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Marcin Anholcer*

ALGORITHM FOR BI-CRITERIA STOCHASTIC GENERALIZED TRANSPORTATION PROBLEM

Abstract

The Generalized Transportation Problem is a variant of the classical Transportation Problem, where the sum of the amounts of goods delivered to the destination points is different from (usually lower than) the total amount sent from the sources. The Stochastic Generalized Transportation Problem (SGTP) is a version with random demand. We present the Bi-Criteria SGTP and propose an algorithm for determining the set of effective solutions.

Keywords: Bi-criteria stochastic generalized transportation problem, Pareto-optimal solution, stochastic programming, equalization method.

1 Introduction

The Generalized Transportation Problem (GTP) can be used in many real-life applications. It is a special case of the Generalized Flow Problem. The theory of generalized flows and chosen solution methods may be found in Ahuja et al. (1993). A polynomial algorithm for the Generalized Minimum Cost Flow Problem was presented in Wayne (2002). Combinatorial algorithms for the Generalized Circulation Problem were presented in Goldberg et al. (1988). Some issues concerning the generalized networks may be also found in Glover et al. (1972).
The Generalized Transportation Problem was considered e.g. in Balas (1966), Balas and Ivanescu (1964) and Lourie (1964). Anholcer and Kawa (2012) considered application of the two-stage GTP in the logistic networks, where complaints are involved in the distribution process. The connection between complaints ratio and the complexity of the resulting logistic network was studied.

* Operations Research Department, University of Economics in Poznań, Al. Niepodległości 10, 61 - 875 Poznań, Poland, e-,mail: *m.anholcer@ue.poznan.pl*

The significance of generalized flows (in particular of the Generalized Transportation Problem discussed in this paper) follows from the fact that they allow to model the situations when the amount of transported goods changes during the transportation process. It is obviously a useful generalization of ordinary flow (in particular transportation) problems and implies many real-life applications. As mentioned above, Anholcer and Kawa (2012) modeled a distribution network involving complaints. In Ahuja et al. (1993), the authors present several possible applications of generalized flows. For example, they may be used in the modeling of conversions of physical entities in financial, mineral and energy networks. Another application may be machine loading. Yet another possible application discussed by the authors is Land Management Problem. Nagurney et al. (2013) discuss in turn the application of generalized flows in the modeling of supplies of medical materials, food, pharmaceuticals and clothes.

Although in the models one often assumes that the demand is fixed, it is more likely that it will be unpredictable. In real-life applications one may assume at most that the distribution of the demand may be somehow estimated. The Stochastic Generalized Transportation Problem (SGTP) is the generalized version of GTP where the demands are given as random variables with known distributions. In such a case we are interested in minimizing the expected value of the total cost. This way a variant of the Nonlinear Generalized Transportation Problem is obtained. The case of the Stochastic Transportation Problem was discussed e.g. by Williams (1963), Cooper (1977), Holmberg and Jörnsten (1984) and Holmberg (1995). More general version of the Nonlinear Transportation Problem (where any convex costs at the destination points are applicable) were analyzed e.g. by Anholcer (2005, 2008a, 2008b), Sikora (1993) and Sikora et al. (1991). In the last papers the Equalization Method was considered. The convergence of the version for Nonlinear Transportation Problem was proved by Anholcer (2005, 2008a). The convergence of the version for Nonlinear and Stochastic GTP was also proved by Anholcer (2013a, 2013b). In Qi (1985) the Forest Iteration Method for the Stochastic Problem was proposed. Finally, a variant of the latter method, the A-Forest Iteration Method for the Stochastic Generalized Transportation Problem (SGTP) was presented in Qi (1987).

Note, that the stochastic versions of the Generalized Transportation Problem and of the Transportation Problem have the objective functions very similar to the Newsvendors Problem. Actually, the Newsvendor Problem can be considered as a special case of the Stochastic Transportation Problem with one source and one destination point (then, of course, the transportation costs can be treated as a constant and omitted). This is worth noticing as the Newsvendor Problem itself has been known at least from the moment of publication of Edgeworth (1888). Since then, the model was generalized and the solution methods were developed – see e.g. Khouja et al. (1996), Chen and Chuang (2000), Yang et al. (2007), Goto (2013).

