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PREFACE

The present volume includes theoretical and application papers from the field of multicriteria decision making. The authors are faculty members of the University of Economics in Katowice, Department of Operations Research, and researchers from Poland and abroad, collaborating with the Department.

The contents of the individual papers are as following.

In the paper ***LQG Monotone Follower Model of Change Control in Turbulent Environment*** (**T. Banek and E. Kozłowski**) a multicriteria approach is taken as a starting point and used in several aspects to create general and coherent quantitative methodology for the change control problem.

In the paper ***Building Personality Profile of Negotiator for Electronic Negotiations*** **J. Brzostowski** and **T. Wachowicz** propose a new approach based on the classification of speech acts contained in message exchanges between negotiators.

In the paper ***Application of an AHP-type Method at Portfolio Management*** **J. Charouz** and **J. Ramík** apply two methodologies: AHP and FVK to the problem of a portfolio manager making a decision on the financial market.

In the paper ***Applying a First Priced Auction Mechanism for Supporting Multi-Bilateral Negotiations*** **P. Fiala** and **T. Wachowicz** apply the notion of equilibrium bidding strategy and use the negotiation strategies proposed in a simple spreadsheet-based negotiation support tool for finding the most satisfying solution in negotiation process.

In the paper ***Multicriteria Performance Comparison of Central European Industrial Firms*** **J. Jablonsky, P. Fiala, T. Trzaskalik** and **M. Nowak** present a modelling approach based on Data Envelopment Analysis and Analytic Network Process, and its application based on the data set resulting from a survey among firms of selected industries.

In the paper ***Multicriterial Examination Timetabling with Uncertain Information*** **B. Gładysz** and **D. Kuchta** propose a new approach, which belongs to the family of robust approaches. A multicriteria approach with the minimization of the examination session days and the maximization of schedule robustness is also considered.

In the paper ***Multiple Criteria Decision Making: from Exact to Heuristic Optimization*** **I. Kaliszewski** and **J. Miroforidis** propose to derive assessments of outcomes to multiple criteria decision making problems instead of just outcomes and carry decision making problem with the former.

In the paper ***On the Properties of Stochastic Multiple-Criteria Comparison Methods in Health Technology Assessment*** **B. Kamiński** and **M. Jakubczyk** compare the decision-theoretic properties of expected net benefit, cost- effectiveness acceptability curve and expected value of choosing the optimal treatment.

In the paper ***Optimization of Public Debt Management in the Case of Stochastic Budgetary Constraints*** **L. Klukowski** presents a stochastic approach to strategic optimization of public debt management in Poland aimed at minimization of serving costs of the debt and costs resulting from stochastic budgetary constraints.

In the paper ***On Multiple Criteria Genetic Approach to Highly Constraint VRPs*** **G. Koloch** and **T. Szapiro** investigate whether real-life Vehicle Routing Problems can be effectively handled by genetic algorithms.

In the paper ***On a Group Multicriteria Method for Project Evaluation*** **L. Kruś** proposes a new method in the practice of funds allocation, supporting multicriteria analysis and selection of projects applying for funds and uses it in the study.

In the paper ***Evolutionary Algorithms with Direct Chromosome Representation in Multi-Criteria Project Scheduling*** **B. Krzeszowska** considers three types of criteria to optimize a project schedule: resource, time and cost allocation, and demonstrates possible applications of evolutionary algorithms.

In the paper ***On Robust Solutions to Multi-Objective Linear Programs*** **W. Ogryczak** shows that the robust solution for proportional upper limits on weights perturbations is the tail β-mean solution for an appropriate β value.

In the paper ***Multiobjective Model for Designing Customized Tourist Tours*** **B. Rodriguez**, **J. Molina** and **R. Caballero** develop a model to solve this problem, taking into account the diverse economic costs, the timing of the different activities, and the tourist's preferences.

In the paper ***Stability of Multicriteria Ranking – a Comparison*** **A. Sielska** considers the stability problem and applies it to rankings of open-end investment funds.

The volume editors would like to thank the authorities of the University of Economics for support in editing the current volume in the series Multiple Criteria Decision Making.

Tadeusz Trzaskalik

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LQG MONOTONE FOLLOWER MODEL OF CHANGE CONTROL IN TURBULENT ENVIRONMENT

Abstract

Change control (ChC) and monotone LQG follower (MFP) are two problems considered in this paper. Analogies and similarities between them allow for introducing a common parametrization, which can be used for translation of results obtained for the follower problem into the jargon of change control. Analogies and similarities between these problems are recognized on two levels: movement's description and criteria of control. That is the first moment when multicriteria approach is taken into account. On the other hand, there are many possible criteria which can, and should, be taken into account in deciding how to apply results coming from solutions of MFP into actions for ChC. That is the second moment for multicriteria approach to work.

Keywords

Stochastic control, change control, monotone follower, LQG problems, incremental value of information, Lagrange multipliers.

Introduction

The change control problem (ChC) has been by now extensively studied in literature (see [3], [4], [5], [6], [10], [11], [12], [14], [15], [16], [21] for example). These studies are qualitative rather then quantitative and till now there is lack of mathematical methods which are necessary for the creation of a general and coherent methodology. This paper is a step in this direction. We take a multicriterial approach as a cornerstone and use it in several aspects. It is particularly convenient when analogies and/or similarities are discussed, because they are, by definition, dependent upon criteria selected. The paper is organized as follows: in the next section we present formulations of two problems, the Linear Quadratic Gaussian (LQG) follower problem (FP) in R^n and the change control problem. From a general cybernetic/behavioral perspective it can be noticed that the problems experience analogies or similarities and create more complex relationships than just a set of random events. Analogies and similarities between these problems are recognized on two levels: movement's description and criteria of control. That is the first moment when multicriterial approach is taken into account. On the other hand, there are many possible criteria which can, and should, be taken into account in deciding how to apply results coming from solutions of MFP into actions for ChC. That is the second moment for multicriterial approach to be taken into account. This is our starting point for a new methodology explained in Subsection 1.2. We sketch here an idea. Mathematical results obtained for the FP in Subsections 2.2, 2.3, are in a quantitative form of optimal control formulae. The optimal control depends however on model parameters and selected criteria. Assuming that parameters and criteria in both problems are the same, optimal control laws can be characterized qualitatively using parametrization introduced in both problems. An optimal solution for the FP is – per analogy – recommended for the ChP. This methodology is applied in the LQG case in Subsection 2.3. Incremental value of information is introduced next and applied in the change control problem using the same approach. In the last section we offer a precise description of the essence of our methodology.

1. Problem Formulation

To make our presentation easier we begin by describing the Follower problem.

1.1. The Follower Problem in R^n

Let (Ω, F, P) be a complete probability space where the random variables $\xi_0, w_1, ..., w_N$, $\theta_1, ..., \theta_N$ are defined. They are assumed to be stochastically independent and such that

$$P(\theta_i = 0) = p = 1 - P(\theta_i = 1)$$

$$P(w_i \in A) = \int_A q(w)dw, \ P(\xi_0 \in A) = \int_A P(d\xi_0)$$

where $i = 1, ..., N$. For $f : R^n \times R^m \to R^n$, a measurable function called the *dynamic function*, let us define a stochastic system, called the Evader (E), via the iterative scheme

$$\xi_{i+1} = f(\xi_i, \varepsilon\theta_{i+1} w_{i+1}) \tag{1}$$

where $\varepsilon > 0$, and the product $\varepsilon\theta_{i+1} w_{i+1}$ models stochastic disturbances occurring in the system. Since $\theta_i = 1$ or 0, the disturbance εw_i occurs in time i or it does not. By allowing ε and p to have bigger or smaller values we can model the intensity of the random disturbances affecting the movement of E. Another system, called the Follower (F), is described by the iterative scheme

$$x_{i+1} = g(x_i, u_i) \tag{2}$$

where $g : R^n \times R^m \to R^n$ is a measurable function. Here, by u_i we denote the control action at the time i. As the Follower is allowed to know $\xi_0, ..., \xi_i, x_0, ..., x_i$ at the time i, only controls of the form

$$u_i = v_i(\xi_0, ..., \xi_i, x_0, ..., x_i) \tag{3}$$

where $v_i : R^{(i+1)n} \times R^{(i+1)n} \to R^m$ are Borel measurable functions, are admissible. For $h : R^n \times R^m \times R^n \to R_+$, a Borel measurable and bounded from below function, let us introduce a cost functional as the optimization criterion

$$J(u) = E\left[\sum_{j=0}^{N-1} h(x_i, u_i, \xi_{i+1})\right] \tag{4}$$

where E denotes expectation with respect to the measure P. The aim of the Follower is to find

$$\min_{u \in U} J(u) \tag{5}$$

where $U = \{u_i = v_i(\xi_0, ..., \xi_i, x_0, ..., x_i); i = 0, ..., N-1\}$.

Example 1. *If* $a, b, c \geq 0$ *and*

$$h(x, u, \xi) = a\|g(x, u) - \xi\|^2 + b\|g(x, u) - x\|^2 + c\|u\|^2$$

then

$$J(u) = E\left[\sum_{j=0}^{N-1} a\|x_{j+1} - \xi_{j+1}\|^2 + b\|x_{j+1} - x_j\|^2 + c\|u_j\|^2\right]$$

(*where* $\|\cdot\|$ *denotes the Euclidean norm in* R^n) *describes a sum of penalties: the first due to the distance between the trajectories of E and F, the second due to the large jumps of F, and the third due to costs of controls. A Follower whose large jumps are costly* ($b >> a$, $b >> c$) *is called monotone.*

1.2. The Change Control Problem

The Change Manager (Project Manager, Steering Committee) has at least two options to steer his organization in the right direction and to put in place necessary and often unpopular changes. He may try to follow a leader (benchmarking), or force implementation of the virtual picture of his organization, an image, which he created as an optimal response for challenges coming from a turbulent environment. The picture, either virtual or real (benchmark), evolves in time in some unpredictable and random fashion. Even more, the intensity of changes in the organization's environment has a strong impact on the variability of pictures under considerations. Hence, the movement of the picture is very similar to those of the Evader from the previous section. However, the implementation of the necessary changes can be done in several different ways. For instance, the changes can be introduced step-by-step, implemented over time, allowing the organization to adapt gradually. Such policy aims at protecting the organization from unnecessary and often costly shocks, the effects of revolutionary changes. It is worth mentioning that too small, time consuming, prudently done, cosmetic or delayed changes are costly as well. The opportunity cost is a special name reserved for this cost in Economics. Hence, one may consider the MFP as a natural candidate to model the ChC problem. However, there are several drawbacks of such a model. Is the space R^n, including its elements, the proper object to model the state space of organizations? From the mathematical point of view, it certainly is not. It is true that some structures, DNA for instance, can be coded as a sequence of numbers. Nevertheless, it is not obvious that for any sequence of numbers one can find DNA which is coded by this sequence. Is any linear combination of structures a structure itself? Again, the answer cannot be affirmative. But structures are only part of a state space description of the organization under consideration. If linear combinations of the elements are not allowed, what kind of mathematical operations are? Perhaps the only thing we can estimate is the distance between the different states of organizations of the same kind or at least very similar. An open question is the completeness of this metric space, say (M,r), i.e., is any Cauchy sequence $m_j \in M$, $r(m_i, m_j) \to 0$, $i, j \to \infty$ convergent to $m_\infty \in M$? In terms of Management Sciences this question would be expressed as follows: is a process a making sequence of smaller (and smaller) changes always convergent to the result which can be identified as a state of the organization? Is it still the organization? If it was law respecting, civilized and honest – will it continue to be civilized, honest and the law respecting? If the answer is affirmative then M is a set of second Baire category, by the Baire-Hausdorff theorem (see [20], p. 11).

If it is negative then either case is possible. But even more important is that the Follower problem in (M, r) seems to be not mathematically tractable. This is demonstrated in the next sections. Given the above, only from the cybernetic perspective and assuming a high abstract level of reasoning, one may consider the MFP in R^n space as a model of the ChC problem. Quoting the great Stefan Banach at this point: "A mathematician is a person who can find analogies between theorems; a better mathematician is one who can see analogies between proofs and the best mathematician can notice analogies between analogies". We have discovered similarities and analogies between the problems. But what can one expect by solving the MFP instead of the ChC problem? Again, similarities and analogies between recommended policies. This is particularly important in the case of ChC. As a rule, such problems are difficult to model mathematically and obtaining quantitative solutions is, in general, hopeless. But the MFP in R^n is mathematically tractable. In simplest cases one can obtain explicit solutions in forms of formulas. These formulas depend explicitly upon problem parameters (for instance, the coefficients ε, p, a, b, c in the previous section) again. If the parameter space of the problem, say $\theta \in \Theta$, (the five-dimensional space of $\theta = (\varepsilon, p, a, b, c)$ in the example) is chosen, then for an optimal control $u^*(\theta)$, and a cost function $h(x, u^*(\theta), \xi)$, $\theta \in \Theta$, one can calculate the expected control energy $E[u^*(\theta)]^2$, the expected cost $E[h(x, u^*(\theta), \xi)]$, etc. Then the pairing $(\theta, u^*(\theta))$ clearly states how to act in ChC, if an element θ of Θ is a true parameter in the ChC Problem. Under these assumptions, quantitative results of the MFP can be characterized qualitatively and translated into the jargon of ChC. Such an approach is our methodological candidate for considering the ChC problems. In the next sections we are going to show how it works. But first we solve the MFP in some generality.

2. Solutions of the MFP

2.1. A linear-quadratic-gaussian case

To be more specific we shall apply the general results obtained so far to a LQG case. Assuming linearity in the right hand side of (1) and (2) we have

$$\xi_{i+1} = A\xi_i + \varepsilon\theta_{i+1}Bw_{i+1} \tag{6}$$

$$x_{i+1} = Cx_i + Du_i \tag{7}$$

where A, B, C, D are matrixes of appropriate dimensions. Denote the independence of random variables $x_1,...,x_n$ by $x_1 \perp ... \perp x_n$. Now we relax slightly the assumption imposed in Section 1.1. and introduce instead.

Condition 2. *Assume that:* (*a*) $\xi_0 \perp \theta_1 \perp ... \perp \theta_N$, (*b*) $\theta_1 \perp ... \perp \theta_N \perp (w_1,...,w_N)$, (*c*) *the conditional distribution of* w_{i+1} *with respect to sub-sigma field* F_i *is* $N(G\xi_i, I)$, *where* G *is an* $m \times n$ *matrix, i.e., for any bounded, Borel measurable function* $h: R^n \to R$, *we have*

$$E\left[h(w_{i+1}) | F_i\right] = (2\pi)^{-m/2} \int h(w) \exp\left[-\frac{1}{2}\left\|w - G\xi_i\right\|^2\right] dw. \tag{8}$$

Remark 3. *By conditioning the Evader's equation we get*

$$E\left[\xi_{i+1} | \xi_i\right] = A\xi_i + (1-p)BG\xi_i. \tag{9}$$

By the Condition (*2*) *we can model an effect of the inner resistances against changes in organizations* $(BG \approx -A)$, *an effect of approbation* $(BG \approx A)$, *or ambivalence* $(G = 0)$. *On the other hand, by putting*

$$C_1 = C + DH. \tag{10}$$

where H *is a* $m \times n$ *matrix, in place of* C *in* (*7*) *we have*

$$x_{i+1} = Cx_i + D\left[u_i + Hx_i\right]. \tag{11}$$

This shows that the substitution (*10*) *is a deterministic version of the Condition* (*2*). *Since* C *in* (*7*) *was arbitrary, we conclude that, to model the above effects, no additional condition for the Follower is needed.*

For a quadratic criterion as in the Example 1

$$J(u) = E\left[\sum_{j=0}^{N-1} a\left\|x_{j+1} - \xi_{j+1}\right\|^2 + b\left\|x_{j+1} - x_j\right\|^2 + c\left\|u_j\right\|^2\right] \tag{12}$$

we have the following

Proposition 4. *Assume:* (*a*) $a, b, c > 0$, (*b*) *Condition* (*2*), (*c*) $\det\left[(a+b)D^T D + cI\right] \neq 0$. *Then*

$$u_j^* = \left[(a+b)D^T D + cI\right]^{-1} D^T \left\{a\left[A + (1-p)\varepsilon BG\right]\xi_j - \left[(a+b)C - bI\right]x_j\right\} \tag{13}$$

is the optimal control for the problem (*6*), (*7*), (*12*).

Proof. See appendix. ■

Conclusion 5. *In the case* $A = C,\ B = D$*, we get*

$$u_j^* = \left[(a+b)B^T B + cI\right]^{-1} B^T \left\{a\left[A + (1-p)\varepsilon BG\right]\xi_j - \left[(a+b)A - bI\right]x_j\right\} \tag{14}$$

Remark 6. u_j^* *is linear with respect to the state* (ξ_j, x_j)*, and system parameters* p, ε*.*

Remark 7. *Our results can be easily extended to cover nongaussian cases.*

Remark 8. *The crucial question is when a LQG Monotone Follower is a good model for Change Control. The most important assumption is linearity of state equations and normality of disturbances. From the equivalence theorem of J. Zabczyk* (*see* [*22*], *Chapter 3, Theorem 3.1.1, p. 26*) *it follows that without loss of generality one can choose as the space* (Ω, F, P)*, the basic probability space* $([0,1), B[0,1), \lambda_{[0,1)})$*, and as the noise, a sequence* $w_1,\ldots,w_N$ *of independent uniformly distributed random variables on* $[0,1)$*, or independent normally distributed random variables on* R*. Hence, it remains only to show linearity. Having a family* $F_{\xi_0,\ldots,\xi_i}(x_0,\ldots,x_i)$*,* $i = 0,\ldots,N$*, of multidimensional distribution functions obtained from observations, one may apply the procedure described in the above-mentioned theorem of Zabczyk and as the result, we obtain a family of functions* $f_k(\xi, w)$*,* $k \in K$ *in* (*1*)*, such that the iterative scheme with this* $f_k(\xi, w)$ *as dynamic function will produce a sequence* $\xi_0,\ldots,\xi_N$ *with the distribution function equal to* $F_{\xi_0,\ldots,\xi_i}(x_0,\ldots,x_i)$*. By selecting members* $f_k(\xi, w)$ *of the set* $\{f_k(\xi, w) : k \in K\}$ *one may choose an element which is* (*1*) *linear, or* (*2*) "*as close as currently possible*" *to be linear. If the first case holds, or an approximation error* (*appropriately defined*) *in the second case is small, then we call a ChC Problem linear.*

2.2. Energy of optimal control

Denote $(a, b, c, p, G, \varepsilon) = \theta \in \Theta$. By introducing matrices

$$L = a\left[(a+b)D^T D + cI\right]^{-1} D^T \left[A + (1-p)\varepsilon BG\right] \equiv L(\theta) \tag{15}$$

$$H = \left[(a+b)D^T D + cI\right]^{-1} D^T \left((a+b)C - bI\right) \equiv H(\theta) \tag{16}$$

we can express the optimal control (13) and its energy in the form

$$u_j^* = L\xi_j + Hx_j \tag{17}$$

$$E_j^* = E\left[\left\|u_j^*\right\|^2 \middle| F_j\right] = x_j^T H^T H x_j + 2x_j^T H^T L\xi_j + \xi_j^T L^T L\xi_j \tag{18}$$

Remark 9. *Since the matrices* L, H *depend on the system parameters* $a, b, c, p, G, \varepsilon$*, so are the optimal control* u_j^* *and its energy* E_j^**. The functions*

$$\theta \in \Theta \to u_j^*(\theta) = L(\theta)\xi_j + H(\theta)x_j \tag{19}$$

$$\theta \in \Theta \to E_j^*(\theta) = x_j^T H^T(\theta)H(\theta)x_j + 2x_j^T H^T(\theta)L(\theta)\xi_j + \xi_j^T L^T(\theta)L(\theta)\xi_j \tag{20}$$

shows this dependence explicitly. Since E_j^* *is a measure of the effort at time j done by the Follower moving in* R^n*, hence – by analogy –* $\theta \to E_j^*(\theta)$ *shows this dependence on* θ *also for Change Control.*

In this section we are going to find a total control energy

$$W_0 = E\left[\sum_{j=0}^{N-1}\left\|u_j^*\right\|^2\right]. \tag{21}$$

Let us denote

$$W_N = 0 \tag{22}$$

$$W_j = E\left[\sum_{i=j}^{N-1}\left\|u_i^*\right\|^2 \middle| F_j\right] = \left\|u_j^*\right\|^2 + E\left[W_{j+1} \middle| F_j\right] \tag{23}$$

Proposition 10. Under the conditions of Proposition 4, we have

$$W_j = E\left[\sum_{i=j}^{N-1}\left\|u_i^*\right\|^2 \middle| F_j\right] = x_j^T Q_j x_j + x_j^T R_j \xi_j + \xi_j^T S_j \xi_j + K_j \tag{24}$$

for $j = 0, 1, ..., N-1$*, where*

$$Q_j = H^T H + (C + DH)^T Q_{j+1}(C + DH), \quad Q_{N-1} = H^T H \tag{25}$$

$$R_j = 2H^T L + 2(C + DH)^T Q_{j+1} DL + (C + DH)^T R_{j+1}\left[A + (1 - p)\varepsilon BG\right] \tag{26}$$

$$R_{N-1} = 2H^T L \tag{27}$$

$$S_j = L^T\left[I + D^T Q_{j+1} D\right]L + L^T D^T R_{j+1}\left[A + (1-p)\varepsilon BG\right] + A^T S_{j+1}\left[A + 2(1-p)\varepsilon BG\right] \tag{28}$$

$$S_{N-1} = 2L^T L \tag{29}$$

$$K_j = \sum_{i=j+1}^{N-1} (1-p)\varepsilon^2 tr\left(S_i BB^T\right), \quad K_{N-1} = 0 \tag{30}$$

Proof. See appendix. ■

Conclusion 11. *The expected total energy of optimal control is given by*

$$E\left[\sum_{i=0}^{N-1} \left\|u_i^*\right\|^2\right] = W_0 = x_0^T Q_0 x_0 + x_0^T R_0 \xi_0 + \xi_0^T S_0 \xi_0 + K_0 \tag{31}$$

where Q_0, R_0, S_0, K_0 *are given in* (*25*)-(*30*).

Remark 12. *All remarks in the previous Remark are also valid for the expected total energy*

$$\theta \in \Theta \to W_0(\theta) = x_0^T Q_0(\theta) x_0 + x_0^T R_0(\theta)\xi_0 + \xi_0^T S_0(\theta)\xi_0 + K_0(\theta) \tag{32}$$

The functions $\theta \in \Theta \to Q_0(\theta), R_0(\theta), S_0(\theta), K_0(\theta)$ *are given in the previous Proposition. Dependences on* θ *are − by analogy − expected to hold for linear Change Control.*

2.3. Size of the jumps

We want to calculate the sum of the expected jumps

$$V_0 = E\left[\sum_{i=0}^{N-1} \left\|x_{i+1} - x_i\right\|^2\right] \tag{33}$$

and its dependence on the systems parameters. Since

$$\left\|x_{i+1} - x_i\right\|^2 = \left\|(C-I)x_i + Du_i^*\right\|^2 = \left\|(C + DH - I)x_i + DL\xi_i\right\|^2 \tag{34}$$

hence

$$V_0 = E\left[\sum_{i=0}^{N-1} \left\|(C + DH - I)x_i + DL\xi_i\right\|^2\right] \tag{35}$$

Dennote

$$V_N = 0 \tag{36}$$

$$V_j = E\left[\sum_{i=j}^{N-1} \left\|(C + DH - I)x_i + DL\xi_i\right\|^2 \middle| F_j\right] = \left\|x_{j+1} - x_j\right\|^2 + E\left[V_{j+1}\middle| F_j\right] \tag{37}$$

Proposition 13. *Under the conditions of Proposition 4, we have*

$$\begin{aligned} V_j &= E\left[\sum_{i=j}^{N-1} \left\|(C + DH - I)x_i + DL\xi_i\right\|^2 \middle| F_j\right] \\ &= x_j^T \widetilde{Q}_j x_j + x_j^T \widetilde{R}_j \xi_j + \xi_j^T \widetilde{S}_j \xi_j + K_j \end{aligned} \tag{38}$$

for $j = 0,1,...,N-1$*, where*

$$\widetilde{Q}_j = (C + DH - I)^T (C + DH - I) + (C + DH)\widetilde{Q}_{j+1}(C + DH)$$

$$\widetilde{Q}_{N-1} = (C + DH - I)^T (C + DH - I)$$

$$\widetilde{R}_j = 2(C + DH - I)^T DL + 2(C + DH)\widetilde{Q}_{j+1} DL + (C + DH)^T \widetilde{R}_{j+1}\left[A + (1-p)\varepsilon BG\right]$$

$$\widetilde{R}_{N-1} = 2(C + DH - I)^T DL$$

$$\widetilde{S}_j = L^T D^T\left[I + \widetilde{Q}_{j+1}\right]DL + L^T D^T \widetilde{R}_{j+1}\left[A + (1-p)\varepsilon BG\right] + A^T \widetilde{S}_{j+1}\left[A + 2(1-p)\varepsilon BG\right]$$

$$\widetilde{S}_{N-1} = L^T D^T DL$$

$$K_j = \sum_{i=j+1}^{N-1} (1-p)\varepsilon^2 tr\left(S_i BB^T\right), \quad K_{N-1} = 0 \tag{39}$$

Proof. See appendix. ■

Conclusion 14. *The expected sum of the jumps is given by the formula.*

$$E\left[\sum_{i=0}^{N-1} \left\|x_{i+1} - x_i\right\|^2\right] = V_0 = x_0^T \widetilde{Q}_0 x_0 + x_0^T \widetilde{R}_0 \xi_0 + \xi_0^T \widetilde{S}_0 \xi_0 + K_0 . \tag{40}$$

Since the matrices H, L are functions of θ, so are the matrices $\widetilde{Q}_0, \widetilde{R}_0, \widetilde{S}_0$ and the scalar K_0. From the Proposition above we get formulas for the expected value of individual jumps and their sum. The functions

$$\theta \in \Theta \to \varphi_{x,y}(\theta) = \left\|(C + DH(\theta) - I)x + DL(\theta)y\right\|^2$$

$$\theta \in \Theta \to \Phi_{x,y}(\theta) = x^T \widetilde{Q}_0(\theta)x_0 + x^T \widetilde{R}_0(\theta)y + y^T \widetilde{S}_0(\theta)y + K_0(\theta)$$

define explicitly dependences of individual jumps sizes and their sum on system's parameters.

In order to answer the question how the recommended changes could or should depend upon the turbulences in the organization's environment, we have to restrict the domain Θ of $\varphi_{x,y}(\theta)$ and $\Phi_{x,y}(\theta)$. Let's fix the coordinates (a,b,c,G) of $\theta = (a,b,c,p,G,\varepsilon)$ and let $(p,\varepsilon) \in [0,1] \times R_+$ be free. Denote

$$\phi_{x,y}^{a,b,c,\alpha}(\cdot,\cdot) = \varphi_{x,y}(a,b,c,\cdot,\alpha,\cdot) \tag{41}$$

$$\Psi_{x,y}^{a,b,c,\alpha}(\cdot,\cdot) = \Phi_{x,y}(a,b,c,\cdot,\alpha,\cdot) \tag{42}$$

Hence (41), (42) are restrictions of $\varphi_{x,y}(\theta)$, $\Phi_{x,y}(\theta)$ on $[0,1] \times R_+$ and are the functions of interest. Hence, by setting

$$\chi = (C + DH - I)x + aD\left[(a+b)D^T D + cI\right]^{-1} D^T Ay\,,$$

$$z = aD\left[(a+b)D^T D + cI\right]^{-1} D^T BGy\,,$$

$$\rho = (p-1)\varepsilon$$

we obtain, after some transformations,

$$\phi_{x,y}^{a,b,c,\alpha}(p,\varepsilon) = \|\chi - \rho z\|^2\,.$$

Conclusion 15. *The expected conditional jumps is a quadratic function* $\rho \in R \to \|\chi - \rho z\|^2$ *of the turbulence parameter* $\rho = (p-1)\varepsilon$. *Its minimum* $\left\|\chi - \frac{\chi^T z}{\|z\|^2} z\right\|^2$ *is achieved at* $\rho = \frac{\chi^T z}{\|z\|^2}$. *Certainly, this minimum also depends on the remaining system parameters, namely* a,b,c,G. *According to our cybernetic/behavioral perspective we claim that the relations describing the LQG Monotone Follower Problem solution given in this section are also valid for the LQG Change Control* (*see Remark 12*).

3. Value of information about Evader's future movement

In any follower problem any information about the evader's future movements can change the pursuer's strategy radically. One may choose, for instance, to aim at the virtual point of intersection of both trajectories, instead of using a pure follower's strategy, etc. Hence, any predictions coming from advisers and/or experts are valuable. But, what is the value? It is particularly interesting, how to price mathematically such information. In this section we introduce the notion of incremental value of information and apply it to the MFP.

Remark 16. *In stochastic optimization problems one can implement at least two approaches to defining the value of information. The first approach, leading to so called Incremental Value of Information* (*IVI*), *was initiated by M.H.A. Davis, M.A.H. Dempster, R.J. Elliott in* [*7*] *and by K. Back, S.R. Pliska in* [*1*] *and uses an idea of R.J.-B. Wets* [*19*]. *The second approach, initiated by T. Banek, R. Kulikowski in* [*2*] *and independently by M. Schweizer, D. Becherer in* [*18*], *is based on the idea that the information can be an object of trade and its value for a particular agent is a consequence of its utility. In this paper we follow the first method. By introducing the Lagrange multiplier we turn the optimization problem into a global minimization one over all controls from* $V \in L_p^m(N \times (\Omega, F, P))$ *which are* ξ^N *– measurable, i.e., take the form* $V = \{v_i(x^i, \xi^N), i = 0,1,...,N-1\}$. *Secondly, the Lagrange multiplier can be interpreted as a price system for small violations of the constraint (the shadow price in* [*1*]), *in our case, small* ξ^N *– measurable perturbations of the controls. That approach is similar to small anticipative* (*allowed to know the future*) *perturbations considered in the paper* [*7*]. *Our price system may perhaps have some practical value for F who has an extra option, for instance;* (*1*) *he can predict the whole future movement of E, i.e.,* ξ^N (*or its part*) *by himself, doing, for instance, extensive* (*and costly*) *research, or* (*2*) *to buy a prognosis of* ξ^N *made by experts. The question of interest for F is: what is the right price for buying* ξ^N*? Our price system tells only how much costs a small violation of the constraint and thus can serve as a linear approximation.*

3.1. Subspace constrains and Lagrange multipliers

Let X be a Banach space with dual space X^*, and let S be a linear subspace of X. We define

$$S^{\perp} = \{x^* \in X^* :< x^*, x >= 0, \forall x \in S\}$$

where $< x^*, x >$ denotes the pairing between $x \in X$ and $x^* \in X^*$.

Let $\phi : X \to R$ be a Frechet differentiable functional and suppose that ϕ achieves its minimum over S at $x_0 \in S$. The Frechet derivative is a map $\phi' : X \to X^*$ such that for $h, x \in X$

$$\phi(x+h) = \phi(x) + \langle \phi'(x), h \rangle + o(\|h\|).$$

Lemma 17. *If ϕ achieves its minimum over S at $x_0 \in S$, then $\phi'(x_0) \in S^{\perp}$.*

Proof. If $\phi'(x_0) \notin S^{\perp}$ then there exists $h \in S$ such that $\langle \phi'(x_0), h \rangle = \delta > 0$. But then $\phi(x_0 - \varepsilon h) = \phi(x_0) - \varepsilon(\delta + o(\varepsilon)/\varepsilon)$ so that $\phi(x_0 - \varepsilon h) < \phi(x_0)$ for small ε.■

Theorem 18. *If $\phi : X \to R$ is Frechet differentiable and achieves its minimum over S at $x_0 \in S$, then there exists $\lambda \in S^{\perp}$ such that Lagrange functional*

$$L(x) = \phi(x) + \langle \lambda, x \rangle$$

is stationary at x_0, i.e., $L'(x_0) = 0$.

Proof. We have only to set $\lambda = -\phi'(x_0)$.■

3.2. Incremental value of information

To apply the above results to our problem, we take X to be the space $L_p^m(N \times (\Omega, F, P))$ of all controls $V = \{v_i(x^i, \xi^N), i = 0,1,..., N-1\}$, i.e., anticipating controls which have access to information about the future movement of Evader F, and S to be a subspace of X of all controls

$U = \{u_i(x^i, \xi^i), i = 0,1,...,N-1\}$. It is clear that S is a linear subspace of X. Then X^* is $L_q^m(N \times (\Omega, F, P))$ space where $q = \frac{p}{p-1}$ and

$$S^{\perp} = \{\lambda \in X^* : E < \lambda, V >= 0, \forall V \in S\}.$$

The relationship between Gateaux and Frechet derivative of ϕ is that if the Gateaux derivative takes the form

$$E \sum \lambda_j v_j \tag{43}$$

for some $\lambda = col(\lambda_0, ..., \lambda_{N-1}) \in X^* = L_q^m(N \times (\Omega, F, P))$, then ϕ is Frechet differentiable and $\phi'(u) = \lambda$. Hence, from (43) we obtain

Theorem 19. *Assume that $\nabla_u h$ is bounded. Under the assumptions of Proposition 4*

$$\lambda_j = \nabla_u h(x_j, u_j^*, \xi_{j+1}) \tag{44}$$

for $j = 0,1,...,N-1$.

Proof. The RHS of (44) is bounded, hence it belongs to $L_q^m(N \times (\Omega, F, P))$ for any $q \geq 0$.■

As it was explained in the proof of Proposition 4, if u_j^* is an optimal control, then the equality

$$E\left[w_j^T \nabla_u h(x_j, u_j^*, \xi_{j+1})\right] = 0$$

must hold for any F_j – measurable function w_j. Comparing this with (43) we get (44). Since u_j^* is Markovian, i.e., it is of the form

$$u_j^* = v_j(x_j, \xi_j, \theta) \tag{45}$$

where θ is a parameter, $\theta \in \Theta$ and $v_j : R^n \times R^n \times \Theta \to R^m$ is some function, hence, from (1), (44), (45) we obtain

$$\lambda_j = \nabla_u h(x_j, v_j(x_j, \xi_j, \theta), f(\xi_j, \varepsilon\theta w_{j+1})) \tag{46}$$

This formula shows dependence of the IVI on $\theta \in \Theta$ explicitly.

3.2.1. A linear case

Taking into account that

$$h(x,u,\xi) = a\|Cx + Du - \xi\|^2 + b\|Cx + Du - x\|^2 + c\|u\|^2$$

we have

$$\begin{aligned}\frac{1}{2}\nabla_u h(x_j, u_j^*, \xi_{j+1}) &= aD^T(Cx_j + Du_j^* - \xi_{j+1}) + bD^T(Cx_j + Du_j^* - x_j) + cu_j^* \\ &= D^T[(a+b)C - bI]x_j - aD^T A\xi_j + [(a+b)D^T D + cI]u_j^* - aD^T \varepsilon\theta_{j+1}Bw_{j+1}\end{aligned} \tag{47}$$

where u_j^* is given by (13).

Corollary 20. *Under the conditions of Propositon 4 we have*

$$\begin{aligned}\frac{1}{2}\lambda_j = &\left\{I - [(a+b)D^T D + cI][(a+b)B^T B + cI]^{-1}\right\}D^T[(a+b)C - bI]x_j \\ &+ a\left\{[(a+b)D^T D + cI][(a+b)B^T B + cI]^{-1} D^T[A + (1-p)\varepsilon aB] - D^T A\right\}\xi_j \\ &- aD^T\varepsilon\theta_{j+1}Bw_{j+1}\end{aligned} \tag{48}$$

Proof. Substitution of (13) into (47) gives (48). Hence, it remains only to show that (50) belongs to $L_q^m(N \times (\Omega, F, P))$ for any $q \geq 0$. Since x_j are deterministic, ξ_j and $\theta_{j+1}Bw_{j+1}$ independent, from (6) it follows that it is enough to have $\theta_{j+1}Bw_{j+1} \in L_q^m(N \times (\Omega, F, P))$. But this is obvious, because θ_j, w_j are independent and Bernoulli, Gaussian distributed. ■

4. Essence

At the end we offer a precise description of the essence of our methodology.

Definition 21. *Let parameter space Θ can be decomposed into disjoint pieces $\Theta_j, j \in J$ a set of indexes, i.e., $\Theta = \bigcup_{j \in J}\Theta_j$, $\Theta_i \cap \Theta_j = 0$, for $i \neq j$. We say that Θ shows an $F/\varepsilon,\sigma$ - distinction with respect to a family $\{\Theta_j : j \in J\}$, if there is a function $F : \Theta \to R^d, d \geq 1$, such that conditions*:

(*a*) $\|F(\Theta_1) - F(\Theta_2)\| \leq \varepsilon, \quad \Theta_1, \Theta_2 \in \Theta_j$ *for any* j

(b) $\qquad \|F(\Theta_1) - F(\Theta_2)\| \geq 1/\delta, \quad \Theta_1 \in \Theta_i, \Theta_2 \in \Theta_j$ *for any* $i \neq j$

hold for some $\varepsilon, \delta \geq 0$*. If* (a) *and* (b) *hold for any* $\varepsilon \geq 0$*, then we call it an* F/δ *− distinction.*

Remark 22. *If* Θ *shows an* $u^*/\varepsilon,\delta$ *− distinction (*u^*/δ *− distinction) with respect to* $\{\Theta_j : j \in J\}$*, where* u^* *is an optimal control for the MFP, then by our cybernetic/behavioral perspective, strategies* v^* *recommended for ChC should − per analogy − have* $v^*/\varepsilon,\delta$ *− distinction (*v^*/δ *− distinction) with respect to the same* $\{\Theta_j : j \in J\}$*. Similarly for energy* E^* *− distinction, size of the jumps* V *− distinction, etc.*

Remark 23. *If* Θ *shows an* $\lambda/\varepsilon,\delta$ *− distinction (*λ/δ *− distinction) with respect to* $\{\Theta_j : j \in J\}$*, where* λ *is a Lagrange multiplier for the MFP, then by our cybernetic/behavioral perspective, a Lagrange multiplier* v *for ChC should − per analogy − has* $v/\varepsilon,\delta$ *− distinction (*v^*/δ *− distinction) with respect to the same* $\{\Theta_j : j \in J\}$*.*

Remark 24. *It is possible to obtain in one ChC/MFP many recommendations with different* $\varepsilon,\delta's$*. It is quite obvious that any recommendation coming from the solutions of the MFP to ChC is as convincing as the value of* $\zeta = \min\left(1/\varepsilon, 1/\delta\right)$*. Hence, the Change Manager should select the recommendations properly and apply them or not, according to the order of* ζ *and his preferences coming from the multicriterial approach.*

Appendix

Proof of Proposition 4.

A standard way for obtaining an optimal control is to take weak variations of $J(u)$, i.e., ε − derivative of $J(u^* + \varepsilon v)$ at $\varepsilon = 0$, where u^* is the optimal control and v an element of U − a set of admissible controls (see [17] for example). The procedure gives the following equalities

$$E\left[aD^T\left(Cx_j + Du_j^* - \xi_{j+1}\right) + bD^T\left(Cx_j + Du_j^* - x_j\right) + cu_j^* \middle| F_j\right] =$$

$$\left[(a+b)D^T C - bD^T\right]x_j - aD^T E\left[\xi_{j+1} \middle| F_j\right] + \left[(a+b)D^T D + cI\right]u_j^* = 0 .$$

Thus

$$u_j^* = \left[(a+b)D^T D + cI\right]^{-1}\left(aD^T E\left[\xi_{j+1} \middle| F_j\right] + \left[bD^T - (a+b)D^T C\right]x_j\right). \tag{49}$$

Since w_i, θ_i are stochastically independent for $i = 1,..., N$, we get

$$\begin{aligned} E\left[\xi_{j+1} \middle| F_j\right] &= \sum_{i=0}^{1} E\left[\xi_{j+1} \middle| F_j, \theta_{j+1} = i\right] P\left(\theta_{j+1} = i\right) \\ &= A\xi_j + (1-p)\varepsilon BG\xi_j = \left[A + (1-p)\varepsilon BG\right]\xi_j . \end{aligned} \tag{50}$$

Substitution of (50) into (49) gives (13). This shows that (13) is the right candidate for an optimal control. But $a, b, c > 0$ implies convexity and non--degeneracy of the problem, hence from the general results of the LQG theory (see R.S. Liptser, A.N. Shiryiaev [13], for instance) follows the sufficiency. ■

Proof of Proposition 10.

From (7) and (13) we have

$$x_{j+1} = Cx_j + Du_j^* = (C + DH)x_j + DL\xi_j . \tag{51}$$

Now, for $j = N-1$ we have

$$\begin{aligned} W_{N-1} &= E\left[\left\|u_{N-1}\right\|^2 \middle| F_{N-1}\right] = x_{N-1}^T H^T H x_{N-1} + 2x_{N-1}^T H^T L\xi_{N-1} + \xi_{N-1}^T L^T L \quad \xi_{N-1} \\ &= x_{N-1}^T Q_{N-1} x_{N-1} + x_{N-1}^T R_{N-1}\xi_{N-1} + \xi_{N-1}^T S_{N-1}\xi_{N-1} . \end{aligned} \tag{52}$$

Assume that (26) is true for $j+1$. Then

$$\begin{aligned} W_j &= E\left[\sum_{i=j}^{N-1}\left\|u_i^*\right\|^2 \middle| F_j\right] = \left\|u_j^*\right\|^2 + E\left[W_{j+1} \middle| F_j\right] \\ &= x_j^T H^T H x_j + 2x_j^T H^T L\xi_j + \xi_j^T L^T L\xi_j \\ &\quad + E\left[x_{j+1}^T Q_{j+1} x_{j+1} + x_{j+1}^T R_{j+}\xi_{j+1} + \xi_{j+1}^T S_{j+1}\xi_{j+1} + K_{j+1} \middle| F_j\right]. \end{aligned} \tag{53}$$

But from (6), (8) and the properties of conditional expectation we have

$$E\left[x_{j+1}^T Q_{j+1} x_{j+1} + x_{j+1}^T R_{j+}\xi_{j+1} + \xi_{j+1}^T S_{j+1}\xi_{j+1} + K_{j+1} \middle| F_j\right]$$
$$= E\left[\left[(C+DH)x_j + DL\xi_j\right]^T Q_{j+1}\left[(C+DH)x_j + DL\xi_j\right] \middle| F_j\right]$$
$$+ E\left[\left[(C+DH)x_j + DL\xi_j\right]^T R_{j+}\xi_{j+1} + \xi_{j+1}^T S_{j+1}\xi_{j+1} + K_{j+1} \middle| F_j\right]$$
$$= x_j^T (C+DH)^T Q_{j+1}(C+DH)x_j + 2x_j^T (C+DH)Q_{j+1}DL\xi_j \tag{54}$$
$$+ \xi_j^T L^T D^T Q_{j+1} DL\xi_j + x_j^T (C+DH)^T R_{j+1}\left[A + (1-p)\varepsilon BG\right]\xi_j$$
$$+ \xi_j^T L^T D^T R_{j+1}\left[A + (1-p)\varepsilon BG\right]\xi_j + \xi_j^T A^T S_{j+1} A\xi_j$$
$$+ 2\varepsilon(1-p)\xi_j^T A^T S_{j+1} BG\xi_j + (1-p)\varepsilon^2 tr\left(S_{j+1}BB^T\right) + K_{j+1}$$

Finally

$$W_j = K_j + x_j^T\left[H^T H + (C+DH)^T Q_{j+1}(C+DH)\right]x_j$$
$$+ x_j^T\left[2H^T L + 2(C+DH)Q_{j+1}DL + (C+DH)^T R_{j+1}\left[A + (1-p)\varepsilon BG\right]\right]\xi_j$$
$$+ \xi_j^T\left[L^T\left[I + D^T Q_{j+1}D\right]L + L^T D^T R_{j+1}\left[A + (1-p)\varepsilon BG\right]\right]\xi_j \tag{55}$$
$$+ \xi_j^T\left[A^T S_{j+1}\left[A + 2(1-p)\varepsilon BG\right]\right]\xi_j$$

what finish the proof. ■

Proof of Proposition 13.

For $j' N+1$, we have

$$V_{N-1} = E\left[\left\|(C+DH-I)x_{N-1} + DL\xi_{N-1}\right\|^2 \middle| F_{N-1}\right]$$
$$= x_{N-1}^T (C+DH-I)^T (C+DH-I)x_{N-1} \tag{56}$$
$$+ 2x_{N-1}^T (C+DH-I)^T DL\xi_{N-1} + \xi_{N-1}^T L^T D^T DL\xi_{N-1}$$
$$= x_{N-1}^T \widetilde{Q}_{N-1} x_{N-1} + x_{N-1}^T \widetilde{R}_{N-1}\xi_{N-1} + \xi_{N-1}^T \widetilde{S}_{N-1}\xi_{N-1}$$

Assume that (40) is true for $j+1$. Then

$$V_j = E\left[\sum_{i=j}^{N-1}\left\|(C+DH-I)x_i + DL\xi_i\right\|^2 \middle| F_j\right] = \left\|(C+DH-I)x_j + DL\xi_j\right\|^2 + E\left[V_{j+1} \middle| F_j\right]$$
$$= x_j^T (C+DH-I)^T (C+DH-I)x_j + 2x_j^T (C+DH-I)^T DL\xi_j + \xi_j^T L^T D^T DL\xi_j \tag{57}$$
$$+ E\left[x_{j+1}^T \widetilde{Q}_{j+1} x_{j+1} + x_{j+1}^T \widetilde{R}_{j+1}\xi_{j+1} + \xi_{j+1}^T \widetilde{S}_{j+1}\xi_{j+1} + K_{j+1} \middle| F_j\right].$$

Using (6), (8) and from conditional expectation properties, we obtain

$$
\begin{aligned}
& E\left[x_{j+1}^T \widetilde{Q}_{j+1} x_{j+1} + x_{j+1}^T \widetilde{R}_{j+1} \xi_{j+1} + \xi_{j+1}^T \widetilde{S}_{j+1} \xi_{j+1} + K_{j+1} \middle| F_j \right] \\
&= E\left[\left((C+DH)x_j + DL\xi_j\right)^T \widetilde{Q}_{j+1} \left((C+DH)x_j + DL\xi_j\right) \middle| F_j \right] \\
&+ E\left[\left((C+DH)x_j + DL\xi_j\right)^T \widetilde{R}_{j+1} \xi_{j+1} + \xi_{j+1}^T \widetilde{S}_{j+1} \xi_{j+1} + K_{j+1} \middle| F_j \right] \\
&= x_j^T (C+DH)^T \widetilde{Q}_{j+1} (C+DH) x_j + 2x_j^T (C+DH)^T \widetilde{Q}_{j+1} DL\xi_j + \xi_j^T L^T D^T \widetilde{Q}_{j+1} DL\xi_j \\
&+ x_j^T (C+DH)^T \widetilde{R}_{j+1} [A + (1-p)\varepsilon BG]\xi_j + \xi_j^T L^T D^T \widetilde{R}_{j+1} [A + (1-p)\varepsilon BG]\xi_j \\
&+ \xi_j^T A^T \widetilde{S}_{j+1} A \xi_j + 2\varepsilon(1-p)\xi_j^T A^T \widetilde{S}_{j+1} BG\xi_j + +(1-p)\varepsilon^2 tr\left(\widetilde{S}_{j+1} BB^T\right) + K_{j+1}.
\end{aligned}
\tag{58}
$$

Finally

$$
\begin{aligned}
V_j &= K_j + x_j^T \left[(C+DH-I)^T (C+DH-I) + (C+DH)^T \widetilde{Q}_{j+1} (C+DH)\right] x_j \\
&+ x_j^T \left[2(C+DH-I)^T DL + 2(C+DH)\widetilde{Q}_{j+1} DL + (C+DH)^T \widetilde{R}_{j+1} [A + (1-p)\varepsilon BG]\right]\xi_j \\
&+ \xi_j^T \left[L^T D^T [I + Q_{j+1}] DL + L^T D^T \widetilde{R}_{j+1} [A + (1-p)\varepsilon BG] + A^T \widetilde{S}_{j+1} [A + 2(1-p)\varepsilon BG]\right]\xi_j
\end{aligned}
$$

what finishes the proof. ■

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BUILDING PERSONALITY PROFILE OF NEGOTIATOR FOR ELECTRONIC NEGOTIATIONS

Abstract

In this work we propose a new mechanism for building the personality profile of a negotiator based on his behavior in past negotiations. The approach is based on the classification of speech acts contained in messages exchanged by negotiators. By assigning to each speech act its type according to our new negotiation context-dependent taxonomy, the mechanism can check the type of speech act received as the response to a particular request. The feature degree can be computed by aggregating the frequency and the strength of different types of responses in different interactions into a compound value. In this work we consider two features: cooperativeness and assertiveness, and show a method for obtaining the degrees of these features.

Keywords

Negotiator's personality profile, communication behavior, speech act taxonomy.

Introduction

In many electronic negotiations the potential players entering the interaction have no prior knowledge about their future counterpart. When the players communicate using instant messaging method the partner is neither seen nor heard by the player. The total lack of knowledge about the partner causes some discomfort for the negotiator, especially when his counterpart is anonymous. Usually the negotiator needs to have basic information about his partner, which allows him to evaluate, for instance, the partner's reliability or honesty. Therefore in this paper we propose to build a personality profile of the negotiator that could be visible for the potential negotiation partners.

Such a profile can contain levels of particular personality features such as cooperativeness and assertiveness. Displaying such an information reveals only small pieces of information important from the negotiation context viewpoint and the players may remain anonymous during their interaction. Some negotiators prefer negotiating with highly cooperative partner while others prefer a more avoiding one. Having the knowledge of the potential partners'bargaining profiles, each negotiator can select the one that meets his expectations best. Moreover, this type of knowledge can be useful for preparation of a negotiation strategy suitable for the chosen type of player.

The problem of determining the type of a player was studied by Ralph Thomas and Kenneth W. Killman [3]. The tool called "Thomas Killman conflict mode instrument" is based on a questionnaire filled out by the potential negotiator. The player is asked to choose between statements matching best his potential negotiation behaviour. Based on his selections the player is fitted into one of the five types of behaviour: competing, collaborating, compromising, avoiding and accomodating. Each of these types of behaviour is determined by the level of cooperativeness and assertiveness. In this paper we propose a new approach for solving a similar problem but without using a questionnaire. We propose to base the determination of particular features on the history of negotiator's behaviour in past negotiations. All speech acts in the messages exchanged between the two parties are classified by the negotiators according to our new negotiation context-dependent speech act taxonomy. The profiling mechanism checks the response of the message receiver to the sender's requests and, based on the types of responses, the feature degree (assertiveness, cooperativeness) is computed. By fusing the partial degrees of a feature over multiple past negotiations we obtain the final degree of a feature that can be displayed for potential future negotiation partners. The Thomas-Killman conflict mode instrument allows for creating a simple profile of a negotiator. However, the questionnaires ask the negotiator general questions about his potential behaviour and do not test it during the actual encounter. The profiling based on the negotiation thread considers only the negotiation context and the actual behaviour of the player. The speech act taxonomy was used in the Negoisst system [4]. Similarly as in the approach we propose that the user be asked to classify his message. However, this knowledge is used for clear specification of the type of speech act to avoid ambiguity but not to create a negotiator's profile.

