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BICRITERIA OPTIMIZATION IN THE NEWSVENDOR PROBLEM WITH EXPONENTIALLY DISTRIBUTED DEMAND[1]

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Abstract

In this paper exponential distribution is implemented as a demand distribution in newsvendor model with two different and conflicting goals. The first goal is the standard objective of maximization of the expected profit. The second one is to maximize the probability of exceeding the expected profit, called survival probability. Using exponential distribution as the demand distribution allows us to obtain the exact solutions. Also for this distribution we can study the monotonicity of survival probability with respect to various model parameters analytically. Additional results are obtained when various sets of the parameters are considered. Finally, the bicriteria index which combines these conflicting objectives is optimized which gives the compromise solution. Moreover, in order to illustrate theoretical results, we present numerical examples and graphs of auxiliary functions.

Keywords: stochastic demand, newsvendor problem, bicriteria optimization.

1 Introduction

There is a great variety of stochastic models in the inventory theory. We refer to the papers of Plewa (2010), Prusa and Hruska (2011), Zipkin (2000) and the references therein. The fundamental inventory stochastic model is the newsvendor problem denoted by NVP. A survey of this topic has been given recently by Quin et al. (2011). In the basic model, the aim is to determine the order quantity which

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maximizes the expected profit. Some authors applied alternate or multiple criteria (Choi, 2012; Gaspars-Wieloch, 2015, 2016; Rubio-Herrero, 2015; Ye and Sun; 2016, Kamburowski, 2014). For instance, instead of the maximization of the expected profit, the maximization of the probability of exceeding the target profit can be used. This is called the survival probability and the corresponding objective is called satisficing or aspiration-level objective. The aspiration-level objective in NVP was first discussed by Kabak and Shiff (1978). Since then the problem was widely studied by Lau (1980) and Li et al. (1991). Recently the NVP has been extended by introducing the bicriteria decision problem. In the extended model the newsvendor incorporates two conflicting goals into the objective function. The first goal is the classic maximization of the expected profit and the second one is the satisficing-level objective. The only decision variable is the order quantity needed to satisfy uncertain demand. Parlar and Weng (2003) consider a bicriteria NVP with a moving target which is the expected profit. In this case two conflicting goals are taken into account together since there is no solution which maximizes both constraints simultaneously. The bicriteria index combines both results by assigning appropriate weights which are numbers between 0 and 1 which sum up to 1. Parlar and Weng (2003) obtained the approximate result which is then applied to the case of normally distributed demand. Arcelus et al. (2012) continued this research for uniform distribution which allows to derive precise analytic results.

It should be noted here that both normal and uniform distributions belong to the class of maximum entropy probability distributions. This class is widely used in practice and in many papers these distributions are applied to model the unknown random demand (see for instance Eren and Maglaras, 2006). The classical entropy maximizing distributions are listed by Perakis and Roels (2008). For more details, we refer also to Eren and Maglaras (2015) and Lim and Shantikumar (2007). The normal distribution is the maximum-entropy distribution on the whole real line with fixed mean and variance. On the other hand, the uniform distribution is a good choice if we only know that the demand has positive mean and support on a finite interval. Yet another distribution which approximates the unknown demand well is the exponential distribution. This is the maximum entropy distribution in the class of continuous distributions with fixed finite mean and support on the positive half-axis $[0, \infty)$ (Andersen, 1970; Harrenoes, 2001).

However, when the coefficient of variation of the demand is large, then using the normal distribution leads to excessive orders and large financial losses may occur, as it was observed by Gallego et al. (2007). For this reason, for products with a large coefficient of variation, they recommended to use another classes of distributions including the exponential distribution.

Another argument to study exponentially distributed demand is to make the model simpler and mathematically tractable. This distribution belongs to the gamma and Weibull family of distributions, which are relatively easy to work with and they often provide good approximation to the actual demand distribution when data are highly variable. The exponential distribution is used in practice to represent interarrival times of customers to a system (times between two independent events) that occur at a constant rate, as well as the time to failure of a piece of equipment. One more feature of the exponential distribution is that its failure rate is constant. More reasonable customer demand distributions such as uniform, normal, gamma and Weibull distributions (Lariviere, 2006) belong to the class of distributions with increasing failure rate.

