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MULTIPLE CRITERIA DECISION MAKING IN THE VALUATION OF REAL OPTIONS

Abstract

Traditional project evaluation is based on discounted cash flow method (DCF) with Net Present Value (NPV) as the main measure. This approach sometimes leads to the abandonment of profitable projects, because the DCF method does not take into account the role of managerial flexibility. The Real Options Valuation (ROV) method takes into account future situations in the valuation, assuming that the project is properly managed. The Project Manager shall have the right to take action as appropriate.

A widely used method for the valuation of real options is the binomial tree method (CRR), proposed by Cox, Ross and Rubinstein. It takes into account one state variable. In many real problems, however, many factors should be considered. This leads to a multi-criteria decision-making problem. This paper presents an extension of the CRR method for several state variables.

Keywords: project management, real options, dynamic programming

1 Introduction

The term real option was proposed by S. Myers (1974, p. 1-25), who noted similarities between financial options and opportunities that arise in project management. An option can be defined as a right, but not an obligation, which means that the holder of that right can determine when to exercise it, depending on the current market situation. This approach was then developed by A.K. Dixit and R.S. Pindyck (1994), and was later discussed by L. Trigeorgis (1993, p. 202-224). The most important element in the Real Options Analysis (ROA) is the valuation (ROV – Real Option Valuation). In ROV methods known from

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financial market were used first, such as the Black-Scholes model (Black, Scholes, 1973, p. 637-654) or the Cox-Ross-Rubinstein model (CRR)(Cox, Ross, Rubinstein, 1979, p. 229-263). Also used was an approach based on Monte Carlo simulations (Boyle, 1977, p. 323-338). The CRR model is based on the binomial tree. This approach was also adopted in the book by Guthrie (2009) on which this study is based.

Traditionally, project evaluation is based on the discounted cash flow method (DCF) with Net Present Value (NPV) as the main measure of effectiveness. When this value is positive, the project is approved, when it is negative, the project is rejected. This approach sometimes leads to the abandonment of profitable projects. The reason for this is that the DCF method does not take into account the role of managerial flexibility. The Project Manager shall have the right to take action as appropriate. This situation is called a real option. Using ROV, we can provide a quantitative measurement of this situation.

The traditional approach in the valuation of Real Options is based on a single factor called the state variable. There are also attempts to take into account many state variables. The first attempt, based on financial options, was made by Boyle (1988, p.1 - 12), who took into account two assets. Mun (2010) described a commercial solution with such possibilities. Guthrie (2009, p. 403) also described problems for which it is necessary to take into consideration several of variables.

This paper presents problems in Real Options valuation with many state variables, which lead to issues considered in multiobjective analysis. This paper presents such multi-criteria problems. The first section presents the Defer Options that may arise in project management. The next section describes a multi-criteria approach in Real Options valuation. The last section is a numerical example.

2 Problem formulation

Many project management methodologies recommend the division of the project into stages. This raises the problem of decision-making, consisting in the choice of the start time of the next steps. For example, in the PRINCE2 (Office of Government Commerce, 2009) methodology, each project must have at least two stages: initialization and actual implementation. We will consider a project consisting of these two phases, each of which takes a period of time. After the initialization of a project we have to decide when to begin its implementation. We can delay the execution by one period. This is a classic example of the defer options, considered by Ingersoll and Ross (1992, p.1-29).

If planned project is static, the decision maker is not able to react to changes in the environment and in the project itself. If only the duration of the project can be extended and the decision maker is allowed to decide freely about the start times of the consecutive stages, a completely new situation arises, which is presented in Figure 1. The decision maker may start the project (decision ***A***), then move from the current state (***initial*** of project) to the last state (***end*** of project). The decision maker may wait (decision ***W***), but then the project will remain in the starting state.

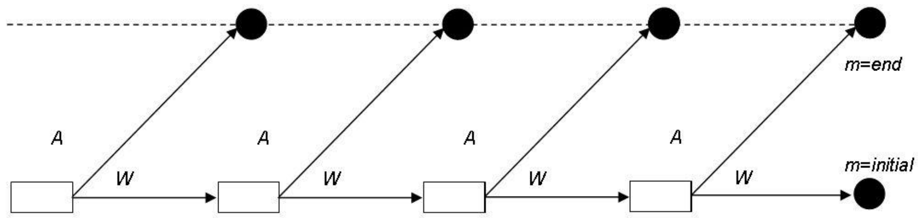

Figure 1. The decision tree

The results of the project, as well as its value, depend on certain factors. If we consider more than one factor that lead to usability design considerations in many areas, the problem is converted from a simple valuation to a multi-criteria evaluation problem.

