = z_{max} = 1.211$.
Step 2. $z_{max} - z_{min} > \varepsilon$, we proceed to step 3.
Step 3. $z = 0.605$. In the optimal solution of the problem (22)-(27), $z_0 = 1$. Thus $z_{min} = 0.605$, $z = z_{max} = 1.211$. We go back to step 2.
Step 2. $z_{max} - z_{min} > \varepsilon$, we proceed to step 3.
Step 3. $z = 0.908$. In the optimal solution of the problem (22)-(27), $z_0 = 1$. Thus $z_{min} = 0.908$, $z = z_{max} = 1.211$. We go back to step 2.
Step 2. $z_{max} - z_{min} > \varepsilon$, we proceed to step 3.
Step 3. $z = 1.059$. In the optimal solution of the problem (22)-(27), $z_0 = 0$ and $v_1 = 1.000$, $v_2 = 0.366$, $v_3 = 0.486$, $v_4 = 0.254$, $v_5 = 0.527$. We save this solution. Moreover, $z_{min} = 0.908$, $z = z_{max} = 1.059$. We go back to step 2.
Step 2. $z_{max} - z_{min} > \varepsilon$, we proceed to step 3.
Step 3. $z = 0.984$. In the optimal solution of the problem (22)-(27), $z_0 = 1$ Thus $z_{min} = 0.984$, $z = z_{max} = 1.059$. We go back to step 2.
Step 2. $z_{max} - z_{min} < \varepsilon$, STOP. The optimal weights are equal to $v_1 = 1.000$, $v_2 = 0.366$, $v_3 = 0.486$, $v_4 = 0.254$, $v_5 = 0.527$. They define consistent PCM of the form

$$B = \begin{bmatrix} 1.000 & 2.735 & 2.059 & 3.941 & 1.899 \\ 0.366 & 1.000 & 0.753 & 1.441 & 0.694 \\ 0.486 & 1.328 & 1.000 & 1.914 & 0.922 \\ 0.254 & 0.694 & 0.523 & 1.000 & 0.482 \\ 0.527 & 1.441 & 1.085 & 2.075 & 1.000 \end{bmatrix}.$$

4. Computational experiments

The algorithm has been implemented in Java and tested for a number of randomly generated problems. The assumed accuracy level was $\varepsilon = 0.001$. The application has been tested on the PC with Intel Core2 Duo CPU (2.20 GHz). For every value of $n = 3,4,\dots,10$ (in real-life problems, the size of the comparison matrix rarely exceeds 10) the elements of A were chosen uniformly at random from the interval $< 1, a_{\max} >$, where $a_{max} \in \{3,5,10\}$. All PC matrices obtained were inconsistent. In every case 100 problems have been solved (which gives the total number of 2400 test problems). The average running times (in milliseconds) are given in the Table 1.

Table 1

Average running times

n	$a_{max} = 3$	$a_{max} = 5$	$a_{max} = 10$
3	0,0117	0,0127	0,0158
4	0,0237	0,0284	0,0297
5	0,0528	0,0550	0,0605
6	0,0979	0,1240	0,1191
7	0,1720	0,1998	0,2047
8	0,2686	0,2934	0,3288
9	0,4215	0,4546	0,5110
10	0,6271	0,6607	0,7507

As we can see, in all the cases the running times are much less than one second, which is acceptable time in real life applications.

Conclusions

The algorithm presented guarantees obtaining the solution for which the objective value is arbitrarily close to the optimal one. Of course this does not mean that the coordinates of vector v are arbitrarily close to their optimal values (distinct local optima may be far from each other even if the objective values are very close). However it is more that gives the heuristic for LSM given by Anholcer et al. [2011], which does not guarantee obtaining the objective value close to the optimal one. On the other hand the algorithm presented is fast and therefore very useful for finding the best consistent approximate of an inconsistent pairwise comparison matrix.

As far as the author knows the method presented here is the first one for the inconsistency measured using the maximum distance G_∞. Further research should focus on looking for the exact method of solving this problem and any methods for other measures (e.g. G_p distance for arbitrary p, including Manhattan distance G_1).

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THE ANALYSIS OF NEGOTIATORS' PREFERENCE CONSISTENCY IN INDIFFERENCE-SURFACE BASED SCORING SYSTEM

Abstract

In this paper we present a new method for analyzing the consistency of preferences of negotiators in building their scoring systems of negotiation offers. The method we propose can be used when the preferences are defined as general examples of full packages with the accompanying utility score, as it is done in the NegoManage negotiation support system in the conjoint analysis approach. During the preference elicitation stage the negotiators identify the indifference surfaces (or indifference sets) to which they also assign sample alternatives and scores. The verification of such the consistency of this assignment is based on the concept of the Jaccard index, that allows for measuring the similarity between fuzzy sets. Since we obtain a characteristics of equivalence sets in the form of probability distributions, which are further treated as fuzzy set membership functions, we can use the distribution characteristics to compute the Jaccard index for every pair of equivalence sets elicited from the negotiator. If these indexes are too high, the corresponding indifference sets should be reconsidered or integrated.

Keywords

Negotiation support, negotiation offers' scoring system, preference elicitation, indifference sets, kernel density estimation.

Introduction

In the process of multiple criteria decision-making (MCDM) the decision makers have to cope with problems of comparing and evaluating very many (usually conflicting) criteria. Such decision-making processes involve, depending on the decision context of the problem, evaluation, prioritization or selection of alternatives. Among many MCDM methods the most popular are: simple additive weighting models (based on multiple attribute utility theory) – [Keeney and Raiffa 1993], AHP [Saaty 1980; Saaty and Alexander 1989], ELECTRE [Roy and Bouyssou 1993] and PROMETHE [Brans 1982]. The MAUT-based models constitute a scoring system allowing for ranking any alternative after the weights and marginal utilities have been elicited. This method fuses single attribute utilities with weights assigned to attributes and results in a final value of utility for the given alternative. The AHP method is based on pairwise comparisons of attributes and alternatives, and results in a ranking of the given alternatives. ELECTRE and PROMETHEE methods are based on an outranking concept and give as a result of analysis an ordering of the given alternatives. The literature review shows quite a lot of examples of using these methods in solving actual business-related decision making problems [see eds. Figuera et al. 2005; Omkarprasad and Sushil 2006; Behzadian et al. 2010]. In the negotiation context, however, it is a simple additive weighting (SAW) model that is most widely used for eliciting negotiators' preferences. All the most popular negotiation support systems (NSSs) such as Inspire [Kersten and Noronha 1999], Negoist [Schoop et al. 2003] and SmartSettle [Thiessen and Soberg 2003] accomplish their decision support function by using the simple additive scoring model (sometimes hybridized with the conjoint analysis approach) for evaluating the negotiation template and building the scoring systems of the negotiation offers used in the actual negotiation phase for evaluation and analysis of the sequence of offers and counteroffers proposed by the parties as the negotiation contact proposals. But the recent negotiation experiments show that NSS users very often misinterpret the SAW scores and find it difficult to assign them to the negotiation options and issues [see Wachowicz and Kersten 2009; Paradis et al. 2010]. Therefore new mechanisms and systems are being built that apply preference elicitation approaches other than the SAW-based ones, such as NegoManage [Brzostowski and Wachowicz 2009, 2010] which allows to determine the scoring systems of negotiation offers deriving from the examples of offers that the negotiator specifies in the prenegotiation phase. The NegoManage system supports the negotiator in all phases of the negotiation process allowing not only for the preference elicitation but also for an exchange of offers and messages, tracking the negotiation history, negotiation profile identification and counterpart evaluation and selection. The preference elicitation engine is a key element of the system. The whole preference elicitation mechanism is based on the concept of the equivalence set that may be specified by the negotiator as a set of alternatives indifferent in terms of preferences. The negotiator also evaluates this set verbally by assigning to it a linguistic value of utility. The whole process of preference analysis requires of the negotiator the specification of the sequence of indifference sets with the corresponding utilities that are the basis for the scoring system of negotiation offers. After the scoring system has been prepared any alternative from the set of feasible alternatives (i.e. those defined in the template) can be evaluated in terms of utility assigned to this alternative. Since the preference analysis approach that we have applied in NegoManage system operates, similarly to the conjoint analysis approach, with complete offers and the corresponding score definitions (a full package must be specified and evaluated by the negotiators) we may face the problem of negotiator consistency in specifying different examples of offers and their scores. It may appear that two very similar packages are assigned to two separate indifference sets that differ much in terms of a linguistic utility evaluation or that the packages assigned to one indifference set differ too much to have assigned the same linguistic utility label. If so, we say that the problem of preference consistency occurs and consequently corrective actions need to be undertaken before the final scoring system is determined and used for the evaluation of offers in the actual negotiation phase.

In this paper we propose a simple mechanism for verifying the consistency of negotiators' preferences that we apply in the NegoManage system. The preference consistency check is based on the concept of the Jaccard index allowing for measuring the similarity between fuzzy sets. Since the NegoManage preference elicitation approach allows to obtain the characteristics of equivalence sets in the form of probability distributions, we may further consider these functions as fuzzy sets membership functions and use the distribution characteristics to compute the Jaccard index for every pair of equivalence sets elicited from the negotiator. The Jaccard indexes measure similarity between the indifference sets defined by the negotiator. If for any two sets the Jaccard index is too high, it is recommended to reconsider these two indifference sets by analyzing both the examples of offers constituting these sets and the values of utility scores assigned to these sets. It may appear that it would be reasonable to join the sets or differentiate their original scores to obtain a more accurate final scoring system of negotiation offers.

The paper consist of four more sections. In Section 1 we introduce the general idea of eliciting preferences of negotiators in the NegoManage system. Then in Section 2 we give a deeper insight into the method of defining the preferences by means of indifference sets that we proposed earlier in the NegoManage NSS and discuss the issue of kernel density analysis required for determining the main characteristics of these sets. We present also briefly the major idea of the Jaccard index (Section 3) and the possibility of inter-change between the two alternative approaches to describing the indifference sets, i.e. probability-based and possibility-based. In Section 4 we show in detail an example of analyzing the preference of the negotiator and measuring its consistency using a very simple negotiation problem where the template consists of three negotiation issues only. We conclude the paper with some comments on the proposed mechanism of preference elicitation and consistency verification.

1. Preference representation in NegoManage system – Indifference sets and their characteristics

In the NegoManage system the negotiator defines preferences by specifying several sets of alternatives, called indifference sets (surfaces), and assigning a degree of utility to each surface. Each indifference set consists of the alternatives that the negotiator considers to be equally good. The degree of utility assigned to the surface is chosen from a linguistic (verbal) scale [see Yevseyeva et al. 2008]. The scale is build on two levels. The first-level scale consists of seven verbal terms. First, the negotiator assigns to the indifference set a level from this scale. The second-level scale allows for stating precisely the degree of utility; namely by choosing a degree between two neighboring terms from the first scale. The second scale consists also of seven verbal terms and leads to an increase in the precision of the utility specification. Each linguistic utility level has its numeric equivalent used during the scoring procedure. However, such sets consisting of alternatives representing a particular level of utility may not be sufficient for deriving a full scoring system of the negotiation offers. There are probably other alternatives that may also belong to this surface, that were not specified by the negotiators in the preference elicitation stage but could easily be built in the actual negotiation phase by changing proportionally the resolution levels of the subsequent negotiation issues (making implicitly trade-offs between the negotiation issues). Initially the negotiators may not have specified all the salient alternatives for a particular indifference set because of lack of time, haste or simply because they subjectively felt that a certain alternative is not important (conveys no important information) in the definition of this indifference surface. Therefore we need to remember that the alternatives that comprise the indifference surfaces are only examples of a particular utility. There are many other alternatives (especially in the continuous negotiation problems) that may also belong to each of these sets. However, the degree of belonging to a surface may be partial since for alternatives not classified directly by the negotiator we may never be sure of their belonging to this surface. To cope with this type of uncertainty we propose to model the level (or chance) of belonging to the surface by using the notion of probability. More precisely, we propose to build a characteristic of an indifference surface in the form of probability distribution. Such a function will assign to each alternative a level of belonging to the particular indifference surface.

After the surfaces have been specified and the utility degrees have been assigned, the NegoManage system performs its most important task, namely the computation of probability distributions for all specified surfaces that will be further used to build a global scoring system. The probability assigned to a particular point of the indifference surface can be interpreted as a chance of actual assignment of this point to the indifference set. To build the characteristic of a surface we use the following straightforward postulate:

> *The closer an alternative under consideration is located to the one that fully belongs to the indifference surface, the higher is the level of probability that we assign to this alternative.*

In other words, for an alternative located in the neighborhood of a fully classified alternative we first compute the distance to the classified alternative and map it into a similarity degree. The similarity degree of the considered alternative to the fully classified alternative is regarded as the probability of belonging to the surface. Based on our postulate we propose to build peaks around the classified alternatives as a preliminary step of building the probability distribution. Such peaks may have a bell shape of the normal distribution curves, and such a peak shape is considered at the current stage of research. However, there are no substantial or experimentally proved reasons for using this type of probability distributions in our approach. We simply use the normal distribution functions as the most commonly used solution in the selection of the predefined shape of the probability function since we lack the relevant information that would allow us to use other types of probability distribution functions. In the next step such peaks are fused together to form an overall multi-modal distribution which is treated as the indifference surface's characteristic together with utility levels assigned to the surfaces by the negotiator. This information about each indifference set is a basis for determining the score of any feasible alternative under evaluation later on in the actual negotiation phase, when the negotiators face the problem of scoring the offers presented by their counterpart as the negotiation compromise proposals.

2. Preference analysis in NegoManage system

Let us now look in detail at the formalism of the preference analysis approach implemented in the NegoManage system and described briefly in the previous section. Let us assume that the negotiator specifies a set of indifferent alternatives in the following form:

$$RS_i = \{\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n\} \tag{1}$$

where the indifference relationship holds between every pair of alternatives: $\overline{a}_i \approx \overline{a}_j$.

The utility value u_i assigned to the *i*-th indifference surface means that all alternatives in this set have this utility value since the alternatives in the set are equivalent in terms of preference, i.e. $\forall \overline{a} \in RS_i \mid u(\overline{a}) = u_i$. To illustrate this, let us consider a simple single-issue case. The focal negotiator decided to form the indifference surface by means of four alternatives.

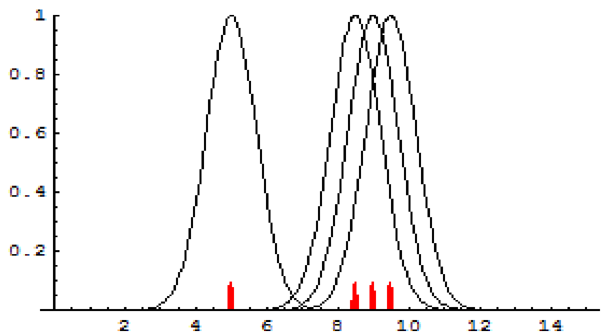

Figure 1. Alternatives and the corresponding peaks for defining the indifference surface (set)

In Figure 1 the peaks for four points are shown (red points indicate four alternatives belonging to the surface) that the negotiator decided to assign to the indifference surface under consideration. The peaks describe the probability distribution that this alternative and the similar ones (the ones in the close neighborhood) belong to the indifference surface under consideration. The concept of Kernel Density Estimation allows for deriving the overall distribution by fusing the peaks using an average operator. Assuming that the kernel is in the shape of normal distribution, the distribution for the surface specified above is of the following form:

$$f_{RS}(x) = \frac{1}{4d}\sum_{i=1}^{4}\exp\left(-\left(\frac{x-m_i}{d}\right)^2\right). \qquad (2)$$

where m_i are the locations of the four points.

As the result of fusing the peaks into a compound distribution we obtain a distribution with two peaks (see Figure 2).

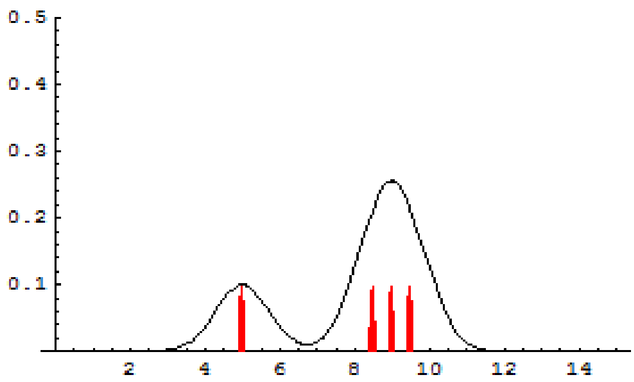

Figure 2. Aggregated peaks for defining the indifference surface (set)

The first peak is located around the first classified alternative and the second peak is located around the group of the other three. What we can observe here is the accumulation of high probability value around the group of three alternatives. We can conclude from this observation that points densely grouped in a small area can accumulate a higher probability in this area than the probability accumulated by other points. Consequently, for other regions represented by single alternatives the probability cumulated in the peaks around a single alternative may be decreased to a level which may be too low as compared to peaks located around dense groups of alternatives. Therefore, we propose to split the set of classified alternatives using hierarchical clustering into groups where the alternatives are close to each other, in order to build peaks over groups of alternatives instead of building peaks over single alternatives. Such a procedure will avoid the accumulation of high probabilities around dense groups of alternatives (see Figure 3).

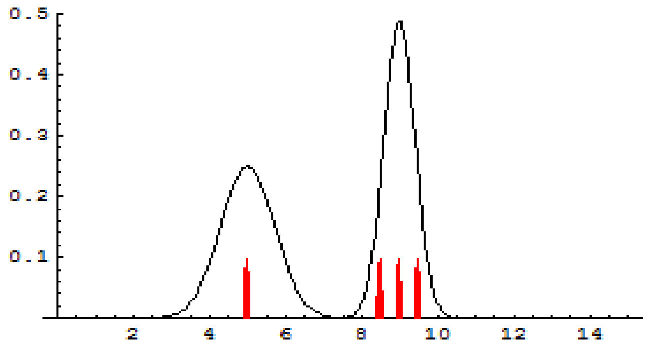

Figure 3. Aggregated peaks for the alternatives grouped within the indifference surface (set)

In a general multi-issue negotiation problem the probability distributions corresponding to the indifference surfaces are of multivariate form. Therefore, first we define the negotiation alternatives as follows. Every alternative $\overline{a}$ is described by a sequence of mappings $g_1, g_2, \ldots, g_m$ in the following way:

$$\overline{a} = (g_1(\overline{a}), g_2(\overline{a}), \ldots, g_m(\overline{a})) . \tag{3}$$

where each mapping g_s maps the alternative $\overline{a}$ into the numerical value of *s*th issue.

The simplest way to cluster the alternatives constituting the indifference set is to use hierarchical clustering [see Hartigan 1975; Hair and Black 1992]. The algorithm is agglomerative which means that at the beginning of the procedure each cluster consists of one alternative. In the next stages of the clustering algorithm the clusters are successively merged together. The number of clusters is decreasing while the size of clusters grows. The merging stops when the maximal distance between the alternatives inside the clusters reaches a selected level. As a result of this algorithm we obtain a split of the indifference surface. Given a split of the set RS_i into k disjoint subsets $M_{i1}, M_{i2}, \ldots, M_{ik}$, the means for all subsets (clusters) are computed: $\overline{m}_{i1}, \overline{m}_{i2}, \ldots, \overline{m}_{ik}$ (for the computation of the mean we use simple average). The multi-modal distribution is built over the indifference surface consisting of kernels determined over subsets M_{ij} [see Parzen 1962]. For the *j*th cluster of the *i*th indifference surface the multi-normal kernel distribution may be calculated:

$$f_{M_{ij}}(\overline{a}) = \frac{1}{(2\pi)^{k/2} |\Sigma_{ij}|^{1/2}} \exp\left(\frac{1}{2}(\overline{a} - \overline{m}_{ij})'\Sigma_{ij}^{-1}(\overline{a} - \overline{m}_{ij})\right). \tag{4}$$

where Σ_{ij} is the covariance matrix.

Let us assume that $M_{ij} = \{\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n\}$. For the estimation of the covariance matrix we use the following estimator:

$$\Sigma_{ij} = \frac{1}{n-1}\sum_{l=1}^{n}(\overline{a}_l - \overline{m}_{ij})(\overline{a}_l - \overline{m}_{ij})'. \tag{5}$$

where the operator ' is the matrix transposition.

Having the distributions $f_{M_{ij}}$ for all clusters M_{ij} the final characteristics in the form of a multi-modal distribution is built and has the following form:

$$f_{RS_l}(\overline{a}) = \frac{1}{k}\sum_{j=1}^{k} f_{M_{ij}}(\overline{a}). \tag{6}$$

The final scoring system consists of the sequence of indifference sets distributions together with its utility (defined in a linguistic form) assigned to the indifference set by the negotiator:

$$(f_{RS_i}, u_i) \quad i \in \{1, \ldots, m\}. \tag{7}$$

During the actual negotiation phase the scoring system is used to evaluate the chosen alternative $\overline{a}$ in the following way. First the degree of belonging to a particular indifference set is computed. In other words, the probability of belonging to a particular indifference set is computed:

$$p(i) = p_i(\overline{a}) = f_{RS_i}(\overline{a}) \quad i \in \{1, \ldots, m\}. \tag{8}$$

The degree of belonging $p_i(\overline{a})$ is computed for all indifference sets. As a result we obtain a discrete probability distribution over a set of indices $i \in \{1, \ldots, m\}$ indexing all indifference surfaces (indifference sets). The distribution $p(i)$ tells us the degree of belonging of the given alternative to any indifference set. In the next step of computation we need to obtain the final utility for the alternative $\overline{a}$. In other words, we look for the indifference set to which the alternative $\overline{a}$ belongs with the highest degree. However, the notion of belonging to a set is not binary here. The alternative may belong to many indifference sets with different values of belonging degree. Therefore, to obtain the indifference set index and the final utility of the alternative $\overline{a}$ we use the concept of von Neumann-Morgenstern expected utility [see Neumann and Morgenstern 1944]. The linguistic utility in NegoManage consists of two linguistic values (v_{1i}, v_{2i}) describing the utility in terms of two integrated scales and these values correspond to numerical interval $[l_i, r_i]$:

$$u_i = (v_{1i}, v_{2i}) \to [l_i, r_i] \quad i \in \{1, \ldots, m\}. \tag{9}$$

These two boundary values of expected utility (lower and upper) are computed as follows:

$$LEU(\overline{a}) = \sum_{i=1}^{n} p_i(\overline{a}) \cdot l_i, \tag{10}$$

$$UEU(\overline{a}) = \sum_{i=1}^{n} p_i(\overline{a}) \cdot r_i. \tag{11}$$

As a result we obtain a pair of utilities describing the final utility interval $[LEU(\overline{a}), UEU(\overline{a})]$ that can be mapped back into linguistic utility to present it to the user. The precise description of interval in numerical form can be used in further computation.

3. Preference consistency and Jaccard index

The consistency of the scoring system is a key issue in the NegoManage system. Since one set contains alternatives ranked with a different degree of utility and a different indifference set contains alternatives ranked with different degree of utility, two difference sets with different utility values should not overlap (should be disjoint). If an alternative belonged to different indifference surfaces, it would mean that different levels of utility have been assigned to the same alternative. Therefore, we assume that the preference structure is fully consistent if all indifference surfaces are disjoint. However, if there is a partial overlap of two indifference surfaces, namely some alternatives partially belong to both surfaces, then a measure needs to be defined indicating the extent to which the condition of separation of two surfaces is violated. This extent is indicated in the simplest possible way by the number of alternatives belonging to both surfaces. However, we need to normalize this value to make the measure of inconsistency universal. The normalization is obtained by dividing the cardinality of the intersection of two surfaces by the cardinality of the union of these surfaces. If the intersection of two surfaces is non-empty, we will measure the preference inconsistency using the Jaccard index described above, given by the following formula:

$$J(A,B)=\frac{m(A\cap B)}{m(A\cup B)}. \tag{12}$$

where m is the cardinality of a set.

In the NegoManage system we have at our disposal the characteristics of surfaces given by probability distributions. We use the concept of probability to describe the degree of belonging of an alternative to the indifference surface. However, this interpretation can be also used if we want to describe the indifference surface in the form of a fuzzy set, namely with the membership degree stating the extent to which an alternative is included in the fuzzy indifference surface. Unfortunately, from a formal point of view, the probability distributions cannot be directly treated as membership functions of a fuzzy surface. One of the reasons for this is the normalization axiom defined in different way for a probability distribution and a possibility distribution (a fuzzy set concept used to describe the plausibility of belonging to the indifference surface in our application context). Namely, the normalization condition for the probability distributions means that the probabilities of all alternatives sum up to 1, and in the case of a possibility distribution the function reaches 1 for some alternative (which is the maximal distribution value). The following formula is an extension of the concept of the Jaccard index for fuzzy sets or possibility distributions:

$$J(A,B)=\frac{m(A\cap B)}{m(A\cup B)}=\frac{\max_{\overline{u}\in\Omega}\min(\mu_A(\overline{u}),\mu_B(\overline{u}))}{\max_{\overline{u}\in\Omega}\max(\mu_A(\overline{u}),\mu_B(\overline{u}))}. \tag{13}$$

where Ω is the space of all feasible alternatives, and μ_A, μ_B are the membership functions of the sets A and B.

To apply the fuzzy Jaccard coefficient in our particular application context, first we need to convert the surface characteristics given in the forms of probability distributions into possibility distributions. Such conversions have been proposed by Dubois et al. [2004]. The most important axiom distinguishing the possibility measures from probability measures is:

$$\forall A\subseteq X \quad \Pi(A)=\sup\{\pi(x), x\in A\}. \tag{14}$$

As we can see, this axiom involves the supremum operation instead of Riemann integration as it is done in the case of probability measures

$$\forall A \subseteq X \quad P(A) = \int_A p(x)dx \,. \tag{15}$$

Probability and possibility measures capture different facets of uncertainty. In the case of two disjoint subsets measured using probability measure, the measure of their union is equal to the sum of their measures. In the case of possibility measures the measure of union of disjoint subsets is the supremum (maximum in the case of finite sets). But some linkages between these two approaches may be distinguished (Dubois et al. 2004):

> *As it turns out, a numerical possibility measure, restricted to measurable subsets, can also be viewed as an upper probability function* [Dubois and Prade 1992]. *Formally, such a real-valued possibility measure P is equivalent to the family P(P) of probability measures such that* $P(\Pi) = \{P, \forall A \; measurable, P(A) \leq \Pi(A)\}$

While converting a probability distribution to a possibility distribution the most important principle to be kept in mind is that introduced by Zadeh [1965], stating that *an event must be possible prior to being probable*. This principle is consistent with the fact that possibility distributions encode upper probability distributions. According to Dubois and Prade [1992] the relationship between the possibility distribution and its probability counterpart is described formally as the order preservation rule

$$\pi(x) < \pi(x') \quad \textit{if and only if} \quad p(x) < p(x') \,. \tag{16}$$

Let us assume that we have a probability distribution defined over a finite set of alternatives: $\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_n$. Moreover, without loss of generality the alternatives are ordered according to the probability values, namely we have the corresponding levels of probability: $p_1 \geq p_2 \geq \ldots \geq p_n$. We want to derive the corresponding possibility distribution satisfying the following assumptions

– $P(A) \leq \Pi(A) \quad \forall A \subseteq X$
– p and π are order-equivalent
– π is maximally specific (any other solution π' is such that $\pi \leq \pi'$).

Under these assumptions there exists a unique possibility distribution that can be obtained as follows

$$\pi_1 = 1,$$

$$\pi_i = \begin{cases} \sum_{j=i,n} p_j & if \quad p_{i-1} > p_i \\ \pi_{i-1} & otherwise \end{cases}. \tag{17}$$

All probability distributions characterizing the indifference surfaces in the NegoManage scoring system are converted to possibility distributions according to the procedure above. After the conversions the indifference surfaces can be compared using the fuzzy Jaccard index (formula 13) by taking for the comparison the obtained possibility distributions.

In the case of the fuzzy Jaccard coefficient the condition of preference consistency is of different nature, since all indifference surfaces described by possibility distributions overlap to some extent. Therefore, we define the so-called soft consistency conditions. The postulate for defining the consistency condition is: The higher the distance between the surfaces on the utility scale, the lower should be the overlap between these surfaces as computed using the fuzzy Jaccard index. Formally, the condition is defined as follows:

Given three indifference surfaces indexed with three values: *i*, *j*, *k*, and the utility scores corresponding to these surfaces: u_i, u_j, u_k, the following implication holds:

$$u_i > u_k \wedge u_i > u_j \wedge (u_i - u_k \geq u_i - u_j) \Rightarrow J(RS_i, RS_k) - J(RS_i, RS_j) \leq t. \tag{18}$$

where *t* is the indifference threshold equal to a small percentage of the utility space (for instance 0.15). This formula means that if the *k*th surface is more distant from the *i*th surface than the *j*th surface is from the *i*th surface, then the overlap of the *i*th and *k*th surfaces should be lower than the overlap of the *i*th and *j*th surfaces. If for all pairs of indifference surfaces the overlap levels in the form of Jaccard coefficients have been computed, we obtain a matrix consisting of the following elements:

$$M(i, j) = J(RS_i, RS_j)$$

Based on the values encoded by this matrix we can check if the preference consistency condition holds according to the formula (18).

4. Example of preference consistency analysis

Let us consider a simple problem of defining the negotiator's preferences and verifying their consistency in the NegoManage system. We assume that during the problem structuring process the negotiators decided to consider three negotiation issues, namely: price, delivery time and warranty. We will illustrate the preference analysis from the buyer's point of view. During the first stage of preference analysis the negotiator specified the following feasible ranges for the three negotiation issues:

– price: [20$, 80$],
– warranty: [2 months, 24 months],
– delivery time: [7 days, 21 days].

The preference analysis system maps the ranges of the issues into [0,1] intervals using the standard normalization formula. For the price, the mapping is:

$$g_1(\overline{a}) = \frac{80 - a_p}{80 - 20} = \frac{80 - a_p}{60}.$$

The mappings corresponding to warranty and delivery time are:

$$g_2(\overline{a}) = \frac{a_g - 2}{24 - 2} = \frac{a_g - 2}{22},$$

$$g_3(\overline{a}) = \frac{21 - a_d}{21 - 7} = \frac{21 - a_d}{14}.$$

After the system performed the mapping of all attributes, the user can use both scales. In the next stage of preference analysis the negotiator specifies the indifference surfaces. As an example, we consider here the first three indifference surfaces. Let us assume that the negotiator specified the first three surfaces (out of thirteen) as follows:

$\mathbf{RS_1}$ = {(0,0,0)}

$\mathbf{RS_2}$ = {(0.25, 0.0, 0.0),(0.0, 0.25, 0.0) ,(0.0, 0.0, 0.25)}

$\mathbf{RS_3}$ = {(0.0, 0.0, 0.5), (0.0, 0.25, 0.25), (0.25, 0.0, 0.25), (0.0, 0.5, 0.0), (0.25, 0.25, 0.0), (0.5, 0.0, 0.0)}

We will show now how the characteristics of the indifference surface are created by the NegoManage system using the example of the third indifference surface RS_3. Let us denote each alternative assigned to this surface by a_i, where i is the consecutive number of the alternative in the surface. Thus we have: a_1= (0.0, 0.0, 0.5), a_2 = (0.0, 0.25, 0.25), a_3 = (0.25, 0.0, 0.25), a_4 = (0.0, 0.5, 0.0), a_5=(0.25, 0.25, 0.0) and a_6=(0.5, 0.0, 0.0). We use hierarchical clustering to split the set RS_3 into clusters. According to the hierarchical clustering algorithm we begin with the initial partition consisting of single-element aggregations:

$$P_1=\{\{a_1\},\{a_2\},\{a_3\},\{a_4\},\{a_5\},\{a_6\}\}.$$

The distance matrix D_1 is computed in the following way. First we compute the means m_i for all defined clusters (in this case the clusters are the single elements a_i). Each element of the distance matrix D_1 is computed in the following way

$$D_1(i,j) = d_e(m_i, m_j), \tag{19}$$

where d_e is the Euclidean distance.
We obtain

$$D_1 = \begin{bmatrix} 0 & 0.35 & 0.43 & 0.7 & 0.75 & 0.86 \\ 0.35 & 0 & 0.25 & 0.35 & 0.43 & 0.61 \\ 0.43 & 0.25 & 0 & 0.43 & 0.35 & 0.43 \\ 0.7 & 0.35 & 0.43 & 0 & 0.25 & 0.5 \\ 0.75 & 0.43 & 0.35 & 0.25 & 0 & 0.25 \\ 0.86 & 0.61 & 0.43 & 0.5 & 0.25 & 0 \end{bmatrix}.$$

The value $D_1(2,3)$=0.25 is the smallest in the matrix D_1 (except for the diagonal elements which are not taken into account). Therefore, the elements a_2 and a_3 are merged in the next step of the clustering algorithm:

$$P_2=\{\{a_1\},\{a_2, a_3\},\{a_4\},\{a_5\},\{a_6\}\}.$$

We proceed this way using the notion of closest neighbor in calculating the distances between the clusters and finally obtain the following sequence of ascending partitions:

$$P_1=\{\{a_1\},\{a_2\},\{a_3\},\{a_4\},\{a_5\},\{a_6\}\},$$

$$P_2=\{\{a_1\},\{a_2, a_3\},\{a_4\},\{a_5\},\{a_6\}\},$$

$$P_3=\{\{a_1\},\{a_2, a_3\},\{a_4, a_5\},\{a_6\}\},$$

$$P_4=\{\{a_1\},\{a_2, a_3, a_4, a_5\},\{a_6\}\},$$

$$P_5=\{\{a_1\},\{a_2, a_3, a_4, a_5, a_6\}\},$$

$$P_6=\{\{a_1, a_2, a_3, a_4, a_5, a_6\}\}.$$

On the fusion level 0.5 we obtain the partition P_5. Over this partition we will span the multi-modal distribution. For the partition P_5 we have two clusters:

$$M_1=\{a_1\},$$

$$M_2=\{a_2, a_3, a_4, a_5, a_6\},$$

with the following means:

$$m_1 = a_1 = (0, 0, 0.5),$$

$$m_2 = 0.2\,(a_2 + a_3 + a_4 + a_5 + a_6) = (0.2, 0.2, 0.1).$$

and the following sigma matrices:

$$\Sigma_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Sigma_2 = \begin{bmatrix} 0.035 & -0.0075 & -0.0075 \\ -0.0075 & 0.015 & 0.015 \\ -0.0075 & 0.015 & 0.015 \end{bmatrix}.$$

To avoid matrix singularity (the first matrix is singular since the first cluster contains only one element) we add a small value to the diagonal elements of the sigma matrices:

$$\overline{\Sigma}_1 = \Sigma_1 + 0.25E$$

$$\overline{\Sigma}_2 = \Sigma_2 + 0.25E$$

where E is the identity matrix.

The resulting probability density functions for two kernels forming the final probability density function are of the following form:

$$f_{M_1}(\overline{a}) = 12.7389\exp\left(\frac{1}{2}(\overline{a}-[0,0.5,0.5])'\begin{bmatrix} 0.025 & 0 & 0 \\ 0 & 0.025 & 0 \\ 0 & 0 & 0.025 \end{bmatrix}(a-[0,0.5,0.5])\right)$$

$$f_{M_2}(\overline{a}) = 6.46301\exp\left(\frac{1}{2}(\overline{a}-[0.2,0.1,0.1])'\begin{bmatrix} 0.06 & -0.0075 & -0.075 \\ -0.075 & 0.04 & 0.015 \\ -0.0075 & 0.015 & 0.265 \end{bmatrix}(a-[0.2,0.1,0.1])\right)$$

The final function for the indifference set RS_3 is represented by:

$$f_{RS_3}(\overline{a}) = \frac{1}{2}(f_{M_1}(\overline{a}) + f_{M_2}(\overline{a})) .$$

Similarly, the probability density functions are calculated for all thirteen remaining equivalence sets.

Having all the indifference sets described by distribution functions we can verify the consistency of their definition provided by the negotiator. As said in the previous section, to check the consistency we will use the Jaccard index, that measures the similarity between two equivalence sets. The higher the similarity between two equivalence sets, the less consistent are the preferences. In our example the matrix of Jaccard indices is:

$$\begin{pmatrix}
0.99 & 0.99 & 0.84 & 0.77 & 0.89 & 0.83 & 0.77 & 0.63 & 0.2 & 0.1 & 0.04 & 0.01 & 0.02 \\
0.99 & 0.99 & 0.84 & 0.7 & 0.95 & 0.67 & 0.49 & 0.29 & 0.07 & 0.02 & 0. & 0. & 0.01 \\
0.84 & 0.84 & 1. & 0.99 & 0.86 & 0.91 & 0.85 & 0.85 & 0.16 & 0.07 & 0.02 & 0. & 0.03 \\
0.77 & 0.7 & 0.99 & 0.99 & 0.88 & 0.91 & 0.97 & 0.9 & 0.28 & 0.2 & 0.07 & 0.02 & 0.07 \\
0.89 & 0.95 & 0.86 & 0.88 & 0.99 & 0.88 & 0.95 & 0.88 & 0.79 & 0.74 & 0.19 & 0.06 & 0.14 \\
0.83 & 0.67 & 0.91 & 0.91 & 0.88 & 0.99 & 0.94 & 0.98 & 0.88 & 0.71 & 0.78 & 0.23 & 0.45 \\
0.77 & 0.49 & 0.85 & 0.97 & 0.95 & 0.94 & 1. & 0.99 & 0.93 & 0.85 & 0.84 & 0.51 & 0.63 \\
0.63 & 0.29 & 0.85 & 0.9 & 0.88 & 0.98 & 0.99 & 1. & 0.88 & 0.85 & 0.89 & 0.64 & 0.77 \\
0.2 & 0.07 & 0.16 & 0.28 & 0.79 & 0.88 & 0.93 & 0.88 & 0.99 & 0.89 & 0.82 & 0.93 & 0.93 \\
0.1 & 0.02 & 0.07 & 0.2 & 0.74 & 0.71 & 0.85 & 0.85 & 0.89 & 0.99 & 0.85 & 0.73 & 0.75 \\
0.04 & 0. & 0.02 & 0.07 & 0.19 & 0.78 & 0.84 & 0.89 & 0.82 & 0.85 & 1. & 0.77 & 0.75 \\
0.01 & 0. & 0. & 0.02 & 0.06 & 0.23 & 0.51 & 0.64 & 0.93 & 0.73 & 0.77 & 1. & 0.96 \\
0.02 & 0.01 & 0.03 & 0.07 & 0.14 & 0.45 & 0.63 & 0.77 & 0.93 & 0.75 & 0.75 & 0.96 & 0.99
\end{pmatrix}$$

All Jaccard indices this matrix are obtained as follows:

$$M(i,j) = J(RS_i, RS_j) = \frac{\max_{\overline{a}\in D}(\min(\pi_i(\overline{a}), \pi_j(\overline{a}))}{\max_{\overline{a}\in D}(\max(\pi_i(\overline{a}), \pi_j(\overline{a}))}$$

where the functions π_i , π_j are the possibility distributions corresponding to two indifference surfaces obtained by the transformation of probability distributions f_{RS_i} , f_{RS_j} also corresponding to the two given surfaces. As we can see from the above matrix the closer two surfaces are located to each other in terms of the utility levels, the higher are the values of the corresponding Jaccard indices. For instance, for the second and first surfaces the Jaccard value is 0.99 which is very high since these surfaces are close to each other. If we take a look at a selected matrix row, we can see that if we move right from the diagonal element the values are weakly decreasing (with an accuracy to the indifference threshold equal to 0.15). Analogously, if we move left along the row from the diagonal element the values are also weakly decreasing (with an accuracy to the indifference threshold equal to 0.15). The same observation holds if we move along a column up or down from the diagonal element. In this example we have defined surfaces preserving the preference consistency condition in terms of crisp definitions of the Jaccard index. If two indifference surfaces in a crisp form are disjoint (consistency condition holds – Jaccard index is equal to 0) its fuzzy counterparts (fuzzy surfaces) should in result have low level of overlap when the fuzzy Jaccard index is used for the comparison of surfaces (fuzzy Jaccard index should be low).

Conclusions

In this paper we presented a straightforward method for checking the negotiator's preference consistency for the preference elicitation method based on the notion of indifference sets, applied in the NegoManage system, that we have built and developed beforehand [see Brzostowski and Wachowicz 2009, 2010]. It seems vital to verify whether the negotiator defines preferences in a coherent and consistent way in every decision problem, but especially when the preference elicitation process has a decompositional character, i.e. the preferences are derived from the examples of the predefined complete packages and evaluated by the negotiator in the prenegotiation phase. In this approach it may very often appear that while defining the examples, the negotiator builds two very similar examples of negotiation offers but assigns them to two indifferent sets with different utility scores. If such a situation occurs, the scoring system of negotiation offers derived from the predefined examples appears imprecise and may result in the false scorings determined for the negotiation offers under evaluation in the actual negotiation phase. If so, the negotiator may feel that the whole scoring system is not adequate to her/his subjective and intrinsic preferences that she/he tried to define in prenegotiation. To avoid such a situation we recommend to check the consistency of preferences just after the definition of the examples of offers and the determination of the characteristics of indifference sets given in the form of distribution functions. We decided to use the simplest possible solution, which is to apply the Jaccard index. Given two sets, the Jaccard index compares the number of alternatives that may be assigned to both sets with the number of alternatives assigned to each set separately. However, while describing the indifference sets we operate with probability distribution functions, therefore we tried to give a rationale to move to the concept of possibility and used the Jaccard index formula defined for the fuzzy sets or possibility distributions. This simple mechanism allows us to find the sets that are too similar and ask the negotiator to revise their definitions. If two similar indifference sets are identified, the negotiator may change their forms by moving or eliminating some sample offers within the sets. She/he may also decide to join these two sets if necessary and assign to them a new value of the linguistic utility. After the negotiator's revision the consistency checkup is conducted once again to verify the impact of these changes on the form and quality of the new scoring system of negotiation offers.

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DEFAULT PREDICTION FOR VARIOUS NATIONAL ECONOMIES THROUGH SYNTHETIC INDICATORS

Abstract

In the current situation, involving a global economic crisis, national economies are under a high pressure. Greek and Irish bailouts have prompted many people to wonder about the economic situation in other countries. The global crisis is causing First World countries need help from institutions such as the IMF or the ECB. The goal of this paper is to analyze the risk that these countries have to be rescued by the economic institutions. In order to prepare this ranking, we are going to use two synthetic indicators. The first one is called Distance Principal Components (DPC) and the other one Goal Programming Synthetic Indicator (GPSI). We develop this indicator taking into account variables from both the public economy and the financial markets. Concerning the public economy, we use variables such as the public debt ratio and its total amount (% of GDP), public revenues, public deficit, real GDP growth and unemployment rate. We strongly believe that the soundness of an economy in the long-term depends on the behavior of these variables. Therefore, if they show a positive trend, other variables exposed to speculation in the financial markets should present a proper behavior as well. With this we mean two variables negotiated in the markets: debt risk premium and credit default swap levels.

This paper will bring easily understandable results that will let us know what the bankruptcy situation is in the rest of the countries analyzed.

Keywords

National Economies, Synthetic Indicators, GPSI indicator.

Introduction

In a world that is totally globalized, the situation of the worldwide economies is of increasing importance. After Greece's and Ireland's rescues by the European Institutions, there is a special worry about a new hypothetical bankruptcy in Europe.

The European Union (EU) has been able to deal with this problem until this moment. Taking into account that Greece's and Ireland's Gross Domestic Product (GDP) represented only around the 4% of the EU GDP in 2009, the problem was not too serious, so that EU institutions could solve it without any external help. In addition, the EU will only ask for help to the International Monetary Fund (IMF) if it is impossible to fix the situation within the EU. It makes no sense to create an Economic and Monetary Union if later they cannot solve their own problems. However, what if a powerful economy like Spain or Italy has to be rescued? Can the EU afford it? That is the reason why the current economic context has such a great importance. It is said that, if there is a new bailout, IMF funds would have to be used. In that case, what we know as 'Eurozone' would be over and we should reconsider the EU as something else than a simple 'geographical Union'.

Through this paper we try to quantify the risk countries have to be rescued. We have focused on important economies from the European Union and the rest of the world. Firstly, because what happens in the EU affects us directly, and secondly, because the non-European economies we have chosen are very powerful and, as we said before, in a globalized world we are strongly influenced by them, it must be said that many companies try to quantify the default risk countries. However, they build the index taking into account only the Credit Default Swap of a country [Kan and Pedersen 2011]. We believe that other variables have to be added to the index because the CDS value is fixed in the financial markets and it is a very volatile variable. The strengths and weaknesses of an economy lie mainly in the real economy. Credit default swaps have existed since the early 1990s. At the beginning they had a marginal role only in the economy. However, in 2003 there was a 'boom' and the market increased tremendously. We think that a variable that is always under negotiation and speculation cannot be a good indicator of the actual state of an economy.

We have chosen a sample of countries. Many of them belong to the Eurozone and EU27 but we also wanted to see how the index works in some countries which do not have the same political economy. We could have increased the number of countries as much as we had wanted but we strongly believe that 20 countries from different parts of the world suffice.

Why Venezuela? The reason why we have chosen Venezuela lies in the particular Venezuelan political situation. Even though Venezuela presents quite good results for the economic variables, the default risk is much higher than in countries with a higher level of debt or deficit. The answer for this is pretty clear and we will study it later. The CDS variable has a huge adverse effect on the index. The fact that Venezuelan political regime is a dictatorship makes the situation gets very unstable. In this context, insurance for this debt is too expensive.

In conclusion, taking Venezuela into consideration we prove that not only is the economic situation important, but the political system also plays a key role in the bankruptcy risk.

We are going to use two kinds of methodologies in order to obtain the results [Nardo et al. 2008]. The first one is called Distance-Principal Components (DPC) – [Blancas et al. 2010b] and the second one is called Goal Programming Synthetic Indicator (GPSI) – [Blancas et al. 2010a].

The rest of the paper is organized as follows. In Section 1, we are going to present aspects related to the basic methodology of the synthetic indicators. In the next section we will present the countries analyzed and the basic indicators we used in our study. The final results using both synthetic indicators are shown in Section 3.

1. Methodological aspects of the syntethic indicators

In this section, we are going to discuss the methodology behind the composite indicators:

We consider an initial system of m indicators to assess a set of n units, where I_{ik} is the value of the i-the unit in the k-th indicator.

We distinguish between positive and negative indicators, depending on the improvement direction ("more is better" or "less is better"). The indicator is considered positive when a higher value represents an improvement in the area. In contrast, the indicator is negative when a higher value represents deterioration.

