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A NEW FUZZY MEASURE FOR THE ANALYTIC HIERARCHY PROCESS WITH THE CHOQUET INTEGRAL

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Abstract

A new fuzzy measure is presented in this paper. Using the assumption that the decision maker is able to provide the pairwise additivity degree between attributes, our method uses Zimmerman's approach to solve the fuzzy multi-objective problem: a simple problem for computing fuzzy density is derived. Having done that, we use this new fuzzy measure to implement an analytic hierarchy process (AHP) with dependent attributes using the Choquet integral. Our identification procedure for fuzzy density is much easier because it reduces the resolution complexity using a linear programming problem rather than the complicated power form used traditionally.

Keywords: fuzzy measure, Choquet integral, linear programming problem, AHP.

1 Introduction

Several methods have been proposed for fuzzy measures. However, the identification of a fuzzy measure could be the most difficult step when fuzzy integrals are applied to solve MCDM problems, because $2^n - 2$ values of the fuzzy measure have to be provided by the decision-maker(s) for an MCDM problem with n criteria (Larbani, Huang, Tzeng, 2011). In earlier reviews by Grabisch (1995), Grabisch et al. (2008), Grabisch and Labreuche (2010), the identification methods are classified into three groups: semantic methods (guessing the fuzzy meas-

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ure on the basis of semantic considerations), learning methods (optimization methods) and methods combining semantic and learning methods. In addition, the main approaches (least square, minimum split and minimum variance) to fuzzy measure identification based on the Möbius transform of capacity and k-additivity are reviewed and their advantages and disadvantages are discussed. The least square method is historically the first approach; it can be regarded as a generalization of the classic multiple linear regression. The goal is to minimize the average quadratic distance between the overall utilities computed by means of the Choquet integral and the desired overall scores provided by the decision-maker(s). However, the objective function is not strictly convex so that the solution is not unique. The maximum split method is based on linear programming; the idea of this approach is to maximize the minimal difference between the overall utilities of objects that have been ranked by the decision-maker(s) using partial weak order. This method is quite simple; but, similarly, it does not have a unique solution. The idea of the minimum variance method is to favor the "least specific" capacity, if any, compatible with the initial preferences of the decision-maker (s). It may lead to a unique solution; however, a unique solution doesn't exist if there are "poor" initial preferences; for example, if the decision maker faces very high positive or negative interaction indices or a very uneven Shapley value.

Since computing the fuzzy density of a fuzzy measure is complicated by its power form, many scholars tried their best to simplify this problem. For example, Lee and Leekwang (1995) developed an identification method for fuzzy measures based on evolutionary algorithms. Wang and Chen (2005) used the sampling method with genetic algorithm, the complexity reduced to the data number of $O(2^n/n)$. Takahagi (2000) proposed an approach based on two types of pairwise comparison. The first one is based on the pairwise comparison values of interaction degrees between criteria. The second one is based on the pairwise comparison values of weights of criteria. Thus, the complexity of data collection can be reduced to $n(n-1)$. In addition, Corrente et al. (2016) showed that the application of the Choquet integral presenting two main problems for the necessity to determine the capacity, which is the function that assigns a weight not only to all single criteria but also to all subset of criteria, and the necessity to express on the same scale evaluations on different criteria. They adopted the recently introduced Non-Additive Robust Ordinal Regression (Greco et al., 2010) for the first problem, which takes into account all the capacities compatible with the preference information provided by the DM; with respect to the second one they build the common scale for the considered criteria using the Analytic Hierarchy Process (AHP). This permits to reduce considerably the number of pairwise comparisons usually required by the DM when applying the AHP.