All the above mentioned arguments justify the use of the exponential distribution as distribution of the demand in the bicriteria newsvendor problem. The exponentially distributed demand with maximization of the probability of exceeding the target profit was studied by Li et al. (1991). The difference between our paper and theirs is that they consider a constant profit goal and a two-product newsvendor, and they do not obtain so many analytical results as we do.

We use the known notions defined in the above mentioned papers but the use of the exponential distribution allows us to obtain precise results and to investigate the obtained solutions more in detail. We can study analytically the monotonicity of the survival probability with varying parameters of the model. The mathematical computations are almost elementary, but we get some additional results for specific combinations of these parameters. It is worth noting here that for the general case an analogous analysis cannot be performed because the equations involved are cubic.

The rest of the paper is organized as follows. Section 2 provides the basic notation and formulation of the single criterion models and the bicriteria newsvendor problem. Next, in Section 3 we study analytically the variability of survival probability with respect to the model parameters. We also present example graphs of the considered functions to illustrate the nature of the solutions. Moreover, we provide a numerical example to illustrate the key properties of the elements of the bicriteria problem. In Section 4 we combine both objectives in one measure called bicriteria index. The solutions can be obtained numerically as well, which is illustrated by a numerical example. The last section concludes the paper.

2 Definition of the bicriteria newsvendor problem

In this section we recall the bicriteria problem in the newsvendor model and derive the optimality conditions for exponentially distributed demand. First we consider the model with the expected profit maximization as the objective. We recall the known results and apply them in the case when the demand is exponentially distributed.

In the newsvendor model we consider a retailer who wants to acquire Q units of a given product to satisfy exponentially distributed demand. First we introduce the following notation. Define:

- $p > 0$ to be the selling price for unit (unit revenue);
- $c > 0$ to be the purchasing cost per unit;
- $s > 0$ to be the unit shortage costs;
- v to be the unit salvage value (unit price of disposing any excess inventory);
- $f(.)$ and $F(.)$ to be the probability density function and the cumulative distribution function of the demand with mean μ.

The standard assumption is $v < c < p$.

In our case the demand is exponentially distributed with the density: $f(x) = \lambda e^{-\lambda x}, x > 0,$ and the cumulative distribution function $F(x) = 1 - e^{-\lambda x}$, $x > 0$, where $\lambda > 0$ is the parameter of this distribution. Then the mean demand is $\mu = \frac{1}{\lambda}$. Define $\pi(Q, x)$ to be the retailer's profit function given by:

$$\pi(Q,x) = \begin{cases} px + v(Q - x) - cQ, if\ x \leq Q \\ pQ - s(x - Q) - cQ, if\ x > Q, \end{cases}$$

where Q is the order quantity and x is the realized demand. Then the expected profit $E(Q)$ is given by:

$$E(Q) = (p - c)\mu - (c - v)(Q - \mu) - (p + s - v)\int_Q^\infty (x - Q)f(x)dx,$$

(Acelus, 2012), which in the exponential case simplifies to:

$$E(Q) = \frac{1}{\lambda}(p - v) - (c - v)Q - \frac{1}{\lambda}(p + s - v)e^{-\lambda Q}.$$

Note that $E(0) = -s/\lambda$ and $E(\infty) = -\infty$. In the expected profit newsvendor model the aim is to maximize $E(Q)$. Thus in this case the condition:

$$E'(Q) = -(c - v) + (p + s - v)e^{-\lambda Q} = 0$$

determines the order quantity maximizing the expected profit, which is given by:

$$Q_E^* = \frac{1}{\lambda}\ln\frac{p + s - v}{c - v}. \quad (1)$$

The feasibility of this solution is proved by the fact that for the second derivative we have:

$$E''(Q) = -\lambda(p + s - v)e^{-\lambda Q} < 0,$$

which implies that the function $E(Q)$ is concave. The shape of the function of $E(Q)$ for the parameters $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$ is shown in Figure 1 below.

