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Helena Gaspars-Wieloch*

A DECISION RULE FOR UNCERTAIN MULTICRITERIA MIXED DECISION MAKING BASED ON THE COEFFICIENT OF OPTIMISM

Abstract

This paper is devoted to multicriteria decision making under uncertainty with scenario planning. This topic has been explored by many researchers since almost all real-world decision problems contain multiple conflicting criteria and a deterministic criteria evaluation is often impossible.

We propose a procedure for uncertain multi-objective optimization which may be applied when a mixed strategy is sought after. A mixed strategy, as opposed to a pure strategy, allows the decision maker to select and perform a weighted combination of several accessible alternatives.

The new approach takes into account the decision maker's preference structure and attitude towards risk. This attitude is measured by the coefficient of optimism on the basis of which a set of the most probable events is suggested and an optimization problem is formulated and solved.

Keywords: multicriteria decision making, uncertainty, mixed strategy, one-shot decision, scenario planning, optimization model, coefficient of optimism, β-decision rule.

1 Introduction

This paper deals with multiple criteria decision making for cases where attribute (criterion) evaluations are uncertain. This topic has been theoretically and practically investigated by many researchers. Durbach and Stewart (2012) provide an impressive review of possible models, methods and tools used to support uncertain multicriteria decision making (e.g. models with scenarios, models using

* Poznań University of Economics and Business, Faculty of Informatics and Electronic Economy, Department of Operations Research, Al. Niepodległości 10, 60-875 Poznań, Poland, e-mail: helena.gaspars@ue.poznan.pl.

probabilities or probability-like quantities, models with explicit risk measures, models with fuzzy numbers). In this paper we propose a method designed for multicriteria decision making with scenario planning and one-shot decision problems. We assume that criteria payoff matrices are dependent. The goal of the new approach is to select an optimal mixed strategy. The procedure takes into consideration decision makers' objective preferences and their attitude towards risk. This attitude is measured by the coefficient of optimism on the basis of which a set of the most probable events is suggested and an optimization problem is formulated and solved.

The paper is organized as follows. Section 2 deals with the main features of multicriteria DMU (decision making under uncertainty) with scenario planning. Section 3 presents a procedure that may be used as a tool in multicriteria optimization under uncertainty for mixed strategy searching. Section 4 provides a case study. Conclusions are gathered in the last section.

2 Uncertain multicriteria decision making with scenario planning

According to the Knightian definition (Knight, 1921), we will assume that DMU is characterized by a situation where the decision maker (DM) has to choose the appropriate alternative (decision, strategy) on the basis of some scenarios (events, states of nature) whose probabilities are not known – uncertainty with unknown probabilities (Courtney et al., 1997; Dominiak, 2006; Groenewald and Pretorius, 2011; Render et al., 2006; Sikora, 2008; Trzaskalik, 2008; von Neumann and Morgenstern, 1944; Walliser, 2008; Williams et al., 1997).

There are many classical and extended decision rules designed for one-criterion DMU (Basili, 2006; Basili et al., 2008; Basili and Chateauneuf, 2011; Ellsberg, 2001; Etner et al., 2012; Gaspars, 2007; Gaspars-Wieloch, 2012, 2013, 2014a, 2014b, 2014c, 2014d, 2014, 2015a, 2015b, 2015c; Ghirardato et al., 2004; Gilboa, 2009; Gilboa and Schmeidler, 1989; Hayashi, 2008; Hurwicz, 1952; Ioan and Ioan, 2011; Marinacci, 2002; Piasecki, 1990; Savage, 1961; Schmeidler, 1986; Tversky and Kahneman, 1992; Wald, 1950) and multicriteria DMU (Aghdaie et al., 2013; Ben Amor et al., 2007; Dominiak, 2006; 2009; Durbach, 2014; Eiselt and Marianov, 2014; Gaspars-Wieloch, 2014e; Ginevičius and Zubrecovas, 2009; Goodwin and Wright, 2001; Hopfe et al., 2013; Janjic et al., 2013; Korhonen, 2001; Lee, 2012; Liu et al., 2011; Michnik, 2013; Mikhaidov and Tsvetinov, 2004; Montibeller et al., 2006; Ram et al., 2010; Ramik et al., 2008; Ravindran, 2008; Stewart, 2005; Suo et al., 2012; Triantaphyllou and Lin, 1996; Tsaur et al., 2002; Urli and Nadeau, 2004; Wang and Elhag, 2006; Wojewnik and Szapiro, 2010; Xu, 2000; Yu, 2002). Nevertheless, the majority of the extended rules refer to the probability calculus (for instance, expected utility maximization, α-maximin expected utility, cumulative prospect theory, Choquet expected utility), which is rather characteristic of DMR – decision making under risk or DMU with known probabilities. Let us recall that according to the Knight's definition uncertainty occurs when we do not know (i.e. we cannot measure) the probabilities of particular scenarios[1] (see complete uncertainty).

