# MULTIPLE CRITERIA PROJECT SCHEDULING WITH PROJECT DELAY, RESOURCE LEVEL AND NPV OPTIMIZATION

#### **Abstract**

One of the most important phases in project management is planning. During this phase tasks are identified and scheduled. A schedule brings information on how tasks should be planned over time during the realization phase of the project. That is why scheduling is a critical issue in project management. The main project scheduling techniques are CPM and PERT. They deliver the schedule with the optimal project finish time and ensure the control of resource usage. In real-life applications the schedule should optimize not only the project finish time but also resource usage and cash flows. In research on the project scheduling problem the mathematical models are used to build an optimal project schedule. Frequently used are one-objective mathematical models for project scheduling. Few papers deal with the multiple objective project scheduling problem. Constraints and objectives in project scheduling are determined by three main issues: time, resource and costs, but only few papers consider all of them.

A zero-one programming formulation has been applied to solve a multiple criteria project scheduling problem in this paper. The purpose of this paper is to present the multiple criteria project scheduling problem with three objectives: project delay minimization, resource usage in each period of time minimization and NPV maximization.

#### **Keywords**

Project scheduling, multiple criteria optimization, zero-one programming.

#### Introduction

In recent years the project management problem became very popular because of its broad real-life applications. One of project definitions states that a project is a set of co-ordinated activities undertaken to meet specific objectives [Brandenburg 2002]. Each project has three main components: activities (tasks to do), resources required to carry out the project tasks,

and precedence relationships, which define the order in which activities should be performed [Kostrubiec 2003]. In summary: a project contains activities, which have an expected duration and resource requirements. They generate costs and cash flows and are constrained by resource limits and precedence relationships. In real-life applications a schedule should hold all those restrictions. That is why scheduling is a critical task in project management.

In project management, a schedule consists of a list of a project's terminal elements with intended start and finish dates. We can say that: "(...) scheduling is to forecast the processing of work by assigning resources to tasks and fixing their start times. (...) The different components of a scheduling problem are tasks, the potential constraints, the resources and the objective..." [Carlier and Chretienne 1988]. "Scheduling concerns the allocation of limited resources to tasks over time. It is a decision-making process that has a goal – the optimization of one or more objectives" [Pinedo 1995]. The main project scheduling techniques are CPM and PERT. CPM calculates the longest path of planned activities to the end of the project and also gives the shortest time of project realization. The Program Evaluation and Review Techniques (PERT) is a method to analyze the tasks involved in completing a given project, especially the time needed to complete each task. Those methods deliver schedules with the optimal project finish time and ensure the control of resource usage. In real-life applications the schedule should optimize not only the project finish time but also resource usage and cash flows.

The purpose of this paper is to analyze the problem of multiple criteria project scheduling problem and to discuss the multiple criteria project scheduling problem with three objectives: project delay minimization, minimization of resource usage in each period of time and NPV maximization. A zero-one programming approach has been applied to model such a problem.

The paper begins with an overview of literature and problem statement. Then, the mathematical model is described and a computational example is presented. The paper finishes with conclusions and ideas for future research.

# 1. Optimization in project scheduling problem – a literature overview

Constraints in project scheduling problem are determined by two main components: time and resources. We can discuss two types of resources: financial and non-financial (human resources and materials). The optimization criteria are determined by three main components: time, resources and economic indicators such as cost or NPV. When we take them into consideration

we can build various optimization models for the project scheduling problem. We can also present various projects depending on the number of objectives, thus we can have a one-objective project scheduling problem and a multiple-criteria project scheduling problem.

Each mathematical model for project scheduling problem needs to include basic constraints: precedence relationship constraints and information about the extent of variables.

In research on project scheduling problems, optimization models with one objective are the most popular. In this case we can build a model containing only basic constraints, in which the project completion time or NPV is optimized, or a model with one constraint in which time, capital or resources are constrained while the project completion time, NPV, cost or resource usage are optimized, or a model with few constraints. A resource constraint is not frequently used in models with resource usage optimization. A multiple objective mathematical model for the project scheduling problem is a combination of mathematical models mentioned above.