The Analytic Hierarchy Process (AHP) is an effective tool for selecting the best alternative in multiple criteria decision making (MCDM) (Liou, Tzeng, 2007; Saaty, 1980; Tzeng et al., 2005; Yang et al., 2008). In multiple criteria decision making (MCDM) fuzzy measures are used to represent interactions between the attributes (Chen, 2001; Chen, Larbani, 2006; Chen, Tzeng, 2001); namely, the aspects of independence, complementarity and redundancy of attributes, which are also the challenges of the AHP (Bortot, Marques Pereira, 2013). Our method of identification is very easy to implement because the optimization problem we solve is linear and the number of its constraints is small compared with the optimization problems in methods cited above.

Our paper is organized as follows: in Section 2, the definitions of a fuzzy measure and λ-fuzzy measure are reviewed. In Section 3, the new fuzzy measure is presented and defined. In Section 4, we propose the Choquet integral AHP which uses the new fuzzy measure. In Section 5, a numerical example is used to illustrate our results. Finally, conclusions and recommendations are in Section 6.

2 Overview of the literature

In this section we will review basic definitions and concepts of fuzzy measure, Choquet integral and AHP.

2.1 Fuzzy measure

Sugeno (1974) presented a theory of fuzzy measures and fuzzy integrals in modeling the human subjective evaluation process (Ishii, Sugeno, 1985; Kambara et al., 1997).

Definition 2.1 (Sugeno, 1974). Let g be a set function defined on the power set $\beta(X)$ of X, and satisfying the following properties:

Property 1. Boundary conditions:
$g : \beta(X)\rightarrow[0,1]$
$g(\phi) = 0$ and $g(X) = 1$

Property 2. Monotonicity:
$\forall A, B\in\beta(X)$ if $A\subseteq B$, then $g(A) \leq g(B)$

Property 3. Continuity:
If $F_k\in\beta(X)$ for $1 \leq k < +\infty$, and the sequence $\{F_k\}$ is monotone (in the sense of inclusion), then $\mathrm{im}_{k\rightarrow+\infty}\, g(F_k) = g(\lim_{k\rightarrow+\infty}F_k)$.
It has to be noted that if X is finite then property 3 can be omitted.

The following are three special fuzzy measures; each measure is defined by certain constraints on g.

(a) Probability measure:

$$A, B \in \beta(X) \text{ and } A \cap B = \phi \rightarrow g(A \cup B) = g(A) + g(B) \tag{1}$$

(b) F-additive measure:

$$A, B \in \beta(X) \text{ and } A \cap B = \phi \rightarrow g(A \cup B) = g(A) \vee g(B) \tag{2}$$

where $a \vee b = max\ \{a,b\}$

(c) λ-measure:

$$A, B \in \beta(X) \text{ and } A \cap B = \phi \rightarrow g(A \cup B) = g(A) + g(B) + \lambda g(A) g(B) \tag{3}$$

where $\lambda \in (-1,\infty]$.

Sugeno constructed the λ-measure as a special case of a fuzzy measure. Here, the measure is based on the parameter λ, which describes the degree of additivity. We have three important types of λ-measures.

1) if $\lambda > 0$, then $g_\lambda(A \cup B) > g_\lambda(A) + g_\lambda(B)$, the measure is superior additive, which implies multiplicative effects between A and B;
2) if $\lambda = 0$, then $g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B)$, the measure is additive;
3) if $\lambda < 0$, then $g_\lambda(A \cup B) < g_\lambda(A) + g_\lambda(B)$, the measure is subadditive, which implies substitutive effects between A and B.

If $X = \{x_1, x_2,\ldots, x_n\}$ i.e. X is finite, the fuzzy measure can be written as (Sugeno, 1974):

$$\begin{aligned} g_\lambda(\{x_1, x_2,\ldots, x_n\}) &= \sum_{i=1}^{n} g_i + \lambda \sum_{i1=1}^{n-1} \sum_{i2=i1+1}^{n} g_{i1} g_{i2} + \ldots + \lambda^{n-1} g_1 g_2 \ldots g_n = \\ &= \frac{1}{\lambda} \left| \prod_{i=1}^{n} (1+\lambda\ g_i) - 1 \right|, \text{ for } -1 < \lambda < \infty \end{aligned} \tag{4}$$

where $g_i = g_\lambda(\{x_i\})$, $i = 1..n$, define the fuzzy density of the fuzzy measure g_λ. The power form is inspired by the utility function of Keeney and Raiffa (1976).