In the DPC composite indicator [Blancas et al. 2010a] we have to normalize the data so that measuring units used for each indicator have no effect on the end result. The procedure involved divides the distance to the anti-ideal point by the difference between the maximum and the minimum values, in the case of positive indicators

$$IN_{ik} = \frac{I_{ik} - Min_k \; I_{ik}}{Max_k \; I_{ik} - Min_k \; I_{ik}}$$

The synthetic indicator, called DPC (distance − principal components), is then defined by the following formula:

$$DPC_i = \sum_{j=1}^{q}\left[VE_j\left(\sum_{k=1}^{m} IN_{ik}\left|Corr_{jk}\right|\right)\right],$$

for i = 1, 2,…, n,

where:

n is the number of units.
m is the number of original indicators.
q is the number of components selected.
VE_j is the variance explained by the j-th component.
$Corr_{jk}$ is the correlation between the j-th component and the k-th indicator.

More details about this composite indicator can be found in [Blancas et al. 2010a]

To define the composite indicator GPSI [Blancas et al. 2010b] we don't need to normalize the basic indicators as in the previous method, as this way this indicator is easier to interpret. We let I_{ij}^{+} denote the value that represents the *i*th unit in the *j*th positive indicator, with $j \in J$, where *J* is the set of positive indicators in the system. In the case of negative indicators, we let I_{ik}^{-} denote the value that provides the *k*th indicator for the *i*th unit considered, with $k \in K$, where *K* is the set of negative indicators included in the initial system. Therefore, $|J| + |K| = m$.

The proposed procedure requires us to identify the improvement direction of each indicator, but without the need to convert all of them into the same type, positive or negative. This facilitates the interpretation and management of the results, as no conversion is required.

With the basic elements of the synthetic indicator defined, the synthetic indicator can be based on the concept of goal used in Goal Programming. This methodology is well-known within the area of Operations Research, and is characterized by an underlying process of optimization that aims at finding the solution that most closely matches the aspiration levels established. Nevertheless, we use the underlying concept of goal rather than the optimizing process [Diaz-Balteiro and Romero 2004a, 2004b]. So, in our case, each unit is compared, for each indicator, with a given predetermined aspiration level. This way, the strength or weakness of this unit with respect to an indicator is established depending on the comparison of the indicator value with the predetermined aspiration level.

In particular, we must set weights, w_j, to state the relative importance of each indicator. Finally, the proposed methodology has to define an aspiration level for each indicator. u_j^+ will be used to refer to aspiration levels of the positive indicators and u_k^- for negative indicators.

The interpretation of the aspiration level differs depending on the indicator type. In the case of positive indicators, the value establishes the minimum level at which a unit is considered to indicate a good situation regarding the aspect evaluated by the indicator. When the indicator is negative, the aspiration level reflects the maximum level that indicates a favourable situation regarding the aspect analysed.

Given the set of aspiration levels, the value that each unit presents in each indicator is compared with the aspiration levels, as in goal programming. We define a goal for each indicator using deviation variables denoted by n and p. For each unit, these variables indicate the difference between the value of an indicator and the corresponding aspiration level. For the ith unit, the goals are represented as follows:

– If the indicator I_j is positive, the goal is formulated as

$$I_{ij}^+ + n_{ij}^+ - p_{ij}^+ = u_j^+ \quad \text{with} \quad n_{ij}^+, p_{ij}^+ \geq 0 \;\; n_{ij}^+ \cdot p_{ij}^+ = 0$$

where n_{ij}^+ is the under-achievement or negative deviation variable and p_{ij}^+ is the over-achievement or positive deviation variable associated with the positive indicator.

– If the indicator I_k is negative, the goal is formulated as

$$I_{ik}^- + n_{ik}^- - p_{ik}^- = u_k^- \quad \text{con} \quad n_{ik}^-, p_{ik}^- \geq 0 \quad n_{ik}^- \cdot p_{ik}^- = 0$$

where n_{ik}^- is the under-achievement or negative deviation variable and p_{ik}^- is the over-achievement or positive deviation variable associated with the negative indicator.

At this point, we propose global measures that serve to evaluate each destination depending on the level of fulfilment of the predetermined aspiration levels. Quantification of the indicators is based on the deviation variables associated with the goals set for each indicator. These measures differ from each other by the degree of compensation for the fulfilment and non-fulfilment of the aspiration levels.

The first component $\left(GPSI^+\right)$ quantifies the strengths displayed by each unit in the concept evaluated, indicating the degree to which the unit fulfils the aspiration levels set. Its definition is based on the aggregation of deviation variables, for which a higher value shows a better relative position: the positive deviation variable for positive indicators $\left(p_{ij}^{+}\right)$ and the negative deviation variable for negative indicators $\left(n_{ik}^{-}\right)$. This aggregation is computed by using the weight of each indicator and normalizing the deviation variables with the corresponding aspiration levels to obtain a correct non-dimensional measure.

Thus, the formulation of this component for the unit *i* is as follows:

$$GPSI_i^{+} = \sum_{j \in J} \frac{w_j p_{ij}^{+}}{u_j^{+}} + \sum_{k \in K} \frac{w_k n_{ik}^{-}}{u_j^{-}} \quad \forall i \in \{1,2,...,n\}$$

The second component enables us to measure the weaknesses of each unit with respect to the indicator system, quantifying the degree to which the units do not fulfil the set of aspiration levels. This is similar to the way in which the first component is determined, by adding the unwanted deviation variables for each type of indicator, normalized and weighted. The formulation of this component for the unit *i* is as follows:

$$GPSI_i^{-} = \sum_{j \in J} \frac{w_j n_{ij}^{+}}{u_j^{+}} + \sum_{k \in K} \frac{w_k p_{ik}^{-}}{u_k^{-}} \quad \forall i \in \{1,2,...,n\}$$

In this way, the ratios that define the components of the vector indicator are a measure of the unfulfilled values described by the initial indicators, normalized as percentages. This first component shows its strengths for each unit without taking its weaknesses. The second component quantifies the degree of weakness shown by each unit without taking into account its strengths.

We can now consider how to achieve such compensation. This leads to the Net Goal Programming Synthetic Indicator $\left(GPSI^{N}\right)$. This indicator aims at assessing each unit, by aggregating its strengths and weaknesses. These components are weighted to take into account situations where the strengths are not given the same importance as weaknesses. That is:

$$GPSI_i^{N} = \lambda\, GPSI_i^{+} - \gamma\, GPSI_i^{-}$$

where λ and γ are relative weights of strengths and weaknesses, respectively.

In this way, the difference between the components of the vector indicator makes it possible to define a compensatory measure. The strengths of the indicators, which are the strengths of each unit, can compensate for the weaknesses in other indicators.

2. Economic Data

In this section we are going to present the countries chosen for this paper and the indicators used. The first step is to define the theoretical framework. Eurostat [2010] has been our main source for data collection but we have also used some data from *'Global Finance 2010'* database. However, we have taken into account some limitations of the theoretical framework, especially in the collecting data stage. That is why we have selected only those indicators which provide rigorous information about the variables to study. Even though Eurostat provides a lot of information about these variables, we have been interested only in those that provide relevant, complete and objective information.

According to what we have explained above, we are going to present the variables selected:

1. Debt-to-Gross Domestic Product (GDP) Ratio: this is one of the indicators of the health of an economy [Cecheti and Zampolli 2010]. It is the amount of federal debt of a country as a percentage of its Gross Domestic Product (GDP). A low debt-to-GDP ratio indicates an economy that produces a large number of goods and services and probably profits that are high enough to pay back debts.

2. Taxes Income-to- GDP Ratio: the percentage of national income that is compulsorily transferred from private pockets to the public exchequer. It is probably the most important variable. I It can be said that the value of the Debt-to-GDP Ratio is irrelevant if the country makes enough money to pay the debt.

3. Government Bond 10 years yield (in basis points, 1/100 of 1%): There is a direct relationship between the yield and the economic uncertainty of a country. It is the interest rate countries have to pay for the bond.

4. GDP real growth rate (annual %): It shows the increase or decrease in value of all final goods and services produced within a nation in a given year, taking into account inflation.

5. Credit Default Swap (in thousands of Euro): A credit default swap (CDS) is an agreement that the seller of the CDS will compensate the buyer in the event of a loan default. The buyer of the CDS makes a series of payments (the CDS “fee” or “spread”) to the seller and, in exchange, receives a payoff if the loan defaults. This might be the most difficult concept to understand.

This number means how much money an investor has to pay to insure 10 Million Euro Bonds. However, it is not as simple as it seems because bounds can be object of speculation in the financial markets.

6. Unemployment rate: This is an indicator of the economic activity. Moreover, less unemployment means less public expenditure and more public income so this is very important variable that affects the economy in a double sense.

7. Deficit-to-GDP Ratio: Nowadays it is a priority for all the countries to reduce the public deficit. This is also a way to reduce the public debt[1].

Next step in our work is to identify the positive or negative sign for each indicator. In this sense, the sense of this paper is to analyze the bankruptcy risk for an economy so that the indicator is considered positive when higher values cause a favorable effect on the 'health' of an economy. By contrast, the indicator will be considered negative when higher values of the indicator entail harmful consequences for an economy. You can find a summary of the nature of the indicators in Table 1.

Table 1

Sign of the indicators

Indicator	Description	Sign
1	Debt-to-Gross Domestic Product (GDP) Ratio	Negative
2	Taxes Income-to- GDP Ratio	Positive
3	Bond 10 years yield (in basis points, 1/100 of 1%)	Negative
4	GDP real growth rate (annual %).	Positive
5	Credit Default Swap (in thousands of Euro)	Negative
6	Unemployment rate	Negative
7	Deficit-to-GDP Ratio	Negative

We have collected data for four different periods (Bloomberg, CMA database, Markit Index, Trading Economics, 2010): first semester 2009, second semester 2009, first semester 2010 and second semester 2010 in order to show the situation in each period.

Table 2 shows all the information related to second semester 2010. Similarly, the same data have been collected for the other periods.

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[1] This information comes from Eurostat database, Global Finance Database and Bloomberg database.

Table 2

2010s2 observed data

	1	2	3	4	5	6	7
SPAIN	63,5	34,7	531	0,2	252,5	20,2	9,3
BELGIUM	101	48,1	399	1,9	201	8,1	6
SWITZERLAND	41	36,5	184	3,1	85	3,6	1,4
GERMANY	75	44,5	315	3,9	53,37	6,6	4,5
PORTUGAL	83,1	38,8	653	1	471,44	10,9	7,3
GREECE	131	36,9	1147	–4,7	912,97	12,9	8
IRELAND	93,6	34,5	845	–0,5	598,67	13,8	17,7
FRANCE	84,2	48,4	351	1,7	104,55	9,7	8
ITALY	118	46,6	480	0,9	208,7	8,6	5,10
USA	92,7	30	348	2,6	39,56	9,4	11,10
NETHERLANDS	66	46	332	1,9	60	4,3	6
POLAND	55,2	37,2	598	4,8	316,6	10	7,4
SWEDEN	41,9	53,7	321	6,9	32,55	7,8	2,2
UK	76,7	40,4	369	2,6	67,63	7,8	10,2
CHINA	19,1	25	399	10,5	72,02	4,1	2,9
BRAZIL	65	23	465	7,5	115	7,2	1,7
VENEZUELA	34,8	14	1275	–1,3	1149	8,6	3,8
MEXICO	45,2	15,2	476	5	118,58	5	3,6
JAPAN	226	35	125	5,3	82,17	4,9	9,6
AUSTRALIA	21,9	40,3	556	3	53,14	5,2	4,6

It can be seen that there is a big difference between the countries. Emerging countries show good values of the variables related to the actual economy. By contrast, there are several countries in the Eurozone that really need to make changes in their economies.

3. Results and discusion

3.1. Results by DPC indicator

Once all the previous steps are completed, we will proceed to put together all the indicators in a common synthetic index according to the DPC method. As the method is based on statistical techniques, in our analysis the weighting given for subsequent variables will be elaborated separately for each variable. Thus, according to authors as Chen et al [2004], using the percentage of total explained variance for each component as the weight is the most frequent option.

To facilitate the managerial use of the information contained in the system, we have obtained DPC composite indicators, the methodology of which presents some advantages. Specifically, the proposed procedure allows the determination of a single common set of objective weights for all units. Furthermore, unlike composite indicators derived using statistical methods, the DPC indicator weights are always positive and allow the identification of the initial indicators that have the most influence on bankruptcy risk. Also, from a practical point of view, the DPC indicator is easier to interpret than other composite indicators obtained with statistical procedures. As mentioned, using initial indicator values to define analogous distances to the anti-ideal situation allows the association of the highest composite indicator values with better sustainability [Blancas et al. 2010a]. Table 3 shows the normalized data from Table 1.

Table 3

2010s2 normalized data

	1	2	3	4	5	6	7
SPAIN	0,215	0,521	0,353	0,322	0,197	1,000	0,485
BELGIUM	0,392	0,859	0,238	0,434	0,151	0,271	0,282
SWITZERLAND	0,106	0,567	0,051	0,513	0,047	0,000	0,000
GERMANY	0,272	0,768	0,165	0,566	0,019	0,181	0,190
PORTUGAL	0,309	0,625	0,459	0,375	0,393	0,440	0,362
GREECE	0,537	0,577	0,889	0,000	0,789	0,560	0,405
IRELAND	0,360	0,516	0,626	0,276	0,507	0,614	1,000
FRANCE	0,315	0,866	0,197	0,421	0,064	0,367	0,405
ITALY	0,478	0,821	0,309	0,368	0,158	0,301	0,227
USA	0,356	0,403	0,194	0,480	0,006	0,349	0,595
NETHERLANDS	0,227	0,806	0,180	0,434	0,025	0,042	0,282
POLAND	0,175	0,584	0,411	0,625	0,254	0,386	0,368
SWEDEN	0,110	1,000	0,170	0,763	0,000	0,253	0,049
UK	0,279	0,665	0,212	0,480	0,031	0,253	0,540
CHINA	0,000	0,277	0,238	1,000	0,035	0,030	0,092
BRAZIL	0,222	0,227	0,296	0,803	0,074	0,217	0,018
VENEZUELA	0,076	0,000	1,000	0,224	1,000	0,301	0,147
MEXICO	0,126	0,030	0,305	0,638	0,077	0,084	0,135
JAPAN	1,000	0,529	0,000	0,658	0,044	0,078	0,503
AUSTRALIA	0,014	0,662	0,375	0,507	0,018	0,096	0,196

Finally, before moving on to discuss the results obtained using DPC method, we will see that the matrix of correlations between the indicators (presented in Table 4) is different from the identity matrix, so that we can continue with our analysis.

Table 4

Indicator correlation matrix

Indicator	1	2	3	4	5	6	7
1	1	0,489	–0,207	0,346	–0,402	–0,001	0,364
2	0,489	1	0,116	0,1042	–0,301	0,116	0,373
3	–0,207	0,116	1	–0,313	0,485	–0,009	0,174
4	0,346	0,104	–0,313	1	–0,134	0,206	0,196
5	–0,402	–0,301	0,485	–0,134	1	0,182	0,162
6	–0,001	0,116	–0,009	0,2069	0,182	1	0,563
7	0,364	0,373	0,1741	0,1967	0,162	0,563	1

Thus, in Tables 5 and 6 we find the values of our synthetic indicator of bankruptcy risk for the main economies calculated by the DPC (as a table and as a graph). We are going to show the evolution of this indicator in four different periods, every semester during the last two years. We should take into account the changes in the economic situation in the past two years due to financial crisis.

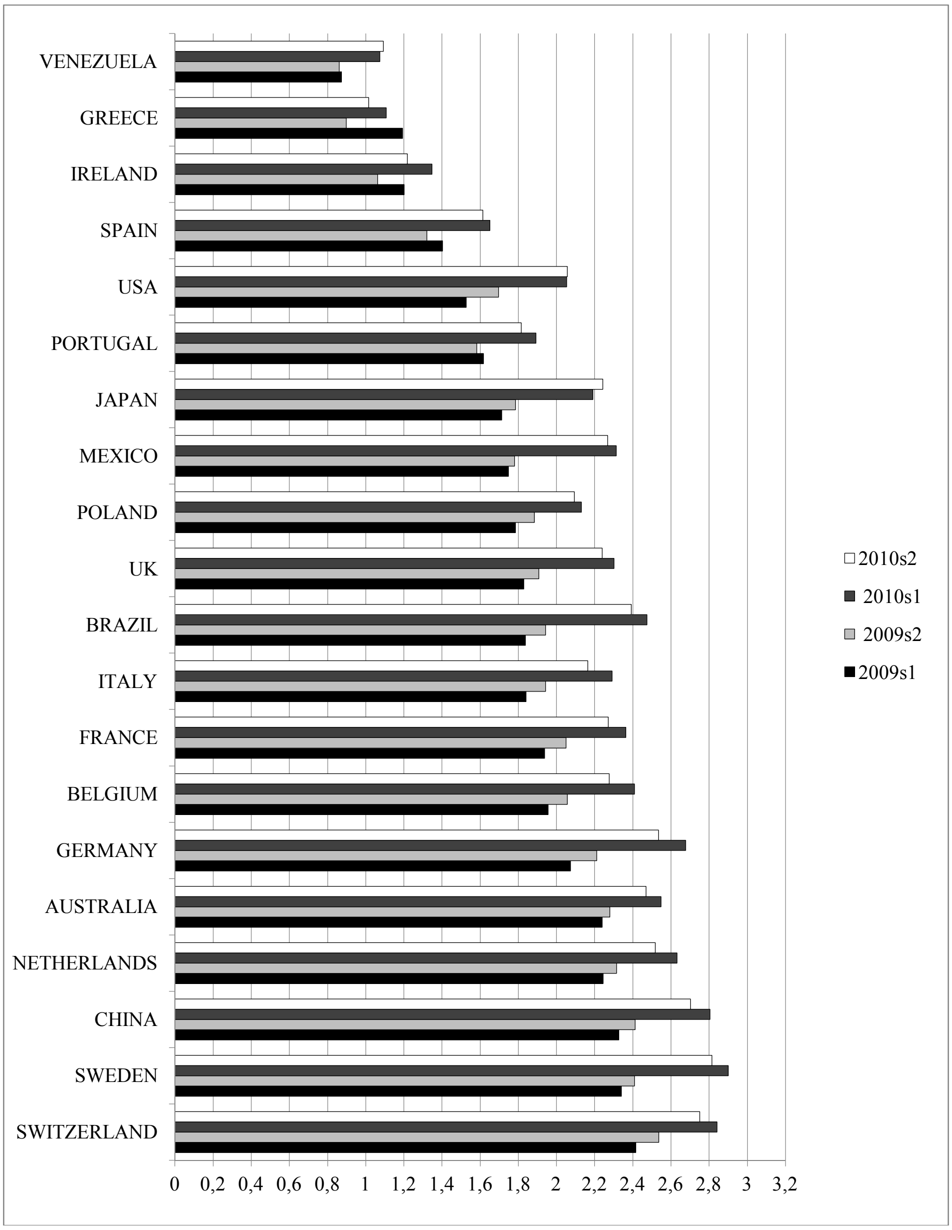
Table 5

DPC Synthetic Index

COUNTRY	2009s1	COUNTRY	2009s2	COUNTRY	2010s1	COUNTRY	2010s2
SWITZERLAND	2,416	SWITZERLAND	2,531	SWEDEN	2,900	SWEDEN	2,8152
SWEDEN	2,3408	CHINA	2,411	SWITZERLAND	2,840	SWITZERLAND	2,750
CHINA	2,326	SWEDEN	2,4085	CHINA	2,804	CHINA	2,702
NETHERLANDS	2,244	NETHERLANDS	2,315	GERMANY	2,677	GERMANY	2,533
AUSTRALIA	2,239	AUSTRALIA	2,279	NETHERLANDS	2,631	NETHERLANDS	2,517
GERMANY	2,074	GERMANY	2,211	AUSTRALIA	2,547	AUSTRALIA	2,4692
BELGIUM	1,956	BELGIUM	2,056	BRAZIL	2,474	BRAZIL	2,392
FRANCE	1,9373	FRANCE	2,049	BELGIUM	2,408	BELGIUM	2,276
ITALY	1,841	ITALY	1,942	FRANCE	2,363	FRANCE	2,271
BRAZIL	1,837	BRAZIL	1,942	MEXICO	2,312	MEXICO	2,267
UK	1,827	UK	1,907	UK	2,301	JAPAN	2,242
POLAND	1,785	POLAND	1,884	ITALY	2,292	UK	2,239
MEXICO	1,748	JAPAN	1,785	JAPAN	2,191	ITALY	2,164
JAPAN	1,712	MEXICO	1,779	POLAND	2,130	POLAND	2,094
PORTUGAL	1,616	USA	1,695	USA	2,052	USA	2,056
USA	1,5263	PORTUGAL	1,582	PORTUGAL	1,893	PORTUGAL	1,815
SPAIN	1,402	SPAIN	1,320	SPAIN	1,651	SPAIN	1,613
IRELAND	1,200	IRELAND	1,063	IRELAND	1,347	IRELAND	1,218
GREECE	1,192	GREECE	0,898	GREECE	1,108	VENEZUELA	1,092
VENEZUELA	0,873	VENEZUELA	0,861	VENEZUELA	1,073	GREECE	1,014

Table 6

DPC Synthetic Index graph



From this information, we can observe two groups standing out: those situated at the top of the table and those situated at the bottom. They are always the same. The case of Venezuela is eye-catching. We are tired of hearing from the news that Mediterranean countries are likely to suffer a default. However, even if this paper confirms that fact, Venezuela presents a higher risk than any other country on the list. This is because of the CDS price and the 10-years--bond-yield. Economic stability depends on both political situation and economic situation. In Venezuela, the political situation penalizes the economy a lot. People will not buy Venezuelan Debt if they are not sure that they will get a return. Who knows how much political and economic situation in Venezuela can change in the next ten years?

As we could anticipate, it is confirmed that countries such as China, Switzerland, Sweden or even Germany have a strong economy. The case of Japan is very interesting. Japanese Public Debt is more than 200% of GDP. However, they do not have to pay too much interest. At the same time, Japan is starting to grow after almost twenty years of economic stagnation. The situation is not as worrying as it was a couple of years ago. Anyway, the recent earthquake and tsunami will have terrible consequences also for the economy.

Lastly, we want to focus on the USA case as well. USA is penalized by a high public debt, high public deficit and taxes incomes below average. By devaluating the dollar, they want people to buy US production but this solution is recommendable in the very short term only, in the long term it could cause inflation and other harmful consequences to the economy. To summarize, there are three different groups of countries: Those whose economies enjoy perfect health, those that really have to apply contracting monetary (if possible) and fiscal policies and, finally, countries in the middle with very different characteristics (the cases of United Kingdom, Brazil, Japan and so on).

3.2. Results by GPSI indicator

We are presenting a new methodology which offers several advantages over existing ones. In particular, it is designed to be practical and to facilitate obtaining easy-to-interpret synthetic indicators. Inspired by goal programming, this method allows us to obtain several synthetic indicators based on information provided by the goals corresponding to each indicator.

The difference between the synthetic measures proposed is reflected by the degree of compensation of fulfillment and non fulfillment of the aspiration levels. In any case, these measures assess each unit, accounting for their strengths and weaknesses, which can be analyzed together or separately. Interpreting the values of the synthetic indicator is easy, because the results are expressed in terms of proximity to the reference situation defining the goals.

The methodology we develop is not a technique based on statistics for the process of weighting the different indicators that will form our composite index of bankruptcy risk. For its development, we will use some previously established steps. In this case it is not necessary to normalize the data but we will use again the positive or negative effect of the indicator (presented in Table 2). However, it will be necessary to add the concept of neutrality of the indicator (when it reaches a specific value, the desired reference level). We have defined what we consider to be the reference level for every indicator. Nowadays, it makes no sense to fix the average point as the desired level because given the delicate situation of the world economy the average point will be a non-desirable point for the governments. The result is a synthetic indicator vector (called $GPSI^{v}$), composed of a two components vector $(GPSI^{+}, GPSI^{-})$. According to Blancas et al. [2010a, p. 10] "the first component of vector synthetic indicator shows the strengths for each unit while ignoring their weaknesses. The second component quantifies the degree of weakness shown by each unit while ignoring their strengths. Neutral indicators are represented by their weaknesses only, because the deviation of variables indicates weakness only". Thus, given the synthetic indicator vector of goal programming $GPSI^{v}$, we note that the comparison is very complicated. In this way, to make the comparison easy-to-interpret we are going to use the Synthetic Index based on Restrictive Goal Programming (*GPSIR*) and the Synthetic Index based on Net Goal Programming (*GPSIN*). The *GPSIR* is based on the idea of distinction of the units that fulfill the levels of reference and, as opposed to the *GPSIv*, it does not compensate strengths and weaknesses. As for the GPIN, it combines strengths and weaknesses, each with a different weighting. The results obtained by using this method can be seen in the next tables.

Table 7

Net GPSI Synthetic Indicator for 1st and 2nd semester 2009

COUNTRY 2009s1	GPSI +	GPSI –	NET GPSI	COUNTRY 2009s2	GPSI +	GPSI –	NET GPSI
CHINA	4,12	0,46	3,67	CHINA	3,9	0,47	3,43
AUSTRALIA	2,36	1,25	1,11	AUSTRALIA	1,91	1,21	0,7
SWITZERLAND	2,45	2,56	–0,11	SWITZERLAND	2,66	2,3	0,36
SWEDEN	2,87	3,56	–0,69	SWEDEN	2,64	3	–0,36
NETHERLANDS	1,8	3,76	–1,96	NETHERLANDS	1,56	3,25	–1,69
BRAZIL	1	3,71	–2,71	BRAZIL	0,84	3,78	–2,94
GERMANY	1,81	4,59	–2,78	GERMANY	1,7	3,94	–2,24
FRANCE	0,88	4,27	–3,38	FRANCE	0,87	4,41	–3,54
USA	0,76	4,4	–3,64	USA	0,88	4,65	–3,77
UK	0,8	4,44	–3,64	UK	0,71	4,56	–3,86
MEXICO	1,12	5,5	–4,39	MEXICO	0,78	5,6	–4,82
BELGIUM	1,27	7,37	–6,1	BELGIUM	0,95	6,92	–5,97
POLAND	0,63	7,06	–6,43	POLAND	0,45	6,98	–6,53
SPAIN	0,01	7,52	–7,5	ITALY	0,93	8,61	–7,68
JAPAN	1,42	9,06	–7,64	JAPAN	1,43	9,72	–8,29
ITALY	1,17	8,88	–7,71	SPAIN	0,03	10,25	–10,22
PORTUGAL	0,49	12,29	–11,8	PORTUGAL	0,39	12,39	–12,01
IRELAND	0,28	16,89	–16,61	IRELAND	0,08	17,07	–16,99
GREECE	0,41	22,57	–22,16	VENEZUELA	0,4	26,16	–25,75
VENEZUELA	0,74	28,01	–27,27	GREECE	0,35	26,18	–25,83

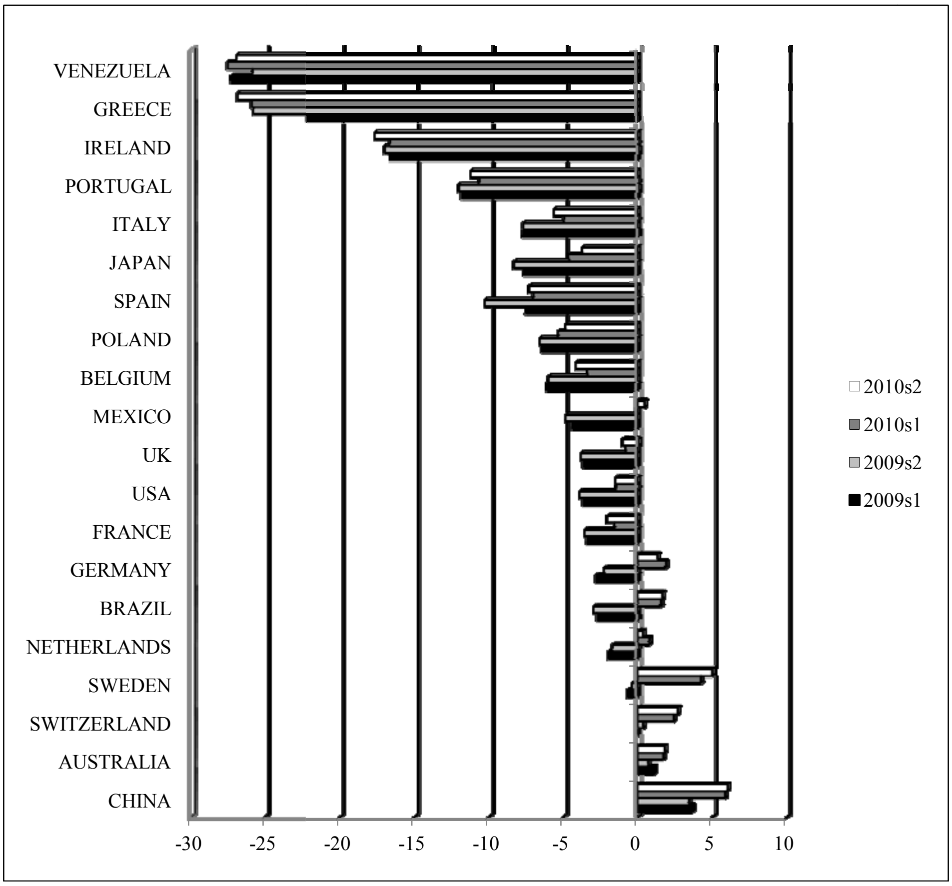
Table 8

Net GPSI Synthetic Indicator for 1st and 2nd semester 2010

COUNTRY 2010s1	GPSI+	GPSI –	NET GPSI	COUNTRY 2010s2	GPSI +	GPSI –	NET GPSI
CHINA	6,36	0,47	5,89	CHINA	6,47	0,47	6
SWEDEN	4,39	0,13	4,26	SWEDEN	5,16	0,14	5,02
SWITZERLAND	2,99	0,58	2,41	SWITZERLAND	3,27	0,58	2,69
GERMANY	3,05	1,18	1,87	AUSTRALIA	2,29	0,49	1,8
AUSTRALIA	2,18	0,49	1,69	BRAZIL	4,18	2,5	1,67
BRAZIL	4,06	2,5	1,55	GERMANY	2,55	1,21	1,34
NETHERLANDS	1,61	0,84	0,76	MEXICO	2,67	2,21	0,46
MEXICO	2,34	2,2	0,13	NETHERLANDS	1,31	0,97	0,34
UK	1,1	1,87	–0,77	UK	0,98	1,99	–1,01
USA	1,21	2,65	–1,44	USA	1,19	2,65	–1,45
FRANCE	0,98	2,52	–1,53	FRANCE	0,79	2,81	–2,02
BELGIUM	1,48	4,83	–3,35	JAPAN	3,28	6,97	–3,69
JAPAN	2,25	6,87	–4,62	BELGIUM	0,82	4,96	–4,13
ITALY	0,89	5,84	–4,95	POLAND	1,85	6,66	–4,81
POLAND	1,22	6,53	–5,31	ITALY	0,88	6,48	–5,6
SPAIN	0,03	7,02	–6,99	SPAIN	0,03	7,3	–7,26
PORTUGAL	0,27	10,89	–10,63	PORTUGAL	0,28	11,47	–11,19
IRELAND	0,03	16,67	–16,64	IRELAND	0,03	17,63	–17,61
GREECE	0,12	26	–25,88	GREECE	0,1	26,95	–26,85
VENEZUELA	0,45	27,96	–27,51	VENEZUELA	0,79	27,68	–26,89

Table 9

Net GPSI Synthetic Indicator Graph



We can clearly state that the situation has improved for almost all the countries. That could mean that we are getting over the economic crisis. Once again, we have the same economies at the top, at the bottom and in the middle of the table. Venezuela is again penalized by the highest CDS price. As we can see, it has much more strengths (0,79 in 2010s2) than the rest of the countries at the bottom but also more weaknesses. The case of China is also striking. Nobody is surprised to see China at the top of this table. The point is that they can keep improving its situation because its problems are not the strengths but the weaknesses. Once again, Switzerland is situated in a very comfortable situation. That is explained by the combination of high public incomes, low deficit, low unemployment and low debt. The tables show that Portugal's situation is very dangerous; it is just behind some countries that have already been rescued. It might be the next if it does not carry out restrictive policies and economic cuts.

3.3. Discusion of results

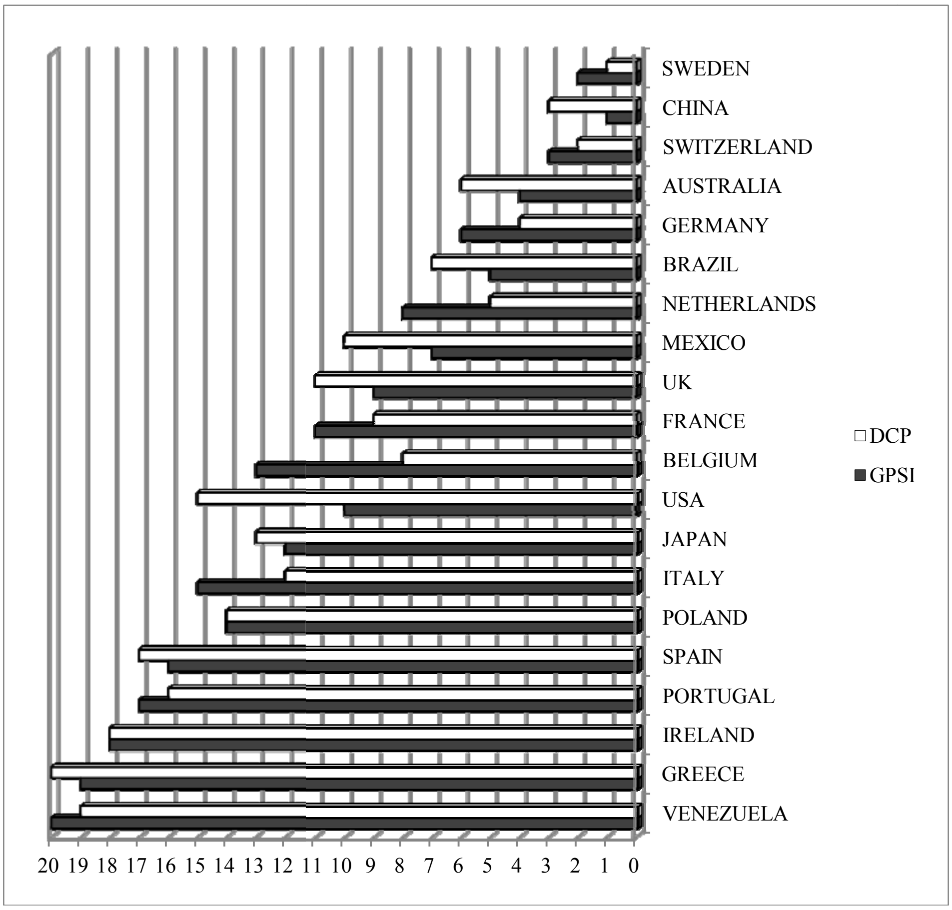
Once the results have been analyzed using both methods, one based on statistical techniques (Distance-Principal Components method), and one based on non-statistical techniques (Goal Programming Synthetic Indicator and its variants) in this section we will discuss the advantages and disadvantages of each by trying to compare the results, in order to find common patterns of behavior among different countries.

Some advantages of the DPC method come from the fact that it represents most of the information provided by the system with a limited number of variables, uncorrelated [Blancas et al. 2010b]. Moreover, comparative analysis is very simple and intuitive. Furthermore, as we said above, the method itself is responsible for providing the weights, without any interference by expert groups (which always brings subjectivity to the analysis).

As regards the GPSI method, and following Blancas et al. [2010a], it has a number of advantages over statistical methods. The first one is that it does not require prior normalization of the data. Moreover, this technique admits a number of indicators lower than the number of observations. Furthermore, there is no a lack of information, since all indicators from the initial system are used to build the synthetic indicator. There are a number of drawbacks, since the analyst is obliged to make decisions, both in the setting of weights and in the aspiration levels for each indicator. Below, we try to summarize and compare the results we have obtained by using each method. We will only focus on the 2010 second semester rankings because they are the most recent information we can analyze.

Table 10

Comparison between DPC and GPSI methods 2010s2 (positions)



Thus, from the information above we can see a number of similarities. Therefore, despite some differences, both methods show reasonably similar results. The two indicators can help to develop a global classification. As we can see, it does not matter which method we use, the countries will keep similar positions. PIIGS (Portugal, Ireland, Italy, Spain and Portugal) present a high bankruptcy risk. It is also remarkable to see the evolution of Japan. Semester by semester, it is improving its average position. Benelux countries remain in a comfortable position in the first half of the table.

As a conclusion from these data, it seems clear which countries are likely to default. 'Piigs' and Venezuela are in a very dangerous situation. However, the causes of this harmful situation depend on the countries. For instance, Venezuela is affected by high prices for the variables negotiated in the financial markets due to its political system, Spain is penalized by the highest unemployment in the UE27 and Greece and Ireland have enormous problems with their bank system and their public debt. The case of Portugal is also complicated. It combines a political problem and a difficult economic situation. As long as politicians do not carry out restrictive policies Portugal will be more vulnerable to default.

Conclusions

Throughout this paper we have analyzed the current problem of bankruptcy risk for the main economies in the world. Before the financial crisis originated in the United States in the summer 2007, it was difficult to imagine that countries such as Greece or Ireland might have to ask for external help. However, this crisis has uncovered the shortages of all economies. As we can see, some countries have been able to recover their GDP and employment rate at the same level as before the crisis. However, countries that already had a structural problem in their economies have been strongly hit by the crisis. Such are the cases of Spain and the property bubble, Ireland and the bank system crisis or Greece. Crisis has only accelerated the process of adjustment.

Thus, throughout this paper we have carried out an assessment of the situation in the EU-27 based on data from Eurostat [2010], by constructing a synthetic index of bankruptcy risk through different methods (the Distance-Principal Components, based on statistical techniques, and goal programming techniques which are not based on statistics), each of them with its pros and cons, keeping in mind that the indicator is not an end in itself but an instrument available to the researcher for better analysis of the situation. Despite the great subjectivity that underlies the construction of such indicators, we have tried to be as explicit as possible in the methodological aspects with intent to make our analysis objective and give it validity and scientific rigor.

To sum up, not only is it important to have a stable economy but it is equally important to convince people that your country has a powerful economy. Otherwise, if there is uncertainty about the economic situation, the financial market will punish that economy. As we have seen, economic stability depends on actual economic variables and those negotiated in the financial markets.

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THE DISCRETE INTERACTIVE MULTIPLE GOAL PROGRAMMING UNDER RISK

Abstract

An Interactive Multiple Goal Programming (IMGP) is a popular method of multicriteria decision aiding. The discrete version of this method was proposed by Habenicht in 1984. In this paper we propose the modification of discrete version IMGP which enables us to take into consideration risk factors and it's also adopted for group decision making.. Risky criteria are described by probability distributions. The aggregation of local judgments making by individual decision makers to the group decision is carried out by voting system. In this paper proposed method is presented. In the last section the proposed method is illustrated by simple numerical example.

Keywords

Multicriteria decision making, decisions under risk, group decision making.

Introduction

In the beginning of the 21st century we observe a significant volatility of the macroeconomic environment, which has a considerable impact on the business world. First of all it is a consequence of rapid technological progress, particularly in the field of information and telecommunication technologies (ICT) and the increasing economic globalization.

In consequence, the influence that these factors exert on economic and business decisions has to be taken into account in the decision-making. The issues related to decision analysis and aiding under incomplete information remain an important part of operational research, in particular of multicriteria decision aiding. Uncertainty implies that in certain situations a person does not possess the information which is quantitatively and qualitatively appropriate to describe, prescribe or predict deterministically and numerically a system, its behavior or other characteristics [Zimmerman 2000].

In the MCDA approach we can find a wide range of methods and techniques to deal with uncertainty: sensitivity analysis [e.g. Rios Insua 1990], fuzzy set approach [e.g. Klir and Fogler 1988], rough set approach [e.g. Greco, Matarazzo and Slowinski 1999], probabilistic models and expected utility [Bazerman 2002; Rosquist 2001], pairwise comparisons based on stochastic dominance [e.g. Martel and Zaras 1995; Nowak 2008]. Rrisk measures as surrogate criteria are also applied [e.g. Millet and Wedley 2002; Jia and Dyer 1996].

In business organizations we observe that very often a Decision Maker (DM) is not a single person but a group of people responsible for making a decision. Thus the modeling of group preferences becomes an important part of the decision making process. The problems of group decision making is discussed, for example, by Ramanathan and Ganesh [1994], Van Den Honert [2001], Herrera, Martınez and Sanchez [2005].

In this paper we propose an interactive procedure which is a modification of the discrete version of Interactive Multiple Goal Programming (IMGP) which enables decision aiding under risk by a group of decision makers. The first section of this paper shortly describes the IMGP algorithm and its main advantages. The second section presents the proposed decision aiding method. A simple numerical example is presented in the last part of this paper.

1. The Interactive Multiple Goal Programming

The IMGP was proposed by Nijkamp and Spronk [1980]. According to this approach, the single criterion problems are solved first. Then, on the basis of the optimal solutions obtained the potency matrix is calculated. The potency matrix consists of the ideal solution and the current one. The DM chooses the criterion on which the current solution should be improved and describes the aspiration level for this criterion. The proper constraint is added to each single criterion problem considered and then those problems are solved again. The DM compares the obtained values in the potency matrixes and decides whether he/she accepts the new solution or not. The procedure is continued till the ideal and the current solutions become equal to each other. Some important advantages are connected with the IMGP. First of all, the DM does not have to give his preference information on an a priori basis but has to consider all kinds of choices and trade-off questions which may be relevant [see Nijkamp

1980, p. 104]. Another important advantage of IMGP is its relatively simply and easy to understand idea. During an interactive procedure the DM has to answer the simple questions:

1 Is the given solution acceptable or not?

2. Which goal value needs to be improved?

3. How much should this goal value be improved at least?

4. Does he accept the consequences of the proposed improvement of the value of the indicated goal variable? [see Nijkamp 1980, p. 250].

The discrete version of IMGP was proposed by Habenicht [1984]. In this case instead of a multicriteria linear programming problem we consider a finite set of alternatives. Each alternative is described by a finite set of attributes. The potency matrixes are calculated on the basis of criteria values within the set of alternatives and in each iteration according to the DM's decisions the set of alternatives is reduced.

2. The discrete IMGP under Risk

In this section we propose an interactive multicriteria decision aiding method which supports multictiteria group decision making under uncertainty for discrete decision problems. Let us assume that:

m – is the number of alternatives,
k – is the number of criteria, all criteria are maximized,
$X_{i,j}$ – is the probability distribution of the j-th criterion of the i-th alternative,
p_j – is the probability at which the j-th criterion is evaluated.

Moreover, let us assume that the matrix

$$X(p_j) = [x_{ij}]_{mk}$$

includes the values of the j-th evaluation criterion for the assumed probability value p_j, and these values guarantee the probability that a particular variable will have a lower value of at least p_j, which is defined as follows:

$$P(X_{i,j} \geq x_{i,j}) = p_j$$

Let $\mathbf{x}_{\mathbf{ideal}}$ denote the **ideal** solution, defined below:

$$x_{ideal}(p_j) = [x_{i,j} : x_{i,j} = \max_{i=1,\dots,m} x_{ij} ; j = 1,\dots,k]$$

Whereas $\mathbf{x_{current}}$ is a **current** solution:

$$x_{current}(p_j) = [x_{i,j} : x_{i,j} = \min_{i=1,\dots,m} x_{ij} ; j = 1,\dots,k]$$

Let P^0 be the initial matrix which consists (for all criteria) of all possible ideal values for different probabilities defined as follows:

$$P^0 = \begin{bmatrix} x_{ideal}(0{,}95) \\ x_{ideal}(0{,}75) \\ x_{ideal}(0{,}50) \\ x_{ideal}(0{,}25) \end{bmatrix}$$

The potency matrix P^r is written as follows:

$$P^r = \begin{bmatrix} x_{ideal}(p_j) \\ x_{current}(p_j) \end{bmatrix}$$

Where the index r = 1, 2, 3,… denotes the number of the algorithm iteration which generated the matrix P.

STEP 1

The DM is presented with the potency matrix P^0. Then, for each criterion, the DM defines the probability value at which he will analyse the values of a given evaluation criterion. The first potency matrix P^1 is calculated and presented to the DM. The DM chooses either to accept the values and move to Step 2 or to correct the adopted probabilities values p_j.

STEP 2

Following the analysis of the potency matrix, the DM chooses the criterion "j" for which the value of the current solution should be improved. He specifies the accepted value of the pessimistic solution of the criterion d_j, which should be greater than the current solution and should be lower than or equal to the ideal solution.

STEP 3

The alternatives that do not fulfil the condition specified by the DM in Step 2 are deleted from the set of the decision alternatives and a new potency matrix P^r is calculated. The DM compares the values in the potency matrixes P^r and P^{r-1} and decides whether he accepts the consequences of his requirements.

3a) If the DM accepts the new solution, we go back to Step 2. The DM can change the required probability values p_j for the particular evaluation criteria and is then presented with the accordingly improved potency matrix.

3b) If the DM rejects the new solution, we restore the deleted alternatives and then go back to Step 2.

STOP CONDITION

The procedure stops when there is only one alternative left in the set of decision alternatives and the DM accepts the solution.

3. The Group decision making

In the case when the decision maker is not a single person but a group of people we need to develop rules which enable us to aggregate individual preferences into a decision accepted by a group of decision makers. We consider the following types of decisions: choosing the criterion on which the current value should be improved, choosing the aspiration level for the selected criterion, changing the probability value at which the criterion is considered and deciding the current solution is accepted or not. We propose the following scheme of group decision making:

1. The group chooses the criterion to be improved by a series of voting,
2. Changing the probability (if desired) by vote,
3. Decision makers describe the aspiration levels individually,
4. Sort aspiration levels from weakest to the strongest,
5. Calculate the potency matrix for the aspiration level considered,
6. Voting on whether the current solution is acceptable or not.

In the first step the decision makers vote for the criterion on which the value should be improved.

If none of the criteria gets the desired number of votes the voting is repeated (without the criterion which gets lowest number of votes) till one criterion gets the necessary number of votes (e.g. 51%).

Next, any decision maker can propose to change the probability at which the chosen criterion is analyzed. He/she proposes the new probability value and the members of the group vote whether they agree with this proposal.

Then each decision maker describes the aspiration level for the chosen criterion. The values entered are sorted started with the weakest one. For the succeeding values of aspiration levels the following potency matrixes are calculated and presented to the decision makers. The decision makers vote whether they accept the obtained solution (i.e. whether they accept the consequences of taking into account the aspiration level considered). If the considered solution is accepted by the group, we consider the next (better) value of the aspiration level till we check all values entered. Or, if the solution is not accepted, the remaining aspiration levels are omitted and we go to the next iteration of IMGP method.

4. The numerical example

Let us assume that the management board which consists of three persons (DM1, DM2, DM3) should choose one of the five investment alternatives A1, …, A5. The following four criteria are taken into consideration:

K1 – NPV: net present value (max),

K2 – MS: market share (max),

K3 – DR: debt ratio (min),

K4 – CR: cover ratio i.e.: the proportion of operating profit to financial costs (max).

The following three tables show the percentile distributions obtained due to Monte Carlo simulation carried out for each alternative[1].