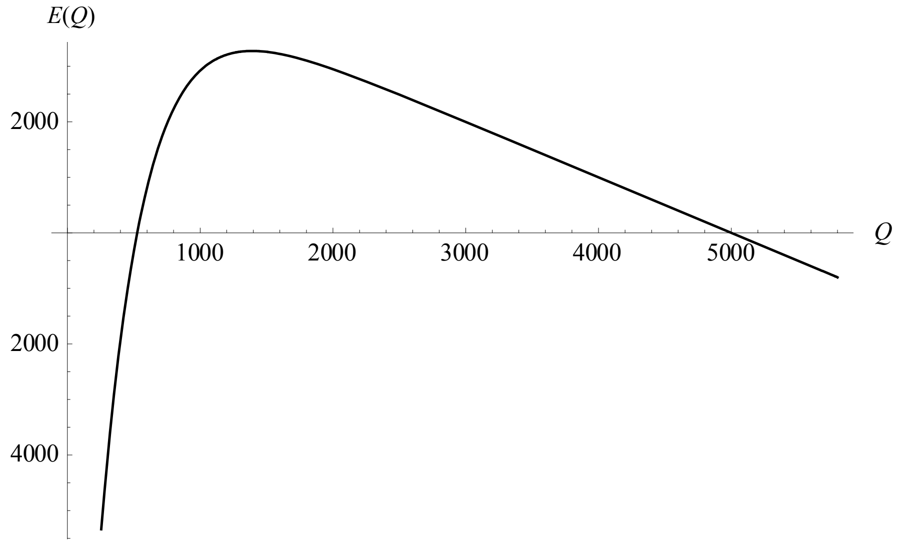

Figure 1. Expected profit function for $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$

In the other approach to the newsvendor model the probability of exceeding the expected profit is maximized instead of the expected profit itself. Let $H(Q)$ be the probability of this event, namely:

$$H(Q) = P\big(\pi(Q) \geq E(Q)\big),$$

which is called the survival probability. Its optimization with respect to Q in the exponential case will be performed in the next section. Let Q_H^* be the optimal order quantity which maximizes $H(Q)$. In the bicriteria newsvendor model both conditions mentioned above are considered together, although these objectives are conflicting with each other. Hence a new measure should be introduced which treats both constraints simultaneously. For this purpose the bicriteria index $Y(Q)$ is defined as:

$$Y(Q) = \frac{w}{E^*} E(Q) + \frac{1-w}{H^*} H(Q).$$

Here $E^* = E(Q_E^*)$ and $H^* = H(Q_H^*)$. Note also that both E^* and H^* are constants in the bicriteria function. The weight $0 \leq w \leq 1$ measures the relative importance of $E(Q)$ and $H(Q)$. If w increases, the risk-aversion decreases and $w = 1$ reflects risk-neutrality. Our aim is to find the order quantity which maximizes the bicriteria index which can be considered as a compromise solution to the bicriteria problem.

3 Optimization of the survival probability for the exponential distribution

Next we give the results for the satisficing-level objective which involves the maximization of survival probability also in the case of exponentially distributed demand. From Parlar and Weng (2003) we know that the survival probability $H(Q)$ can be written as:

$$H(Q) = \int_{D_1(Q)}^{D_2(Q)} f(x)dx,$$

where for exponentially distributed demand with parameter λ the limit functions $D_1(Q)$ and $D_2(Q)$ are given by $D_1(Q) = \max\{0, k(Q)\}$ with:

$$k(Q) = \frac{(c-v)Q + E(Q)}{p-v} = \frac{1}{\lambda}\left(1 - \frac{p+s-v}{p-v}e^{-\lambda Q}\right)$$

and:

$$D_2(Q) = \frac{(p+s-c)Q - E(Q)}{s} = \frac{1}{s}\left((p+s-v)Q - \frac{p-v}{\lambda} + \frac{p+s-v}{\lambda}e^{-\lambda Q}\right).$$

To calculate the survival probability it is necessary to analyse the behaviour of the limit functions which is done in the next subsection.

3.1 The analysis of the limit functions

First we recall some properties of the limit functions such as their monotonicity or their zeroes. The expressions presented below are easily obtained from Parlar and Weng (2003), but we need them for the exponential distribution in the following study.