1. The approach

To build a profile of the negotiator we use the whole description of the previous negotiation threads. Therefore, we assume all the negotiations to be conducted by means of an electronic negotiation system (ENS) in which the negotiators have individual user accounts. The ENS records all the negotiation threads in the database that can be used for all required analysis. Two types of knowledge are used for building the personality profile. The first one is the thread of speech acts communicated and the second one is the thread of offers exchanged between the players. The characteristics of the negotiator can be determined based on his behaviour during negotiation. Similarly as in the tool of Thomas Killman, in this work we consider two features of a negotiator: cooperativeness and assertiveness. The method of measuring the degree of the cooperativeness of the agent being evaluated is based on the classification of the speech acts uttered as a response to the speech act of his partner. Deriving from the existing taxonomies ([2], [5], [6]) we propose our own Negotiation Content Dependent Taxonomy – NCDT – (see Section 2) that allows to structure any single message exchange during the negotiation process and classifies it as a particular type of forward or backward communication act. Then, by analyzing each communication thread, we examine the backward communication acts (responses) and consider how they match the forward communication acts (requests). For instance, a positive response of the negotiator being evaluated to the request of his partner increases the degree of cooperativeness and a negative response of the negotiator to the request of his partner decreases the degree of cooperativeness. In the case of assertiveness the situations is analogous but the negotiator is evaluated as a sender of a speech act. If he receives positive responses to his requests then his assertiveness degree is increased. If he receives negative responses his assertiveness is decreased. This rule is based on the postulate that a communication which causes the counterpart to perform the actions desired is considered to be assertive.

2. Classification of speech acts

To classify a speech act contained in a message, we need a taxonomy of speech acts. The first speech act taxonomy was proposed by John Searle [5]. This taxonomy divides the speech acts into five types, namely: assertives, directives, commissives, expressives, declarations. The types of speech acts have the following meaning:

- **assertives** – speech acts that commit a speaker to the truth of the proposition expressed,
- **directives** – speech acts that are to cause the hearer to take a particular action, e.g. requests, commands and advice,
- **commissives** – speech acts that commit a speaker to some future action, e.g. promises and oaths,
- **expressives** – speech acts that express the speaker's attitudes and emotions towards the proposition, e.g. congratulations, excuses and thanks,
- **declarations** – speech acts that change the reality in accordance with the declaration proposed, e.g. baptisms, verdicts, or pronouncing someone husband and wife.

This taxonomy takes into consideration different types of intentions of the speaker. Another taxonomy is the Verbal Response Mode taxonomy developed while studying therapist interventions in psychotherapy [6]. This taxonomy takes into consideration three criteria: source of experience, presumption about experience and frame of reference. The taxonomy is presented in the Table 1.

Table 1

Verbal Response Mode speech act taxonomy

Source of experience	Presumption about experience	Frame of reference	VRM Mode	Description
1	2	3	4	5
Speaker	Speaker	Speaker	Disclosure (D)	Reveals thoughts, feelings, perceptions or intentions. E.g., I like pragmatics.
		Other	Edification (E)	States objective information. E.g., He hates pragmatics.
	Other	Speaker	Advisement (A)	Attempts to guide behaviour; suggestions, commands, permission, prohibition. E.g., Study pragmatics!
		Other	Confirmation (C)	Compares speaker's experience with other's; agreement, disagreement, shared experience or belief. E.g., We both like pragmatics.

Table 1 contd.

1	2	3	4	5
Other	Speaker	Speaker	Question (Q)	Requests information or guidance. E.g., Do you like pragmatics?
		Other	Acknowledgement (K)	Conveys receipt of or receptiveness to other's communication; simple acceptance, salutations. E.g., Yes.
	Other	Speaker	Interpretation (I)	Explains or labels the other; judgements or evaluations of the other's experience or behaviour. E.g., You're a good student.
		Other	Reflection (R)	Puts other's experience into words; repetitions, restatements, clarifi-cations. E.g., You dislike pragmatics.

Source: [5].

The Searle and Stiles taxonomies give an insight into the issue of speech act classification, but they do not consider some important factors. For instance, does the speech act constitute a response to a previous utterance or not? This kind of criterion was considered in the speech act classification proposed by Mark Core and James Allen [2]. The authors divide the speech act types into two groups: forward communicative functions and backward communicative functions. The latter group contains all speech acts constituting responses to the previous speech acts of the interlocutor. The former contains all the remaining speech acts. The DAMSL Annotation Scheme has the following form:

1. Forward Communicative Functions
   - Statement
     - Assert
     - Reassert
     - Other-Statement
   - Influencing Addressee Future Action
     - Open-option
     - Directive

        - Info-Request
        - Action-Directive
    - Committing Speaker Future Action
        - Offer
        - Commit
    - Performative (informing)
    - Other Forward Function
2. Backward Communicative Functions
    - Agreement
        - Accept
        - Accept-Part
        - Maybe
        - Reject-Part
        - Reject
        - Hold
    - Understanding
        - Signal-Non-Understanding
        - Signal-Understanding
            - Acknowledge
            - Repeat-Rephrase
            - Completion
    - Answer
    - Information-Relation

We use this type of taxonomy to develop our own taxonomy suited to the negotiation context. The additional characteristic feature of a negotiation treated as a discourse is the usage of logical arguments. In this sense these are statements supported by an argumentation line, and its aim is to convince the negotiation partner about its truthfulness. Moreover, the partner may respond with an Accept or Reject but the Reject may be of the form of a counter-argument, treated as an opposite statement supported by an argumentation line. Apart from introducing the statement of the argument type, we structure the taxonomy in the way presented in Table 2.

Table 2

The new negotiation context dependent taxonomy – NCDT

Direction of a speech act	Intention of a speech act	The issue of discourse	Description
1	2	3	4
Forward Communicative Function	inform interlocutor	perform action	**IPA** Informing the partner about performing an action or intending to perform an action
		Give information	**IGI** Informing the partner about facts or beliefs without intention to discuss them
	request from interlocutor	perform action	**RPA** Requesting the partner to perform an action
		give information	**RGI** Requesting the partner to give information (Asking a question)
		accept belief	**RAB** Requesting the partner to accept the belief stated
Backward Communicative Function	respond to IPA	positive	Thanking the partner for the action performed
		negative	Disapproving the action performed by the partner
		not understood	Signalling not understanding the speech act
		ignored	Not responding to the signal given
	respond to IGI	positive	Thanking the partner for the information given
		negative	Disapproving the information revelation
		not understood	Signalling not understanding the speech act
		ignored	Not responding to the signal given
	respond to RPA	positive	Informing about performing the requested action
		negative	Refusing to perform the requested action
		not understood	Signalling not understanding the speech act
		ignored	Not responding to the signal given
	respond to RGI	positive	Revealing the requested Information
		negative	Refusing to reveal the requested information
		not understood	Signalling not understanding the speech act
		ignored	Not responding to the signal given

Table 2 contd.

1	2	3	4
	respond to RAB	positive	Accept the statement presented in the speech act
		negative	Deny the statement and/or give counterargument
		not understood	Signalling not understanding the speech act
		ignored	Not responding to the signal given

We will now illustrate the relationships between the different taxonomies in Table 3.

Table 3

Comparative analysis of the different types of speech act taxonomies

NCDT	DAMSL AS	Stiles	Searles
1	2	3	4
IPA	Assert, Reassert, Offer	Disclosure	Assertives, Commissives,
	Commit, Performative		Declarations
IGI	Assert, Reassert, Performative	Disclosure, Edification	Assertives, Declarations
RPA	Directive: Action--Directive	Advisement	Directives
RGI	Directive: Info-Request	Question	Directives
RAB	Assert	Disclosure, Interpretation, Reflection	Assertives
positive response to IPA	Understanding: Acknowledge	Acknowledgement	Expressives
negative response to IPA	Information-Relation	Disclosure	Assertives, Expressives
not understood IPA	Signal-Non--Understanding	Disclosure	Assertives, Expressives
positive response to IGI	Understanding: Acknowledge	Acknowledgement	Expressives
negative response to IGI	Information-Relation	Disclosure, Edification, Confirmation	Assertives, Expressives
not understood IGI	Signal-Non--Understanding	Disclosure	Assertives, Expressives
positive response to RPA	Offer, Commit	Disclosure, Confirmation	Commissives

Table 3 contd.

1	2	3	4
negative response to RPA	Reject	Disclosure, Confirmation	Commissives
not understood RPA	Signal-Non-Understanding	Disclosure	Assertives, Expressives
positive response to RGI	Answer	Disclosure, Edification Confirmation	Assertives
negative response to RGI	Reject, Information--Relation, Assert	Disclosure, Confirmation	Assertives, Expressives
not understood RGI	Signal-Non--Understanding	Disclosure	Assertives, Expressives
positive response to RAB	Accept	Disclosure, Confirmation	Assertives
negative response to RAB	Reject, Assert	Disclosure, Confirmation	Assertives, Expressives
not understood RAB	Signal-Non--Understanding	Disclosure	Assertives, Expressives

Similarly to the DAMSL taxonomy, our new taxonomy splits the speech acts into forward communicative functions and backward communicative functions. This division is important in the negotiation context because the negotiation discourse is a process of exchanging messages that are usually different types of requests or different types of responses to previous requests such as: requesting information or requesting the next proposal. In the negotiation context three important issues occur quite often: gathering information during interaction, requesting proposal from the partner and attempting to convince the partner to accept certain beliefs. By considering these three issues we can distinguish three types of intentions of the requesting player. The remaining two types of intentions are: informing about performed action and giving information to the other party but not responding to the partner's question. The speech act types mentioned above constitute five types of forward communicative function speech acts. All the backward communicative function speech act types are responses to these five types. Therefore, we distinguish five groups of responsive speech act types which are further divided into four types. The four types denote four possible ways of responding to a forward communicative speech act: positive, negative, not understood, ignored. The first two are active responses to the speech act. The positive backward communicative function speech act constitutes a cooperative way of reacting to the partner's speech act. These positive responses include: accepting the partner's statement, confirming performing the action requested by the partner, approving the partner's action, giving the requested information and thanking the partner for the activity performed or information given. The negative responses are opposite to the positive responses and include: refusing to give information or perform an action, denying the partner's claim and disapproving the belief stated.

3. The assessment of negotiators' communication behavior

As said in the Introduction we can determine the type of behaviour based on the relationship between the forward communicative function speech act of one party and the response to this speech act in the form of backward communicative function speech act of the other party. When the negotiator using forward communicative speech acts receives positive backward communicative function speech acts with high frequency and high strength, he can be considered highly assertive. At the same time, the responding party can be considered highly cooperative. When in an analogous situation the sender receives negative backward communicative function speech acts with high frequency and high strength, he can be considered lowly assertive and his partner can be considered lowly cooperative (competitive). Let us denote by $\overline{a}_{i,j}^{\alpha\to\beta} = \overline{a}^{\alpha\to\beta}(i,j)$ an atomic speech act uttered by the speaker α to the speaker β. The number i denotes the consecutive number of a message in the whole communication thread. The number j denotes the number of speech act contained in the message. The communication thread is of the following form:

$$\overline{a}_{1,1}^{\alpha\to\beta}, \overline{a}_{1,2}^{\alpha\to\beta}, \dots, \overline{a}_{1,k_1}^{\alpha\to\beta}, \overline{a}_{2,1}^{\beta\to\alpha}, \overline{a}_{2,2}^{\beta\to\alpha}, \dots, \overline{a}_{2,k_2}^{\beta\to\alpha}$$

In the above thread the number of speech acts contained in the consecutive message is k_i, where i is the number of the message. Each atomic speech act is encoded in the following way

$$\overline{a}_{i,j}^{\alpha\to\beta} = (n_{i,j}, t_{i,j}, d_{i,j}, \overline{r}_{i,j})$$

where:

- $n_{i,j}$ denotes the intention of the speech act ($n_{i,j} \in \{1,...,7\}$, according to Table 2 there are seven possible intentions),
- $t_{i,j}$ denotes either the issue of discourse or the type of speech act depending on the intention of the speech act ($t_{i,j} \in \{1,...,5\}$, according to Table 2 there are either 2 possible issues of discourse for the first type of intention with 3 possible issues of discourse for the second type of intention or 4 possible types of response in the case of five remaining types of intentions),
- $d_{i,j}$ is the degree of importance specified by the sender of a speech act in the case of forward communicative function or the degree of response satisfaction specified by the receiver of a speech act in the case of backward communicative function (the value of d can be specified on a finite point scale, for instance $d_{i,j} \in \{1,...,7\}$).
- $\overline{r}_{i,j}$ identifies the forward communicative function speech act to which the current speech act $\overline{a}_{i,j}^{\alpha\to\beta}$ responds. For all forward communicative function speech acts the value of $\overline{r}_{i,j}$ is (0,0) which means that it does not constitute a response to any other speech act.

For the sake of further formalization we introduce functions mapping the speech acts into the particular components. These functions, defined below, will be called projections because they project the whole vector encoding a speech act onto a chosen axis (intention – p_1, issue of discourse or type – p_2, importance – p_3, matching requesting speech act – p_4):

$$p_1(\overline{a}_{i,j}^{\alpha\to\beta}) = n_{i,j} \tag{1}$$

$$p_2(\overline{a}_{i,j}^{\alpha\to\beta}) = t_{i,j} \tag{2}$$

$$p_3(\overline{a}_{i,j}^{\alpha\to\beta}) = d_{i,j} \tag{3}$$

$$p_4(\overline{a}_{i,j}^{\alpha\to\beta}) = \overline{r}_{i,j} \tag{4}$$

Let us consider a simple example of a communication thread:

$$\overline{a}_{1,1}^{\alpha\to\beta}, \overline{a}_{2,1}^{\beta\to\alpha}.$$

The above thread consists of two messages containing single speech acts which are further specified in the following way:

$$\overline{a}_{1,1}^{\alpha\to\beta} = (2,2,6,(0,0)),$$

$$\overline{a}_{2,1}^{\beta\to\alpha} = (6,1,4,(1,1)).$$

This means that the speaker α is sending one message to the speaker β containing one speech act (2,2,6,(0,0)), where the intention of the speech act is denoted by 2 meaning that it is a request and the issue of discourse is denoted by 2 that corresponds to the "give information" issue. The degree of importance specified is 6. Therefore, the message $\overline{a}_{1,1}^{\alpha\to\beta}$ is a question that is highly important to the speaker α. The speaker β is sending one message to the speaker α containing one speech act (6,1,4,(1,1)), where the intention of the speech act is denoted by 6 corresponding to the speech act "response to RGI" and the type of response is denoted by 1 meaning that it is a positive response. The degree of response satisfaction specified by the speaker α is 4, and because $\overline{r}_{2,1} = (1,1)$, this speech act responds to the speech act $\overline{a}_{1,1}^{\alpha\to\beta}$. Therefore, the message $\overline{a}_{2,1}^{\beta\to\alpha}$ is an answer to the question posed in the previous message by the speaker α.

4. Building the negotiator personality profile

The cooperativeness degree of a negotiator can be computed in the following way. All pairs of matching speech acts in terms of forward communicative speech acts with backward communicative function speech acts responding to them are considered in the computation of the degree of cooperativeness. As said before, the positive responses of the speaker β to the requests of the speaker α increase the value of cooperativeness, the negative responses decrease the value of cooperativeness (increase the value of competitiveness), the "not understood" type responses can be considered neutral (no change in value) and the responses of the type "ignored" can be considered either neutral or decreasing the value of cooperativeness. In the case of assertiveness the situation is analogous but the types of responses of the receiver influence the feature degree of the sender, while in the case of cooperativeness the types of responses of the receiver influence his own feature degree. The four possible types of response to a request are: positive, negative, not understood, and ignored. In the compound feature degree computation the types of response contribute with different sign and strength. We will define a function m by assigning to each type of response a multiplier. The positive response can be assigned a multiplier of value 1($m(1)=1$) meaning that the strength of response will be multiplied by this value resulting in an overall positive score of response. The negative response can be assigned a multiplier of value -1 ($m(2)=-1$) meaning that the strength of response will be multiplied by this value resulting in an overall negative score of the response. In the case of the neutral response (not understood) the multiplier value can be assigned the value 0 ($m(3)=0$) because this type of response does not influence the features considered. The ignored type of response can be considered to be either neutral or competitive, therefore the possible multiplier value is in the range $[-1;0]$ ($m(4)\in[-1;0]$). The strength of response is computed as an aggregate of the importance degree $d_{i,j}$ of the request $\overline{a}_{i,j}^{\alpha\to\beta}$ and the response satisfaction degree $d_{k,m}$ in the responding speech act $\overline{a}_{k,m}^{\beta\to\alpha}$, and it can be a product. For a given communication thread the feature degree can be computed by summing all the feature degrees corresponding to single pairs of request and response. Let us consider the set Λ_f^{α} of all forward communicative speech acts in the whole communication thread uttered by the speaker α to the speaker β, and the set Λ_f^{β} of all backward communicative speech acts in the whole communication thread uttered by the speaker β to the speaker α:

$$\Lambda_f^{\alpha} = \left\{\overline{a}_{i,j}^{\alpha\to\beta} \mid n_{i,j} = p_1\left(\overline{a}_{i,j}^{\alpha\to\beta}\right)\in[1;2]\right\},$$

$$\Lambda_f^{\beta} = \left\{\overline{a}_{i,j}^{\beta\to\alpha} \mid n_{i,j} = p_1\left(\overline{a}_{i,j}^{\beta\to\alpha}\right)\in[3;7]\right\}.$$

The degree of cooperativeness of the negotiator β and the degree of assertiveness of the negotiator α is computed in the following way:

$$\begin{aligned}&\deg^{\alpha}(\mathit{Assertiveness}) = \deg^{\beta}(\mathit{Cooperativeness}) = \\ &= \sum_{\overline{a}\in\Lambda_f^{\beta}} m\left(p_2(\overline{a})\right)\times p_3(\overline{a})\times p_3\left(\overline{a}^{\alpha\to\beta}\left(p_4(\overline{a})\right)\right)\end{aligned} \tag{5}$$

The assertiveness of β and the cooperativeness of α can be computed similarly analogously:

$$\deg^{\beta}(Assertiveness) = \deg^{\alpha}(Cooperativeness) = \\ = \sum_{\overline{a}\in \Lambda_f^{\alpha}} m\left(p_2(\overline{a})\right)\times p_3(\overline{a})\times p_3\left(\overline{a}^{\beta\to\alpha}\left(p_4(\overline{a})\right)\right). \qquad (6)$$

The values $p_3(\overline{a})$ and $p_3\left(\overline{a}^{\beta\to\alpha}\left(p_4(\overline{a})\right)\right)$ are the degrees of importance of the backward communicative function speech act $(\overline{a})$ and its corresponding forward communicative function speech act $\overline{a}^{\beta\to\alpha}\left(p_4(\overline{a})\right)$. In other words, these values are importances of a request and a matching response. We can treat these values as degrees of inclusion of a speech act in a fuzzy set of important speech acts. Therefore the degree of importance of the pair "request, response" is a fuzzy conjunction of these two degrees (the product realizes the conjunction operator). The value $m\left(p_2(\overline{a})\right)$ is a multiplier determined on the basis of the type of the speech act $(\overline{a})$. As said before, if the speech act is a positive response then the multiplier is positive and if it is negative then the multiplier is negative. The degrees of a feature for different negotiations are aggregated to form the final compound value of a feature.

Conclusions and future work

In this paper we have proposed a new mechanism for building the negotiator personality (bargaining) profile on the basis of its behavior during the negotiation process. All the speech acts uttered by the negotiators are classified according to the new negotiation context-dependent taxonomy (NCDT). The degrees of personality features are determined on the basis of the types of responses of the speech acts receiver. The values of feature degrees for different interactions are fused to form the overall degree of a feature that can be displayed for future negotiation partners as a component of negotiator's bargaining profile. The parties approaching negotiations could then select the partners whose character and attitude assure the best negotiation climate and bring closer to the most satisfying agreement. The knowledge of the bargaining profiles of the parties can be also used by the electronic negotiation system to accomplish its mediation function. The ENS can analyze the profiles of the negotiating parties and, on the basis of the data of the previous negotiation threads, suggest to them the most efficient negotiation strategies that will lead to a mutually satisfying agreement. Many arbitration procedures can be adopted to realize such a mediation function of the ENS [6].

The profiling mechanism proposed has been already included in the conceptual model of the ENS supporting all negotiation phases called NegoManage [1]. In the further study the mechanism will be implemented and tested. The mechanism will be extended to cope with different types of personality features.

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APPLICATION OF AN AHP-TYPE METHOD AT PORTFOLIO MANAGEMENT

Abstract

The article deals with an application of the methodology of Analytic Hierarchy process (AHP) and also with its newly developed modification named FVK to portfolio management. The method AHP was published already in 1980s whereas FVK is a newly created tool expanding application possibilities of the AHP. Both methods are based on the definitions of decision criteria and variants in a logical hierarchy. First, we decompose the decision problem analytically from the upper to the lowest level and then we perform a synthesis by evaluating the decision variants and eliciting the best one. Here, we apply this multi-criteria methodology to the problem of a portfolio manager making a decision when selecting the best possible instrument on the financial market. Using a case study we demonstrate how appropriate application of the above mentioned methods could show a clear way for finding a satisfactory solution of this problem.

Keywords

Multi-criteria decision making; Analytic hierarchy process; Portfolio management.

Introduction

In this paper we propose an application of the Analytic hierarchy process (AHP) to a partial task in portfolio management. Any portfolio manager who works in an asset management company deals with the problem of converting his portfolio into cash. There is a need to decide, which instrument available on the market is the best one to invest in. Of course, there are specific areas, e.g. law, contract, internal requirements (criteria), which make the problem difficult. This problem can be viewed as a multicriteria decision making (MCDM) problem and Analytic Hierarchy Process (AHP) seems to be a suitable method for solving it.

Apart from the classical portfolio selection based on the Markowitz theory, see for example [1], there are studies applying AHP to portfolio mix, see e.g. [10]. This paper represents an appropriate approach to e.g. a pension fund portfolio. This may be also a problem of pair-wise comparison, see e.g. [11]. Another application of AHP close to our approach can be found in [12]. In this paper we show that AHP results could be comparable to those obtained by mean-variance optimization. In fact our specific approach can be viewed as a development of the idea given in [10].

In [4] a new MCDM method is presented which, in some sense, is an extension of AHP allowing for using triangular fuzzy inputs and feedback between criteria. The result of our work should answer the question: whether the software tool FVK created in [4] can be used as an alternative to the well known SW Expert Choice (EC) for solving our portfolio problem.

Let us start with the basic characteristics of a decision making (DM) model. Any DM model should satisfy the following characteristics:

– should be easy to compose,
– should be intuitive (it is not always the case),
– should be flexible in all elements,
– should comply with common sense,
– should include instructions for compromise,
– should be comprehensible.

It is important that even a poor problem design (its mathematical model) brings useful insight into a detail of the problem. The logic of MCDM is based on the goal identification, elements incorporated and influencing the output. In the next stage we shall deal with the time horizon, scenarios and limiting factors, see [7].

Some studies on analytical thinking led to the development of such models in 1970s, see e.g. [5] and the references therein. It was at that time that the method for DM support called the AHP was developed. The author Thomas L. Saaty – an American professor – and his co-workers and successors found many applications for the method. For example, in everyday life (e.g. a new car purchase, a choice of carrier, and so on) or in decision making problems in society or institutions (general elections, marketing strategies, political decisions, project selection etc.) For more information, see [2] or ([5], [6]).

Since its inception, the AHP has become one of the most widely used tools for MCDM. The procedures of the AHP involve the following steps, see ([5], [6]):

- Define the problem, objectives and outcomes.
- Decompose the problem into a hierarchical structure with decision elements (criteria, detailed criteria and alternatives).
- Apply the pair-wise comparison method resulting in pair-wise comparison matrices.
- Apply the principal eigenvalue method to estimate the relative weights of the decision elements.
- Check the consistency of pair-wise comparison matrices to ensure that the judgments of decision makers are consistent.
- Aggregate the relative weights of decision elements to obtain an overall rating for the alternatives.

1. Description of AHP

Here, we consider a three-level hierarchical decision system: On the first level we consider a decision goal G; on the second level, n independent evaluation criteria: C1, C2, ..., Cn are considered such that $\sum_{i=1}^{n} w(C_i) = 1$, where $w(C_i)$ is a positive real number – the weight, usually interpreted as a relative importance of the criterion C_i subject to the goal G. On the third level, m alternatives (variants) of the decision outcomes $V_1, V_2, ..., V_m$ are considered; again $\sum_{r=1}^{m} w(V_r, C_i) = 1$, where $w(V_r, C_i)$ is a non negative number – the weight of the alternative V_r subject to the criterion C_i, $i = 1, 2, ..., n$. It is advantageous to put the above mentioned weights into a matrix form.

Let $\mathbf{W}_1$ be the $n\times1$ matrix (weighing vector of the criteria), i.e. $W_1 = \begin{bmatrix} w(C_1) \\ \vdots \\ w(C_n) \end{bmatrix}$, and $\mathbf{W_3}$ be $m\times n$ matrix:

$$W_3 = \begin{bmatrix} w(C_1, V_1) & \cdots & w(C_n, V_1) \\ \vdots & \cdots & \vdots \\ w(C_1, V_m) & \cdots & w(C_n, V_m) \end{bmatrix} \tag{1}$$

The columns of this matrix are evaluations of alternatives according to the given criteria. Moreover, in matrix $\mathbf{W}_3$ the sums of columns are assumed to be equal to one (this property is called stochasticity, for more details see [5]). The following matrix product

$$\mathbf{Z} = \mathbf{W}_3\mathbf{W}_1 \tag{2}$$

is an $m \times 1$ matrix – the resulting vector of weights of the alternatives – – expressing the relative importance of the alternatives. From formula (2) we get the weights in the following way

$$z_j = \sum_{i=1}^{n} w(C_i)w(C_i, V_j),\ j = 1,2,\ldots,m. \tag{3}$$

The weights $w(C_i)$, and $w(C_i,V_j)$ will be denoted in the following text simply as w_k; they are obtained from the pair-wise comparison matrix. An element of the pair-wise comparison matrix serves as a relative evaluation element from the given hierarchy level to a given element from the dominant level. Each pair of elements is evaluated on a specific scale, see below. A starting point for the calculation of weights is a pair-wise comparison matrix $\boldsymbol{S} = \{s_{ij}\}$. The value s_{ij} expresses the relative importance of elements x_i to element x_j, with respect to the superior element, in other words the ratio of w_i and w_j:

$$s_{ij} = \frac{w_i}{w_j},\ i,j = 1,2,...,m. \tag{4}$$

As the weights w_k are not known in advance, (it is our goal to find them), we use for their determination additional information about the numbers s_{ij}, from the basic scale {1,2,...,9}, i.e.

$$s_{ij} \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}. \tag{5}$$

It follows from (4) that the pair-wise comparison matrix $\boldsymbol{S}$ is reciprocal, which means that

$$s_{ij} = \frac{1}{s_{ji}}. \tag{6}$$

In AHP, the vector $\boldsymbol{w}$ of weights w_k is calculated by a specific method based on the principal eigenvector of the pair-wise comparison matrix $\boldsymbol{S} = \{s_{ij}\}$. The following equation holds:

$$\boldsymbol{S}\,\boldsymbol{w} = \lambda_{\max}\,\boldsymbol{w}, \tag{7}$$

where $\lambda_{\max}$ is the maximal eigenvalue of matrix $\boldsymbol{S}$.

The values (called also intensities) from 1 to 9 in the evaluation scale (5) which are used in pair-wise comparisons can be interpreted qualitatively as follows:

Pair-wise comparison of elements x_i and x_j – number scale	Intensity of relative importance of element x_i to element x_j – word scale
1	x_i and x_j are equally important
3	x_i is more important than x_j
5	x_i is strongly more important than x_j
7	x_i is very strongly more important than x_j
9	x_i is absolutely more important than x_j

The numbers 2, 4, 6, 8 and their reciprocals are used to facilitate a compromise between slightly different judgments. Some authors also use rational numbers to form ratios from the above scale values, see [3] or [4].

2. Application to Portfolio Management – Case Study

The main task of a portfolio manager is asset allocation, that is, the selection of new assets for a new investment. Moreover, the portfolio manager has to make predictions about the price development of each asset class and, consequently, sell some of his positions and make new investments. The trickiest part of his work is to close some losing positions. It may happen when the loss reaches a specified value, which is not bearable for the owner of the portfolio any more. This is called realization of Stop-Losses. By the word "trickiest" we mean the effect given by cutting off any recovery possibility of the price.

Nevertheless, the main motivation for portfolio management is a possibility of its diversification. Financial instruments are divided into several categories, i.g. cash, bonds, equities and others. The prices movements at asset allocation could take different directions, or, they do not have the same drift, which is reflected by correlation. There are other possible diversification styles: we distinguish credit, geographic, currency and other diversification styles depending on different characteristics of the issuer, see e.g. [1].

Since in portfolio management it is necessary to make daily decisions concerning substitution of matured instruments for some new allocations, it may be useful to apply the AHP. Here, we illustrate the application of this MCDM technique on the following practical problem.

In Table 1 we consider four instruments, which are available for sale on the financial market:

Table 1

Financial instruments

ISIN	Name	1. volatility	2. rating	3. duration	4. liquidity
CZ0002000219	Ceskomoravska Hypotecni Bank	0,03	A	0,8491	low
XS0212596240	Deutsche Bank AG	0,05	AA	0,0381	good
XS0215579946	Tesco PLC	0,08	A	1,0991	worse
CZ0001000863	Czech Republic Government Bond	0,01	A	0,4916	the best

Source: Authors.

Table 1 contains preselected instruments (bonds), considered by a portfolio manager for his investment activity. For all financial instruments we consider some characteristics – evaluation criteria. In particular, we consider 4 evaluation criteria: Crit1 – volatility, Crit2 – rating, Crit3 – duration and Crit4 – liquidity.

Volatility is one of the most popular characteristic of a financial instrument. Sometimes it is considered as a risk. We can simply say: the more volatile the price of some instrument, the higher the risk of loss. Some conservative models consider equity of volatility at the level of 30%. Bond prices have lower volatility which is given by the fact that the investment in such an instrument is not risky, of course, from the point of view of volatility. A usual expected volatility level of bonds is between 0% and 10%. Moreover, the bonds are in fact the right to get back the money invested – nominal value plus the coupon, which is usually paid through the life of the bond.

Here, we use a well known historical approach for volatility calculation. First, we calculate the changes of asset returns by the formula:

$$R_{i,t} = P_{i,t} \frac{-P_{i,t\text{-}1}}{P_{i,t\text{-}1}} = \frac{P_{i,t}}{P_{i,t\text{-}1}} - 1.$$

Next, the expected value of returns is calculated by the following formula:

$$E(R_i) = \frac{1}{N}\sum_{t=1}^{N} R_{i,t} .$$

The sample variance of returns is calculated as follows:

$$\sigma_i^2 = \frac{1}{N-1} \cdot \sum_{t=1}^{N} \left[R_{i,t} - E(R_i) \right]^2 ,$$

and the sample standard deviation of returns is calculated as:

$$\sigma_i = \sqrt{\sigma_i^2}$$

This is considered as the volatility (risk). Here, historical prices are used; however, there exist elaborated models for volatility prediction, e.g. Vasicek´s model, EWMA model or GARCH models, see [9].

The second criterion is *rating* of a given issuer or issue. Here, we use the rating format given by Moody's scale in a simplified form without increasing signs (+) and decreasing signs (-). A higher number of A-symbols indicates more positive information about the credit profile of issuer. On the lower levels of the scale, instead of symbols A, symbols B and C can be used, but issuers or issues rated under BBB are considered as speculative investments.

The third criterion is *duration*. The bond price function $f(x)$ is approximated by the *Taylor's expansion*. The first member of this expansion is called the duration, i.e.:

$$f(x + \Delta x) = f(x) + f^{'}(x) \cdot \Delta x .$$

The price after certain time is calculated with help of the first member of Taylor's expansion in the following way:

$$P_1 = P(y + \Delta y) = P(y) + P'(y) \cdot \Delta y$$

where y is a yield to maturity, P is a price at the beginning of time period and P_1 is a price of bond after the change of interest rates.

Modifying the equation by subtraction and division of the starting price P we get:

$$P_1 \frac{-P}{P} = \frac{\Delta P}{P} = \frac{1}{P} \cdot \frac{dP}{dy} \Delta y,$$

The right side of the equation

$$\frac{1}{P} \cdot \frac{dP}{dy} = \frac{1}{P} \cdot \sum - t \cdot CF_t (1 + y)^{-t-1} = -MD$$

is called the modified duration, where CF_t is the expected cash flow an owner of the bond will receive till the maturity of the bond. The negative sign of MD is a reflection of the reverse relationship between the yield curve represented here by y and the price of the bond.

The modified duration is expressed by the Macauloy's duration as follows:

$$D = \frac{\frac{dP}{P}}{\frac{dy}{(1+y)}} = \frac{1}{P} \cdot \sum t \cdot CF_t (1+y)^{-t} \ ,$$

and, consequently, we obtain:

$$MD = \frac{1}{1+y} D \ .$$

The above formulae show that the results reflect the cash flows weighted by time. The bonds, which do not pay coupons, have the duration equal to their time to maturity. Portfolio managers usually use the second expressed duration, which is a MD with the positive sign, because they consider this number as an average time to maturity of their portfolio. The MD is the parameter of a portfolio, which is usually requested by contract and must be watched after.

The fourth criterion is *liquidity*. Here, the empirical approach is used: In Table 1 the relative evaluation is carried out by pair-wise comparison.

2.1. Solving the Problem by AHP and Expert Choice

Now, we shall solve the problem by the special SW tool named Expert Choice (EC), see [13], based on the AHP theory. The original data of our problem are given in Table 1. For evaluating the liquidity criterion which is given in ordinal expressions as well as the other qualitative criterion rating we use pair-wise comparison on the Saaty's scale mentioned earlier in Section 1. We proceed simirarily for evaluating relative importance of all individual criteria. Table 2 shows the pair-wise comparison matrix of the criteria importance given by a portfolio manager.

Table 2

Pair-wise comparison matrix of importance of the individual criteria

Criteria	**Crit 1**	**Crit 2**	**Crit 3**	**Crit 4**	
Crit 1	1	1/3	1/2	1/2	**- volatility**
Crit 2	3	1	3	2	**- rating**
Crit 3	2	1/3	1	2	**- duration**
Crit 4	2	1/2	1/2	1	**- liquidity**

Source: Authors.

Table 3 contains the weights of criteria calculated by the well known eigenvector method mentioned earlier, see Eq. (7). It is clear that rating and duration are the most important criteria.

Table 3

Relative importance of the criteria obtained by pair-wise comparison

Criteria	Weights
Volatility	0,079
Rating	0,526
Duration	0,246
Liquidity	0,149

Source: Authors.

Table 4 shows the pair-wise comparison matrix of liquidity.

The values of the other criteria are calculated explicitly from the original data in Table 1.

Table 4

Pair-wise comparison matrix of Liquidity

Zn=	Var 1	Var 2	Var 3	Var 4	
Var 1	1	1/5	1/3	1/4	**- Ceskomoravska hypotecni banka**
Var 2	5	1	2	1/2	**- Deutsche Bank AG**
Var 3	3	1/2	1	1/2	**- Tesco PLC**
Var 4	4	2	2	1	**- Czech Republic Government bond**

Source: Authors.

Table 5 shows the result of calculation of each variant and criterion in the final, normalized form, i.e. the sum of all numbers in each column is equal to 1.

Table 5

Weights of criteria and weights of variants

Variant	Volatility	Rating	Duration	Liquidity
V1	0,201	0,200	0,039	0,088
V2	0,121	0,400	0,864	0,197
V3	0,075	0,200	0,030	0,231
V4	0,603	0,200	0,067	0,484

Source: Authors.

Table 6 shows the result of the final synthesis calculated as weighting average (3) using both the calculation method called the Distributive mode and the calculation method called the Ideal mode. In the Distributive mode, all values of each criterion (i.e. in each column) are normalized, i.e. divided by the sum of the values of the respective criterion, see Table 5, whereas in the Ideal mode, all values of each criterion (i.e. in each column) are divided by the maximal value of the respective criterion, i.e. the highest value of each criterion is then equal to 1. In both modes the resulting ranking of the variants is identical. For more details, see [5].

Table 6

Final synthesis by AHP

Distributive mode	Weights	Rank	Ideal mode	Weights	Rank
V1	0,144	4	V1	0,161	4
V2	0,462	1	V2	0,416	1
V3	0,153	3	V3	0,173	3
V4	0,242	2	V4	0,250	2

Source: Authors.

Summarizing the results in Table 6, we can see a clear dominance of variant V2 over all other variants. Variant V4, which is ranked as the second best, has significantly lower weight. The weights of V1 and V3 are very similar to each other, and significantly lower than V4. Consequently, the best choice from the given variants is V2, hence available cash should be invested into variant V2.

2.2. Solving the Problem by FVK

In this part we solve the same problem as in section 2.1 by an alternative method. The AHP method was published as early as in 1980s, and now it is considered a “classical” methodology; on the other hand, FVK is a newly created tool expanding application possibilities of the AHP. The acronym FVK stands for Fuzzy Multicriteria Method (in Czech language). Here, we compare and discuss the results obtained by both methods.

When comparing the AHP and FVK we find out some significant differences:

– In FVK the vector of weights w_k is calculated from the pair-wise comparison matrix $\boldsymbol{S} = \{s_{ij}\}$ by the geometric mean as follows:

$$w_k = \frac{\left(\prod_{j=1}^{n} s_{kj}\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} s_{ij}\right)^{1/n}} \tag{8}$$

– FVK reduces some disadvantages of the principal eigenvector method used in AHP (see [5]),
– FVK allows for reflecting criteria interdependency, which is not considered in the classical AHP.
– FVK enables to use fuzzy evaluations, specifically by triangular fuzzy numbers (i.e. triangular membership functions). Hence, FVK is convenient in situations, where the decision maker has vague information for evaluation (here we will not use this feature).

All results presented below have been calculated by a software tool named FVK. This software application has been created as an add-on for MS Excel 2003 within the GACR project No. 402060431, see [3].

Table 7 shows the criteria weights calculated by (8); they are calculated from pair-wise comparison matrix in Table 2. As compared with Table 3 the weights in Table 7 are different; however, the order of the importance of the criteria is the same.

Table 7

Weights of criteria by FVK

Criteria	Weights
Volatility	0,119065
Rating	0,456456
Duration	0,238131
Liquidity	0,186347

Source: Authors.

Table 8 shows the final weights of the variants and the ranking according to FVK. Again, the best variant is V2; however, the variants on the third and the fourth place interchanged their positions.

Table 8

Final synthesis by FVK

Zn=	Weights	Rank
Var 1	0,176003	3
Var 2	0,396322	1
Var 3	0,127531	4
Var 4	0,300144	2

Source: Authors.

In the AHP we assume that the decision criteria are mutually independent. In practice, it is, however, not the case. Generally, the criteria are frequently interdependent, one criterion directly or indirectly influences the other one, e.g. rating strongly influences liquidity etc. On the other hand, FVK enables also to reflect influences between the criteria, which enables a deeper analysis of convenient alternatives. The influences (interdependences) between the criteria are evaluated also by pair-wise comparison,

The values in the pair-wise comparison matrix evaluating the influences between Crit 1 and other criteria (see Table 9) can be interpreted as follows: Crit 2 influences Crit 1 two times (2) more than Crit 3. Crit 2 influences Crit 1 four times (4) more than Crit 4. Crit 3 influences Crit 1 three times (3) more than Crit 4, etc.

Table 9

Pair-wise comparison matrix (influences between volatility and other criteria)

Crit 1	Crit 2	Crit 3	Crit 4	
Crit 2	1	2	4	- rating
Crit 3	1/2	1	3	- duration
Crit 4	1/4	1/3	1	- liquidity

Source: Authors.

In Table 10 influences of Crit 2 – Rating by other criteria is presented:

Table 10

Pair-wise comparison matrix (influences between rating and other criteria)

Crit 2	Crit 1	Crit 3	Crit 4	
Crit 1	1	2	3	- volatility
Crit 3	1/2	1	1	- duration
Crit 4	1/3	1	1	- liquidity

Source: Authors.

In Table 11 influences of Crit 3 – Duration by other criteria is presented:

Table 11

Pair-wise comparison matrix (influences between duration and other criteria)

Crit 3	Crit 1	Crit 2	Crit 4	
Crit 1	1	1	1	- volatility
Crit 2	1	1	1	- rating
Crit 4	1	1	1	- liquidity

Source: Authors.

In Table 12 influences of Crit 4 – Liquidity by other criteria is presented:

Table 12

Pair-wise comparison matrix (influences between liquidity and other criteria)

Crit 4	Crit 1	Crit 2	Crit 3	
Crit 1	1	1/2	2	- volatility
Crit 2	2	1	5	- rating
Crit 3	1/2	1/5	1	- duration

Source: Authors.

In the last table Table 13 the final weights and the corresponding ranking of the variants is presented. In comparison to the previous case, the weights of the criteria are calculated by FVK, particularly by the method of geometric mean taking into account interdependences (infuences) between the criteria, see [3,4].

Table 13

Final evaluation of variants according FVK

Zn=	Weights	Rank
Var 1	0,192401	3
Var 2	0,371611	1
Var 3	0,111567	4
Var 4	0,324421	2

Source: Authors.

When comparing the results obtained by FVK with those obtained earlier by AHP we conclude: The best variant is again Variant 2 and the second-best is again Variant 4. However, Variant 1, ranked in the case of AHP as the fourth, is now located on the third place. In this particular example, from the viewpoint of the investor, who is focused on the top variants, both AHP and FVK supply equivalent results. In general, we should, however, be careful as the results obtained by these methods could be different, particularly in case of strong interdependences between criteria.

Conclusion

In this paper we tried to show that an application of MCDM methods in portfolio management may be useful. Here, we applied the classical Saaty's AHP and, at the same time, the newly developed modification of AHP named FVK extending the application power of AHP as well as reducing some of its theoretical shortages.

In the AHP we assume that the decision criteria are mutually independent; however, it is usually not the case. Generally, the criteria are interdependent: one criterion either directly or indirectly influences the other one. The new method, FVK, enables also to reflect influences between the criteria, which enables a deeper analysis of all convenient alternatives. The influences (interdependences) between the criteria are evaluated also by pair-wise comparison.

A comparison of the results obtained by FVK with those obtained earlier by AHP, in this particular application, shows that from the viewpoint of the investor, both methods give more or less equivalent results. In general, we should, however, be careful as the results obtained by these methods could differ, particularly in case of strong interdependences between criteria.

By the help of MCDM methods, the portfolio manager is able to acquire quick information (feedback) about advantages of the asset allocation into some specific product. Consequently, every specific requirement of a contract can be reflected by the methods applied. For example, liquidity evaluation could be derived from the liquidity spread. On the one hand, this approach is much more dependent on input data; on the other hand, the suggested modification could increase the objectivity of the model. Further extensions could be made by the implementation of ex-ante volatility, see [8]. Moreover, the rating inputs taken from the external rating agencies could be derived also from the rating models developed within the project BASEL II.

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APPLYING A FIRST PRICE AUCTION MECHANISM FOR SUPPORTING MULTI-BILATERAL NEGOTIATIONS

Abstract

In this paper we consider a multi-bilateral negotiation problem from the perspective of all involved parties that we call the seller and the buyers. We model the negotiation process as a sequentially repeated first price auction. Since we consider a multi-issue negotiation, we do not operate with a bidding price as a single evaluation criterion but with a utility of the package (negotiation offer). To construct the optimal negotiation strategy we apply the notion of equilibrium bidding strategy. The parties' negotiation strategies are represented as vectors of bids for successive negotiation phases. The negotiation strategies are then used by a simple spreadsheet-based negotiation support tool for finding the most satisfying solution of the negotiation process. The software acts as a simple agent that converts the strategies into the values of the bids and then into the negotiation offers that maximize the payoffs of the buyer. The compromise is represented by the first bid that satisfies the current seller's aspiration level.

Keywords

Negotiation, utility, additive scoring system, first price auction, equilibrium strategy, negotiation support system

Introduction

Many economic situations can be described as a multi-bilateral negotiations. Selling a house, applying for a job, obtaining a building contract – all of these require interaction with many potential buyers or sellers who can submit various offers and change them during the negotiation process.

This makes the negotiation situation very uncertain. The party that negotiates with a multitude of counterparts does not know their negotiation strategies and can not foresee which of them will propose the most satisfying offer and when. That is because the shapes of the counterparts' concession curves can be very different. The ones that decrease fast at the beginning of the negotiations may never cross a negotiator's reservation level, and vice versa, the slowly decreasing ones may later quickly reach a negotiator's aspiration level. On the other hand, the party that competes with another for buying some goods or winning the contract from the sole seller cannot foresee the decisions of other competitors. By submitting an offer with tough conditions he risks that the offers of other competitors will meet the seller's aspiration level and they will win the contract. By submitting an offer with attractive conditions he increases the chance of winning the contract, but on very poor conditions for himself.

Negotiators can handle this uncertainty by applying multiple criteria decision making methods and tools that will support them in the evaluation and selection of their offers. Usually in negotiation processes this kind of support is given by a software negotiation support system (NSS). There are many negotiation support systems nowadays used in training, teaching and simulation of two-parties negotiations such as INSPIRE [4], Negoisst [9] or NegoCalc [11]But there are also NSSs solving real world problems such as RAINS [2] – used in negotiating air pollution limits within the Europeans countries, FamillyWinner [1] – used for solving divorce negotiation in Australia or SmartSettle [10] – used for structuring and analyzing negotiations between Canadian First Nations and the government of the Alberta Province. Moreover, the supply chain support systems proposed by IBM, SAP, Oracle or Ariba contain also simple components supporting negotiation between the cooperating companies. The multi-bilateral negotiation, however, are not so frequently considered in the research that leads to the construction of the method and software systems dedicated to support all the involved parties. They are usually structured and modeled as auctions with the single attribute of price. While the price usually is not a single criterion negotiators use to evaluate an offer, it is still important to develop a multi-bilateral negotiation methodology.

In this paper we focus on supporting the buyer parties in a multi-bilateral negotiation with a sole seller. We will build a mechanism that will help the buyers in selection of offers for subsequent negotiation phases based on their negotiation strategy formulated before in the pre-negotiation phase and assuming that the negotiation process is conducted by the negotiation support platform. This assumption is far form unrealistic since lots of transactions are conducted nowadays by means of electronic tools, such as auction services, e-shops etc., but it is methodologically required, since we need information about the preferences of all involved parties. The mechanism we apply is derived from the first price auction mechanism proposed for sequential first price symmetric auction based on a private value model [7]. Using the negotiators' aspiration levels declared in their evaluation spaces it computes the equilibrium bids [8] evaluated in the sellers' negotiation space and then transforms them into the best (the most preferable) offers evaluated again in the buyers' negotiation spaces.

The structure of this paper is the following. In the first section we give a brief statement of the multi-bilateral negotiation problem. Then in section 2 we introduce the basics: the first price symmetric auction based on a private value model with the idea of determining the equilibrium bids that will be used in the supporting algorithm we describe in section 3. In section 4 a short example is given to show the method of application of the mechanism for solving a hypothetical negotiation problem. To solve this problem we use a simple spreadsheet-based software support tool we had programmed to show the ease of software implementation of the mechanism proposed.

1. Multi-bilateral negotiation problem statement

To define the negotiation problem we will consider one seller offering a single good or service (a contract to win) and many buyers bidding for the object being sold. Furthermore, we assume this contract to be described multi-attributively, i.e. there is a list of negotiation issues whose resolution levels need to be agreed during the negotiation process between the seller and a single buyer. To evaluate the offers negotiators will use an additive scoring system with cardinal utility payoffs [3]. The application of this system requires a predefined list of all resolution levels (options) that could be used in the construction of the offers (packages of options). As the number of the options is usually big, only the most important of them are identified in the pre-negotiation phase (salient options). The algorithm of building an additive scoring system of the offers consists of two steps:

1. Distribution of scoring points between k negotiation issues to establish the importance of them (weights).
2. Assigning the scores to the options within each issue so that the least preferred option receives the score of 0, while the most preferred, the score of the issue weight w_i. All other options receive the scores from the interval $[0; w_i]$.

The score of the offer is the sum of the scores of the options that constitute the offer. A brief example of building the offers' scoring system for the negotiation between an employee and an employer is shown in Table 1.

Table 1

Scoring the issues and offers for a three-issue negotiation

Issue	Issue rating	Option	Option rating
Salary	50	3000 USD	0
		4000 USD	10
		5000 USD	40
		6000 USD	50
Holidays	30	20 days	0
		25 days	10
		30 days	30
Insurance	20	By employer	0
		By employee	20

From now on, we will denote the buyer's i scoring system as the function $s_i : R^k \to R$, transforming the vector of k options defined for the offer under evaluation into a scalar score.

We assume further that negotiators exchange the offers sequentially. The buyer submits his proposal, which can be accepted or rejected by the seller. If the proposal is rejected, the seller can propose his own counteroffer, which can be accepted or rejected by the buyer. Since the problem is multi-bilateral for the seller, he does not have to submit his own offers but can request another proposal from the buyer. The sub-process of submitting the offer and a possible counteroffer will be called a negotiation round.

In the multi-bilateral negotiation problem considered the buyers compete with each other by submitting at the subsequent negotiation rounds their proposals for agreement (offers). We assume that the first offer that exceeds the seller's reservation level defined for a particular negotiation round wins the contract*. The buyers compete with each other to win the contract by submitting the offer that gives relatively high score to the seller, which means they need to make concessions, i.e. to lower the score they achieve, but simultaneously they want to maximize their outcome, which means they are not willing to make huge concessions. In such a situation the problem of selection of the most competing offers (i.e. the offers that maximize the seller's score for the assumed buyer's payoff) arises. We will assume therefore that the negotiators do not define the exact offers (packages) for each negotiation round but prepare their negotiation strategies as the vectors of their aspiration levels, i.e. the scores they wish to achieve in subsequent negotiation rounds. In other words, they define their concession paths that say how much (in terms of utility scores) they can give in at each negotiation round. The idea of defining the strategies, transforming them into the offers and wining the contract is shown in Figure 1.

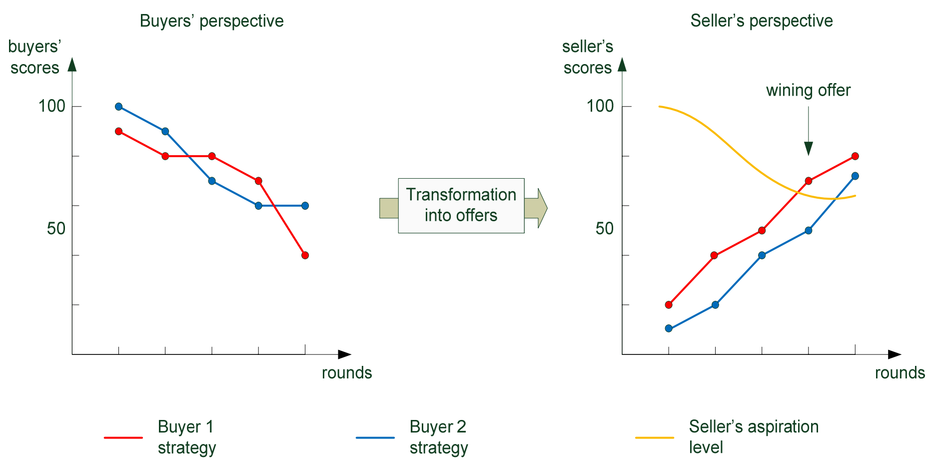

Figure 1. Definition of the strategies and their consequences in winning the contract

With the multi-bilateral negotiation problem defined as described above, each buyer needs a support for transforming his aspiration levels into offers that will best satisfy the seller, taking into consideration the fact that he is acting

* It is also possible that the seller's counteroffer will be accepted by the buyer as a negotiation agreement, but since the construction of this offer has not required any effort by the buyer's party it is trivial for us and we will not consider it in our analysis.

in a competing environment with many buyers. We will propose such a supporting mechanism in Section 3, derived from the first price auction theory and the equilibrium bid theory which we review briefly in the following section.