Some existing procedures are dedicated to searching for an optimal pure strategy, other are designed for searching for an optimal mixed strategy. In the case of pure strategies, the DM chooses and completely executes only one alternative. On the other hand, a mixed strategy implies that the DM selects and performs a weighted combination of several accessible alternatives, see e.g. bonds portfolio construction, cultivation of different plants (Guzik, 2009; Ignasiak, 1996; Officer and Anderson, 1968; Puppe and Schlag, 2009; Sikora, 2008). This paper will deal with the latter case.

We recognize both types of uncertainties: internal (related to DM's values and judgments) and external (related to imperfect knowledge of the consequences of action), but in this paper we focus on the latter (Durbach and Stewart, 2012; Stewart, 2005).

Durbach and Stewart (2012) state that uncertainties become increasingly so complex that the elicitation of measures such as probabilities, belief functions or fuzzy membership functions becomes operationally difficult for DMs to comprehend and virtually impossible to validate. Therefore, in such contexts it is useful to construct scenarios which describe possible ways in which the future might unfold. Hence, MDMU+SP (multicriteria decision making under uncertainty with scenario planning) will be considered in this paper. Scenario planning, used within the framework of DMU (Pomerol, 2001), is a technique for facilitating the identification of uncertain and uncontrollable factors which may influence the effects of decisions in the strategic management context. The construction of scenarios is described e.g. in (Dominiak, 2006; Van der Heijden, 1996). The result of the choice made under uncertainty with scenario planning depends on two factors: which decision will be selected and which scenario will occur.

The discrete version (the set of alternatives is explicitly defined and discrete in number) of MDMU+SP consists of n decisions ($D_1, \ldots, D_j, \ldots, D_n$), each evaluated on p criteria $C_1, \ldots, C_k, \ldots, C_p$ and on m mutually exclusive scenarios ($S_1, \ldots, S_i, \ldots, S_m$). The problem can be presented by means of p payoff matrices (one for each criterion) and $p \times n \times m$ evaluations. Each payoff matrix contains

[1] Of course, we are aware of the fact that many researchers apply the alternative approach according to which each non-deterministic (with known and unknown probabilities) decision problem is treated as an uncertain problem, while risk is understood as the possibility that some adverse circumstances might happen (see. e.g. Ogryczak and Sliwiński, 2009).

$n \times m$ evaluations, say a_{ij}^{k}, which denote the performance of criterion C_k resulting from the choice of decision D_j and the occurrence of scenario S_i. We assume that the distribution of payoffs related to a given decision is discrete.

Existing decision rules differ from each other with respect to the DM's attitude towards risk which can be measured, for instance, by the coefficient of pessimism (α) or the coefficient of optimism (β). Note that in this context we do not treat risk as a situation where the probability distribution of each parameter of the decision problem is known, but we admit the possibility that some adverse circumstances might happen (Dominiak, 2006, 2009; Fishburn, 1984).

It is worth emphasizing that some rules can be applied when the DM intends to perform the selected strategy only once. Others are recommended for people considering multiple realizations of the chosen variant. In the first case, the alternatives are called one-shot (one-time) decisions; in the second case, multi-shot decisions. This paper focuses on one-shot decision problems which are commonly encountered in business, economics and social systems (Guo, 2010, 2011, 2013, 2014; Liu and Zhao, 2009).