Table 1

Net Present Value in mln EUR

Centiles	A1	A2	A3	A4	A5
1,00%	30	25	16	23	25
5,00%	30	25	16	23	25
10,00%	30	25	17	23	25
25,00%	30	25	19	25	25
50,00%	33	29	25	32	31
75,00%	41	43	30	36	37
90,00%	46	49	34	40	42
95,00%	48	52	35	40	45
99,00%	49	54	35	41	45

[1] An examples of the use of Monte Carlo simulation in strategic decision making can be found in Dominiak [1998, 1999].

Table 2

Market share

Centiles	A1	A2	A3	A4	A5
1,00%	21%	14%	18%	21%	20%
5,00%	21%	15%	18%	22%	22%
10,00%	21%	15%	18%	22%	22%
25,00%	21%	15%	19%	22%	22%
50,00%	22%	17%	22%	24%	24%
75,00%	27%	18%	23%	26%	26%
90,00%	28%	20%	24%	27%	27%
95,00%	28%	22%	25%	27%	27%
99,00%	29%	23%	25%	28%	28%

Table 3

Debt ratio

Centiles	A1	A2	A3	A4	A5
1,00%	29%	39%	35%	44%	28%
5,00%	29%	39%	35%	44%	28%
10,00%	30%	40%	35%	45%	30%
25,00%	31%	41%	35%	46%	32%
50,00%	34%	45%	37%	47%	34%
75,00%	35%	46%	38%	48%	37%
90,00%	36%	47%	39%	49%	39%
95,00%	36%	47%	39%	49%	39%
99,00%	36%	47%	39%	49%	39%

Table 4

Cover ratio

Centiles	A1	A2	A3	A4	A5
1,00%	2,0	1,1	1,4	1,4	1,6
5,00%	2,1	1,1	1,5	1,4	1,7
10,00%	2,1	1,1	1,5	1,4	1,7
25,00%	2,1	1,1	1,5	1,4	1,8
50,00%	2,3	1,2	1,6	1,5	1,9
75,00%	2,4	1,3	1,7	1,6	2,1
90,00%	2,5	1,4	1,8	1,8	2,3
95,00%	2,5	1,4	1,8	1,9	2,3
99,00%	2,5	1,4	1,9	1,9	2,4

The initial potency matrix is presented in Table 5.

Table 5

The initial matrix P^0

Probability	K1	K2	K3	K4
25%	43	27%	0,49	2,1
50%	33	24%	0,47	2,1
75%	30	22%	0,46	2,3
90%	30	22%	0,45	2,4

Looking at the values from Table 5, the decision makers determine the probabilities at which they want to analyze the criteria. Let us assume that for the criterion K1 the members of the management board determined the probabilities: 0.95, 0.99, and 0.99 respectively. Thus the first criterion will be analyzed at the probability level equal to 0.99. In the same way the probabilities for other criteria were determined. Let us assume that they are equal to: 0.95, 0.90, and 0.99 respectively. Then the first potency matrix is presented to the DMs:

Table 6

Potency Matrix P^1

Criterion:	K1	K2	K3	K4
Probability: / Solution:	0,99	0,95	0,9	0,99
Ideal	30	22%	0,36	2,0
Current	16	15%	0,49	1,1

The decision makers vote which criterion should be improved. Let us assume that DM1 voted for K1, DM2 voted for K2 and DM3 voted for K1. Thus K1 was chosen to be improved. In the next step the decision makers describe the desired value of the chosen criterion in the solution. They propose the following values for K1: 25, 16, 20. First we check the lowest value. Since the desired value of K1 is equal to 16, which is the current solution, the solution remains unchanged. Then we check what happens when K1>=20. In this case we obtain:

Table 7

Potency Matrix P^{1-1}

Criterion:	K1	K2	K3	K4
Probability: / Solution:	0,99	0,95	0,9	0,99
Ideal	30	22%	0,36	2,0
Current	23	15%	0,49	1,1

All decision makers accept this solution, therefore we check the last value K1>=25. In this case we get the following potency matrix:

Table 8

Potency Matrix P^{1-2}

Criterion:	K1	K2	K3	K4
Probability: / Solution:	0,99	0,95	0,9	0,99
Ideal	30	22%	0,36	2,0
Current	25	15%	0,47	1,1

Let us assume that the DMs again accept the solution and we can go to the second iteration. The P^2 matrix is equal to $P^{1\text{-}2}$ matrix. The decision makers chose the criterion that will be improved. They voted as follows: DM1-K3, DM2-K4, DM-K4. The fourth criterion is chosen to be improved. The decision makers determined the following aspiration levels: 1.2, 1.4, 1.5. When the constraint K3>=1.2 is added, we get the following potency matrix:

Table 9

Potency Matrix $P^{2\text{-}1}$

Criterion:	K1	K2	K3	K4
Probability: / Solution:	0,99	0,95	0,9	0,99
Ideal	30	22%	0,36	2,0
Current	25	21%	0,39	1,6

The decision makers accepted the consequences and the new solution. The values 1.4 and 1.5 don't change the last solution so we start the next iteration. The P^3 matrix is equal to $P^{2\text{-}1}$. DM2 proposed to change the probability at which K2 is analyzed to the level of 0.99. DM1 did not agree but DM3 did, thus the revised P^3 matrix is calculated and presented to the DMs:

Table 10

Potency Matrix P^3

Criterion:	K1	K2	K3	K4
Probability: / Solution:	0,99	0,99	0,9	0,99
Ideal	30	21%	0,36	2,0
Current	25	20%	0,39	1,6

First DM votes that K1 should be improved but DM2 and DM3 decided to improve the value of K2. They wanted to increase the accepted market share to at least 21%. The following potency matrix is obtained:

Table 11

Potency Matrix P^4

Criterion:	K1	K2	K3	K4
Probability: / Solution:	0,99	0,99	0,9	0,99
Ideal	30	21%	0,36	2,0
Current	30	21%	0,36	2,0

Each decision maker accepted the above solution. We can see that the ideal solution is equal to the current one and the set of alternatives consists of one object. Thus the decision aiding procedure stops. As the final decision according to DM's preferences, A1 should be chosen.

Conclusions

In this paper we proposed an interactive method for decision support in discrete multi criteria problems under risk. The proposed procedure is based on the discrete version of Interactive Multiple Goal Programming. The important part of the proposed method is the scheme of aggregating local judgments, made by individual decision makers, into a group decision made by a voting system.

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DESIGN OF OPTIMAL LINEAR SYSTEMS BY MULTIPLE OBJECTIVES

Abstract

Traditional concepts of optimality focus on valuation of already given systems. A new concept of designing optimal systems is proposed. Multi-objective linear programming (MOLP) is a model of optimizing a given system by multiple objectives. In MOLP problems it is usually impossible to optimize all objectives simultaneously in a given system. An optimal system should be tradeoff-free. As a methodology of optimal system design, De Novo programming for reshaping feasible sets in linear systems can be used. Basic concepts of the De Novo optimization are summarized. Possible extensions, methodological and actual applications are presented. The supply chain design problem is formulated and solved by De Novo approach.

Keywords

Optimization of given systems, design of optimal systems, multiple objectives, De Novo Programming, trade-offs free.

Introduction

Traditional concepts of optimality focus on valuation of already given systems. A new concept of designing optimal systems was proposed [Zeleny 1990 and others]. Mathematical programming under multiple objectives has emerged as a powerful tool to assist in the process of searching for decisions which satisfy best a multitude of conflicting objectives. Multi-objective linear programming (MOLP) is a model of optimizing a given system by multiple objectives. As a methodology of optimal system design, De Novo programming for reshaping feasible sets in linear systems can be used. The goal of this paper is to popularize the De Novo concept and present the literature review on it. The De Novo concept has been introduced by Milan Zeleny [see Zeleny 1990]. Basic concepts of the De Novo optimization are summarized. The paper presents approaches for solving the Multi-objective De Novo linear programming (MODNLP) problem, its possible extensions, methodological and actual applications, and an illustrative example. The approach is based on a reformulation of the MOLP problem by given prices of resources and a given budget. Searching for meta-optimum with a minimal budget is used. The instrument of optimum-path ratio is used for achieving the best performance for a given budget. Searching for a better portfolio of resources leads to a continuous reconfiguration and reshaping of system boundaries. Innovations bring improvements to the desired objectives and result in a better utilization of available resources. These changes can lead to beyond tradeoff-free solutions. Multi-objective optimization can be taken as a dynamic process. Possible extensions, methodological and real applications are presented. A supply chain design is formulated and solved by the De Novo approach.

1. Optimization of given systems

Multi-objective linear programming (MOLP) is a model of optimizing a given system by multiple objectives. In MOLP problems it is usually impossible to optimize all objectives simultaneously in a given system. Trade-off means that one cannot increase the level of satisfaction for an objective without decreasing it for another one. Trade-offs are properties of an inadequately designed system and thus can be eliminated through designing a better one. The purpose is not to measure and evaluate tradeoffs, but to minimize or even eliminate them. An optimal system should be tradeoff-free.

The multi-objective linear programming (MOLP) problem can be described as follows

$$\begin{aligned} \text{“Max”} \quad & z = Cx \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned} \tag{1}$$

where C is a (k, n)-matrix of objective coefficients, A is a (m, n)-matrix of structural coefficients, b is an m-vector of known resource restrictions, x is an n-vector of decision variables. In MOLP problems it is usually impossible to optimize all objectives in a given system. For multi-objective programming problems the concept of non-dominated solutions is used [see for example Steuer 1986]. A compromise solution is selected from the set of non--dominated solutions.

Two subjects, the Decision Maker and the Analyst, have been introduced due to classification of methods for solving MOLP problems by information mode:

– Methods with a priori information.

The Decision Maker provides global preference information (weights, utility, goal values,…). The Analyst solves a single objective problem.

– Methods with progressive information – interactive methods.

The Decision Maker provides local preference information. The Analyst solves local problems and provides current solutions.

– Methods with a posteriori information.

The Analyst provides a non-dominated set. The Decision Maker provides global preference information on the non-dominated set. The Analyst solves a single objective problem.

Many methods from these categories have been proposed. Most of them are based on trade-offs. The next part is devoted to the trade-off free approach.

2. Designing optimal systems

Multi-objective De Novo linear programming (MODNLP) is a problem for designing an optimal system by reshaping the feasible set. By given prices of resources and a given budget, the MOLP problem (1) can be reformulated as a MODNLP problem (2).

$$\begin{aligned} \text{“Max”} \quad & z = Cx \\ \text{s.t.} \quad & Ax - b \leq 0 \\ & pb \leq B \\ & x \geq 0 \end{aligned} \tag{2}$$

where b is an m-vector of unknown resource restrictions, p is an m-vector of resource prices, and B is the given total available budget.

From (2) follows

$$pAx \leq pb \leq B$$

By defining an n-vector of unit costs $v = pA$ we can rewrite the problem (2) as

$$\begin{aligned} \text{“Max”} \quad & z = Cx \\ \text{s.t.} \quad & vx \leq B \\ & x \geq 0 \end{aligned} \tag{3}$$

Solving single objective problems

$$\begin{aligned} \text{Max } & z^i = c^i x \quad i = 1, 2, \ldots, k \\ \text{s.t. } & vx \le B \\ & x \ge 0 \end{aligned} \tag{4}$$

z^* is a k-vector of objective values for the ideal system with respect to B.

The problems (4) are continuous “knapsack” problems, the solutions are

$$x_j^i = \begin{cases} 0, j \ne j_i \\ B/v_{j_i}, j = j_i \end{cases}$$

where

$$j_i \in \left\{ j \in (1, \ldots, n) \middle| \max_j (c_j^i / v_j) \right\}$$

The meta-optimum problem can be formulated as follows

$$\begin{aligned} \text{Min } & f = vx \\ \text{s.t. } & Cx \ge z^* \\ & x \ge 0 \end{aligned} \tag{5}$$

Solving the problem (5) provides the solution:

$$x^*$$

$$B^* = vx^*$$

$$b^* = Ax^*$$

The value B^* identifies the minimum budget to achieve z^* through solutions x^* and b^*.

The given budget level $B \le B^*$. The optimum-path ratio for achieving the best performance for a given budget B is defined as

$$r_1 = \frac{B}{B^*}$$

The optimum-path ratio provides an effective and fast tool for the efficient optimal redesign of large-scale linear systems. Optimal system design for the budget B:

$$x = r_1 x^*, \quad b = r_1 b^*, \quad z = r_1 z^*$$

If the number of criteria k is less than that of variables n, we can individually solve the problem individually and obtain synthetic solutions. Shi [1995] defined the synthetic optimal solution as follows: $x^{**} = (x_{j_1}^1, ..., x_{j_k}^k, 0,..,0) \in R^n$, where $x_{j_q}^q$ is the optimal solution of [1995]. For the synthetic optimal solution a budget B^{**} is used. One can define six types of optimum-path ratios [Shi 1995]:

$$r_1 = \frac{B}{B^*},\ r_2 = \frac{B}{B^{**}},\ r_3 = \frac{B^*}{B^{**}},$$

$$r_4 = \frac{\sum_i \lambda_i B_i^j}{B},\ r_5 = \frac{\sum_i \lambda_i B_i^j}{B^*},\ r_6 = \frac{\sum_i \lambda_i B_i^j}{B^{**}}.$$

Optimum-path ratios are different. It is possible to establish different optimal system designs as options for the decision maker.

3. Extensions

The following extensions of De Novo programming (DNP) are possible:

– Fuzzy DNP.
– Interval DNP.
– Complex types of objective functions.
– Continuous innovations.

Fuzzy De Novo Programming (FDNP) uses instruments as fuzzy parameters, fuzzy goals, fuzzy relations, and fuzzy approaches [Li and Lee 1990].

Interval De Novo programming (IDNP) combines the interval programming and De Novo programming, allowing uncertainties represented as intervals within the optimization framework. The IDNP approach has the advantages in constructing an optimal system design via an ideal system by introducing the flexibility toward the available resources in the system constraints [Zhang et al. 2009].

Complex types of objective functions are defined. The generalization of the single objective Max $(cx - pb)$ to the multi-objective form appears to be the right function to be maximized in a globally competitive economy [Zeleny 2010].

The search for a better portfolio of resources leads to continuous reconfiguration and "reshaping" of system boundaries. Innovations bring improvements to the desired objectives and the better utilization of available resources. The technological innovation matrix $T = (t_{ij})$ is introduced. The elements in the structural matrix A should be reduced by a technological progress. The matrix T should be continuously explored. The problem (2) is reformulated as an innovation MODNLP problem (6)

$$\begin{aligned} &\text{"Max"} \quad z = Cx \\ &\text{s.t.} \;\; TAx - b \le 0 \\ &\qquad pb \le B \\ &\qquad x \ge 0 \end{aligned} \tag{6}$$

The multi-objective optimization can be then seen as a dynamic process in three time horizons:

1. Short-term equilibrium:
   - trade-off,
   - operational thinking.
2. Mid-term equilibrium:
   - trade-off free,
   - tactical thinking.
3. Long-term equilibrium:
   - beyond trade-off free,
   - strategic thinking.

The process is illustrated by example 1.

Example 1

The MOLP problem is formulated as follows:

$$\begin{aligned} &\text{Max} \quad z_1 = x_1 + x_2 \\ &\text{Max} \quad z_2 = x_1 + 4x_2 \\ &3x_1 + 4x_2 \le 60, \\ &x_1 + 3x_2 \le 30, \\ &x_1 \ge 0, x_2 \ge 0. \end{aligned}$$

The MODNLP problem is formulated as follows:
Input: $p = (0.5, 0.4)$ $B = 42$,
unit costs $v = pA = (1.9, 3.2)$.

$$\text{Max } z^i = c^i x \quad i = 1, 2, \ldots, k \qquad z_1^* = 22.11,\ z_2^* = 52.50,$$
$$\text{s.t.} \quad vx \le B$$
$$x \ge 0$$

$$\text{Min} \quad f = vx \qquad x_1^* = 11.98,\ x_2^* = 10.13$$
$$\text{s.t.} \quad Cx \ge z^* \qquad B^* = vx^* = 55.17$$
$$x \ge 0 \qquad b^* = Ax^* \quad b_1^* = 76.48,\ b_2^* = 42.39$$

$$r_1 = \frac{B}{B^*} = 0.761$$

Optimal system design for B: $x = r_1 x^*$, $b = r_1 b^*$, $z = r_1 z^*$,
$x_1 = 9.12$, $x_2 = 7.71$, $b_1 = 58.23$, $b_2 = 32.25$, $z_1 = 16.83$, $z_2 = 39.96$.

The innovation MODNLP problem is formulated as follows:
Input: $p = (0.5, 0.4)$ $B = 42$,
the technological innovation matrix $T = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.7 \end{bmatrix}$,
unit costs $v = pTA = (1.48; 2.44)$,

$z_1^* = 28.38$, $z_2^* = 68.85$,

$x_1^* = 14.89$, $x_2^* = 13.49$,

$B^* = vx^* = 54.95$,

$r_1 = 0.764$,

$x_1 = 11.38$, $x_2 = 10.31$,

$z_1 = 21.69$, $z_2 = 52.62$.

The solutions in different time horizons are represented in Figure 1.

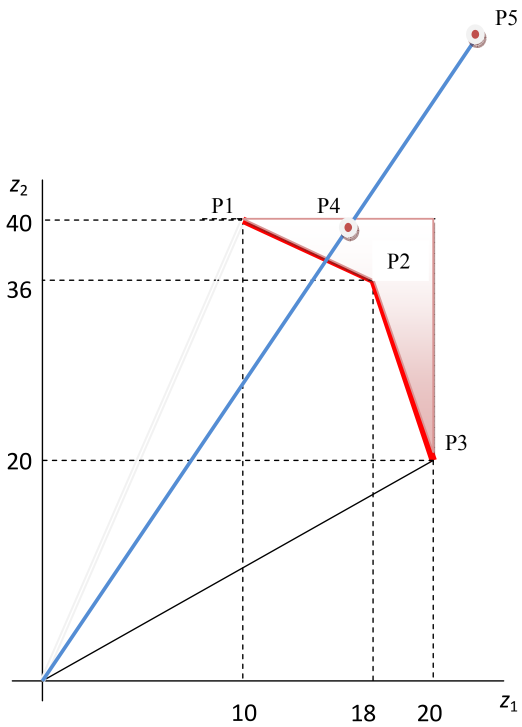

Figure 1. Solutions for the illustrative example

Figure 1 shows the non-dominated frontier (P1-P2-P3) for the MOLP problem, the solution (point P4) of the MODNLP problem and the solution (point P5) of the innovative MODNLP problem. The solution of the MODNLP problem is not fully trade-off free in this example. The solution of the innovative MODNLP problem shows the beyond trade-off free trajectory.

4. Applications

The tradeoff-free decision making has a significant number of method-logical applications. All such applications have the tradeoff-free alternative in common:

- Compromise programming – minimize distance from the ideal point.
- Risk management – portfolio selection – tradeoffs between investment returns and investment risk.
- Game theory – win-win solutions.
- Added value – value for the producer and value for the customer – both must benefit.

There are real applications of the De Novo approach. For example, the production plan for an actual production system is defined taking into account financial constraints and given objective functions [Babic and Pavic 1996]. The paper [Zhang et al. 2009] presents an Inexact DNP approach for the design of optimal water-resources-management systems under uncertainty. Optimal supplies of good-quality water are obtained with different revenue targets of municipal–industrial–agricultural competition under a given budget taken into account.

In the next part a supply chain design problem is formulated. Supply chain management has generated a substantial amount of interest from both managers and researchers. Supply chain management is now seen as a governing element in strategy and as an effective way of creating value for customers. A supply chain is defined as a system of suppliers, manufacturers, distributors, retailers and customers where material, financial and information flows connect participants in both directions [see for example Fiala 2005]. There are many concepts and strategies applied to the design and management of supply chains. The fundamental decisions to be made during the design phase are the location of facilities and the capacity allocated to these facilities. An approach to designing an economically optimal supply chain is to develop and solve a mathematical programming model. A mathematical program determines the ideal locations for each facility and allocates the activity to each facility so that the costs are minimized and the constraints of meeting the customer demand and the facility capacity are satisfied. A general form of the model for the supply chain design is given below.

Model

Our model of a supply chain consists of 4 layers with m suppliers, $S_1, S_2, \dots S_m$, n potential producers, $P_1, P_2, \dots P_n$, p potential distributors, $D_1, D_2, \dots D_p$, and r customers, $C_1, C_2, \dots C_n$.

The following notation is used:

a_i = annual supply capacity of supplier i,

b_j = annual potential capacity of producer j,

w_k = annual potential capacity of distributor k,

d_l = annual demand of customer l,

f_j^P = fixed cost of potential producer j,

f_k^D = fixed cost of potential distributor k,

c_{ij}^S = unit transportation cost from S_i to P_j,

c_{jk}^P = unit transportation cost from P_j to D_k,

c_{kl}^D = unit transportation cost from D_k to C_l,

t_{ij}^S = unit transportation time from S_i to P_j,

t_{jk}^P = unit transportation time from P_j to D_k,

t_{kl}^D = unit transportation time from D_k to C_l,

x_{ij}^S = number of units transported from S_i to P_j,

x_{jk}^P = number of units transported from P_j to D_k,

x_{kl}^D = number of units transported from D_k to C_l,

y_j^P = bivalent variable for build-up of fixed capacity of producer j,

y_k^D = bivalent variable for build-up of fixed capacity of producer k.

With this notation the problem can be formulated as follows:
The model has two objectives. The first one expresses the minimizing of total costs. The second one expresses the minimizing of total delivery time.

$$\text{Min } z_1 = \sum_{j=1}^{n} f_j^P y_j^P + \sum_{k=1}^{p} f_k^D y_k^D + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^S x_{ij}^S + \sum_{j=1}^{n}\sum_{k=1}^{p} c_{jk}^P x_{jk}^P + \sum_{k=1}^{p}\sum_{l=1}^{r} c_{kl}^D x_{kl}^D$$

$$\text{Min } z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}^S x_{ij}^S + \sum_{j=1}^{n}\sum_{k=1}^{p} t_{jk}^P x_{jk}^P + \sum_{k=1}^{p}\sum_{l=1}^{r} t_{kl}^D x_{kl}^D$$

Subject to the following constraints:

– the amount sent from the supplier to the producers cannot exceed the supplier's capacity

$$\sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1, 2, ..., m,$$

– the amount produced by the producer cannot exceed the producer's capacity

$$\sum_{k=1}^{p} x_{jk} \le b_j y_j, \ \ j = 1, 2, ..., n,$$

– the amount shipped from the distributor should not exceed the distributor's capacity

$$\sum_{l=1}^{r} x_{kl} \le w_k y_k, \ \ k = 1, 2, ..., p,$$

– the amount shipped to the customer must equal the customer's demand

$$\sum_{k=1}^{p} x_{kl} = d_l, \ \ l = 1, 2, ..., r,$$

– the amount shipped out of producers cannot exceed units received from suppliers

$$\sum_{i=1}^{m} x_{ij} - \sum_{k=1}^{p} x_{jk} \ge 0, \ j = 1, 2, ..., n,$$

– the amount shipped out of the distributors cannot exceed quantity received from the producers

$$\sum_{j=1}^{n} x_{jk} - \sum_{l=1}^{r} x_{kl} \ge 0, \ \ k = 1, 2, ..., p,$$

– binary and non-negativity constraints

$$Y_{j,k} \in \{0,1\},$$
$$x_{ij}, x_{jk}, x_{kl} \ge 0, \ \ i = 1, 2, ..., m, \ j = 1, 2, ..., n, \ \ k = 1, 2, ..., p, \ \ l = 1, 2, ..., r.$$

The formulated model is a multi-objective linear programming problem. The problem can be solved by an MOLP method.

The De Novo approach can be useful in the design of the supply chain. Only a partial relaxation of constraints is adopted. Producer and distributor capacities are relaxed. Unit costs for capacity build-up are computed:

$$p_j^P = \frac{f_j^P}{b_j} = \text{cost of unit capacity of potential producer } j,$$

$$p_k^D = \frac{f_k^D}{w_k} = \text{cost of unit capacity of potential distributor } k.$$

Variables for build-up capacities are introduced:

u_j^P = variable for flexible capacity of producer j,

u_k^D = variable for flexible capacity of producer k.

The constraints for non-exceeding producer and distributor fixed capacities are replaced by the flexible capacity constraints and the budget constraint:

$$\sum_{k=1}^{p} x_{jk} - u_j^P \leq 0, \;\; j = 1, \; 2, \; ..., \; n,$$

$$\sum_{l=1}^{r} x_{kl} - u_k^D \leq 0, \;\; k \; = \; 1, \; 2, \; ..., p,$$

$$\sum_{j=1}^{n} p_j^P u_j^P + \sum_{k=1}^{p} p_k^D u_k^D \leq B.$$

Example 2

An example of the supply chain with 3 potential producers, 3 potential distributors, and 3 customers was tested. Data are presented in Tables 1, 2 and 3.

Table 1

Unit transportation costs

c_{ij}^P	D_1	D_2	D_3	c_{jk}^D	C_1	C_2	C_3
P_1	5	3	8	D_1	3	1	4
P_2	3	6	2	D_2	6	7	2
P_3	8	4	5	D_3	5	4	8

Table 2

Unit transportation time

c_{ij}^P	D_1	D_2	D_3	c_{jk}^D	C_1	C_2	C_3
P_1	4	2	3	D_1	6	1	2
P_2	3	2	2	D_2	3	2	5
P_3	1	5	3	D_3	1	4	2

Table 3

Capacity and costs for producers and distributors

	P_1	P_2	P_3	D_1	D_2	D_3
Capacity	250	300	200	300	200	300
Costs	150	200	180	50	60	90
unit cost	0.60	0.67	0.90	0.17	0.30	0.30

Customer demand: $d_1 = 100$, $d_2 = 150$, $d_3 = 200$.

We get the ideal objective values z^* by solving single objective problems. The interactive method STEM is used for finding a compromise non-dominated solution. The De Novo approach is used for the supply chain design. The results are compared in Table 4.

Table 4

Results for supply chain design

	Max z_1	Max z_2	Compromise	De Novo
x_{11}^P	0	0	0	0
x_{12}^P	200	0	50	0
x_{13}^P	0	0	0	0
x_{21}^P	250	0	200	250
x_{22}^P	0	0	0	0
x_{23}^P	0	250	100	100
x_{31}^P	0	200	100	100
x_{32}^P	0	0	0	0
x_{33}^P	0	0	0	0
x_{11}^D	100	0	0	0
x_{12}^D	150	150	150	150
x_{13}^D	0	50	150	200
x_{21}^D	0	0	0	0
x_{22}^D	0	0	0	0

Table 4 contd.

	Max z_1	Max z_2	Compromise	De Novo
x_{23}^D	200	0	50	0
x_{31}^D	0	100	100	100
x_{32}^D	0	0	0	0
x_{33}^D	0	150	0	0
u_1^P	250	0	250	0
u_2^P	300	300	300	350
u_3^P	0	200	200	100
u_1^D	300	300	300	350
u_2^D	200	0	200	0
u_3^D	0	300	300	100
z_1	2660	4670	3830	3644
z_2	2900	1350	1800	1700
B	460	520	730	444

The De Novo approach provides a better solution in both objectives and also with lower budget thanks to flexible capacity constraints. The capacity of supply chain members has been optimized with regard to flows in the supply chain and to the budget.

Conclusions

De Novo programming is used as a methodology of optimal system design for reshaping feasible sets in linear systems. The MOLP problem is reformulated by given prices of resources and a given budget. Searching for a better portfolio of resources leads to a continuous reconfiguration and reshaping of systems boundaries. Innovations bring improvements to the desired objectives and the better utilization of available resources. These changes can lead to beyond tradeoff-free solutions. Multi-objective optimization can be regarded as a dynamic process. The De Novo approach has been applied to the supply chain design problem; it provides a better solution than traditional approaches applied to fixed constraints. The De Novo programming approach is open for further extensions and applications.

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Dorota Górecka

SENSITIVITY AND ROBUSTNESS ANALYSIS OF SOLUTIONS OBTAINED IN THE EUROPEAN PROJECTS' RANKING PROCESS

Abstract

After entering the European Union on 1 May 2004 Poland has become eligible to benefit from the EU Structural Funds and the Cohesion Fund and projects co-financed by these means have become a crucial instrument supporting restructuring and modernization of Polish economy. Total financial assistance granted to Poland for the previous (2004-2006) and present (2007-2013) programming periods amounts to over 80 billion euro. An efficient allocation of these subsidies depends, among other things, on proper choice of projects that are going to be co-financed, which can be made with the aid of such multi-criteria techniques as ORESTE, EVAMIX, PROMETHEE II, EXPROM II or modified BIPOLAR method.

In the paper sensitivity and robustness analysis of solutions obtained with the help of the above-mentioned methods will be carried out. It will enable to show the influence of the information delivered by the decision-makers and choices made by them during the decision aiding process on the final European projects' ranking. In a real-life example concerning this issue 16 applications for project co-financing by the European Regional Development Fund submitted to Measure 1.2 Environmental protection infrastructure in one of Polish voivodships in the programming period 2004-2006 will be used.

Keywords

Multi-criteria decision aiding methods: ORESTE, EVAMIX, PROMETHEE II, EXPROM II, modified BIPOLAR method, stochastic dominance rules, sensitivity and robustness analysis.

Introduction

European regional policy is currently one of the crucial factors in strengthening the socio-economic development of Poland and other European Union countries, especially those that entered the EU in 2004 and 2007, whose economies have lagged far behind the economies of the old Member States of EU-15 and whose needs in the areas of environment, infrastructure, research and innovation, industry, services and SMEs are truly significant [Górecka 2011b].

Regional policy helps to reduce disparities between countries, increase the regions' competitiveness and attractiveness, improve the employment prospects and support innovation and development of the knowledge society as well as environmental protection. Moreover, it strengthens cross-border co-operation through financing concrete projects for regions, towns and their inhabitants.

In the previous programming period 2000-2006 over 233 billion EUR was earmarked for all regional instruments for the 15 old Member States. Moreover, around 24 billion EUR was allocated for the 10 new Member States for years 2004-2006, not to mention 22 billion EUR granted for pre-accession aid. In the present programming period 2007-2013 cohesion policy benefits from total allocation of about 347 billion EUR, which represents nearly 36% of the entire Union's budget.

Because of the enormous amount of money devoted to the structural aid it is crucially important to allocate the means in the most effective way possible. And that depends, among other things, on the proper choice of projects to be co-financed. In order to help the decision-makers in this challenging and difficult task multi-criteria decision aiding techniques, which refers to making decision in the presence of multiple, usually conflicting criteria, should be applied as evaluation of the European projects requires taking into account many diverse aspects: economic, financial, environmental, ecological, technical, technological, social and legal [Górecka 2011b].

Sensitivity and robustness analysis of the obtained solutions to the changes of the parameters of the preference model is in the case of projects applying for co-financing from the European Union funds quite a risky undertaking – in the extreme case it may lead even to undermining the decisions taken as a result of the proceedings conducted, i.e. to contesting the list of projects selected for funding. However, such analysis will be carried out for the purposes of this paper, primarily to demonstrate the importance of the quality of both the information acquired from the participants of the decision-making process and choices made by the decision-makers during the decision-aiding process, and the extent of their influence on the final ranking of projects [Górecka 2011a[1]].

[1] This publication is devoted to the sensitivity and robustness analysis of solutions obtained with the help of the following MCDA techniques: PROMETHEE II with stochastic dominance rules, EXPROM II with stochastic dominance rules and modified BIPOLAR method with stochastic dominance rules.

1. Sensitivity and robustness analysis

When solving real decision-making problems decision-makers and analysts encounter problems related to the imperfection of knowledge. This deficiency has several different causes but invariably leads to assigning arbitrary values to the certain parameters of models and algorithms used in the decision-making process. In this case parameters are very broadly defined and include both the data in the classical sense of the word and information about values and beliefs of the participants of the decision-making process as well as information regarding technical issues related to the algorithms operation. In the multi-criteria methods based on the outranking relation, among which the ELECTRE and PROMETHEE methods stand out, doubts resulting from the imperfection of available data may concern both parameters related to the modelling of preferences (weights, thresholds or categories profiles) and technical parameters such as, for example, the cutting level λ [Figueira et al. 2005, p. 149]. Since it is difficult to expect that the participants of the decision-making process will easily define the values of parameters, therefore each of their permissible combinations should be treated as a "working hypothesis". The problem is that different "working hypothesis" may lead to different results [Dias and Climaco 1999, p. 74].

In practice, a reference system composed of central values of the parameters is often defined and on this basis the calculations are carried out, whose results are used to prepare recommendations for the decision-makers. Subsequently the sensitivity of the solution to changes in the values of the parameters is examined. This analysis is usually performed for each parameter separately (ignoring possible interdependencies among them). It allows you to define the scope of the changes in the values of the parameters which make no impact on the solution designated earlier and also specify these parameters, whose values, when varying from the central positions, particularly strongly affect the outcome [Figueira et al. 2005, p. 149-150].

As an alternative for the sensitivity analysis the robustness analysis of the obtained solutions to changes in the values of the parameters may be considered. In this case the problem is defined as follows: assuming that the role of the analyst is to build such recommendations that will prove correct for the possibly wide range of the parameter values, we want to obtain information on the solutions proposed, depending on the values of the parameters. Thus, we are interested in whether and how the solution of the problem will be changing with modification of the parameters within the sets of their admissible values.

The concept of robustness was introduced by Roy [Roy and Hugonnard 1982, p. 301-312; Roy 1998, p. 141-160] who has formulated a definition of the robust conclusion describing it as a formalized premise that is true for all plausible combinations of parameter values. Dias and Climaco, starting with the definition given by Roy, have distinguished the following types of the robust conclusions [Dias and Climaco 1999, p. 75]:

– an absolute robust conclusion – a premise intrinsic to one of the examined variants, which is valid for all acceptable combinations of parameter values; in the case of additive aggregation model the absolute conclusion may be "for example as follows: "the assessment of the variant a_i is less than 0,5";
– a relative binary robust conclusion – a premise concerning a pair of variants, which is true for all possible values of parameters; for example: "a_i outranks a_j with credibility greater than 0,8";
– a relative unary robust conclusion – a premise concerning one variant but referring to others, binding for each admissible combination of parameter values; for example: "a_i is placed on one of the top three positions in the ranking".

2. The proposed procedure of appraising and selecting European projects

Meeting the need to improve the system of evaluation and selection of applications for project co-financing by the European Union funds and taking into account advantages and disadvantages of different multi-criteria decision aiding methods[2], the procedure composed of the following elements has been proposed to aid the choice of European projects:

– identification of the participants of the decision-making process;
– selection of the criteria and determination of their weights with the help of:

[2] [See Górecka 2010, p. 105-108].

  - Analytic Hierarchy Process [Saaty 2006; Saaty and Vargas 1991],
  - REMBRANDT system [Lootsma et al. 1990, p. 293-305; Olson et al. 1995, p. 522-531],
  - revised Simos' method [Figueira and Roy 2002, p. 317-326],

  (depending on the preferences of the decision-makers);
- establishing indifference, preference and veto thresholds for each of the criteria;
- building a table of assessments (evaluation matrix) of the projects participating in the contest;
- application of:
  - ORESTE method [Roubens 1982, p. 51-55],
  - EVAMIX method [Voogd 1982, p. 221-236],
  - PROMETHEE II method with stochastic dominance rules[3],
  - EXPROM II method with stochastic dominance rules[4],
  - modified BIPOLAR method[5] with stochastic dominance rules [Górecka 2009, p. 223-230],
  - EVAMIX method with stochastic dominance rules[6] [Górecka 2010, p. 120-122]

  (depending on the available data and the expectations and preferences of the decision-makers);
- taking final decision.

3. Case study

The proposed procedure was employed in the simulation of the process of appraising and ranking European projects carried out with the use of applications for project co-financing by the European Regional Development Fund submitted to Measure 1.2 *Environmental protection infrastructure* in one of Polish voivodships in the programming period 2004-2006. Measure 1.2 was implemented within the framework of the Priority 1 *Development and modernisation of the infrastructure to enhance the competitiveness of regions* of the Integrated Regional Operational Programme.

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[3] The indifference threshold has been introduced to the technique. [See Górecka 2009, p. 218-223, 263-277]. Compare with the original approach presented in Nowak [2005, p. 193-202].
[4] See Appendix A.
[5] Original version of BIPOLAR method was proposed in Konarzewska-Gubała [1991].
[6] See Appendix B.

Sixteen infrastructure projects were considered[7]. They concern the protection of surface waters, waste management and flood control and include:
- construction and modernisation of wastewater and rainwater collection networks and wastewater treatment plants,
- implementation of a system of communal waste management, i.a. construction of a sorting and composting plants and recultivation of landfills,
- modernisation of dikes.

Five experts – specialists in the field of environmental protection infrastructure – scored them[8] from 0 (the lowest evaluation) to 10 (the highest evaluation) taking into account 11 criteria[9] presented in Table 1.

Table 1

Preference model
(with weighting coefficients established by means of REMBRANDT system)

No.	Criteria	Weights	Indifference thresholds	Preference thresholds	Veto thresholds	
					ELECTRE	BIPOLAR
1	**2**	**3**	**4**	**5**	**6**	**7**
1	Total cost	0,12	2	3	7	3
2	Efficiency	0,19	1	3	6	3
3	Influence on the environment	0,15	2	4	7	3
4	Influence on the employment	0,05	3	4	9	2
5	Influence on the inhabitants' health	0,14	3	5	8	2
6	Influence on the investment attractiveness	0,07	2	4	8	2

[7] They are denoted by letters from A to T.

[8] In order to keep the classified data confidential while enabling an objective evaluation, the descriptions of the projects were truncated and standardised.

[9] All criteria are treated as quality criteria, even if it is possible to use them as quantitative criteria as in the case of total cost or efficiency. This is due to the specificity of the applications, in which the influence of the projects was often described in very complex and diverse manner and by means of incomparable data. The reason for treating efficiency as a quality criterion is that in the programming period 2004-2006 the guidelines for the preparation of the documents by applicants were not very precise and allowed them to provide a free and sometimes even creative financial analysis and benefit cost analysis (BCA). In many cases an inappropriate financial analysis methodology was applied and in economic analyses not all transfers, corrections and benefits were taken into consideration. Therefore, the appraisal of the projects was very often intuitive and based on the expertise and experience of specialists. In the case of total cost, in turn, their reliability and validity was to be assessed. An exception is made for EVAMIX method (with and without stochastic dominance rules), in which case the total cost is treated as a quantitative criterion.

Table 1 contd.

1	2	3	4	5	6	7
7	Influence on the tourist attractiveness	0,06	2	5	8	2
8	Validity of the technical solutions	0,08	1	3	7	2
9	Sustainability and institutional feasibility of the project	0,06	1	3	8	2
10	Complementarity with other projects	0,04	2	4	8	2
11	Comprehensiveness	0,04	2	4	8	2

The above-mentioned set of 11 criteria was constructed as follows: a list of the criteria (based on the data available in the considered applications considered for project co-financing and information contained within official documents related to the EU funds as well as on the criteria applied in the programming period 2004-2006 and the aims of regional development strategy) was presented to five specialists in the field of environmental protection infrastructure and European Union funds who could accept or reject each of them. They had also a possibility to add their own criteria to the preliminary list.

To obtain the essential data to use AHP method and REMBRANDT system, each of the five aforementioned experts in the scope of environmental protection infrastructure and the EU funds was asked to compare criteria pair--wise using the 1-to-9 Saaty's scale [Saaty 2006, p. 73]. As a result two different vectors of weighting coefficients have been produced. The third one was formed as a result of the application of the modified Simos' procedure. In this case the role of decision-maker was assumed by the author of the paper.

The experts were also asked to determine values of indifference, preference and veto thresholds within the meaning of ELECTRE method. Two extreme opinions were disregarded and with the remaining three the arithmetic mean was calculated. It was subsequently rounded to the nearest integer. Veto thresholds within the meaning of BIPOLAR method were established by the author of this paper.

Table 2 provides a summary of the results yielded by means of six multi-criteria techniques enumerated in the previous section of this paper. For comparison, the table includes also ranking of the projects obtained with the help of the arithmetic mean of the weighted sums of points assigned by experts, i.e. the method functioning so far in the system of evaluation and selection of the applications for project co-financing by the European Union funds.

Table 2

Rankings of the projects obtained using different MCDA methods

No.	ORESTE	EVAMIX	PROMETHEE II	EXPROM II	modified BIPOLAR method	EVAMIX with stochastic dominance rules	Arithmetic mean of the weighted sums of points	No.
1	C	C	P	P	C	C	C	1
2	P	D	C	C	D	D	D	2
3	D	M	D	D	M	G	P	3
4	M	G	G	M	G	M	M	4
5	R	P	M	K	R	P	G	5
6	G	H	K	G	T	T	T	6
7	T	T	T	R	E	R	R	7
8	H	R	R	H	N	H	H	8
9	K	K	H	N	P	E	K	9
10	E	E	N	T	H	L	E	10
11	L	L	B	B	F	K	N	11
12	N	N	E	E	K	N	B	12
13	B	B	F	S	B	F	L	13
14	S	F	S	F	S	B	F	14
15	F	S	A	A	L	S	S	15
16	A	A	L	L	A	A	A	16

The rankings presented in Table 2 show the sensitivity of the solutions to choice of the decision-aiding technique: depending on the method used to support the decision-making process and on the amount of available financial resources, different projects would receive subsidies.

The orders of the projects in the rankings are not in agreement. However, in spite of that it is possible to determine the set of projects which are the best (C, D, M and G) and the other one containing projects which are the worst (L, A, S, B and F). Project P may be regarded as controversial since on the one hand it is classified at the forefront of rankings in the case of PROMETHEE II and EXPROM II methods combined with stochastic dominance rules, but on the other hand, it is characterised by a very low appraisal of one of the criteria (namely influence of the project on the employment), which was clearly caught by the modified BIPOLAR method thanks to the veto procedure applied in this technique.

In this context it is worth mentioning that the ranking obtained with the help of arithmetic mean of the weighted sums of points granted by experts coincides fairly well with the results obtained using different multi-criteria decision aiding techniques. This is not surprising as high-quality projects should be classified at the top of the rankings and weak projects should be ranked low regardless of the method used. However, the assumptions of multi-criteria decision aiding methods based on the outranking relation are more congruent with reality than those of the method consisting in calculating weighted mean. Hence, they can definitely improve the procedure of appraising and selecting projects applying for co-financing from the European Union taking into account uncertainty and imprecision accompanying all the decision-making problems. Moreover, they can exclude – at least partly – the possibility of compensation a bad evaluation on one criterion by a good one on the other and limit – thanks to the earlier determination of the preference model – the risk of the manipulation of the outcomes. They prove correct especially in the case of projects with high appraisals with respect to some criteria and very low appraisals with respect to the others[10].

4. Sensitivity and robustness analysis of the solutions obtained

In this part of the paper we will present a sensitivity and robustness analysis of solutions obtained by applying ORESTE, EVAMIX, PROMETHEE II, EXPROM II and the modified BIPOLAR method (see Table 2).

In the first step of the analysis the ranges of variations of indifference and preference thresholds, which do not result in modification of the rankings, were determined using optimization tools integrated with Excel. The analysis was carried out separately for each of the thresholds provided that they satisfy the condition $q_k \le p_k \le v_k$ in the case of PROMETHEE II and EXPROM II methods with stochastic dominance rules and the condition $q_k \le p_k$ in the case of the modified BIPOLAR method. The results are displayed in Tables 3 and 4. They indicate that the results obtained for each MCDA technique considered are least sensitive to variations of the values of the thresholds for the criterion No. 5.

[10] [See Górecka and Pietrzak 2012].

Table 3

Ranges of variations of the indifference thresholds values

No.	Criteria	q_{min}			q original	q_{max}		
		PROMETHEE II	EXPROM II	BIPOLAR		BIPOLAR	EXPROM II	PROMETHEE II
1	Total cost	1,974	1,596	1,788	2	2,225	2,143	2,132
2	Efficiency	0,946	0,945	0,077	1	1,272	1,002	1,067
3	Influence on the environment	1,501	1,641	1,715	2	2,214	2,201	2,106
4	Influence on the employment	2,843	2,998	3,000	3	3,655	3,306	3,079
5	Influence on the inhabitants' health	2,122	2,196	2,412	3	5,000	5,000	5,000
6	Influence on the investment attractiveness	1,432	1,998	1,539	2	2,180	2,246	2,081
7	Influence on the tourist attractiveness	0,702	1,998	1,420	2	2,159	2,907	2,080
8	Validity of the technical solutions	0,930	0,995	0,000	1	1,379	1,115	1,330
9	Sustainability and institutional feasibility of the project	0,954	0,979	0,372	1	2,529	1,300	3,000
10	Complementarity with other projects	1,702	0,877	1,415	2	2,836	2,002	2,248
11	Comprehensiveness	1,812	1,996	1,509	2	3,733	3,383	2,735

Table 4

Ranges of variations of the preference thresholds values

No.	Criteria	p min			p original	p max		
		PROMETHEE II	EXPROM II	BIPOLAR		BIPOLAR	EXPROM II	PROMETHEE II
1	Total cost	2,954	2,998	2,606	3	3,411	3,159	3,154
2	Efficiency	2,796	2,774	2,527	3	3,192	3,003	3,040
3	Influence on the environment	2,200	3,002	3,334	4	4,647	5,612	6,036
4	Influence on the employment	3,907	4,000	3,345	4	4,428	4,200	4,064
5	Influence on the inhabitants' health	3,000	3,000	3,000	5	10,000	8,000	8,000
6	Influence on the investment attractiveness	3,163	3,995	2,500	4	4,450	5,211	4,169
7	Influence on the tourist attractiveness	3,418	4,995	3,464	5	5,380	8,000	5,116
8	Validity of the technical solutions	2,358	2,986	2,298	3	4,720	3,257	3,926
9	Sustainability and institutional feasibility of the project	2,806	2,625	2,177	3	5,732	4,403	8,000
10	Complementarity with other projects	3,941	3,416	3,312	4	4,901	4,001	4,246
11	Comprehensiveness	3,618	3,990	3,492	4	7,647	6,423	8,000

The analysis of robustness of the solutions to the changes of the weighting coefficients of evaluation criteria has been performed using the approach proposed by Hyde, Maier and Colby [Hyde et al. 2005, p. 278-290). Its essence consists in determining for each pair of variants (a_i, a_j) the minimum admissible modification of criteria weights that is required to alter the total values of two selected variants such that rank equivalence occurs. This smallest change in the values of the criteria weights is obtained by solving an optimisation problem, in which the objective function is formulated as follows:

$$\min d_E = \sqrt{\sum_{k=1}^{n}(w_k - w'_k)^2}\ ,\quad k = 1,...,n.$$

The aim is therefore to minimise a distance metric that provides the numerical measure to the amount of dissimilarity between the initial weights of the criteria w_k and the optimised criteria weights w'_k. The Euclidean distance has been selected as the most commonly used.