2. First price auctions and equilibrium bids

Following Milgrom and Weber [7] we consider the first price auction with n potential bidders (buyers). Let $X = (X_1, ..., X_n)$ be a vector of random information variables describing the private information of each buyer according to the value of the bidding object. Its value can be also determined by external factors described as a vector $S = (S_1, ..., S_m)$ of m different factors influencing the auction process. Each buyer i has his own evaluation of the bidding object, which is a random variable $V_i = v_i(S, X)$, where $v : R^{m+n} \to R$. The payoff of the buyer i can be described then as

$$g_i = \begin{cases} v_i - b_i & \text{if the buyer } i \text{ wins the auction} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where:

v_i is the ith buyer's evaluation of the bidding object,

b_i is the ith buyer's actual bid for the bidding object.

We will assume that each buyer knows the distribution of the bidding object evaluations $f(x)$ of all his competitors, and that $m = 0$, i.e. there are no external factors influencing the auction analyzed. Furthermore, we assume that the bidders are risk neutral and their decisions are reflected in an increasing decision function, which is also a random variable $B : R^n \to R$. The buyer i wins the auction if:

$$B(v_j) < b_i, \text{ for } j = 1, ..., i-1, i+1, ..., n \tag{2}$$

Following Riley and Samuelson [8] we can build now a formula for determining the optimal bid of buyer i, which is in fact an equivalent of the equilibrium strategy of this bidder. This formula can be denoted as:

$$b(v_i) = v_i - \frac{\int_{p_0}^{v_i} [F(x)]^{n-1} dx}{[F(v_i)]^{n-1}}, \quad \text{for} \quad i = 1, ..., n, \tag{3}$$

where:

$F(.)$ is the distribution function of the buyers' bidding object evaluations,
p_0 is the auction starting price.

The buyer's optimal offer can be computed with the formula (3) if $v_i \geq p_0$, otherwise the bid will be lower than the auction starting price p_0 and formally will not be taken into consideration as the auction proposal.

3. Buyers supporting mechanism

We will apply the first price auction approach described in section 2 for supporting the buyers in the process of construction of the negotiation offers. We propose a supporting mechanism for a multi-bilateral negotiation problem assuming that the whole negotiation process is conducted by means of software (electronic) support system, such as an e-auction service, which gathers the information about the users, including their preferences evaluation data and their scoring systems. We will regard each negotiation round as a separate first price auction. As we assumed in Section 1, the buyers define their strategies as the payoffs they want to achieve in the subsequent negotiation rounds. We will use these payoffs as the buyer's evaluation of the bidding object in each negotiation round. Therefore the negotiation strategy of the bidder i can be defined now as

$$N_i = \left(v_i^1,\ldots,v_i^R\right) \tag{4}$$

where:
v_i^r is the payoff of the bidder i he wishes to achieve for the negotiation agreement settled in the round r.

Usually, to each desired payoff v_i^r there correspond several alternative offers that we can construct using the offer scoring system of the buyer i. The problem now arises: which of these offers should be chosen by the buyer as the bidding offer to best satisfy the seller. Since we have assumed that the negotiation support is given by means of electronic negotiation support system, acting as a facilitator, we know both the buyer's and the seller's preferences. Therefore within each negotiation round the system will look for an offer $\hat{a}_i^r \in A$, A being the set of all feasible offers, that gives the buyer i

the aspired payoff v_i^r and simultaneously maximizes the payoff of the seller. Knowing the seller's scoring system, the negotiation support system will consider the negotiation strategy of the buyer i as the vector

$$N_i^{seller} = \left(s_{seller}(\hat{a}_i^1),..., s_{seller}(\hat{a}_i^R)\right) \tag{5}$$

The offers $\hat{a}_i^r$ can be presented directly to the seller as the negotiation proposals, but the negotiation theory says [5] that to finish negotiation with a satisfying agreement, the concessions in the negotiation rounds should be made gradually. Therefore, we will not offer as much as $s_{seller}(\hat{a}_i^r)$ to the seller, but a little less, taking into account the competing environment of many buyers. For each $s_{seller}(\hat{a}_i^r)$ the optimal bid will be determined using the equilibrium strategy approach shown in the equation (3)[*] and we obtain the vector of optimal bids

$$O_i^{seller} = \left(b_{seller}^1,..., b_{seller}^R\right) \tag{6}$$

where:

b_{seller}^r is the payoff the seller should receive in the negotiation round r.

The problem we are facing now is quite opposite to the one we had while finding the offers $\hat{a}_i^r$. To each payoff b_{seller}^r there usually correspond several negotiation offers. Therefore, the system needs to find an offer $\breve{a}_i^r \in A$ that gives the seller the assumed payoff b_{seller}^r and simultaneously maximizes the payoff of the buyer i. If $s_i(\breve{a}_i^r) > s_i(\hat{a}_i^r)$ then the system will recommend to the buyer i the alternative $\breve{a}_i^r$ as the one corresponding to his initial v_i^r, otherwise the recommendation will be the offer $\hat{a}_i^r$ since it simply dominates $\breve{a}_i^r$.

The final product of the mechanism we propose is hence the list of offers (optimal bids) determined for the negotiation strategy defined by the buyer supported. The key steps of the entire procedure of supporting the negotiator i are presented as an algorithm in Figure 2.

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[*] Since the negotiation has been conducted by means of a software system, the number of the bidders required to make the calculations is determined automatically by the system, which will count the number of registered auction participants.

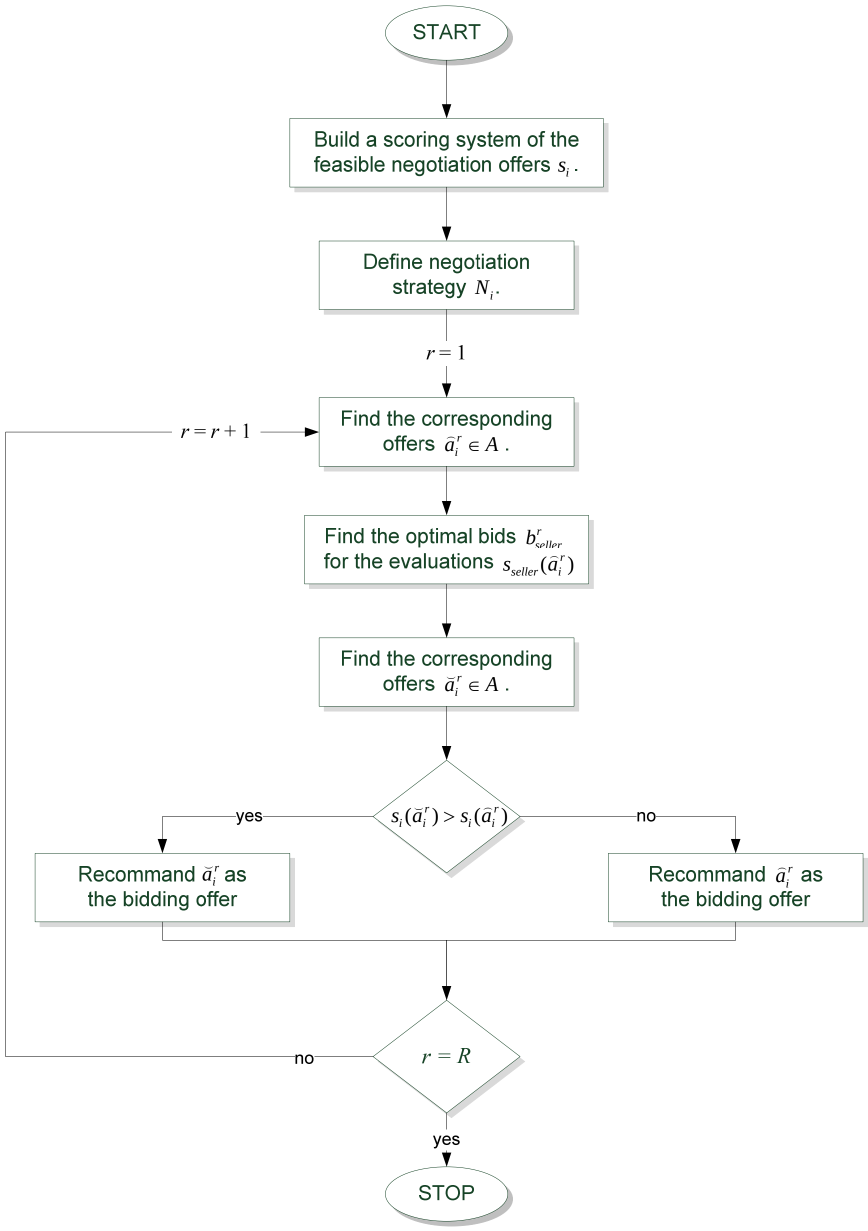


Figure 2. An algorithm for supporting *i*th buyer

Example

In this section we will present an application of the above mechanism to solve a hypothetical multi-bilateral negotiation problem. We will assume a situation with one seller and four buyers and will show the supporting process for one selected buyer party. We assume then that there is one contract to win in the negotiations supported –a business contract for supplying parts for production. Within the contract the parties need to agree on the resolution levels for four issues: unit price of the parts, time of delivery, time of payment and return conditions. We assume further that the parties agreed on some salient options for all the issues (the considered problem is discrete), which is required to apply an additive scoring system for offer evaluation.

Step 1

The first step of the supporting algorithm (see Figure 2) is conducted by means of the spreadsheet based negotiation support system called NegoCalc [11]. As described in section 1 the supported negotiator (a buyer) needs to assign weights to the negotiation issue first and then distribute the scores among all the salient options within each issue. These two steps are realized by means of Preference Elicitation Engine of the NegoCalc system (see Figure 3).

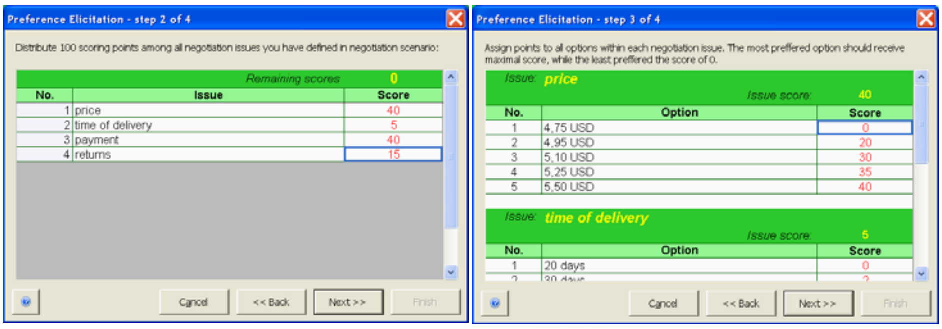
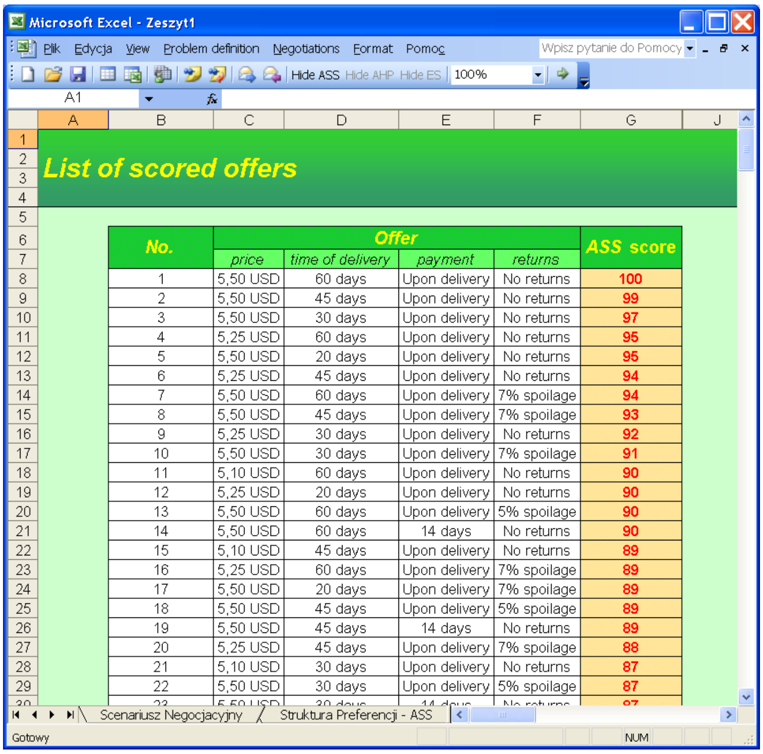


Figure 3. Building the offer scoring system (scoring issues and options) in NegoCalc

After preference elicitation the offer scoring system is ready, which allows to build the list of offers with the corresponding scores (see Figure 4).



List of scored offers

No.	Offer				ASS score
	price	time of delivery	payment	returns	
1	5,50 USD	60 days	Upon delivery	No returns	100
2	5,50 USD	45 days	Upon delivery	No returns	99
3	5,50 USD	30 days	Upon delivery	No returns	97
4	5,25 USD	60 days	Upon delivery	No returns	95
5	5,50 USD	20 days	Upon delivery	No returns	95
6	5,25 USD	45 days	Upon delivery	No returns	94
7	5,50 USD	60 days	Upon delivery	7% spoilage	94
8	5,50 USD	45 days	Upon delivery	7% spoilage	93
9	5,25 USD	30 days	Upon delivery	No returns	92
10	5,50 USD	30 days	Upon delivery	7% spoilage	91
11	5,10 USD	60 days	Upon delivery	No returns	90
12	5,25 USD	20 days	Upon delivery	No returns	90
13	5,50 USD	60 days	Upon delivery	5% spoilage	90
14	5,50 USD	60 days	14 days	No returns	90
15	5,10 USD	45 days	Upon delivery	No returns	89
16	5,25 USD	60 days	Upon delivery	7% spoilage	89
17	5,50 USD	20 days	Upon delivery	7% spoilage	89
18	5,50 USD	45 days	Upon delivery	5% spoilage	89
19	5,50 USD	45 days	14 days	No returns	89
20	5,25 USD	45 days	Upon delivery	7% spoilage	88
21	5,10 USD	30 days	Upon delivery	No returns	87
22	5,50 USD	30 days	Upon delivery	5% spoilage	87

Figure 4. List of scored offers in NegoCalc

Step 2

Having his preferences elicited the buyer needs to define his negotiation strategy. Let us assume that it is defined as follows: $N_i = (90;85;80;70;60;40)$. It means that the buyers is going to make a maximal concession of 10 points in the first negotiation round (within his first offer), 15 points in the second, etc. He would be then willing to accept as the negotiation agreement, for instance, the offer $a_1 = [5{,}50\ \text{USD}; 60\ \text{days}; 14\ \text{days}; \text{no returns}]$ in the first negotiation round. He introduces his negotiation strategy into the multi-bilateral negotiation support system (MB-NSS) (an add-in to the NegoCalc system) which is shown in Figure 5.

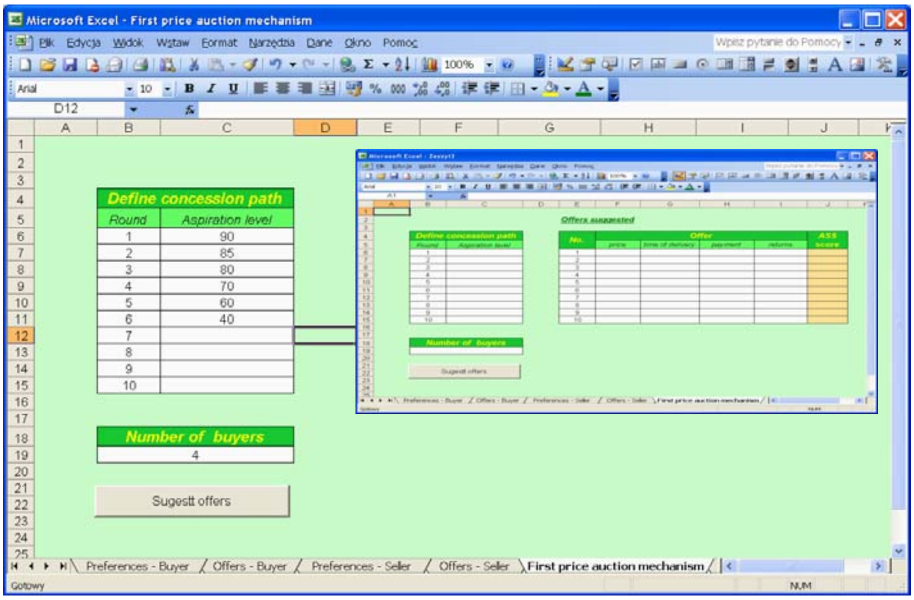


Figure 5. Defining the negotiation strategy in the MB-NSS

Step 3

The MB-NSS finds the offers corresponding to the first aspiration level of the buyer ($v_1^1 = 90$) and from the set of alternatives obtained it selects the one that maximizes the seller's payoff, $\hat{a}_1^1 = [4{,}95\text{ USD}; 30\text{ days}; 14\text{ days}; 3\%\text{ spoilage}]$, $s_{seller}(\hat{a}_1^1) = 52$. In Figure 6 there is a list of all alternatives satisfying the buyer at the level of 90 and their evaluation in the seller's scoring space (the scores of the seller are given in column I).

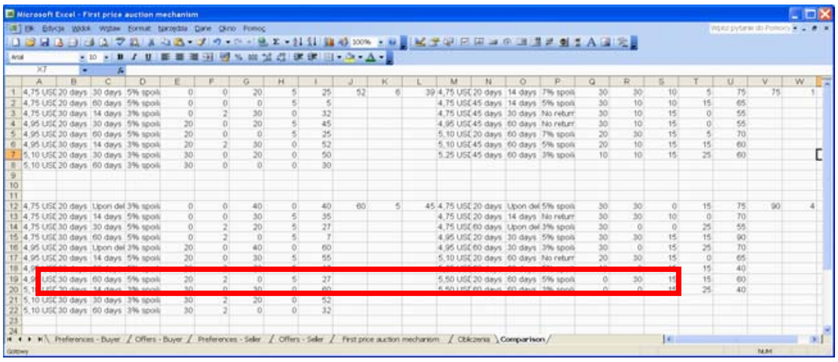


Figure 6. The analysis of the corresponding offers

Step 4

We use the evaluation of the maximizing offer $s_{seller}(\hat{a}_1^1) = 52$ to determine the optimal bid in terms of the seller's payoffs (equation 3), which, assuming there are four auction participants, equals $b_{seller}^1 = 39$.

Step 5

The system finds the offers corresponding to the seller's score $b_{seller}^1 = 39$. All the offers giving the score no better then b_{seller}^1 should be identified (see the list of 7 offers in Figure 7).

Step 6

The system finds the best corresponding offer which maximizes the buyer's payoff $\breve{a}_1^1 = [4{,}75\,\text{USD}; 20\,\text{days}; 14\,\text{days}; 7\%\,\text{spoilage}]$ (see Figure 7).

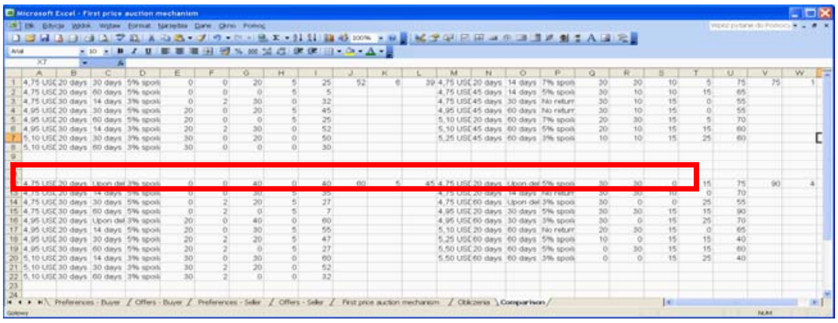


Figure 7. Corresponding offers and $\breve{a}_1^1$

Step 7

We have obtained $s_1(\breve{a}_1^1) = 75 < s_1(\hat{a}_1^1) = 90$; therefore the system will recommend $\hat{a}_1^1$ as the bidding offer.

As we can see, the supported mechanism proposed did not make any improvement in the first round of negotiation. The offer $\hat{a}_1^1$ corresponding directly to the buyer's aspiration level $v_1^1 = 90$ was recommended as the final bidding offer. Such a situation can appear during the negotiation process since the buyer's and the seller's scoring systems are not directly opposite, i.e. an increase of the scores assigned for the successive options in the buyer's scoring system is not equal to the decrease of the scoring points for these options in the seller's scoring system (and *vice versa*).

If we consider the second round of the negotiation process above we will find, however, that the mechanism proposed gives a significant improvement in the formulation of the bidding offer. We have $v_1^2 = 85$ and from the set of the corresponding alternatives (see the red rectangle in Figure 8) we select $\hat{a}_1^2 = [4{,}95\,\text{USD}; 20\,\text{days}; \text{upon delivery}; 3\%\,\text{spoilage}]$, $s_{seller}(\hat{a}_1^2) = 60$. For $s_{seller}(\hat{a}_1^2)$ we obtain the optimal bid $b_{seller}^2 = 45$ and from the corresponding offers (see the green rectangle in Figure 8) we select $\breve{a}_1^2 = [4{,}95\,\text{USD}; 20\,\text{days}; 30\,\text{days}; 5\%\,\text{spoilage}]$ with the buyer's payoff $s_1(\breve{a}_1^2) = 90$. As we can see, thanks to the supporting mechanism the buyer will have recommended the offer that gives the seller the same payoff as for the declared aspiration level v_1^2, but simultaneously he will give himself a value greater than the aspiration level v_1^2 not allowing for leaving any gains on the negotiation table.

The mechanism repeats the steps of the algorithms, which finally leads to the identification of the full negotiation proposals corresponding to the negotiation strategy declared (Figure 9).

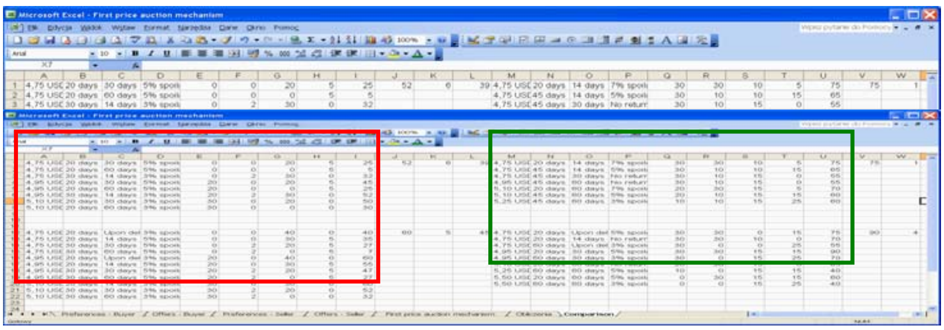
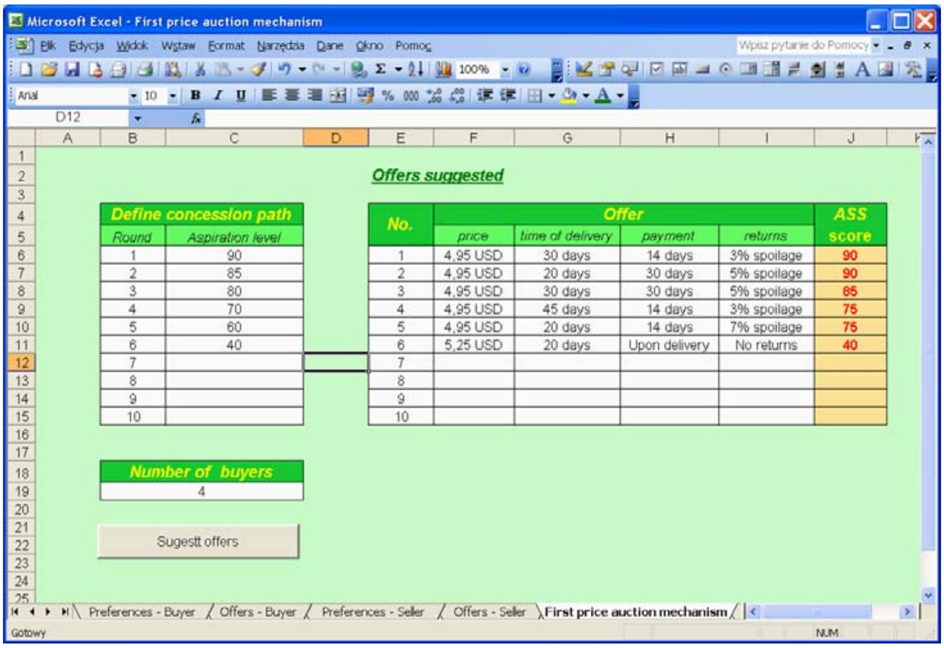


Figure 8. Finding the optimal bidding offer



Round	Aspiration level
1	90
2	85
3	80
4	70
5	60
6	40
7	
8	
9	
10	

No.	Offer: price	time of delivery	payment	returns	ASS score
1	4,95 USD	30 days	14 days	3% spoilage	90
2	4,95 USD	20 days	30 days	5% spoilage	90
3	4,95 USD	30 days	30 days	5% spoilage	85
4	4,95 USD	45 days	14 days	3% spoilage	75
5	4,95 USD	20 days	14 days	7% spoilage	75
6	5,25 USD	20 days	Upon delivery	No returns	40
7					
8					
9					
10					

Figure 9. Recommendation of negotiation offers for full negotiation strategy

As we can see in the negotiation rounds number 2-5 the recommendations of the supporting mechanism allow for improvements for the buyer's party. Instead of having the payoffs: 85, 80, 70, 60 the buyer can assure for himself the outcomes: 90, 85, 75, 75 while offering to the seller the same level of payoffs as in the former case.

Summary

In this paper we have proposed a comprehensive proactive mechanism for supporting the selection of offers congruent with the buyers' negotiation strategy, defined as the maximal concession paths for successive negotiation rounds, and best satisfying the seller. Our approach can be implemented if the whole multi-bilateral negotiation process is conducted by means of an electronic negotiation system or is managed by an external facilitator. That is because we assumed we know the exact form of the distribution function of the values of the buyers' offers and the preferences of all parties (i.e. offer scoring systems). This information is confidential, and it is not transferred from the buyers to the seller or among the buyers but is only used to maximize the parties' payoffs. While building the supporting mechanism we have combined two different supporting elements. The first was a simple searching algorithm, which had to transform the total score of the offer (of the buyer or the seller respectively) into the set of corresponding offers and then select within the set the offers that maximize the payoff of the counterpart. We use this procedure to assure that the negotiators are not going to consider the non--efficient solutions and leave the gains on the negotiation table. The second element was the first price auction mechanism for determining the equilibrium (optimal) bid. In the competing environment of many buyers (bidders) the participants need to find a balance between their own profits from the winning the auction (here, the contract) and the risk of losing it. Therefore they need to declare how much they are willing to give in every negotiation round, but, to leave some extra point for themselves, if possible. The repeated first price auction mechanism fits precisely the situation we described here in multi-bilateral negotiation situation.

The mechanism we have proposed may be applied for the electronic commodity exchange or electronic auction services, especially when the bidding objects are described multi-attributively. Nowadays many business transactions are conducted via Web-based services, but they require the users to track the bidding or transaction process step by step, make decisions, prepare argumentations, etc. The software implementation of the mechanism we propose could make the transaction process a little bit easier and less involved for a decision maker, since it requires only preference elicitation and strategy formulation in the pre-negotiation phase while the actual negotiation phase can be conducted automatically.

Acknowledgements

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MULTICRITERIA PERFORMANCE COMPARISON OF CENTRAL EUROPEAN INDUSTRIAL FIRMS

Abstract

The paper presents a modeling approach for productivity comparison of Central European firms. The approach is based on Data Envelopment Analysis and Analytic Network Process. The proposed model consists of two basic parts. The first one estimates the importance of branches within the countries and the second one evaluates the performance of the firms within branches. The results of both parts are synthesized and the productivity of the countries is estimated. The evaluation is based on the data set resulting from a survey among firms from selected industries.

Keywords

AHP, ANP, data envelopment analysis, multiple criteria decision making, efficiency.

1. Formulation of the problem

The main aim of the paper is to propose a methodological framework for evaluation of performance and identification of productivity gaps between selected Central European countries accessing the European Union and developed industrial western European economies. The paper describes and discusses issues and results of the international project focusing on this subject of study.

The proposed approach starts with efficiency evaluation of selected firms of different industrial branches that are very important for all the countries included into the study. Then the results from the first step are aggregated and the efficiencies of the branches are derived. The last step consists in the aggregation of the results from the previous step according to the economic strength of the branches within the countries; finally, the relative productivity measures for all countries are derived. Due to the hierarchical nature of the process mentioned the problem can be expressed as an AHP or ANP model.

Our aim is to compare the efficiency and performance of Central European firms, branches and countries by different models and to try to identify the sources of inefficiencies of the units evaluated. To receive appropriate data sets for the evaluation, a questionnaire was prepared and distributed to hundreds of firms in the countries participating in the study. Almost one thousand letters with the request to fill out the questionnaire were distributed in each of the participating countries (Czech Republic, Poland, Hungary, East and West Germany) to the firms of selected branches. The most important branches in all of the attending countries (building, meat processing, furniture, freight transport, etc.) have been taken into account. The questionnaire used in our survey has the following structure:

1. General information about the firm
   - turnover,
   - pre-tax profit or loss,
   - fixed and variable costs,
   - estimated market share,
   - information concerning the basic features of the production process, such as number of products or services, rate of automatization of the production process, share of the intermediate consumption, etc.
2. Information related to the personnel and capital of the firm
   - structure of personnel (management, administration, workers),
   - labor costs,
   - qualification of personnel and the cost spent on the improving of qualification,
   - size of floor space,
   - investments into fixed assets.
3. Information related to the management, organization and structure of the firm
   - the number of hierarchies in the organizational structure of the form,
   - the main roles and tasks of the management.

4. Information related to innovations of products and/or production processes
   - the number of hierarchies in the organizational structure of the form,
   - the level of substantial innovations or introductions of new products/services,
   - costs spent on product/service innovations.
5. Information related to networking activities of the firm
   - the rate of co-operation with the customers and suppliers,
   - the level and importance of the use of current communication technologies (e-mail, www, e-business).

The paper is organized as follows. The next section contains a brief description of the basic models that can be used for performance evaluation by taking into account several characteristics influencing the efficiency. Section 3 describes an AHP model that derives the efficiency scores for firms, branches and countries of the study and presents some results on the reduced data set. Section 4 brings a discussion on the possibilities of modeling the problem by the analytic network process. The last section contains a summary of results and a discussion of future research.

2. Performance evaluation models

Within the process of analysis of performance and productivity of countries it is necessary to take into account the performance of production units operating in these countries. As production units, important firms in different economic branches can be taken. Their productivity depends on many factors that can be divided into two basic groups – inputs and outputs. Inputs can be characterized as sources used by the firm during the process of producing outputs. Then, the measure of productivity of firms can be derived by a comparison of outputs and inputs. Usually it is true that higher outputs and/or lower inputs lead to higher productivity measure. The knowledge of productivity measures of firms can be used for estimation of productivity measures of economic branches (according to the size of the firms including in the survey and other factors). Similarly, the importance of the branches within the selected country together with performance measures of branches can lead to estimation of productivity measure of the country.

One of the important problems within the above mentioned process is the evaluation of productivity (efficiency, performance) of the firms with respect to information about their inputs and outputs substantially influencing the productivity. In this section we will not discuss the selection of main factors (inputs and outputs) for productivity comparison but we will mention some of the basic models and techniques that can be used in the evaluation. It is clear that the evaluation is based on the comparison of multiple inputs and outputs. That is why one of the methodological tools available for this purpose is multiple criteria decision making.

Many multiple criteria decision making methods are available; they are usually based on computation of utility measures of evaluated units by means of weighting of the criteria. The most often used methods are WSA, ELECTRE and PROMETHEE class methods and the AHP. The last method is not only a technique for evaluation of units but it can be also profitably used for hierarchical modeling of large and complex decision situations. That is why it can be a nice tool for our purposes. Our aim is not to describe the methods listed in detail. Below we give just the brief characteristics of the AHP, WSA and PROMETHEE II (one of the methods from the PROMETHEE class methods).

The AHP is based on the possibility to express decision problems as hierarchical structures. The hierarchy representing a decision problem always consists of several levels. The first (topmost) level defines a main goal of the decision problem and the last (lowest) level describes usually the decision units. The levels between the first and the last levels can contain secondary goals, criteria and subcriteria of the decision problem. The number of the levels is not limited, but in the typical case it does not exceed four or five. Figure 1 shows a very simple three-level hierarchy, which can represent the standard decision problem – the evaluation of n units $X_1, X_2, \ldots, X_n$, by k criteria $Y_1, Y_2, \ldots, Y_k$.

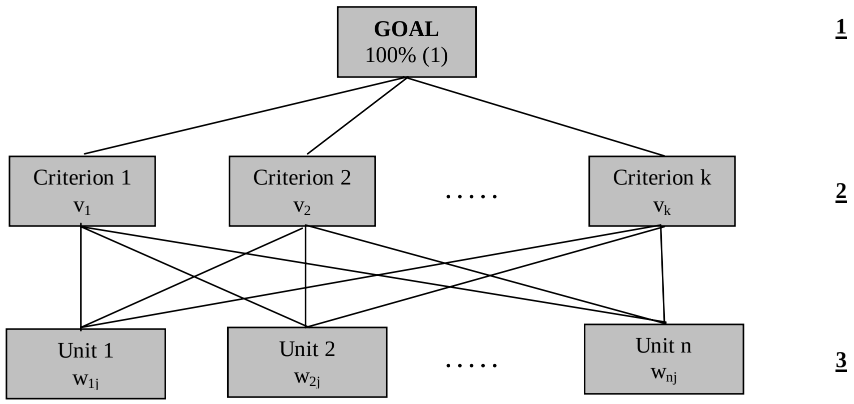

Figure 1. Three-level hierarchy

The decision maker expresses their preferences or compares the importance of the elements on the given level with that of the element on the preceding level. The information resulting from the decision maker's judgements on the given level of the hierarchy is synthesised onto the local priorities. They can express, e.g. the relative importance of the criteria (weight coefficients – in Fig. 1 denoted by v_j, i=1,2,...,k) or preference indices of the units with respect to the given criterion (w_{ij}, i=1,2...,n, j=1,2,...,k). In the standard AHP model the decision maker's judgements are organised into pairwise comparison matrices at each level of the hierarchy. The judgements are point estimates of the preference between two elements of the level. Let us denote the pairwise comparison matrix $A = \{a_{ij} | a_{ji} = 1/a_{ij}, a_{ij} > 0, i,j=1,2,...,k\}$, where k is the number of elements of the particular level. Saaty (1990) proposes to use for preference expression a_{ij} integers in the range 1 through 9, where 1 means that the i-th and the j-th elements are equally important and 9 means that the i-th element is absolutely more important than the j-th element. The local priorities are derived by solving the following eigenvector problem

$$A.v = \lambda_{\max} v,$$
$$\sum_{i=1}^{k} v_i = 1,$$

where λ_{max} is the largest eigenvalue of A and v is the normalised right eigenvector belonging to λ_{max}.

The WSA (weighted sum approach) method is based on the principle of utility maximization. The normalized criterion values are aggregated by means of weights and the utility of each evaluated unit is derived. The complete ranking of all the units is received by their utilities.

The PROMETHEE II method works with six basic types of preference functions. They are used for measuring the intensity of preferences of all the pairs of units with respect to the given criterion. The partial pairwise intensities are aggregated by means of weights of the criteria specified by the decision maker and the global preferences between pairs of units are derived. The complete ranking of all the units is obtained by their descending ordering according to their net flows computed from the global preferences.

Multiple criteria decision making techniques are often based on the definition of the utility of units by means of several basic principles, e.g. aggregation of normalized criterion values. Another methodological framework that can be used for the evaluation of performance of decision making units is Data Envelopment Analysis (DEA). The essential characteristic of the DEA model is the reduction of the multiple inputs and multiple outputs using weights computed by the model. This model searches weights that define a virtual unit with the best (not worse) characteristics with respect to the evaluated unit. That means the virtual unit is the unit with lower inputs and higher outputs as compared to the evaluated unit. The unit is called efficient if there does not exist any set of weights that defines the virtual unit with the properties mentioned. Otherwise the unit is not efficient and the virtual inputs and outputs are target values for reaching the efficiency. The formulation of the DEA models leads to a linear fractional programming problem that can be simply transformed into the standard linear programming problem. Data envelopment analysis is a rising area. Many DEA models based on different assumptions have been formulated. Information about them can be found e.g. in Cooper et al. [1].

3. The AHP model

Because of hierarchical structure of the above-discussed problem of the evaluation of performance firms, branches and countries, we propose a simple two step AHP model with the following basic levels:

1. *Countries.* In our study four former Soviet-block countries (Czech Republic, Poland, Hungary and East Germany) on the one hand and one highly developed Western country (West Germany) on the other hand were included on this level. Generally, it is possible to assume we have h items (countries) on this level.
2. *Branches.* The most important branches of industry and services in the region discussed were taken into account (machine building industry, meat processing, freight transport, building industries, furniture, textile industry, etc.). The number of branches in the model will be denoted by m.

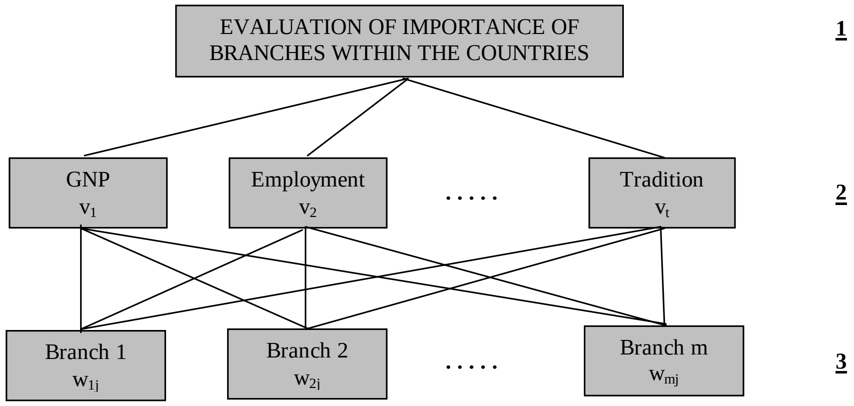

Figure 2. First step – evaluation of importance of branches within the countries

3. *Firms.* Selected more important firms of the branches listed above from all the countries were addressed by the questionnaire (its structure was presented in the introductory section of the paper) and the data from the questionnaires returned were analyzed. An identical number of firms for all the branches and countries was considered. The total number of firms in the study is $m.n$, where n is the number of firms for all the countries from a branch. That means, the number of firms from a branch for the given country is $d=n/h$ (supposing we have h countries).
4. *Criteria influencing the efficiency of firms* (inputs and outputs). The criteria used in the analysis correspond to the items of the questionnaire. As the basic inputs fixed and variable costs, labor costs, available floor space, investments, etc., can be considered, while the output characteristics are turnover, profit, market share, etc. The total number of criteria ($r+s$) consists of the number of inputs (r) and the number of outputs (s).
5. *Criteria influencing the position of the branches within the countries* (e.g. GNP, employment, tradition of the branch in the country, etc.). The number of elements on this level is t.

The proposed AHP model contains the following three steps:

1. **Estimation of the relative strength of the branches within countries.**
For each country the AHP model presented in Figure 2 is solved. It is the standard three-level AHP model with the units (branches) being evaluated by the criteria influencing their strength. The results of this model assign to the i-th branch its relative importance within the k-th country expressed by the value p_{ik}, i=1,2,...,m, k=1,2,...,h

$$p_{ik} = \sum_{j=1}^{t} w_{ij}, \; i = 1,2,...,m, k = 1,2,...,h,$$

$$\sum_{i=1}^{m} w_{ij} = v_j, \; j = 1,2,...,t,$$

$$\sum_{j=1}^{t} v_j = 1.$$

These formulas show that the sum of values p_{ik} over all the branches is equal to unity for all the countries k=1,2,...,h.

2. **Evaluation of performance of the firms within branches.**
The hierarchical model for this step is presented in Figure 3. This model is solved for all the branches separately, that means we have to analyze m similar AHP models. As the result of each of these models we obtain

$$q_{ik} = \sum_{j=1}^{r+s} u_{ij}, \; i = 1,2,...,n, k = 1,2,...,m.$$

The values of q_{ik} express the relative performance of the i-th firm from the k-th branch. Due to the principle of dividing of preferences from the higher hierarchical level to the lower level the sum of the values of q_{ik} over all the firms i=1,2,...,n is equal to unity for all the branches k=1,2,...,m.

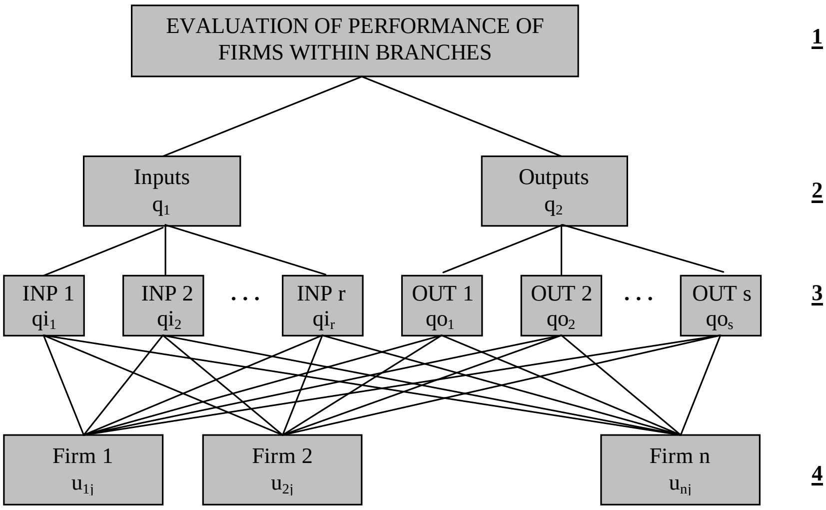

Figure 3. Second step – evaluation of performance of firms

3. **Synthesis of the results from the previous two steps.**

   The productivity score for the countries can be derived from the results of the previous two steps. Let us denote the productivity score for the k-th country as P_k, k=1,2,...,*h*. This characteristic can be computed as follows:

$$P_k = \sum_{j \in C_k} \sum_{i=1}^{m} p_{ik} q_{ji}, \quad k = 1,2,\ldots,h,$$

   where C_k is the set of indices of firms of the k-th country. The set of indices of firms within any branch is {1,2,...,*n*}. We split this set as follows: $C_1 = \{1,2,\ldots,d\}$, $C_2 = \{d+1, d+2,\ldots,2d\},\ldots,$ $C_h = \{n-d+1, n-d+2,\ldots,n\}$. Because of the relations presented above the sum of P_k over all the countries equals unity.

Table 1

Input and output characteristics of the firms

Branch	Country	Fixed costs	# of workers	Floor space	Investments	Turnover	Market share
		mil. Euro	#	sq. m	mil. Euro	mil. Euro	%
Weights of inp/outp		0.33676	0.14181	0.06124	0.08301	0.29733	0.07986
Building	CZ	11.743	164	6600	0.171	12.314	55
Building	CZ	2.257	308	10000	0.486	11.571	10
Meat	CZ	5.468	458	20129	0.010	12.943	10
Meat	CZ	3.657	316	27000	0.914	6.229	80
Transport	CZ	9.143	80	5000	0.600	14.543	10
Transport	CZ	5.743	421	18652	0.286	16.029	10
Building	PL	2.251	37	8537	0.184	9.043	2
Building	PL	0.285	85	29400	0.284	6.599	80
Meat	PL	1.000	100	3000	0.168	13.233	100
Meat	PL	1.611	95	3000	0.057	3.771	10
Transport	PL	2.281	366	18848	0.258	15.288	2
Transport	PL	3.544	235	24000	0.204	5.724	5
Building	HU	2.789	49	1101	0.974	30.567	2
Building	HU	1.800	198	2500	0.818	22.362	18
Meat	HU	3.047	559	40000	2.493	21.817	3
Meat	HU	2.376	74	4385	0.074	2.645	80
Transport	HU	1.886	316	14300	1.800	13.800	5
Transport	HU	1.000	79	45000	0.010	8.114	60
Building	GW	12.271	220	11000	1.534	86.920	40
Building	GW	1.790	78	1200	0.041	17.282	15
Meat	GW	7.005	85	22000	0.562	16.873	30
Meat	GW	0.665	75	5600	0.153	11.248	5
Transport	GW	6.136	80	3500	0.511	13.294	10
Transport	GW	0.782	57	1400	0.818	8.896	20
Building	GE	1.023	62	1500	0.015	3.272	20
Building	GE	1.841	111	2900	0.010	5.317	35
Meat	GE	6.382	88	21000	0.662	12.976	20
Meat	GE	4.244	77	19000	3.375	31.189	30
Transport	GE	4.286	65	1600	0.162	4.421	30
Transport	GE	2.301	132	5900	3.630	11.862	40

Our approach will be illustrated on the small example with 5 countries (Czech Republic, Poland, Hungary, Germany East and West), 3 branches (building industries, meat processing industry and freight transport) and 2 firms from each branch and country, i.e. the total number of firms in this example is 30. Each firm is described by 4 inputs (fixed costs, number of workers, floor space and investments) and 2 outputs (turnover and market share). The inputs and outputs specific for all the firms are listed in Table 1. The results of the AHP model will be compared to the DEA analysis and results of the WSA and PROMETHEE II methods.

The first step of our approach consists in the evaluation of the importance of branches within the countries. For each of the five countries we applied the model in Figure 2 with three criteria (GNP, employment, tradition) and three branches and asked an expert to perform pairwise comparisons in this model. The results (p_{ik}, i=1,2,3, j=1,2,...,5) are summarized in Table 2.

Table 2

National importance coefficients of branches

	CZ	PL	HU	GW	GE
Building	0.425	0.508	0.343	0.447	0.447
Meat proc	0.212	0.242	0.442	0.191	0.191
Transport	0.363	0.250	0.215	0.352	0.352

In the second step we compute the performance scores for all the firms within their branches according to the model presented in Figure 3. The input and output weights derived by pairwise comparisons are listed in the second row of Table 1. These values are used for the computation of performance scores q_{ik} of firms i=1,2,...,10, j=1,2,3 in Table 4. For computational reasons we show in Table 3 the pairwise comparison matrix for fixed costs of building industry firms together with the preferences u_{ij} only.

Table 3

Pairwise comparisons of building firms with respect to fixed costs

	CZ1	CZ2	PL1	PL2	HU1	HU2	GW1	GW2	GE1	GE2	u_{ij}
CZ1	1.000	0.200	0.200	0.111	0.250	0.167	1.000	0.167	0.125	0.167	0.0174
CZ2	5.000	1.000	1.000	0.333	2.000	0.500	6.000	0.500	0.333	0.500	0.0712
PL1	5.000	1.000	1.000	0.333	2.000	0.500	6.000	0.500	0.333	0.500	0.0712
PL2	9.000	3.000	3.000	1.000	5.000	4.000	9.000	4.000	3.000	4.000	0.2924
HU1	4.000	0.500	0.500	0.200	1.000	0.500	5.000	0.500	0.333	0.500	0.0523
HU2	6.000	2.000	2.000	0.250	2.000	1.000	8.000	1.000	0.500	1.000	0.1050
GW1	1.000	0.167	0.167	0.111	0.200	0.125	1.000	0.143	0.125	0.143	0.0158
GW2	6.000	2.000	2.000	0.250	2.000	1.000	7.000	1.000	0.500	1.000	0.1035
GE1	8.000	3.000	3.000	0.333	3.000	2.000	8.000	2.000	1.000	2.000	0.1677
GE2	6.000	2.000	2.000	0.250	2.000	1.000	7.000	1.000	0.500	1.000	0.1035

The values q_{ik}, i=1,2,...,10, j=1,2,3 in Table 4 express the relative performance of the firms from one of the selected branches. We can see that the most efficient among building firms is the second Polish firm whereas the least efficient are both the Czech firms. Similar conclusions can be drawn from other columns of the following table (meat processing industry and freight transport):

Table 4

Performance of the firms

	Building	Meat pr.	Transport
CZ1	0.06038	0.05839	0.07977
CZ2	0.05753	0.05904	0.07770
PL1	0.08527	0.15543	0.08517
PL2	0.15189	0.09211	0.05227
HU1	0.11389	0.07912	0.10182
HU2	0.09178	0.10589	0.16178
GW1	0.11803	0.08534	0.08972
GW2	0.11382	0.16481	0.16562
GE1	0.11814	0.06439	0.09490
GE2	0.08928	0.13547	0.09124

The results contained in Table 4 can be synthesized by branch weights in Table 2. The final results are presented in the first column of Table 5. The AHP model shows that the West Germany firms reach highest performance whereas the Czech firms reach the lowest one. Apart from the results given by the AHP model, Table 5 contains the average performance scores of the countries computed by other approaches – DEA, WSA and PROMETHEE II. All the results were standardized to the unity sum. By comparison of all the results it can be seen that the AHP model is very close to the DEA, which is a special technique for efficiency evaluation. Other approaches more or less differ in their results as compared to the AHP and DEA models.

Table 5

Productivity scores of the countries

	AHP	**DEA**	**WSA**	**PROM**
CZ	0.13217	0.12419	0.15532	0.14959
PL	0.21474	0.20700	0.22017	0.19244
HU	0.20899	0.21585	0.20483	0.23229
GW	0.24130	0.23245	0.21123	0.23249
GE	0.19641	0.22051	0.20845	0.19319

4. The generalized ANP model

The generalized model for productivity measurement of Central European countries is based on the Analytic Network Process (ANP) approach. The ANP approach [6], Saaty [7] is used for the local performance measuring of units and also for comparison of the global performance of units. The structure of the ANP model is described by clusters of elements connected by their dependence on one another. A cluster groups elements that share a set of attributes. At least one element in each of the clusters is connected to some element in another cluster. These connections indicate the flow of influence between the elements (Figure 4).

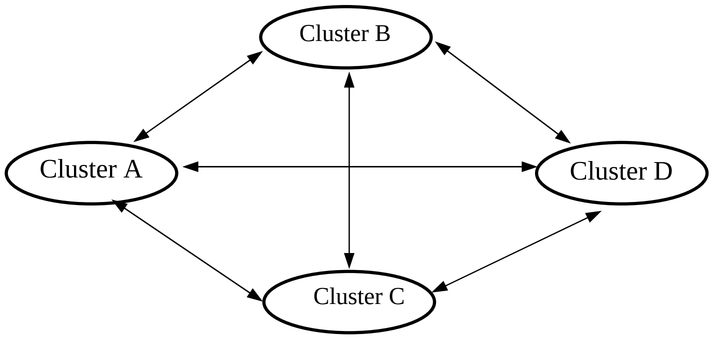

Figure 4. Flows of influence between the elements

Pairwise comparisons are needed for all the connections in the performance model - they are considered as inputs for the computation of the global performance of network production systems. A supermatrix is a matrix of all elements by all elements. The weights from the pairwise comparisons are placed in the appropriate column of the supermatrix. The sum of each column corresponds to the number of comparison sets. The weights in the column corresponding to the cluster are multiplied by the weight of the cluster. Each column of the weighted supermatrix sums to one and the matrix is column stochastic. Its powers can stabilize after several iterations to a limited supermatrix. The columns of each block of the matrix are identical and we can read off the global priority of business units.