A problem with only basic constraints in which NPV is maximized has been solved by Russell [Russell 1970]. In this paper the author assumed that the cost is generated at the moment when the project starts and income is generated when some groups of activities are finished.

There are two types of mathematical models for the project scheduling problem with one constraint: the time constrained project scheduling problem and the resource constrained project scheduling problem. In the case of the resource constrained project scheduling problem we can differentiate between problems with non-financial resources and those with capital constraints.

The project scheduling problem with time constraints where NPV is maximized has been presented in the paper by Vanhoucke, Demeulemeester and Herrorlen [Vanhoucke et al. 2002]. In the problem described in the paper cash flows were generated at the time when each activity was finished.

There are two types of the resource constrained project scheduling problems: resource constrained project scheduling problem with time optimization and resource constrained project scheduling problem with NPV optimization.

The resource constrained project scheduling problem with time optimization was discussed by Shouman, Ibrahim, Khater and Forgani [Shouman et al. 2006] and the problem with NPV optimization was presented by Icmeli and Erenguc [Icmeli and Erenguc 1996]. The resource constrained project scheduling problem was also described in Talbot's paper [Talbot 1982]. The author presented this problem with time-resource tradeoffs.

Doersch and Patterson [Doersch and Patterson 1977] proposed a financial resources (capital) constrained project scheduling problem in their paper. They assumed that the capital is limited at the project start time. The capital availability changes throughout the project duration. Activities generate cash flows (outflows and inflows), which have influence on capital availability.

Vanhoucke, Demeulemeester and Herroelen [Vanhoucke et al. 2001] described a time- and resource-constrained project scheduling problem with NPV maximization.

A resource- and time-constrained project scheduling problem was also presented by Bartusch, Mohring and Readermacher [Bartusch et al. 1988]. A vector containing the finish time of each activity is minimized. The authors assumed that an activity should start in the "time window", which is the time between the earliest and the latest start times.

Bianco, Dell'Olmo and Speranza [Bianco et al. 1998] described resource-constrained project scheduling problems with financial and non-financial resources. Each activity can be executed in several ways. Additionally, each activity generates a given cost. The project budget is limited. Additionally, activities using the same resource cannot be scheduled at the same time. The project completion time is optimized in this problem.

Gaspars-Wieloch [Gaspars-Wieloch 2008] presented a paper on time and cost analysis for the project scheduling problem. The author considered a few mathematical models from the literature on this problem. In the models considered both time and cost can be a constraint and an objective function.

A multiple criteria project scheduling problem was described by Viana and de Sousa [Viana and Sousa 2000]. The authors proposed a mathematical model in which: project completion times are minimized, project delay is minimized and disruptions in resource usage are minimized. Renewable and nonrenewable resources are constrained in the problem. A binary variable is used in the model. We define  $x_{ijt} = 1$  when the operation j of the activity i is finished in time t. Otherwise,  $x_{ijt} = 0$ .

Leu and Yang [Leu and Yang 1999] considered a multiple-criteria resource-constrained project scheduling problem with time optimization, cost optimization and resource usage optimization. Also Hapke, Jaszkiewicz and Słowiński [Hapke et al. 1998] described that problem in their paper.

# 2. Project scheduling problem – basic elements of the model

The problem presented in this paper can be formulated as follows: there is a project to be scheduled. By 'scheduling' we understand setting the start and finish times of each activity. For each activity, resource requirements and budget are specified. Resource availability and precedence relationships are constrained.

The following example (Figure 1) describes the problem presented in this paper. We have a project containing 9 activities. The project is presented by an AOA (Activity On Arc) network. For each activity, its duration, required resources and net cash flows generated are shown in Table 1.