2.2 The Choquet integral

Consider a fuzzy measure space $(X, \beta(X), g)$ with $X = \{x_1, x_2, \ldots, x_n\}$. The Choquet integral of a function h: $X \rightarrow [0,1]$ with respect to g is defined as follows (Yang et al., 2008; Sugeno, 1974):

$$\int_X h(X) \bullet g(H) = \bigvee_{j=1}^{n} \left[h(x_j) \wedge g(H_j) \right] \tag{5}$$

where $h(x_j) \geq h(x_{j+1})$ for $1 \leq j \leq n-1$, $a \wedge b = \min\{a,b\}$ and $Hj = \{x_1, x_2, \ldots, x_j\}$, $j = 1, 2, \ldots, n$. When the Choquet integral is used to describe an MCDM problem, a value of the function h can be thought of as the performance of a particular attribute for an alternative, and g represents the decision maker's subjective evaluation of the importance of the attributes. The Choquet integral of h with respect to g gives the overall evaluation of an alternative. Furthermore, we have (Sugeno, 1974):

$$\int hdg = h(x_n)g(\mathrm{H}_n) + [h(x_{n-1}) - h(x_n)]g(\mathrm{H}_{n-1}) + \ldots + [h(x_1) - h(x_2)]g(\mathrm{H}_1) =$$
$$= h(x_n)[g(\mathrm{H}_n)\text{-}g(\mathrm{H}_{n-1})] + h(x_{n-1})[g(\mathrm{H}_{n-1})\text{-}g(\mathrm{H}_{n-2})] +\ldots +h(x_1)g(\mathrm{H}_1) \quad (6)$$

where $\mathrm{H}_1 = \{x_1\}$, $\mathrm{H}_2 = \{x_1, x_2\},\ldots$, $\mathrm{H}_n = \{x_1, x_2, \ldots, x_n\} = X$.

The basic concept of equation (6) can be illustrated as shown in Figure 1.

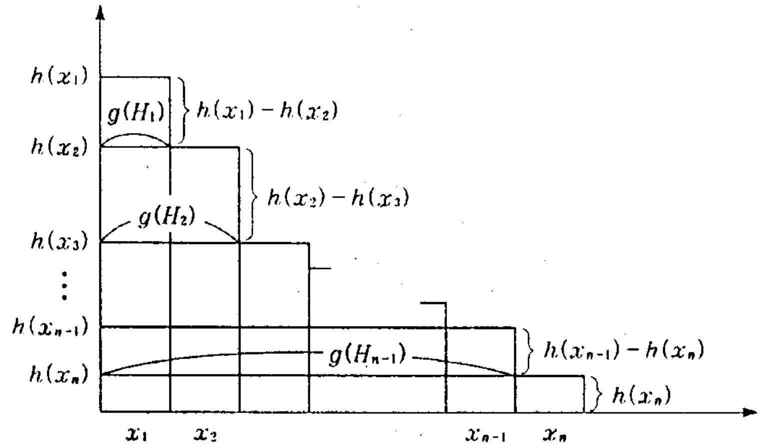

Figure 1. The Choquet integral

The Choquet integral is a powerful tool to measure the subjective human evaluation (Ishii, Sugeno, 1985; Kambara et al., 1997). The main reason for that is that it is not necessary to assume the independence between the attributes in the Choquet integral model.