In the generalized model we take into account countries, branches, firms and criteria as clusters and different types of connections in the system. There are dependencies and feedback among elements and clusters. The whole system is more properly represented as a network system. We state some examples of dependencies in the system. There are dependencies among countries resulting from foreign trade. The branches are interconnected and the flows can be modeled by input-output models. The questionnaire contains questions about networking activities of firms as rates of co-operation with customers and suppliers. The dependencies and feedback should be expressed by appropriate measures.

We used the alpha version of the ANP software package Super Decisions developed by Creative Decisions Foundation (CDF) for experiments in testing the possibilities of the expression and performance evaluation of the network system (Figure 5). Figure 5 contains an example of 4 clusters of our performance evaluation model and basic dependencies among them. It presents only an introductory idea for the ANP performance evaluation model. The model of the real situation is too complex and it is supposed that it will be elaborated as part of future research.

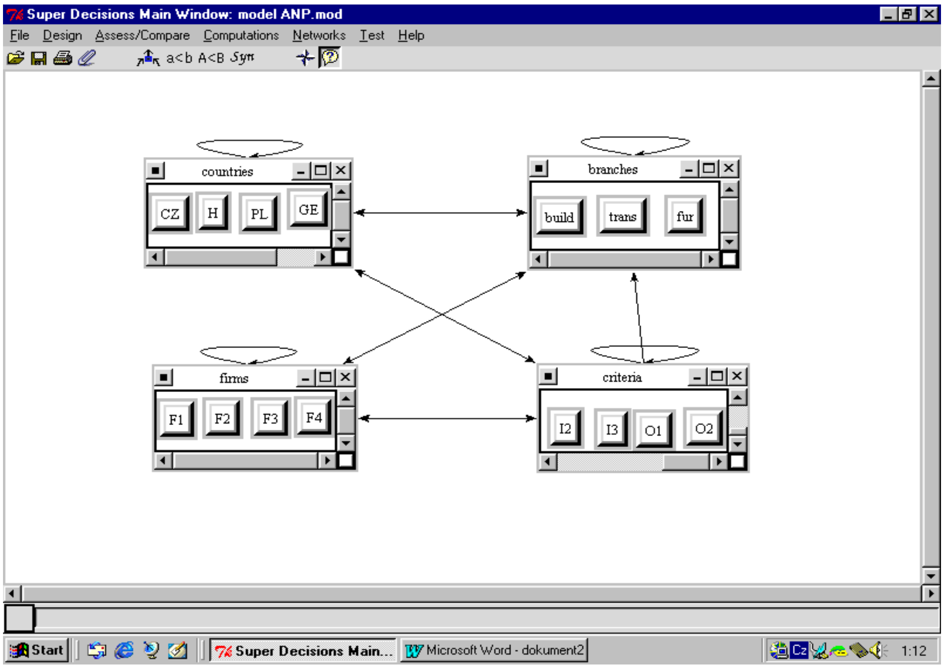

Figure 5. Generalized ANP model

The ANP approach seems better for applications in performance analysis than standard AHP models because it allows to model dependencies among basic elements of the model influencing the performance. In our model there are strong linkages and feedbacks among countries, branches and performance criteria. These relations are important between pairs of clusters on the one hand and among elements of clusters on the other hand.

Conclusions

The analysis and design of production systems has been an active area of research. Performance models help to understand the behaviour of business systems and to provide guidelines to improve their performance.

The AHP model presented in Section 3 offers a simple approach to the estimation of the performance scores of the countries. The possibility to use qualitative and hardly measurable characteristics is its advantage in comparison with other techniques. Small-scale example shows the basic principle of the approach but its results cannot be generalized. A large study taking into account a huge number of firms from much more branches is being prepared and it will be the aim of our future research.

Individual units are interconnected into a network system by material, financial and information flows. The network system is responsible for global performance whereas each unit is responsible for local performance. The ANP approach seems an appropriate method for performance measuring of network production systems. Future research will be oriented towards more detailed and sophisticated network models and methodology of performance measuring of network systems.

Acknowledgements

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Dorota Kuchta

MULTICRITERIAL EXAMINATION TIMETABLING WITH UNCERTAIN INFORMATION

Abstract

We consider examination timetabling at a university. This problem has been widely treated in the literature (e.g. [1]. [8], [9]); however, we propose a new approach, which belongs to the family of robust approaches. The main obvious assumption is that two examination sessions sharing at least one student cannot be scheduled at the same time. This scheduling problem will be stated as a graph coloring problem. The stability of the solution scheduled is desirable in the sense that it remains valid also when, unexpectedly, some additional students want to take the exams, for example those who failed in earlier examination sessions. This stability is defined as the robustness of examination scheduling. In [6], [10] a probabilistic robustness measure has been proposed. We propose a fuzzy approach, similarly as in [3]. We consider three different schedule robustness measures: mean value of the fuzzy number of examination conflicts considered in [3], and two new measures, put forward in this paper: the cardinality of the fuzzy number of session conflicts and the possibility that the fuzzy number of session conflicts is 0. We also consider a multicriterial approach with the minimization of the examination session days and the maximization of schedule robustness.

Keywords

Scheduling, timetabling, fuzzy number, coloring graph, robustness.

Introduction

In the paper we will use the following notions.

A fuzzy number (set) in space $\Re$, denoted by a capital letter with a tilde (e.g. $\widetilde{A}$), is defined as a set of pairs $\{(\mu_A(x), x)\}$, where $x \in \Re$ and $0 \le \mu_A(x) \le 1$ is a membership function describing to what degree $\widetilde{A}$ equals x [7], [11].

We will use discrete normal fuzzy numbers (e. g. fuzzy numbers defined on $N \cup \{0\} \subset \Re$ for which $\mu_A(x) = 1$ at least for one x). For discrete fuzzy numbers we will use the notation

$$\widetilde{A}_i = x_1 / \mu_{A_i}(x_1) + x_2 \mu_{A_i}(x_2) + \ldots + x_n \mu_{A_i}(x_n)$$

where $\{x_1, .., x_n\}$ are those numbers from $N \cup \{0\} \subset \Re$ for which the membership function takes a non-zero value.

Addition of two fuzzy numbers $\widetilde{A}$, $\widetilde{B}$ can be defined as follows:

$$\mu_{A+B}(z) = \sup_{z=x+y} \min(\mu_A(x), \mu_B(y)) \tag{1}$$

and scalar multiplication of a fuzzy number and scalar addition to a fuzzy number as:

$$\mu_{rA}(z) = \sup_{z=rx} \mu_A(rx) \tag{2}$$

$$\mu_{r+A}(z) = \sup_{z=r+x} \mu_A(rx).$$

According to these definitions, if $r = 0$ then $r\widetilde{A}$ is a crisp number equal to 0, $r + \widetilde{A} = \widetilde{A}$, and if $r = 1$ then $r\widetilde{A} = \widetilde{A}$.

The membership function $\mu_A(x)$ of the fuzzy number $\widetilde{A} = \sum_{i=1}^{n} \widetilde{A}_i$ of the sum of *n* normal discrete fuzzy numbers $\widetilde{A}_i = 0 / \mu_{A_i}(0) + 1 / \mu_{A_i}(1)$ takes on the form [5]:

$$\widetilde{A} = \sum_{i=1}^{n} \widetilde{A}_i = \sum_{k=0}^{n} k / \mu_A(k) = \sum_{k=0}^{n} k / \min(\lambda_k, \Lambda_k) \tag{3}$$

where:

$\lambda_0, \lambda_1, \ldots, \lambda_n$ – $\mu_{A_i}(0)$ are sorted in a non-decreasing way, with the number 1 at the end of the sequence,

$\Lambda_0, \Lambda_1, \ldots, \Lambda_n$ – $\mu_{A_i}(1)$ are sorted in a non-increasing way, with the number 1 at the beginning of the sequence.

From (3) it follows that

$$\mu_A(0) = \min_{k=0,1,\ldots,n}(\lambda_k) = \lambda_0 \tag{4}$$

Now we assume that the normal fuzzy numbers $\widetilde{A}_i = 0/\mu_{A_i}(0) + 1/\mu_{A_i}(1)$ are ordered in the following way:

$$\begin{cases} \mu_{A_1}(0) \le \mu_{A_2}(0) \ldots \le \mu_{An}(0), \\ \mu_{A_1}(1) \mu_{A_2}(1) \ldots \ge \mu_{An}(1), \end{cases}$$

where for n_1 fuzzy numbers $\mu_{A_i}(0)$<1, for n_2 fuzzy numbers $\mu_{A_i}(1) < 1$ and for $n - n_1 - n_2$ fuzzy numbers $\mu_{A_i}(0) = \mu_{A_i}(1) = 1$. In this case the fuzzy number $\widetilde{A} = \sum_{i=1}^{n} \widetilde{A}_i$ can be expressed as:

$$\widetilde{A} = \sum_{i=1}^{n} \widetilde{A}_i = \sum_{i=0}^{n_1 - 1} i/\mu_{A_{i+1}}(0) + \sum_{n_1}^{n - n_2 - 1} i/1 + \sum_{i=n-n_2}^{n} i/\mu_{A_i}(1) \tag{5}$$

In the literature there are many definitions of the cardinality of a fuzzy set. We will use the following one [7]: the cardinality of a fuzzy set $A = x_1/\mu_A(x_1) + x_2/\mu_A(x_2) + \ldots + x_n/\mu_A(x_n)$, denoted by $Card(A)$, is the real number equal to

$$Card(A) = \sum_{i=1}^{n} \mu_A(x_i). \tag{6}$$

The cardinality of the fuzzy set defined by (3) takes the form:

$$\begin{aligned} Card(A) &= \sum_{i=1}^{n} \mu_A(x_i) = \sum_{i=1}^{n_1} \mu_{A_i}(0) + (n - n_1 - n_2 + 1)\cdot 1 + \sum_{i=n-n_2+1}^{n} \mu_{A_i}(1) = \\ &= \sum_{i=1}^{n} \mu_{A_i}(0) + \sum_{i=1}^{n} \mu_{A_i}(1) - (n-1) = 1 + \sum_{i=1}^{n} \left(\mu_{A_i}(0) + \mu_{A_i}(1) - 1\right). \end{aligned} \tag{7}$$

The mean value of a normal fuzzy set $A = 0/\mu_A(0) + 1/\mu_A(1)$ equals [2], [5]:

$$E(A) = \frac{1}{2}\left(1 + \mu_A(1) - \mu_A(0)\right) \tag{8}$$

By the linearity of the mean, the mean of the weighted sum of fuzzy sets equals:

$$E\left(\sum_{i=1}^{n} w_i A_i\right) = \sum_{i=1}^{n} w_i E(A_i) \tag{9}$$

1. Robust schedules

We consider examination timetabling at a university. We assume that there are so called “standard students” and “non standard students”. Standard students are these who study according to the basic study program and non standard ones are those who repeat examinations or have an individual study program. Two examinations sharing at least one standard student cannot be scheduled on the same day. Taking into account examination conflicts, we construct the graph $G = (V, E)$, whose vertices $V = \{1, 2, \ldots, m\}$ represent the examinations *E1, E2,…, Em* and whose edges (i, j) are included in the edge set *E* if the examinations *Ei* and *Ej* share at least one standard student.

In order to schedule examinations in no more than *C* days, a *C*-colouring problem can be stated. We can construct an examination schedule by solving the following binary linear model:

$$\begin{aligned} &\sum_{c=1}^{C} x_{ic} = 1 \quad for \quad i \in V, \\ &x_{ic} + x_{jc} \le 1 \quad for \quad (i, j) \in E, \quad c = 1, 2, \ldots, C, \\ &x_{ij} = 0, 1. \end{aligned} \tag{10}$$

Moreover, the stability of a schedule found by solving the problem (10) is desirable in the sense that it remains valid also when non standard students examination conflicts are taken into account. Let $G' = (V, E')$ be the complementary graph set, where the set of edges $E' = (V \times V) \setminus (E \cup I)$ (where: $I = \{(i, j) \in G : i = j\}$) represent all non standard students examination conflicts. In [10] the authors consider a the validity of a schedule found by solving the problem (10) taking as a measure the probability that such solution remains valid after random complementary edges from E’ have been added to the edge set *E*. The validity of the solution of the problem (10) taking into account the non standard students examination conflicts can be defined as the robustness of this solution. In [3] the mean of the non standard students examination conflicts (a fuzzy number) was assumed to be the robustness measure of (10).

Further we assume that for each edge (i, j) in *E'* the fuzzy number

$$\widetilde{A}_{(i,j)} = 0/\mu_{A_{(i,j)}}(0) + 1/\mu_{A_{(i,j)}}(1)$$

determining whether students who take the exams *Ei*, *Ej* will turn up, is known. This fuzzy number is 1 if the examinations *Ei* and *Ej* have at least one non standard student in common and 0 if they have no non-standard students in common. Let $\widetilde{A}$ be the fuzzy number determining the number of non standard students examination conflicts.

Let us now consider two measures (criteria) of the robustness of the session schedule for exams taken by non-standard students. One of them will be the possibility that the fuzzy number of non standard students examination conflicts $\widetilde{A}$ is 0 – $\mu_A(0)$. We can construct the best robust solution solving the following binary linear programming model. According to (3) and (4) the model takes the following form:

$$m_o \to \max$$

subject to:

$$\begin{aligned}
&\sum_{c=1}^{C} x_{ic} = 1 \quad \text{for} \quad i \in V, \\
&x_{ic} + x_{jc} \le 1 \quad \text{for} \quad (i,j) \in E, \quad c = 1,2,...,C, \\
&z_c \le \sum_{i=1}^{n} x_{ic} \le M z_c \quad \text{for} \quad c = 1,2,...,C, \\
&2y_{ij} \le x_{ic} + x_{jc} \le 1 + y_{ij} \quad \text{for} \quad (i,j) \in E', \quad c = 1,2,...,C, \\
&m_0 \le 1 - y_{ij} + \mu_{A_{(i,j)}}(0) y_{ij} \quad \text{for} \quad (i,j) \in E', \\
&m_0 \le 1
\end{aligned} \tag{11}$$

$x_{ic} = 0,1$, for i = 1,2,…,n, c = 1,2,…,C,

$y_{ij} = 0,1$ $(i,j) \in E^{'}$,

$z_c = 0,1$ for c = 1,2,…,C.

$m_o \in \Re^+$,

M – a big number

where:

$$x_{ic} = \begin{cases} 1 & if \quad vertex \quad i \quad is \quad colored \quad c, \\ 0 & if \quad vertex \quad i \quad isn't \quad colored \quad c, \end{cases}$$

$$y_{ij} = \begin{cases} 1 & if \quad vertices \quad i,j \quad are \quad equally \quad colored, \\ 0 & if \quad vertices \quad i,j \quad aren't \quad equally \quad colored, \end{cases}$$

$$z_c = \begin{cases} 1 & if \quad color \quad c \quad is \quad used, \\ 0 & if \quad color \quad c \quad isn't \quad used, \end{cases}$$

In the model the first and the second conditions ensure that the vertices connected with an arc are colored with different colors. The third condition expresses the requirement that if a color is used, then at least one vertex has to be colored with it (if the day c has been taken into account in the session schedule, then at least one exam has to be scheduled on that day. The fourth condition sets y_{ij} equal to 1 if the exams E_i and E_j for non standard students conflict for the given schedule. The fifth and the sixth condition together with the maximization of m_o define $m_0 = \mu_A(0) = \min_{k=1,\ldots,n}(\lambda_k)$.

Another robustness measure of the solution of (10) is the cardinality of the fuzzy set of non-standard students examination conflicts. Taking into account (7), the 0-1 linear model determining the most robust solution with this criterion takes on the following form:

$$1 + \sum_{(i,j)\in E'} \left(\mu_{A(i,j)}(1) + \mu_{A(i,j)}(0) - 1 \right) y_{ij} \to \min$$

subject to:

$$\begin{aligned}
&\sum_{c=1}^{C} x_{ic} = 1 \quad \text{for} \quad i \in V\,, \\
&x_{ic} + x_{jc} \le 1 \quad \text{for} \quad (i,j) \in E, \quad c = 1,2,\dots,C\,, \\
&z_c \le \sum_{i=1}^{n} x_{ic} \le M z_c \quad \text{for} \quad c = 1,2,\dots,C\,, \\
&2y_{ij} \le x_{ic} + x_{jc} \le 1 + y_{ij} \quad \text{for} \quad (i,j) \in E', \quad c = 1,2,\dots,C\,, \\
&x_{ic} = 0,1 \;, \quad \text{for } i = 1,2,\dots,n, \; c = 1,2,\dots,C, \\
&y_{ij} = 0,1 \qquad (i,j) \in E'\,, \\
&z_c = 0,1 \qquad \text{for } c = 1,2,\dots,C.
\end{aligned} \tag{12}$$

M – a big number

where:

$$x_{ic} = \begin{cases} 1 & \textit{if vertex } i \textit{ is colored } c, \\ 0 & \textit{if vertex } i \textit{ isn't colored } c, \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \textit{if vertices } i,j \textit{ are equally colored}, \\ 0 & \textit{if vertices } i,j \textit{ aren't equally colored}, \end{cases}$$

$$z_c = \begin{cases} 1 & \textit{if color } c \textit{ is used}, \\ 0 & \textit{if vertex } c \textit{ isn't used}, \end{cases}$$

1.1. Example

The examinations for seven courses *E1*, *E2*, *E3*, *E4*, *E5*, *E6*, *E7* for a particular program at the university must be scheduled within no more than C = 5 days. The graph $G = (V, E)$ for the standard students examination conflicts is shown in Figure 1. The vertices $V = \{1, 2, 3, 4, 5, 6, 7\}$ represent the examination numbers. If the examinations *Ei* and *Ej* share at least one standard student, the edge (i, j) has been included in graph *G*. The adjacency matrix is:

$$M_G = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ & 0 & 1 & 1 & 0 & 0 & 1 \\ & & 0 & 0 & 1 & 0 & 0 \\ & & & 0 & 1 & 1 & 1 \\ & & & & 0 & 1 & 0 \\ & & & & & 0 & 0 \\ & & & & & & 0 \end{pmatrix}$$

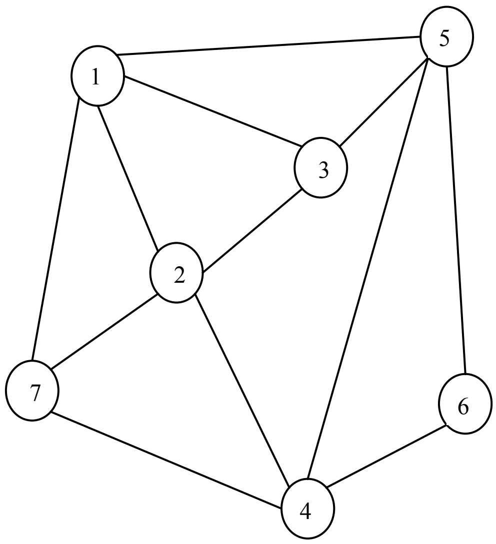

Figure 1. The graph G = (V,E) for the standard students examination conflicts

The matrix of the complementary graph $G' = (V, E')$, where a set of edges $E' = (V \times V) \setminus E \setminus \cup I$ represent all possible non standard students examination conflicts is:

$$M_{G'} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 1 & 1 & 0 \\ & & 0 & 1 & 0 & 1 & 1 \\ & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 1 \\ & & & & & 0 & 1 \\ & & & & & & 0 \end{pmatrix}$$

Using information from past years, we determine the possibility of an examination conflict for non-standard students:

$$\widetilde{A}_{G'} = \begin{pmatrix} 0 & 0 & 0 & \widetilde{A}_{(1,4)} & 0 & \widetilde{A}_{(1,6)} & 0 \\ & 0 & 0 & 0 & \widetilde{A}_{(2,5)} & \widetilde{A}_{(2,6)} & 0 \\ & & 0 & \widetilde{A}_{(3,4)} & 0 & \widetilde{A}_{(3,6)} & \widetilde{A}_{(3,7)} \\ & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & \widetilde{A}_{(5,7)} \\ & & & & & 0 & \widetilde{A}_{(6,7)} \\ & & & & & & 0 \end{pmatrix}$$

In the above matrix, $\widetilde{A}_{(i,j)}$ is a fuzzy number equal to 1 if the examinations Ei and Ej have at least one non standard student in common and 0 if the examinations Ei and Ej have no non-standard students in common. These fuzzy numbers take the following forms:

$\widetilde{A}_{(1,4)} = 0/1 + 1/0{,}6$ $\quad\widetilde{A}_{(1,6)} = 0/1 + 1/0{,}3$ $\quad\widetilde{A}_{(2,5)} = 0/0{,}6 + 1/1$

$\widetilde{A}_{(2,6)} = 0/0{,}8 + 1/1$ $\quad\widetilde{A}_{(3,4)} = 0/1 + 1/0{,}5$ $\quad\widetilde{A}_{(3,6)} = 0/0{,}4 + 1/1$

$\widetilde{A}_{(3,7)} = 0/1 + 1/0{,}7$ $\quad\widetilde{A}_{(5,7)} = 0/0{,}1 + 1/1$

$\widetilde{A}_{(6,7)} = 0/0{,}1 + 1/0{,}4$.

The most robust schedule according to the model (11) with the robustness criterion $\mu_A(0) \to \max$ is the following one:

Day 1: *E3*, *E4*,
Day 2: *E7*,
Day 3: *E1*, *E6*,
Day 4: E5,
Day 5: *E2*.

The fuzzy number of the number of non standard students examination conflicts takes the form: $\widetilde{A} = 0/1 + 1/0{,}5 + 2/0{,}3$, Figure 2a. For this solution $\mu_A(0) = 1$, while $Card(\widetilde{A}) = 1{,}8$.

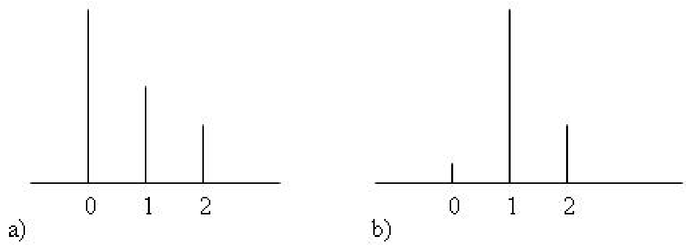

Figure 2. The form of the fuzzy number of the number of conflicts for examination schedule for one-criterial problems

And for the robustness criterion $Card(\widetilde{A}) \to \min$ the session schedule according to the model (12) is as follows:

Day 1: *E5*, *E7*,
Day 2: *E3*,
Day 3: *E1*, *E6*,
Day 4: *E4*,
Day 5: *E2*,

and the fuzzy number of the number of non standard students examination conflicts takes on the form: $\widetilde{A} = 0/0{,}1 + 1/1 + 2/0{,}3$, Figure 2b. For this solution we have $\mu_A(0) = 0{,}1$ and $Card(\widetilde{A}) = 1{,}4$.

The schedules generated have high robustness according to one criterion and low robustness according to the other one.

2. A multicriteria approach

Let us consider a multicriteria optimization problem, where our goal is to construct a schedule with a small number of session days and at the same time one which would be as robust as possible. The following robustness criteria are assumed:

- The possibility that the non standard students examination conflicts are zero,
- The power of the fuzzy number of the non standard students examination conflicts,
- The mean value of the fuzzy number of the non standard students examination conflicts, (8) (9).

The corresponding multicriteria programming model takes on the following form

$$\begin{aligned}
&F_1\left(x_{11},\dots,x_{iC},y_{ij}\in E',z_1,\dots,z_C\right)=\sum_{c=1}^{C} z_c \to \min\\
&F_2\left(x_{11},\dots,x_{iC},y_{ij}\in E',z_1,\dots,z_C\right)=m_o \to \max\\
&F_3\left(x_{11},\dots,x_{iC},y_{ij}\in E',z_1,\dots,z_C\right)=1+\sum_{(i,j)\in E'}\left(\mu_{A_{(i,j)}}(1)+\mu_{A_{(i,j)}}(0)-1\right)y_{ij} \to \min\\
&F_4\left(x_{11},\dots,x_{iC},y_{ij}\in E',z_1,\dots,z_C\right)=\frac{1}{2}\sum_{(i,j)\in E'}\left(1+\mu_{A_{(i,j)}}(1)-\mu_{A_{(i,j)}}(0)\right)y_{ij} \to \min
\end{aligned}\tag{13}$$

with the constraints of (11).

To solve (13), we suggest to use the method of objectives prioritizing.

2.1. Example

Let us consider the problem from section 2.1. We assume the following hierarchy of objectives: F_1, F_2, F_3, F_4.

We will get a schedule with the minimal number of session days by minimizing the function $F_1 = \sum_{c=1}^{C} z_c$ with the constraints of (13) with an additional one: $m_0 = 0$. This schedule is:

Day 1: *E2*, *E5*,

Day 2: *E1*, *E4*,

Day 3: *E3, E6*, *E7*.

The fuzzy number of the number of non standard students examination conflicts takes the form: $\widetilde{A} = 0/0{,}4 + 1/0{,}6 + 2/1 + 3/0{,}7 + 4/0{,}6 + 5/0{,}4$, Figure 3a. The robustness of this schedule according to the measures we consider is $F_1 = \mu_A(0) = 0{,}4$, $F_2 = Card(\widetilde{A}) = 3{,}7$, $F_2 = E(\widetilde{A}) = 1{,}95$.

The decision maker has decided that he can accept four session day (but not more) if this results in higher robustness. Adding to the constraints of (13) the constraint $\sum_{c=1}^{C} z_c \le 4$ we solve our problem by taking as the optimization criterion the second objective in the hierarchy $F_2 = m_o \to \max$. We get the following schedule:

Day 1: *E1*, *E4*,

Day 2: *E3*, *E7*,

Day 3: *E2*, *E6*,

Day 4: *E5*.

The fuzzy number of the number of non standard students examination conflicts takes the form: $\widetilde{A} = 0/0{,}8 + 1/1 + 2/1 + 3/0{,}7 + 4/0{,}6$, Figure 3b. The criteria take on the following values: $F_1 = \mu_A(0) = 0{,}8$, $F_2 = Card(\widetilde{A}) = 3{,}1$, $F_2 = E(\widetilde{A}) = 1{,}25$. It is a four-day schedule whose robustness is better from the point of view of all the robustness measures considered here.

Let us now assume that the decision maker has decided that he would be satisfied with a schedule for which the possibility that there are no non standard students examination conflicts is 0.6 (not less), but which has a smaller number of non standard students examination conflicts. We check whether it is possible to minimize the objective $F_3 = 1 + \sum_{(i,j)\in E'} \left(\mu_{A_{(i,j)}}(1) + \mu_{A_{(i,j)}}(0) - 1 \right) y_{ij}$ with the constraints from (13) with the two additional ones: $\sum_{c=1}^{C} z_c \le 4$ and $m_o \ge 0{,}6$.

The solution of this problem is:

Day 1: E2, E5,

Day 2: E1, E6,

Day 3: E3, E4,

Day 4: E7.

The fuzzy number of the number of non standard students examination conflicts takes the form: $\widetilde{A} = 0/0{,}6 + 1/1 + 2/1 + 3/0{,}5 + 4/0{,}3$, Figure 3c. The values of the criteria are: $F_1 = \mu_A(0) = 0{,}6$, $F_2 = Card(\widetilde{A}) = 2{,}4$, $F_2 = E(\widetilde{A}) = 1{,}1$.

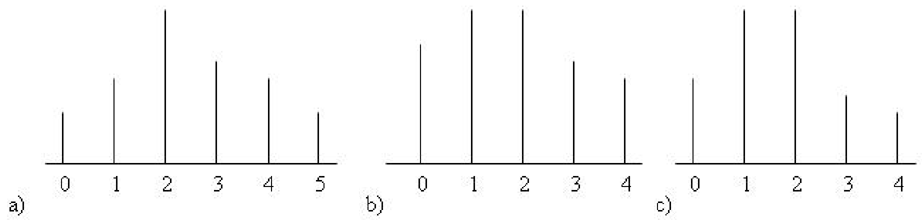

Figure 3. The form of the fuzzy number of the number of conflicts for examination schedule for multicriterial problem

In some cases it might be interesting to consider the fourth criterion in the hierarchy: $F_4 = \frac{1}{2} \sum_{(i,j)\in E'} \left(1 + \mu_{A_{(i,j)}}(1) - \mu_{A_{(i,j)}}(0) \right) y_{ij} \to \min$. For the example under consideration it did not bring anything new.

Conclusions

In the paper [10] the authors showed that the problem (11) is NP-Complete for the criterion function $F_0(y_{ij}) = \sum_{(i,j)\in E'} p_{ij} \cdot y_{ij} \to \min$ for non negative penalties p_{ij}. In our case the following inequality holds true: $0 \le p'_{ij} = \left(\mu_{A_{(i,j)}}(1) + \mu_{A_{(i,j)}}(0) - 1 \right) \le 1$. We can use the function

$f\left(p_{ij}^{'}\right)=\frac{p_{ij}}{\max\left(p_{ij}\right)}$ as a polynomial mapping function which maps the problem with non negative penalties p_{ij} to the problem with penalties $0 \le p_{ij}^{'} \le 1$. This implies that the fuzzy robust examination scheduling problem minimizing the power of the fuzzy number of the non standard students examination conflicts is NP-Complete. So, only a small-size fuzzy robust examination scheduling problem can be solved by means of binary linear programming models; for large-size problems some heuristics have to be applied to obtain appropriate solutions. To solve a probability robust examination scheduling problem a genetic algorithm has been proposed [10]. It can be also used to solve fuzzy robust examination scheduling problem. However, it seems that problems of optimal examination schedule have small dimensions and for such problems the solution of (13) is obtained very quickly.

Future research will deal with other graph problems such as assignment, map coloring, open shop problems. We will also analyze models, in which the non standard students examination conflicts are modeled as fuzzy variables.

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MULTIPLE CRITERIA DECISION MAKING: FROM EXACT TO HEURISTIC OPTIMIZATION

Abstract

We propose to derive *assessments of outcomes* to Multiple Criteria Decision Making problems, instead of just outcomes, and carry decision making processes with the former. In contrast to earlier works in that direction, which to calculate assessments have made use of subsets of the *efficient set* (*shells*), here we provide formulas for calculations of assessments based on the use of *upper and lower approximations* (*upper and lower shells*) of the efficient set, derived by *evolutionary optimization*. Hence, by replacing shells, which are to be in general derived via optimization, with pairs of upper and lower shells, the need of exact optimization methods can be eliminated from Multiple Criteria Decision Making.

Keywords

Multiple criteria decision making, evolutionary optimization, parametric outcome assessments.

Introduction

Decision making, whether in economic or social domain, calls for multi aspect deliberations. The field of *Multiple Criteria Decision Making* (where “criteria” stands for “aspects”) provides methodologies and supporting tools to cope with decision problems.

For a class of “complex” decision problems, where because of scale, bulk of data, and/or intricate framing a formal model is requested, efficient variants, and among them the most preferred variant (the decision), can be derived with the help of exact optimization methods. This in turn requires that the model has to be tied to an exact optimization package, which certainly precludes popular, lay and widespread use on Multiple Criteria Decision Making (MCDM) methods.

In a quest for simpler MCDM tools than those offered by now, it was proposed in Kaliszewski [3], [4] that the decision maker (DM) instead of evaluating exact *outcomes* (i.e. vectors of variant criteria values) would evaluate *assessments of outcomes*, provided with sufficient (and controlled) accuracy. Once the *most preferred outcome assessment* is derived, the closest (in a sense) variant is determined.

For an *efficient outcome* (i.e. outcome of an efficient variant) assessment calculations a subset of efficient variants (a *shell*) has to be known. As a shell can be derived (by exact optimization methods) prior to starting the decision process, replacing outcomes by their assessment relives MCDM from a *direct* dependence on exact optimization methods and packages.

In Miroforidis [7] it has been recently proposed to replace shells by somewhat weaker constructs, namely *lower shells* and *upper shells* and formulas for assessments of *weakly efficient outcomes* (i.e. outcomes of *weakly efficient variants*) have been derived. As lower and upper shells can be derived by *evolutionary optimization*, replacing shells by pairs of lower and upper shells leads to replacement of exact optimization methods (required to derive shells) by their evolutionary (*bona fide* heuristic) counterparts. This, in consequence, eliminates from MCDM the need of exact optimization methods and packages completely.

In this paper, on the base of the concept of lower and upper shells, we derive formulas for assessments of *properly efficient outcomes* (i.e. outcomes of *properly efficient variants*). These bounds subsume as a special case formulas derived in Miroforidis [7].

The outline of the paper is as follows. In Section 1 we provide basic definitions and notation. In Section 2 we derive formulas for assessments of properly efficient outcomes using lower and upper shells. Final Section concludes.

1. Definitions and notation

Let x denote a (decision) variant, $\mathcal{X}$ a variant space, X_0 a set of *feasible variants*, $X_0 \subseteq \mathcal{X}$. Then the underlying model for MCDM is formulated as:

$$\begin{gathered} \text{“}max\text{”}\, f(x) \\ x \in X_0 \subseteq \mathcal{X}, \end{gathered} \tag{1}$$

where $f: \mathcal{X} \to \mathcal{R}^k$, $f = (f_1,...,f_k)$, $f_i : X \to R$, are objective (criteria) functions, $i = 1,...,k$, $k \geq 2$; "*max*" denotes the operator of deriving all efficient variants in X_0 according to the definition of efficiency given below.

In MCDM to compare feasible variants x one makes use of their outcomes $f(x)$. Relations between outcomes in *outcome space* $\mathcal{R}^k$ induce relations between variants in variant space $\mathcal{X}$.

Below we make use of the following notation: $y = f(x)$.

Element $\bar{t}$ of T, $T \subseteq \mathcal{R}^k$, is:

- *efficient in* T, if $t_i \geq \bar{t}_i$, $i = 1,...,k$, $t \in T$, implies $t = \bar{t}$,
- *weakly efficient in* T, if there is no $t \in T$, such that $t_i > \bar{t}_i$, $i = 1,...,k$,
- *properly efficient in* T [1], if it is efficient and there exists a finite number $M > 0$ such that for each i we have

$$\frac{t_i - \bar{t}_i}{\bar{t}_j - t_j} \leq M$$

for some j such that $t_j < \bar{t}_j$, whenever $t \in T$ and $t_i > \bar{t}_i$.

Variant $\bar{x} \in A \subseteq \mathcal{X}$ is called *efficient* (*weakly efficient*, *properly efficient*) in A if $\bar{y} = f(\bar{x})$ is efficient (weakly efficient, properly efficient) in $f(A)$. We denote the set of efficient variants of X_0 by N.

We define on $\mathcal{X}$ the *dominance relation* $\succ$,

$$x' \succ x \Leftrightarrow f(x') >> f(x),$$

where $>>$ denotes $f_i(x') \geq f_i(x)$, $i = 1,...,k$, and $f_i(x') > f_i(x)$ for at least one i. If $x' \succ x$, then we say that x is *dominated* by x' and x' is *dominating* x.

The following definitions of lower and upper shells come from [7].

Lower shell is a finite nonempty set $S_L \subseteq X_0$, elements of which satisfy

$$\forall_{x \in S_L} \neg \exists_{x' \in S_L} x' \succ x. \tag{2}$$

By condition (2) all elements of shell S_L are efficient in S_L.

For a given lower shell S_L we define *nadir point* $y^{nad}(S_L)$ as

$$y_i^{nad}(S_L) = \min_{x \in S_L} f_i(x), \ i = 1,...,k.$$

Upper shell is a finite nonempty set $S_U \subseteq \mathcal{X} \setminus X_0$, elements of which satisfy[*]

$$\forall_{x \in S_U} \neg \exists_{x' \in S_U} \; x \succ x' \,, \tag{3}$$

$$\forall_{x \in S_U} \neg \exists_{x' \in N} \; x' \succ x \,, \tag{4}$$

$$\forall_{x \in S_U} \; f_i(x) > y_i^{nad}(S_L), \;\; i = 1,\dots,k \,. \tag{5}$$

Below we make use of a selected element of outcome space $\mathcal{R}^k$, denoted y^*, defined as

$$y_i^* = \hat{y}_i + \varepsilon, \quad i = 1,\dots,k \,,$$

where ε is any positive number and $\hat{y}$ is the *utopian element* of $\mathcal{R}^k$, calculated as

$$\hat{y}_i = \max_{y \in f(X_0) \cup f(S_U)} y_i \,, \;\; i = 1,\dots,k \,,$$

and we assume that all these maxima exist.

We assume that all efficient outcomes are ρ-properly efficient, i.e. they can be derived by solving the optimization problem

$$\min_{y \in f(X_0)} \max_i \lambda_i ((y_i^* - y_i) + \rho e^k (y^* - y)) \,, \tag{6}$$

where $\lambda_i > 0, \; i = 1,\dots,k$, and $\rho > 0$ (cf. e.g. [8], [6], [2], [4]).

By condition (3) all elements of upper shell S_U are efficient in S_U. We also assume that they all are ρ-properly efficient in S_U, i.e. they can be derived by solving the optimization problem

$$\min_{y \in f(S_U)} \max_i \lambda_i ((y_i^* - y_i) + \rho e^k (y^* - y)) \,, \tag{7}$$

where $\lambda_i > 0, \; i = 1,\dots,k$, and $\rho > 0$ has the same value as for ρ-properly efficient outcomes (elements of $f(X_0)$) defined above.

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[*] In [7] condition (5) has the form $f(x) \gg y^{nad}(S_L)$. We have had to strengthen this condition to deal with proper efficiency in formula (11) below [5].

2. Parametric bounds on outcomes

An outcome which is not derived explicitly (i.e. it is not an *explicit outcome*) but is only designated by selecting vector λ for the purpose to solve the optimization problem (6), is called an *implicit outcome*.

We use lower and upper shells of N to calculate parametric bounds on implicit outcomes, with weights λ as parameters.

We are aiming at the following. Suppose vector of weights λ is given. Let $y(\lambda)$ denote an implicit properly efficient outcome of $f(X_0)$, which would be derived if optimization problem (6) were solved with that λ. Let $L(y(\lambda))$ and $U(y(\lambda))$ be vectors of lower and upper bounds on components of $y(\lambda)$, respectively. These bounds form an *assessment* $[y(\lambda)]$ of $y(\lambda)$,

$$[y(\lambda)] = \{L(y(\lambda)), U(y(\lambda))\} .$$

To simplify notation we put $L(y(\lambda)) = L(\lambda)$ and $U(y(\lambda)) = U(\lambda)$.

To calculate bounds (assessments) one needs to know a pair of lower and upper shells. As can be seen below, computational costs to calculate such bounds are negligible as compared to derivation of efficient outcomes by exact optimization methods.

Formulas we show may at the first glance look complicated, but in fact they consist of no more than operations of adding and taking maxima over finite sets of numbers.

Proofs of formulas can be found in [5].

Let $\overline{L}_i$ and $\overline{U}_i$ be such that for each $y \in f(X_0)$ the following holds

$$\overline{L}_i \le y_i \le \overline{U}_i , \; i = 1,...,k . \tag{8}$$

2.1. Lower Bounds

Below we give a formula to calculate lower bounds on outcome components. For a given vector of weights λ, $\lambda_i > 0$, $i = 1,...,k$, let $y(\lambda)$ be an implicit properly efficient outcome, which would be derived if optimization problem (6) were solved with that λ.

For a given lower shell S_L the lower bounding formula is

$$y_i(\lambda) \geq L_i(S_L, \lambda)$$

$$\max\{y_i^* - (\lambda_i(1+\rho))^{-1} \max_{y \in f(S_L)}[\max_j \lambda_j((y_j^* - y_j) \quad (9)$$

$$+ \rho e^k(y^* - y))] + \frac{\rho}{1+\rho}\sum_{j \neq i}^{k}(y_j^* - U_j(\lambda)), \overline{L}_i\}, \ i = 1,...,k,$$

where $U_j(\lambda)$ are such that $y_j \leq U_j(\lambda), \ j = 1,...,k, \ j \neq i$. One possible selection of $U_j(\lambda)$ is $\overline{U}_j, \ j = 1,...,k, \ j \neq i$, where $\overline{U}_j$ is defined by (8). Here we extend notation $L(\lambda)$ to $L(S_L, \lambda)$ to stress dependence of lower bounds on lower shells S_L.

Putting $\rho = 0$ in (9) we get the lower bounding formula for weakly efficient outcomes, derived in [7].

2.2. Upper Bounds

Below we give a formula to calculate upper bounds on outcome components. For a given vector of weights $\lambda, \ \lambda_i > 0, \ i = 1,...,k$, let $y(\lambda)$ be, as previously, an implicit properly efficient outcome, which would be derived if optimization problem (6) were solved with that λ.

Suppose that an upper shell S_U is given. To calculate upper bounds on components of efficient outcomes, for each element $\overline{y}$ of $f(S_U)$ we have to know vector $\lambda, \ \lambda_i > 0, \ i = 1,...,k$, such that $\overline{y}$ solves optimization problem (6) on $f(S_U)$ for that λ. To stress the association between λ and $\overline{y}$ we denote $\lambda = \lambda(\overline{y})$.

It is easy to show that any ρ-properly efficient element $\overline{y}$ of $f(S_U)$ solves optimization problem (6) on $f(S_U)$ with $\lambda = \overline{\lambda}(\overline{y})$, where

$$\overline{\lambda}_i(\overline{y}) = ((y_i^* - \overline{y}_i) + \rho e^k(y^* - \overline{y}))^{-1}, \ i = 1,...,k. \quad (10)$$

Indeed, for $\overline{\lambda}_i(\overline{y}), \ i = 1,...,k$, we clearly have, by the definition of y^*,

$$\overline{\lambda}_i(\overline{y}) > 0,$$

and

$$\overline{\lambda}_i(\overline{y})((y_i^* - \overline{y}_i) + \rho e^k(y^* - \overline{y})) = 1.$$

Since all elements of S_U are ρ-properly efficient in S_U, $\bar{y}$ is a solution of optimization problem (7) for some $\lambda_i > 0,\ i = 1,...,k$, i.e. for all $y \in S_U$

$$\max_i \lambda_i((y_i^* - y_i) + \rho e^k (y^* - y)) \geq \max_i \lambda_i((y_i^* - \bar{y}_i) + \rho e^k (y^* - \bar{y})).$$

Hence, for some j

$$\lambda_j((y_i^* - y_j) + \rho e^k (y^* - y)) \geq \lambda_j((y_j^* - \bar{y}_j) + \rho e^k (y^* - \bar{y})),$$

and

$$(y_j^* - y_j) + \rho e^k (y^* - y) \geq (y_j^* - \bar{y}_j) + \rho e^k (y^* - \bar{y}).$$

Thus,

$$\bar{\lambda}_j(\bar{y})((y_j^* - y_j) + \rho e^k (y^* - y)) \geq \bar{\lambda}_j(\bar{y})((y_j^* - \bar{y}_j) + \rho e^k (y^* - \bar{y}))$$

In consequence, for each $y \in S_U$ we have

$$\max_i \bar{\lambda}_i(\bar{y})((y_i^* - y_i) + \rho e^k (y^* - y)) \geq \max_i \bar{\lambda}_i(\bar{y})((y_i^* - \bar{y}_i) + \rho e^k (y^* - \bar{y})) = 1.$$

Hence, $\bar{y}$ is a solution of (6) on $f(S_U)$ with $\lambda = \bar{\lambda}_i(\bar{y})$.

For a given upper shell S_U the upper bounding formula is

$$y_i(\lambda) \leq U_i(S_U, \lambda)$$

$$\min\{\min_{\bar{y} \in f(s_U)}[\min_{l \in I(\lambda)}\{y_l^* + \frac{\rho}{1+\rho}\sum_{j \neq l}^k y_j^* - \bar{\lambda}_l(\bar{y})^{-1}(1+\rho)^{-1} \tag{11}$$

$$- \frac{\rho}{1+\rho}\sum_{j \neq i}^k L_j(\lambda)\}]\}, \bar{U}_i\},\ i = 1,...,k,$$

where $I(\lambda)$ is a subset of indices $\{1,2,...,k\}$ such that $l \in I(\lambda)$ if $t^l = \min\{t^1,...,t^k\}$, where

$$t^i = ((\tau_i + \rho e^k \tau)\bar{\lambda}_i(\bar{y}))^{-1},$$

τ is defined by formula

$$\tau = y^* - \bar{y}, \tag{12}$$

and $L_j(\lambda)$ are such that $y_j(\lambda) \geq L_j(\lambda),\ = 1,...,k,\ j \neq l$. One possible selection of $L_j(\lambda)$ is $\bar{L}_j,\ j = 1,...,k,\ j \neq i$, where $\bar{L}_j$ is defined by (8). Here we extend notation $U(\lambda)$ to $U(S_U, \lambda)$ to stress dependence of lower bounds on upper shells S_U.

Putting $\rho = 0$ in (11) we get the upper bounding formula for weakly efficient outcomes, derived in [7].

Concluding remarks and directions for further research

The obvious advantage of replacing shells, which are to be derived by solving optimization problems, with their lower and upper counterparts S_L and S_U, which can be derived, as in [7], by evolutionary computations, would be complete elimination of exact optimization from MCDM.

The open question is the quality (tightness) of assessments when $S_L \not\subset N$, $S_U \not\subset N$. This question imposes itself on the same question with respect to assessments derived with $S_L = S_U \subset N$, addressed in Kaliszewski [3], [4]. However, if S_L and S_U derived by evolutionary computations are "close" to N there should be no significant deterioration in the quality of assessments. Indeed, preliminary experiments with some test problems reported in [7], confirm such expectations.

To make condition (4) of the definition of upper shells operational one has to replace N by S_L, for obviously N is not known (for details cf. [7]), but with such a replacement formulas (9) and (11) remain valid (though in principle they become weaker).

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ON THE PROPERTIES OF STOCHASTIC MULTIPLE-CRITERIA COMPARISON METHODS IN HEALTH TECHNOLOGY ASSESSMENT

Abstract

To ensure the optimal usage of scarce resources in the assessment of health technologies two criteria are used: costs and effectiveness of available options. For each treatment the evaluations of these criteria are obtained from clinical trials, cost and utility studies and therefore are given as random variables. In our research we compare the decision theoretic properties of expected net benefit, cost-effectiveness acceptability curve and expected value of perfect information methods of choosing the optimal treatment.

Keywords

Cost-effectiveness acceptability curves; net benefit; willingness to pay; uncertainty.

Introduction

In the paper we consider the problem of comparing a given finite set of available therapies (cf. [9], [16]). We assume that the decision maker bases her decision on two criteria: the expected costs and effects of the therapies, but she is not able to measure these parameters with certainty (cf. 6). Instead, she has some estimates available, e.g. results of clinical trials, cost studies or utility of health state evaluations (cf. 20).

It is usually assumed that the decision maker compares the therapies *incrementally*, i.e. calculates the ratio of increment of the expected cost to the increment of the expected effect when switching from a worse and cheaper therapy to a more effective and expensive one (dominated and extendedly--dominated therapies are excluded). This ratio is called the incremental cost-effectiveness ratio (henceforth $ICER$). When both the numerator and denominator are positive, $ICER$ is interpreted as expected additional cost that needs to be incurred to obtain an additional unit of expected effect. It is then compared with the threshold value, named willingness-to-pay, henceforth WTP (cf. [3]), representing societal preferences. If $ICER$ is below this value then therapy switch is recommended. For the theoretical foundation of this approach see Garber and Phelps [8].

As the decision maker knows only the estimates of expected costs and effects, the $ICER$ is also given with uncertainty. Analyzing this uncertain $ICER$ presents statistical difficulties due to several causes. When expected costs and effects estimates are normally distributed, so are their increments, and the ratio is vulnerable to a so-called Hodgson paradox, i.e. it can have a Cauchy distribution without any mean value or variance defined (cf. [12]). Moreover, for some data and methods (e.g. Fieller's method) the calculated confidence intervals can be empty, constitute a set of disconnected intervals, or encompass the whole real line (cf. [2], [13]). Another problem is that the negative $ICER$ loses its interpretation (can mean that the therapy is either dominated or dominant), so confidence intervals containing negative values are meaningless.

The solution to the above problems proposed in the literature and analyzed in this paper is to analyze the so-called net benefit, i.e. the difference between the expected effect expressed in monetary terms (using WTP as the monetary value of a unit of effect) and the expected cost (cf. [18], [19], [16]). Then the decision maker can compare the net benefits of all available therapies and choose the therapy offering the biggest net benefit; we shall call this therapy to be *cost-effective*. The uncertainty of expected cost and effect results in the uncertainty of net benefit. However, as the calculation of net benefit involves only multiplication and addition, this parameter has better statistical properties than $ICER$, while remaining equivalent in terms of decision making process when no uncertainty is present (cf. [15]).

The expected net benefit (ENB) approach does not directly take into account the stochastic nature of expected cost and effect estimate. Therefore two additional measures have been proposed in the literature: cost-effectiveness acceptability curve ($CEAC$) and expected value of perfect information ($EVPI$).

The $CEAC$ approach gives the probability that the true net benefit of the therapy is the highest among all alternatives considered (cf. [20]), and, therefore, the probability that the therapy is cost-effective. The $EVPI$ criterion measures the expected value of removing uncertainty from the decision making problem – in other words, how much at maximum should a decision maker be willing to pay for perfect information about the true expected costs and effects of the options compared (cf. [3], [5]).

The objective of the paper is to analyze the decision-theoretic properties of maximization of ENB and $CEAC$ and minimization of $EVPI$ criteria for the selection of optimal therapy in health technology assessment. This paper continues the work of Jakubczyk and Kamiński [14] formalizing some of their ideas, developing the properties of $EVPI$ criterion and introducing uncertainty of WTP assessment.

In the next section we present the notation used throughout the paper, introduce the choice rules used in health technology assessment and present the properties of choice rules usually demanded in decision analysis. In the third section we analyze the properties of choice criteria introduced for a given value of societal willingness-to-pay. In the fourth section we present the analysis in the case of random value of willingness-to-pay (representing the uncertain elicitation of societal preferences). The last section is a summary.

1. The model of therapy comparison

In this section we present a general notation used in the paper. First we describe the set of alternatives and decision maker's uncertainty. Then we formalize the decision making process by defining the choice function – a method of choosing one of the alternatives and some often required properties. Finally we introduce three choice functions based on the ENB, $CEAC$ and $EVPI$ criteria.

Throughout the whole paper we analyze the decision maker’s choosing from a given set of n therapies represented by the set $I = \{1,2,\ldots,n\}$. Each therapy i is associated with its expected cost and effect. The true values of these are not known; instead, the decision maker knows their distributions (resulting from the estimation procedure) defined for cost and effect, respectively, by the random variables C_i and E_i. We assume that these random variables are independent across therapies, i.e. any subset of $\{C_1,\ldots,C_n,E_1,\ldots,E_n\}$ with random variables of different indices (e.g. not containing simultaneously variables C_i and E_i) is independent. We do not assume that C_i is independent from E_i.