Decisions are made based on the observation of the change of the factors. These factors vary stochastically according to a certain random process. The idea behind the CRR method is to cover a possible future state variable binomial tree as shown in Figure 2. It meets a role of scenario possible changes in the value of the state variable. At each stage, we consider only the possibility of an increase or decrease in value. This procedure simplifies the decision-making process.

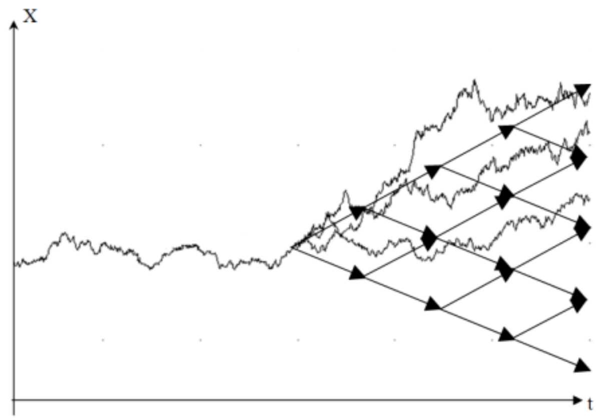

Figure 2. The binomial tree covering a stochastic process

Guthrie (2009, p. 168) considers one-factor models. This paper expands these discussion, by binding results of the project with two factors. There are used methods of multiobjective dynamic programming.

3 Method of evaluation

Each of the two factors called state variables, may be increased u-times and fall d-times in each period. This assumption leads to the tree of possible state variable values, which consists of nodes marked with indices (i, n) where i is the number of falls and n is the period number.

With each node a state variable and cash flow are connected. We denote it by:

- $X_k(i, n)$ – k-th state variable in period n
- $Y_m(i, j, n)$ – cash flow in period n (where m is state of project).

Given are: the number of periods N, the present value of each state variable $X_k(0,0)$, and also coefficients u, d. The value of u can be obtained from historical data by the calibration procedure (Guthrie, 2009, p. 263).

The proposed procedure consists of the following steps:

Step 1 – Build the decision tree (D-Tree)

We identify the possible states of the project, which may be different phases or specific stages. We also identify the possible decisions that we consider. Taking such a decision leads to a transition from one state to another. Finally, we identify all possible transitions. The result of this step is a D-Tree, shown in Figure 1.

Step 2 – Build the lattice of state variables (X-Tree)

We identify quantifiable economic magnitudes, on which the result of the project may depend (state variables). The method currently proposed does not include the correlation between these variables (we assume that such correlation does not exist).

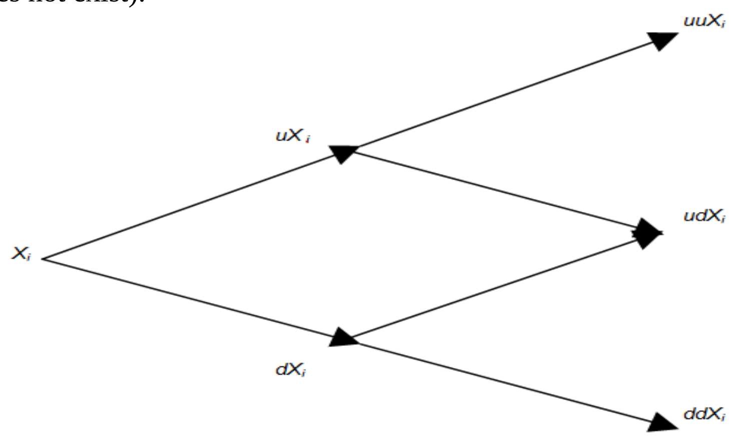

Figure 3. The binomial tree of state variables

The tree starts from a known present value of the state variables. Based on the history of changes of this magnitude, the values *u* and *d* can be determined (this is done in the calibration process – Guthrie, 2009, p. 324). A tree of possible changes of state variable is built, as a possible scenario of the situation; it is presented in Figure 3. Calibration it is an appropriate selection of the number of steps and the choice of parameters (*d*, *u*), so as to best meet the future value of the variable state.

