MULTIPLE CRITERIA DECISION MAKING '08

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MULTIPLE CRITERIA DECISION MAKING '08

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In the paper ***MCDM with Relative Importance of Criteria: Application to Configuration Design of Vehicles*** **V.Y. Blouin, B.J. Hunt** and **M.M. Wiecek** present a preference model based on convex cones. Preferences quantifies as allowable tradeoffs are elicited from the decision maker and used for the construction of a new preference.

In the paper ***Optimized Star Plot as Decision Aids: Applications of Maximum Resolution Topology*** **S.C.K. Chu, J.K. Ho and S.S. Lam** review the background of extending a star plot to a topological model, report goal programming formulation of the problem, provide details of maximum resolution topology DSS and its applications.

In the paper ***Multicriteria Decision Aiding Procedure under Risk and Uncertainty*** **C. Dominiak** proposes a new procedure. Scenario planning technique is used to deal with uncertainty and Monte Carlo simulation is applied to deal with risk factors.

In the paper ***A Study of Distributed Evolutionary Algorithms for Multi-Objective Optimisation*** **A. Essabri, M. Gzara and T. Loukil** propose a new parallel evolutionary algorithm for multi-objective optimisation named "parallel multi-objective evolutionary algorithm with Multi-Front Equitable Distribution".

In the paper ***Prediction of Bankruptcy Based on the Mathematical Programming*** **S. Imen, J.M. Martel and C. Habib** develop a method based on mathematical programming and classify firms into three categories: non-failed, failed and uncertain.

In the paper ***Modified Nominal Group Technique: What and How*** **R. Islam** shows how a traditional brainstorming technique can be integrated with the analytic hierarchy process in generating and subsequently prioritizing a large number of ideas.

In the paper process ***Technology Assessment for New Production Line Development – Analytic Network Process Approach*** **J. Michnik** studies the problem of choosing right technology by manufacturing enterprise. The use of Analytic Network Process is proposed as a supporting tool for the decision under several conflicting and interrelated criteria.

In the paper ***Models of Criteria Systems of Building Design Contract*** **S. Mitkus** and **E.Trinkūnienė** three models are analysed and the best model for creation of multicriteria evaluation technique is determined.

In the paper ***Value-Focused Development of a Multiobjective Watershed-Management Plan in Hawaii*** **M. Ridgley and D. Mills** describe a year-long effort that applied value-focused thinking and the AHP to the challenge of facilitating public participation in the development of a watershed-management plan for the island of Hawaii.

In the paper ***Applying reference Sets in Content-Based Interactive Image Retrieval*** **A.M.J. Skulimowski and P. Rotter** propose a new approach to image search based on preference information in form of reference images provided by the user interacting with an intelligent search system.

In the paper ***The Use of the Reference MCDM Methods to Define the Second Stochastic Dominance Effective Portfolios*** **O. Sobotka** presents application of stochastic dominance rules to portfolio selection problem with diversification possibilities.

In the paper ***Dynamic Stochastic Problems of Profit Maximization with Partially Ordered Criteria Space*** **T. Trzaskalik** and **S. Sitarz** consider exemplary stochastic dynamic programming profit maximization processes.

In the paper ***The CLARA Method — a New Approach to Expert Verbal Classification*** **L. Ustinovichius** and **G. Shevchenko** present a novel algorithm CLARA for ordering multi-criteria alternatives.

In the paper ***Dynamic, Fuzzy and AHP Procedures in a Multi-Criteria Decision Process: An Application to Ecosystem Management*** **L. Zadnik Stirn** construct a hierarchical, discrete, dynamic and multi-criteria decision support model to solve the problem.

In the paper ***Ranking by the Rough Approximation of a Preference Relation for an Extrusion Process – a Robustness Study*** **K. Zaras, L.N. Kiss, S. Massebeu** and **C. Fonteix** present an extrusion process example is studied to illustrate a new methodology in the field of decision engineering, which is based on the rough set approach.

The volume editor would like to thank the authorities of the Karol Adamiecki University of Economics for support in editing the current volume in the series ***Multiple Criteria Decision Making***.

Tadeusz Trzaskalik

Vincent Y. Blouin

Brian J. Hunt

Margaret M. Wiecek

MCDM WITH RELATIVE IMPORTANCE OF CRITERIA: APPLICATION TO CONFIGURATION DESIGN OF VEHICLES*

Abstract

In this article, a preference model of relative importance of criteria is presented and applied to a vehicle configuration design problem formulated as a multi-objective program. The model is based on convex cones and extends the classical notion of Pareto optimality. Preferences quantifies as allowable tradeoffs are elicited from the decision maker and used for the construction of a new preference. In effect, the Pareto set is reduced which facilitates the process of choosing a final preferred solution.

Configuration design of mechanical systems corresponds to finding the placement of a set of components such that performance criteria are optimized while satisfying design constraints. The presented configuration design problem involves designing a midsize truck for optimum vehicle dynamic behavior, survivability, and maintainability in the presence of decision maker's preferences that are included *a posteriori*. A set of Pareto solutions is first generated with a multi-objective genetic algorithm, and the Pareto solutions are screened according to the preferences quantified as allowable tradeoffs. The model extracts preferred designs from the Pareto set producing a short list of "strong" or "privileged" designs, which is a useful feature when preferences are unknown.

* This research builds upon the work of Dr Yi Miao whose effort is gratefully acknowledged. This research was supported in part by the Automotive Research Center (ARC), a U.S. Army TACOM Center of Excellence for Modeling and Simulation of Ground Vehicles at the University of Michigan and by the National Science Foundation, Grant number DMS-0425768. The views presented here do not necessarily reflect those of our sponsors whose support is gratefully acknowledged.

Keywords

Multi-criteria decision-making, preferences, Pareto optimality, relative importance, tradeoffs, Pareto set, configuration design, vehicle design.

Introduction

In the operations research as well as engineering literature on multi-criteria decision- making (MCDM), relative importance of criteria has predominantly been modeled with lexicographic ordering or weights applied to criteria with a higher weight denoting greater importance. Among many others who worked with weights, Podinovskii seems to have mastered this technique in a series of papers [22, 23, 24]. However, it has been observed that weights do not always effectively model relative importance of criteria [16] and methods that relax or even eliminate weights have also been proposed. Weight-free optimization of the most important criterion in the presence of additional constraints generated by the other criteria was proposed by Ignizio [8], Papalambros and Wilde [20], and others. Yakowitz and Lane [31] proposed a method based on a set of weights for every criterion rather than a unique weight. Noghin [18] and Noghin and Tolstykh [19] used weights to construct a vector representing the relative importance and augmenting the Pareto concept of optimality.

Theoretical foundations of the preference model used in this paper were developed by Hunt [6] and Hunt and Wiecek [7]. The approach uses tradeoffs rather than weights to augment Pareto optimality. Tradeoffs play a fundamental role in MCDM providing a direct link between the criteria and attainable outcomes. Kaliszewski and Michalowski [9, 10] developed a methodology to generate Pareto solutions with a priori-selected bound on tradeoffs. They also proposed psychologically stable solutions defined to be the solutions that attain satisfactory criterion values and have acceptable tradeoffs [11].

Our work takes a similar approach by allowing the decision maker (DM) to set a priori bounds on tradeoffs and model relative importance of criteria using these bounds. The approach augments Pareto optimality. In particular, not only are criteria expected to improve (or remain unchanged) but, additionally, a group of relatively more important criteria are allowed to improve at the expense of decaying relatively less important criteria. The augmentation reduces the Pareto set and therefore facilitates the choice of a single final solution. While the model is theoretically grounded in convex analysis, it is numerically easily available in the form of a matrix, which is used to pre-multiply the vector of criterion functions. The model is applicable to decision--making situations under the following axioms: (1) the DM recognizes relative importance of criteria; (2) the Pareto set is large; (3) a small subset of preferred solutions is determined in agreement with the assumed relative importance from which a final preferred solution will be selected.

This paper illustrates the applicability of the relative importance model to a vehicle configuration design problem which has been selected due to inherent presence of multiple criteria and the ongoing interest in solving complex design problems within the engineering community. In contrast to other approaches in engineering, the designer is not automatically led to a unique final design but rather he/she retains the right to choose and exercises this right within a subset of preferred designs. The model extracts preferred designs from the Pareto set producing a short list of "strong" or "privileged" designs, which is a useful feature when preferences are unknown.

The remainder of the paper includes five sections. In Section 1, we briefly review the conventional MCDM framework governed by the concept of Pareto optimality. In Section 2, we develop a model of relative importance of criteria which is based on the convex-cone approach to multi-objective programming originally proposed by Yu [32]. Sections 3 and 4 present our application. The vehicle configuration design problem is described in Section 3. In Section 4, a design methodology developed by incorporating the preference model into the configuration design problem is described and applied to the design of the FMTV truck to enhance vehicle dynamics performance, survivability, and maintainability. Significance of the method in engineering design is also discussed. Finally, Section 6 concludes the paper.

1. Preliminaries

In MCDM we typically consider the following conventional multi-objective program (MOP)

$$\begin{aligned} &\text{minimize} && \boldsymbol{f}(\boldsymbol{x}) = [f_1(\boldsymbol{x}),\ldots,f_m(\boldsymbol{x})]^{\mathrm{T}} \\ &\text{subject to} && \boldsymbol{x} \text{ in } S \subseteq R^n \end{aligned} \tag{1}$$

where $S \subseteq R^n$ is the set of feasible solutions in the decision (solution, design) space R^n and each criterion function $f_i(\boldsymbol{x})$, $i = 1,\ldots,m$, is real-valued. The image

$\boldsymbol{y} = f(\boldsymbol{x}) \in R^m$ of a solution $\boldsymbol{x} \in S$ is called an outcome and R^m is referred to as the objective (outcome, performance) space. The set of all attainable outcomes is defined as $Y = \{\boldsymbol{y} \in R^m : \boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}), \boldsymbol{x} \in S\}$. In the first *optimization* stage of MCDM, problem (1) is solved for Pareto solutions according to the notion of Pareto-optimality. A feasible solution $\boldsymbol{x}^1$ is said to be *Pareto* if there exists no other feasible solution $\boldsymbol{x}^2$ such that $f_i(\boldsymbol{x}^2) \leq f_i(\boldsymbol{x}^1)$ for all $i = 1,\ldots,m$, and $f_j(\boldsymbol{x}^2) < f_j(\boldsymbol{x}^1)$ for at least one j. Note that if such a solution $\boldsymbol{x}^2$ exists, then outcome $f(\boldsymbol{x}^2)$ is said to dominate outcome $f(\boldsymbol{x}^1)$. The image of a Pareto solution is called a *Pareto* outcome. In other words, an attainable outcome $\boldsymbol{y}^1$ is said to be Pareto if there exists no other attainable outcome $\boldsymbol{y}^2$ and a non-zero $\boldsymbol{d} \in R^m$, $\boldsymbol{d} \leq \boldsymbol{0}$ such that $\boldsymbol{y}^2 = \boldsymbol{y}^1 + \boldsymbol{d}$ [32]. Let $E(S, f, \text{Par})$ and $N(Y, \text{Par})$ denote the set of all Pareto solutions and Pareto outcomes of (1), respectively. In the second *decision-making* stage of MCDM, a preferred solution is selected from among Pareto solutions based on preferences elicited from the DM. This preferred solution is the final result of the MCDM process.

According to Pareto optimality used in problem (1), which is a minimization problem, during the search for Pareto outcomes one examines those directions in the outcome space, $\boldsymbol{d} \in R^m$, along which at least one criterion value decreases (improves) while the others remain unchanged, i.e., $\boldsymbol{d} \leq \boldsymbol{0}$. In this context, we define *decay* and *improvement* of criteria, and the notion of *preferred direction* in the outcome space.

Definition 1.1. For MOP (1), criterion *decay* occurs when the value of a criterion function increases and criterion *improvement* occurs when the value of a criterion function decreases.

Definition 1.2. For MOP (1) with Pareto optimality, a direction $\boldsymbol{d} \in R^m$ along which criterion values improve or remain unchanged is referred to as *preferred direction.*

In other words, for (1), the preferred directions $\boldsymbol{d} \in R^m$ yield the inequalities

$$d_i \leq 0, \text{ for all } i = 1,\ldots,m \tag{2}$$

or, in an equivalent matrix notation:

$$\boldsymbol{Id} \leqq 0 \tag{3}$$

where $\boldsymbol{I}$ denotes the $m \times m$ identity matrix. Alternatively, the collection of all preferred directions forms the set

$$R^m_{\leqq} = \{\boldsymbol{d} \in R^m : \boldsymbol{Id} \leqq \boldsymbol{0}\} \tag{4}$$

known as the *Pareto cone of preferred directions* or *Pareto preference cone* where the matrix ***I*** is referred to as the *Pareto preference matrix* [29]. Figure 1 depicts the Pareto preference cone for $m = 2$. Using this perspective, the Pareto sets are often denoted as $E(S, f, R_{\leqq}^{m})$ and $N(Y, R_{\leqq}^{m})$.

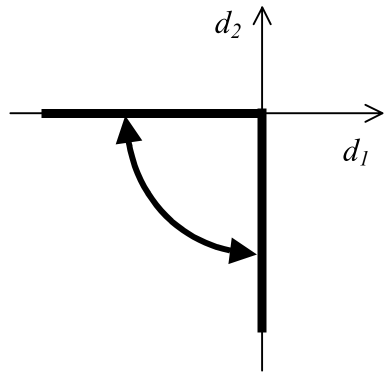

Figure 1. Pareto preference cone in R^2

2. Model of relative importance of criteria

In this section, we present an approach to modeling relative importance of criteria and give its simple description in order to lay the groundwork for the application presented in the subsequent section. For the detailed development and complete derivations of the model, we refer the reader to Hunt [6].

2.1. Relative importance preference cone

We assume that every DM follows the Pareto optimality implying that every direction in the Pareto cone is a preferred direction. However, we also assume that the indices of all criteria $\{1,\ldots,m\}$ are divided into two groups: the set of indices M corresponding to a relatively more important group of criteria that are not allowed to decay and the set of indices L corresponding to a relatively less important group of criteria that are allowed to decay. Even though the criteria represented by L are allowed to decay, if they also improve or remain unchanged then we consider ourselves fortunate.

We introduce the concept of an *allowable tradeoff* to quantify decay and improvement between two criteria.

Definition 2.1. An *allowable tradeoff* between criteria i and j, $i, j \in \{1,\ldots,m\}$, $i \neq j$, denoted a_{ij}, is the largest amount of decay in criterion i considered allowable to the DM to gain one unit of improvement in criterion j. Also, $a_{ij} \geq 0$ for all i and j, $i \neq j$.

The values of the allowable tradeoffs a_{ij} depend on the DM's preferences. If $a_{ij} = 0$ for all $i, j = 1,\ldots,m$, $i \neq j$, then the DM follows only the notion of Pareto optimality and only applies the Pareto preference cone. If the DM is willing to allow tradeoffs between criteria (i.e., there exists at least one nonzero allowable tradeoff), then additional preferred directions are appended to the Pareto cone to construct a new preference cone. An experienced DM may have previous knowledge of and experience with the decision problem to guide the assignment of allowable tradeoff values but, in general, assigning specific values to a_{ij} may be difficult. In Section 4 we show how this issue may be resolved.

The sets L and M are constructed such that $L \cup M = \{1,\ldots,m\}$ and $L \cap M = \emptyset$. The DM is required to define an allowable tradeoff a_{ij} for every pair of criteria i and j, $i,j \in \{1,\ldots,m\}$, $i \neq j$, such that $i \in L$ and $j \in M$. As assumed above, all directions in the Pareto preference cone are always preferred to the DM and are always contained in a new preference cone. However, to reflect the relative importance of criteria, other directions also become preferred and are appended to the Pareto cone to yield this new cone.

Definition 2.2. Consider (1) and let L and M be the set of indices of relatively less important and relatively more important criteria, respectively, defined by the DM. Define a set of preferred directions as

$$W = \{\boldsymbol{d} \in R^m : d_i \leq 0 \text{ for all } i \in M \text{ and } -d_k \geq \sum_{i \in M} a_{ki} d_i \text{ for each } k \in L\}$$

Note that the components d_i, $i \in M$, of directions $\boldsymbol{d} \in$ W are only allowed to be non-positive because the corresponding criteria are considered relatively more important and are never allowed to decay. The components d_k, $k \in L$, of directions $\boldsymbol{d} \in$ W are allowed to be positive to represent decay in the corresponding criteria. However, these components are also allowed to be non--positive to represent improvement or no change in the corresponding criteria, which is an attractive occurrence in an effort to minimize the criteria as prescribed in (1). Definition 2.2 also reveals that the total amount of decay allowed for each criterion indexed by $k \in L$ is bounded from below by the value of the expression:

$$\sum_{i \in M} a_{ki} d_i \leq 0 \tag{5}$$

which is always non-positive and equal to a cumulative allowable decay based upon the allowable tradeoff values and the total amount of improvement obtained in the criteria in M. If $d_k \leq 0$ for some $k \in L$, representing improvement or no change in criterion k, then $-d_k \geq 0$ and the inequality

$$-d_k \geq \sum_{i \in M} a_{ki} d_i \tag{6}$$

always holds. However, if $d_k > 0$ for some $k \in L$, representing decay in criterion k, then $-d_k < 0$ and we require that this decay be bounded by the desired expression so that inequality (6) holds again.

Definition 2.3 Consider (1) and let L and M be the sets of indices of relatively less important and relatively more important criteria, respectively, defined by the DM. The *relative importance preference cone* is defined as:

$$C^A = R^m_{\leqq} \cup W \tag{7}$$

In [6] it is shown that cone (7) can be represented in the inequality form:

$$C^A = \{\boldsymbol{d} \in R^m : \boldsymbol{Ad} \leqq \boldsymbol{0}\} \tag{8}$$

where A is an $m \times m$ relative importance preference matrix with entries Aij in row i and column j defined as:

$$\begin{array}{ll} A_{ii} = 1 & \text{for all } i \in \{1,...,m\} \\ A_{ij} = a_{ij} & \text{for all } i \in L \text{ and } j \in M \\ A_{ij} = 0 & \text{otherwise.} \end{array} \tag{9}$$

The structure of $\boldsymbol{A}$ depends on the definition of the sets of criterion indices L and M. For example, for a three-criteria problem, let $L = \{2\}$ and $M = \{1, 3\}$. Then $\boldsymbol{A}$ has the following form:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

To observe the relationship between the structure of this matrix and the structure of the cone, we analyze the system of linear inequalities from the cone representation in (8). The first and last inequalities in (8), corresponding to the first and last rows of matrix $\boldsymbol{A}$, enforce that the first and third criterion may only improve or remain unchanged which is required since $M = \{1, 3\}$. The second inequality in (8), corresponding to the second row of matrix $\boldsymbol{A}$, enforces that the second criterion is allowed to decay since $L = \{2\}$

and the decay is controlled by the lower bound implied by this inequality. The structure of the cone implied by this preference can be easily presented graphically when considering two criteria at a time. For the criteria 1 and 2, the model reduces to

$$d_1 \le 0$$
$$a_{21}d_1 + d_2 \le 0$$

which is depicted in Figure 2.

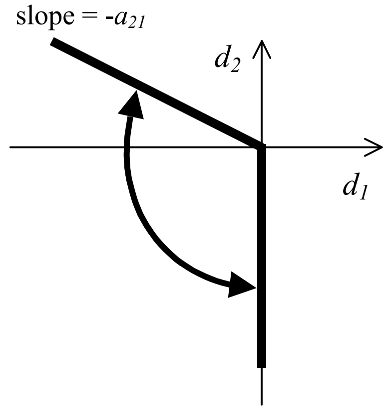

Figure 2. Relative importance preference cone in R^2

2.2. Integration of relative importance with conventional MOPs

In order to integrate the preference model given by (7-9) with the MOP given by (1) we make use of the following results from the literature. In multi-objective programming with general cones representing preferences rather than the Pareto cone, the solution sets are typically referred to as the *efficient set* $E(S, f, C)$ and the *nondominated set* $N(Y, C)$ for a cone C in R^m.

Theorem 1 [26]: Let C_1 and C_2 be cones in R^m. If $C_1 \subseteq C_2$ then $N(Y, C_2) \subseteq N(Y, C_1)$.

Theorem 2: Let C be a convex and pointed cone in R^m represented by $C = \{\boldsymbol{d} \in R^m\!: \boldsymbol{Ad} \leqq \boldsymbol{0}\}$. Then

(i) [18]: $E(S, \boldsymbol{f}, C) = E(S, \boldsymbol{Af}, R_{\leqq}^{m})$;

(ii) [7]: $\boldsymbol{A}[N(Y, C)] = N(\boldsymbol{A}[Y], R_{\leqq}^{m})$.

Based on Theorem 1, we observe that since $R_{\leqq}^{m} \subseteq C^{A}$, then $N(Y, C^{A}) \subseteq N(Y, R_{\leqq}^{m})$ and the Pareto set of (1) is reduced to a subset of efficient solutions with respect to the cone C^{A}. Theorem 2 requires that the cone C be pointed. In [6] it is shown that cone C^{A} given by (8-9) is always pointed. In part (i), this theorem reveals that the efficient set with respect to the cone C^{A} is equal to the Pareto set of an MOP related to (1) and formulated as:

$$\begin{array}{ll} \min & \boldsymbol{g}(\boldsymbol{x}) = [g_1(\boldsymbol{x}),\ldots,g_m(\boldsymbol{x})]^{\mathrm{T}} = \boldsymbol{A}\,[f_1(\boldsymbol{x}),\ldots,f_m(\boldsymbol{x})]^{\mathrm{T}} \\ \text{s.t.} & \boldsymbol{x} \in S \end{array} \tag{11}$$

where $g_i(\boldsymbol{x}) = \boldsymbol{A}_i f_i(\boldsymbol{x})$ and $\boldsymbol{A}_i$ is the i-th row of matrix $\boldsymbol{A}$, $i = 1,\ldots,m$.

In view of this results we propose two methods to implement the preference model and find $E(S, f, CA)$. Let a matrix $\boldsymbol{A}$ be given.

(1) In the first one-step method, we find the set $E(S, \boldsymbol{Af}, R_{\leqq}^{m})$, i.e., solve (11) for its Pareto solutions. To do so, we may apply any suitable method from the literature.

(2) In the other two-step method, we first find the Pareto set $\mathrm{E}(S, \boldsymbol{f}, R_{\leqq}^{m})$ again applying any suitable method. In the next step, we find the Pareto set $E(E(S, \boldsymbol{f}, R_{\leqq}^{m}), \boldsymbol{Af}, R_{\leqq}^{m})$, i.e., we solve an MOP:

$$\begin{array}{ll} \min & \boldsymbol{A}\,[f_1(\boldsymbol{x}),\ldots,f_m(\boldsymbol{x})]^{\mathrm{T}} \\ \text{s.t.} & \boldsymbol{x} \in E(S, \boldsymbol{f}, R_{\leqq}^{m}) \end{array} \tag{12}$$

To accomplish this, we screen the set $E(S, \boldsymbol{f}, R_{\leqq}^{m})$ for the solutions that fulfill the preference quantified by $\boldsymbol{A}$.

In general, both methods are applicable to MCDM problems. The one--step method is especially suitable for decision-making situations in which a priori preferences are known and the resulting solutions are immediately sought. On the other hand, the two-step method is flexible and allows the DM to explore relationships between the preferences given a priori and possibly changed later and the resulting changes in the reduction of the Pareto set. Since DM's preferences are not always known in engineering design problems, we apply the two-step method to the vehicle configuration design problem presented in this paper. We first generate the Pareto set and then identify $E(S, \boldsymbol{f}, C^{A})$ and $N(Y, C^{A})$ of the design problem. For brevity, in the remaining sections we refer to these solutions as *preferred solutions* (*decisions, designs*) and to their images as *preferred outcomes*.

3. Vehicle configuration design

Configuration design of mechanical systems consists of finding optimal locations and orientations of a set of mechanical components such that design constraints are satisfied and performance criteria are met or exceeded. In the literature, it is sometimes referred to as packing, packaging, and layout optimization. Due to the complexity of such problems, algorithms that guarantee to find optimal design solutions are computationally prohibitive. Thus, researchers have directed their effort toward generating acceptable solutions in a reasonable amount of time. Recent surveys of configuration design methods are available in [1, 2].

As the demand on performance of mechanical systems increases, the number of criteria to be included in a configuration design problem is generally large leading to multi-objective programs of type (1). To deal with the multi-criteria aspect of configuration design problems, the most popular approach is to aggregate the criteria into a single objective function with a weighted sum. Drawbacks of this approach have been highlighted in several publications [4, 12, 16]. In particular, to generate the efficient set, the execution of the algorithm must be repeated with different values of the weights without a clear control over the exploration of the design space and objective space.

Other research efforts focused on developing techniques that can generate the efficient set (or its good approximation) with a single execution of the algorithm. Miao et al. [14, 15] developed an approach based on a rank-based genetic algorithm (GA), NSGA-II [5]. They applied this algorithm to the configuration design problem of the US Army family of medium tactical vehicles (FMTV) with three conflicting criteria: vehicle dynamics, survivability, and maintainability. An approximation of the Pareto set was successfully generated with high diversity in both the solution (design) space and the objective space, which is the result of an efficient and thorough exploration of the feasible space.

Once the Pareto set has been approximated, it is then the responsibility of designers and DMs to choose a Pareto solution (design) that is acceptable based on their preferences. Since many methods have been proposed in engineering to facilitate the designers' task of eliciting preferences and incorporating them into the design problem, we refer to more recent or effective efforts. Messac [13] introduced physical programming, in which designers express preferences by specifying ranges of desirability to each criterion. Narayanan and Azarm [17] proposed an interactive method to iteratively narrow down the Pareto solution set to a size small enough for DMs to easily select the final solution. At each iteration, preferences are captured by ranking a limited set of solutions. The advantage of such an interactive method is that DMs can view a few sample points from the efficient set before zooming into a region of interest. Wan and Krishnamurty [28] recognized the possibility of establishing preference inconsistencies in utility theory and proposed a preference learning process based on a dynamic interactive procedure to help generate a set of acceptable outcomes for the problem and a set of rules for selection of a reduced set of outcomes. Finally, preference methods based on ranking of criteria and their integration in genetic algorithms have been reported by Cvetkovic and Parmee [3] and Park and Koh [21] among others.

The present paper builds upon the work of Miao et al. [15] that allows us for the incorporation of the preference model of relative importance of criteria into their configuration design methodology.

3.1. Vehicle and components

The FMTV is a midsize truck widely used in the US Army for transportation of equipment and personnel. A recent study focused on the redesign of the FMTV to accommodate the integration of a fuel cell auxiliary power unit (APU) [15]. The purpose of the APU is to provide power to non-propulsive applications such as cabin cooling, radio, and other electronic devices. While the integration of the APU increases the overall power efficiency of the vehicle, additional components of significant size must be installed on the vehicle. These components, shown in Figure 3, are the main fuel cell unit (APU), reformer, accumulator, reservoir, and pump. The redesign problem is then to find the optimum placement of these components as well as that of the already present components, i.e. engine, transmission, fuel tank, and axles – on the chassis.

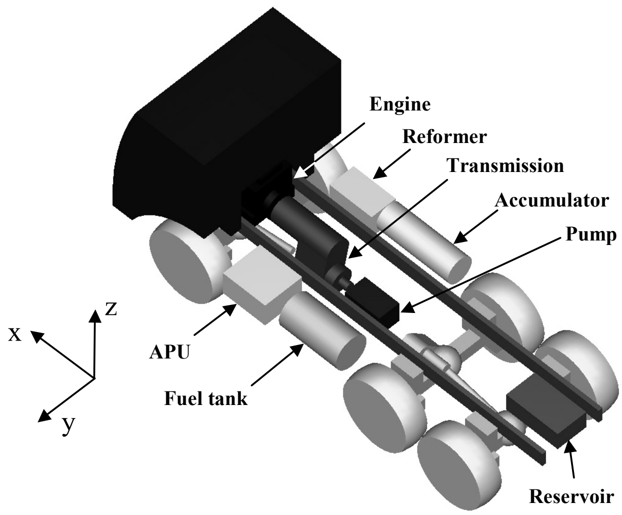

Figure 3. FMTV medium tactical vehicle and its components

Table 1

Predefined packaging sequence of vehicle components

	Component	Relative to	Degree of Freedom[a]
1	First Axle	Ground	x
2	Second Axle	First Axle	–
3	Third Axle	Second Axle	–
4	Chassis	Ground	z
5	Engine	Ground	x, z
6	Transmission	Engine	x′
7	Pump	Transmission	x′
8	Fuel tanks	Ground	x, y, z
9	Accumulator	Ground	x, y, z
10	Reservoir	Ground	x, y, z
11	APU[b]	Ground	x, y, z
12	Reformer	Ground	x, y, z

[a] x, y, z are longitudinal, transversal, and vertical directions, respectively and x′ is coordinate with respect to engine.
[b] auxiliary power unit.

A predefined packaging sequence, presented in Table 1, is used to ensure coherence of the vehicle configuration as well as to define 21 design variables for the optimization problem. Each object is allowed to translate according to the degree of freedom listed in Table 1. The location of each object is known by the location of its reference point, which is generally the center of gravity of the object. Object rotations are not considered in this problem. Note that allowing rotations is possible but would significantly increase complexity without adding value to this research effort.

3.2. Design criteria and constraints

Three conflicting design criteria are identified for this problem: vehicle dynamics performance, survivability, and maintainability. In addition, constraints are defined on the vehicle ground clearance, functional arrangements, and overlap between objects. A detailed description of the criteria and constraints briefly described below is available in [15].

The *vehicle dynamics index*, to be minimized, is an overall performance index that quantifies the dynamic behavior of the vehicle in a standard maneuver. More specifically, for this type of vehicle the index characterizes the risk of rollover and loss of control during a J-turn maneuver. An eight--degree-of-freedom vehicle dynamics model is used to numerically simulate the behavior of the vehicle [33]. In order to capture the complexity of vehicle dynamics, eight performance indices are computed during a numerical simulation. The overall vehicle dynamics index is then calculated as a weighted sum of the eight performance indices. Note that tradeoffs between these eight performance indices are not captured since the above weights are defined prior to the execution of the algorithm.

Survivability is a criterion that quantifies the ability of the vehicle to survive attacks from bullets and explosives. Since the vehicle components have different survivability priorities, a set of weights is defined and the overall vehicle survivability is calculated as the weighted sum of the survivability values of all components. The survivability of an object is defined as the level of protection provided by other objects along selected directions (bottom, rear and both sides). Each direction is associated with a weight corresponding to its importance in the overall survivability of the vehicle. The survivability of the vehicle must be maximized. However, a survivability index inversely proportional to the survivability of the vehicle is defined and minimized.

Maintainability of an object is defined as the number of objects that must be removed before direct removal of this object along selected directions. This quantification is very similar to the survivability described earlier except that it is based on the number of overlapping objects rather than the projected overlap area. To account for various maintenance priorities, an integer weight is defined for each object and the overall maintainability is defined as the weighted sum of the maintainability of all objects. The maintainability of the vehicle must be maximized. However, a maintainability index inversely proportional to the maintainability of the vehicle is defined and minimized.

Since minimizing the dynamic performance index has the tendency to lower the center of gravity of the vehicle, a *ground clearance constraint* is defined. Ground clearance comprises three terms: front, rear, and between axles. If an object is lower than the chassis, the lowest point of this object is identified and is used to calculate the angle defined with the lowest point of the wheels. The minimum value among the three angles is used as the over all ground clearance of the vehicle.

Several *functional constraints* are defined in the definition of the design variables. For instance, the transmission must be aligned with the engine output in the x-direction. This requirement is enforced by defining the design variable as the location of the transmission in the x-direction relative to that of the engine (refer to Table 1).

Finally, a *zero-overlap constraint* is imposed to ensure that none of the components collide and the configuration is spatially acceptable. Overlap is calculated as the sum of all interference volumes between every pair of vehicle components.

3.3. Problem formulation

The configuration design problem is formulated mathematically as follows:

$$
\begin{array}{ll}
\text{minimize} & [VDI(\boldsymbol{x}), SI(\boldsymbol{x}), MI(\boldsymbol{x})] \\
\text{subject to} & \boldsymbol{x} \in X \\
& GC(\boldsymbol{x}) \geq \alpha \\
& Overlap(\boldsymbol{x}) = 0
\end{array}
\tag{13}
$$

where $\boldsymbol{x}$ is the vector of 21 design variables (degrees of freedom) listed in Table 1; X is the set of feasible values for the design variables determined by their lower and upper bounds; $VDI(\boldsymbol{x})$, $SI(\boldsymbol{x})$, $MI(\boldsymbol{x})$ are the vehicle dynamics, survivability, and maintainability indices, respectively; $GC(\boldsymbol{x})$ and α are the ground clearance and its lower bound, respectively; and $Overlap(\boldsymbol{x})$ is the total overlap defined as the sum of all overlap volumes between pairs of objects

4. Implementation with relative importance of criteria

We now apply the developed preference model to problem (13). We first discuss the numerical solvability of this problem and then present the results for the case of known allowable tradeoffs and the case of unknown allowable tradeoffs.

4.1. Computational considerations

A rank-based genetic algorithm, also referred to as multi-objective GA (MOGA) [5], is used to find an approximation of the Pareto set of (13). The algorithm was implemented in a C++ computer code based on Galib [27] and reported in [15]. The individuals that are Pareto when considering the entire population have a rank equal to 1. The individuals that are Pareto when considering the population without the individuals of rank 1 have a rank equal to 2, etc. By minimizing the rank, the population evolves towards an *observed* Pareto set, which is an approximation of the *true* Pareto set for the problem.

Since the stochastic nature of a GA can lead to possibly large differences between results of different runs, MOGA was executed several times and in this paper we report the results of two different runs to illustrate the issues related to non-repeatability.

Approximation of the true nondominated set. Each of the two executions of MOGA generates a set of Pareto designs of (14): 238 and 317 designs, respectively, whose outcomes are shown as dots in Figures 4 and 5. These two sets of Pareto outcomes correspond to two approximations of the *true* Pareto set.

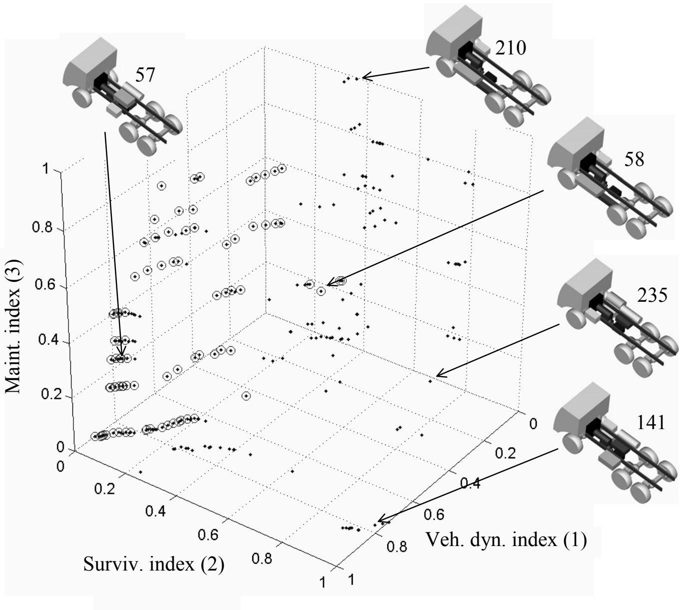

Figure 4. Pareto outcomes (dots) and preferred outcomes (circles) in case 1 of run 1: $L = \{1,3\}$, $M = \{2\}$, $a_{12} = 0.6$, $a_{32} = 0.4$

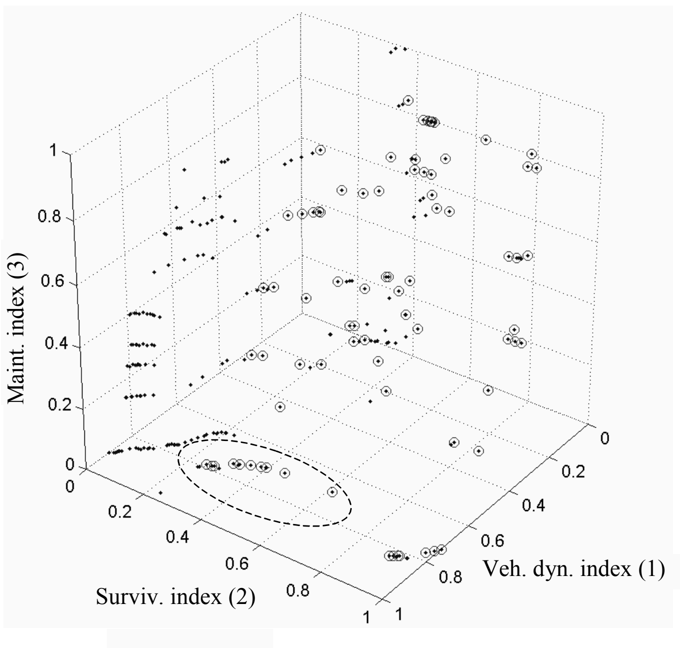

Figure 5. Pareto outcomes (dots) and preferred outcomes (circles) for case 2 of run 1: $L = \{2\}$, $M = \{1,3\}$, $a_{21} = 0.6$, $a_{23} = 0.4$

As seen in Figures 4 and 5, the two Pareto sets are significantly different. While it is virtually impossible to determine the goodness of each approximation compared to the true Pareto set, it is possible to compare them to each other. To do so, the two sets are combined and screened for overall Pareto outcomes. From the first run, 47 percent of the outcomes are Pareto when combining both runs, compared to 86 percent for the second run. This means that the second set of outcomes is a better approximation. Note that in both cases MOGA was terminated using the same convergence criteria, which is not enough to ensure the same quality of approximation of the true Pareto set. Note that a formal study of the goodness of these Pareto sets might be performed using metrics such as those proposed by Sayin [25] and Wu and Azarm [30] among others.

Normalization of criteria. After the two executions of the algorithm, all criterion values are normalized with respect to the minimum and maximum values of each set so that all normalized criterion values fall between 0 and 1. The normalization is not required but allowed by the definition of allowable tradeoffs. In fact, it is up to the DM to relate either to the normalized or the -non-normalized values of the criteria when selecting allowable tradeoff values. For example, using mass (measured in *kilograms*) and cost (measured, for instance, in *dollars*) as two conflicting criteria, there is a difference between dealing with *kilograms* rather than *units of mass* and *dollars* rather than *units of cost*. Some DMs may prefer to consider allowing decay of *x kilograms* to gain one *dollar*. Others may prefer to normalize mass and cost and consider allowing decay of *y units of mass* to gain one *unit of cost*. While the two statements are theoretically equivalent, their cognitive meaning may differ depending on the DM's experience and understanding of the problem.

4.2. Identification of preferred designs with known preferences

Assume that the preferences are known, that is the sets L and M, and allowable tradeoffs a_{ij} can be defined unambiguously. Two cases are considered for illustration purposes. In the first case, vehicle dynamics and maintainability are the more important criteria and the DM is willing to allow 0.6 and 0.4 units of decay in survivability to gain one unit of improvement in vehicle dynamics and in maintainability, respectively, i.e., $L = \{2\}$ and $M = \{1, 3\}$, with $a_{21} = 0.6$ and $a_{23} = 0.4$. Then, the preference matrix $\boldsymbol{A}$ is written:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0.6 & 1 & 0.4 \\ 0 & 0 & 1 \end{bmatrix}$$

and according to (12), the new criteria to be minimized are $g_1 = VDI(\boldsymbol{x})$, $g_2 = 0.6VDI(\boldsymbol{x}) + SI(\boldsymbol{x}) + 0.4MI(\boldsymbol{x})$, and $g_3 = MI(\boldsymbol{x})$. The screening process described in Section 3 is then performed on the two Pareto sets to extract two sets of preferred outcomes. Table 2 lists significant results and Figures 4 and 5 show the Pareto outcomes and the preferred outcomes as dots and circles, respectively.

Table 2

Identification of preferred outcomes with known preferences

Run[a]	Case[b]	M[c]	N_P[d]	VDI[e]			SI			MI			N_I[f]
				Min	Mean	Max	Min	Mean	Max	Min	Mean	Max	
1	–	0	238	0	0.50	1	0	0.30	1	0	0.33	1	-
	1	{2}	86	0.09	0.61	0.97	0	0.11	0.42	0.06	0.32	0.74	126
	2	{1,3}	78	0	0.30	0.83	0.12	0.50	1	0	0.36	0.84	210
2	–	0	317	0	0.36	1	0	0.27	1	0	0.38	1	-
	1	{2}	100	0.06	0.36	0.89	0	0.09	0.26	0	0.44	0.78	163
	2	{1,3}	96	0	0.26	0.86	0.12	0.37	1	0	0.86	0.33	301

[a] Execution of MOGA.
[b] Preference case.
[c] Set of more important criteria (Note: M = 0 means no preferences). In case 1, $a_{12} = 0.6$, $a_{32} = 0.4$. In case 2, $a_{21} = 0.6$, $a_{23} = 0.4$.
[d] Number of preferred outcomes (Number of Pareto nondominated outcomes if M = 0).
[e] Min, mean, and max values of veh. dyn. index in the set of preferred outcomes.
[f] In case 1, number of Pareto outcomes with SI < Max^{SI}. In case 2, number of Pareto outcomes with VDI < Max^{VDI} and MI < Max^{MI}.

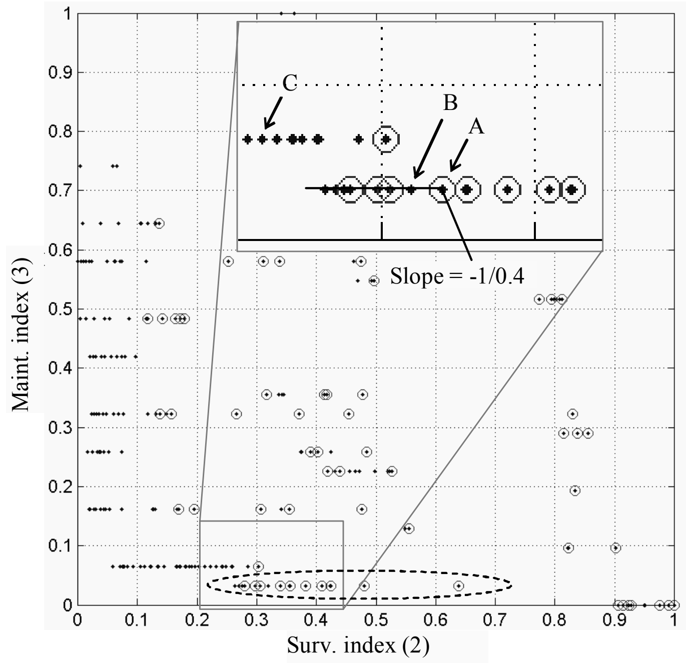

Figure 6. Projection of entire Pareto set of run 1 on two-dimensional space (SI, MI), Pareto outcomes (dots), preferred outcomes (circles), $L = \{2\}$, $M = \{1,3\}$, $a_{21} = 0.6$, $a_{23} = 0.4$

In order to further explain how preferred outcomes are identified, the distribution of Pareto outcomes in the three-dimensional objective space must be closely examined by considering the two-dimensional projections. Figure 6 shows the projection of the Pareto set of run 1 on the two-dimensional space (SI, MI). As explained in Section 3.2, the maintainability criterion assumes integer values. Therefore the normalized maintainability index can only assume a finite number of equally spaced values between 0 and 1. In this context, note the special structure of the projected Pareto set in Figure 6 showing groups of outcomes aligned horizontally. The Pareto outcomes inside the dashed oval in Figure 5 are also highlighted in Figure 6 by a dashed oval. Since they are the only outcomes with MI = 0.0323, they can be isolated from the other outcomes and projected on the two-dimensional space (VDI, SI) shown in Figure 7. Consider the outcome labeled "A" in Figure 7 and place the two-dimensional cone defined by $a_{21} = 0.6$ at "A". One can realize that there is no other outcome inside the cone. Therefore, "A" is a preferred outcome in the two-dimensional space (VDI, SI). Similarly, by placing the cone defined by $a_{23} = 0.4$ at "A" in Figure 6, one can verify that there is no other outcome inside the cone. Therefore, "A" is a preferred outcome in the two-dimensional space (SI, MI). Since "A" is preferred in both (VDI, SI) and (SI, MI), it is a preferred outcome for the overall three-dimensional problem. Comparatively, the outcome labeled "B" in Figure 7 is dominated by outcome "A", therefore "B" is not a preferred outcome. Note that the outcome labeled "C" in Figure 6 is not dominated by any outcome in the two- dimensional space (SI, MI). However, it is dominated in the space (VDI, SI) but not shown in Figure 7 since it lies in a plane defined by a higher MI value.

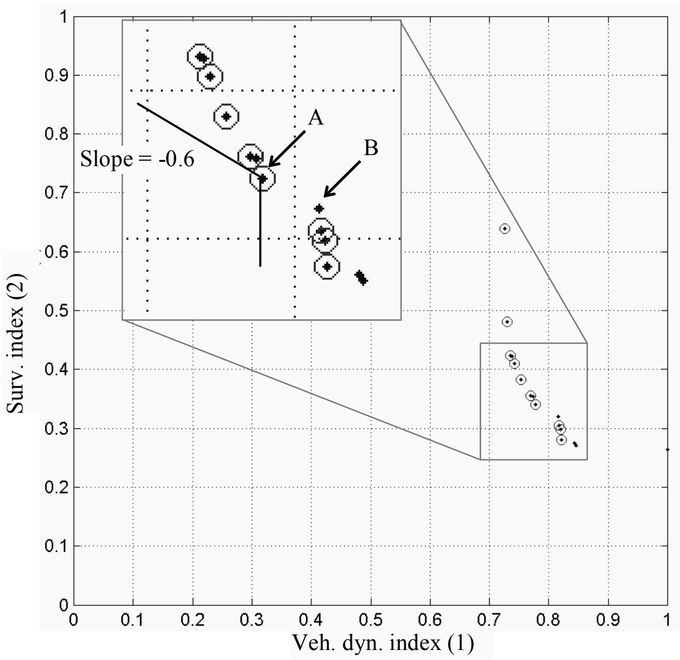

Figure 7. Projection on two-dimensional space (VDI, SI) of Pareto outcomes (dots) and preferred outcomes (circles) highlighted by dashed oval in Figure 6, $L = \{2\}$, $M = \{1,3\}$, $a_{21} = 0.6$, $a_{23} = 0.4$

In the second preference case, survivability is the more important criterion and the decision maker is willing to allow 0.6 and 0.4 units of decay in vehicle dynamics and in maintainability, respectively, to gain one unit of improvement in survivability, i.e., $L = \{1,3\}$ and $M = \{2\}$, with $a_{12} = 0.6$ and $a_{32} = 0.4$.

Based on the examination of Figures 4 and 5 and Table 2, several remarks can be made:

1. The set of Pareto outcomes is reduced significantly in both runs and both preference cases. In run 1, out of 238 outcomes, 86 (36%) and 78 (33%) are preferred in the first and second preference cases, respectively.
2. As expected, since the two preference cases have conflicting meanings, outcomes from different regions of the Pareto set are identified as preferred outcomes. In Figure 4, the preferred outcomes tend to have a low survivability index, which is due in part to the fact that survivability is the more important criterion. As explained in the following remark, however, this is not the only reason.
3. In Figure 5, since the more important criteria are vehicle dynamics and maintainability, one may expect the preferred outcomes to have a low vehicle dynamics index and low maintainability index. However several preferred outcomes, such as those highlighted by the dashed oval in Figure 5, have a high vehicle dynamics index. Therefore, it is not because the values of the more important criteria are large that an outcome cannot be preferred. Rather, this depends on the relative distribution of the Pareto outcomes in the three-dimensional objective space.
4. For preference case 1, not all outcomes with a low survivability index are preferred. This is due to the fact that these outcomes are dominated by at least one other outcome. Table 2 shows that in case 1 of run 1, from the set of preferred outcomes, the maximum SI value is $\mathrm{Max}^{SI} = 0.42$. In the region defined by $SI \leq 0.42$, there are 126 outcomes of which 86 are preferred.
5. In Figure 4, several truck configurations are provided to show the differences in the design space between regions of the outcome space. Configurations that have a high vehicle dynamics index tend to have high center of gravity by raising the height of the chassis and placing heavy components higher. For example, outcomes 57 and 141 correspond to such designs and differ by their survivability index while outcome 210 has a very low vehicle dynamic index partly because of low center of gravity at the expense of maintainability.

4.3. Sensitivity to allowable tradeoff values

The allowable tradeoff values a_{ij} have an impact on the number of preferred outcomes. Graphically, this is simply explained by the fact that as the allowable tradeoff values a_{ij} increase, the related preference cone opens up and the chance for a design to lie inside the cone increases. In order to illustrate this sensitivity, the allowable tradeoff values are varied incrementally between 0 and 3 for two cases: (L = {2}, M = {1,3}) and (L = {1,3}, M = {2}). The sensitivity is graphically represented in Figure 8 for run 1. A similar sensitivity, not shown, was found for run 2.

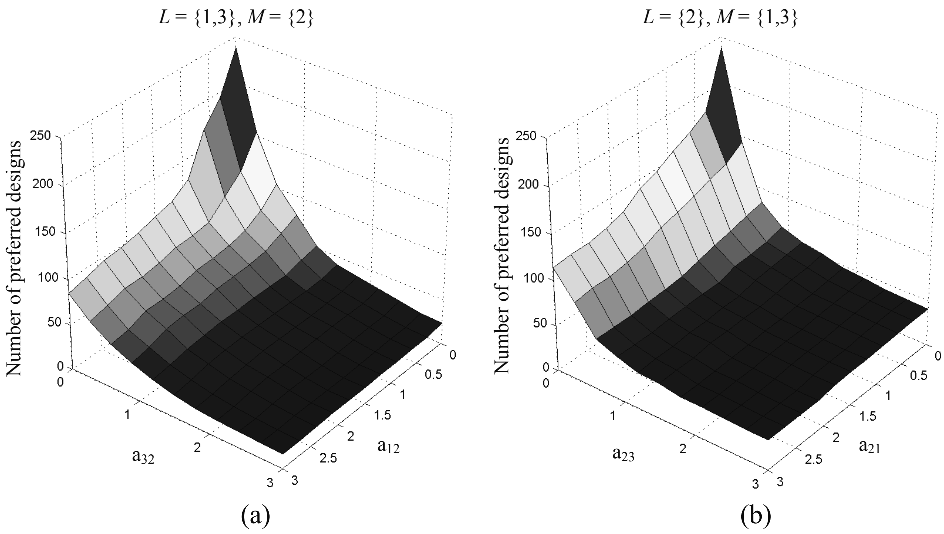

Figure 8. Effect of allowable tradeoff values on reduction of set of preferred outcomes in run 1 for two preference cases: (a) = {1,3}, *M*={2}, (b) *L* = {2}, *M* = {1,3}

Case 1 has a greater impact on the reduction than case 2 for large values of allowable tradeoffs (the set of preferred outcomes is reduced to 16 in case 1 and 33 in case 2 for all $a_{ij} = 3$). However, case 2 is more effective in reducing the set for low values of allowable tradeoffs.

One can notice that both cases show an asymmetric behavior. It appears that a_{12} does not have as much impact on the reduction of the set of preferred outcomes as a_{32} in case 1 for low values of allowable tradeoffs.

4.4. Identification of strong designs with unknown preferences

If *L*, *M*, and/or a_{ij} are not explicitly known, we introduce the term *strong design,* to describe designs that prevail for any combination of *L* and *M* and for any value of a_{ij} and propose a method to identify them. The image of a strong design in the outcome space is referred to as a *strong outcome*.

The idea is to consider multiple preference cases and vary allowable tradeoff values. In the configuration design problem, the permutation of the three criteria leads to six preference cases: ($L = \{1, 2\}$, $M = \{3\}$), ($L = \{1, 3\}$, $M = \{2\}$), ($L = \{2, 3\}$, $M = \{1\}$), ($L = \{1\}$, $M = \{2, 3\}$), ($L = \{2\}$, $M = \{1, 3\}$), and ($L = \{3\}$, $M = \{1, 2\}$). For each case, the two allowable tradeoff values a_{ij} are set to three different values: 0.6, 0.8, 1.0. The screening process identifies the preferred outcomes for each case and all outcomes are then ranked based on the number of cases for which they are preferred. The strong outcomes are then those that are preferred for the greatest number of preference cases.

Note that the two allowable tradeoff values can vary independently, which means that a large number of cases could be considered. In this paper, we selected to keep the two values equal in all cases. The results are reported in Table 3 and shown in Figures 9 and 10.

Table 3

Strong designs identified by considering six preference cases

Run 1						Run 2					
$a_{ij} = 0.6$		$a_{ij} = 0.8$		$a_{ij} = 1.0$		$a_{ij} = 0.6$		$a_{ij} = 0.8$		$a_{ij} = 1.0$	
DN	N_c	DN	N_c	DN	N_c	DN	N_c	DN	N_c	DN	N_c
206	6	206	5	206	5	250	6	250	6	250	6
159	6	159	5	221	4	260	5	315	4	298	4
121	6	121	5	182	4	256	5	298	4	257	4
116	6	116	5	89	4	172	5	260	4	228	4
104	6	89	5	238	3	315	4	257	4	148	4
89	6	104	4	236	3	300	4	228	4	130	4
54	6	54	4	…	…	298	4	174	4	128	4
221	5	…	…	159	3	257	4	172	4	111	4
…	…	221	4	54	3	240	4	148	4	86	4
182	4	182	4	121	2	237	4	130	4	75	4
236	4	236	4	116	2	228	4	…	…	…	…
238	3	238	3	104	2	…	…				

DN: Design number.
N_c: Number of preference cases for which outcome is preferred.

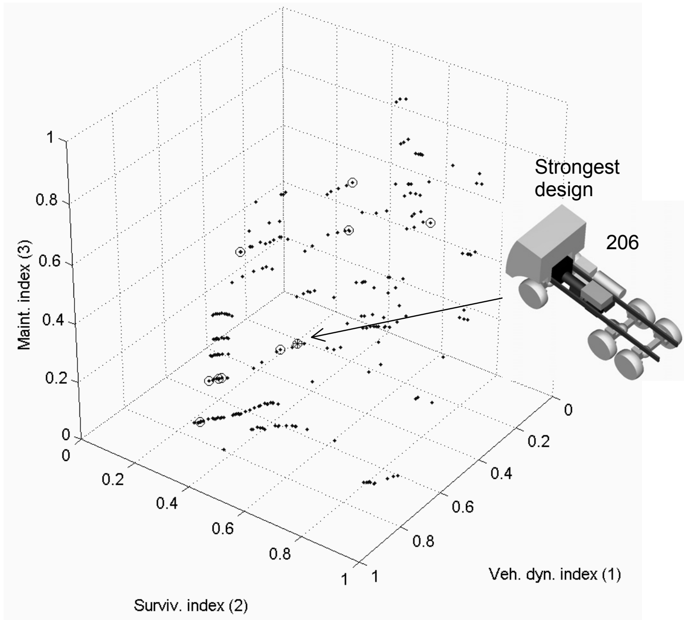

Figure 9. Strong outcomes (top ten) in run 1 identified by considering six preference cases

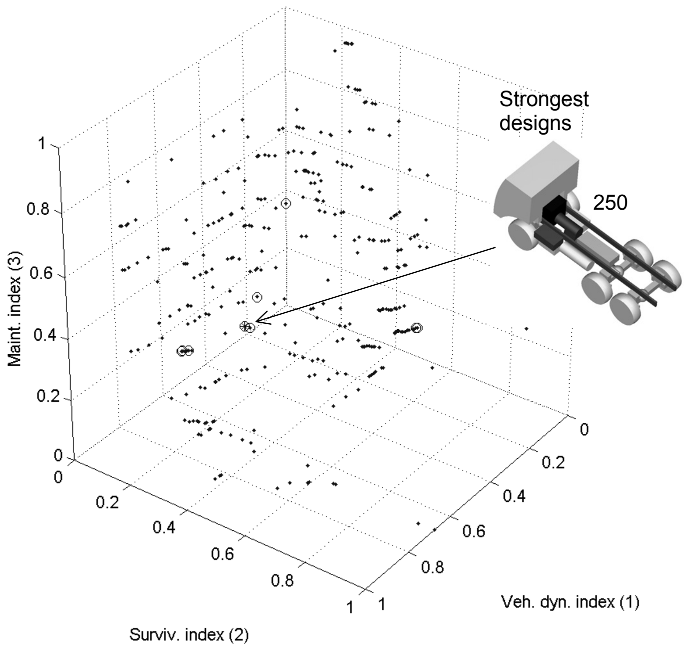

Figure 10. Strong outcomes (top ten) in run 2 identified by considering six preference cases

Based on Figures 9 and 10, it appears that the strong outcomes are not concentrated in a small region of the outcome space. On the contrary, they are evenly distributed within the Pareto set. This is due to the fact that the screening process works on the observed Pareto set rather than the true Pareto set. In particular, the observed outcomes' proximity to the true Pareto set interacting with the assumed preference may both determine whether they are extracted or not. This remark directly concerns managers and decision makers who, because of the complexity of real-world problems, only have access to approximations as opposed to the desired true outcomes of the multi-criteria optimization problem.