For each i we define a probability space (Ω_i, F_i, P_i) that describes the distribution of the two-dimensional random variable (C_i, E_i). We also define a probability space (Ω, F, P) that is a product space for all i. Thus it represents the whole uncertainty present in the problem.

In this paper we analyze the choice based on the comparison of net benefit, where the equivalent of a unit of effect in monetary terms (WTP) is denoted for brevity by k. Therefore, throughout most of the paper we do not directly analyze C_i and E_i, but define a new random variable $NB_{i,k} = kE_i - C_i$ denoting the expected net benefit of the therapy i given that the value of WTP is equal to k.

We assume that all random variables are continuously distributed and denote cumulative distribution function of $NB_{i,k}$ as $\Phi_{i,k}(\cdot)$ and its density function as $\phi_{i,k}(\cdot)$. Notice that $\{NB_{1,k}, NB_{1,k},\ldots, NB_{n,k}\}$ is the set of independent random variables.

We now define ENB, $CEAC$ and $EVPI$ measures for the therapy i:

$$\begin{aligned} ENB_{i,k} &= E(NB_{i,k}), \\ CEAC_{i,k} &= \Pr(NB_{i,k} = \max_{t\in I}\{NB_{t,k}\}), \\ EVPI_{i,k} &= E(\max_{t\in I}\{NB_{t,k}\} - NB_{i,k}). \end{aligned} \tag{1}$$

Let us notice that $CEAC$ and $EVPI$ methods are related to the concept of Savage's regret criterion – that is the difference between the selected therapy i net benefit $NB_{i,k}(\omega)$ and the optimal therapy net benefit $\max_{t\in I}\{NB_{t,k}(\omega)\}$ averaged out over all $\omega \in \Omega$. They differ in the measure of regret. The $CEAC$ criterion assumes constant regret (equal to 1) if the therapy is not optimal and the $EVPI$ criterion assumes that regret is proportional to the expected net benefit loss. Both of these can be rationalized for the decision maker in specific situations. If the decision maker cares only to make a decision that will be confirmed to be optimal *a posteriori* when enough evidence is gathered to remove uncertainty from the problem then she should use the $CEAC$ approach. When the decision maker is mostly concerned not with probability but the expected monetary value of making sub-optimal decision then she should use the $EVPI$ approach.

Before moving to the definition of choice functions based on the on ENB, $CEAC$ and $EVPI$ criteria we outline the desired properties of such functions in a decision-theoretic approach.

We formalize the decision making process by introducing a *choice function* as a representation of a method of making a choice. For a given set of alternatives $I \neq \emptyset$ let us define a regular choice function T following Hammond [10]:

$$T : 2^I \to 2^I ,$$
$$\forall \emptyset \neq I' \subset I : \emptyset \neq T(I') \subset I .$$

Therefore a regular choice function selects a non-empty subset of the given set of alternatives. Henceforth we consider only regular choice functions, and call them simply choice functions.

Regularity does not imply that the choice function have the intuitive properties usually required. One of these properties is *coherence*. We will call a choice function T coherent, if:

$$\forall \emptyset \neq I'' \subset I' \subset I : I'' \setminus T(I'') \subset I' \setminus T(I').$$

Coherence means that if an alternative is not selected out of a smaller subset of alternatives (I''), it will not be selected when additional alternatives are available (and the bigger set I' is considered). It is often required that a “nice” choice function be coherent as otherwise it is open to manipulation – adding irrelevant (not chosen) alternatives can change the outcome. This property is also referred to as α-property or basic contraction consistency, cf. Sen [17].

If choice function T is coherent then it generates a pre-order $\leq$ in I, such that $\forall I' \subset I : T(I') = \{x \in I' : \forall y \in I' : y \leq x\}$ (cf. [11]).

Additionally we will call choice functions T' and T'' equivalent if $\forall \emptyset \neq I' \subset I : T'(I') = T''(I')$.

Now using those definitions let us introduce choice functions stemming from ENB, $CEAC$ and $EVPI$ measures in health technology assessment given a set of therapies $I' \subset I$.

Table 1

Health technology evaluation measures and choice functions associated with them

Measure	Choice function
ENB	$T_k^{ENB}(I') = \arg\max_{i \in I'}\{ENB_{i,k}\}$
$CEAC$	$T_k^{CEAC}(I') = \arg\max_{i \in I'}\{CEAC_{i,k}\}$
$EVPI$	$T_k^{EVPI}(I') = \arg\min_{i \in I'}\{EVPI_{i,k}\}$

In the next section we will analyze the properties of the choice functions introduced above.

2. Properties of choice functions for fixed *WTP*

In this section we first analyze the coherence properties of ENB, $CEAC$ and $EVPI$ and their conditions for their equivalence for fixed WTP (k parameter). Let us start with the comparison of the of ENB and $EVPI$ criteria.

Proposition 1.

The choice functions T_k^{ENB} and T_k^{EVPI} are coherent and equivalent.

Proof

First we will show the equivalence of those two choice functions. Notice that for $I' \subset I$:

$$EVPI_{i,k} = E\left(\max_{t \in I'}\{NB_{t,k}\} - NB_{i,k}\right) = E\left(\max_{t \in I'}\{NB_{t,k}\}\right) - ENB_{i,k} .$$

But this implies that:

$$T_k^{EVPI}(I') = \arg\min_{i \in I'}\{EVPI_{i,k}\} = \arg\min_{i \in I'}\left\{E\left(\max_{t \in I'}\{NB_{t,k}\}\right) - ENB_{i,k}\right\}.$$

Notice that $E\left(\max_{t\in I'}\{NB_{t,k}\}\right)$ is constant given I' so:

$$T_k^{EVPI}(I') = \arg\min_{i\in I'}\{-ENB_{i,k}\} = \arg\max_{i\in I'}\{ENB_{i,k}\} = T_k^{ENB}(I').$$

This implies that these choice functions are equivalent. Hence, to prove their coherence it is enough to check that $T_k^{ENB}(I')$ is coherent. To show this consider any $I'' \subset I' \subset I$. Assume that $i \in I'' \setminus T(I'')$. This implies that there exists $j \in I''$ such that $ENB_{i,k} < ENB_{j,k}$. However $i, j \in I'$, so i will not be an element of $T(I')$, as ENB does not depend on a set of available alternatives. This implies that T_k^{ENB} is coherent.

Although the $CEAC$ criterion, similarly to $EVPI$, is also based on the regret concept, it has different properties than the ENB and $EVPI$ approaches. Jakubczyk and Kamiński ([14]) showed that the choice function T_k^{CEAC} is not coherent and therefore not equivalent to T_k^{ENB} and T_k^{EVPI}. The following example illustrates this issue.

Example 1

Consider the following three distributions of ENB:

$$ENB_{1,k} \sim N(0,10);$$

$$ENB_{2,k} \sim N(1,1);$$

$$ENB_{3,k} \sim N(1,1).$$

Let us consider the following sets $I'' = \{1,2\}$ and $I' = \{1,2,3\}$. Using the properties of normal distribution we can calculate that for the set I'': $CEAC_{1,k} \cong 46\%$ and $CEAC_{2,k} \cong 54\%$. Therefore $T_k^{CEAC}(I'') = \{2\}$. However, for the set I' we get: $CEAC_{2,k} = CEAC_{3,k} \cong 28\%$ and $CEAC_{1,k} \cong 44\%$. Therefore $T_k^{CEAC}(I') = \{1\}$. So T_k^{CEAC} is not coherent.

But if T_k^{CEAC} is not coherent then it is also not equivalent to T_k^{ENB} and T_k^{EVPI} that are coherent.

□

Fenwick *et al.* ([7]) show that $CEAC$ and ENB methods are equivalent for two therapies if the distributions of NB_i are symmetric. The above--presented proof shows that this property does not hold for more than two therapies.

We have shown that in general $CEAC$ method can give different results than ENB and $EVPI$ criteria. However, there are cases when those methods give the same recommendations. Jakubczyk and Kamiński ([14]) postulated that if one option dominates the other in the sense of first-order stochastic dominance, then this option has a greater probability of being cost-effective (even in the case of a choice from more than two options). We prove this assertion in the following proposition.

Proposition 2.

If there exists such $i \in I$ that $NB_{i,k}$ first order stochastically dominates $NB_{j,k}$ for $j \neq i$ then T_k^{CEAC}, T_k^{ENB} and T_k^{EVPI} are equivalent.

Proof

Define random variables X_t that have the same distribution as $NB_{t,k}$ but are independent from all $NB_{t,k}$ variables. We have for all $j \in I \setminus \{i\}$:

$$CEAC_{j,k} = \Pr\left(NB_{j,k} > \max_{t\in I\setminus\{j\}}\{NB_{t,k}\}\right) =$$

$$= \Pr\left(X_j > \max\left\{X_i, \max_{t\in I\setminus\{i,j\}}\{NB_{t,k}\}\right\}\right) <$$

$$< \Pr\left(X_i > \max\left\{X_j, \max_{t\in I\setminus\{i,j\}}\{NB_{t,k}\}\right\}\right) <$$

$$< \Pr\left(NB_{i,k} > \max_{t\in I\setminus\{i\}}\{NB_{t,k}\}\right) = CEAC_{i,k}$$

Therefore $T_k^{CEAC}(I) = \{i\}$.

However, first order stochastic dominance of $NB_{i,k}$ over $NB_{j,k}$, $j \neq i$ implies that $\forall j \neq i : E(NB_i) > E(NB_j)$ [1]. Therefore also $T_k^{ENB}(I) = T_k^{EVPI}(I) = \{i\}$.

□

In the above analysis we have shown that for fixed WTP approaches using ENB and $EVPI$ are coherent and equivalent. On the other hand $CEAC$ criterion is not coherent and not equivalent to the above two. Therefore one can conclude that $CEAC$ method should not be used as a basis for decision support in health technology assessment.

Setting the value of WTP is rather a matter of consensus than estimation (even though theoretical models have been proposed – e.g. [8]). Some methods encompass using the price of a referential therapy (usually dialysis) or referring to gross domestic product *per capita* (e.g. setting WTP to be three times greater). Due to these informal methods in applied research the value of WTP is treated as given approximately, and therefore can be analyzed as a randomly distributed variable. The next section explores these issues.

3. Properties of choice functions for random *WTP*

Now let us assume that the decision maker does not know k (WTP) with certainty, but assumes that it has a continuous random distribution (independent from C_i and E_i). In this section we abandon the analysis of $CEAC$ method as not recommended and concentrate on ENB method only ($EVPI$ method is also not analyzed as it was shown in Section 2 to be equivalent to ENB).

We will consider two approaches of the decision maker. In the first one we assume that the decision maker prefers the option with the highest probability of being chosen by the ENB criterion given the uncertainty of the evaluation k. Formally, we define the evaluation of *probability of the expected net benefit* ($PENB$) maximization of the alternative i as follows:

$$PENB_i = \Pr\left(i \in T_k^{ENB}(I)\right), \tag{2}$$

where the probability is taken over the distribution of k. The choice rule associated with this criterion is $T_k^{PENB}(I') = \arg\max_{i \in I'} \{PENB_{i,k}\}$.

In the second approach we assume that the decision maker maximizes the expected value of net benefit including the uncertainty of k. Formally, we define the evaluation of the *total expected net benefit* ($TENB$) of the alternative i as follows:

$$TENB_i = E\left(NB_{i,k}\right), \tag{3}$$

where the expectation is taken over the distribution of k, C_i and E_i. The obvious choice rule associated with this criterion can be defined as $T_k^{TENB}(I') = \arg\max_{i \in I'} \{TENB_{i,k}\}$.

We will show that the $TENB$ choice rule is coherent while $PENB$ is not (which also implies that they are not equivalent).

Proposition 3.

The choice function T^{TENB} is coherent, while the choice function T^{PENB} is not.

Proof

First we show the coherence of T^{TENB}. Consider any $I'' \subset I' \subset I$. Assume that $i \in I'' \setminus T(I'')$. This implies that there exists $j \in I''$ such that $E\left(NB_{i,k}\right) < E\left(NB_{j,k}\right)$. However $i, j \in I'$, so i is not an element of $T(I')$, as $E\left(NB_{i,k}\right)$ does not depend on the set of available alternatives. This implies that T^{TENB} is coherent.

To prove the second part of the proposition consider the following counterexample:

$E(C_1) = 1$ and $E(E_1) = 1$;

$E(C_2) = 2$ and $E(E_2) = 2$;

$E(C_3) = 3.5$ and $E(E_3) = 3$;

and assume that k has uniform distribution over the set $[0.1;2]$.

Let us consider the sets $I'' = \{1,2\}$ and $I' = \{1,2,3\}$ (all the calculations are illustrated in Figure 1).

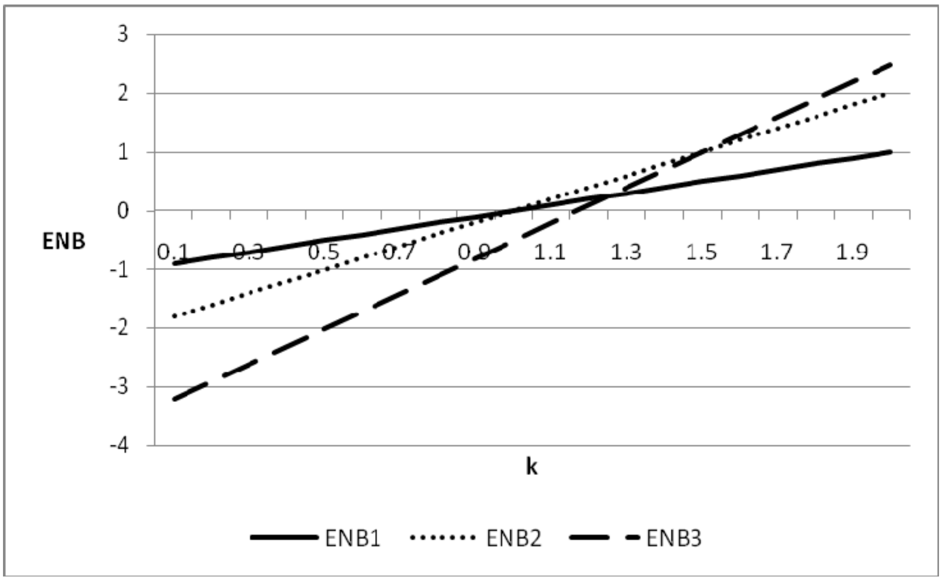

Figure 1. Presentation of $ENB_{i,k}$ as a function of k

We start with the analysis of the set I''. Notice that $E(E_1 - kC_1) > E(E_2 - kC_2)$ for $k \in [0.1;1]$. Therefore in this case $E(E_1 - kC_1)$ and $PENB_2 \cong 53\%$ and consequently $T^{PENB}(I'') = \{2\}$.

Now let us move on to the analysis of the set I'. Notice that $E(E_1 - kC_1)$ is optimal for $k \in [0.1;1]$, but $E(E_2 - kC_2) > E(E_3 - kC_3)$ for $k \in [1;1.5]$. Therefore in this case $E(E_1 - kC_1)$ and $PENB_2 = PENB_3 \cong 26.5\%$ and, consequently, $T^{PENB}(I') = \{1\}$. This implies that T^{PENB} is not coherent.

□

Summing up, the criterion of maximal fraction of good choices ($PENB$) again proves not to be coherent. Therefore it is recommended to use the criterion of averaging out the uncertainty of k ($TENB$).

4. Discussion

In this paper we analyzed the formal properties of methods of comparison used in the applied health technology assessment taking into account cost, effectiveness and willingness-to-pay criterions. These methods encompassed expected net benefit, cost-effectiveness acceptability curves and expected value of perfect information.

The basic conclusion from the paper is that, for given societal willingness-to-pay, minimizing the expected value of perfect information is equivalent to maximizing the expected net benefit of an option. Both of these methods are coherent and therefore robust to manipulation through adding irrelevant alternatives. Conversely, maximizing the probability of making the best choice, i.e. using the cost-effectiveness acceptability curves, do not yield coherent choices. These properties hold when we consider indeterminacy in WTP valuation. Again, maximizing the probability of making the best choice is a non-coherent method, while maximizing the expected net benefit of a choice is coherent.

In general, the choice method used in decision making should, on one hand, result from the preferences of the decision maker, but on the other, from the verification of statistical properties. Otherwise the decision making process may be prone to (possibly unintended) manipulation. It should be required that the decision maker is aware of the properties of the choice criterion so that she can structure her decision problem properly, i.e. choose the set of options compared in some preceding phase.

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Leszek Klukowski

OPTIMIZATION OF PUBLIC DEBT MANAGEMENT IN THE CASE OF STOCHASTIC BUDGETARY CONSTRAINTS

Abstract

The paper presents a stochastic approach to strategic optimization of public debt management in Poland, aimed at minimization of two criterions: servicing costs of the debt and costs resulting from stochastic budgetary constraints. The main results comprise: formulation of the problem, determination of necessary components (parameters, forecasts, etc.) and the method of problem solution. The results show the complexity of the problem and gains from its implementation (budgetary savings). The paper is based on research conducted in Polish Ministry of Finance [6].

Keywords

Optimization of debt management, multiple criteria deficit and surplus, stochastic budgetary constraints.

Introduction

Decision problems, which appear in optimization of public debt management, are typically of stochastic nature. Their main stochastic components are: forecasts of interest rates and constraints of budgetary requirements. Risk resulting from interest rates is discussed broadly in the literature (see e.g. [1], [2]). The random character of budgetary requirements is of similar importance, because changes of their level together with the non-linear form of the criterion function and constraints can influence optimal solutions in unexpected ways. The range of methods, which take into account the stochastic form of the constraints, is quite extensive. However, some empirical limitations, e.g. computation time, mathematical complexity, knowledge of necessary functions, parameters, etc., limit the feasible set in this area. The approach used in the paper combines mathematical simplicity with the main features of the actual problem. It exploits the idea of goal programming with stochastic constraints expressing budgetary requirements. The constraints indicate surplus or deficit, which result in certain costs. They are incorporated into the criterion function together with servicing costs of the debt. The random constraints generate additional decision variables and increase the size of the problem – proportionally to the sizes of sets of values of the random variables.

The aim of this paper is to present a complete solution of the stochastic problem based on empirical data, i.e.: the formulation of the task, an algorithm for its solution and empirical results.

The paper consists of five sections. The main results – formulation of optimization problem, determination of its components (i.e. functions, parameters, forecasts) and empirical results (an example of optimal solution) are presented in Sections 1-3. The last section summarizes the results.

1. Formulation of optimisation task

The problem examined in the paper can be stated as follows:

To determine the optimal portfolio of treasury securities (bonds):

- aimed at minimizing of the criterion function comprising: servicing costs of securities and costs of deficit/surplus resulting from stochastic constraints of budgetary requirements – in three years period,
- under constraints on: risk level and other features of the debt.

The optimization task for the problem can be formulated as an extension of the deterministic approach [5], i.e. without costs of deficit and surplus, resulting from stochastic budgetary requirements. The deterministic task, formulated for the set of bonds issued in Poland (in the year 2001), can be written as follows (with budgetary constraints only):

$$\min_{x_{it}} \left\{ \sum_{t=1}^{3} \sum_{i=1}^{K} x_{it} (M - d^{(it)}(x_{it})) \varphi^{(it)}(x_{it}) \right\}, \tag{1}$$

$$\sum_{i=1}^{K} x_{i1} (M - d^{(i1)}(x_{i1})) = \mathrm{A}_1, \tag{2}$$

$$\sum_{i=1}^{K} x_{i2} (M - d^{(i2)}(x_{i2})) = \mathrm{A}_2, \tag{3}$$

$$\sum_{i=1}^{\kappa} x_{i3}(M - d^{(i3)}(x_{i3})) - M\, x_{1,1} = \mathrm{A}_3\,, \tag{4}$$

$$x_{it}^{\min} \le x_{it} \le x_{it}^{\max} \quad (i=1, ..., \kappa;\ t=1, 2, 3), \tag{5}$$

where:

x_{it} (i=1, ...,κ; t=1, 2, 3) – sale of i-th bond in year t – decision variable, κ – number of bonds issued,

$d^{(it)}(x_{it})$ – average discount of i-th bond corresponding to sale level x_{it},

$\varphi^{(it)}(x_{it})$ – compound rate of return (CRR) of i-th bond, corresponding to sale level x_{it} (8 – years investment horizon),

A_t – budgetary requirement (in capital constraint), in year t,

M – nominal value of one bond (1000 of Polish zlotys).

The set of bonds issued in 2001 comprises three fixed rates bonds (two-years – x_{1t}, five-years – x_{2t}, ten-years – x_{3t}) and one ten-years variable rate bond x_{4t}. The constraint (4) includes the term $M\,x_{11}$, which reflects the amount of redemption of the two-year bond, issued in the first year of the period; it increases budgetary requirements in the third year. The investment horizon (8 years) in compound rate of return [4] has been determined as a median in redemption schedule; it is clear that the median exceeds the optimization period (three years).

Stochastic level of budgetary requirements indicates the replacement of the vector $\mathbf{A}' = [\mathrm{A}_1, \mathrm{A}_2, \mathrm{A}_3]$ (symbol $\mathbf{A}'$ – means transposed vector) with the vector of random variables $\mathbf{\Lambda}' = [\Lambda_1, \Lambda_2, \Lambda_3]$. The distribution functions of the variables Λ_t (t=1, 2, 3) can be written in the form:

$$P(\Lambda_t = A_{tr}) = p_{tr} \qquad (r = 1, ..., s_t;\ s_t \ge 1)\,, \qquad \sum_{r=1}^{s_t} p_{tr} = 1, \tag{6}$$

where:

A_{tr} (t=1, 2, 3; r=1, …, s_t) – an element of the value set of the random variable Λ_t; at least one value $s_t\ (1 \le t \le 3)$ satisfies $s_t \ge 2$.

The random variables Λ_t, incorporated into the constraints (2) – (4), indicate the possibility of discrepancy in capital constraint, i.e. the realization of the random value A_{tr} can be different from the deterministic value A_t. The case when $\sum_{i=1}^{K} x_{it}(M - d^{(it)}(x_{it}))$ is lower than the actual capital requirement means deficit, while the opposite case, surplus. These situations can generate some costs; deficit – the necessity of extra borrowing under higher rates, surplus – the necessity of deposits with rates lower than the profitability of bonds issued. For simplicity, the costs of deficit and surplus are assumed constant (for any level and structure of bonds issued in year *t*). Moreover, it is assumed that:

$$\gamma_t\,,\ \eta_t \geq 0\,, \tag{7}$$

$$\gamma_t + \eta_t > 0\,, \tag{8}$$

where:

γ_t $(t = 1, 2, 3)$ – cost of deficit,
η_t $(t = 1, 2, 3)$ – cost of surplus.

The variables expressing deficit y_{tr} and surplus z_{tr}, included in a set of decision variables, are defined as follows $(t = 1, 2, 3;\, r = 1, ..., s_t)$:

$$y_{tr} = \max\{A_{tr} - \sum_{i=1}^{K} x_{it}(M - d^{(it)}(x_{it}))\,,\ 0\}\,, \tag{9}$$

$$z_{tr} = \max\{\sum_{i=1}^{K} x_{it}(M - d^{(it)}(x_{it})) - A_{tr}\,,\ 0\}\,. \tag{10}$$

The cost resulting from the deficit y_{tr} is equal to $\gamma_t y_{tr}$, while the cost resulting from the surplus, to $\eta_t z_{tr}$. Each of the values y_{tr} or z_{tr} appears with the probability p_{tr} and therefore the expected value of the cost of incorrect capital level equals $\sum_{t=1}^{3}\sum_{r=1}^{s_t} p_{tr}(\gamma_t y_{tr} + \eta_t z_{tr})$. This expression is added to the criterion function (1) as the second criterion. It is clear that the terms expressing the costs of deficit and surplus have to be compatible with the term expressing servicing costs of the debt. Therefore the costs of deficit and surplus have to be precisely determined – also with the possibility of different values of individual levels of budgetary requirements.

The random level of budgetary requirements implies modifications of the feasible set of the task (1)-(5); the differences: $y_{tr} - z_{tr}$ are added to left hand sides of the inequalities (2)-(4). It is also reasonable to include the costs resulting from the deficit and surplus into constraints for servicing costs of the debt. Taking into account the modifications, the task (1) – (5) assumes the form:

$$\sum_{t=1}^{3}\sum_{i=1}^{\kappa} x_{it}\,(M\text{-}d^{(it)}(x_{it}))\,\varphi^{(it)}(x_{it}) + \sum_{t=1}^{3}\sum_{r=1}^{s} p_r\,(\gamma_t\, y_{tr} + \eta_t\, z_{tr}) \to \min\,, \qquad (11)$$

$$\sum_{i=1}^{\kappa} x_{i1}\,(M - d^{(i1)}(x_{i1})) + y_{1r} - z_{1r} = A_{1r} \qquad (r=1, \ldots, s_1), \qquad (12)$$

$$\sum_{i=1}^{\kappa} x_{i2}\,(M - d^{(i2)}(x_{i2})) + y_{2r} - z_{2r} = A_{2r} \qquad (r=1, \ldots, s_2), \qquad (13)$$

$$\sum_{i=1}^{\kappa} x_{i3}\,(M - d^{(i3)}(x_{i3})) + y_{3r} - z_{3r} - M \cdot x_{1,1} = A_{3r} \qquad (r=1, \ldots, s_3), \qquad (14)$$

$$x_{it}^{\min} \le x_{it} \le x_{it}^{\max} \qquad (i=1, \ldots, \kappa;\ t=1, 2, 3), \qquad (15)$$

(y_{tr}, z_{tr} – defined in (9), (10)).

Additionally, the stochastic task generates $2\prod_{t=1}^{3} s_t$ variables and constraints. Thus, the complexity of the stochastic task increases in comparison with the deterministic one.

2. Determination of components of stochastic task

The parameters and functions necessary to formulate the numerical form of the problem (11) – (15) comprise:

a) the probability functions of the random variables Λ_t,
b) the rates γ_t, η_t,
c) the functions $d^{(it)}(x_{it})$ and $\varphi^{(it)}(x_{it})$,
d) feasible sets (intervals) for decision variables x_{it}.

The functions $d^{(it)}(x_{it})$, $\varphi^{(it)}(x_{it})$ and the feasible intervals appear also in the deterministic form of the problem.

2.1. Parameters of stochastic constraints

The parameters of stochastic constraints together with the cost of deficit and surplus are of crucial importance for empirical results. They are typically determined on the basis of experts opinions or with the use of statistical methods (probability functions, forecasts). The parameters, costs and functions have been determined in the following way: deficit rates – on the basis of compound rate of return of bonds from previous years, surplus rates – on the basis of credit and deposit spread. The values of these parameters are presented in Table 1.

The probability functions of budgetary requirements have been determined on the basis of budget realizations from previous years. The number of possible levels of the requirements has been assumed to be three in each year – minimal, medium and maximal – with the same probability of each level in consecutive years (see Table 2). Such a number allows to avoid a large size of optimization problem (number of variables and constraints). The number of levels of the requirements can be increased, if necessary.

Table 1

Rates of shortage and surplus

	2002	2003	2004
Rate of shortage	0,1011	0,1004	0,0952
Rate of surplus	0,0101	0,0100	0,0095

Table 2

Variants of budgetary requirements in the years 2002–2004 and their probability functions

Year	Variant I (r=1)	Variant II (r=2)	Variant III (r=3)
2002	61 719 000 000	63 719 000 000	59 719 000 000
2003	60 596 000 000	62 696 000 000	58 496 000 000
2004	56 554 000 000	58 854 000 000	54 254 000 000
Probab. function	0,5	0,3	0,2

2.2. Forecasting of CRR functions

The compound rate of return of treasury bonds (symbol $\varphi^{(it)}(x_{it})$ (i=1, …, 4) in the formula (11)) assumes a nonlinear form, with parameters determined by the results of the auctions [7]. The prediction of the functions is not an easy problem; the method used in the paper rests on two basic assumptions:

– there exists a typical shape (pattern) of the function of each type of bond,
– the forecast of each function $\varphi^{(it)}(x_{it})$ (i=1, ..., 4; t=1, 2, 3) can be expressed as the product of the pattern and the forecast of interest rate in the year t=1, 2, 3.

Thus, the forecast of each function has been obtained in the following way:

– to predict interest rates for the years t=1, 2, 3,
– to determine the pattern of compound rate of return of each bond,
– to determine the product of rate and product of each bond, with adjustment to expected demand level (for details see [6]).

The patterns of compound rate of return have been determined on the basis of data from previous years, with the use of two methods of classification: the first one – based on a statistical pairwise algorithm [3] and the second – based on the Kohonen neuronal network (SPSS Neuronal Connecting® 2.2 has been used). The empirical results of both approaches are similar.

It is clear that the components of the stochastic task, based on estimates, forecasts and experts' opinions, include imprecise variables. Such variables require careful analytical research, because they can influence significantly the optimal solution. However, the application of such data does not weaken the practicability of the optimization approach. The optimal solution provides a broad set of information for decision maker, especially resulting from the properties of the criterion function and constraints. The results of optimization can be applied in other decision models, e.g. ones based on game theory [7].

It should be stressed that the optimal solution of a stochastic task is not comparable with the deterministic one, because of difference in assumptions; the deterministic solution does not take into account costs of surplus and deficit and is solved for one level of budgetary requirements.

3. Empirical results

The example presented in this section is based on actual functions and empirical data.

Each component $x_{it}(M\text{-}d^{(it)}(x_{it}))\varphi^{(it)}(x_{it})$ (i=1, …, 4) of the criterion function is non-linear and non-convex (for 8-year investment horizon), but it is convergent to a convex piecewise linear function under weak conditions ([7], Chapter 4). Empirical researches shows that the polynomial approximation obtained with the use of the least squares method provides a convex form of the approximated components and appropriate precision. The functions expressing capital of bonds, i.e. $x_{it}(M\text{-}d^{(it)}(x_{it}))$, are piecewise linear concave functions. They can be also approximated in the same way. An alternative approach is to approximate the components of the criterion function with the use of a piecewise linear function, without approximation of capital constraints. However, this increases considerably the number of decision variables of the task, which typically includes non-linear constraints that make the solution of the problem more complicated. Therefore, a polynomial approximation, indicating a moderate number of variables, has been applied. The parameters of the approximated criterion function (polynomial form) are presented in Table 3.

Table 3

Parameters of polynomial approximations of the criterion functions
for Polish treasury bonds (2002 year)

Power of polynomial	Type of bond			
	2-year (x_{1t})	5-year (x_{2t})	10-year (fixed rate) (x_{3t})	10-year (variable rate) (x_{4t})
0 (constant)	1474,20	-6023,09	692,43	-273959,70
1	79,20	91,52	77,88	100,46
2	1,17	2,01E-08	2,69E-08	×
3	-2,06E-15	-2,89E-15	-5,81E-15	×
4	2,31E-22	2,69E-22	1,57E-21	×
5	-1,53E-29	-1,51E-29	-3,43E-28	×
6	6,32E-37	5,31E-37	5,19E-35	×
7	-1,67E-44	-1,16E-44	-4,93E-42	×
8	2,85E-52	2,54E-52	2,77E-49	×
9	-3,01E-60	-1,13E-60	-8,39E-64	×
10	1,79E-68	3,53E-69	1,05E-64	×
11	-4,61E-77	×	×	×

The approximated form of the task can be written as follows:

– the criterion function:

$$\sum_{t=1}^{3}\sum_{i=1}^{4}\sum_{k=0}^{m_{it}} a_{itk}\, x_{it}^{k} + \sum_{t=1}^{3}\sum_{r=1}^{s} p_r\,(\gamma_t\, y_{tr} + \eta_t\, z_{tr}) \rightarrow \min\,,$$

where:

x_{it}^{k} – variable x_{it} to the k-th power,

a_{itk} – polynomial coefficient of the variable x_{it} in the k-th power,

m_{it} – the degree of the polynomial for the variable x_{it},

– the constraints:
  – intervals for decision variables:

$$x_{it}^{\min} \le x_{it} \le x_{it}^{\max} \quad (i = 1, ..., 4;\ t = 1, ..., 3),$$

values $x_{it}^{\min}$ and $x_{it}^{\max}$ (in thousands) in the table below,

	x_{1t}	x_{2t}	x_{3t}	x_{4t}
$x_{it}^{\min}\ (t = 1,2,3)$	20000	30000	5000	1100
$x_{it}^{\max}\ (t = 1,2,3)$	35000	50000	12000	2000

– budgetary requirements for the individual values of surplus and shortage (i.e. y_{tr} and z_{tr}):

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i1}} b_{i,1,k}\, x_{i,1}^{k} + y_{1,1} - z_{1,1} = 61\ 719\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i1}} b_{i,1,k}\, x_{i,1}^{k} + y_{1,2} - z_{1,2} = 63\ 719\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i1}} b_{i,1,k}\, x_{i,1}^{k} + y_{1,3} - z_{1,3} = 59\ 719\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i2}} b_{i,2,k}\, x_{i,2}^{k} + y_{2,1} - z_{2,1} = 60\ 596\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i2}} b_{i,2,k}\, x_{i,2}^{k} + y_{2,2} - z_{2,2} = 62\ 696\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i2}} b_{i,2,k}x_{i,2}^{k}+y_{2,3}-z_{2,3}=58\ 496\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i3}} b_{i,3,k}x_{i,3}^{k}+y_{3,1}-z_{3,1}-Mx_{1,1}=69\ 054\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i3}} b_{i,3,k}x_{i,3}^{k}+y_{3,2}-z_{3,2}-Mx_{1,1}=71\ 354\ 000\ 000,$$

$$\sum_{i=1}^{4}\sum_{k=0}^{n_{i3}} b_{i,3,k}x_{i,3}^{k}+y_{3,3}-z_{3,3}-Mx_{1,1}=66\ 754\ 000\ 000,$$

where:

b_{itk} – coefficients of polynomial (similar as a_{itk} in the criterion function),

n_{it} – a power of the polynomial for the variable x_{it} (from the range 9 – 11 for the individual variables),

– servicing costs (in the years 2003 – 2006):

$$85\,x_{2,1}+60\,x_{3,1}+112{,}5\,x_{4,1}+0{,}5(0{,}1011\,y_{1,1}+0{,}0101\,z_{1,1})+0{,}3(0{,}1011\,y_{1,2}+ 0{,}0101\,z_{1,2})+0{,}2(0{,}1011\,y_{1,3}+0{,}0101\,z_{1,3})\le 21\ 000\ 000\ 000,$$

$$1000\,x_{1,1}+\sum_{k=0}^{1} b_{1,1,k}\,x_{1,1}^{k}+85\,x_{2,1}+60\,x_{3,1}+104{,}1\,x_{4,1}+85\,x_{2,2}+60\,x_{3,2}+ 104{,}1\,x_{4,2}+0{,}5(0{,}1004\,y_{2,1}+0{,}01\,z_{2,1})+0{,}3(0{,}1004\,y_{2,2}+0{,}01\,z_{2,2})+ +0{,}2(0{,}1004\,y_{2,3}+0{,}01\,z_{2,3})\le 27\ 000\ 000\ 000,$$

$$1000\,x_{1,2}-\sum_{k=0}^{1} b_{1,2,k}\,x_{1,2}^{k}+85\,x_{2,1}+60\,x_{3,1}+98{,}3\,x_{4,1}+85\,x_{4,2}+60\,x_{3,2}+98{,}3\,x_{4,2}+ +85\,x_{2,3}+60\,x_{3,3}+98{,}3\,x_{4,3}+0{,}5(0{,}0952\,y_{3,1}+0{,}0095\,z_{3,1})+0{,}3(0{,}0952\,y_{3,2}+0{,}0 095\,z_{3,2})+0{,}2(0{,}0952\,y_{3,3}+0{,}0095\,z_{3,3})\le 31\ 000\ 000\ 000,$$

– the share of fixed-rate bonds in the total sale of bonds in each year:

$$0{,}75\le \sum_{i=1}^{3}x_{it}\Big/\sum_{i=1}^{4}x_{it}\le 0{,}985 \qquad (t=1, 2, 3),$$

– the share of variable-rate bonds in the total sale of bonds in each year:

$$0{,}015 \le x_{4t} / \sum_{i=1}^{4} x_{it} \le 0{,}25 \qquad (t=1, 2, 3),$$

– average maturity of bonds issued in each year:

$$3{,}5 \le (2\,x_{1,t} + 5\,x_{2,t} + 10(\,x_{3,t} + x_{4,t}\,))\,/\sum_{i=1}^{4} x_{it} \;\le 5{,}4 \qquad (t=1, 2, 3),$$

– average duration of fixed rate-bonds issued in each year:

$$3{,}0 \le (2\,x_{1,t} + 4{,}2\,x_{2,t} + 7{,}5\,x_{3,t}\,)/\sum_{i=1}^{3} x_{it} \;\le 4{,}3 \qquad (t=1, 2, 3),$$

– constraint of the expression including semivariance and semicovariance matrix (see Klukowski 2003, chapt. 6):

$$[z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t}]\mathbf{Q}[z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t}]' \le 0{,}005 \qquad (t=1, 2, 3).$$

The numerical solution of the stochastic task has been obtained with the use of *solver* procedure from Excel system. The value of the criterion function corresponding to the optimal solution equals 18 673 631 500; the optimal values of variables are presented in Table 4 (sale of bonds) and Table 5 (shortage and surplus). Servicing costs of the debt assume the values (in the period from 2003 to 2006 respectively): 18 862 224 981; 22 116 427 533; 22 354 043 699; 22 247 884 741. The values of the remaining constraints are presented in Table 6.

Summary and conclusions

The paper presents an application of the multiple criteria optimization approach in the area of public debt management, under assumption about stochastic constraints of budgetary requirements.

The "quality" of debt management with the use of optimisation tools exceeds significantly the "traditional" approach. In particular, it provides budgetary savings, increases transparency of the decision process, reduces employment costs and speeds up decisions. Moreover, experience shows that computation time (with the use of *solver* procedure from Excel worksheet) is acceptable for the assumed task size (number of variables and constraints). It seems possible to solve more complex tasks – without simplifications made – e.g. aggregation of bonds in a one-year period. However, up to now, the optimisation approach has not been applied in Poland.

Table 4

Optimal solution of the stochastic task (sale of bonds)

Type of the bond	Absolute values in the year			Relative values (%) in the year		
	2002	2003	2004	2002	2003	2004
2-year bond (x_{1t})	20000	35000	35000	27,1	46,4	38,3
5-year bond (x_{2t})	45820	31328	43957	62,2	41,5	48,0
10-year (fixed) bond (x_{3t})	6770	8030	10520	9,2	10,6	11,5
10-year (variable) bond (x_{4t})	1105	1139	2000	1,5	1,5	2,2

Table 5

Values of shortage (y_{it}) and surplus in the optimal solution z_{it}

Probability	2002		2003		2004	
	shortage	surplus	shortage	surplus	Shortage	surplus
0,5	0	0	0	2100	0	2300
0,3	2000	0	0	0	0	0
0,2	0	2000	0	4200	0	4600

Table 6

Values of the remaining constraints in the optimal solution

	Year 2002	Year 2003	Year 2004
Share of fixed-rate bonds	0,985	0,985	0,978
Share of variable-rate bonds	0,015	0,015	0,022
Average maturity	4,72	4,22	4,54
Duration	3,90	3,52	3,734
Risk (quadric of semivariance and semicovariance matrix)	0,0039	0,0047	0,0042

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Tomasz Szapiro

ON MULTIPLE CRITERIA GENETIC APPROACH TO HIGHLY CONSTRAINT VRPs

Abstract

The literature provides numerous examples of either rich* or multi-criteria Vehicle Routing Problems (VRPs). Practitioners claim, however, that real-life problems need effective methods for VRPs which are both rich and multiobjective. In the paper we investigate whether such problems can be efficiently handled by standard metaheuristics – genetic algorithms. The answer is affirmative. Additionally, the analysis conducted supports the thesis that it is purposeful to adjust components of metaheuristics so that they take advantage of the multiobjective nature of the problems they solve.

Keywords

Multiple Criteria Optimization, Genetic Algorithms, Vehicle Routing Problems.

Introduction

The Vehicle Routing Problem (VRP henceforth) consists in determination of the optimal transportation plan to be performed by a fleet of vehicles in order to serve a collection of clients. It was introduced by Danzig and Ramser [4] and can be perceived as an extension of the classical transportation task. Customers in the VRP are geographically distributed, hence, this problem can also be perceived as an extension of the traveling salesman problem. This indicates that VRPs are NP-hard problems – their complexity growths rapidly with the number of clients to be served, which makes them far more complex

* VRPs are called rich if they impose an elaborate restriction structure on numerous objects involved in the problem.

than classical transportation tasks. Toth and Vigo [17] point out that VRPs are among the most intensively studied combinatorial optimization problems. This is mainly due to practical needs – effective transportation management can save a considerable proportion of a logistic company's distribution costs as a consequence of better resource allocation: fewer trucks and fewer drivers travel together a shorter total distance, costs of storing goods decrease, more deliveries are performed on time and customers don't wait long for their orders.

VRPs constitute a diverse family of problems. However, they can all be defined in the language of graphs. Let us define a graph G = (V, E) with a set of vertices (which represent clients) $V = \{v_1, \ldots, v_n\}$ and a set of edges (which represent routes) $E = \{(v_1, v_j) : i \neq j,\ v_1, v_j \in V\}$. Selected vertices (a subset V_d of V) represent depots – places where vehicles are located and commodities are stored. Clients place orders for commodities to be delivered to them by the available fleet of vehicles. If there is only one depot and one vehicle with unlimited capacity and all clients can be visited only once, the VRP boils down to the traveling salesperson problem*. This constitutes a starting point for extensions. Assume there are more vehicles, though they are still identical. One can impose constraints on their capacity. This is the capacitated VRP. If customers impose constraints on time intervals within which they can be served, this is a VRP with Time Windows. Both capacity and time window constraints complicate the matter, since the feasibility of solutions has to be verified within each iteration of the procedure**. Going a step further, one assumes that consumers can both demand and supply commodities, which means that vehicles have to both deliver goods and pick them up along the route. Such VRPs, which depend on further assumptions, are called VRPs with Backhauls or VRPs with Pickup and Delivery. A further complication comes from the fact that vehicles can vary with respect to capacity and other technical specifications. Such problems are called Heterogeneous or Mixed Fleet VRPs. At the end of this (selective) list of VRPs, an instance, in which customers can place multiple orders, we have. This is known as a VRP with Multiple Orders.

Enlisted VRPs (summarized in Table 1) add a significant amount of complexity to the simplest VRP framework. However, as we put forward in the next section, these VRPs are still not complex enough for practitioners, who request further extensions – both in the single and in the multicriteria directions.

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* Under the assumption that the graph G is connected.
** If this procedure does not allow to explore the search space through infeasible solutions.

Table 1

Typical VRP characteristics

1. Size of available vehicle	– One or more vehicle, limited or unlimited fleet
2. Vehicle capacity constraints	– Limited or unlimited capacity
3. Type of available fleet	– Homogeneous or heterogeneous fleet Special vehicle types – e.g. fridge
4. Housing of vehicle	– Single depot, multiple depots
5. Time restrictions	– With and without time windows
6. Consumers actions	– Declare demand or demand and supply

1. Routing problems and multiple objectives

The classes of VRPs which we referred to in the last section are subjects of intensive studies. This list covers the most important VRP instances in the deterministic framework. Practitioners, nonetheless, indicate a need for development of these frameworks, so that they take into account such elements of the problem as drivers and their characteristics*; elaborate, more detailed constraints on vehicles and customers; hierarchical treatment of customers; optimal positioning of transported goods in vehicles etc. These issues generate additional complexity in the already existing VRPs, either by the inclusion of more time consuming feasibility verification routines (enrichment of restriction structure), or by the need of inclusion of subroutines that solve, on-the-fly, subproblems added to the original framework. These extensions are mainly of a single criteria nature, or, at least, can be naturally defined without referring to multicriteria concepts.

The second direction along which practitioners requirement to extend the VRP formulation reflects *explicitly* the multicriteria nature of real-life VRPs. Jozefowicz, Semet and Talbi [8] point out, in their state-of-the-art survey, that multi-objective routing problems are utilized mainly for three purposes:

1. To extend classic academic problems in order to improve their practical application.
2. To generalize classic problems,

* In fact, in practice the treatment of drivers is very similar to the treatment of vehicles.

3. To study real-life cases in which the objectives have been clearly identified by decision-makers and are dedicated to a specific real-life problem or application.

We will concentrate on the first and the last points of this list. Here are the examples:

1. Lee and Ueng [10] propose a VRP in which the balance of route lengths is considered in order to increase fairness of solutions. When drivers compare their schedules and discover disparities, they complain. Since drivers constitute a vital element of the transportation company, their welfare is important.
2. Sessomboon et al. [15] add objectives to a VRP with time windows to improve customer satisfaction with regard to delivery dates.
3. Ribeiro and Lourenco [14] take into account diverse objectives, including cost, balancing, and marketing. They claim that the relationship between the customer and the driver is very important for improving sales and the reputation of the company.
4. Several researchers have studied the multi-objective traveling salesman problem in which several costs are associated to each edge. Problems of this kind are used to model networks for which two or more objectives must be computed simultaneously (e.g. the cost of the solution and the time required to execute the orders placed).
5. Chitty and Hernandez [3] define a dynamic stochastic VRP in which the total mean transit time and the total variance of transit time are minimized simultaneously.
6. Murata and Itai [12] define a bi-objective VRP which seeks to minimize both the number of vehicles and the maximum routing time.
7. El-Sherbeny [6] worked on a problem with eight objectives defined by the company. The fleet was heterogeneous, consisting of both covered and uncovered trucks. There was no capacity constraint.
8. Bowerman et al. [2] consider a problem where a set of students living in different areas must have access to a public schoolbus to take them from their residences to their school and back. The problem is to find a collection of routes that will ensure a fair distribution of services to all eligible students. The authors proposed a multi-objective model with four objectives: minimization of the total length of routes, minimization of the total student walking distance, fair distribution of the load (i.e., the number of students transported), and fair division of the total distance traveled among the buses.

9. Corberan et al. [1] and Pacheco and Marti [13] examined methods for transporting students from home to school and back. The transportation had to be accomplished as safely as possible, while still considering economic aspects as well as comfort. The time constraint ensured that students would not spend too much time on the bus and that there would be no glaring inequities between the first student picked up on the tour and the last one.
10. In Lacomme et al. [9] trash has to be collected and delivered to a waste treatment facility. The trucks leave the factory at 6 a.m. and have to return to the factory before a given hour since the workers have to sort the waste afterwards. The authors consider two objectives: the minimization of the total route length and the minimization of the longest route.
11. Zografos and Androustsopoulos [18] have proposed modeling hazardous product distribution as a bi-objective routing problem in which objectives of minimizing the route length and minimizing risk are considered simultaneously.
12. Doerner et al. [5] attempt to deal with the fact that developing countries frequently face a dilemma engendered by a growing population and very restrictive budget limitations for healthcare expenditures. The purpose of the study is to propose cost-effective routing for mobile healthcare facilities, thus providing access to health services for a large proportion of the population. The problem involves selecting the stops and the routing for the mobile facility, while also considering the following three objectives: (1) efficiency of workforce deployment, as measured by the ratio between the time spent on medical procedures and total time spent, including travel time and facility setup time, (2) average accessibility, as measured by the average distance that the inhabitants need to walk to reach the nearest stop on the tour, and (3) coverage, as measured by the percentage of inhabitants living within a given maximum walking distance to a tour stop.

The examples listed reflect the practitioners' growing need for the analysis of multicriteria VRPs, since the specification of such VRPs better corresponds to what they encounter in everyday business or policy activity. VRPs involving diverse types of heterogeneous objects (e.g. vehicles, drivers, customers, loading spaces), on which a rich structure of restrictions is imposed and many type of interactions are allowed to arise between the objects involved with reference to the notion of time, distance and geographical location, are called rich VRPs. Practitioners lead us therefore to the consideration of what is called a rich VRP with multiple objectives. In the simulation section we focus our attention on instances of such problems. Before we proceed to the solution method and simulations, multicriteria representation of VRPs considered will be presented.

2. MCDM Problem Formulation

Let us consider, for simplicity, the bi-objective case. We start from the assumption that solutions to multiobjective VRPs (i.e. *transportation plans*) are represented by collections ϕ of vectors $\mathbf{x_i}$ from the space $\mathbf{R}^n$, $\mathbf{x_i} \in \mathbf{R}^n$, $\phi = \{\mathbf{x_i}, I = \{1,2,...,p\}\}$. Since vectors $\mathbf{x_i}$ can be stacked to form a single vector $\mathbf{x} \in \mathbf{R}^N$, where N equals pn, we assume that the solutions are represented by vectors $\mathbf{x} \in \mathbf{R}^N$. The set $\mathbf{X}_d$ of feasible solutions is assumed to be bounded and constrained:

$$\mathbf{X}_d = \{\mathbf{x} \in \mathbf{R^N} \mid \mathbf{g(x) = b}\},$$

$$\mathbf{b} \in \mathbf{R^k},$$

$$\mathbf{g} : \mathbf{R^N} \to \mathbf{R^k},$$

Let us assume that the evaluation of the transportation plan **x** is done with respect to m criteria f_i, i.e. the functions $f_i : \mathbf{X}_d \to \mathbf{R}$, i=1,2, …,m. The mapping $\mathbf{f}(\mathbf{x}) = [f_1, f_2] : \mathbf{X}_d \to \mathbf{R}^2$ is called a bi-objective function and the direct image $\mathbf{Y}_d = \mathbf{f}(\mathbf{X}_d)$ is called a feasible bi-evaluation space.

Let us consider the set $\mathcal{C}$ of convex cones $\mathcal{C}$, $\mathcal{C} = \{ \mathbf{y} \in \mathbf{R}^2 : \mathbf{y} \in \mathcal{C} \Rightarrow \mathbf{ay} \in \mathcal{C}, a > 0\}$ and the family $\mathcal{S} : \mathbf{R}^2 \to \mathcal{C}$. The family $\mathcal{S}$ defines the preference structure as follows. Given $\mathbf{y} \in \mathbf{R}^2$, the dominance relation $\rho_\mathbf{y}$ for the evaluation **y** is defined by the formula:

$$\mathbf{y} \rho_\mathbf{y} \mathbf{y'} \Leftrightarrow \mathbf{y'} - \mathbf{y} \in \mathcal{S}(\mathbf{y}) \Leftrightarrow \mathbf{y'} \in \mathbf{y} + \mathcal{S}(\mathbf{y});$$

the symbol "+"stands here for the vector sum of a vector and a set. Thus, by moving the cone $\mathcal{S}(\mathbf{y})$ to the point **y**, we obtain the *dominance set* $\mathbf{Y}^{\mathbf{pref(y)}} = \mathbf{y} + \mathcal{S}(\mathbf{y})$ *for* **y.** The dominance set consists of such evaluations **y'** which remain in relation $\rho_\mathbf{y}$: $\mathbf{y} \rho_\mathbf{y} \mathbf{y'}$. If $\mathbf{Y}^{\mathbf{pref(y)}} \cap \mathbf{Y}_d = \{\mathbf{y}\}$, then **y** is said to be nondominated in the set $\mathbf{Y}_d$. The set $\mathbf{Y}_d^{ND}(\mathbf{Y}, \mathcal{S})$ denotes the set of all nondominated evaluations (of transportation plans) for all evaluations from the set $\mathbf{Y}_d$.