Marler and Arora (2004) divide multi-objective optimization concepts and methods into three categories: (a) methods with a priori articulation of preferences (MPAP), (b) methods with a posteriori articulation of preferences (MPSAP) and (c) methods with no articulation of preferences (MNAP). In MPAP the user indicates the relative importance of the objective functions or desired goals (by means of parameters which are coefficients, exponents, or constraint limits) before running the optimization algorithm (Chang, 2011; Churmann and Ackoff, 1954; Gaspars-Wieloch, 2011; Lotfi et al., 1997). MPSAP entail selecting a single solution from a set of mathematically equivalent solutions. This means that the DM imposes preferences directly on a set of potential final solutions. In this paper we propose an MPAP procedure with the application of weights for each attribute.

As mentioned before, the decision rule presented in this paper allows the DM to find an optimal mixed strategy, but it is worth emphasizing that the existing one-criterion and multicriteria procedures for mixed strategies are related more to game theory, i.e. game between players (Czerwiński, 1969; Gilboa, 2009; Grigorieva, 2014; Lozan and Ungureanu, 2013; Luce and Raiffa, 1957; Voorneveld et al., 1999; 2000), than to game against nature (which constitutes a neutral opponent). Therefore, the creation of an approach for uncertain multiobjective mixed decision making with scenario planning (or scenario-based MMDM) seems vital and desirable.

3 β-decision rule for uncertain multicriteria mixed decision making

When preparing a decision rule for uncertain multicriteria mixed decision making, one should answer two main questions: (1) how should DM's preferences (concerning the attitude towards risk and the importance of particular criteria) be taken into account?, and (2) how should criteria be aggregated and how should they be combined with scenarios?

Possible rules for 1-criterion mixed strategies are as follows:

(a) Bayes' rule (the DM performs the selected plan many times) – the optimization model maximizes the average income.

(b) Wald's rule (the DM performs the chosen decision only once and behaves cautiously, the minimal guaranteed benefit is maximized) – the solution of such a problem ensures that even if the least attractive scenario takes place, the income of the DM will not be lower than y^*, i.e. the maximized minimum guaranteed revenue.

(c) Hurwicz's rule (the DM performs the selected plan only once and declares the level of pessimism or optimism) – the optimization model takes into consideration only extreme payoffs connected with a given alternative, not the frequency of the occurrence of intermediate ones (see Gaspars-Wieloch, 2012; 2014a; 2014c), which may lead to quite illogical recommendations.

The last two approaches treat nature as a conscious opponent who is altering strategies depending on the outcomes, which is strongly criticized by (Milnor, 1954; Officer and Anderson, 1968).

In connection with the fact that we analyze only one-shot decisions and that solutions recommended by the rule should vary depending on the DM's attitude towards risk, Hurwicz's rule seems the most appropriate. Nevertheless, due to some drawbacks connected with this procedure, we will refer to another method – the β-decision rule, originally designed for one-criterion mixed decision making (Gaspars-Wieloch, 2014b). In this method the number of scenarios considered in the optimization model depends on the level of the DM's optimism. If $\beta = 0$, then all states of nature are taken into account, since the DM intends to be well prepared for the uncertain future. Meanwhile, if $\beta > 0$, then the initial set of possible scenarios is appropriately reduced to a smaller set of events, because the most pessimistic states of nature may be omitted in the analysis (i.e. they are the least probable). When $\beta = 0$, the mixed strategy recommended by the β-decision rule is the optimal solution generated by the problem formulated according to Wald's rule. In Gaspars-Wieloch (2014b) it is suggested to assign the status to a given event on the basis of two measures connected with the outcomes of that state of nature. Nevertheless, the method of determining the set of the most probable scenarios may be different.