2.3 The Analytic Hierarchy Process (AHP)

Saaty (1980) introduced a method of computing relative weights using a positive pairwise comparison matrix using the eigenvector method: let **P** be the positive pairwise comparison matrix with respect to n attributes:

$$\mathbf{P} = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} \cdots & \frac{w_2}{w_n} \\ \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} \cdots & \frac{w_n}{w_n} \end{bmatrix} \quad (7)$$

where $\frac{w_a}{w_b}$ represents the relative importance of the a-th attribute over the b-th attribute, where a, $b \in \{1, 2, \ldots, n\}$. Multiplying **P** by the relative importance vector: $\mathbf{W} = (w_1, w_2, \ldots, w_n)^t$, we get the following equation:

$$\mathbf{PW} = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} \cdots & \frac{w_2}{w_n} \\ & & \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} \cdots & \frac{w_n}{w_n} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ w_n \end{bmatrix} \tag{8}$$

In general, the value of $\frac{w_a}{w_b}$ is subjectively given by the decision maker. Saaty (1980) uses the maximal eigenvalue: $EV_{\max}$ to find the solution **W** of equation (8) as shown in the following equation:

$$(\mathbf{P} - EV_{\max}\,\mathbf{I})\mathbf{W} = \mathbf{0} \tag{9}$$

where **I** is the identity matrix. A set of linear equations for $w_1, w_2, \ldots, w_n$ can be obtained from equation (9); the final values of $w_1, w_2, \ldots, w_n$ are computed using the normalization condition:

$$w_1 + w_2 + \ldots + w_n = 1 \tag{10}$$

3 A new fuzzy measure with variable additivity degree

In this section we propose a generalization of the λ-measure (Larbani et al., 2011). When the degree of additivity λ depends on the sets considered i.e.:

$$A, B \in \beta(X) \text{ and } A \cap B = \phi \rightarrow g(A \cup B) = g(A) + g(B) + \lambda_{AB}$$

where $\beta(X)$ is the set of all subsets of a set $X = \{x_1, x_2, \ldots, x_n\}$ of attributes and λ_{AB} is the additivity degree between subsets A and B.

Procedure 3.1

Step 1. Assume that the decision maker is able to provide a pairwise evaluation of the interdependence of the attributes, i.e. for each pair of different attributes i and j the decision maker is able to assign/guess their additivity degree λ_{ij} with $-1 \leq \lambda_{ij} \leq 1$ and $0 \leq \max\lambda_{ij} \leq 1$. Here λ_{ij} plays the role of the correlation coefficient in the traditional statistics. Actually, this is a simple idea if we trace it back to the traditional statistics. In the traditional statistics, a positive correlation means that the two variables are synergistic: An increasing effect of one variable leads to a similarly increasing effect of another; conversely, a negative correlation means that the two variables may be substitutive. The decision maker is encouraged to determine these λ_{ij} by his/her arbitrary perception.

Step 2. The decision maker can give only one fuzzy estimation of the value of the density g_i for each attribute x_i. Without loss of generality and for the ease of presentation, we assume that the decision maker's fuzzy estimation of each density g_i is a fuzzy value number $\tilde{g}_i = [a_i, b_i]$, with $0 \le a_i \le b_i \le 1$. It should be noted here that the decision maker has the freedom to choose a_i, b_i according to his/her experience and knowledge about the given attribute. Here we assume $g_i = (1 - \alpha)a_i + \alpha b_i$, $0 \le \alpha \le 1$. Therefore density g_i, $i = 1, \ldots, n$ of the fuzzy measure can be determined by solving the following optimization problem:

$$\begin{gathered}\text{Max }(g_1, g_2, \ldots, g_n)\\ \text{such that } 0 \le g_i + g_j + \lambda_{ij} \le 1, \text{ for all } i, j \text{ with } i \ne j:\\ \sum_{1}^{n} g_i + \max_{\substack{i,j\in\{1..n\},\\ i\ne j}} \lambda_{ij} = 1\\ g_i = (1 - \alpha)a_i + \alpha b_i,\ i = 1, \ldots, n.\end{gathered} \tag{11}$$

where α represents the achievement level of the fuzzy numbers $\tilde{g}_i$, $i = 1, \ldots, n$, the larger the better. In fact, any density that satisfies the constraints of problem (11) can be taken as a feasible solution; however, we look for the maximal value of the objective function in this problem. Now the fuzzy measure can be determined on the basis of the density obtained from problem (11). When the fuzzy multi-objective approach of Zimmerman (1985) is applied, problem (11) can be reduced to problem (12):