Step 3 – Build the tree of the project values (V-Tree)

The calculation of the V-Tree is based on the principle of optimality, formulated in Bellman's paper (Bellman, 1957). In our case, where decisions are made based on more than one factor, we use the multiobjective dynamic programmming principle of optimality, where we want to find the set of noninferior (efficient) solutions. In our case this principle can be formulated as follows (Li, Haimes, 1989, p. 471-483):

"Each noninferior control sequence has the property that, whatever the initial state, the existing control subsequence must constitute noninferior policy with respect to this initial state".

The application of this principle leads to backward induction in which we consider the sets of efficient (noninferior or nondominated) solutions, in this case the values at the *k*-1 stage:

$$\{V(k-1)\} = \sup_{d_k}\left\{e^{-r\Delta t_m} E\left[V\ (k, d_k)\right]\right\} \quad k = n,...,1 \tag{1}$$

In our case we consider a project evaluation based on many state variables, which is therefore presented as a vector of values:

$$\mathbf{V}_m(i^1, i^2, ..., i^s, n) = \begin{bmatrix} f_1(X_1(i^1, i^2, ..., i^s, n), V_1(i^1, i^2, ..., i^s, n+1)) \\ f_2(X_2(i^1, i^2, ..., i^s, n), V_2(i^1, i^2, ..., i^s, n+1) \\ ... \\ f_s(X_s(i^1, i^2, ..., i^s, n), V_s(i^1, i^2, ..., i^s, n+1) \end{bmatrix} \tag{2}$$

Since we consider two state variables, the resulting V-Tree grows in two dimensions. We denote the present value of the project, which is dependent on two state variables, by:

$V_{(s)}(i, j, n)$ – utility value of project in period n,

where:

i – number of falls of first state variable,

j – number of falls of second state variable.

The calculation of the V-Tree starts from the project results in a final. We assume that the final value of the project is a function of state variables:

$$\mathbf{V}^m(i,j,n)=\begin{bmatrix} f_1^e(X_1(i,j,n)\ ) \\ f_2^e(X_2(i,j,n)\ ) \end{bmatrix} \tag{3}$$

On their basis, moving from the final set, we calculate the value of the project in previous nodes. Trees are constructed for each state project. The calculation of the value is done by backward induction. Knowing the value of the project after its completion (which is usually equal to the state variable or can be calculated using the correct formula for this variable), we calculate the values of the project in the preceding nodes.

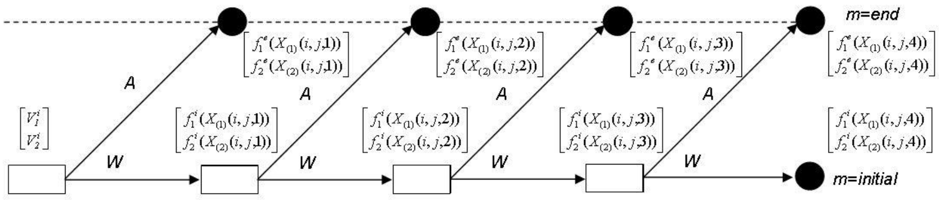

Figure 4. The V-Tree

We will consider two possibilities:

- the case of financial factors, when the present value is the discounted expected value of subsequent values:

$$V_1\ (i,j,n)=\left(\pi_u^1\cdot V_1(i,j,n+1)+\pi_d^1\cdot V_1(i+1,j,n+1)\ \right)e^{-r\Delta t} \tag{4}$$

$$V_2\ (i,j,n)=\left(\pi_u^2\cdot V_2(i,j,n+1)+\pi_d^2\cdot V_2(i,j+1,n+1)\ \right)e^{-r\Delta t} \tag{5}$$