The dominance relation in the feasible evaluation set $\mathbf{Y}_d$ which is defined by the cone family $\mathcal{S}$ can be transferred to the feasible set of transportation plans $\mathbf{X}_d$. Let us consider the transportation plan $\mathbf{x} \in \mathbf{X}_d$ and the relation $\wp_{\mathbf{x}} \subset \mathbf{X}_d \times \mathbf{X}_d$. defined by the formula:

$$\mathbf{x} \wp_{\mathbf{x}} \mathbf{x'} \Leftrightarrow \mathbf{f(x)} \rho_{f(x)} \mathbf{f(x')}.$$

If $\mathbf{x} \wp_{\mathbf{x}} \mathbf{x'}$, then we say that the transportation plan **x'** is preferred to the plan **x.** The transportation plan is said to be *efficient* when its evaluation is not dominated. The set of efficient transportation plans is denoted by X_d^E; $\mathbf{X}_d^E = \mathbf{f}^{-1}(Y_d^{ND})$. For simplicity we consider constant preference defined by $\mathcal{S}_0 : \mathbf{R}^2 \rightarrow \mathcal{C} : \mathcal{S}_0(\mathbf{y}) = \mathbf{R}^- \times \mathbf{R}^-$, $\mathbf{y} \in \mathbf{Y}_d$. Thus we have:

$$\mathbf{y} \rho_{\mathbf{y}} \mathbf{y'} \Leftrightarrow \mathbf{y'} - \mathbf{y} \in \mathcal{S}_0(\mathbf{y}) \Leftrightarrow \mathbf{y'} - \mathbf{y} \in \mathbf{R}^- \times \mathbf{R}^- \Leftrightarrow y_1' \le y_1 \wedge y_2' \le y_2.$$

Thus we assume that both evaluations are to be minimized (e.g. cost of routes and the number of time windows violated).

The bi-criteria problem is to find a unique, most preferred transportation plan with respect to the dominance relation $\wp_{\mathbf{x}}$, i.e. it requires identification of an efficient solution from the set X_d^E, or to determine the entire efficient frontier. We conduct simulations for both of these cases. The set $\langle \mathbf{f}, \mathbf{g}, \mathbf{b}, \mathcal{S}_0 \rangle$ represents the problem in the sense that **g** and **b** represent actions and technological issues (here, transportation infrastructure), the functions **f** represent the evaluation of transportation plans and the family of cones $\mathcal{S}_0$ describes the decision maker's preference. However, there may exist infinitely many maximal elements with respect to the preference defined above*. Thus the natural definition of the solution of the problem – the efficient set – although satisfactory from the formal point of view, sometimes may not be sufficient from the practical point of view as the decision maker is left with many solutions. Moreover, the method of presentation of X_d^E to the decision maker is not an obvious issue.

When a solution method has to yield a unique solution, it has to involve a procedure which contracts the X_d^E to a unique solution. The MCDM theory lists a series of approaches aimed at the identification of unique solutions (using e.g. weights, trade-offs, lexicographic orders, reference points and others). Let us focus our attention on weighting approach to show general problems which

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* The definition of the solution can be rephrased using the relation theory terminology: viz. the option **x** is a solution if the set $\mathbf{Y}_d$ of feasible outcomes contains the maximal element with respect to the partial order $\le$ in $\mathbf{R}^2$ (Yu, 1995).

are not specific to the approach in scalar evaluated transportation tasks. Leaving aside very serious doubts on the decision maker's ability to define reliable weights, we finally arrive at scalar objective functions.

3. Genetic Approach

Not only rich, but also standard VRPs are most often approached by means of optimization metaheuristics: simulated annealing, taboo search, ant colony optimization techniques and genetic algorithms. To solve the multi-criterial VRP we employ the genetic approach, which turned out to be useful in this respect. As pointed out by Tan et al. [16], genetic algorithms tend to be stable over a wide range of VRP instances, which gives rise to the assumption that they should also give satisfactory results when applied to diverse rich multicriteria VRPs (since these constitute a mixture of diverse VRP classes). Secondly, genetic algorithms give better results than simulated annealing and taboo search over a wide range of diverse VRP problem sets, while being positioned on average between these heuristics in terms of computation time.

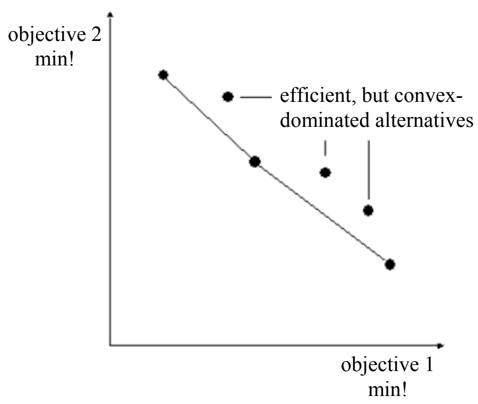

Figure 1. Efficient, convex-dominated solutions

Source: M.J. Geiger.

The genetic algorithm maintains a population of candidate members over many generations. Population members (solutions) are encoded as variable length strings of integers. A selection mechanism chooses parents who go to the reproduction phase, i.e. crossover and mutation, which in turn produces children who replace the parents. We use the tournament selection. Each selection round is proceeded by the evaluation of the current population of solutions. Geiger (2001) points out that, if the objectives considered are integrated with a weighted sum to get a scalar evaluation (as it is the case in our study), efficient but convex-dominated alternatives are difficult to obtain, see Figure 1.

This is a significant drawback of standard selection mechanisms. In the multicriterial case, the decision maker very often (especially if long-term plans are to be implemented) wants to have a possibility of choosing from a set of good alternatives by means of procedures which are external to the optimization algorithm and often involve expert judgement. These alternatives can come from the final population of the algorithm. It is desirable that these alternatives contain efficient, yet convex-dominated solutions. The presence of such solutions is also crucial for sufficient diversity of populations. To overcome this problem, a selection operator which provides a self-adoption technique is implemented. In this approach, we use dominance information of the individuals of the population by calculating for each individual i the number of alternatives n_i by which this individual is dominated. For a population consisting of N alternatives we have: $0 <= n_i <= N-1$. Individuals that are not dominated by others receive a higher fitness value than individuals that are dominated. We calculate fitness values for all individuals using linear normalization. Individuals with the lowest values of n_i ($n_i = 0$) receive the highest corresponding value of $f_i = f_{max}$ and the individual with the highest value $n_{max} = \max\{n_i, i = 1,..., N\}$ receives the lowest value of $f_i = f_{min}$ for $f_{max} > f_{min} > 0$. For all other individuals the following condition is imposed: $f_i = f_{max} - ((f_{max} – f_{min}) / n_{max})n_i$.

An efficient reproduction mechanism, i.e. selection, crossover and mutation, is largely responsible for the performance of the algorithms. Conventional single/double point crossover operations are relevant to orderless strings. They make a cut point (or points) on both of the strings. A crossover is then completed by swapping substrings after the cut point (or between two cut points) in both strings. In VRPs, where routes are crossed over, each integer value appears only once in a string, and such a procedure produces invalid offsprings, for they can have duplicated values in the resulting strings. To prevent such offsprings from being constructed, we use a set of ordered crossover operators, see Tan et al. [16]. The one that turned out to be especially useful is a so called permutation crossover.

Apart from the above part-and-parcel components of genetic algorithms, we used their extensions: breaks for local search (typical for the intensification phase of taboo search algorithms), variable mutation probability and (in some experiments) controlled proportion of feasible and infeasible solutions in evolving populations, as well as initialization of populations by means of deterministic heuristics such as push-forward-insertion heuristic and interchange procedures.

4. Simulations and results

We have run efficiency tests on multiple sets of data simulated from predefined probability distributions. Here are the findings which we believe are most important:

1. The procedure works efficiently for all tested distributions over a wide range of parameter values by which these distributions are parametrized.
2. For the majority of problem classes it is possible to define instances (by a suitable parametrization of probability distributions) which makes the procedure highly time consuming, hence ineffective for practitioners.
3. For implementations which allow populations to contain infeasible solutions, the convergence process is on average longer, but results produced are on average better.
4. For such implementations it may, however, often be the case that convergence to a feasible region is not achieved after a significant number of iterations.
5. Best results are achieved when evolving populations are controlled with respect to the proportion of feasible and infeasible solutions.
6. The employment of selection technique which refers to the concept of dominance relation produces results which are more useful in practice.

The above findings are intuitive and consistent with common knowledge about genetic algorithms. The first of them supports the thesis that such algorithms are powerful tools of optimization, even for very complex problems, such as rich multicriteria VRPs. The second finding constitutes a warning. It refers to something similar to what is called in the literature the deceptive problem – a problem for which a given heuristic method (here a genetic algorithm) tends to fail in finding satisfactory solutions within a reasonable time. Fortunately, parametrisations for which the second case occurs do not happen very often and seem to be implausible in real-life scenarios. Findings 3 and 5 indicate that the inclusion of infeasible solutions in evolving populations does not have to increase the computing time excessively, and, if reasonably modeled, such mixed populations produce superior results. Finally, point 6 is an example of the advisability of developing components of metaheuristics which explicitly take into account the multiobjective structure of problems being solved.

Since, to our best knowledge, there are no reference data sets on rich VRPs (not to mention multicriterial ones), which we could use to compare our implementation with other implementations, we provide results obtained on our simulated data. Table 2 presents runtime needed to conduct 200 iterations of the algorithm for problems varying from 10 to 100 clients*. Evolving populations always consist of 50 solutions. Computations were conducted on a Celeron 1.5 Gh CPU with 512 Mb RAM.

The second column presents average time consumption over simulated data sets with different parametrisations of probability distributions. The third column reports worst-case run times. Averages do not take into account instances for which computations didn't terminate within 3600 seconds. Such instances are indicated in the third column by inf-ties. The results reported are comparable to the benchmark results reported in other studies, e.g. in Tan et al. [16]. Our simulations produce visibly longer, but still practically acceptable runtimes. Longer runtimes are due to our problems' being much richer than the ones considered in the paper referred to.

Table 2

Time efficiency of the algorithm

Problem size	Average run time (seconds)	Worst runtime (seconds)
10	85.16	431.11
50	240.96	inf
100	1863.91	inf

* In reported instances it is assumed that each customer places only one order. These can, however, be considered as instances in which there are less customers than assumed, but some of them place multiple orders.

Concluding Remarks

In the paper we investigated if rich VRPs – computationally greedy problems made even more complex by the introduction of multiple criteria – can still be effectively (ie. within reasonable time) solved by means of standard techniques – genetic algorithms. We implemented an algorithm capable of handling such VRPs. The answer is affirmative. Our results also confirm the thesis that it is useful to adapt components of optimization heuristics (here of a genetic algorithm) so that they explicitly take advantage of the multi objective structure of problems they are designed to solve. Further work will cover the following two topics. First, we will concentrate on the issue of enhancing crossover operators to the multiobjective framework. Second, we will try to experiment with the structure of rich VRPs, so that their complexity is lowered by the use of multiobjective approach in single criteria problems with nested structure of complex restrictions, e.g. in the case of Loading VRPs.

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Lech Kruś

ON A GROUP MULTICRITERIA METHOD FOR PROJECT EVALUATION

Abstract

Experiences with a real-life case study are presented. The case study deals with the allocation of EU structural funds in the capital region of Mazovia in Poland. A new method in the practice of the funds allocation, supporting multicriteria analysis and selection of projects applying for the funds, has been proposed and used in the study. According to the method, an interactive procedure has been implemented in which a group of experts formulates the multicriteria decision making problem, carries out the multicriteria analysis of the projects, and finally creates a ranking of the projects.

Keywords

Multicriteria analysis, group methods, computer-based support, EU structural funds.

Introduction

The structural funds of the European Union are the financial instruments used to implement the policy for support of multi-dimensional development, enhancement of economic and social cohesion, reducing differences of regional development standards and restructuring and modernizing the economies of those member states whose development level is below the average development level in the European Union.

In the 2007-2013 programming perspective, Poland may take advantage of the support within the framework of the following structural funds: the European Regional Development Fund (ERDF), the European Social Fund (ESF), the Cohesion Fund, the European Agricultural Fund for Rural Development (EAFRD), and the European Fisheries Fund (EFF).

The European Regional Development Fund (ERDF) is meant for financing undertakings in the regions with the development level substantially lagging behind the average for the EU, as well as in the regions with major restructuring activities in industry and employment. The funds are addressed particularly to financing investment in infrastructure and environmental protection, development of small and medium enterprises, creation of new jobs through investment in manufacturing, research and development activities. Potential beneficiaries are territorial self-government units, their unions and associations, entrepreneurs (small and medium), government administration bodies, national and landscape parks, National Forestry and its organizational units, R&D units, (other) units of the public finance sector with legal entity, non-governmental organizations, business support institutions, housing associations and housing cooperatives, as well as water law companies.

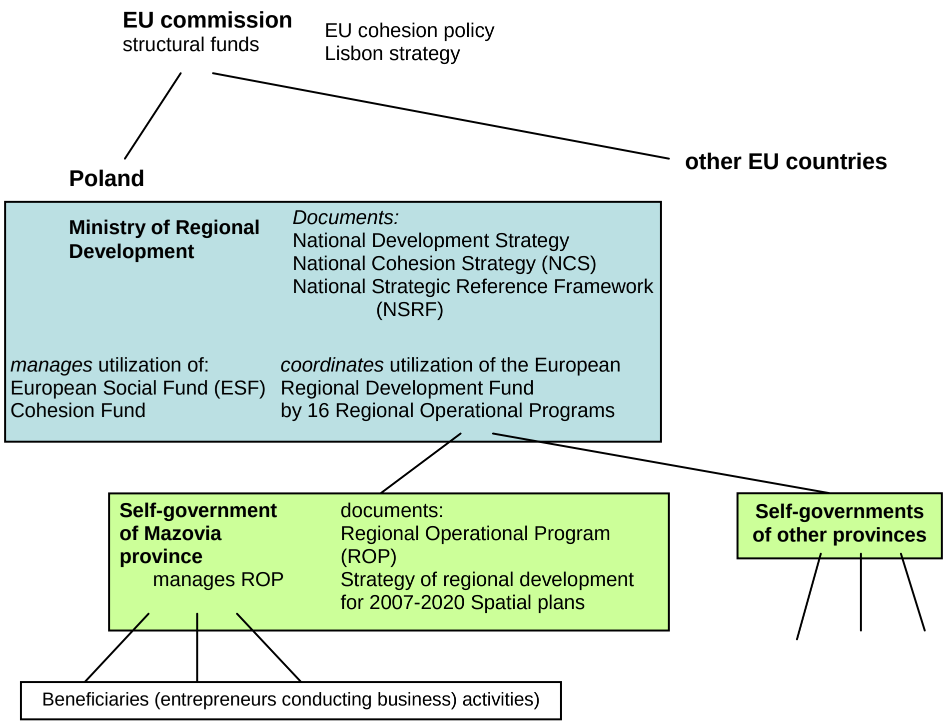

Figure 1. Decision making units allocating and supervising utilization of the EU structural funds

Utilization of the ERDF is coordinated in Poland by the Ministry of Regional Development (see Figure 1). It is done according to the documents such as the National Development Strategy (NDS) for Poland, the National Strategic Reference Framework, and the National Cohesion Strategy adopted by the EU Commission. The Ministry allocates the funds among regions – provinces being administrative units, called voivodships in Poland. The funds are allocated among beneficiaries on the regional level by the self-governments of voivodships within the Regional Operational Programs (ROP), negotiated and approved by the EU Commission. The Ministry, having the consent of the EU Commission, decided that the most important projects for regional development (called key projects) can be submitted and co-financed within the ROP prior to the beginning of standard competitions for other projects.

The paper deals with the Regional Operational Program (ROP) of the capital Mazovian Voivodship for the years 2007-2013. A case study has been organized to support selection of the key projects from a list of projects submitted. The paper describes experiences with the case study.

There exists a rich bibliography on multicriteria analysis, ranking and group methods. Advance ordinal and cardinal approaches have been developed. The respective reviews can be found in [5], [25], [26], [29]. A proposal including application of the outranking method for ordering projects is given by Górecka [4]. On the other hand, in the practice of the UE funds allocation, we deal with hundreds of projects applying, a limited number of experts assessing the projects and very limited time for the assessment and selection process. The experts – assessors obtain evaluation sheets with predefined criteria and propose values for the criteria within given ranges of points. Usually, different experts can understand the criteria in different ways. Finally, the classical weight method is still used to assess the projects. This case study has been organized with the idea that the experts should be involved in the whole MCDM process starting from its formulation. A relatively simple evaluation method, acceptable by the experts was looked for, which could improve the typical defects of the weight method.

A multicriteria group method – new in the practice of EU funds – supporting analysis, assessment and selection of the key projects has been proposed and implemented within the study. The method enables evaluation and ranking of projects on the basis of assessments made by a group of independent experts. The method includes full procedure of activities of the experts, starting from a formal definition of the multicriteria decision making problem, and leading to the final selection of the key projects. An implementation of the procedure is presented in the paper.

1. Procedure

In 2006, the Self-Government of the Mazovian Voivodship the competition for the key projects co-financed from the EU structural funds within the Regional Operational Program of the voivodship for 2007-2013. More than 150 projects applied for the competition. The list of the key projects had to be prepared together with the respective justification. The projects not qualified as the key projects could apply again in the standard competitions organized later.

I. Formulation of the mutiobjective decision making problem (panel sessions of experts)

- Lecture introducing the experts into MCDM problems.
- Initial formulation and analysis of the decision making problem (brainstorming).
- Analysis of objectives and specification of criteria (brainstorming).
- Formulation of acceptability conditions (brainstorming).
- Analysis of logical relations among criteria, importance of the criteria, specification of weights assigned to the criteria (Delphi).
- Definition of the reference point and the reservation point in the space of criteria (Delphi).

⇩

II. Assessment, ranking and selection of the key projects

Individual assessment of projects made by each expert. Rejection of non acceptable projects and assignment of values for the criteria of the projects initially accepted.

Interactive session – common analysis of the individual opinions, discussion of arguments and reaching a consensus about the values assigned to the criteria.

Ranking of the projects using three norms: l_1, l_2, l_∞ in the space of criteria.

Final selection of the key projects.

⇩

III. Preparation of the expertise for the final decision maker

Figure 2. Scheme of the procedure

A procedure, schematically shown in Figure 2, has been proposed and approved. The figure presents activities performed by a group of experts, leading to the preparation of the list of the selected key projects. It consists of three main stages.

The first stage deals with formulation of the multicriteria decision making problem (MCDM). It started with a lecture introducing the experts to MCDM problems. The proper formulation of the problem requires the specification of the following key components (see [2]):

– Decision making unit. It is the decision maker and possibly a collection of men and machines acting as an information processor and generating the decision. In general, it can be the single or the group decision maker, system analysts, computing and graphical instruments.

– Set of objectives and their hierarchy. The objective defines the state of the system required by the decision maker.

– Set of criteria (attributes), relations objectives – criteria, the scales on which the criteria are measured. The values of the criteria measure the degrees of the attainment of the objectives.

– Decision situation that defines the problem structure and the decision environment of the decision problem. The description of a decision situation should include the specification of input information required and accessible, set of alternatives, constraints, decision variables, relations: decision variables – criteria, and finally the states of the decision environment.

– Decision rule. The rule includes processing of the input information, analysis, value judgment, decision generation and implementation.

These elements were considered and specified during the case study.

The following work of experts was organized in the form of a panel session with application of the brainstorming technique or the Delphi method, referred to in brackets. At the end of the first phase the experts were asked to define the best and the worst key projects in their opinion. These projects, considered as points in the space of criteria, refer, respectively, to the reference and the reservation point concepts in multicriteria analysis.

The second phase deals with the assessment method based on the cardinal approach to multicriteria group decision making. It includes individual assessments of projects made by the experts, joint analysis of the individual opinions to reach a consensus, ranking and final selection of the projects. The ranking is based on the distance of a given project measured to the reference point in the multicriteria space. Different norms are used to measure the distance. A special session was organized to make the final selection of the key projects.

The third phase refers to formal preparation of the expertise including the above-mentioned list of the recommended key projects, and the description of the implemented method and argumentation.

2. Multiobjective decision problem

2.1. Decision making unit and specification of objectives

The decision unit was the Board of the Self-Government of the Mazovian Voivodship, responsible for the final decision. The decision was prepared by the Department of Strategy and Regional Development of the Board and by the Mazovian Bureau for Regional Development.

The meaning of the “key projects” had to be specified first as the basis for the formulation of objectives. The working team has been organized; it consisted of experts from the Department of the Strategy and Regional Development of the Government, experts from the Mazovian Bureau for Regional Planning in Warsaw and an adviser responsible for group multicriteria decision support. Working sessions have been organized in which the brainstorming technique was used ([6]; [22]). The technique enables free and unlimited presentation of proposals but with strictly defined rules of analysis and evaluation of the proposals.

The team of experts decided that as the key projects such projects should be selected which substantially realize the directions of the activities specified in the development strategy of the province, taking into account: the directions of the spatial management defined in the spatial plan of the province, the competitiveness of the province in the international and the national contexts, the effects of synergy with other socio-economic spheres, and innovativeness. The acceptability conditions have been specified. The projects that do not effect structural, socio-economic and spatial changes in the region, or belong to other operational programs or have local character or do not fulfill the objectives of the Regional Operational Program for 2007-2013, should be rejected.

2.2. Input information, documents

The main objectives of the cohesion policy, taking into account the socio-economic conditions in Poland, are included in the document entitled “National Strategic Reference Framework for 2007-2013”. The document, elaborated according to the EU directives, defines support directions for funding available from the EU budget in the forthcoming seven years within the European Regional Development Fund and the Cohesion Fund. It is a reference instrument for the development of operational programs. According to the document, the regional development programs have been elaborated, negotiated and adopted by the EU Commission. In the voivodships other documents are also prepared, such as development strategies, spatial management plans and others.

The team analyzed the respective documents and decided that the assessment of projects should be made according to the objectives and the directions of activities given in the Development Strategy of the Mazovia Province till 2020, according to the objectives and priorities of the Regional Operational Program of the voivodship for 2007-2013, and to the specifications given in the Plan of Spatial Management of the Mazovia Province. The documents as well as the application questionnaires formed the information base for the project assessment.

2.3. Features of the decision problem

It has been found that the set of the objectives, which should be taken into account, is really complex. The Development Strategy of the Province till 2020 presents a hierarchical system including an overall objective, strategic and indirect objectives, and directions of activities. The Regional Operational Program (ROP) for 2007-2013 includes also a hierarchical set of objectives, priorities and directions of activities. The criteria respective to the objectives have qualitative character. The projects submitted within the different priorities are hardly comparable.

It has been found that the information included in the existing questionnaires is very limited. These questionnaires were elaborated earlier.

The decision had to be prepared in a very short time. The entire process, including preparation of the method, organization of the interactive sessions, assessment of all the projects, derivation of the ranking and the final list of the key projects had to be conducted in 10 days. The team had no earlier experience in such work.

3. Specification of criteria, reference and reservation projects

The experts have been informed how they should understand the meaning of objectives and criteria. The objective defines the required state of the system that the DM would like to achieve. The criteria specified for an objective measure (on a numerical scale) the degree to which the objective is achieved.

The criteria should fulfill the following requirements ([8]). The values of the criteria should define the achievement level of the respective objective in a unique and sufficient way. Each criterion should be comprehensive and measurable. A set of criteria should be:

- complete, i.e. all pertinent aspects of the decision problem are represented by the criteria,
- operational, i.e. it can be utilized in a meaningful manner in the ensuing analysis,
- decomposable, i.e. simplification of the evaluation process is possible by breaking up the decision process into stages,
- not redundant, i.e. no aspect of the decision problem is accounted for (by criteria) more than once,
- minimal – there is no other complete set of criteria representing the same problem with a smaller number of elements.

An interactive multi-round session has been organized in which experts worked according to the "brainstorming" technique. Proposals of criteria were generated to cover all the objectives specified in the Development Strategy of the Province and in the Regional Operational Program. The requirements presented above have been checked as well as accessibility of information from the application questionnaires. Finally, after analysis and discussion of all the objectives and their hierarchy, the following set of criteria has been specified, and unanimously accepted by all the experts:

K1. The degree of realization of the activity directions specified in the development strategy and in the spatial plan of the voivodship.

K2. The influence of the project on the competitiveness of the voivodship in the national and international context.

K3. Effects of synergy with other socio-economic spheres.

K4. Innovativeness of the project.

In the case of a large number of objectives specified in the above documents, the criteria have to be defined in an aggregated way. The experts have agreed on a method of checking the application sheets to evaluate the criteria of the projects assessed in the similar way.

Next, the experts were asked to define, according to their preferences, the best possible "key project", treated later as the reference project and the worst one, treated as the reservation project. They had also to analyze the logical relations of the criteria, to set the weights assigned to the criteria and to set the interval scales. The modified version of the Delphi method has been applied. The original Delphi method has been elaborated in the Rand Corporation, see

Linston, Turoof [16]. In the version implemented, the work of the group of experts was organized in the form of multi-round interactive sessions. In the consecutive rounds the experts' proposals were presented together with the respective argumentation. The proposals were jointly analyzed and discussed, especially in the case of divergent evaluations. On this basis, each expert could correct his opinion in the next round taking into account the arguments of other experts.

The weights assigned to the criteria have been fixed as follows: K1: 50%, K2: 20%, K3: 20%, K4: 10%.

The experts have defined the properties characterizing the best possible, in their opinion, key project. They specified when each criterion could be reached at the maximal level. The hypothetical project having all criteria at the maximum possible level was assumed as the reference project. The experts specified also the case when the particular criteria could be at the possible minimum level. This case refers to the hypothetical reservation project.

4. Project evaluation and ranking

An original method, which extends the cardinal approach described by Hwang, Yoon [6], has been proposed to the experts. In comparison with the classical approach, the concept of the reference point was used in place of the ideal point, several ways of measuring the distance to the reference point were applied and the Delphi method was used to find a consensus in the case of divergent opinions of experts. The reference point approach has been proposed and developed in the case of multicriteria analysis ([27], [28], [20], [21]). The reference point and the reference set concepts are developed by Konarzewska-Gubała ([9], [10]) in the case of multicriteria group decision support. It is also used in the methods supporting multicriteria cooperative decisions ([11], [12], [13]).

The method proposed enables the group, multicriteria judgment of projects in the case of qualitative criteria. The interval scales are used. Experts evaluate projects by assigning values for criteria using the scales. The experts' evaluations are discussed, corrected and set with use of the Delphi method. Each project is represented by a point in the space of criteria K1–K4. The ranking of projects is based on the distance to the reference point. Different ways of measuring the distance, compared also to the classical weight method have been proposed to the experts.

4.1. Idea of the evaluation method

We assume that the experts have equal power and their evaluations have equal importance. Each expert evaluates each criterion for a given project by proposing a value from a given scale interval. Values given by the experts are normalized. Let n be the number of experts, m – the number of evaluated projects, p – the number of criteria. The following steps are performed.

Step 1

Each expert k assigns a value a^{k}_{ij} to the project i for the criterion j. The normalized individual values are calculated:

$$d^k_{ij} = a^k_{ij} / \sqrt{\sum_{i=1}^{m} (a^k_{ij})^2} \text{ , where } k=1..n,\ i=1,..,m,\ j=1,..,p.$$

The values are aggregated in the matrix

$$C = [c_{ij}] = \sum_{k=1}^{n} d^k_{ij} / n \ .$$

A vector of weights is given: $W=\{w_1,..,w_p\}$, such that $\sum_{j=1,\dots,p} w_j = 1$.

The collective values are derived in the matrix

$$F=[f_{ij}]=[c_{ij}w_j],\ i=1,..,m,\ j=1,..,p.$$

Step 2

The reference project defined by the experts in Section 3 is considered as the reference point in the space of criteria:

$$A^* = \{f^*_1,..f^*_p\},$$

and the reservation project, as the point:

$$A^- = \{f_1,..f_p\}.$$

Step 3

The importance ("value") of each project is derived on the basis of the distance between this project and the reference one. The distance can be measured in different ways. Three measures have been proposed to the experts and then considered by them.

The distance measured according to the norm l_1:

$$s_{i1} = \sum_{j=1}^{p} \mid f^*_j - f_{ij} \mid \text{, where } i= 1,...m, \tag{1}$$

– according to the Euclidean norm l_2:

$$s_{i2} = \sqrt{\sum_{j=1}^{p} (f_{ij} - f^*_j)^2}\ , \qquad (2)$$

– according to the Chebyshev norm l_∞:

$$s_{i\infty} = max\ (|f^*_1\text{-}f_{i1}|,...,|f^*_p\text{-}f_{ip}|). \qquad (3)$$

Step 4

The distance of a project i to the reference one is normalized to the 10-points scale.

$$Gi\text{=}10\times(1\text{-}si/s),\quad 0\leq Gi \leq 10,\ i = 1,..m, \qquad (4)$$

where s is the distance of the point A^- (reservation) to the reference point A^*. A greater value of Gi means that the project i is better. The project equivalent to the reference one gets 10 points, while to the reservation one - 0 points. It can be shown that in the case considered, the evaluation of projects with use of the the norm l_1 coincides with the evaluation obtained by the classical methods of weights.

4.2. Implementation

The above general idea of the method has been presented to and discussed with the experts. In the proposal, the values a_{ij}^k can be assigned by each expert in his own individual, arbitrarily assumed interval scale for each criterion. The normalized values d_{ij}^k are used in further steps of the procedure. The normalization can be done after all projects have been evaluated by a given expert. This means that the evaluations of the same project given by the different experts can not be compared before. The experts asked for the possibility to compare their evaluations at the earlier stages of the procedure and they all agreed to use the same scale. They decided to use the scale of 10 points for each criterion, assuming 10 points for any criterion on the reference project level and 0 points to for any criterion on the reservation level. The first criterion was divided into two subcriteria: K1a – the degree of realization of the activity directions defined in the development strategy of the province (assessed on the scale of 0-7 points), and K1b – the degree of realization of the directions of the spatial management defined in the spatial plan of the province (0-3 points). The experts decided that these sub-criteria are additive.

Initially, the experts evaluated several projects. The different rankings of the projects according to the norms (1), (2), (3) and according to the classical weights method were derived and presented to the experts. Figs 3, 4, and 5 illustrate the methods of ranking. The set of projects is shown in each figure as a set of points in the space of two weighted criteria. The reference and the reservation points are shown. The continuous lines represent sets of projects being at the same distance to the reference point, i.e. being in the same position in the ranking.

The classical method of weights is shown in Figure 3. Selection of the key projects means that a border line of distance to the reference point has to be assumed. The projects below the line are rejected. Our real problem is considered in a four dimensional space. The border is defined in this case by a hyperplane. The weight method is very popular and often applied in practice due to its simplicity and practicality. The question arises: Does it really reflect the preferences of experts? Let us look at the project with a low value of the criterion k_2 and a very high value of the other criterion (the project in question is indicated in Figure 3). This project would be higher in the ranking than projects with balanced values of all criteria. Is this really correct according to the intuition of the experts? The weight method is justified if the criteria are additive. In general, the description of the experts' preferences may be nonlinear. The rankings derived with use of the norms l_2 and $l\infty$ serve as examples of such nonlinear descriptions of the preferences. Of course, it is also possible to use other nonlinear descriptions.

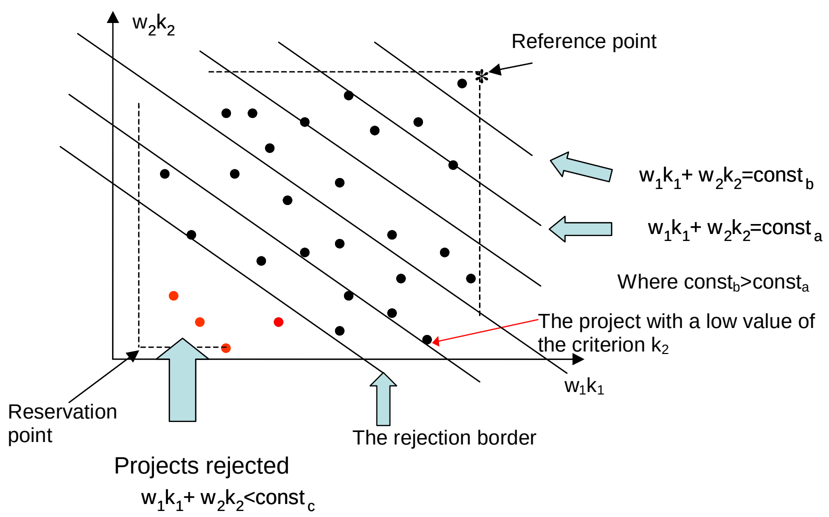

Figure 3. The evaluation and ranking of projects according to the classical weight method

The experts decided that the key projects should be selected using the Euclidean norm. The rankings defined with use of the norm $l\infty$ and by the weight method were derived for the sake of comparison.

In practice, in typical implementations, each project is assessed by five-seven or a larger number of experts. Once the values are given by the experts, the extreme values are rejected and the mean value is derived as the joint one. In the case study considered, the time for the entire procedure was very limited. All the projects had to be analyzed and evaluated in a few days. The team of experts consisted of seven specialists. In the solution applied, each project was analyzed and assessed independently by the experts from the Department for the Strategy and Regional Development of the Self-Government and from the Bureau for the Regional Planning of the Mazovian Voivodship. The experts checked whether a given project satisfied acceptability conditions mentioned in Section 2.1, and if so, made the assessment according to the assumed set of criteria. The assessments were treated as introductory. A special interactive session was organized after the individual assessments had been made. In the session, the projects and the introductory opinions were analyzed again by all the experts,

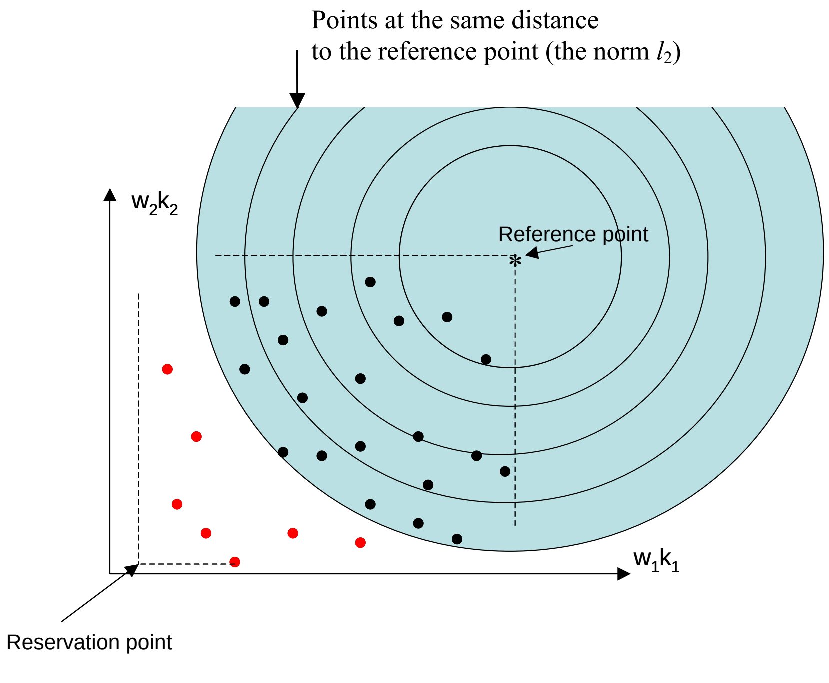

Figure 4. The evaluation and ranking of projects according to the distance to the reference point (the distance measured by the Euclidean norm l_2)

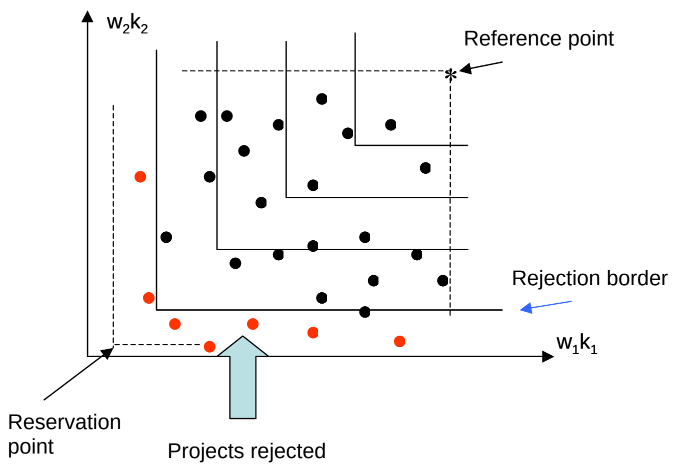

Figure 5. The evaluation and ranking of projects according to the distance to the reference point (the distance measured by the norm $l\infty$)

especially in the case of divergent introductory opinions. The opinions could be corrected after the discussion and the negotiation of arguments according to the Delphi method. The experts were supported during the session by a computer-based system.

The system takes as inputs the experts' opinions. On this basis it produces evaluations of projects, derives the distance of each project to the reference point according to the Euclidean norm, and also according to the l_1 and $l\infty$ norms. It generates the respective ranking lists. The system works interactively. Experts can correct their opinions on-line, obtain corrected results, analyze project evaluations and observe changes in the ranking lists.

The entire evaluation process was carried on under the confidentiality conditions required by the Ministry of Regional Development supervising the competition. The application sheets are confidential and could be analyzed by experts on premises only. Detailed information about individual evaluations, discussions, and preliminary scores is also confidential. The experts accepted the procedure proposed and performed their work without difficulties. From the operational point of view, the individual assessments were made in the same way as in the traditional method. Only aggregated scores and analyzed variants of the ranking list were derived not by hand, but by the computer-based system. Only in the case of 10% of the projects, the individual opinions differed significantly. In this case, the experts had to present their argumentations during the final session and to discuss their opinions to reach a consensus. In all the cases, they reached a consensus. A special discussion was needed to decide where to make the rejection border in the ranking list of all the projects. The projects with scores near the border discussed were additionally analyzed, so that the final decision was justified, and accepted unanimously.

The resulting list of the key projects established and approved by the team of experts, and the ranking list of all the projects have been presented and recommended to the Board of the Self-Government of the Mazovian Voivodship. On the basis of the list and the opinions of the experts, the indicative investment plan has been elaborated and accepted by the Board of the Self-Government of Mazovia. The list of the key projects is presented on the website of the Self-Government.

Conclusions

A specially prepared group multicriteria method, original in the practice of EU funds allocation, has been applied to make the ranking and selection of the key projects. The ideas of different approaches have been used including the brainstorming techniques, the Delphi method and the extended cardinal approach to the group multicriteria decision making. To make the ranking, the positions of the projects in the multidimensional space of criteria are analyzed. On the basis of the experts' opinions the distance of each project to the reference key project is derived. The projects closest to the reference one are selected as the key projects. It has been found that the experts, when comparing several different measures of distance, have not selected the classical weight method but the nonlinear measure based on the Euclidean norm.

The weight method, frequently used, is justified under the assumption that all criteria are additive in the preference relation. In general, the assumption can be not fulfilled, but in practical implementations, it is frequently even not checked.

In this case study, the experts could make a choice. They did not approve the weight method, but selected and approved a non-linear description of their preferences according to the Euclidean norm for measuring the distance of each project to the reference „key" project.

The method has been elaborated and implemented by the commission from the Mazovian Bureau for Regional Planning in Warsaw [14]. The final list of the selected key projects was the basis for the indicative investment plan elaborated and accepted by the Board of the Self-Government of the Mazovia Voivodship.

In future work applications of the bipolar reference system ideas proposed by Konarzewska-Gubała [9] and developed by Trzaskalik [26] and of the interactive approach to ordinal regression multiple criteria ranking using a set of additive value functions [5] are planned.

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Bogumiła Krzeszowska

EVOLUTIONARY ALGORITHM WITH DIRECT CHROMOSOME REPRESENTATION IN MULTI-CRITERIA PROJECT SCHEDULING

Abstract

In recent years project scheduling problems became popular because of their broad real-life applications. In practical situations it is often necessary to use multi--criteria models for the evaluation of feasible schedules.

Constraints and objectives in project scheduling are determined by three main issues: time, resource and capital; but few papers consider all of them. In research on project scheduling the most popular is the problem with one objective. There are only few papers that consider the multi-objective project scheduling problem.

This paper considers the multi-criteria project scheduling problem. There are three types of criteria used to optimize a project schedule: resource allocation, time allocation and cost allocation. An evolutionary algorithm with direct chromosome representation is used to solve this problem. In this representation a chromosome is a sequence of completion times of each activity.

The purpose of this paper is to demonstrate how evolutionary algorithms can be used in multi-criteria project scheduling. The paper begins with an overview of previous literature and problem statement; after that there is direct chromosome representation description and at the end final results.

Keywords

Project scheduling, multi-criteria analysis, evolutionary algorithms, multi-criteria scheduling.

Introduction

A project is a unique set of co-ordinated activities, with definite starting and finish times, undertaken by an individual or organization to meet specific objectives within defined schedule, cost and performance parameters [1].

A project is also defined as a temporary (with defined beginning and end) endeavor undertaken to create a unique product or service [2]. The planning, monitoring and control of all aspects of a project to achieve the project objectives on time and to specified cost, quality and performance is called project management [1]. Each project has three main components: activities, resources and precedence relationships [4].

Activities are tasks to do. They build a project. An activity has an expected duration, an expected cost, and resource requirements.

Resources are required to carry out the project tasks. They can be people, equipment, facilities, funding, or anything else capable to perform an activity required for the completion of a project. Resources may be renewable or non-renewable. Renewable resources are available in each period without being depleted. Non-renewable resources are depleted as they are used.

Precedence relationships define the order in which activities should be performed. This order is specified. A precedence relationship is always assigned to activities based on the dependencies of each activity. There are four recognized precedence relationships: Finish-to-Start, Finish-to-Finish, Start-to-Start, Start-to-Finish.

Scheduling concerns the allocation of limited resources to tasks over time. It is a decision-making process that has a goal – the optimization of one or more objectives [9].

The project schedule lists times planned for performing activities. Any schematic display of the logical relationships of project activities can be presented as a network. There are two types of networks for project scheduling problems: AON (Activity On Node) and AOA (Activity On Arc). In an AON network activities are represented by nodes, and they are linked by the precedence relationship to illustrate the sequence in which activities should be performed. In an AOA network activities are presented by arrows. The tail of the arrow represents the start and the head represents the finish of the activity. Activities are connected at nodes to illustrate the sequence in which activities should be performed [10].

A project scheduling problem includes many types of constraints. Type of constraints and optimization criteria are determined by three main components: time, resources and capital. When we take them into consideration we can build various schedule optimizing models. The most popular are problems with one objective. We can build models without constraining time, cost or resources; in those models we can optimize project completion time or cash flows. We can also consider problems with one type of constraint, where we can optimize resource allocation or costs with constrained time, or optimize project completion time or costs with constrained resources allocation or else optimize project NPV with limited costs. Many authors consider problems with two constraint types. It is possible to build and solve a model with three types of constraints but so far there are no studies on it [4]. It is obvious that apart from those constraints the model can have other constraints, e.g. related to a precedence relationship.

Early efforts in project scheduling focused on minimizing the overall project duration (makespan). Scheduling problems have been studied extensively for many years to determine exact solutions by using methods from the field of operation research.

Due to the necessity of using multi-criteria models for evaluation of feasible schedules in practical situations, several methods have been proposed for multiple-criteria project scheduling. So far there are only a few papers that discuss multiple-criteria project scheduling problem.

Vina and de Sousa [11] solve a multiple-criteria project scheduling problem with three objectives. The first objective aims at minimizing project completion time, the second one, at minimizing the mean weighted lateness of activities, the third one at minimizing the sum of the violation of resource availability. They use also some constraints: to ensure that each activity is processed exactly once, that resource consumption of renewable and nonrenewable resources does not exceed the available quantities and that precedence conditions are fulfilled. They presented two heuristics in their paper: Pareto simulated annealing (PSA) and multiobjective taboo search (MOTS).

Lova, Maroto and Tormos [7] presented a multicriteria heuristic algorithm for multiproject scheduling with two phases. It starts from a feasible multiproject schedule and it improves lexicographicly two criteria: one of time type and one of no time type. In the first phase it obtains a good schedule for the multiproject with time criterion. In the second phase the multiple project schedule is improved with a no time criterion. In their paper the authors use the following time criteria: minimizing mean project delay, minimizing multiproject duration and no time criteria: minimizing project splitting, minimizing in-process inventory, maximizing resource leveling and minimizing idle resources.

Leu and Yang [6] proposed model with two optimization directions: minimizing project completion time and minimizing costs. To solve this problem they used GA- based multicriteria method.

Hapke, Jaszkiewicz and Słowiński [3] presented an interactive search for multi-criteria project scheduling. Their approach consists of two stages. In the first stage, a large representative sample of approximately non-dominated schedules is generated by the PSA method. In the second stage, an interactive search method is used. They use three criteria in their paper: minimizing project completion time, minimizing total project cost and minimizing the average deviation from the average resource usage.

The problem presented in this paper is a multi-criteria project scheduling problem in which the following three types of criteria are used to optimize the project schedule: project completion time, resource smoothness (presented as regularity of resource usage) and cost smoothness. To solve this problem an evolutionary algorithm with direct chromosome representation is used. In this representation a chromosome is a sequence of completion times of each activity.

This paper is organized as follows. Section 1 describes the problem of multiple-criteria project scheduling problem. The problem is stated. Section 2 introduces the evolutionary algorithms for project scheduling problems. The scheme of the algorithm and the description of the direct chromosome representation are presented. In Section 3, the results of the application of evolutionary algorithms for the scheduling problem is presented. Conclusions and ideas for future work are in find Section.

1. Problem statement

A multiple-criteria project scheduling problem is presented in this paper. The goal of this problem is to schedule project tasks so that they meet the constraints and optimize the schedule with respect to time, resource usage and costs generated. This problem can be formulated as follows.

We assume that:

- there is a project to schedule,
- project contains activities,
- project is presented on an AON network,
- each activity is described by a triple: duration, costs and resources,
- precedence relationships are of the Finish-to-Start type,
- costs are generated when an activity starts,
- there is one type of resources,
- resources are needed throughout the duration of an activity,
- we do not allow idle times.

The following notation is used:

$k = 1,...,T$ – set of periods,
$i = 1,...,I$ – set of activities,
S_i – the start time of activity i,
F_i – the finish time of activity i,
d_i – duration of activity i,
r_i – amount of resources required by activity i (in this case we assume that we have one type of resources),
c_i – cost generated by activity i.

The criteria functions can be presented as the following objectives:

1. $F_i \to \min$

2. $\max_{k=1,...,T} \sum_{i=1}^{I} r_{ik} \to \min$ for $i = 1,...,I$, $k = 1,...,T$

3. $\max_{k=1,...,T} \sum_{i=1}^{I} c_{ik} \to \min$ for $i = 1,...,I$, $k = 1,...,T$

The constraints are presented as follows:

4. $F_{i+1} \le F_i - d_i$

5. $r_{ik} \ge 0$ for $i = 1,...,I$, $k = 1,...,T$
   $c_{ik} \ge 0$ for $i = 1,...,I$, $k = 1,...,T$
   $F_i \ge 0$ for $i = 1,...,I$, $k = 1,...,T$.

The (1) goal is to minimize the total time it takes to process all tasks; to minimize the finish time of task i. In the criterion (2) we are minimizing the maximum resource usage in each time. This objective takes care of smoothness in resource allocation. The criterion (3) is minimizing the maximum cost level in each period. It takes care of smoothness in capital allocation. Criteria 2 and 3 are most often connected. Cost is resource usage expressed in money. Often in project management we consider separately the resource usage directly connected with project and the cost, which is understood in a wider sense. Additionally, there is also a formula expressing the precedence relations (4), and constraints about nonnegative variables (5).

2. Evolutionary algorithm for project scheduling problem

An evolutionary algorithm with direct chromosome representation is used in this paper. Below is a description of approach used.

Algorithm steps

In this paper we use a classical scheme of evolutionary approach, but with change in selection. The non-dominated solution goes automatically to the next generation. The scheme of the algorithm is presented below:

1. $t \to 0$
2. Set population $P(t)$
3. Evaluate $P(t)$
4. If condition of finish is fulfilled then end
5. $t \to t+1$
6. Chose $P(t)$ from $P(t-1)$
7. Change $P(t)$ using crossover and mutation
8. Evaluate $P(t)$ and go to 4.

The algorithm starts with a population generated randomly. In the next step the individuals are evaluated. If the condition of finish is fulfilled then we can finish the algorithm. After evaluating, the individuals are selected to breed a new generation. At first to the next generation the non-dominated solutions are selected. This population is changing by crossover and mutation operations, then the individuals are evaluated again.

Chromosome representation

An evolutionary algorithm with direct chromosome representation is used to solve this problem. In this representation a chromosome is a sequence of completion times of each activity [5].

For the problem presented in this paper, in which we have eleven activities, a chromosome looks as follows:

$$(F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}).$$

This is an example of the chromosome presented as a sequence of completion times of each activity.

(2, 5, 6, 4, 5, 11, 13, 8, 7, 18, 22).

This chromosome represents the earliest completion times for each activity. It is also acceptable because it satisfies all the constraints.

Based on this chromosome we can build a schedule which represents start and finish times for each activity and resources required by all activities (Figure 2). Resources are needed throughout the duration of an activity.

In the same way we can present a schedule for activities and generated costs. In this approach the costs are generated at the moment when an activity starts.

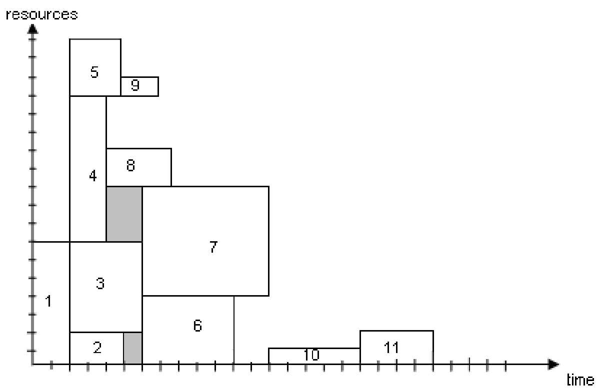


Figure 1. Example of Project schedule

Evaluation (fitness)

The fitness function has three components.

The first component of the fitness function measures the quality of the chromosome related to project completion time. The second component of the fitness function measures the quality of the chromosome related to the smoothness in resource allocation. The third component of the fitness function measures the quality of the chromosome related to the smoothness in cost generating. The costs are generated when an activity starts.

The schedule must satisfy the precedence constraints, so the finish times for activities should satisfy the following inequalities:

$F_2 \geq F_1 + 3$, $F_7 \geq F_3 + 7$,

$F_3 \geq F_1 + 4$, $F_8 \geq F_4 + 4$,

$F_4 \geq F_1 + 2$, $F_9 \geq F_5 + 2$,

$F_5 \geq F_1 + 3$, $F_{10} \geq \max(F_6, F_7, F_8, F_9) + 5$,

$F_6 \geq F_2 + 5$, $F_{11} \geq F_{10} + 4$.

There is a penalty for a chromosome that does not meet those inequalities.

Crossover operation

In this approach the classical crossover operation is used. A single crossover point on both parents organism strings is selected. All data beyond that point in either organism string are swapped between the two offspring organisms.

Mutation operation

Mutation is a genetic operator that alters one or more gene values in a chromosome from its initial state. In this paper only one gene value is changed. The new value is generated as a random variable from all the possible finish times.

To compute this problem, the author of this paper wrote a program in the programming language C.