In all these papers only the costs were taken under consideration. It is more likely, however, that the Decision Maker will be interested also in the minimization of the total time of the transportation process. This leads us to the bi-criteria problem.

Various versions of the Bi- and Multi-criteria Transportation Problems were analyzed e.g. by Aneja and Nair (1979), Gupta and Gupta (1983), Shi (1995), Li (2000), Basu and Acharya (2002), Khurana and Arora (2011), Kesavarz and Khorram (2011) and Kumar et al. (2012). The Generalized Transportation Problem in the multi-criteria version was studied e.g. by Gen et al. (1999).

In this paper we present a method for determining the set of effective solutions of Bi-criteria Stochastic Generalized Transportation Problem. In section 1 the problem is formulated. In sections 2-5 the algorithm is presented. In section 6 computational experiments are described. Section 7 contains main conclusions and final remarks.

2 Problem formulation

In the ordinary Generalized Transportation Problem, uniform good is transported from m supply points to n destination points. The amount of good delivered to the demand point j from the supply point i is equal to $r_{ij}x_{ij}$, where x_{ij} is the amount of good that leaves the supply point i and r_{ij} is the *reduction ratio*. The unit transportation costs c_{ij} are constant, the demand b_j of every demand point j has to be satisfied and the supply a_i of any supply point i cannot be exceeded. Hence, the model has the following form:

$$\min\left\{f(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}\right\},$$

s. t.

$$\sum_{i=1}^{m} r_{ij}x_{ij} = b_j, j = 1, \dots, n,$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i, i = 1, \dots, m,$$

$$x_{ij} \geq 0, i = 1, \dots, m, j = 1, \dots, n.$$

In the Stochastic GTP (SGTP), the demands b_j are continuous random variables X_j with density functions φ_j. We will assume that for every $j = 1, \dots, n$ and for every $x > 0$,

$$\varphi_j(x) > 0.$$

The unit surplus cost $s_j^{(1)}$ and the unit shortage cost $s_j^{(2)}$ are defined for every destination point j. As the total amount of good delivered to any destination cannot be less than 0, the function of expected extra cost for destination j takes the form:

$$f_j(x_j) = s_j^{(1)} \int_0^{x_j} (x_j - t)\varphi_j(t)dt + s_j^{(2)} \int_{x_j}^{\infty} (t - x_j)\varphi_j(t)dt,$$

which can be easily transformed to the form:

$$f_j(x_j) = s_j^{(2)}\left(E(X_j) - x_j\right) + \left(s_j^{(1)} + s_j^{(2)}\right) \int_0^{x_j} \Phi_j(t)dt,$$

where Φ_j is the cumulative distribution function of the demand at destination j.

Finally, the SGTP takes the form:

$$\min\left\{f(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{j=1}^{n} f_j(x_j)\right\},$$

s. t.

$$\sum_{i=1}^{m} r_{ij}x_{ij} = x_j, j = 1, \dots, n,$$

$$\sum_{j=1}^{n} x_{ij} \le a_i, i = 1, \dots, m,$$

$$x_{ij} \ge 0, i = 1, \dots, m, j = 1, \dots, n.$$

It is straightforward to see that each function f_j is twice differentiable and strictly convex. This means that the Equalization Method described by Anholcer (2013a) (dedicated to convex functions) may be applied in order to solve this type of problem (see Anholcer, 2013b).

The second criterion that we are interested in is the time. For every pair (i, j) of supply point *i* and destination point *j* an integer delivery time t_{ij}, not depending on the amount of transported good, is defined. We assume that all the deliveries may be performed simultaneously. This implies that the transportation process finishes when the last delivery is finished. Thus, the second objective is to minimize the maximum time over all the deliveries. It means that the bi-criteria problem (BSGTP) takes the form:

$$\min\left\{f(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{j=1}^{n} f_j(x_j)\right\},$$

$$\min\left\{t(x) = \max_{x_{ij}>0} t_{ij}\right\},$$

s. t.

$$\sum_{i=1}^{m} r_{ij}x_{ij} = x_j, j = 1, \dots, n,$$

$$\sum_{j=1}^{n} x_{ij} \le a_i, i = 1, \dots, m,$$
$$x_{ij} \ge 0, i = 1, \dots, m, j = 1, \dots, n.$$

Usually the minima of both objective functions are different. Our goal is to find the set of the effective (Pareto-optimal) solutions.