- the case of other factors, when the present value is the expected value of subsequent values:

$$V_1\ (i,j,n)=\pi_u^1\cdot V_1(i,j,n+1)+\pi_d^1\cdot V_1(i+1,j,n+1) \tag{6}$$

$$V_2\ (i,j,n)=\pi_u^2\cdot V_2(i,j,n+1)+\pi_d^2\cdot V_2(i,j+1,n+1) \tag{7}$$

Subsequent values are weighted by the probability of achieving those values. If we denote by *r* the risk free interest rate, we can calculate them from the formulas (Seydel, 2009, p. 15):

$$\pi_u^l=\frac{e^{r\Delta t}-d^l}{u^l-d^l} \tag{8}$$

for the growth of the l-th state variable,

$$\pi_d^l = \frac{u^l - e^{r\Delta t}}{u^l - d^l} \tag{9}$$

for the fall of the l-th state variable in the case of using Geometric Brownian Motion (GBM) and

$$\pi_u^l = \frac{e^{r\Delta t} E\left(X_l(-, n+1)\right) - X_l(i+1, n+1)}{X_l(i, n+1) - X_l(i+1, n+1)} \tag{10}$$

for the growth of the l-th state variable,

$$\pi_d^l = 1 - \pi_u^l \tag{11}$$

for the fall of the l-th state variable in the case of using Brownian Motion (BM) (G. Guthrie, 2009, p. 280).

Step 4 – Determine effective transitions (decisions)

The presented procedure allows not only to determine the possible cash flow, but also to identify the best decisions. As we are in the area of multi-criteria decision analysis, these will not be the optimal decisions but only the effective ones. The best decision is one for which we obtain the 'supremum' of the discounted expected value:

$$d_k^* = \arg\sup_{d_k} \left\{ e^{-r\Delta t_m} E\left[V\,(k, d_k)\right] \right\} \quad k = n, \ldots, 1 \tag{12}$$

In the multicriteria case under consideration, we can obtain a set of efficient decisions for which there is no worse decision at any stage:

$$\neg\exists d_k \quad e^{-r\Delta t_m} E\left[V\,(k, d_k)\right] \succ e^{-r\Delta t_m} E\left[V\,(k, d_k^*)\right] \quad k = n, \ldots, 1 \tag{13}$$

In our case, we have at each stage two decisions as shown in Figure 1:

$$\text{m} = \{\text{A, W}\} \tag{14}$$

and also two criteria values:

- V_1 – dependent on first state value (X_1),
- V_2 – dependent on second state value (X_2).

Using the scalarization approach (Trzaskalik, 1988, p. 64), we can simplify the calculation to a simple comparison of the two values obtained as the sum of the weighted components of the vector **V**.

The values determined by (4)-(7) must be calculated for each decision, so we introduce a superscript denoting the relevant decision:

$$\mathbf{V}^A(i,j,n) = \begin{bmatrix} V_1^A(i,j,n) \\ V_2^A(i,j,n) \end{bmatrix} \tag{15}$$

for the decision **A̲ct** and

$$\mathbf{V}^W(i,j,n) = \begin{bmatrix} V_1^W(i,j,n) \\ V_2^W(i,j,n) \end{bmatrix} \tag{16}$$

for the decision **W̲ait**.

Because we assume that the project is properly managed, a favorable decision will be chosen.

$$\begin{aligned} &\text{IF } \mathbf{V}^W(i,j,n) \succ \mathbf{V}^A(i,j,n) \quad \text{THEN } \mathit{Wait}; \mathbf{V}^m = \mathbf{V}^W \\ &\qquad\qquad\qquad\qquad\qquad\qquad\quad \text{ELSE } \mathit{Act}; \mathbf{V}^m = \mathbf{V}^A \end{aligned} \tag{17}$$

The only problem is to determine the preferred vector evaluation. Using the scalarization approach (Trzaskalik, 1988, p. 64), we denote:

$$Q^W(i,j,n) = \sum_l w_l V_l^W(i,j,n) \tag{18}$$

$$Q^A(i,j,n) = \sum_l w_l V_l^A(i,j,n) \tag{19}$$

where: $w_l \geq 0$ are weights of assessments, hence the calculations are simplified to:

$$\begin{aligned} &\text{IF } Q^W(i,j,n) > Q^A(i,j,n) \quad \text{THEN } \mathit{Wait} \\ &\qquad\qquad\qquad\qquad\qquad\qquad\quad \text{ELSE } \mathit{Attempt} \end{aligned} \tag{20}$$

which gives not only effective decisions but also allows to calculate the value of the cash flow associated with the project.

4 Numerical example

We will consider a three-month project of social nature. The project may begin in any quarter of 2013. After its implementation financing in the amount of 10 million euros will be obtained. Costs were estimated at 41 million PLN. In addition, if the project proves to be purposeful further co-operation of the financing institution will be possible. The purposefulness the project depends on the development of the level of unemployment. If it remains high, the project will be deemed purposeful. If the unemployment rate drops, its implementation will be useless.