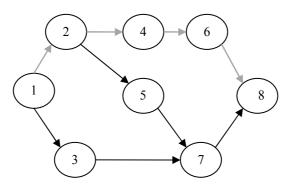


Figure 1. Activity network for example 1

For given durations and precedence relationships, the earliest start and finish times and the latest start and finish times were computed using the critical path method. Those times will be compared with the results obtained by using the mathematical model proposed in this paper.

Table 1

Example 1. Data

Activity	Duration	ES	EF	LS	LF	Slack	Critical tasks?	Renewable resources	Net cash flow
1	2	3	4	5	6	7	8	9	10
1-2	2	0	2	0	2	0	YES	2	-4
1-3	4	0	4	1	5	1	NO	1	-3

Table 1 contd.

1	2	3	4	5	6	7	8	9	10
2-4	1	2	3	2	3	0	YES	2	-1
2-5	2	2	4	5	7	3	NO	3	1
3-7	3	4	7	5	8	1	NO	4	3
5-7	1	4	5	7	8	3	NO	2	5
4-6	4	3	7	3	7	0	YES	1	7
6-8	3	7	10	7	10	0	YES	3	8
7-8	2	7	9	8	10	1	NO	2	10

Additionally, resource usage in each period is limited to 5 and the project duration time is limited to 15.

The following assumptions were made for the formulation of our mathematical model:

- project contains j = 1, ..., J activities,
- project duration is constrained to T (t = 0, ..., T),
- project is represented by AOA network,
- precedence relationships are Finish-to-Start type  $(S_{ij}$  set of predecessors i of activity j),
- $d_i$  activity j's duration,
- $F_i$  finish time of activity j,
- $F_{ij}$  finish time of predecessor i of activity j,
- -k = 1,..., K set of renewable resources,
- $r_{ik}^r$  amount of renewable resource k required by activity j.

Only renewable resources are taken into consideration. We assume that the amount of nonrenewable resources needed for the project execution is constant and is not limited for the period of time, but for the project. That is why we do not need to consider them in the model. If we do not have the necessary amount of non-renewable resources the project cannot be completed. Renewable resources are constrained in each period.

#### 2.1. Variables

The following binary variable is used in the model considered:

$$x_{it} = \{0, 1\} \quad (j = 1, ..., J, t = 1, ..., T)$$

where  $x_{jt} = 1$  when an activity j is finished in time t, otherwise  $x_{jt} = 0$ . In the problem considered we have  $j \times t$  variables. In the problem represented in the example 1 we have 9 activities and 15 time units for project execution, so the number of variables is  $9 \times 15$ , which is 135:

$$x_{1,1}, x_{1,2}, x_{1,3}... x_{J,T}.$$

# 2.2. An activity execution constraint

Because a binary variable is used in the model formulated, we have to ensure that each activity will be executed only once. We will write this constraint as the following equation:

$$\sum_{t=1}^{T} x_{jt} = 1 \qquad (j = 1, ..., J, t = 1, ..., T)$$

In the equation above we add the variables in each time unit for each activity. If the sum is equal 1 we are sure, that an activity *j* is finished only once.

In example 1 this constraint is formulated as follows (for the activity I-2):

$$x_{l,l+} x_{l,2+} x_{l,3+} x_{l,4+} x_{l,5+} x_{l,6+} x_{l,7+} x_{l,8+} x_{l,9+} x_{l,10+} x_{l,11+} x_{l,12+} x_{l,13+} x_{l,14+} x_{l,15} = 1.$$

# 2.3. Precedence relationships

The project scheduling problem considered in this paper has been presented by an AOA network. This network allows to consider only finish-to-start precedence relationships between activities. This type of precedence relationships can be formulated as follows:

$$F_{j} - d_{j} \ge F_{ij}$$
  $(j = 1, ..., J, t = 1, ...T, I \in S_{ij})$ 

In this case a successor can start only when its predecessor is finished.