3 Algorithm – main idea

Let S denote the set of the feasible solutions of BSGTP. The problem may be rewritten as

$$\min f(x),$$
$$\min t(x),$$

s. t.

$$x \in S.$$

Let T_1 be the minimum value of $t(x)$ over S and let T_2 be the minimum value of $t(x)$ over the set of solutions x where $f(x)$ is minimized. Obviously both T_1 and T_2 are integers. The following observation holds true.

Theorem 1

A solution $x' \in S$ of BGSTP is Pareto-optimal if and only if it satisfies the following conditions:

$t(x') = T$ for some integer T, where $T_1 \le T \le T_2$,

there exist integers i and j, $1 \le i \le m$, $1 \le j \le n$, such that $T = t_{ij}$,

x' is the optimal solution of the problem

$$\min f(x),$$

s. t.

$$x \in S,$$
$$t(x) = T.$$

Proof

Assume that the solution x is Pareto-optimal. Then there is no solution x'' such that either $f(x'') \le f(x')$ and $t(x'') < t(x')$, or $f(x'') < f(x')$ and $t(x'') \le t(x')$. It follows that $t(x') \ge T_1$ by the definition of T_1. On the other hand $t(x') \le T_2$, as otherwise by the definition of T_2 there exists a point x'' with $t(x'') = T_2$, where $f(x)$ reaches its minimum value over S and so $f(x'') \le f(x')$ and $t(x'') < t(x')$, a contradiction. Of course at every point $x \in S$ we have $t(x) = t_{ij}$ for some i and j, $1 \le i \le m$, $1 \le j \le n$. Finally, x' has to be the optimum of the above problem as otherwise any of its optimal solutions x'' would satisfy $f(x'') < f(x')$ and $t(x'') \le t(x')$, a contradiction.

Assume now there is a point $x'' \in S$ such that $f(x'') \le f(x')$ and $t(x'') < t(x')$. As by decreasing the value of T we cannot obtain a better solution to the problem from the condition (iii), this means that $f(x'') = f(x')$ and in consequence $x'' = x'$ (as the objective function is strictly convex, there is a unique optimum for each value of *T*). Finally assume that there is a point $x'' \in S$ such that $f(x'') < f(x')$ and $t(x'') \leq t(x')$. It would mean that the problem from condition (iii) has a better solution on a subset of its set of feasible solutions than on the whole set of feasible solutions. That is impossible, a contradiction. This implies finally that x' is Pareto-optimal.

The solution strategy is then as follows. We start with finding the solution $x^{(1)}$ minimizing the value of $t(x)$. Let $T_1 = T^{(1)} = t\left(x^{(1)}\right)$. Then in iteration k, given $T^{(k)}$, we define the value of $T^{(k+1)}$ as the smallest t_{ij} such that $t_{ij} > T^{(k)}$. Then we solve the problem from point (iii) of Theorem 1 and find the solution $x^{(k+1)}$. If $x^{(k+1)} \neq x^k$, then it is the next Pareto-optimal solution of BSGTP. Finally, if $x^{(k+1)}$ is also an optimal solution of SGTP, then we stop the procedure, as $x^{(k+1)} = T_2$. In the three following sections we are going to present the details of the algorithm. In section 3 we briefly describe the solution method of the GTP with Time Criterion. In section 4 we present the modified version of the Equalization Method for the SGTP with an additional time constraint. Finally, in section 5 we describe the main algorithm in a detailed way.

4 Generalized Transportation Problem with Time Criterion

The GTP with time criterion has the following form:

$$\min\left\{t(x) = \max_{x_{ij}>0} t_{ij}\right\},$$

s. t.

$$\sum_{i=1}^{m} r_{ij}x_{ij} = x_j, j = 1, \dots, n,$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i, i = 1, \dots, m,$$

$$x_{ij} \geq 0, i = 1, \dots, m, j = 1, \dots, n.$$

It is straightforward to see that the optimal solution of this problem is

$$x_{ij} = 0, i = 1, \dots, m, j = 1, \dots, n,$$

and the minimum value of the objective function is:

$$T_1 = t_{\min}(x) = 0.$$

This solution will be the initial solution for the modified Equalization Method at the first step of the main algorithm. The initial solution in all other steps will be the optimum obtained by the Equalization Method in the previous step.