X-Trees determined on the basis of the observations of variables in 2012 are presented in Table 1 for the exchange rate EUR/PLN and Table 2 for the level of unemployment. For the first state variable we use GBM, for the second, BM.

Table 1

X-Tree for EUR/PLN exchange rate

X_1	n				
i	0	1	2	3	4
0	4,0946	4,2727	4,4585	4,6525	4,8548
1		3,9239	4,0946	4,2727	4,4585
2			3,7604	3,9239	4,0946
3				3,6036	3,7604
4					3,4534

Table 2

X-Tree for unemployment rate

X_2	n				
i	0	1	2	3	4
0	10,4	10,7	11,0	11,3	11,6
1		10,1	10,4	10,7	11,0
2			9,8	10,1	10,4
3				9,5	9,8
4					9,2

In the following tables the final values obtained by the project are shown. The final value for the first state value is calculated as project profit :

$$f_1^e(X_1(i,j,n)\ ) = M \cdot X_1(i,j,n) - K, \tag{21}$$

where:

M = 10 M EUR
K = 41 M PLN

The final value of the second state value is calculated as project utility. If unemployment is greater than 10%, this value is 100, otherwise it is 0:

$$f_2^e(X_2(i,j,n)\ ) = \begin{cases} 100 & X_2(i,j,n) > 10{,}0\% \\ 0 & X_2(i,j,n) <= 10{,}0\% \end{cases} \tag{22}$$

The calculated final values are presented in Tables 3 to 6.

Table 3

Final values for n = 4

$(f_1^e; f_2^e)$	n = 4				
i, j	0	1	2	3	4
0	(7,5; 100)	(7,5; 100)	(7,5; 100)	(7,5; 0)	(7,5; 0)
1	(3,6; 100)	(3,6; 100)	(3,6; 100)	(3,6; 0)	(3,6; 0)
2	(-0,1; 100)	(-0,1; 100)	(-0,1; 100)	(-0,1; 0)	(-0,1; 0)
3	(-3,4; 100)	(-3,4; 100)	(-3,4; 100)	(-3,4; 0)	(-3,4; 0)
4	(-6,5; 100)	(-6,5; 100)	(-6,5; 100)	(-6,5; 0)	(-6,5; 0)

Table 4

Final values for n = 3

$(f_1^e; f_2^e)$	n = 3			
i, j	0	1	2	3
0	(5,5;100)	(5,5; 100)	(5,5; 100)	(5,5; 0)
1	(1,7; 100)	(1,7; 100)	(1,7; 100)	(1,7; 0)
2	(-1,8; 100)	(-1,8; 100)	(-1,8; 100)	(-1,8; 0)
3	(-5,0; 100)	(-5,0; 100)	(-5,0; 100)	(-5,0; 0)

Table 5

Final values for n = 2

$(f_1^e; f_2^e)$	n = 2		
i, j	0	1	2
0	(3,6; 100)	(3,6; 100)	(3,6; 0)
1	(-0,1; 100)	(-0,1; 100)	(-0,1; 0)
2	(-3,4; 100)	(-3,4; 100)	(-3,4; 0)

Table 6

Final values for n = 1

$(f_1^e; f_2^e)$	n = 1	
i, j	0	1
0	(1,7; 100)	(1,7; 100)
1	(-1,8; 100)	(-1,8; 100)

Using backward induction, from the equations (4)-(7), we calculate the vectors of values for each decision stage. The calculations used the values r = 4%, π_u^1 = 0,3725, π_d^1 = 0,6275, π_u^2 = 0,5 , π_d^2 = 0,5. The results are presented in Tables 7 to 8.