Criterion 1	D_1	D_j	D_n
S_1	a^1_{11}	a^1_{1j}	a^1_{1n}
S_i	a^1_{i1}	a^1_{ij}	a^1_{in}
S_m	a^1_{m1}	a^1_{mj}	a^1_{mn}

Criterion k	D_1	D_j	D_n
S_1	a^k_{11}	a^k_{1j}	a^k_{1n}
S_i	a^k_{i1}	a^k_{ij}	a^k_{in}
S_m	a^k_{m1}	a^k_{mj}	a^k_{mn}

Criterion p	D_1	D_j	D_n
S_1	a^p_{11}	a^p_{1j}	a^p_{1n}
S_i	a^p_{i1}	a^p_{ij}	a^p_{in}
S_m	a^p_{m1}	a^p_{mj}	a^p_{mn}

Figure 1. Payoff matrices

Now, let us check how scenarios should be combined with criteria. According to Durbach, Stewart (2012); Michnik (2013) and Stewart (2005) MDMU+SP models can be divided into two classes. The first one (A) includes two-stage models in which evaluations of particular alternatives are estimated with respect to scenarios and criteria in two separate stages. Class A contains two subclasses: A-CS and A-SC. Subclass A-CS is the set of approaches considering decisions separately in each scenario and setting an $n \times m$ table giving the aggregated (over attributes) performance of alternative D_j under scenario S_i. These evaluations are then aggregated over scenarios. In subclass A-SC the order of aggregation is reversed – performances are generated across scenarios and then measures are calculated over criteria. The second class (B) consists of one-stage procedures considering combinations of scenarios and attributes (scenario-criterion pairs) as distinct meta-criteria. In our research we will apply an A-CS model since we assume that payoff matrices are dependent, which means that if scenario S1 occurs and decision D1 is selected, then the performance of the particular criteria is as follows: $a_{11}{}^1$, $a_{11}{}^2$, …, $a_{11}{}^p$ (Figure 1).

To adapt the β-decision rule for uncertain one-criterion mixed decision making to multicriteria analysis, it is necessary to combine that procedure with a multiobjective method. At first glance, there are many approaches dedicated to MDU (Trzaskalik, 2014), e.g.:

a) additive methods, such as SAW, SMART or SMARTER (Churmann and Ackoff, 1954; Edwards and Barron, 1994),
b) AHP, REMBRANDT, ANP (Saaty, 1980; 1996; Lootsma, 1993),
c) MACBETH, ZAPROS (Bana e Costa and Chagas, 2004; Larichev and Moshkovich, 1995),
d) ELECTRE (Roy and Bouyssou, 1993),
e) PROMETHEE (Brans et al., 1984),
f) TOPSIS, VIKOR, BIPOLAR (Hwang and Yoon, 1981; Opricovic, 1998; Konarzewska-Gubała, 1989).

However, it is worth noting that, due to the construction of the β-decision rule, it would be desirable if the chosen method fulfilled the following conditions:

a) it is not time-consuming since it constitutes only a stage in the whole procedure,
b) it may be applied when payoff matrices for each criterion are dependent,

c) it is applicable to problems with criteria defined in different scales and units,
d) it generates a synthetic measure for each pair: decision/scenario.

Therefore, the only methods satisfying all conditions aforementioned are SAW, SMART, SMARTER and TOPSIS. Here, the β-decision rule will be combined with SAW (Simple Additive Weighting Method).

Hence, the β-decision rule for multicriteria mixed decision making includes the following steps:

Step 1: Given a set of potential decisions and payoff matrices for each criterion, define an appropriate value of the parameter $\beta \in [0,1]$ according to your level of optimism and choose weights w^k for each attribute ($k = 1,\ldots,p$):

$$\sum_{k=1}^{p} w^k = 1 \tag{1}$$

Step 2: If necessary, normalize the evaluations (use Equation (2) for maximized criteria and Equation (3) for minimized criteria):

$$a(n)_{ij}^k = \frac{a_{ij}^k - \min\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}} \left\{a_{ij}^k\right\}}{\max\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}} \left\{a_{ij}^k\right\} - \min\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}} \left\{a_{ij}^k\right\}} \qquad k = 1,\ldots,p,\ i = 1,\ldots,m,\ j = 1,\ldots,n \tag{2}$$

$$a(n)_{ij}^k = \frac{\max\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}} \left\{a_{ij}^k\right\} - a_{ij}^k}{\max\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}} \left\{a_{ij}^k\right\} - \min\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}} \left\{a_{ij}^k\right\}} \qquad k = 1,\ldots,p,\ i = 1,\ldots,m,\ j = 1,\ldots,n \tag{3}$$