Table 4

Non-normalized criterion values of selected designs

Run	Outcome	VDI	SI	MI
1	Min	12.3	-18.4	-47
	206	22.9	-17.5	-42
	Max	35.0	-12.9	-16
2	Min	13.4	-20.4	-47
	250	22.5	-19.4	-41
	Max	36.1	-11.6	-10

In both runs, only one outcome is consistently at the top of the list. These are outcomes number 206 in run 1 and number 250 in run 2, whose non--normalized criterion values are listed in Table 4. Note that the two truck configurations, shown in Figures 9 and 10, are fairly different. By examining the range of variation of each criterion, one can verify that the two outcomes are similar in the outcome space while different in the design space. Several other outcomes appear at the top of the list in both cases, however, less consistently. For instance, outcomes 159 and 221 in run 1 and outcomes 257 and 228 in run 2 are considered strong designs. This discussion may represent a sample analysis a designer would perform when choosing a final configuration design for the FMTV. The two strong outcomes, 206 and 250, resulting from the a posteriori analysis with unknown preferences, can be considered candidates for the final preferred outcome. The choice between these two "privileged" outcomes may be dictated by other considerations (e.g., cost) that have not been included in multi-objective program (13).

Conclusion

A model of relative importance of criteria has been presented and applied to the configuration design of a midsize truck for optimum vehicle dynamic behavior, survivability, and maintainability. The presented application, although related to a specific design problem, demonstrates the special feature of the model that a small subset of preferred solutions is determined in agreement with the assumed relative importance from which a final preferred solution is selected. The model therefore offers an alternative to scalarizing approaches that convert MOPs into scalar optimization problems with tradeoff information available only at their optimal solutions, and to genetic algorithms that produce approximations of the true Pareto set from which the choice of a final preferred solution is difficult.

The paper also highlights the issues related to the stochastic nature of methods for finding Pareto solutions. Two sets of Pareto outcomes were first generated with a multi- objective genetic algorithm. These sets were then screened according to the new preference model. It is shown that for known preferences this model can be used to extract preferred outcomes from the Pareto set. In addition, a method has been proposed to produce a short list of “strong” solutions for the case where preferences are unknown or non-quantifiable, which generally results from DM’s lack of experience and understanding of the problem or lack of consensus between multiple DMs.

Finally, the versatility of the proposed model allows preferences to be also included in the problem *a priori* to directly obtain a set of preferred solutions and thus offers promise for practical use in real-world engineering and other problem. Indeed, a majority of the issues discussed here in relation to engineering design, remain relevant for applications in business, management, and other domains of human life.

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OPTIMIZED STAR PLOT AS DECISION AIDS: APPLICATIONS OF MAXIMUM RESOLUTION TOPOLOGY*

Abstract

The traditional star plot has been a longstanding means of presenting multi-variate data. Its early-days use can be traced back to "star symbol plot" of automobile data for large rays to represent favorable characteristics. Another pioneering use in clinical data is a graphical way of summarizing patient's evolving responses. Since the early 1970s and with the coined name of Kiviat plot or graph, wide-spread use in visualizing computer and program performance has become industrial standard among software engineers. It is ever more so in the modern advance of computer graphics, transformed into popular evaluation tools such as 2/3-D Kiviat graph and 3-D Kiviat tube. Its importance amidst forward technological strides remains largely in its ease of visualization, qualitatively on the basis of the shape of a star plot.

In recent years we have staged a series of studies, by focusing of its analysis and topology, resulting in usefulness in the following extensions. First, a (canonical) star plot topology for high-dimensional data visualization is applied to data records of, specifically, multi-attribute dichotomies. Our project on data analysis of on-line auction markets provides such generic sample usage for dimensions identified in constructing a multi-attribute dichotomy to help discern relative empirical advantages to buyers and sellers.

The second stage, of data and optimization modeling aspects, bases on the deeper observation that the areas of the plot for the two parts of a dichotomy may be used quantitatively as an aggregate measure of their relative dominance. An optimization GP model is developed to determine a topology – the geometry and the arrangement of dimensions – that maximizes the resolution of this measure with respect to a given set of reference dichotomies. The outcome of this modeling phase

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is what we call an MRT (or Maximum Resolution Topology), that in the sense of maximally discriminating its dichotomy of a set of multi-attribute data records, it is an overall best representation (accompanied by an "optimized" visualization).

The third stage is the coding of MRT construction integrated into a spreadsheet-style decision support system (MRT-DSS). Its ease of use has been promising and robust for diverse applications. Samples of these will conclude the paper as illustrations.

Keywords

Star plot, data visualization, multi-attribute dichotomy, optimization modeling, goal programming, maximum resolution topology (MRT), decision support system, applications for exploratory data analysis.

Introduction

The traditional star plot has been a long-standing means of presenting multivariate data [1]. Its early-days use can be traced back to "star symbol plot" of automobile data for large rays to represent favourable characteristics. Another pioneering use in clinical data is a graphical way of summarizing patient's evolving responses [4]. Since the early 1970s and with the coined name of Kiviat plot or graph [19; 20; 21], wide-spread use in visualizing computer and program performance has become industrial standard among software engineers [25]. It is ever more so in the modern advance of computer graphics, transformed into popular evaluation tools such as 2-D and 3-D Kiviat graphs and 3-D Kiviat tube [5]. One particularly important use in computer hardware/software system monitoring is its real-time visualization of parallel programs and performance [6; 7]. Other applications abound, for example, in petroleum industry and geology [26]. Its importance amidst forward technological strides remains largely in its ease of visualization [22], qualitatively on the basis of the shape of a star plot [25].

The rest of this paper is organized as follows. Section 1 reviews the background of our first stage work on extending a star plot to a topological model. Section 2 reports our second stage work on goal programming (GP) formulation. Section 3 provides details of our third stage work on the maximum resolution topology (MRT) DSS and its applications. Section 4 gives concluding remarks with respect to our current spectrum of application projects and future studies.

1. Visualization and resolution

1.1. Star plot and visualization

Our visualization (and pre-optimization) model is based on "canonical" star plot for displaying multivariate data with an arbitrary number of dimensions [1]. Each data point is plotted as a star-shaped figure (or star graph) with one ray for each dimension. As the resulting shapes depend on the configuration of the dimensions, we subsequently analyse the observations along the dimensions in an effort to present a visual model (see a generic example in Figure 1 of the shape of on-line auction markets [8]).

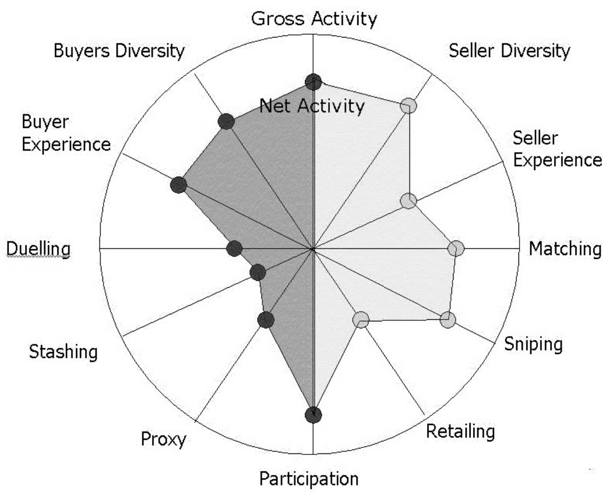

Figure 1. Star plot visualization for on-line auction market

This further investigation leads to generalization from conventional star plot for visualization to a topological model with extended features of dichotomous resolution.

1.2. Topology and resolution

A topological model for a high dimensional data set is a simultaneous graphical display of all its relevant attributes, which provides a geometrical shape as a descriptive, visual statistics of the underlying construct engendering the data. In particular, when various dimensions can be identified to form a multi-attribute dichotomy, the area spanned by the two halves of the topological model can be used as a measure of the relative dominance of the two parts of the dichotomy. Using a reference subset of prejudged cases, the configuration of the dimensions and the angles among them can be optimized in a GP [24] model for a topology that maximizes the resolution of such dichotomies.

In recent years we have staged such a series of studies, by focusing on its analysis and topology, resulting in usefulness in the following extensions. First, a (canonical) star plot topology for high-dimensional data visualization is applied to data records of, specifically, multi-attribute dichotomies. Our project on data analysis of on-line auction markets provides such generic sample usage for dimensions identified in constructing a multi-attribute dichotomy to help discern relative empirical advantages to buyers and sellers. (See Figure 1 above for the dimensions on the right being favourable for buyers versus those on the left for sellers).

The second stage, of data and optimization modelling aspects, bases on the deeper observation that the areas of the plot for the two parts of a dichotomy may be used quantitatively as an aggregate measure of their relative dominance. A GP model is developed to determine the topology – the geometry and the arrangement of dimensions – that maximizes the resolution of this measure with respect to a given set of reference dichotomies. The outcome of this modelling phase is what we call an MRT (for Maximum Resolution Topology), that in the sense of maximally discriminating its dichotomy of a set of multi-attribute data records, it is an overall best representation, accompanied by an "optimized" visualization [2]. (See Figure 2 below for such an "optimized" star graph [17] for the same generic example as in Figure 1).

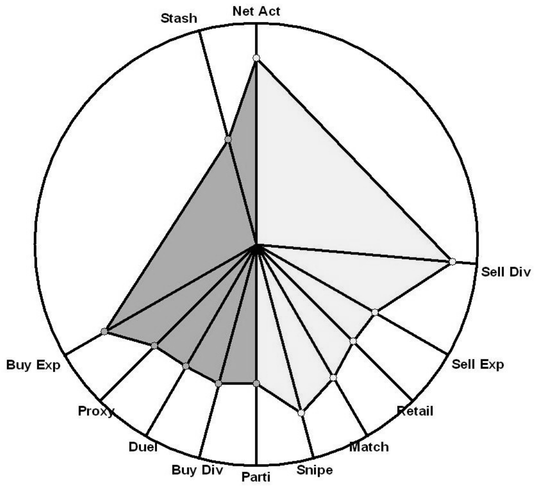

Figure 2. Maximum resolution topology for on-line auction market

The third stage is the coding of MRT construction integrated into a spreadsheet-style decision support system (MRT-DSS). Its ease of use has been promising and applications abound in diverse fields, including diffusion of innovation [11], investment climate and business environment [12], marketing research and customer relations management [13], and medical diagnostics. The implementation of the MRT optimization model as an easy-to-use, spreadsheet-based DSS provides a robust tool for such applications.

2. Modelling and optimization

Subject to the constraints of preserving the prejudged dominance in the reference subset of dichotomies, an optimal topology – the arrangement of dimensions on, and geometry of, the star graph (i.e. configuration of attributes and angles between adjacent pairs) is sought that maximizes the discriminating power, or *resolution*, as measured by the sum of absolute differences in right and left areas for the reference subset. Such an optimal configuration will be called a *maximum resolution topology* (MRT). For any given configuration of the attributes, maximization of the discriminating power can be formulated as a linear program (LP). However, LP produces extreme-point solutions, which may reduce some of the angles between attributes to zero, thus collapsing the corresponding sector areas of the graph. To avoid such degeneration, maximization with bounded deviation of the angles is modelled as a goal program in [16], as summarized below.

2.1. Goal programming formulation

In the star graph for a multi-attribute dichotomy, denote the *angles* between attributes x_{i-1} and x_i by α_i, $i \in I$, and those between attributes y_{j-1} and y_j by β_j, $j \in J$, for all $|K|$ records in the reference set. The sector areas are similarly denoted by A_i A_i and B_j. Let the *weights* be

$$a_i \equiv \text{Sin}\alpha_i, \quad i \in I; \quad \text{and} \quad b_j \equiv \text{Sin}\beta_j, \quad j \in J$$

And we have $0 \le a_i \le 1$, $i \in I$ and $0 \le b_j \le 1$, $j \in J$. Hence for the k^{th} record, the sector area between attributes x_{i-1} and x_i, and that between attributes y_{j-1} and y_j, are given respectively by

$$A_i^k (= A_i^k(x)) \equiv \frac{1}{2} x_{i-1}^k x_i^k \text{Sin } \alpha_i = \frac{x_{i-1} x_i}{2} a_i, i \in I$$

$$B_j^k (= B_j^k(y)) \equiv \frac{1}{2} y_{j-1}^k y_j^k \text{Sin } \beta_j = \frac{y_{j-1} y_j}{2} b_j, j \in J$$

With $A^k = \Sigma A_i^k$ denoting the area of the right part of the dichotomy, and $B^k = \Sigma B_j^k$ that of the left part, of record k, $k \in K$, the $|K|$ records are partitioned according to prejudgment in the reference set, such that $K \equiv K^+ \cup K^-$, for "right" and "left" dominance, respectively.

The MRT-GP with decision variables a_i, $i \in I$ and b_j, $j \in J$ is then given by [16]

Max	TtlMRT
Min	TtlDev
Min	TtlVar

Subject to

$$A^k - B^k = D^k, \quad k \in K^+ \quad \text{and} \quad B^k - A^k = E^k, \quad k \in K^-$$
$$\sum_{i \in I} a_i = 1; \quad \sum_{j \in J} b_j = 1;$$
$$a_i \geq 0, i \in I; \quad b_j \geq 0, j \in J$$
$$\text{TtlMRT} = \sum_{k \in K^+} D^k + \sum_{k \in K^-} E^k$$
$$\text{TtlDev} = \frac{1}{|I|}\sum_{i \in I}\left[\text{PDevX}(i) + \text{NDevX}(i)\right] + \frac{1}{|J|}\sum_{j \in J}\left[\text{PDevY}(j) + \text{NDevY}(j)\right]$$
$$\text{TtlVar} = \frac{1}{|I|}\sum_{i \in I}\left[\text{PVarX}(i) + \text{NVarX}(i)\right] + \frac{1}{|J|}\sum_{j \in J}\left[\text{PVarY}(j) + \text{NVarY}(j)\right]$$

(Deviation bounds)

$$\frac{1}{|I|} - F * \frac{1}{|I|} \leq a_i \leq \frac{1}{|I|} + F * \left(1 - \frac{1}{|I|}\right), \quad i \in I; \qquad \frac{1}{|J|} - F * \frac{1}{|J|} \leq b_j \leq \frac{1}{|J|} + F * \left(1 - \frac{1}{|J|}\right), \quad j \in J$$

(Deviation goal constraints)

$$a_i - \text{PDevX}(i) + \text{NDevX}(i) = \frac{1}{|I|}, \quad i \in I; \qquad b_j - \text{PDevY}(j) + \text{NDevY}(j) = \frac{1}{|J|}, \quad j \in J$$

(Variation/smoothing goal constraints)

$$a_i - a_{i+1} - \text{PVarX}(i) + \text{NVarX}(i) = 0, \quad i \in I \quad (a_{m+1} \equiv a_0)$$
$$b_j - b_{j+1} - \text{PVarY}(j) + \text{NVarY}(j) = 0, \quad j \in I \quad (b_{n+1} \equiv b_0)$$

The $0 < F < 1$ parameter in the MRT-GP formulation above is the fraction of deviation from equal weights allowed for the weight variables a and b. PDev, Ndev, PVar, NVar stand for positive deviation, negative deviation, positive variation, negative variation, respectively, in conventional GP terminology [24].

2.2. Sample illustration of MRT computation

Table 1 records two numerical instances (34 eBay and 20 eGDI data records) of MRT-GP computation. The canonical star plot values are the equal-weight (i.e. angle) total area difference, given by the definition:

$$\sum_{k\in K^+}[A^k - B^k] + \sum_{k\in K^-}[B^k - A^k]\left|\left\{a_i = \frac{1}{|I|}, \forall i; b_j = \frac{1}{|J|}, \forall j\right\}\right. = 1.8682$$

for eBay dataset and 0.6725 for eGDI dataset. The respective optimal values for the first objective function TtlMRT are increased to 3.1498 and 1.1725. Hence there are gains in discriminating power of 68.60% and 74.35%, respectively. As MRT consider both configuration of attributes *and* angles, we illustrate in Table 1 the less desirable effects of optimizing only *one* of the two aspects of the resolution topology. Numerically, it can be seen that GP optimization with respect to the angles as decision variables accounts for the substantially larger share of improved resolution in both instances.

Table 1

Sample performance of MRT-GP results

	Canonical star plot	Configuration ONLY: attributes permutation	Weight ONLY: angles optimization	Optimized MRT
34 eBay data records	1.8682	1.8684 (0.01%)	2.7100 (45.06%)	3.1498 (68.60%)
20 eGDI data records	0.6725	0.7125 (5.95%)	0.7499 (11.51%)	1.1725 (74.35%)

3. Applications of maximum resolution topology

3.1. Decision support system: MRT-DSS

To facilitate the computation of a maximum resolution topology (MRT) for a given set of data from a multi-attribute dichotomy, an easy-to-use decision support system (DSS) has been built on Excel spreadsheet software. Such an MRT-DSS system has both its front end and report routine integrated in the same Excel spreadsheet workfile, into which the input data records can be placed (for example, imported from a database); and outputs of values and MRT-star plots displayed. The functions of MRT-DSS are grouped in a pull-down menu as shown in Figure 3.

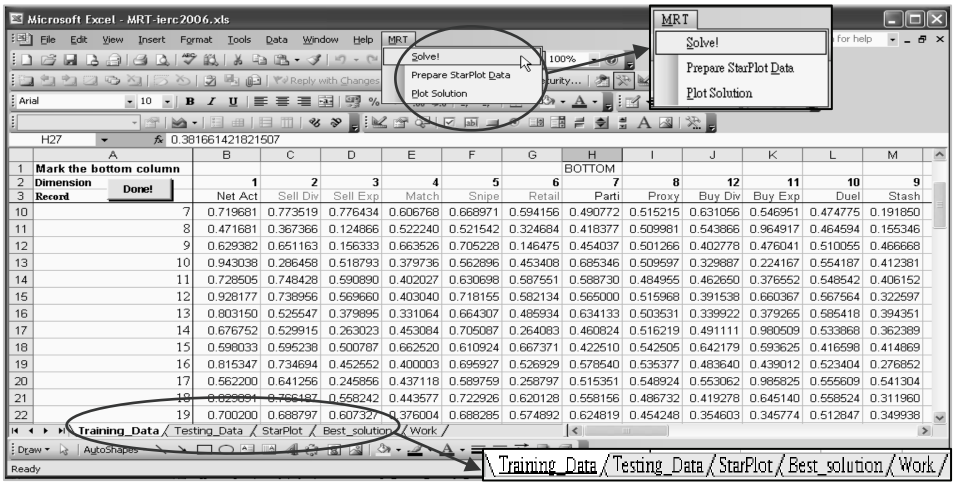

Figure 3. Worksheets and menu of MRT-DSS

To find the solution, the user only needs to copy and paste the records of training data (the “reference set”) to the ‘Training Data’ worksheet and click the ‘Solve!’ item button on the ‘MRT’ menu. MRT-DSS will permute over all possible configurations and dynamically generate the input data for each configuration. The training data will be passed to an LP solver (LINGO [23]) to find the solution based on the MRT-GP model. MRT-DSS will store the solution of each configuration on the ‘Work’ worksheet, as well as the best solution on the ‘Best solution’ worksheet. It will also keep the optimal MRT configuration and angles in the ‘StarPlot’ worksheet for preparing the test data for plotting.

By completing the training of MRT-DSS and obtaining the optimal configuration, the system can then be used to evaluate new cases of the model. With data copied to the ‘Testing Data’ worksheet, the ‘Prepare StarPlot Data’ item dichotic on the ‘MRT’ menu is selected. MRT-DSS will arrange and store the data in the ‘StarPlot’ worksheet. It will also compute for each test case the areas of the right (A) and left (B) parts of the dichotomy and their difference (A-B) as shown in Figure 4. The user can easily evaluate the test cases based on these numerical results. To visualize and further analyse a particular data record, the user can choose the ‘Plot Solution’ item on the ‘MRT’ menu to draw its StarPlot diagram under the maximum resolution topology. By inspecting and comparing records under the optimal configuration and angles in the diagrams, and by studying the right-left differentials provided by MRT-DSS, substantial topological analysis can be performed for insight into the model under study.

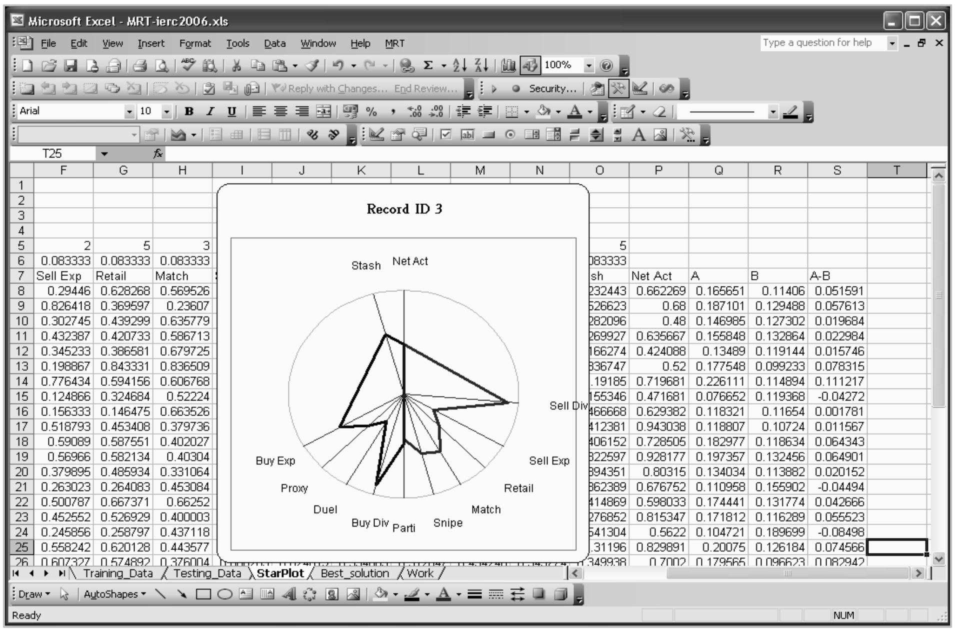

Figure 4. Using MRT-DSS to evaluate and visualize test dataset

3.2. Applications of MRT-DSS

To demonstrate the robust application of MRT-DSS in diverse fields, the results of various published studies are summarized below.

3.2.1. Comparative study of on-line auction markets

As exploratory data mining, 34 data sets with the bidding records of approximately 500 auctions each are used as the reference subset. The market categories comprise automobiles, business software, camcorders, coffee tables, coins, cordless phones, desktop PCs, digital cameras, gift certificates, handbags, laptop PCs, lodging, necklaces, personal digital devices, rings, travel and vacation packages, and wristwatches. MRT-DSS produced the optimal configuration of the twelve dimensions in the topological model for on-line auctions as shown in Figure 2. In [17], a comparative study of four markets in five countries was conducted. The results are summarized here for illustrative purposes. The countries are Australia (AU), Canada (CA), France (FR), United Kingdom (UK), and United States (US). The markets are: classical CDs, Star Wars toys and games; diamond rings, and digital cameras. The star plots for the twenty cases are shown in Figure 5.

The seven cases boxed in Figure 5 are classified as “right-dominant” (buyer’s market). The other thirteen cases are classified “left-dominant” (seller’s market). It was observed that on-line auction markets for digital cameras, a hi-tech product of common value, tend to be favourable to buyers across nations. By contrast, the other markets of particular items catering to more subjective preferences and tastes tend to favour sellers on eBay.

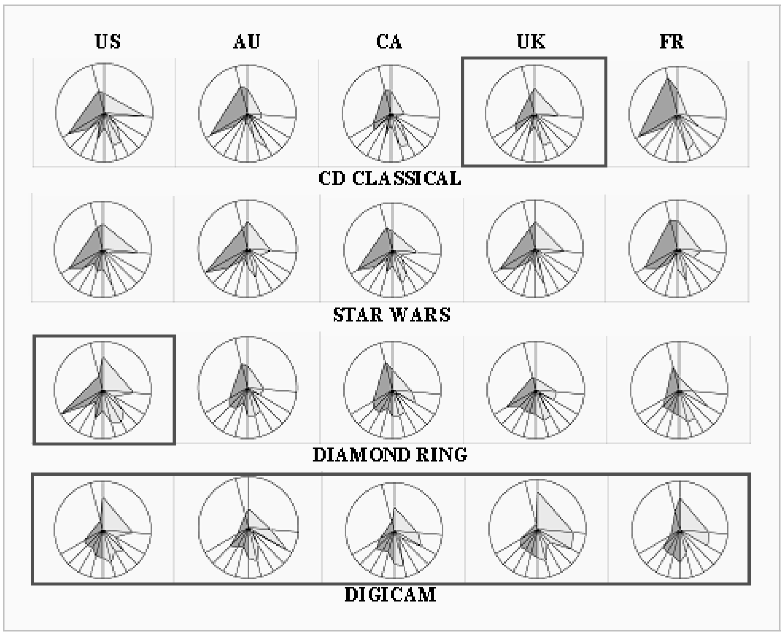

Figure 5. MRT plots of 20 markets in comparative study

3.2.2. Global Diffusion of the Internet

The Global Diffusion of the Internet (GDI) framework has been used to study the progress in the adoption of this communication medium in close to 30 countries since 1997. Six dimensions are used to measure the essential features of the status of the Internet in a country. Collectively, they cover the bundle of requisite technologies, from infrastructure to end use applications, to capture the multifaceted evolution of the Internet experience in different countries. They also fall into a supply-demand dichotomy:

Demand Attributes	Supply Attributes
Pervasiveness	Organizational Infrastructure
Sectoral Absorption	Connectivity Infrastructure
Sophistication of Use	Geographic Dispersion

Using a reference subset of 24 cases in 1999, an MRT is obtained and used to calibrate subsequent cases [11]. For example, China, India, Pakistan, and Turkey in 2000 are plotted in Figure 6, showing that China was demand-dominant while the other three were supply-dominant in GDI.

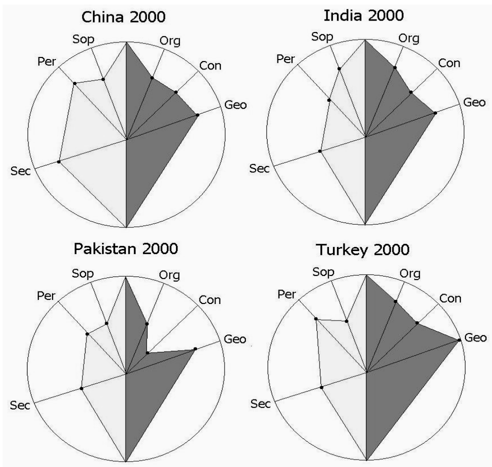

Figure 6. MRT plot for GDI of four countries in 2000

3.2.3. Investment climate indicators

The Investment Climate Surveys conducted by the World Bank measure specific constraints facing firms in over 50 countries, and relate them to measures of firm performance, growth, and investment. By casting ICI as a multi-attribute dichotomy, an MRT is sought to classify countries as to whether the investment climate is constrained primarily by policies or by resources [12].

Policy Attributes	*Resource Attributes*
Policy Uncertainty	Electricity
Regulation	Finance
Tax rate	Labour Skills

Based on the initial plots, the five countries with the highest policy dominance in constraints on investment climate are Belarus, Moldova, Romania, Poland, and Brazil. The five with the highest resource dominance in constraints on investment climate are Bangladesh, Senegal, Zambia, Uganda, and Eritrea. These are used as the reference subset to derive an optimal MRT. The resulting plots for four populous countries are shown in Figure 7.

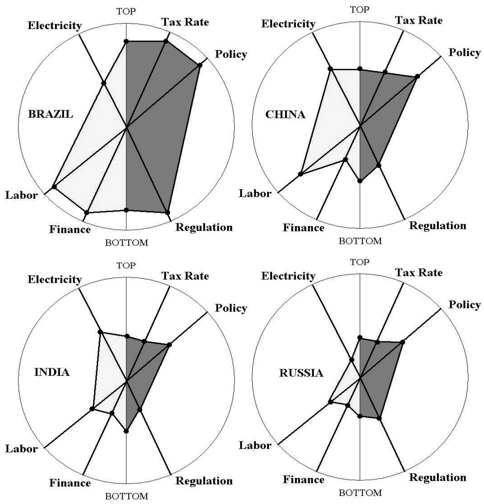

Figure 7. MRT plot for ICI of four countries

3.2.4. Customer relations management

In customer relations management (CRM), consumers are often surveyed for their attitude toward various attributes of products and services for insight into their perception of price, quality and value. The results of analysis can be used in market segmentation for more effective design and promotion of future offerings. In [13] a survey was presented that asked customers to rate the importance of the attributes directly. The maximum resolution dichotomy (or MRD) methodology was applied as a DSS tool in market research to classify customers according to the MRT of whether they tend to be cost- or benefit-focused.

We consider customer attitude surveys that ask for the rating, on some given scale of relative importance, of various cost and benefit attributes of a product, service, or experience. Typically, a number of questions relate to different aspects of cost, including direct pricing, and indirect opportunity costs. The other questions, not necessarily in the same number, relate to benefits in terms of perceived quality, and customer satisfaction. A high importance rating of a cost attribute means the customer is cost-conscious in that regard, and a high actual cost is likely to detract from overall satisfaction. Conversely, a high importance rating of a benefit attribute means the customer is particular about its value and is likely to pay for improved quality there.

As an example, we use the case of a major league sports team and a survey of its fans' attitude toward the following attributes of their entertainment experience at a home game.

On a scale of 1 to 7 (most important), how do you rate the importance of:
Question 1: the view of the game from your seat?
Question 2: amenities available in the stadium?
Question 3: the team's performance in the games?
Question 4: cost of tickets?
Question 5: cost of transportation and/or parking?
Question 6: cost of concessions?

Of the 369 respondents, 58% were classified as benefit-focused, 33.3% as cost-focused, and 8.7% as cost-benefit neutral. The ratio of benefit-focused respondents to cost-focused respondents is 1.74. Management may use this indicator to, for example, devote more effort into quality improvement instead of price reduction.

3.2.5. On-line auction markets in tourism

Since the value of an empty seat on an airliner vanishes once the flight takes off, just as an empty hotel room by check-out time, most travel- and tourism-related services can be regarded as perishable commodities. Because of that, the concept of dynamic pricing to balance supply and demand in a free market appeals to both economists and consumers. To the former, this means market efficiency. To the latter, it suggests potential bargains as expiration looms. However, suppliers must overcome the stigma of brand dilution if they let know that lower than set prices are acceptable. In [14], a comprehensive, though by no means exhaustive survey found on-line auction markets in travel and tourism in a fledgling stage of development. On eBay.com, for example, there were around 2000 active listings in December 2006, while the comparable number was over 190,000 for Antiques, and over 390,000 for Consumer Electronics. Yet there is already significant data available for formal analysis to track the growth of markets in travel and tourism.

The MRD methodology is applied to the Travel categories on eBay in the early 2006. The sub-category of Vacation Packages and Lodging are used. Under Lodging, we have Vacation Rental, Hotel, and Bed and Breakfast. The last two are grouped as one.

As quantitative measures, we can use an index:

$$I_B = (\text{Right Area} - \text{Left Area}) / (\text{Right Area} + \text{Left Area})$$

which is positive for a buyer's market, and negative for a seller's market. This index ranges from -0.29 to +0.43 with an average of 0.11 for the 34 cases in the reference set. For the three Travel cases, we have the indices below:

Market	VacRental	Hotel+B&B	VacPack
I_B	0.18	0.21	0.64

Our results show that actual conditions are also favourable to buyers, and to extents that are above the reference average index of 0.12. In particular, Vacation Packages, with an index of 0.64 is remarkable.

3.2.6. Inter-brand comparisons

In [15], the MRD methodology is applied to four brands of digital cameras on eBay in the first quarter of 2006. The specific auction categories used were:

Cameras & Photo> Digital Cameras> Point & Shoot> 4.0 to 4.9 Megapixels> *Brand*; where *Brand* was Nikon, Sony, Kodak, and Canon, respectively.

It should be apparent that with the high number of dimensions and their categorical measures, inter-brand comparison of subtle differences will be rather difficult. This is where our maximum resolution dichotomy methodology can become useful for gaining further insight into such market data. For the four brands of digital cameras, we have the indices below:

Brand	**Sony**	**Canon**	**Kodak**	**Nikon**
I_B	0.20	0.39	0.53	0.55

All four MRDs exhibit a right-dominant topology, indicating that market conditions are favourable to buyers for these brands. On a relative scale, Nikon ranks the highest at 0.55, closely followed by Kodak at 0.53. Even at 0.39 and 0.20, respectively, both Canon and Sony are significantly above the average of 0.1 for the reference set used to derive the MRT. This is consistent with more general and extensive observations that for hi-tech consumer products of so--called common values, eBay auctions tend to be buyer's markets. Cost of entry and operation being low compared to brick-and-mortar stores, competition is keen among sellers. They must compete on price and service to establish credibility and gain market share.

Concluding remarks

Our series of studies ([8; 9; 10; 16; 3; 17; 18] in this order) have progressed from an initial key study on topology analysis of on-line auction markets [8], to optimization modelling towards maximum resolution topology for multi-attribute dichotomies [16], to its spreadsheet MRT-DSS implementation [18], and finally to a wide spectrum of applications [11; 12; 13; 14; 15]. A concise summary from the data mining applications perspective of this tool is given below.

The potential usefulness of MRT-DSS has been demonstrated in diverse applications. In customer relations management and marketing research, it is of common interest to gain insight into consumers' attitude toward the costs and benefits associated with a product, service, or experience. Surveys on the multitude of attributes lead to high-dimensional data, for which meaningful aggregate measures remain a major challenge in data mining. As a contribution to the modelling of market segments, our methodology helps to rationally classify customers as to whether they are benefit-focused or cost--focused. The cost-benefit survey framework is cast as a multi-attribute dichotomy, with a cost side and a benefit side [13]. Similarly, as a contribution to the rational classification of countries as to whether their investment climate is constrained primarily by policies or by resources, the Investment Climate Indicators (ICI) framework used by the World Bank is cast as a multi-attribute dichotomy, with a policy-constraints side and a resource-constraints side. As an extension of the star plot to display multi-dimensional data, the areas spanned by the policy-side and resource-side attributes of a data instance in ICI suggest an aggregate measure of the relative dominance of the corresponding parts [12]. Future work includes expounding the application of this dichotomy as a significant output of data mining for CRM and ICI, as well as refining the selection of the reference set by, for example, incorporating results from focus group studies, as well as expert judgment from researchers of specific markets and economies.

The inter-brand comparative study of on-line auction markets was conducted for four brands of digital cameras. Potential sellers and buyers may ask whether eBay auctions are good places to sell and buy digital cameras, respectively. The auctioneer (eBay) may wish to determine how such markets are performing, and whether to apply strategies such as discounted fees to promote activity and participation. Do answers to such questions apply broadly to all brands, or do they depend on the brand? Our findings help to discern aggregate, operational characteristics for the market among brands [15]. Similarly, while travel- and tourism-related markets are still in a fledgling stage of development, our study showed that actual conditions are also favourable to buyers, and that further progress and growth will depend on the reaction of sellers, and their willingness to put their offerings to the test of the free market. Our results demonstrated the potential usefulness of our approach as a tool in data mining for future study and analysis of on-line auctions [14]. Indeed, in recognition of the potential wealth of information in its transactional data, eBay.com itself has begun marketing both the content and analysis of its databases commercially. Third-party vendors, known as eBay market data resellers, are also appearing to offer services in related market analysis. Our MRT-DSS is an addition to data mining tools that may be particularly useful in this context. Finally, it should be remarked that our example applications, while diverse, are quite generic. Other potential cases abound, notably in medical diagnostics, risk profiling in both financial and security issues, just to name a few.

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MULTI-CRITERIA DECISION AIDING PROCEDURE UNDER RISK AND UNCERTAINTY

Abstract

Decision making under uncertainty is a very important area of decision theory. Uncertainty implies that in certain situations a person does not have the information which would adequately describe, prescribe or predict a system, its behavior or other characteristics, deterministically and numerically. Thus uncertainty relates to a state of the human mind, i.e., lack of complete knowledge about something.

In this paper we propose an interactive multicriteria decision aiding procedure which enables to take into consideration together uncertainty and risk factors. The uncertainty factors we consider when we don't know the probabilities of the states of nature. The risk factors are applied when we are able to estimate the probability distributions.

The proposed procedure uses scenario planning technique to deal with uncertainty and Monte Carlo simulation to deal with risk factors.

Proposed decision aiding procedure is illustrated by the complete numerical example.

Keywords

Multicriteria decision aiding, risk, uncertainty. Monte Carlo simulation, scenario planning.

Introduction

Rapid technological progress, particularly in the field of information and telecommunication technologies (ICT), and the increasing economic globalization, taking place at the turn of the 20^{th} and 21^{st} centuries, result in a significant volatility of the macroeconomic environment, which has a considerable impact on the business world.

In consequence, the influence that these factors exert on economic and business decisions has to be taken into account in the decision-making. The issues relating to decision analysis and aiding under incomplete information remain important part of operational research, in particular of multi--criteria decision aiding.

Uncertainty implies that in the certain situation a person does not possess the information which quantitatively and qualitatively is appropriate to describe, prescribe or predict deterministically and numerically a system, its behavior or other characteristics [26]. Thus uncertainty relates to a state of the human mind i.e. lack of complete knowledge about something [24].

In earlier works term "Risk" was applied to the situations in which probabilities of outcomes are known objectively, recently term "Risk" means a chance of something bad happening [10]. The term "Uncertainty" is applied to the problems in which exist alternatives with several possible outcomes. The sources of uncertainty may be divided into two main groups: internal sources of uncertainty and external ones. Internal sources of uncertainty are created by imprecision of human judgments concerned with specification of preferences or values or to assessment of consequences of actions [24]. In the MCDA approach we can find a wide range of methods and techniques to deal with uncertainty created by internal factors: sensitivity analysis (e.g. [21]), fuzzy set approach (e.g. [16, 3]), rough set approach (e.g. [12]). External uncertainty refers to lack of knowledge about the consequences of our choices [24]. For those types of problems the following methods are applied: probabilistic models and expected utility (e.g. [14, 1, 22]), pair wise comparisons based on stochastic dominance (e.g. [4, 17]). The risk measures as surrogate criteria are also applied (e.g. [18, 23, 13]). In such problems where we have to take into account external uncertainty the scenario planning may be applied (e.g. [15, 11, 20, 25]).

While considering the traditional division of the issues relating to decision making under incomplete information into the issues relating to decision making under uncertainty and the issues relating to decision making under risk, one can notice that both cases have so far been treated independently (both in scientific literature and in business practice), e.g. decision situations have been analysed as under uncertainty or as under risk.

We think, based on the previously conducted research, that management (especially strategic management) comprises a number of decision-related areas, where uncertainty factors and risk factors should be considered jointly and decisions should be evaluated based on many criteria. In such situations, however, decision aiding requires the development of an appropriate methodology for decision analysis and aiding.

This paper discusses the proposal of the multi-criteria decision aiding procedure under uncertainty and risk, which is a modification of the method presented in the paper [9]. The multi-criteria decision aiding method which has been developed takes into account both uncertainty factors and risk factors.

To incorporate uncertainty factors, the scenario-based approach was adopted, while the Monte Carlo simulation* was applied to deal with risk factors. The decision aiding process was carried out with the use of the interactive method, which allowed to take into account individual preferences of a decision maker (DM) without the necessity of making prior assumptions about them.

To analyse numerical (computational) problems, we created a numerical example, which is the main part of the paper. It was based on the data assumed and illustrates the possibilities of practical applications of the presented decision aiding methodology. Spreadsheets were created to aid decision making based on this method and to test its practical applications (including numerical problems).

The results presented in the paper indicate that the proposed approach to decision aiding, incorporating many evaluation criteria and uncertainty and risk factors, can be effectively implemented in a spreadsheet supplemented with the simulation device and can become a useful tool to aid real life decision problems.

1. Decision aiding under uncertainty and risk**

This section of the paper deals with the multi-criteria decision aiding procedure under risk and uncertainty. The procedure uses the scenario-based method to incorporate uncertainty factors and the Monte Carlo simulation to reflect risk factors. To compare and aid the process of selecting alternatives, we developed a multi-criteria interactive decision aiding method. The method comprises eight main stages, listed below:

1. Formulation of potential decision alternatives.
2. Determination of the evaluation criteria for each alternative.
3. Identification of uncertainty factors.
4. Planning of the scenarios of the environment development.

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* Monte Carlo simulation can be found in [2].
** This part of the paper presents the modified procedure, discussed in the [9].

5. Identification of risk factors.
6. Development of strategic financial plans.
7. Performing the Monte Carlo simulation.
8. Selection of the alternative with the use of the interactive decision aiding method.

The stages of the procedure created, aiding the selection of a decision alternative under uncertainty and risk, are discussed below in detail.

1.1. The formulation of decision alternatives

The first stage involves an analysis which aims to construct the set of potential decision alternatives. Let us assume that the set of alternatives is finite and it will be denoted as follows:

$$W = \{w_1, ..., w_M\}$$

1.2. The determination of the evaluation criteria

Next, the evaluation criteria for alternatives are determined. They should allow to compare decision alternatives and reflect the goals of the Decision Maker (DM).

Let us assume that the evaluation criteria can have a directional, point or interval character. They can measure both quantitative characteristics (then they are measures on a ratio scale) and qualitative attributes (noted on an ordinal scale).

To simplify the notation and improve its clarity, we assume, further in the paper, that all the criteria have a directional character and should be maximised. (Minimised, point or interval criteria can be taken into account after their simple transformations).

The set of evaluation criteria for the alternatives is denoted by:

$$K = \{k_1, ..., k_J\}$$

1.3. The identification of uncertainty factors

Based on the results of the analysis of the economic macro-environment, the third stage involves identifying uncertainty factors which may have an impact on the values of the evaluation criteria for the decision alternatives which we are considering. These are the factors which remain beyond DM's control and the probability of their occurrence cannot be objectively determined. In practice, they are mainly legislative factors (the introduction or modification of business-related legislation), social factors (change in fashion or lifestyle, etc.), and technological factors (new technical or technological developments).

Next, we determine the set of potential future values for each uncertainty factor. At this stage we can use heuristic techniques, such as "brain storming" or the "Delphi method". The following notation is used:

C – the number of uncertainty factors,

N^z – a set of potential values for factor z (z = 1,…,C) with elements denoted as follows:

$$N^z = \{n_1^z,...,n_{wz}^z\}$$

where **wz** denotes the number of analyzed values of factor **z**.

1.4. The planning of the scenarios of the environment development

Taking into consideration the set of values of uncertainty factors determined at the previous stage, we plan the scenarios of the economic environment development. The set of scenarios should include all the situations considered. As a result, the set of scenarios can be specified as:

$$S = \{s_1,...s_{ls}\} = N^1 x N^2 x ... N^Z$$

It is, then, the Cartesian product of the sets of potential values of all uncertainty factors. The number of scenarios **ls** equals the product of the number of values which can be taken by each uncertainty factor: w1x w2 x ….x wz. The examples of scenarios created for strategic analysis can be found in the papers [7, 9].

1.5. The identification of risk factors for each alternative

Stage 5 involves identifying the risk factors for each alternative. These are the factors which have an impact on the values of evaluation criteria and such that the probability distribution for their values in the future can be assessed. In practice, the factors will mainly include such characteristics of the alternatives as investment costs, demand, selling prices, per-unit costs of production, sales costs and costs of management and administration.

In relation to risk factors, we need to collect additional information which will allow to determine the probability distributions of their occurrence. The sources of data on the unknown parameters of a financial plan can be:

– the results of the statistical analysis of historical data,
– the forecasts based on statistical econometric models incorporating the error distribution of a forecast,
– expert opinions.

The BestFit module, part of the Decision Tools Suite package, can be applied to estimate the probability distributions based on historical data. The module allows to find the distribution and parameters with the best fit to historical data and to cooperate directly with the MS Excel spreadsheet. The examples of estimates of probability distributions for risk factors can be found in the paper [6, 7].

1.6. The development of strategic financial plans

Next, a strategic financial plan is developed for each situation (i.e. for each pair: decision alternative/scenario). The financial plan is the basis for the calculation of the evaluation criteria. Thus, the number of financial plans which have to be developed is M x 1s (the number of alternatives x the number of scenarios).

The starting point for the development of a financial plan for the situation considered is the creation of sales forecasts (including the alternative and the scenario of the environment development) and the investment costs plan. Based on this model, operating costs, divided into fixed costs and variable costs, are estimated. This allows to create profit and loss account forecasts on an operating level. Then, based on additional assumptions about the indices of working capital turnover (inventories and receivables) and payables due dates, we determine the demand for working capital and stabilise the balance. The balance sheet forecasts allow to create cash-flow statement forecasts with the use of the indirect method. The examples of strategic financial plans can be found in the paper [8].

1.7. The conduct of the Monte Carlo simulation

Based on the estimates of probability distributions for risk factors (discussed in 1.5) and the models of strategic financial plans (discussed in 1.6), we conduct the Monte Carlo simulation, which generates the distributions of the values of evaluation criteria for each scenario of the environment development. If the financial plan models are created in the MS Excel spreadsheet, we can use the @Risk module, part of the Decision Tools Suite package by Palisade, to carry out the simulation. An example of the simulation carried out with the use of the @Risk module can be found in the papers [5, 7].

As a result of the simulation, we obtain an evaluation vector for each alternative. The components of the vector are the distributions of the evaluation criteria variables:

$$X_i^k = [X_{i,1}^k, ..., X_{i,ls}^k]$$

denotes the vector which consists of distribution functions of the *k*-th evaluation criterion for the *i*-th alternative for the next scenarios, whereas the matrix:

$$X^k = [X_{ij}^k]_{M,\mathrm{ls}}$$

includes the value distributions of the *k*-th evaluation criterion for all alternatives and scenarios (k = 1,…,ls).

1.8. The interactive method used for the comparison of alternatives*

Following the calculation of the values of evaluation criteria for each decision alternative, we carry out the multi-criteria analysis which aims to indicate the most favourable alternative of the strategy in the light of the assumed evaluation criteria and DM's preferences or, at least, to select the decision alternatives which are definitely the worst and should be rejected.

Now, we present the proposal of the interactive multi-criteria decision aiding with the use of the scenario-based method. A decision aiding procedure allows DM to evaluate trade-offs both between the evaluation criteria and between the outcomes that are certain and the outcomes that are possible in favourable conditions. Moreover, we assume that during the decision aiding process we will not expect DM to define his preferences a priori, but only to provide this information during the decision-making process, as a result of the analysis and assessment of the solution proposals. Let us assume that the matrix:

$$E(x^k) = [E(x_{ij}^k)]_{M,\mathrm{ls}}$$

* The idea of proposed method is based on the concept of Interactive Multiple Goal Programming presented in [19].

includes the expected values of the *k*-th evaluation criteria for the subsequent scenarios. Moreover, let us assume that the matrix:

$$x^k(p_k) = [x_{ij}^k]_{M,\text{ls}}$$

includes the values of the *k*-th evaluation criterion for the subsequent scenarios calculated for the assumed probability value p_k, and these values guarantee the probability that a particular variable will have a lower value of at least p_k, which may be defined as below:

$$P(X_{i,j}^k \geq x_{i,j}^k) = p_k$$

Furthermore, let us assume that for probability p_k:

$$\overline{x}_i^k = \min_{j=1,\dots,ls} x_{ij}^k$$

means the worst value of the k-th criterion for the i-th decision alternative whose probability is p_k. Let $\mathbf{x_{io}}$ denote the "**ideal optimistic**" solution, defined below:

$$x_{io} = [x_{io},_k : x_{io,k} = \max_{i=1,\dots,M} \max_{j=1,\dots,ls} x_{ij}^k ; k = 1,\dots,J]$$

Whereas $\mathbf{x_{ip}}$ is an "**ideal pessimistic**" solution:

$$x_{ip} = [x_{ip},_k : x_{ip,k} = \max_{i=1,\dots,M} \overline{x}_{ij}^k ; k = 1,\dots,J]$$

Vector $\mathbf{x_{arp}}$, defined below, is referred to as a "**current solution**":

$$x_{arp} = [x_{arp},_k : x_{arp,k} = \min_{i=1,\dots,M} \overline{x}_{ij}^k ; k = 1,\dots,J]$$

Potency matrix P^r is noted as follows:

$$P^r = \begin{bmatrix} x_{io} \\ x_{ip} \\ x_{arp} \end{bmatrix}$$

where index r = 1,2,3,… denotes the number of the algorithm iteration which generated matrix P.

Let us also assume that matrix P^0 is constructed in the way similar to the one discussed above, but with the use of the matrix of expected values of each criterion $E(x^k)$. The decision aiding procedure can be described in three main steps:

Step 1

DM is presented with potency matrix P^0 calculated on the basis of the expected values. Then, for each criterion *k*, DM defines the probability value at which he will analyse the values of a given evaluation criterion. The first potency matrix P^1 is calculated and presented to DM. DM chooses either to accept the values and move to Step 2 or to correct the adopted values of probabilities p_k.

Step 2

Following the analysis of the potency matrix, DM chooses the criterion for which the value of the current (pessimistic) solution should be improved. He specifies the accepted value of the pessimistic solution of criterion d_k, which fulfills the condition $x_{arp}^k < d^k \leq x_{ip}^k$ for the specified probability of realizing p_k.

DM can change the required values of probabilities p_k for particular evaluation criteria and is then presented with the accordingly improved potency matrix.

Step 3

The alternatives that do not fulfil the condition specified by DM in Step 2 are deleted from the set of the decision alternatives and a new potency matrix P^r is calculated. DM compares the values in potency matrix P^r and P^{r-1} and evaluates whether he accepts the consequences of his requirements.

a) If DM accepts the new solution, we go back to Step 2.
b) If DM rejects the new solution, we restore the deleted alternatives and then go back to Step 2.

Stop condition

The procedure stops when there is only one alternative left in the set of decision alternatives and DM accepts the solution.

2. The numerical example

Now we present the numerical example which illustrates the application of the proposed multi-criteria decision aiding procedure under uncertainty and risk to the selection of a company's investment alternative. The example has been developed based on the assumed data.

2.1. The formulation of the problem and decision alternatives

Let us assume that we consider the case of a consumer electronics manufacturer. The company specialises in a narrow segment of this market (characterised by high growth dynamics) and is a market challenger (it has the second largest market share). The main competitor, the market leader, builds its strategy on systematic innovation in the functionality of the products, which forces other market participants to introduce similar solutions.

The management board recognizes the need to modernize the production facilities in order to facilitate the application of the latest technological developments in the manufactured goods.

As a result of preliminary technical and technological analyses, several opportunities to modernize the production plant have been identified. Firstly, the modernisation is carried out in stages, over the span of four years, and financed with the company's own funds. The second alternative involves the purchase a new production line in addition to the existing one and its launch within the first year. The third alternative is the construction of an entirely new production line as a replacement of the existing one and its launch within the first year. Due to high investment costs, the last project may be financed through the increase in the company's equity capital or with a long-term loan.

In order to provide basis for comparison with the current situation, we also consider the possibility that the company chooses not to make the investment and continues to operate in an unchanged manner. The set of decision alternatives consists of five elements:

$$W = \{w_1, \ldots, w_5\}$$

briefly characterized below:

Alternative 1: w_1 – refraining from the investment.

Alternative 2: w_2 – the modernisation of the production plant is carried out in stages, over the span of four years, with the capital outlays in the four years amounting to PLN 2.3m, PLN 3.1m, PLN 3.1m, PLN 3.8m, respectively. The full capacity will be reached after the completion of the investment.

Alternative 3: w_3 – the modernization of the production plant involving the installation of a new production line in addition to the existing one within the first year. The estimated capital outlay amounts to approximately PLN 10.2m and will partly be financed with a long-term loan.

Alternative 4: w_4 – the purchase of a new production line as the replacement of the existing one within the first year. The total capital outlay is estimated at PLN 20.2m, with PLN 19m financed with a long-term loan and the rest coming from the company's own funds.
Alternative 5: w_5 – the purchase of a new production line as the replacement of the existing one as in the previous alternative. The capital outlay will be financed through the increase in equity capital (PLN 18m) and the rest will come from the company's own funds.

2.2. The determination of the evaluation criteria

Alternatives will be compared based on the analysis of the five-year period from 2007 to 2011. Let us assume that five evaluation criteria have been determined in order to compare the decision alternatives. Thus, the set of criteria may be noted as follows:

$$K = \{k_1,...,k_5\}$$

The characteristics of the adopted criteria are:
Criterion 1: k_1 – **SALES**: the level of sales in 2011. The criterion is used to assess the market position of the company at the end of the analysed period. The higher the value of this criterion is, the better.
Criterion 2: k_2 – **NPV**: the updated net present value of the project calculated at the discount rate of 12%. The outlays are covered by the value of own capital at the beginning of the analysed period, the cash-flow values in the next years based on the cash-flow forecast, and the residual value equal to the value of net assets at the end of the last year. The criterion is a commonly used measure of investment profitability. The higher the value of this criterion is, the better.
Criterion 3: k_3 – **ROE:** return on equity in 2011 calculated by dividing net income by equity capital. The criterion reflects the expected return on shareholder capital after the completion of the project. The higher the value of this criterion is, the better.
Criterion 4: k_4 – **MAX_DR:** maximum debt ratio in the whole analysed period calculated as the ratio of the sum of short- and long-term loans to total assets. The criterion is a measure of financial risk of the project. The lower the value of this criterion is, the better.
Criterion 5: k_5 – **MIN_IC:** minimum interest coverage ratio in the analysed period calculated as the ratio of operating income to financial costs (in this case, interest expense). The criterion also measures the financial risk of the investment (loss of liquidity in the analysed period). The higher the value of this criterion is, the better.

2.3. The identification of uncertainty factors

Let us assume that the analysis conducted allowed to identify two main uncertainty factors: market growth dynamics (in the given market segment) and the behaviour of the main competitor.

Thus, the number of uncertainty factors is C=2 and N^1 denotes the set of potential values of a factor relating to market growth dynamics, while N^2 denotes the set of values relating to the behaviour of the competition.

Let us assume that there are two alternatives relating to the situation on the market: stable high market absorption dynamics and a significant slowdown in these dynamics (due to market saturation). We can write:

$$N^1 = \{n_1^1, n_2^1\}$$

where:

n_1^1 denotes stable market growth dynamics,

n_2^1 denotes a slowdown in market growth dynamics.

Let us assume that while considering the potential impact of the competition, we also identified two possible situations. One situation is when the main competitor completes the development of and successfully launches a new product with improved functionality, which will lead to a decrease in sales of other manufacturers. The other situation assumes that the project of the main competitor is not completed successfully, which will not adversely affect the position of other manufacturers. Thus, we can write:

$$N^2 = \{n_1^2, n_2^2\}$$

where:

n_1^2 denotes lack of the negative impact from the main competitor,

n_2^2 denotes the negative impact due to a new product launched by a competitor.

2.4. The planning of the scenarios of the environment development

The next stage involves the planning of four scenarios of the environment development based on the characteristic factors defined and the sets of values adopted for these factors. The scenarios are presented below in Table 1.

Table 1

List of scenarios

Factor / Scenario	N_1	N_2
S_1	n_1^1	n_1^2
S_2	n_2^1	n_1^2
S_3	n_1^1	n_2^2
S_4	n_2^1	n_2^2

We can see that Scenario 1 assumes stable high market growth dynamics and the lack of a negative impact from the main competitor. Scenario 2 also includes the lack of a negative impact from the competition, but it also predicts a less optimistic market growth. Scenario 3 assumes that high market growth dynamics are accompanied by a negative impact from the competition. Scenario 4 is definitely the least favourable: it assumes both a slowdown in market growth dynamics and a negative impact from the main competitor.

2.5. The identification of risk factors

Let us assume that during further analysis of the decision alternatives the following risk factors have been identified:
- projected market absorption,
- projected market share,
- investment costs,
- main operating costs.

Export opinions are used to assess the probability distributions for particular factors.

2.5.1. Market absorption

The next two tables present the projected market sizes in consecutive years. The projections are delivered by independent experts.

Table 2

Projected market absorption for scenarios S1 and S3

Market Forecast	2007	2008	2009	2010	2011
Expert 1	1 620 000 000	1 765 800 000	1 907 064 000	2 021 487 840	2 102 347 354
Expert 2	1 580 000 000	1 738 000 000	1 877 040 000	1 989 662 400	2 089 145 520
Expert 3	1 520 000 000	1 702 400 000	1 821 568 000	1 912 646 400	1 950 899 328
Expert 4	1 550 000 000	1 689 500 000	1 790 870 000	1 880 413 500	1 918 021 770
Expert 5	1 580 000 000	1 722 200 000	1 842 754 000	1 934 891 700	1 973 589 534
Expert 6	1 650 000 000	1 831 500 000	1 978 020 000	2 096 701 200	2 201 536 260
Expert 7	1 490 000 000	1 594 300 000	1 674 015 000	1 707 495 300	1 741 645 206
Expert 8	1 490 000 000	1 609 200 000	1 721 844 000	1 825 154 640	1 916 412 372
Expert 9	1 570 000 000	1 727 000 000	1 865 160 000	1 977 069 600	2 075 923 080
Expert 10	1 585 000 000	1 727 650 000	1 848 585 500	1 941 014 775	2 038 065 514
Average	**1 563 500 000**	**1 710 755 000**	**1 832 692 050**	**1 928 653 736**	**2 000 758 594**
Standard deviation	**52 283 947**	**69 465 173**	**87 790 583**	**108 178 070**	**129 017 321**

Table 3

Projected market absorption for scenarios S2 and S4

Market Forecast	2007	2008	2009	2010	2011
Expert 1	1 450 000 000	1 464 500 000	1 493 790 000	1 508 727 900	1 523 815 179
Expert 2	1 408 500 000	1 408 500 000	1 422 585 000	1 436 810 850	1 451 178 959
Expert 3	1 415 500 000	1 429 655 000	1 443 951 550	1 443 951 550	1 443 951 550
Expert 4	1 404 300 000	1 432 386 000	1 489 681 440	1 534 371 883	1 565 059 321
Expert 5	1 530 000 000	1 560 600 000	1 607 418 000	1 655 640 540	1 688 753 351
Expert 6	1 488 000 000	1 562 400 000	1 609 272 000	1 657 550 160	1 674 125 662
Expert 7	1 310 000 000	1 336 200 000	1 349 562 000	1 363 057 620	1 376 688 196
Expert 8	1 380 000 000	1 421 400 000	1 464 042 000	1 493 322 840	1 493 322 840
Expert 9	1 350 000 000	1 404 000 000	1 446 120 000	1 489 503 600	1 504 398 636
Expert 10	1 380 000 000	1 435 200 000	1 463 904 000	1 493 182 080	1 508 113 901
Average	**1 411 630 000**	**1 445 484 100**	**1 479 032 599**	**1 507 611 902**	**1 522 940 759**
Standard deviation	**64 569 016**	**69 460 059**	**79 227 360**	**91 780 522**	**97 948 489**

We assume that, in further analysis, the projected market size will be described by normal distributions, respectively for each scenario, and the parameters will be given in the last two rows of Tables 2 and 3.