5 Modified Equalization Method

Given time *T*, we will solve the problem using the modified version of the Equalization Method (see Anholcer 2013a and 2013b). Let us introduce *m* additional variables $x_{i,n+1}$. Let $c_{i,n+1} = 0$, $r_{i,n+1} = 1$ for $i = 1, \dots, m$ and let $f_{n+1}(x_{n+1}) \equiv 0$. Then the SGTP with time constraint can be rewritten as :

$$\min \left\{ f(x) = \sum_{i=1}^{m} \sum_{j=1}^{n+1} c_{ij} x_{ij} + \sum_{j=1}^{n+1} f_j(x_j) \right\},$$

s. t.

$$\sum_{i=1}^{m} r_{ij} x_{ij} = x_j, j = 1, \dots, n+1,$$

$$\sum_{j=1}^{n+1} x_{ij} = a_i, i = 1, \dots, m,$$

$$x_{ij} = 0, i = 1, \dots, m, j = 1, \dots, n, t_{ij} > T,$$

$$x_{ij} \geq 0, i = 1, \dots, m, j = 1, \dots, n, t_{ij} \leq T.$$

The KKT optimality conditions can be formulated as

$$c_{ij} + r_{ij} f_j'(x_j) \geq u_i, i = 1, \dots, m, j = 1, \dots, n+1, t_{ij} \leq T, x_{ij} = 0,$$

$$c_{ij} + r_{ij} f_j'(x_j) = u_i, i = 1, \dots, m, j = 1, \dots, n+1, t_{ij} \leq T, x_{ij} > 0.$$

The following version of the Equalization Method converges to the KKT point of the SGTP with Time Constraint (the proof is similar to the proofs that can be found in Anholcer (2013a, 2013b), so it will be omitted). As it is a convex programming problem, the resulting point is the optimal solution. We calculate two measures of accuracy, α and $\alpha^\star$ in order to be able to decide if the optimal solution of the SGTP with time constraint is also the optimum of the underlying SGTP with no additional constraints.

Algorithm 1: The Equalization Method for SGTP with time constraint

Input: the initial solution x, the accuracy level ε and the maximum acceptable delivery time T.

Output: the optimal solution $x^\star$ and the global accuracy $\alpha^\star$.

1. *Initial solution*. Given the initial solution, calculate the sums of deliveries to every destination point:

$$x_j = \sum_{i=1}^{m} r_{ij} x_{ij}, j = 1, \dots, n,$$

And the partial derivatives:

$$k_{ij} = c_{ij} + r_{ij} f_j'(x_j), i = 1, \dots, m, j = 1, \dots, n,$$

$$k_{i,n+1} = 0, i = 1, \dots, m.$$

Go to step 2.

2. *Checking the optimality*. For every i calculate:

$$v_i = \min\{k_{ij} | j = 1, \dots, n+1, t_{ij} \le T\},$$
$$v_i^\star = \min\{k_{ij} | j = 1, \dots, n+1\},$$
$$w_i = \max\{k_{ij} | j = 1, \dots, n+1, x_{ij} > 0, t_{ij} \le T\} - v_i,$$
$$w_i^\star = \max\{k_{ij} | j = 1, \dots, n+1, x_{ij} > 0,\} - v_i^\star.$$

Let $j^{\star\star}(i)$ be the index j such that $k_{ij} = v_i$ and let $j^\star(i)$ be the index j for which $k_{ij} - v_i = w_i$. Compute:

$$\alpha = \max\{w_i | i = 1, \dots, m\},$$
$$\alpha^\star = \max\{w_i^\star | i = 1, \dots, m\}.$$

Let $i^\star$ be the index i for which $w_i = \alpha$. If $\alpha < \varepsilon$, then STOP. Return the solution and the value of $\alpha^\star$.

Otherwise go to step 3.