A set of constraints takes the following form:

$$\sum_{k=1}^{n} w_k = \sum_{k=1}^{n} w'_k = 1,\ k = 1,...,n,$$

$$w_k, w'_k > 0,$$

$$w_k^d \le w'_k \le w_k^g\ ,\quad k = 1,...,n,$$

where w_k^d and w_k^g are the lower and upper limits, respectively, of the values of the weighting coefficients assigned to each of the evaluation criteria f_k.

Applying the optimised criteria weighs should cause the total values of two variants being assessed to be equal, thus we have in addition:

$$\phi'(a_i) = \phi'(a_j).$$

As a result of solving the non-linear programming task presented above the values of the minimum Euclidean distance for all pairs of variants are obtained. They can be presented in the form of a matrix.

In some situations one of the variants is always classified higher in ranking than the other, regardless of the values of the modified parameters. In this case, the ordering of these two variants – because of the insensitivity to variation of parameters – is called robust. Much more often, however, we have to deal with the situation, in which there are at least a few different combinations of the weighting coefficients for which $\phi'(a_i) = \phi'(a_j)$. Setting the smallest overall modification of the criteria weights allowing two variants to achieve the same position in ranking, enables determining whether their ordering is robust or not. Large values of the minimum Euclidean distance mean that one of the variants is generally better than the other, regardless of the values of the parameters changing within the range given by the decision--maker. If, on the other hand, the minimum Euclidean distances are small, minor changes in the values of the parameters will cause rank equivalence of variants being considered, thus their ordering may be concluded to be sensitive to the criteria weights [Hyde et al. 2005, p. 281-282].

In the analysis performed the eight highest-ranked European projects in the orderings obtained with the help of EVAMIX method without stochastic dominance, PROMETHEE II technique with stochastic dominance rules and the modified BIPOLAR method have been taken into account. It has been assumed that the values of the weighting coefficients for evaluation criteria are within the following limits:

Table 5

The permissible range of variability in the values
of the weighting coefficients for project evaluation criteria

No.	Criteria	Coefficients of importance		
		w_{min}	$w_{original}$	w_{max}
1	Total cost	0,05	0,12	0,20
2	Efficiency	0,10	0,19	0,20
3	Influence on the environment	0,10	0,15	0,20
4	Influence on the employment	0,03	0,05	0,10
5	Influence on the inhabitants' health	0,10	0,14	0,20
6	Influence on the investment attractiveness	0,03	0,07	0,10
7	Influence on the tourist attractiveness	0,03	0,06	0,10
8	Validity of the technical solutions	0,03	0,08	0,10
9	Sustainability and institutional feasibility of the project	0,03	0,06	0,10
10	Complementarity with other projects	0,03	0,04	0,10
11	Comprehensiveness	0,03	0,04	0,10

The values of the minimum Euclidean distance d_E for pairs of considered projects contained in Tables 6, 7 and 8 signify that the final rankings are not robust to changes in the criteria weights – in some cases only small modifications of the starting values are required for rank equivalence between the two examined variants.

The results of using the distance-based analysis approach for 28 pairs of projects also indicate that although the obtained solutions are sensitive to variations of input parameter values, the orderings of some projects are robust. For the acceptable ranges of weighting coefficients given in Table 5 in the case of:

– EVAMIX method without stochastic dominance projects C and D (not necessarily in that order) will always be superior to projects H, T and R (no feasible changes in criteria weights could be found); furthermore project C will be also superior to projects M, G and P;

– PROMETHEE II method combined with stochastic dominance rules project C will always be ranked higher than projects D, G, M, K, T and R;
– modified BIPOLAR method projects C, D and M (not necessarily in that order) will always be classified higher in ranking than projects T, E and N.

Table 6

A minimum Euclidean distance matrix for pairs of European projects consisting of the 8 highest-ranked variants using EVAMIX method without stochastic dominance

Projects	C	D	M	G	P	H	T	R
C		0,0706	–	–	–	–	–	–
D			0,1308	0,0554	0,0621	–	–	–
M				0,0010	0,0282	0,1304	0,1284	0,1719
G					0,0188	0,0980	0,0995	0,1188
P						0,0625	0,0656	0,0966
H							0,0015	0,0161
T								0,0122
R								

Table 7

A minimum Euclidean distance matrix for pairs of European projects consisting of the 8 highest-ranked variants using PROMETHEE II method with stochastic dominance

Projects	P	C	D	G	M	K	T	R
P		0,0395	0,1437	0,1245	–	0,1117	0,1591	–
C			–	–	–	–	–	–
D				0,0179	0,1023	0,0308	0,1235	0,1072
G					0,0007	0,0088	0,0896	0,0322
M						0,0136	0,0878	0,0859
K							0,0444	0,0371
T								0,0015
R								

Table 8

A minimum Euclidean distance matrix for pairs of European projects consisting of the 8 highest-ranked variants using the modified BIPOLAR method

Projects	C	D	M	G	R	T	E	N
C		0,0201	0,0437	–	0,1381	–	–	–
D			0,1353	0,0506	0,1242	–	–	–
M				0,0185	0,1060	–	–	–
G					0,0484	0,0901	0,0704	0,0721
R						0,0088	0,0104	0,0460
T							0,0048	0,0135
E								0,0112
N								

In order to show the impact of changes in the weights of evaluation criteria on the final rankings of projects obtained using ORESTE method, EVAMIX method with stochastic dominance rules and EXPROM II method with stochastic dominance rules, calculations with the aid of these techniques have been made again but for preference models in which the criteria weights obtained by means of REMBRANDT system (model I) have been replaced by the weights obtained by applying the Analytic Hierarchy Process (model II) and the weights obtained with the help of the revised Simos' procedure (model III). In both cases the modification of the vector of weights led to alterations in rankings, which indicates that the original solutions are not robust with respect to the variations of parameters. The values of parameters are determined on the basis of data provided by the participants of the decision-making process, thus the information and its skilful use is extremely important in the process of evaluation and selection of the European projects.

Table 9

Positions obtained by projects as a result of the application of ORESTE, EVAMIX and EXPROM II methods with different preference models

Projects	Variants of the preference model								
	I (with the weights obtained by means of the REMBRANDT system)			II (with the weights obtained by means of the AHP method)			III (with the weights obtained by means of the revised Simos' method)		
	ORESTE	EVAMIX	EXPROM	ORESTE	EVAMIX	EXPROM	ORESTE	EVAMIX	EXPROM
A	16	16	15	16	16	15	16	16	15
B	13	14	11	13	14	12	12	14	9
C	1	1	2	1	1	2	1	1	1
D	3	2	3	3	2	3	2	2	4
E	10	9	12	10,5	9	11	11	11	12
F	15	13	14	14	13	13	15	13	16
G	6	3	6	6	3	5	6	3	6
H	8	8	8	8	8	9	8	7	7
K	9	11	5	9	11	6	9	10	3
L	11	10	16	10,5	10	16	10	9	14
M	4	4	4	4	4	4	4	4	5
N	12	12	9	12	12	10	13	12	11
P	2	5	1	2	6	1	3	8	2
R	5	7	7	5	7	8	5	6	8
S	14	15	13	15	15	14	14	15	13
T	7	6	10	7	5	7	7	5	10

Table 9 contains ranks attributed to the 16 analysed European projects as a result of the utilisation of three different MCDA methods with three different preference models. It should be noted that rankings obtained during the analysis are similar. This observation can be confirmed by the Spearman rank correlation coefficients presented in Table 10. These coefficients, calculated separately for each of three considered MCDA techniques, indicate the existence of strong correlation dependencies between the obtained orderings of projects. However, the order of the projects in the rankings is not the same, and – depending on the method of determining the criteria weights and the available allocation of financial resources – different projects would be co-financed.

Table 10

Spearman rank correlation coefficients

ORESTE			
Method	REMBRANDT	AHP	Simos’
REMBRANDT	1,000	0,996	0,991
AHP	0,996	1,000	0,990
Simos’	0,991	0,990	1,000
EVAMIX			
Method	REMBRANDT	AHP	Simos’
REMBRANDT	1,000	0,997	0,974
AHP	0,997	1,000	0,982
Simos’	0,974	0,982	1,000
EXPROM			
Method	REMBRANDT	AHP	Simos’
REMBRANDT	1,000	0,974	0,962
AHP	0,974	1,000	0,924
Simos’	0,962	0,924	1,000

Conclusions

The results of the case study as well as the sensitivity and robustness analysis undertaken in the framework of it have clearly illustrated that the output of MCDA methods depends significantly on the data input. Therefore, for the proper choice of projects that are going to be co-financed it is extremely important to determine the values of the parameters of the preference model consciously and precisely. It is an essential condition for the effective and efficient utilisation of the European Union funds.

A key decision parameter in the models used in the paper, on which the preference structure is based, is the vector of criteria weights. Conducted research has shown that the solutions obtained with the help of different multi--criteria decision aiding techniques are not robust to the modifications of this parameter – it turned out that changes in weighting coefficients affect the rankings of examined projects. Thus, on the one hand identification of the most critical (most sensitive to the variations of the values) criteria weights is extremely beneficial, and on the other – the assignment of importance weightings to each criterion is a crucial step within the methods considered. As there are many different techniques of criterion weighting, the choice of one of them may be directed by the simplicity of its application, explanation and interpretation.

Appendix A

APPLICATION OF THE EXPROM METHOD WITH STOCHASTIC DOMINANCE RULES TO THE EUROPEAN PROJECTS' SELECTION

EXPROM is a modification and extension of PROMETHEE method[11] that was proposed in Diakoulaki and Koumoutsos [1991]. It is based on the notion of ideal and anti-ideal solutions and enables the decision-maker to rank variants on a cardinal scale. Assuming that all criteria are to be maximized, the ideal and anti-ideal solutions' values are defined as follows:

– ideal variant: $f_k(a^*) = \max\limits_{a_i \in A} f_k(a_i)$,

– anti-ideal variant: $f_k(a_*) = \min\limits_{a_i \in A} f_k(a_i)$[12],

where $A = \{a_1, a_2, ..., a_m\}$ is finite set of m variants and $F = \{f_1, f_2, ..., f_n\}$ is set of n criteria examined.

After introducing stochastic dominance rules to EXPROM method the procedure of ordering projects consists of the following steps[13]:

1. Identifying stochastic dominances for all pairs of projects with respect to all criteria[14]. Because all criteria are measured on ordinal scale the ordinal stochastic dominance approach proposed in Spector et al. [1996] is applied:

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[11] The idea of PROMETHEE methodology is presented in Brans and Vincke [1985] and a description of PROMETHEE techniques can be found in Brans et al. [1986].

[12] The values can be also defined independently from the examined variants, representing – in the case of an ideal solution – some realistic goals and in the case of an anti-ideal solution – a situation that should be avoided.

[13] The PROMETHEE method with stochastic dominance rules was proposed by Nowak. A detailed description of this method is presented in Nowak [2005].

[14] According to the results of experiments presented in Kahneman and Tversky [1979] it is assumed that the decision-maker(s) is (are) risk-averse.

Definition 1: Ordinal First-Degree Stochastic Dominance (OFSD):

$$X_k^i \text{ OFSD } X_k^j \text{ if and only if } \sum_{l=1}^{s} p_{kl}^i \le \sum_{l=1}^{s} p_{kl}^j \text{ for all s = 1,..., z,}$$

where:

X_k^i – distribution of the evaluations of project a_i with respect to criterion f_k,

p_{kl} – probability of obtaining given evaluation by the project in case of criterion f_k.

Definition 2: Ordinal Second-Degree Stochastic Dominance (OSSD):

$$X_k^i \text{ OSSD } X_k^j \text{ if and only if } \sum_{r=1}^{s} \sum_{l=1}^{r} p_{kl}^i \le \sum_{r=1}^{s} \sum_{l=1}^{r} p_{kl}^j \text{ for all s = 1,..., z.}$$

For modelling preferences the ordinal almost stochastic dominances are also used[15]:

Definition 3: Ordinal Almost First-Degree Stochastic Dominance (OAFSD):

$$X_k^i \ \varepsilon_1^* - OAFSD \ X_k^j \text{, if for } 0 < \varepsilon_1^* < 0{,}5$$

$$\sum \left(\sum_{l=1}^{s_1} p_{kl}^i - \sum_{l=1}^{s_1} p_{kl}^j \right) \le \varepsilon_1^* \left\| X_k^i - X_k^j \right\| \text{ for all } s_1 = 1,..., z,$$

where: $s_1 = \left\{ s : \sum_{l=1}^{s} p_{kl}^j < \sum_{l=1}^{s} p_{kl}^i \right\}$, $\left\| X_k^i - X_k^j \right\| = \sum \left(\left| \sum_{l=1}^{s} p_{kl}^i - \sum_{l=1}^{s} p_{kl}^j \right| \right)$,

ε_1^* – allowed degree of OFSD rule violation, which reflects the decision-maker's preferences; $\varepsilon_1^* \ge \varepsilon_1$, where ε_1 – the actual degree of OFSD rule violation.

[15] Almost stochastic dominances were proposed in Leshno and Levy [2002].

Definition 4: Ordinal Almost Second-Degree Stochastic Dominance (OASSD):

$$X_k^i \;\; \varepsilon_2^* - OASSD \;\; X_k^j \text{, if for } 0 < \varepsilon_2^* < 0{,}5$$

$$\sum\left(\sum_{l=1}^{s_2} p_{kl}^i - \sum_{l=1}^{s_2} p_{kl}^j\right) \le \varepsilon_2^* \left\| X_k^i - X_k^j \right\| \quad \text{for all } s_2 = 1,\ldots,z \quad \text{and } \mu_k^i \ge \mu_k^j \,,$$

$$\text{where } s_2 = \left\{ s_1 : \sum_{r=1}^{s_1}\sum_{l=1}^{r} p_{kl}^j < \sum_{r=1}^{s_1}\sum_{l=1}^{r} p_{kl}^i \right\},$$

$$\left\| X_k^i - X_k^j \right\| = \sum\left(\left|\sum_{l=1}^{s} p_{kl}^i - \sum_{l=1}^{s} p_{kl}^j\right|\right),$$

μ_k^i and μ_k^j – average performances (expected values of the evaluations' distributions) of the projects a_i and a_j on the criterion f_k,

ε_2^* – allowed degree of OSSD rule violation, which reflects the decision--maker's preferences; $\varepsilon_2^* \ge \varepsilon_2$, where ε_2 – the actual degree of OSSD rule violation.

2. Calculation of concordance indices for each pair of projects (a_i, a_j):

$$c(a_i, a_j) = \sum_{k=1}^{n} w_k \varphi_k(a_i, a_j)$$

where:

$$\sum_{k=1}^{n} w_k = 1$$

$$\varphi_k(a_i, a_j) =$$

$$= \begin{cases} 1 \quad \text{if} \quad X_k^i SD\, X_k^j \quad \text{and} \quad \mu_k^i > \mu_k^j + p_k[\mu_k^i], \\ \dfrac{\mu_k^i - q_k[\mu_k^i] - \mu_k^j}{p_k[\mu_k^i] - q_k[\mu_k^i]} \quad \text{if} \quad X_k^i SD\, X_k^j \quad \text{and} \quad \mu_k^j + q_k[\mu_k^i] < \mu_k^i \le \mu_k^j + p_k[\mu_k^i], \\ 0 \quad \text{otherwise}, \end{cases}$$

w_k – coefficient of importance for criterion f_k,

$q_k[\mu_k^i]$, $p_k[\mu_k^i]$ – indifference and preference threshold for criterion f_k respectively.

3. Calculation of discordance indices for each pair of projects and for each criterion:

$$d_k(a_i,a_j)=$$
$$=\begin{cases} 1 \quad \textit{if} \;\; X_k^j SD X_k^i \quad \text{and} \quad \mu_k^j > \mu_k^i + v_k[\mu_k^i], \\ \dfrac{\mu_k^j - p_k[\mu_k^i] - \mu_k^i}{v_k[\mu_k^i] - p_k[\mu_k^i]} \quad \text{if} \;\; X_k^j SD X_k^i \quad \text{and} \quad \mu_k^i + p_k[\mu_k^i] < \mu_k^j \le \mu_k^i + v_k[\mu_k^i], \\ 0 \quad \text{otherwise,} \end{cases}$$

where $v_k[\mu_k^i]$ – veto threshold for criterion f_k.

4. Calculation of credibility indices for each pair of projects (a_i,a_j):

$$\sigma(a_i,a_j)=c(a_i,a_j)\prod_{k\in D(a_i,a_j)}\frac{1-d_k(a_i,a_j)}{1-c(a_i,a_j)}$$

where: $D(a_i,a_j)=\{k: d_k(a_i,a_j)>c(a_i,a_j)\}$.

5. Determination of strict preference indices for each pair of projects (a_i,a_j):

$$\pi(a_i,a_j)=\nu(a_i,a_j)\cdot\sum_{k=1}^{n} w_k\pi_k(a_i,a_j),$$

where:

$$\nu(a_i,a_j)=\begin{cases} 1, & \text{if} \quad \forall k: d_k(a_i,a_j)\le c(a_i,a_j), \\ 0, & \text{if} \quad \exists k: d_k(a_i,a_j)> c(a_i,a_j), \end{cases}$$

$$\pi_k(a_i,a_j)=\begin{cases} \dfrac{(\mu_k^i-\mu_k^j)-p_k[\mu_k^i]}{(\mu_k^*-\mu_{k*})-p_k[\mu_k^i]} & \text{if} \quad \varphi_k(a_i,a_j)=1, \\ 0 \quad \text{otherwise,} \end{cases}$$

$$\mu_k^* = \max_{a_i\in A}\mu_k^i \quad \text{and} \quad \mu_{k*} = \min_{a_i\in A}\mu_k^i .$$

The aim of the strict preference function $\pi_k(a_i, a_j)$ is to differentiate the state of the strict preference found to be valid for more than one pair of projects at a given criterion f_k. Their values belong to the interval $[0, 1]$ and $\pi_k(a_i, a_j) = 0$ denotes weak preference or indifference between two projects.

6. Calculation of total preference index for each pair of projects (a_i, a_j):

$$\omega(a_i, a_j) = \min\left\{1;\ \sigma(a_i, a_j) + \pi(a_i, a_j)\right\}.$$

The total preference index gives an accurate measure of the intensity of preference of project a_i over a_j for all the criteria. It combines two aspects: a subjective one, expressed by the credibility index and referring only to the relation between two examined projects, and an objective one, expressed by the strict preference index and representing the relation between two considered projects with regard to other projects examined.

7. Calculation of outgoing flow $\phi^+(a_i)$ and incoming flow $\phi^-(a_i)$ for each project:

$$\phi^+(a_i) = \frac{1}{m-1}\sum_{j=1}^{m}\omega(a_i, a_j)$$

$$\phi^-(a_i) = \frac{1}{m-1}\sum_{j=1}^{m}\omega(a_j, a_i)$$

In EXPROM I a final partial ranking is obtained as follows:

$$\begin{cases} a_i P a_j, & \text{if } \begin{cases} \phi^+(a_i) > \phi^+(a_j) & \text{and} \quad \phi^-(a_i) < \phi^-(a_j) & \text{or} \\ \phi^+(a_i) = \phi^+(a_j) & \text{and} \quad \phi^-(a_i) < \phi^-(a_j) & \text{or} \\ \phi^+(a_i) > \phi^+(a_j) & \text{and} \quad \phi^-(a_i) = \phi^-(a_j); \end{cases} \\ a_i I a_j, & \text{if } \quad \phi^+(a_i) = \phi^+(a_j) \quad \text{and} \quad \phi^-(a_i) = \phi^-(a_j); \\ a_i R a_j, & \text{if } \begin{cases} \phi^+(a_i) > \phi^+(a_j) & \text{and} \quad \phi^-(a_i) > \phi^-(a_j) & \text{or} \\ \phi^+(a_i) < \phi^+(a_j) & \text{and} \quad \phi^-(a_i) < \phi^-(a_j); \end{cases} \end{cases}$$

where P, I and R stand for preference, indifference and incomparability, respectively.

In EXPROM II a final complete ranking is constructed according to the descending order of the net flows $\phi(a_i)$, where $\phi(a_i) = \phi^+(a_i) - \phi^-(a_i)$.

Appendix B

APPLICATION OF THE EVAMIX METHOD WITH STOCHASTIC DOMINANCE RULES TO THE EUROPEAN PROJECTS' SELECTION

In EVAMIX method, proposed by H. Voogd, the qualitative and quantitative data are distinguished and the final appraisal score of a given variant is the result of a combination of the evaluations calculated separately for the qualitative and quantitative criteria.

After introducing stochastic dominance rules to EVAMIX method the procedure of ordering projects consists of the following steps:

1. Determination of the qualitative dominance measures for the ordinal criteria:

$$\alpha_{ij} = \left[\sum_{k \in O} \left\{ w_k \operatorname{sgn}\left(\mu_k(a_i) - \mu_k(a_j) \right) \right\}^c \right]^{\frac{1}{c}}, \quad c = 1, 3, 5...,$$

where:

c – an arbitrary scaling parameter, for which any positive odd value may be chosen; the higher the value of the parameter, the weaker the influence of the deviations between the evaluations for the less important criteria;

O – a set of qualitative (ordinal) criteria[16];

$$\operatorname{sgn}\left(\mu_k(a_i) - \mu_k(a_j) \right) = \begin{cases} 1 & \text{if } X_k^i \; SD \; X_k^j \text{ and } \mu_k(a_i) > \mu_k(a_j), \\ -1 & \text{if } X_k^j \; SD \; X_k^i \text{ and } \mu_k(a_j) > \mu_k(a_i), \\ 0 & \text{otherwise,} \end{cases}$$

[16] It is assumed that all the criteria are maximized.

X_k^i – distribution of the evaluations of project a_i with respect to criterion f_k,

$\mu_k(a_i)$ – average performance (expected value of the distribution of evaluations) of the project a_i on the criterion f_k,

and SD denotes stochastic dominance relation (OFSD, OSSD, OAFSD, OASSD).

2. Calculation of the quantitative dominance measures for the cardinal criteria:

$$\gamma_{ij} = \left[\sum_{k \in Q} \left\{ w_k \left(\mu_k(a_i) - \mu_k(a_j) \right) \right\}^c \right]^{\frac{1}{c}}, \quad c = 1, 3, 5...,$$

if $(F_{ik}\ SD\ F_{jk}$ and $\mu_k(a_i) > \mu_k(a_j))$

or $(F_{jk}\ SD\ F_{ik}$ and $\mu_k(a_j) > \mu_k(a_i))$; otherwise $\gamma_{ij} = 0$,

where:

Q – a set of quantitative (cardinal) criteria[17],

F_{ik} – distribution function representing evaluations of project a_i with respect to criterion f_k.

3. Standardization of the dominance measures as follows:

$$\delta_{ij} = \alpha_{ij} \left(\sum_{i=1}^{m} \sum_{j=1}^{m} \left| \alpha_{ij} \right| \right)^{-1},$$

$$\sigma_{ij} = \gamma_{ij} \left(\sum_{i=1}^{m} \sum_{j=1}^{m} \left| \gamma_{ij} \right| \right)^{-1}.$$

4. Calculation of the overall dominance measure q_{ij} for each pair of projects:

$$q_{ij} = w_O \delta_{ij} + w_Q \sigma_{ij},$$

where:

w_O – the sum of weights of quantitative criteria,
w_Q – the sum of weights of qualitative criteria.

[17] It is assumed that all the criteria are maximized.

5. Determination of the final appraisal score u_i for each project as follows:

$$u_i = \frac{1}{m}\sum_{j=1}^{m} q_{ij} \ .$$

6. Ranking projects according to the descending order of the final appraisal scores.

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Josef Jablonsky

MULTICRITERIA ANALYSIS OF CLASSIFICATION IN ATHLETIC DECATHLON

Abstract

Man's decathlon is an athletic contest which consists of 10 events, four of them measured in seconds and the remaining six in meters. Each athlete (alternative) is described by his 10 results (criteria). Current system of classification is based on aggregation of results using utility (scoring) functions which are defined exactly for each event. This system has been used since 1984 and the aim of the paper is to analyse it with respect to current conditions. The current method of classification is compared with several alternative methods which better reflect current top results of 10 events included in men decathlon. Proposed methods are applied to real results, specifically to the Olympic Games 2008.

Keywords

Multiple criteria decision making, decathlon, utility function.

Introduction

Decathlon is an athletic contest with a very long tradition which consists of 10 different events. Since ancient Greece it has been regarded as a measure of universality of athletes and is often called „king athletic event". For the first time a kind of athletic multicontest was scheduled in the 3rd Summer Olympic Games in Saint Louis in 1904. In 1914 the decathlon was established by the International Amateur Athletic Federation in the form in which it is known now. Since that year the decathlon consists of 10 events (3 sprints, 1 long distance run, 3 jump events and 3 throw events) and the order of all events within a contest is fixed. The contest is always scheduled in two consecutive days as follows:

1st day:

– 100 meters run,
– Long jump,
– Shot put,

– High jump,
– 400 meters run,

2^{nd} day:

– 110 meters hurdles run,
– Discus throw,
– Pole vault,
– Javelin throw,
– 1500 meters run.

Principles of decathlon, its history and the current system of scoring are described in detail in an official IAAF paper [Diack 2004].

In general, ranking of contestants in decathlon is a multiple criteria decision problem (MCDM) with athletes attending the contest as alternatives and 10 criteria. The aim of the problem is to evaluate the alternatives (athletes) and rank them from the best to the worst. As it is clear, criterion values in this problem are not comparable – they are given either in seconds or in centimeters and moreover the values in seconds (centimeters) are incomparable directly. That is why the results of athletes in individual events are aggregated using so-called scoring tables which assign points to the performances in individual events. The athletes are ranked according to the total points achieved.

1. Current system of performance evaluation in decathlon

The scoring tables went over time through several evolution steps – [for details see: Diack 2004]. Their last change is dated 1984 and the tables constructed according to the following principles still stand today:

1. The scoring tables should differ from those used for individual event scoring.
2. The scores for different events should be comparable, so that equal skill levels in different events are rewarded with equal point levels.
3. The scoring tables should be one of the following:
   – linear in all events, or
   – slightly progressive in all events.
4. The tables should be applicable to all levels of performance, from youth to professional.
5. Men and women should have different tables.
6. Specialists' performances should be the basis for the scores in the tables.
7. The tables should be applicable in the future.

8. The total scores using the new tables for top world-class athletes should remain approximately the same.
9. As much as possible, the new tables should ensure that a specialist in one event cannot be better than top performances in the other events.

The current scoring tables were designed according to the above-mentioned principles by Czech mathematician dr. Trkal. They assign points based on performances of athletes using the following formulas:

$$u_{ik} = \mathrm{INT}[a_k(b_k - p_{ik})^{c_k}],\ k \in \mathrm{R},\ i \in \mathrm{A}, \tag{1}$$

$$u_{ik} = \mathrm{INT}[a_k(p_{ik} - b_k)^{c_k}],\ k \in \mathrm{T},\ i \in \mathrm{A}, \tag{2}$$

where

R is the index set of running events,
T is the index set of throw (jump) events,
A is the index set of athletes attending the contest,
u_{ik} is the number of points achieved by i-th athlete in the k-th event,
p_{ik} is the performance of the i-th athlete in the k-th event measured in seconds for running events and in centimeters for throw and jump events,
INT(x) is the integer part of x, and
a_k, b_k, c_k are specific parameters for the k-th event. The fixed values of parameters a, b and c are presented in Table 1 – [source: Diack 2004].

Table 1

Parameters of scoring functions

Event	a_k	b_k	c_k
100 meters	25.4347	18.00	1.81
Long jump	0.14354	220.00	1.40
Shot put	51.39	1.50	1.05
High jump	0.8465	75.00	1.42
400 meters	1.53775	82.00	1.81
110 m hurdles	5.74352	28.50	1.92
Discus throw	12.91	4.00	1.10
Pole vault	0.2797	100.00	1.35
Javelin throw	10.14	7.00	1.08
1500 meters	0.03768	480.00	1.85

It is not clear how the parameters for scoring functions are derived. Explanation of b_k parameters is not difficult – it is the performance rewarded by zero points. c_k parameters express the degree of progressiveness of the scoring function. The most progressive functions are those for all running events. On the contrary the functions for throw events are almost linear. Explanation of a_k parameters is not clear at all.

The total number of points for each athlete U_i is given as a simple sum of points rewarded in all events, i.e.

$$U_i = \sum_{k=1}^{10} u_{ik}, \qquad i = 1,2,...,n.$$

The presented approach has been applied since 1985 without any changes. Not every system is ideal. This one has several questionable features too. The most important one is that the number of points awarded in the events is not uniform – some of the events are rewarded by a higher number of points than the other ones. The differences are clearly shown in Table 2. This table presents current world records (WR) in all events and their appropriate numbers of points given by (1) and (2), as well as the average number of points in particular events of TOP100 historic performances in men decathlon – [source: Diack 2004; Westera 2006] and the author's own calculations.

Table 2

Differences in point rewards

Event	*WR*	u_{wr}	u_{100}
100 meters	9.58	1202	917
Long jump	895	1312	970
Shot put	23.12	1295	815
High jump	245	1244	859
400 meters	43.18	1156	899
110 m hurdles	12.87	1126	946
Discus throw	74.08	1383	808
Pole vault	614	1277	901
Javelin throw	98.48	1331	810
1500 meters	206	1218	711

2. Alternative definitions of scoring functions

Decathlon is a multiple criteria decision making problem with the aim to rank all alternatives (athletes). There are many methods based on different principles that can be used for multicriteria evaluation of alternatives. AHP/ANP, PROMETHEE class methods, ELECTRE class methods and aggregation using utility functions are the most often applied ones but for decathlon ranking only the last-mentioned approaches are applicable. That is why we suggest a modification of the current scoring functions in order to take into account differences in awarding points in decathlon events. Similar re-definitions of scoring functions were discussed by several researchers, e.g. [Cox and Dunn 2002; Cheng et al. 2003; Westera 2006; Taborski 2008]. The following four modifications of scoring functions (M1 – M4) are further discussed:

M1 – The scoring functions (1) and (2) remain unchanged but the parameter a_k is modified for all events to unify the number of points for world records on the level 1250 points (the sum of u_{WR} in Table 2 is approx. 1250). This model preserves the rate of progression of the utility function (parameter c_k) and the bounds (parameter b_k) that correspond to zero points performances.

M2 – Similar to the previous case – except that the parameter a_k is modified to approximate the point rewards of average TOP100 performances to the same level. The average value of the last column in Table 2 is approx. 864 and, that is why for the first event the parameter a_k is reduced by approx. 5.8% and for the last event it is increased by approx. 21.5%.

M3 – Linear utility functions with lower bound (zero points) on the same level as in formulas (1) and (2), i.e. parameter b_k, and upper bound (1250 points) for the current world record (it is not expected that an athlete can beat current world record in any event).

M4 – Linear utility function with the lower bound as in the previous case. The number of points is given by the following formula (for events measured in seconds):

$$u_{ik} = \frac{b_k - p_{ik}}{b_k - q_{ik}} 864 \,,$$

where q_{ik} is the performance rewarded by TOP100 average points (Table 2).

Table 3 presents original value of a_k parameter and its modifications in models M1 and M2 given by own calculations.

Table 3

Parameters for alternative scoring functions

Event	a_k	M1 – a_k	M2 – a_k
100 meters	25.4347	26,4500	23,9646
Long jump	0.14354	0,1368	0,1279
Shot put	51.39	49,6100	54,4797
High jump	0.8465	0,8511	0,8514
400 meters	1.53775	1,6630	1,4779
110 m hurdles	5.74352	6,3800	5,2457
Discus throw	12.91	11,6700	13,8048
Pole vault	0.2797	0,2737	0,2682
Javelin throw	10.14	9,5250	10,8160
1500 meters	0.03768	0,0387	0,0458

3. Re-calculation of Olympic Games 2008 results

The alternative definitions of scoring functions presented in previous section are applied to the data set of decathlon results in Olympic Games 2008 in Beijing. The criterion matrix, i.e. the performances of first 15 athletes in descending ranking, is presented in Table 4 (source: official web pages of IAAF – www.iaaf.org). All running performances are given in seconds, long and high jump and pole vault in centimetres and remaining two events in meters (discus and javelin).

Table 4

Performances of first 15 athletes in Olympic Games 2008

	100 m	Long	Shot	High	400 m	110 m	Disc	Pole	Javel	1500 m
1	2	3	4	5	6	7	8	9	10	11
1	10.44	778	16.27	199	48.92	13.93	53.79	500	70.97	306.59
2	10.96	761	14.39	211	47.30	14.21	44.58	500	60.23	267.47
3	10.90	733	14.49	205	47.91	14.15	44.45	470	73.98	269.17
4	11.07	737	16.53	208	50.91	14.47	50.04	500	64.01	301.56
5	11.26	708	15.42	196	49.51	14.21	45.17	500	65.40	269.29
6	11.21	768	14.78	211	49.54	14.71	45.50	480	63.93	289.63

Table 4 contd.

1	2	3	4	5	6	7	8	9	10	11
7	10.53	756	15.15	196	47.70	14.37	48.39	430	51.59	268.94
8	11.12	729	13.23	205	49.65	14.37	45.39	520	60.21	272.90
9	10.85	704	15.09	199	47.96	14.08	50.91	460	51.52	271.62
10	10.80	770	13.67	199	48.47	14.71	40.41	480	60.27	266.77
11	11.15	704	14.36	211	50.90	14.51	49.35	480	67.07	287.03
12	11.02	723	16.26	202	51.56	15.51	47.43	510	62.57	281.34
13	10.89	729	14.79	196	48.98	14.06	39.83	480	67.16	289.60
14	11.19	719	13.78	199	49.99	14.73	44.09	470	71.44	277.96
15	10.64	707	15.82	196	49.66	13.90	36.73	470	65.60	300.49

Table 5 compares the total number of points and ranking of all athletes derived in standard way – formulas (1) and (2) – which is denoted as M0, with four alternative definitions M1 – M4 presented in the previous section.

Table 5

Comparison of original and alternative approaches

	M0	R	M1	R	M2	R	M3	R	M4	R	PII
1	8791	1	8787	1	8762	1	9674	1	8803	1	1
2	8551	2	8584	2	8556	2	9441	3	8520	3	2
3	8527	3	8551	3	8555	3	9450	2	8569	2	3
4	8328	4	8319	4	8324	4	9338	4	8497	4	4
5	8253	5	8273	5	8288	5	9248	5	8392	5	6
6	8241	6	8247	7	8235	7	9245	6	8376	7	11
7	8238	7	8272	6	8246	6	9176	10	8282	11	8
8	8220	8	8242	8	8227	8	9197	9	8306	9	5
9	8205	9	8241	9	8227	8	9171	11	8292	10	12
10	8194	10	8227	10	8197	11	9153	12	8247	13	9
11	8178	11	8180	11	8200	10	9206	8	8369	8	7
12	8154	12	8139	13	8184	12	9207	7	8377	6	10
13	8118	13	8151	12	8109	13	9124	13	8243	14	14
14	8055	14	8065	14	8082	14	9100	14	8258	12	13
15	7992	15	8035	15	7971	15	9017	15	8141	15	15

The results Presented are completed by ranking given by one of the most often used MCDM methods for evaluation of alternatives which is PROMETHEE II method even though it is clear that this kind of methods is not suitable for evaluation of decathlon athletes. The main reason is that this method compares each pair of alternatives with respect to all criteria.

That is why the final ranking depends on mutual relations of pairs of alternatives and rank reversal is not eliminated. This is unacceptable for decathlon purposes and the only method which can be used in this context is an application of utility functions. For all criteria (events) a linear preference function with a sufficiently high preference threshold was used in application of PROMETHEE method. Principles of PROMETHEE class methods are generally known. They were proposed by Brans and Vincke [1985]. Their basic description as well as information about original software support for MCDM problems including PROMETHEE class methods can be found in [Jablonsky 2007].

Table 6

Average point rewards given by models

Event	M0	M1	M2
100 meters	876	911	825
Long jump	899	856	800
Shot put	786	759	833
High jump	824	829	829
400 meters	850	919	817
110 m hurdles	925	1027	845
Discus throw	782	707	837
Pole vault	860	842	825
Javelin throw	794	746	847
1500 meters	674	691	819
MIN	674	691	800
MAX	925	1027	847

The comparison of results shows a close similarity of rankings given by original formulas and by models M1 and M2, i.e. by models based on utility functions with the same progression level as the original ones. Only a few rank reversals appear. On the other hand the linear utility functions (models M3 and M4) generate a quite different results as compared to the original ranking.

Table 6 shows differences in average point rewards for all events and for original model M0 and models M1 and M2. It is clear that the models M0 and M1 on the one hand and model M2 on the other hand are significantly different. Minimum and maximum average rewards differ by around 300 points in the first case and only by fewer than 50 points in the model M2. A similar relations holds for models M3 and M4 not presented in Table 6.

Conclusions

Analysis of decathlon results is a very interesting multiple criteria decision making problem of high importance. The paper presents current system of evaluation based on aggregation of performances in events into final point score and proposes several new definitions of scoring functions. The most promising definition has been introduced as model M2. It solves the problem of high differences in point rewards and preserves the current level of progression in individual events. Moreover, results of the model M2 keep the current final level of points and seem to be comparable to the current standard system.

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REAL AND VIRTUAL PARETO SET UPPER APPROXIMATIONS

Abstract

This paper deals with the problem of the derivation of lower and upper approximations of an efficient element set.

We consider the case where upper approximations cannot be derived as criteria mapping images of infeasible variants.

Keywords

Multiobjective optimization, evolutionary algorithms, lower Pareto set approximations, upper Pareto set approximations.

Introduction

Under assumption that all criteria are of the type "the more the better" each *outcome* (i.e. the image of an admissible variant under the criteria mapping) lies "below" the Pareto set (the set of efficient outcomes) or is an element of this set. A number of such outcomes form a *lower approximation* of the Pareto set.

By analogy, we consider an *upper approximation* of the Pareto set, i.e. a set of elements of the outcome (criteria) space which lie "above" the Pareto set.

Having pairs of lower and upper approximations is of interest for two reasons. First, provided that elements of a lower and upper approximation are uniformly distributed along the Pareto set, we are in position to assess the maximal error one makes when representing an efficient outcome y by an outcome y' taken from the lower approximation and dominated by y [Kaliszewski 2008; Miroforidis 2008, 2010; Kaliszewski et al. 2011, 2012]. That is important in cases where deriving elements of the Pareto set is computationally costly and working with lower approximations of the Pareto set instead of the Pareto set itself is a rational option. Second, given a pair of lower and upper approximations lower and upper bounds on components of any efficient outcome pointed to by the *Decision Maker* preferences can be easily calculated. How the Decision Maker preferences point to efficient outcomes will be explained below.

In this paper we discuss problems arising when deriving upper approximations and we illustrate our considerations by an illustrative example.

In our earlier papers we have proposed to conduct interactive multiple criteria decision processes with outcome assessments instead of outcomes themselves [Miroforidis 2008, 2010; Kaliszewski, Miroforidis 2010a, b; Kaliszewski et al. 2011]. By an outcome assessment we mean lower and upper bounds on values of outcome components. Such an approach stemmed from a variety of *Multiple Criteria Decision Making* problems where efficient outcomes are given implicitly by a set of constraints and therefore have to be derived by solving optimization problems. To provide for versatility of such an approach we have adapted it to employ *evolutionary calculations* (*Evolutionary Multiobjective Optimization*) driven by Decision Maker preferences revealed in the course of interactive decision processes.

We have founded our approach on two constructs, namely on *lower approximations*, i.e. finite subsets of feasible variants, and *upper approximations*, i.e. finite subsets of infeasible variants with some specific properties. The formal definitions of both constructs are given in the next section.

Of interest are lower and upper approximations which are tight, i.e. their images under the criteria mapping are close, in a sense, to the set of efficient outcomes. Tight approximations provide for tight bounds in outcome assessments mentioned above. Moreover, images of tight lower and upper approximations represent sets of efficient outcomes within measurable accuracy. Except for our earlier papers we are aware of only one paper attempting to exploit a similar concept, namely Legriel et al. [2010].

Evolutionary Multiobjective Optimization (EMO) methods and algorithms are dedicated to deriving tight lower approximations and that subject is represented by numerous publications, see e.g. Deb [2001], Coello Coello et al. [2002]. In contrast to this, deriving lower and upper approximations is a novel concept.

A lower approximation always exists as long as the set of feasible variants is nonempty. However, the existence of an upper approximation is not guaranteed in general. In this work we consider the case where upper approximations cannot be derived as images of infeasible variants under criteria mappings.

The outline of the paper is as follows. In Section 1 we provide basic definitions and notation. In Section 2 we briefly outline the concept of approximating the set of efficient outcomes with the help of *lower shells* and *upper shells*. In Section 3 we address the case where upper shells do not exist and propose how to deal with that case to have our concept of outcome assessments still workable. Section 4 concludes.

1. Definitions and notation

Let x denote a (decision) variant, X a space of variants, X_0 a set of *feasible variants*, $X_0 \subseteq X$. Here we assume that X and X_0 *are infinite.* Then the underlying model for MCDM is formulated as:

$$\begin{aligned} &\text{“}max\text{”}\, f(x) \\ &x \in X_0 , \end{aligned} \tag{1}$$

where $f: X \to R^k$, $f = (f_1,...,f_k)$, $f_i : X \to R$, $i = 1,...,k$, $k \geq 2$, are criteria functions; ”*max*” denotes the operator of deriving all efficient variants (as defined below) in X_0.

Element $\bar{t}$ of T, $T \subseteq R^k$, is:

– *efficient in* T, if $t_i \geq \bar{t}_i$, $i = 1,...,k$, $t \in T$, implies $t = \bar{t}$,
– *weakly efficient in* T, if there is no $t \in T$, such that $t_i > \bar{t}_i$, $i = 1,...,k$.

Variant $\bar{x} \in X_0$ is called efficient (weakly efficient) in X_0 if $\bar{y} = f(\bar{x})$ is efficient (weakly efficient) in $f(X_0)$. Elements of $f(X_0)$ are called outcomes.

We denote the set of efficient variants of X_0 by N. Elements of $f(N)$ are called efficient outcomes for, by the definition, they are efficient in $f(X_0)$.

We define on X – the *dominance relation* $\prec$,

$$x' \prec x \Leftrightarrow f(x') << f(x),$$

where $<<$ denotes $f_i(x') \leq f_i(x)$, $i = 1,...,k$, and $f_i(x') < f_i(x)$ for at least one i. If $x' \prec x$, then we say that x' is dominated by x and x is dominating x'.

2. Lower and upper shells

In this paper we are concerned with specific lower and upper approximations of the set of efficient outcomes stemming from the concept of lower shell and upper shell [cf. also Kaliszewski 2008; Kaliszewski Miroforidis 2010a, b; Kaliszewski et al. 2010, 2011].

The following definitions of lower and upper shells come from [Miroforidis 2008, 2010].

Lower shell is a finite nonempty set $S_L \subseteq X_0$, elements of which satisfy

$$\forall_{x \in S_L} \neg \exists_{x' \in S_L} x \prec x' . \tag{2}$$

Nadir point y^{nad} is defined as

$$y_i^{nad} = \min_{x \in N} f_i(x), \ i = 1,...,k.$$

Upper shell is a finite nonempty set $S_U \subseteq X \setminus X_0$, elements of which satisfy

$$\forall_{x \in S_U} \neg \exists_{x' \in S_U} x' \prec x, \tag{3}$$

$$\forall_{x \in S_U} \neg \exists_{x' \in N} \ x \prec x', \tag{4}$$

$$\forall_{x \in S_U} \ y^{nad} \leq f(x), \tag{5}$$

where the last inequality means $y_i^{nad} \leq f_i(x), \quad i = 1,...,k$ [1].

To illustrate the concept of lower and upper shells, in Figure 1 we present an example of the images of lower and upper shells under criteria mapping derived for a problem described in Kaliszewski, Miroforidis, [2010b]. The problem is as follows

$$\text{“}max\text{”} \ (f_1(x), f_2(x))$$

where $f_1(x) = -x_1^2 + x_2$, $f_2(x) = \dfrac{1}{2}x_1 + x_2 + 1$,

subject to $x \in X_0$, where X_0 is defined as

---

[1] Since for *N* is not known (if otherwise, there is no need to approximate *N*) this definition is not operational and in Kaliszewski et al. [2010] we have shown how to overcome this by a somewhat weaker constructs than upper shells, with no direct reference to *N*. But if upper shells do not exist those we weaker constructs exist neither.

$$\frac{1}{6}x_1 + x_2 - \frac{13}{2} \le 0,$$
$$\frac{1}{2}x_1 + x_2 - \frac{15}{2} \le 0,$$
$$\frac{5}{x_1} + x_2 - 30 \le 0,$$
$$0 \le x_i \le 7,\ i = 1, 2.$$

3. The case of nonexistence of S_U

The existence of upper shells is not in general guaranteed. A collection of problems, selected from the EMO literature, for which upper shells do not exist, has been identified in Kaliszewski, Miroforidis [2010a].

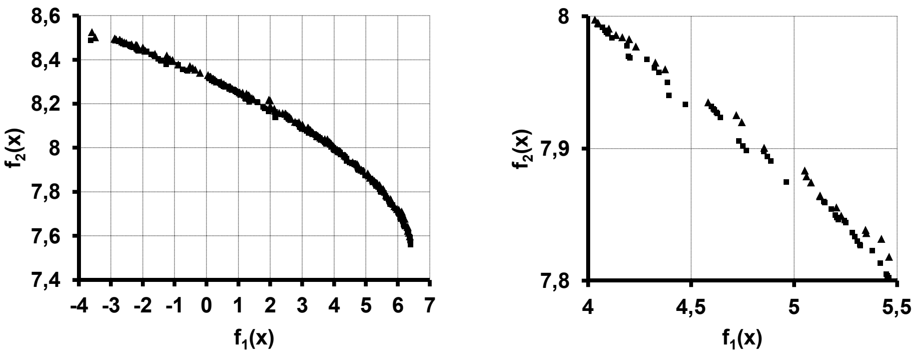

Figure 1. The images of elements of a lower shell (squares) and an upper shell (triangles) under the criteria mapping for the example problem of Section 3: left – full view, right – window $4 \le f_1(x) \le 5.5$, $7.8 \le f_2(x) \le 8.0$

The nonexistence of upper shells means that there is no $x \in X,\ x \notin X_0$, such that $f(x') << f(x)$ for some $x' \in X_0$. However, it does not mean that there does not exists $y \in R^k$, such that $f(x') << y$ for some $x' \in X_0$. In formulas for bounds on outcome components elements x of S_U appear only indirectly, via elements $f(x),\ x \in S_u$. Therefore we can replace elements $f(x),\ x \in S_u$, with elements $y \in R^k$ having the same property as $f(x),\ x \in S_u$, regardless existence of x such that $y = f(x)$.

To implement this concept we are to define an appropriate counterpart of the notion upper shell. We shall call such a construct a *virtual upper shell*.