In example 1 activity 2-4 can start when activity 1-2 is finished, so the set  $S_{ij}$  of predecessors i of activity j has only one element. The precedence relationship for this case can be formulated as follows:

$$F_2 - d_2 \ge F_1$$
.

In the case of precedence relationships we use activity finish time. We can calculate activity finish times by using the formula:  $F_j = \max\{ \bigvee_{t=1}^{T} (t \times x_{jt}) \}$ . So, the finish time for the activity *1-2* is:

$$F_{1} = \max\{1 \times x_{1,1}, 2 \times x_{1,2}, 3 \times x_{1,3}, 4 \times x_{1,4}, 5 \times x_{1,5}, 6 \times x_{1,6}, 7 \times x_{1,7}, 8 \times x_{1,8}, 9 \times x_{1,9}, 10 \times x_{1,10}, 11 \times x_{1,11}, 12 \times x_{1,12}, 13 \times x_{1,13}, 14 \times x_{1,14}, 15 \times x_{1,15}\}.$$

# 2.4. Project completion time optimization

The time criterion is frequently used in the literature. Many solutions for project completion time optimization are considered, e.g. each activity finish time minimization, last activity finish time minimization. In some cases project delays (delay is the difference between the planned and the actual finish time of an activity) minimization is also used.

In our model the project delay (not activity delays) is minimized. A delay is a situation when an activity is finished later than the latest finish time determined by the critical path method (a time given by the decision maker can also be used). Activity delays are summed up and reduced with theirs predecessors delays.

Mathematically, this criterion can be formulated as follows:

$$\sum_{j=1}^{J} \max\{0, F_j - LF_j\} - \sum_{i=1}^{I} \max\{0, F_i - LF_i\} \rightarrow \min(j = 1, ..., J, t = 1, ..., T)$$

The criterion of project completion time minimization for the example 1 is illustrated below.

$$x_{13} + 2x_{14} + 3x_{15} + 4x_{16} + 5x_{17} + 6x_{18} + 7x_{19} + 8x_{110} + 9x_{111} + 10x_{112} + 11x_{113} + 12x_{114} \\ + 13x_{115} + x_{26} + 2x_{27} + 3x_{28} + 4x_{29} + 5x_{210} + 6x_{211} + 7x_{212} + 8x_{213} + 9x_{214} + 10x_{215} + \\ x_{34} + 2x_{35} + 3x_{36} + 4x_{37} + 5x_{38} + 6x_{39} + 7x_{310} + 8x_{311} + 9x_{312} + 10x_{313} + 11x_{314} + 12x_{315} + x_{48} + \\ 2x_{49} + 3x_{410} + 4x_{411} + 5x_{412} + 6x_{413} + 7x_{414} + 8x_{415} + x_{59} + 2x_{510} + 3x_{511} + 4x_{512} + 5x_{513} + 6x_{514} \\ + 7x_{515} + x_{69} + 2x_{610} + 3x_{611} + 4x_{612} + 5x_{613} + 6x_{614} + 7x_{615} + x_{78} + 2x_{79} + 3x_{710} + 4x_{711} + 5x_{712} + \\ 6x_{713} + 7x_{714} + 8x_{715} + x_{811} + 2x_{812} + 3x_{813} + 4x_{814} + 5x_{815} + \\ x_{911} + 2x_{912} + 3x_{913} + 4x_{914} + 5x_{915} = \\ (2x_{14} + 3x_{15} + 4x_{16} + 5x_{17} + 6x_{18} + 7x_{19} + 8x_{110} + 9x_{111} + 10x_{112} + 11x_{113} + 12x_{114} \\ + 13x_{115} + 2x_{27} + 3x_{28} + 4x_{29} + 5x_{210} + 6x_{211} + 7x_{212} + 8x_{213} + 9x_{214} + 10x_{215} + \\ 2x_{35} + 3x_{36} + 4x_{37} + 5x_{38} + 6x_{39} + 7x_{310} + 8x_{311} + 9x_{312} + 10x_{313} + 11x_{314} + 12x_{315} + \\ 2x_{49} + 3x_{410} + 4x_{411} + 5x_{412} + 6x_{413} + 7x_{414} + 8x_{415} + 2x_{510} + 3x_{511} + 4x_{512} + 5x_{513} + 6x_{514} + 7x_{515} + 2x_{610} + 3x_{611} + 4x_{612} + 5x_{613} + 6x_{614} + 7x_{615} + 2x_{79} + 3x_{710} + 4x_{711} + 5x_{712} + 6x_{713} + 7x_{714} + 8x_{715} + 2x_{812} + 3x_{813} + 4x_{814} + 5x_{815} + 2x_{912} + 3x_{913} + 4x_{914} + 5x_{915})$$