$$\begin{gathered}\text{Max } \alpha\\ \text{such that } 0 \le g_i + g_j + \lambda_{ij} \le 1, \text{ for all } i, j \text{ with } i \ne j:\\ \sum_{1}^{n} g_i + \max_{\substack{i,j\in\{1..n\},\\ i\ne j}} \lambda_{ij} = 1\\ g_i = (1-\alpha)a_i + \alpha b_i,\ i = 1, \ldots, n.\end{gathered} \tag{12}$$

Proposition 3.1. The set function defined by:

$$g(A) = \sum_{i\in A} g_i + \max_{\substack{i,j\in A\\ i\ne j}} \lambda_{ij} \tag{13}$$

for all subsets of X such that Card(A) ≥ 2 and $g(\varnothing) = 0$, is a fuzzy measure.

Proof. By construction we have $g(\varnothing) = 0$ and $g(X) = 1$. Let us now prove that given two subsets A and B of X such that $A \subset B$, we have $g(A) \le g(B)$. Since $A \subset B$, we have:

$$\sum_{i\in A} g_i \le \sum_{i\in B} g_i \text{ and } \max_{\substack{i,j\in A\\ i\ne j}} \lambda_{ij} \le \max_{\substack{i,j\in B\\ i\ne j}} \lambda_{ij}$$

Adding these two inequalities we get:

$$g(A) = \sum_{i\in A} g_i + \max_{\substack{i,j\in A\\ i\ne j}} \lambda_{ij} \le g(B) = \sum_{i\in B} g_i + \max_{\substack{i,j\in B\\ i\ne j}} \lambda_{ij}$$

Now it remains to prove that for any subset A of X we have $0 \le g(A) \le 1$. If A has one or two elements, the inequality $0 \le g(A) \le 1$ holds according to the constraints of problem (11). Assume now that A has more than three elements. Let $\{i, j\}$ be any two elements in A; then, according to the first part of this proof, we have:

$$g(A) \ge g(\{i, j\}) = g_i + g_j + \lambda_{ij} \ge 0$$

On the other hand, we have $A \subset X$, therefore $g(A) \le g(X) = 1$, hence $g(A) \le 1$. Given two subsets A and B of X such that $A \cap B = \varnothing$, we have:

$$g(A\cup B) = \sum_{i\in A\cup B} g_i + \max_{\substack{i,j\in A\cup B\\ i\ne j}} \lambda_{ij} = \sum_{i\in A} g_i + \sum_{i\in B} g_i + \max_{\substack{i,j\in A\cup B\\ i\ne j}} \lambda_{ij} =$$

$$= g(A) + g(B) + \max_{\substack{i,j\in A\cup B\\ i\ne j}} \lambda_{ij} - \left(\max_{\substack{i,j\in A\\ i\ne j}} \lambda_{ij} + \max_{\substack{i,j\in B\\ i\ne j}} \lambda_{ij}\right)$$

Hence A and B are independent if and only if the degree of additivity in $A \cup B$ is equal to the sum of degrees of additivity within A and within B, that is:

$$A \text{ and } B \text{ are independent} \Leftrightarrow \max_{\substack{i,j\in A\cup B\\ i\ne j}} \lambda_{ij} = \left(\max_{\substack{i,j\in A\\ i\ne j}} \lambda_{ij} + \max_{\substack{i,j\in B\\ i\ne j}} \lambda_{ij}\right)$$

To summarize we give a procedure for an effective implementation of our method of identification.

Procedure 3.2

Step 1. Ask the decision maker to provide/guess the pairwise degrees of additivity λ_{ij}, if attribute i and attribute j are mutually substitutive then λ_{ij} should be less than zero; while if attribute i and attribute j are mutually complementary then λ_{ij} should be larger than zero, and the fuzzy evaluations of the density are $\tilde{g}_i$, $i = 1, \ldots, n$ of attributes.