Virtual upper shell is a finite nonempty set $VS_U \subseteq R^k \setminus f(X_0)$, elements of which satisfy

$$\forall_{y \in VS_U} \neg \exists_{y' \in VS_U} \quad y' << y, \tag{7}$$

$$\forall_{y \in VS_U} \neg \exists_{x \in N} \quad y << f(x), \tag{8}$$

$$\forall_{y \in VS_U} \quad y^{nad} \leq y. \tag{9}$$

In the algorithm presented below we operationalize the condition (8) replacing it by

$$\forall_{y \in VS_U} \neg \exists_{x \in S_L} \quad y << f(x). \tag{8'}$$

The following EMO-type algorithm derives virtual upper shells. It builds directly on the logic of algorithm PDAE/M proposed in Miroforidis [2010], cf. also Kaliszewski et al. [2011, 2012].

To limit the domain of searching through the set $R^k \setminus f(X_0)$, we assume existence of bounded set (box)

$$R^k_{DEC} = \{y \in R^k \mid Y_i^L \leq y \leq Y_i^U,\ i = 1,...,k\},$$

such that $f(X_0) \subseteq \mathrm{int}(R^k_{DEC})$.

Algorithm PDAE/M_VS$_U$

1. 1 $j := 0$, $S_L^j := \emptyset$, $VS_U^j := \emptyset$.
2. Generate randomly $\eta \geq 1$ elements of X_0 and derive from those elements S_L^0.
3. $S := S_L^j$; for each element $\bar{x}$ of S perform Steps 4-6.
4. Select element $x' \in X_0$ such that $\neg x' \prec \bar{x}$. Select element $y' \in R^k_{DEC}$ such that $f(\bar{x}) << y'$.

5. If $\neg\exists x \in S_L^j : x' \prec x$ then

5.1. $S_L^j := S_L^j \cup \{x'\}$,

5.2. $S_L^j := S_L^j \setminus \{x \in S_L^j \mid x \prec x'\}$.

6. If $\neg\exists x \in X_0 : f(x) = y'$ and $\neg\exists y \in VS_U^j : y << y'$ and

$\neg\exists x \in S_L^j : y' << f(x)$ then

6.1. $VS_U^j := VS_U^j \cup \{y'\}$,

6.2. $VS_U^j := VS_U^j \setminus \{y \in VS_U^j \mid y' << y\}$.

7. If $j = j^{\max}$ then STOP, otherwise $j = j + 1$ and go to 3.

Step 1 initializes, whereas in Step 2 an initial lower shell is derived from a number of elements of X_0.

Step 3 specifies that an attempt to modify S_L^j has to be made at each of its elements. It has been found in Miroforids [2010] that such a deterministic strategy, as opposed to random selection of elements to be modified, reduces clustering of elements in S_L^j and thus produces much more uniform lower approximations of N.

The evolutionary multiobjective optimization principle is realized in Step 4 via the mutation operation. In this step element x' is selected in the following process:

4.1. $x' := \bar{x}$.

4.2. $i = rndInt(1, m)$.

4.3. If $i = rndInt(0,1) \le 0.5$ then

$$x_i' := x_i' + (X_i^U - x_i')(1 - rnd(0,1)^{2(1 - \frac{j}{j^{\max}})});$$

otherwise

$$x_i' := x_i' + (x_i' - X_i^L)(1 - rnd(0,1)^{2(1 - \frac{j}{j^{\max}})}).$$

4.4. If $x' \neg\prec \bar{x}$ then go to 5; otherwise go to 4.1.

Function $rndInt(a,b)$ returns an integer number from the range [a,b] with uniform distribution. Function $rnd(a,b)$ returns a random real number from the range [a,b] with uniform distribution. The presented method of mutations and the strategy of decreasing mutation range have been taken form the literature [cf. e.g. Michalewicz 1996].

In Step 5 an attempt is made to modify the current lower shell S_L^j with the newly generated element x'.

Similarly, in Step 6 the same attempt is made with respect to the current virtual upper shell VS_U^j. Here the tricky point is to verify whether for given y' there exists $x \in X_0$ such that $f(x) = y'$. If yes then no amendment of VS_U^j is made[2]. The existence of $x \in X_0$ such that $f(x) = y'$ can be verified by solving the optimization problem $\min \| y' - f(x) \|$ subject to $x \in X_0$ by an evolutionary optimization algorithm.

In Step 7 the stopping rule is checked, where $j^{\max}$ is the limit for the number of iterations of algorithm PDAE/M_VS$_U$.

We illustrate the concept of lower shells and virtual upper shells with Figure 2, where we present the image of a lower shell under the criteria mapping and a virtual upper shell derived for the problem DTLZ1a from Deb et al. [2001], as follows.

$$\text{“}max\text{”}\ (f_1(x, g),\ f_2(x, g))$$

where $f_1 = 0.5x_1(1 + g)$, $f_2 = 0.5(1 - x_1)(1 + g)$, all objective functions are to be minimized, and

$$g = 100\ [5 + \sum_{i=2}^{6}\ (x_i - 0.5)^2 - cos(2\pi(x_i - 0.5))]$$

$$X_0 = \{x \mid x_i \in [0, 1],\ i = 1,..., 6\}.$$

Set N is made up of elements in which $x_2,..., x_6 = 0.5$ and $x_1 \in [0, 1]$. Elements of $f(S_L)$ and VS_U were derived in 60 iterations of algorithm PDAE/M_VS$_U$.

[2] If there exists $x \in X_0$ such that $f(x) = y'$ then x is a suitable element for Step 5 for $f(\bar{x}) << y' = f(x')$ entails $x' \neg\prec \bar{x}$. To exploit this fact the order of Step 5 and Step 6 should be reversed. However, in this paper we do not investigate this variant of the algorithm.

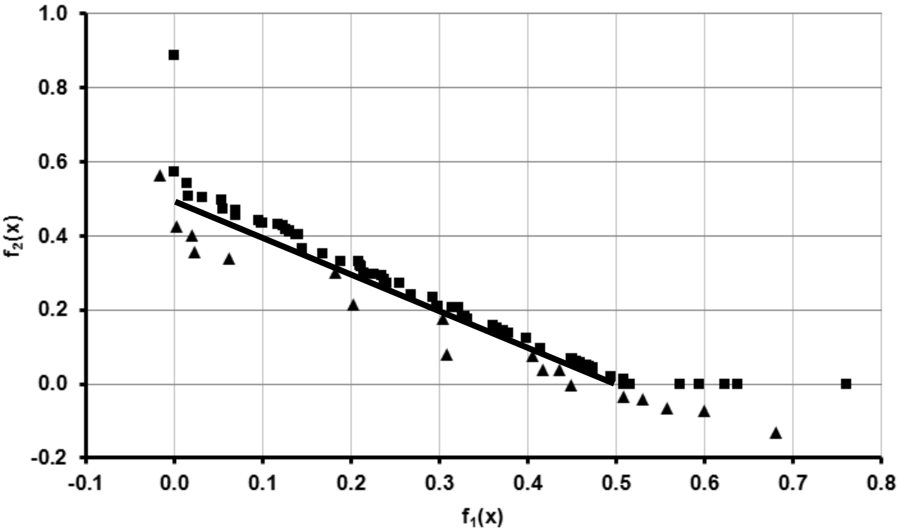

Figure 2. The images of elements of a lower shell (squares) under the criteria mapping and elements of a virtual upper shell (triangles) for the DTLZ1a problem. Set $f(N)$ is represented by the continuous line

4. Concluding remarks and directions for further research

In this paper we have proposed how to derive upper approximations of the Pareto set when upper shells do not exist. To this aim we have introduced the concept of virtual upper shells and we have shown on a numerical example that the idea is perfectly viable.

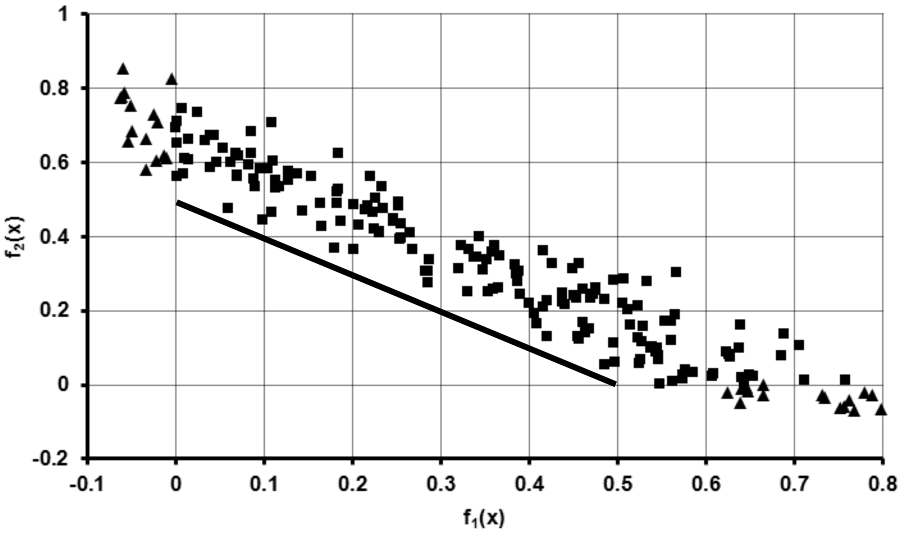

Figure 3. The images of feasible variants (squares) and infeasible variants (triangles) of total 200 variants generated randomly under the criteria mapping for the DTLZ1a problem. Set *f*(*N*) is represented by the continuous line

With virtual upper shells in place we are in the position to derive, for any instance of problem (1), an approximation of the Pareto set in the form of a pair of a lower approximation $f(S_L)$, where S_L is a lower shell, and an upper approximation in the form of VS_U.

In our previous papers we addressed the problem of efficiency of algorithms we proposed to derive S_U. The question of efficiency of the algorithm we proposed in this work to derive VS_U has been left to further research.

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ANALYSIS OF INCENTIVE COMPATIBLE MULTICRITERIA DECISIONS FOR A PRODUCER AND BUYERS PROBLEM

Abstract

The paper deals with analysis of incentive compatible multicriteria decisions within a computer-based multiagent framework. This general question is discussed on an example of a market decision problem, where a producer is introducing a product and some buyers are considering the purchase of the product. Decisions of the producer and buyers are multicriterial. Each of the buyers is seeking a product variant according to his own preferences. The producer decides which variant of the product is introduced to the market. In order to incentivize the decisions, one of his criteria takes into account an aggregated satisfaction of the buyers. A multiagent computer-based system has been constructed for supporting mulicriteria analysis made by buyers and by the producer. Selected results of an interactive session made with use of the system are presented and analyzed.

Keywords

Incentive compatible decision mechanisms, multiagent systems, multicriteria optimization.

Introduction

The paper relates to a wider direction of research dealing with analysis of incentive compatible multicriteria decision mechanisms with use of multi-agent computer-based systems. In the research the situations are analyzed in which there are agents with their own interests and trying to realize their own egoistic goals. The effects of their decisions depend also on decisions of other agents. Each agent has his own private information and, in general, he does not like to reveal the information to others. The subject of the research includes decision mechanisms leading to motivation compatibility. The mechanisms can be constructed by harmonization of agents' activity to secure the effecttiveness of the whole system. The incentive compatibility in market mechanisms was analyzed previously by Toczyłowski [2003, 2009]. The ideas developed in the papers have inspired the research presented here.

In this paper an analysis of incentive compatible decisions is performed on an example of a two stage mechanism in which a producer and his potential buyers participate. In the first stage, each buyer makes independent multicriteria analysis of a possible variant of a product and selects the variant preferred according to his individual criteria. In the example presented, each buyer minimizes a cost criterion and maximizes a criterion defined by the usefulness of the product. In the second stage, the producer also performs multicriteria analysis in the set of possible variants of the product but with respect to his criteria, including a profit criterion. The reputation of the product on the market has been assumed as one of important criteria of the producer. The reputation is expressed by an aggregated measure of satisfaction of buyers with the variant offered by the producer.

A special multiagent computer-based system has been designed. It enables problem formulation and supports multicriteria analysis made by buyers and by a producer. The system has been implemented using Optimization Software for Operations Research Applications [AIMMS]. Information about the AIMMS environment can be found on www.aimms.com and [Bisschop and Roelofs 2009]. Details referring to the functionality of the system, its implementation and user instructions can be found in the Eng. diploma thesis [Skorupiński 2010]. The system secures the confidentiality of information of users playing roles of buyers and a producer. The producer has no access to the information introduced by the buyers, nor to results of their analysis made with the use of the system.

We could imagine that the system is at the disposal of an institution, whom both the producer and the buyers trust. The institution secures the confidentiality of the individual information and performs market analysis of the new product among potential buyers. It also supports the producer in the selection of the product variant which would be favorable with respect to his criteria but would also have a good reputation on the market.

This paper includes a mathematical formulation of multicriteria optimazation tasks for buyers and for the producer. The tasks are solved by the system during multicriteria analysis with use of the reference point method [Wierzbicki 1986; Wierzbicki et al. 1993, 2000]. A question arises how to define and derive buyers satisfaction levels with respect to the variant of the product offered on the market. Then, how to calculate a cumulative reputation of the product variant on the market. Some proposals are presented.

Series of interactive sessions have been made with use of the system. Different results have been obtained showing possible behaviors of buyers and of the producer, as well as relations among solution variants chosen by them. The results of one of the sessions are presented in the paper. In the final remarks, directions of further research are discussed.

1. Mathematical description

A producer is going to offer a new variant of his product to a set L of buyers. The variants of the product that can be produced are described by a vector of decision variables $x \in D \subset R^n$, where D is a set of admissible vectors of the variables. The set D is not given explicitly. We assume that it is given by a set of linear constraints of the form: $A\, x^{\mathrm{T}} \le b,$ where A and b are matrix and vector of coefficients respectively.

The analysis of the product variants is performed in two stages.In the first stage each buyer can generate, review and analyze nondominated product variants in his space of criteria, using the reference point method [Wierzbicki 1986; Wierzbicki et al. 1993, 2000]. The following optimization tasks are formulated:

$$\max_x \{[\phi(r,a,y)] : x \in D \subset R^n\}, r \in R^n, a \in R^n,$$

where Φ denotes a scalarizing achievement function, r and a are vectors of controlling parameters. The vectors r and a play the roles of the reservation and aspiration points, respectively. The criteria y are selected variables of the vector x. They include buyers' criteria, such as:

e – economic attributes of the product, including purchasing and operating cost covered by the buyer,

u – usefulness of the product due to quality, technological advantage, reliability.

A nondominated solution is derived for reservation and aspiration points given by a buyer, solving the optimization problem:

$$\max z + \varepsilon \sum_{k \in \bar{X}} z_k \ ,$$

subject to constraints of the reference point method:

$$\begin{aligned}
&z \le z_k, \forall k \in \overline{X}, \\
&z_k \le \gamma(x_k - r_k)/(a_k - r_k), \forall k \in \overline{X}, \\
&z_k \le (x_k - r_k)/(a_k - r_k), \forall k \in \overline{X}, \\
&z_k \le \beta(x_k - a_k)/(a_k - r_k) + 1, \forall k \in \overline{X},
\end{aligned}$$

and constraints of admissible values of the variables x:

$$A\,x \le b.$$

In the formulation z, z_k, x denote variables, $\overline{X}$ is a set of criteria indexes, $\gamma > 1$ and $0 < \beta < 1$ are parameters of the achievement function applied in the above formulation.

Analysis is performed by each buyer in a number of iterations. In each iteration a given buyer assumes the reservation and aspiration points according to the reference point method. The computer-based system solves the above problem and calculates the respective variant, nondominated in the set D.

We have assumed that the reservation point of each buyer is not selected arbitrarily but is defined on the basis of the BATNA concept, similarly as in the assumptions of the procedures supporting cooperative decisions [Kruś 2002, 2004, 2008]. The BATNA concept (Best Alternative to Negotiated Agreement) is widely applied in negotiations [Fisher and Ury 1981; Raiffa 1982]. It means the best alternative a negotiating party can obtain if negotiations will not succeed. In our case, it relates to a product, which is already accessible on the market and can be compared to the variants of the product offered by the producer. We assume that each buyer is interested in a variant proposed by the producer if the variant is better than that defined by his BATNA. The BATNA concept is important for the calculation of the buyer satisfaction, proposed further in the paper.

For a reservation point given in this way each buyer assumes some number of different aspiration points. The system derives the respective nondominated variants, so the buyer can compare the variants according to his preferences. Finally, he is asked to indicate his preferred variant. The above actions are made independently by all the buyers with use of the system. The system stores information about the variants indicated by all the buyers, what completes the first stage.

The criteria of the producer include a profit obtained from the product variant offered on the market and a reputation among buyers accepting the variant offered. The profit equals sales revenues minus total expenses corresponding to the product variant. The profit criterion y_{profit} is calculated by:

$$y_{profit} = (p_e x_e - p_u x_u)\sum_{l \in L} v_l \ ,$$

where v_l is a binary variable indicating which buyer accepts the product variant offered. A simplifying assumption has been made that the revenues are in proportion to the variable x_e denoting costs covered by the buyer, and the producer's expenses are in proportion to the usefulness x_u, with coefficients p_e and p_u, respectively.

The reputation is defined as an aggregated measure of satisfaction levels of buyers accepting the variant proposed by the producer. The satisfaction level of a given buyer is calculated for the product variant offered by the producer when the buyer has already performed the multicriteria analysis, and has indicated the preferable nondominated variant. A buyer can not accept a given variant if the variant is dominated by the buyer's reservation point.

An interval scale has been assumed to measure the satisfaction level of each buyer. The scale has to be normalized with respect to different buyers and should be manipulation-free. The interval scales are constructed based on two uniquely defined points. The Celsius temperature scale defined by the temperature of ice thawing and the temperature of water boiling serves as an example. We have assumed that the satisfaction level of a buyer is measured, based on his reservation point (with a lower level $s_{lo} = 0$) and on the accessible variant preferred by him (with the upper level $s_{up} = 100$). Of course, an arbitrary variant may have assigned a satisfaction level lower than 0, or greater than 100. Discussions of different types of scales and their applicability to measuring can be found in [Torgerson 1958; Coombs, Dawes and Tversky 1970].

In our research, we also discussed other definitions of the scale and of different ways of calculating the satisfaction level. It seems natural to take the aspiration point chosen by a given buyer as a variant with the maximum satisfaction level equal to 100. In this case, the buyer can manipulate the distance of the aspiration point to the reference point. Increasing the distance, he could influence the producer's decisions, increasing his importance in comparison to other buyers.

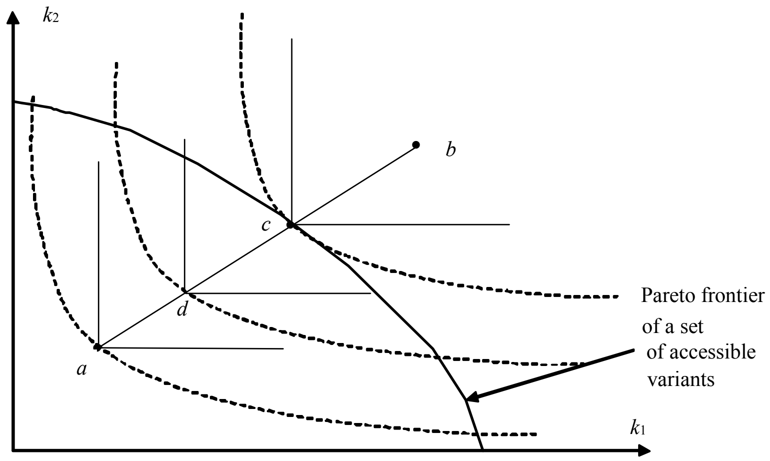

Figure 1. Indifference sets of a function measuring a buyer's satisfaction level

Figure 1 presents an example illustrating how the buyer satisfaction level is derived. Two maximized criteria are considered. The satisfaction level is defined by a scalar function defined in the space of the criteria. In a general case, it is a nonlinear utility function. Indifference sets of the function are presented by dashed lines in the space of criteria k_1, k_2. In the present version of the computer-based system, we assumed a specific variant of the function defined by the frontiers of the shifted positive cone drawn with thin continuous lines. In further research, other forms of the utility functions will be discussed including problems of its estimation and implementation in the system. The points presented in Figure 1 denote: *a* – reservation point, *b* – aspiration point indicated by a buyer after his multicriteria analysis, *c* – chosen preferred accessible variant. According to the scale assumed, all the points on the continuous lines point *d* have the satisfaction level equal to

$$s = (s_{up} - s_{lo}) \cdot |a, d| \,/\, |a, c|.$$

Let the producer offer a variant characterized by a point in the space of criteria k_1, k_2. We can construct the respective line representing the indifference set, derive the point *d* and calculate the satisfaction level according to the above formula.

The maximized reputation criterion is calculated as:

$$y_{reputation} = \sum_{l \in L} s_l ,$$

where $y_{reputation}$ denotes a value of the reputation, L is the set of all buyers and s_l is the satisfaction level of the buyer $l \in L$.

The producer performs multicriteria analysis in the space of his own criteria, assuming reservation and aspiration points, respectively. The computer--based system derives and stores respective nondominated solutions. The producer can review the solutions generated and select the preferred one. The system derives the nondominated solution by solving the following optimization problem for the given reservation and aspiration points:

$$\max z + \varepsilon \sum_{i \in \overline{Y}} z_i$$

subject to the constraints due to the reference point method:

$$\begin{aligned}
& z \le z_i , \ \ \forall i \in \overline{Y}, \\
& z_i \le \gamma (y_i - r_i)/(a_i - r_i), \ \ \forall i \in \overline{Y}, \\
& z_i \le (y_i - r_i)/(a_i - r_i), \ \ \forall i \in \overline{Y}, \\
& z_i \le \beta (y_i - a_i)/(a_i - r_i) + 1, \ \ \forall i \in \overline{Y},
\end{aligned}$$

to the reputation criterion

$$y_{reputation} \le \sum_{l \in L} s_l ,$$

$$\begin{aligned}
& s_l \le (s_{up} - s_{lo}) f_{l_k} , \forall l \in L^+ , k \in \overline{X}, \\
& f_{l_k} \le (x_k - \hat{r}_{l_k})/(\hat{x}_{l_k} - \hat{r}_{l_k}) + M(1 - v_l), \forall l \in L^+ , k \in \overline{X}, \\
& (x_k - \hat{r}_{l_k})/(\hat{x}_{l_k} - \hat{r}_{l_k}) \ge -M(1 - v_l), \forall l \in L^+ , k \in \overline{X}, \\
& s_l \ge \varepsilon^* - M(1 - v_l), \forall l \in L^+ , \\
& v_l \le 0, \ \ \forall l \in L^- , \\
& f_{l_k} \le s_d + M v_l , \forall l \in L \ ,
\end{aligned}$$

to the profit criterion

$$y_{profit} \le \sum_{l \in L} w_l ,$$

$$w_l \le Mv_l, \forall l \in L,$$
$$w_l \le p_e x_e - p_u x_u + M(1 - v_l), \forall l \in L,$$
$$p_e x_e - p_u x_u - w_l + Mv_l \le M, \forall l \in L,$$

to the model constraints of the admissible variants of the product:

$$\mathrm{A\ x} \le \mathrm{b}.$$

In the above relations, w_l, v_l, f_{l_k} denote additional variables, $\bar{Y}$ is a set of indexes of the producer criteria, L^+ is a set of buyers for which there exists a product variant better than that defined by the reservation point, L^- the set of buyers for which such a variant does not exists. M is a large positive number, $\hat{x}_{l_k}, \hat{r}_{l_k}$ denote the components of the accessible solution selected by the buyer l, and of his reservation point, respectively. Not all buyers from the set L^+ can be interested in the variant offered by the producer. It has been assumed that a buyer is interested in the variant of the product if the level of his satisfaction is at least ε^* value greater than the level of his reservation point.

2. Analysis of some results

Computing experiments and a series of sessions have been performed with use of the system. In the first experiments, the system was intensively tested. Next, interactive sessions were carried on by a producer and by several buyers. It was interesting to check how preferences of buyers impacted the decisions of the producer maximizing his profit but also attaching an importance to the reputation of his product. On the other hand, the producer's decisions impacted the satisfaction levels of particular buyers. In one of the experiments presented here the users of the system played the roles of a producer and of 8 buyers.

Figure 2 illustrates the first stage of the algorithm. It presents the results of multicriteria analysis performed by one of the buyers. The selected reservation and aspiration points as well as the respective nondominated solutions are presented in the space of buyer's criteria: e (minimized cost), and u (maximized usefulness).

All the nondominated points shown in the figure have been derived by the reference point method, but only selected reservation points, aspiration points and the corresponding nondominated points are presented.

Each buyer, assuming different reservation and aspiration points, can derive a representation of the set of Pareto optimal variants. He is asked to select the preferred variant and the respective aspiration point.

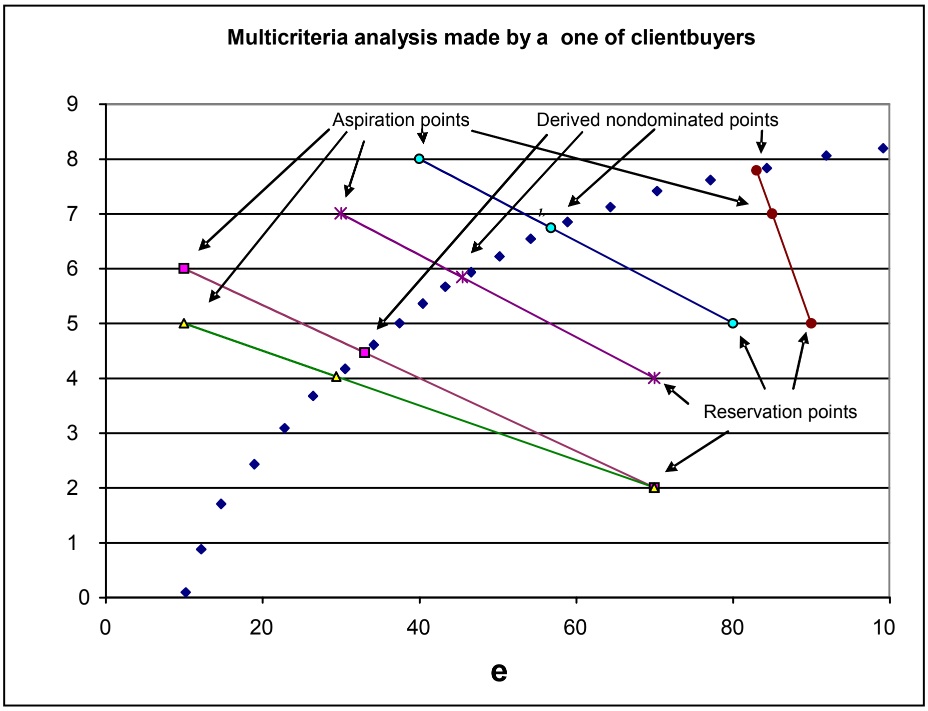

Figure 2. Illustration of multicriteria analysis made by one of the buyers.

The producer can start analysis when all the buyers have already selected their preferred variants. It is performed in the second stage of the algorithm. In the experiment, different preferences of the buyers have been assumed. They are represented by different reservation and aspiration points, and different preferred variants selected by each of the eighth buyers, as shown in Figure 3. Note that buyer 1 prefers a low cost (*e* – economic attributes) and a low usefulness (*u* criterion) of the product variant, while buyer 8 prefers a variant characterized by a high usefulness and a high cost.

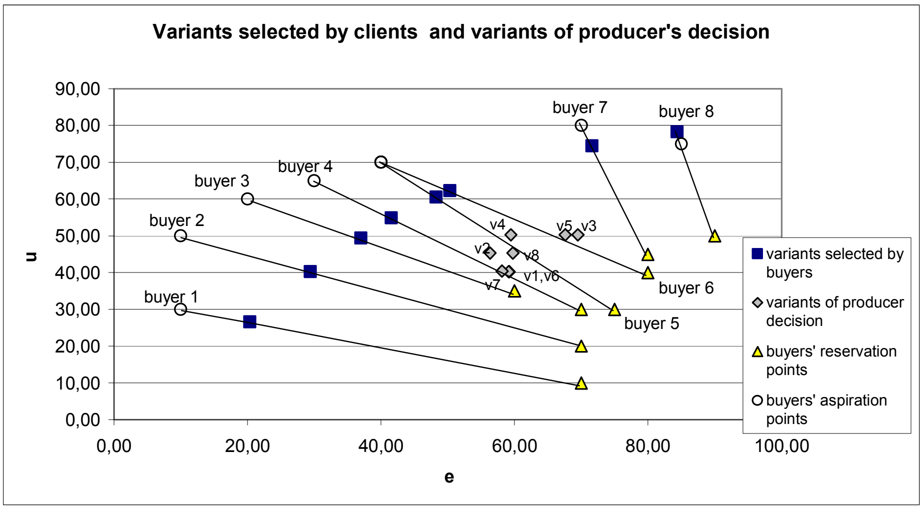

Figure 3. Results of a session with eight buyers

The producer has no access to information related to the particular buyers, their analysis, decisions or preferences. The computer-based system derives the values of the producer's criteria: the reputation of the variant among the buyers, and the profit, depending on the product variant considered by the producer to be offered to the buyers. Multicriteria analysis is performed by a producer using the reference point method analogously as in the case of buyers. The producer can make a representation of the set of Pareto solutions, can compare different nondominated variants and can select the preferred variant. Several nondominated variants derived by the system for different reservation and aspiration points in the space of producer's criteria are presented in Figure 4.

It was interesting to check the effects of a producer's preferences on his choice of the variant offered to buyers; which buyers accept the variant; what is their satisfaction levels and the resulting reputation criterion. A simulation was made assuming eighth different preferences of the producer resulting in eighth different variants of the producer's decisions.

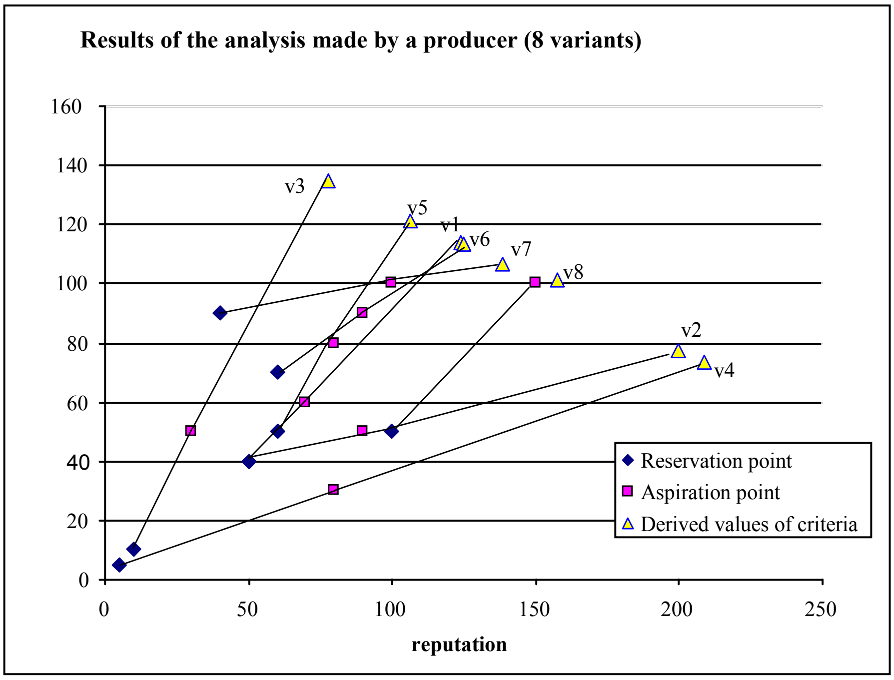

Figure 4. Different variants of producer's decision in the case of eight buyers

The variants differing with respect to reservation points, aspiration points and the respective nondominated solutions are shown in Table 1. One can see a variant with low reputation and a relatively high profit (variant 3), and on the other hand a variant with high reputation and a small profit (variant 4). The decision variables describing each variant are presented, i.e. *e* – economic attributes (cost criterion for buyers) and *u* – the usefulness. All the variants analysed are presented also in Figure 3, in the space of buyers' criteria: *e* and *u*. Therefore one can compare each variant of the producer's decision with the variants preferred by the buyers. In the last column of the table one can find the number of buyers accepting the given variant of the product.

Table 1

Reservation and aspiration points assumed by a producer and respective nondominated solutions (criteria and decision variables)

Variant	Analysis made by a producer				Decision variables		
	Criterion	Reservation point	Aspiration point	Derived values of criteria	e	u	Number of satisfied buyers
1	reputation profit	50 40	70 60	123,97 113,98	59,22	40,22	6
2	reputation profit	50 40	90 50	199,59 77,42	56,36	45,29	7
3	reputation profit	10 10	30 50	77,85 134,54	69,5	50,28	7
4	reputation profit	5 5	80 30	208,95 73,32	59,45	50,28	8
5	reputation profit	60 50	80 80	106,55 121,24	67,60	50,28	7
6	reputation profit	60 70	90 90	125,08 113,42	59,13	40,22	6
7	reputation profit	40 90	100 100	138,74 106,45	58,13	40,39	6
8	reputation profit	100 50	150 100	157,51 101,33	59,77	45,29	7

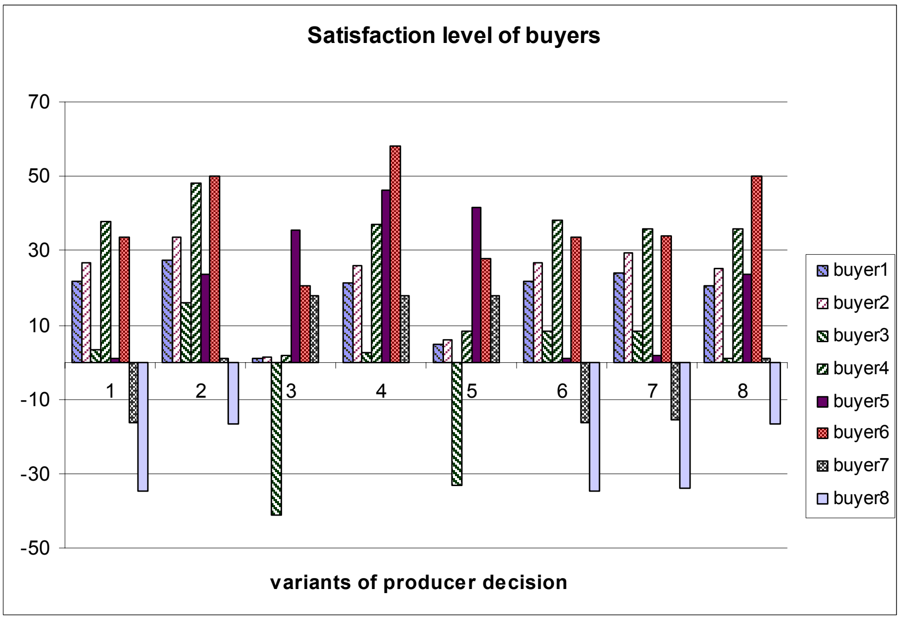

Figure 5. Buyers' satisfaction levels dependent on the producer's decision

Figure 5 shows satisfaction levels of buyers depending on the variants of product offered by the producer. Negative values of the level mean that the respective variant is not accepted by the respective buyers. Variants 1, 6, 7 are not accepted by buyers 7 and 8. Variant 3 and 5 are not accepted by buyer 3. Variant 3 gives the greatest profit to the producer in the set of variants analysed here. The greatest number of buyers is interested in variant 4. This variant has the greatest reputation among buyers but it gives the lowest profit to the producer.

Conclusions

A mathematical model describing the producer and buyers problem has been proposed. It includes formulations of optimization tasks solved during the multicriteria analysis conducted by the buyers and by the producer. The optimization tasks have been implemented in a specially designed multiagent computer-based system.

An original proposal for the derivation of satisfaction levels of individual buyers is presented. On this basis, the reputation can be calculated. It is one of the producer's criteria. It reconciles the producer's and buyers' interests. The buyer's satisfaction level is derived with use of the BATNA concept and with use of an assumed form of the buyer's utility function. In further research, different ways of the derivation will be analyzed. In particular, different forms of the utility function and interactive procedures for scaling the function with use of information obtained from buyers will be discussed.

The multiagent computer-based system enables buyers and producers to perform multicriteria analysis in two stages. An experiment with human users of the system, playing the roles of a producer and 8 buyers, illustrates the method. It is shown how the variant proposed to the buyers depends on the producer's preferences and how it is seen by the buyers.

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Bogumiła Krzeszowska

MULTIPLE CRITERIA PROJECT SCHEDULING WITH PROJECT DELAY, RESOURCE LEVEL AND NPV OPTIMIZATION

Abstract

One of the most important phases in project management is planning. During this phase tasks are identified and scheduled. A schedule brings information on how tasks should be planned over time during the realization phase of the project. That is why scheduling is a critical issue in project management. The main project scheduling techniques are CPM and PERT. They deliver the schedule with the optimal project finish time and ensure the control of resource usage. In real-life applications the schedule should optimize not only the project finish time but also resource usage and cash flows. In research on the project scheduling problem the mathematical models are used to build an optimal project schedule. Frequently used are one-objective mathematical models for project scheduling. Few papers deal with the multiple objective project scheduling problem. Constraints and objectives in project scheduling are determined by three main issues: time, resource and costs, but only few papers consider all of them.

A zero-one programming formulation has been applied to solve a multiple criteria project scheduling problem in this paper. The purpose of this paper is to present the multiple criteria project scheduling problem with three objectives: project delay minimization, resource usage in each period of time minimization and NPV maximization.

Keywords

Project scheduling, multiple criteria optimization, zero-one programming.

Introduction

In recent years the project management problem became very popular because of its broad real-life applications. One of project definitions states that a project is a set of co-ordinated activities undertaken to meet specific objectives [Brandenburg 2002]. Each project has three main components: activities (tasks to do), resources required to carry out the project tasks, and precedence relationships, which define the order in which activities should be performed [Kostrubiec 2003]. In summary: a project contains activities, which have an expected duration and resource requirements. They generate costs and cash flows and are constrained by resource limits and precedence relationships. In real-life applications a schedule should hold all those restrictions. That is why scheduling is a critical task in project management.

In project management, a schedule consists of a list of a project's terminal elements with intended start and finish dates. We can say that: "(…) scheduling is to forecast the processing of work by assigning resources to tasks and fixing their start times. (…) The different components of a scheduling problem are tasks, the potential constraints, the resources and the objective…" [Carlier and Chretienne 1988]. "Scheduling concerns the allocation of limited resources to tasks over time. It is a decision-making process that has a goal – the optimization of one or more objectives" [Pinedo 1995]. The main project scheduling techniques are CPM and PERT. CPM calculates the longest path of planned activities to the end of the project and also gives the shortest time of project realization. The Program Evaluation and Review Techniques (PERT) is a method to analyze the tasks involved in completing a given project, especially the time needed to complete each task. Those methods deliver schedules with the optimal project finish time and ensure the control of resource usage. In real-life applications the schedule should optimize not only the project finish time but also resource usage and cash flows.

The purpose of this paper is to analyze the problem of multiple criteria project scheduling problem and to discuss the multiple criteria project scheduling problem with three objectives: project delay minimization, minimization of resource usage in each period of time and NPV maximization. A zero-one programming approach has been applied to model such a problem.

The paper begins with an overview of literature and problem statement. Then, the mathematical model is described and a computational example is presented. The paper finishes with conclusions and ideas for future research.

1. Optimization in project scheduling problem – a literature overview

Constraints in project scheduling problem are determined by two main components: time and resources. We can discuss two types of resources: financial and non-financial (human resources and materials). The optimization criteria are determined by three main components: time, resources and economic indicators such as cost or NPV. When we take them into consideration we can build various optimization models for the project scheduling problem. We can also present various projects depending on the number of objectives, thus we can have a one-objective project scheduling problem and a multiple-criteria project scheduling problem.

Each mathematical model for project scheduling problem needs to include basic constraints: precedence relationship constraints and information about the extent of variables.

In research on project scheduling problems, optimization models with one objective are the most popular. In this case we can build a model containing only basic constraints, in which the project completion time or NPV is optimized, or a model with one constraint in which time, capital or resources are constrained while the project completion time, NPV, cost or resource usage are optimized, or a model with few constraints. A resource constraint is not frequently used in models with resource usage optimization. A multiple objective mathematical model for the project scheduling problem is a combination of mathematical models mentioned above.

A problem with only basic constraints in which NPV is maximized has been solved by Russell [Russell 1970]. In this paper the author assumed that the cost is generated at the moment when the project starts and income is generated when some groups of activities are finished.

There are two types of mathematical models for the project scheduling problem with one constraint: the time constrained project scheduling problem and the resource constrained project scheduling problem. In the case of the resource constrained project scheduling problem we can differentiate between problems with non-financial resources and those with capital constraints.

The project scheduling problem with time constraints where NPV is maximized has been presented in the paper by Vanhoucke, Demeulemeester and Herrorlen [Vanhoucke et al. 2002]. In the problem described in the paper cash flows were generated at the time when each activity was finished.

There are two types of the resource constrained project scheduling problems: resource constrained project scheduling problem with time optimization and resource constrained project scheduling problem with NPV optimization.

The resource constrained project scheduling problem with time optimization was discussed by Shouman, Ibrahim, Khater and Forgani [Shouman et al. 2006] and the problem with NPV optimization was presented by Icmeli and Erenguc [Icmeli and Erenguc 1996]. The resource constrained project scheduling problem was also described in Talbot's paper [Talbot 1982]. The author presented this problem with time-resource tradeoffs.

Doersch and Patterson [Doersch and Patterson 1977] proposed a financial resources (capital) constrained project scheduling problem in their paper. They assumed that the capital is limited at the project start time. The capital availability changes throughout the project duration. Activities generate cash flows (outflows and inflows), which have influence on capital availability.

Vanhoucke, Demeulemeester and Herroelen [Vanhoucke et al. 2001] described a time- and resource-constrained project scheduling problem with NPV maximization.

A resource- and time-constrained project scheduling problem was also presented by Bartusch, Mohring and Readermacher [Bartusch et al. 1988]. A vector containing the finish time of each activity is minimized. The authors assumed that an activity should start in the "time window", which is the time between the earliest and the latest start times.

Bianco, Dell`Olmo and Speranza [Bianco et al. 1998] described resource--constrained project scheduling problems with financial and non-financial resources. Each activity can be executed in several ways. Additionally, each activity generates a given cost. The project budget is limited. Additionally, activities using the same resource cannot be scheduled at the same time. The project completion time is optimized in this problem.

Gaspars-Wieloch [Gaspars-Wieloch 2008] presented a paper on time and cost analysis for the project scheduling problem. The author considered a few mathematical models from the literature on this problem. In the models considered both time and cost can be a constraint and an objective function.

A multiple criteria project scheduling problem was described by Viana and de Sousa [Viana and Sousa 2000]. The authors proposed a mathematical model in which: project completion times are minimized, project delay is minimized and disruptions in resource usage are minimized. Renewable and nonrenewable resources are constrained in the problem. A binary variable is used in the model. We define $x_{ijt} = 1$ when the operation j of the activity i is finished in time t. Otherwise, $x_{ijt} = 0$.

Leu and Yang [Leu and Yang 1999] considered a multiple-criteria resource-constrained project scheduling problem with time optimization, cost optimization and resource usage optimization. Also Hapke, Jaszkiewicz and Słowiński [Hapke et al. 1998] described that problem in their paper.

2. Project scheduling problem – basic elements of the model

The problem presented in this paper can be formulated as follows: there is a project to be scheduled. By 'scheduling' we understand setting the start and finish times of each activity. For each activity, resource requirements and budget are specified. Resource availability and precedence relationships are constrained.

The following example (Figure 1) describes the problem presented in this paper. We have a project containing 9 activities. The project is presented by an AOA (Activity On Arc) network. For each activity, its duration, required resources and net cash flows generated are shown in Table 1.

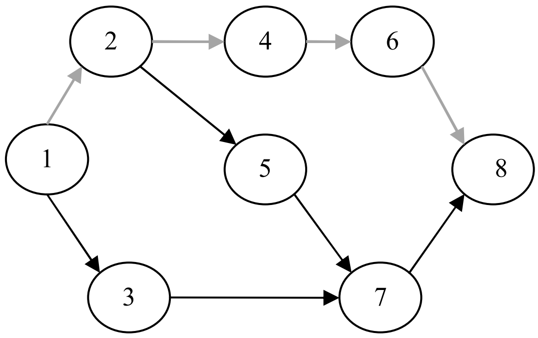

Figure 1. Activity network for example 1

For given durations and precedence relationships, the earliest start and finish times and the latest start and finish times were computed using the critical path method. Those times will be compared with the results obtained by using the mathematical model proposed in this paper.

Table 1

Example 1. Data

Activity	Duration	ES	EF	LS	LF	Slack	Critical tasks?	Renewable resources	Net cash flow
1	2	3	4	5	6	7	8	9	10
1-2	2	0	2	0	2	0	YES	2	–4
1-3	4	0	4	1	5	1	NO	1	–3

Table 1 contd.

1	2	3	4	5	6	7	8	9	10
2-4	1	2	3	2	3	0	YES	2	–1
2-5	2	2	4	5	7	3	NO	3	1
3-7	3	4	7	5	8	1	NO	4	3
5-7	1	4	5	7	8	3	NO	2	5
4-6	4	3	7	3	7	0	YES	1	7
6-8	3	7	10	7	10	0	YES	3	8
7-8	2	7	9	8	10	1	NO	2	10

Additionally, resource usage in each period is limited to 5 and the project duration time is limited to 15.

The following assumptions were made for the formulation of our mathematical model:

– project contains $j = 1,\dots, J$ activities,
– project duration is constrained to T $(t = 0,\dots, T)$,
– project is represented by AOA network,
– precedence relationships are Finish-to-Start type (S_{ij} – set of predecessors i of activity j),
– d_j – activity j`s duration,
– F_j – finish time of activity j,
– F_{ij} – finish time of predecessor i of activity j,
– $k = 1,\dots, K$ – set of renewable resources,
– r^r_{jk} – amount of renewable resource k required by activity j.

Only renewable resources are taken into consideration. We assume that the amount of nonrenewable resources needed for the project execution is constant and is not limited for the period of time, but for the project. That is why we do not need to consider them in the model. If we do not have the necessary amount of non-renewable resources the project cannot be completed. Renewable resources are constrained in each period.

2.1. Variables

The following binary variable is used in the model considered:

$$x_{jt} = \{0,1\} \quad (j = 1,\dots, J, \quad t = 1,\dots, T)$$

where $x_{jt} = 1$ when an activity j is finished in time t, otherwise $x_{jt} = 0$. In the problem considered we have $j \times t$ variables. In the problem represented in the example 1 we have 9 activities and 15 time units for project execution, so the number of variables is 9×15, which is 135:

$$x_{1,1},\ x_{1,2},\ x_{1,3}\dots\ x_{J,T}.$$

2.2. An activity execution constraint

Because a binary variable is used in the model formulated, we have to ensure that each activity will be executed only once. We will write this constraint as the following equation:

$$\sum_{t=1}^{T} x_{jt} = 1 \qquad (j = 1, \ldots, J,\ t = 1, \ldots, T)$$

In the equation above we add the variables in each time unit for each activity. If the sum is equal 1 we are sure, that an activity j is finished only once.