2.5.2. Market share

Expert opinions on the projected market share for Alternative 1 are presented in the next two tables.

Table 4

Expert opinion on projected market share for Alternative 1, Scenarios 1 and 3

Market share	2007	2008	2009	2010	2011
Expert 1	30,00%	30,00%	29,00%	27,00%	27,00%
Expert 2	31,00%	30,00%	28,00%	26,00%	26,00%
Expert 3	29,00%	28,00%	26,00%	25,00%	25,00%
Expert 4	32,00%	32,00%	30,00%	28,00%	28,00%
Expert 5	30,00%	30,00%	30,00%	30,00%	28,00%
Expert 6	31,00%	31,00%	30,00%	29,00%	29,00%
Expert 7	30,00%	30,00%	29,00%	28,00%	28,00%
Expert 8	28,00%	27,00%	27,00%	25,00%	23,00%
Expert 9	32,00%	32,00%	32,00%	29,00%	29,00%
Expert 10	31,00%	31,00%	30,00%	28,00%	28,00%
Minimum	**28,00%**	**27,00%**	**26,00%**	**25,00%**	**23,00%**
Mode	**30,00%**	**30,00%**	**30,00%**	**28,00%**	**28,00%**
Maximum	**32,00%**	**32,00%**	**32,00%**	**30,00%**	**29,00%**

Table 5

Expert opinion on projected market share for Alternative 1, Scenarios 2 and 4

Market share	2007	2008	2009	2010	2011
Expert 1	30,00%	27,00%	26,00%	22,00%	22,00%
Expert 2	31,00%	27,00%	25,00%	21,00%	21,00%
Expert 3	29,00%	25,00%	23,00%	20,00%	20,00%
Expert 4	32,00%	29,00%	27,00%	22,00%	22,00%
Expert 5	30,00%	27,00%	27,00%	23,00%	23,00%
Expert 6	31,00%	28,00%	27,00%	24,00%	24,00%
Expert 7	30,00%	27,00%	24,00%	24,00%	23,00%
Expert 8	28,00%	24,00%	22,00%	20,00%	18,00%
Expert 9	32,00%	29,00%	27,00%	23,00%	24,00%
Expert 10	31,00%	28,00%	27,00%	23,00%	23,00%
Minimum	**28,00%**	**24,00%**	**22,00%**	**20,00%**	**18,00%**
Mode	**30,00%**	**27,00%**	**27,00%**	**23,00%**	**23,00%**
Maksimum	**32,00%**	**29,00%**	**27,00%**	**24,00%**	**24,00%**

We assumed that triangle distributions would be used to describe changes in market share in the future. The parameters of the distributions are presented in the last three rows of the tables containing the expert opinions.

2.5.3. Investment costs

Probability distributions for investment costs in the scenarios are presented below:

Table 6

Probability distributions for investment costs, Alternative 2

Investment cost	2007	2008	2009	2010
1 800 000 - 2 000 000				
2 000 001 - 2 200 000	0,05			
2 200 001 - 2 400 000	0,80			
2 400 001 - 2 600 000	0,15			
2 600 001 - 2 800 000		0,05		
2 800 001 - 3 000 000		0,10	0,15	
3 000 001 - 3 200 000		0,15	0,50	
3 200 001 - 3 400 000		0,55	0,20	
3 400 001 - 3 600 000		0,15	0,10	
3 600 001 - 3 800 000			0,05	0,10
3 800 001 - 4 000 000				0,50
4 000 001 - 4 200 000				0,35
4 200 001 - 4 400 000				0,05
	1,00	**1,00**	**1,00**	**1,00**

Table 7

Probability distributions for investment costs, Alternative 3

Investment cost	2007	2008	2009	2010
9 600 000 - 9 800 000				
9 800 001 - 10 000 000	0,05			
10 000 001 - 10 200 000	0,70			
10 200 001 - 10 400 000	0,20			
10 400 001 - 10 600 000	0,05			
SUMA	**1,00**	**0,00**	**0,00**	**0,00**

Table 8

Probability distributions for investment costs, Alternatives 4 and 5

Investment cost	2007	2008	2009	2010
19 600 000 - 19 800 000				
19 800 001 - 19 000 000	0,10			
20 000 001 - 20 200 000	0,55			
20 200 001 - 20 400 000	0,25			
20 400 001 - 20 600 000	0,10			
SUMA	**1,00**	**0,00**	**0,00**	**0,00**

2.5.4. Selected operating costs

The next tables present the parameters of the triangle distributions for main items of operating costs for Alternative 1. They show the estimated contributions of particular items in relation to total sales. We assume they were estimated based on the expert opinions similarly to the way presented in the previous section.

Table 9

Parameters of triangle distributions – Alternative 1

Materials and energy	2007	2008	2009	2010	2011
Min	64,50%	64,50%	64,50%	64,50%	64,50%
Mode	65,00%	65,00%	65,00%	65,00%	65,00%
Max	65,50%	65,50%	65,50%	65,50%	65,50%

Table 10

Parameters of triangle distributions – Alternative 1

External services	2007	2008	2009	2010	2011
Min	9,00%	9,00%	9,00%	9,00%	9,00%
Mode	10,00%	10,00%	10,00%	10,00%	10,00%
Max	11,00%	11,00%	11,00%	11,00%	11,00%

Table 11

Parameters of triangle distributions – Alternative 1

Payroll	2007	2008	2009	2010	2011
Min	13,50%	13,50%	13,50%	13,50%	13,50%
Mode	15,00%	15,00%	15,00%	15,00%	15,00%
Max	17,00%	17,00%	17,00%	17,00%	17,00%

Table 12

Parameters of triangle distributions – Alternative 1

Other expenditures	2007	2008	2009	2010	2011
Min	2,50%	2,50%	2,50%	2,50%	2,50%
Mode	3,00%	3,00%	3,00%	3,00%	3,00%
Max	5,00%	5,00%	5,00%	5,00%	5,00%

2.6. The development of strategic financial plans

The next stage involves creating financial forecasts for the years 2007--2011, based on the assumptions discussed in 2.1 for each situation (e.g. for each pair: a decision alternative – a scenario). Table 12 presents the sales forecasts for Alternative 1 – Scenario 1. The numerical values, included in the tables, were calculated using the expected distribution values of risk factors.

Table 13

Market forecast

MARKET FORECAST	2006	2007	2008	2009	2010	2011
Total market (PLN)	1 563 500 000,00	1 563 500 000,00	1 710 755 000,00	1 832 692 050,00	1 928 653 735,50	2 000 758 593,74
Market share	30,00%	30,00%	29,67%	29,33%	27,67%	26,67%
Sales revenues	**469 050 000,00**	**469 050 000,00**	**507 523 983,33**	**537 589 668,00**	**533 594 200,16**	**533 535 625,00**

The next two tables present the profit and loss account forecast, the balance sheet forecast and the cash flow forecast for a particular situation.

Table 14

Profit and loss account forecast for Alternative 1 – Scenario 1

Profit and Loss Account	2006	2007	2008	2009	2010	2011
Net sale revenues and net sale revenues equivalents including	**469 050 000,0**	**469 050 000,0**	**507 523 983,3**	**537 589 668,0**	**533 594 200,2**	**533 535 625,0**
Net revenues from the sale of finished products	469 050 000,0	469 050 000,0	507 523 983,3	537 589 668,0	533 594 200,2	533 535 625,0
Net revenues from the sale of merchandise and raw materials		0,0	0,0	0,0	0,0	0,0
Operating expenses	**455 711 450,0**	**459 041 705,0**	**496 596 160,1**	**525 943 274,9**	**522 043 298,8**	**521 986 123,6**
Depreciation	1 200 000,0	1 200 000,0	1 200 000,0	1 200 000,0	1 200 000,0	1 200 000,0
Materials and energy	304 882 500,0	304 882 500,0	329 890 589,2	349 433 284,2	346 836 230,1	346 798 156,2
External services	46 905 000,0	46 905 000,0	50 752 398,3	53 758 966,8	53 359 420,0	53 353 562,5
Taxes and charges	2 000,0	2 000,0	2 000,0	2 000,0	2 000,0	2 000,0
Payroll	70 357 500,0	71 139 250,0	76 974 470,8	81 534 433,0	80 928 453,7	80 919 569,8
Social security and other benefits	18 292 950,0	18 496 205,0	20 013 362,4	21 198 952,6	21 041 398,0	21 039 088,1
Other expenditures by kind	14 071 500,0	16 416 750,0	17 763 339,4	18 815 638,4	18 675 797,0	18 673 746,9
Cost of merchandise and raw materials sold	0,0	0,0	0,0	0,0	0,0	0,0
Gross profit/(loss)	**13 338 550,0**	**10 008 295,0**	**10 927 823,2**	**11 646 393,1**	**11 550 901,4**	**11 549 501,4**
Other operating revenues	0,0	0,0	0,0	0,0	0,0	0,0
Other operating expenses	0,0	0,0	0,0	0,0	0,0	0,0
Operating profit/(loss)	**13 338 550,0**	**10 008 295,0**	**10 927 823,2**	**11 646 393,1**	**11 550 901,4**	**11 549 501,4**
Financial revenue	0,0	0,0	0,0	0,0	0,0	0,0
Financial expenses	4 100 000,0	3 800 000,0	3 500 000,0	3 500 000,0	3 500 000,0	3 500 000,0
Gross profit/(loss) on business activities	**9 238 550,0**	**6 208 295,0**	**7 427 823,2**	**8 146 393,1**	**8 050 901,4**	**8 049 501,4**
Extraordinary profits	0,0	0,0	0,0	0,0	0,0	0,0
Extraordinary loses	0,0	0,0	0,0	0,0	0,0	0,0
Profit/(loss) before taxation	**9 238 550,0**	**6 208 295,0**	**7 427 823,2**	**8 146 393,1**	**8 050 901,4**	**8 049 501,4**
Corporate income tax	1 755 324,5	1 179 576,1	1 411 286,4	1 547 814,7	1 529 671,3	1 529 405,3
Net profit/(loss)	**7 483 225,5**	**5 028 719,0**	**6 016 536,8**	**6 598 578,4**	**6 521 230,1**	**6 520 096,2**

Table 15

Balance sheet forecast for Alternative 1 – Scenario 1

	BALANCE SHEET-ASSETS	31.12.2006	31.12.2007	31.12.2008	31.12.2009	31.12.2010	31.12.2011
A.	**Non-current assets**	**23 456 000,0**	**22 256 000,0**	**21 056 000,0**	**19 856 000,0**	**18 656 000,0**	**17 456 000,0**
I.	Intangible assets and legal values	0,0	0,0	0,0	0,0	0,0	0,0
II.	Fixed tangibles	23 456 000,0	22 256 000,0	21 056 000,0	19 856 000,0	18 656 000,0	17 456 000,0
III.	Long-term debtors	0,0	0,0	0,0	0,0	0,0	0,0
IV.	Long-term investments	0,0	0,0	0,0	0,0	0,0	0,0
V.	Long-term prepayments	0,0	0,0	0,0	0,0	0,0	0,0
B.	**Current assets**	**111 073 058,8**	**111 301 777,8**	**123 861 923,4**	**135 836 291,3**	**143 002 595,3**	**150 714 556,0**
I.	Inventory	45 602 083,3	45 602 083,3	49 342 609,5	52 265 662,2	51 877 213,9	51 871 519,1
II.	Short-term debtors	65 145 833,3	65 145 833,3	70 489 442,1	74 665 231,7	74 110 305,6	74 102 170,1
III.	Short-term investments	325 142,2	553 861,1	4 029 871,8	8 905 397,5	17 015 075,8	24 740 866,8
IV.	Short-term prepayments	0,0	0,0	0,0	0,0	0,0	0,0
	Total assets	**134 529 058,8**	**133 557 777,8**	**144 917 923,4**	**155 692 291,3**	**161 658 595,3**	**168 170 556,0**

	BALANCE SHEET-EQUITY AND LIABILITIES	31.12.2006	31.12.2007	31.12.2008	31.12.2009	31.12.2010	31.12.2011
A.	**Equity**	**28 383 225,5**	**33 411 944,5**	**39 428 481,2**	**46 027 059,6**	**52 548 289,7**	**59 068 385,9**
I.	Share capital	15 000 000,0	15 000 000,0	15 000 000,0	15 000 000,0	15 000 000,0	15 000 000,0
II.	Outstanding share capital contributions	0,0	0,0	0,0	0,0	0,0	0,0
III.	Shares not distributed	0,0	0,0	0,0	0,0	0,0	0,0
IV.	Reserve capital	5 900 000,0	13 383 225,5	18 411 944,5	24 428 481,2	31 027 059,6	37 548 289,7
V.	Revaluation reserve	0,0	0,0	0,0	0,0	0,0	0,0
VI.	Other reserve capital	0,0	0,0	0,0	0,0	0,0	0,0
VII.	Profit (loss) brought forward	0,0	0,0	0,0	0,0	0,0	0,0
VIII.	Net profit (loss)	7 483 225,5	5 028 719,0	6 016 536,8	6 598 578,4	6 521 230,1	6 520 096,2
IX.	Net profit (loss) write-offs	0,0	0,0	0,0	0,0	0,0	0,0
B.	**Creditors and provisions**	**106 145 833,3**	**100 145 833,3**	**105 489 442,1**	**109 665 231,7**	**109 110 305,6**	**109 102 170,1**
I.	Provisions	0,0	0,0	0,0	0,0	0,0	0,0
II.	Long-term creditors	0,0	0,0	0,0	0,0	0,0	0,0
III.	Short-term creditors	106 145 833,3	100 145 833,3	105 489 442,1	109 665 231,7	109 110 305,6	109 102 170,1
IV.	Accruals and deferred income	0,0	0,0	0,0	0,0	0,0	0,0
	Total Equity and Liabilities	**134 529 058,8**	**133 557 777,8**	**144 917 923,4**	**155 692 291,3**	**161 658 595,3**	**168 170 556,0**

The forecasts of the financial statements are the basis for the calculation of the values of evaluation criteria for each situation.

The values of the evaluation criteria for each situation, calculated based on the financial forecasts (taking into account the expected values of risk factors), are presented below.

Table 16

Expected criteria values (prior to the simulation)

	SALES	NPV	ROE	MAX DR	MIN IC
A1S1	533 494 018,7	19 633 186,2	43,43%	26,28%	1,12
A1S2	433 436 732,6	19 128 726,6	30,55%	26,66%	0,97
A1S3	433 415 591,0	19 119 371,2	30,70%	26,66%	0,97
A1S4	329 963 072,0	18 032 295,5	17,20%	29,34%	0,70
A2S1	586 782 731,2	38 145 671,6	131,34%	26,32%	1,48
A2S2	513 517 013,2	33 520 180,8	110,84%	27,96%	1,29
A2S3	446 709 796,3	30 624 731,8	92,41%	29,51%	1,16
A2S4	390 877 844,4	26 874 278,8	76,86%	29,79%	1,04
A3S1	600 226 508,4	93 922 519,1	168,49%	31,76%	2,26
A3S2	533 505 008,6	83 463 420,5	146,17%	31,76%	2,25
A3S3	456 802 806,1	75 883 142,8	120,12%	33,45%	2,00
A3S4	406 093 192,0	67 573 228,9	103,49%	33,44%	2,00
A4S1	647 080 104,3	79 024 211,5	179,26%	37,81%	1,50
A4S2	586 908 045,1	74 015 995,2	159,56%	37,81%	1,50
A4S3	492 461 449,2	61 787 671,7	128,70%	39,63%	1,32
A4S4	446 718 674,9	57 892 385,2	113,68%	39,63%	1,32
A5S1	646 964 175,9	89 465 354,1	82,02%	25,46%	1,86
A5S2	586 906 598,6	84 539 122,9	73,05%	25,46%	1,86
A5S3	492 451 952,9	72 311 774,9	58,97%	26,69%	1,65
A5S4	446 744 810,2	68 419 802,2	52,17%	26,69%	1,64

2.7. The conduct of the Monte Carlo simulation

The next stage involves conducting 20 Monte Carlo simulations (one for each situation). The simulations are conducted based on the financial statement forecasts, created at the previous stage, and the probability distributions for risk factors, determined in Section 2.5.

The Monte Carlo simulations are carried out with the use of financial forecast models, created in the Excel spreadsheet, and the test version of the @Risk package, available on the www.palisade.com website.

Each simulation involves performing 1,000 iterations. As a result, we generate the value distributions for the evaluation criteria for each situation. Table 17 presents the value distributions for the evaluation criteria for Alternative 1 – Scenario 1.

Table 17

Distributions for the evaluation criteria for Alternative 1 – Scenario 1

A1-S1	SALES	NPV	ROE	MAX DR	MIN IC
Minimum	393 744 352,0	-15 590 017,0	-59,14%	23,41%	-2,5
Maximum	691 684 864,0	57 601 560,0	134,97%	29,61%	4,1
Mean	533 494 018,7	19 633 186,2	43,43%	26,28%	1,1
Standard Deviation	42 587 777,0	12 843 824,8	33,28%	1,05%	1,1
Variance	1,81372E+15	1,64964E+14	0,110723463	0,000110836	1,289306274
Skewness	0,021893336	0,029807905	-0,053627716	0,329533618	-0,016696687
Kurtosis	3,082068542	2,813936299	2,699544732	2,991281803	2,818343389
Number of Errors	0	0	0	0	0
Mode	499 693 152,0	26 488 429,2	52,10%	25,47%	0,8
5,0%	464 402 624,0	-2 345 538,3	-11,33%	24,69%	-0,7
10,0%	479 295 168,0	3 025 462,3	-0,14%	24,96%	-0,3
15,0%	489 719 232,0	6 063 398,0	7,32%	25,20%	-0,1
20,0%	496 788 512,0	8 838 973,0	14,91%	25,38%	0,1
25,0%	502 238 560,0	11 071 894,0	20,67%	25,50%	0,3
30,0%	509 138 976,0	12 636 161,0	26,17%	25,67%	0,5
35,0%	517 696 512,0	14 558 067,0	31,38%	25,80%	0,7
40,0%	523 695 360,0	16 038 949,0	35,43%	25,93%	0,8
45,0%	528 360 672,0	17 979 706,0	39,16%	26,06%	1,0
50,0%	534 068 896,0	19 655 728,0	43,69%	26,22%	1,1
55,0%	539 301 248,0	21 107 544,0	46,88%	26,38%	1,2
60,0%	544 892 736,0	22 852 978,0	51,55%	26,51%	1,4
65,0%	550 030 144,0	24 650 686,0	56,19%	26,66%	1,5
70,0%	556 483 264,0	26 549 442,0	61,12%	26,79%	1,7
75,0%	561 892 096,0	28 362 112,0	67,30%	26,95%	1,9
80,0%	569 238 912,0	30 585 984,0	73,11%	27,12%	2,1
85,0%	576 150 400,0	32 901 004,0	79,79%	27,41%	2,3
90,0%	587 610 240,0	36 355 424,0	87,41%	27,68%	2,6
95,0%	603 567 872,0	40 801 132,0	97,05%	28,08%	3,1

2.8. The selection of the alternative with the use of the interactive decision aiding method

According to the decision aiding procedure, the first step involves constructing the matrices which include the expected values of evaluation criteria for each situation. The matrices are presented in Tables 18-22. The last two columns of the tables show the maximum and minimum values for each alternative. These values are used to construct potency matrix P0.

Table 18

Expected values – Criterion 1

SALES	S1	S2	S3	S4	MAX	MIN
A1	533 494 018,7	433 436 732,6	433 415 591,0	329 963 072,0	**533 494 018,7**	**329 963 072,0**
A2	586 782 731,2	513 517 013,2	446 709 796,3	390 877 844,4	**586 782 731,2**	**390 877 844,4**
A3	600 226 508,4	533 505 008,6	456 802 806,1	406 093 192,0	**600 226 508,4**	**406 093 192,0**
A4	647 080 104,3	586 908 045,1	492 461 449,2	446 718 674,9	**647 080 104,3**	**446 718 674,9**
A5	646 964 175,9	586 906 598,6	492 451 952,9	446 744 810,2	**646 964 175,9**	**446 744 810,2**

Table 19

Expected values – Criterion 2

NPV	S1	S2	S3	S4	MAX	MIN
A1	19 633 186,2	19 128 726,6	19 119 371,2	18 032 295,5	**19 633 186,2**	**18 032 295,5**
A2	38 145 671,6	33 520 180,8	30 624 731,8	26 874 278,8	**38 145 671,6**	**26 874 278,8**
A3	93 922 519,1	83 463 420,5	75 883 142,8	67 573 228,9	**93 922 519,1**	**67 573 228,9**
A4	79 024 211,5	74 015 995,2	61 787 671,7	57 892 385,2	**79 024 211,5**	**57 892 385,2**
A5	89 465 354,1	84 539 122,9	72 311 774,9	68 419 802,2	**89 465 354,1**	**68 419 802,2**

Table 20

Expected values – Criterion 3

ROE	S1	S2	S3	S4	MAX	MIN
A1	43,43%	30,55%	30,70%	17,20%	**43,43%**	**17,20%**
A2	131,34%	110,84%	92,41%	76,86%	**131,34%**	**76,86%**
A3	168,49%	146,17%	120,12%	103,49%	**168,49%**	**103,49%**
A4	179,26%	159,56%	128,70%	113,68%	**179,26%**	**113,68%**
A5	82,02%	73,05%	58,97%	52,17%	**82,02%**	**52,17%**

Table 21

Expected values – Criterion 4

MAX DR	S1	S2	S3	S4	MAX	MIN
A1	26,28%	26,66%	26,66%	29,34%	**29,34%**	**26,28%**
A2	26,32%	27,96%	29,51%	29,79%	**29,79%**	**26,32%**
A3	31,76%	31,76%	33,45%	33,44%	**33,45%**	**31,76%**
A4	37,81%	37,81%	39,63%	39,63%	**39,63%**	**37,81%**
A5	25,46%	25,46%	26,69%	26,69%	**26,69%**	**25,46%**

Table 22

Expected values – Criterion 5

MIN IC	S1	S2	S3	S4	MAX	MIN
A1	1,12	0,97	0,97	0,70	**1,1**	**0,7**
A2	1,48	1,29	1,16	1,04	**1,5**	**1,0**
A3	2,26	2,25	2,00	2,00	**2,3**	**2,0**
A4	1,50	1,50	1,32	1,32	**1,5**	**1,3**
A5	1,86	1,86	1,65	1,64	**1,9**	**1,6**

Based on the values presented above, matrix P^0 is constructed and presented to DM.

Table 23

Potency matrix P^0

	SALES	NPV	ROE	MAX_DR	MIN IC
IDEAL OPTIMISTIC	647 080 104,3	93 922 519,1	179,26%	25,46%	2,3
IDEAL PESSIMISTIC	446 744 810,2	68 419 802,2	113,68%	26,69%	2,0
CURENT SOLUTION	329 963 072,0	18 032 295,5	17,20%	39,63%	0,7

After analyzing the values in potency matrix P0 and taking into account his attitude to risk, DM makes a decision about what level of probability of achieving the values of particular criteria will allow further analysis and aiding of the decision-making process.

Let us assume that DM accepted the following values (respectively): 0.80, 0.95, 0.90, 0.95, 0.80. Based on the accepted probability levels and percentile distributions, we construct matrices which contain such criteria values for each situation that the probability of their occurrence is not lower than the value defined by DM.

For example, while analysing the data in Table 24, we can see that the selection of Alternative 1 and the occurrence of Scenario 1 with the probability of 0.80 results in sales not lower than PLN 496,788,512. The values corresponding to all the criteria are presented in the five tables below.

Table 24

Criterion 1 for probability = 0.80

SALES	S1	S2	S3	S4	MAX	MIN
W1	496 788 512,0	402 001 632,0	400 295 936,0	305 389 920,0	**496 788 512,0**	**305 389 920,0**
W2	548 861 504,0	476 524 384,0	415 883 648,0	361 734 432,0	**548 861 504,0**	**361 734 432,0**
W3	565 915 456,0	499 633 376,0	429 154 624,0	382 371 872,0	**565 915 456,0**	**382 371 872,0**
W4	600 737 216,0	549 728 192,0	460 752 864,0	419 316 448,0	**600 737 216,0**	**419 316 448,0**
W5	604 107 776,0	549 510 848,0	460 077 440,0	418 510 656,0	**604 107 776,0**	**418 510 656,0**

Table 25

Criterion 2 for probability = 0.95

NPV	S1	S2	S3	S4	MAX	MIN
W1	-2 345 538,3	457 942,1	1 515 949,5	3 478 451,0	**3 478 451,0**	**-2 345 538,3**
W2	16 524 271,0	14 949 031,0	13 699 345,0	11 910 183,0	**16 524 271,0**	**11 910 183,0**
W3	72 767 864,0	62 551 192,0	56 990 408,0	50 871 920,0	**72 767 864,0**	**50 871 920,0**
W4	58 890 912,0	54 390 064,0	44 201 056,0	42 151 648,0	**58 890 912,0**	**42 151 648,0**
W5	68 109 808,0	65 586 956,0	55 191 764,0	53 117 816,0	**68 109 808,0**	**53 117 816,0**

Table 26

Criterion 3 for probability = 0.90

ROE	S1	S2	S3	S4	MAX	MIN
W1	-0,14%	-3,52%	-4,75%	-9,57%	**-0,14%**	**-9,57%**
W2	84,07%	70,97%	55,46%	46,49%	**84,07%**	**46,49%**
W3	119,93%	102,14%	82,98%	70,11%	**119,93%**	**70,11%**
W4	129,90%	113,82%	91,14%	79,19%	**129,90%**	**79,19%**
W5	58,12%	53,30%	42,03%	37,34%	**58,12%**	**37,34%**

Table 27

Criterion 4 for probability = 0.95

MAX_DR	S1	S2	S3	S4	MAX	MIN
W1	28,08%	28,67%	28,77%	32,33%	**32,33%**	**28,08%**
W2	28,10%	29,91%	31,39%	31,89%	**31,89%**	**28,10%**
W3	33,87%	33,92%	35,57%	35,49%	**35,57%**	**33,87%**
W4	39,84%	39,99%	41,93%	42,02%	**42,02%**	**39,84%**
W5	26,88%	26,85%	28,31%	28,11%	**28,31%**	**26,85%**

Table 28

Criterion 5 for probability = 0.80

MIN IC	S1	S2	S3	S4	MAX	MIN
W1	0,15	0,15	0,17	-0,01	**0,2**	**0,0**
W2	0,41	0,38	0,30	0,28	**0,4**	**0,3**
W3	1,18	1,15	0,96	1,04	**1,2**	**1,0**
W4	0,46	0,58	0,54	0,53	**0,6**	**0,5**
W5	0,76	0,70	0,63	0,67	**0,8**	**0,6**

Based on the values presented in Tables 24-28, the first potency matrix P1 is generated and presented to DM.

Table 29

Potency matrix P^1

ITERATION 1	SALES	NPV	ROE	MAX_DR	MIN IC
Probability	**0,80**	**0,95**	**0,90**	**0,95**	**0,80**
IDEAL OPTIMISTIC	604 107 776,0	72 767 864,0	129,90%	26,85%	1,2
IDEAL PESSIMISTIC	419 316 448,0	53 117 816,0	79,19%	28,31%	1,0
CURRENT SOLUTION	305 389 920,0	-2 345 538,3	-9,57%	42,02%	0,0

While analyzing the values from the first potency matrix, we can say that choosing the best alternative in terms of the value of Criterion 1: SALES with the probability of 0.8, sales will be not lower than PLN 419m irrespective of the scenario which will develop. In the case of the most favourable scenario, there is an 80% chance that sales will not be lower than PLN 604m (if the most favourable solution is selected). Finally, choosing any decision alternative we know that, irrespective of the scenario which will develop, there is an 80% chance that sales will not be lower than PLN 305m.

DM uses the similar reasoning for each criterion (analyzed independently of the others).

Let us assume that after the analysis of the values in matrix P1, DM decided that further solutions should exclude the alternatives which gave less than a 95% chance that NPV was positive.

Fulfilling this condition (according to the procedure presented in Part 1 of this paper) means that further analysis does not include Alternative 1 and the next potency matrix P^2 is generated (Table 30).

Table 30

Potency matrix P^2

ITERATION 2	SALES	NPV	ROE	MAX_DR	MIN IC
Probability	**0,80**	**0,95**	**0,90**	**0,95**	**0,80**
IDEAL OPTIMISTIC	604 107 776,0	72 767 864,0	129,90%	26,85%	1,2
IDEAL PESSIMISTIC	419 316 448,0	53 117 816,0	79,19%	28,31%	1,0
CURRENT SOLUTION	361 734 432,0	11 910 183,0	37,34%	42,02%	0,3

When we compare the values from matrices P1 and P2 , we can see that the introduction of DM's condition has not led to the worsening of the ideal optimistic values or the ideal pessimistic values of the remaining criteria. Moreover, it has improved the current solutions for the remaining criteria. Let us assume then that DM accepts this solution.

Let us also assume that DM analyzed the values in potency matrix P2 and decided that the solution should yield a 95% chance that the maximum debt ratio was not higher than 35%.

After this condition is satisfied, Alternative 4 is deleted from the set of decision alternatives and potency matrix P3 is calculated.

Table 31

Potency matrix P^3

ITERATION 3	SALES	NPV	ROE	MAX_DR	MIN IC
Probability	**0,80**	**0,95**	**0,90**	**0,95**	**0,80**
IDEAL OPTIMISTIC	604 107 776,0	72 767 864,0	119,93%	26,85%	1,2
IDEAL PESSIMISTIC	418 510 656,0	53 117 816,0	70,11%	28,31%	1,0
CURRENT SOLUTION	361 734 432,0	11 910 183,0	37,34%	35,57%	0,3

DM compares the values from tables P2 and P3 and decides that the introduction of the last criterion has decreased the ideal optimistic value of the ROE index to 119.93%; he accepts this change. The ideal optimistic values of the remaining criteria have not fallen.

While analyzing the ideal pessimistic values, DM notices the decrease in the values of the criteria of Sales, ROE and MAX_DR, but the changes are still relatively insignificant. Let us assume that DM accepts them.

In the following two iterations DM wants to increase the expected value of ROE to 40% (with the probability of 0.90). As a result, we reject Alternative 5. Then, in the fifth iteration, DM chooses to reject these alternatives which do not guarantee 80% chance of at least 90% coverage of interest expense with operating income (MIN_IC>=0.9). We delete Alternative 2.

Table 32

Potency matrix P^4

ITERATION 4	SALES	NPV	ROE	MAX_DR	MIN IC
Probability	**0,80**	**0,95**	**0,90**	**0,95**	**0,80**
IDEAL OPTIMISTIC	565 915 456,0	72 767 864,0	119,93%	28,10%	1,2
IDEAL PESSIMISTIC	382 371 872,0	50 871 920,0	70,11%	31,89%	1,0
CURRENT SOLUTION	361 734 432,0	11 910 183,0	46,49%	35,57%	0,3

DM accepts the consequences of the requirements introduced. There is only Alternative 3 left in the set of decision alternatives and it is the indication of a final decision. Analyzing the values from matrix P^5 we can see that the selection of Alternative 3 gives an 80% chance of sales not lower than PLN 382m in the case of the unfavourable development of the environment and PLN 565m if the scenario is favourable.

Table 33

Potency matrix P^5

ITERATION 5	SALES	NPV	ROE	MAX_DR	MIN IC
Probability	**0,80**	**0,95**	**0,90**	**0,95**	**0,80**
IDEAL OPTIMISTIC	565 915 456,0	72 767 864,0	119,93%	33,87%	1,2
IDEAL PESSIMISTIC	382 371 872,0	50 871 920,0	70,11%	35,57%	1,0
CURRENT SOLUTION	382 371 872,0	50 871 920,0	70,11%	35,57%	1,0

This alternative also gives a chance to obtain NPV not lower than PLN 50m with probability of 0.95 in the case of unfavourable external conditions and as much as PLN 73m otherwise.

Irrespective of the external conditions, there is a 90% chance that ROE will not be lower than 70.11%, and in the most favourable situation it will not be less than 119%. Moreover, the values of the two other criteria are also satisfying for DM.

Finally, DM is presented the potency matrix which includes the expected values for this solution.

Table 34

Expected values for the solution

ITERATION 5	SALES	NPV	ROE	MAX_DR	MIN IC
IDEAL OPTIMISTIC	600 226 508,4	93 922 519,1	168,49%	31,76%	2,3
CURRENT SOLUTION	406 093 192,0	67 573 228,9	103,49%	33,45%	2,0

Let us assume that DM accepts the outcomes, so the decision aiding procedure stops. In the light of the analysis of the scenarios, the Monte Carlo simulation and DM's preferences, Alternative 3 should be suggested as the final solution.

Conclusion

The paper discusses the proposal of the multi-criteria decision aiding procedure under uncertainty and risk. The proposal uses the scenario method and the Monte Carlo simulation. The scenario-based method takes into account the influence of uncertainty factors. The risk factors which have an impact on the values of the evaluation criteria are described with probability distributions and the Monte Carlo simulation is used to generate the probability distributions for evaluation criteria.

The main component of the procedure proposed is the multi-criteria interactive decision-aiding method under risk and uncertainty. The method allows DM to aid the decision-making process while taking into consideration his preferences. It is notable that DM is not required to define his preferences prior to the decision aiding process (e.g. as criteria weights). DM is only asked to assess the proposals of the solutions developed in the process and indicate the directions for their improvement. This allows to take into account DM's preferences in terms of the relations between the criteria and his attitude to risk (when he defines the expected values in the subsequent iterations of the algorithm and the probability used to calculate the values in potency matrices).

The procedure proposed was implemented with the use of the MS Excel spreadsheet and the additional @Risk module for the Monte Carlo simulation. The numerical example illustrates the selection aiding process for an investment alternative. We consider five decision alternatives. Two uncertainty factors, each having two possible values, are taken into account. As a result, we need to analyse four scenarios of the environment development. Moreover, we consider seven risk factors.

We developed financial forecasts for each situation (the pair of the alternative and the scenario) and their models were recorded in the spreadsheet. Based on the spreadsheets, we conducted 20 simulations, 1,000 iterations each. As a result, we received 100 probability distributions for the evaluation criteria (20 situations x 5 criteria).

Based on the results of the previous stages of the procedure, we used the multi-criteria interactive method that we created to carry out the decision aiding process.

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A STUDY OF DISTRIBUTED EVOLUTIONARY ALGORITHMS FOR MULTI-OBJECTIVE OPTIMISATION

Abstract

Most popular Evolutionary Algorithms for single multi-objective optimisation are motivated by the reduction of the computation time and the resolution larger problems. A promising alternative is to create new distributed schemes that improve the behaviour of the search process of such algorithms. In the multi-objective optimisation problems, more exploration of the search space is required to obtain the whole or the best approximation of the Pareto front. Almost all proposed Parallel Multi-Objective Evolutionary Algorithms (PMOEAs) are based on the specialisation concept which means dividing the objective and/or the search space then assigning each part to a processor. One processor called the organiser or the coordinator is usually charged to direct the whole algorithm. In this paper, we present a new parallel scheme of multi-objective evolutionary algorithms which is based on a clustering technique. This new parallel algorithm is implemented and compared to three PMOEAs which are cone-separation [1], Divided Range Multi-Objective Genetic Algorithm (DRMOGA) [8] and a Parallel Strength Pareto Evolutionary Algorithm (PSPEA) based on the island model without migration.

Keywords

Parallel computing, multi-objective optimisation, evolutionary algorithms, parallel genetic algorithms, clustering algorithms.

Introduction

Evolutionary Algorithms (EAs) are adaptive methods that have proven successful in the resolution of several optimisation problems. They are based on the genetic evolution process of Darwin. EAs make evolve a set of solutions, called population of individuals. A new population is produced while selecting parents among the most excellent individuals of the "present generation" to perform crossover and mutation. The new population will contain a bigger proportion of features from the best individuals of the previous generation. The search is thus guided towards the most promising regions of the search space. In Multi-objective Optimisation Problems (MOP), a set of conflicting criteria have to be simultaneously optimised. The aim is to find a set of non dominated solutions rather than one solution in the single objective optimisation case. EAs are particularly suitable to solve MOP. They perform well global search, since they simultaneously explore different regions of the search space. To obtain a set of diversified non dominated solutions, Multi-Objective Evolutionary Algorithms (MOEAs) integrate techniques such as elitism and diversity (crowding, sharing) (NSGA-II [4], SPEA-II [15], NPGA-II [6]). These techniques have proven successful in deriving good Pareto optimum solutions. However, their high computing time constitutes a major drawback. Parallel computing has been applied to MOEAs so as to accelerate solving problems [2]. Moreover, parallelism offers a best exploration of the search space by the cooperation between populations evolving with different genetic operators. Several approaches have been proposed to parallelise both EAs and MOEAs. In the single objective case, Parallel Evolutionary Algorithms (PEAs) exploit the intrinsic parallelism in the algorithm. In fact, fitness evaluation, crossover and mutation operators can be performed independently on different individuals and thus can be easily distributed. In the multi-objective case, most Parallel Multi-Objective Evolutionary Algorithms (PMOEAs) are based on the algorithmic "divide to conquer" principle: divide the objective/search space among available processors while favouring migration between sub-populations. Each processor will concentrate its search in a specific region of the search space (specialisation).

In this paper, we propose a new parallel evolutionary algorithm for multi--objective optimisation named „[…] parallel multi-objective evolutionary algorithm with Multi-Front Equitable Distribution" (MFED). MFED, which is based on the island model, uses a clustering technique to divide the global population into sub-populations. MFED is implemented and compared to three

PMOEAs: cone-separation [1], Divided Range Multi-Objective Genetic Algorithm (DRMOGA) [8] and a Parallel Strength Pareto Evolutionary Algorithm (PSPEA) based on the island model without migration.

This paper is organised as follows: Section 1 gives common parallelisation approaches of EAs. Section 2 deals with PMOEAs. Then, in Section 3, the MFED is described. Test results and comparisons are presented and analysed in Section 4. Finally, conclusions will be highlighted.

1. Parallel evolutionary algorithms

Parallel architectures permit to get some very satisfactory results in parallelising EAs. Parallel genetic algorithms are roughly classified into three categories: master-slave population model, island model, and cellular model [2].

Master-slave model

The master-slave model is based on a simple parallelisation of the fitness calculation stage or of the recombination/evaluation steps. In the first case, a single station (called master) manages the algorithm itself (selection/replacement and genetic operators), and sends the performance computation to other stations (called slaves). In the second case, the master centralises the population and manages the selection and the replacement steps. It sends the sub-populations to the workers that execute the recombination and evaluation steps. This model is useful only for a small number of processors and high fitness computing time [1].

Island model

The island model, which is also called coarse-grained model or distributed model, divides the population into small sub-populations. Each of them will evolve in a certain processor, following a traditional diagram to which a stage of migration is added. In other words, every sub-population transmits its good individuals towards the neighbouring sub-populations in a "common pool" (this choice depends on the relative cost of communications between processors). Then, every sub-population receives individuals which are sent by neighbours, or which already exist in the central pool.

The island model modifies the basic genetic algorithm and introduces some new parameters (i.e. the migration strategies and the topology of the network) [2]. It becomes particularly interesting when the number of processors is lower than the population size [1].

This model is better adapted than the precedent one to parallel machines. It is a very popular model since it is very easy to apply on a local network with standard workstations. Moreover, it offers the possibilities that every subpopulation can evolve using some different parameters. {?}

Cellular model

The cellular model distributes a unique population among several processors (in general on massively parallel machines). On each processor some individuals (often only one) evolve. Then, the selection, replacement and crossover operations are outperformed between "neighbourhood individuals" for the topology of the processor network. This model is particularly suitable for massively parallel computers with a fast local intercommunication network.

A detailed discussion of parallelisation approaches for EAs is found in [1, 3].

2. Parallel multi-objective evolutionary algorithms

MOEAs look for a whole set of Pareto-optimal solutions. Therefore, they require more exploration of the search space and more computation to characterise the Pareto-optimal front than the single objective EAs. For that reason, several works ([11], [5], [1], [8], etc.) discuss the manner to parallelise them. Since the resolution of MOP aims at finding a set of different trade-off solutions between the objectives, the most natural parallel scheme was to assign different parts of the search/fitness space to different processors. Each process will focus its computation on a specific region (explore one region) of the search space to characterise one area of the global Pareto optimal front. That's why most proposed PMOEAs are based on the island model and deal with the manner of dividing the search space and/or the fitness space between the different processors. In this section we limit our focus on DRMOGA [8] and Cone Separation [1] in order to compare them to our new parallel model.

DRMOGA

Hiroyasu et al. [8] have developed a Divided Range Multi-Objective Genetic Algorithm (DRMOGA) based on an island model where the Pareto--optimum solutions, which are close to each other, are collected by one subpopulation. All individuals are gathered in the master process and are again divided among the different processors. Each sub-population receives a set of N/m individuals (N is the population size and m is the number of processors) selected according to the value of the objective function considered f_i (the objectives are considered in turn). Figure 1 shows the division of the population according to the objective f1 in the case of a bi-objective optimisation problem and three processors. An improvement version of DRMOGA uses a sharing operation for Pareto-optimum solutions when the number of the frontier solutions exceeds a given size.

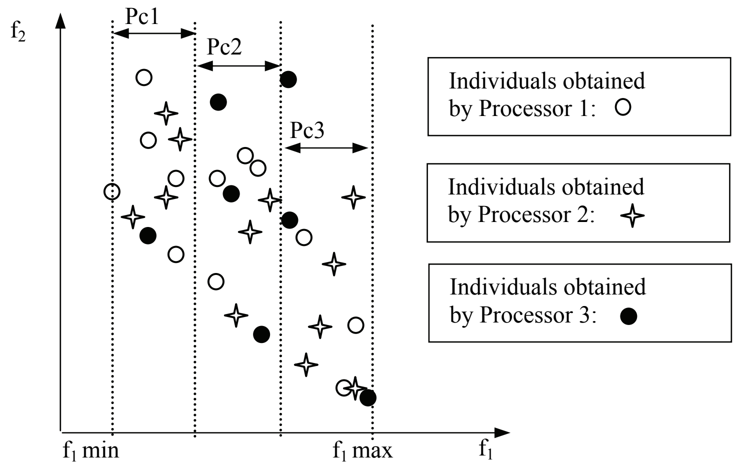

Figure 1. DRMOGA: distribution of the population according to the first objective with 3 processors

Cone separation

Branke et al. [1] normalise the fitness values and then they partition the fitness space into equal cones. In the bi-objective case, the population is within the unit square after normalisation. They start from the reference point (1,1) and divide the 90° angle that encompasses the non-dominated front into equal parts (see Figure 2). The fitness space is renormalised at regular intervals leading to a migration step of individuals to processors specialised on the cone to which they belong.

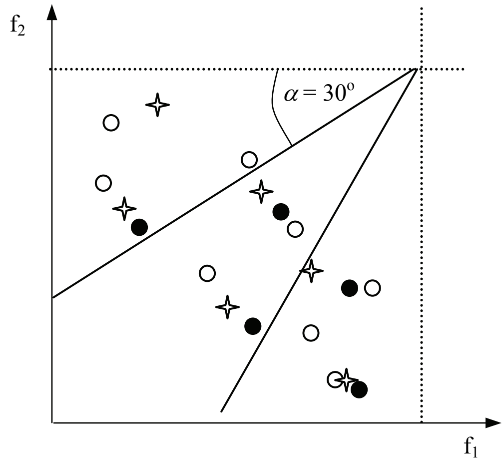

Figure 2. Cone-separation with 2 objectives functions problem and 3 processors

3. Parallel Multi-Objective Evolutionary Algorithms with multi-front equitable distribution

Most PMOEAs are based on the specialisation concept where neighbouring solutions in the search/objective space live and progress on the same island. Thus, these models authorise only crossover of individuals close to each other. This fact may result in a lack of diversity among sub-populations, in a stagnation of the search and thus in a rapid convergence. Moreover, in the case of a multi-objective optimisation problem for which the Pareto Optimal Front (POF) is discontinuous, individuals of the same sub-population may belong to extreme zones of two continuous portions of the Pareto Front (PF).

Our new algorithm is an island model. As the previous PMOEAs, it is based on the algorithmic principle “divide to conquer”. Yet, it is different from the others in the fact that no sub-population focuses its computation on a specific region of the search/fitness space. Each processor focuses its computation over all promising regions of the search space that are already discovered within the global population. In fact, MFED functions as a multi-start optimisation procedure where an elitist MOEA evolves on each processor with its proper starting sub-population. These sub-populations cooperate through a recombination/distribution mechanism. This parallel model divides the global population among the available processors so that each one receives a representative set of solutions from the global population. Each sub-population uses its proper genetic operators (crossover and mutation). In this way, diversification among sub-populations will be maintained.

The main algorithm consists of several elitist MOEAs. One processor, the organiser, has the responsibility of collecting individuals from the other processors and then redistributing them.

Every processor k (k=0, 1,…, p) constitutes its first n Pareto fronts found (n is a parameter) as follows: the non-dominated individuals of the sub--population P_k of the processor k constitute the first front F_{1k}. The set of non-dominated individuals in $P_k \backslash F_{1k}$ makes up the second front F_{2k}. This scheme is repeated until the first n PFs are created. After that the processor sends them to the organiser.

The organiser gathers individuals sent by all processors (the organiser and the others) in order to create the first n global Pareto fronts. The redistribution mechanism is described as follows: each global Pareto front GF_i ($i \in \{1,...,n\}$) is first partitioned into NC clusters (NC is a parameter). After that, every cluster is redistributed with equity between the available processors. Each processor will receive at least one individual from each cluster.

Let's consider a cluster CL of |CL| individuals. If $p \le |CL|$ then the processor k (k=0,1, ..., p-1) receives from CL all individuals j (j=0, ..., |CL|-1) such that $j\%p = k$. Otherwise the processor k will receive the individual j such that $k\%|CL|=j$.

In this way, each processor receives a good approximation of the first n global Pareto fronts which is different from the sets received by the others. We maintain diversity in each sub-population and especially in the Pareto front. After the redistribution process, the size of each sub-population will be increased by crossover so that this sub-population will be equal to its initial size N/p, N is the population size.

We have used an agglomerative clustering algorithm that begins with each individual representing a single cluster. At each step, the distance between each two different clusters is calculated as the maximal Euclidean distance between two individuals from these clusters. Then, the nearest two clusters with respect to the distances calculated are merged. These steps are repeated until the number NC of desired clusters is obtained.

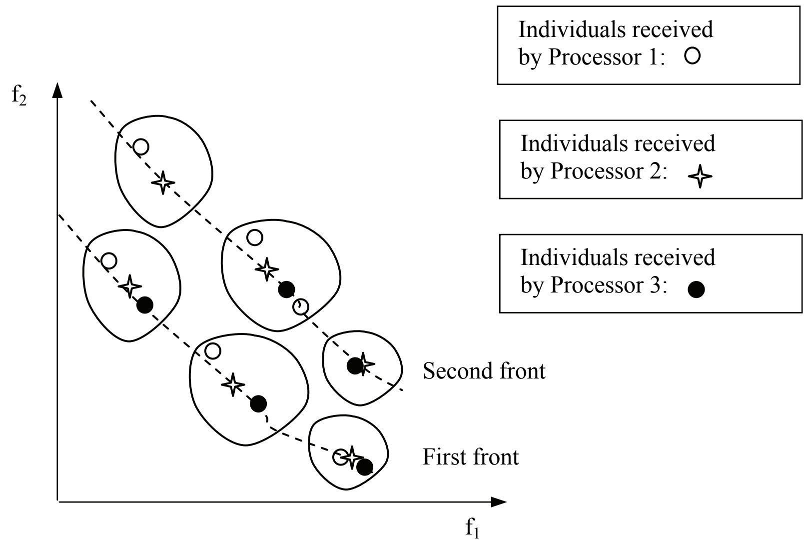

Figure 3. Equitable distribution of the first and the second front

The pseudo-code of the algorithm is the following:

I. **Parameters:**

N: population size,
NC: number of clusters,
T: maximum number of generations,
C: interval of migration,
n: number of fronts to distribute.

II. **Step 1. Initialization:** randomly generate an initial sub-population P_0. The global population size is N. The size of each sub-population is N/P (p is the number of processors)

Set *t*=0.

III. **Step 2. Evolutionary computation:**

1. If t≠0 then increase the size of the sub-population by crossover.
2. Run a Multi-objective genetic algorithm for C generations.

IV. **Step 3. Recombination/redistribution of the first *n* global fronts:**

1. Each process sends its first *n* Pareto fronts to the organiser.
2. The organiser process:

- Creates the first n global fronts,
- divides the i^{th} global front into NC clusters (i=1, …, n),
- distributes each cluster with equity between all the processors.

V. **Step 8. Termination:** if $t=T$, stop the algorithm, else go to Step 2.

4. Experimental results

In this section, we study the performance of the MFED and the following PMOEAs: DRMOGA, Cone-Separation and PSPEA which is a standard island (no migration). PSPEA is a parallel version with an independent SPEA running on each processor. The four parallel schemes studied are based on the Strength Pareto Evolutionary Algorithm (SPEA) [14] which is one of the most popular elitist MOEAs. It utilises an external population (the archive) in order to preserve diversity and prunes it when a predetermined size is exceeded. Four test problems [14, 15], which include: convex (ZDT1), non-convex (ZDT2), discontinuous (ZDT3) and non-uniform (ZDT6) Pareto Optimal Front (POF), are chosen for this comparative study (see appendix A). To solve the test functions, we have used bit coding for representing individuals. 20-bit length is used for each design variable of the problems studied. Since each sub-population evolves with its proper genetic operators, we have implemented three crossover operators (one point, two points and uniform crossover) and two mutation operators (one point and mapping mutation).

The parameters of the elitist MOEA are:

- Population size (N): 300 individuals distributed among the available processors.
- Archive size (N): 200 individuals.
- Maximum number of generations (T): 1000.
- Crossover probability (Pc): 0.86.
- Mutation probability (Pm): 0.1
- C: interval of the global clustering: 20 (distribution every 20 generations).
- Number of processors: 4.

All algorithms are implemented using C++ language on a local network with Pentium IV 2.4 GHz 80 Gb computers under Windows XP. The communication between the processors has been supported by the freely available MPICH (Message Passing Interface) parallelisation library.

To compare the results derived from each algorithm, we use the following metrics:

1. The spacing indicator gives a good indication of how evenly the solutions are distributed in the objective space. It is defined as:

$$S = \left[\frac{1}{n} \sum_{i=1}^{n} (d_i - \overline{d})^2 \right]^{1/2}$$

where:

$$d_i = \min_{(k \in PO) \wedge (k \neq i)} \left(\sum_{j=1}^{m} \left| f_j^i - f_j^k \right| \right)$$

$$\overline{d} = \frac{\sum_{i=1}^{n} d_i}{n}$$

where n is the number of solutions in the current front PO, d_i is the Euclidean distance in the objective space between the solution i in PO and its nearest solution in POF.

The smaller the spacing is, the more regularly distributed the solutions in PO are:

2. The S-metric (Hypervolume) [17] gives the size of the objective space enclosed by POF and a reference point Z^{ref}.
3. The generational distance [15] measures how far the elements in the set of non-dominated vectors found (PO) are from those in the Pareto optimal set (POF). For instance, it represents how far the current front PO is from the optimal front POF. Its value is defined as:

$$GD = \sqrt{\frac{\sum_{i=1}^{n} (d_i)^2}{n}}$$

where n is the number of individuals in the current front PO and d_i is the Euclidean distance between any of the individuals and the nearest individual from the Pareto-optimal front.

Ten trials are launched for each configuration. Then, the min, max and average for each performance metric are given.

ZDT1 problem

The results of the four algorithms are similar with respect to the S metric (Table 1). DRMOGA and Cone Separation result in a better distribution of solutions in the PO set found since they produce the smallest values for the spacing (Table 1). Indeed, the reproduction of neighbouring individuals in these two parallel models, which are based on the specialisation concept, generates close individuals in the objective space. The approximation of the true

Pareto front obtained by Cone Separation is the nearest one to POF since it obtains the least value for the generational distance (Table 2). MFED is less effective than the other models concerning the spacing and the generational distance. This stems from the fact that each sub-population in MFED performs computation on all promising regions of the search/objective space discovered and thus converges less rapidly than the other algorithms. Due to the easiness of this problem (the Pareto front is convex and uniform), the three algorithms PSPEA, DRMOGA and Cone Separation have produced results close to each other.

Table 1

Results for ZDT1 problem: S metric and Spacing

Algorithm	Smetric			Spacing		
	Min	Mean	Max	Min	Mean	Max
PSPEA	0,65201	0,654164	0,654164	0,0019373	0,00367523	0,00367523
DRMOGA	0,645368	0,6483987	0,650027	0,00365695	0,00512045	0,00627473
Cone separation	0,649734	0,6512663	0,652182	0,00204366	0,0031909	0,00410994
MFED	0,652839	0,6542778	0,655483	0,0104054	0,01245304	0,0167373

Table 2

Results for ZDT2 problem: Generational distance

Algorithm	Generational distance		
	Min	Mean	Max
PSPEA	5,23E-05	8,62E-05	0,00016884
DRMOGA	3,32E-05	4,63E-05	7,54E-05
Cone separation	1,28E-05	1,44E-05	1,65E-05
MFED	0,00014301	0,000646334	0,00232182

ZDT2 problem

The results for the S metric (Table 3) and the generational distance (Table 4) for the ZDT2 problem show that the MFED outperforms the other parallel algorithms according to these two metrics. MFED achieves a good approximation of the true Pareto optimal front POF. Its results for the spacing metric are the worst when compared to those of DRMOGA and Cone Separation. We can conclude that the non-dominated solutions found by MFED are not uniformly distributed in the objective space. This may be due to the non--convexity of the POF so that some regions in the objective space are not as well explored as the others. On average, PSPEA, DRMOGA and Cone Separation provide similar results for the S metric (Table 3) but cone separation slightly improves the spacing.

Table 3

Results for ZDT2 problem: S metric and Spacing

Algorithm	Smetric			Spacing		
	Min	Mean	Max	Min	Mean	Max
PSPEA	0,4871	0,4892946	0,4871	0,0051399	0,0047197	0,0058635
DRMOGA	0,474847	0,4832458	0,49057	0,0037625	0,00682405	0,0104535
Cone separation	0,4889	0,4935705	0,496057	0,00232241	0,00360916	0,00574457
MFED	0,318422	0,3216687	0,325326	0,00932995	0,01202236	0,0147275

Table 4

Results for ZDT2 problem: Generational distance

Algorithm	Generational distance		
	Min	Mean	Max
PSPEA	0,00255696	0,003288901	0,00360972
DRMOGA	0,0049013	0,00587651	0,0069419
Cone separation	1,2775E-05	0,004449959	0,00515236
MFED	0,00022007	0,000747621	0,00158096

ZDT3 problem

Table 5

Results for ZDT3 problem: S metric and Spacing

Algorithm	Smetric			Spacing		
	Min	Mean	Max	Min	Mean	Max
PSPEA	0,740543	0,8400583	0,86043	0,00378297	0,0087262	0,0121672
DRMOGA	0,318422	0,4476161	0,863358	0,00232241	0,00679382	0,0209619
Cone separation	0,841538	0,8473562	0,85312	0,00486804	0,00789774	0,0167653
MFED	0,778603	0,7791379	0,779501	0,00366007	0,00673456	0,00869159

Table 6

Results for ZDT3 problem: Generational distance

Algorithm	Generational distance		
	Min	Mean	Max
PSPEA	0,00085425	0,001182975	0,00140321
DRMOGA	0,00221447	0,00309332	0,0044729
Cone separation	0.00153496	0.00179118	0.00201776
MFED	1,6994E-07	2,51092E-07	3,90953E-07

The problem ZDT3 is discontinuous, so the spacing is not significant. As shown in Table 5, the experimental results show that DRMOGA is capable of providing better quality of solutions with respect to the S metric than all the other algorithms. The results of PSPEA and Cone Separation on the three considered metrics are very close (Table 6 and Table 5). From Table 6, we conclude that MFED clearly performs better with respect to the generational distance. Since the Pareto front is discontinuous, the division of the objective space into cones in Cone-Separation may result in empty cones, so that it reduces the performance and the convergence of this algorithm. However, MFED is less affected by the discontinuity of the POF than the other methods.