3. *Changing the solution*. Let:

$$\delta^-(\lambda) = f'_{j^\star}(x_{j^\star}) - f'_{j^\star}(x_{j^\star} - \lambda)$$

and

$$\delta^+(\lambda) = f'_{j^{\star\star}}(x_{j^{\star\star}} + \lambda) - f'_{j^{\star\star}}(x_{j^{\star\star}}).$$

Let $\lambda^\star$ be the solution to the equation:

$$\delta^-(\lambda) + \delta^+(\lambda) = w_{i^\star}.$$

If

$$\lambda^\star > x_{i^\star j^\star},$$

then set

$$\lambda^\star := x_{i^\star j^\star},$$

Adjust the solution and the derivatives according to the formulae:

$$k_{ij^\star} := k_{ij^\star} - r_{ij^\star}\delta^-(\lambda^\star), i = 1, \dots, m,$$
$$k_{ij^{\star\star}} := k_{ij^{\star\star}} + r_{ij^{\star\star}}\delta^+(\lambda^\star), i = 1, \dots, m,$$
$$x_{i^\star j^\star} := x_{i^\star j^\star} - \lambda^\star,$$
$$x_{i^\star j^{\star\star}} := x_{i^\star j^{\star\star}} + \lambda^\star.$$

Go back to step 2.

Remark: There are special cases, in which we are able to derive a simple formula for the root of the equation:

$\delta^-(\lambda) + \delta^+(\lambda) = w_{i^\star}$

(see Anholcer, 2013a, 2013b for the details). In other cases we use the one-dimensional Newton method to find the value $\lambda^\star$.

6 The main algorithm

The main method has the following form:

Algorithm 2. Method for finding the set of Pareto-optimal solutions of BSGTP.

Input: BSGTP, accuracy level ε.

Output: finite list L of Pareto-optimal solutions sorted by increasing delivery time.

1. *Initial solution*. Solve the GTP with Time Criterion as in Section 3. Initialize list L. Add x at the end of L. Create the list L_T of distinct delivery times t_{ij} in increasing order.
2. *Finding next Pareto-optimal solution*. Let x be the last element of L. Let T be the first element of L_T. Remove T from L_T. Solve the SGTP with Time Constraint using the modified Equalization Method presented in section 4. Let $x^\star$ be the optimal solution obtained and $\alpha^\star$ the global accuracy level. If $x^\star \neq x$, then add $x^\star$ at the end of L.
3. *Checking the stopping criterion*. If $\alpha^\star < \varepsilon$, then STOP, the list L contains all the Pareto-optimal solutions of BSGTP. Otherwise go back to step 2.

The presented method is convergent, which follows from two facts. First, the modified Equalization Method converges to the KKT point. Second, the number of iterations is finite because the number of distinct delivery times is finite.

7 Computational experiments

Several test problems were randomly generated and solved with the proposed method. All the demands have exponential distribution *Exp*(λ), where the values of λ were chosen uniformly at random from the interval <0.5, 0.6). The unit transportation costs were chosen from the interval <5, 10), the surplus costs from the interval <1, 2), the shortage costs from the interval <5, 10), the reduction ratios from the interval <0.8, 0.9) and the supply of each source point from the interval <10, 20). The delivery times were chosen from the set of integers {1, 2, …, 10}. The algorithm was implemented in Java SE and run on a personal computer with Intel(R) Core(TM) i7-2670 QM CPU @2.20 GHz. 1000 randomly generated problems of eight sizes were solved for (m, n) = (10, 10), (10, 20), (30, 30), (30, 60), (50, 50), (50, 100), (100, 100) and (100, 200) – 8000 test problems in total. The running times in milliseconds (average, standard deviation, minimum and maximum) are presented the Table 1.

Table 1.

Running times in milliseconds

Problem size	10×10	10×20	30×30	30×60	50×50	50×100	100×100	100×200
AVG	4.64	10.81	118.15	401.52	713.55	2733.14	7143.04	34136.65
ST DEV	14.16	23.57	148.17	317.26	615.66	1592.90	3837.50	12030.73
MIN	0.00	0.62	18.70	117.10	125.00	859.00	2185.00	14619.00
MAX	337.74	610.91	2569.40	3304.40	5538.00	11966.00	32605.00	91246.00

As we can see, the algorithm is very fast – the running times are expressed in seconds or even milliseconds in the case of the problems of reasonable size.