In example 1 this constraint is formulated as follows (for the activity *1-2*):

$$x_{1,1}+ x_{1,2}+ x_{1,3}+ x_{1,4}+ x_{1,5}+ x_{1,6}+ x_{1,7}+ x_{1,8}+ x_{1,9}+ x_{1,10}+ x_{1,11}+ x_{1,12}+ x_{1,13}+ x_{1,14}+ x_{1,15} = 1.$$

2.3. Precedence relationships

The project scheduling problem considered in this paper has been presented by an AOA network. This network allows to consider only finish-to-start precedence relationships between activities. This type of precedence relationships can be formulated as follows:

$$F_j - d_j \geq F_{ij} \quad (j = 1, \ldots, J,\ t = 1, \ldots T,\ I \in S_{ij})$$

In this case a successor can start only when its predecessor is finished.

In example 1 activity *2-4* can start when activity *1-2* is finished, so the set S_{ij} of predecessors i of activity j has only one element. The precedence relationship for this case can be formulated as follows:

$$F_2 - d_2 \geq F_1 .$$

In the case of precedence relationships we use activity finish time. We can calculate activity finish times by using the formula: $F_j = \max\{\forall_{t=1,..,T}(t \times x_{jt})\}$. So, the finish time for the activity *1-2* is:

$$F_1 = max\{1\times x_{1,1},\ 2\times x_{1,2},\ 3\times x_{1,3},\ 4\times x_{1,4},\ 5\times x_{1,5},\ 6\times x_{1,6},\ 7\times x_{1,7},\ 8\times x_{1,8},\ 9\times x_{1,9},\ 10\times x_{1,10},\ 11\times x_{1,11},\ 12\times x_{1,12},\ 13\times x_{1,13},\ 14\times x_{1,14},\ 15\times x_{1,15}\}.$$

2.4. Project completion time optimization

The time criterion is frequently used in the literature. Many solutions for project completion time optimization are considered, e.g. each activity finish time minimization, last activity finish time minimization. In some cases project delays (delay is the difference between the planned and the actual finish time of an activity) minimization is also used.

In our model the project delay (not activity delays) is minimized. A delay is a situation when an activity is finished later than the latest finish time determined by the critical path method (a time given by the decision maker can also be used). Activity delays are summed up and reduced with theirs predecessors delays.

Mathematically, this criterion can be formulated as follows:

$$\sum_{j=1}^{J} \max\{0, F_j - LF_j\} - \sum_{i=1}^{I} \max\{0, F_i - LF_i\} \to \min \ (j = 1,\ldots,J,\ t = 1,\ldots,\ T)$$

The criterion of project completion time minimization for the example 1 is illustrated below.

$x_{13}+2x_{14}+3x_{15}+4x_{16}+5x_{17}+6x_{18}+7x_{19}+8x_{110}+9x_{111}+10x_{112}+11x_{113}+12x_{114}$
$+13x_{115}+x_{26}+2x_{27}+3x_{28}+4x_{29}+5x_{210}+6x_{211}+7x_{212}+8x_{213}+9x_{214}+10x_{215}+$
$x_{34}+2x_{35}+3x_{36}+4x_{37}+5x_{38}+6x_{39}+7x_{310}+8x_{311}+9x_{312}+10x_{313}+11x_{314}+12x_{315}+x_{48}+$
$2x_{49}+3x_{410}+4x_{411}+5x_{412}+6x_{413}+7x_{414}+8x_{415}+x_{59}+2x_{510}+3x_{511}+4x_{512}+5x_{513}+6x_{514}$
$+7x_{515}+x_{69}+2x_{610}+3x_{611}+4x_{612}+5x_{613}+6x_{614}+7x_{615}+x_{78}+2x_{79}+3x_{710}+4x_{711}+5x_{712}+$
$6x_{713}+7x_{714}+8x_{715}+ x_{811}+2x_{812}+3x_{813}+4x_{814}+5x_{815}+$
$x_{911}+2x_{912}+3x_{913}+4x_{914}+5x_{915}$ -
$(2x_{14}+3x_{15}+4x_{16}+5x_{17}+6x_{18}+7x_{19}+8x_{110}+9x_{111}+10x_{112}+11x_{113}+12x_{114}$
$+13x_{115}+2x_{27}+3x_{28}+4x_{29}+5x_{210}+6x_{211}+7x_{212}+8x_{213}+9x_{214}+10x_{215}+$
$2x_{35}+3x_{36}+4x_{37}+5x_{38}+6x_{39}+7x_{310}+8x_{311}+9x_{312}+10x_{313}+11x_{314}+12x_{315}+$
$2x_{49}+3x_{410}+4x_{411}+5x_{412}+6x_{413}+7x_{414}+8x_{415}+2x_{510}+3x_{511}+4x_{512}+5x_{513}+6x_{514}+7x_{51}$
${}_{5}+2x_{610}+3x_{611}+4x_{612}+5x_{613}+6x_{614}+7x_{615}+2x_{79}+3x_{710}+4x_{711}+5x_{712}+6x_{713}+7x_{714}+8$
$x_{715}+2x_{812}+3x_{813}+4x_{814}+5x_{815}+2x_{912}+3x_{913}+4x_{914}+5x_{915})$
$\to min$

The latest finish time for the activity *1-2* is 2. The activity *1-2* is delayed when it is finished later than in the second unit of time (that is why we have 0 x_{11} and 0 x_{12}). If the activity *1-2* is finished in the third unit of time its delay will be 1 ($1x_{13}$), when it is finished in the fourth unit of time its delay is 2 ($2x_{14}$), and so on.

2.5. Resource level optimization

In the next criterion a resource usage level is optimized. Resource level optimization is not discussed in the literature frequently. In some papers a criterion in which the difference between resources available and required is minimized.

In our model the maximum resource usage level is minimized in each unit of time. It is described by the following objective function.

$$\max_{t=1,\dots,T}[\sum_{j=1}^{J} r_{jk}^{r} \cdot x_{jt}] \rightarrow \min \quad (j = 1, \dots, J,\ \ t = 1, \dots, T,\ \ k = 1, \dots, K)$$

The resource level minimization objective function for example 1 is illustrated below.

$$Max\{(2x_{11}+x_{21}+2x_{31}+3x_{41}+4x_{51}+2x_{61}+x_{71}+3x_{81}+2x_{91}),$$
$$(2x_{12}+x_{22}+2x_{32}+3x_{42}+4x_{52}+2x_{62}+x_{72}+3x_{82}+2x_{92}),$$
$$(2x_{13}+x_{23}+2x_{33}+3x_{43}+4x_{53}+2x_{63}+x_{73}+3x_{83}+2x_{93}),$$
$$(2x_{14}+x_{24}+2x_{34}+3x_{44}+4x_{54}+2x_{64}+x_{74}+3x_{84}+2x_{94}),$$
$$(2x_{15}+x_{25}+2x_{35}+3x_{45}+4x_{55}+2x_{65}+x_{75}+3x_{85}+2x_{95}),$$
$$(2x_{16}+x_{26}+2x_{36}+3x_{46}+4x_{56}+2x_{66}+x_{76}+3x_{86}+2x_{96}),$$
$$(2x_{17}+x_{27}+2x_{37}+3x_{47}+4x_{57}+2x_{67}+x_{77}+3x_{87}+2x_{97}),$$
$$(2x_{18}+x_{28}+2x_{38}+3x_{48}+4x_{58}+2x_{68}+x_{78}+3x_{88}+2x_{98}),$$
$$(2x_{19}+x_{29}+2x_{39}+3x_{49}+4x_{59}+2x_{69}+x_{79}+3x_{89}+2x_{99}),$$
$$(2x_{110}+x_{210}+2x_{310}+3x_{410}+4x_{510}+2x_{610}+x_{710}+3x_{810}+2x_{910}),$$
$$(2x_{111}+x_{211}+2x_{311}+3x_{411}+4x_{511}+2x_{611}+x_{711}+3x_{811}+2x_{911}),$$
$$(2x_{112}+x_{212}+2x_{312}+3x_{412}+4x_{512}+2x_{612}+x_{712}+3x_{812}+2x_{912}),$$
$$(2x_{113}+x_{213}+2x_{313}+3x_{413}+4x_{513}+2x_{613}+x_{713}+3x_{813}+2x_{913}),$$
$$(2x_{114}+x_{214}+2x_{314}+3x_{414}+4x_{514}+2x_{614}+x_{714}+3x_{814}+2x_{914}),$$
$$(2x_{115}+x_{215}+2x_{315}+3x_{415}+4x_{515}+2x_{615}+x_{715}+3x_{815}+2x_{915})\} \rightarrow min$$

The resources required are multiplied by the binary variable and summed up in each time unit. Then the maximum value is chosen. Resources are used only when the variable is 1.

2.6. NPV optimization

The next criterion presented in this paper is the NPV maximization. Cash flow depends on activity duration and finish time.

This problem is frequently discussed in the literature. An example is considered in Icmeli and Erenguc's paper [Icmeli and Erenguc 1996]. There, cash flows are generated in each unit of time of activity duration. This problem can be formulated as follows:

$$\sum_{i=1}^{J}[\sum_{t=1}^{d_j}[cf_{jt} \cdot e^{\alpha(d_j-t)}] \cdot e^{-\alpha F_j}] \rightarrow \max.$$

In our paper we assume that cash flows are generated by activities at the end of their durations, so the criterion can be formulated as follows:

$$\sum_{j=1}^{J} cf_j \cdot e^{-\alpha F_j} \rightarrow \max \qquad (j = 1,\dots,J,\ t = 1,\dots,T)$$

In example 1 this criterion has the following form:

$$cf_1 \cdot e^{-\alpha F_1} + cf_2 \cdot e^{-\alpha F_2} + cf_3 \cdot e^{-\alpha F_3} + cf_4 \cdot e^{-\alpha F_4} + cf_5 \cdot e^{-\alpha F_{51}} + cf_6 \cdot e^{-\alpha F_6}$$
$$+ cf_7 \cdot e^{-\alpha F_7} + cf_8 \cdot e^{-\alpha F_8} + cf_9 \cdot e^{-\alpha F_9} \rightarrow \max.$$

3. Multiple objective project scheduling problem

We can build various one-objective optimization models for the project scheduling problem using the criterion and constraints considered above. In some cases the objective function can be presented as a constraint, e.g. a resource constraint can be formulated as follows:

$$\max_{t=1,\dots,T}[\sum_{j=1}^{J} r_{jk}^{r} \cdot x_{jt}] \le R_{kt}.$$

The resource-constrained project scheduling problem with time optimization is dealt with in many papers. In this paper the resource constraint is not considered because a resource type criterion is used. Resources are rarely considered as both criterion and constraint in the same problem.

The multiple objective project scheduling problem containing all issues important in project management (presented in Section 2) – time, resources and NPV – can be formulated as follows:

$$\sum_{j=1}^{J} \max\{0, F_j - LF_j\} - \sum_{i=1}^{I} \max\{0, F_i - LF_i\} \to \min \tag{1}$$

$$(j = 1, \ldots, J,\ \ t = 1, \ldots, T)$$

$$\max_{t=1,\ldots,T} [\sum_{j=1}^{J} r_{jk}^{r} \cdot x_{jt}] \to \min \quad (j = 1, \ldots, J,\ \ t = 1, \ldots, T,\ \ k = 1, \ldots, K) \tag{2}$$

$$\sum_{j=1}^{J} cf_j \cdot e^{-\alpha F_j} \to \max \qquad (j = 1, \ldots, J,\ \ t=1, \ldots, T) \tag{3}$$

with the following constrains:

$$\sum_{t=1}^{T} x_{jt} = 1 \quad (j = 1, \ldots, J,\ \ t = 1, \ldots, T) \tag{4}$$

$$x_{jt} = \{0, 1\} \quad (j=1, \ldots, J, t=1, \ldots, T) \tag{5}$$

$$F_j - d_j \geq F_{ij} \quad (j = 1, \ldots, J,\ \ t = 1, \ldots T,\ \ i \in S_{ij}) \tag{6}$$

There are many methods for solving a multiple objective problem. We can solve this problem by using the weighted method (then we will obtain a one-objective problem) or we can use methods dedicated to the multiple objective optimization.

If we solved this problem as a three separate one-objective problems we would obtain three very different schedules. By solving it as a multiple-objective optimization problem we will obtain a set of non-dominated solutions. Below are examples of non-dominated solutions. We denote the time criterion by C1, the resource criterion by C2, and the NPV criterion as C3.

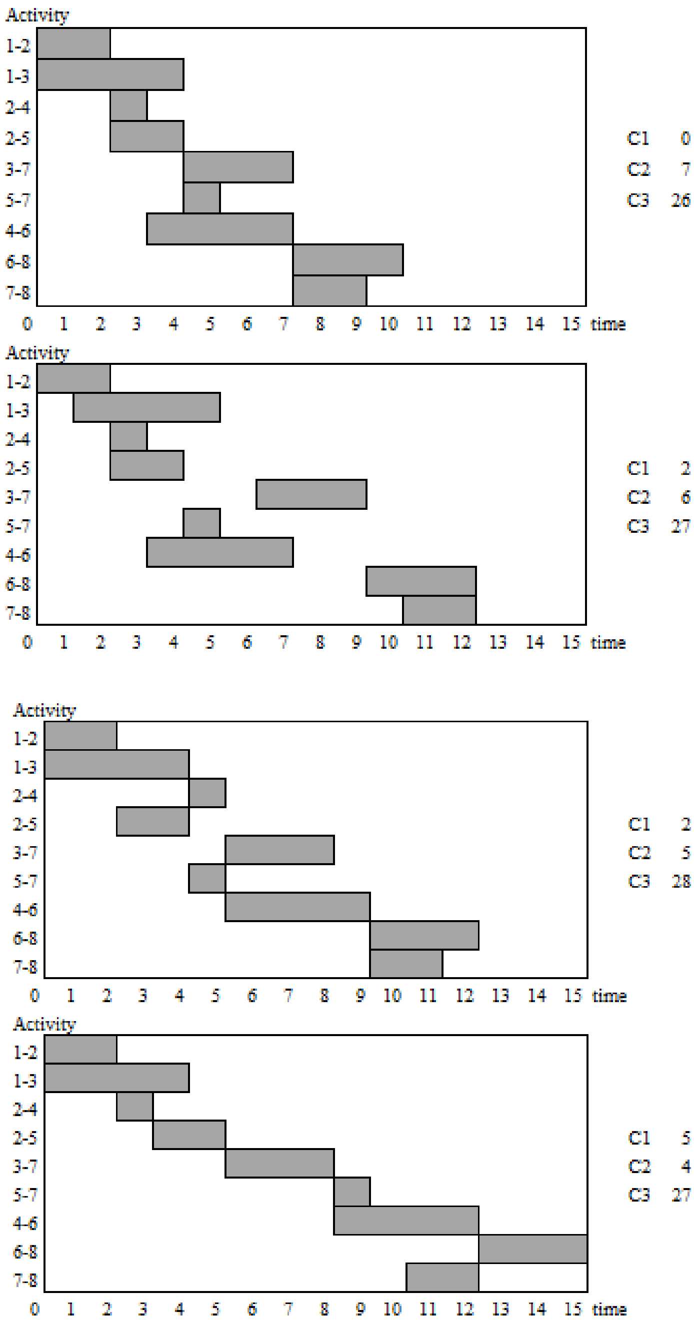

Figure 2. Non-dominated solutions for example 1

Four non-dominated solutions are presented above (Figure 2). We can see that the value of time criterion is between 0 (project finished on time) and 5 (project 5 time units delayed), of resource criterion is between 4 and 7 (the maximum level of resource usage) and of NPV is between 26 and 27 (Table 2).

Table 2

Example 1. Non-dominated solutions – criteria values

	S1	S2	S3	S4
C1	0	2	2	5
C2	7	6	5	4
C3	26	27	28	27

Project delay and resource usage are strongly connected with each other. When resource usage level is decreasing then project delay is increasing.

The result of multiple objective problem is a set of non-dominated solutions. In this case four of them were identified. But in the cases of larger projects the number of non-dominated solutions can be much larger. Then the preferential information of the decision maker about the schedules should be considered and one schedule should be chosen.

Conclusions

The type of constrains and optimization criteria in the project scheduling problem are determined by three main components: time, resource and capital.

By using multiple criteria optimization models in the project scheduling problem we can create an optimal project schedule because it is expressed not only in terms of time, but also in terms of resource usage or project's NPV.

A zero-one programming approach for project scheduling problem has been presented in this paper.

An advantage of this approach is its form. A binary variable is easy to use and adapt to include new objectives related to the needs. The indicators of project schedule obtained from objective functions deliver clear information about project realization, e.g. an objective function in time optimization gives a concrete number, which is the project delay.

A disadvantage of model proposed is its large number of variables, namely $j \times t$. In the case of larger projects or larger planning horizon the number of variables will be huge.

In future research other mathematical models for project scheduling problem should be considered, e.g. mathematical model in which, variables present finish times of activities or mathematical model with binary variables in which $x_{jt} = 1$ when an activity j last in period t.

Algorithms for solving this problem should be considered. The objective functions are nonlinear, so heuristic methods should be considered as a method of solution. Binary variables enable to use genetic algorithms.

In future research Activity-On-Node network should be considered. When representing the project scheduling problem by an AON network we can use not only finish-to-start precedence relationship but also other types of precedence relationships, such as: start-to-finish, start-to-start or finish-to-finish.

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Jerzy Michnik

WHAT KINDS OF HYBRID MODELS ARE USED IN MULTIPLE CRITERIA DECISION ANALYSIS AND WHY?*

Abstract

Currently, a growing number of papers presenting the hybrid models in the field of MCDA is observed. In this paper we try to explain the reasons for the growing popularity of hybrid models. The most important question considers the reasons for the use of these, rather complicated, structures. We also try to find out the differences between various models which are called hybrid.

Keywords

Discrete models, hybrid models, hybrid solvers, integrated models, multiple criteria decision analysis (MCDA), mixed models.

Introduction

If one reviews MCDA literature, it seems that the term "hybrid" became popular in the beginning of the 21st century. However, it is clear that mixed methods had been used earlier for many years without being named as such. The authors used other terms such as integrated or mixed models. In many cases, there was no particular emphasis put on the use of two or more different MCDA methods in a single model. Hence, it is very likely that many more papers that use a mix of MCDA methods have been published. Nevertheless, they are not easy to detect because the term "hybrid" does not appear in the title, key words or anywhere else in the text. This is why this review is limited only to papers that can be found with the key "hybrid".

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Perhaps the term "hybrid" was borrowed by the MCDA/MCDM literature from the field of computer programming, mainly from the domain of programs that solve optimization problems. In a natural way, these programs are strongly related to MCDM (and MCDA) methods and sometimes are an integral part of the multiple criteria methodology.

Most search results for the keyword "hybrid" in the literature databases refer to programs or algorithms. Nowadays, many different heuristics (or meta-heuristics) became popular tools for solving hard problems. There is also a large group of papers in which only one MCDA method is applied but the authors use the term hybrid to point out that they include some other auxiliary or complementary techniques for special purposes. In our terminology such models are not really hybrid, so we will call them "pseudo-hybrids".

In the next section we discuss the various examples of the use of the term "hybrid" in the literature related to MCDA. It appears that this term is used in different context in MCDA. As our main interest is in situations in which two or more MCDA methods are combined together into a single model, we distinguish that kind of models from all other hybrids. In this paper we assume that an MCDA method is one which can independently solve an MCDA problem.

Consequently, we classify the models into three categories:
– the models outside the strict MCDM/MCDA methodology,
– the models with unjustified use of the term hybryd,
– the models that present really hybridized MCDA methodology.

We will call the first category "hybrid solvers", the second, "pseudo--hybrids", and the third, "real MCDA hybrids".

In view of the growing popularity of hybrid models in MCDA literature, this paper presents an attempt to answer the following questions:
– Why are hybrid models developed?
– What kinds of hybrid models occur in the MCDA field?
– What are the theoretical and practical reasons for hybrid models?
– What kinds of MCDA problems can be solved with hybrid models?

1. Hybrid models in the scientific literature

1.1. Hybrid solvers

As was said in Introduction, we divided the existing variety of papers in which the term hybrid is used into three categories. We start our brief review with the examples of "hybrid solvers", hybridization that is closely related to MCDM but is essentially outside the strict MCDM. This category comprises mixtures of various computation techniques which serve as solving tools for complicated MCDM/MCDA problems. It must be said that this category is very rich but we limit ourselves to a few examples only, as such models lie outside the field of our main interest.

The computationally difficult problem of vehicle routing with time windows became the starting point to develop a hybrid algorithm that used specialized genetic operators and variable-length chromosome representation to accommodate the sequence-oriented optimization [Tan et al. 2005]. It has been proved that this algorithm leads to better or competitive solutions when compared with the best known ones from literature.

In the paper entitled "A hybrid method for solving multi-objective global optimization problems" the authors presented their new hybrid method msPESA (mixed spreading PESA) which combines some aspects of PESA (Pareto Envelope-based Selection Algorithm), NSGA-II (non-dominated sorting genetic algorithm II), and LS (local search) – [Gil et al. 2006]. They also compared their method with other methods testing them with five two-objective problems and three three-objective problems.

Improving technical ability of meta-heuristics applied to solve multiple objective problems was the aim of a novel approach based on hybridizing simulated annealing and tabu search [Baños et al. 2007].

The combination of elements from simulated annealing and a variable neighborhood search was a basis for other hybrid meta-heuristics [Behnamian et al. 2009]. The authors use their method to solve the bi-objective scheduling problem. They say: "(…) using a hybrid meta-heuristic is to raise the level of generality so as to be able to apply the same solution method to several problems".

In order to get a fast method for calculation of the Pareto optimal set for multiple objectives the genetic algorithm has been combined with an activity analysis as a local search method [Whittaker et al. 2009]. The authors incorporated a biophysical model and an economic model in the integrated optimization, and presented its application in the evaluation of agricultural and environmental policy.

1.2. Pseudo-hybrids

The next category of models we will call "pseudo-hybrid". The reason for such a term comes from the observation that the models consist of only one authentic MCDA method which is usually assisted by one or more complementary tools which are not essentially multiple criteria methods. Here we present a selection of models which have been classified to this category.

The first example is a continuous method and a combination of goal programming and fuzzy set theory (El-Wahed and Abo-Sinna 2001). The fuzzy membership function has been used to determine the weights of objectives and the degree of conflict among the objectives. The developed method has been used to solve a real-life problem.

The next example is the paper entitled "A hybrid model of fuzzy and AHP for handling public assessments on transportation projects" (Arslan 2008). It presents the application of MCDA method to the decision concerning the choice of the type of public transport in a metropolitan city in Turkey. In this paper the decision model is based on only one MCDA method: AHP. This method is supported by the fuzzy logic for processing of incomplete and imprecise data.

Three different techniques – AHP, Monte Carlo simulation and fuzzy expert system – have been used together in (S. Li and J.Z. Li 2009). Among them only AHP belongs to the MCDA field. In fact, the authors did not call their method "MCDA hybrid", they used the term "a hybrid intelligent decision support system or approach" instead.

The next paper differs from the previous examples as it employs one MCDA technique, DEMATEL (Decision Making Trial and Evaluation Laboratory), accompanied by structural equation modeling (SEM), the method taken from multivariate analysis [Tzeng et al. 2007]. It uses the DEMATEL technique to improve the causal model built with the aid of SEM. Then, this hybrid model has been used for selecting the most important factor affecting web-advertising.

1.3. Real MCDA hybrids

The third category, which we call "real MCDA hybrid" is the focus of our consideration. To this category we assign all models in which two or more MCDA methods are combined together.

We start our review with the hybrid consisting of ELECTRE and AHP [Rudolphi and Haider 2003]. According to the authors, such an approach enables to balance the frequently conflicting goals of visitor management and ecological integrity. A case study from the West Coast Trail in Pacific Rim National Park Reserve, BC, Canada, has been presented. From ELECTRE the model took the concordance and discordance analysis and also the definition of the different types of thresholds for criteria. From the AHP method the hierarchical structure and pair-wise comparisons have been adopted. The reason for such a hybrid model was that the two methods complement each other.

The next paper has also aimed at solving the real-life problem of the Armenian energy sector [Goletsis et al. 2003]. The authors have developed a hybrid of ELECTRE III and PROMETHEE to get an integrated methodology for group decision making in project ranking. The procedure has been divided into two steps. In the first step the weights and thresholds for pseudo-criteria have been assigned according to the ELECTRE III technique. In the second stage the flows have been calculated in a way similar to the PROMETHEE method.

In the paper "The assessment of the information quality with the aid of multiple criteria analysis" [Michnik and Lo 2009] the authors have not used the term hybrid explicitly, however they combined two MCDA methods together to asses and improve the information quality in a firm. The AHP method has been utilized to get the hierarchy of criteria and the corresponding weights. To compare the decision alternatives, the modified version of ELECTRE method has been used. The two main techniques have been accompanied by two auxiliary tools: the stochastic dominances (for group decision making) and fuzzy measures (uncertainty of human judgment).

Both DEMATEL and ANP are methods that have been designed to solve the problem of interrelations between elements of a complex system. However, the authors of two papers [Ou Yang et al. 2008; Y.-C. Chen et al. 2010] have presented the opinion that ANP does not cope well enough with the weighting and interrelations between clusters. Hence, they have proposed to supplement the ANP method with DEMATEL to solve the dependence and feedback in real-world problems.

A quite different methodology has been developed in a recently published paper [Y. Chen et al. 2011]. In this case a hybrid approach means an incorporation of OWA's (Ordered Weighted Averaging) way of aggregation into TOPSIS (technique for order performance by similarity to ideal solution). The aim of such a procedure was to facilitate the methodology comprising many ideal and anti-ideal points (e.g. given by a group of decision makers).

An interesting case of a real hybrid is a paper in which two MCDA methods: DEMATEL and AHP have been supplied by fuzzy integral and factor analysis [Tzeng et al. 2007]. The experiments done by the authors have shown that this hybrid model is capable of producing effective evaluation of e-learning programs.

Conclusions

Our search in the literature databases shows the existence of various kinds of hybrid models that appear in the wide range of MCDA discipline. A close look at the papers that explicitly refer to the term "hybrid" led us to the idea of dividing all models into a few categories that contain more uniform models.

Many examples of hybrid models refer to the hybrid solving methods, mostly from artificial intelligence. They have been designed to assist the MCDM/MCDA models with efficient and fast computing algorithms. We called this category "hybrid solvers".

This category, which perhaps is the richest one, contains models that use one MCDA method complemented by one or more auxiliary techniques. Usually, the main task of such supplementary techniques is to support the core MCDA method in solving the more complicated problems that arise from real-world problems. It comes from the observation that the classical MCDA models usually restricted by many assumptions appeared to be too idealized for the practical applications. To cope with a variety of qualitative and quantitative criteria, uncertainty conditions and other specific features of elements of the MCDA model, other techniques are incorporated into the model. It is often called hybrid, however as the auxiliary methods are not able to solve autonomously the MCDA problem, for this kind of models we use the term "pseudo-hybrid".

In our opinion, a model which deserves the name "real MCDA hybrid" is one in which at least two independent MCDA methods have been used. It can be expected that there are plenty of examples of real hybrid MCDA models because in many cases, especially in the past, the authors did not use explicitly the term "hybrid". In all cases, the authors advocate such a construct in order to deal with more complicated problems and real-world applications.

It is not easy to find any fundamental theoretical reasons for the use of hybrid models. The methods chosen to form a hybrid are driven mostly by a practical goal. However, their sort, range and number are – to some extent – a subjective choice depending on experience, intuition, and inclination of the authors.

The question "why do we need hybrid models?" is important from the point of view of future directions of MCDA development. The overview of current literature shows that the reason for using hybrid models is better approximation of the reality. In such models two or more MCDA methods complement each other and perform different tasks.

Another important issue is "how should a reasonable hybrid model be constructed?". Some general suggestions emerge from the analysis of various existing hybrids:

1. The second, third, … MCDA method may be added to the model only if they really are complementary to the core MCDA method and they really extend the ability of the hybrid in comparison to the single MCDA method.
2. The methods compounded together to make a hybrid should not be very similar to each other and should not be designated to do the same task.
3. In any case the subtle balance between the complexity of the hybrid model and its ability to solve the problem should be carefully kept. In other words, the gain acquired from the employment of another method should outweigh the disadvantages of the more complicated structure.

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MULTICRITERIA METHODS FOR EVALUATING COMPETITIVENESS OF REGIONS IN V4 COUNTRIES

Abstract

Regional competitiveness is the source of national competitiveness. This paper presents multi-criteria decision making methods for evaluation of the regional competitiveness and regional differences and disparities. Specific indicators reflect the economic productivity of the region in form of factors of production inside of the region. The technology for the evaluation of regional competitiveness is based on the application of two methods of multi-criteria decision making. The first one is the method of Ivanovic deviation, the second one is the well known DEA. The results of the applications of the methods are compared on the basis of the competitiveness of the NUTS2 regions (V4 – Visegrad Four countries) in the EU within the period of 7 years (2000-2006). In particular, the disparities between the Czech and Polish NUTS2 regions are discussed.

Keywords

Regional competitiveness, regional disparities, multi-criteria methods, Ivanovic deviation, DEA.

Introduction

This paper deals with multi-criteria decision making methods for evaluating the regional competitiveness and regional differences and disparities. Specific indicators reflect economic productivity of the region in form of factors of production and/or efficiency inside the region (effect of one-regional unit) and are revitalized by the capacity of actual employment in the region. In particular, we deal with the following indicators: Gross domestic product and Labour productivity per person employed, Gross fixed capital formation, Total intramural R&D expenditure, Income of households, Employment rates.

The technology of evaluation of regional competitiveness is based on the application of two methods of multi-criteria decision making. The first one is the method of Ivanovic deviation, the second one is the well known Data Envelopment Analysis – DEA. The results of both methods will be compared.

1. Method of Ivanovic deviation

There does not exist a "universal" methodology for assessing the degree of regional non-competitiveness. An "alternative way" for evaluating regional competitiveness is to define a group of specific economic indicators of efficiency [see: Melecký and Nevima 2010]. The basic idea is to assess the internal sources of regional competitiveness in detail [see: Krugman 1994]. The evaluation of the competitiveness through five specific indicators have been proposed and discussed in [Nevima and Ramik 2009].

The classical weighted average methods (WA) proved to be irrelevant to the problem of regional competitiveness as the usual assumption of independent criteria is not satisfied. That is why we were looking for other suitable methods. Here, we present an application of two methods of this kind: Ivanovic deviation and DEA.

To overcome the problem of dependent criteria, we propose the technique of evaluation of regional competitiveness called Ivanovic deviation (ID) [see: Nevima and Ramik 2009]. This method is a technique of multi-criteria decision--making and its purpose here is to assess the ranks of the regions, too. In comparison with the simple averaging [see e.g.: Ramík and Perzina 2008], it takes into account the importance and mutual dependence of the decision--making criteria, i.e. six specific indicators ranked by their relative importance, that is: Gross domestic product (GDP), Labour productivity per person employed (LP), Gross fixed capital formation (THFK_EUR), Total intramural R&D expenditure (GERD), Income of households (INDIC_NA) and Employment rates (Y15_MAX). This ranking is done by an expert evaluation; here, GDP is the most important indicator as it reflects the total economic efficiency of the region and it also includes the level of production. The second most important criterion is LP, the labour productivity per person employed. THFK_EUR is the gross fixed capital – an indicator of connections of expenditures for the creation of the fixed assets. These assets are also included in the regional production. GERD could be interpreted as the total R&D expenditures. INDIC_NA is the income of households and Y15_MAX is the criterion of employment rate. In this method, the weight of each criterion is based on its relative importance – the ranking takes into account the correlation coefficients with the previous (i.e. more important) criteria. Then the weighted distance of the current variant to the ideal (fictitious) one is calculated as follows [see: Nevima and Ramik 2009]:

$$I_j = \frac{\left|x_1^f - x_{1j}\right|}{s_1} + \sum_{i=2}^{n} \frac{\left|x_i^f - x_{ij}\right|}{s_i} \prod_{k=1}^{i-1} (1 - |r_{ki}|), \tag{1.1}$$

where:

x_i^f – value of *i*-th criterion of ideal (fictitious) variant (i.e. region),

x_{ij} – value of *i*-th criterion *j*-th variant,

r_{ki} – correlation coefficient *i*-th a *k*-th criterion (i.e. specific coefficient),

s_i – standard deviation *i*-th criterion calculates:

$$s_i = \sqrt{\frac{1}{m} \sum_{j=1}^{m} \left(x_i^j\right)^2 - \left(\bar{x}_i\right)^2}\ , \tag{1.2}$$

where *m* – total value of variants, *n* – total number of criteria.

The approach based on the application of the Ivanovic deviation seems more relevant as compared to the results of the method of simple averaging. As we know the importance of the criteria and correlations (i.e. dependences) among the criteria, we are able to determine the "distance" to the ideal region in a more realistic way. Then the final rank of regions corresponds to the different economic importance of individual criteria (i.e. specific indicators of efficiency). Thanks to this fact we consider the final rank as another contribution of this alternative approach to the evaluation of regional competitiveness of the NUTS2 regions in the V4 countries, see Table 1 and Figures 3, 4 and 5.

2. DEA

Data Envelopment Analysis (DEA) is a relatively new data-oriented approach for evaluating the performance of a set of peer entities called Decision Making Units (DMUs) converting multiple inputs into multiple outputs. Here, we applied DEA to all 35 central European NUTS2 regions in Visegrad Four countries (V4). Recent years have seen a great variety of applications of DEA for use in evaluating the performances of many different kinds of entities engaged in many different activities in many different contexts in many different countries [see: Cooper et al. 2000]. These DEA applications have used DMUs of various forms to evaluate the performance of entities, such as hospitals, US Air Force wings, universities, cities, courts, business firms, and others, including the performance of countries, regions, etc.

As pointed out in [Cooper et al. 2000], DEA has also been used to supply new insights into activities (and entities) that have previously been evaluated by other methods. Since DEA in its present form was first introduced in 1978 [see: Charnes et al. 1978], researchers in a number of fields have quickly recognized that it is an excellent and easily used methodology for modeling operational processes for performance evaluations. In their original study [Charnes et al. 1978], DEA is described as a "mathematical programming model applied to observational data that provides a new way of obtaining empirical estimates of relations – such as the production functions and/or efficient production possibility surfaces – that are cornerstones of modern economics".

In most management or social science applications the theoretically possible levels of efficiency are not known. Our model is based on the inputs and outputs, which must be chosen carefully with regard to their definition in economic theory. This fact is vital for us to perceive the efficiency as a "mirror" of competitiveness. Moreover, here we present only one version of the DEA model, that is, the most popular input oriented CCR model and also the output--oriented CCR model [see: Charnes et al. 1978]. For more detailed analysis of efficient regions (coefficient of efficiency is equal to one) we applied DEA super-efficiency models [see e.g.: Cooper et al. 2000].

Now we introduce criteria for selecting inputs and outputs used in the DEA model as applied to efficiency of NUTS2 regions in V4 (i.e. Czech Republic, Slovakia, Poland, Hungary). It is evident that the overall performance of the regional economy affects the number of people employed in various sectors, their skills and working age (15-55 years). Therefore, we selected the criterion of employment rate and that of the creation of the THFK_EUR (Gross Fixed Capital Formation). This criterion includes, in general, investment activity of domestic companies and fixed assets of foreign companies, which are the "engine" of the innovation competitiveness. The total intramural R&D expenditure (GERD) is considered for the future development of the region. The third input included is the net disposable income of households (INDIC_NA). In terms of competitiveness the disposable income plays an important role, especially because it directly reflects the purchasing power of the region [see: Nevima and Ramik 2010]. The last input indicator is the employment rate (Y15_MAX).

There are two outputs in our DEA model [see: Zhu 2002]. The outputs are measured by GDP in purchasing parity standards and labor productivity per person employed. The GDP is the most important macroeconomic aggregate, and if it is measured per region, we can take into account a limited number of inputs for its calculation. Similarly, the labor productivity (LP) may be taken into account, as it shows what amout of production was created by economically active people or employed persons.

In Figure 1 and 2 we compare the Czech and Polish region super-efficiency and in Table 2 it is evident that the best results are traditionally achieved by economically powerful "capital" regions being efficient during the whole period 2000-2006. It is clear that whereas in the Czech regions the regional disparities between the capital region and the other NUTS2 regions diminish within the given period, in Poland the disparities in economic efficiency between the capital region and the other NUTS2 regions increase within the given period. Hence, the tendency in Poland is opposite to that in the Czech Republic.

Conclusions

The paper aims at presenting multi-criteria approaches to evaluating competitiveness (efficiency) and disparities of the European regions (NUTS2). This evaluation was based on the applications of two models (Ivanovic deviation and DEA) calculating an "efficiency index" of each region. Since no universal methodological approach to regional competitiveness exists, this paper should be understood as a contribution to the discussion of quantitative measurement of competitiveness at the regional level.

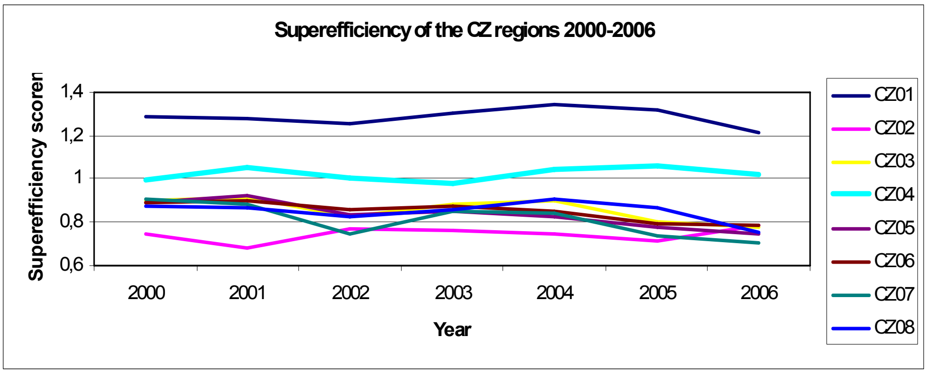

Figure 1. Superefficiency of the Czech NUTS2 regions

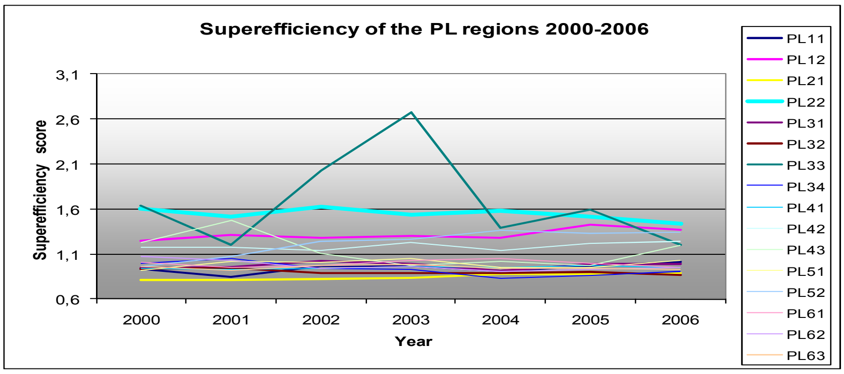

Figure 2. Superefficiency of the Polish NUTS2 regions

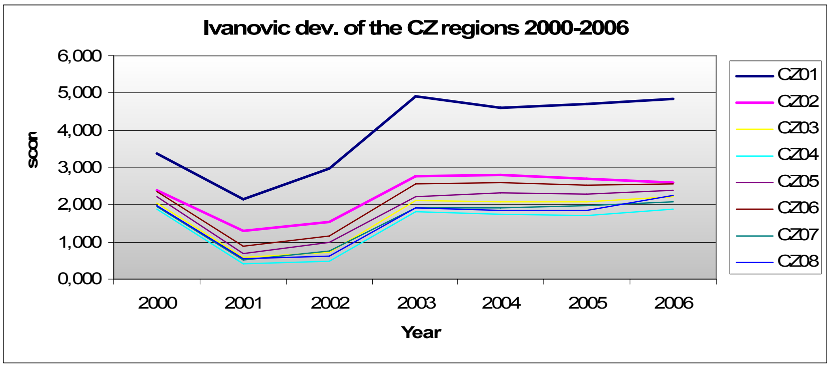

Figure 3. Ivanovic deviation of the Czech NUTS2 regions

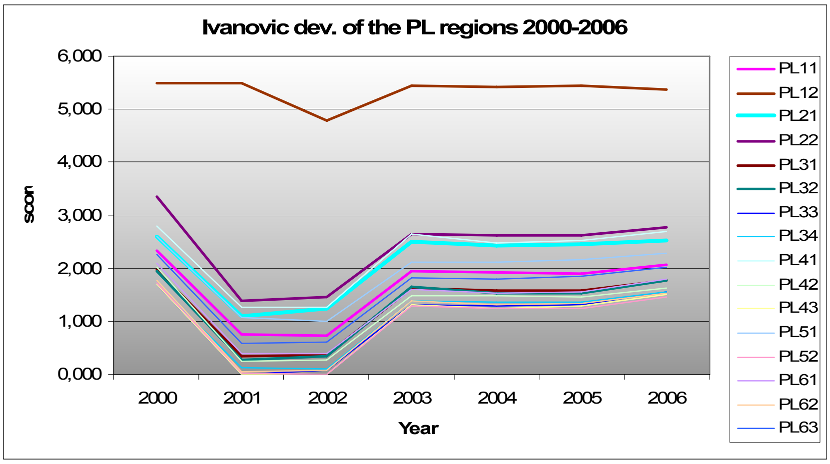

Figure 4. Ivanovic deviation of the Polish NUTS2 regions

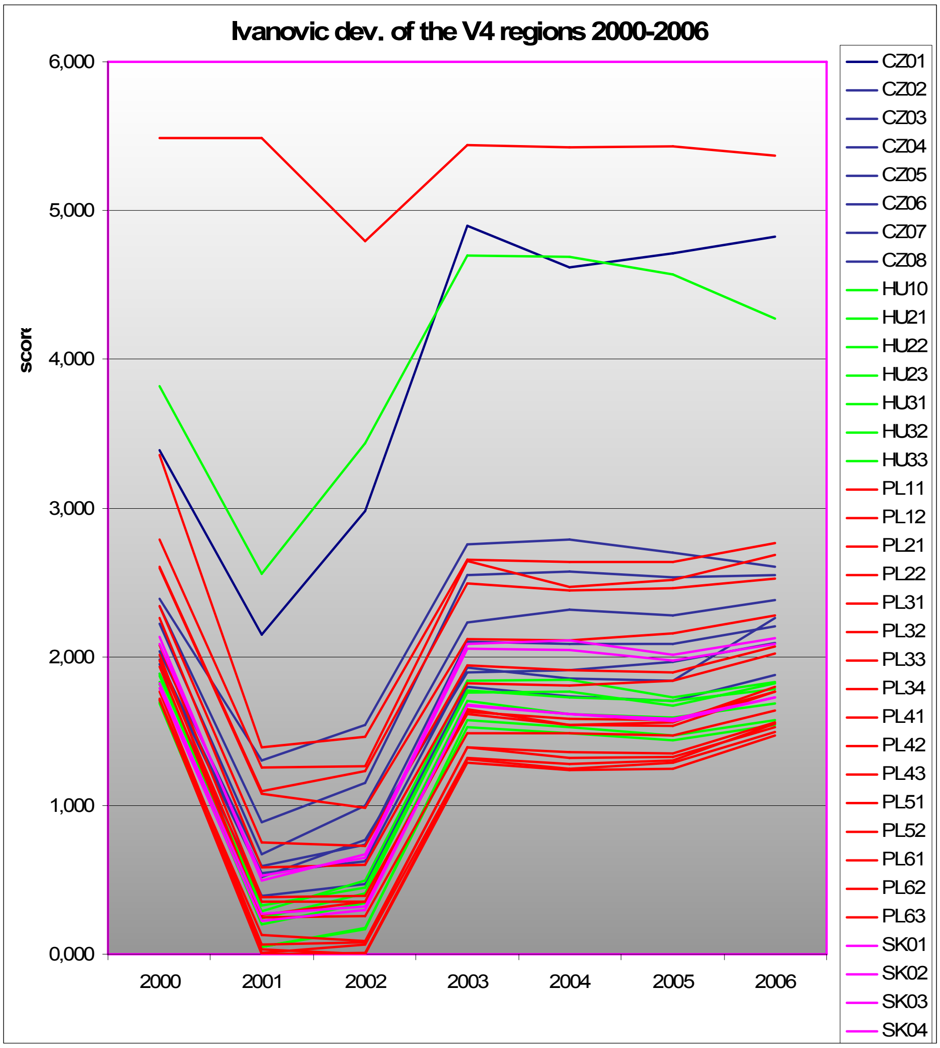

Figure 5. Ivanovic deviation of the V4 regions

Table 1

Application of Ivanovic deviation in NUTS 2 regions

Code	Region	2000	2001	2002	2003	2004	2005	2006
CZ01	Praha	3,388	2,150	2,981	4,898	4,617	4,714	4,829
CZ02	Strední Cechy	2,392	1,302	1,543	2,753	2,786	2,702	2,604
CZ03	Jihozápad	2,035	0,595	0,732	2,100	2,088	2,085	2,204
CZ04	Severozápad	1,883	0,393	0,469	1,801	1,735	1,706	1,877
CZ05	Severovýchod	2,220	0,675	1,001	2,226	2,321	2,276	2,380
CZ06	Jihovýchod	2,343	0,884	1,154	2,547	2,575	2,529	2,550
CZ07	Strední Morava	1,953	0,522	0,766	1,896	1,907	1,964	2,089
CZ08	Moravskoslezsko	1,984	0,540	0,625	1,925	1,855	1,838	2,261
HU10	Közép-Magyarország	3,816	2,555	3,437	4,699	4,691	4,567	4,275
HU21	Közép-Dunántúl	1,866	0,284	0,493	1,841	1,846	1,724	1,830
HU22	Nyugat-Dunántúl	1,888	0,325	0,443	1,761	1,765	1,673	1,824
HU23	Dél-Dunántúl	1,694	0,045	0,175	1,526	1,484	1,441	1,543
HU31	Észak-Magyarország	1,688	0,046	0,170	1,570	1,524	1,469	1,576
HU32	Észak-Alföld	1,812	0,236	0,408	1,777	1,722	1,701	1,779
HU33	Dél-Alföld	1,830	0,197	0,342	1,699	1,616	1,588	1,685
PL11	Lódzkie	2,341	0,752	0,731	1,939	1,912	1,897	2,070
PL12	Mazowieckie	5,486	5,489	4,796	5,442	5,427	5,435	5,370
PL21	Malopolskie	2,601	1,092	1,233	2,490	2,441	2,459	2,527
PL22	Slaskie	3,353	1,388	1,464	2,649	2,633	2,634	2,761
PL31	Lubelskie	1,971	0,352	0,353	1,628	1,580	1,577	1,767
PL32	Podkarpackie	1,934	0,265	0,352	1,646	1,541	1,535	1,767
PL33	Swietokrzyskie	1,712	0,007	0,066	1,322	1,280	1,301	1,548
PL34	Podlaskie	1,706	0,127	0,086	1,389	1,362	1,353	1,562
PL41	Wielkopolskie	2,788	1,254	1,266	2,643	2,471	2,521	2,685
PL42	Zachodniopomorskie	2,015	0,248	0,256	1,485	1,485	1,467	1,637
PL43	Lubuskie	1,707	0,000	0,010	1,314	1,250	1,289	1,498
PL51	Dolnoslaskie	2,599	1,080	0,985	2,113	2,111	2,154	2,279
PL52	Opolskie	1,714	0,034	0,000	1,288	1,235	1,248	1,467
PL61	Kujawsko-Pomorskie	2,079	0,381	0,388	1,613	1,540	1,561	1,796
PL62	Warminsko-Mazurskie	1,766	0,060	0,082	1,393	1,320	1,326	1,529
PL63	Pomorskie	2,260	0,587	0,602	1,824	1,806	1,838	2,019
SK01	Bratislavský kraj	2,131	0,492	0,668	2,086	2,106	2,010	2,125
SK02	Západné Slovensko	2,082	0,529	0,645	2,054	2,042	1,974	2,086
SK03	Stredné Slovensko	1,819	0,268	0,321	1,672	1,614	1,578	1,726
SK04	Východné Slovensko	1,792	0,227	0,298	1,674	1,617	1,566	1,724

Table 2

Superefficiency of NUTS2 regions in V4 countries

Code	Region	2000	2001	2002	2003	2004	2005	2006
CZ01	Praha	1,285	1,275	1,256	1,304	1,341	1,319	1,217
CZ02	Strední Cechy	0,749	0,681	0,768	0,761	0,747	0,716	0,784
CZ03	Jihozápad	0,901	0,918	0,824	0,883	0,903	0,803	0,776
CZ04	Severozápad	0,999	1,051	1,005	0,979	1,047	1,059	1,022
CZ05	Severovýchod	0,893	0,927	0,835	0,847	0,828	0,778	0,744
CZ06	Jihovýchod	0,892	0,902	0,855	0,875	0,851	0,796	0,790
CZ07	Strední Morava	0,910	0,884	0,743	0,848	0,842	0,734	0,705
CZ08	Moravskoslezsko	0,874	0,865	0,824	0,861	0,906	0,866	0,750
HU10	Közép-Magyarország	1,232	1,292	1,234	1,166	1,132	1,128	1,184
HU21	Közép-Dunántúl	0,954	0,955	0,813	0,850	0,898	0,898	0,863
HU22	Nyugat-Dunántúl	1,283	1,012	1,039	1,102	0,997	0,989	0,992
HU23	Dél-Dunántúl	1,090	1,038	0,948	0,936	0,970	0,909	0,886
HU31	Észak-Magyarország	0,926	1,065	0,946	0,926	1,016	0,996	0,904
HU32	Észak-Alföld	0,939	0,929	0,848	0,819	0,813	0,736	0,716
HU33	Dél-Alföld	1,102	1,009	0,890	0,831	0,835	0,769	0,756
PL11	Lódzkie	0,931	0,842	0,974	0,968	0,908	0,969	1,016
PL12	Mazowieckie	1,243	1,315	1,280	1,306	1,281	1,426	1,367
PL21	Malopolskie	0,809	0,816	0,825	0,837	0,878	0,879	0,887
PL22	Slaskie	1,610	1,516	1,622	1,541	1,579	1,514	1,441
PL31	Lubelskie	0,941	0,956	1,029	0,995	0,917	0,989	0,992
PL32	Podkarpackie	0,939	0,945	0,887	0,892	0,887	0,902	0,867
PL33	Swietokrzyskie	1,634	1,200	2,026	2,679	1,395	1,589	1,200
PL34	Podlaskie	0,993	1,049	0,939	0,924	0,830	0,859	0,902
PL41	Wielkopolskie	0,936	0,927	0,935	0,966	0,950	0,969	0,956
PL42	Zachodniopomorskie	1,169	1,180	1,138	1,231	1,140	1,223	1,246
PL43	Lubuskie	1,235	1,477	1,103	0,975	1,022	0,978	1,202
PL51	Dolnoslaskie	0,917	1,015	1,003	1,049	0,954	0,957	1,028
PL52	Opolskie	0,961	1,079	1,242	1,266	1,362	1,334	1,341
PL61	Kujawsko-Pomorskie	1,009	0,954	0,969	1,031	1,050	1,001	0,952
PL62	Warminsko-Mazurskie	1,078	1,030	0,943	0,962	0,906	0,926	0,948
PL63	Pomorskie	0,958	0,919	0,974	0,986	0,944	0,944	0,933
SK01	Bratislavský kraj	1,681	1,742	1,831	1,800	1,758	1,839	1,876
SK02	Západné Slovensko	1,030	1,085	1,036	1,006	0,958	1,024	1,076
SK03	Stredné Slovensko	0,884	0,941	0,979	0,965	0,910	0,920	0,965
SK04	Východné Slovensko	0,939	1,033	1,080	0,981	0,901	0,959	0,910

Acknowledgements

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MULTIPLE CRITERIA EVALUATION OF PROJECT GOALS

Abstract

This text is focussed on the quantitative evaluation of project SMART goals using the ANP method. This approach should be used in the project initiation phase. The very first step in all projects: business, home, or education, is to define goals and objectives. It is important to develop several goals that will enable us to be successful. Goals should be SMART – S – specific, significant, stretching, M – measurable, meaningful, motivational, manageable, A – agreed upon, attainable, achievable, acceptable, action-oriented, R – realistic, relevant, reasonable, rewarding, results-oriented, resourced, T – time-based, timely, tangible, trackable.