ZDT6 problem

Table 7

Results for ZDT6 problem: S metric and Spacing

Algorithm	Smetric			Spacing		
	Min	Mean	Max	Min	Mean	Max
PSPEA	0,702722	1,4089039	2,25275	0,0438337	0,19234355	0,548809
DRMOGA	1,00986	1,611229	2,07032	0,0344967	0,22391992	0,834136
Cone separation	1,72002	1,722573	1,72408	0,00286583	0,01388675	0,0669603
MFED	4,48067	4,482684	4,48369	0,00898641	0,11755475	0,608804

Table 8

Results for ZDT6 problem: Generational distance

Algorithm	Generational distance		
	Min	Mean	Max
PSPEA	0,599989	1,2432687	2,09514
DRMOGA	1,04722	1,520106	1,92963
Cone separation	0,0119125	0,01499197	0,0173161
MFED	5,9604E-08	0,012355877	0,0707847

Table 7 shows that cone separation, DRMOGA and PSPEA largely outperform MFED with respect to the S metric. Since ZDT6 has a non-uniform POF, the spacing metric is not very significant. Nevertheless, the spacing value obtained by Cone Separation is remarkably lower than the others (Table 7). Cone separation generates a set of non-dominated solutions which are near to the true Pareto front (Table 8) and fairly dispersed in the objective space. For ZDT6, the only difficulty is that the POF has a non-uniform distribution of its points. The performance of MFED is very much affected by the equitable distribution of the fronts. So, the distribution decreases the convergence of each sub-population.

Conclusions

This paper proposes a new parallel model of genetic algorithms for multi--objective optimisation problems which is based on the clustering technique. In MFED, at regular intervals, each island receives an initial sub-population that is different from those received by the others. Then it performs a search over all the promising regions found in the search space. The idea of the equitable distribution insures the reception of a set of representative individuals of these promising regions, by each processor. At the same time, MFED increases the convergence to the true Pareto front. Thanks to the use of different genetic operators in each island, the diversity all over the population is maintained. Experiments have been carried on four PMOEAs (MFED, DRMOGA, PSPEA and Cone Separation) while using a well known multi-objective benchmark function set that covers a wide range of difficulties (discontinuity, non-convexity, non-uniformity) in finding the Pareto front. Experimental results have shown that the performance of each model depends on the shape of the search space. The MFED outperforms the other algorithms on the ZDT3 problem which has a discontinuous POF, while DRMOGA and Cone Separation perform better on the ZDT1 problem with convex POF. But, MFED converges less rapidly than the others.

On the one hand, the PMOEAs that are based on the specialisation concept ensure a good performance vis à vis some difficulties or vis à vis some performance criteria. On the other hand, MFED that doesn't limit each island to a specific region of the search space, (each island perform a global exploration), succeeds where the other algorithms fail. Thus, the two division/re-distribution mechanisms are complementary and essential for improvement of results in different types of problems.

Appendix

The benchmark functions set used in this work addresses the following minimisation problem:

1. ZDT2.

$$f_1(\overrightarrow{x}) = x_1$$

$$f_2(\vec{x}) = g(x)[1-(f_1(x)/g(x))^2]$$

$$g(x) = 1+\frac{9}{n-1}\sum_{i=2}^{n} x_i$$

with

$$x_i \in [0,1],\ n = 30$$

2. ZDT6.

$$f_1(\overrightarrow{x}) = 1-\exp(-4x_1)\sin^6(6\Pi x_1)$$

$$f_2(\vec{x}) = g(x)[1-(f_1(x)/g(x))^2]$$

$$g(x) = 1+9((\sum_{i=2}^{n} x_i)/9)^{0.25}$$

with

$$x_i \in [0,1],\ n = 10$$

3. ZDT3.

$$f_1(\overrightarrow{x}) = x_1$$
$$f_2(\overrightarrow{x}) = g(x)[1 - \sqrt{f_1(x)/g(x)} - ((f_1(x)/g(x))\sin(10\Pi f_1)]$$
$$g(x) = 1 + \frac{9}{n-1}\sum_{i=2}^{n} x_i$$
with
$$x_i \in [0,1],\ n = 30$$

4. ZDT1.

$$\min \quad f_1(x) = x_1$$
$$\min \quad f_2(x) = 1 - \sqrt{x_1 / g(x)}$$
$$g(x) = 1 + 9\left(\frac{\sum_{i=2}^{n} x_i}{n-1}\right)$$
$$x_i \in [0,1] \quad \forall i \in \{1,2,.....,30\}$$

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PREDICTION OF BANKRUPTCY BASED ON THE MATHEMATICAL PROGRAMMING

Abstract

The concern about the prediction of bankruptcy is not recent. Since the 1960s, several methods have been developed. The most popular is certainly the Multiple Discriminant Analysis. In this paper, we are developing a method based on Mathematical Programming. The global performance of every firm is evaluated by a multicriteria index and the firms are classified into three categories: the non-failed firms, failed firms and those about which we are uncertain. The minimization of the latter constitutes the main objective of our mathematical program.

Keywords

Mathematical programming, multicriteria index, classification, predicting bankruptcy.

Introduction

The problem of classification and segregation of different firms into disconnected groups is frequently used in decision-making. The problem of binary classification, which is set down in this context, is a problem in which the number of groups is limited to two. This type of classification is appropriate to numerous problems such as the differentiation between a good and a bad customer or between failed and non-failed firms. Zopounidis and Doumpos [39] showed that discrimination problems are very common in several fields of finance, including predicting bankruptcy, credit granting, corporate mergers and acquisitions, country risk evaluation, venture capital investments, portfolio selection and management, etc.

"The definition of business failure varies across different studies depending on purposes and scopes of studies" Gu [18]. But generally, bankruptcy refers to a situation in which the firm has negative net worth as well as insufficient liquidity to meet current liabilities. "For more than 30 years, researchers from all over the world work on the problem of business failure prediction. The problem of predicting bankruptcy timely and correctly, is of great importance for financial institutions" Tsakonas et al. [36].

Multivariate Discriminant Analysis (MDA) remains undoubtedly the best-known model and the most often used in this field. It allows us to formulate the problem mathematically, to analyze the set of variables simultaneously, to reduce the dimension of the space of analysis from n (the number of independent variables) to g-1 (where g is the number of groups) and to assign the new firms to the groups defined. It defines a synthetic variable Z capable of summarizing the information included in the set of variables. Z is a linear combination of n ratios which separates the two types of firms (non-failed firms and failed firms). Z is written as follows:

$$Z = a_0 + a_1R_1 + a_2R_2 + ...+ a_nR_n$$

Z_1 and Z_2 are gravity centers of the two classes. The MDA looks for Z_1 and Z_2 which are the most distant possible and such that the uncertainty zones be the narrowest in the sample of reference and validation.

Altman is the first who had used the MDA to predict bankruptcy, but later on, several works attempted to adapt this analysis to their environments such as: Collongues [6], Conan and Holder [7] and the case of the Banque of France [4], etc.

Despite the success achieved by the MDA, the method has been criticized in numerous studies such as: Joy and Tollefson [20], Scott [32], Malecot [23, 24, 25] and Eisenbeis [11, 12]. The latter raised problems concerning the applicability of the MDA.

Eisenbeis [11] summarized the difficulties with the MDA in seven points:

1. Violation of the assumption of multivariate normal distribution of the variables.
2. Use of linear instead of quadratic discriminant functions when group dispersions are unequal.
3. Unsuitable interpretation of the role of independent variables.
4. Reduction in dimensionality.
5. Group definition.
6. Inappropriate choice of a priori probabilities and costs of misclassification.
7. Problems in estimating classification error rates.

Similar difficulties have been pointed out by Malecot [23], but he tried to classify them according to their origin: statistical limit (the absence of the normality of variables, the unequal group dispersions, etc. and the limit of the transposition. Whereas Joy and Tollefson [20] revealed other difficulties related to the form of the discriminant function, the cross-validation and sampling.

The other popular functional form used by bankruptcy researchers is the logit model and the probit model. Laitinen & Laitinen [22] proposed a combined use of logistic regression and the Taylor series expansion. A comparison of the logit model and the quadratic interval logit model is described in Tseng and Lin [37]. Doganay et al. [9] proposed an integrated early warning system by combining multiple regression, discriminant analysis, logit and probit.

For the last few years, several theoretical studies have been devoted to the use of Mathematical Programming (MP) techniques to solve the classification problems. Several experiences and studies showed that MP can overclassify statistical techniques of discriminations. Glover, Keene and Duea [16] mention the following advantages:

– The MP methods are free from underlying parametric assumptions.
– Various objectives and more complex problems are easily accommodated.
– Misclassification costs can be easily incorporated into the model.
– Some MP methods, especially Linear Programming, lend themselves to sensitivity analyses.

The approaches of Mathematical Programming applied to the discrimination problems try to build a discriminant function or a hyperplane which separates the two groups in order to classify a new entity in the corresponding group. The hyperplane corresponds to the following equation:

$$AX = b$$

where A is the set of alternatives.

The objective is to determine the weighting vector X and a scalar b, so that we assign, as correctly as possible, the individuals of group 1 to the one side of the hyperplane and the individuals of group 2 to the other side. Therefore, the goal is to minimize the weighted sum of boundary violation.

Mathematical programming methods were employed early to solve the discriminant problem. Grinold is one of the pioneers who formulated a MP for failure prediction in 1972. More recently different clustering and discrimination methodologies have been developped, based on mathematical programming. Among these models are MSD (minimize the sum of distances) presented by Freed and Glover [14], MMD (minimize the maximum distance) proposed by Freed and Glover [15] and MIP (mixed-integer programming). Various combinations of these basic methods have been proposed in literature.

With the technological and scientific development, other discrimination models appeared and were applied to the problems of bankruptcy prediction. Among these models we can mention: the Recursive Partitioning Algorithm, rough sets [28], the expert system, the neural networks [1, 30, 35, 31, 2], the genetic algorithm [19, 28, 13] and the Multicriteria Decision methods such as ELECTRE TRI [38, 33], UTADIS [10], MINORA, PROMETHEE [5, 11] (for more details see [8]). Kumar and Ravi [21] presented a review of the work done, during the 1968-2005, in the application of statistical and intelligent techniques to solve the bankruptcy prediction problem faced by banks and firms.

By relying on the Multicriteria Index (MI) suggested by Martel et al. [27] in order to compare the performance-size of mutual funds and by taking advantage of the competitiveness and the flexibility of Mathematical Programs, we have formulated a mathematical program which integrates this index to assess the global performance of the firms. The latter allows us to predict the bankruptcy of the firms by classifying them into two categories: failed firms and non-failed firms. Martel and Khoury [26] proposed a procedure of binary classification of firms which refers to this index.

Our paper includes 5 sections. In the second section there is a brief description of the Multiciteria Index. Our MP model of bankruptcy prediction will be presented in the third section. An empirical illustration is given in the fourth section and the last one summarizes the content of the paper.

1. A Multicriteria Index (MI) of financial performance

As the Multicriteria Index (MI) of Martel et al. [27] is inspired by PROMETHEE (Preference Ranking Organisation MEthods for THe Enrichment Evaluation), we start by presenting basically this procedure as developed by Brans et al. [5]. This method classifies a set of alternatives on the basis of pairwise comparisons. It defines a preference function for every criterion.

Suppose:

$$a, a' \in A$$

where A is the set of the alternatives.

The preference function, denoted $P_j(a, a')$, expresses the decision-maker's preference for the alternative a over a' according to the criterion C_j. This function is defined as follows:

$$P_j(a, a') = \begin{cases} 0 & \text{if} \quad C_j(a) \leq C_j(a') \\ H_j(d_j(a, a')) & \text{if} \quad C_j(a) > C_j(a') \end{cases}$$

where $d_j = C_j(a) - C_j(a')$.

Every criterion function $H_j(d_j(a, a'))$ (see Figure 1) takes its values between 0 and 1. In other words the decision-maker's preference varies from indifference (0) to the strict preference (1) for each criterion.

Brans et al. [5] proposed six types of functions (H_j) that correspond to different types of criteria (Usual Criterion, Quasi-Criterion, Criterion With Linear Preference, Level-Criterion, Criterion With Linear Preference and Indifference Area and Gaussian Criterion).

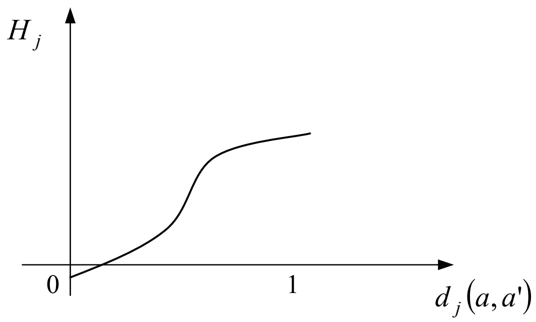

Figure 1. Criterion function

Following the definition of the criterion function, we are applying an aggregation procedure which consists of the following steps:

1. We calculate for each alternative a a global preference (the performance) index of a over a' taking into account all criteria. Therefore for every pair of alternatives (a, a'), we calculate:

$$P(a, a') = \sum_j W_j P_j(a, a')$$

where W_j is the relative importance of every criterion C_j, with $\Sigma W_j = 1$.

2. We use this index to calculate the out flows and the in flows for each alternative *a*. The out flows represents the preference (the performance) of the alternative *a* over other alternatives:

$$P^{+}(a) = \sum_{e \neq e'} P(a, a')$$

The in flows represent the preference (the performance) of the set of other alternatives over the alternative *a*:

$$P^{-}(a) = \sum_{e \neq e'} P(a', a)$$

3. Finally, we use the notion of net flow to evaluate the performance of the alternative *a*:

$$P(a) = P^{+}(a) - P^{-}(a)$$

Then *a* is preferred to (or is more "performing" than) *a'* if $P(a) > P(a')$.

When the number of pairwise comparison raises, Martel and al. [27] and Martel and Khoury [26] introduced the notion of ideal and anti-ideal. Thus, Martel and Khoury [27] have proposed a procedure that allows an absolute evaluation of each alternative by comparing it with two fictitious firms: one is ideal (a^*) and the other anti-ideal (a_*).

$$V(a^*) = \{ C_1^* \ldots\ldots\ldots C_j^* \ldots\ldots\ldots\ldots C_m^* \}$$

where $C_j^* = Max\left[Max\ C_j(a), \overline{C_j} + 2\sigma_j\right]$

for a criterion to maximize

and $C_j^* = Min\left[Min\ C_j(a), \overline{C_j} - 2\sigma_j\right]$

for a criterion to minimize,

where $\overline{C}_j$ and σ_j are the average and the gap type of assessments according to C_j, respectively.

We suppose that each criterion is measured on a cardinal scale. We added and reduced $2\sigma_j$ since we work with a sample of an ordered population and we use a confidence interval. We could use + or $-2.5\sigma_j$ or even + or $-3\sigma_j$.

So, the ideal firm is constructed by forming the vector of the best assessments with respect to each of the criteria. However, the anti-ideal firm is the combination of the worst assessments with respect to each of the criteria.

$$V(a_*) = \{ C_{*1} \ldots\ldots\ldots C_{*j} \ldots\ldots\ldots\ldots C_{*m} \}$$

where $C_{*j} = Min\left[Min\ C_j(a), \overline{C_j} - 2\sigma_j\right]$

for a criterion to maximize,

and $C_{*j} = Max\left[Max\ C_j(a), \overline{C_j} + 2\sigma_j\right]$

for a criterion to minimize.

For every firm $a / a \in A$, Martel and Khoury [26] constructed a MI that satisfies the following inequality:

$$MI(a_*) \leq MI(a) \leq MI(a^*)$$

By relying on PROMETHEE II and the pairwise judgments between the alternatives
a^*, a and a_*, we can determine their net flow $P(a^*)$, $P(a)$ and $P(a_*)$. And we get:

$$P(a^*) = 1 + \sum W_j H_j(d^*)$$

$$P(a) = \sum W_j \left[H_j(d_*) - H_j(d^*)\right]$$

$$P(a_*) = -\left[1 + \sum W_j H_j(d_*)\right]$$

where

$P_j(a^*, a) = H_j(d^*)$, $P_j(a, a_*) = H_j(d_*)$ and $d^* = C_j(a^*)\text{-}C_j(a)$, $d_* = C_j(a)\text{-}C_j(a_*)$ for a criterion to maximize; $d^* = C_j(a)\text{-}C_j(a^*)$, $d_* = C_j(a_*)\text{-}C_j(a)$ for a criterion to minimize.

To facilitate the comparison between firms, Martel and Khoury [26] normalized the global performance $P(a)$ of every firm. The index thus obtained, called Multicriteria index (MI), varies between 0 and 1:

$$MI(a) = \frac{P(a) - P(a_*)}{P(a^*) - P(a_*)}$$

So for every firm, Martel and Khoury [26] calculated its MI in order to assign it to a predefined category. Then, they determined two thresholds MI^* and MI_* permitting to partition the set of the firms into three categories:

– “Failed firms”,
– “Non-failed firms”,
– “We don't know”.

The rule is that if $MI(a) > MI^*$ the firm is assigned to the non-failed category, if $MI(a) < MI_*$ the firm is assigned to the failed category and if the $MI(a) \in [MI_*, MI^*]$ then the firm is assigned to the category "we don't know".

Finally, Martel and Khoury [26] presented an *ad hoc* method (a sort of "trial and error") for the determination of the coefficients (W_j) in order to minimize the numbers of misclassified firms while trying to reduce the width of the uncertainty zone which is limited by $[MI_*, MI^*]$. This method includes several iterations before obtaining the "best" classification. So it is a quite heavy process which offers no guarantee of success. It is mainly at this level that the MP can be very useful.

2. A model for predicting bankruptcy

Inspired by the work of Martel and Khoury [26] we have developed a model for predicting bankruptcy based on mathematical programming. As MI presented in the previous section generates a non-linear mathematical program whose linearization may be long and complex, we have decided to adopt a new form of normalization.

Let's suppose that:

$$x = P\left(a^*, a\right) = \sum_j W_j P_j\left(a^*, a\right)$$

$$z = P(a, a_*) = \sum W_j P_j(a, a_*)$$

For all j we have $W_j \geq 0, \sum W_j = 1$, $P_j(a^*, a) = H_j(d_j(a^*, a))$ and $P_j(a, a_*) = H_j(d_j(a, a_*))$

With $0 \leq H_j(d_j(a, a')) \leq 1$, we can deduce that $0 \leq x \leq 1$ and $0 \leq z \leq 1$,

Knowing that:

$$P(a) = P(a, a_*) - P(a^*, a) = z - x$$

we concluded that: $-1 \leq P(a) \leq 1$.

So, we can do a simple linear normalization by adding a unit and by dividing by 2. We get:

$$0 \leq 1/2(P(a) + 1) \leq 1$$

where: $P(a) = \sum W_j \left[H_j(d_*) - H_j(d^*)\right]$

This normalised performance will play the role of the score in our classification model. So, we will try to determine two critical values of performance: P_L (lower performance) and P_U (upper performance) which will limit the uncertainty zone $[P_L, P_U]$.

In order to minimize the number of misclassified firms and the uncertainty zone, we have been inspired by the extension of "DEA-Discriminant Analysis" (DEA-DA) developed by Sueyoshi [34]. Yet, our method can be perceived as a sort of combination of two programs: the MSD (Minimize the sum deviation) and the MMO (Minimize the Misclassified Observations), composed of two steps. The first one minimizes the uncertainty zone by minimizing the maximal deviation s. So this step can be formulated as follows:

Min s

Subject to

$$[1/2(P(a_i)+1)] - d + s \geq 0 \qquad \text{for all} \quad i \in G_1$$

$$[1/2(P(a_i)+1)] - d - s \leq 0 \qquad \text{for all} \quad i \in G_2$$

$$\sum_{j=1}^{k} W_j = 1$$

With $P(a_i) = \sum W_j \left[H_j(d_*) - H_j(d^*)\right]$

G_1: group of non-failed firms.
G_2: group of failed firms.
d: a positive critical score.
s: maximal deviation.
W_j: a vector of coefficients.

The application of the first step to a sample of firms allows us to classify them into three categories; one for non-failed firms, one for failed firms and one where we are not certain. The last one will be limited by $(d^* + s^*)$ and $(d^* - s^*)$ (these are the corresponding values P_U and P_L). If $s^* = 0$ there is no uncertainty zone and all the firms are correctly classified and therefore the program stops at this step. Otherwise we move to the second step to minimize the number of misclassified firms (those belonging to the uncertainty zone).

$$Min \sum_{i \in D_1 \cup D_2} X_i$$

Subject to

$$[1/2(P(a_i)+1)] - P_c(a) - P_i + M X_i = 0 \qquad \text{for all} \quad i \in D_{1m}$$

$$[1/2(P(a_i)+1)] - P_c(a) + \varepsilon - M X_i + N_i = 0 \qquad \text{for all} \quad i \in D_2$$

$$\sum_{j=1}^{k} W_j = 1$$

With $P(a_i) = \sum W_j \left[H_j(d_*) - H_j(d^*)\right]$

Where X_i is a binary variable which is equal to:

– 0 if the corresponding firm is well classified,

– 1 if the corresponding firm is misclassified.

$P_c(a)$: a critical performance.

P_i: a positive deviation.

N_i: a negative deviation.

W_j: the coefficient of the criterion C_j.

M: a large value.

ε : A quite small value.

$D_1 = \left\{ i \in G_1 / [1/2(P(a_i)+1)] \le d^* + s^* \right\}.$

$D_2 = \left\{ i \in G_2 / [1/2(P(a_i)+1)] \ge d^* - s^* \right\}.$

This last step allows us the establishments of credit to predict with less risk the probability of bankruptcy of firms belonging to the uncertainty zone.

To predict the financial situation of a new firm, we should calculate its performance $P(a)$ by using the W_j generated in the first step and we compare the classification thresholds (d^*+s^*) and $(d^* - s^*)$ with $[1/2(P(a)+1)]$.

If $[1/2(P(a_i)+1)] > d^* + s^*$ then the firm a_i is non-failing,

If $[1/2(P(a_i)+1)] < d^* - s^*$ then the firm a_i is failing.

And if $(d^* - s^* \le [1/2(P(a_i)+1)] \le d^* + s^*)$ then the firm a_i belongs to the uncertainty zone and the "decision-maker" should calculate the performance of the firm a_i by using the W_j generated in the second step. Then:

if $[1/2(P(a_i)+1)] > P_c^*$ the firm a_i can be predicted as non-failing,

if $[1/2(P(a_i)+1)] < P_c^*$ the firm a_i can be predicted as failing.

3. An empirical illustration

Our sample is made up of fifty four firms involved in the Saving system (this sample has been extracted from [26]). Three of them have been eliminated because they proved to be aberrant. The remaining fifty-one firms have been divided into two subsamples. The first subsample is composed of forty four firms is used for the determination of the value of parameters of the model and the second subsample of control composed of seven firms to validate the obtained results.

First, Martel and Khoury [26] considered twelve Financial Ratios, but they kept only three which were significant following the application of the software STEPDISC of SAS. These three ratios are:

X_2: TOTAL DEBT/TOTAL ASSET.

X_4: ACTIVE SHORT-TERM/PASSIVE SHORT-TERM.

X_8: LOG 10 (TOTAL ASSET).

Where X_2 should be minimized, X_4 maximized, and X_8 maximized. So for our illustration we have relied on these three variables already found significant.

The application of the first step of our method to this sample (Annex 1), gave an s^* equal to 0.000089 thus different to zero. Hence our sample will be discriminated in three categories according to the values of s^* and d^*. Having obtained a value d^* equal to 0.500099, we can classify our sample as follows:

The firms for which $[1/2(P(a_i)+1)] > 0.500188$ are non-failing firms,

The firms for which $[1/2(P(a_i)+1)] < 0.50001$ are failing firms,

Whereas the firms for which $[1/2(P(a_i)+1)] \in [0.50001\,;\,0.500188]$ have to go through the second step of our method in order to predict their financial situation.

Table 1 classifies the different firms of our sub-sample according to their performance and thus we can see the different categories: the firms which are in bold belong to the uncertain category; those which are above (1) are the non-failing firms whereas those which are below (0) are the failing firms.

Table 1

Result of the application of the first step of our method

FIRMS	CATEGORY	1/2(P(*a*)+1)
36	1	0,996960
12	1	0,995436
24	1	0,993684
50	1	0,983583
40	1	0,906997
38	1	0,852619
44	1	0,679543
20	1	0,544961
48	1	0,532709
34	1	0,529385
28	1	0,512472
52	1	0,503516

Table 1 contd.

FIRMS	CATEGORY	1/2(P(*a*)+1)
54	1	0,502765
42	1	0,500368
22	1	0,500295
25	**0**	**0,500188**
18	**1**	**0,500129**
14	**1**	**0,500103**
46	**1**	**0,500089**
1	**0**	**0,500083**
21	**0**	**0,500080**
10	**1**	**0,500019**
30	**1**	**0,500010**
26	**1**	**0,500010**
16	**1**	**0,500000**
49	0	0,499991
47	0	0,499987
41	0	0,499986
15	0	0,499981
45	0	0,499961
53	0	0,499913
17	0	0,499879
51	0	0,499850
37	0	0,499850
11	0	0,499637
35	0	0,499333
29	0	0,499194
19	0	0,499167
23	0	0,499066
39	0	0,498638
9	0	0,498471
27	0	0,497531
43	0	0,424236
33	0	0,000005

The performances of the firms are calculated with the weight provided by the application of the first step of our model ($W_1 = 0.994396$; $W_2 = 0.000050$; $W_3 = 0.005554$), where W_1: the coefficient of the ratio X_2, W_2: the coefficient of the ratio X_4 and W_3: the coefficient of the ratio X_8.

By applying the second step of our method to the firms belonging to the uncertainty zone and by refering to the discriminating threshold (critical performance) $P_c(a)$ equal to 0.499318, we obtain two misclassified firms (as shown in Table 2, firms 26 and 30 are really non-failing but predicted as failing).

Table 2

Result of the application of the second step of our method

FIRMS	CATEGORY	$1/2(P(_a)+1)$
10	1	0,501369
18	1	0,500788
46	1	0,500622
14	1	0,499995
16	1	0,499318
21	0	0,498318
1	0	0,498317
26	**1**	**0,497338**
25	0	0,495333
30	**1**	**0,465743**

The coefficients provided by the application of the second step of our model are: $W_1 = 0.924542$; $W_2 = 0.006659$ and $W_3 = 0.068799$.

To validate the results of our model, we have applied the coefficients obtained from the first step of our model to the sub sample test made up of seven firms. These firms, their normalized performances as well as their classification are given in Table 3.

Table 3

Classification of firms of sub sample test

FIRMS	CATEGORY	1/2(P(a)+1)
8	1	0,732815
2	1	0,635040
6	1	0,610619
4	1	0,583682
7	0	0,490657
3	0	0,480655
5	0	0,305725

According to the rule of classification provided by the first step of our model the firms 8, 2, 6 and 4 have been predicted as non-failed whereas the firms 7, 3 and 5 have been predicted as failing. For this sub sample, we notice that the seven firms have been well classified and none of them belongs to the zone of uncertainty.

Conclusion

This paper presented a new linear programme to solve a multi-criteria classification problem. In contrast to other discriminating linear programmes the proposed one provided not simply a description of the alternatives, but also performing information that helps in the identification of the most and the least performed alternatives. The concepts introduced by the PROMETHEE methods were used to develop an appropriate multi-criteria index for the classification of alternatives.

"In most of the existing approaches, the classification of the alternatives is determined on the basis of their comparison to some reference profiles (fictitious alternatives) that define the boundaries of the classes. Nevertheless, the definition of these reference profiles, as class boundaries, is not always clear" [10]. Therefore we used the techniques of linear programming to define the boundaries of classes and the weight W_j of the criterion C_j.

The proposed method consists of two steps. The first seeks to determine the narrowest uncertainty zone whereas the second seeks to minimize the number of misclassified firms belonging to this zone. Thus the proposed model provides in its first step two discriminating thresholds which permitted to classify firms into three categories and in the second step classifies again the firms belonging to the uncertainty zone.

This model applied to a sample of 44 firms, 22 of which are non-failed and 22 failed, generated results which seem promising. Indeed, we have managed to classify the two groups of firms obtaining only two misclassified ones. Finally, to validate the results obtained, we have used a sub sample of control and all the firms have been perfectly classified.

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Rafikul Islam

MODIFIED NOMINAL GROUP TECHNIQUE: WHAT AND HOW

Abstract

Continuous improvement in quality of products/services, more often than not, requires implementation of new ideas in the systems. Consequently, generation of good ideas is regarded as a crucial task in quality management. This paper shows how a traditional brainstorming technique can be integrated with the analytic hierarchy process in generating and subsequently prioritizing a large number of ideas. The working of the integrated technique has been shown by means of two exercises: (1) Identifying important factors to improve quality in teaching, and (2) Identifying important factors to improve quality in academic institutions administration. The method can be applied in many similar situations.

Keywords

Nominal group technique, analytic hierarchy process, quality in higher education, total quality management.

Introduction

In today's highly competitive business world, national/multinational companies increasingly demand that their employees find new and better ideas so that the jobs are done in better ways. The most common way to generate ideas is to take the relevant people in a room and brainstorm. Each mind is filled with ideas that can be valuable to decision making and problem solving. Brainstorming is one way to access this information, experience, and judgment.

The history of brainstorming dates back to 1954, when Osborn published his seminal work [8]. With illustrative examples, he explained how brainstorming could be used to help groups generate ideas. Osborn's central theme was that group can generate more ideas if their members concentrate on producing whatever ideas come into their minds while avoiding evaluation of their own and others' ideas. However, it is to be remembered that simply bringing people together does not assure maximum participation and quality group decision. Brainstorming sessions are often more storm than brain. It has been observed that the sessions are dominated by only a few individuals who impose their ideas upon the majority. To overcome the difficulties in this traditional brainstorming technique, researchers have developed a number of structured variants of it including Delphi technique [6] and Nominal Group Technique (henceforth NGT) [2]. The NGT has alleviated many of the difficulties present in the traditional brainstorming technique. Before proceeding further, a brief description of NGT is provided.

1. A brief description of NGT

In business today, it is necessary to stimulate employees to generate fresh, creative, and productive ideas for the benefit of the organization. NGT is a management tool that is being increasingly used to generate a large number of ideas. The technique is helpful in identifying problems, exploring solutions and establishing priorities among the solutions generated. It structures group interactions to elicit the information and judgments of individual participants and to promote the development of a consensus among all group members. The technique has the following steps (the steps are also shown in Figure 1):

1. Enunciation of the statement of the question pertaining to the issue.
2. Silent generation of ideas in writing.
3. Round-robin recording of ideas.
4. Serial [consecutive?] discussion of the ideas.
5. Voting to select the most important ideas.
6. Discussion and reaching consensus on the selected ideas.

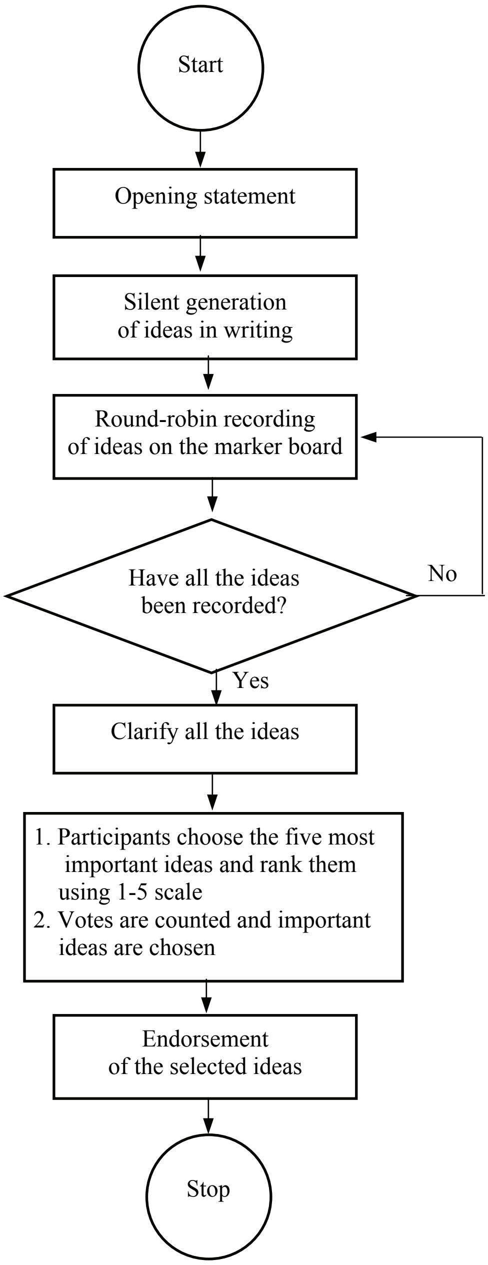

Figure 1. Steps in Nominal Group Technique

For a successful nominal group session, the following rules should be observed:

- No criticism on any idea during the session.
- The more unusual and original the idea, the better.
- While generating ideas, quantity not quality is the primary objective.
- Dissecting, modifying and commingling of ideas is desirable.
- Anonymity of input.
- Defer in-depth evaluation until all the inputs are displayed.

Although the nominal group technique has been applied in manifold areas it has received criticisms from many researchers. To alleviate its limitations, a number of modifications have been proposed. A brief account of proposed modifications has been provided in the following.

2. Previous modifications of NGT

Since the introduction of NGT, a number of modifications of the technique have been proposed. Fox [3] proposed to use 3×5 cards to provide all the ideas by one person at one time instead of round-robin recording of ideas. Although it ensures anonymity of the participants, its shortcoming is that one cannot get stimulated by other's ideas. To increase group member participation, Bartunek and Murninghan [1] suggested one of the two possible voting procedures: (1) Vote for an idea at one time with a minimum number of votes for selection. (2) Vote as described in (1) but eliminate the ideas with only few votes prior to the additional voting. In addition to the above, NGT has been combined with other methodologies. Some of the integrated methods are: NGT and Multi-attribute utility theory [11]; NGT and Multi-dimensional scaling [4]. Also in numerous studies, NGT has been compared with Delphi technique.

One major issue pertaining to NGT in its existing form is that it assigns ordinal weights to the most important ideas. For example, 5 is assigned to the most important idea, 4 for the second most important one, and so on. This weighting scheme means that the difference (5-4) between the quality of most important and the second most important is same as the difference (2-1) between the 4^{th} most and the 5^{th} most. However, in reality, this may not be true. In order to overcome this difficulty, we propose to integrate NGT with the Analytic Hierarchy Process (AHP), a popular multi-criteria decision making method. The revised technique has been called Modified Nominal Group Technique (MNGT).

3. Proposed modification of the NGT

In the fifth step of NGT, the participants need to find out and rank the 5 most important ideas. In the existing framework of the methodology, there is no specific guideline to rank the best 5 ideas, rather they (the participants) have to do it by a holistic approach. The main objective of this paper is to show how Analytic Hierarchy Process (AHP) [10, 12] can be integrated with NGT to alleviate the above drawback.

Specifically, we propose to choose the five most important five ideas from the pool of ideas as it is done in the NGT. But unlike NGT, we compare these important ideas in a pairwise fashion, i.e., each idea in the chosen five is compared with all others one-by-one using Saaty 1-9 absolute scale. This will result in a pairwise comparison matrix from which the necessary priorities can be computed. Later, we have provided the advantages of this integrated method to generate and prioritize ideas in a nominal group setting.

To show the working of the integrated method, we have conducted two exercises.

Exercise 1

Teaching is an essential part in any academic institution. The quality of out-going students depends largely on the quality of teaching in the classroom. The problem of improving quality in teaching in a classroom is long-standing [5, 7, 9]. The topic has drawn considerable interest from many researchers. With the development of newer technologies, research will continue on the topic. Staying on the same issue, we conducted a nominal group session. Thirteen (final year) students from the author's undergraduate class on Quality Management and two Master's of Management students took part in the session. In the following, all the steps plus the proposed modification have been described:

Step 1. As the facilitator of the session, I (the author) posed the following question at the start of the session, "what factors contribute to quality teaching in a university classroom?"

Step 2. The participants were given 10 minutes to generate ideas on the issue.

Step 3. The whole session lasted about 85 minutes. Due to time constraint, I conducted only 3 rounds of round-robin recording of ideas. The ideas are shown in Table 1.

Table 1

Factors for quality teaching in an academic institution

No.	Factor	MNGT Weights	MNGT Ranking
1.	Study materials and lecture should be well coordinated	0.061	
2.	Avoid bias	0.035	
3.	Lecturer should be a responsible person	0.398	8
4.	Lecturer should have relevant and in-depth knowledge	0.559+0.567+0.191+0.579+0.567+ +0.495+0.352+0.355+0.461= =4.126	1
5.	Use relevant and clear visual aids	0.032+0.055+0.174+0.053 =0.314	10
6.	Equipment provided and used		
7.	2-way communication	0.541+0.239+0.288+0.133+ +0.106=1.307	4
8.	Create conducive environment	0.083	
9.	Use of teaching aid, e.g. PowerPoint slides with OHP		
10.	Lecturer should make class interesting	0.044+0.249+0.063+0.057+0.047= =0.460	7
11.	Fun learning environment		
12.	Lecture should be delivered in such a manner that students can understand	0.092+0.045+0.139=0.276	12
13.	Encourage creativity and openness	0.079+0.101=0.180	15
14.	Attitude of the students	0.205	13
15.	Smaller number of students	0.048+0.041=0.089	
16.	Respect each other	0.038	
17.	Flexibility of the lecturer		
18.	Encourage students to participate	0.062+0.067+0.066=0.195	14
19.	Time management	0.145+0.048+0.037+0.085= =0.315	10
20.	Efficient and effective delivery of knowledge	0.297+0.477+0.106+ +0.221+0.380=1.481	3
21.	Use simple examples	0.146	
22.	Relate subject to the practical problems	0.079	
23.	Lecturer should gauge students' proficiency level		
24.	Lecturer should be able to recognize all the students in the class	0.047	
25.	Lecturer should have proper control over the class		
26.	Proper planning on the lecturer's part	0.205+0.079+0.235=0.519	6
27.	Reasonable duration of the class		
28.	Lecturer should be able to convince the students with his /her ideas		
29.	Give some group work		

30.	Lecturer should ask thought provoking, interesting questions		
31.	Effective communication skills	0.438+0.556+0.447+ +0.311+0.439+0.313=2.504	2
32.	Proper choice of time slot	0.038+0.062=0.100	
33.	Lecturer should be aware about students' proficiency level	0.124	
34.	Personality of the lecturer	0.393	9
35.	Lecturer is well prepared	0.061+0.216+0.166+0.269= =0.712	5
36.	Students centered approach in teaching	0.084+0.218=0.302	11
37.	Lecturer should discuss the answers of the mid-term and quiz question papers		
38.	Comfortable class room	0.060	
39.	Variety of teaching methods	0.121+0.102+0.175=0.398	9
40.	Deliver lectures at a reasonable voice and speed		
41.	No interruption during the lecture		

Legend: A = Lecturer should have relevant and in-depth knowledge, B = 2-way communication, C = Choice of proper time slot, D = Variety of teaching methods, E = Respect for each other, 'O' = quality teaching.

Step 4. A few ideas on the table were clarified, so that all the participants had proper understanding about them. The purposes of this step are to (1) ensure that all the participants have proper understanding of all the ideas, and (2) make sure that the meaning of a particular idea is the same to all (i.e., no idea should be construed differently by different participants). In particular, it was agreed that the idea 'Efficient and effective delivery of knowledge' did not include 'effective communication skills'. 'Avoid bias' (No. 2) means that the lecturer should be fair in dealing with all the students. The idea No. 11 means that the lecturer should be witty.

Step 5. In this step (where modification is proposed), each participant is required to select the five best ideas and rank them in order of importance. Usually, a 1-5 scale is adopted to perform the task. The most important idea is assigned a rating of 5 and the least important, 1. The three intermediate ideas receive 4, 3, and 2, respectively. Instead of doing so, the task can be performed in two stages, namely: (i) out of the 41 ideas on the board (visible to all), choose the 5 most important ideas but do not rank them as above, (ii) using the Saaty 1-9 ratio scale, compare these 5 ideas pairwise. In the following, we provide one participant's pairwise comparison matrix and the weights of the five ideas:

Cards from all the thirteen participants were collected and the weights of the ideas were calculated on an individual basis. The weights were written on the board. The overall weight of an idea was calculated by adding the individual weights obtained from the participants. For example, the overall weight for '2-way communication' is 1.307 (0.541+0.239+0.288+0.133+0.106). The most important ideas selected are shown in Table 2. It is to be noted that each participant is required to select only five best ideas and compare them using AHP, irrespective of the total number of ideas on the master list.

Step 6. A few minutes were spent to discuss the selected ideas.

Table 2

Top 10 ranked factors for quality teaching

No.	Factor	Absolute Weight	Relative Weight	Requirement in Percentage	Rank
1.	Lecturer should have relevant and in-depth knowledge	4.126	0.338	33.8	1
2.	Effective communication skill	2.504	0.205	20.5	2
3.	Efficient and effective delivery of knowledge	1.481	0.121	12.1	3
4.	2-way communication	1.307	0.107	10.7	4
5.	Lecturer is well prepared	0.712	0.058	05.8	5
6.	Proper planning on the lecturer's part	0.519	0.042	04.2	6
7.	Lecturer should make class interesting	0.460	0.038	03.8	7
8.	i. Lecturer should be responsible ii. Variety of teaching methods	0.398	0.033	03.3	8
9.	Personality of the lecturer	0.393	0.032	03.2	9
10.	Time management	0.315	0.026	02.6	10
	Total	12.215	1.000	100	

Exercise 2

Basically, staff members of an academic institution are divided into two categories: academic staff and administrative staff. Academic staff are involved in teaching and research, while administrative staff are mainly involved in administering the institution. However, there are some academic staff who are also involved in administration; mainly they hold high positions on the administrative ladder, e.g., vice-chancellor, deputy vice-chancellor, dean, etc.

In the previous section, we have dealt with quality in teaching which is pertinent to academic staff. In this section, we will deal with administration. Specifically, our objective is to identify the factors that contribute to quality in administration. Towards this end, we have again applied the modified nominal group technique to generate the factors. In this exercise, 12 students participated. Since the details of various steps are provided in the previous section, here we provide only the output of the session. After performing all the 5 steps, we obtained the factors with their corresponding weights, as shown in Table 3.

Table 3

Factors for quality administration in an academic institution

No.	Factor	MNGT Weights	MNGT Ranking
1.	Motivated administrative staff	0.184+0.474+0.285+0.242+ +0.256+0.074 = 1.515	2
2.	Good reward system	0.096+0.500+0.396+0.451+0.142= = 1.585	1
3.	Well-equipped administration		
4.	Good communication	0.060	
5.	Have fun	0.068	
6.	Good relationship among people of various divisions	0.117+0.191 = 0.308	13
7.	High responsibility	0.270+0.273+0.301+0.255 = 1.099	4
8.	No communication gap between teachers and students	0.079+0.417+0.088 = 0.584	7
9.	Clearly stated Vision and Mission statements	0.372+0.078 = 0.450	12
10.	Full utilization of resources		
11.	Courtesy	0.023+0.144 = 0.167	15
12.	Quick process of application forms		

13.	Good leadership qualities	0.144	
14.	Quick in response	0.111	
15.	High employees' involvement		
16.	Effective registration in each semester		
17.	Qualified/efficient staff	0.493	11
18.	Effective system for receiving students' feedback		
19.	Friendly/helpful staff	0.091+0.214+0.327+0.543 = 1.175	3
20.	No technical problems during pre-registration		
21.	Proper planning		
22.	Employee empowerment	0.072+0.092 = 0.164	
23.	Knowledge of students' needs		
24.	Sufficient equipments provided to keep all the department' records updated	0.167	15
25.	Strong support from upper level management		
26.	Training facilities to the employees		
27.	Awareness among employees regarding quality improvements		
28.	Long term planning	0.055	
29.	Timely communication of grades	0.214+0.103+0.190 = 0.507	10
30.	Secured student records/files	0.045	
31.	Good facilities		
32.	Enough staff	0.484+0.207 = 0.691	6
33.	Clear policy towards quality		
34.	Robust policies	0.123+0.164 = 0.287	14
35.	Ensure trust	0.394+0.121+0.067 = 0.582	8
36.	Positive attitude towards teamwork	0.121	
37.	Top management commitment to quality	0.113	
38.	Full knowledge about all facilities	0.252+0.188+0.365 = 0.805	5
39.	Sufficient information to the students		
40.	Ethical behavior	0.040	
41.	Reduced absenteeism	0.119	
42.	Rapid maintenance process		
43.	Completeness in service	0.060+0.177+0.195+0.072+0.044 = = 0.548	9
44.	No gap between actions and words		

From the overall weights of the factors, we select the most important 10, which are shown in Table 4.

Table 4

Top 10 ranked factors of quality administration

No.	Factor	Absolute Weight	Relative Weight	Requirement in Percentage	Rank
1.	Good reward system	1.585	0.174	17.4	1
2.	Motivated administrative staff	1.515	0.167	16.7	2
3.	Friendly/helpful staff	1.175	0.129	12.8	3
4.	High responsibility	1.099	0.121	12.1	4
5.	Full knowledge about all facilities on campus	0.805	0.088	8.8	5
6.	Enough staff	0.691	0.076	7.6	6
7.	No communication gap between teachers and students	0.584	0.064	6.4	7
8.	Ensure trust	0.582	0.064	6.4	8
9.	Completeness in service	0.548	0.062	6.2	9
10.	Timely communication of grades	0.507	0.056	5.6	10
	Total	9.091	1.000	100	

Overall, the participants' view is that the administrative staff play a crucial role in realizing comprehensive excellence in an academic institution. For this matter, the staff must be motivated in discharging their duties and responsibilities. University's top management should implement a 'good reward system' in order to motivate its administrative staff. Since it is extremely important to have 'friendly/helpful' staff, especially at the counters, these staff must be provided with sufficient and relevant training. Training should not be an occasional job, it should be imparted on the continuous basis. According to the findings, staff should be trained to ensure the following:

– Adequate knowledge of the system within which the staff is working.
– Provide complete service to the students.
– Courteous behavior.
– Minimize application processing time.
– Satisfy customers, especially students, by fulfilling their needs promptly.

4. Advantages of applying AHP in step 5 of NGT

1. In the traditional NGT, the five most important ideas are selected by using the 1-5 ordinal scale. In this procedure, merit or superiority of one idea is not judged with respect to the other four ideas separately. Consequently, relative weights are not obtained. On the other hand, in AHP ideas are compared in a pairwise fashion, i.e., one idea is compared with each of the other ideas separately. This increases the exactness of the results and gives the relative superiority of one idea over another.
2. In the traditional NGT, the important ideas are assigned 5, 4, 3, 2, and 1 leaving no room for equal weights. But in the modified NGT, if the participants feel that two ideas are equally important, then they can enter 1 in the appropriate cell of the pairwise comparison matrix.
3. In the traditional NGT, two distinct ideas can receive the same weight: 2+1+1+1 (from four persons) and 5 (from a single person). In this case, neither is regarded superior over the other. In the MNGT, chances of having tie are minimal due to the usage of cardinal weights.
4. In NGT, there is a very high chance that a large number of ideas will receive the same overall weight, whereas in MNGT this chance is minimal.
5. Following NGT, let us assume that the ranking made by two participants for the five ideas A, B, C, D, and E are respectively (5, 4, 3, 2, 1) and (4 ,5, 3, 2, 1). It is to be noted that exactly the same ranking has been assigned to the idea D. Following the MNGT, the weights of the ideas for the same participants could be (0.53, 0.23, 0.15, 0.05, 0.04) and (0.28, 0.35, 0.16, 0.09, 0.07). So, for the second participant, the idea E has received more weightage than the weightage assigned to D by the first participant. So, ultimately, 'E' may emerge superior than 'D'. But in NGT, 'E' will remain inferior as compared to D.
6. In NGT, the weights of 5 most important ideas are 5, 4, 3, 2, and 1. Therefore, the relative weights are 0.333, 0.267, 0.2, 0.133, and 0.067. In all cases, this relative standing remains constant for the five best ideas selected by all the participants. This fact is contrary to human perception about relative weights of two different entities. MNGT overcomes this difficulty.

5. Further possible applications of MNGT in managing quality in higher education

There are numerous situations in an institution of higher learning, where MNGT can be applied. This paper has described only two applications. Following is a list of further possible applications of MNGT in an academic setting:

1. SWOT analysis: Identify strengths, weaknesses, opportunities and threats for certain department/unit/institution.
2. Solve problem such as 'why are enrollments decreasing in the business (for example) courses' or 'how to increase enrollments in business courses?'
3. What should be the vision, mission, goals for the department/unit/institution?
4. What is the most crucial problem the department is facing?
5. What are the suggestions to improve the working conditions in the department?
6. How can the surplus budget of a certain financial year be utilized?
7. What are the ways through which a local university can generate funds?
8. How can the overall communication be improved in the institution?
9. What are the issues that are to be resolved in order to ensure that the students leave the institution with a 'good' feeling?
10. What are the ways to check the high turnover in an institution?
11. How can campus security be improved?
12. How can food services on the campus be improved?
13. What measures of performance would be appropriate for the department?

Conclusion

Nominal group technique is a powerful idea generation technique that has been used by practitioners in diverse areas. Its power is further enhanced by the integration with the popular multi-criteria decision making method, the analytic hierarchy process. The integration alleviates some of the difficulties present in the traditional NGT. The working of the integrated technique has been shown by means of two examples in academic setting. In addition to this, many areas in an academic environment have been identified where the method can be applied. Although the applications described came only from the academic area, the technique can obviously be applied in many other areas as well. We hope that the integrated method will draw due attention of the practitioners in those areas.

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Jerzy Michnik

TECHNOLOGY ASSESSMENT PROCESS FOR NEW PRODUCTION LINE DEVELOPMENT – ANALYTIC NETWORK PROCESS APPROACH

Abstract

In a modern industry technology assessment have become an important factor of successful technological development. In this paper the problem of choosing right technology by manufacturing enterprise is studied. The use of Analytic Network Process is proposed as a supporting tool for the decision under several conflicting and interrelated criteria.

Keywords

Technology assessment, technology development, ANP.

Introduction

A modern enterprise working in industrial field faces a large number of technical and organizational challenges. The production of an even relatively simple article very often needs quite complicated technology. Rapidly changing and highly competitive environment demands fast and optimal response. While making decisions it is necessary to consider several criterions including technology, finance, personnel, environment and many others.

Technology Assessment (TA) is being increasingly viewed as an important tool in the shift towards technology development. However, this is a wide concept, evolving on different levels: national, industrial and corporate. This article aims to provide some practical guidance for choosing technology when the new production line is developed.

We consider the decision situation when the managers have to make several decisions in order to choose a technology being the part of new production line. Those decisions can be considered – to some extent – as independent, as very often there are various alternatives which are, in principle, replaceable. They all satisfy the basic demands but differ in many details and features. It is important to find the exhaustive set of criteria characterizing a technological solution which help to make the best decision. To get better insight into the problem criteria are usually further divided into a number of sub-criteria which describe the alternatives more accurately.

The number of criteria, sub-criteria and their interrelations are mapped into the framework based on Analytic Network Process (ANP) [6]. As a result we can practically support the decision concerning a choice of suitable technology for the new production line. All calculations were performed with the aid of "Super Decisions" software (ver. 1.6.0) [9].

1. Technology assesment process

1.1. Literature review

There are many empirical studies on TA [7, 10, 11]. Linstone's perspective [3] considers the dimensions of science and technology to be the kind of threat we may confront – countries, groups, or individuals with aggressive goals, intense commitment, rational or irrational selection of tools and strategies, preference for high risks, and unconventional tactics. Reference [4] presents a conceptualizing sustainability assessment and uses the concept of "integrated assessment" to achieve the impact of strategic assessment. Smits and Leijten [8] focus on TA as a process consisting of analyses of technological development and its consequences and of debate in with respect to these consequences. It provides information that could help the company involved in developing their strategies. Coates thinks that TA is a class of policy studies which systematically examines the effects on society that may occur when a technology is introduced, extended or modified. It emphasizes those consequences that are unintended, indirect or delayed. Cetron and Connor [1] attempt to establish an early warning system to detect, control, and direct technological changes and developments so as to maximize the public good while minimizing the public risks. Daddario [2] proposes a form of policy research which provides a balanced appraisal to the policy maker. Ideally, it is a system to ask the right questions and obtain correct and timely answers. It identifies policy issues, assesses the impact of alternative courses of action and presents findings. It is a method of analysis that systematically appraises the nature, significance, status, and merit of a technological program. Reference

[5] defines a framework by the hierarchical structure of the enterprise to assess the impact of manufacturing technology on the productivity and competitiveness of the enterprise.

1.2. Design of production line and choice of technology

When the new production line is under consideration, the management faces several partial decisions which involve a complex analysis of technical, financial and others aspects. In our analysis we focus on the particular choice of technology for an auxiliary system which is a part of a production line. Such a kind of decision can be considered as independent and the main goal is to find the best suitable alternative. Following the discussions with experts, four important criteria have been found: technical merits, investment cost, operating costs, environmental impact.

A more detailed analysis leads to a decomposition of each criterion to a number of subcriteria which characterize the available solutions more accurately. This part of the decision model is strongly case sensitive, which means that both the meaning and the number of sub-criteria and alternatives depend on the real life problem which is under the consideration. It is shown in Section 3 how this detailed analysis works in practice.

2. Analytic network process as a multiple criteria decision tool [6]

ANP is defined as a multiple criteria theory of measurement used to derive priority scales of absolute numbers from individual judgments which represent the relative influence, of one of two elements over the other in a pairwise comparison process on the third element in the system, with respect to an underlying control criterion. In the ANP one provides the judgment by answering two kinds of questions with regard to strength of dominance:

1. Given a criterion, which of the two elements is more dominant with respect to a criterion?
2. Which of the two elements influences the third element more with respect to a criterion?

It is assumed that there is a system of N clusters, whereby the elements in each cluster interact or have an impact on or are themselves influenced by some or all of the elements of that cluster or of another cluster with respect to a property governing the interactions of the entire system. The cluster is denoted by $C_h, h=1,...,N$ and it has n_h elements that are denoted $e_{h1}, e_{h2}, ..., e_{hn_h}$.

A priority vector is derived from paired comparisons matrix by normalizing its columns and taking the geometric mean form rows (in the same way as in AHP). It represents the impact of a given set of elements in a cluster on another element in the system. Each priority vector is entered as a part of some column of a supermatrix. The supermatrix (denoted by *W*) represents the influence priority of an element on the left of the matrix on the element at the top of the matrix. The structure of the supermatrix is presented in equation (1). Each element W_{ij} of the supermatrix *W* is a matrix itself. W_{ij} represents the influence of the elements from cluster *i* on the elements from cluster *j*.

$$W = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1N} \\ W_{21} & W_{22} & \cdots & W_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ W_{N1} & W_{N2} & \cdots & W_{NN} \end{bmatrix}, \quad W_{ij} = \begin{bmatrix} w_{11}^{ij} & w_{12}^{ij} & \cdots & w_{1n_j}^{ij} \\ w_{21}^{ij} & w_{22}^{ij} & \cdots & w_{2n_j}^{ij} \\ \vdots & \vdots & \vdots & \vdots \\ w_{n_i1}^{ij} & w_{n_i2}^{ij} & \cdots & w_{n_in_j}^{ij} \end{bmatrix} \tag{1}$$

In the next step the supermatrix is transformed into a weighted supermatrix, i.e. to the matrix, whose columns sums to unity. Initially the supermatrix columns are made up of several eigenvectors which, in normalized form, sum to one and hence that column sums to the number of nonzero eigenvectors. The weighted supermatrix (called also ‘a column stochastic matrix’) can be obtained by weighting the initial supermatrix with a cluster matrix. The cluster matrix contains eigenvectors representing the priorities of clusters with respect to the general control criterion (in most cases it will be a main goal).

In the end the limit matrix is derived by raising the weighted matrix to an arbitrarily high power. This procedure sums up the influences along paths of different length in the underlying network and determines the overall priorities.

3. Illustrative example

3.1. Density measurement technology

As a practical example of technology assessment we have chosen the density measurement of construction material which will be the product of a newly designed production line. Three modern methods of measuring the density of solid state are considered:

– beta-ray based method with the radioactive isotope as the source of beam,
– X-ray based method,
– microwaves based method.

The above methods differ with respect to all criteria considered. Technically they are different in gauge scale and precision. They are produced by different suppliers and their user interfaces present final data differently. They also have different additional characteristics. For example: both beta-ray and X-ray devices scan the sample locally with the beam according to predefined scheme; the microwave device advantage is that it can make image of the whole sample volume.

In the beta-ray device, the radioactive isotope source should be replaced periodically according to its decay constant (which measures the speed of the decay). A worn out radioactive source must be utilized properly as it is dangerous for the environment. The X-ray source does not produce radioactive waste but it needs a high voltage power supply to produce an X-ray beam. These technical differences influence the environment in different ways and also result in different operating costs.

3.2. ANP Model

As it was discussed in subsection 1.2, the main goal is the choice of the best technological solution for the new production line. This decision requires the analysis of four important criteria which have been found to be: technical merits, investment cost, operating costs, and environmental impact. Following the AHP ideas the main goal is the first hierarchy level. The criteria take place at the second level of the hierarchy. At the next level each criterion has been split into a number of sub-criteria. In the example discussed the following subcriteria have been developed:

– Technical Merits: gauge scale, gauge precision, user interface, additional advantages.
– Investment Cost: purchase price, installation costs, training costs.
– Operating Costs: energy consumption, water consumption, other media consumption, maintenance, guarantee.
– Environmental Impact: labor environment, natural environment.