Table 7

Decision values for n = 3

$\left(\mathbf{V}^A(i,j,n)\right)^T$ $\left(\mathbf{V}^W(i,j,n)\right)^T$	n = 3			
i, j	0	1	2	3
0	(5,0; 100)* (0; 100)	(5,0; 100)* (0; 100)	(5,0; 50)* (0; 50)	(5,0; 0)* (0; 0)
1	(1,3; 100)* (0; 100)	(1,3; 100)* (0; 100)	(1,3;50)* (0;50)	(1,3; 0)* (0; 0)
2	(-2,1;100) (0; 100)*	(-2,1; 100) (0; 100)*	(-2,1; 50) (0; 50)*	(-2,1; 0) (0; 0)*
3	(-5,3; 100) (0; 100)*	(-5,3; 100) (0; 100)*	(-5,3; 50) (0; 50)*	(-5,3; 0) (0; 0)*

Table 8

Decision values for n = 2

$\left(\mathbf{V}^A(i,j,n)\right)^T$ $\left(\mathbf{V}^W(i,j,n)\right)^T$	n = 2		
i, j	0	1	2
0	(3,1; 100)* (2,7; 100)	(3,1; 100)* (2,7; 75)	(3,1; 50)* (2,7; 25)
1	(-0,5; 100) (0,5; 100)*	(-0,5; 100)? (0,5; 75)?	(-0,5; 50)? (0,5; 25)?
2	(-3,8; 100) (0; 100)*	(-3,8; 100)? (0; 75)?	(-3,8; 50)? (0; 25)?

The dominant elements are marked with an asterisk. There are no such elements in nodes (1,1), (1,2), (2,1) and (2,2). The definitive decision can be calculated using the preference structure obtained by weights in the scalarization approach. Assume that current revenues are more important than the possibility of implementing a similar project in the future, so we may take w_1 = 0,9 and w_2 = 0,1. Then we have:

Q^A (1,1,2) = 0,9 ·(- 0,5) + 0,1 · 100 = 9,05!

Q^W (1,1,2) = 0,9 · 0,5 + 0,1 · 75 = 7,95

Q^A (1,2,2) = 0,9 ·(- 0,5) + 0,1 · 50 = 4,55!

Q^W (1,2,2) = 0,9 · 0,5 + 0,1 · 25 = 2,95

Q^A (2,1,2) = 0,9 ·(- 3,8) + 0,1 · 100 = 6,58

Q^W (2,1,2) = 0,9 · 0,0 + 0,1 · 75 = 7,5!

Q^A (2,2,2) = 0,9 ·(- 3,8) + 0,1 · 50 = 1,58

Q^W (2,2,2) = 0,9 · 0,0 + 0,1 · 25 = 2,5!

By comparing the calculated values we get the preferred choice. This time the preferred element is marked with an exclamation mark.

Table 9

Decision values for n = 1

$\left(\mathbf{V}^A(i,j,n)\right)^T$ $\left(\mathbf{V}^W(i,j,n)\right)^T$	n = 1	
i, j	0	1
0	(1,39; 100)* (1,14; 100)	(1,39; 50)? (0,83; 75)?
1	(-2,15; 100)? (0;91, 24)?	(-2,15;50) (-0,18; 58,7)*

Once again, this time for the nodes (0,1) and (1,0), we calculate the preferred decisions:

Q^A (0,1,1) = 0,9 ·1,39 + 0,1 · 50 = 6,25

Q^W (0,1,1) = 0,9 · 0,83 + 0,1 · 75 = 8,25!

Q^A (1,2,2) = 0,9 ·(- 2,15) + 0,1 · 100 = 8,07

Q^W (1,2,2) = 0,9 · 0 + 0,1 · 91,24 = 9,12!

And finally for n = 0 we have $\left(\mathbf{V}^{A}(i,j,n)\right)^{T}$ = (-0,49; 100) and $\left(\mathbf{V}^{W}(i,j,n)\right)^{T}$ = (0,35; 78,85) which gives us:

Q^{A} (0,0,0) = 0,9 ·(-0,49) + 0,1 · 100 = 9,56

Q^{W} (0,0,0) = 0,9 · 0,35 + 0,1 · 78,85 = 8,2

The best decision is to start the project at the beginning of 2013. Although this approach brings a small loss in the implemented project, it also raises hopes for future profitable projects.

5 Conclusions

The present paper outlines the valuation method of development projects in which real option situations occur. The method proposed takes into account the dependence of the project on two independent random factors, which are called state variables. Our procedure is based on binomial trees and uses a multicriteria dynamic programming method. The numerical example shows the need of computer implementation of the method. The calculations performed are straightforward but tedious.

The method discussed here, as shown in the example, allows not only to make the right decisions about the beginning the project, but also to support decision making during the project's implementation. It does this by determining the appropriate start times of the subsequent phases.

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