8 Final remarks

The algorithm presented above allows to find quickly the set of Pareto-optimal solutions of the Bi-criteria Stochastic Generalized Transportation Problem, where one criterion is the expected total cost of all the deliveries and the other one, the maximum delivery time Given the upper bound on the delivery time, the subproblem takes the form of SGTP with additional Time Constraint. This can be solved by a variant of Equalization Method, that can be shown to be convergent to the KKT point (see Anholcer 2013a, 2013b). As there are only finitely many possible delivery times, also the method dedicated for BGSTP presented above is also convergent. In consequence, we are able to find the set of all the Pareto-optimal solutions in finite time.

While formulating the problem, we made some assumptions. Let us discuss two of them.

First, we assumed that the delivery times must be integers. Although this is likely to be the case in the real world, we can remove this assumption. Even if the times are arbitrary real numbers, their number is finite and so the algorithm is still capable of delivering the list of effective solutions.

Then, we assumed that the density functions have to be positive. This implies that the objective function $f(x)$ is strictly convex and the subproblem, which has the form of SGTP with Time Constraint, has an unique optimal solution. This was in turn used in the proof of Theorem 1. However, we can omit this assumption as we can consider the equivalence relation:

$$x^{(1)} \cong x^{(2)} \Leftrightarrow f\left(x^{(1)}\right) = f\left(x^{(2)}\right).$$

Then, the set of the effective solutions consists of a finite number of the equivalence classes of $\cong$, and the presented method finds the representatives of all those classes. This is of course acceptable in real-life applications.

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References

Ahuja R.K., Magnanti T.L., Orlin J.B. (1993), Network Flows. Theory, Algorithms and Applications, Prentice Hall.

Aneja Y.P., Nair K.P.K. (1979), *Bicriteria Transportation Problem*, Management Science 25, p. 73-78.

Anholcer M. (2005), *Zbieżność metody wyrównań dla nieliniowych zadań alokacji*, in: *Z prac Katedry Badań Operacyjnych*, Ed. K. Piasecki, W. Sikora, Zeszyty Naukowe Akademii Ekonomicznej w Poznaniu 64, p. 183-198.

Anholcer M. (2008a), Analiza porównawcza wybranych algorytmów rozwiązywania nieliniowych zadań alokacji dóbr jednorodnych, Wydawnictwo Akademii Ekonomicznej, Poznań.

Anholcer M. (2008b), Porównanie działania wybranych algorytmów rozwiązywania nieliniowych zadań alokacji, in: Metody i zastosowania badań operacyjnych '2007, Ed. D. Kopańska-Bródka , Prace Naukowe Akademii Ekonomicznej w Katowicach, Katowice, p. 9-25.

Anholcer M. (2013a), On the Nonlinear Generalized Transportation Problem, preprint.

Anholcer M. (2013b), Algorithm for Stochastic Generalized Transoprtation Problem, submitted.

Anholcer M., Kawa A. (2012), *Optimization of Supply Chain via Reduction of Complaints Ratio*, Lecture Notes in Computer Science, Vol. 7327/2012, pp.622-628.

Balas E. (1966), *The Dual Method for the Generalized Transportation Problem*, Management Science, Vol. 12, No. 7, Series A, Sciences, p. 555-568.

Balas E., Ivanescu P.L. (1964), *On the Generalized Transportation Problem*, Management Science, Vol. 11, No. 1, Series A, Sciences, p. 188-202.

Basu M., Acharya D.P. (2002), *On quadratic fractional generalized solid bi-criterion transportation problem*, The Korean Journal of Computational & Applied Mathematics 10 (1-2), p. 131-143.

Bazaraa M.S., Sherali H.D., Shetty C.M. (1993), *Nonlinear Programing. Theory and Algorithms*, John Wiley & Sons Inc., New York – Chichester – Brisbane – Toronto – Singapore.

Chen M.S., Chuang C.C. (2000), *An extended newsboy problem with shortage-level constraints*, International Journal of Production Economics 67 (2000), p. 269-337.

Cooper L., LeBlanc L. (1977), *Stochastic Transportation Problems and Other Network Related Convex Problems*, Naval Research Logistics Quarterly 24 (2), p. 327-337.

Edgeworth, F.Y. (1888), *The Mathematical Theory of Banking*, Journal of the Royal Statistical Society 53, p. 113-127.