$$\rightarrow min$$

The latest finish time for the activity I-2 is 2. The activity I-2 is delayed when it is finished later than in the second unit of time (that is why we have  $0 x_{II}$  and  $0 x_{I2}$ ). If the activity I-2 is finished in the third unit of time its delay will be  $1 (1x_{I3})$ , when it is finished in the fourth unit of time its delay is  $2 (2x_{I4})$ , and so on.

# 2.5. Resource level optimization

In the next criterion a resource usage level is optimized. Resource level optimization is not discussed in the literature frequently. In some papers a criterion in which the difference between resources available and required is minimized.

In our model the maximum resource usage level is minimized in each unit of time. It is described by the following objective function.

$$\max_{t=1,...,T} \left[ \sum_{i=1}^{J} r_{jk}^{r} \cdot x_{jt} \right] \rightarrow \min \left( j = 1,..., J, \ t = 1,..., T, \ k = 1,..., K \right)$$

The resource level minimization objective function for example 1 is illustrated below.

$$\begin{aligned} & Max\{(2x_{11} + x_{21} + 2x_{31} + 3x_{41} + 4x_{51} + 2x_{61} + x_{71} + 3x_{81} + 2x_{91}), \\ & (2x_{12} + x_{22} + 2x_{32} + 3x_{42} + 4x_{52} + 2x_{62} + x_{72} + 3x_{82} + 2x_{92}), \\ & (2x_{13} + x_{23} + 2x_{33} + 3x_{43} + 4x_{53} + 2x_{63} + x_{73} + 3x_{83} + 2x_{93}), \\ & (2x_{14} + x_{24} + 2x_{34} + 3x_{44} + 4x_{54} + 2x_{64} + x_{74} + 3x_{84} + 2x_{94}), \\ & (2x_{15} + x_{25} + 2x_{35} + 3x_{45} + 4x_{55} + 2x_{65} + x_{75} + 3x_{85} + 2x_{95}), \\ & (2x_{16} + x_{26} + 2x_{36} + 3x_{46} + 4x_{56} + 2x_{66} + x_{76} + 3x_{86} + 2x_{96}), \\ & (2x_{17} + x_{27} + 2x_{37} + 3x_{47} + 4x_{57} + 2x_{67} + x_{77} + 3x_{87} + 2x_{97}), \\ & (2x_{18} + x_{28} + 2x_{38} + 3x_{48} + 4x_{58} + 2x_{68} + x_{78} + 3x_{88} + 2x_{98}), \\ & (2x_{19} + x_{29} + 2x_{39} + 3x_{49} + 4x_{59} + 2x_{69} + x_{79} + 3x_{89} + 2x_{99}), \\ & (2x_{110} + x_{210} + 2x_{310} + 3x_{410} + 4x_{510} + 2x_{610} + x_{710} + 3x_{810} + 2x_{910}), \\ & (2x_{111} + x_{211} + 2x_{311} + 3x_{411} + 4x_{511} + 2x_{611} + x_{711} + 3x_{811} + 2x_{911}), \\ & (2x_{112} + x_{212} + 2x_{312} + 3x_{412} + 4x_{512} + 2x_{612} + x_{712} + 3x_{812} + 2x_{912}), \\ & (2x_{113} + x_{213} + 2x_{313} + 3x_{413} + 4x_{513} + 2x_{613} + x_{713} + 3x_{813} + 2x_{913}), \\ & (2x_{114} + x_{214} + 2x_{314} + 3x_{414} + 4x_{514} + 2x_{614} + x_{714} + 3x_{814} + 2x_{914}), \\ & (2x_{115} + x_{215} + 2x_{315} + 3x_{415} + 4x_{515} + 2x_{615} + x_{715} + 3x_{815} + 2x_{915})\} \rightarrow min \end{aligned}$$