Note that:

$$k(0) = -\frac{s}{\lambda(p-v)} < 0$$

and:

$$k'(Q) = \frac{p+s-v}{p-v}e^{-\lambda Q} > 0.$$

Moreover, for the second derivative of the function k we have:

$$k''(Q) = -\lambda\frac{p+s-v}{p-v}e^{-\lambda Q} < 0,$$

which implies that $D_1(Q)$ is concave and increasing. Let Q_0 be such that $k(Q_0) = 0$. Then:

$$Q_0 = \frac{1}{\lambda}\ln\frac{p+s-v}{p-v},$$

which implies that $D_1(Q)$ is equal to 0 in the interval $(0, Q_0)$. Moreover, the lower limit function tends to $\frac{1}{\lambda}$ as $Q \to \infty$.

Next for the upper limit we have:

$$D_2(0) = \frac{1}{\lambda},$$
$$D_2'(Q) = \frac{p+s-v}{s}\left(1 - e^{-\lambda Q}\right) > 0,$$
$$D_2''(Q) = \lambda \frac{p+s-v}{s} e^{-\lambda Q} > 0$$

and the upper limit $D_2(Q)$ tends to infinity as $Q \to \infty$. Therefore, the upper limit function is a convex increasing function of Q. Taking into account that:

$$D_2'(Q) - D_1'(Q) = 0$$

for $Q = Q_0$, we infer that, rather surprisingly, on the interval $[Q_0, \infty)$ the difference $D_2(Q) - D_1(Q)$ is minimized also at Q_0.

Examples of graphs of the limit functions $D_1(Q)$ and $D_2(Q)$ are presented below.

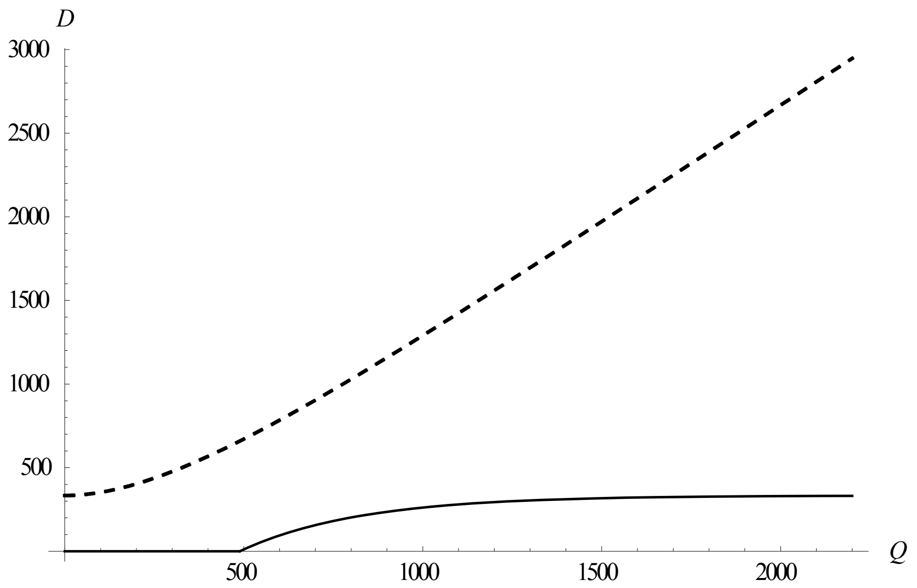

Figure 2. Limit functions $D_1(Q)$ and $D_2(Q)$ for $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$

From the expressions for the limit functions we observe in Figure 2 that the graph of the upper limit always lies under the graph of the lower one. Moreover, the minimum distance between these limits occurs for the point identical with a zero of $k(Q)$, which is the function corresponding to the lower limit.

Using these facts in the next subsection we solve the problem of optimization of the survival probability function $H(Q)$.

3.2 Optimization of survival probability $H(Q)$

First we investigate the variability of the survival probability function when the demand is exponentially distributed. It is known that $H(0) = 1 - e^{-1}$ and $H(\infty) = e^{-1}$. We stress the fact that while for uniformly distributed demand the minimum distance between the limit functions corresponds to the minimum probability $H(Q)$, this is not the case for the normal or the exponentially distributed demand. However, for the exponential distribution under some conditions on the parameters of the model, the order quantity which minimizes the distance between the limit functions is simultaneously the quantity which optimizes the survival probability function $H(Q)$. The following theorem provides satisfactory conditions which assure the existence of the maximum of $H(Q)$.