3. Example and results

In this section an example has been solved. The goal was to schedule tasks of activity and find non-dominated solutions. First, the example is presented, then one iteration of the evolutionary algorithm with direct chromosome representation is shown and at the end of this section, the results are discussed.

3.1. Example

Example 1.

A schedule for the project presented in Table 1 should be created. In this project we have eleven activities. For each task its predecessor, duration, costs generated and resources required are presented.

Table 1

Example 1 tasks

Activity	Predecessor	Activity duration	Amount of resources required by an activity	Costs generated by an activity
1	-	2	7	2
2	1	3	2	3
3	1	4	5	3
4	1	2	8	5
5	1	3	3	6
6	2	5	4	3
7	3	7	6	3
8	4	4	2	4
9	5	2	1	4
10	6, 7, 8, 9	5	2	2
11	10	4	2	1

The problem is presented as an AON network (Figure 2) in which activities are on nodes and relationships between them are on arrows. Each activity is determined by three parameters: duration, resources needed for this activity and costs generated by this activity.

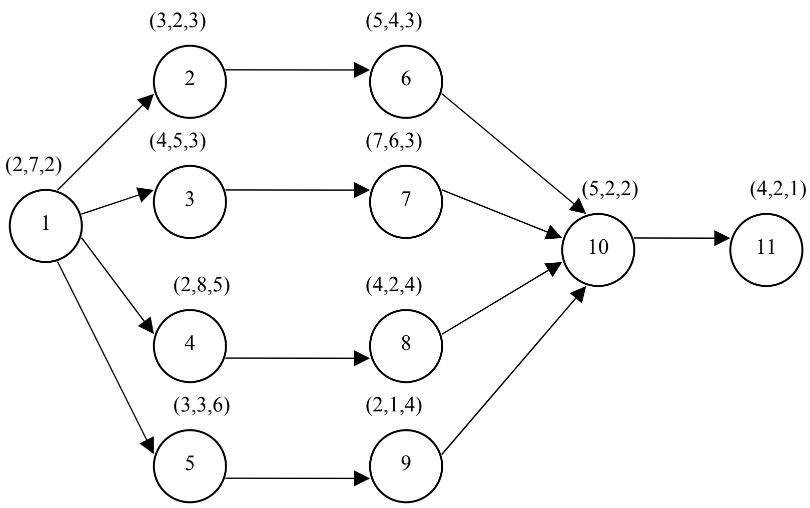


Figure 2. Example 1 network

3.2. Computations

The following EA parameters have been set:

- number of generations: 200,
- crossover rate: 0,9,
- mutation rate: 0,05,
- population size: 20.

Below we show one iteration of evolutionary algorithm with direct chromosome representation.

3. $t \rightarrow 0$
4. Set population $P(t)$

The computation starts with a randomly generated population of 20 individuals:

Individual 1 = (2, 5, 6, 4, 5, 11, 13, 8, 7, 18, 22),
Individual 2 = (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12),
Individual 3 = (2, 5, 6, 4, 3, 7, 4, 2, 8, 12, 30),
Individual 4 = (7, 3, 2, 2, 5, 11, 13, 8, 7, 18, 22),
Individual 5 = (12, 11, 14, 16, 18, 20, 8, 4, 22, 35),
Individual 6 = (5, 6, 8, 5, 20, 16, 18, 17, 17, 22, 26),
Individual 7 = (21, 25, 16, 23, 27, 13, 30, 32, 37, 41, 21),
Individual 8 = (2, 5, 9, 11, 14, 19, 26, 30, 32, 37, 41),
Individual 9 = (2, 5, 6, 15, 13, 10,13,19, 15, 24, 28),
Individual 10 = (16, 23, 27, 13, 32, 36, 4, 7, 12, 15, 16, 14),
Individual 11 = (2, 5, 9, 11, 9, 16, 18, 20, 18, 25, 29),
Individual 12 = (5, 11, 17, 21, 9, 16, 18, 18, 22, 27, 31),
Individual 13 = (21, 12, 11, 9, 8, 7, 16, 18, 10, 21),
Individual 14 = (20,21,19, 22, 18, 17, 16, 37, 19, 11, 23),
Individual 15 = (2, 5, 6, 8, 5, 12, 15, 12, 8, 20, 24),
Individual 16 = (2, 4, 40, 12, 14, 19, 3, 9, 13, 27, 42),
Individual 17 = (2, 5, 6, 20, 16, 18, 13, 24, 18, 29, 33),
Individual 18 = (1, 3, 7, 8, 10, 12, 13, 19, 21, 23, 27),
Individual 19 = (2, 2, 5, 4, 3, 27, 9, 9, 11, 16, 37),
Individual 20 = (2, 6, 8, 4, 7, 12, 15, 16, 14, 21, 25)

3. Evaluate $P(t)$

In this population 6 individuals meet the constraints (individuals: 1, 8, 9, 15, 17 and 20), from those two are non-dominated (individual 1 and individual 8).

4. If the condition of finish is fulfilled then end.

There are 200 generations, so the condition of finish isn`t fulfilled, so we move to the next step.

5. $t \to t+1$

6. Chose $P(t)$ from $P(t-1)$

To the next generation we chose non-dominated individuals first: 1 and 8. Then we fill the population with 18 individuals from the population $P(t-1)$ chosen using proportional selection.

7. Change $P(t)$ using crossover and mutation

Crossover is made with rate 0,9:

Two individuals have been randomized to do crossover. From individuals 11 and 12 after crossover in point 7 we can have two offsprings:

Individual 11` = (2, 5, 9, 11, 9, 16, 18, 18, 22, 27, 31)

Individual 12` = (5, 11, 17, 21, 9, 16, 18, 20, 18, 25, 29)

Individual 11` became a chromosome that meets the constraints.

The mutation rate is 0,05. One mutation point is selected.

Individual 19 after mutation:

Individual 19 = (2, 2, 5, 4, 3, 27, 9, 9, 11, 16, 37),

Individual 19` = (2, **6**, 5, 4, 3, 27, 9, 9, 11, 16, 37).

3.3. Results

After 200 generations nine non-dominated and satisfying the constraints solutions have been found. The solutions are presented in Table 1. For each solution the triple: (time, resource, cost) is presented.

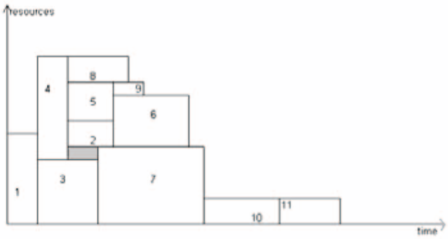
Table 2

Solutions

Non-dominated solution no.	Individual	Level of		
		time	resources	costs
1	(2, 7, 6, 4, 7, 12, 13, 8, 9, 18, 22)	22	13	13
2	(2, 5, 6, 8, 5, 13, 15, 12, 7, 20, 24)	24	12	12
3	(2, 6, 6, 4, 7, 12, 13, 10, 15, 21, 25)	25	10	9
4	(2, 5, 6, 4, 7, 10, 13, 11, 12, 18, 22)	22	15	12
5	(2, 5, 9, 11, 15, 16, 23, 22, 18, 28, 33)	33	8	6
6	(2, 5, 9, 7, 10, 12, 16, 14, 12, 21, 25)	25	13	8
7	(2, 5, 6, 8, 11, 20, 15, 15, 13, 25, 29)	29	9	9
8	(2, 6, 6, 8, 8, 13, 15, 17, 11, 22, 26)	26	11	6
9	(2, 5, 12, 8, 6, 14, 19, 19, 18, 24, 28)	28	10	6

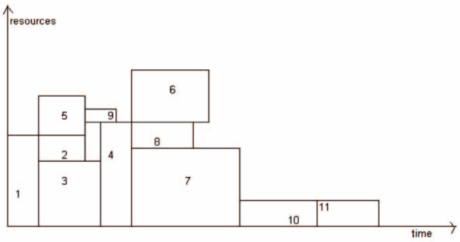
The schedules for solutions are presented below.

Solution 1.



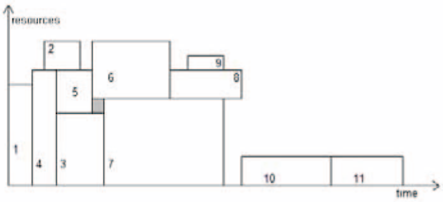
(time, resource, cost) = (22, 13, 13)
Individual = (2, 7, 6, 4, 7, 12, 13, 8, 9, 18, 22)

Solution 2.



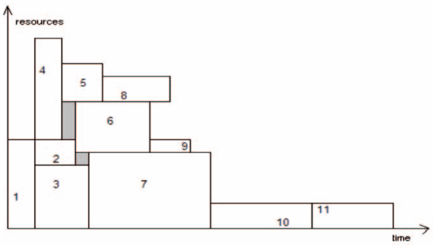
(time, resource, cost) = (24, 12, 12)
Individual = (2, 5, 6, 8, 5, 13, 15, 12, 7, 20, 24)

Solution 3.



(time, resource, cost) = (25, 10, 9)
Individual = (2, 6, 6, 4, 7, 12, 13, 10, 15, 21, 25)

Solution 4.

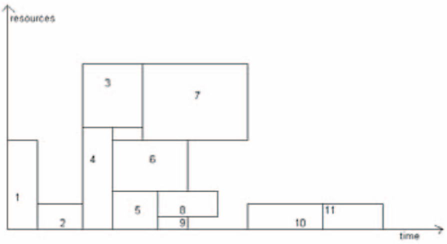


(time, resource, cost) = (22, 15, 12)
Individual = (2, 5, 6, 4, 7, 10, 13, 11, 12, 18, 22)

Solution 5.

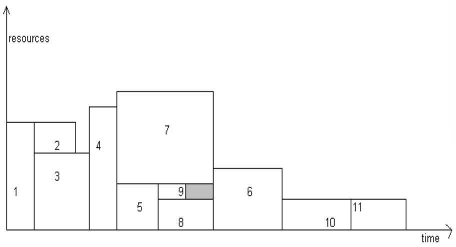
(time, resource, cost) = (33, 8, 6)
Individual = (2, 5, 9, 11, 15, 16, 23, 22, 18, 28, 33)

Solution 6.



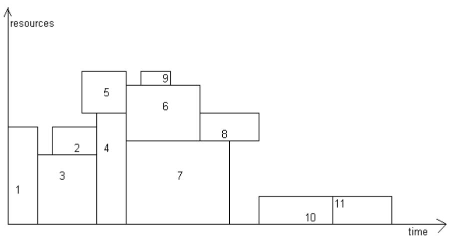
(time, resource, cost) = (25, 13, 8)
Individual = (2, 5, 9, 7, 10, 12, 16, 14, 12, 21, 25)

Solution 7.



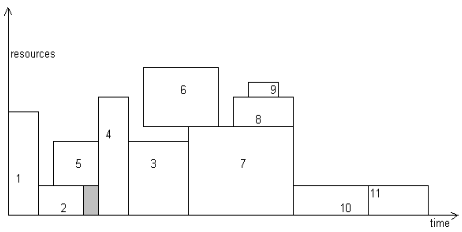
(time, resource, cost) = (29, 9, 9)
Individual = (2, 5, 6, 8, 11, 20, 15, 15, 13, 25, 29)

Solution 8.



(time, resource, cost) = (26, 11, 6)
Individual = (2, 6, 6, 8, 8, 13, 15, 17, 11, 22, 26)

Solution 9.



(time, resource, cost) = (28, 10, 6)
Individual = (2, 5, 12, 8, 6, 14, 19, 19, 18, 24, 28)

The fourth solution is represented by the chromosome: (2, 5, 6, 4, 7, 10, 13, 11, 12, 18, 22). This solution is the best solution because of the time criterion. The shortest time to finish the whole project is 22. It is impossible to finish this project faster without making other criteria worse. The resources in this solution are 15. This is the highest amount of resources needed in one period. The highest amount of costs generated in this project in one period is 12. The same finish time is in the first solution. In this solution the level of costs generated is lower, but the resource level is higher. In the second solution the cost and resource levels are lower, but reduction of these parameters leads to longer project completion time.

The fifth solution is represented by the chromosome: (2, 5, 9, 11, 15, 16, 23, 22, 18, 28, 33). This solution is the best solution because of resource and cost criteria. The resources in this solution are 8 and costs are 6, but the project is finished after 33. The lowest cost level occurs also in the eighth solution, the finish time is better – it is 26 – but the resource level is higher it is 11. A similar situation occurs in the ninth solution, where the finish time is 28 and resource level is 10. The seventh solution also has low cost and resource levels – 9 each, and late finish time 29.

The third solution is represented by the chromosome: (2, 6, 6, 4, 7, 12, 13, 10, 15, 21, 25). This is the most interesting solution because it satisfies all criteria in some way. The project completion time is 25, resource level is 10 and cost level is 9. The same completion time occurs in sixth solution. In this case the cost level is lower but it leads to higher resource level. As we can observe in other solutions, an improvement with respect to one criterion leads to worse levels of other parameters. This situation can be observed in all our solutions.

Conclusion and future works

An evolutionary algorithm with direct chromosome representation for solving a multiple-objective project scheduling problem has been described. It is based on the classical evolutionary algorithm with change in selection where a non-dominated solution automatically goes to the next generation.

This method can be adapted to include new objectives related to needs. It can be also adapted to new chromosome representations and a new algorithm scheme. This approach needs a better solution for indication non-dominated solutions.

It can be useful to try other chromosome presentation to solve a multiple--objective project scheduling problem, eg. permutation without repetition (or with repetition) chromosome representation, priority rule representation, disjunctive graph based representation or random key representation.

We should consider also how to decide which non-dominated individual should be chosen and implemented. It would be useful to use other multicriteria methods, e.g. one of the Electre group method. The first Electre method was proposed in 1966. Since then many adapting techniques have been proposed: to choose the best option (Electre I and Electre IS), to sort solutions (Electre TRI) and to order decision options (Electre II, Electre III, Electre IV) [8].

For further work the elitist evolutionary algorithms can be very useful. This approach uses an archive containing non-dominated solutions previously found (it uses external non-dominated set). At each generation, non-dominated individuals are copied to the external non-dominated set. For each individual in this set, a strength value is computed. The fitness is computed according to the strengths of external non-dominated solutions that dominate it [2].

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ON ROBUST SOLUTIONS TO MULTI-OBJECTIVE LINEAR PROGRAMS

Abstract

In multiple criteria linear programming (MOLP) any efficient solution can be found by the weighting approach with some positive weights allocated to several criteria. The weights settings represent preferences model thus involving impreciseness and uncertainties. The resulting weighted average performance may be lower than expected. Several approaches have been developed to deal with uncertain or imprecise data. In this paper we focus on robust approaches to the weighted averages of criteria where the weights are varying. Assume that the weights may be affected by perturbations varying within given intervals. Note that the weights are normalized and although varying independently they must total to 1. We are interested in the optimization of the worst case weighted average outcome with respect to the weights perturbation set. For the case of unlimited perturbations the worst case weighted average becomes the worst outcome (max-min solution). For the special case of proportional perturbation limits this becomes the conditional average. In general case, the worst case weighted average is a generalization of the conditional average. Nevertheless, it can be effectively reformulated as an LP expansion of the original problem.

Keywords

Multiple criteria, linear programming, robustness, conditional average.

Introduction

In multi-objective linear programming (MOLP) any efficient solution can be found by the weighting approach with some positive weighting of criteria. The weights settings represent preferences and inevitably involve impreciseness and uncertainties causing that the resulting weighted average performance may be lower than expected.

Several approaches have been developed to deal with uncertain or imprecise data in optimization problem. The approaches focused on the quality or on the variation (stability) of the solution for some data domains are considered robust. The notion of robustness applied to decision problems was first introduced by Gupta and Rosenhead [2]. Practical importance of the performance sensitivity against data uncertainty and errors has later attracted considerable attention to the search for robust solutions. Actually, as suggested by Roy [18], the concept of robustness should be applied not only to solutions but, more generally to various assertions and recommendations generated within a decision support process. The precise concept of robustness depends on the way the uncertain data domains and the quality or stability characteristics are introduced. Typically, in robust analysis one does not attribute any probability distribution to represent uncertainties. Data uncertainty is rather represented by non-attributed scenarios. Since one wishes to optimize results under each scenario, robust optimization might be in some sense viewed as a multiobjective optimization problem where objectives correspond to the scenarios. However, despite of many similarities of such robust optimization concepts to multiobjective models, there are also some significant differences [3]. Actually, robust optimization is a problem of optimal distribution of objective values under several scenarios [9] rather than a standard multiobjective optimization model.

A conservative notion of robustness focusing on worst case scenario results is widely accepted and the min-max optimization is commonly used to seek robust solutions. The worst case scenario analysis can be applied either to the absolute values of objectives (the absolute robustness) or to the regret values (the deviational robustness) [6]. The latter, when considered from the multiobjective perspective, represents a simplified reference point approach with the utopian (ideal) objective values for all the scenario used as aspiration levels. Recently, a more advanced concept of ordered weighted averaging was introduced into robust optimization [16], thus, allowing to optimize combined performances under the worst case scenario together with the performances under the second worst scenario, the third worst and so on. Such an approach exploits better the entire distribution of objective vectors in search for robust solutions and, more importantly, it introduces some tools for modeling robust preferences. Actually, while more sophisticated concepts of robust optimization are considered within the area of discrete programming models, only the absolute robustness is usually applied to the majority of decision and design problems.

In this paper we focus on robust approaches to the weighted averages of criteria where the weights are imprecise. Assume that the weights may be affected by perturbations varying within given intervals. Note that the weights are normalized and although varying independently they must total to 1. We are interested in the optimization of the worst case weighted average outcome with respect to the weights perturbation set. For the case of unlimited perturbations the worst case weighted average becomes the worst outcome (max-min solution). For the special case of proportional perturbation limits this becomes the tail average. In general case, the worst case weighted average is a generalization of the tail average. Nevertheless, it can be effectively reformulated as an LP expansion of the original problem.

The paper is organized as follows. In the next section we recall the tail mean (conditional min-max) solution concept providing a new proof of the computational model which remains applicable for more general problems related to the robust solution concepts. Section 2 contains the main results. We show that the robust solution for proportional upper limits on weights perturbations is the tail β-mean solution for an appropriate β value. For proportional upper and lower limits on weights perturbation the robust solution may be expressed as optimization of appropriately combined the mean and the tail mean criteria. Finally, a general robust solution for any arbitrary intervals of weights perturbations can be expressed with optimization problem very similar to the tail β-mean and thereby easily implementable with auxiliary linear inequalities.

1. Solution concepts

Consider a decision problem defined as an optimization problem with m linear objective functions $f_i(\mathbf{x}) = \mathbf{c}^i\mathbf{x}$. They can be either maximized or minimized. When all the objective functions are minimized the problem can be written as follows:

$$\min \{ \; (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})) \; : \; \mathbf{x} \in Q \; \} \tag{1}$$

where $\mathbf{x}$ denotes a vector of decision variables to be selected within the feasible set $Q \subset R^q$, of constraints under consideration and $\mathbf{f}(x) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x}))$ is a vector function that maps the feasible set Q into the criterion space R^m. Let us define the set of attainable outcomes

$$A = \{\mathbf{y} \; : \quad y_i = f_i(\mathbf{x}) \; \forall \, i, \quad \mathbf{x} \in Q\} \tag{2}$$

Model (1) only specifies that we are interested in minimization of all objective functions f_i for $i = 1, 2, \ldots, m$. In order to make the multiple objective model operational for the decision support process, one needs to assume some solution concept well adjusted to the decision maker's preferences. The solution concepts are defined by aggregation functions $a : R^m \to R$. Thus the multiple criteria problem (1) is replaced with the (scalar) minimization problem

$$\min\{a(\mathbf{f}(x)) \ : \quad \mathbf{x} \in Q\} \tag{3}$$

The most commonly used aggregation is based on the weighted mean where positive importance weights w_i ($i = 1, 2, \dots, m$) are allocated to several objectives

$$a(\mathbf{y}) = \sum_{i=1}^{m} y_i w_i \tag{4}$$

The weights are typically normalized to the total 1

$$\bar{w}_i = w_i / \sum_{i=1}^{m} w_i \quad \text{for} \quad i = 1, 2, \dots, m \tag{5}$$

Note that, in the case of equal weights (all $w_i = 1$), all the normalized weights are given as $\bar{w}_i = 1/m$. Due to positive weights, every optimal solution to the weighted mean aggregation (i.e. problem (3) with the aggregation function (4)) is an efficient solution of the original multiple criteria problem. Moreover, in the case of MOLP problems for any efficient solution $\mathbf{x} \in Q$ there exists a weight vector such that $\mathbf{x}$ is an optimal solution to the corresponding weighted problem [19].

Exactly, for the weighted sum solution concept is defined by minimization of the objective function expressing the mean (average) outcome

$$\mu(\mathbf{y}) = \sum_{i=1}^{m} \bar{w}_i y_i$$

but it is also equivalent to minimization of the total outcome $\sum_{i=1}^{m} w_i y_i$. The min-max solution concept is defined by minimization of the objective function representing the *maximum* (worst) outcome

$$M(\mathbf{y}) = \max_{i=1,\dots,m} y_i$$

and it is not affected by the objective weights at all.

A natural generalization of the maximum (worst) outcome $M(\mathbf{y})$ is the (worst) tail mean defined as the mean within the specified tolerance level (amount) of the worst outcomes. For the simplest case of equal weights, one may simply define the tail mean $\mu_{\frac{k}{m}}(\mathbf{y})$ as the mean outcome for the k worst-off objectives (or rather k/m portion of the worst objectives). This can be mathematically formalized as follows. First, we introduce the ordering map $\Theta : R^m \rightarrow R^m$ such that $\Theta(\mathbf{y}) =$

$(\theta_1(\mathbf{y}), \theta_2(\mathbf{y}), \ldots, \theta_m(\mathbf{y}))$, where $\theta_1(\mathbf{y}) \geq \theta_2(\mathbf{y}) \geq \cdots \geq \theta_m(\mathbf{y})$ and there exists a permutation τ of set I such that $\theta_i(\mathbf{y}) = y_{\tau(i)}$ for $i = 1, 2, \ldots, m$. The use of ordered outcome vectors $\Theta(\mathbf{y})$ allows us to focus on distributions of outcomes impartially. Next, the linear cumulative map is applied to ordered outcome vectors to get $\bar{\Theta}(\mathbf{y}) = (\bar{\theta}_1(\mathbf{y}), \bar{\theta}_2(\mathbf{y}), \ldots, \bar{\theta}_m(\mathbf{y}))$ defined as

$$\bar{\theta}_k(\mathbf{y}) = \sum_{i=1}^{k} \theta_i(\mathbf{y}), \quad \text{for} \quad k = 1, 2, \ldots, m. \tag{6}$$

The coefficients of vector $\bar{\Theta}(\mathbf{y})$ express, respectively: the largest outcome, the total of the two largest outcomes, the total of the three largest outcomes, etc. Hence, the *tail $\frac{k}{m}$-mean* $\mu_{\frac{k}{m}}(\mathbf{y})$ is given as

$$\mu_{\frac{k}{m}}(\mathbf{y}) = \frac{1}{k}\bar{\theta}_k(\mathbf{y}), \quad \text{for} \quad k = 1, 2, \ldots, m. \tag{7}$$

According to this definition the concept of tail mean is based on averaging restricted to the portion of the worst outcomes. For $\beta = k/m$, the tail β-mean represents the average of the k largest outcomes.

For any set of weights and and tolerance level β the corresponding tail mean can be mathematically formalized as follows [9,11]. First, we introduce the left-continuous right tail cumulative distribution function (cdf):

$$F_{\mathbf{y}}(d) = \sum_{i=1}^{m} \bar{w}_i \kappa_i(d) \quad \text{where} \quad \kappa_i(d) = \begin{cases} 1 & \text{if } y_i \geq d \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

which for any real (outcome) value d provides the measure of outcomes greater or equal to d. Next, we introduce the quantile function $F_{\mathbf{y}}^{(-1)}$ as the right-continuous inverse of the cumulative distribution function $F_{\mathbf{y}}$:

$$F_{\mathbf{y}}^{(-1)}(\beta) = \sup \{\eta : F_{\mathbf{y}}(\eta) \geq \beta\} \quad \text{for} \quad 0 < \beta \leq 1.$$

By integrating $F_{\mathbf{y}}^{(-1)}$ one gets the (worst) tail mean:

$$\mu_\beta(\mathbf{y}) = \frac{1}{\beta} \int_0^\beta F_{\mathbf{y}}^{(-1)}(\alpha) d\alpha \quad \text{for } 0 < \beta \leq 1. \tag{9}$$

Minimization of the tail β-mean

$$\min_{\mathbf{y} \in A} \mu_\beta(\mathbf{y}) \tag{10}$$

defines the tail β-mean solution concept. When parameter β approaches 0, the tail β-mean tends to the largest outcome ($M(\mathbf{y}) = \lim_{\beta \to 0_+} \mu_\beta(\mathbf{y})$). On the other hand, for $\beta = 1$ the corresponding tail mean becomes the standard mean ($\mu_1(\mathbf{y}) = \mu(\mathbf{y})$).

Note that, due to the finite distribution of outcomes y_i ($i = 1, 2, \ldots, m$) in our MOLP problems, the tail β-mean is well defined by the following optimization

$$\mu_\beta(\mathbf{y}) = \frac{1}{\beta} \max_{u_i} \{ \sum_{i=1}^{m} y_i u_i \ : \quad \sum_{i=1}^{m} u_i = \beta, \ 0 \leq u_i \leq \bar{w}_i \ \forall\, i \}. \tag{11}$$

The above problem is a Linear Program (LP) for a given outcome vector $\mathbf{y}$ while it becomes nonlinear for $\mathbf{y}$ being a vector of variables as in the β-mean problem (10). It turns out that this difficulty can be overcome by an equivalent LP formulation of the β-mean that allows one to implement the β-mean problem (10) with auxiliary linear inequalities. Namely, the following theorem is valid [15]. Although we introduce a new proof which can be further generalized for a family of robust solution concepts we consider.

Theorem 1 *For any outcome vector* $\mathbf{y}$ *with the corresponding objective weights* w_i*, and for any real value* $0 < \beta \leq 1$*, the tail* β*-mean outcome is given by the following linear program:*

$$\mu_\beta(\mathbf{y}) = \min_{t, d_i} \{ t + \frac{1}{\beta} \sum_{i=1}^{m} \bar{w}_i d_i \ : \quad y_i \leq t + d_i, \ d_i \geq 0 \ \forall\, i \}. \tag{12}$$

Proof. The theorem can be proven by taking advantage of the LP dual to problem (11). Introducing dual variable t corresponding to the equation $\sum_{i=1}^{m} u_i = \beta$ and dual variables d_i corresponding to upper bounds on u_i one gets the LP dual (12). Due to the duality theory, for any given vector $\mathbf{y}$ the tail β-mean $\mu_\beta(\mathbf{y})$ can be found as the optimal value of the LP problem (12). □

Following Theorem 1, the tail β-mean solution can be found as an optimal solution to the optimization problem:

$$\min_{\mathbf{y}, \mathbf{d}, t} \{ t + \frac{1}{\beta} \sum_{i=1}^{m} \bar{w}_i d_i \ : \quad \mathbf{y} \in A; \quad y_i \leq t + d_i, \ d_i \geq 0 \ \forall\, i \}, \tag{13}$$

or in a more compact form:

$$\min_{\mathbf{y}, t} \{ t + \frac{1}{\beta} \sum_{i=1}^{m} \bar{w}_i (y_i - t)^+ \ : \quad \mathbf{y} \in A \ \},$$

where $(.)^+$ denotes the nonnegative part of a number.

For the special case of equal weights ($w_i = 1/m$ for all $i \in I$) and $\beta = k/m$ one gets the tail k/m-mean. Model (13) takes then the form:

$$\min_{\mathbf{y},t} \{t + \frac{1}{k} \sum_{i=1}^{m} (y_i - t)_+ \ : \quad \mathbf{y} \in A\} \tag{14}$$

where $(.)_+$ denotes the nonnegative part of a number and r_k is an auxiliary (unbounded) variable. The latter, with the necessary adaptation to the location problems, is equivalent to the computational formulation of the k–centrum model introduced in which is the same as the computational formulation of the k-centrum introduced in [14]. Hence, Theorem 1 and model (13) providing an alternative proof of that formulation generalize the k-centrum formulation of [14] allowing to consider weights and arbitrary size parameter β but preserving the simple structure and dimension of the optimization problem. Within the decision under risk literature, and especially related to finance application, the β-mean quantity is usually called tail VaR, average VaR or conditional VaR (where VaR reads after Value-at-Risk) [17].

2. Robust solutions

The weighted mean solution concept is usually very attractive solution concept due to maximizing the system efficiency taking into account objective importance. It is defined as

$$\min_{\mathbf{y} \in A} \{\sum_{i=1}^{m} \bar{w}_i y_i\} \tag{15}$$

However, in practical problems the objective weights may vary. Therefore, a robust solution is sought which performs well under uncertain objective weights.

The simplest representation of uncertainty depends on a number of predefined scenarios $s = 1, \ldots, r$. Let $\bar{w}_i^s$ denote the realization of weight i under scenario s. Then one may seek a robust solution by minimizing the mean outcome under the worst scenario

$$\min_{\mathbf{y} \in A} \max_{s=1,\ldots,r} \{\sum_{i=1}^{m} \bar{w}_i^s y_i\} = \min_{\mathbf{y} \in A} \{z : z \geq \sum_{i=1}^{m} \bar{w}_i^s y_i \ \forall \ s\}$$

or by minimizing the maximum regret [1]

$$\min_{\mathbf{y} \in A} \max_{s=1,\ldots,r} \{\sum_{i=1}^{m} \bar{w}_i^s y_i - \bar{b}^s\} = \min_{\mathbf{y} \in A} \{z : z \geq \sum_{i=1}^{m} \bar{w}_i^s y_i - \bar{b}^s \ \forall \ s\}$$

where $\bar{b}^s$ represent the best value under scenario s

$$\bar{b}^s = \min_{\mathbf{y}\in A} \{\sum_{i=1}^{m} \bar{w}_i^s y_i\}.$$

Frequently, uncertainty is represented by limits (intervals) on possible values of weights varying independently rather than by scenarios for all the weights simultaneously. We focus on such representation to define robust solution concept. Assume that the objective weights $\bar{w}_i$ may be affected by perturbations varying within intervals $[-\delta_i, \Delta_i]$. Note that the weights are normalized and although varying independently they must total to 1. Thus the objective weights belong to the hypercube:

$$\mathbf{u} \in W = \{(u_1, u_2, \ldots, u_m) \ : \quad \sum_{i=1}^{m} u_i = 1, \ \bar{w}_i - \delta_i \leq u_i \leq \bar{w}_i + \Delta_i \ \forall \ i\}.$$

Alternatively one may consider completely independent perturbations of unnormalized weights w_i and normalize they later to define set W. Focusing on the mean outcome as the primary system efficiency measure to be optimized we get the robust mean solution concept

$$\min_{\mathbf{y}} \max_{\mathbf{u}} \{\sum_{i=1}^{m} u_i y_i \ : \quad \mathbf{u} \in W, \ \mathbf{y} \in A\}. \tag{16}$$

Further, taking into account the assumption that all the constraints of attainable set A remain unchanged while the importance weights are perturbed, the robust mean solution can be rewritten as

$$\min_{\mathbf{y}\in A} \max_{\mathbf{u}\in W} \sum_{i=1}^{m} u_i y_i = \min_{\mathbf{y}\in A} \{\max_{\mathbf{u}\in W} \sum_{i=1}^{m} u_i y_i\} = \min_{\mathbf{y}\in A} \mu^w(\mathbf{y}) \tag{17}$$

where

$$\begin{aligned} \mu^w(\mathbf{y}) &= \max_{\mathbf{u}\in W} \sum_{i=1}^{m} u_i y_i \\ &= \max_{u_i} \{\sum_{i=1}^{m} y_i u_i \ : \quad \sum_{i=1}^{m} u_i = 1, \ \bar{w}_i - \delta_i \leq u_i \leq \bar{w}_i + \Delta_i \ \forall \ i\} \end{aligned} \tag{18}$$

represents the worst case mean outcome for given outcome vector $\mathbf{y} \in A$.

Note that in the case of $\delta_i = \Delta_i = 0$ (no perturbations/uncertainty at all) one gets the standard mean outcome $\mu^w(\mathbf{y}) = \sum_{i=1}^{m} y_i \bar{w}_i$ thus the original mean solution concept. On the other hand, for the case of unlimited perturbations ($\delta_i = \bar{w}_i$ and $\Delta_i = 1 - \bar{w}_i$) the worst case mean outcome

$$\mu^w(\mathbf{y}) = \max_{u_i}\{\sum_{i=1}^m y_i u_i \ : \quad \sum_{i=1}^m u_i = 1,\ 0 \le u_i \le 1\ \forall\, i\} = \max_{i=1,\dots,m} y_i$$

becomes the worst outcome thus leading to the min-max solution concept.

For the special case of proportional perturbation limits $\delta_i = \delta \bar{w}_i$ and $\Delta_i = \Delta \bar{w}_i$ with positive parameters δ and Δ, one gets

$$\mu^w(\mathbf{y}) = \max_{u_i}\{\sum_{i=1}^m y_i u_i : \sum_{i=1}^m u_i = 1,\ \bar{w}_i(1-\delta) \le u_i \le \bar{w}_i(1+\Delta)\ \forall\, i\} \quad (19)$$

In particular, when lower limits are relaxed ($\delta = 1$) this becomes the classical tail mean outcome [12,15] with $\beta = 1/(1+\Delta)$. Thus the tail mean represents the robust mean solution concept for proportionally upper bounded perturbations.

Theorem 2 *The tail β-mean represents a concept of robust mean solution (17) for proportionally upper bounded perturbations $\Delta_i = \Delta \bar{w}_i$ with $\Delta = (1-\beta)/\beta$ and relaxed the lower ones $\delta_i = \bar{w}_i$ for all $i \in I$.*

Proof. For proportionally bounded upper perturbations $\Delta_i = \Delta \bar{w}_i$ and $\delta_i = \bar{w}_i$ the corresponding worst case mean outcome (18) can be expressed as follows

$$\begin{aligned}\mu^w(\mathbf{y}) &= \max_{u_i}\{\sum_{i=1}^m y_i u_i : \sum_{i=1}^m u_i = 1,\ 0 \le u_i \le \bar{w}_i(1+\Delta)\ \forall\, i\} \\ &= (1+\Delta)\max_{u_i'}\{\sum_{i=1}^m y_i u_i' : \sum_{i=1}^m u_i' = \frac{1}{1+\Delta},\ 0 \le u_i' \le \bar{w}_i\ \forall\, i\} \\ &= (1+\Delta)\mu_{\frac{1}{1+\Delta}}(\mathbf{y})\end{aligned}$$

which completes the proof. □

As the tail mean is easily defined by auxiliary LP constraints, the same applies to the robust mean solution concept for proportionally bounded upper perturbations and relaxed the lower ones.

Corollary 1 *The robust mean solution concept (17) for proportionally bounded upper perturbations $\Delta_i = \Delta \bar{w}_i$ and relaxed the lower limits $\delta_i = \bar{w}_i$ for all $i \in I$ can be found by simple expansion of the optimization problem with auxiliary linear constraints and variables to the following:*

$$\min_{\mathbf{y},\mathbf{d},t} \{t + (1+\Delta)\sum_{i=1}^m \bar{w}_i d_i : \quad \mathbf{y} \in A; \quad y_i \le t + d_i,\ d_i \ge 0\ \forall\, i\}. \quad (20)$$

Example 1 Consider the following MOLP problem with two objectives:

$$\min \{(x_1, x_2) : 3x_1 + 5x_2 \geq 36,\ x_1 \geq 2,\ x_2 \geq 3\}. \tag{21}$$

The efficient set for this problem is

$$\{(x_1, x_2) : 3x_1 + 5x_2 = 36,\ x_1 \geq 2,\ x_2 \geq 3\},$$

i.e. the entire line segment between vertices $(2, 6)$ and $(7, 3)$, including both vertices.

Let us assume that the DM preferences has been recognized as represented by equal weights $\bar{w}_1 = \bar{w}_2 = 0.5$ although the weights may actually vary around this values thus belonging to the hypercube:

$$W = \{(u_1, u_2) : \quad u_1 + u_2 = 1,\ 0 \leq u_1 \leq 0.5(1+\Delta),\ 0 \leq u_2 \leq 0.5(1+\Delta)\}$$

for some $\Delta > 0$. The ideal weights $\bar{w}$ generate the best efficient solution in the vertex $(2, 6)$. However, for weights $(0.35, 0.65)$ one gets rather the vertex $(7, 3)$ as the best solution. Hence, it is quite natural to look for a robust solution which is based on the worst weights within the set W. Following Corollary 1, such a robust solution can be found by solving the expanded LP problem:

$$\begin{aligned} \min\{t + (1+\Delta)(0.5d_1 + 0.5d_2) : \ & 3x_1 + 5x_2 \geq 36, \\ & x_1 \geq 2,\ x_2 \geq 3, \\ & x_1 \leq t + d_1,\ x_2 \leq t + d_2, \\ & d_1 \geq 0,\ d_2 \geq 0\}. \end{aligned} \tag{22}$$

In our case, due to only to outcomes and equal weights, one can easily notice that for any (x_1, x_2) the best values of auxiliary variables are defined as $t = \min\{x_1, x_2\}$, $d_1 = x_1 - t$ and $d_2 = x_2 - t$. Hence, $d_1 + d_2 = \max\{x_1, x_2\} - \min\{x_1, x_2\} = |x_1 - x_2|$ and the auxiliary variables can be eliminated leading to the ordered weighted objective [13]

$$\begin{aligned} t + (1+\Delta)(0.5d_1 + 0.5d_2) &= 0.5(1+\Delta)\max\{x_1, x_2\} \\ &\quad +0.5(1-\Delta)\min\{x_1, x_2\} \\ &= 0.5(1+\Delta)\theta_1(\mathbf{x}) + 0.5(1-\Delta)\theta_2(\mathbf{x}) \end{aligned}$$

or alternatively to its cumulated form

$$\begin{aligned} t + (1+\Delta)(0.5d_1 + 0.5d_2) &= \Delta\max\{x_1, x_2\} + 0.5(1-\Delta)(x_1 + x_2) \\ &= \Delta\bar{\theta}_1(\mathbf{x}) + 0.5(1-\Delta)\bar{\theta}_2(\mathbf{x}). \end{aligned}$$

Hence, our robust optimization problem (22) can be simplified to the following form:

$$\min\{\Delta \max\{x_1, x_2\} + (1-\Delta)0.5(x_1+x_2) : 3x_1+5x_2 \geq 36,\ x_1 \geq 2,\ x_2 \geq 3\}$$

thus representing a convex combination of the original weighted optimization and the minimax optimization models. One may easily verify that for $\Delta = 0.1$ the optimal vertex $(2,6)$ remains the corresponding robust solution. On the other hand, for $\Delta = 0.5$ the minimax point $(4.5, 4.5)$ becomes the corresponding robust solution.

Certainly, in the case of unequal weights or especially for more than two criteria the robust optimization problem cannot be simply expressed as a combination of the original weighted aggregation with minimax criterion. Nevertheless, the LP formulation (20) can be effectively solved. □

In the general case of proportional perturbation limits (19) the robust mean solution concept cannot be directly expressed as an appropriate tail β-mean. It turns out, however, that it can be expressed by the optimization with combined criteria of the tail β-mean and the original mean as shown in the following theorem.

Theorem 3 *The robust mean solution concept (17) for proportionally bounded perturbations $\Delta_i = \Delta \bar{w}_i$ and $\delta_i = \delta \bar{w}_i$ for all $i \in I$ is equivalent to the convex combination of the mean and tail β-mean criteria minimization*

$$\min_{\mathbf{y}\in A} \mu^w(\mathbf{y}) = \min_{\mathbf{y}\in A} (1+\Delta)[\lambda \mu_\beta(\mathbf{y}) + (1-\lambda)\mu(\mathbf{y})] \tag{23}$$

with $\beta = \delta/(\Delta+\delta)$ and $\lambda = (\Delta+\delta)/(1+\Delta)$.

Proof. For proportionally bounded perturbations $\Delta_i = \Delta \bar{w}_i$ and $\delta_i = \delta \bar{w}_i$ the corresponding worst case mean outcome (18) can be expressed as follows

$$\begin{aligned}
\mu^w(\mathbf{y}) &= \max_{u_i}\{\sum_{i=1}^m y_i u_i : \sum_{i=1}^m u_i = 1,\ \bar{w}_i(1-\delta) \leq u_i \leq \bar{w}_i(1+\Delta)\ \forall\, i\} \\
&= (1+\Delta) \max_{u'_i}\{\sum_{i=1}^m y_i u'_i : \sum_{i=1}^m u'_i = \frac{1}{1+\Delta},\ \bar{w}_i \frac{1-\delta}{1+\Delta} \leq u'_i \leq \bar{w}_i\ \forall\, i\} \\
&= (1+\Delta) \max_{u''_i}\{\sum_{i=1}^m y_i u''_i : \sum_{i=1}^m u''_i = \frac{\delta}{1+\Delta},\ 0 \leq u''_i \leq \bar{w}_i \frac{\Delta+\delta}{1+\Delta}\ \forall\, i\} + \\
&\quad + (1-\delta)\sum_{i=1}^m y_i \bar{w}_i
\end{aligned}$$

$$= (\Delta + \delta) \max_{u_i'''} \{ \sum_{i=1}^{m} y_i u_i''' : \sum_{i=1}^{m} u_i''' = \frac{\delta}{\Delta + \delta},\ 0 \le u_i''' \le \bar{w}_i \ \forall\, i\} +$$
$$+ (1 - \delta)\mu(\mathbf{y})$$
$$= (1 + \Delta)[\frac{\Delta + \delta}{1 + \Delta}\mu_{\frac{\delta}{\Delta+\delta}}(\mathbf{y}) + \frac{1 - \delta}{1 + \Delta}\mu(\mathbf{y})]$$

which completes the proof. □

Following Theorems 1 and 3, the robust mean solution concept (17) can be expressed as an LP expansion of the original mean problem.

Corollary 2 *The robust mean solution concept (17) for proportionally bounded perturbations $\Delta_i = \Delta\bar{w}_i$ and $\delta_i = \delta\bar{w}_i$ for all $i \in I$ can be found by simple expansion of the mean problem with auxiliary linear constraints and variables to the following problem:*

$$\min_{\mathbf{y},\mathbf{d},t} \{ \sum_{i=1}^{m} \bar{w}_i y_i + \frac{\Delta + \delta}{1 - \delta} t + \frac{(\Delta + \delta)^2}{\delta(1 - \delta)} \sum_{i=1}^{m} \bar{w}_i d_i : \quad \mathbf{y} \in A; \quad y_i \le t + d_i,\ d_i \ge 0 \ \forall\, i\}. \tag{24}$$

In general case of arbitrary intervals of weights perturbations, the worst case mean outcome (18) cannot be expressed as a tail β-mean or its combination. Nevertheless, the structure of optimization problem (18) remains very similar to that of the tail β-mean (11). Note that problem (18) is an LP for a given outcome vector $\mathbf{y}$ while it becomes nonlinear for $\mathbf{y}$ being a vector of variables. This difficulty can be overcome similar to Theorem 1 for the tail β-mean.

Theorem 4 *For any arbitrary intervals $[-\delta_i, \Delta_i]$ (for all $i \in I$) of weights perturbations, the corresponding worst case mean outcome (18) can be given as*

$$\mu^w(\mathbf{y}) = \min_{t, d_i^u, d_i^l} \{ \ t + \sum_{i=1}^{m} (\bar{w}_i + \Delta_i) d_i^u - \sum_{i=1}^{m} (\bar{w}_i - \delta_i) d_i^l : \quad t + d_i^u - d_i^l \ge y_i,\ d_i^u, d_i^l \ge 0 \quad \forall\, i\}. \tag{25}$$

Proof. The theorem can be proven by taking advantages of the LP dual to (18). Introducing dual variable t corresponding to the equation $\sum_{i=1}^{m} u_i = 1$ and variables d_i^u and d_i^l corresponding to upper and lower bounds on u_i, respectively, one gets the following LP dual to problem (18). Due the duality theory, for any given vector $\mathbf{y}$ the worst case mean outcome $\mu^w(\mathbf{y})$ can be found as the optimal value of the LP problem (25). □

Following Theorem 4, the robust mean solution concept (17) can be expressed similar to the tail β-mean with auxiliary linear inequalities expanding the original constraints.

Corollary 3 *For any arbitrary intervals* $[-\delta_i, \Delta_i]$ *(for all* $i \in I$*) of weights perturbations, the corresponding robust mean solution (17) can be given by the following optimization problem:*

$$\begin{aligned}\min_{\mathbf{y},t,d_i^u,d_i^l} \{ \quad & t + \sum_{i=1}^{m} (\bar{w}_i + \Delta_i) d_i^u - \sum_{i=1}^{m} (\bar{w}_i - \delta_i) d_i^l : \\ & \mathbf{y} \in A; \quad t + d_i^u - d_i^l \geq y_i,\ d_i^u, d_i^l \geq 0 \quad \forall\, i\}.\end{aligned} \tag{26}$$

Actually, there is a possibility to represent general robust mean solution (17) with optimization problem even more similar to the tail β-mean and thereby with lower number of auxiliary variables than in (26).

Theorem 5 *For any arbitrary intervals* $[-\delta_i, \Delta_i]$ *(for all* $i \in I$*) of weights perturbations, the corresponding robust mean solution (17) can be given by the following optimization problem*

$$\begin{aligned}\min_{\mathbf{y},t,d_i} \{ \quad & \sum_{i=1}^{m} (\bar{w}_i - \delta_i) y_i + \bar{\delta} t + \sum_{i=1}^{m} (\Delta_i + \delta_i) d_i : \\ & \mathbf{y} \in A; \quad t + d_i \geq y_i,\ d_i \geq 0 \quad \forall\, i\}\end{aligned} \tag{27}$$

where $\bar{\delta} = \sum_{i=1}^{m} \delta_i$.

Proof. Note that the worst case mean (18) may be transformed as follows

$$\begin{aligned}&\mu^w(\mathbf{y}) = \max_{u_i}\{\sum_{i=1}^{m} y_i u_i : \sum_{i=1}^{m} u_i = 1,\ \bar{w}_i - \delta_i \leq u_i \leq \bar{w}_i + \Delta_i\ \forall\, i\} \\ &\max_{u_i'}\{\textstyle\sum_{i=1}^{m} y_i u_i' : \sum_{i=1}^{m} u_i' = \sum_{i=1}^{m} \delta_i,\ 0 \leq u_i' \leq \Delta_i + \delta_i\ \forall\, i\} + \\ &\qquad + \sum_{i=1}^{m} y_i(\bar{w}_i - \delta_i).\end{aligned} \tag{28}$$

Next, replacing the maximization over variables u_i with the corresponding dual we get

$$\mu^w(\mathbf{y}) = \min_{t,d_i} \{(\sum_{i=1}^{m}\delta_i)t + \sum_{i=1}^{m}(\Delta_i + \delta_i)d_i : t + d_i \geq y_i,\ d_i \geq 0\,\forall\, i\} + \sum_{i=1}^{m}(\bar{w}_i - \delta_i)y_i$$

Further, minimization over $\mathbf{y} \in A$ leads us to formula (27) which completes the proof. □

For a special case of arbitrary upper bounds Δ_i and completely relaxed lower bound we get the following result.

Corollary 4 *For any arbitrary upper bounds Δ_i and and relaxed the lower ones $\delta_i = \bar{w}_i$ (for all $i \in I$) on weights perturbations, the corresponding robust mean solution (17) can be given by the following optimization problem*

$$\min_{\mathbf{y},t,d_i} \{t + \sum_{i=1}^{m} (\Delta_i + \bar{w}_i)d_i : \ \mathbf{y} \in A; \ t + d_i \geq y_i, \ d_i \geq 0 \quad \forall\, i\}. \tag{29}$$

Note that optimization problem (29) is very similar to the tail β-mean model (13). Indeed, in the case of proportional upper limits $\Delta_i = \Delta\bar{w}_i$ (for all $i \in I$) problem (29) simplifies to (20) as stated in Corollary 1.

Concluding remarks

For multiple objective linear programming problems with objective weights the mean solution concept is well suited for system efficiency maximization. However, real-life objective weights inevitably involve errors and uncertainties and thereby the resulting performance may be lower than expected. We have analyzed the robust mean solution concept where weights uncertainty is represented by limits (intervals) on possible values of weights varying independently. Such an approach, in general, leads to complex optimization models with variable coefficients (weights).

We have shown that in the case of the weighted multiple objective linear programming problem the robust mean solution concepts can be expressed with auxiliary linear inequalities, similarly to the tail β-mean solution concept [15] based on minimization of the mean in β portion of the worst outcomes. Actually, the robust mean solution for proportional upper limits on weights perturbations turns out to be the tail β-mean for an appropriate β value. For proportional upper and lower limits on weights perturbation the robust mean solution may be sought by optimization of appropriately combined the mean and the tail mean criteria. Finally, a general robust mean solution for any arbitrary intervals of weights perturbations can be expressed with optimization problem very similar to the tail β-mean and thereby easily implementable with auxiliary linear inequalities.

Our analysis has shown that the robust mean solution concept is closely related with the tail mean which is the basic equitable solution concept. It corresponds to recent approaches to the robust optimization based on the equitable optimization ([7], [16], [5]). Further study on equitable solution concepts and their relations to robust solutions seems to be a promising research direction.

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MULTIOBJECTIVE MODEL FOR DESIGNING CUSTOMIZED TOURIST TOURS

Abstract

The tourist sector has undergone many changes in recent years due to technological progress and demographic change. On the one hand, there have been immense advances in communication technology and easy access to the Internet, which have led to the globalization of tourist information and greater numbers of tourists being able to access information on a huge number of products. On the other hand, a substantial change has taken place in tourist preferences and behaviour, with a move away from standard trips to other more personalized options, where customer preferences are taken into account. They are not only looking for sun and beach activities, but are also interested in culture and heritage, thus distributing their time between cultural visits and relaxation and leisure. When planning a tour, the tourist's objectives may be in conflict because, among other factors, the most important attractions are usually the most expensive, and thus the tourist is clearly faced with a multiobjective problem. We develop a model to solve this problem, taking into account the diverse economic costs (transport, the cost of different activities, lodging, etc), the timing of the different activities, and his/her particular preferences.

Keywords

Tourism, tourist tours planning, multi-criteria selection of tourist tours packages.

Introduction

The tourist sector of the economy has experienced immense growth and various changes in recent decades, including a substantial transformation in tourist preferences and behaviour that has had several consequences.

We specifically focus on the fact that planning a trip has become a complex task because tourists are increasingly more demanding and require customized trips instead of standardized ones ([15], [11]). Several factors are involved in this issue.

First, tourist interests have evolved from traditional activities, such as sun and sand, to new ones, such as business tourism, cultural tourism, leisure and entertainment tourism, rural tourism, health tourism and religious tourism [5]. Thus, instead of having one main interest, there are large numbers of tourists who have several, and thus have to divide their time among different activities, which complicates planning the trip.

Second, changes in patterns of tourist behaviour affect trip planning. On the one hand, there are increases in short vacations, with tourists travelling on weekends and holidays, instead of during the summer vacation, such that he/she needs a personalized plan that fits his/her needs. Furthermore, tourists tend to book at the last moment, so they have little time for planning.