The resources required are multiplied by the binary variable and summed up in each time unit. Then the maximum value is chosen. Resources are used only when the variable is 1.

#### 2.6. NPV optimization

The next criterion presented in this paper is the NPV maximization. Cash flow depends on activity duration and finish time.

This problem is frequently discussed in the literature. An example is considered in Icmeli and Erenguc's paper [Icmeli and Erenguc 1996]. There, cash flows are generated in each unit of time of activity duration. This problem can be formulated as follows:

$$\sum_{i=1}^{J} \left[ \sum_{t=1}^{d_j} \left[ c f_{jt} \cdot e^{\alpha(d_j - t)} \right] \cdot e^{-\alpha F_j} \right] \rightarrow \max.$$

In our paper we assume that cash flows are generated by activities at the end of their durations, so the criterion can be formulated as follows:

$$\sum_{j=1}^{J} cf_{j} \cdot e^{-\alpha F_{j}} \to \max \qquad (j = 1, ..., J, t = 1, ..., T)$$

In example 1 this criterion has the following form:

$$\begin{split} cf_1 \cdot e^{-\alpha F_1} + cf_2 \cdot e^{-\alpha F_2} + cf_3 \cdot e^{-\alpha F_3} + cf_4 \cdot e^{-\alpha F_4} + cf_5 \cdot e^{-\alpha F_{51}} + cf_6 \cdot e^{-\alpha F_6} \\ + cf_7 \cdot e^{-\alpha F_7} + cf_8 \cdot e^{-\alpha F_8} + cf_9 \cdot e^{-\alpha F_9} &\to \max. \end{split}$$

# 3. Multiple objective project scheduling problem

We can build various one-objective optimization models for the project scheduling problem using the criterion and constraints considered above. In some cases the objective function can be presented as a constraint, e.g. a resource constraint can be formulated as follows:

$$\max_{t=1,...,T} \left[ \sum_{j=1}^{J} r_{jk}^{r} \cdot x_{jt} \right] \leq R_{kt}.$$

The resource-constrained project scheduling problem with time optimization is dealt with in many papers. In this paper the resource constraint is not considered because a resource type criterion is used. Resources are rarely considered as both criterion and constraint in the same problem.

The multiple objective project scheduling problem containing all issues important in project management (presented in Section 2) – time, resources and NPV – can be formulated as follows:

$$\sum_{j=1}^{J} \max\{0, F_j - LF_j\} - \sum_{i=1}^{J} \max\{0, F_i - LF_i\} \to \min$$

$$(j = 1, ..., J, t = 1, ..., T)$$
(1)

$$\max_{t=1,...,T} \left[ \sum_{i=1}^{J} r_{jk}^{r} \cdot x_{jt} \right] \to \min \quad (j = 1,..., J, \quad t = 1,..., T, \quad k = 1,..., K)$$
 (2)

$$\sum_{j=1}^{J} cf_{j} \cdot e^{-\alpha F_{j}} \to \max \qquad (j = 1, ..., J, t = 1, ..., T)$$
(3)

with the following constrains:

$$\sum_{t=1}^{T} x_{jt} = 1 \quad (j = 1, ..., J, \quad t = 1, ..., T)$$
(4)

$$x_{jt} = \{0, 1\} \quad (j=1, ..., J, t=1, ..., T)$$
 (5)

$$F_j - d_j \ge F_{ij}$$
  $(j = 1, ..., J, t = 1, ...T, i \in S_{ij})$  (6)

There are many methods for solving a multiple objective problem. We can solve this problem by using the weighted method (then we will obtain a one-objective problem) or we can use methods dedicated to the multiple objective optimization.