The bottom level comprises the alternatives considered. They are shortly denoted: Beta-ray, X-ray and Microwave. The overall structure of the ANP model is presented in Fig. 1. The main blocks of the ANP structure (GOAL, OPERATING COSTS, …, ALTERNATIVES) are called clusters (or components). They comprise one or more nodes which are represented in our model by "Technology Assessment" – the only node in the GOAL cluster, subcriteria grouped in the clusters they belong to, and three alternatives contained in the ALTERNATIVES cluster.

At first sight the model looks as it was pure AHP, but the interaction of environmental impact and operating costs cannot be mirrored by the AHP and a more general structure is required. It is not enough to compare the alternatives with respect to their environmental impact and with respect to their operating costs. We need also to judge the water, energy and other media consumption in respect to their environmental impact. And this influence may be different from mere financial assessment.

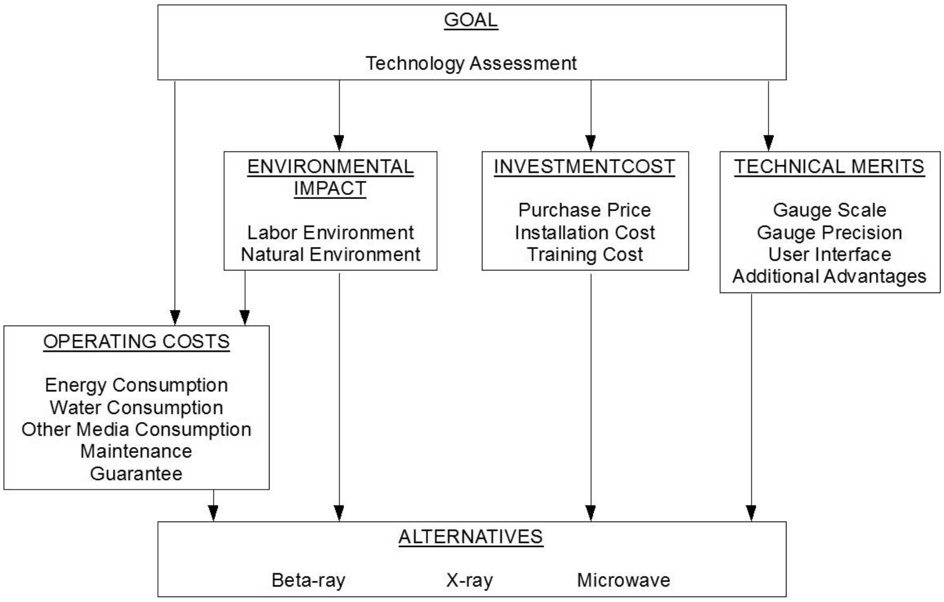

Figure 1. Technology Assessment – the ANP model structure

3.3. Numerical results for the ANP Model

At the beginning the pairwise comparisons of criteria with respect to the goal have been performed. From that data, priorities of criteria have been derived. The eigenvectors representing these priorities form columns of the cluster matrix which is presented in Table 1.

Table 1

Cluster Matrix for Density Measurement Technology Assessment

	Goal	Operating Costs	Investment Costs	Environmental Impact	Technical Merits	Alternatives
Goal	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Operating Costs	0.225353	0.000000	0.000000	0.500000	0.000000	0.000000
Investment Costs	0.178104	0.000000	0.000000	0.000000	0.000000	0.000000
Environmental Impact	0.094196	0.000000	0.000000	0.000000	0.000000	0.000000
Technical Merits	0.502346	0.000000	0.000000	0.000000	0.000000	0.000000
Alternatives	0.000000	1.000000	1.000000	0.500000	1.000000	0.000000

Then a substantial number of pairwise comparisons should be performed. First of all they comprise assessment of all alternatives with respect to all sub-criteria which makes altogether $14 \times 3 = 42$ comparisons. With addition of 3 comparisons of media consumption with respect to natural environment there are 45 comparisons in total. The resulting priority eigenvectors are placed in columns of the unweigted supermatrix (Tables 2-4).

Table 2

Unweighted Supermatrix for Density Measurement Technology Assessment

	Choice of Technology	Energy Con-sumption	Guarantee	Maintenance	Other Media Consumption	Water Con-sumption
Choice of Technology	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Energy Consumption	0.322192	0.000000	0.000000	0.000000	0.000000	0.000000
Guarantee	0.077448	0.000000	0.000000	0.000000	0.000000	0.000000
Maintenance	0.341793	0.000000	0.000000	0.000000	0.000000	0.000000
Other Media Consumption	0.101721	0.000000	0.000000	0.000000	0.000000	0.000000
Water Consumption	0.156846	0.000000	0.000000	0.000000	0.000000	0.000000
Installation Costs	0.199991	0.000000	0.000000	0.000000	0.000000	0.000000
Purchase Costs	0.600012	0.000000	0.000000	0.000000	0.000000	0.000000
Training Costs	0.199997	0.000000	0.000000	0.000000	0.000000	0.000000
Labor Environment	0.500000	0.000000	0.000000	0.000000	0.000000	0.000000
Natural Environment	0.500000	0.000000	0.000000	0.000000	0.000000	0.000000
Additional Advantages	0.087717	0.000000	0.000000	0.000000	0.000000	0.000000
Gauge Precision	0.462131	0.000000	0.000000	0.000000	0.000000	0.000000
Gauge Scale	0.293755	0.000000	0.000000	0.000000	0.000000	0.000000
User Interface	0.156396	0.000000	0.000000	0.000000	0.000000	0.000000
Beta-ray	0.000000	0.238476	0.310814	0.428567	0.249299	0.249299
Microwave	0.000000	0.625026	0.195800	0.142851	0.593647	0.593647
X-ray	0.000000	0.136498	0.493386	0.428582	0.157054	0.157054

Table 3

Unweighted Supermatrix for Density Measurement Technology Assessment

	Installation Costs	Purchase Costs	Training Costs	Labor Environment	Natural Environment
Choice of Technology	0.000000	0.000000	0.000000	0.000000	0.000000
Energy Consumption	0.000000	0.000000	0.000000	0.000000	0.625013
Guarantee	0.000000	0.000000	0.000000	0.000000	0.000000
Maintenance	0.000000	0.000000	0.000000	0.000000	0.000000
Other Media Consumption	0.000000	0.000000	0.000000	0.000000	0.136500
Water Consumption	0.000000	0.000000	0.000000	0.000000	0.238487
Installation Costs	0.000000	0.000000	0.000000	0.000000	0.000000
Purchase Costs	0.000000	0.000000	0.000000	0.000000	0.000000
Training Costs	0.000000	0.000000	0.000000	0.000000	0.000000
Labor Environment	0.000000	0.000000	0.000000	0.000000	0.000000
Natural Environment	0.000000	0.000000	0.000000	0.000000	0.000000
Additional Advantages	0.000000	0.000000	0.000000	0.000000	0.000000
Gauge Precision	0.000000	0.000000	0.000000	0.000000	0.000000
Gauge Scale	0.000000	0.000000	0.000000	0.000000	0.000000
User Interface	0.000000	0.000000	0.000000	0.000000	0.000000
Beta-ray	0.139642	0.249310	0.527836	0.249299	0.113002
Microwave	0.332525	0.157056	0.139648	0.593647	0.651929
X-ray	0.527833	0.593634	0.332516	0.157054	0.235069

Table 4

Unweighted Supermatrix for Density Measurement Technology Assessment

	Additional Advantages	Gauge Precision	Gauge Scale	User Interface	Beta-ray	Microwave	X-ray
Choice of Technology	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Energy Consumption	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Guarantee	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Maintenance	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Other Media Consumption	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Water Consumption	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Installation Costs	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Purchase Costs	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Training Costs	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Labor Environment	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Natural Environment	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Additional Advantages	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Gauge Precision	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Gauge Scale	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Additional Advantages	Gauge Precision	Gauge Scale	User Interface	Beta-ray	Microwave	X-ray
User Interface	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Beta-ray	0.166667	0.189711	0.319618	0.330897	0.000000	0.000000	0.000000
Microwave	0.666667	0.547216	0.121957	0.288919	0.000000	0.000000	0.000000
X-ray	0.166667	0.263073	0.558425	0.380184	0.000000	0.000000	0.000000

According to the ANP procedure described in Section 2 cluster matrix has been used to produce the weighted (column stochastic supermatrix). As there is not enough room here the presentation of this matrix is omitted. The last step in ANP is to derive the limit supermatrix by taking arbitrarily high power of the weighted supermatrix. This matrix comprises in its columns the final priority eigenvectors of the problem. The content of the limit supermatrix is presented in Tables 5-7.

Table 5

Limit Supermatrix for Density Measurement Technology Assessment

	Choice of Technology	Energy Consumption	Guarantee	Maintenance	Other Media Consumption	Water Consumption
Choice of Technology	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Energy Consumption	0.043155	0.000000	0.000000	0.000000	0.000000	0.000000
Guarantee	0.008625	0.000000	0.000000	0.000000	0.000000	0.000000
Maintenance	0.038064	0.000000	0.000000	0.000000	0.000000	0.000000
Other Media Consumption	0.012917	0.000000	0.000000	0.000000	0.000000	0.000000
Water Consumption	0.020243	0.000000	0.000000	0.000000	0.000000	0.000000
Installation Costs	0.017602	0.000000	0.000000	0.000000	0.000000	0.000000
Purchase Costs	0.052811	0.000000	0.000000	0.000000	0.000000	0.000000
Training Costs	0.017603	0.000000	0.000000	0.000000	0.000000	0.000000
Labor Environment	0.023275	0.000000	0.000000	0.000000	0.000000	0.000000
Natural Environment	0.023275	0.000000	0.000000	0.000000	0.000000	0.000000
Additional Advantages	0.021776	0.000000	0.000000	0.000000	0.000000	0.000000
Gauge Precision	0.114724	0.000000	0.000000	0.000000	0.000000	0.000000
Gauge Scale	0.072925	0.000000	0.000000	0.000000	0.000000	0.000000
User Interface	0.038825	0.000000	0.000000	0.000000	0.000000	0.000000
Beta-ray	0.131134	0.238476	0.310814	0.428567	0.249299	0.249299
Microwave	0.189201	0.625026	0.195800	0.142851	0.593647	0.593647
X-ray	0.173847	0.136498	0.493386	0.428582	0.157054	0.157054

Table 6

Limit Supermatrix for Density Measurement Technology Assessment

	Installation Costs	Purchase Costs	Training Costs	Labor Environment	Natural Environment
Choice of Technology	0.000000	0.000000	0.000000	0.000000	0.000000
Energy Consumption	0.000000	0.000000	0.000000	0.000000	0.208338
Guarantee	0.000000	0.000000	0.000000	0.000000	0.000000
Maintenance	0.000000	0.000000	0.000000	0.000000	0.000000
Other Media Consumption	0.000000	0.000000	0.000000	0.000000	0.045500
Water Consumption	0.000000	0.000000	0.000000	0.000000	0.079496
Installation Costs	0.000000	0.000000	0.000000	0.000000	0.000000
Purchase Costs	0.000000	0.000000	0.000000	0.000000	0.000000
Training Costs	0.000000	0.000000	0.000000	0.000000	0.000000
Labor Environment	0.000000	0.000000	0.000000	0.000000	0.000000
Natural Environment	0.000000	0.000000	0.000000	0.000000	0.000000
Additional Advantages	0.000000	0.000000	0.000000	0.000000	0.000000
Gauge Precision	0.000000	0.000000	0.000000	0.000000	0.000000
Gauge Scale	0.000000	0.000000	0.000000	0.000000	0.000000
User Interface	0.000000	0.000000	0.000000	0.000000	0.000000
Beta-ray	0.139642	0.249310	0.527836	0.249299	0.118512
Microwave	0.332525	0.157056	0.139648	0.593647	0.421729
X-ray	0.527833	0.593634	0.332516	0.157054	0.126425

Table 7

Limit Supermatrix for Density Measurement Technology Assessment

	Additional Advantages	Gauge Precision	Gauge Scale	User Interface	Beta-ray	Microwave	X-ray
Choice of Technology	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Energy Consumption	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Guarantee	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Maintenance	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Other Media Consumption	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Water Consumption	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Installation Costs	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Purchase Costs	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Training Costs	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Labor Environment	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Natural Environment	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Additional Advantages	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Gauge Precision	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Gauge Scale	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Additional Advantages	Gauge Precision	Gauge Scale	User Interface	Beta-ray	Microwave	X-ray
User Interface	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Beta-ray	0.166667	0.189711	0.319618	0.330897	0.000000	0.000000	0.000000
Microwave	0.666667	0.547216	0.121957	0.288919	0.000000	0.000000	0.000000
X-ray	0.166667	0.263073	0.558425	0.380184	0.000000	0.000000	0.000000

The final ranking of alternatives with respect to main goal can be found in the "Choice of Technology" column (the numbers in parentheses represent the normalized scores of alternatives):
1. Microwave (0.3829) 2. X-ray (0.3518) 3. Beta-ray (0.2654)

Conclusions

The important and complicated problem of technology assessment was analyzed in the particular case of arrangement of a new production line. Analysis shows that four main important criteria should be taken into account in the decision process: technical merits, investment cost, operating costs, environmental impact. Usually these criteria are subject to further analysis and several subcriteria are derived. At this stage the problem becomes case sensitive and specific features of the problem determine the content of analysis.

To help the decision process, the technology assessment problem was structured with the aid of Analytic Network Process. The advantage of ANP is that it models interactions of different aspects of the problem while most popular methods usually treat them as independent. In the case of technology assessment an example of such an interaction is the interaction of environmental impact and operating costs. They are connected by energy, water and other media consumption. The use of resources can be evaluated differently from the financial and the environmental points of view.

ANP includes also several pairwise comparisons of criteria, subcriteria and alternatives. These comparisons result in a number of priority vectors organized in a supermatrix and give a numerical assessment of alternatives. Finally, a ranking of alternatives is achieved which reflects the whole analysis process.

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MODELS OF CRITERIA SYSTEMS OF BUILDING DESIGN CONTRACT

Abstract

A number of multicriteria decisions must be made during construction investment processes. A number of support systems for multicriteria tasks of construction investment processes are available. Some of them are reviewed in this article. The effectiveness of the construction investment process is greatly influenced by the appropriate formulation of a building design contract (BDC). In order to formulate an efficient BDC, multicriteria techniques for evaluation and comparison of BDCs must be created. Beside technical, organisational and economic aspects of construction, legal aspects of a BDC must be also analysed in order to prepare such techniques. Therefore, legal decision making systems are also reviewed in the article. A conclusion can be made from the review that legal decision making systems for BDSs are not available currently. One of the main tasks in the creation of multicriteria support systems is the formation of a criteria system. Three models of criteria systems of BDSs are analysed in the article, and the best model for creation of multicriteria evaluation technique is determined. On the basis of this model, the importance of criteria should be determined and a multicriteria decision support system should be created in further research stages.

Keywords

Decision support systems, building design contracts, multiple criteria evaluation.

Introduction

Construction is a complicated process with a number of stages, which must be appropriately adjusted and managed. The entity that commissions construction must make different multi-aim decisions at various construction stages.

Most problems encountered during construction depend upon the selected contractor. Therefore, selection of a contractor is a very important stage in the implementation of an investment project. Patrick Sik-Wah Fong and Sonia Kit-Yung Choi [1] have analysed methods of contractor selection and noted that some methods are non-exhaustive and tend to be biased: there is a lack of opportunities to evaluate abilities of a contractor and to meet time, price, quality and security requirements at the same time. These authors have analysed possibilities to apply the Analytic Hierarchy Process (AHP) Method in contractor selection according to various criteria.

Architects and designers are no less important in the construction process. F.K.T. Cheung et al. [2] claim that price cannot be the only criterion influencing the selection of an architect. Authors have compiled a questionnaire and made an expert research, which helped to determine criteria that influence the selection of an architect and the importance of the criteria. An architect was selected using AHP method. The system for selection of architects was created on the basis of the model formed during the research.

Multicriteria methods may be used not only for selection of contractors. E.K. Zavadskas, L. Ustinovičius and A. Stasiulionis [3] have analysed possibilities to apply *Electre III* method in evaluation of effectiveness of investment to commercial buildings. Authors note that while evaluating effectiveness of investment to commercial buildings, the total effect of various criteria must also be evaluated: the amount of construction works in commercial buildings, trends, legal issues and available construction solutions.

J. Antuchevičienė [4] notes that rural buildings are an important part in Lithuania's economic potential. The author has introduced a system of criteria specially designed for reconstruction of rural buildings. The priority of rebuilding may be determined using the author's model, and multicriteria analysis methods may be used for evaluation. When information is incomplete or under-defined, methods based on the uncertainty theory are offered.

While analysing multicriteria building evaluation from the sustainable development perspective, J. Šaparauskas [5] reviewed guides, manuals, recommendations, databases, software and internet tools. The author offered an evaluation system on the basis of the analysis performed. Software based on MCDM-23 (multi-criteria decision-making) method was used for evaluation, and projects of individual houses *Kedras* and *Vasaris* were compared to test the principles.

One more important issue in construction is the selection of construction materials. E.K. Zavadskas, A. Kaklauskas and V. Trinkūnas [6, 7] have analysed systems of e-trading for construction materials and goods and have offered the model of an internet decision support system for trading in construction materials. The model is based on the determination of criteria which define construction materials and goods, on importance of the criteria and on application of multicriteria evaluation methods. A pilot internet decision support system for trading in construction materials was created on the basis of the model suggested by the authors.

The construction industry is among the most important branches in each country's economy. This is witnessed by the attempts of various authors to increase the effectiveness of construction solutions. Most of the above-mentioned authors solve various issues related to construction investment process. However, such an important question as the evaluation of BDCs remains unanalysed or almost unanalysed. Even when a contractor is selected and the price and terms of work agreed, at least several contract variants are still available. Selection of the most favourable variant is a multicriteria problem, and a technique must be created for its solution.

In order to create a multicriteria evaluation technique for BDCs, it is necessary to create a system of criteria characterizing BDCs, to determine the importance of the criteria and to select and adjust appropriate multicriteria evaluation methods. The system of BDC provisions is analysed in this article, contract provisions which may be considered criteria of BDCs are determined and models of BDC criteria systems are created.

1. Legal decision support systems

Many and various systems to facilitate contract making and legal issue solving have already been created. Two different types of rules were used in the system by Meldman: general norms defined in claims and special norms taken from precedent cases [8]. Disputable situations are immediately compared to precedents and the system determines a precedent that is closest to the violation of the civil law.

TAXADVISOR [9] used *EMYCIN* system to assist lawyers in land tax administration. The audit company *Ernst and Young* has created three legal expert systems: *VATIA*, *Latent Damage Adviser* and *THUMPER*.

In the *VATIA* (Value Added Tax Intelligent Assistant) [10] system, attention is paid mainly to VAT calculation. With the help of *VATIA* system auditors could analyse VAT payments of a client.

Latent Damage Adviser [11] was created on the basis of the 1986 Latent Damage Act (Australia). With the help of this system, experts of latent damage could solve some difficulties with less effort; however, it was too complex for non-experts, because they were not knowledgeable in abundant interrelated rules, which are characteristic to this sphere of law. The law is scarcely commented, complex and difficult to understand.

The *THUMPER* [12] system was designed for the employees of *Ernst and Young* who specialize in general taxation issues. With the help of this system information about taxes applied could be retrieved and activities regarding taxes could be planned. Three abstract legal models were implemented in the *THUMPER* system:

– The farthest level: consumer problems.
– Middle level: expert explanations and legislation.
– The level which represents legislation and legal cases.

One of the first *Rand* Corporation expert systems is *LDS*, which helps lawyers to solve inheritance disputes. The *LDS* system consists of laws, court cases and law principles; lawyers use this information operatively when they are preparing claims in inheritance cases.

SAL [13] is another system created by the *Rand* Corporation; it is also used to solve inheritance issues. Knowledge about losses, liability of the defendant, liability of the complainant, the main property distribution characteristics such as type of parties and legal mastery of the opponent were used in the *SAL* system. These two systems are important in that they represent first steps of IT in property distribution solutions.

WIRE IQ (Wire Intelligent Quantum) [14] is an Internet decision support system, which enables lawyers, insurers and reinsurers perform quantitative analysis for claims in property distribution and personal damages rapidly. In 1999, Douglas and Toulson analysed value determination structure in torts, property distribution and personal damage. A rule-based system must be the basis in this process. Claims are detailed (claim type, complainant, age, gender, salary, etc.) and included in the system. Rules used in the system help to determine the value of litigious property or tort. *WIRE IQ* database consists of thousands of records including disputes on property distribution and claims on damage remuneration. The system analyses variants, performs comparative analysis, selects precedents and forecasts.

Although the above-mentioned systems have been created by different authors, at different times and for different tasks, it is possible to distinguish one common feature: information and the sequence of problem solutions are specified basis of certain principles. To reveal peculiarities of contract agreements and to determine the system of criteria defining BDCs, it is expedient to classify and to model provisions of such contracts and to perform their systematic analysis.

2. The system of provisions in building design contracts

From the philosophical perspective, a system is a sum of interrelated and interconnected elements, which are integral and united. A system is more than a mechanic unity of its constituent elements. Interconnected elements of the system bring new quality to the totality. The whole system and the relations among elements change when elements are modified, added or removed. Each system may be an element of another macrosystem, and each element of a system may be a microsystem.

Such a complex thing as a BDC cannot be analysed without regarding it as a system with its own elements and its own relations among elements. The view of a BDC as a system is especially important when creating techniques allowing for a multicriteria evaluation of BDCs and a comparison of BDC variants.

After the analysis of the contents of BDCs, the conclusion can be made that the smallest element of a BDC as a system is a contract provision. While analysing a BDC (like any system), different models of systems can be formed depending upon research aims. Therefore, it is necessary to determine which model of a BDC best suits the aims specified in the article.

2.1. The model of a BDC provisions system based on importance of provisions

One of the most important elements of a contract is its contents which include the system of contract provisions. One of the main principles of the civil law is followed in the formulation of contract provisions: contract freedom. On the basis of this principle, parties have a right to make contracts independently and to determine their provisions.

Provisions have different importance in a contract. Lithuanian legal doctrine divides all contract provisions into essential and non-essential. The model of the system of BDC provisions is formed on the basis of this classification; its principal scheme is showed in Figures 1-3.

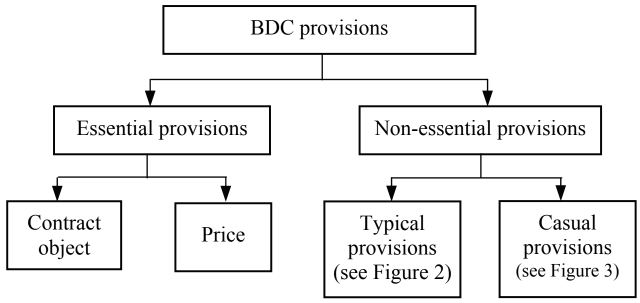

Figure 1. The principal scheme of the model of BDC provision system based on the importance of provisions

Essential contract provisions are those that are necessary and sufficient to formulate a contract which would create rights and liabilities to the parties after coming into force. Essential contract provisions have two features: necessity and sufficiency. Necessity is understood in the following way: the contract is not valid until the parties agree on all essential contract provisions. If an agreement is not achieved, it is considered that parties have pre-contract relations. Sufficiency is understood in the following way: when the parties agree on all essential contract provisions, the BDC is considered valid although adjustment of non-essential provisions is postponed. The analysis of the Civil Code shows that three essential contract provisions are distinguished in a BDC: the contract object, the contract price and the fulfilment terms. When these provisions are adjusted, the BDC is considered valid and creates rights and liabilities to its parties. It is not required that parties agree on all possible contract provisions in all cases. The parties may agree on non-essential contract provisions later at the request of the interested party; non--essential provisions may also be determined by the court in accordance with contract specifications, dispositive legal norms, traditions, legal principles, interrelations of the parties, etc.

It is important to note that essential contract provisions may be determined not only by law but also by the parties of the contract. Suppose that the parties agree that the term of intermediary completion must also be considered an essential contract provision. In this case the term of intermediary completion becomes an essential provision and will have the same importance on contract validity as the essential provisions described in the law.

Non-essential contract provisions are all other provisions that are not essential. This means that non-essential contract provisions are neither necessary nor sufficient for the contract. Their presence or absence does not influence the validity of the contract. If all non-essential provisions are adjusted but at least one essential provision is not adjusted, then the contract is not valid. Two types of secondary contract provisions may be distinguished: typical and casual.

Typical provisions are the provisions set by laws, which become mandatory for parties due to the fact of contract making. They differ from essential provisions in that it is not necessary to adjust them: if the parties agree on all essential provisions, then upon making the contract they adjust the typical contract provisions as well. When a BDC is signed they are automatically included in the contents. Typical BDC provisions can be imperative and dispositive.

Imperative contract provisions are determined by imperative legal norms and are mandatory for contract parties whether included in the contract or not. Parties cannot neither change nor cancel these provisions. For example, the Civil Code, Chapter 6.702, Part 2 sets a typical imperative BDC provision: “The contractor shall have no right to transfer the result of the work to third persons without the consent of the customer”.

BDC provisions may also be determined by dispositive legal norms; for example, the Civil Code, Chapter 6.703, Part 2 includes a dispositive norm which sets a typical BDC provision that the contractor shall be obliged at the demand of the client to correct without compensation the defects of the technical documentation if not specified otherwise in the BDC. This is a typical dispositive BDC provision and need not be adjusted by the parties; it will be valid ipso facto (due to the making of the contract). Dispositive legal norms may be changed by the parties in their contract upon agreement. In this case the provisions agreed by the parties will be superior over the provisions set by dispositive legal norms. If the parties have not changed the contents of dispositive legal norms upon agreement or have not discussed legal relations regulated by dispositive norms, then legal relations between the parties are regulated by dispositive legal norms. Thus dispositive legal norms are valid when contract parties do not specify otherwise.

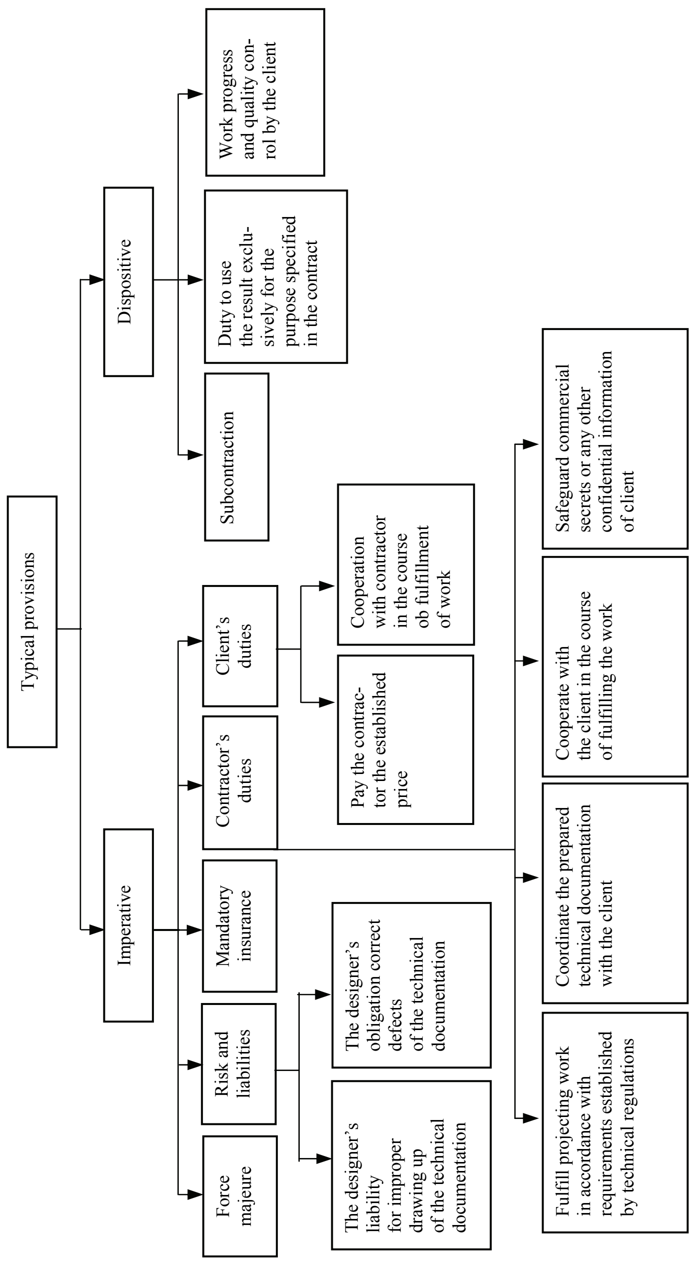

Figure 2. The model of typical BDC provisions

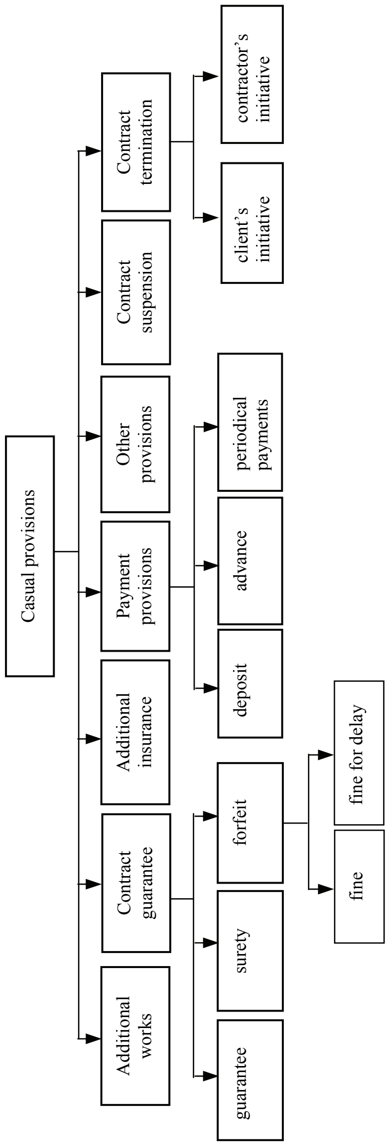

Figure 3. The model of casual BDC provisions

Casual contract provisions are non-essential and determined by the parties and not by laws. They are not automatically included in a BDC as typical provisions. Absence of such provisions does not influence the validity of the contract, because they are determined upon mutual agreement between the contractor and the client. It is important to note that regardless of the group to which a certain contract provision is attributed according to the provided classification, all these provisions are equally obligatory when the BDC comes into force, and all contract provisions have legal power over the parties.

Such a classification of BDC provisions helps to determine the hierarchy of provisions, and even non-experts can see legal significance and importance of provisions. This way they can clearly see what legal outcomes will be when some contract provisions are not discussed, in which cases BDC provisions discussed in the Civil Code shall be applicable and in which cases BDC provisions will be valid.

2.2. The model of the BDC provision system based on grouping of provisions into general and special

BDC provisions may also be divided into two main groups according to types of contracts for which they are typical. One group includes contract provisions that are characteristic only of a BDC. The other group would include contract provisions that are characteristic of other types of contracts as well.

The model of the contract provision system based on this classification does not specify essential and non-essential provisions. The model of the BDC provision system based on grouping of provisions into general and special is shown in Figures 4-6.

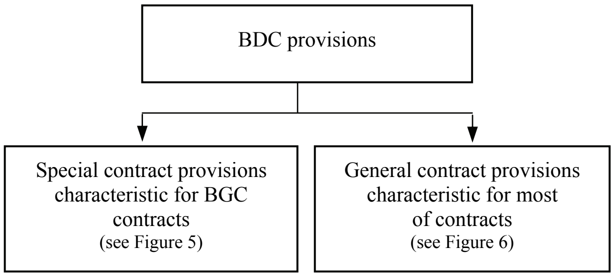

Figure 4. The principal scheme of the model of the BDC provision system based on grouping of provisions into general and special

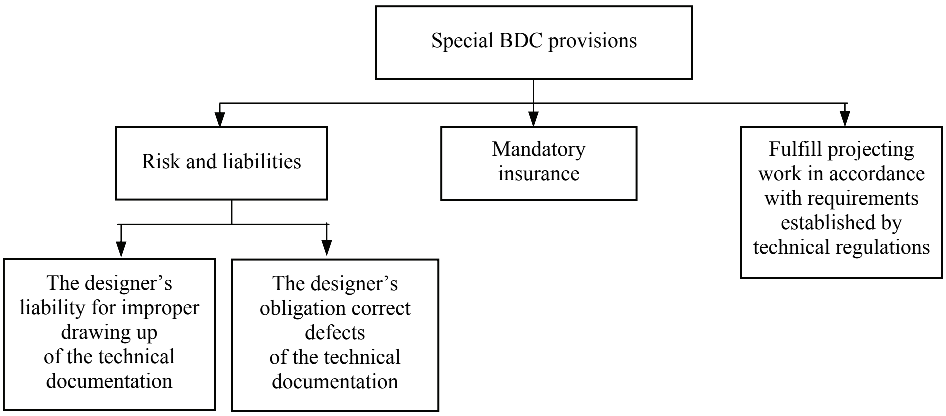

Figure 5. The model of special BDC provisions

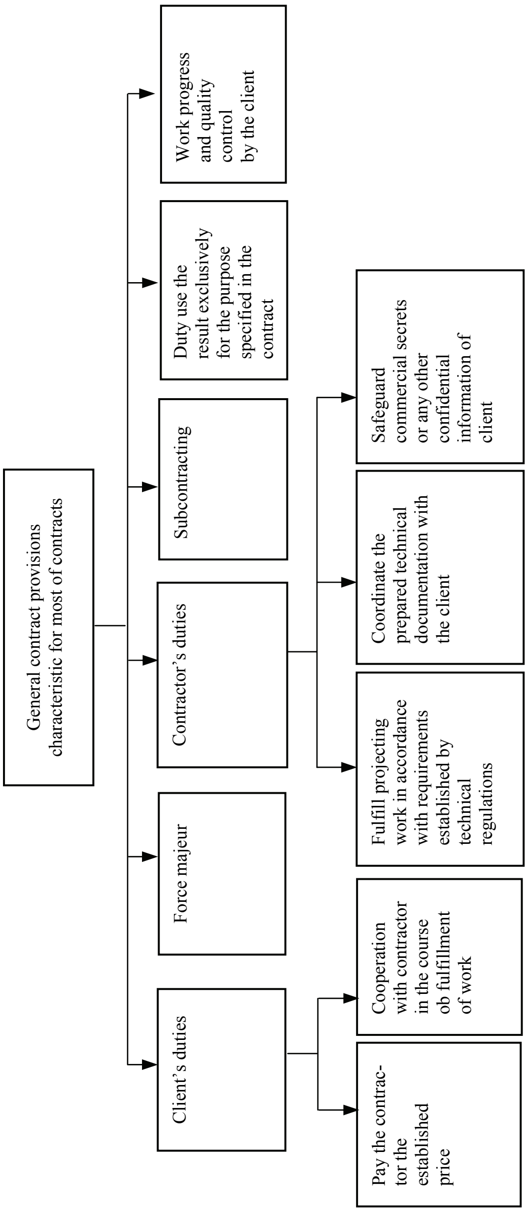

Figure 6. The model of general BDC provisions

2.3. The model of the system of BDC provisions based on functions of provisions

The system of BDC provisions may be also modelled with respect to functions of provisions. All BDC provisions have certain functions. For example, contract provisions regulating guarantees, surety and forfeit have liability guarantee function. All provisions regulating this function may be grouped into a separate subsystem. Other contract provisions may be similarly grouped into subsystems. The model of the system of BDC provisions formed on the basis of this principle is shown in Figure 7.

We think that this model is the most suitable for the creation of the multi-criteria evaluation technique for BDCs. This conclusion can be made due to the following reasons:

- experts can more easily evaluate the importance of contract provisions when the provisions are grouped according to their functions,
- legal power of all BDC provisions is equal regardless of the group they are attributed to according to any of classifications analysed; however, the latter classification shows best the actual operation and functions of a BDC.

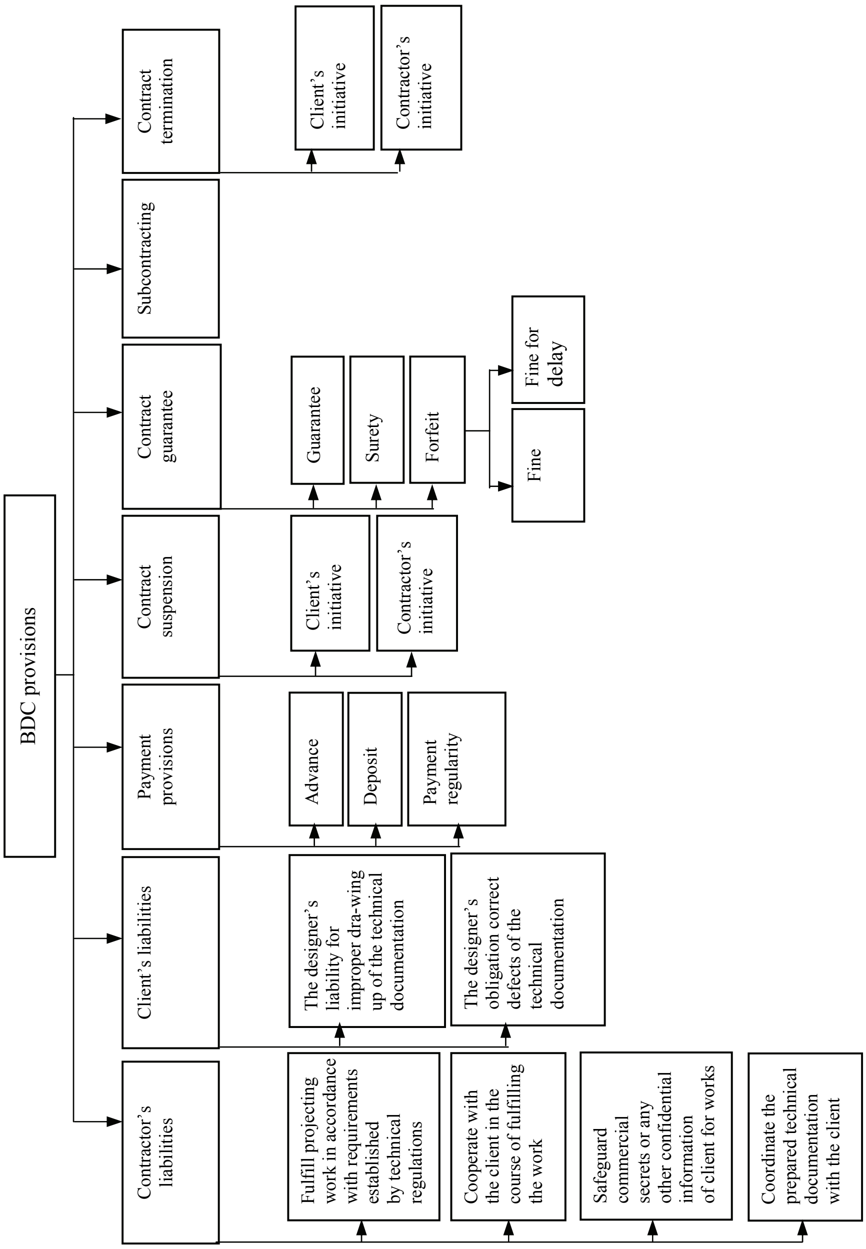

Figure 7. The model of BDC provisions based on functions

Conclusions

1. Construction is an expensive, long-term and complex process during which various problems occur and multicriteria decision making methods must be applied. Various authors offer different multicriteria decision making methods for problem solving at various stages of a construction investment process: selection of a contractor and architects, evaluation of priority for building reconstruction, evaluation of buildings from the point of view of sustainable development, making of decisions related to building maintenance, selection of construction materials, etc.

2. Problem solving in many systems analysed is made by determining criteria which influence the solution and by applying special multicriteria decision making methods. Criteria and their number usually depend on the nature of the problem being solved. This also influences the selection of mathematical methods.

3. Currently multicriteria methods and models are available to increase effectiveness of solutions of various issues related to construction; however, insufficient attention is still paid to making, evaluation and comparison of BDCs. For the construction process to be effective and well-run, the BDC must be well formulated. The model of the BDC provision system is created in order to successfully solve this problem.

4. Currently there are many systems facilitating contract making and legal issue solving; they help to determine precedents and civil law violations, to consult on tax structure, to solve disputes on inheritance, etc. Although these systems are created by various authors, at different time and for different tasks, it is possible to distinguish one common feature: information and the sequence of problem solutions are detailed on the basis of certain principles. In order to reveal peculiarities of contraction agreement making, it is expedient to make a scheme showing BDC provisions and their relationships in detail.

5. After the analysis of BDCs and laws regulating their making, three different models of BDC provision systems were created:

– The model of the BDC provision system based on the importance of provisions. This model is useful for persons who are not knowledgeable in civil law, because BDC provisions are divided according to their legal importance in this model, i.e. outcomes are shown when some provisions are not included in the contract. However, persons who are knowledgeable in law know this classification very well.

- The model of the BDC provision system based on grouping of provisions into general and special. Shows the differences of a BDC as compared with other contract types regulated by the CC. The model has one drawback: it is difficult to determine the importance of provisions and to apply it in decision making.
- The model of the BDC provision system based on functions of provisions. This model helps to determine the importance of contract provisions. Thus this model makes it possible to create an internet-based legal BDC multi-criteria decision support system.

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VALUE-FOCUSED DEVELOPMENT OF A MULTIOBJECTIVE WATERSHED-MANAGEMENT PLAN IN HAWAII

Abstract

This paper describes a year-long effort that applied value-focussed thinking and the AHP to the challenge of facilitating public participation in the development of a watershed-management plan for the island of Hawaii. Given the intended audience of this volume, the paper focusses on the multiobjective methodology (value-focused thinking and the AHP) we used to guide the participants in their quest. However, we also offer commentary on the role and caveats of using such methodologies in and for facilitation of public participatory processes, and on the nature of consensus and how it is often construed and used in faciliation.

1. Getting started: background and context

To relatively recent arrivals, the last thought conjured up by the dry rocky terrain of the "Big Island" of Hawaii's western "Kona" coast is that of flood hazard. Kona's watersheds are forming on a geologically young landscape, with drainage channels and catchments poorly defined and their boundaries sometimes hard to delineate. Add to this the infrequent rainfall, the high infiltration into the porous lava substrate, and the often bright and blinding desert-like sun, and the newcomer can be excused for ignoring flood potential.

Yet Kona has a well-documented history of floods that periodically ravaged the area for nearly a century until the early 1980s. Those who have lived there for several decades, let alone those whose families have been there for generations, can recount the damage that occurred during those events, and there is growing concern among them that many activities in the watershed, especially the pell-mell development in coastal regions over the past 20-30 years, are making economies and communities ever more vulnerable to flood damage. Unfortunately, the collective human memory tends to focus on recent experience, and so where flooding and other potentially harmful phenomena have not occurred for a while, one tends to overlook or minimize their likelihood and significance.

It was this background and context that catalyzed a handful of long-time residents to group together in 2002 to initiate a concerted effort to reduce the risk of catastrophic flooding. Their efforts led to a funding proposal and subsequently a small grant from the Division of Forestry and Wildlife (DOFAW) of the Hawaiʻi Department of Land and Natural Resources, awarded to the Kona Soil and Water Conservation District (KSWCD) of the U.S. Natural Resources Conservation Service (NRCS). The grant was to support the development of a plan to confront flood hazard, and the KSWCD hired us, through Visions & Decisions LLC, to guide the process and facilitate the involvement and interaction of the community, local and state officials, and other interested stakeholders.

There are seven watersheds comprising the North and South Kona districts, with all but the Waiaha watershed extending outside the jurisdictional boundaries of those districtss. These watersheds extend from the forested *mauka* lands on Mauna Loa and Hualalai to the sea (*mauka-makai*). Topographic ridges, or "drainage divides", separate individual watersheds. Ecosystems found within watershed boundaries range from rain forests to dry land forests to the marine environment, comprising diverse communities of plants, animals, and other living things that depend on the availability of soil and water. Since upland land cover influences flood regimes lower in the catchment, it was clear to us from the beginning that the stated concern with flood protection would likely need to be approached within the larger framework of watershed management.

The NRCS often uses a planning method known across the U.S. as Coordinated Resource Management, or CRM, which served as the model used by the KSWCD in the development of its grant proposal for this project. The proposal outlined the following guidelines for this work [3], which are extracted from the CRM guidelines:

– Watershed-management plan will be a voluntary program within this District.
– All interested and concerned agencies, organizations, and interest groups will be involved.

- Facilitation will be by a neutral party and will focus on common goals.
- The group will address two items by consensus at the start of the project. These are ground rules and common goals. The purpose of ground rules is to open lines of communication and create middle ground among all the stakeholders.
- Representatives of agencies and organizations will have authority to speak and make decisions for their respective entities. This will avoid wasted time to gain approval and confusion created due to misunderstanding.
- Management decisions will be made by consensus.
- Focus will be on what management practices are needed to improve natural resources, not agency policies or positions implemented in the past.
- Develop an understanding among committee members to build trust and commitment.
- Watershed management goals and objectives will be developed and prioritized in order to develop action plans.
- The neutral facilitator will monitor the process and ensure flexibility of the plan to address changes of land ownership, weather, and change of topography of this geologically young land area.

Visions & Decisions (V&D) led participants in the meetings through a process similar to these steps. Although similar, our approach differed in some key ways. We discuss the nature and significance of those differences in the final section of the paper, but three should be highlighted at this point. First, the shoe-string budget available, and the fact that we were based on the island of Oahu and not on the Big Island, meant that we would not be available *continuously* to facilitate and meet in person all who might want to participate, or to ensure the monitoring and response to the changes identified above. We thus made clear from the beginning that although we would guide and facilitate the process, a steering committee and Big Island residents would be responsible for the work required for process implementation and plan preparation. Second, and related to the first, since we would have no control over who attended meetings and participated, we could guarantee neither the representation of all interested stakeholders nor the participation of actual decision makers or their surrogates. Third, we pointed out that the term *consensus* more often than not is ill-defined, rarely if ever equates with unanimity and often yields “agreement” only under fatigue, peer pressure, or other kinds of duress, and that, therefore, given the time and budgetary constraints we faced and the divergent viewpoints and interests likely to be involved, ours would not be a process dependent on achieving consensus.

2. Methodology

The process we proposed and which was followed and executed during the year-long (2003-2004) project is an example of value-focussed thinking [1] within which was embedded the Analytic Hierarchy Process (AHP). Value-focussed thinking uses one's concerns and desires to articulate specific objectives, goals, and criteria that not only indicate *what* in a given situation needs to be improved but also *how* to improve it. The "how" involves specifying actions to take, either by identifying ones already existing and available, or by guiding one in the design of new ones. The identification and design follow logically from the values-derived objectives.

The AHP [5, 6] is a multicriterion prioritization methodology widely used in business and seeing ever more applications in planning and resource management. It uses a hierarchical structure to decompose a decision problem into relevant criteria, and a pairwise comparison methodology that leads to the prioritization both of criteria and of alternatives (actions and projects) [6]. This hierarchical structure, formally termed an *analytic hierarchy*, may be identical to the *objectives hierarchy* or *value tree* that plays the central role in value--focussed thinking, but it may also incorporate "means" elements that denote cause-effect relations*. Elements throughout the analytic hierarchy are termed criteria, with those in the bottom-most level also called "alternatives". It is the prioritization of these alternatives that is the overall objective of an AHP analysis**.

In the AHP, prioritization (tantamount to weighting) is effected through a pairwise comparison of the subcriteria ("children") of each non-alternative criterion ("parent") in the hierarchy. Comparisons are made on a 1-to-9 positive, fundamental scale indicating the relative dominance of one member of the pair over the other in determining, contributing to, or exemplifying the quality represented by the parent criterion. "Dominance" is usually in terms of "importance", "preference", or "likelihood". Each set of comparisons yields a positive reciprocal matrix, the components of the normalized right eigenvector

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* In the literature of decision analysis (e.g. [2]), objectives hierarchies (value trees) are hierarchies of values and not cause-effect hierarchies; cause-effect hierarchies are termed "means-ends objectives networks" [1]. Analytic hierarchies, more general, subsume both and thus may have elements of both.

** Note that the "alternatives" in an AHP hierarchy need not be choices or alternative courses of action. They may in fact be criteria that need to be prioritized or weighted. Thus, all alternatives are criteria of the hierarchy, but only the bottom-most criteria are alternatives.

of which represent the best approximation of the weights or priorities of the corresponding subcriteria. Multiplying up through the hierarchy yields the overall priority of each alternative. Since such priorities exist on a ratio scale, they may legitimately be multiplied and divided by priorities for criteria from other analytic hierarchies. As seen below, this will prove important for this analysis. Details on the AHP are readily found in the voluminous literature on it [5].

Figure 1 depicts graphically the value-focussed procedure we followed, illustrating clearly how values, or value-laden interests, can be used to identify watershed-management actions. The goal of this process was to generate a broad range of alternative actions for the steering committee and other stakeholders to evaluate (via the AHP) and select for implementation in the Kona region. It consisted of the following steps:

Step 1. Identify the fundamental objectives and subobjectives (designated as "O_i" in Figure 1) for watershed-management, as defined in the value tree.

Step 2. Identify the problems, obstacles, or difficulties that could potentially prevent an objective from being achieved ("P_i" in Figure 1). A particular problem P_i may be associated with more than one objective O_i.

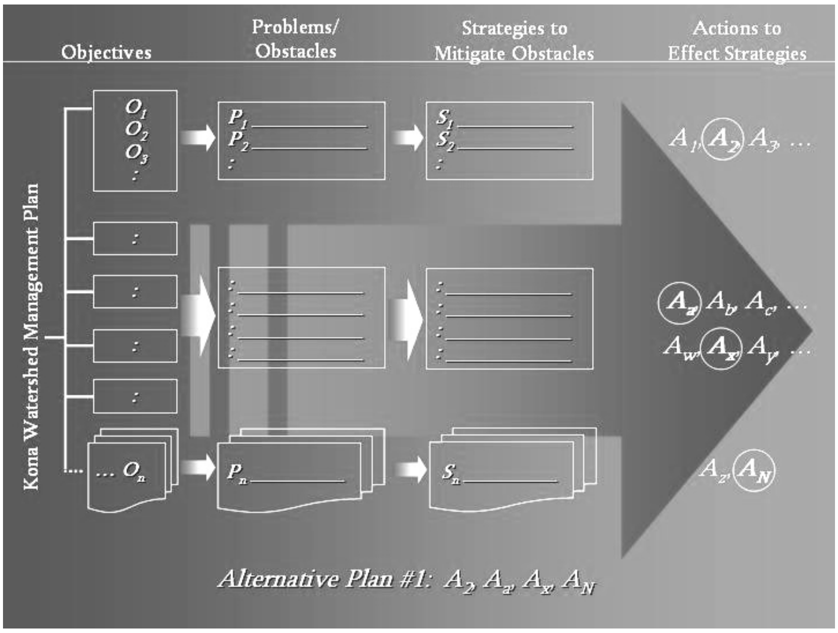
Example: Private landowners refuse or resist the construction of flood--control and drainage works on their property.

Step 3. Identify any strategies ("S_i") that could be implemented to overcome the problems, obstacles, or difficulties. Again, it is possible that a particular strategy may respond to more than one obstacle.

Example: For the example under Step 2, one strategy is to increase public awareness of potential flood hazard and work with landowners top encourage their cooperation to reduce the potential for flood damage or risk.

Step 4. Alternatively: Identify specific actions "A_i" corresponding to each strategy that could be implemented to overcome or mitigate the obstacles. Some actions may address multiple objectives.

Step 5. This final step involves selecting various actions to be combined into several alternative watershed management plans for ultimate evaluation, as exemplified by the hypothetical "Alternative Plan #1" at the bottom of Figure 1. Since the procedure yields a large number of possible actions, this final step uses the rating procedure of the AHP to help evaluate the actions.



Note: A conceptual model showing how the planning process progressed from problem definition and identification of watershed-management objectives (as stated in the value tree [Figure 2]) to identification of actions to achieve those objectives. The variables are:

O_i = watershed-management objectives listed in the value tree, $i = 1,2,\ldots,n$,

P_i = problems or obstacles that could prevent the associated objective from being achieved, $i = 1,2\ldots,n$,

S_i = strategies to overcome problems or obstacles, $i = 1,2,\ldots,n$,

A_i = actions to effect strategies, $i = 1,2,\ldots,N$.

The list of actions developed using this process can be treated as a "menu" for formulating alternative plans, or sets of actions, for watershed management. "Alternative Plan #1" illustrates how any combination of alternatives A_1 through A_N can be combined into a watershed--management plan, allowing for a high degree of alternative evaluation and selection in the planning process.

Figure 1. Process for identifying actions

3. Implementing the methodology

3.1. Planning committees and public participation

If Step 1 of a value-focussed procedure is to identify and structure the criteria and objectives reflecting the values of the parties in a problematic situation, then Step 0 must be to determine who those parties are and establish a framework and process for their participation and interaction.

At the start of the planning process, the grant coordinator and others from the Kona community formed a core planning group, which V&D designated as the Kona Watershed-Management Plan Steering Committee. A core group of five to seven individuals participated or contributed in some way throughout the planning process, with other members of the steering committee participating on an *ad hoc* basis. A second, larger group of more than 40 stakeholders was identified to participate in stakeholder meetings; some of the stakeholders were also identified as members of the steering committee.

A series of eight steering committee and stakeholder meetings were held between January and November 2003. Attendance at the steering committee meetings varied, ranging from five to ten individuals, with regular attendance and participation by the "core" group. The first stakeholder meeting was held in late April, and attended by nine individuals, including some of the "core" group from the steering committee. A second meeting of stakeholders and the steering committee was held at the end of July, with two stakeholders attending although all of them had been invited. One additional meeting was convened in June to begin coordinating with individuals whom the steering committee identified as technical and local experts who could contribute to the planning process in several ways, but particularly, to assist in the development and evaluation of flood- and watershed-management actions identified during the planning process.

To elicit watershed-management concerns from the community at large, three community meetings were convened in north, central, and south Kona on weekends in late March. Except for members of the steering committee, KSWCD board members, and NRCS and DOFAW staff already involved in the planning process, only four community members attended. Despite the low participation, the input of the participants and the conversations that stemmed from their concerns was critical for furthering the steering committee's work on the watershed-management plan.

3.2. Challenges related to public participation

Although the KSWCD strived to involve over 40 stakeholders and a larger group of citizens in all stages from problem definition to identifying and ranking watershed-management actions, the process was challenged by low participation. For example, except for the core group noted above, successive steering committee meetings were often attended by different people who either missed one or more previous meetings or were new to the process. Consequently, in several meetings more time than was anticipated was spent reiterating the overall watershed planning goals and the spirit and intent of the procedure to involve stakeholders in the planning and decision processes and explaining the planning process to newcomers in sufficient detail to enable them to participate on the same level as the regular participants. Other explanations included recapping what had been accomplished in prior meetings, reiterating the desire to continue moving forward, and encouraging new participants to accept the work completed to date and agree to contribute to the forward progress. While newcomers' input was welcomed and essential for contributing to a richer outcome for the process and to engage a broader public, the timeframe for developing this plan prevented ongoing debate of the planning method, the questioning of the motivations for watershed-management planning in Kona, and critique of accomplishments by the core participants in prior meetings.

Another challenge concerns the representation of community interests in the plan development. The steering committee and stakeholders who participated in most meetings remained largely self-selected and committed to the implementation of a watershed-management plan in Kona. To encourage community participation, the KSWCD purchased advertising space once a month in the Sunday edition of the *West Hawai'i Today* newspaper to publish succinct summaries of the watershed-planning process, to provide progress reports and explain the watershed-management objectives, and to invite the general public to attend steering committee meetings or volunteer to help in other ways. Eight advertisements were run from May through December, and during that time only one person contacted the NRCS office expressing interest to become involved.

3.3. Problem definition

In the first three committee meetings and in the community meetings in March, the facilitation team focused the steering committee for the first three months on problem definition by eliciting participants' concerns about Kona's watersheds. Conflict arose in the very first meeting when it became clear that despite the initiators' overwhelming worries about flood protection, the project's funder, DOFAW, was primarily concerned with "watershed management", including conservation and promotion of native ecosystems. Whereas the former typically focusses attention on the lower part of a catchment, the latter focusses on the uplands. We were able to mitigate the problem by explaining that "flood protection" and "flood-hazard mitigation" could easily be subobjectives of the higher-order "watershed management" objective.

Steering committee and community meeting participants were asked to identify their watershed-management concerns and objectives for watershed management for the Kona region. In an iterative process, we structured the participants' concerns and objectives into a value tree. Six versions of the value tree were created, reflecting continual refinement throughout the planning process, including the period after the problem definition phase, with each version integrating both clarifications of previous value statements and new concerns into the structure. The final version (Figure 2), integrates many natural and man-made aspects of natural resource management, including hazard abatement and public safety, assuring economic sustainability, assuring preservation of biodiversity and ecosystem management, facilitating plan implementation through favorable institutional and policy structures, and sociocultural elements such as preserving the quality of life and cultural resources in the Kona region.

