RE-CALCULATION OF HAPPY PLANET INDEX USING DEA MODELS

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Abstract

Happy Planet Index (HPI) is an aggregated index that measures the extent to which each nation produces long and happy lives per unit of environmental input. The HPI uses global data on life expectancy, experienced well-being, and ecological footprint to rank countries. The last HPI report was published in 2012 and it contains data for 151 countries from all continents. The aim of the paper is to re-calculate the HPI using DEA models and other multiple criteria decision making techniques and compare the results obtained results. MCDM methods evaluate alternatives (countries) according to the set of criteria with respect to given preferences. Most of them allow ranking of alternatives according to aggregated indices defined by various methods. DEA models compare the countries with the best performers in the data set and measure the efficiency of transformation of multiple inputs into multiple outputs. Even though they are based on different principles than MCDM methods they allow ranking of evaluated units according to their efficiency or super-efficiency scores. The paper analyzes both methodological approaches and compares their results.

Keywords: Data envelopment analysis, MCDM, Happy Planet Index, efficiency

1 Introduction

There are many attempts to compare the level of development of world countries from different points of view. The best-known and oldest characteristic is the human development index (HDI) which has been published by the United Nations Development Programme (UNDP) since 1990. It is an aggregated measure that is based on four criteria: life expectancy at birth, adult literacy rate, combined enrolment ratio, and GDP per capita.

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A multiple criteria decision making problem (MCDM) consists in the selection of a “best” (compromise) alternative or, more generally, ranking of all alternatives. In a narrow sense one of the characteristics of the MCDM problem is the presence of the decision maker’s preferences that can be given in several quite different ways. The most common way how the DM’s expresses his/her preferences is the selection of the set of criteria and the specification of their weights. In a broader sense MCDM problems are any problems where a set of alternatives is evaluated with respect to the given set of criteria. This set of criteria and their weights can be determined by a discussion in a group of DMs or by any authority or institution. In this case the DM is not present in the construction of the final solution but the problem remains an MCDM one. The calculation of the HDI belongs to a broad group of problems of the above-mentioned nature. Several attempts to re-calculate the HDI were published in the past. They are based either on using other MCDM methodology than the one used by the UNDP, or on data envelopment analysis (DEA) models. An interesting attempt to re-calculate the HDI using DEA models can be found e.g. in Mahlberg and Obersteiner (2001) and Despotis 2005).

One of the newest global indicators of countries is the Happy Planet Index (HPI). This index was introduced in 2006 by the New Economic Foundation (NEF); it is a global measure of sustainable human well-being and environmental impact. The value of the HPI is influenced positively by the level of experienced well-being and life expectancy and negatively by the ecological footprint. The aim of the present paper is to discuss a possibility of recalculation of the HPI using MCDM and DEA models and to compare the results obtained with standard methodology. Section 2 of the paper contains detailed information about the calculation of the HPI. Section 3 formulates DEA models suitable for analysis of the HPI and Section 4 presents results obtained by various modelling approaches. The final section contains conclusions and directions for future research.

2 Happy Planet Index

The information about the HPI and its calculation are taken from the HPI 2012 Report (Abdallah et al., 2012). The HPI is an efficiency measure which expresses the level which long and happy lives achieve per unit of environmental impact. It is based on the following data sources:

1. *Life expectancy at birth* (further denoted x). This figure expresses the number of years an infant born in that country could expect to live if prevailing patterns of age-specific mortality rates at the time of birth in the country stay the same throughout the infant’s life. The calculation of HPI 2012 uses data published in 2011 Human Development Report.
2. *Experienced well-being* (y). The data for the average level of well-being in the countries are taken from the survey of the Gallup World Poll which uses samples of around 1000 individuals aged more than 15 years from

each country. They assign grades from 0 (worse living conditions) to 10 (best living condition) and the final well-being country index is a simple average of all responses.

3. *Ecological footprint* (z). It is a measure expressed in g ha (so called global hectares) per capita of human demand on nature. Ecological footprint index measures the amount of land required to sustain the country's consumption pattern. It includes the land required to provide the renewable resources that people use, the area occupied by infrastructure, and the area necessary to absorb CO_2 emissions. More information about calculation of this composite index can be found in Boruckea et al. (2013).

In general, the HPI is calculated as the ratio:

$$\mathrm{HPI} = \frac{x \cdot y}{z}.$$

The HPI cannot be simply calculated using the formula above because x, y, and z are given on different scales with different variances. This formula is only very general and for comparison purposes the HPI is calibrated to reach values from 0 to 100. The calculation is divided into two steps:

Calculation of the Happy Life Years index (w). This index composes first two elements of the HPI, i.e. life expectancy and well-being, as follows:

$$w = \frac{x(y+\alpha)}{10+\alpha},$$

where α = 2.93 is a constant added to y (experienced well-being) to unify the level of variance of both characteristics.