Gen M., Ida K., Choi J. (1999), *Improved Genetic Algorithm for Bicriteria Generalized Transportation Problem*, Research Reports Ashikaga Institute of Technology 28, pp. 297-303.

Glover F., Klingman D., Napier A. (1972), *Basic Dual Feasible Solutions for a Class of Generalized Networks*, Operations Research, Vol. 20, No. 1, p. 126-136.

Goldberg A.V., Plotkin S.A., Tardos E. (1988), *Combinatorial Algorithms for the Generalized Circulation Problem*, SFCS'88 Proceedings of the 29th Annual Symposium on Foundations of Computer Science, p. 432-443.

Goto H. (2013), *Multi-Item Newsvendor Problem with an Equality Resource Constraint*, Asia-Pacific Journal of Operational Research 30, p. 1250041-1 - 1250041-17.

Gupta B., Gupta R. (1983), *Multi-criteria Simplex Method for a linear multiple-objective transportation problem*, Indian Journal of Pure and Applied Mathematics 14 (2), p. 222-232.

Holmberg K., Jörnsten K.O. (1984), *Cross Decomposition Applied to the Stochastic Trans-portation Problem*, European Journal of Operational Research 17, p. 361-368.

Holmberg K (1995), Efficient Decomposition and Linearization Methds for the Stochastic Transportation Problem, Computational Optimization and Applications 4, p. 293-316.

Keshavarz E., Khorram E. (2011), *A Fuzzy Bi-criteria Transportation Problem*, Computers and Industrial Engineering 61 (4), p. 947-957.

Khouja M., Mehrez A., Rabinowitz G. (1996), *A two-item newsboy problem with substitutability*, International Journal of Production Economics 44, p. 267-275.

Khurana A., Arora S.R. (2011), Fixed Charge Bi-criterion Indefinite Quadratic Transportation Problem with Enhanced Flow, Revista Investigación Operacional 32 (133-145).

Kumar S.K., Lal I.B., Lal S.B. (2012), *Fixed-charge Bi-criterion Transportation Problem*, International Journal of Computer Application 2 (1), p. 39-42.

Li J. (2000), A Dynamic Transportation Model with Multiple Criteria and Multiple Constraint Levels, Mathematical and Computer Modelling 32, p. 1193-1208.

Lourie J.R. (1964), *Topology and Computation of the Generalized Transportation Problem*, Management Science, Vol. 11, No. 1, Series A, Sciences, pp. 177-187.

Nagurney A., Yu M., Masoumi A. H., Nagurney L. S. (2013), *Networks Against Time. Supply Chain Analytics for Perishable Products*, Springer Briefs in Optimization, Springer.

Qi L. (1985), *Forest Iteration Method for Stochastic Transportation Problem*, Mathematical Programming Study 25, p. 142-163.

Qi L. (1987), The A-Forest Iteration Method for the Stochastic Generalized Transportation Problem, Mathematics of Operations Research Vol. 12, No. 1, p. 1-21.

Shi Y. (1995), A Transportation Model with Multiple Criteria and Multiple Constraint Levels, Mathematical and Computer Modelling 21 (4), p. 13-28.

Sikora W. (1993), *Modele i metody optymalnej dystrybucji dóbr*, Zeszyty naukowe - seria II, Prace doktorskie i habilitacyjne, Akademia Ekonomiczna w Poznaniu, Poznań.

Sikora W., Runka H., Pyrzyński D. (1991), *Optymalizacja przepływów w sferze dystrybucji dóbr jednorodnych*, Grant no. H\12\209\90-2, Akademia Ekonomiczna w Poznaniu, Poznań.

Şen A., Zhang A. X. (1999), *The newsboy problem with multiple demand classes*, IEE Transactions 31, p. 431-444.

Yang P.C., Wee H.M., Chung S.L., Kang S.H. (2007), *Constrained Optimization of a Newsboy Problem with Return Policy Using KKT Conditions and GA*, New Trends in Applied Artificial Intelligence, Lecture Notes in Computer Science Volume 4570, p. 227-237.

Wayne K.D. (2002), *A Polynomial Combinatorial Algorithm for Generalized Minimum Cost Flow*, Mathematics of Operations Research, Vol. 27, No. 3, p. 445-459.

Williams A.C. (1963), *A Stochastic Transportation Problem*, Operations Research 11.