Objectives Hierarchy for Kona Watershed-Management Plan

(NB: "Cost" objectives are shaded)

Objective 1: Minimize Threats to Public Safety

1.1 Minimize threats to life
1.2 Minimize threats to capital

Objective 2: Support a Robust Economic Base

2.1 Ensure sustainability of irreplaceable economic resources
2.11 Min loss of agricultural land
2.12 Min loss of unique rural landscapes
2.13 Min loss of unique or noteworthy ecosystems (marine and terrestrial)
2.2 Ensure adequate infrastructure
2.21 Minimize water shortages
2.22 Ensure adequate wastewater treatment capacity
2.3 Minimize Kona's financial cost
2.31 Minimize initial investment costs to Kona
2.31.1 Initial financial costs
2.31.2 Initial staffing effort
2.32 Minimize recurrent/future costs to Kona
2.32.1 Ongoing and recurrent OM&R costs
2.32.2 Ongoing and recurrent staffing costs

Objective 3: Minimize Threats to Ecosystems

3.1 Restore native biodiversity
3.2 Reestablish native forest cover
3.3 Minimize contamination
3.4 Minimize alteration of natural water balance

Objective 4: Maximize Implementability

4.1 Minimize conflicts with existing regulations
4.2 Minimize political and public resistence
4.3 Minimize delay of implementation

Objective 5: Maintain valued sociocultural conditions and traditions

5.1 Maintain agricultural/rural character and lifestyle
5.2 Minimize aesthetic impacts
5.21 Minimize loss of open space
5.22 Minimize loss of the historic built environment
5.3 Minimize adverse impacts on Hawaiian culture
5.31 Minimize loss of Hawaiian aquaculture (fish ponds)
5.32 Minimize loss of Hawaiian agriculture (taro)
5.33 Minimize changes to integral *ahupua'a* management

Figure 2. Objectives Hierarchy for Kona Watershed-Management Plan

3.4. Identification of watershed-management actions

From April through November 2003, the planning process involved the steering committee, some KSWCD board members, a larger group of stakeholders, some technical experts, and self-selected members of the community to develop a list of actions (means) that would enable the KSWCD to make progress towards the watershed-management objectives, as articulated in the value tree (Figure 2). Actions were identified by a systematic process that drew upon the fundamental watershed-management objectives listed in the value tree, with a goal of developing a "menu" of actions that could be integrated into different alternative plans for evaluation and selection by the community. To complement the list of actions developed in this process, actions identified in steering committee and community meetings during problem definition were included on the menu.

Using this method the steering committee, some stakeholders (self--selected), and the facilitators developed a suite of 24 broad watershed--management strategies and an accompanying list of 114 actions to be considered in the final phase of the planning process. This list was organized into a matrix, and included in the final report [4]. The report summarizes who will be responsible for implementing or whose support is needed to implement each action, the objectives that would be satisfied by each action, and new money in capital and recurring operating budgets necessary for implementation, if any.

3.5. Prioritization of objectives and rating actions

The watershed-management objectives shown in Figure 1 were decomposed into two hierarchies, a risk-reduction "benefits" or "effectiveness" hierarchy, and a "cost" hierarchy. The elements comprising the "cost" hierarchy are those shaded in Figure 1. Although the prioritization of the objectives and subcriteria was done with the usual pairwise-comparison procedure typical of AHP, as previously described, the large number of actions required us to use the ratings procedure instead of pairwise comparison. That procedure has one define ratings classes (intensity categories) for each of the lower-most criteria, and then assign each alternative to the most appropriate class for each subcriterion. One then compares pairwise the *ratings classes* with respect to the subcriterion in question, deriving an importance weight for each class. The alternative then receives the weight of each class to which it is assigned, those weights are themselves weighted by the product of the respective criterion priorities, and then that weighted sum becomes the score for each alternative.

Since it was not possible to carry out such an analysis for all seven watersheds comprising the N. and S. Kona districts within the project's budgetted financial and calendar limits, the team chose the Waiaha Watershed as the single case for which to conduct a complete analysis. That would serve as a model for the remaining analyses that would be done later by the KSWCD and community alone.

V&D led the Steering Committee through a group AHP evaluation of the actions using Team Expert Choice (TeamEC^{TM}) decision software [7]. As described above, the benefit and cost criteria were first prioritized with respect to their relative importance for the Waiaha watershed. (Since different watersheds have different attributes and problems, the criteria would be weighted differently for each one. For example, the planning priorities for the Kiilae watershed will likely be different from the planning priorities in the Kiholo watershed where the climate is drier and dry land forests dominate). Then each of the 114 actions identified in this planning process was rated and prioritized on both "benefit" and "cost" dimensions. The resulting benefit and cost scores for each action were used to compute the benefit-cost ratio and therefore determine the actions that would contribute the most to achieving the watershed-management objectives. The actions with the highest benefit-cost ratios are considered to be the "best-choice." These benefit-cost ratios can be used to guide decisions about which actions or projects to undertake, what type(s) of grants or other financial assistance to apply for, and how to combine actions into different watershed-management strategies. While the actions with the highest benefit-cost ratio would be preferable to lower-ranked actions, the ratings method preserves all of the actions introduced to the decision process and therefore allows a high degree of flexibility in choosing among alternatives. Although the benefit-cost ratio implies a strict order of preference among the proposed actions, there is in fact a whole suite of very good ("efficient" or "Pareto optimal" in technical terms) projects from which one may comfortably choose. As in the benefit-cost ratio, this indicates that it would be ill-advised to select projects or actions with higher costs unless they also exhibit higher benefits.

Summary and conclusion

We believe this project has demonstrated the effectiveness of applying value-focussed thinking and the AHP to the facilitation of community participation in resource management. The endeavor succeeded in organizing a diverse set of people with divergent interests, helping them articulate their concerns clearly and precisely, leading them to identify and design actions and policies that can help them achieve their objectives, and enabling them to prioritize those actions on the basis of the participants' values, the actions' relative effectiveness, and the resources available. Moreover, these accomplishments were attained within a tight deadline, demanding logistics, and a laughably tiny budget.

We regard the development and acceptance of the value tree as the crucial and most important achievement. It made very clear and precise what at first were nebulous or ambiguous views and claims, in the process dissolving in many cases what appeared to be incompatible stances. Although the funders' (DOFAW) and the project initiators' (steering group's) concerns seemed initially at loggerheads, use of the criteria and objectives in the value tree enabled the identification of actions that adressed both sets of issues. The value tree also provided a clear record of what had been agreed on, facilitating summary and recap to late arrivals, stemming "attendance drift", and documenting the process for third parties such as potential funders and later grant recipients.

These benefits notwithstanding, we cannot claim that the intervention was wholly successful; it encountered a number of pitfalls and registered what we regard as some failures along the way. Most of these relate to the challenges of facilitating public participation in politically contentious decision-making. As we discussed above, despite considerable efforts on the team's part, public participation remained scant. Although this obviously *can* jeopardize the comprehensiveness of the concerns addressed, it *need not* necessarily do so, as the full range of concerns can be elicited from even a small group of informed people. More problematic is the burnout of the few who shoulder the many and ongoing activities required to bring the process to a close. In addition, those community volunteers who shoulder such burdens are rarely decision makers who have authority to commit resources to implement the decisions made. When bonafide decision makers did attend the meetings (e.g. state congressmen), they came with the intention of pushing their agendas, frequently trying to derail the process by contesting something long earlier decided or questioning the MCDM methodology. As many MCDM practitioners working in the public sphere have observed, a major strength of MCDM – making values clear and precise – can be a disdvantage when decision makers and stakeholders stand to lose power or persuasion if what they claim as facts are seen instead as value-laden viewpoints. We sensed this to be the case on more than one occasion.

Finally, facilitation of public interaction is *hard* work that requires consummate skill to do well in politically charged or sensitive situations. One reason is because public participation itself is hard work, requiring energy and commitment by a public that has other things to do and often wonders why officials and others paid to plan and manage resources are not taking care of it themselves. Although we are convinced that we made many errors that other facilitators would've not made, and that we ourselves have much to learn and perfect as regards facilitation, we also believe that knowledge of MCDM can help facilitators do better in these and other cases. Many times we have observed clumsy facilitation by people who indiscriminately mixed and confused "priorities" with "objectives", "criteria" with "ranks", "ranks" with "weights", and even positions and principles. Using the terms incorrectly and inconsistently not only confuses their clientele, it undermines the confidence the latter have in them. In addition, we think the primacy given by most facilitators to *consensus* is not only misplaced but counterproductive. Insisting on it before progress can be made, frequently means that little progress is made at all, with seemingly endless discussion frustrating many initial participants, causing them to lose interest and stop participating, with those that remain claiming in the end that "consensus" had been reached. We have our doubts, and feel that approaches that can clearly identify tradeoffs, priorities, and corresponding actions are preferable.

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APPLYING REFERENCE SETS IN CONTENT-BASED INTERACTIVE IMAGE RETRIEVAL

Abstract

The search for graphical objects in multimedia databases is a challenging field of current research and an emerging area of application of multicriteria decision theory. It is characterised by co-existence of qualitative, quantitative, and graphical criteria and gradual approximation of preference structures during the search. Here, we propose a new approach to image search based on preference information in form of reference images provided by the user interacting with an intelligent search system. Such information can be used in image retrieval systems with relevance feedback for complex graphical objects such as leisure facilities, human faces etc. Reference sets can be combined with any other method of content-based image retrieval (CBIR), resulting in a refined search. Computational experiments have proven that the proposed approach is computationally efficient. Finally, we provide a real-life illustration of the methods proposed: an image-based hotel selection procedure.

Keywords

Multimedia databases, content-based image retrieval, relevance feedback, multicriteria decision support, preference elicitation, reference sets.

Introduction

Multimedia technologies have been developing rapidly over the last years yielding a large number of databases containing graphical data. Tools for content-based search of graphical objects have been the subject of intensive research (cf. e.g. [1]), but their performance is still unsatisfactory for many applications. Up to now, the popular Internet search engines have been only text- or quantitative-information-based, including those that search for images. Moreover, only a few existing content-based image retrieval systems, like MARS [4, 18], MindReader [4], or VisualSeek [22] allow for an interaction with the user during the search process. The general idea of an interactive search applied therein consists in changing search parameters based on the user's assessment of the relevance of images presented by the search system in consecutive iterations of the search process.

In this paper we propose new methods of content-based image retrieval, based on elicitation of users' preferences from their interactive feedback. The new features of the approach presented here can be summarized as follows:

1. The result of the search can be either a single image satisfying best the user's expectations, a subset of mutually non-comparable images from a database closest to a set of hypothetically most desired objects, or a (partial) ranking of images.
2. Pairwise and n-to-1 comparisons of images are allowed at each iteration of search as well as an individual and group relevance assessment.
3. The characteristic low-level features of images are first calculated in the background, then their monotonicity with respect to the users' assessments is tested and the CP-net updated.
4. New images to be assessed are retrieved from the multimedia database using the partial preference information gathered so far.
5. The users can generate specific graphical queries, which play the role of reference sets (cf. [20]), and speed-up the search process.
6. One or more users can interact with the retrieval system on the same query at one time.

In Section 1, we outline existing relevance-feedback-based image retrieval methods and point out their limitations. We will show that the use of reference sets in the MCDM framework enhances the relevance feedback approach. The method which we propose in Section 2, in contrast to algorithms presented in our previous work [13], is appropriate for images that cannot be recognized by matching sub-objects and the relations between them directly. Based on the binary image data, high-level features are calculated and a subset of these features serves then as the user's criteria. The decision-making method uses reference sets, the idea introduced and investigated by Skulimowski [19, 20]. As a real-life example, we have applied the above method to hotel selection based on their photographic images.

The preference elicitation method used here should ultimately lead to a gradual approximation of a simple deterministic user-specific utility function, which represents the degree to which the images in the database correspond to the user's preferences. Recall that by a *deterministic utility function* we will call a real function $v{:}D \rightarrow I\!R$, which defines the linear order $\leq_v$ in the set of alternatives assessed (here: the set D of images in the database) and conforms to the partial order $\lesssim_p$ derived from the user's preferences, i.e.

$$u_1 \leq v\ u_2 \ \Leftrightarrow^{\mathrm{df}} v(u_1) \leq v(u_2) \tag{1}$$

and

$$u_1 \lesssim_p u_2 \Rightarrow u_1 \leq_v u_2$$

where u_1 and u_2 are images in the database just surveyed. Let us note that the opposite implication may not be true when the same value of v is assigned to non-comparable elements in D. The satisfaction by $\leq_v$ of the linear order properties results directly from the definition of v:

1. Completeness: $\forall\ u_1, u_2 \in D$: $u_1 \leq_v u_2\ \vee u_2 \leq_v u_1$
2. Reflexivity: $u_1 \leq_v u_1$
3. Transitivity: $u_1 \leq_v u_2\ \wedge u_2 \leq_v u_3\ \Rightarrow\ u_1 \leq_v u_3$.

The above conditions will be referred to as the utility theory axioms. In the sequel we will presuppose that a higher value of v denotes higher user satisfaction, therefore $u_1 \leq_v u_2$ denotes that the solution u_2 is at least as good as u_1 (weakly preferred). Level sets of a utility function will be called *indifference sets* with respect to v.

Since the relation $\leq_v$ linearly orders the set of images while a total ranking of images is rarely sought, finding v is both difficult and superfluous in most decision problems. Instead, the user wishes to select one or a few image objects $u_{c1}, u_{c2}, \ldots, u_{ck}$, out of the set of objects D such that:

$$\forall u \in \{u_{c1}, u_{c2}, \ldots, u_{ck}\}\ \ \forall x \in D \setminus \{u_{c1}, u_{c2}, \ldots, u_{ck}\}\ v(x) \leq v(u) \tag{2}$$

(the subset selection problem).

The latter property means that the above axioms of the utility function 1-3 need to be fulfilled only on a subset of D×D, i.a. it does not need to be fulfilled on $\{u_{c1}, u_{c2}, \ldots, u_{ck}\}$, which can consist of elements non-comparable with respect to v. However, since the database (including web) search engines present the result of search sequentially in a predefined order, the user always gets a ranking, even if he/she did not wish so. Therefore in image retrieval problems the total ordering introduced by the utility function v on the set of surveyed objects $D_1 \subset D$ guarantees that the presentation of results is coherent with one's preferences. Moreover, the gradual approximation of v during the search process may contribute to the numerical efficiency of the selection algorithm.

1. Interactive image retrieval methods

In this section, we review methods of interactive image retrieval, point out their advantages and limitations, and give some references to existing systems which apply interaction with a user in the search process. We will refer to the method proposed by Rui, Huang and Mehrotra [17] as a typical approach to image retrieval that may be interpreted in terms of a multicriteria decision making theory. In the next subsection we will propose modifications of this method, which aim to extend its application to complex objects (i.a. with non--homogenous colour and texture). Then we will compare the performance of methods based on the approach described below with the method based on neural networks proposed in Section 2.

The selection of object features to be used for calculation of similarity between images plays a crucial role in systems for image recognition and retrieval. Much work has been done on finding the features which are most useful for recognition, i.e. those that give high similarity of objects from the same class and low similarity of objects belonging to different classes. The methodology which consists of a choice of a specific similarity measure and a scalarization method (e.g. by weighted sum of several similarity measures) before recognition (retrieval) process is referred to as the *isolated approach* [6]. In the case of image retrieval, unlike image recognition, interaction with the user is possible and even desired. Some research has, therefore, been done with a view to modifying similarity measures during the retrieval process, based on information provided by the user in interactive feedback. It is assumed that users do not have any specialised knowledge on image analysis so, in interactive feedback, they only need to provide evaluation of individual images in the form of grades which express the *relevance* of images. In each iteration, the system presents to the user several images, and the relevance information given by the user is a starting point for upgrading similarity function parameters. Therefore images presented by the system in the next iteration correspond better to the user's preferences – in other words, to what the user is looking for. Besides the parameters of similarity function, descriptors of a query object can also be modified. Starting values are calculated based on an image provided by the user (who wants to find other images similar to the one/ones he already has) or randomly chosen in the first iteration, if a query image was not provided. The term *virtual query* means the set of descriptors corresponding to a system's guess about the image the user is looking for. The concept described above is referred to as *relevance feedback* and it is depicted in Figure 1.

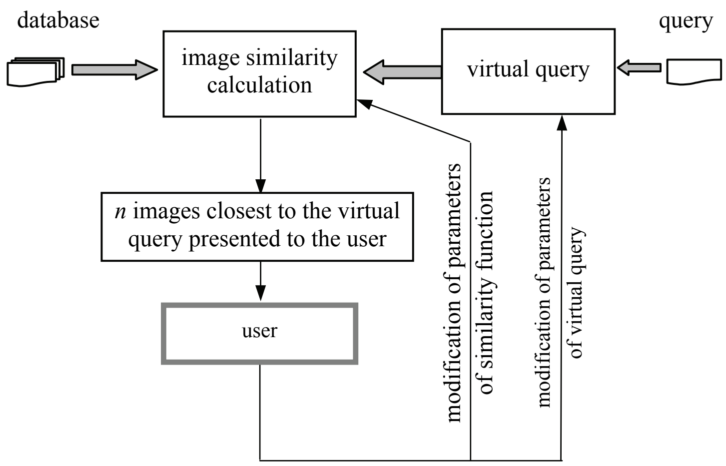


Figure 1. A typical image retrieval system with relevance feedback

Rui, Huang and Mehrotra [17] proposed an approach, where functions describing the similarity of objects are defined at three levels: 1° object – area with homogeneous colour and texture, 2° feature – e.g. colour or texture, 3° feature representation – e.g. colour histogram or average value of Gabor transform for a given area. They assume that the user's utility function is a linear combination of preferences concerning image features (like colour, texture or shape) – for example shape is k_1 times more relevant than texture. Moreover, they assume that preferences for a specific image feature are a linear combination of similarities of feature representations – for example for shape, the Hausdorff distance (cf. e.g. [14, 11]) is k_2 times more relevant than similarity of Fourier descriptors. Coefficients (in our case k_1 and k_2) can be modified in every iteration of algorithm based on *relevance feedback*, provided by the user.

Based on the assumptions given above, the distance between the query object q and the model m can be expressed as a linear combination of functions Ψ_i, which define the distance for feature representation i:

$$d(q,m) := \sum_{i=1}^{I} u_i \Psi_i (q_i, m_i, P_i) \tag{3}$$

where q and m denote the query object and model, q_i and m_i are representations (vectors, with different dimensions for different i) and P_i is a set of parameters of metric in the *i-th* representation space. For example, if the by weighted

Euclidean distance is used as a scalarizing function, P_i are weights $w_{i,j}$ of components of representation i. The calculation of parameters of similarity functions can thus be formulated as the following minimization problem:

$$\sum_k \sum_{i=1}^{I} \pi_k u_i \Psi_i(q_i, m_i^{(k)}, P_i) \longrightarrow \min_{u_i, q_i, P_i} \tag{4}$$

where π_k defines the degree of k-th image relevance for the user, which is positive for *relevant*, zero for *indifferent* and negative for *non-relevant* images (i.e. images with negative relevance, which are examples of what the user is not looking for).

When optimal parameters u_i*, q_i*, P_i* are selected based on (4), the object sought is a solution to the optimization problem:

$$\rho(k) := \sum_{i=1}^{I} u_i^* \Psi_i(q_i^*, m_i^{(k)}, P_i^*) \longrightarrow \min_k \tag{5}$$

K objects with the smallest value of ρ are presented during an interactive procedure to the user, who can again assign to them a degree of relevance in order to recalculate optimal search parameters according to (4) and perform the next iteration of the algorithm.

The formulation of the above problem (3)-(5) corresponds obviously to the distance scalarization problem, well-known in the MCDM theory. In the relevance feedback approach to image retrieval the assignment of weights and scale coefficients is purely heuristic and the researchers clearly have not yet used any virtues of multicriteria analysis. Many authors refer to the Rocchio formula (cf. e.g. [4, 6, 8, 17, 21]). The idea proposed by Rocchio in [11] is based on moving a virtual query towards the centre of gravity of *relevant* objects (in the descriptors' space) and in the opposite direction to the centre of gravity of *non-relevant* objects:

$$q_i' = \alpha q_i + \beta \frac{1}{\# M_R} \sum_{n \in M_R} m_i^n - \gamma \frac{1}{\# M_{NR}} \sum_{n \in M_{NR}} m_i^n \tag{6}$$

where α, β and γ are parameters determining what part of the modified query *q'* comes from the original query (if provided), *relevant* and *non-relevant* objects – provided by user feedback.

The Rocchio formula defines how to modify descriptors of the query object but does not solve the problem of how to find parameters of similarity function. This has been done by further heuristics methods, cf. [11]. Ishikawa, Subramanya and Faloutsos in [4] gave analytical solution of the problem, but only for a specific class of similarity functions. Nevertheless, the concept of relevant, non-relevant objects and the successful use of direction of improvement between the sets of such objects, together with the direct correspondence to the distance scalarization problem of (3)-(5), gives a hint of the use of reference sets, as described in [19] and [20].

The methods described above are based on the assumption that the user is looking for an object with pre-specified values of descriptors and his/her utility function is monotonically decreasing with the distance between the vector of descriptors of a query and retrieved object. The choice of a distance influences the choice of a utility function and it is very limited: indifferent sets are (non-dominated) parts of spheres in the selected metric. The assumption that they have such a shape is technical and does not follow from other properties of the image retrieval problem, thus cannot be regarded as justified. In real-life problems the utility function may be non-convex – depending on the structure of preferences. For example, if the user wants to find one of several objects (the case of several queries combined with *OR*), for every query component there is a corresponding local maximum of the utility function.

Above we have presented the typical approach to the image retrieval problem with relevance feedback. This methodology has several drawbacks, which may lead to inconsistent selection processes, specifically:

– the assumption of linearity of user's preferences is not justified; on the contrary, the experiments indicate that in most cases these preferences are nonlinear,
– in the methods cited above, search results not only depend on the ordinal structure of ranks assigned by the user to objects, but also on their values. This is incoherent with the basic assumptions of utility theory,
– the assumption that any object the user is looking for can be represented by a single point in feature space does not always correspond to real-life situations.

Therefore in the subsequent sections we propose an extension of the relevance feedback approach by using the specific graphical queries originating from the reference sets method in MCDA.

2. Image retrieval methods based on reference sets

Reference sets (cf. [19], [20]) have been originally designed to support industrial design and financial decisions. However, as we will show below, they are very well suited as a selection supporting tool in interactive image retrieval processes. Recall that *reference sets* are defined as sets of points in the criteria space with similar levels of utility. Skulimowski defines four basic types of reference sets in the monograph [19], cf. also [20]:

– A_0 – bounds of optimality – upper (in case of maximisation of criteria) borders of area where optimisation of criteria makes sense.
– A_1 – *target points* – goals of optimisation.
– A_2 –*status quo solutions* – existing solutions, which should be improved in optimisation process or lower bounds of the set of satisfactory solutions.
– A_3 – *anti-ideal point* – solutions to avoid.

The above sets can be further split into subclasses. All or only a few classes of reference sets may occur in a decision problem, while the consistency of problem formulation imposes a set of conditions to be fulfilled by the reference sets (cf. [20]).

The reference sets are always defined in the context of a multicriteria optimization problem, i.e.:

$$(F\colon D \rightarrow E) \rightarrow max \qquad (7)$$

where $F=(F_1,\ldots,F_N)$ are criteria to be optimised, E is the space of criteria values ordered by a partial order "$\leq$" which is consistent with the preference structure (1).

Let us recall that the solutions to 00 are called "Pareto-optimal". We will show below analogies between decision support systems based on reference sets and image retrieval systems with relevance feedback. It should be noted that images in a database can be seen as elements of the set of feasible solutions. Therefore, we will redefine the interpretation of reference sets in the context of image retrieval:

– A_0 is a set of graphical queries provided by the user. We assume that the goal of the user is to find an image which is most similar to one of his queries. When the user cannot provide a query, then $A_0=\varnothing$.
– A_1 is a set of reference images ranked by the user as *relevant at the most desired level*.
– A_2 is a set of images ranked by the user as *relevant*.
– A_3 is a set of images ranked by the user as *irrelevant*.
– A_4 is a set containing images ranked by the user as *anti-relevant*, i.e. characterised by attribute values opposite to those sought.

Moreover, we assume that the vector criterion F in 0 need not be *a priori* known to the user, as the explicit user preferences constitute the primary background information. The present approach bases on an assumption that the criteria can be constructed gradually using the preference information elicited during the search process. Thus even the number of relevance criteria cannot be assumed to be a priori known as various classes of graphical objects may be characterised by different sets of features and coefficients.

2.1. Elimination of dominated solutions

In an image retrieval system with a variable number of criteria, not all dominated solutions can be rejected, because some of them can become non--dominated (Pareto-optimal) in next iterations of the search process. For example, when the new criterion F_3 is added in the problem $(F_2, F_2,)\rightarrow$max, a previously dominated solution *b* with F_3(b)> F_3(a) for all a∈D will become non-dominated. In order to avoid a premature elimination of solutions which are temporarily dominated, in our algorithm we will eliminate only solutions dominated by images assigned by the user to sets A_3 or A_4.

Sets A_1 to A_4 can change during the search process. In every iteration, *K* solutions are presented (e.g. *K*=12) and assigned by the user to one of sets Ai. We assume that solutions in *i*-th iteration are at least as good as in previous iterations, therefore the solution assigned to set Ai cannot be later assigned to A_j for j<i – therefore we can eliminate solutions dominated by images from sets A3 or A4 because they cannot be assigned in the future to A1 or A2. The opposite situation is also possible: objects originally ranked as *relevant* among *K* randomly chosen images can be later ranked as neutral.

2.2. Image feature and selection of criteria

Criteria used for ranking images according to user preferences are modified in every iteration based on user's evaluation of images and are calculated based on subset of image features ς. The selection of image features depends on the class of images; we present the feature set for hotel selection in Section 3.3.

Let us denote by u_i <A u_j the fact that solution u_i has been assigned by the user to a reference set with an index higher than u_j. Features *f* for which it holds:

$$u_i <_A u_j \Leftrightarrow f(u_i) < f(u_i) \quad (8)$$

will be called *monotonically increasing* with respect to the user's preferences. Features for which holds:

$$u_i <_A u_j \Leftrightarrow f(u_i) > f(u_i) \tag{9}$$

will be called *monotonically decreasing*. Sets of features monotonically increasing and decreasing will be denoted by $\varsigma\uparrow$ and $\varsigma\downarrow$, respectively.

As criteria, we will select features from the set $\varsigma\uparrow$ and a decreasing function of features from the set $\varsigma\downarrow$. Utility function will then be calculated based on two criteria: distance from the set A_1 (or A_0, if it has been defined by providing virtual queries) and distance from the set A_4. Therefore utility function can be expressed as:

$$v(u) = 1 / [d(u, A_1) + h(d(u, A_4))] \tag{10}$$

where h is a decreasing function. For implementation, we used $h(x):=1/(x+\varepsilon)$.

A resulting image retrieval algorithm with reference sets is presented below.

Algorithm 1. (interactive image retrieval with reference sets)

Step 1 Present to the user the set $S(i)$ of images (i is the number of the iteration), ordered according to ranking based on recently calculated information about user preferences. In the first iteration, the set $S(i)$ is chosen randomly from the database.

Step 2 The user assigns elements of the set $S(i)$ to the reference sets.

Step 3 Calculate the set of features monotonically increasing $\varsigma\uparrow$ and monotonically decreasing $\varsigma\downarrow$.

Step 4 Calculate criteria values based on $\varsigma\uparrow$ and $\varsigma\downarrow$, estimation of utility function v and calculation of utility of images from the set $S(i)$.

Step 5 Check if $\forall\ u_1, u_2 \in S(i)\ u_i <_A u_j \Rightarrow v(u_1) > v(u_2)$. If the condition is not fulfilled, the user should redefine reference sets, and we return to Step 2.

Step 6 Assign images dominated by the elements of $\{A_3(i) \cup A_4(i)\}$ to the set of dominated solutions.

Step 7 Calculate utility for all images in database.

Step 8 Rank all images in database based on utility function.

Step 9 Assign $i=i+1$, return to Step 1.

■

3. An example of real-life image retrieval with reference sets

To evaluate the above-presented method, we have developed an interactive system Scene Retrieval for Matlab environment. Tests have been done for image-based hotel search. The set of image features depends on a specific application and class of images. The feature set for our application is presented in Table 1.

Table 1

Set of image features for favourite hotel retrieval

No.	Elementary criteria/feature description
1	area of hotel divided by area of image
2	area of forest divided by area of image
3	area of meadow divided by area of image
4	area of sea divided by area of image
5	area of swimming pool divided by area of image
6	area of beach divided by area of image
7	area of forest divided by area of hotel
8	area of meadow divided by area of hotel
9	area of sea divided by area of hotel
10	area of swimming pool divided by area of hotel
11	area of beach divided by area of hotel
12	number of segmented parts of image recognized as parts of hotel
13	width of hotel divided by width of image
14	height of hotel divided by height of image
15	height of hotel divided by its width
16	width of forest areas divided by width of image
17	height of forest areas divided by height of image
18	value of feature17 divided by value of feature 16

In Figure 2, we can see 6 out of 137 images of Greek hotels, available at www.dilos.com.

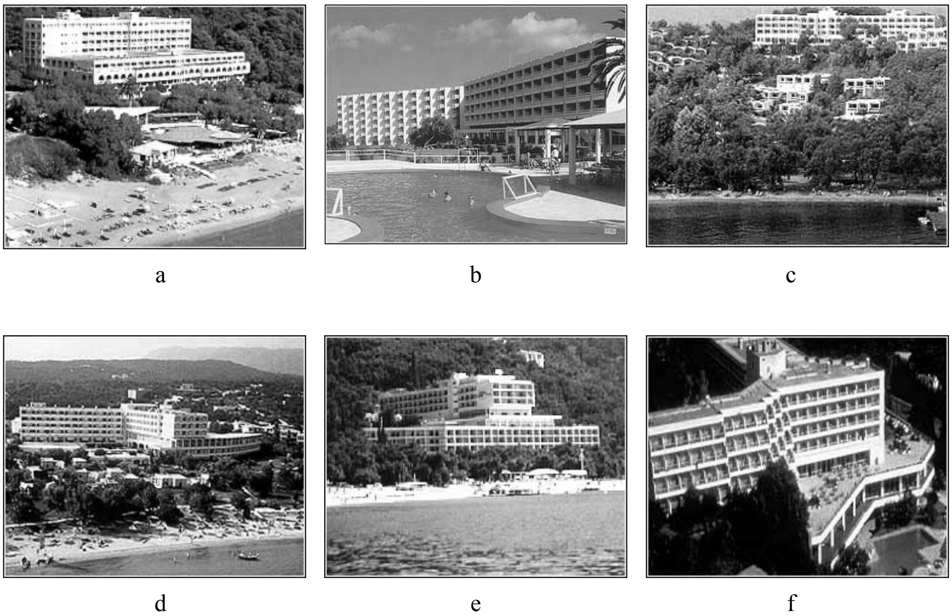


Figure 2. Images of hotels presented to the user in the first iteration of search algorithm

Let us assume that the user – intentionally or not – selects a hotel when the image contains a lot of greenery (the main user criterion) and with small buildings (additional criterion), therefore images are assigned:

– to the set A_1 if the image contains forest and small buildings,
– to the set A_2 if the image contains forest and big buildings,
– to the set A_3 if the image contains no forest (or a small area of forest) and small buildings,
– to the set A4 if the image contains no forest (or a small area of forest) and larger buildings.

In a single iteration, six images are presented to the user. This number, determined from the point of view of efficiency of the decision-making process, is a result of experiments aiming at minimizing the overall reflexion time of the user. Images presented in the first iteration are shown in Figure 2. Reference sets A_1,...,A_4 assigned by the user to these images are shown in Table 2.

Table 2

Preferences of the user, expressed by assignment of images to reference sets

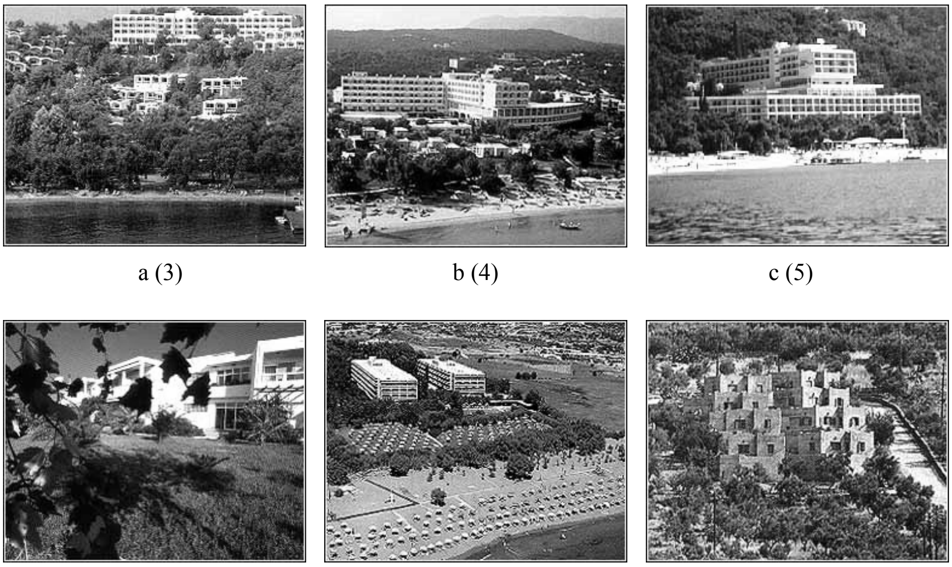
No. of image	No. of reference set	Filename
1 (a)	2	1811.jpg
2 (b)	4	1814.jpg
3 (c)	1	1818.jpg
4 (d)	1	1824.jpg
5 (e)	2	1831.jpg
6 (f)	2	1836.jpg

Based on data presented in Table 2, the Algorithm 2 automatically found features that change monotonically with a change of utility value. Features monotonically increasing (i.e. with smaller value for higher level of user's satisfaction) are: 1 and 14 and features monotonically decreasing are 2, 7, 16 and 17 – cf. Table 1. The ranking of 6 images with the lowest value of utility function is presented in Table 3 and in Figure 3.

Table 3

Results of search: 6 images with the lowest value of estimated utility

No. in ranking	No. in database	Utility	Filename
2	3	0.0053	1818.jpg
3	4	0.0494	1824.jpg
4	5	0.8899	1831.jpg
5	111	0.9782	1939.jpg
6	33	1.0472	1143.jpg
8	36	1.2404	1156.jpg



a (3) b (4) c (5)

d (111) e (33) f (36)

Figure 3. Search results – 6 images with the lowest value of utility function in brackets: numbers of images in the database

Experiments, such as the one described above, and others, which can be found in [12], show that the system is able to elicit user's preferences based on his/her assessment of several exemplary images. Preferred image features are calculated correctly (cf. Table 4) and retrieved images correspond to user's expectations, therefore we can claim that the method proposed can be useful for interactive image retrieval systems.

Table 4

Image features preferred by the user, automatically calculated by the system
(reference to the example in Figures 2-3)

Image features preferred by the user	– large area of forest (the main criterion) – small size of buildings (additional criterion)
Preferred image features calculated by the system based on 6 examples	Low value of: – area of hotel divided by area of image – height of hotel divided by height of image High value of: – area of forest divided by area of image – area of forest divided by area of hotel – width of forest areas divided by width of image – height of forest areas divided by height of image

Table 4 contd.

Preferred image features calculated by the system based on 12 examples	Low value of: – width of hotel divided by width of image – height of hotel divided by its width High value of: – width of forest areas divided by width of image – height of forest areas divided by height of image

The above preference elicitation algorithm is just a single component of the entire image retrieval system. Image analysis is another crucial part and overall usefulness of the above method depends on correct classification of image objects (cf. [15]). The performance of an image retrieval system is therefore dependent on the low-level feature detection and image recognition methods and strongly depends on the properties of the class of images where the search is performed.

Final remarks

In the above approach to the image retrieval, which most likely appears as a problem of selecting an image from a multimedia database, we have successfully applied the reference sets – a MCDM tool originally designed for other types of decision-making problems. Conversely, it may turn out that the methods of visual information extraction might be used in multicriteria decision-making problems in other areas of application. In particular, when the set of feasible alternatives may be characterized by a set of elementary features, they might be implicitly extracted as used as pre-criteria by a decision support system. Based on users' feedback, an automatic elaboration of *ceteris paribus* (CP) nets (cf. e.g. [2]) for each pre-pre-criterion might be possible and – in turn – might support an interactive search for a compromise solution. As a potential field of application of such methods one can mention the situation where a complex technical system is to be chosen by a person or a group of decision makers without an adequate technical knowledge.

The most straightforward application of relevance feedback methods enhanced by the reference sets approach presented here is web visual object search systems. At the time when this paper was written (May 2007) the commonly used systems did not allow to define graphical queries directly, which indicated a lack of adequate image search mechanisms. A prototype

(Matlab) implementation of the algorithms here presented ([21]) points out that the use of MCDM methods is very promising in the field of multimedia databases.

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Olena Sobotka

THE USE OF THE REFERENCE MCDM METHODS TO DEFINE THE SECOND STOCHASTIC DOMINANCE EFFECTIVE PORTFOLIOS*

Abstract

The paper is devoted to the application of stochastic dominance rules to portfolio selection problem with diversification possibilities. The approach based on multi-criteria decision making methodology, proposed by W. Ogryczak, is considered. The paper describes the application of the reference methods to define the set of the SSD effective portfolios and to choose the portfolio according to the general model of preference under risk.

Keywords

Second stochastic dominance, effective portfolios, compromise programming, bi-reference procedure of multi-criteria optimization.

Introduction

The portfolio selection problem is one of the classical problems of decision theory under risk. The selection of portfolio can be done according to the investor's risk preferences described by the certainty equivalent function – the utility function in the expected utility theory or the distortion function in the dual theory of the decision under risk. These certainty equivalent functions are implicit and not available before the decision process. That is why the stochastic dominance (SD) concept has been widely applied to portfolio selection problems in the last decades. The theoretical attractiveness of SD lies in its non-parametric orientation. SD criteria do not require the full specification of decision-maker's risk preferences, but rather rely on general preference assumptions [4].

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But practical application of SD rules to portfolio problems with diversification possibilities is difficult, because these rules are based on pairwise comparison of distribution functions of linear combinations of random variables. This problem can be modeled using the multi-criteria optimization problem. Such approach, proposed by Ogryczak [7], is considered in this paper.

First, the motivation for the use of the multi-criteria optimization methods to define the SSD effective portfolios is presented. We analyze the consistence of the preference structure among the criteria of the multi-criteria problem generating the SSD effective portfolios with the preferences under risk. We consider different models of the risk preferences.

Then the definition of the set of the SSD effective portfolios by the methods of the compromise programming, proposed by M. Zeleny [13], is described.

The selection of the SSD effective portfolio according to the models of the risk preferences is also possible. For this aim, we use the bi-reference procedure of multi-criteria optimization, proposed by W. Michalowski and T. Szapiro [5].

1. The models of preference under risk

The portfolio selection problem is considered, as follows. Let us denote the returns of n assets, comprising the investment universe, by $\mathbf{r} = (r_1, r_2, ..., r_n)$. The returns are the random variables with cumulative distributions functions $G_{r_i}(t) = \Pr\{r_i \le t\}, t \in \mathbf{R}, (i = 1,...,n)$. The investor may diversify between the assets and the decision vector $\mathbf{x} = (x_1, x_2, ..., x_n)$ is a vector of portfolio weights. Portfolio weights belong to the portfolio possibilities set $\mathbf{X} = \{\mathbf{x} \in \mathbf{R}^{\mathbf{n}}_{+} : \sum_{i=1}^{n} x_i = 1\}$. The return of portfolio $R = \sum_{i=1}^{n} r_i x_i$ as a linear combination of random variables is a random variable too with cumulative distribution function $G_R(t) = \Pr\{R \le t\}, t \in \mathbf{R}$.

The random returns of portfolios are compared using the investor's risk preferences described by the function of the certainty equivalent – the utility function u(t) in the expected utility theory or the distortion function w(t) in the dual theory of the decision under risk.

The utility function u(t) assigns to the value $t \in [a, b]$ the probability u(t) with which the lottery, where the gain b is given with probability u(t) or gain a is given with probability (1-u(t)), is equivalent to the receiving the certainty value t. The utility function u(t) is non-decreasing on t and for a risk-averse decision-maker this function is concave on t.

If the investor's preferences are described by the utility function u(t), the optimal portfolio is the one maximizing the Neumann-Morgenstern's expected utility function of the portfolio return:

$$U(R) = Eu(R) = \int_a^b u(t)dG_R(t) \to \max \tag{1}$$

In the dual theory of decision under risk [12] the certainty equivalent is the distortion function w(t). Distortion function w(p) assign to the probability value $p \in [0,1]$ the part of the lottery gain, which received certainly is equivalent to the participation in lottery where gain is given with probability p. The distortion function w(p) is non-decreasing on p and $w(0) = 0$, $w(1) = 1$. The risk-averter's distortion function is convex on p.

If the distortion function describes the investor's preferences, the optimal portfolio is the one which maximizes the Yaari's functional:

$$W(R) = \int_a^b w(G_R^*(t))\,dt = \int_0^1 (G_R^*)^{-1}(p)\,dw(p) \tag{2}$$

where:

$G^*_R(t) = \Pr\{R > t\} = 1 - G_R(t)$ $t \in \boldsymbol{R}$ survival function of R,

$(G_R^*)^{-1}(p) = \inf\{t \mid G^*_R(t) \le p\}$ $0 \le p \le 1$ – inverted function of $G^*_v(t)$.

The utility function and the distortion function are complementary descriptions of the decision-maker's attitude to the risk and are explored together in the modern theories of decision under risk. Rank-dependent Expected Utility Theory (RDUT) [8] and Cumulative Prospect Theory (CPT) [11], using the combination of the utility function and the distortion function in the model of decision-maker's preferences, allow to describe the preferences under risk more flexibly.

The most characteristic feature of this theories is the consideration of the rank-dependence and reference-dependence of the preferences under risk [9, 1]. The rank-dependence of the decision-maker's preferences is the non-linear perception of the probabilities – the overweighting of large probabilities and underweighting of small probabilities. The reference-dependence of the decision-maker's preferences means that the losses (the negative deviations from the status-quo) are perceived differently than gains. The aversion to the loss weights significantly more than the attraction of a corresponding gain, this feature called loss aversion.

2. Stochastic dominance concept and portfolio selection problem

In practical applications full information about the preference function is not usually available and this is the reason for using the stochastic dominance criteria that rely on a set of general assumptions rather than a full specification of the preference function.

Stochastic dominance criteria allows to divide the set of feasible decisions into efficient and inefficient sets depending on general assumption about attitude to risk. Uncertain returns are compared by pointwise comparison for some performance function constructed from distributions functions [4].

The first stochastic dominance (FSD) criterion assumes that the decision-maker prefers more to less. The return of portfolio $\mathbf{x}'$ dominates the return of portfolio $\mathbf{x}''$ in the sense of first stochastic dominance if and only if

$$G_{R'}(t) \le G_{R''}(t), \forall t \in R$$

$$\text{or} \quad G_{R'}^{(-1)}(p) \ge G_{R''}^{(-1)}, \forall p \in [0;1]$$

where $G_R^{(-1)}(p) = \inf\{t \mid G_R(t) \ge p\}$ is a p-quantile function of random variable R.

The second stochastic dominance (SSD) criterion assumes risk aversion. The return of portfolio $\mathbf{x}'$ dominates the return of portfolio $\mathbf{x}''$ in the sense of second stochastic dominance if and only if

$$G_{R'}^{(2)}(t) = \int_{-\infty}^{t} G_{R'}(\alpha)d\alpha \le \int_{-\infty}^{t} G_{R''}(\alpha)d\alpha = G_{R''}^{(2)}(t), \forall t \in R$$

$$\text{or} \quad G_{R'}^{(-2)}(p) = \int_{0}^{p} G_{R'}^{(-1)}(\alpha)d\alpha \ge \int_{0}^{p} G_{R''}^{(-1)}(\alpha)d\alpha = G_{R''}^{(-2)}(p), \forall 0 \le p$$

A portfolio is efficient if its return is nondominated.

An SSD efficient portfolio is preferred to an inefficient portfolio within all risk-averse preference models where larger returns are preferred.

3. SSD consistent portfolio diversification as a multi-criteria optimization problem

The diversification of portfolio makes an infinite number of choice alternatives.

Stochastic dominance consistent diversification is possible within multiple-criteria optimization methodology [7]. This approach based on point-wise approximation of the stochastic dominance conditions to a set of criteria for multi-criteria optimization problem.

This approach, based on the quantile stochastic dominance conditions, allows for taking into consideration non-expected utility theories of choice under risk.

In this approach the finite set of tolerance levels of probability $0 < \lambda_1 < \lambda_2, ..., \lambda_K = 1$ is selected and the criteria $G_R^{(-2)}(\lambda_k)$ are maximized for k=1,…,K. If the joint probability function of returns: $p_j = \Pr\{r_1 = r_{1j}, r_2 = r_{2j}, ..., r_n = r_{nj}\} \forall j = 1,...,m$, is known, this multiple criteria problem can be modeled as a problem of maximization of the worst conditional means of portfolio return m_{λ_k} for k=1,…, K [7]:

$$
\begin{aligned}
&\max\{m_{\lambda_1}, m_{\lambda_2}, .. m_{\lambda_K}\} \\
&m_{\lambda_k} = \lambda_k q_k - \sum_{j=1}^{m} d_{kj}^{-} p_j, \mathrm{k} = 1,...,\mathrm{K} \\
&d_{kj}^{-} \geq q_k - \sum_{i=1}^{n} r_{ij} x_i, \mathrm{k} = 1,...,\mathrm{K},\ \mathrm{j} = 1,...,\mathrm{m} \\
&d_{kj}^{-} \geq 0, \mathrm{k} = 1,...,\mathrm{K}, \mathrm{j} = 1,...,\mathrm{m} \\
&\mathrm{x} \in X
\end{aligned}
\tag{3}
$$

This multiple criteria model is consistent with the SSD relation in the sense that the set of efficient solutions of this multi-criteria problem is the set of the portfolios with nondominated returns in the sense of second stochastic dominance.

Using the multi-criteria optimization methods we can not only generate the SSD efficient portfolios, but also to choose the best portfolio according to the decision-maker's preferences. When choosing the solution of the multi--criteria problem (3) we consider the preferences among the criteria of the problem (3). These preferences are consistent with the Yaari (2) model of preferences under risk.

The Yaari's functional (2) can be rewrite in the form [1]:

$$W(v)=\int_0^1 (G_R^*)^{-1}(p)dw(p)=w'(0)G_R^{(-2)}(1)+\int_0^1 G_R^{(-2)}(1-p)dw'(p) \tag{4}$$

From (4) we can see that the Yaari's functional can be approximated by the linear combination of the worst conditional means of portfolio returns m_{λ_k} with the positive coefficients $dw'(1-\lambda_k)$ and expected value of portfolio returns with coefficient $w'(0)$. The coefficients $dw'(p)$ are positive, because the function $w(p)$ is convex, if it presents risk aversion.

The coefficients representing the preferences among the criteria of the problem (3) characterize the form of the Yaari's functional, describing the preferences under risk. That is why the choice of the final solution of the problem (3) using the multi-criteria methods is consistent with the decision-maker's preferences under risk.

To define the set of efficient portfolios and choose the best portfolio according to the decision-maker's preferences the multi-criteria optimization methods based on the idea of the reference point are useful. These methods generate the efficient solutions, in which criteria values vector is closest to the vector of the desired (reference) values of criteria.

It is possible to define the set of the SSD efficient portfolios by applying to problem (3) the method of compromise programming proposed by M. Zeleny [13]. This method is based on the idea of the reference point.

The best portfolio according to the decision-maker's preferences can be found using the interactive methods to solve the problem (3). One of the interactive methods is a bi-reference procedure of multi-criteria optimization [5].

4. The definition of the SSD efficient portfolios using the reference multi-criteria optimization methods

To define the set of the SSD efficient portfolios we apply to the multi--criteria problem (3) the method of compromise programming, proposed by M. Zeleny [13] (described in [2, 3]). This method allows to define the set of effective solutions of the multi-criteria problem, in which criteria values vectors are closest to the vector of the reference (desired) values of criteria according to the set of metrics. This set of solutions is called the set of compromise solution.

This set of efficient solutions can be defined by solving the two-criteria problem:

$$\begin{aligned}
&\min \sum_{k=1}^{n} v_k (a_k - f_k(\mathbf{x})) \\
&\min(\max\{v_k (a_k - f_k(\mathbf{x})) : k = 1,\dots,n\}) \\
&\mathbf{x} \in \mathbf{X}
\end{aligned} \tag{5}$$

where:

$f_k(\mathbf{x})$ – k-th criterion of the multi-criteria problem (k=1,…, n),

a_k – reference value of the k-th criterion (k=1,…,n),

v_k – weight of the k-th criterion (k=1,…,n).

Using the parametric method to solve the two-criteria problem (5), we can define the corner points of the set of compromise solutions.

Applying this methods to problem (3) we have to choose the set of reference values of criteria. It is reasonable to assume that a reference point is a certain portfolio return of the desired value. Then, if the desired value of portfolio return is y^*, than the reference value of the k-th criterion in the problem (3) is $\lambda_k y^*$.

The two-criteria problem defining the set of compromise solutions of the problem (3) is the problem (6).

The weights v_k used in the problem (5) to normalize the criteria values are not necessary to use in the problem (6).

Solving the problem (6) by the parametric method we can define corner points of the set of the SSD efficient portfolios, whose returns are closest to the reference return y^*.

By varying the value of the reference return we can define the corner points of the set of the SSD efficient portfolios in the area of the reference return.

$$\max \sum_{i=1}^{m} (m_{\lambda_k} - \lambda_k y^*)$$
$$\max(\min\{(m_{\lambda_k} - \lambda_i y^*) : k = 1,..., K\})$$
$$m_{\lambda_k} = \lambda_k q_k - \sum_{j=1}^{m} d_{kj}^{-} \mathrm{p}_j, \; \mathrm{k} = 1,..., \mathrm{K}$$
$$d_{kj}^{-} \geq q_k - \sum_{\mathrm{i}=1}^{n} r_{ij} x_i, \mathrm{k} = 1,..., \mathrm{K}, \; \mathrm{j} = 1,..., \mathrm{m} \tag{6}$$
$$d_{kj}^{-} \geq 0, \mathrm{k} = 1,..., \mathrm{K}, \mathrm{j} = 1,..., \mathrm{m}$$
$$\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{x}_{\mathrm{i}} = 1;$$
$$\mathrm{x}_{\mathrm{i}} \geq 0, \mathrm{i} = 1,..., n$$

To define the best portfolio with respect to the decision-maker's risk preferences we can use the interactive multi-criteria decision making methods. Using the interactive methods we do not need to define the preference function in explicit form, but investigate the decision-maker's preferences trying to generate the most acceptable effective solution.

One of the interactive methods based on the idea of reference point is a bi-reference procedure of multi-criteria optimization [5]. In this method the structure of preference is specified by two sets of reference points (worst and ideal values of criteria), that is why it can be used to search for the best portfolio according to the decision-maker's risk preferences.

When the worst and the ideal values of the criteria are identified, the improvement direction from the worst to the ideal points is constructed. A trial solution is found by moving from a current solution along the improvement direction, while maximizing the step size. For a trial solution the decision-maker is requested to divide the set of criteria to three categories: those to be improved, those to be unchanged, and those which may be relaxed. Based on this partition new sets of worst and ideal values of criteria are constructed, a new improvement direction is calculated and a new trial solution is found. The method terminates when two trial solutions are reasonably similar.

By varying the set of the ideal values of portfolio and the partition of the set of criteria we can realize different strategies of the best solution search, modeling the rank-depending and the reference-depending risk preferences.

5. Illustrative example

Consider the diversification among the 4 investment projects. The joint probabilities of the project's returns are estimated and given in Table 1.

Table 1

Probability	Returns (%)			
	project A	project B	project C	project D
p_j	r_{1j}	r_{2j}	r_{3j}	r_{4j}
0,1	–200	90	50	350
0,1	–105	90	10	150
0,3	25	85	10	–100
0,3	75	–39	0	–120
0,2	100	–150	0	50

We have to define the portfolio weights $x_i \geq 0, i = 1,..4, \sum_{i=1}^{4} x_i = 1$ which maximize the random return of the portfolio: $R = \sum_{1}^{4} r_i x_i \xrightarrow[\mathbf{x}]{} \max$

Assuming risk-aversion of the decision-maker's preferences, we applied the second stochastic dominance rule to define the effective portfolios.

We formulated the multi-criteria problem for maximizing the worst conditional means of portfolio return (3) for the set of the probability levels $\lambda = \{0{,}1;\ 0{,}4;\ 0{,}6;\ 0{,}9;\ 1\}$ and solved it using the M. Zeleny's method of the compromise programming (6).

By warying the value of the reference return y^* from 6 to 10, we defined the corner SSD effective portfolios with expected portfolio return around 10. This corner effective portfolio is presented in Table 2.

Table 2

Expected value of portfolio return (%)	SSD effective portfolios (decisions weights)			
	X_1	X_2	X_3	X_4
1	2	3	4	5
9,80	0,09	0,02	0,89	0
10,21	0,14	0,03	0,83	0

Table 2 contd.

1	2	3	4	5
10,48	0,15	0,01	0,84	0
10,34	0,15	0,03	0,82	0
10,54	0,16	0,01	0,83	0
10,19	0,15	0,05	0,79	0,01
10,37	0,16	0,03	0,8	0,01
9,72	0,2	0,1	0,66	0,04
9,79	0,25	0,11	0,57	0,07

As a final decision we can select any portfolio presented in Table 2.

To generate the portfolio in accordance with the decision-maker's risk preferences we used the bi-reference procedure, proposed by W. Michalowski and T. Szapiro.

We selected the set of the worst values of the criteria of the multi-criteria problem maximizing the worst conditional means of the portfolio return:

λ	0,1	0,4	0,6	0,9	1
$m_\lambda^W(0)$	-6	0	0	5,4	8

Selecting the portfolio by the bi-references procedure, we performed three search strategies, modeling reference-depending and rank-depending risk preferences.

The first strategy is to model the aversion to worst returns. We improved the values of the worst conditional means for levels from 0,1 to 0,4, relaxing the values for the other levels.

The second strategy modeled the aversion to the worst returns and the aversion to not receiving the best returns. We improved the values of the worst conditional means for the levels 0,1 and 1, relaxing the values for the other levels.

The third strategy modeled loss-aversion, when the losses were the returns less than 0. We looked for the portfolio with the positive value of the worst conditional mean for the level 0,1 by improving the value for the level 0,1 and relaxing the values for other levels. Then this solution was improved by fixing the achieved value for level 0,1 and improving the value for other levels.

The portfolios selected by the strategies are presented in Table 3.

Table 3

	x_1	x_2	x_3	x_4
Strategy 1	0,10	0,03	0,87	0
Strategy 2	0,21	0,14	0,65	0
Strategy 3	0,14	0,08	0,78	0

The values of the worst conditional means of the selected portfolios are presented in Figure 1.

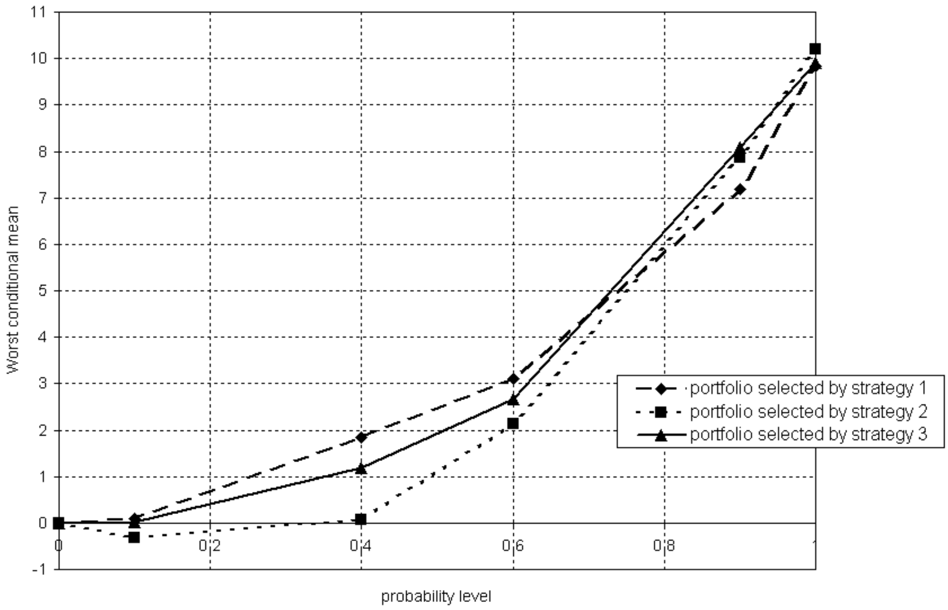

Figure 1. The worst conditional means of the returns of the portfolios selected by the three strategies

Conclusions

The SSD-consistent portfolio selection can be modeled using the multi--criteria decision making approach. The consistency of the preferences for the criteria of the multi-criteria problem generating the SSD-effective portfolios with the preferences under risk allows to select the SSD-effective portfolio using the multi-criteria optimization methods. The reference methods are useful for this problem.

By applying the method of the compromise programming, proposed by M. Zeleny, we defined the corner SSD-effective portfolios with returns around the reference value. By using the interactive bi-references multi-criteria optimization method, proposed by W. Michalowski and T. Szapiro, we selected the best portfolios with respect to the decision-maker's preferences under risk, modeling the rank-dependence and the reference-dependence of the preferences.

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Sebastian Sitarz

DYNAMIC STOCHASTIC PROBLEMS OF PROFIT MAXIMIZATION WITH PARTIALLY ORDERED CRITERIA SPACE

Abstract

Stochastic dynamic programming (DP) is a strong mathematical tool allowing modeling and solving many multiperiod decision processes. Multiple objective and dynamics characterize many sequential decision problems. In the paper we consider returns in partially ordered criteria set as a way of generalization of single criterion DP models to multiobjective case.