Step 3: Compute the aggregated measure $A(n)_{ij}$ for each pair: decision/scenario (according to the methodology of SAW):

$$A(n)_{ij} = \sum_{k=1}^{p} w^k \cdot a(n)_{ij}^k \qquad i = 1,\ldots,m,\ j = 1,\ldots,n \tag{4}$$

Step 4: Find M^* (the maximum aggregated value computed according to the max-max rule) and calculate y^* which is the maximized minimum guaranteed aggregated value computed on the basis of Wald's model (Equations 5-8):

$$y \to \max \tag{5}$$

$$\sum_{j=1}^{n} A(n)_{ij} x_j \geq y \qquad i = 1,\ldots,m \tag{6}$$

$$\sum_{j=1}^{n} x_j = 1 \tag{7}$$

$$x_j \geq 0 \qquad j = 1,\ldots,n, \tag{8}$$

where x_j is the share of alternative D_j in the mixed strategy and n stands for the number of decisions.

Step 5: Find the set of the most probable scenarios (K) with the aid of Equations (9)-(13):

$$S_i \in K \Leftrightarrow \left(\underset{A(n)_{ij}\,(j=1,\ldots,n)}{\exists} A(n)_{ij} \geq r_\beta \right) \vee \left(d_i \geq d_\beta \right) \tag{9}$$

$$r_\beta = \beta(M^* - y^*) + y^* \tag{10}$$

$$d_\beta = \beta(d_{\max} - d_{\min}) + d_{\min} \tag{11}$$

$$d_i = \sum_{j=1}^{n} d_{ij} \qquad i = 1,\ldots,m \tag{12}$$

$$d_{ij} = m - \max_i \left\{ p(A(n)_{ij}) \right\} \qquad i = 1,\ldots,m, j = 1,\ldots,n \tag{13}$$

where K is the set of the most probable events, $A(n)_{ij}$ is the synthetic value of normalized payoffs connected with decision D_j and event S_i. r_β is the expected level of the aggregated outcome dependent on β (Equation 10). d_{ij} denotes the number of aggregated values related to alternative D_j which are worse than $A(n)_{ij}$. The symbol m still denotes the number of scenarios and $p(A(n)_{ij})$ is the position of the value $A(n)_{ij}$ in the non-increasing sequence of all synthetic evaluations connected with decision D_j (if $A(n)_{ij}$ has the same value as other evaluations of a given alternative, then it is recommended to choose the farthest position of this value in the sequence – see Equation 13). d_i is the total number of "dominance cases" related to state S_i (Equation 12), d_{max} and d_{min} are the biggest and the smallest number of "dominance cases", respectively (Equation 11).

As can be seen, scenario S_i may belong to K if and only if it contains at least one aggregated payoff not lower than r_β (Equations 9 and 10) or if its number of "dominance cases" is sufficiently close to d_{max} (Equations 9 and 11). The scenario with d_{max} and with at least one aggregated payoff equal to M^* might be treated as the best state of nature (the most optimistic), but in many decision problems such an event does not exist.

Step 6: Solve the following optimization problem:

$$\sum_{i \in K} \max\{g_i, 0\} \rightarrow \min \tag{14}$$

$$\sum_{j=1}^{n} A(n)_{ij} x_j = r_\beta - g_i \quad i \in K \tag{15}$$

$$\sum_{j=1}^{n} x_j = 1 \tag{16}$$

$$x_j \geq 0 \qquad j = 1,\ldots,n \tag{17}$$

where g_i is the deviation from r_β of the aggregated income achieved by the DM if scenario S_i occurs. The optimal solution represents the multi-criteria mixed strategy reflecting the DM's level of optimism.