Calculation of the HPI. In the second stage, the constant γ = 5.67 is subtracted from w to ensure that the country with an average well-being score of 0 or a life expectancy of 25 or lower achieves the HPI score of 0, and the constant β = 4.38 is added to ecological footprint to ensure that its coefficient variance is equal to that of index w. Finally, the HPI scores are calculated according to the following formula:

$$\mathrm{HPI} = \frac{\delta(w-\gamma)}{z+\beta},$$

where δ = 7.77 is the constant that ensures that the country with average well-being score of 10, average life expectancy of 85 years, and ecological footprint of 1.78 g ha per capita (equivalent to one planet living) achieves the HPI score of 100.

3 DEA models for HPI re-calculation

DEA models are a general tool for evaluation of efficiency and performance of the set of decision making units. The re-assessment of the HPI or other indices of world countries is a very specific problem. The main aim of the present paper is to propose a DEA based methodology for the calculation of the HPI and to compare the results with commonly used methodology based on a simple aggregation of the criteria.

Let us suppose that the set of decision making units (DMUs) contains n elements. The DMUs are evaluated by m inputs and r outputs with input and output values x_{ij}, $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$ and y_{kj}, $k = 1,2,\ldots,r$, $j = 1,2,\ldots,n$, respectively. The efficiency score θ_q of the DMU$_q$ can be expressed as the weighted sum of outputs divided by the weighted sum of inputs with weights reflecting the importance of single inputs/outputs v_i, $i = 1,2,\ldots,m$ and u_k, $k = 1,2,\ldots,r$ as follows:

$$\theta_q = \frac{\sum_{k=1}^{r} u_k y_{kq}}{\sum_{i=1}^{m} v_i x_{iq}}.$$

The conventional CCR DEA model formulated by Charnes et al. (1978) consists in the maximization of the efficiency score θ_q of the DMU$_q$ subject to constraints that efficiency scores of all other DMUs are lower than or equal to 1. The linearized form of this model with output orientation is as follows:

Minimize

$$\theta_q = \sum_{i=1}^{r} v_i x_{iq}$$

$$\begin{aligned} \text{subject to} \quad & \sum_{k=1}^{m} u_k y_{kq} = 1, \\ & \sum_{k=1}^{r} u_k y_{kj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,2,\ldots,n, \\ & u_k, v_i \ge \varepsilon, \qquad k = 1,2,\ldots,r, i = 1,2,\ldots,m. \end{aligned} \tag{1}$$

If the optimal value of model (1) $\theta^*_q = 1$ then the DMU$_q$ is CCR efficient and it is lying on the CCR efficient frontier, otherwise $\theta^*_q > 1$ and the unit is not efficient. The value θ^*_q expresses the rate of increase of outputs needed to reach the efficient frontier.

Model (1) is the CCR output oriented model with the assumption of constant returns to scale. The appropriate model with variable returns to scale (VRS) is as follows:

Minimize

$$\theta_q = \sum_{i=1}^{r} v_i x_{iq} + \mu$$

subject to

$$\begin{aligned}
&\sum_{k=1}^{m} u_k y_{kq} = 1, && \\
&\sum_{k=1}^{r} u_k y_{kj} - \sum_{i=1}^{m} v_i x_{ij} + \mu \le 0, && j = 1,2,\ldots,n, \\
&u_k, v_i \ge \varepsilon, && k = 1,2,\ldots,r, i = 1,2,\ldots,m, \\
&\mu - \text{free}.
\end{aligned} \qquad (2)$$

Many other modifications of the conventional DEA models have been formulated in the literature. One of the most interesting is the slack based model (SBM) – Tone (2001) which is formulated as follows:

Minimize

$$\rho_q = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{iq}}{1 + \frac{1}{r}\sum_{k=1}^{r} s_k^+ / y_{kq}},$$

subject to

$$\begin{aligned}
&\sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{iq}, && i = 1,2,\ldots,m, \\
&\sum_{j=1}^{n} y_{kj}\lambda_j - s_k^+ = y_{kq}, && k = 1,2,\ldots,r, \\
&\lambda_j \ge 0, \quad j = 1, 2,\ldots, n. \\
&s_k^+, s_i^- \ge 0, \quad i = 1, 2,\ldots, m, k = 1, 2,\ldots, r,
\end{aligned} \qquad (3)$$

where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ is a vector of weights of the DMUs, $\mathbf{s}^+ = (s_1^+, s_2^+,\ldots, s_r^+)$ is a vector of surplus variables, $\mathbf{s}^- = (s_1^-, s_2^-,\ldots, s_m^-)$ is a vector of slack variables and ρ_q is efficiency score of the DMU_q. Tone's model is a non-radial model that measures the efficiency using relative slack and surplus variables only. The efficiency score ρ_q equals 1 for efficient units (all slack and surplus variables equal 0) and is lower than 1 for inefficient ones. The model (3) is not linear in objective function but can be simply transformed into a LP problem – see Tone, (2001) for more details.