Step 2. Solve problem (12). If it has no solution then return to Step 1. The decision maker has to enlarge the interval of fuzzy evaluations $\tilde{g}_i$, $i = 1, \ldots, n$ or/and reevaluate the additivity values λ_{ij}. If (12) has a solution, go to Step 3.

Step 3. Let g_i, $i = 1, \ldots, n$ be the solution of problem (12) with the largest α, then identify the fuzzy measure by formula (13).

4 The Choquet Integral AHP of the new fuzzy measure

In this section we will compare the results from the traditional AHP (Zeleny, 1982) and the Choquet Integral AHP with the new fuzzy measure. Assume now that the traditional AHP pairwise comparison matrix **P** in (7) is given. The Choquet Integral AHP is defined by the triplet (X, **P**, g_i), where X is the set of attributes, **P** is the pairwise comparison matrix and g_i is the computed fuzzy density. Now we will show how the attribute weights are calculated in the Choquet Integral AHP. We will use the Choquet Integral to calculate the new weights of the attributes in order to take into account the interdependence of attributes. Let us assume, for the time being, that the λ-fuzzy measure g describing the interdependence of attributes is known (identification will be performed in the next section). Let w_i represent the normalized weight of attribute i from (10) and let $h(x_i) = w_i$. Let $H_1 = \{x_1\}$, $H_2 = \{x_1, x_2\},\ldots,$ $H_n = \{x_1, x_2,\ldots, x_n\} = X$ and $h(x_1) \geq h(x_2) \geq \ldots \geq h(x_n)$. If this ordering does not hold, one can reorder the attributes accordingly. Assume for the time being that the λ-fuzzy measure g is known. Now, according to (6) we have:

$$\int hdg = h(x_n)[g(H_n) - g(H_{n-1})] + h(x_{n-1})[g(H_{n-1}) - g(H_{n-2})] + \ldots + h(x_1)g(H_1)$$

Let us consider the vector $\mathbf{W}^f = (w_1^f, w_2^f, \ldots, w_n^f)$ with the following components (Chen, 2001):

$$\begin{aligned} w_1^f &= h(x_1)g(H_1) \\ w_2^f &= h(x_2)\,[g(H_2) - g(H_1)] \\ &\vdots \\ w_n^f &= h(x_n)\,[g(H_n) - g(H_{n-1})] \end{aligned} \tag{14}$$

This Choquet Integral gives an aggregated evaluation of the effect of interdependence of the attributes on the weights $h(x_i) = w_i$, $i = \overline{1,n}$ given by the traditional AHP where attributes are assumed to be independent. It is then natural to write the new weight of any attribute i as $h(x_i)[g(H_i) - g(H_{i-1})]$ based on the Choquet Integral (6); that is, as the corresponding term in the Choquet Integral. Furthermore, the coefficient $[g(H_i) - g(H_{i-1})]$ can be interpreted as the effect of the interdependence of the attributes on the weight $h(x_i) = w_i$ of attribute i.

Definition 4.1. We define the vector $\mathbf{W}^{f'} = (w_1^{f'}, w_2^{f'}, \ldots, w_n^{f'})$ of weight attributes in the Choquet Integral AHP as the normalized vector of $\mathbf{W}^f$. Thus, we have:

$$w_1^{f'} + w_2^{f'} + \ldots + w_n^{f'} = 1 \tag{15}$$

where $w_i^{f'} = \dfrac{w_i^f}{\sum_{j=1}^{n} w_j^f}$, $i = 1, \ldots, n$.

Now, we will use a numerical example to show how our ideas work. We use Procedures 3.1 and 3.2 to compute each g_i, then apply equations (14)-(15) to get our modified weights.

5 A numerical example

Here we propose two applications of our new fuzzy measure. The first one consists in adjusting the weights in the traditional AHP, the second one, in finding the dependency (λ_{ij}) in a decision process.