Third, another factor that influences these difficulties is the multitude of alternatives available regarding each element of a journey. This information is accessible to tourists, given the new technologies, and they can compare prices, features, etc. Searching and studying all this information involves a high time-cost, and he/she also has to coordinate and schedule these activities. These new tourist activities have arisen in recent years in the attempt to meet the needs of diverse types of tourists ([11], [25], [27]). Among such offers, we can highlight accommodation, catering, transport, additional activities (cultural, leisure parks, etc) or tourist intermediaries, and each of these groups provide various options to satisfy different types of tourists.

As noted, new technologies make planning a customized trip easier. First, they offer easy access to a large amount of tourist information, provided by suppliers and users, thereby facilitating tourist information searches. The tourist can obtain detailed information on tourist destinations, the activities in those destinations, updated tariffs, timetables, etc. The new technologies also provide various tools that assist the tourist during the Web-based purchasing process such that he/she can make the booking.

However, with so much information available, it is too complex for the tourist to study all the possible alternatives. In addition, studying and searching for the best alternative would not guarantee choosing the best option, since his/her objectives may be in conflict [18]. On the one hand he/she may wish to minimize costs, but on the other hand, he/she may wish to maximize the utility provided by the activities.

Thus, we consider that tourists require assistance in decision-making regarding the various alternatives when planning a trip. This is an opportunity to improve the sector, and is addressed by the development and implementation of a tool to facilitate the organization of a customized trip. This system would benefit both the tourist, because he/she would obtain the trip best suited to his/her needs, and also the travel agents, as it would enable them to offer added value to tourists, thus motivating the present work.

The aim of this paper, therefore, was to develop a model that would help the tourist to plan his/her trip, by helping him/her to choose the best alternative. We provide a detailed itinerary that includes all the activities at each time during the trip, by considering the tourist's wishes and any conflicts between his/her objectives. We also take into account the various constraints – such as the time available for the activities, the duration of each activity, the time spent on route from one activity to another and the budget, among others – in order to choose the most appropriate tourist route.

Various systems have been described in the literature, which have attempted to help tourists plan a customized trip; however, they have certain limitations, as they do not take into consideration all the elements needed to offer each tourist the most appropriate option. Thus, the present paper proposes a new system that takes into account these gaps.

Some support systems developed for the tourism sector have only attempted to facilitate the search for tourist information [4]. Other systems, when providing information, consider the tourism offer at that time [12]. Several systems provide recommendations regarding the destination or activities ([10], [21], [22], [3], [2], [24]).

Other systems take tourist location into account by means of global positioning systems, to indicate which activities are nearer and guide him/her towards a specific tour, whereas other systems also consider user profile [26]. Yet others consider the tourist's context ([28], [17], [23]), whereas some systems consider all these items at the same time ([19], [20], [13], [14]). Finally, some systems use multi-criteria techniques to consider the various issues that may arise when planning a trip ([7], [8], [9]).

As mentioned, this study attempts to meet the requirements and drawbacks of the systems and methods analyzed, to offer tourists a system that actually helps them choose the best alternative when planning a trip, and engaging in a series of activities during their stay. This system is so generic that it can be used for any tourist and can be applied to specific segments by incorporating a suitable database.

1. Model formulation

We formulate a model that can be applied to any tourist who wants to spend a certain number of days, N, in a specific area doing some activities (there are a number of possible activities, M). Thus, we help him/her to make a decision among various alternative tours.

Activities are classified into 3 groups – accommodation, restaurants, and visits – which are denoted by A_1, A_2 and A_3; the latter group includes museums, monuments, beaches, leisure, walks and other visits.

The set of alternatives is formed by the different itineraries that the tourist can follow. Each itinerary is composed of different tourist routes to be followed each day; by "tourist route" we mean an ordered set of activities that the tourist will do during the day. This set of activities is formed by the decision variables x_{ijt} ($i, j = 1, 2,\ldots, M; t = 1, 2,\ldots, T$), which are binary variables that take value 1 if the tourist moves from activity i to activity j on day t, and value 0 otherwise:

$$x_{ijt} = \begin{cases} 1 \ \textit{if tourist moves from activ. } i \textit{ to activ. } j \textit{ on day } t \\ 0 \ \textit{otherwise} \end{cases} \tag{1}$$

$$i, j = 1,\ldots.M; \quad t = 1,\ldots..N$$

We also define some auxiliary variables to simplify the model: we denote by y_{jt} the number of times the tourist does activity j during day t, that is:

$$y_{\mathrm{jt}} = \sum_{i=1}^{M} x_{ijt} \quad t = 1,\ldots..N \tag{2}$$

and we denote by y_j the number of times the tourist does activity j during the entire tour.

$$\mathrm{y}_{\mathrm{j}} = \sum_{t=1}^{N} y_{jt} \qquad j = 1,\ldots.M \tag{3}$$

1.1. Objectives and constraints

We now define the tourist objectives and the constraints that the model must fulfil:

Objectives:

Regarding the objectives, we take into account: minimizing the cost of transport from one activity to another, minimizing the cost of the activities, maximizing the utility of the activities for the tourist, and adjusting the time dedicated to each type of visit to the preferences of the tourist.

– First, we minimize the cost of transport from one activity to another. This cost depends on the distance between the place of activity and the means of transport; we assume the tourist travels by car. Thus, this objective is equivalent to minimizing the distance measured in kilometres during the tour, and is formulated as follows:

$$Min \sum_{i=1}^{M} \sum_{i=1}^{M} \sum_{t=1}^{N} d_{ij} x_{ijt} \quad (4)$$

  where d_{ij} represents the distance from activity *i* to activity *j*.

– The second objective is to minimize the cost of the activities. The formuation for this objective is:

$$Min \sum_{j=1}^{M} \sum_{t=1}^{N} c_j y_{jt} \quad (5)$$

  where c_j is the cost of activity *j*. This cost can be broken down as follows: accommodation cost, restaurant cost, and visit cost, and could be equal to zero in the case of free activities, e.g., visiting a park or going to the beach.

– The third objective is to maximize tourist satisfaction with the activities, which is calculated by aggregating the relevance of the activity and the tourist's preferences. The formulation for this objective is as follows:

$$Max \sum_{j=1}^{M} \sum_{t=1}^{N} u_j y_{jt} \quad (6)$$

  where u_j is the utility of activity *j*. Relevance is measured by the importance of the activity in the media, quality, etc. Regarding tourist preferences, and given the tourist does not have complete information, we assume that the tourist places a value on some characteristic of the activity. For example, each activity can be classified into subtypes; museums can be art museums, history museums, etc.

– The final objective is to adjust the time dedicated to each type of visit to the preferences of the tourist. We minimize the distance between "Real Time of Visit" and "Desired Time of Visit". The formulation for this objective is:

$$Min \quad \left| ttar_{3k} - ttad_{3k} \right| \qquad k = \{1,\dots w\} \tag{7}$$

where we denote by $ttar_{3k}$ an auxiliary variable that indicates the real time dedicated to the type of visit k and denote by $ttad_{3k}$ an auxiliary variable that indicates the time the tourist wishes to dedicate to the type of visit k. We define these variables as follows:

Real Time of Visit:

$$ttar_{3k} = \sum_{j \in A_{3k}} \sum_{t=1}^{N} ta_j \, y_{jt}\,, \quad k = \{1,\dots w\}, \quad j = \{1,\dots M\} \tag{8}$$

where ta_j is the duration of activity j. The duration of activities depends on the average duration of the activity, the decision-maker's preferences, and a rest period.

Desired Time of Visit:

$$ttad_{3k} = pd_{3k} \; Tv \qquad k = \{1,\dots w\} \tag{9}$$

where pd_{3k} is the percentage of the total time dedicated to visits that the tourist wishes to dedicate to the type of visit k; and Tv is the total time dedicated to visits during the tour. This is defined as:

$$Tv = \sum_{j \in A_3} ta_j \, y_j \tag{10}$$

Once the objectives have been determined, we formulate the constraints of the model.

There are two types of constraints: permanent constraints, which are independent of the decision-maker, that is, they must be fulfilled anyway; and the decision-maker's constraints, that is, when the tourist wishes the tour to have certain characteristics.

Permanent constraints:

- If a tourist does activity j ($j = 1, 2,\ldots, M$) on day t ($t = 1, 2,\ldots, T$), the tourist must finish activity j during day t, unless activity j is accommodation, in which case the tourist will stop activity j the following day.

$$\sum_{i=1}^{M} x_{ijt} = \sum_{k=1}^{M} x_{jkt} \qquad \forall j \begin{cases} \notin A_1 \\ \neq initial \ \ point \ \ , \ \ t = 1,\ldots N \\ \neq end \ \ point \end{cases} \tag{11}$$

- Regarding accommodation, the tourist will leave the day after arrival, and therefore:

$$\sum_{i=1}^{M} x_{ijt} = \sum_{k=1}^{M} x_{jk,t+1} \quad \forall j \in A_1 \quad t = 1,\ldots N-1 \tag{12}$$

  This constraint does not affect to the last day of the tour, because the tourist arrives to the accommodation, but he/she does not leave for any other activity. In this case, accommodation will be the end point.

- On the first day, the tourist starts from an initial point indicated by him/her (airport, train station, accommodation from a previous stage, etc). From this point, he/she leaves for a given activity, but does not have to return, so we must add the following constraint:

$$\sum_{j=1}^{M} x_{ijt} = 1 \quad i = initial \ \ point, \ \ t = 1 \tag{13}$$

- Likewise, an endpoint is a point of arrival but not a point of departure, therefore it does not fulfil the previous constraints and we must add the next constraint:

$$\sum_{i=1}^{M} x_{ijt} = 1 \quad j = end \ \ point, \ \ t = N \tag{14}$$

- The tourist must seek accommodation each day when a route with an overnight stay is planned. Accommodation is selected in a previous stage (as described below) based on tourist satisfaction:

$$y_{jt} = 1 \qquad \forall j \ set \ \ accommodation \ \ for \ day \ \ t \tag{15}$$

- The maximum number of times that the tourist can do an activity during the entire tour is indicated by "*Numrepetitions*". This number depends on each type of activity, e.g., a church will be visited only once:

$$\sum_{t=1}^{N} y_{jt} \leq Num\, repetitions_j, \quad j = 1,\ldots M \tag{16}$$

– Regarding restaurants, different circumstances can occur: depending on the tour timetable, we may plan one, two or no meals in a day, but, if possible, a meal will be planned:

$$\sum_{j\in A_2} y_{jt} = 2 \qquad t \in D_{ac}\,;\, D_{ac} = \{t : I_t 13:00 \; y \; F_t 22:00\} \tag{17}$$

$$\sum_{j\in A_2} y_{jt} = 1 \qquad t \in D_a\,;\; D_a = \{t : I_t 13:00 \; y\, 15:00 \le F_t\, 20:00\} \tag{18}$$

$$\sum_{j\in A_2} y_{jt} = 1 \qquad t \in D_c\,;\; D_c = \{t : I_t 15:00 \; y \; F_t 22:00\} \tag{19}$$

$$\sum_{j\in A_2} y_{jt} = 0 \qquad t \in D_s\,;\; D_s = \left\{ \begin{array}{c} t:\, I_t \ge 15:00 \;\; y \;\; F_t \le 21:00, \\ ó \;\;\; F_t \le 13:00 \end{array} \right\} \tag{20}$$

D_{ac} being the days when it is possible to plan both lunch and dinner; D_a, the days when it is possible to plan only lunch; D_c, the days when it is possible to plan only dinner; and D_s, the days when it is not possible to plan any meal. We denote by I_t the time when the tourist wants to start the route on day *t*, and by F_t, the time he/she wants to finish, defined as the time of arrival at the accommodation.

Each activity has a timetable, and the tourist must follow this schedule. We define auxiliary variables, HI_{jt}, as the starting time of activity *j* ($j = 1, 2,\ldots, M$) on day *t* ($t = 1, 2,\ldots, T$), and HF_{jt} as the finishing time of this activity.

– The finishing time of an activity is equal to the starting time of this activity plus its duration; it occurs if this activity is done, if not it is equal to zero, the constraint being as follows:

$$\mathrm{HF_{jt}} = \mathrm{HI_{jt}} + ta_j\, y_{jt}\,, \qquad \begin{cases} j \notin A_1 \\ j \neq \text{initial point} \end{cases} \qquad t = 1,\ldots..N \tag{21}$$

ta_j being the duration of the activity *j*. We use the variable y_{jt}, defined in expression (2) as the number of times that activity *j* is done on day *t*:

$$\mathrm{HF_{jt}} = \mathrm{HI_{jt}}\,, \qquad \begin{cases} j \notin A_1 \\ j \neq \text{initial point} \end{cases} \qquad t = 1,\ldots..N \tag{22}$$

in expression (30), $HI_{jt} = 0$ for activities not carried out.

This is true for any activity except for accommodation and the initial point. In the case of accommodation, we define the start time and the end time according to the preferences of the tourist. In the case of the initial point, the tourist has not arrived at this point from another activity, so it is considered by expression (3) as an activity not carried out, and the start time will be equal to zero by expression (30). In such cases we cannot add this constraint, and introduce the following.

– If the tourist has indicated the initial point, the start time of the tour is defined by him/her, or we assume that the start time is the first day, accommodation being the initial point. The start time is defined as the finish time of the initial point:

$$HF_{jt} = star\ time\ tour\ , \qquad j = initial\ \ point \qquad t = 1 \tag{23}$$

– The finish time of the tour is the start time of the end point, and is indicated by the tourist, or we assume the finish time of the last day:

$$HI_{jt} = end\ time\ \ tour\ , \qquad j = end\ \ point \qquad t = N \tag{24}$$

– The start and the end times of accommodation are indicated by the tourist. He/she can indicate at what time he/she wants to start each day, I_t, and at what time he/she wants to finish, F_t; and if the tourist does not indicate either of these, we assign values 9:00 and 21:00.

  – The start time of the route on day t is equal to the finish time of accommodation, except on the first day because he/she does not end accommodation.

$$HF_{jt} = I_t\, y_{jt} \quad j = set\ accommodation,\ t = 2,.....N \tag{25}$$

  – The time when the tourist wants to finish the route on day t will be the start time of accommodation on day t.

$$HI_{jt} = F_t\, y_{jt} \quad j = set\ accommodation,\ t = 1,.....N \tag{26}$$

– The start time of activity j ($j=1,\ldots M$) on day t ($t=1,\ldots N$) should be greater than or equal to the time of finishing the previous activity i, ($i=1,\ldots M$), plus travel time from i to j.

$$\mathrm{HI}_{jt} \geq \mathrm{HF}_{it} + td_{ij} - 1000000 \quad (1 - x_{ijt}) \qquad i, j = 1,...M \quad t = 1,.....N \tag{27}$$

where td_{ij} is the spent time going from activity i to activity j.

If the tourist does not go from activity *i* to activity *j*, this constraint will not affect the model, so we have incorporated the variable x_{ijt} in constraint (27).

– The start time of activity *j* should be greater than or equal to the opening time of this activity, e_{jt} being the opening time of activity *j* on day *t*:

$$HI_{jt} \geq e_{jt} y_{jt} \qquad j \notin A_1 \qquad t = 1,\ldots..N \tag{28}$$

– The start time of activity *j* should be less than or equal to the "last visit time" of the activity *j* on day *t*:

$$HI_{jt} \leq l_{jt} y_{jt} \qquad j \notin A_1 \qquad t = 1,\ldots..N \tag{29}$$

where l_{jt} is the "last visit time" of activity *j*, that is, the difference between the closing time (c_{jt}) and the duration of the activity, $l_{jt} = c_{jt} - ta_j$.

Tourist constraints:

Each tourist has preferences regarding the duration of the tour, free time, type of accommodation, type of visits, etc.

– The tourist may determine if he/she wants some free days:

$$\sum_{j=1}^{M} y_{jt} = 0, \quad t = set \ \ day\, by \ \ tourist \tag{30}$$

– The tourist may want to specify some activities, and the system will be forced to offer these activities.

$$\sum_{t=1}^{T} y_{jt} \geq 1, \quad j = set\ activity\ by\ \ tourist \tag{31}$$

– He/she may also specify what activities he/she does not want to do:

$$\sum_{t=1}^{T} y_{jt} = 0,\ j = set\ activity\ by\ tourist\ to\ not\ do \tag{32}$$

– And he/she may indicate what activities are his/her favourites:

$$u_j = Max \ \ value \ \ utility, \ \ j = set \ \ activity \ \ by \ \ tourist \tag{33}$$

"$Max\, value \ \ utility$" being the maximum value of any activity according to its characteristics and tourist preferences.

– The tourist may specify some types of visits that he/she prefers, and the system will be prevented from offering other types of visits.

$$y_j = 0, \forall j \in A_{3k} \ \ \ k = unwanted\ type \ \ k = \{1,\ldots w\}, t = 1,\ldots N \tag{34}$$

– Regarding accommodation, he/she may indicate the following: locations where he/she wishes to stay, type of accommodation, accommodation category, minimum category, and minimum services.

– The tourist may indicate preferences regarding restaurants: type of restaurant, category, and type of cuisine.

– Regarding museums, monuments and other visits, the tourist may indicate what subtype he/she wishes to visit.

– The tourist may indicate preferences regarding beaches: composition, type of sand, bathing conditions, level of urbanization, average width, average length, density of use, minimum services and other characteristics.

– We also consider the maximum time on the route that the tourist wants to spend going from one activity to another, and this is denoted by the parameter "*tdMax*":

$$td_{ij} x_{ijt} \leq tdMax, \; j = 1,...,M; \; i = 1,...,M, \; t = 1,...N \quad (35)$$

If the tourist does not indicate this amount of time, we assume *tdMax* = 4 hours.

– The tourist may indicate whether he/she prefers a relaxed tour, with a rest period (approximately 10% of the duration of activity) between one activity and another, or whether he/she prefers to do as many activities as possible.

$$ta_j = tar_j + m \, tar_j, \; j = 1,...,M; \; i = 1,...,M, \; t = 1,...N \quad (36)$$

"*m*" being the percentage used.

2. Model resolution

We studied the model and concluded that it corresponds to a Multi-objective Assignment and Routing Problem. It is an assignment problem as the tourists must choose which activities to do each day from among all the activities on offer, that is, they have to assign activities to days; it is a routing problem as these activities have to be ordered for each day; and it is a multiobjective problem since, among other difficulties, it involves choosing a satisfactory alternative from among the multiple objectives of the decision-maker ([1], [16]).

The complex character of the problem makes it very difficult to solve using exact methods, and therefore we chose other methods, known as metaheuristics. We used a metaheuristic method based on Tabu Search. This search method itself is based on the concept of memory taken from the field of Artificial Intelligence. By means of this procedure, and once all the information required from the decision-maker has been collected, an approximation to the set of efficient solutions is obtained [6].

This procedure provides a very large set of alternatives, and therefore an interactive and iterative process is required that gradually reduces the set of solutions obtained, using the information provided by the decision maker. This guides the search for an efficient frontier area where the most suitable solution will be found, yielding a set of solutions tailored to tourist preferences.

Conclusions

In a world where tourist information is widespread and readily available, each tourist may plan his/her own trip. However, this involves an important cost in terms of effort and time, due to the wide range of tourist products in the market and to conflict among the objectives of the tourist.

Therefore, a system is required which facilitates the tourist's decision-making process, and which also offers him/her the alternative best suited to his/her needs.

We have met this need by creating a tourist aid system, which could act as an efficient tool within the tourist sector, that is, for the tourist, travel agencies or official bodies. The system facilitates the tourist's decision-making process; and travel agencies and official institutions can use this system to offer an additional service to the tourist. Similarly, this work may serve as a methodological tool in other areas of interest.

Future lines of work include developing an interactive method to guide the decision-maker in finding an appropriate solution, as mentioned; developing a computer implementation that incorporates the various stages of the process and, by means of an interface, also collects the information needed from the decision-maker and shows him/her the solutions; and fine-tuning the model to match reality as far as possible.

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Agata Sielska

STABILITY OF MULTICRITERIA RANKINGS – A COMPARISON

Abstract

Set of weights describing decision maker's preferences plays a crucial role in construction process of multicriteria rankings, because even small changes of preferences may lead to different results. Moreover, the stability of rankings is important due to the fact that weights are only an approximation of decision maker's preferences. Most popular approaches to rankings sensitivity usually focus on the case in which no changes occur after the modification of eights or parameters. Such analysis does not give an insight into the way ranking is changed when the given weight intervals are exceeded. Spearman's rank correlation coefficient can provide information on that case, but it does not inform directly about the scales of shifts inside the ranking. Stability analysis of investment funds rankings constructed by three multicriteria outranking methods (PROMETHEE, WSA and TOPSIS) is presented. Similarity of rankings is assessed on the basis of Spearman's correlation coefficient and chosen stability definitions. Simulation includes different weight sets describing decision maker's preferences concerning such criteria as parameters of distribution of return rates, purchase and management costs and customers' convenience.

Keywords

Multicriteria rankings, PROMETHEE method, WSA method, TOPSIS method.

Introduction

In outranking approach a decision-maker's preferences are usually introduced to the analysis in form of parameters and, in most cases, weighting vectors. They are subject to change in time, hence such an assessment involves constructing new rankings after each change. The process is time-consuming and computationally demanding. It can be also bothersome to a decision maker, because of the necessity of interviewing in order to set up the new relative importance of criteria. Moreover, usually we do not know which changes of preferences will result in a change of hierarchy i.e. in which cases a new ranking should be constructed.

It should be also considered that weighting vectors used to construct rankings are only an approximation of the decision-maker's preferences and do not reflect them exactly. Therefore it is necessary to perform a stability and sensitivity analysis.

Stability can be defined in many ways. When assessing volatility of a ranking resulting from the changes of weights or parameters, we usually focus on the case in which no shifts occur i.e. ranking is preserved after modification of the weighting vector. After conducting such an analysis it is still unknown which changes in ranking can occur when the weights exceed the given boundaries [21]. Spearman's rank correlation coefficient [19] can give some insight into that problem, but it does not inform directly on the scales of shifts inside the ranking.

The case in which ranking is preserved can be generalized by defining *k*-stability [10]. It is assumed that maintenance of the strict order of alternatives with highest evaluations is the issue of greatest importance for the decision-maker. However, there can exist problems in the case of which the decision-maker can be interested in selecting first n alternatives (i.e. alternatives that have the highest evaluations in a given ranking), but the order in this group does not play a crucial role (e.g. selecting 5 best alternatives for a further assessment by other methods). The definition proposed in [10] does not inform about the way the ranking is going to change when the change actually occurs. It informs only that some alternatives ranked among *k* objects with highest evaluations will change their positions. The scale and character of the change remain unknown.

We considered the stability problem by introducing the definition of stability of order *s*, according to which a ranking R_B is called stable of order *s* if $\max_i(d_i) = s$, where $d_i = d_{i,B} - d_{i,Z}$ is the difference of ranks of alternative *i* in the initial ranking R_B and ranking R_Z constructed using modified weights.

Stability of rankings of open-end investment funds constructed by three different multicriteria methods was analyzed on the basis of the above-mentioned definition. The results obtained indicate that the stability of rankings depends on the method applied, however this finding can result from the strong assumptions about decision-makers' preferences [18]. We intend to verify this hypothesis by simulation of the ranking stability with different assumptions and applying different weighting vectors.

The paper is organized as follows. In section 1 three multicriteria methods used to construct rankings are described. In section 2 decision criteria are proposed. The final section is focused on the presentation of simulation results and comments.

1. Multicriteria rankings

Multicriteria rankings are applied to problems in which a decision-maker is to rank the set A consisting of N alternatives a_i, $i=(1,\ldots,N)$ taking into consideration the values acquired by criteria $f_1,\ldots f_K$.

Three methods considered in the study represent different approaches towards solving multicriteria problems. We discuss them in detail in the following section.

1.1. SAW

The SAW method (Simple Additive Weighting) [9] allow for comparison of criteria by normalising evaluations. The alternatives evaluated are ranked according to the decreasing value of benefit function:

$$u_j = \sum_{i=1}^{K} w_i e_{ij}$$

where:

w_i – the weight associated with criterion f_i,
e_{ij} – the standardized evaluation of alternative a_j for criterion f_i.

The method of normalisation is not specified; usually the following formulas are used:

$$e_{ij} = \frac{f_i(a_j)}{\max_j f_i(a_j)}$$ for maximized criteria (the higher evaluation the better),

$$e_{ij} = \frac{f_i(a_j)}{\min_j f_i(a_j)}$$ for minimized criteria (higher evaluation of the criterion means also worse evaluation).

Or, respectively:

$$e_{ij} = \frac{f_i(a_j) - \min_j f_i(a_j)}{\max_j f_i(a_j) - \min_j f_i(a_j)}$$

$$e_{ij} = \frac{\max_j f_i(a_j) - f_i(a_j)}{\max_j f_i(a_j) - \min_j f_i(a_j)}$$

Let us notice that using the SAW method it is possible to compensate lower evaluations on some criteria by higher evaluations on others. In some cases, e.g. investment problems, this feature can be seen as beneficial, as it is possible to compensate costs (e.g. management fees) by higher returns. The method is also intuitive for the decision-maker.

1.2. TOPSIS

In the case of the TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution) [7] a ranking is constructed on the basis of the distance from two reference points: the ideal and the negative ideal solutions of the multicriteria problem. The reference points are defined as follow:

$$F^* = \left(f_1^*, f_2^*, \ldots, f_K^*\right),$$

where $f_i^* = \max_{a \in A} f_i(a)$

$$F^- = \left(f_1^-, f_2^-, \ldots, f_K^-\right),$$

where $f_i^- = \min_{a \in A} f_i(a)$.

The alternatives are ranked according to the decreasing value of the function:

$$D_p(a) = \frac{d_p^-(a)}{d_p(a) + d_p^-(a)}$$

where:

$$d_p(a) = \left(\sum_{i=1}^{K} w_i^p \left(f_i - f_i(a)\right)^p \right)^{\frac{1}{p}}$$

$$d_p^-(a) = \left(\sum_{i=1}^{K} w_i^p \left(f_i(a) - f_i^-\right)^p \right)^{\frac{1}{p}}$$

The idea of the TOPSIS method is intuitive and comprehensible for the decision-maker, as in the case of the SAW method. This method can be also regarded as more flexible than SAW, due to the fact that decision-maker's preferences can be introduced not only in the form of weights but also in the form of the parameters *p* used to calculate the distance measure.

It should be noted that it is indirectly assumed that approaching the ideal solution and moving away from the negative ideal one are of equal importance for a decision-maker.

1.3. PROMETHEE II

The PROMETHEE methods ([1], [2]) seem to be the most flexible and easy to adjust to the preferences of the decision-maker. However, similarities of rankings constructed by applying the SAW and PROMETHEE II methods were reported in previous studies ([6], [18]).

Criteria are normalised by applying the so-called generalized criteria (see [5]), which admit values between 0 and 1. We used the Gaussian criterion (*), in which case the preference function does not involve determining other parameters and making additional assumptions concerning decision-maker's preferences.

$$P_i(d_i(\cdot)) = \begin{cases} 0 & d_i(\cdot) \le 0 \\ 1 - \exp\left(-\dfrac{d_i(\cdot)^2}{2\sigma_i^2}\right) & d_i(\cdot) > 0 \end{cases}, \qquad (*)$$

where:

$\forall_{a_1, a_2 \in A} \; d_i(a_1, a_2) = f_i(a_1) - f_i(a_2)$ is the difference between the evaluations of the pair of alternatives a_1, a_2 on criterion f_i ,

σ_i – standard deviation of evaluation of criterion f_i .

The values of the function $P_i()$ are used to determine the aggregated preference indices:

$$\begin{cases} \pi(a_1, a_2) = \sum_{i=1}^{K} P_i(a_1, a_2) w_i \\ \pi(a_2, a_1) = \sum_{i=1}^{K} P_i(a_2, a_1) w_i \end{cases}$$

The value of the index $\pi(a_1, a_2)$ expresses the degree to which the alternative a_1 is preferred to alternative a_2. For a given alternative $a^* \in A$ the positive and negative outranking flows are defined respectively as follow:

$$\varphi^+(a^*) = \frac{1}{N-1}\sum_{a \in A}^{K} \pi(a, a^*)$$

and

$$\varphi^-(a^*) = \frac{1}{N-1}\sum_{a \in A}^{K} \pi(a^*, a)$$

The positive outranking flow for an alternative a^* is interpreted as the degree to which a^* outranks other alternatives. The greater value it acquires, the better alternative a^* is in comparison with others. The high value of the negative outranking flow expresses, in turn, the weakness of the alternative examined.

The net outranking flow (**) is defined as the balance of the positive and negative outranking flows and is the basis of the final ranking. A positive value of this flow means that the given alternative is in the group of dominating and dominated objects.

$$\varphi(a) = \varphi^+(a) - \varphi^-(a) \qquad (**)$$

The alternatives are sorted according to the decreasing net flow values.

2. Criteria

In the literature it is suggested to evaluate funds assuming that their results are affected by such indicators as historical means of returns, risk, participation costs, size of the fund and minimum value of the initial investment ([3], [11], [12], [15]). Investors are assumed to be interested in the quality of services [4] and the manager's reputation [13]. Funds can be classified [14] on the basis of the i.a. rates of return, Sharpe ratio ([16], [17]), Treynor ratio [20] and VaR measure based on the quantiles [8].

We consider a set consisting of 47 open-end stock funds operating on Polish market. The data reflect their performance in the period from January to July 2008.

In the set of criteria we included nine indicators associated with the financial results of funds, costs connected with management of the assets and customer's convenience.

Three groups of criteria are listed below.

1. Criteria associated with the distribution of the return rates that inform about the expected return rate and risk. In this group we took into consideration the following measures: *expected value, standard deviation, skewness, kurtosis* and *0.05 percentile of the return rate*.

2. Criteria associated to fees and minimum values of inputs required by the fund. From the decision-maker's point of view these criteria can be treated as obstacles and costs. Minimum values of inputs mean that the decision-maker is unable to invest individually chosen amount of capital at time he/she chooses. Even though the fund may offer shares it is impossible for the decision-maker to purchase them or invest only the amount of capital he/she intends to invest. Criteria from this group include: *management fee, minimum required value of the first input, minimum required value of the next input.*

3. *Number of methods of placing orders* – this criterion describing different facilities for the decision-maker. The funds analysed offered different methods of placing orders: directly in the customer service point, by transfer, phone, fax or Internet.

3. Empirical analysis

In the empirical analysis we applied 14 scenarios presented in the Table 1. The decision-maker's preferences were described by weighting vectors. In the first scenario S1 we assumed no preference among the criteria (i.e. equal weight for each criterion). The following scenarios are based on equal (S2) and differentiated (S3-S14) weights for the groups of criteria and individual criteria as well.

Table 1

Values of weights describing decision-maker's preferences in analysis scenarios

		Expected value	Standard deviation	Skew-ness	Kur-tosis	Per-centile 0.05	Minimum required value of the first input	Minimum required value of the next input	Manage-ment fee	Number of methods of placing orders
		C1	C2	C3	C4	C5	C6	C7	C8	C9
		Max	Min	Max	Min	Max	Min	Min	Min	Max
Values of weights in scenarios	S1	1	1	1	1	1	1	1	1	1
	S2	0.067	0.067	0.067	0.067	0.067	0.11	0.11	0.11	0.33
	S3	2	2	2	2	2	1	1	1	1
	S4	2	2	2	2	2	2	2	2	1
	S5	1	1	1	1	1	1	1	1	2
	S6	1	1	1	1	1	2	2	2	2
	S7	1	1	1	1	2	2	2	2	1
	S8	2	2	2	2	2	1	1	1	2
	S9	3	3	3	3	3	1	1	1	2
	S10	3	3	3	3	3	2	2	2	1
	S11	1	1	1	1	1	2	2	2	3
	S12	1	1	1	1	1	3	3	3	2
	S13	2	2	2	2	2	3	3	3	1
	S14	2	2	2	2	2	1	1	1	3

After constructing rankings by applying weights listed above we assessed similarity of the results by calculating Spearman's rank correlation coefficient. The majority of the coefficients calculated acquire high values which indicate a considerable similarity between rankings constructed by applying different methods. A similarity between the PROMETHEE II and SAW methods is noticeable. Minimum and maximum values of correlation coefficients for rankings constructed by using the same weights vectors are presented in Table 2.

Table 2

Minimum and maximum values of correlation coefficients for rankings constructed by applying different methods in all scenarios

	PROMETHEE/TOPSIS	PROMETHEE/SAW	TOPSIS/SAW
Min r_s	0.7484	0.9744	0.7516
Max r_s	0.7978	0.9940	0.8171

The minimum values of rank correlation coefficients for rankings constructed by applying the same method in different scenarios are presented in Table 3. It can be noticed that results obtained by the TOPSIS method are most dependent on the values of weights.

Table 3

Minimum values of correlation coefficients for given methods

	PROMETHEE II	SAW	TOPSIS
Min r_s	0.8019	0.8231	0.6911

Once rankings had been constructed in each scenario the weight for each criterion was successively changed as follows:

$$w_i = \begin{cases} w_i \cdot \left(1 + \dfrac{j}{10}\right) & i = k \\ & ; \\ w_i & i \neq k \end{cases}$$

where:

$j \in \overline{1,90}$ – number of iteration.

After each change the weights were normalized and a new ranking was constructed. In each iteration we compared the new ranking with the initial one by calculating the stability of order measure.

The results obtained in all scenarios are similar. The orders of stability of the SAW and PROMETHEE II methods are similar. Their difference average, depending on the values of weights, 2-3.5. The stability of the TOPSIS method is noticeably lower.

Figures 1-3 illustrate stability of rankings for the S13 scenario. The results are representative, because in all scenarios, the methods acted in a similar way. A considerable similarity of the PROMETHEE II and SAW methods can be noticed. Those two methods can be regarded as least sensitive to weight changes. There is also a substantial difference between them and TOPSIS. In the case of *skewness* and *kurtosis* criteria the difference mentioned is the least and the values of order of stability measure, comparable for all three methods. For the *management fee* criterion the order of stability of the TOPSIS method was, in turn, lower than for others.

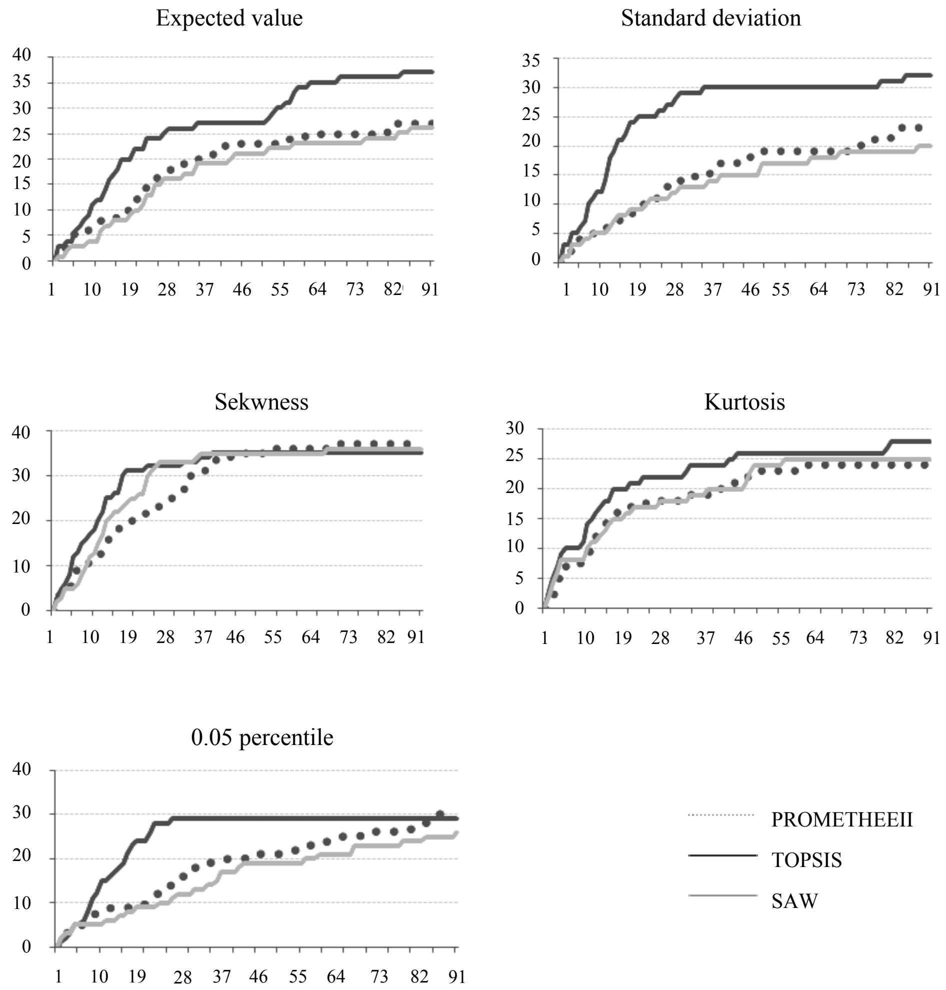

Figure 1. Order of stability in all iterations for the methods compared and the first group of criteria

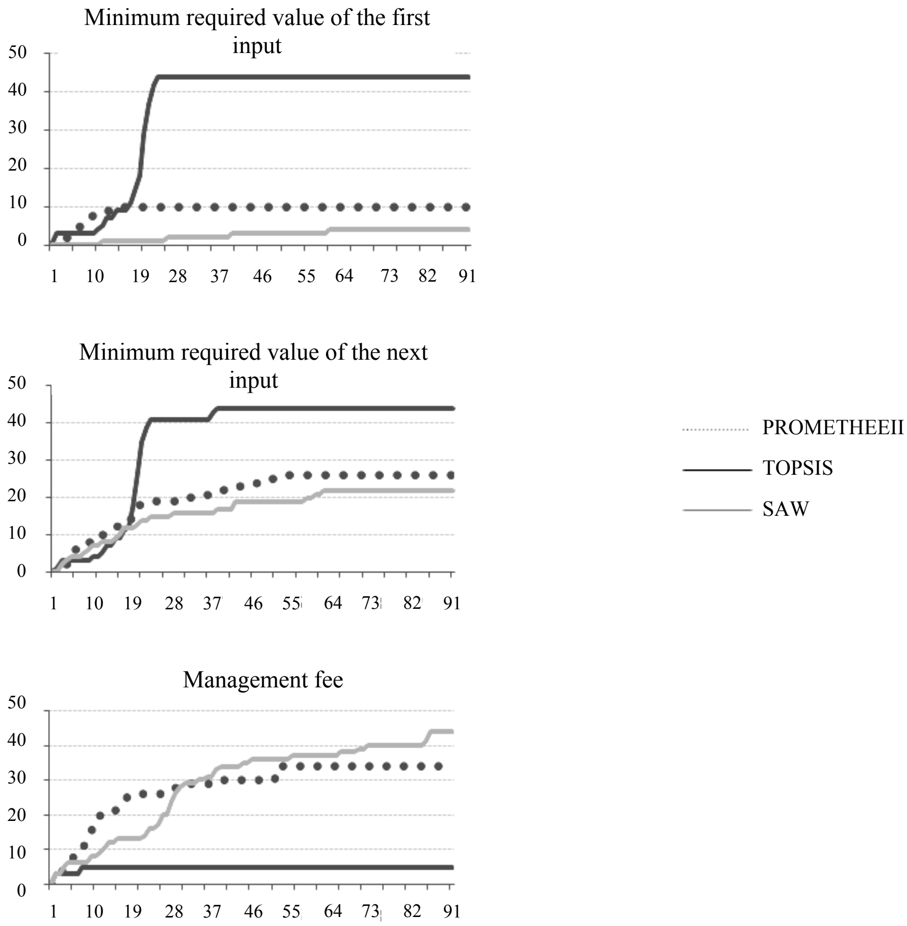

Figure 2. Order of stability in all iterations for the methods compared and the second group of criteria

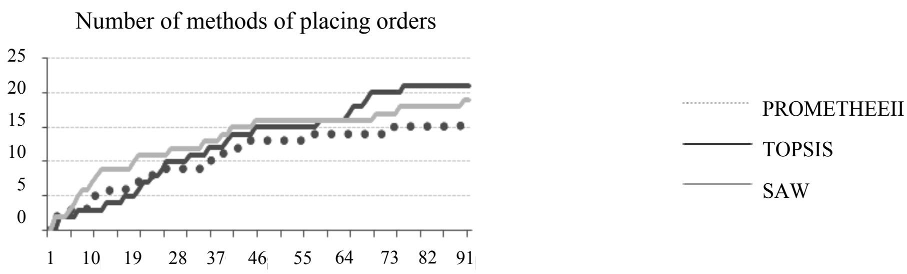

Figure 3. Order of stability in all iterations for the methods compared and the *number of methods of placing orders* criterion

The only exception among the scenarios analysed was the *management fee* criterion. For weights assumed in scenarios S2, S3, S8-S10 and S14, the order of stability of the TOPSIS method was higher and comparable with the other methods (see Figure 4), but the similarity of the SAW and PROMETHEE II methods was preserved. The scenarios in which the above-mentioned changes were noticed have one feature in common – low weights for criteria from the second group (except for S2 scenario).

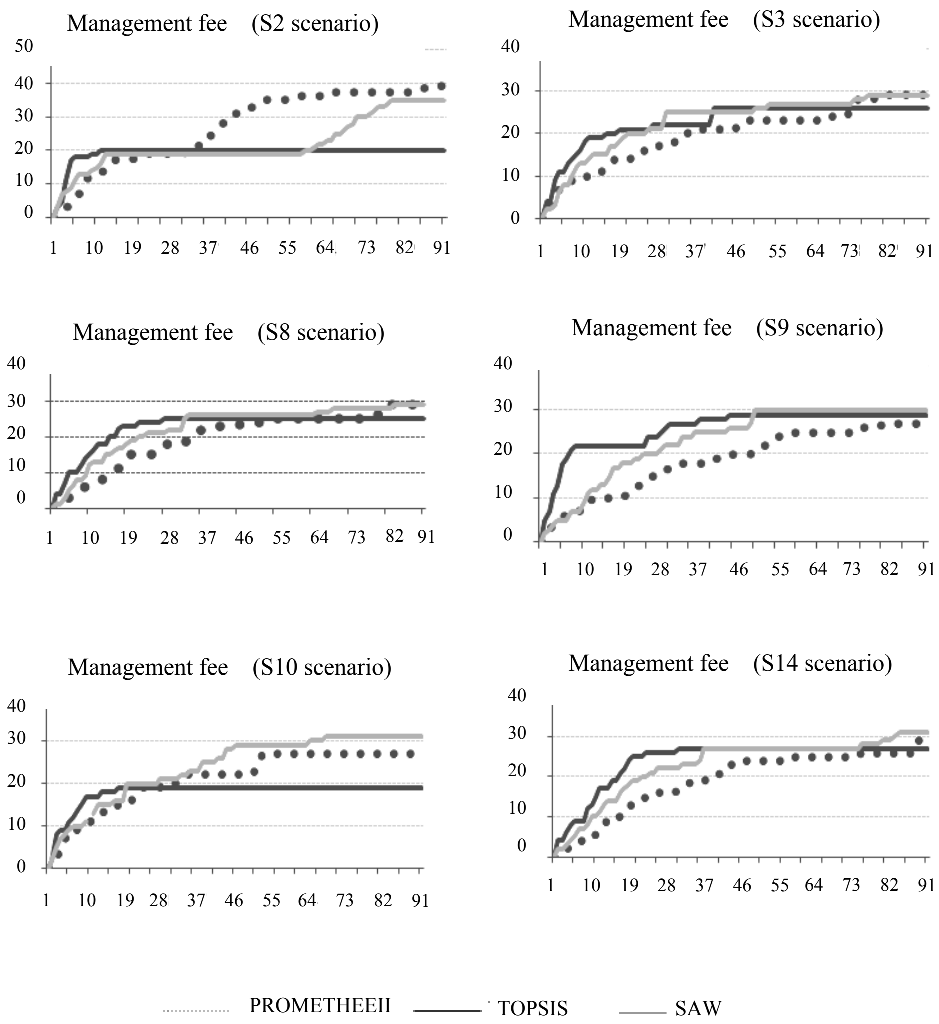

Figure 4. Order of stability in all iterations for the methods compared and the *management fee* criterion

The similarity of the SAW and PROMETHEE II methods is confirmed by the evaluation of the r_s coefficients. The lowest values of the coefficients for rankings constructed on the basis of modified weights and initial rankings (depending on criteria) are presented in Table 4. Taking into consideration the results presented, it can be noticed that for most criteria the rankings are not prone to change substantially due to modifications of weights. In the case of the TOPSIS method the similarity measured in this way is the smallest.

Table 4

Minimum values of rank correlation coefficients for given methods

	PROMETHEE II	TOPSIS	SAW
Expected value	0.811	0.569	0.827
Standard deviation	0.804	0.473	0.794
Skewness	0.385	0.156	0.378
Kurtosis	0.798	0.665	0.804
Percentile 0.05	0.630	0.248	0.641
Minimum required value of the first input	0.954	0.817	0.986
Minimum required value of the next input	0.931	0.744	0.897
Management fee	0.798	0.657	0.810
Number of methods of placing orders	0.768	0.772	0.782

In the next step we assessed the stability of rankings using the approach presented in [10]. By modifying the weights in the same way as before we examined how many alternatives with highest ranks remain on their positions in the initial ranking. For the sake of simplicity, in this case we present only those modifications that occur in the initial iterations, because of the substantial changes of stability defined in this way as the number of iterations increase.

In the case when no changes in the ranking occur it can be noticed that almost all modifications lead to instability. The criteria *0.05 percentile* and *minimum required value of the first input* are the only exceptions, as in these cases in the majority of scenarios no shifts occurred in the initial iterations. In general, increasing values of these two criteria did not lead to shifts among highly classified alternatives.

With the exception of a few particular cases, similarities between the SAW and PROMETHEE II methods are noticeable, but they are not as distinct as in the case of the previously applied approach. The numbers of alternatives with highest evaluations that preserve their ranks in the chosen sample scenarios are compared in Table 5.

Table 5

Number of alternatives with highest evaluations that preserve their ranks in 3 first iterations
for SAW and PROMETHEE II methods

		S5			S6			S7			S10			S12			S13		
		j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3
PROMETHEE II	C1	25	13	13	7	5	5	8	8	5	13	13	3	39	6	6	19	6	6
	C2	12	12	11	16	7	7	19	19	9	3	3	3	5	5	5	15	7	6
	C3	8	0	0	11	10	6	8	8	7	0	0	0	12	12	9	6	5	5
	C4	12	12	12	15	11	6	13	11	11	3	3	3	47	17	10	19	12	6
	C5	47	47	47	47	47	47	47	47	47	47	37	37	47	47	47	47	47	47
	C6	16	12	12	16	16	16	20	20	20	22	22	22	14	14	14	47	20	20
	C7	9	9	9	20	5	5	8	8	8	30	14	14	16	9	9	18	18	7
	C8	12	10	8	6	4	0	4	4	0	3	3	3	8	8	7	5	4	4
	C9	16	14	11	14	7	7	8	8	5	15	3	3	21	5	5	15	15	11
SAW	C1	10	10	10	15	5	5	9	6	6	14	1	1	47	7	6	20	6	6
	C2	10	10	10	10	8	5	9	9	9	1	1	1	6	6	6	7	7	5
	C3	8	0	0	11	10	10	9	9	8	0	0	0	12	12	12	6	6	5
	C4	13	13	9	15	13	13	12	12	12	1	1	1	47	6	6	7	7	7
	C5	38	38	38	47	43	43	38	17	17	47	47	23	47	44	44	47	36	36
	C6	15	15	15	15	15	14	17	17	17	47	20	20	13	13	14	16	16	16
	C7	9	9	9	13	13	9	17	17	17	19	19	16	21	7	6	7	7	7
	C8	9	8	8	8	8	4	10	6	6	1	1	1	12	10	5	5	5	5
	C9	10	10	10	10	10	5	9	7	5	14	1	1	14	13	13	16	7	7

The assessment of the stability of the methods considered determined by the means of this approach does not allow to draw conclusions. In some cases (see Tables 6-7) the results are marked only for a given set of criteria, whereas modifications of weights associated to other criteria lead to different outcomes. The results obtained in some scenarios (see scenarios S6 and S4) suggest that in rankings constructed using the TOPSIS method more alternatives with high evaluations remain in their positions when weights are modified. In the following tables we present results for scenarios with most regularities.

Table 6

Number of alternatives with highest evaluations that preserve their ranks in first iterations for PROMETHEE II and TOPSIS methods

	PROMETHEE II									TOPSIS								
	S6			S9			S11			S4			S5			S6		
	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3
C1	7	5	5	8	8	5	6	6	6	13	11	11	13	12	12	14	14	14
C2	16	7	7	19	19	9	14	6	6	7	7	7	15	13	8	16	8	3
C3	11	10	6	8	8	7	12	8	5	9	3	3	3	3	3	16	10	8
C4	15	11	6	13	11	11	15	15	5	13	13	13	28	13	13	16	16	16
C5	47	47	47	47	47	47	47	47	47	22	17	17	11	11	11	28	19	19
C6	16	16	16	20	20	20	16	16	16	22	17	17	11	11	11	28	28	28
C7	20	5	5	8	8	8	8	8	8	22	22	22	12	2	2	32	10	10
C8	6	4	0	4	4	0	5	5	4	8	8	8	11	4	3	16	16	16
C9	14	7	7	8	8	5	13	6	6	13	13	12	15	11	11	19	14	14

Table 7

Number of alternatives with highest evaluations that preserve their ranks in first iterations values in chosen scenarios for SAW method

	S3			S4			S5			S6			S7		
	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3
C1	9	6	6	11	6	6	10	10	10	15	5	5	9	6	6
C2	6	6	0	6	6	6	10	10	10	10	8	5	9	9	9
C3	11	11	4	21	0	0	8	0	0	11	10	10	9	9	8
C4	6	6	6	6	6	6	13	13	9	15	13	13	12	12	12
C5	47	47	47	29	29	29	38	38	38	47	43	43	38	17	17
C6	20	20	20	16	16	16	15	15	15	15	15	14	17	17	17
C7	6	6	6	6	6	6	9	9	9	13	13	9	17	17	17
C8	16	15	0	8	8	8	9	8	8	8	8	4	10	6	6
C9	9	6	6	11	6	6	10	10	10	10	10	5	9	7	5

Concluding remarks

We have compared the stability and similarity of rankings of open-end stock funds constructed by applying different multicriteria methods based on different stability and similarity measures.

The simulation results presented in the paper indicate a considerable similarity between the SAW and PROMETHEE II methods. Similarities reported in the earlier studies turn out to be independent of the values of weights. The rankings constructed by applying those two methods are similar with regard to rank correlation coefficient and order of stability measure.

Nonetheless, the stability assessment based on the approach suggested in [10] does not lead to clear conclusions. In some of the cases considered the similarity of the SAW and PROMETHEE II methods is still noticeable. However, they lack regularities and their results are more dependent on the values associated to weights. Moreover, the higher flexibility of the TOPSIS method is not noticeable in this case. It can suggest that shifts in open-end investment funds rankings obtained when applying the TOPSIS method occur among the alternatives in middle and low positions in the ranking.

The results presented in the paper imply that the SAW and PROMETHEE II methods with the Gaussian generalized criterion lead to similar rankings and respond to the modifications of the weighting vector in a very similar way. The rankings obtained by their application are also less dependent on the values of the weights than those constructed by the TOPSIS method.

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