In the present paper, on the basis of theoretical findings, described in our previous papers we consider exemplary stochastic DP profit maximization processes. Because of the lack of space we omit the general, formal description of such a process and concentrate on explanation, how the theory of DP models in partially ordered criteria space works. Both in level-volume and velocity-volume process we will consider formulated problems step by step, first as single criterion problems and next as bi-criteria ones. Conclusions are presented in the last section.

Keywords

Stochastic dynamic programming, multiobjective dynamic optimization, profit maximization, partially ordered criteria space.

Introduction

Stochastic dynamic programming (DP) is a strong mathematical tool for modeling and solving many multiperiod decision processes. There are many stochastic DP applications in different fields. One of the most important of them is the profit maximization problem. Different aspects of this problem have been considered in literature. Recently Teunter [12] proposed a stochastic DP algorithm for determining the optimal disassembly and recovery strategy, the quality-dependent recovery options and associated profits for assemblies. Zhang and Piplani [19] utilized yield management technique and stochastic DP modeling to achieve maximization of expected profit in Make-To-Order manufacturing companies. Jonker et al. [13] present a joint optimization approach addressing the segmentation of customers into homogeneous groups and determining the optimal policy towards each segment. They propose a stochastic DP procedure based on the long-run maximization of expected average profit. Kamrad and Siddique [4] consider supply chain contracts as the producer's profit maximization problem with respect to the supplier's reaction and analyze risk reduction in a unique framework as a stochastic DP problem. Sboui et al. [11] present a profit maximization stochastic DP model for supply chain management.

Many sequential decision problems are characterized by multiple objective and dynamics. Research extending the principle of optimality formulated in Bellman [1] to multiobjective case made it possible to apply the vector principle of optimality to deterministic, stochastic and fuzzy problems. A review of multiobjective dynamic programming (MODP) models was done by Li and Haimes [6] and more recently by Trzaskalik [13].

Changeable hierarchy problems belong to the most challenging issues in MODP. Period criteria for separate periods and multiperiod criteria for the whole process can be distinguished. A period criterion is called important in a given period if it is considered in the evaluation of the process in that period. The following questions can be asked: how multiperiod criteria depend on period criteria and how to define preference structure in such a case? The notion of importance of criteria and the definition of preference structure was introduced and elaborated by Trzaskalik [16, 15, 14].

Another way of generalization of single criterion DP models is to consider returns in partially ordered criteria space. First attempts were done by Mitten [7], Sobel [9], Steinberg and Parks [10], Henig [2]. More recently, discrete DP problems with partially ordered criteria space were considered by Trzaskalik and Sitarz [18, 17]. It is worth noticing that MODP is based on the Pareto concept of optimality that determines a partial order in the criteria space, so each MODP model is also a DP model with returns in partially ordered criteria space. On the other hand, there exist single criterion DP problems with returns in partially ordered criteria space, which obviously are not MODP problems. Examples can be found below.

In the present paper we will consider stochastic problems of profit maximization as examples of DP problems in partially ordered criteria space. We will consider a situation in which important criteria for consecutive periods depend on the progress of the process until now, and, in particular, on the cumulated values of criteria from the beginning of the process.

The problem is stated as follows. We consider an investment multiperiod decision process. Decisions can be made at the beginning of consecutive periods. Probabilistic distributions of period returns are known. Two situations are of interest.

In the *level-volume* case we assume that the decision maker applies two criteria:

1a. The profit should be greater or equal to a given level (*level criterion*).

2. The profit should be as big as possible (*volume criterion*).

In the *velocity-volume* case it is assumed that the decision maker's criteria are as follows:

1b. The profit should be greater or equal to a given level as soon as possible (*velocity criterion*).

2. The profit should be as big as possible (*volume criterion*).

Such problems are examples of multiperiod, multiobjective processes, whose sets of important criteria depend on cumulated values of profit from the beginning of the process. We will define the set of important criteria as a function of these values. Such problems are close to real-life problems considered by decision makers in financial assessment of investments.

In the present paper, on the basis of theoretical findings described in our previous papers [17, 18] we will consider examples of stochastic DP profit maximization processes. Because of the lack of space we will omit the general, formal description of such processes and concentrate on explanation of how the theory of DP models in partially ordered criteria space works. Both in the level-volume and velocity-volume process we will consider the problems step by step, first as single criterion problems and next as bi--criteria ones.

The paper consists of six sections. In Section 1 we will describe the considered process and in particular, its dynamics and outcomes. In Section 2 we will consider the level-volume case and in Section 3, the velocity-volume case. In Section 4 we will review all the process realizations from the point of view of criteria considered and problems solved. Conclusions are presented in section 5.

1. Description of the process

Let us consider an example of the multiperiod decision process, presented in Figure 1. The nodes of the graph correspond to the states of the process and the arcs correspond to the feasible period decisions. Paths in the graph leading from the initial states of the process to its end correspond to process realizations. We assume that the transition functions of the process are deterministic. This means that if in a given state a decision is made, then the state of the process at the beginning of the next period is determined by means of the appropriate transition function. Outcomes (profits) of the process in subsequent periods are realizations of discrete random variables with given distributions. The values on arcs are intepreted as probabilities of profits equal to 0, 1, 2,... For instance, if at the beginning of period 4 the process is in the state 0 and we take decision 0, the probability of profits are realizations of period random variable $\xi_4(0,0)$. We have the following probabilities (see Figure 1):

$$P[\xi_4(0,0)=0] = 0.1 \quad P[\xi_4(0,0)=1] = 0.2 \; P[\xi_4(0,0)=2] = 0.7$$

We denote this probability distribution as (0.1, 0.2, 0.7).

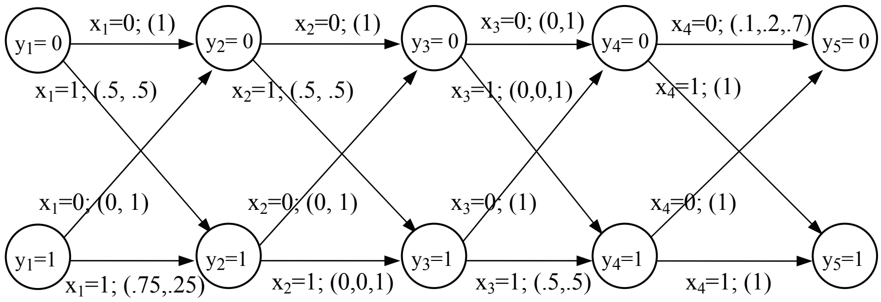

Figure 1. The graph of the process under consideration

Let us consider a process realization starting in the state y_1 and consisting of states and decisions: y_1, x_1, y_2, x_2, y_3, x_3, y_4, x_4, y_5. Since the transition functions are deterministic, it is sufficient to consider the states only and to omit the decisions, so we denote this realization as $d = (y_1, y_2, y_3, y_4, y_5)$. The total outcome for the realization d is a realization of random variable $\xi(d)$, which is the sum of realizations of random variables $\xi_t(y_t, x_t)$ for t=1,...,4, hence

$$\xi(d) = \xi_1(y_1, x_1) + \xi_2(y_2, x_2) + \xi_3(y_3, x_3) + \xi_4(y_4, x_4)$$

For instance, for the process realization $d^0 = (0, 0, 0, 0, 0)$ we have

$$\xi(d^0) = \xi_1(0,0) + \xi_2(0,0) + \xi_3(0,0) + \xi_4(0,0)$$

It is easy to find the probability distribution $p(d^0)$ for $\xi(d^0)$. We obtain:

$$P[\xi(d^0)=0]=0 \qquad P[\xi(d^0)=1]=0.1$$
$$P[\xi(d^0)=2]=0.2 \qquad P[\xi(d^0)=3]=0.7$$

We denote the set of all process realization as D and the set of all discrete probability distributions for all the realizations of the process as $\Xi(D)$.

Let us consider the second example of the realization of the process, for instance $d^4 = (0, 0, 1, 0, 0)$ (for the numbering of realizations see Table 1). The probability distribution for $\xi(d^4)$ is as follows:

$$P[\xi(d^4)=0] = 0.05 \quad P[\xi(d^4)=1] = 0.15$$
$$P[\xi(d^4)=2] = 0.45 \quad P[\xi(d^4)=3] = 0.35$$

We can compare process realizations according to the FSD (first stochastic dominance) rules (see [8]). Let us compare d^0 and d^4. For the realizations considered we obtain the following cumulated values:

1. Realization d^0:

$$c_0(d^0) = P[\xi(d^0)=0] = 0$$
$$c_1(d^0) = P[\xi(d^0)=0] + P[\xi(d^0)=1] = 0.1$$
$$c_2(d^0) = P[\xi(d^0)=0] + P[\xi(d^0)=1] + P[\xi(d^0)=2] = 0.3$$
$$c_3(d^0) = P[\xi(d^0)=0] + P[\xi(d^0)=1] + P[\xi(d^0)=2] + P[\xi(d^0)=3] = 1$$
$$c_k(d^0) = 1 \text{ for } k \geq 4$$

2. Realization d^4:

$$c_0(d^4) = P[\xi(d^4)=0] = 0.05$$
$$c_1(d^4) = P[\xi(d^4)=0] + P[\xi(d^4)=1] = 0.2$$
$$c_2(d^4) = P[\xi(d^4)=0] + P[\xi(d^4)=1] + P[\xi(d^4)=2] = 0.65$$
$$c_3(d^4) = P[\xi(d^4)=0] + P[\xi(d^4)=1] + P[\xi(d^4)=2] + P[\xi(d^4)=3] = 1$$
$$c_k(d^4) = 1 \text{ for } k \geq 4$$

Since for each k=0,1,2,... we have $c_k(d^0) \leq c_k(d^4)$ and $c_k(d^0) \neq c_k(d^4)$, it means that $\xi(d^0)$ dominates $\xi(d^4)$ according to the first degree stochastic dominance rule. We denote it as $p(d^0)$ FSD $P(d^4)$.

2. Level-volume case

2.1. Level criterion

The level criterion is important as long as the cumulated profit is less than 2. Because we analyze the process *ex ante*, before it has started, we are only interested in process realizations, whose cumulated probability distributions have the form $(r_0, r_1, r_2, \ldots,)$, and

$$r_0 = r_1 = 0 \tag{1}$$

Probability distributions for all these realizations dominate (according to FSD rule) the "weakness" probability distribution $\overline{p} = (0, 0, 1)$.

For any process realization the multiperiod level criterion function F^L takes one of the following values:
1 – assumed level of profit will be reached,
0 – assumed level of profit will not be reached.
It is possible to find the value $F^L(d)$ applying the formula

$$F^L(d) = \begin{cases} 1, & \text{if} \quad p(d)\,\text{FSD}\,\overline{p} \quad \text{or} \quad p(d) = \overline{p} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Depending on the value $F^L(d)$, each realization d is classified to one of two classes. Let $D(i)=\{d\colon F^L(d)=i\}$ $(i=0,1)$ be the set of all process realizations from the class i. We have $D(0) \cup D(1)=D$, $D(0) \cap D(1) = \varnothing$.

The preference structure can be described as follows: each process realization from D(1) dominates any realization from D(0). Realizations belonging to the same class are equally preferred. Let D^L denote the set of efficient realizations. We have $D^L = D(1)$. The set D^L can be obtained by means of the forward procedure for dynamic process with partially ordered criteria space (see [17]). Process realizations belonging to D^L are marked in Table 1.

2.2. Volume criterion

Volume criterion is important in all the periods considered. The multiperiod volume criterion function F^V has the form:

$$F^V(d) = p(d) \tag{3}$$

Let d^i, d^j, $d^i \neq d^j$ be feasible process realizations. d^i dominates d^j iff $p(d^i)$ FSD $p(d^j)$. A realization d^V is efficient if there doesn't exist any other realization d such that:

$$p(d)\ \text{FSD}\ p(d^V) \tag{4}$$

The set D^V of all efficient realizations can be obtained by means of forward dynamic procedure. Process realizations belonging to D^V are marked in Table 1 (column 12).

2.3. Bi-criteria case

The criteria space is defined as the product {0, 1} × Ξ(D). The vector multiperiod criterion function has the form $F^{LV} = [F^L, F^V]'$. For each process realization d we have $F^{LV}(d) = [i, p(d)]'$ ($i \in \{0,1\}$, $p(d) \in \Xi(D)$). The preference structure is defined as follows. A realization d^{LV} is efficient if

$$\neg \underset{d}{\exists} \quad i \geq i_{LV} \wedge p(d) \text{ FSD } p(d^{LV}) \wedge (i \neq i_{LV} \vee p \neq p^{LV}) \qquad (5)$$

The set D^{LV} of all the efficient realizations can be obtained by means of the forward dynamic procedure. Process realizations belonging to D^{LV} are marked in Table 1 (column 13).

Let us notice that some maximal elements from the criteria space are generated by more than one process realization.

3. Velocity-volume case

3.1 Velocity criterion

The velocity criterion is important as long as the cumulated profit is less than 2. The moment of achievement of the required level of profit is important – the sooner the better. Similarly as before, we are interested only in those process realizations for which the condition (1) is fulfilled.

For any process realization, the multiperiod velocity (speed) criterion function F^S takes one of the following values:
4 – assumed level of profit will be reached at the end of the period 1,
3 – assumed level of profit will be reached at the end of the period 2,
2 – assumed level of profit will be reached at the end of the period 3,
1 – assumed level of profit will be reached at the end of the period 4,
0 – assumed level of profit will not reached.
Depending on the value $F^S(d)$, each realization d is classified to one of five classes. Let $D(i)=\{d: F^S(d)=i\}$ ($i=0,...,4$) be the set of all process realizations from the class i. We have $\cup_{i=0...4}D(i)=D$, $\cap_{i=0,...,4}D(i)=\varnothing$.

The preference structure is described as follows: for i>j each process realization from D(i) dominates any process realization from D(j). Realizations belonging to the same class are equally preferred. Efficient realizations D^S belong to the highest non-empty class. The set D^S can be obtained by means of dynamic forward procedure. Process realizations belonging to D^S are marked in Table 1 (column 14).

3.2. Volume criterion

The set of efficient realizations with respect to the volume criterion is obtained in the same way as in Section 2.2.

3.3. Bi-criteria case

The criteria space is defined as the product $\{0, 1, 2, 3, 4\} \times \Xi(D)$. The vector multiperiod criterion function has the form $F^{SV} = [F^S, F^V]'$. For each realization $d \in D$ we have $F(d) = [i, p(d)]'$. A realization d^{SV} is efficient if condition (4) is fulfilled. The set D^{SV} of all the efficient realizations can be obtained by means of the forward dynamic procedure. Process realizations belonging to D^{SV} are marked in Table 1 (column 15).

4. Review of process realizations

The set of process realizations and the results obtained for the considered process are shown in Table 1. Its structure is as follows:
column 1 – number of realization,
column 2 – trajectory (sequence of states),
column 3 – probability of profit at the level 0,
column 4 – probability of profit at the level 1,
column 5 – probability of profit at the level 2,
column 6 – probability of profit at the level 3,
column 7 – probability of profit at the level 4,
column 8 – probability of profit at the level 5,
column 9 – value of the level criterion,
column 10 – value of the velocity criterion,
column 11 – efficient realizations for the level criterion,
column 12 – efficient realizations for the volume criterion,
column 13 – efficient realizations for the level-volume case,
column 14 – efficient realizations for the velocity criterion,
column 15 – efficient realizations for the velocity-volume case.

Table 1

Process realizations and results

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	00000	0	0		0,1	0,2	0,7							
1	00001	0	0		1									
2	00010	1	2			1				x				
3	00011	1	2			1				x				
4	00100	0	0	0,05	0,15	0,45	0,35							
5	00101	0	0	0,5	0,5									
6	00110	0	0	0,25	0,5	0,25								
7	00111	0	0	0,25	0,5	0,25								
8	01000	1	2			0,05	0,15	0,45	0,35	x	x			
9	01001	1	2			0,5	0,5			x				
1	01010	1	2				0,5	0,5		x	x	x		x
11	01011	1	2				0,5	0,5		x	x	x		x
12	01100	1	3	0	0	0,05	0,15	0,45	0,35	x	x	x	x	x
13	01101	1	3	0	0	0,5	0,5			x			x	
14	01110	1	3	0	0	0,25	0,5	0,25		x			x	
15	01111	1	3	0	0	0,25	0,5	0,25		x			x	
16	10000	1	2	0	0	0,1	0,2	0,7		x				
17	10001	1	2	0	0	1				x				
18	10010	1	2	0	0	0	1			x				
19	10011	1	2	0	0	0	1			x				
20	10100	0	0	0	0,05	0,15	0,45	0,35						
21	10101	0	0	0	0,5	0,5								
22	10110	0	0	0	0,25	0,5	0,25							
23	10111	0	0	0	0,25	0,5	0,25							
24	11000	1	2	0	0	0,075	0,175	0,575	0,175	x				
25	11001	1	2	0	0	0,75	0,25			x				
26	11010	1	2	0	0	0	0,75	0,25		x				
27	11011	1	2	0	0	0	0,75	0,25		x				
28	11100	1	3	0	0	0,075	0,175	0,575	0,175	x			x	
29	11101	1	3	0	0	0,75	0,25			x			x	
30	11110	1	3	0	0	0,375	0,5	0,125		x			x	
31	11111	1	3	0	0	0,375	0,5	0,125		x			x	

Conclusions

It is worth comparing the number of efficient realizations for single-criterion processes with the number of efficient realizations in bi-criteria cases (see Table 2).

Table 2

The number of efficient realizations

Process	Criterion 1	Criterion 2	Bi-criteria case
level-volume	22	4	4
velocity-volume	8	4	3

The number of efficient realizations in the bi-criteria case (both in level--volume and in velocity-volume problems) is less or equal to the number of efficient realizations in the single-criterion case. Such situations occur infrequently in multiobjective programming. We can explain this by recalling that our single criteria problems have outcomes in partially ordered criteria spaces which usually contain more than one maximal element (contrary to single-objective mathematical programming problems, whose criterion space is an ordered set and usually there exists a unique optimal solution). Additional explanation for the case under consideration can be found in the construction of criteria and their dependences. If the required level of profit is reached faster, cumulated profits can be bigger than profits cumulated later.

The presented solution can be extended to any multiperiod process with finite number of periods, states and decisions.

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THE CLARA METHOD – A NEW APPROACH TO EXPERT VERBAL CLASSIFICATION

Abstract

In project development it is hardly possible to get exhaustive and accurate information. As a result, the situations occur, the consequences of which can be very damaging to the project. Inaccurate evaluation of the strategy related to capital investment and project implementation is one of the reasons why such estimates are not required in practice. Instead, a classification approach may be used for this purpose. Classification is a very important aspect of decision making. In the present paper, a novel algorithm CLARA is offered for ordering multicriteria alternatives. It differs from the existing similar methods in the wider range of application allowing it to be used with various scales of criteria evaluation, a random number of the solution classes, incomplete order on the criteria scales as well as in the considerably rarefied space of the alternatives. The suggested algorithm is more effective in terms of the time spent by an expert. A comparative calculation of the efficiency of the algorithms used in classifying the objects in the order of their significance has shown that CLARA is much more effective than CLANSH and other algorithms. At the same time, its general effectiveness was found to be lower than that of the algorithms CYCLE which has a narrower scope of application.

Keywords

Expert system, decision-making, verbal analysis methods; methods of solving multi-criteria classification problems.

Introduction

In practical use the task of getting expert knowledge can often be formulated similarly to the task of classification, because experts sort objects (alternatives, states of object) through classes of decision [2,6,7]. For example an engineer entity to be classified may have different origin. They can be different physical objects, choice cases or conditions of some object.

Describing the method of assigning an object to a certain class of decisions is complicated because of non-verbality of the strategy used by the expert [5]. Anyway, these non-verbal skills are effectively and promptly used, when the expert solves a task of classification in his sphere of knowledge. Classification is a very important aspect in decision making [1, 10, 12, 13]. One of the tasks preparing a basis for classification is the setting of numerous criteria (attributes), which are capable of describing any object. The scale of all criteria is formed by defining a finite set of possible values. If in certain task the scale of values of one or more criteria is infinite, it can be changed to a finite one by limiting it to a finite set of intervals. Finally, on the basis of expert knowledge classification of definite intervals and its components must be organized i.e. rules must be formulated according which any object can be assigned to one of the predefined classes [11]. The projects classified are described by assessing various efficiency criteria that can be expressed both qualitatively and quantitatively [16].

The purpose of this paper is to demonstrate how multiple criteria can be used in the analysis of facility location problems. The paper begins with an overview, explains the most popular multiple objective analysis methods used in various countries (ORKLASS, DIFKLASS, CIKL etc.), and demonstrates their applications to real-life problems. To solve the classification problem, a method called CLARA (CLAssification of Real Alternatives) has been developed. This method can be used to classify a complete set, or a specified number of objects of the set, with minimal involvement of experts [10,13,18].

1. The data of the problem

The problem may be formally represented in the following way:

1. G is the property satisfying the target criterion of the problem.
2. $K = \{K_1, K_2, ..., K_Q\}$ is a set of evaluating criteria of an object.
3. $S_q = \{k_1^q, ..., k_{w_q}^q\}$ for q=1,...,Q is a set of estimates based on the criterion K_q, w_q is the number of graduation marks on the scale of the criterion K_q; the scales are arranged in the order of distinctness of the property G.
4. $Y = S_1 \times ... \times S_Q$ is the state space of the objects to be classified. Every object is described by a number of estimates based on the criteria $K_1, ..., K_Q$. In this way, a set of alternatives $\{y_1, y_2, ..., y_L\}$ is defined, where $L = |Y| = \prod_{q=1}^{Q} w_q$ is the cardinality of a set Y, (the number of alternatives).

5. $C = \{C_1, C_2, ..., C_M\}$ is a set of classes to be obtained by breaking down the set Y^a, which should be arranged in the ascending order of distinctness of the property G (in the class C_{n+1} this property is more distinct, while in the class C_n it is less distinct).
6. $Y^a \subseteq Y$ is a set of *admissible* real objects.

Since the estimates based on each criterion are ordered, then the scale showing the order of classes S_q can be compared with the numerical scale $B^q = \{1, 2, ..., w_q\}$, where $b_i^q < b_j^q$, if b_i^q is less preferable for a decision maker (DM) than b_j^q.

The information of the DM preferences determines the relationships of rigorous preference (or dominance) P^0 in the set Y:

$$P^0 = \{(y_i, y_j) \in Y \times Y \mid \forall q \in K\ b_i^q \geq b_j^q\ \wedge \exists q^0 : b_i^{q^0} > b_j^{q^0}\}$$

implying that the alternative $x \in Y$ is dominant over the alternative $y \in Y$.

On the other hand, it is known that the classes of solutions are ordered for the DM. It means that any alternative from the class *n+1* is more preferable for the DM than any alternative from the class *n.* This is shown by the following binary preference relationship in the set Y:

$$P^1 = \{(y^i, y^j) \in Y \times Y / y^i \in Y^k, y^j \in Y^l, \quad k > l\}$$

It can be assumed that none of the vector estimates in the set Y, dominating over the given one, should be assigned to a less preferable class. This statement is known as the “hypothesis of distinctness”. It can be formally expressed as follows:

$$(y_i, y_j) \in P^0 \Rightarrow (y_j, y_i) \notin P^1 \tag{1}$$

Definition. Partition of a set of vector estimates Y into the M ordered classes is consistent if the condition (1) is *satisfied* for any $y_i, y_j \in Y$.

Based on the preferences of the decision maker, a consistent representation of $F: Y^a \rightarrow \{Y_l\}, l = 1, 2, ..., M$, has to be constructed, such that:

$$Y^a = \bigcup_{l=1}^{M} Y_l;\ Y_l \bigcap Y_k = 0$$

where $k \neq l$, Y_l is a set of the vector estimates from Y, assigned to the class C_l.

2. The analysis of verbal decision methods for classification of alternatives

Many widely known methods for solving multi-criteria classification problems are presented in Table 1 [2, 6, 7, 8, 9]. In this chapter some most frequently used verbal ordinal classification methods are considered. All these methods belong to the Verbal Decision Analysis group and have the following common features:

1. The attribute scale is based on verbal description unchanged in the process of solution, when verbal evaluation is not converted into the numerical form or score.
2. An interactive classification procedure is performed in steps, where the DM is offered an object of analysis (a course of treatment, for example). The object is presented as a small set of rankings. The DM is familiar with this type of description, therefore he/she can make the classification based on his/her expertise and intuition.
3. When the DM has decided to assign an object to a particular class, the decisions are ranked on the dominance basis. This provides the information about other classes of objects related to it by the relationship of dominance. Thus, an indirect classification of all the objects can be made based on a single decision of the DM.
4. A set of objects dominating over an object considered is referred to as a domination cone. A great number of objects have been classified many times. This ensures error-free classification. If the DM makes an error, violating this principle, he/she is shown the conflicting decision on the screen and is prompted to adjust it.
5. In general, a comprehensive classification may be obtained for various numbers of the DM decisions and phases in an interactive operation. The efficiency of multi-criteria classification technique is determined on the basis of the number of questions for the DM needed to make the classification. This approach is justified because it takes into consideration the cost of the DM's time and the need for minimizing classification expenses [2, 9, 11].

Let us consider several most commonly used methods in more detail.

ORCLASS [4, 6]. This method (Ordinal CLASSification) allows us to build a consistent classification, to verify the information and to obtain general decision rules. The method relies on the notion of the most informative alternative, allowing a great number of other alternatives to be implicitly assigned to various classes. ORCLASS takes into account the possibilities and limitations of the human information processing system.

Method assessment: The main disadvantage of the method is its low effectiveness due to the great number of questions to DM needed for building a comprehensive classification.

Table 1

Verbal analysis methods

The type of method	The purport of method	Notes
ORCLASS	This method is used for classifying different types of loans [6,7,8]	By deficiency algorithm appears its the large number of questions for DM to do the classifications [6]
DIFCLASS	This method was the first to use dynamic construction of chains covering alternative space for selecting questions to DM (decision maker) [7 9]	The area of DIFCLASS application is restricted to tasks with binary criteria scales and two decision classes [7]
CYCLE	The CYCLE algorithm makes it possible to effectively build the complete non--contradictory bases of expert knowledge for the subject areas by complete order of the scales of criteria [8, 9]	The methods can be successfully applied to classify investment projects when the decision classes and the criteria used are thoroughly revised [9]
CLANSH	The CLANSH method makes it possible to build the wheel bases of expert know-ledge, when assumption about the presence of linear order of many estimations with respect to each of the criteria is substituted by assumption about the presence of the incoherent transitive binary relation [8,13]	
STEPCLASS	System realizes technological approach to the structurization subject area and to the development of the decisive rules of expert and guarantees completeness and consistency [9]	

DIFCLASS [4]. This method was the first to use dynamic construction of chains covering *Y* space for selecting questions to DM. However, the area of DIFCLASS application is restricted to tasks with binary criteria scales and two decision classes.

CYCLE [6]. CYCLE (Chain Interactive Classification) algorithm overcomes the restrictions of DIFCLASS, generalizing the idea of dynamic chain construction to the area of ordinal classification task with arbitrary criteria scales and any number of decision classes. The chain here means an ordered sequence of vectors $\langle x_1,\ ..., x_d \rangle$, where $(x_{i+1}, x_i) \in P$ and vectors x_{i+1} and x_i differ in one of the components.

Method assessment: As comparisons demonstrate, the idea of dynamic chain construction allows us to get an algorithm close to optimal by a minimum number of questions to DM necessary to build a complete classification. The application of ordinal classification demonstrates that problem formalization as well as introduction of classes and criteria structuring allow to solve classification problems by highly effective methods.

The method can be successfully applied to classification of investment projects when the decision classes and the criteria used are thoroughly revised.

3. A method of constructing a comprehensive order classification

At the first stage, the alternatives of the set Y are numbered in the specified order. In this case, $y_i > y_j \Rightarrow i < j$. This preliminary numbering ensures that a particular alternative is considered when all the alternatives dominant over it had been already analysed.

The use of the hypothesis of distinctness (1) allows us to considerably reduce the number of questions to an expert, required to make the classification.

Let us denote by G^i a set of class numbers $Y_l (1 \le l \le M)$, admissible for the vector estimate $y_i \in Y$. Before questioning the DM (an expert), $G^i = \{1,2, \ldots,\mathrm{M}\}$ is assumed for $\forall y_i \in Y$, because we do not have any information about the expert's preferences. Finally, it is required that all G^i consist of only one element.

Suppose that the expert decided that the vector estimate $y_i \in Y$ should belong to the class $Y_l (1 \le l \le M)$ in accordance with its global quality. Following the hypothesis of distinctness, in this case a vector estimate – described by a number of the criteria values, which are not less preferable for an expert – cannot belong to a less preferable class.

Similarly, a vector estimate, described by a number of the criteria values which are not more preferable that those of y_i, cannot belong to a more preferable class.

Consequently, the data, related only to one vector estimate of *Y,* which were elicited from an expert, can result in the reduction of the sets G^i, corresponding to other vector estimates. In this way, in a particular case, vector estimates can be assigned to a particular class of vector estimates without being submitted to an expert.

It is necessary to take into consideration the possibility of assigning a particular vector to a particular class. The indicator p_{il} (assessing the possibility of assigning the vector y_i to the class Y_i) shows the proximity of the vector considered to the members of this class because the vectors of the same class usually form compact groups in multidimensional space. To calculate p_{il}, the normalized distance between the vector y_i and the center of the class C_k can be used.

Relying on two indicators, p_{il} and G^i, a unified quantitative estimate of the informativity of any not estimated state Φ can be obtained:

$$\Phi_i = f(\{p_{il}, g_{il} \mid l \in G^i\}) \tag{3}$$

where *f* is a certain real function, g_{il} is the number of vectors from Y whose membership in a particular class becomes known (i.e. the cardinal number of the corresponding set of the class numbers G^i is equal to one) if the expert assigns the vector y_{il} to the class Y_l

This concept underlies a multistage procedure of carrying on a dialogue which can be generally described in the following way. A subset of the alternatives Y_g for which the set G^i of the admissible classes contains more than one element is determined. If Y_g is empty, go to stage 7.

1. The indicator p_{il} is calculated for all the alternatives from Y_g and g_{il} is determined for $\forall l \in G^i$.
2. The indicators p_{il} are found from the formula.
3. Based on the above indicators, the amount of information of the vector $y_i - \Phi_i$ is determined.
4. $y_i \in Y_g : \Phi_i = \max_{y_j \in Y_g} \Phi_j$ is determined.

5. The above vector is submitted to an expert to be assigned to one of the classes.
6. The sets G^i are modified in accordance with the class of the vector as specified by the expert. Go to stage 1.
7. The procedure is completed.

In the ORCLASS method, mathematical expectation of the number of classified vectors is used as a function of informativity (2) for developing a comprehensive classification:

$$\Phi_i^{abcde} = \sum_{i=G^l} p_{il} g_{il} \tag{5}$$

To classify a specified subset, the number of actual alternatives, whose classes become known when a particular choice is made by an expert, should be maximized. This implies that, in calculating the indicators, g_{il}, only the alternatives belonging to Y^a should be taken into account. Thus, to achieve the specified aim, the way of determining the informativity should be changed. Similarly to the formula (3), in the following expression:

$$\Phi_i = \sum_{i \in G^i} p_{il} g_{il}^a \tag{7}$$

g_{il}^a is the number of vectors from Y^a, whose membership of a particular class becomes known when an expert refers the vector y_i to the class Y_l. The coefficients g_{il} in the informativity formula are considered a random quantity Γ_i with the probabilities of realizing the l-th value of p_{il}. Then, $\Phi_i = M\Gamma_i$, where $M\Gamma_i$, is mathematical expectation of the random quantity Γ_i. The spread of the random quantity Γ_i about its mean value $M\Gamma_i$ is the mean square deviation $\sigma_i = \sqrt{D\Gamma_i} = \sqrt{M(\Gamma_i - M\Gamma_i)^2} = \sqrt{M\Gamma_i^2 - (M\Gamma_i)^2}$, where $D\Gamma_i$ is the variance Γ_i However, relative rather than absolute deviation is important for this analysis. In fact, large deviations may be allowed for large values of Φ_i. Therefore, the following function can be used to express the informativity:

$$\tilde{\Phi}_i = \frac{\Phi_i}{1 + n\dfrac{\sigma}{\Phi_i}} = \frac{\Phi_i}{1 + n\dfrac{\sqrt{\sum_{l \in G_i} p_{il}(g_{il}^a)^2 - \Phi_i^2}}{\Phi_i}}, \quad n \geq 0 \tag{5}$$

The following notation is used in this formula:

Φ_i is the informativity in the sense of mathematical expectation g_{il}^{a} (4),

σ / Φ_i indicates relative deviation of the indicators g_{il}^{a} from their mean value,

A unity is added to the denominator for it to be not less than one (i.e. to be more than zero in all cases), n is an empirical multiplier in the case of relative deviation referred to as *the significance level of variance*. This multiplier allows us to specify the effect of deviation on informativity. It is clear that, when n=0, $\widetilde{\Phi}_i = \Phi_i$, i.e. informativity is determined without taking into account the variance. This helps to avoid some "risky" situations, when an alternative is offered to the expert for evaluation with the varying numbers of indirectly classified alternatives (depending on the expert's decision).

4. CLARA. Classification algorithms

The CLARA algorithm (Classification of Real Alternatives) is based on the dichotomy of the chains of alternatives, beginning with the longest chain. This concept, first used in the DIFCLASS algorithm [6] and then in CLANSH [18], has been adapted for rarefied spaces Y. Moreover, the CLARA algorithm uses a new idea of the adaptive dichotomy allowing us to determine the boundaries between classes of solutions and perform classifications much faster.

A general block-diagram of the CLARA algorithm is presented in Figure 1.

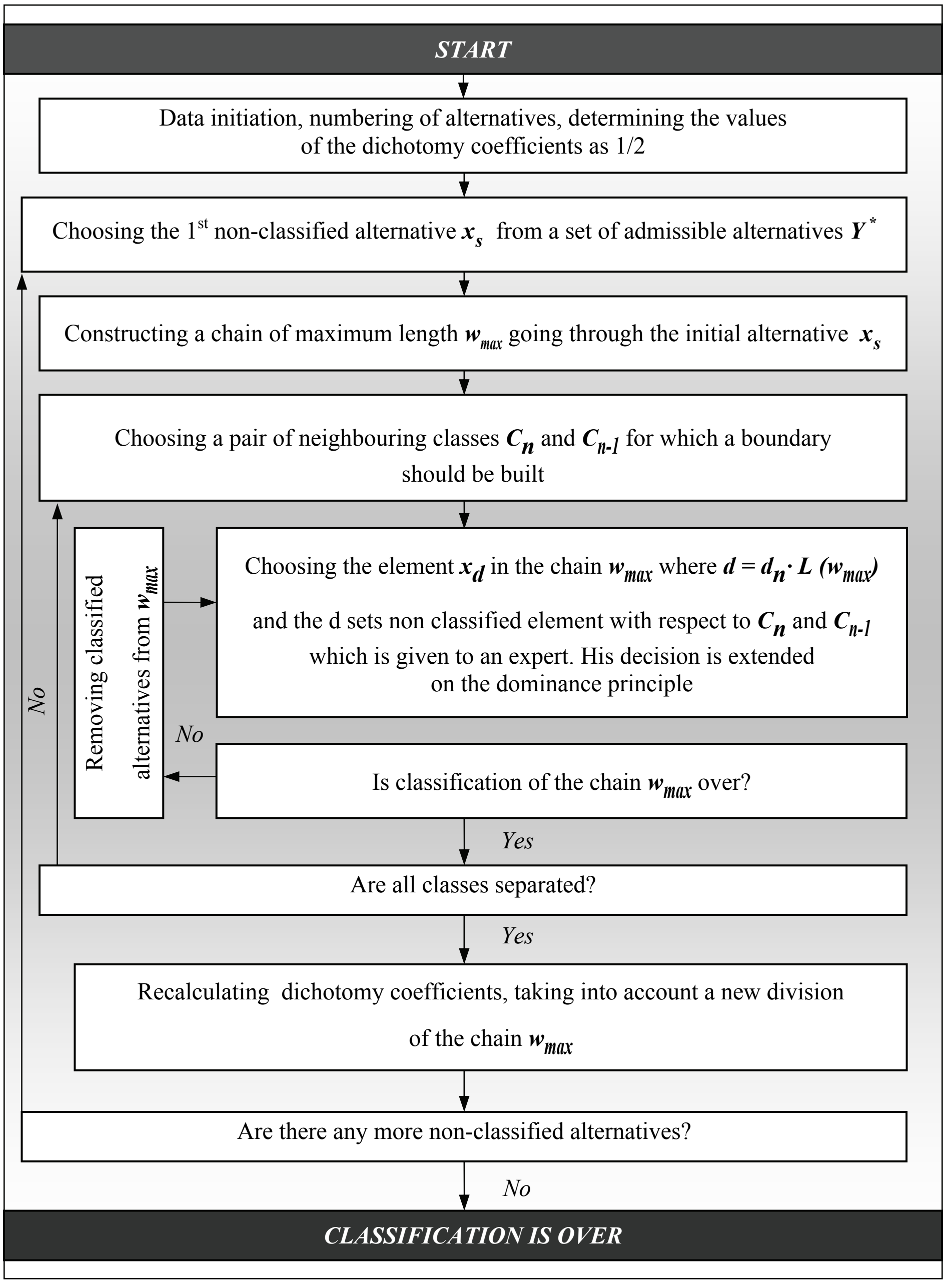

Figure 1. A general block-diagram of the CLARA algorithm

4.1. The main stages of gaining expert knowledge by using the CLARA method

The knowledge is gained by carrying on a dialogue with an expert. First, the main operations are outlined:

1. Discussing the statement of the problem. Defining the properties of G.
2. Generating a set of criteria K by an expert.
3. Constructing the scales for criteria evaluation. Preliminary analysis: checking if the estimates are (partially) arranged in the descending order of the distinctness of the property G.
4. Defining a set of ordered classes of solutions C by an expert [12].

The second stage of applying the method – expert-made classification – involves submitting to an expert the possible combinations of the attribute values for analysis. This is a time-consuming procedure because the number of combinations is usually large. This may entail expert's errors. Therefore, the method allows to define some simple problems within the original classification problem by considering only two values of any attribute. Then, the results obtained are included in the original problem, and the expert solves this partially solved problem on the full scale [13].

In the process of classification it may become clear that some combinations of the criteria values cannot be practically realized. In this case, the objects to which they refer are excluded from the analysis by an expert.

The classification is over, when all the objects included in the analysis (the set Y*) are assigned to a particular classes.

At the third stage of analysis the boundaries of classes are verified again because the mistakes the expert could made during previous stages. Since class boundaries are the key factors in making classifications, every class specified by the expert should be verified. For this purpose, every boundary element is offered to the expert again for checking. At the fourth stage, the boundaries of the classes are converted into the expert rules of solution of the form:

$$ab^{***} + p_n^{k_1[x_1]}\text{, except } \{abcde, \ldots, abpqr\} \tag{6}$$

So that every alternative follow one rule, where *ab**** is *a fixed part* of the rule, while $p_n^{k_1[x_1]}$ is the *rearrangeable part* of the rule. Here, n is equal to the number of asterisks, k_i is the number of estimates x_i involved in the rearrangement. The third part is activated if a set of alternatives described by a template is not completely rearrangeable, and to achieve this, the number of elements should be small. Then, the missing elements are simply listed.

The rules described are introduced into the system when solving the problem on a large scale. They simplify the solution considerably by reducing the classification space.

The decision rules of a particular class can be represented as a two-level tree (Figure 2):

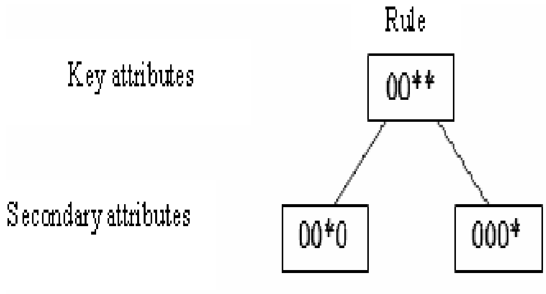

Figure 2. Decision rules of a particular class

Here the values of key attributes are found at the higher level, while the combinations of the values of secondary attributes are found at the lower level [12].

The rules described comply with inexplicit expert knowledge. The rules are submitted to an expert for approval. Some rules may be too complicated. In this case, the procedure of identifying the zone of superficial knowledge might be needed because complicated rules often indicate that knowledge is not stable [12]. For this purpose, it is necessary to go back to the second stage of method application (Figure 3).

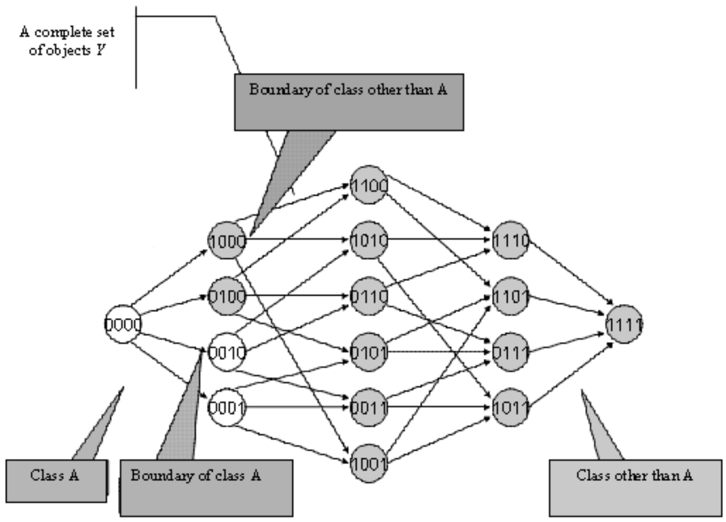

Figure 3. Example of method application

Conclusion

In project development, it is hardly possible to get exhaustive and accurate information. As a result, the situations occur, the consequences of which can be very damaging to the project.

Very often investment decision-making and research planning are referred to as non-structured problems. Since the essential characteristics of such problems are qualitative, they can hardly be used in the analysis. On the other hand, the quantitative models are not sufficiently reliable.

Non-structured problems have the following common characteristics. They are unique decision-making problems, i.e. every time the decision-maker is faced with an unknown problem or with one having new features compared to the previously considered case. These problems are associated with the uncertainty of the alternatives to be evaluated, caused by the lack of information for making a decision. The evaluation of the alternatives is of qualitative nature, being usually expressed verbally (in statements). Very often, experts cannot measure qualitative variables against an absolute scale where the level of quality does not depend on the alternatives. When the uncertainty is high, experts can only compare the alternatives qualitatively, based on particular criteria.

The CLARA algorithm (Classification of Real Alternatives) is based on the dichotomy of the alternatives chains, beginning with the longest chain. This concept was first used in the DIFCLASS algorithm and then in CLANSH.

Investment risk in construction can be evaluated efficiently enough using the CLARA method. This method allows to classify all possible construction investment projects presented by evaluations on the predefined criteria into several accurately defined classes reflecting the project risk level. CLARA method contains an algorithm to achieve the minimal amount of the DM questions. Moreover, the CLARA algorithm uses a new idea of the adaptive dichotomy allowing us to determine the boundaries between classes of solutions and to make classifications much faster.

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Lidija Zadnik Stirn

DYNAMIC, FUZZY AND AHP PROCEDURES IN A MULTI-CRITERIA DECISION PROCESS: AN APPLICATION TO ECOSYSTEM MANAGEMENT

Abstract

In this paper a multi-criteria, fuzzy and long term management problem in the frame of an ecosystem is introduced. As an aid to solve this problem, a hierarchical, discrete, dynamic and multi-criteria decision support model (DSM) is generated. In the DSM the process is defined in terms of time periods, states, decisions and weighted values of objective functions. For subjective and uncertain variables fuzzy methods are used. Further, SWOT analysis, analytic hierarchy process and analytic network process are employed to evaluate the conflicting objective functions by several decision makers. In the subsequent chapters, the problem is considered as a discrete, multi-objective, and dynamic problem which is presented in a form of a network. Finally, the optimal policy is determined by Bellman's iteration method for the solution of sequential decision processes. To illustrate the problem and the DSM developed some computational experiences are presented.

Keywords

Ecosystem management, multiple criteria problem, decision support model, discrete dynamic programming, fuzzy parameters, AHP, ANP, cumulative utility.

Introduction

Ecological crisis and social demands for the environment play an active role in many public discussions. These issues, along with the existing economic criteria, have become a key part of the modern concept of any ecosystem (for example, arable land, forest, recreational land, etc.) management. This concept includes the protection and use of the ecosystem. The state or the owner of an ecosystem should maintain and use it according to the principle of sustainability, while the society as a whole benefits from the amenity value of that ecosystem. Therefore, the ecosystem management problem consists of decisions on how to schedule the activities for an existing ecosystem over a long time horizon. These decisions include the management of the ecosystem while maximizing the expected profit for the landowner, and guaranteeing the present and future needs of the society as a whole, referring to ecological and social objectives, which is clearly shown by Venn diagram in Figure 1. Consequently, in ecosystem management, the decision-maker is challenged with a large-scale and complex decision problem which is long-term, dynamic, multi-objective and stochastic. Traditional approaches to this problem employ methods which are not based on newly developed multiple-criteria decision-making methods but rather on data processing and/or public survey results which rank choices by level or degree of importance [1, 8, 18, 19, 4, etc.].

Taking into account the multiple criteria ecosystem management problem, we have generated a discrete dynamic, multiple-criteria, fuzzy and hierarchical decision support model (DSM) which is intended as an aid for decision makers participating in ecosystem management process.

In the DSM, the process is defined in terms of time periods, states, decisions and weighted values of objective functions [3, 21, 22, 26]. Because the state, decisions and objectives of the system are also described with subjective and uncertain variables we use for these uncertainties and imprecision fuzzy methods [28]. Further, SWOT analysis, analytic hierarchy process and analytic network process are used to evaluate the conflicting objective functions by landowners, experts and the public [2, 13, 16, 25, 5, 27]. As soon as the assessment of decisions in accordance with different objectives is completed, we elicit the "total" evaluation of each decision, also assigned as "return", joint utility or reward [6], i.e., an acceptable trade-off between conflicting objective functions, using the composite utility values of a decision defined through analytic hierarchy process. The problem is then considered as a discrete, multi-objective, and dynamic problem which is presented in the form of a network. Finally, the optimal policy, the one that maximizes the utility over all time periods, is determined by Bellman's iteration method for the solution of sequential decision processes [3]. To illustrate the problem and the DSM developed we present some computational experiences from a case study in Slovenia [11, 24].

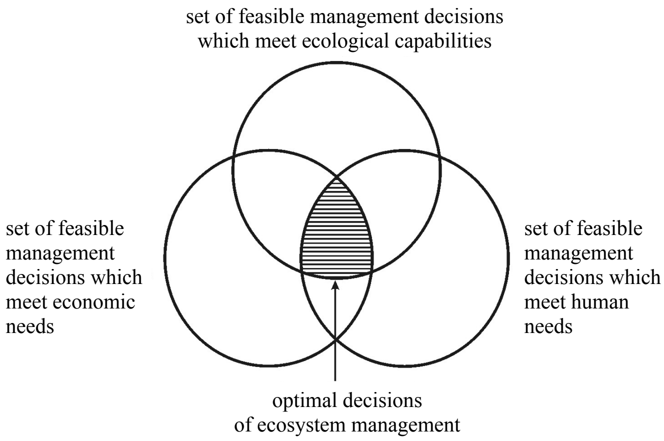

Figure 1. Venn diagram of optimal ecosystem management

1. A multiple-criteria decision support model (DSM) based on dynamic, fuzzy and AHP procedures – methodology

1.1. Discrete dynamic programming

Thus, in the problem presented, we are acquainted with a complex system which has to be led by a sequence of decisions from the existing state to the goal state, over several phases or periods according to multiple criteria. The common idea of generating the DSM is presented in Figure 2 which shows the period i with one decision.

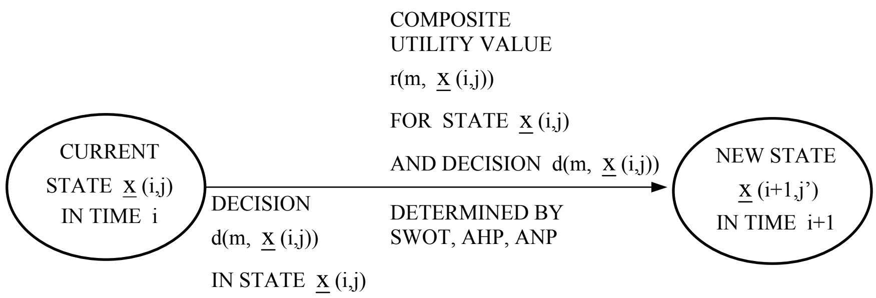

Figure 2. The time period i in the DSM

The planning horizon is divided into periods, defined by a finite and discrete time variable i (i=0,1,2,…, i, i+1, ..,I). Thus, the time variable i defines the time periods (1, 2, …, i, i+1, … ,I) in which the decisions are made.

Each time i is associated with a number of states. The state of a given system at each time i is described in terms of properties (components) of the elements which comprise the system. These are represented by variables (parameters) $s_1, s_2, ... s_s$, (for example: area, the level of conservation, the amount of products, labor force, machinery, financial resources, ecological conditions, recreational possibilities, number of visitors, etc.). Some parameters cannot be expressed precisely since we deal not only with parameters defined by numerical variables but also with subjective assessments and value judgments. Thus, fuzzy logic is used for descriptive (linguistic) parameters [28], i.e., a fuzzy set and its membership function are derived for each linguistic parameter. The limit values for each individual linguistic parameter are defined by means of certain rules – as "acceptable" values. In Figure 3, an example with a triangular membership function with limit values x_1, x_2 and x_3 is presented. The fuzzy and non-fuzzy parameters which define the possible state of the system at time i form a state vector $\underline{x}(i,j) = x(i, s_1, s_2, ... , s_s) \in X(i)$, where X(i) is the set of all possible state vectors at the time i. We suppose that there is a finite number of such vectors (j=1,2,…,J) at each time i. Thus, in practice, the set X(i) is a finite and discrete set of state vectors $\underline{x}(i,j)$. At this point, a considerable attention must also be paid to the determination of the goal state of the system $\underline{x}(I,j^*)=x^*(I, s_1^*, s_2^*, ... , s_s^*)$ which is designed to meet all the demands for optimal management of the system with which the owner, decision maker, experts, the public, etc. reach a compromise. In practice, the development of a goal state is a very important and difficult part of the decision process. The determination of the goal state requires an extremely high level of expertise, knowledge of environmental, economic and social issues, and above all a compromise among all parties involved in a decision process. It may even happen that for a particular system there does not exist any goal state which can fulfill the demands which, in general, are in conflict. In such a situation, the parties involved have to come to an agreement through a long set of negotiations to allow a goal state of the system to be defined. In such cases the Delphi method is often used to reach the agreement [14].

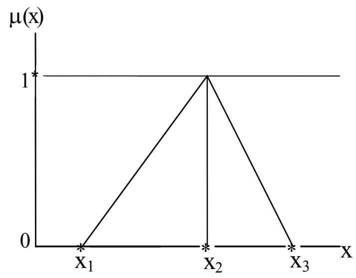

Figure 3. Triangular membership function for linguistic parameter with limit values x_1, x_2 and x_3

The decision maker can influence the existing state of the system, described by the state vector $\underline{x}(i,j) = x(i, s_1, s_2, \dots, s_s) \in X(i)$, by invoking management decisions, expressed by decision variables $d(m,\underline{x}(i,j))$ (Figure 2). The decisions $d(m,\underline{x}(i,j))$ are determined on the basis of technical, financial, environmental and other possibilities [23]. The set of feasible decisions is further constrained by several environmental factors. Thus, at each time i, the state vector $\underline{x}(i,j)$ is associated with a finite discrete set of decisions $D(\underline{x}(i,j))$; $d(m,\underline{x}(i,j)) \in D(\underline{x}(i,j))$, $m=1,2,\dots,M$. The decisions are mutually exclusive for any given time i and its accompanying state $\underline{x}(i,j)$.

The decision variables are also called control variables, since the effect of the decision $d(m,\underline{x}(i,j))$ is to move the system from a time i and state $\underline{x}(i,j)$ to another state $\underline{x}(i+1,j')$ at the next time i+1 (Figure 2):

$$\underline{x}(i+1, j') = f(\underline{x}(i,j), d(m,\underline{x}(i,j))) \tag{1}$$

The transition function f is defined empirically [21].

1.2. Value of decision

To control the initial state vector over time periods towards the goal state, the objectives are used (Figure 2). They link state and decision variables. The objectives encapsulate several factors, represented by different attributes which cannot be easily identified, measured or quantified. Therefore, assessing the objectives according to the decision and the existing state is inherently a complex undertaking. Thus, SWOT analysis, fuzzy analytic hierarchy process (AHP) and analytic network process (ANP) are introduced to determine a composite utility value within a discrete dynamic programming process (Figure 2).

1.2.1. SWOT analysis

SWOT analysis means analysis of comparative strengths and weaknesses of a decision in relation to competitive decisions, and opportunities and threats which the decision under consideration may face. SWOT analysis is, as such, a systematic study and identification of those aspects of the decision that best suit, in our case, sustainability and maximum expected profit, referring to ecological objectives, and respecting the public's acceptance of the decision examined. SWOT should be based on logic and relational thinking so that the selected decision improves the decision's strengths and opportunities and at the same time reduces the weaknesses and threats [2].

Strength is a distinct superiority (competitive advantage) of technical knowledge, financial resources, skill of the people, the image of products and services, access to best network, of discipline and morale. Weakness is the incapability, limitation and deficiency in resources such as technical, financial, manpower, skills, image and distribution patterns of the decision under examination. It refers to constraints of and obstacles to the decision. Corporate weaknesses and strengths are a matter of how the decision can achieve the best results compared to other, similar competitive decisions. Weaknesses and strengths of the decision present internal forces and factors which have to be studied and assessed with the goal to evaluate and rank the decision under consideration. Opportunities and threats are the external factors of the decision examined. These factors are changing together with governmental, industrial, monetary and market policies, including the changes of legal and social environment. An environmental opportunity is an area in which a particular decision would enjoy a competitive advantage. A proper analysis of the environment, identification of new market, new and improved customer groups and new relationship could present an opportunity for the decision. A threat is an unfavorable environment for the decision. Increased bargaining power of users and suppliers, quick change of government policy, rules and regulations may pose a serious threat to the decision undertaken [27].

SWOT analysis is very important for decision making nowadays. Such an analysis can be undertaken effectively through brainstorming session with the participation of experts, owners and users of the environment, land, firm, etc. involved in the decision. SWOT analysis has many advantages. Within

SWOT, internal and external factors are analyzed and summarized in order to attain a systematic decision situation. There are also several shortcomings of using SWOT. SWOT results in the creation of a list of internal and external factors, and groups the factors in strength, weakness, opportunity and threat groups, but it is not able to identify or analytically determine the most significant factor or group in relation to the examined decision. In order to get a quantitative information, to yield analytically determined priorities for the factors and groups included in SWOT analysis and to make them commensurable, Pesonen et al. [13] suggested to integrate the analytic hierarchy process (AHP) with SWOT analysis.

1.2.2. AHP and ANP analysis

In problems dealing with multiple and conflictive objectives (goals, factors) of the decisions, and above all with objectives of different importance, Saaty's analytic hierarchy process (AHP) is employed to determine the best decision. AHP can incorporate mixed data that may include both qualitative and quantitative judgments, and is capable of analyzing multiple factors (www.decisionlens.com, 19 June 2008). AHP is based on a gradual mutual comparison of two objectives (pairwise comparison) at the same level. A scale from 1 to 9 is used for making the comparison, where, for example, 1 means that two objectives are of equal importance, 3 means that judgments slightly favour one objective over another, …, 9 means that favouring one objective over another is of the highest possible order of affirmation, 2, 4, 6 and 8 are intermediate values, while the reciprocals of these values tell that if objective k has one of reasonable assumptions of the above non-zero numbers assigned to it when compared with objective j, then j has the reciprocal value when compared with k. Comparisons between individual objectives are gathered in a pairwise comparison matrix A. Each objective k is associated with a weight w_k. The weight ratio of the objectives k and j is written as intensity of importance:

$$a_{kj} = \frac{w_k}{w_j} \tag{2}$$

The matrix $A = [a_{kj}]$, $(k = 1, 2, ... K, j = 1, 2, ... K)$ if there are K objectives. By entering the estimated values a_{kj} into the matrix we get the pairwise comparison matrix A. The matrix A is a square, positive and reciprocal matrix, its diagonal values equal 1 and symmetrical values are inverse. Since, in practice, we never encounter perfectly consistent estimations [16], we proved the consistency as described in Winston [20], using the consistency index. Further, the vector of weights $w = (w_1, w_2, \ldots, w_K)$ is calculated by squaring the matrix A several times to the satisfactory exponent, i.e., A, A^2, $(A^2)^2$, etc. Then the lines are summed up, and finally the values are normalized [20]. The vector of weights $w = (w_1, w_2,\ldots,w_K)$ is therefore scaled between 0 and 1, $\sum w_k = 1$, and calculated by the following equation:

$$w_k = \sum_{j=1}^{K} a_{kj} / (\sum_{k=1}^{K} (\sum_{j=1}^{K} a_{kj})) \tag{3}$$

The procedure to be followed for the evaluation of the decision is presented in a hierarchical structure [25]. The hierarchy is organized around the concept of objectives (in our case SWOT groups: strengths, weaknesses, opportunities, threats), and attributes (in our case SWOT factors), within a two--level hierarchy (Figure 4). The first level is viewed as objective/group level. These groups are not directly measurable by themselves, but are presented by factors which are found at the second level. The factors define the cumulative effect of the SWOT group. Further, because the evaluation of a decision, as presented in Figure 4, involves the interaction and dependence among the levels of objectives and attributes, the analytic network process (ANP) is introduced into the model to solve the problem. ANP is the framework that allows to include all the factors and criteria, tangible and intangible, which have bearing on making a best decision. The key concept of the ANP is that influence does not necessarily have to flow only downwards, as is the case with the hierarchy in the AHP. Influence can flow between any two factors in the network. The ANP allows both interaction and feedback within clusters of elements (inner dependence) and between clusters (outer dependence). Such feedback best captures the complex effects of interplay, especially when risk and uncertainty are involved. The ANP, developed by Saaty [15], provides a way to input judgments and measurements to derive ratio scale priorities for the distribution of influence among the factors and groups of factors in the decision. Thus, the AHP is a special case of the ANP. ANP models have two parts: the first controls the interactions in the system under study; the second determines the degree of impact or influence between the criteria and attributes, i.e. the pairwise comparisons (www.decisionlens.com, 19 June 2008) which are then gathered in a so-called unweighted supermatrix. It must be stochasticized into columns. This is performed so that the blocks of the unweighted supermatrix are weighted using the corresponding priorities of the clusters. In this way we obtain a column stochastic matrix, called weighted supermatrix.