Most of the DEA applications consider solely inputs or resources used by a DMU and desirable outputs that are the results of input utilization. In this case higher values of outputs lead to higher efficiency (when a fixed level of inputs is used). Nevertheless, this assumption is rarely acceptable and one or several outputs in the model are undesirable (e.g. environmental impact, pollutions, tax payments, etc.). Various approaches were proposed in the past for dealing with undesirable outputs. The easiest way is to transform the undesirable output into a desirable one by subtracting the original values from a given upper (worse) bound. In evaluation of the HPI there are two main desirable outputs: life expectancy and well-being, and one undesirable output: ecological footprint.

Conventional DEA models, e.g. the model (2), optimize the efficiency of the evaluated unit using adjustment of the weights of the inputs and outputs. The weights are limited by the infinitesimal constant ε (e.g. 10^{-8}) only. That is why some of the weights may be reaching their lower bounds, i.e. they are very small which may be unacceptable for decision makers. Various ways of restricting weights in DEA models have been proposed. This question is very important because inappropriate restrictions can easily lead to infeasible solutions of the model.

Another important task in applications of DEA models for ranking of DMUs consists in ranking of efficient units. All efficient units have maximum efficiency score 1 and cannot be ranked using the conventional models. For discrimination among them various so called super-efficiency models were proposed in the past. More information about them can be found e.g. in Cooper at al. (2000) and Tone (2002).

In the numerical experiments described in the next section of the paper there were applied all modifications of the DEA models mentioned above, i.e. models with weight restrictions, super-efficiency models and models with undesirable outputs.

4 DEA and MCDM analysis of the HPI

As described above, the HPI consists of three indicators (criteria). The calculation of this index can be regarded as a conventional MCDM problem. In the numerical experiments described below we use WSA and TOPSIS. Their common feature is that these three methods do not require any additional information from DM except weights of criteria. Apart from that, various modifications of DEA models are applied. All calculations using DEA models were performed on the modified data set that assigns 0 to basal and 1 to ideal alternative. MCDM methods in our experiments use the original data set described in detail in Table 1. This table contains, apart from the three main indicators, information about GDP because it was used as an additional indicator in some calculations presented below (the ideal value for GDP is set up to \$ 60 000/cap and to each of the few countries with a higher value of GPD a maximum value, i.e. 1, is assigned). Numerical experiments are performed using the software package *Sanna* which implements MCDM methods, and the *DEA Excel Solver* (Jablonsky and Dlouhy, 2010). Both applications can be downloaded from the author's web page. The results of the numerical experiments for both modeling approaches are described below.

Table 1

Characteristics of the original data set

	Life expectancy [years]	Well-being [points]	Ecological footprint [g ha/cap]	GDP [$/cap]
Minimum	47.8	2.81	0.54	347
Lower quartile	63.1	4.38	1.39	2308
Median	73.2	5.18	2.13	8274
Upper quartile	76.5	6.22	4.26	20 545
Maximum	83.4	7.77	11.68	86 124
Mean value	69.83	5.39	3.07	14 582
Standard dev.	9.77	1.17	2.16	16 168
Basal	25	0	12	0
Ideal	85	10	0	60 000

The *Sanna* application implements most of the MCDM methods. For the numerical experiments two simple methods are used. The weights of all three criteria for all methods are supposed to be identical, i.e. 1/3. This corresponds to the practice used in the original definition of the HPI. The applied methods are:

1. WSA, which uses a simple linear utility function for the aggregation of preferences.
2. TOPSIS, which uses a different way of normalization of the original criterion matrix; that is why no prior normalization is necessary. The main idea of this method is a minimization of the distances from both basal and ideal alternatives.

Table 2 presents the results obtained by both approaches. Due to the limited space only rankings of the first and the last three countries (according to the original HPI definition) are presented together with results for the Czech Republic, Slovakia and Poland. The information in Table 2 is completed by the average and maximum differences in rankings obtained by the appropriate method.