Consider the following management problem: an enterprise always faces many negative impacts (factors), which could lead to a reduction in productivity. For example, decreased productivity may result from improper human resource management, financial management, management of technology (MOT), operations management, etc. However, since the available resources: time and money are limited when tackling the negative impacts (factors) of an enterprise, the manager wants to rank these possible causes of decreased productivity; the cause with the highest priority could be tackled first.

Table 1: Comparison matrix with respect to three causes

$\frac{w_a}{w_b}$	**Poor management of human resources (IHR)**	**Poor management of innovative technologies (IIT)**	**Poor management of manufacturing operations (IMO)**
Poor management of human resources (IHR)	1	$\frac{1}{3}$	$\frac{1}{2}$
Poor management of innovative technologies (IIT)	3	1	3
Poor management of manufacturing operations (IMO)	2	$\frac{1}{3}$	1

Assuming that a manager finds three possible causes of decrease in productivity in an enterprise, namely: poor management of human resources (IHR), poor management of innovative technologies (IIT) and poor management of manufacturing operations (IMO). The pairwise comparison matrix with respect to these three causes is shown in Table 1; the AHP is used to rank them. However, they are correlated with each other in practice. For example, IHR and IIT could happen at the same time and have an adverse effect on the enterprise – these two causes may have mutual multiplicative effects ($0 < \lambda < \infty$). Thus, the fuzzy integral AHP is applied in this example to remove this limitation of the traditional AHP. Two approaches: the traditional AHP and the Choquet integral AHP with a new fuzzy measure, are proposed and their results are compared. First, by applying the procedures from Section 2.3, we can obtain the weights of the traditional AHP: $\mathbf{W}^{T3}$= [0.16(IHR), 0.59(IIT), 0.25(IMO)]^t. Next, we assume $\lambda_{ij} = 1$ for $i = j$, $\lambda_{ij} = 0.2$ for $i \neq j$, $0.2 \leq g_1 \leq 0.3$, $0.2 \leq g_2 \leq 0.3$ and $0.3 \leq g_3 \leq 0.4$ to solve problem (12). Thus, we get $\alpha = 0.33$, $g(\mathrm{H}_1) = 0.23$, $g(\mathrm{H}_2) = 0.86$, $g(\mathrm{H}_3) = 1$; therefore, $\mathbf{W}^{f'}_{T3}$ = [0.08(IHR), 0.84(IIT), 0.08(IMO)]^t. Comparing these two models, we see that the traditional AHP doesn't have such a significant impact on IIT as does the Choquet integral AHP with a new fuzzy measure. In other words, this new model has the ability to emphasize the major cause and tends to ignore the less important causes.

The actual relationship between fuzzy densities is an interesting problem, worth exploring further. To summarize, according to our new model, when the relationship between fuzzy densities is assumed, it is possible to trace back the hidden interaction between attributes. In addition, computational difficulties are reduced because our model is the linear problem (12) instead of problem (4).

6 Conclusions and recommendations

In the well-known traditional AHP, the relative weights from Satty (1980) define the core of the problem. The AHP technique is widely and commonly used to choose the best alternative with many competing attributes; however, the interdependencies among the competing attributes are seldom considered (Bortot, Marques Pereira, 2013). Since the substitutive and multiplicative effects, i.e., additivity degrees between attributes surely influence the final decision, a process to implement the AHP accommodating such a realistic situation should be developed. We successfully propose and validate a new fuzzy measure, which is quite simple when it is compared with the traditional fuzzy measure expressed in power form. The way of finding fuzzy density is also simple in this paper.

More advanced topics should be discussed in near future, for example, what is the right/correct assumption of the relationship between fuzzy densities? An evolutionary scheme may be useful to solve this complicated problem by arranging the collected data into a training set and a validation set. Furthermore, how to implement a large scale AHP with many levels in the real world on the basis of the results of this paper? And how to use the Choquet integral AHP in machine learning, knowledge acquisition or data mining? Such problems are interesting.

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