Theorem 1
If the demand distribution in the NVP is an exponential distribution with parameter λ, then the following statements hold.

a. If for some parameters p, s, v and some $\lambda > 0$, we have:
$$a(Q) < b(Q), for\ Q > Q_0, \tag{2}$$
where $a(Q) = e^{-\lambda(D_2(Q)-D_1(Q))}$ and $b(Q) = \frac{s}{p-v}\frac{e^{-\lambda Q}}{1-e^{-\lambda Q}}$, then condition (1) is satisfied for any $\lambda > 0$.
b. If (2) holds, then the survival probability function $H(Q)$ attains the maximum value at:
$$Q_H^* = \frac{1}{\lambda}\ln\frac{p+s-v}{p-v} \tag{3}$$
and the maximal survival probability is given by the formula:
$$H^* = 1 - \left(\frac{p-v}{p-v+s}\right)^{\frac{p-v+s}{s}}$$

Proof of Theorem 1. For simplicity let $a = p - v$. Then the expressions for the limit functions simplify to:
$$k(Q) = \frac{1}{\lambda}\left(1 - \frac{a+s}{a}e^{-\lambda Q}\right)$$
and:
$$D_2(Q) = \frac{1}{s}\left((a+s)Q - \frac{a}{\lambda} + \frac{a+s}{\lambda}e^{-\lambda Q}\right).$$
Note that $a > 0$ and both the limit functions and the survival probability do not depend directly on the parameters p and v but only on their difference. Then we calculate:
$$H'(Q) = e^{-\lambda Q\left(\frac{a+s}{s}+1\right)}e^{-\frac{(a+s)^2}{as}e^{-\lambda Q}} - \frac{s}{a}\frac{e^{-\lambda Q}}{1-e^{-\lambda Q}}$$

and we infer that the sign of the derivative does not depend on the parameter of the exponential distribution which proves (a).

Moreover, the variability of the function $H(Q)$ is as follows. First, it is increasing on the interval $(0, Q_0)$ since the lower limit $D_1(Q)$ is equal to 0 on $(0, Q_0)$ and the upper limit $D_2(Q)$ is increasing on this interval. It suffices to note that condition (2) is equivalent to the statement that $H(Q)$ is decreasing on (Q_0, ∞). Combining these facts, we get statements (b) and (c) of the theorem.

We illustrate the results of Theorem 1 with example graphs. In Figure 3 we present the graph of $H(Q)$ for the case when the vector of the model parameters is $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$, and then the survival probability is decreasing for all $Q > Q_0$. To illustrate the possibility of non-monotonicity of the survival probability for $Q > Q_0$, consider the following set of the model parameters $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 1)$. The graph of $H(Q)$ for this case is shown in Figure 4.

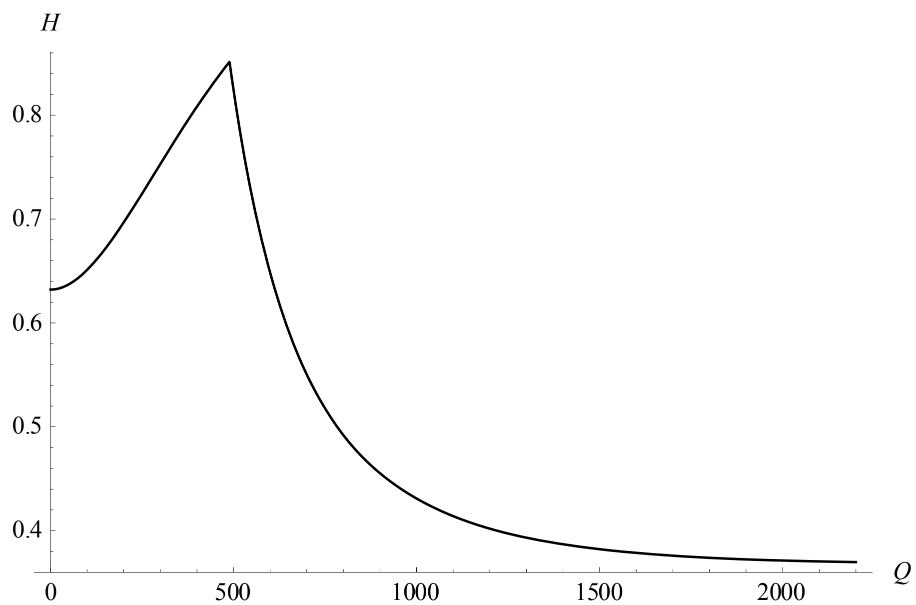

Figure 3. Survival probability for $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$