Table 2

MCDM re-calculation

HPI	Country	WSA	TOPSIS
1	Costa Rica	1	1
2	Vietnam	8	12
3	Colombia	6	5
:			
71	Poland	63	55
89	Slovakia	78	72
92	Czech Republic	82	101
:			
149	Qatar	151	151
150	Chad	149	140
151	Botswana	150	143
Average difference		10,8	12,1
Maximum difference		34	44

Table 3 shows the same information as the previous table but it contains results obtained by the application of DEA models. Due to the limited space only five different experiments are presented. They are described as follows:

1. DEA model (1) with one dummy input (identical for all countries) and three outputs. Efficient countries are ranked according to their super-efficiency measure (Andersen and Petersen model).
2. DEA model (1) with weight restrictions. The weights can be restricted in different ways – either by absolute lower and/or upper bounds or by their ratios. The results in the second column correspond to the relative restrictions – all pairs of weights can differ by 50% of their values only.
3. SBM model (3). The efficient countries are ranked according to the SBM super-efficiency measures – see (Tone, 2002).
4. The fourth column contains results obtained using the common set of weights (CSW) – see (Despotis, 2005). The weights of three outputs are the results of the following linear optimization model:
   Minimize
   $$z = \sum_{j=1}^{n} d_j / n$$
   subject to (4)

$$\sum_{k=1}^{r} u_k y_{kj} + d_j = \theta_j, \qquad j = 1,2,\ldots,n,$$
$$u_k \geq \varepsilon, d_j \geq 0, \qquad k = 1,2,\ldots,r, \; j = 1,2,\ldots,n,$$

where θ_j is the efficiency score of the DMU_j. The model minimizes the sum of deviations from efficiency scores using the weights of the outputs. The optimal weights of the model (4) are: u_1 = 0.465, u_2 = 0.157, u_3 = 0.588. They are applied in a similar way as when the WSA method is used.

5. The data set was extended by the fourth output (GDP per capita) and the impact of this change was analyzed. The conventional DEA model (2) was applied to the extended data set.

Table 3

Re-calculation using DEA models

HPI	Country	CCR DEA	CCR w WR	SBM	CSW	CCR +GDP
1	Costa Rica	1	1	1	1	3
2	Vietnam	8	4	5	2	10
3	Colombia	17	8	17	8	16
:						
71	Poland	108	74	75	74	112
89	Slovakia	125	93	91	95	126
92	Czech Rep.	105	89	100	103	108
:						
149	Qatar	95	149	151	151	58
150	Chad	139	148	144	142	139
151	Botswana	151	151	147	144	141
Average difference		29.6	9.2	12,5	13,7	30,0
Maximum difference		105	33	103	43	136

The results presented in Table 2 and 3 can be explained from several points of view. The main conclusions are:

1. The MCDM methods based on similar principles as the original definition give similar results even though the differences in rankings for some countries are quite large.
2. DEA models without weight restrictions are hardly usable for the given problem. This is because the efficiency score is based on optimal

weights of the evaluated units which can differ significantly (very small values for some criteria and large values for the others).

3. DEA models with weight restrictions give much better results than models without them (in our case even better than WSA and TOPSIS methods).
4. The extension of the model by the fourth output (GDP) does not affect significantly the results.
5. The application of a common set of weights is a compromise between the conventional DEA model and the WSA method. The results are quite close to WSA method.

5 Conclusions

The selection of a compromise alternative or ranking of alternatives in the case of multiple criteria depends not only on the DM's preferences but it is also influenced by the application of a suitable methodology. The final result depends on the DM's preferences and the selection of the method for the analysis. Unfortunately it is very difficult or even impossible to determine the most appropriate method for a given problem. That is why it can be interesting to apply various evaluation methods and compare their results. One of the aims of the present paper was to compare the original definition of the HPI with two MCDM methods and several DEA models.

The results presented in the previous sections show that the ranking of a large number of alternatives according to few criteria depends not only on the weights of the criteria but also on many other factors. The simple CCR DEA model with one dummy input and all the remaining criteria as outputs does not give acceptable results in comparison to the standard HPI definition. The differences in rankings are very high, which results from the nature of the DEA models that optimize weights of the outputs to maximize the efficiency of the evaluated unit. This can lead to unacceptably high differences in weights of the output pairs. Much more results are obtained when the model with weight restrictions is applied. Then the final ranking gets closer to the original HPI very significantly. These experiments show that the DEA models can be used to define the final ranking of countries (or other alternatives) according to given criteria.

Future research in this field is open. There are many country indices related to various areas of human activity. The data used for their calculation are often given with a certain level of uncertainty; to work with them, a methodology for dealing with imprecise data is needed. Other directions of research can involve real discretionary and/or non-discretionary inputs instead of one dummy input.

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