In our paper we make complex decisions about satisfying project SMART goals based on the ANP method using Super Decisions Software. As criteria we used a general SMART (SMARTER) model, as sub-criteria we use S, M, A, R, T sub-goals and as alternatives different project schedules are applied. We experiment with their mutual dependencies and we try to propose the best methodology for evaluating projects using the Analytic Network Process.

Keywords

Project management, project proposal, evaluation of project goals achievement, Analytic Network Process, Super Decisions Software.

Introduction

Modern project management uses many methods, techniques and tools for evaluating the quality of a project, both in the phase of proposal and in the phase of realization. Any project proposal should look very nice but a deeper study of its aim, time schedule, and resource allocation can detect whether it is likely to fail/to succeed. The majority of methods used for project evaluation are not based on quantitative approaches; sophisticated mathematical methods of multiple criteria evaluation of alternatives are used only very rarely.

The BOSCARD (Background, Objectives, Scope, Constraints, Assumptions, Risks and Deliverables) is a tool used to provide the terms-of-reference for the newly proposed project [Haughey 2011]. It is used in the phase of project initiation. What future events may impact the project? For forecasting the future and customizing the project schedule the Delphi Step by Step technique can help. The MoSCoW method (Must have this, Should have this if at all possible, Could have this if it does not affect anything else, Won't have this time but Would like in the future) is applied when establishing a clear understanding of the customers' requirements and their priorities [Clegg and Barker 2004]. The PEST is a strategic planning tool for evaluating the possible impact of Political, Economic, Social, and Technological factors on a project. The RACI model (Responsibility, Accountability, Consultation, and Information) is a straightforward tool used for identifying roles and responsibilities and avoiding confusion over those roles and responsibilities during a project [Smith 2005]. SWOT analysis (Strengths, Weaknesses, Opportunities, Threats) is a well known strategic planning tool used to evaluate the strengths, weaknesses, opportunities, and threats to a project [Armstrong 2006]. It involves specifying the objective of the project and identifying the internal and external factors that are favourable and unfavourable to achieving that objective.

The tool we consider in this paper is called SMART Goals evaluation. Project goals should be SMART [Doran 1981], which very briefly means: S – specific, significant, M – measurable, manageable, A – agreed, action-oriented, R – realistic, relevant, resourced, T – time-based, trackable. SMARTI project adds I – Integrated criteria to SMART goals, SMARTER project is moreover E – Ethical, Excitable, Enjoyable, Engaging, Ecological and R – Rewarded, Reassess, Revisit, Recordable.

For our paper it is more important to evaluate a completed project, final proposals, or project baselines (schedules) whether the SMART goals have been achieved or not. These goals are hard to measure; they have no final quantitative features. That is why we first tried to apply the Analytic Hierarchy Process [Saaty 1980, 1999] for comparing a finite set of projects with respect to general SMART goals (criteria) and individual SMART specifications (sub-criteria). Upon receiving the AHP results we decided to abandon this approach and apply the Analytic Network Process [Saaty 2001, 2003] for this evaluation. In the AHP each element in the hierarchy is considered to be independent of all the others, the ANP does not require independence among elements. It is very hard to make complex decision on satisfying project SMART goals without applying the ANP method because SMART specifications (sub-criteria) are not independent of each other. The majority of them are judged from sometimes very various points of views and one judgment strongly influences the others. R – goals (sub-criteria) "Realistic" and "Relevant" are typical examples of this dependency.

According to the survey [White and Fortune 2001] three crucial success factors mentioned most frequently by respondents were:

– Clear goals.
– Support from senior management.
– Adequate funds/resources.

That's why we focus mostly on project goals.

1. The ANP process as a tool for SMART goals evaluation

Multiple criteria decision models are used by many industries to quantify, compare, and manage their performance. The Analytic Network Process is one of the most effective tools in cases where the interactions among qualitative and quantitative factors generate a hierarchical or a network structure. Isik at all [2007] presented a conceptual performance measurement framework that takes into account company-level factors (objectives, strategies, resources) as well as project-level (risks, opportunities) and market-level factors (competition, demand).

As a tool for SMART goals achievement evaluation the hierarchy evaluation by the Analytic Network Process (Saaty 2001, 2003) should be used. Two types of the ANP model are theoretically defined: the Feedback System model and the Series System model. The Series System model usually consists of a tree, where the root is a model goal; branches of various levels have the meaning of criteria or sub-criteria of various levels respectively and finally the leaves represent a set of alternatives. Branches and leaves together determine the so-called model clusters (criteria, sub-criteria, project proposals). A crucial role for the project proposal evaluation plays the Feedback System model, where the clusters are linked one by one into a complex network system. We assume that all sub-criteria (within S, M, A, R and T criteria) influence and interact with each other and in the same way all the criteria are interconnected, too. It means that the hierarchy structure can be transformed into a network structure and the ANP feedback model seems to be a very suitable tool for solving this problem. The ANP super-matrices (non-weighted, weighted, limits) with possible cluster interactions and influences have to be defined and calculated and the most suitable project proposal will be selected according to the synthesis through addition of all the control criteria. The computation itself should be made using, for instance, the SuperDecisions software.

$$\mathbf{W} = \begin{array}{c} \\ C \\ S \\ P \end{array} \begin{array}{c} \begin{array}{ccc} C & S & P \end{array} \\ \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{W_C} \\ \mathbf{W_S} & \mathbf{W_{S*}} & \mathbf{0} \\ \mathbf{0} & \mathbf{W_A} & \mathbf{0} \end{pmatrix} \end{array}$$

where:

– W_C is the matrix of criteria weights with respect to projects,
– W_S is the matrix of sub-criteria weights with respect to criteria,
– W_{S*} is the matrix of sub-criteria weights with respect to each other,
– W_A is the matrix of project weights with respect to sub-criteria.

1.1. The ANP Criteria Level

The criteria level in the ANP process includes the general SMART goals. The goal is a general statement about a desired outcome with one or more specific objectives that define in precise terms what is to be accomplished within a designated time frame. The goal may be performance-related, developmental, a special project, or some combination [Sheid 2011].

– **S-criteria** evaluate who, what, when, where, why and how provides a project.
– **M-criteria** include a numeric or descriptive measurement of a project.
– **A-criteria** consider the resources needed and set a realistic goal.
– **R-criteria** ensure the goal is consistent with the mission of a project.
– **T-criteria** set a realistic deadline.

The project's scope, goals and sub-goals should be clearly outlined, taking into consideration cost, time and quality factors. The project should also be within the capacity of the project team and with incentive and encouragement to push the project forward to reach a more general goal.

1.2. The ANP Sub-criteria Level

Many meanings of the S, M, A, R, T letters are known from the literature. For example, A-criteria should be divided into Actionable, Attainable, Ambitious, Aspirational, Accepted/Acceptable, Aligned, Accountable, Agreed, Adapted, As-if-now, Adjustable, Adaptable etc. [RapidBI 2011].

For the ANP process analysis we have decided to use the following sub-criteria. We don't aspire to actual project evaluation, our aim is to propose a methodology of using the ANP process in this type of problems. Any other sub-criteria can be set or applied.

- **Specific** – What exactly are we going to do, with or for whom? "Specific" in the context of developing objectives refers to an observable action, behaviour or achievement.
- **Significant** – Significant goals are the ones that will make a positive difference in reality.
- **Measurable** – A method or procedure allowing the tracking and recording the project behaviour or progress must exist.
- **Meaningful –** Realization of a project must have a meaning. The goal must be very important.
- **Manageable –** The project must be easy to manage!
- **Achievable –** It must be possible for the project to be done in the timeframe/in this political climate/with this amount of money.
- **Action Oriented –** The plan of "attack" to make each goal real.
- **Relevant –** The project goal being set with an individual is something that can impact, change or be important to the organization
- **Realistic –** It must be an objective toward which you are both willing and able to work.
- **Resourced –** The goal or target being set is something that must have relevant resources allocated to be satisfied.
- **Time Based –** Every project task must have clearly stated a finish and/or a start date.
- **Trackable –** All goals should be trackable so you can see what your progress is. In terms of Project Management, you are tracking progress of project tasks in time, earned value, work etc.

1.3. The ANP Alternatives Level

As the alternatives level the actual projects or project proposals are set. Criteria and sub-criteria weights differ from project to project, according to the different project types and scopes. But these differences are not very distinguished; every project must be built according to similar rules and principles. Until now this part of the ANP process has not been included in our approach.

2. Network Model for Criteria and Sub-criteria Weights

As a tool for setting a dependency network among criteria and sub--criteria, SuperDecisions® software has been used. One hierarchic level underneath the goal node, SMART criteria level as a unique cluster (there are no dependencies – relations among them) starts the Analytic Network Model. Weights of criteria were set identically to 0,2.

The next level, consisting of subcriteria divided into clusters, is a crucial element of the process evaluating importance of each of them within the ANP process. These relations have been set according to the authors' experiences with managing various types of projects. Very often the project managers correlate the time frame of a project and its specificity (originality). The more specific the project, the more time it needs, and the less trackable it is. The most crucial are relations within the clusters A and R. Sometimes, the achievement of certain project goals excludes the achievement of others, while the achievement of one goal accelerates the achievement of another one. Also, a relevant goal must be realistic to achieve. Similar relationships have been observed within and among other clusters (Figure 1). These current weights are based on expert evaluation and calculated using Saaty's pairwise comparisons matrix – as integral part of SuperDecision® Software.

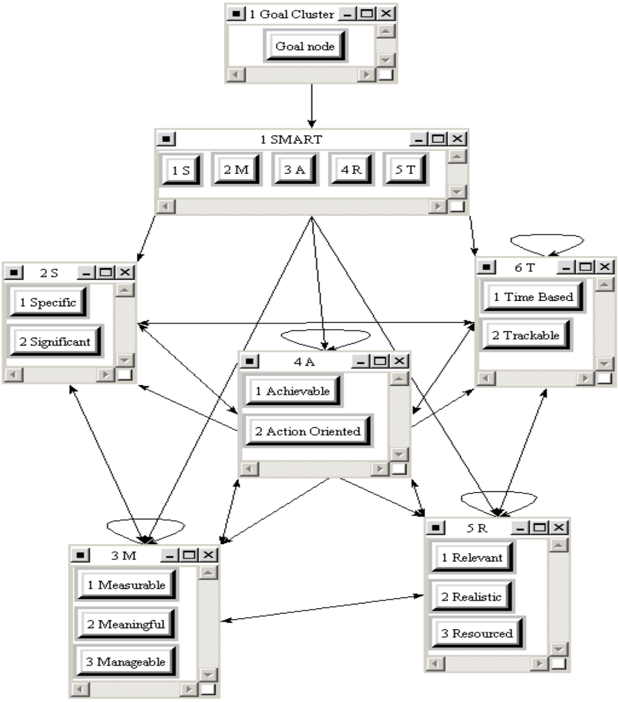

Figure 1. SMART Project Criteria Network (SuperDecisions Software)

2.1. The ANP Model Results

The first ANP result, un-weighted super-matrix for equal criteria weights, gives a good idea about clusters, established connections and their evaluation by weights.

	1 Goal Cluster	1 SMART				
	Goal node	1 S	2 M	3 A	4 R	5 T
Goal node	0	0	0	0	0	0
1 S	0,2	0	0	0	0	0
2 M	0,2	0	0	0	0	0
3,A	0,2	0	0	0	0	0
4 R	0,2	0	0	0	0	0
5 T	0,2	0	0	0	0	0
1 Specific	0	0,6	0	0	0	0
2 Significant	0	0,4	0	0	0	0
1 Measurable	0	0	0,4	0	0	0
2 Meaningful	0	0	0,3	0	0	0
3 Manageable	0	0	0,3	0	0	0
1 Achievable	0	0	0	0,6	0	0
2 Action Oriented	0	0	0	0,4	0	0
1 Relevant	0	0	0	0	0,3	0
2 Realistic	0	0	0	0	0,4	0
3 Resourced	0	0	0	0	0,3	0
1 Time Based	0	0	0	0	0	0,6
2 Trackable	0	0	0	0	0	0,4

Figure 2. First part of un-weighted matrix – Goal node and criteria

	2 S		3 M			4 A		5 R			6 T	
	1 Specific	2 Significant	1 Measurable	2 Meaningful	3 Manageable	1 Achievable	2 Action Oriented	1 Relevant	2 Realistic	3 Resourced	1 Time Based	2 Trackable
Goal node	0	0	0	0	0	0	0	0	0	0	0	0
1 S	0	0	0	0	0	0	0	0	0	0	0	0
2 M	0	0	0	0	0	0	0	0	0	0	0	0
3,A	0	0	0	0	0	0	0	0	0	0	0	0
4 R	0	0	0	0	0	0	0	0	0	0	0	0
5 T	0	0	0	0	0	0	0	0	0	0	0	0
1 Specific	0	0	0	0,7	0	1	1	0,5	0,7	1	1	1
2 Significant	0	0	0	0,3	0	0	0	0,5	0,3	0	0	0
1 Measurable	0,7	0	0	0	1	0,6	0	0,4	0	0	0,8	0,7
2 Meaningful	0	1	0	0	0	0	0	0,6	0,8	0	0	0
3 Manageable	0,3	0	0	0	0	0,4	0	0	0,2	1	0,2	0,3
1 Achievable	0,6	0,7	0	1	0,8	0	1	1	0,8	0	0	0
2 Action Oriented	0,4	0,3	1	0	0,2	0	0	0	0,2	1	1	1
1 Relevant	0,3	0,4	0	0,4	0	0,25	0	0	0	0	0	0
2 Realistic	0,5	0,5	0	0,4	0	0,5	0	0,7	0	0	1	0
3 Resourced	0,2	0,1	1	0,2	1	0,25	0	0,3	1	0	0	1
1 Time Based	0,6	1	0	1	0,5	0,4	0,4	0	0	0,3	0	1
2 Trackable	0,4	0	1	0	0,5	0,6	0,6	0	0	0,7	1	0

Figure 3. Second part of un-weighted matrix – Sub-criteria

Next, the ANP results and the limit matrix are used to calculate the final weights. The limit calculation gives the following weights for SMART sub-criteria (Figure 4). As supposed, the most important sub-criteria are those, usually mentioned first within S, M, A, R, T – Specific, Measurable, Action Oriented, Resourced and Trackable.

Subcriteria	Weight
1 Specific	0,159
2 Significant	0,008
1 Measurable	0,096
2 Meaningful	0,017
3 Manageable	0,064
1 Achievable	0,102
2 Action Oriented	0,129
1 Relevant	0,021
2 Realistic	0,059
3 Resourced	0,107
1 Time Based	0,101
2 Trackable	0,136

Figure 4. Sub-criteria limit weights

Figure 5 shows the sub-criteria weights in graphical form. We tried to calculate some typical cases based on different input assumptions but these criteria weight values remained very similar or the same. It is surprising that the “Significant” sub-criteria have the lowest limit weight. Analyzing this situation we have discovered that sometimes project managers do not understand what the expressions "Significant project, significant goal" mean. Often, they assume that every project is significant and therefore they have unrealistic expectations with regard to the values of this sub-criteria weight.

Figure 5. Sub-criteria limit weights – chart view

Conclusions

The aim of the paper was to analyse quality and quantity of various criteria frequently used while evaluating a project within a project management process. We have chosen the SMART approach to evaluate the fulfilment of project goals.

- The methodology used seems to be useful for the analysis of various projects according to more or less differing criteria.
- The ANP method allows description and research of complex dependencies among the important project criteria from various points of view. Network dependencies are typical for this problem.
- Our future research will be focused on criteria weights and on actual project proposal assessment. These weights have to be estimated by experts' judgement, because the set of SMART criteria requires the soft system approach.

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DEMATEL, ANP AND VICOR BASED HYBRID METHOD APPLICATION TO RESTORATION OF HISTORICAL ORGANS

Abstract

The aim of the paper is to perform ex-post analysis for of a portable organ restoration applying a hybrid method, which combines DEMATEL, ANP and VICOR multicriteria approaches. The analysis of results and comparison with the earlier research based in the scheme of ELECTRE I method is included.

Keywords

Portable organ reconstruction, hybrid multicriteria method, DEMATEL, ANP, VICOR.

Introduction

The goal of the paper [Trzaskalik-Wyrwa et al. 2006] was to determine the best way of renovation of a historic positive organ, found several years ago in the Podlasie region (part of Poland). Portable organs were very popular musical instrument in Poland in the seventeenth and eighteenth centuries. Its popularity was due above all to the ease of handling and possibility of easy transportation. Unfortunately, only several copies of this once so common instrument are nowadays extant in Poland.

The following decision problem arose: what is the best way to reconstruct the found instrument, taking into account a variety of criteria. In [Trzaskalik--Wyrwa et al. 2006], this issue has been presented as a multi-criteria decision--making problem and solved by means of ELECTRE I method. Reconstruction of the instrument was performed using the received. recommendation.

ELECTRE I method requires the decision maker to specify criteria weights and subsequent variant ratings. However, interactions among criteria are not analyzed. This possibility can be found in ANP method. Anyway, applying ANP in the source version is numerically troublesome. This is why a hybrid method which combines elements of three multicriteria approaches: DEMATEL [Fontela and Gabus 1974], ANP [Saaty 1999] and VIKOR [Opricovic and Tzeng 2007], allows to overcome numerical difficulties, emerging when performing the calculations using only the ANP method. A description of that hybrid method can be found in [Liu et al. 2012; Tzeng et al. 2007; Tzeng and Huang 2011]. This method is also presented in our paper.

The aim of the research is to perform ex-post analysis for the rediscovered instrument reconstruction by applying hybrid method mentioned above. We want to find out, how to take into account the mutual influence of criteria and whether these mutual influences will affect the selection of the final solution.

The paper is divided into four parts. In chapter 1, we present brief considerations of the decision-making problem, fully described in [Trzaskalik-Wyrwa et al. 2006]. The criteria considered and the decision variants are described. The second chapter includes a description of the hybrid method, wchich combines elements of DEMATEL, ANP, and VICOR. The third chapter presents the data provided by the expert (co-author of this paper – Małgorzata Trzaskalik-Wyrwa). Some of them (the evaluation of alternatives due to subsequent criteria) were used previously [Trzaskalik-Wyrwa et al. 2006], others (the specification of the mutual influence of criteria) have been prepared by the expert for the purposes of this study. An application of the hybrid method and detais of numerical calculations are presented. The fourth chapter contains an analysis of the results and compare them with the earlier results, obtained in [Trzaskalik-Wyrwa et al. 2006] by means of ELECRE I method.

1. Restoration of historical portable organ as a multicriteria decision process

1.1. Decision criteria

We consider a division of the values of historical organs into four groups: historic, artistic, musical and utilitarian values. We will describe the values constituting each of the four groups [Trzaskalik-Wyrwa et al. 2006].

Historic values determine the character of the object as a document and its influence on the development of historical knowledge. Among the values of this group are **scientific values**, due to the fact that an organ is an historic object, requiring a scholarly description. Also in this group are **technical values**, determining the ingenuity of the construction, the quality of the workmanship and the scientific value of its current condition. Also **historic emotional values**, perceived not only by scientists and scholars, but also by the public at large, belong here.

The **ownership values**, i. e., values stemming from the ownership of the original item (without hypothetical additions) are connected with honest approach of the conservators to the historic object, in which that what is preserved should be emphasised above all, as opposed to that what we think might have been there.

The group of **artistic values** is related to the perception of historic organs as works of art, and this is connected with the instrument's case. To this group belong **historic-artistic values**, determining whether the solutions chosen by the builders are typical or atypical as well as the importance of the original, its copy or its hypothetical reconstruction. **Artistic qualities** affect the public independently of the current fashion or style. The **artistic effect** of the case of historic organ should match musical impressions received by the audience from the musical compositions heard by it.

Musical values become apparent during a musical performance. We deal here with the issue of style (**historical musical value**) and of sound (**musical quality**). All of them taken together may reinforce the **musical influence** on the amateur listener. It can happen that the regaining of musical value and the preservation of the original technical solutions are conflicting goals. In such case we face the problem of **utilitarian values** of the historic instrument. The notions of live organ and dead organ are related to this group of values. A **musically dead organ** is an instrument that nowadays cannot fulfil its function of a musical instrument. A **live instrument** is an instrument capable of being used in musical performance, affecting the audience in various ways. Like any historic object, an organ as a piece of furniture can be also visually dead – not suitable for being exhibited, or else visually alive (independently of its musical "vitality") – beautiful, but unplayable.

1.2. Decision alternatives

On the basis of research and evaluation of the condition of the individual parts of the instrument (or their lack) 12 renovation treatments of the rediscovered instrument have been suggested. They are decision alternatives, described below [Trzaskalik-Wyrwa et al. 2006].

Alternative A1

Preservation of the instrument as a non-functional, visually unattractive object (“destrukt”) and its exhibition in the form of a group of museum exhibits.

Alternative A2

Integration of the elements of the instrument using racks necessary to place the individual elements in proper places.

Alternative A3

Integration of the parts of the instrument with full completion of the construction elements of the case (without covering the “windows” with reconstructed wood carved ornaments) according to their former shape as concluded from the elements preserved; completion of the missing parts of the mechanism. The pipes remain secured, but do not play.

Alternative A4

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome and covering of the “windows” by a neutral filling (canvas, wooden grill). The pipes remain secured, but do not play.

Alternative A5

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome. Hypothetical reconstruction of the wood carved ornaments filling out the “windows” (on the basis of comparative analysis – it is impossible to achieve the historical truth). The pipes remain secured, but do not play.

Alternative A6

Integration of the parts of the instrument with full completion of the construction elements of the case (without covering the “windows” by reconstructed wood carved ornaments) according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Bringing the extant pipes to working condition and reconstruction of the missing pipes, so as to match the sound capabilities of the extant pipes.

Alternative A7

Integration of the parts of the instrument with full completion of the construction elements of the case (without covering the “windows” by reconstructed wood carved ornaments) according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Exhibition of the extant historic pipes in a display case without giving them their former technical functionality. Reconstruction of the entire sound system according to preserved models.

Alternative A8

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome and covering the "windows" by a neutral filling (canvas, wooden grill). Bringing the pipes to a working condition and reconstruction of the missing pipes, so as to match the sound capabilities of the extant pipes.

Alternative A9

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome and covering the "windows" by a neutral filling (canvas, wooden grill). Exposition of the extant historical pipes in a display case without bringing them to a working condition. Reconstruction of the whole sound system according to preserved models.

Alternative A10

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved and completion of the missing parts of the mechanism. Reconstruction of the polychrome. Hypothetical reconstruction of the wood carved ornaments filling out the "windows" (on the basis of comparative analysis – it is impossible to achieve historical truth). Bringing the pipes to a working condition and reconstruction of the missing pipes so as to match the sound of the sound capabilities of the preserved pipes.

Alternative A11

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved and completion of the missing parts of the mechanism. Reconstruction of the polychrome. Hypothetical reconstruction of the wood carved ornaments filling out the "windows" (on the basis of comparative analysis – it is impossible to achieve historical truth). Exhibition of the preserved historic pipes in a display case without bringing them to a working condition. Reconstruction of the whole sound system according to preserved models.

Alternative A12

Preservation of the instrument in its non-functional, visually unattractive condition (as a "destrukt"). Making of an accurate copy. The evaluation focuses on the values of the copy, which is presented to the public.

2. The hybrid method

The hybrid method [Liu et al. 2012; Tzeng et al. 2007; Tzeng and Huang 2011] is a combination of:

– DEMATEL – applied to clarify relation between components,
– ANP – applied to determine the relationship between the criteria (in limited supermatrix),
– VIKOR – applied to obtain the index values in gaps.

Let A be a finite set of decision alternatives:

$$A = \{ A_1, A_2, \ldots, A_l \}$$

C – a set of criteria, divided into n categories (called here aspects, dimensions, clusters):

$$C = \{ C_1, C_2, \ldots, C_n \}$$

where:

$C_i = \{c_{i1}, c_{i2}, \ldots, c_{im_i}\}$, $i = 1, \ldots, n$ – is a subset of criteria in i-th aspect

and $\boldsymbol{F}$ – matrix of values of the j-th alternative in the k-th criterion:

$$\boldsymbol{F} = [f_{kj}] ,\ j = 1, 2, \ldots, l,\ k = 1, 2, \ldots, M$$

where $M = \sum_{i=1}^{n} m_i$

We assume that the criteria are defined so that a higher the value of the criterion is preferred to a lower one. Each criterion is assigned a positive number which reflects the valid contribution of that criteria.

The considered method is divided into following steps:

Step 1: Develop the structure of the problem.
The problem is broken down to a level structure.

Step 2: Develop the total influence matrix.
Based on the DEMATEL method, interactions between the aspects are explained to construct the map of the direct impact. This step is divided into three sub-steps:

Step 2a: Identify the average influence matrix $\boldsymbol{A}$

The initial matrix $\boldsymbol{A} = \left[a_{ij}^h\right]_{n \times n}$ is calculated, using experts' evaluations, where a_{ij}^h denotes the influence of i-th factor on j-th factor in h-th expert's opinion. If i-th element affects j-th element directly, then $a_{ij}^h \neq 0$; otherwise, $a_{ij}^h = 0$. W obtain:

$$\boldsymbol{A} = \left[a_{ij}\right]_{n \times n} \quad a_{ij} = \frac{1}{H}\sum_{h=1}^{H} a_{ij}^h \tag{1}$$

where H denotes the number of experts and $h = 1, 2, ..., H$. In particular, we can use expertise of one expert, then influence matrix $\boldsymbol{A}$ is obtained directly:

$$\boldsymbol{A} = \left[a_{ij}\right]_{n \times n} \tag{2}$$

Step 2b: Calculate the normalized influence matrix $\boldsymbol{X}$

We normalize the matrix $\boldsymbol{A}$, applying (3) and (4). The diagonal in normalized matrix is equal to 0, and the maximum sum of each row or column is equal to 1:

$$\boldsymbol{X} = s\boldsymbol{A} \tag{3}$$

where:

$$s = \min\left\{\frac{1}{\max\limits_{1 \le i \le n}\sum_{j=1}^{n} a_{ij}}, \frac{1}{\max\limits_{1 \le j \le n}\sum_{i=1}^{n} a_{ij}}\right\} \tag{4}$$

Step 2c: Compute the total-influence matrix $\boldsymbol{T}$

The total-influence matrix $\boldsymbol{T}$ can be obtained according to (5) ($\boldsymbol{I}$ denotes the identity matrix):

$$\boldsymbol{T} = \boldsymbol{X} + \boldsymbol{X}^2 + \ldots + \boldsymbol{X}^k = \boldsymbol{X}(\boldsymbol{I} - \boldsymbol{X})^{-1}, \text{ when } \lim_{k \to \infty} \boldsymbol{X}^k = [0]_{n \times n} \tag{5}$$

The proof of this relationship can be found in [Tzeng and Huang 2011].

Step 2d: Set a threshold value α and obtain the normalized α-cut total--influence matrix $\boldsymbol{T}^{\alpha}$

We have total-influence matrix $\boldsymbol{T}$ in the form:

$$\boldsymbol{T} = \begin{bmatrix} t_{11} & \dots & t_{1j} & \dots & t_{1n} \\ \vdots & & \vdots & & \vdots \\ t_{i1} & \dots & t_{ij} & \dots & t_{in} \\ \vdots & & \vdots & & \vdots \\ t_{n1} & \dots & t_{nj} & \dots & t_{nn} \end{bmatrix} \tag{6}$$

The α-cut total-influence matrix $\boldsymbol{T}^{\alpha}$ will be given by Eq. (7)

$$\boldsymbol{T}^{\alpha} = \begin{bmatrix} t_{11}^{\alpha} & \dots & t_{1j}^{\alpha} & \dots & t_{1n}^{\alpha} \\ \vdots & & \vdots & & \vdots \\ t_{i1}^{\alpha} & \dots & t_{ij}^{\alpha} & \dots & t_{in}^{\alpha} \\ \vdots & & \vdots & & \vdots \\ t_{n1}^{\alpha} & \dots & t_{nj}^{\alpha} & \dots & t_{nn}^{\alpha} \end{bmatrix} \tag{7}$$

where if $t_{ij} < \alpha$ then $t_{ij}^{\alpha} = 0$ else $t_{ij}^{\alpha} = t_{ij}$

The α-cut total-influence matrix $\boldsymbol{T}_{\alpha}$ needs to be normalized by dividing by the value:

$$d_i = \sum_{j=1}^{n} t_{ij}^{\alpha}, \ i = 1, 2, \dots, n \tag{8}$$

Finally we obtain $\boldsymbol{T}_{\mathrm{D}}$ as follow:

$$\boldsymbol{T}_D = \begin{bmatrix} t_{11}^{\alpha}/d_1 & \dots & t_{1j}^{\alpha}/d_1 & \dots & t_{1n}^{\alpha}/d_1 \\ \vdots & & \vdots & & \vdots \\ t_{i1}^{\alpha}/d_i & \dots & t_{ij}^{\alpha}/d_i & \dots & t_{in}^{\alpha}/d_i \\ \vdots & & \vdots & & \vdots \\ t_{n1}^{\alpha}/d_n & \dots & t_{nj}^{\alpha}/d_n & \dots & t_{nn}^{\alpha}/d_n \end{bmatrix} = \begin{bmatrix} t_{11}^{D} & \dots & t_{1j}^{D} & \dots & t_{1n}^{D} \\ \vdots & & \vdots & & \vdots \\ t_{i1}^{D} & \dots & t_{ij}^{D} & \dots & t_{in}^{D} \\ \vdots & & \vdots & & \vdots \\ t_{n1}^{D} & \dots & t_{nj}^{D} & \dots & t_{nn}^{D} \end{bmatrix} \tag{9}$$

Step 3: Compare all criteria to form the initial supermatrix.

The initial supermatrix can be obtained in two ways:

A. Initial (unweighted) supermatrix can be obtained by pairwise comparison of all criteria as in AHP method [Tzeng and Huang 2011, p. 161]:

$$
\boldsymbol{W} = \begin{array}{cc|cccc|}
 & & C_1 & C_2 & \cdots & C_n \\
 & & c_{11}\ c_{12}\ \cdots\ c_{1m_1} & c_{21}\ c_{22}\ \cdots\ c_{2m_2} & \cdots\ \cdots\ \cdots\ \cdots & c_{n1}\ c_{n2}\ \cdots\ c_{nm_n} \\
\hline
C_1 & \begin{matrix} c_{11} \\ c_{12} \\ \cdots \\ c_{1m_1} \end{matrix} & \boldsymbol{W}_{11} & \boldsymbol{W}_{12} & \cdots & \boldsymbol{W}_{1n} \\
C_2 & \begin{matrix} c_{21} \\ c_{22} \\ \cdots \\ c_{2m_2} \end{matrix} & \boldsymbol{W}_{21} & \boldsymbol{W}_{22} & \cdots & \boldsymbol{W}_{2n} \\
\cdots & \begin{matrix} \cdots \\ \cdots \\ \cdots \end{matrix} & \cdots & \cdots & \cdots & \cdots \\
C_n & \begin{matrix} c_{n1} \\ c_{n2} \\ \cdots \\ c_{nm_n} \end{matrix} & \boldsymbol{W}_{n1} & \boldsymbol{W}_{n2} & \cdots & \boldsymbol{W}_{nn}
\end{array} \tag{10}
$$

B. We can repeat the steps 2a-2d, as in [Liu et al. 2012], on initial influence matrix for all criteria

$$
\boldsymbol{A} = \left[a_{ij}\right]_{\sum_{i=1}^{n} m_i \times \sum_{i=1}^{n} m_i}
$$

Then we obtain the matrix $\boldsymbol{T}_C$ and $\boldsymbol{W} = (\boldsymbol{T}_C)^{\mathrm{T}}$

Finally we receive unweighted supermatrix $\boldsymbol{W}$ in the form:

$$
\boldsymbol{W} = \begin{bmatrix}
\boldsymbol{W}_{11} & \cdots & \boldsymbol{W}_{1j} & \cdots & \boldsymbol{W}_{1n} \\
\vdots & & \vdots & & \vdots \\
\boldsymbol{W}_{i1} & \cdots & \boldsymbol{W}_{ij} & \cdots & \boldsymbol{W}_{in} \\
\vdots & & \vdots & & \vdots \\
\boldsymbol{W}_{n1} & \cdots & \boldsymbol{W}_{nj} & \cdots & \boldsymbol{W}_{nn}
\end{bmatrix} \tag{11}
$$

Step 4: Obtain the weighted supermatrix.

The normalized $\boldsymbol{T}_{\mathrm{D}}$ is multiplied by unweighted supermatrix $\boldsymbol{W}$ to obtain weighted supermatrix $\boldsymbol{W}^{\alpha}$. The results are shown in Eq. (12).

$$\boldsymbol{W}^{\alpha} = \boldsymbol{T}_D \times \boldsymbol{W} = \begin{bmatrix} t_{11}^D \times \boldsymbol{W}_{11} & \cdots & t_{i1}^D \times \boldsymbol{W}_{1j} & \cdots & t_{1n}^D \times \boldsymbol{W}_{1n} \\ \vdots & & \vdots & & \vdots \\ t_{i1}^D \times \boldsymbol{W}_{i1} & \cdots & t_{ij}^D \times \boldsymbol{W}_{ij} & \cdots & t_{in}^D \times \boldsymbol{W}_{in} \\ \vdots & & \vdots & & \vdots \\ t_{n1}^D \times \boldsymbol{W}_{\mathrm{n1}} & \cdots & t_{nj}^D \times \boldsymbol{W}_{nj} & \cdots & t_{nn}^D \times \boldsymbol{W}_{nn} \end{bmatrix} \tag{12}$$

Step 5: Obtain the limit supermatrix.

The ANP weights of each criterion can be obtained from limit supermatrix:

$$\boldsymbol{W}^{lim} = \lim_{k \to \infty} \left(\boldsymbol{W}^{\alpha}\right)^k \tag{13}$$

The evaluation of the total preference is performed by means of VIKOR method, which can be divided into following steps:

Step 6: Check the best value f_k^* and the worse value f_k^-.

f_k^* represents the positive-ideal point, that means the expert gives the scores of the best value (aspired levels) for each criterion and f_k^- represents the negative-ideal point, that means the expert gives the scores of the worst values for each criterion. Those values can be computed by the traditional approach, using Eqs. (14) and (15) to obtain the results:

$$f_k^* = \max_i f_{ki}, \quad i = 1, 2, \ldots, l \tag{14}$$

$$f_k^- = \min_i f_{ki}, \quad i = 1, 2, \ldots, l \tag{15}$$

or by setting aspire levels vector as in Eq. (16)

$$f^* = \left(f_1^*, f_2^*, \cdots, f_M^*\right) \tag{16}$$

and setting worst value vector as in Eq. (17)

$$f^- = \left(f_1^-, f_2^-, \cdots, f_M^-\right) \tag{17}$$

Step 7: Calculate the mean of group utility S_i and the maximal regret Q_i.

The S_i represents the ratios of distance to the positive-ideal, it means the synthesized gap for all criteria. The Q_i represents the maximal gap-ratios (regret) of normalized distance to the aspired level in all criteria, that is, the maximal gap for prior improvement. Those values can be computed respectively by Eqs. (18) and (19):

$$S_i = \sum_{k=1}^{M} w_k r_{ki} \tag{18}$$

$$Q_i = \max_k \{w_k r_{ki} \mid k = 1, 2, \ldots M\} \tag{19}$$

where:

w_k – represents the influential weights of the k-th criterion from previous step,

$r_{ki} = \dfrac{\left|f_k^* - f_{ki}\right|}{\left|f_k^* - f_k^-\right|}$ – represents the gap-ratios (regret) of normalized distance to the aspired level point

Step 8: Obtain the comprehensive indicator R_i .

The values can be computed using Eq. (20).

$$R_i = v\left(S_i - S^*\right)/\left(S^- - S^*\right) + (1-v)\left(Q_i - Q^*\right)/\left(Q^- - Q^*\right) \tag{20}$$

where:

$S^* = \min_k S_k$ or we can set $S^* = 0$ (the aspired level)

$S^- = \max_k S_k$ or we can set $S^- = 1$ (the worst situation)

$Q^* = \min_k Q_k$ or we can set $Q^* = 0$ (the aspired level)

$Q^- = \max_k Q_k$ or we can set $Q^- = 1$ (the worst situation)

Therefore, when $S^* = 0$ and $S^- = 1$, and $Q^* = 0$ and $Q^- = 1$, we can re-write the Eq. (21) as:

$$R_i = vS_i + (1-v)Q_i \tag{21}$$

The coefficient $\nu = 1$ represents situation where only the average gap (average regret) is considered. Coefficient $\nu = 0$ represents situation where only the maximum gap to the prior improvement is considered. Generally the coefficient is adjusted according to the situation. In the most situations we can use $\nu = 0{,}5$.

Step 9: Rank the alternatives, sorting by the value of $min\{ R_i \mid i = 1,2,\ldots,l \}$.

3. Application of the hybrid method to the problem of restoration of historical organs

According to the expert's evaluations, we will consider the set of values, gathered in Table 1.

Table 1

Value Evaluation by Criteria

Criteria	*A1*	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*	*A8*	*A9*	*A10*	*A11*	*A12*
Historical-scientific value	10	8	6	6	6	6	6	6	6	6	6	0
Historical-technical value	4	6	10	8	8	6	10	6	10	6	10	0
Emotional value	10	10	10	8	6	10	6	6	4	4	2	0
Ownership value	10	10	10	9	5	8	5	9	4	5	0	0
Historical-artistic value	0	2	4	6	6	4	4	6	6	8	8	0
Artistic quality	0	0	2	4	8	2	2	4	4	8	8	8
Artistic influence	2	2	6	8	10	6	6	8	8	10	10	10
Historical-musical value	0	0	0	0	0	10	4	10	4	10	4	4
Musical quality	0	0	0	0	0	8	10	8	10	8	10	10
Musical influence	0	0	0	0	0	8	10	8	10	8	10	10
Visual-utilitarian value	2	4	6	8	10	6	6	8	8	10	10	10
Musical-utilitarian value	0	0	0	0	0	8	10	8	10	8	10	10

Step 1: According to the literature review and expert experiences, an value evaluation system including four dimensions and 12 criteria is established, as given in Table 2.

Table 2

The structure of evaluation criteria

Aspects/Dimensions		Criteria
C_1 Historical Values	c_{11}	Historical-scientific value
	c_{12}	Historical-technical value
	c_{13}	Emotional value
	c_{14}	Ownership value
C_2 Artistic values	c_{21}	Historical-artistic value
	c_{22}	Artistic quality
	c_{23}	Artistic influence
C_3 Music values	c_{31}	Historical-musical value
	c_{32}	Musical quality
	c_{33}	Musical influence
C_4 Utilitarian values	c_{41}	Visual-utilitarian value
	c_{42}	Musical-utilitarian value

Step 2: The ratings for each criterion's relationship to sustainable development using a five-point scale ranging from 0 (no effect) to 4 (extremely influential) were collected.

Table 3

Influence between aspects

		C_1	C_2	C_3	C_4
Historical Values	C_1	x	2	2	3
Artistic values	C_2	3	x	0	3
Music values	C_3	3	0	x	2
Utilitarian values	C_4	0	4	4	x

Step 2a: Identify the average influence matrix ***A***

As it was difficult to the decision maker to determine the influence between aspects themselves, we calculated the influence as rounded average of all influences between the criteria in the aspects. The result is presented in Table 4.

Table 4

Influence matrix

		C_1	C_2	C_3	C_4
Historical Values	C_1	2	2	2	3
Artistic values	C_2	3	2	0	3
Music values	C_3	3	0	3	2
Utilitarian values	C_4	0	4	4	0

Step 2b-2d: Total-influential dimensions matrix $\boldsymbol{T}_D$.

We used $\alpha = 0{,}1$ so it was necessary to normalize the resulting matrix. Result is presented in Table 5.

Table 5

Total-influential aspects (dimensions) matrix $\boldsymbol{T}_D$.

	C_1	C_2	C_3	C_4
C_1	0,00	0,41	0,28	0,31
C_2	0,32	0,00	0,32	0,36
C_3	0,28	0,41	0,00	0,31
C_4	0,29	0,43	0,29	0,00

Step 3: Compare all the criteria to form the initial supermatrix.
Because only direct impacts of the criteria were available, we use method **B**, repeating steps 2a-2d to the matrix presented in Table 6.

Table 6

Influential matrix ***A*** on criteria

	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}
c_{11}	0	3	1	4	4	4	1	4	1	1	2	2
c_{12}	3	0	1	4	2	1	0	2	1	1	2	2
c_{13}	2	2	0	3	2	2	1	2	1	2	4	4
c_{14}	4	4	4	0	4	4	4	4	4	4	3	3
c_{21}	3	3	3	4	0	4	4	2	2	2	4	0
c_{22}	2	2	2	2	3	0	2	0	0	0	4	0
c_{23}	1	1	1	4	4	4	0	0	0	0	4	0
c_{31}	3	3	2	4	0	0	0	0	4	4	0	4
c_{32}	1	1	0	2	0	0	0	4	0	4	0	4
c_{33}	1	1	0	2	0	0	0	4	4	0	0	4
c_{41}	0	0	0	0	4	4	4	0	0	0	0	0
c_{42}	0	0	0	0	0	0	0	4	4	4	0	0

The result of repeating steps 2a-2b is shown in Table 7. Now we use $\alpha = 0$ but it is also necessary to normalize the resulting matrix.