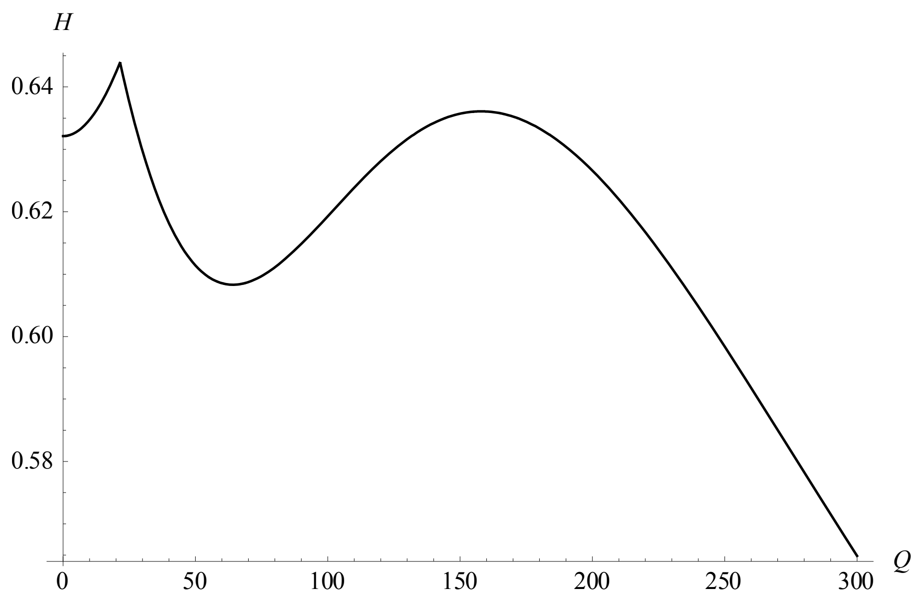

Figure 4. Survival probability for $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 1)$

We conclude this subsection with a short discussion of Theorem 1.

First, the answer to the question whether condition (2) is satisfied or not does not depend on the parameter λ of the demand distribution. The value of maximal probability H^* does not depend on λ either.

Second, if condition (2) is satisfied, then comparing equations (1) and (3) we see that the order quantity Q_H^* optimizing the survival probability is strictly smaller than Q_E^* which optimizes the expected profit.

Third, in the special case when $p - v = s$, we get the optimal solution $Q_H^* = \frac{1}{\lambda}\ln 2$ and the maximal survival probability $H^* = 0.75$, so these quantities do not depend on the model parameters. Indeed, in this case the derivative $H'(Q)$ is negative for any $Q > Q_0$. To prove this statement, in the expression for $H(Q)$ we substitute $e^{-\lambda Q} = z$, where $0 < z < \frac{1}{2}$, and define an auxiliary function:

$$g(z) = H\left(-\frac{1}{\lambda}\log z\right) = e^{-2z-1} - z^2 e^{1-2z}.$$

The function $g(z)$ is increasing which is shown in Figure 5. Moreover $-\frac{1}{\lambda}\log z$ is a decreasing function of z which implies that the survival probability $H(Q)$ is decreasing.