Both sides of condition (15) present the true aggregated revenue obtained if the shares of a particular mixed strategy equal x_1, x_2, …, x_n and scenario S_i takes place. The aim of the optimization model (Equation 14) is to minimize, within the set K, the sum of all deviations of the true aggregated payoffs from the expected one. Note that only positive deviations are disadvantageous since then the expected revenue exceeds the true aggregated income.

Let us call the aforementioned procedure β-MMDM, i.e. the β decision rule for multicriteria mixed decision making.

4 Case study

The method suggested in this paper will be illustrated by means of the following example. Let us assume that the DM intends to find the optimal mixed strategy on the basis of two objectives C1 and C2, which are both maximized. There are four possible decisions: D1, D2, D3 and D4. The DM is not able to define exact evaluations of both criteria, but thanks to scenario planning the list of possible states of nature (S1, S2, S3, S4, S5) has been generated. Table 1 presents payoff matrices of the analyzed decision problem.

To find the most appropriate strategy with the aid of β-MMDM, in the first step the DM is asked to declare the coefficient of optimism, let us say $\beta = 0.7$, and to set weights for each attribute, e.g. $w^1 = 0.4$ and $w^2 = 0.6$.

Table 1: Payoff matrices – initial evaluations

	C1				C2			
No	D1	D2	D3	D4	D1	D2	D3	D4
S1	2.5	4.0	4.5	3.0	20	22	15	21
S2	1.3	2.5	3.5	3.0	32	18	19	17
S3	1.6	3.0	4.3	2.0	29	19	16	18
S4	1.7	3.0	2.0	2.5	28	15	23	24
S5	1.5	3.5	4.2	4.0	30	17	16	24

Step 2 is optional, but in our case it is obligatory because the evaluations are defined in different scales. The normalized values are computed in Table 2 (Equation 2).

Table 2: Payoff matrices – normalized values

	C1				C2			
No	D1	D2	D3	D4	D1	D2	D3	D4
S1	0.38	0.84	1.00	0.53	0.29	0.41	0.00	0.35
S2	0.00	0.38	0.69	0.53	1.00	0.18	0.24	0.12
S3	0.09	0.53	0.94	0.22	0.82	0.24	0.06	0.18
S4	0.13	0.53	0.22	0.38	0.76	0.00	0.47	0.53
S5	0.06	0.69	0.91	0.84	0.88	0.12	0.06	0.53

In step 3 we refer to the A-CS approach and to SAW. The aggregated normalized values are given in Table 3 (Equation 4).

Table 3: Aggregated measures $A(n)_{ij}$

No	D1	D2	D3	D4
S1	0.326	0.585	0.400	0.424
S2	0.600	0.256	0.416	0.283
S3	0.532	0.354	0.410	0.193
S4	0.509	0.213	0.370	0.468
S5	0.554	0.346	0.398	0.655

In step 4 we find $M^* = \max\{0.600;0.585;0.416;0.655\} = 0.655$ and $y^* = 0.418$ according to the following model:

$$y \to \max$$
$$0.326x_1 + 0.585x_2 + 0.400x_3 + 0.424x_4 \geq y$$
$$0.600x_1 + 0.256x_2 + 0.416x_3 + 0.283x_4 \geq y$$
$$0.532x_1 + 0.354x_2 + 0.410x_3 + 0.193x_4 \geq y$$
$$0.509x_1 + 0.213x_2 + 0.370x_3 + 0.468x_4 \geq y$$
$$0.554x_1 + 0.346x_2 + 0.398x_3 + 0.655x_4 \geq y$$
$$x_1 + x_2 + x_3 + x_4 = 1$$
$$x_1, x_2, x_3, x_4 \geq 0$$

In step 5 parameters r_β (Equation 10) and d_β (Equations 11-13) are calculated in order to find the most probable scenarios:

$$r_\beta = 0.7(0.655 - 0.418) + 0.418 = 0.5838$$
$$d_\beta = 0.7(10 - 4) + 4 = 8.2$$

Table 4 contains the values of “dominance cases” and the sum of “dominance cases” for each state of nature ($d_{max} = \max\{8;10;8;4;10\} = 10$, $d_{min} = \min\{8;10;8;4;10\} = 4$).