Table 7

Total-influential criteria matrix $\boldsymbol{T}_C$

	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}
c_{11}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0
c_{12}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0
c_{13}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0
c_{14}	0	0	0	0	0,068	0,14	0,257	0,043	0,23	0,191	0,072	0
c_{21}	0	0	0	0,099	0	0,135	0,248	0,042	0,223	0,184	0,069	0
c_{22}	0	0	0	0,106	0,071	0	0,267	0,045	0,239	0,198	0,075	0
c_{23}	0	0	0	0,121	0,08	0,165	0	0,051	0,272	0,225	0,085	0
c_{31}	0	0	0	0,096	0,064	0,132	0,242	0	0,217	0,18	0,068	0
c_{32}	0	0	0	0,117	0,078	0,16	0,294	0,05	0	0,219	0,082	0
c_{33}	0	0	0	0,112	0,075	0,153	0,282	0,048	0,253	0	0,079	0
c_{41}	0	0	0	0,099	0,066	0,136	0,249	0,042	0,223	0,185	0	0
c_{42}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0

Step 4: We obtain the weighted supermatrix by multiplying matrixes $(\boldsymbol{T}_C)^{\mathrm{T}}$ and $\boldsymbol{W}$ presented in tables 5 and 7. The result is presented in Table 8.

Table 8

Weighted supermatrix W^{α}

	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}
c_{11}	0	0	0	0	0	0	0	0	0	0	0	0
c_{12}	0	0	0	0	0	0	0	0	0	0	0	0
c_{13}	0	0	0	0	0	0	0	0	0	0	0	0
c_{14}	0	0	0	0	0,041	0,044	0,05	0,027	0,032	0,031	0,031	0,029
c_{21}	0,02	0,02	0,02	0,022	0	0	0	0,021	0,025	0,024	0,024	0,022
c_{22}	0,041	0,041	0,041	0,045	0	0	0	0,042	0,051	0,049	0,049	0,046
c_{23}	0,075	0,075	0,075	0,082	0	0	0	0,078	0,094	0,09	0,09	0,084
c_{31}	0,011	0,011	0,011	0,012	0,017	0,019	0,021	0	0	0	0,013	0,012
c_{32}	0,058	0,058	0,058	0,064	0,092	0,099	0,113	0	0	0	0,069	0,065
c_{33}	0,048	0,048	0,048	0,053	0,076	0,082	0,093	0	0	0	0,057	0,054
c_{41}	0,019	0,019	0,019	0,02	0,03	0,032	0,036	0,019	0,024	0,022	0	0
c_{42}	0	0	0	0	0	0	0	0	0	0	0	0

Step 5: By multiplying weighted supermatrix W^{α} we obtain the limit supermatrix $W^{\lim}$ presented in Table 9.

Table 9

Limit supermatrix W^{lim}

	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}
c_{11}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
c_{12}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
c_{13}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
c_{14}	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126
c_{21}	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058
c_{22}	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119
c_{23}	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220
c_{31}	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036
c_{32}	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189
c_{33}	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157
c_{41}	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094
c_{42}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000

The weights obtained are shown in Table 10.

Table 10

The evaluation criteria

Aspects/Dimensions		Criteria	Weight
C_1 Historical Values	c_{11}	Historical-scientific value	0,000
	c_{12}	Historical-technical value	0,000
	c_{13}	Emotional value	0,000
	c_{14}	Ownership value	0,126
C_2 Artistic values	c_{21}	Historical-artistic value	0,058
	c_{22}	Artistic quality	0,119
	c_{23}	Artistic influence	0,220
C_3 Music values	c_{31}	Historical-musical value	0,036
	c_{32}	Musical quality	0,189
	c_{33}	Musical influence	0,157
C_4 Utilitarian values	c_{41}	Visual-utilitarian value	0,094
	c_{42}	Musical-utilitarian value	0,000

Steps 7-9: The results are presented in Table 11.

Table 11

Vikor Method Evaluation of Alternatives

Dimensions/Criteria		Weight	f^*	f	*A1*	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*	*A8*	*A9*	*A10*	*A11*	*A12*
C_1. Historical Values																
c_{11}	Historical-scientific value	0	10	0	0	0	0	0	0	0	0	0	0	0	0	0
c_{12}	Historical-technical value	0	10	0	0	0	0	0	0	0	0	0	0	0	0	0
c_{13}	Emotional value	0	10	0	0	0	0	0	0	0	0	0	0	0	0	0
c_{14}	Ownership value	0,126	10	0	0	0	0	0,013	0,063	0,025	0,063	0,013	0,076	0,063	0,126	0,126
C_2. Artistic values																
c_{21}	Historical-artistic value	0,058	10	0	0,058	0,046	0,035	0,023	0,023	0,035	0,035	0,023	0,023	0,012	0,012	0,058
c_{22}	Artistic quality	0,119	10	0	0,119	0,119	0,096	0,072	0,024	0,096	0,096	0,072	0,072	0,024	0,024	0,024
c_{23}	Artistic influence	0,22	10	0	0,176	0,176	0,088	0,044	0	0,088	0,088	0,044	0,044	0	0	0
C_3. Music values																
c_{31}	Historical-musical value	0,036	10	0	0,036	0,036	0,036	0,036	0,036	0	0,021	0	0,021	0	0,021	0,021
c_{32}	Musical quality	0,189	10	0	0,189	0,189	0,189	0,189	0,189	0,038	0	0,038	0	0,038	0	0
c_{33}	Musical influence	0,157	10	0	0,157	0,157	0,157	0,157	0,157	0,031	0	0,031	0	0,031	0	0
C_4. Utilitarian values																
c_{41}	Visual-utilitarian value	0,094	10	0	0,076	0,057	0,038	0,019	0	0,038	0,038	0,019	0,019	0	0	0
c_{42}	Musical-utilitarian value	0	10	0	0	0	0	0	0	0	0	0	0	0	0	0
				S_i	0,811	0,78	0,638	0,552	0,492	0,351	0,341	0,24	0,255	0,168	0,183	0,23
				Q_i	0,189	0,189	0,189	0,189	0,189	0,096	0,096	0,072	0,076	0,063	0,126	0,126
$\nu = 0,5$				R_i	0,5	0,485	0,414	0,371	0,341	0,223	0,218	0,156	0,165	0,116	0,155	0,178
Rank					**12**	**11**	**10**	**9**	**8**	**7**	**6**	**3**	**4**	**1**	**2**	**5**

Conclusions

According to the hybrid method considered in the paper alternative *A10* was classified as the best one.

We will compare results obtained in the present research by means of hybrid method with the results obtained previously applying ELECTRE I method.

Table 12

Ranks

No.	Metoda Electre	No.	Hybryd method	No.	Hybryd method
1.	*A8, A10*	1.	*A10*	7.	*A6*
2.	*A6, A9, A11*	2.	*A11*	8.	*A5*
3.	*A7*	3.	*A8*	9.	*A4*
4.	*A3, A12*	4.	*A9*	10.	*A3*
5.	*A1, A2, A4, A5*	5.	*A12*	11.	*A2*
		6.	*A7*	12.	*A1*

In both rankings alternative *A10* was classified as the best one. The alternative *A11* was classified as the second in the hybrid method and was better than the alternative *A8*, classified in ELECTRE I method into the first class and recommended for further realization. The rest of the alternatives were classified similarly in the both methods. It is less important, because the considered decision problem was formulated as the best alternative choice problem.

It is seen that taking into account the mutual influence of criteria causes a change of recommendation. When applying ELECRE I method, the decision maker could choose between two alternatives: *A8* and *A10*. After the analysis of these alternatives the decision maker concluded that the alternative *A10* is better. When applying the hybrid method we obtained a ranking in which alternative *A10* was the best one. The alternative *A8*, recommended previously, was placed in the new ranking at the third position, so its chance to be recommended on the basis of the hybrid method is small.

The expert's ex post opinion (several years after reconstruction of the instrument) seems interesting. In perspective, it is seen that earlier choice of the alternative *A11* (which was second in the new ranking) would be better because of the possibility of the use of the instrument in musical performances. It is connected with the revision of criteria values for decision alternatives.

The alternative *A11* recommends reconstruction of the whole sound system according to preserved models. This solution gives the possibility of uncomprising use of the new pipes to obtain satisfactory level of sound. The conservation of several original pipes and the adjustment to them the rest of reconstructed pipes to their loudness caused an additional adverse result (among other problems) that the instrument plays too softly, and the "historical" timbre makes up for this insufficiently.

Also, the problem of hypothetical shape of wood-carver's decorations in upper box windows (not hitherto reconstructed) could be positively solved now, as not causing damage to historical substance and, at the same time considerably increasing visual attraction of the monument.

The knowledge obtained during the decision process and later can be used in conservation works in the future.

The recommendation of the alternative *A8* (first compared with the alternative *A10* while performing ELECTRE I analysis) was prepared by the expert on the merits of the case. The merits and arguments from the field of historical objects restoration should be the most important – the ranking (of course significant) is of auxiliary importance.

A detailed analysis of the hybrid method assumptions and justification of joint applications of DEMATEL, ANP and VICOR methods is a separate problem. Such an analysis has not been performed yet and it should be presented in future research.

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MULTICRITERIA EVALUATION OF FUZZY NET PRESENT VALUE[1]

Abstract

In this paper it is shown how to assess the degree of influence of various factors on the value of the project (NPV). The assessment is based on grouping and ranking of cash flows linked to various factors. The formulas are generated both for crisp and fuzzy net present value analysis. The projects are then evaluated on the basis of at least two criteria: the NPV and the risk (positive or negative) linked to the factors which have most influence on the project's NPV whose change may change the NPV considerably. In applications, fuzzy present values of different factors are calculated and compared for two different cases.

Keywords

Fuzzy Logic, Fuzzy Net Present Value, Ranking Fuzzy Numbers.

Introduction

Investment decisions are strategic decisions, which directly affect the position of a firm in the market. One of the most important criteria used to select an investment project is its worth for the decision makers. The discounted cash flows analyses (mostly net present value (NPV) analysis) are preferred to evaluate the investment projects. The NPV of a project, calculated before the beginning of the project, is the most widely used criterion to evaluate a project – it is generally accepted that the higher the NPV, the better the project.

However, things are not as simple that. It is widely known that investment decisions are always exposed to a high degree of uncertainty

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and risk. Thus, the NPV cannot be the only project selection criterion, because it may change and be in reality, once the project is implemented, substantially different from its planned value. That is why each investment project has to be evaluated taking into account more than just this one criterion. It is widely agreed that the risk of the project has to be taken into account too.

If risk is understood both in the negative (as a possible threat) and positive (as a possible chance), it is good to know where risk lies in a given investment project and which factor can change or be changed in such a way that its influence on the NPV of the project would be high. Each investment project is influenced by several factors, which may be dependent or independent of the company in question (behavior and situation of the customers and suppliers, payment conditions, prices, "make or buy" decisions etc.). The aim of the paper is to show how these factors and their possible influence on the project value can be identified. Each project should then be evaluated on the basis of its NPV and of the risk of a change in the NPV, linked to various factors influencing the value of the project.

What is more, as each investment project is a long-term project and its parameters are always connected with risk and uncertainty, we consider here the fuzzy approach to the estimation of the project's parameters. Fuzzy numbers allow us to model the incomplete knowledge about cash flows in the future.

The outline of the paper is as follows: first we present the classical approach to the evaluation of investment projects, but extending the classical definition of the NPV so that different factors influencing it, as well as their degree of influence, may be identified. Then we briefly describe the fundamentals of the fuzzy logic [Zadeh 1965; Ross 1995]. Finally the fuzzy approach to investment project evaluation is presented [Chio and Park 1994; Kuchta 2000; Zhang et al. 2011; Sorenson and Lavelle 2008], into which we incorporate the proposed approach taking into account those factors which influence the NPV. Subsequently, we present one of several possible approaches to fuzzy numbers ranking, which will be needed while estimating the "force" of individual factors influencing the NPV. The approach chosen permits to adjust the method to the attitude of the decision maker (pessimistic or optimistic). The paper concludes with two numerical examples.

1. Net Present Value Analysis

One of the most used discounted cash flow analysis method is net present value analysis which is calculated by adding up the present values of all cash flows into or out of the project.

The basic formula of present value of a single future payment ($PV(F)$) at the end of n^{th} year from now is given in Eq. 1.1 where F stands for the amount of the payment and i stands for the compound interest rate.

$$PV(F) = F\frac{1}{(1+i)^n} \tag{1.1}$$

The formula of net present value of a cash flow series ($NPV(F_1, \dots F_m)$) which has m different payments is calculated by Eq. 1.2 where F_j stands for the amount of the payment and n_j stands for the time period of the payment.

$$\begin{aligned} NPV(F_1, \dots F_m) = -I + \left(F_1\tfrac{1}{(1+i)^{n_1}}\right) + \left(F_2\tfrac{1}{(1+i)^{n_2}}\right) + \\ + \dots + \left(F_m\frac{1}{(1+i)^{n_m}}\right) \end{aligned} \tag{1.2}$$

Sometimes the initial investment of the project can be distributed over several years. Eq. 1.3 gives NPV when the initial investment is distributed over z years:

$$\begin{aligned} NPV = -I_0 - \left(I_1\frac{1}{(1+i)^{n_1}}\right) - \left(I_2\frac{1}{(1+i)^{n_2}}\right) - \dots - \left(I_z\frac{1}{(1+i)^{n_z}}\right) + \\ + \left(F_1\frac{1}{(1+i)^{n_1}}\right) + \left(F_2\frac{1}{(1+i)^{n_2}}\right) + \dots + \left(F_m\frac{1}{(1+i)^{n_m}}\right) \end{aligned} \tag{1.3}$$

The NPV is influenced by several factors. Sometimes small changes in some cash flow factors could be introduced – or occur independently of us – which result in a substantially better or worse NPV. For example, if the NPV is strongly influenced by labor costs, the decision maker could prefer to hire lower qualified workers or to outsource the work. If the factors influencing strongly the NPV are the payment conditions offered to the customers, the deadlines of the payments could be changed. Sometimes the NPV may be very sensitive to the situation of one of the customers or the suppliers. At that point, the decision maker may want to know the degree of influence of the situation of a given customer or supplier, of the chosen resources, suppliers, payment conditions etc. on the net present value of the project.

We propose, thus, to reorder the cash flow from Eq. (1.3) into groups of influence: each group consists of cash flows which depend on one specific factor (e.g. the situation of one customer, the decision to hire a certain workforce etc.). Of course, we assume that such reordering is possible – that it is possible to classify the cash flows into such groups which are influenced mainly by one factor. It is a limiting assumption, but less limiting than that – generally assumed – about the independence of the cash flows in subsequent years – the flows in various years are usually not independent, as the choice of one supplier, of the payment conditions or of resources used has its consequences for several cash flows in different years.

Thus let F_{jl} ($j = 1,\ldots, n$, $l = 1,\ldots, k$) denote the groups of cash flows occurring in the j-th year influenced by the l-th factor. The cash flows dependent on the l-th factor in the whole project are represented in Eq. 1.4, and the present value of these flows are calculated by Eq. 1.5 (we assume here that the investment is excluded from the influence of the factors, which is not a limiting assumption – the approach will be identical to the one that includes those factors).

$$F_l = F_{1l} + F_{2l} + F_{3l} + \cdots + F_{nl} \qquad l = 1, \ldots k \tag{1.4}$$

$$PV^l = \sum_{j=1}^{n} \frac{F_{jl}}{(1+i)^{n_j}} \qquad l = 1, \ldots k \tag{1.5}$$

As shown in Eq. 1.7, the NPV of the project is equal to the sum of the NPVs of the cash flows linked to the individual factors and to the PV^I, which is the present value of the initial investment given by Eq. 1.6.

$$PV^I = PV(I_0) + PV(I_1) + \cdots + PV(I_z) \tag{1.6}$$

$$NPV = -PV^I + PV^1 + PV^2 + PV^3 + \cdots + PV^k \tag{1.7}$$

Let us assume that we have the following inequality:

$$PV^1 > PV^2 > PV^3 > \cdots > PV^k \tag{1.8}$$

If the “greater-than-relations” stand for substantial differences between values $PV^1, PV^2, PV^3, \ldots, PV^k$, then a unitary change (intended by us or independent of us) in the values $PV^1, PV^2, PV^3, \ldots, PV^k$ leads to a different change in the NPV of the project. Thus, if Eq. 1.8 holds, the cash flows F_1 constitutes the main risk source (positive or negative) for the project, F_2 the next one etc. Now we have to evaluate, how easy it is for F_1 to change. If we can easily increase it, then F_1 constitutes a chance. If our environment can easily decrease it, then it constitutes a negative risk. The same analysis may be applied to F_2, F_3 etc.,

For another project we may have another ranking:

$$PV^k > PV^{k-1} > PV^{k-2} > \cdots > PV^1 \tag{1.9}$$

In such a case we would have to start our analysis of the project from F_k, F_{k-1} etc. Thus for one project the main positive or negative risk may be due to such factors as the inventory level or the payment conditions, while for another project the main risk factor may be the choice of suppliers, of resources etc. These factors may be more or less susceptible to change – and this information has to influence our project evaluation apart from the basic criterion, the NPV.

2. Fuzzy Logic

Fuzzy set theory was introduced by Zadeh in 1965. A fuzzy set is defined as a class of objects with a continuum of grades of membership, which is characterized by a membership function that assigns to each object a grade of membership ranging between zero and one. A fuzzy set A in U is characterized by a membership function $\mu_A(x)$ which associates with each point in U a real number in interval $[0, 1]$, with the value of $\mu_A(x)$ at x representing "the grade of membership" of x in A [Zadeh 1965]. We can also interpret $\mu_A(x)$ as the possibility degree of x being the actual value of a magnitude which is not known to us for the moment

The fuzzy number type most often used is a so-called triangular fuzzy number, in short TFN. The membership function for a triangular fuzzy number $\widetilde{M} = (m_l, m_m, m_r)$, characterized by three crisp parameters $m_l < m_m < m_r$ is given in Eq. 2.1.

$$\mu_M(x) = \begin{cases} 0 & \\ 1 + \dfrac{x - m_m}{m_m - m_l} & \\ 1 - \dfrac{x - m_m}{m_r - m_m} & \\ 0 & \end{cases} \quad \text{if} \quad \begin{array}{c} x \le m_l \\ m_l < x < m_m \\ m_m \le x < m_r \\ x \ge m_r \end{array} \tag{2.1}$$

Algebraic operations for *TFN*s $\widetilde{M} = (m_l, m_m, m_r)$ and $\widetilde{N} = (n_l, n_m, n_r)$ are given by the following formulas with the order of addition, subtraction, multiplication, division and multiplication by a scalar [Chen et al. 1992]:

$$\widetilde{M} \oplus \widetilde{N} \cong (m_l + n_l, m_m + n_m, m_r + n_r) \tag{2.2}$$

$$\widetilde{M} \ominus \widetilde{N} \cong (m_l - n_r, m_m - n_m, m_r - n_l) \tag{2.3}$$

$$\tilde{M} \otimes \tilde{N} \cong \begin{cases} (m_l n_l, m_m n_m, m_r n_r) & \tilde{M} \geq 0, \tilde{N} \geq 0 \\ (m_l n_r, m_m n_m, m_r n_l) \quad \text{if} & \tilde{M} \leq 0, \tilde{N} \geq 0 \\ (m_r n_r, m_m n_m, m_l n_l) & \tilde{M} \leq 0, \tilde{N} \leq 0 \end{cases} \tag{2.4}$$

$$\tilde{M} \oslash \tilde{N} \cong \begin{cases} \left(\frac{m_l}{n_r}, \frac{m_m}{n_m}, \frac{m_r}{n_l}\right) & \tilde{M} \geq 0, \tilde{N} \geq 0 \\ \left(\frac{m_r}{n_r}, \frac{m_m}{n_m}, \frac{m_l}{n_l}\right) \quad \text{if} & \tilde{M} \leq 0, \tilde{N} \geq 0 \\ \left(\frac{m_r}{n_l}, \frac{m_m}{n_m}, \frac{m_l}{n_r}\right) & \tilde{M} \leq 0, \tilde{N} \leq 0 \end{cases} \tag{2.5}$$

$$\lambda \otimes \tilde{M} \cong \begin{cases} (\lambda m_l, \lambda m_m, \lambda m_r) \\ (\lambda m_r, \lambda m_m, \lambda m_l) \end{cases} \text{if} \quad \begin{matrix} \lambda \geq 0 \\ \lambda \leq 0 \end{matrix} \quad \forall \lambda \in \mathcal{R} \tag{2.6}$$

The support of a fuzzy number $\tilde{M} = (m_l, m_m, m_r)$ is the interval $[m_l, m_r]$ – thus the domain on which the membership function takes on positive values, together with its boundary. The support of $\tilde{M}$ will be denoted as $\overline{M}$.

3. Fuzzy Net Present Value

Fuzzy present value of a single future payment ($\widetilde{PV}(F)$) occurred at the end of n^{th} year from now is given in Eq. 3.1 where $\tilde{F}$ stands for fuzzy amount of the payment and i stands for the compound interest rate.

$$\widetilde{PV}(\tilde{F}) = \frac{\tilde{F}}{(1+i)^n} \tag{3.1}$$

Kuchta [2000] defined the general formula of fuzzy net present value as given in Eq. 3.2, where $\tilde{F}_i$ denotes net cash flows in the time period i and ĩ denotes the fuzzy interest rate.

$$\widetilde{NPV} = -\tilde{I} + \sum_{i=0}^{n} \frac{\tilde{F}_i}{(1+\tilde{\imath})^i} \tag{3.2}$$

The formula of fuzzy net present value of a project ($\widetilde{NPV}$) which has m different payments and has an initial investment at the beginning of the project is calculated by Eq. 3.3.

$$\widetilde{NPV} = -\tilde{I} + \left(\tilde{F}_1 \frac{1}{(1+\tilde{\imath})^{n_1}}\right) + \left(\tilde{F}_2 \frac{1}{(1+\tilde{\imath})^{n_2}}\right) + \cdots + \left(\tilde{F}_m \frac{1}{(1+\tilde{\imath})^{n_m}}\right) \quad (3.3)$$

Eq. 3.4 gives $\widetilde{NPV}$ of a project which has m different payments and the investment distributed over z years:

$$\widetilde{NPV} = -\tilde{I}_0 - \left(\tilde{I}_1 \frac{1}{(1+\tilde{\imath})^{n_1}}\right) - \left(\tilde{I}_2 \frac{1}{(1+\tilde{\imath})^{n_2}}\right) - \cdots - \left(\tilde{I}_z \frac{1}{(1+\tilde{\imath})^{n_z}}\right) +$$
$$+ \left(\tilde{F}_1 \frac{1}{(1+\tilde{\imath})^{n_1}}\right) + \left(\tilde{F}_2 \frac{1}{(1+\tilde{\imath})^{n_2}}\right) + \cdots + \left(\tilde{F}_m \frac{1}{(1+\tilde{\imath})^{n_m}}\right) \quad (3.4)$$

Eq. 3.5 and Eq. 3.6 represent fuzzy equivalents of Eqs. 1.4 and 1.5.

$$\tilde{F}_l = \tilde{F}_{1l} + \tilde{F}_{2l} + \tilde{F}_{3l} + \cdots + \tilde{F}_{nl} \quad (3.5)$$

$$\widetilde{PV}^l = \sum_{j=1}^{n} \frac{\tilde{F}_{jl}}{(1+\tilde{\imath})^{n_j}} \quad (3.6)$$

As shown in Eq. 3.8, $\widetilde{NPV}$ of the project is equal to the sum of the $\widetilde{PV}$s due to individual factors because of the linearity of $\widetilde{NPV}$ and the definition of the addition of fuzzy numbers, where $\widetilde{PV}^I$ denotes the present value of the initial investment which is given in Eq. 3.7:

$$\widetilde{PV}^I = \widetilde{PV}(I_0) + \widetilde{PV}(I_1) + \cdots + \widetilde{PV}(I_z) \quad (3.7)$$

$$\widetilde{NPV} = -\widetilde{PV}^I + \widetilde{PV}^1 + \widetilde{PV}^2 + \widetilde{PV}^3 + \cdots + \widetilde{PV}^k \quad (3.8)$$

To be able to perform in the fuzzy case the type of analysis illustrated in the crisp case by Eqs. 1.8 and 1.9, we have to be able to rank fuzzy numbers. The ranking of fuzzy numbers is not unambiguous and there are several methods which can be used to obtain it. The choice depends on the decision maker, on his preferences and attitude (he may be a pessimist or an optimist or someone “in between”). In the following section we present one method only – which allows us to differentiate between the pessimistic and optimistic attitudes of the decision maker, but other ranking methods may be also used, without modifying the proposed approach.

4. Ranking Method for Fuzzy Numbers

In decision-making problems, having the fuzzy data leads to fuzzy numbers as final solutions. A fuzzy number represents many possible real numbers that have different membership values. It is not easy to compare the fuzzy numbers to determine which alternatives are preferred. Many authors have proposed fuzzy ranking methods that can be used to compare fuzzy numbers [Chen et al. 1992].

According to Kahraman and Tolga [2009] the fuzzy ranking method of Dubois and Prade (1978) which will be used in our paper is one of the most cited ranking methods. Dubois and Prade (1978) proposed four indices to assess the position of a fuzzy number $\widetilde{N}$ relative to the position of a fuzzy number $\widetilde{M}$ to find out if $\widetilde{N}$ is smaller than $\widetilde{M}$ or not, out of which we chose two.

$$\Pi_M(]N,+\infty)) = Poss(x > N|x \text{ } is \text{ } M) = \underset{u}{sup} \min\left(\mu_M(u), \underset{\substack{v \\ v\geq u}}{inf} 1 - \mu_N(v)\right) \tag{4.1}$$

$$= \underset{u}{sup} \underset{\substack{v \\ v\geq u}}{inf} \, min \, (\mu_M(u), 1 - \mu_N(v))$$

$$\mathcal{N}_M([N,+\infty)) = Ness(x \geq N|x \text{ } is \text{ } M) = \underset{u}{inf} \max\left(1 - \mu_M(u), \underset{v\leq u}{sup} \, \mu_N(v)\right) = \tag{4.2}$$

$$= \underset{u}{inf} \underset{\substack{v \\ v\leq u}}{sup} \max(1 - \mu_M(u), \mu_N(v))$$

The interpretation of the above indices, introduced by Dubois and Prade [1978], is as follows:

- $\Pi_M(]N,+\infty))$ is large if the values included in the support $\overline{N}$ and close to its upper bound are smaller or equal to the values in the support $\overline{M}$ and close to its upper bound
- $\mathcal{N}_M([N,+\infty))$ is large if the values included in the support $\overline{N}$ and close to its lower bound are smaller or equal to the values in the support $\overline{M}$ and close to its lower bound.

Thus, the two indices give the possibility to compare two fuzzy numbers according to the attitude of the decision maker (he may be a pessimist or an optimist – the first index corresponds to the positive attitude, we believe the flows will be rather rather be high, the second index corresponds to the pessimistic attitude).

5. Applications

In this section we will illustrate by means of examples how investment projects can be evaluated on the basis of different criteria.

Example 1: A manufacturer wants to decide to invest in a project which has 5 years of useful life. The company has five customers $C_i, i = 1,...5.$ and three suppliers $S_i, i = 1,...3.$ Supplier S_1 delivers materials for the production for customers C_1 and C_2, supplier S_2 delivers materials for the production for Customer C_3, and supplier S_3 delivers materials for the production for Customers C_4 and C_5. The sales (revenues) of the project for each customer are given as TFNs in Table 5.2. The profits before depreciation for each customer are given as TFNs in Table 5.3. The accounts payable for each customer are planned to be equal to the value of sales in 2 months for customer C_1, 1 month for customer C_2, 3 months for customer C_3, 1 month for customer C_4 and 3 months for customer C_5. The inventory of the project is planned to equal two months' sales, and liabilities of the suppliers are planned to be equal to the value of sales in 2 months, 1 month and 3 months, respectively for each supplier $S_i, i = 1,...3.$ There will be no salvage value of the assets after 5 years. The interest rate is taken as $\tilde{\imath} = (8,10,12)\%$.

This is the basic information about the project; because of the uncertainty it is given in the form of fuzzy numbers.

Table 5.1

The planned payments for the assets

Year	Payment for the assets
0	(450000, 500000, 550000)
1	(270000, 300000, 330000)
2	(90000, 100000, 110000)
3	(90000, 100000, 110000)
4	None
5	None

As mentioned earlier, it is assumed that the assets are not dependent on any factors such as customers, resource choice, payment conditions etc. Thus they will be taken into account totally, as in Eq. 3.8. Then we have the data for different customers, including the sales (thus the revenue which is not necessarily cash in the same period, Table 5.2), the profit before depreciation (Table 5.3 – as depreciation is linked to the assets, it will not be distributed among factors, thus not among customers either).

Table 5.2

The sales of the project for each customer

Year \ Sales	Customer C_1 (x1000$)	Customer C_2 (x1000$)
0	–	–
1	(950, 1028, 1116)	(500, 520, 540)
2	(1023, 1087, 1251)	(622, 654, 686)
3	(1110, 1190, 1270)	(734, 772, 810)
4	(1230, 1310, 1390)	(760, 790, 820)
5	(1400, 1460, 1520)	(850, 910, 970)
Year \ Sales	Customer C_3 (x1000$)	Customer C_4 (x1000$)
0	–	–
1	(1104, 1200, 1296)	(780, 830, 880)
2	(1840, 2000, 2160)	(910, 960, 1050)
3	(2208, 2400, 2592)	(1100, 1230, 1350)
4	(2208, 2400, 2592)	(1150, 1250, 1350)
5	(2208, 2400, 2592)	(1200, 1330, 1460)
Year \ Sales	Customer C_5 (x1000$)	
0	–	
1	(1090, 1130, 1170)	
2	(1220, 1270, 1320)	
3	(1760, 1850, 1940)	
4	(1935, 2046, 2157)	
5	(2100, 2330, 2560)	

Table 5.3

The profits before depreciation for each customer

Year	Profits before depreciation	
	Customer C_1 (x1000$)	Customer C_2 (x1000$)
0	–	–
1	(–30, 10, 40)	(–35,–10, 15)
2	(206, 219, 232)	(135, 150, 165)
3	(590, 622, 654)	(340, 367, 394)
4	(921, 992, 1063)	(560, 600, 640)
5	(935, 987, 1039)	(580, 610, 630)

Year	Profits before depreciation		
	Customer C_3 (x1000$)	Customer C_4 (x1000$)	Customer C_5 (x1000$)
0	–	–	–
1	(–48, 5, 58)	(–35, 4, 43)	(–45, 12, 69)
2	(313, 330, 347)	(200, 210, 220)	(300, 327, 354)
3	(836, 880, 924)	(550, 590, 640)	(798, 875, 952)
4	(1235, 1300, 1365)	(885, 930, 975)	(1121, 1286, 1451)
5	(1130, 1190, 1250)	(900, 950, 1000)	(1240, 1275, 1310)

The payment and inventory conditions above given determine the change of accounts payable, liabilities and inventory level in each year. All these data together allow for calculating the cash flows. The cash flows (with the investment payment excluded) occurring in a time period can be calculated by the Eq. 5.1 below, which represents the indirect calculation method of cash flow in a given year:

$$\tilde{F}_t = \tilde{P}_t \quad + \Delta\tilde{L}_t - \Delta\widetilde{AP}_t - \Delta\tilde{I}_t \tag{5.1}$$

where $\tilde{F}_t$ denotes total cash flows of the project in year *t*, $\tilde{P}_t$ denotes profit before depreciation in year *t*, $\Delta\tilde{L}_t$ denotes change in year *t* in the liabilities, $\Delta\widetilde{AP}_t$ denotes change in year *t* in the accounts payable, $\Delta\tilde{I}_t$ denotes change in year *t* in the inventory and $\tilde{C}_t$ denotes payments for the newly purchased fixed assets in year *t*.

Then we have:

$$\tilde{P}_t = \widetilde{PC}_{1_t} + \widetilde{PC}_{2_t} + \widetilde{PC}_{3_t} + \widetilde{PC}_{4_t} + \widetilde{PC}_{5_t} \tag{5.2}$$

where $\widetilde{PC}_{i_t}$ denotes the profit before depreciation in year *t* from the sales to customer i.

$$\Delta\tilde{L}_t = \widetilde{LS}_{i_t} = \widetilde{LS}_{1_t} - \widetilde{LS}_{1_{t-1}} + \widetilde{LS}_{2_t} - \widetilde{LS}_{2_{t-1}} + \widetilde{LS}_{3_t} - \widetilde{LS}_{3_{t-1}} \qquad (5.3)$$

Where $\widetilde{LS}_{i_t}$ denotes the liabilities in year *t* of seller i.

Similarly we have:

$$\Delta\widetilde{AP}_t = \widetilde{APC}_{1_t} - \widetilde{APC}_{1_{t-1}} + \widetilde{APC}_{2_t} - \widetilde{APC}_{2_{t-1}} + \widetilde{APC}_{3_t} - \widetilde{APC}_{3_{t-1}} + \\ + \widetilde{APC}_{4_t} - \widetilde{APC}_{4_{t-1}} + \widetilde{APC}_{5_t} - \widetilde{APC}_{5_{t-1}} \qquad (5.4)$$

Where $\widetilde{APC}_{i_t}$ denotes the accounts payable in year *t* of customer i.

$$\tilde{F}_t = \tilde{P}_t + \Delta\tilde{L}_t - \Delta\widetilde{AP}_t - \Delta\tilde{I}_t - \tilde{C}_t = \\ = \sum_{i=1}^{5} \widetilde{PC}_{i_t} + \sum_{i=1}^{3} \left(\widetilde{LS}_{i_t} - \widetilde{LS}_{i_{t-1}}\right) - \sum_{i=1}^{5} \left(\widetilde{APC}_{i_t} - \widetilde{APC}_{i_{t-1}}\right) \qquad (5.5)$$

The fuzzy net present value of the project is found by applying Eq. 3.4 to the values from Tables 5.1 and 5.4. Then we have $\widetilde{NPV} = (4950189, 7947617, 11289530)\$$.

Table 5.4

Total cash flows of the project

YEAR	CASH FLOWS TOTAL
0	(–550000, –500000, –450000)
1	(–1476333, –1082000, –699333)
2	(357333, 825000, 1277667)
3	(2262333, 2977000, 3718333)
4	(4121000, 5053833, 5986667)
5	(4060833, 4929667, 5788500)
$\widetilde{NPV}$	**(4950189, 7947617, 11289530)**

However, we want to see how the project is influenced by different factors. We start by choosing customers' situations as factors. We assume that it is not sensible to isolate flows year by year, because the yearly flows are not independent – if something happens in one year, it will have its consequences in the other years too. However, we think it is sensible to isolate the flows by customers – the customers may be assumed to be more independent from each other and we can imagine that we or the environment may influence, in a substantial way, only one customer at a time (but throughout all the years). Therefore, we group the flows not according to the years, but to the customers. The cash flows in the time period *t* for each customer C_i are calculated by Eq. 5.6.

$$\widetilde{FC}_{\iota_t} = \widetilde{PC}_{i_t} + \Delta\widetilde{LSC}_{i_t} - \Delta\widetilde{APC}_{i_t} \tag{5.6}$$

where $\widetilde{FC}_{\iota_t}$ denotes cash flows of the project for customer i in year t, $\widetilde{PC}_{i_t}$ denotes profit before depreciation for customer i in year t, $\Delta\widetilde{LSC}_{i_t}$ denotes change in year t in the liabilities of the supplier (suppliers) of the product for customer i, $\Delta\widetilde{APC}_{i_t}$ denotes change in year t in the accounts payable for customer i.

The present value of the total cash flows linked to each customer is found from Eq. 5.7.

$$PV(C_i) = \sum_{t=1}^{n} \frac{\sum_{i=1}^{m} \widetilde{FC}_{\iota_t}}{(1+i)^t} = \sum_{t=1}^{n} \frac{\sum_{i=1}^{m}\left(\widetilde{PC}_{i_t} + \Delta\widetilde{LSC}_{i_n} - \Delta\widetilde{APC}_{i_t}\right)}{(1+i)^t} \tag{5.7}$$

Here are the values $PV(C_i),\ i = 1, \dots, 5$

Table 5.5

The values $PV(C_i), i = 1, \dots, 5$

Year	Present value of cash flows	
	Customer C_1 (x1000$)	Customer C_2 (x1000$)
0	–	–
1	(–217560, –146667, –83951)	(–77381, –48485, –18519)
2	(71880, 172865, 268490)	(81447, 114738, 150463)
3	(344620, 454420, 589156)	(214246, 268345, 328117)
4	(521760, 663889, 825439)	(336931, 408784, 490142)
5	(476639, 597326, 737752)	(302155, 372553, 447483)

Year	Present value of cash flows		
	Customer C_3 (x1000$)	Customer C_4 (x1000$)	Customer C_5 (x1000$)
0	–	–	–
1	(–442857, –359091, –272222)	(–53571, 3636, 62963)	(–232143, –160303, –85802
2	(–65104, 52342, 178612)	(111607, 173554, 240055)	(172725, 250964, 334934)
3	(374871, 560982, 767371)	(322081, 443276, 585451)	(432762, 584773, 753082)
4	(662845, 887917, 1144441)	(490938, 635203, 799345)	(606496, 856044, 1141011)
5	(532246, 738896, 981401)	(445430, 589875, 758850)	(547756, 762284, 1014069)

Now we want to analyze the influence of the situation of each customer on the NPV of the project. We use the ranking method for fuzzy numbers described in the previous section.

Table 5.6

The possibility and necessity indices of the present values of cash flows determined for customers

Cases \ Indices	$\Pi_M(]N, +\infty))$	$\mathcal{N}_M([N, +\infty))$
$\widetilde{PV}_{C1} > \widetilde{PV}_{C2}$	1	1
$\widetilde{PV}_{C2} > \widetilde{PV}_{C1}$	0	0
$\widetilde{PV}_{C1} > \widetilde{PV}_{C3}$	0.3	0.5
$\widetilde{PV}_{C3} > \widetilde{PV}_{C1}$	0.7	0.5
$\widetilde{PV}_{C1} > \widetilde{PV}_{C4}$	0.42	0.4
$\widetilde{PV}_{C4} > \widetilde{PV}_{C1}$	0.58	0.6
$\widetilde{PV}_{C1} > \widetilde{PV}_{C5}$	0.05	0.16
$\widetilde{PV}_{C5} > \widetilde{PV}_{C1}$	0,95	0.84
$\widetilde{PV}_{C2} > \widetilde{PV}_{C3}$	0	0.05
$\widetilde{PV}_{C3} > \widetilde{PV}_{C2}$	1	0.95
$\widetilde{PV}_{C2} > \widetilde{PV}_{C4}$	0	0
$\widetilde{PV}_{C4} > \widetilde{PV}_{C2}$	1	1
$\widetilde{PV}_{C2} > \widetilde{PV}_{C5}$	0	0
$\widetilde{PV}_{C5} > \widetilde{PV}_{C2}$	1	1
$\widetilde{PV}_{C3} > \widetilde{PV}_{C4}$	0.62	0.42
$\widetilde{PV}_{C4} > \widetilde{PV}_{C3}$	0.38	0.58
$\widetilde{PV}_{C3} > \widetilde{PV}_{C5}$	0.28	0.23
$\widetilde{PV}_{C5} > \widetilde{PV}_{C3}$	0.72	0.77
$\widetilde{PV}_{C4} > \widetilde{PV}_{C5}$	0.1	0.24
$\widetilde{PV}_{C5} > \widetilde{PV}_{C4}$	0.9	0.76

We can now rank the discounted flows influenced by each customer in the pessimistic (when we assume that rather small flow values will occur) and in the optimistic cases. We assume that the corresponding relation is true if the respective possibility (necessity) index is greater than or equal to 0.7. Then in the optimistic case (the possibility measure) we get the following partial order: $\widetilde{PV}_{C5} > \widetilde{PV}_{C3} > \widetilde{PV}_{C1} > \widetilde{PV}_{C2}$, $\widetilde{PV}_{C5} > \widetilde{PV}_{C4}$, and in the pessimistic case the following one: $\widetilde{PV}_{C5} > \widetilde{PV}_{C1} > \widetilde{PV}_{C2}, \widetilde{PV}_{C5} > \widetilde{PV}_{C3} > \widetilde{PV}_{C2}$, $\widetilde{PV}_{C5} > \widetilde{PV}_{C4}$. Thus in both cases customer C_5 is responsible for the greatest flow contributing to the NPV of the project.

Let us now suppose that we have another project with the same NPV, but in this project customer C_2 has most influence on the NPV of the project. Now the selection of one of the two projects will be depending on the stability of the customer's situation, on the probability of their buying less, paying too late, etc. and also on the probability of the success of our endeavors influencing them and make them buying more, paying less, using cheaper suppliers, using less inventory etc.

We might also group the flows in different groups, e.g. flows due to accounts payable, liabilities and inventory level. In this way we might see which factor: the payment conditions offered by us to the customers, the payment conditions given to us by the suppliers, our decision about keeping a certain amount of inventory, has the most influence on the project's NPV and what the risk (positive or negative) connected to this factor is. This would constitute an additional criterion, apart from the NVP, to select an investment project.

Example 2: A company decides to manufacture a new product. One of two different machines X and Y, with different production volumes, can be used to manufacture the product. The new product needs two kinds of raw materials (R_A and R_B). The production requirements for the machines and cost and revenues for the product are given in Tables 5.7 and 5.8. The interest rate is taken as $i = (8,10,12)\%$.

Table 5.7

Parameters of the machines

	Machine X	Machine Y
Initial Investment	(1250000,1430000,1610000)$	(1240000,1320000,1400000)$
Production capacity per year	(1620000, 1800000, 1980000) units	(1140000, 1200000, 1260000) units
Work power per month	(3360, 3600, 3840) hours	(2016, 2160, 2304)hours
Raw material A per product unit	2	2
Raw material B per product unit	1	1
Energy cost per year	(11760,12000,12240)$	(8340,8400,8460)$
Useful life	10 years	10 years

Table 5.8

Costs and revenues generated by the new product

	Cost or Revenue
Labor cost per hour	(4.5, 5, 5.5)$
Cost of Raw material A per product	(0,8,0,9,1)$
Cost of Raw Material B per product	(0.9, 1, 1.1)$
Price of the product per unit	(9, 10, 11)$

Net annual cash flows and fuzzy present values of the cash flows for each factor are calculated and given in Table 5.9.

Table 5.9

Cash Flows and Present Values of the Factors

	TOTAL CASH FLOWS	
	Machine X	Machine Y
1	2	3
Net annual cash flows (total)	(8408640, 12930000, 17703120)	(6332868, 8620800, 10992588)
$\widetilde{PV}$ of total cash flows	(18660829, 49014873, 114812908)	(14054183, 32679615, 71292009)
$\widetilde{NPV}$ of the project	(17050829, 47584873, 113562908)	(12654183, 31359615, 70052009)
Annual Energy Cost	(12240, 12000, 11760)	(8460, 8400, 8340)
$\widetilde{PV}$ of Energy Costs ($\widetilde{PV}_E$)	(76269, 45489, 27164)	(54089, 31843, 18775)
Annual Labor Cost	(21120, 18000, 15120)	(12672, 10800, 9072)
$\widetilde{PV}$ of Labor Costs ($\widetilde{PV}_L$)	(98060, 68234, 46870)	(58836, 40940, 28122)
Annual raw material cost	(6138000, 5040000, 4050000)	(3906000, 3360000, 2850000)
$\widetilde{PV}$ of total raw material costs ($\widetilde{PV}_{RT}$)	(26266120, 19105565, 13621724)	(18483566, 12737044, 8668370)
Annual cost of raw material A	(3960000, 3240000, 2592000)	(2520000, 2160000, 1824000)
$\widetilde{PV}$ of raw material A costs ($\widetilde{PV}_{RA}$)	(16810317, 12282149, 8788209)	(11829482, 8188099, 5592496)
Annual cost of raw material B	(2178000, 1800000, 1458000)	(1386000, 1200000, 1026000)

Table 5.9 contd.

1	2	3
$\widetilde{PV}$ of raw material B costs ($\widetilde{PV}_{RB}$)	(9455803, 6823416, 4833515)	(6654084, 4548944, 3075873)
$\widetilde{PV}$ of initial investment ($\widetilde{PV}_I$)	(1610000, 1430000, 1250000)	(1400000, 1320000, 1240000)

To determine which machine has higher $\widetilde{NPV}$, the possibility indices of ranking cases are calculated and given in Table 5.10.

Table 5.10

Possibility and necessity indices for Fuzzy Net present Values of Two Machines

Indices / Cases	$\Pi_M(]N, +\infty))$	$\mathcal{N}_M([N, +\infty))$
$\widetilde{NPV}(X) > \widetilde{NPV}(Y)$	0.78	0.71
$\widetilde{NPV}(Y) > \widetilde{NPV}(X)$	0.22	0.29

We can see that the decision maker, regardless of his pessimistic or optimistic attitude, should decide to invest in Machine X. However, we want to evaluate this decision also according to other criteria, because the opposite relation ($\widetilde{NPV}(Y) > \widetilde{NPV}(X)$) is also true to some extent (this is the feature of fuzzy values, which are based on uncertainty and incomplete knowledge), thus we may still consider the choice of machine Y − if there is a too high negative risk linked to Machine X or a high positive risk (chance) linked to Machine Y.

To determine the importance of the factors which affect fuzzy net present value of the project “buy Machine X”, the possibility and necessity indices are calculated. We get only the values 0 or 1 which means the numbers don’t have intersections and the ranking is exactly known. We get the following ranking: $\widetilde{PV}_{RT} > \widetilde{PV}_{RA} > \widetilde{PV}_{RB} > \widetilde{PV}_I > \widetilde{PV}_L > \widetilde{PV}_E$. Thus the factors linked to the raw materials have the greatest influence here. If we judge that the prices of raw materials may change considerably and in the unfavorable direction, we may want to see what is the influence of this factor on the NPV of the project “buy machine Y” and we may found out that this project is biased in the first place by other types of risk and/or offers new chances. In such a situation the fuzzy NPV and the fuzzy ranking shown in Table 5.10 would not constitute the only criterion to choose a machine.

Conclusions

The fuzzy net present value (NPV) method is one of the most preferred investment analysis methods which can deal with the uncertainty of forecasting cash flows of investment projects. In the fuzzy net present value method the worth of an investment is defined by a fuzzy number and the decision maker has to decide whether to invest in the project by considering this number. In reality all the factors influencing fuzzy net present value of an investment should also be taken into account (e.g. the credibility of customers or suppliers, the cash flows resulting from work power, raw material selection, payment conditions etc.), together with their variability/stability. The decision maker should analyze, apart from the NPV, what is the factor which influences it most and how probable (possible) are changes in the flows linked to this factor. The final decision in the evaluation of a project should be made on the basis of NPV and the structure of its components dependent on individual factors. In the present paper we propose a method to perform such an evaluation. Further research is needed to propose a methodology of identifying different factors and verify the independence between selected groups of cash flows.

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