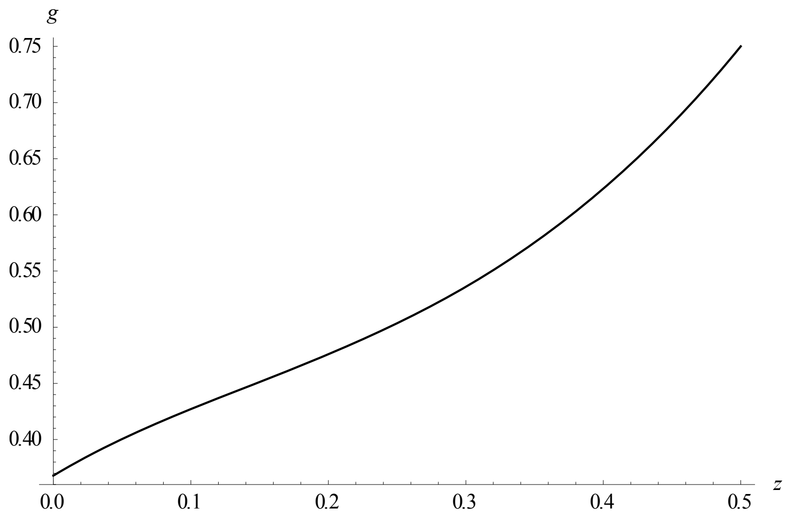

Figure 5. Auxiliary function $g(z)$

In the next subsection we investigate the monotonicity of the survival probability with respect to changes in the values of parameters.

3.3 Sensitivity analysis

The optimal order quantity in the expected profit model is different from the optimal order quantity in the aspiration-level model. In this section we study the sensitivity of the survival probability and the order quantity maximizing it with respect to the changes of the selling price, salvage value and shortage cost. For general distributions this appears to be a rather challenging problem, but for exponential distributions a full analytical study can be performed. The results are presented in the following theorem.

Theorem 2

Let A be the set of triples (p, s, v) which satisfy condition (2). For the exponentially distributed demand and any parameters (p, s, v) belonging to the set A the following statements hold.

a. Q_H^* is a decreasing function of selling price p, an increasing function of shortage cost s, and an increasing function of salvage value v.
b. H^* is a decreasing function of p, an increasing function of s and an increasing function of v.
c. Q_E^* increases if p increases, increases as s increases, increases if v increases and decreases as c increases.
d. E^* is an increasing function of p, a decreasing function of s, an increasing function of v and a decreasing function of c.

Proof of Theorem 1. For simplicity we write the expressions in terms of $a = p - v$. Then:

$$H^* = 1 - \left(\frac{a}{a+s}\right)^{\frac{a+s}{s}}$$

and:

$$Q_H^* = \frac{1}{\lambda}\ln\frac{a+s}{a}.$$

Let $x = \frac{s}{a+s}$ with the values from the interval $(0,1)$. The function $H(x) = 1 -$ $- (1-x)1x$ is increasing as x increases from 0 to 1, which proves claim (b). Statements (a), (c) and (d) are straightforward.

Now, we illustrate Theorem 2 with a numerical example. The values of the optimal solution in the classic newsvendor model and the aspiration-level model are calculated separately, taking into account the varying parameters v, c, p and s, one at a time. The parameter of the exponential distribution which modelled the random demand is assumed to be $\lambda = 0.003$. We solve the problem for $v = 11, 14, 15$, $c = 16, 17, 18$, $p = 25, 30, 35$ and $s = 20, 50, 80$. The base data values are $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$.

Table 1: Sensitivity analysis for varying parameters v, c, p and s

Parameter	Q_E^*	Q_H^*	H^*	E^*
$v = 11$	874.89	429.889	0.831	292.219
14	1165.503	472.355	0.846	2335.662
15	1391.462	488.779	0.851	3275.204
$c = 16$	1391.462	488.779	0.851	3275.204
17	1160.413	488.779	0.851	2012.507
18	1025.258	488.779	0.851	924.225
$p = 25$	1364.782	597.253	0.884	1635.218
30	1391.462	488.779	0.851	3275.204
35	1416.165	417.588	0.827	4917.168
$s = 20$	1185.116	282.433	0.851	3481.551
50	1391.462	488.779	0.918	3275.204
80	1517.959	615.276	0.943	3148.708

From Table 1 we conclude that the order quantity maximizing the survival probability increases from 429.9 to 488.8 as the salvage value increases from 11 to 15, which confirms statement (a) of Theorem 2. Next, as the unit shortage cost increases from 20 to 80, the order quantity with satisficing-level objective also increases, from 282.4 to 615.3. But if the selling price increases from 25 to 35 this order quantity decreases from 597.2 to 416.6. A similar analysis can be performed for the remaining quantities.