Table 4: “Dominance cases“

No	D1	D2	D3	D4	d_i
S1	0	4	2	2	8
S2	4	1	4	1	10
S3	2	3	3	0	8
S4	1	0	0	3	4
S5	3	2	1	4	10

Hence, there are three scenarios with at least one value not lower than r_β, i.e. S1, S2 and S5. Additionally, we note that the sum of “dominance cases” for events S2 and S5 is not lower than d_β. That means that the set K contains three elements: K={S1, S2, S5}, see Equation (9).

The optimal multicriteria mixed strategy ($x_1 = 0.9489$, $x_2 = 0$, $x_3 = 0$, $x_4 = 0.0511$) is established on the basis of the optimization model formulated below (step 6):

$$\max\{g_1,0\} + \max\{g_2,0\} + \max\{g_5,0\} \rightarrow \min$$
$$0.326x_1 + 0.585x_2 + 0.400x_3 + 0.424x_4 = 0.5838 - g_1$$
$$0.600x_1 + 0.256x_2 + 0.416x_3 + 0.283x_4 = 0.5838 - g_2$$
$$0.554x_1 + 0.346x_2 + 0.398x_3 + 0.655x_4 = 0.5838 - g_5$$
$$x_1 + x_2 + x_3 + x_4 = 1$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Thus, the DM should invest 94.89% of his funds in decision D1 and 5.11% in decision D4. The deviation degrees for scenarios S1, S2 and S5 are: $g_1 = 0.2528$, $g_2 = 0$, $g_5 = 0.0246$. The deviation for event S1 is the largest, but note that this state of nature does not satisfy the second condition of disjunction (9), which means that this is the least probable scenario among all scenarios belonging to K.

In this paper, the set K is formed in the paper on the basis of two criteria (the expected aggregated income r_β and the number of "dominance cases" d_β). Nevertheless, this is only a suggestion – one may choose other indices. Here, we will explain why it is recommended to consider both r_β and d_β, not only the first criterion. When the maximum aggregated payoff M^* is much higher than the remaining payoffs in the matrix, then, even for low values of β, index r_β becomes so high that only the scenario offering M^* meets the criterion r_β . This means that in such cases, regardless of the level of optimism, only one state of nature is treated as the most probable, which is not reasonable. The cardinality of the set K depends on the coefficient of optimism. The higher β is, the fewer elements the set K contains. However, it is worth emphasizing that when $\beta = 1$, the set of the most probable scenarios does not need to contain exactly one element.

5 Conclusions

In this paper, we propose a procedure for uncertain multiobjective optimization which may be applied when a mixed strategy is sought. The new approach (β-MMDM, i.e. β-decision rule for multicriteria mixed decision making) takes into account the decision maker's preference structure and attitude towards risk. This attitude is measured by the coefficient of optimism on the basis of which a set of the most probable events is suggested and an optimization problem is formulated and solved. The β-decision rule (a procedure originally designed for scenario-based one-criterion mixed decision making) is combined with the Simple Additive Weighting Method. Hence, according to the classification described in (Michnik, 2012), the β-MMDM is not a typical MCDA (multicriteria decision analysis) hybrid, since only one of its components involves multiobjective optimization (i.e. SAW), while the other one is related to one-criterion decision problems. The new decision rule has at least four significant advantages. First, it recommends different mixed strategies depending on the DM's level of optimism (in contradiction to Wald's rule or max-max rule). Second, it involves game against nature, while the existing multicriteria mixed decision making procedures are designed for games with another player). Third, is does not treat nature as a conscious opponent who is altering strategies depending on the outcomes. Fourth, it is suitable for problems with criteria defined in different scales and units. Future research should deal with the coefficient of optimism, i.e. the method of estimation of that parameter and its impact on the final decision.

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