Then, the weighted supermatrix is raised to limit powers until the weights converge and remain stable (limit supermatrix).

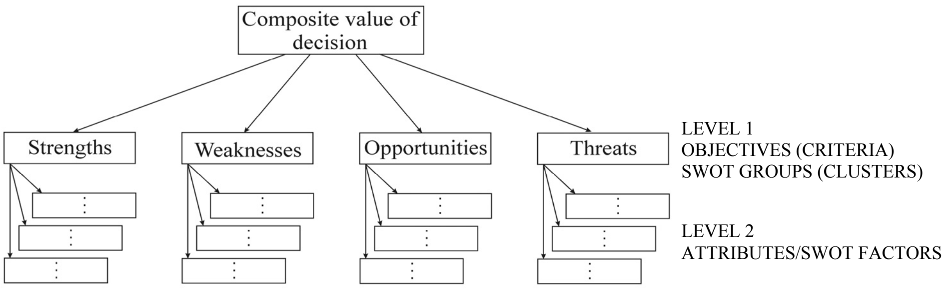

Figure 4. The hierarchy of objectives and attributes for composite value of the decision

In the numerical example of this study, the ANP process was simplified and it was used only on the objective level (Figure 4) for interdependencies among SWOT groups. The judgments for the weighted supermatrix were in this case obtained in the interview with the experts. Finally, the relative importance weights regarding the perception of independency and dependency in SWOT groups by decision makers, experts, general public, etc., i.e. all who participated in interviews, was expressed by synthesizing the AHP and ANP results in the sense of two levels (independent and dependent).

1.2.3. Cumulative effect – composite utility value of decision

Hence, the impact of factors on the group to which they belong must be aggregated. That is, the composite value of objective/group is measured based on a number of attributes/factors. The factors define suitable reference conditions for a group and therefore constitute the primary source of data or information for assessing the composite value of the decision because the latter reflects the cumulative effects of all SWOT groups (Figure 4). In this paper, a simple method of aggregation involving the linear combination of all factors and groups is used. This method was chosen because of its simplicity and transparency. The cumulative effect is in this method, namely, aggregated by simply adding the individual effects of all factors on the first level and groups on the second level. In the aggregation process, we also consider the fact that some factors, respectively some groups, must be viewed as relatively more significant as the others. Therefore, the aggregation of the effects of all factors, respectively groups, must also take into account their weights.

Some of the impacts of the factors on groups are only subjective judgments. Thus, they need to be defined in the interview with experts [26]. The impact values are normalized between 0 and 1 and reflect varying degrees of favourability to the group. In other words, the extent or impact of the factor on the SWOT group may be difficult or impossible to evaluate. It can only be judged in terms of the degree to which they lead to a favourable value of the group, in terms of the membership function f(x). Factors close to one imply being close to "favourable composite value of the group". In the paper, we use a linear function of a trapezoidal form (Figure 5), but some complex functions may also be used:

$$f(x) = \left\{ \begin{array}{ll} \frac{x}{a} & \text{for } 0 \le x \le a \\ 1 & \text{for } x \ge a \end{array} \right\} \quad \text{if factors represent strengths or opportunities} \qquad (4)$$

and

$$f(x) = \left\{ \begin{array}{ll} 1 & \text{for } 0 \le x \le b \\ \frac{1}{c-b}(-x+c) & \text{for } b \le x \le c \\ 0 & \text{for } x \ge c \end{array} \right\} \quad \text{if factors represent weaknesses or threats} \qquad (5)$$

where a, b, c are parameters representing limits or threshold values of the factors with regard to their favourability to the SWOT group, and x is the current value of the factor, i.e., the mean value obtained from surveys.

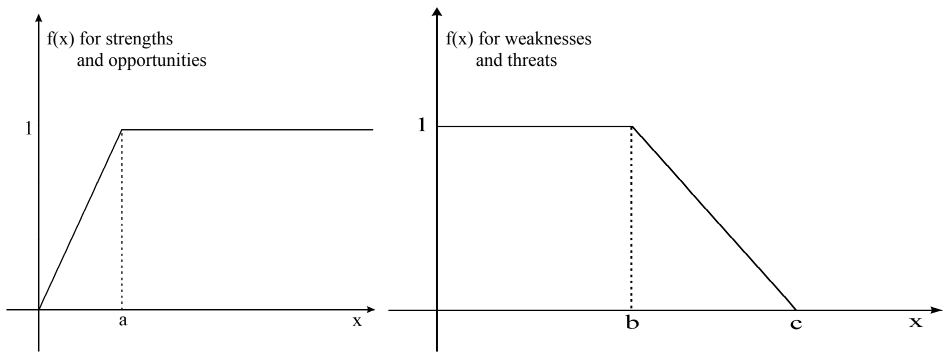

Figure 5. Impact (membership) functions f(x) of SWOT factors

In the next step, the analysis involves estimating the cumulative impacts of attributes (SWOT factors) that are calculated as a sum over all products of the impact function f(x) of the attribute (equation 4 and equation 5) and its relative importance (weight), obtained from AHP as w_k for the attribute k which has the value x:

$$c_n = \sum w_x f(x) \tag{6}$$

c_n provides the cumulative impacts of all factors on the favourability of objective n, i.e. the SWOT group n, to the composite value of the decision $d(m, \underline{x}(i,j))$. The values of c_n are also between 0 and 1. Its value close to 1 implies that the objective n is favourable to the composite value of the decision, while its low value implies that the objective contributes poorly to the composite value of the decision.

At the second level, the cumulative impacts of objectives, i.e. SWOT groups (strengths, weaknesses, opportunities, threats) for the decision $d(m, \underline{x}(i,j))$ are calculated by combining the values from both levels (Figure 4 and Figure 2):

$$CV = r(m, \underline{x}(i,j)) = \sum s_n c_n \tag{7}$$

where s_n are the weights of SWOT groups obtained by AHP and ANP.

1.2.4. Graph of the process and Bellman's principle of optimality

As soon as the decision-maker (analyst) determines the time periods for the described system, for each time period the possible states, for each state the possible decisions, the transition function expressed by (1), the objectives, assessed by CV (formula 7), he/she is able to show all these elements of DSM in form of a network [3, 20]. In the network (Figure 6), the states are designated by nodes (circles). The transitions from the state $\underline{x}$(i,j) in the time period i to the state $\underline{x}$(i+1,j') in the next time period under the decision d(m,$\underline{x}$(i,j)) are designated by the connection of two nodes, while the supposed goal state is presented as the final node. At the end of the connection of the nodes the composite value CV of belonging decision is noted. The optimal sequence of decisions for multi-objective problem subject to (1) with regard to the generated network is found recursively using the theory of discrete dynamic programming based on Bellman's principle of optimality, and Bellman's recursive equations [3].

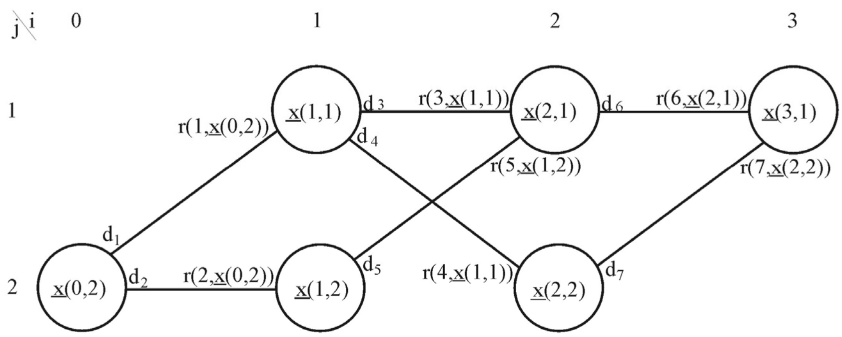

Figure 6. A network to demonstrate the multi-objective dynamic procedure

2. Numerical example for the presented problem and model

The presented DSM is illustrated with the management problem of a rural area which lies in the western part of Slovenia and covers a total area of 384 ha (forests, meadows, trails, recreational areas). This area is open to the general public. A trail was built to attract more visitors (young and adult) to this area in order to educate them about nature in many ways. The trail is also used for recreation (walking, running, …) and commercial uses (transportation of all kinds of material). Thus, the area is very important for the owners, experts, scientists, and the general public from the economic, ecological and social point of view. For the sake of simplicity, however, and with the aim to serve only for illustration, the management of the area is presented and treated here in a restricted way. We will consider here only three possible decisions (scenarios, alternatives):

– d_1, which increases the economic and recreational development of the area,
– d_2, which intends to increase the knowledge about nature and ecological awareness of the public,
– d_3, which emphasizes the preservation of nature [23, 24].

All three decisions pursue sustainable development of the area, maximum profit, ecological objectives and respect the public's acceptance of the management decision. The decisions are competitive and only one of them could be selected in one time period. Three time periods were observed (Figure 7). The present state of the area was determined according to the description of the area by Papež [11]. To evaluate the decisions according to the current state of the area SWOT/AHP/ANP were used.

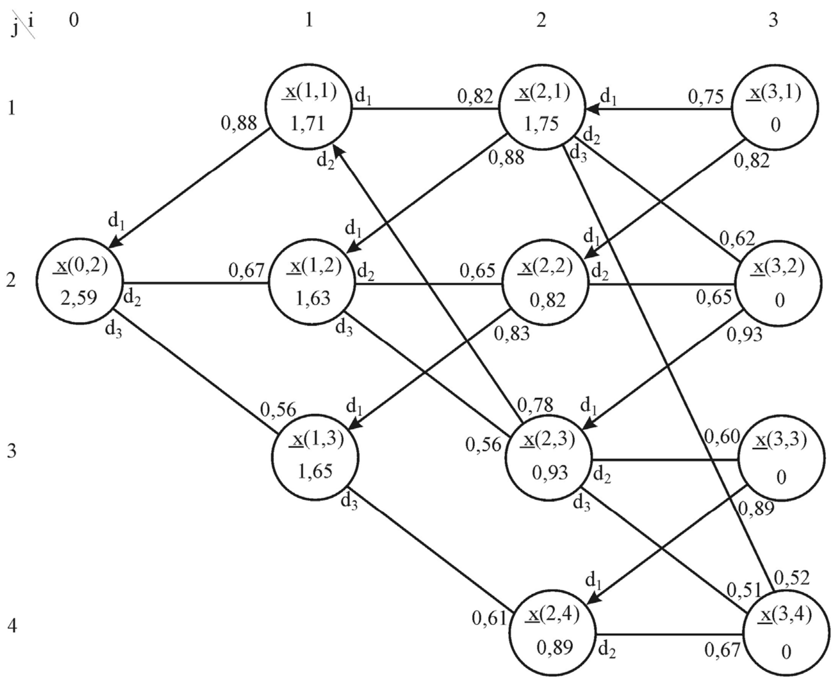

Figure 7. The network for 3 periods, states, decisions and values of decisions with the optimal sequence of decisions determined by Bellman's principle of optimality

For each of the three decisions regarding the state in which the decision should be undertaken the SWOT factors were generated. Because the factors should take into account the socio-economic, educational and environmental effects of the decision, as well as engineering feasibility and match the characteristics related to the location, physical size and level of operation, i.e. physical aspects, natural resources, land use, socio-economic, demographic, institutional, local and regional development conditions, a critical concern in the identification of all the actors involved in the decision under consideration was expressed. The actors were identified by a variety of means, such as educational degrees, professional memberships, peer recognition and even self-proclamation. Two types of actors were identified as potentially useful in generating the SWOT factors. The first type belongs to the representatives of a sub-population whose attitudes or actions influence the project under consideration. These types of participants are used, for example, in surveys and in the Delphi-like methods. The second type of participants has extensive special knowledge and experience about the research topic of concern. Discussion, conferences, brainstorming, the Delphi method, and similar methods were used. Here we summarize only the SWOT factors for d_1, undertaken in the state $\underline{x}(0, j$ (describes the current state of the area)) at the beginning of the first period (i=0). As presented in Figure 8, two SWOT factors were generated for SWOT group strengths (employees with knowledge in natural sciences, available financial resources), three factors for SWOT group weaknesses (lack of knowledge in advertising, lack of seasonal workers, low market prices of products from the area), three factors for SWOT group opportunities (location close to bigger cities – public interest for the area, governmental and institutional support, good cooperation with other areas) and finally two factors for SWOT group threats (increased competition in products and recreation, changes in financial policy). Similar factors were generated for the other two alternatives according to different area states.

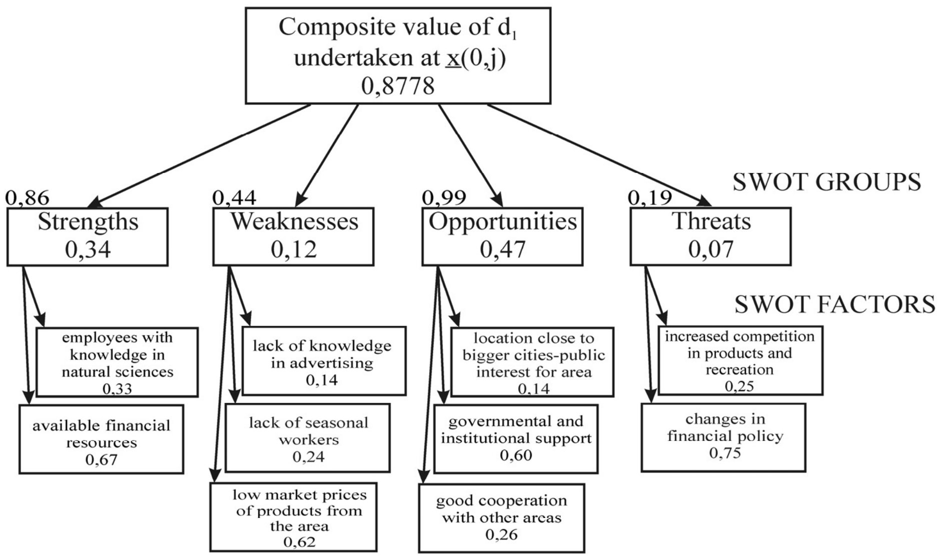

Figure 8. SWOT factors and groups for decision d_1 and their AHP priorities

Further, in order to assess this decision by the SWOT/AHP model the experts were asked to make their judgments by pairwise comparisons between four SWOT groups, and within SWOT groups, i.e., pairwise comparisons of two factors of strengths, pairwise comparisons of three factors of weaknesses, pairwise comparisons of three factors of opportunities, and pairwise comparisons of two factors of threats. The estimates from their pairwise comparisons are given in matrices, where group strengths are denoted by S, weaknesses by W, opportunities by O, threats by T, employees with knowledge in natural sciences by EK, available financial resources by AF, lack of knowledge in advertising by LK, lack of seasonal workers by LW, low market prices of products from the area by LM, location close to bigger cities – public interest for the area by PI, governmental and institutional support by GI, good cooperation with other areas by GC, increased competition in products and recreation by IC and changes in financial policy by CF. Here we show the data and results of the relative weights $w_{SWOT(AHP)}$ for four SWOT groups, and relative weights w_x for all SWOT factors, according to the AHP theory:

$$\begin{array}{c} \\ \mathrm{S} \\ \mathrm{W} \\ \mathrm{O} \\ \mathrm{T} \end{array} \begin{array}{c} \begin{array}{cccc} \mathrm{S} & \mathrm{W} & \mathrm{O} & \mathrm{T} \end{array} \\ \begin{bmatrix} 1 & 6 & 1/3 & 4 \\ 1/6 & 1 & 1/2 & 2 \\ 3 & 2 & 1 & 5 \\ 1/4 & 1/2 & 1/5 & 1 \end{bmatrix} \end{array} \to \cdots \to \begin{bmatrix} 0{,}34 \\ 0{,}12 \\ 0{,}47 \\ 0{,}07 \end{bmatrix} = w_{SWOT(AHP)} \qquad \begin{array}{c} \\ \mathrm{EK} \\ \mathrm{AF} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{EK} & \mathrm{AF} \end{array} \\ \begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix} \end{array} \to \cdots \to \begin{bmatrix} 0{,}33 \\ 0{,}67 \end{bmatrix} = w_{xS}$$

$$\begin{array}{c} \\ \mathrm{LK} \\ \mathrm{LW} \\ \mathrm{LM} \end{array} \begin{array}{c} \begin{array}{ccc} LK & LW & LM \end{array} \\ \begin{bmatrix} 1 & 1/2 & 1/4 \\ 2 & 1 & 1/3 \\ 4 & 3 & 1 \end{bmatrix} \end{array} \to \cdots \to \begin{bmatrix} 0{,}14 \\ 0{,}24 \\ 0{,}62 \end{bmatrix} = w_{xW} \qquad \begin{array}{c} \\ \mathrm{PI} \\ \mathrm{GI} \\ \mathrm{GC} \end{array} \begin{array}{c} \begin{array}{ccc} PI & GI & GC \end{array} \\ \begin{bmatrix} 1 & 1/2 & 1/4 \\ 2 & 1 & 5 \\ C4 & 1/5 & 1 \end{bmatrix} \end{array} \to \cdots \to \begin{bmatrix} 0{,}14 \\ 0{,}60 \\ 0{,}26 \end{bmatrix} = w_{xO}$$

$$\begin{array}{c} \\ \mathrm{IC} \\ \mathrm{CF} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{IC} & C\mathrm{F} \end{array} \\ \begin{bmatrix} 1 & 1/3 \\ 3 & 1 \end{bmatrix} \end{array} \to \cdots \to \begin{bmatrix} 0{,}25 \\ 0{,}75 \end{bmatrix} = w_{xT}$$

The results of AHP are shown in Figure 8 within the group and factor boxes.

Next, we consider interdependencies among groups. When we think about SWOT groups, we cannot concentrate only on one group, but must consider the other groups with it. Therefore, we need to examine the impact of all the groups on each group by using pairwise comparisons. The relationship of interdependence among groups is shown in Figure 9. We obtain the weights through expert group interviews [24]. The interdependence matrix of groups is assigned as $w_{SWOT(ANP)}$, where, for example, we see that strength's degree of relative impact for weakness is 0.2, the weaknesses' degree of relative impact of threat is 0.1, and the opportunities' degree of relative impact for threat is 0.4.

$$\begin{array}{c} \\ S \\ W \\ O \\ T \end{array}\begin{array}{c} \begin{array}{cccc} S & W & O & T \end{array} \\ \begin{bmatrix} 1 & 0.2 & 0 & 0 \\ 0 & 0.5 & 0 & 0.1 \\ 0 & 0.3 & 1 & 0.4 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} \end{array} = w_{SWOT(ANP)}$$

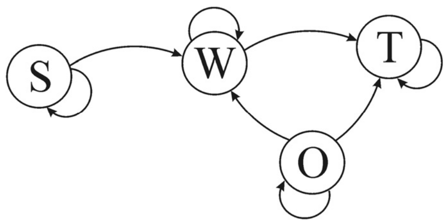

Figure 9. Interdependent relations among SWOT groups

The interdependence priorities of the criteria (SWOT groups) s_n are then obtained by combining the results from AHP and ANP, i.e., $w_{SWOT(AHP)}$ and $w_{SWOT(ANP)}$:

$$s_n = w_{SWOT(ANP)} \cdot w_{SWOT(AHP)} = \begin{bmatrix} 1 & 0.2 & 0 & 0 \\ 0 & 0.5 & 0 & 0.1 \\ 0 & 0.3 & 1 & 0.4 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 0.34 \\ 0.12 \\ 0.47 \\ 0.07 \end{bmatrix} = \begin{bmatrix} 0.364 \\ 0.067 \\ 0.534 \\ 0.035 \end{bmatrix}$$

Finally the overall priority of the decision d_1 is calculated. Questionnaires containing twenty questions about the importance of the internal and external factors of the area and response forms were distributed to fifty respondents (experts, investors, representatives of NGO's, residents and visitors). The five--point Likert scale was used, where in questions on strengths and opportunities 1 means that the question is of absolutely no importance to the respondent, while 5 means that the question is extremely important to the respondent, and vice versa for the questions on weaknesses and threats. The questionnaire, the answers, and the statistics of the answers are published in detail in Zadnik [24]. Table 1 summarizes the average scores of the factors (x values) according to the decision and SWOT groups, the values a, b and c, which represent the limit values of the factors with regard to their lowest and their highest observed values (given by the experts), f(x) for factors, calculated on the basis of formulas (4) and (5), and weights w_x. Given the data in Table 1 (f(x) and w_x), and using formula (6), the impacts of SWOT factors on the SWOT groups are calculated: $c_{1(S)} = 0.86$ for strengths, $c_{2(W)} = 0.44$ for weaknesses, $c_{3(O)} = 0.99$ for opportunities and $c_{4(T)} = 0.19$ for threats. These impacts can be found in Figure 8 above the SWOT groups. Further, using the weights s_n for SWOT groups and formula (7) the composite value CV = $r(d_1, \underline{x}(0,j))$ for the decision d_1 in state $\underline{x}(0,j)$ is calculated. It amounts to 0.8778 (Figure 8) ≈ 0.88.

The composite values for the other decisions which are under consideration in the corresponding state of the area were determined according to the same procedure and are given in Figure 7 which presents the dynamic problem during 3 periods, all possible states, decisions and values of decisions in the form of a network. Figure 7 also demonstrates the optimal sequence of decisions over all three periods according to Bellman's principle of optimality.

The optimal sequence is shown in bold in Figure 7, i.e., in the current state $\underline{x}(0,2)$ the decision d_1 has to be chosen and the state $\underline{x}(1,1)$ is reached. Then, with the optimal decision d_2 the state $\underline{x}(2,3)$ is reached. In the third period d_1 is suggested to be optimal. It transforms the area state $\underline{x}(2,3)$ to the final state $\underline{x}(3,2)$, which for this area is also the compromise goal state according to all demands of owners, experts and the public.

Table 1

Calculation of c_n for SWOT groups obtained from 50 respondents (equation (6))

	Factor values, limit values, function, weight / Factor	**X**	**A**	**b**	**c**	**f(x)**	**w_x**
S	Employees with knowledge in natural sciences	2.37	4	–	–	0.59	0.33
S	Available financial resources	4.08	4	–	–	1	0.67
W	Lack of knowledge in advertising	3.97	–	1	4	0.01	0.14

Table 1 contd.

	Factor values, limit values, function, weight / Factor	X	A	b	c	f(x)	w_x
W	Lack of seasonal workers	3.29	–	1	4	0.24	0.24
W	Low market prices of products from the area	2.13	–	1	4	0.62	0.62
O	Location close to bigger cities – public interest for the area	2.93	3	–	–	0.97	0.14
O	Governmental and institutional support	4.01	3	–	–	1	0.60
O	Good cooperation with other areas	3.25	3	–	–	1	0.26
T	Increased competition in products and recreation	3.50	–	0.5	3	0	0.25
T	Changes in financial policy	2.35	–	0.5	3	1	0.75

Concluding remarks

Theoretical as well as computational aspects of determining the optimal sequence of decisions/scenarios for the management of an existing system were introduced through a DSM. The problem is obviously of great complexity. The conclusion we can draw from the methodology and calculations presented is that the problem can be readily solved by means of SWOT analysis, analytic hierarchy process, analytic network process and interviews with experts.

The approach presented in this combination of methods is relatively new, as it encompasses the SWOT analysis, analytic hierarchy process, analytic network process, and analysis of the surveys.

The decision support models are, in general, concerned with how to choose from a set of decisions. Therefore, they usually fall short in framing the problem and setting the goals. In the DSM presented, this drawback is overcome by interaction with decision makers, experts, residents and visitors. The interviews conducted with economists, sociologists, politicians, environmentalists, community activists, NGOs, and other experts that emerge through a snowball sampling technique in the case study area, identified the objectives of the individuals or groups and provided input for the DSM generated. The results obtained by the use of the DSM proposed confirm the expectations of the decision makers (experts, residents, NGOs, etc.), as their preferences regarding the area are of economic and educational nature. Further, the optimal decisions derived from the application of the DSM are also consistent with the EU legislative framework. A substantial component of the legislation is devoted to public participation in decision making, particularly with regard to spatial planning and environmental matters. Further, it outlines the protection of air, soil, water, as well as the promotion of education. The results obtained in the application of the DSM developed confirm that the model presented is appropriate for practical use.

The paper shows a simplified computational example. In further research, it is planned to present an application of a more real-world character and extensive problem. In this context it could be argued that users might not be inclined to use sophisticated methods because of the complicated and extensive calculations. However, recent surveys indicate that the use of mathematical models is becoming more prevalent with the availability of commercial software packages such as Expert Choice, Decisionlens, Excel, etc., which also holds true for the DSM presented.

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RANKING BY THE ROUGH APPROXIMATION OF A PREFERENCE RELATION FOR AN EXTRUSION PROCESS – A ROBUSTNESS STUDY

Abstract

In this paper an extrusion process example is studied to illustrate a new methodology in the field of decision engineering, which is based on the rough set approach. Rough sets are used to aid the Decision Maker in choosing the best point in the Pareto's region, which is the zone of the non-dominated solutions. The rough set approach allows us to make a rough approximation of a preference relation on a sample of experimental points chosen from the Pareto set. These approximations are done to induce the decision rules, which can be afterwards applied on whole sets of the potential points. To give the final recommendation the concept of technical robustness is suggested.

Keywords

Rough sets, decision rules, multi-criteria problem, robustness, genetic algorithms.

Introduction

In applications of the rough set approach to the model of the Decision Maker (DM) preferences in the multi-criteria problem, the starting point is his global evaluation of the subset chosen actions as examples. So, after getting the preferential information in terms of exemplary comparisons, it would be natural to build the preference model in the form of “if..., then...” decision rules.

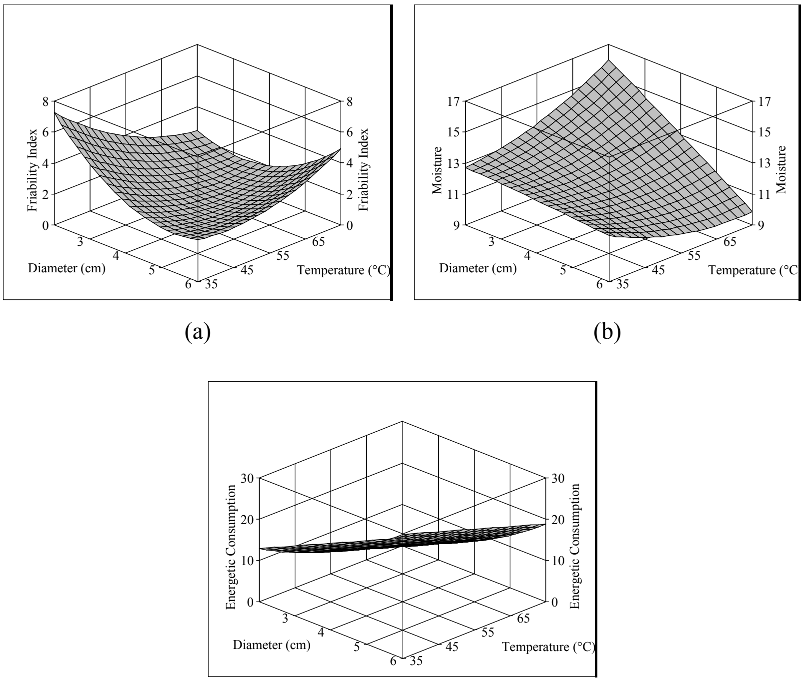
The consequence of the ambiguity of these evaluations is that some rules are non-deterministic, i.e., they cannot be univocally described by means of "granules" of the representation of preferential information. A formal framework for dealing with the granularity of information; called Rough Set theory has been given by Pawlak [9, 7]. The Rough Sets theory assumes a representation of the information in a table whose rows correspond to objects and its columns to attributes. If a description of objects by a set of attributes is supplied by the DM, these attributes are called decisions, and the remaining ones conditions, and all together these form the decision table. This decision table is a particularly appropriate form for the description of decision sorting problems (see [8]). As was shown by Greco, Matarazzo and Slowinski [4], a direct application of the Rough Sets approach to multi-criteria decision analysis is not possible when the ordinal properties of the criteria have to be taken into account. In this paper, and following this way of reasoning, a decision table is when the objects are pairs of actions considered. For each pair of actions, the partial evaluations of the preferences with respect to each attribute are given. The attributes taken from the multi-criteria problem can be called conditions. For a chosen subset of pairs of actions, a global preference relation from the total order is supposed to be given by the DM or expert. In this table, a global preference relation will be noted as the decision attribute. In the original theory of Rough Sets the rough approximations are defined using an indiscernibility relation on the pairwise comparison table. In this paper following the Greco, Matarazzo and Slowinski [6] approach, an indiscernibility relation was substituted by a dominance relation.

This paper is structured as follows. The problem under study is formulated in Section 1. After a brief presentation of the industrial application, this section deals with the determination of the Pareto set, which is a prerequisite to obtain all possible actions. The aim is to eliminate all points, which are not optimal in the multi-criteria problem. The remaining optimal points are determined with an evolutionary approach by adapting a genetic algorithm. The Decision Maker has to choose the best action in this zone, so, in Section 2, a multi-criteria method based on the theory of rough sets is developed to support him in his choice. A few experimental points are required in this method and it is supposed that the DM is able to express his preferences in relation to these points. From these preferences the method of rough sets allows us to build the set of rules and they are applied on whole sets of points in the Pareto zone to determine the total ranking of the points in Section 3. In Section 4, the robustness of the proposed solution is discussed. The solution has to take into account a possible evolution of the input parameters and a possible change in the decision parameters. A robustness study is proposed for the application of an extrusion process, which is a didactic example. So, the results can be easily represented in two dimensions.

1. Determination of the Pareto set by an evolutionary approach

1.1. Description of the industrial process

Let us consider an industrial application concerning the problem of food granulation for cattle. The goal is to propose the best recommendation for the working conditions of an extrusion process. A pulverulent product is converted into granules due to the conjugated effects of heat, moisture and pressure. The industrialist would like to simultaneously minimise three characteristics: moisture, the friability of his product and the energy consumption of the process. Two discretized input parameters are taken into account in this study: the flour temperature T (between 35°C and 75°C) and the drawplate profile D (between 2 cm and 6 cm). All three attributes are described by functions represented in Figure 1.



(a) (b)

(c)

Figure 1. Evaluations of friability index, moisture and energy consumption vs temperature and drawplate profile

These evolutions are derived from Courcoux, Qannari, Melcion and Morat [1]. The modelling of these attributes depends on the information at our disposal and they are generally incomplete or vague. In this paper, discussed application is limited to the deterministic case. In the case of stochastic or determinist evaluations the modelling can be done using the Stochastic Dominances (see [12, 13, 14]).

1.2. Multiobjective optimisation algorithm

In a real-world problem, like our extrusion process, an optimal working condition does not generally exist due to the multi-criteria aspect. In the case of the multi-criteria analysis of a process, all possible points have to be determined first. These points, which are called Pareto-efficient solutions, are in fact a continuous set, which is not always possible to determine analytically. The concept of the Pareto dominance is used which implies that a point dominates another if it is better for all criteria. So, the set of the non--dominated points forms the Pareto set.

A multiobjective optimisation algorithm is developed to obtain the Pareto domain sampled by a set of points. The aim of this part is not to immediately find the preferred solution but to exclude all conditions which are not interesting, i.e. not optimal in a multi-criteria sense. An extension of the traditional genetic algorithm is then used to deal with discretized data by introducing the dominance concept. Genetic algorithms are an optimisation method which is inspired by an analogy using the evolution of populations [3]. The approach consists, after a random initialisation of points, of an evaluation of them, a selection of the best points and a recombination of these ones until a convergence of the algorithm occurs. Some thousands of points can randomly initialise the algorithm. The evaluation of each point is determined by the calculation of a dominance function which counts how many times a point is dominated by the others [2]. The non-dominated points are kept and are recombined to replace the dominated points. The procedure evaluation – recombination is applied until all points are non-dominated. This method allows us to obtain a set of points, which corresponds to Pareto efficient solutions and to also discretize a continuous problem with these points. The Pareto set is represented in Figure 2 by 5000 grey points, which define precisely the continuous multi-criteria optimal zone. This obtained domain is the prerequisite before the preference analysis of the process.

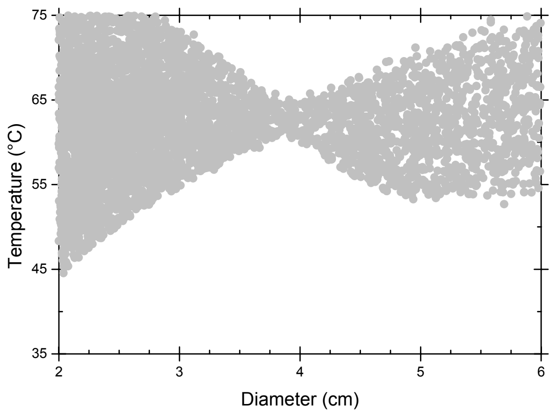


Figure 2. Pareto set of the extrusion process for temperature and drawplate profile parameters

2. Rough sets preference analysis

2.1. Algorithmic design

The basis for rough sets preference analysis is the building of a decision table for a sample of experimental points B chosen from the optimal Pareto's zone. Let X be a set of the output process attributes (in our example: moisture, the friability index and the energy consumption) describing the performance of points. Let C be the set of condition attributes describing the pairs of points, and D the decision attribute. The decision table is defined as 4-tuple, $(T = \langle H, C \cup D, V_C \cup V_D, g\rangle)$, where $H \subseteq BxB$ is a finite set of pairs of points, $C \cup D$ is the union of two sets of attributes, called condition and decision attributes, $V_C \cup V_D$ is the union of the values of the functions which are defined as follows [5]: g: $H \times (C \cup D) \rightarrow V_C \cup V_D$ is a total function where $V_C = \cup V_k$ (criterion k). This function is such that:

$$g\left[\left(a_i, a_j\right), k\right] = \begin{cases} 1, & \text{if } X_k(a_i) < X_k(a_j) \\ \\ 0, & \text{if } X_k(a_i) \geq X_k(a_j) \end{cases} \tag{1}$$

$$(\text{if criterion } k \text{ is to minimise}) \ \forall C_k \in C \ \text{ and } \ \forall (a_i, a_j) \in H$$

where a_i is the one point of the temperature-profile parameter combination, i, j = 1, ..., n, and $X_k(a_i)$ is the value of the performance of the point a_i in relation to the attribute $X_k \in X$ where k=1,..., m (in our example m = 3) and C_k is the condition attribute associated with the criterion k. The decision table can also take only two values $g[(a_i, a_j),D]$ on $H \subseteq B \times B$:

$$g\left[\left(a_i, a_j\right), D\right] = \begin{cases} P, & \text{if } a_i \text{ is preferred to } a_j; \\ N, & \text{if } a_i \text{ is not preferred to } a_j \end{cases} \tag{2}$$

Then, we suppose that the DM is able to express his preferences on the small number of points. The appeal of this approach is that the DMs are typically more confident exercising their evaluations than explaining them. In general, the decision table can be presented as follows in

Table 1

General presentation of the decision table

		X_1	X_2	…..	X_m	**D**
	.	.	.	…..	.	.
H_P	(a_i, a_j)	$g[(a_i, a_j),1]$	$g[(a_i, a_j),2]$	…..	$g[(a_i, a_j),m]$	$g[(a_i, a_j),D]=P$
	.	.	.	…..	.	.
	.	.	.	…..	.	.
H_N	(a_s, a_t)	$g[(a_s, a_t),1]$	$g[(a_s, a_t),2]$	…..	$g[(a_s, a_t),m]$	$g[(a_s, a_t),D]=N$
	.	.	.	…..	.	.

Let $Q \subseteq C$ be a subset of condition attributes. In relation to the subset Q of C attributes (see Zaras (2001)) we can define the Multi-attribute Stochastic Dominance noted MSD_Q for a reduced number of attributes which can be defined as follows.

Definition 1

$$\text{ai MSDQ aj if and only if } g[(ai, aj),k] = 1 \text{ for all Ck} \in Q \tag{3}$$

where $Q \subseteq C\{1, ..., m\}$

The problem is how to identify the subsets Q from which we could approximate global preferences? To answer this question we use the idea of the approximation, which was taken from the rough sets of Greco et al. [6] where an indiscernibility relation was substituted by a dominance relation.

For each pair of alternatives $(a_i, a_j) \in H$ in the decision table we can identify Multi-Attribute Dominance MSD_Q and complementary Multi-Attribute Non-dominance not MSD_{C-Q} where C-Q is the set difference and where this second dominance can be defined as follows:

Definition 2

ai nonMSDC-Q aj , if and only if g[(ai,aj),k] = 0 for all Ck $\in$ C –Q (4)

These dominances are similar to the P-positive dominance and P-negative dominance suggested by Greco et al. [6] for ordinal data.
According to them these dominances satisfy the following property:

Property 1

If $(a_i, a_j) \in MSD_Q$ (not MSD_{C-Q}) then $(a_i, a_j) \in MSD_R$ (not MSD_{C-R}) for each $R \subseteq Q \subseteq C$ (5)

In this approach we propose to approximate the P-global preference relation by the MSD_Q relation. Usually this approximation in the rough set methodology is done by $Q_*(P)$-lower and $Q^*(P)$-upper approximations. According to Greco et al. [6] these approximations can be defined as follows:

$$Q_*(P) = \bigcup_{Q \subseteq C} \left\{ \left(MSD_Q \cap H \right) \subseteq P \right\}$$
$$Q^*(P) = \bigcap_{Q \subseteq C} \left\{ \left(MSD_Q \cap H \right) \supseteq P \right\} \tag{6}$$

The Q-boundary (doubtful region) of a set of preferences P is defined as follows:
The set $BN_Q(P)$ contains the MSD_Q which introduces uncertainty in the deduction of the decision rules using the subset of attributes Q.

$$BNQ(P) = Q^*(P) - Q^*(P) \tag{7}$$

If $BN_Q(P) \neq \emptyset$, then P is a rough set. Taking into account property 1, we obtain a certain number of the MSD_Q, which verifies the condition of the lower approximation.
Analogously, we can approximate a non-preference denoted by the letter N in the decision table by the Multi-Attribute non-dominance for a reduced number of attributes not MSD_Q:

$$Q_*(N) = \bigcup_{Q \subseteq C} \left\{ \left(not \;\; MSD_Q \cap H \right) \subseteq N \right\}$$
$$Q^*(N) = \bigcap_{Q \subseteq C} \left\{ \left(not \;\; MSD_Q \cap H \right) \supseteq N \right\} \tag{8}$$

We can induce a generalised description of the preferential information contained in a given decision table in terms of decision rules.

2.2. Application

We apply this algorithmic design in the case of our granulation process with the following discretized input parameters: temperature T (between 35 C^o and 75 C^o) and drawplate profile D (between 2 cm and 6 cm) values on the one hand, and the three output attributes: X_1: the friability index, X_2: the moisture of the granules and X_3: the energy consumption on the other hand. The criteria role has been attributed to minimise all three output attributes. To illustrate the application of the Rough Set approach, we began by showing in Table 2 the subset of five points chosen by a human expert and their evaluations in relation to each of the three attributes.

Table 2

Evaluations of the five chosen actions for three attributes

Actions	X_1	X_2	X_3
a_1	2.0495	13.361	7.8915
a_2	2.0925	14.998	5.4695
a_3	3.3122	14.096	4.7477
a_4	2.9819	10.128	20.871
a_5	4.3177	10.225	17.916

Figure 3 shows us the localisation of these five points in relation to the Pareto domain which was determined analytically from the equations (13).

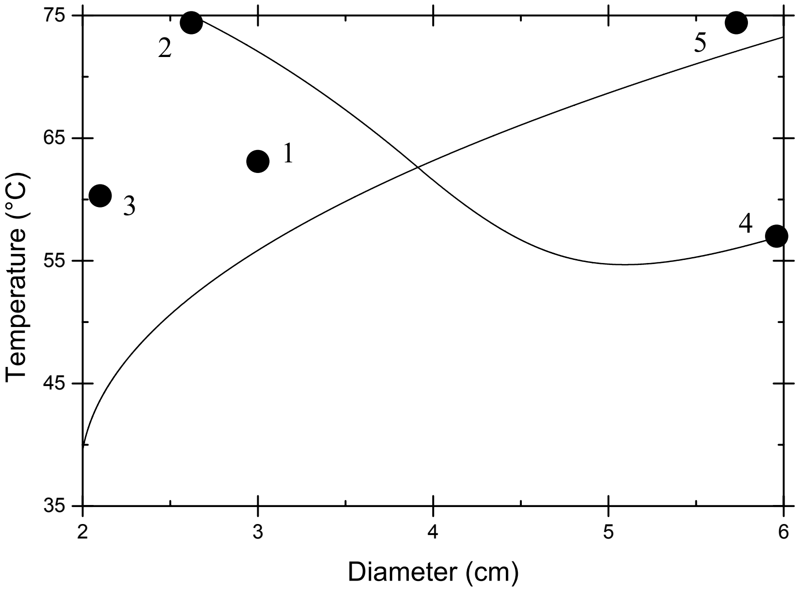


Figure 3. The subset of five points chosen by a human expert and the Pareto domain which was determined analytically

These points have been ordered by the DM from the best to the worst. Then, we can add the decision attribute D to Table 3 which makes a dichotomic partition of the set of pair points according to the values $V_D = P$, which means preference and $V_D = N$, which means non preference.

Table 3

Decision table for the five chosen points

	Pairs	X_1	X_2	X_3	D
H_P	(a_1, a_2)	1	1	0	**P**
	(a_1, a_3)	1	1	0	P
	(a_1, a_4)	1	0	1	P
	(a_1, a_5)	1	0	1	P
	(a_2, a_3)	1	0	0	P
	(a_2, a_4)	1	0	1	P
	(a_2, a_5)	1	0	1	P
	(a_3, a_4)	0	0	1	P
	(a_3, a_5)	1	0	1	P
	(a_4, a_5)	1	1	0	P

Table 3 contd.

	Pairs	X_1	X_2	X_3	D
H_N	(a_2, a_1)	0	0	1	**N**
	(a_3, a_1)	0	0	1	N
	(a_4, a_1)	0	1	0	N
	(a_5, a_1)	0	1	0	N
	(a_3, a_2)	0	1	1	N
	(a_4, a_2)	0	1	0	N
	(a_5, a_2)	0	1	0	N
	(a_4, a_3)	1	1	0	N
	(a_5, a_3)	0	1	0	N
	(a_5, a_4)	0	0	1	N

According to definition (6), from Table 4 we obtain one dominance relation which intersects with five pairs of points and which verifies the condition of the lower approximation.

Table 4

Dominance which intersects with five pairs of points in the decision table

MMD_Q /H	(a_1, a_4)	(a_1, a_5)	(a_2, a_4)	(a_2, a_5)	(a_3, a_5)
Q = {X_1, X_3}	x	x	x	x	x

From this we can induce the minimal decision rule, which can be formulated as follows:

Rule 1

$$\text{If ai MMDQ aj then ai P aj , with Q} = \{X1, X3\} \tag{9}$$

The meaning of this rule is that if the point a_i in the Pareto's zone is better that the point a_j with respect to friability index and energy consumption, then the point a_i is preferred to point a_j. In the same way we can do the approximation of non-preference N by the Multi-Attribute non-dominance not MMD_Q. According to the definition (9) from Table 5, we obtain one non-dominance relation which intersects with five pairs of points and which verifies the condition of lower approximation.

Table 5

Non-dominance which intersects with five pairs of points in the decision table

Not MMD_Q /H	(a_4, a_1)	(a_5, a_1)	(a_4, a_2)	(a_5, a_2)	(a_5, a_3)
$Q = \{X_1, X_3\}$	x	x	x	x	x

Based on the same principle as in the case of preference choosing, we can induce the rule as follows:

Rule 2

If ai not MMDQ aj then ai N aj , with Q = {X1, X3} (10)

The last step of the suggested methodology is to apply the rules to order the whole set of points.

3. Ranking deduction from rough approximation

3.1. Algorithmic design

The overall binary preference relation noted (P) is identified if Rule 1 is fulfilled between points. If the second rule is fulfilled, the overall non--preference is identified which is noted (N). In general, it is possible to induce decision rules being propositions of the following type:

1. D++ decision rule, which is a proposition of the type:
   if ai MMDQ aj then ai P aj.
2. D-- decision rule, which is a proposition of the type: (11)
   if ai notMMDQ aj then aj N ai.

The final set of decision rules is the set of minimal decision rules. According to Greco et al. [6] a D_{++} decision rule [D_{+-} decision rule] a_i MMD_Q a_j $\rightarrow a_i$ P a_j

[if ai not MMDQ aj $\rightarrow$ ai N aj] will be called *minimal* if there is not any other rule (12)

ai MMDR aj $\rightarrow$ ai P aj [if ai not MMDR aj $\rightarrow$ ai N aj] such that R$\subseteq$ Q.

For each point a_i let

$SC^{++}(a_i)$ = card({ $a_j \in A$: there is at least one D_{++} − decision rule stating that a_i P a_j }),

$SC^{+-}(a_i)$ = card({ $a_j \in A$: there is at least one D_{++} – decision rule stating that a_j P a_i }),
$SC^{-+}(a_i)$ = card({ $a_j \in A$: there is at least one D_{--} – decision rule stating that a_j N a_i }),
$SC^{--}(a_i)$ = card({ $a_j \in A$: there is at least one D_{--} – decision rule stating that a_i Na_j }).
To each $a_i \in A$ we assign a score, called *Net Flow Score* (see [6]),
$SNF(a_i) = SC^{++}(a_i) – SC^{+-}(a_i)+ SC^{-+}(a_i)- SC^{--}(a_i)$.

The final recommendation is the best point established by ranking 5000 points in decreasing order of the value of the SNF(a_i) on A. The rough sets approach gives us a clear recommendation: the best combination of two input parameters is temperature 74.85 C^o and drawplate profile diameter 2.80 cm. This combination of input parameters assumes the following values of three output attributes, the friability index = 1.89, the moisture of granules = 14.84, and the energetic consumption = 6.02. Figure 4 shows the results plotted *quintile* by *quintile* and the best recommendation (black diamond) for a better understanding.

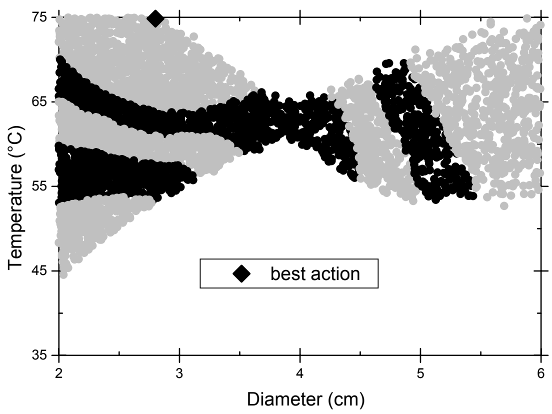


Figure 4. Total ranking of the Pareto set presented *quintile* by *quintile*

The first 1000 actions are represented in grey around the best action, the second in black, etc. From this figure, we can notice that the best point is on the border of the Pareto's zone. Then, this remark leads us to study the robustness of this best recommendation.

4. A robustness study

Robustness is connected to the fact that the decision-aid methods often contain parameters whose values have to be chosen (more or less arbitrary) by the user. In our example we have two input parameters: temperature T (between 35°C and 75°C) and drawplate profile D (between 2 cm and 6 cm). For different instances of parameter values the zone Pareto represents the feasible part of the result obtained by application of the genetic algorithm taking into account the three criteria for minimisation: the friability index, the moisture of granules and the energy consumption. The case of the robust solution was discussed by Roy and Bouyssou [10] and Roy [11]. Roy [11] suggests three kinds of assertions in order to establish recommendations: *a perfectly robust* where the domain of the result for all parameter values is well known, *an approximately robust* where a perfectly robust conclusion is not necessarily identified and *a pseudo-robust* conclusion which is a more or less formal statement concerning all parts of the result for all parameter values. In our example, the points in the Pareto's zone are approximately robust because of the variation of the working conditions of the process, which can put them out of the optimal zone.

5.1. Technical robustness

Therefore, we propose the concept of technical robustness defined as stability of the result when technical parameters have small variation. In our example, a point on the border of the Pareto set could be considered as robust where no variation of the working conditions of the process is observed. In the real world of the industrial process this condition is rarely verified. This is why we suggest the hypothesis that the robustness of the points in the Pareto's zone increases with the distance of the point from the boundary. We call this concept the technical robustness because this analysis is made before the rough set approximation of the preference, which allows us to do the final recommendation. So, another criterion may be defined to take into account the robustness of the solution. Maximise the distance between a point and the border can be considered as a supplementary objective when the Pareto's zone is already found. This concept allows us to decrease the impact of a possible instability of the system.

From a theoretical point of view the borderline of the Pareto zone is represented by three non-dominated branches determined by the locus of points where isocriteria of the attributes are tangent *pairwise*:

$$D = \frac{0.005T^2 - 0.135335T + 7.52324}{-0.0105T + 0.9656} \qquad for\ \ D \le 3.91\,cm \ \ and \ \ T \ge 62.6\ ^oC$$

$$D = \frac{4.2125*10^{-4}T^2 - 0.0322555T + 0.9161}{-4.175*10^{-4}T + 0.166196} \qquad \begin{matrix} for\ \ 2 \le D \le 6\,cm \\ and\ \ 35 \le T \le 75\ ^oC \end{matrix} \tag{13}$$

$$3.0198*10^{-4}T^2 + (-0.04227664 + 0.00295924D)T + (0.712418 + 0.1116062D - 0.0268384D^2 = 0$$

$for\ D \ge 3.91\,cm\ and\ T \le 62.6\ ^oC$

In Figure 3 the border of the theoretical Pareto domain is showed by the bold line with the five points chosen by the human expert. The distance between a point and the border is calculated in terms of temperature because the drawplate profile is considered stable and can have standard characteristics. For given drawplate profile D_0 two distances are calculated between the temperature T_0 and the lower border temperature $T_1(D_0)$ and the higher border temperature $T_2(D_0)$. These distances are maximised in relation to zero in such a way that:

$$d_1 = \text{Max}\ \{T_0 - T_1(D_0), 0\};\ d_2 = \text{Max}\ \{T_0 - T_2(D_0), 0\} \tag{14}$$

The negative value of the difference between temperatures shows us that the point is out of the Pareto domain. Finally, the value of the distance which is taken to the technical robustness analysis is as follows:

$$d = \text{Min}\ \{d_1; d_2\} \tag{15}$$

In our granulation process, the rough sets preference analysis is applied by taking into account a fourth attribute X_4: the technical robustness, which is a distance d to maximise. Table 6 shows the five points chosen by the human expert, their respective four conditional attributes and ranking (it is the same ranking as previously). So, our rules changed and another total ranking are deducted.

Table 6

Evaluations of the five chosen actions for four attributes

Actions	X_1	X_2	X_3	X_4
$\mathbf{a_1}$	2.0495	13.361	7.8915	7.2253
a_2	2.0925	14.998	5.4695	0.6671
a_3	3.3122	14.096	4.7477	16.674
a_4	2.9819	10.128	20.871	0.0625
a_5	4.3177	10.225	17.916	0.0000

Then, we can add the decision attribute D to Table 7 which makes a dichotomic partition of the set of pair points.

Table 7

Decision table for the five chosen points and for four conditional attributes

	Pairs	X_1	X_2	X_3	X_4	D
H_P	(a_1, a_2)	1	1	0	**1**	**P**
	(a_1, a_3)	1	1	0	0	P
	(a_1, a_4)	1	0	1	1	P
	(a_1, a_5)	1	0	1	1	P
	(a_2, a_3)	1	0	0	0	P
	(a_2, a_4)	1	0	1	1	P
	(a_2, a_5)	1	0	1	1	P
	(a_3, a_4)	0	0	1	1	P
	(a_3, a_5)	1	0	1	1	P
	(a_4, a_5)	1	1	0	0	P
H_N	(a_2, a_1)	0	0	1	**0**	**N**
	(a_3, a_1)	0	0	1	1	N
	(a_4, a_1)	0	1	0	0	N
	(a_5, a_1)	0	1	0	0	N
	(a_3, a_2)	0	1	1	1	N
	(a_4, a_2)	0	1	0	0	N
	(a_5, a_2)	0	1	0	0	N
	(a_4, a_3)	1	1	0	0	N
	(a_5, a_3)	0	1	0	0	N
	(a_5, a_4)	0	0	1	0	N

According to definition (6), from Table 8 we obtain one dominance relation which intersects with seven pairs of points and which verifies the condition of the lower approximation.

Table 8

Dominance which intersects with seven pairs of points in the decision table

MMD_Q/H	(a_1, a_2)	(a_1, a_4)	(a_1, a_5)	(a_2, a_4)	(a_2, a_5)	(a_3, a_5)	(a_4, a_5)
$Q = \{X_1, X_4\}$	x	x	x	x	x	x	x

From this we can induce the minimal decision rule, which can be formulated as follows:

Rule 3

$$\text{If } a_i \text{ MMD}_Q\, a_j \text{ then } a_i \text{ P } a_j \text{, with } Q = \{X_1, X_4\} \tag{16}$$

The meaning of this rule is that if the point a_i in the Pareto's zone is better that the point a_j with respect to friability index and to the distance from the boundary, then the point a_i is preferred to point a_j. In the same way we can do the approximation of non-preference N by the Multi-Attribute non--dominance not MMD_Q. According to the definition (9) from Table 9, we obtain one non-dominance relation which intersects with seven pairs of points and which verifies the condition of lower approximation.

Table 9

Non-dominance which intersects with seven pairs of points in the decision table

MMD_Q /H	(a_2, a_1)	(a_4, a_1)	(a_5, a_1)	(a_4, a_2)	(a_5, a_2)	(a_5, a_3)	(a_4, a_5)
$Q = \{X_1, X_4\}$	x	x	x	x	x	x	x

Based on the same principle as in the case of preference choosing, we can induce the rule as follows:

Rule 4

$$\text{If } a_i \text{ not MMD}_Q\, a_j \text{ then } a_i \text{ N } a_j \text{, with } Q = \{X_1, X_4\} \tag{17}$$

The last step of the robustness analysis is to apply the rules 3 and 4 to order once again the whole set of points.

The rough approximation then gives a new recommendation: 66.00°C as the best temperature and 2.82 cm is the best drawplate profile. This combination of input parameters assumes the following values for the four output attributes: $X_1 = 1.99$, $X_2 = 13.86$, $X_3 = 6.62$ and $X_4 = 7.53$. We can notice that this action is different when compared to the one found in the previous part.

But, the most important information from this ranking is that the best recommendation is a robust one, because a little variation of the temperature (e. g. difficulty to control the temperature in the granules) does not alter the product quality, the point stays in the Pareto set. Figure 5 shows the results of this new total ranking plotted *quintile* by *quintile* and the best recommendation (black diamond).

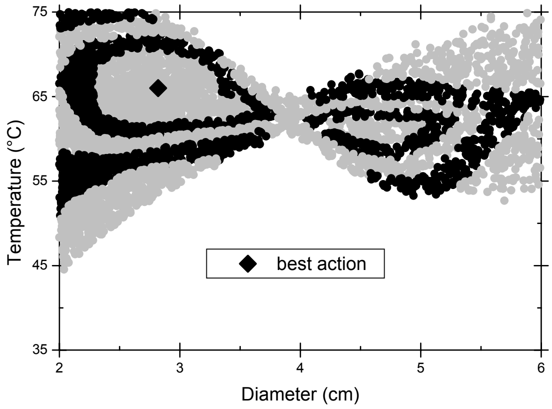


Figure 5. Total ranking of the Pareto set by taking into account the technical robustness

We can notice that the first *quintile* is represented in two parts in the middle right and the middle left of the Pareto's zone. This leads to confirm the importance of a criterion like the technical robustness.

Conclusion

The Rough Set approach has been used for the analysis of preferential information concerning multi-attribute choices in the Pareto's zone. This information is given by the DM as set of pairwise comparisons among some reference points. Taking into account these preferences, we deduce the rules. These rules represent the preference model of the DM, which can be applied to a whole set of points in the Pareto's zone.

After obtaining of the total ranking of the actions, the robustness of the results has been studied. The technical robustness has been added as another criterion to take into account a possible degradation of the working condition. So, this study has been able to propose a very good action, which can be defined as a robust one from technical points of view.

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