4 Optimal bicriteria index

In this section we give the solution to the optimization of the bicriteria index as well as some numerical examples for various values of weight w. Since the function $Y(Q)$ is continuous on the interval (Q_H^*, Q_E^*), it attains its maximum value. The derivative of the bicriteria index is equal to:

$$Y'(Q) = \frac{w}{E^*} E'(Q) + \frac{(1-w)}{H^*} H'(Q).$$

In order to optimize $Y(Q)$ it suffices to find Q such that $Y'(Q) = 0$ and then to prove that $Y''(Q) < 0$ for all $Q > Q_H^*$. If this is the case, then we get a unique Q_Y^* which maximizes the bicriteria index and satisfies the inequality $Q_H^* \leq Q_Y^* \leq Q_E^*$; we write $Y^* = Y(Q_Y^*)$. Note that if the second derivative satisfies $H''(Q) > 0$, then the second derivative $Y''(Q)$ is negative for weights w such that:

$$w > \frac{E^* H''(Q)}{E^* H''(Q) - H^* E''(Q)}$$

for all $Q > Q_H^*$.

In the following subsection a numerical example is given using the same base values of the parameters as in the previous example.

4.1 Sensitivity analysis

This subsection is dedicated to show the results of a numerical example. Note that the value of Q_Y^* is found here numerically. We examine the sensitivity of the optimal solution with respect to weight w. The base values are also $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$. For these parameters to ensure the negativity of $Y''(Q)$ the weight w has to be greater than 0.5. For $w \leq 0.5$ we take $Q_Y^* = Q_H^*$.

Table 2: Sensitivity analysis for various values of weight w

w	Q_E^*	Q_H^*	Q_Y^*	Y^*
0.0	1391.462	488.779	488.779 = Q_H^*	1.0
0.1	1391.462	488.779	488.779 = Q_H^*	0.885
0.2	1391.462	488.779	488.779 = Q_H^*	0.77
0.3	1391.462	488.779	488.779 = Q_H^*	0.665
0.4	1391.462	488.779	488.779 = Q_H^*	0.54
0.5	1391.462	488.779	488.779	0.425
0.6	1391.462	488.779	1339.517	0.783
0.7	1391.462	488.779	1359.011	0.837
0.8	1391.462	488.779	1372.915	0.891
0.9	1391.462	488.779	1383.344	0.946
1.0	1391.462	488.779	1391.462 = Q_E^*	1.0

We observe that as weight w increases, both the optimal order quantity maximizing the bicriteria index and the bicriteria index itself increase. In this case greater values of w correspond to the customer who is less risk-averse and therefore the expected profit model has an increasing influence on the bicriteria model. Hence the optimal value Q_Y^* is closer to the optimal order quantity Q_E^* of the expected profit model.

5 Conclusions

The paper is devoted to the bicriteria optimization in the newsvendor problem. One has to find the optimal order quantity which fulfils two goals. One objective is the classic optimization of the expected profit while the second one deals with the maximization of the probability of exceeding the expected profit. The assumed criteria are conflicting and there do not exist any solutions which optimize both criteria simultaneously.

We solve the bicriteria newsvendor problem with the exponential distribution as the distribution of the random demand. This distribution is widely used in many areas and in some situations it approximates the stochastic demand very well. The motivations for using this kind of distribution are explained in the introduction. The advantage of modelling the demand by an exponentially distributed random variable is the possibility of analytic derivation of exact solutions to the problem under some weak assumptions on the parameters of the model. In the paper of Arcelus et al. (2012) uniform distribution is studied which allows to find precise solutions of the optimization problem. The authors use the notions introduced by Parlar and Weng (2003) for the general distribution. They suggest to consider the problem with other demand distributions, which increases our knowledge about the bicriteria problem. Note that the solution of the bicriteria newsvendor problem presented in Parlar and Weng (2003) gives only an approximated optimal order quantity of the aspiration-level objective. In our case the order quantity maximizing the probability considered is given explicitly. Additionally, we derive the monotonicity of the solution with respect to the parameters of the model analytically. Even though the mathematics used is basic, we get some interesting results with various sets of the model parameters. It appears that the existence of a solution does not depend on the parameter of the demand distribution. To illustrate the general problem, the graphs of the expected profit, the probability of exceeding the expected profit and the limit functions are presented. Moreover, the numerical examples concerning the sensitivity of the model parameters are also given. The values of the optimal order quantities maximizing the bicriteria index are obtained numerically with Mathematica software.

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