If we solved this problem as a three separate one-objective problems we would obtain three very different schedules. By solving it as a multiple-objective optimization problem we will obtain a set of non-dominated solutions. Below are examples of non-dominated solutions. We denote the time criterion by C1, the resource criterion by C2, and the NPV criterion as C3.

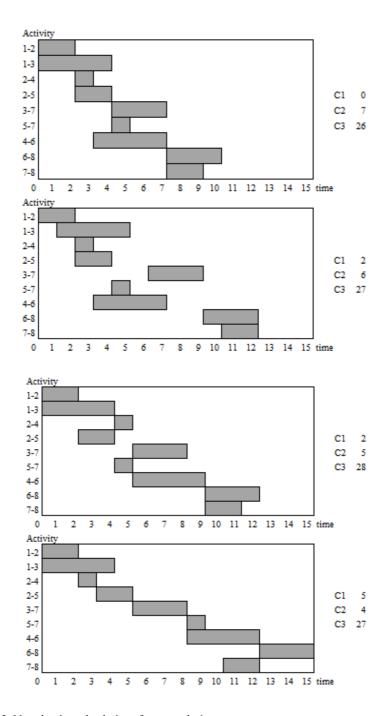


Figure 2. Non-dominated solutions for example 1

Four non-dominated solutions are presented above (Figure 2). We can see that the value of time criterion is between 0 (project finished on time) and 5 (project 5 time units delayed), of resource criterion is between 4 and 7 (the maximum level of resource usage) and of NPV is between 26 and 27 (Table 2).

Table 2

Example 1. Non-dominated solutions – criteria values

	S1	S2	S3	S4
C1	0	2	2	5
C2	7	6	5	4
C3	26	27	28	27

Project delay and resource usage are strongly connected with each other. When resource usage level is decreasing then project delay is increasing.

The result of multiple objective problem is a set of non-dominated solutions. In this case four of them were identified. But in the cases of larger projects the number of non-dominated solutions can be much larger. Then the preferential information of the decision maker about the schedules should be considered and one schedule should be chosen.

### **Conclusions**

The type of constrains and optimization criteria in the project scheduling problem are determined by three main components: time, resource and capital.

By using multiple criteria optimization models in the project scheduling problem we can create an optimal project schedule because it is expressed not only in terms of time, but also in terms of resource usage or project's NPV.

A zero-one programming approach for project scheduling problem has been presented in this paper.

An advantage of this approach is its form. A binary variable is easy to use and adapt to include new objectives related to the needs. The indicators of project schedule obtained from objective functions deliver clear information about project realization, e.g. an objective function in time optimization gives a concrete number, which is the project delay.

A disadvantage of model proposed is its large number of variables, namely  $j \times t$ . In the case of larger projects or larger planning horizon the number of variables will be huge.

In future research other mathematical models for project scheduling problem should be considered, e.g. mathematical model in which, variables present finish times of activities or mathematical model with binary variables in which  $x_{it} = I$  when an activity j last in period t.

Algorithms for solving this problem should be considered. The objective functions are nonlinear, so heuristic methods should be considered as a method of solution. Binary variables enable to use genetic algorithms.

In future research Activity-On-Node network should be considered. When representing the project scheduling problem by an AON network we can use not only finish-to-start precedence relationship but also other types of precedence relationships, such as: start-to-finish, start-to-start or finish-to-finish.

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