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MULTIPLE CRITERIA DECISION MAKING
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BI-CRITERIA STOCHASTIC GENERALIZED TRANSPORTATION PROBLEM: EXPECTED COST AND RISK MINIMIZATION[1]

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Abstract

In the paper we consider a bi-criteria version of the Stochastic Generalized Transportation Problem, where one goal is the minimization of the expected total cost, and the second one is the minimization of the risk. We present a model and a solution method for this problem.

Keywords: Stochastic Generalized Transportation Problem, Bi-criteria Stochastic Generalized Transportation Problem, expected cost, variance of the cost, Equalization Method, branch and bound.

1 Introduction

The Generalized Transportation Problem (GTP) and its generalizations can be used in many real-life applications, in particular in modeling of transportation of perishable products, see e.g. Nagurney et al. (2013). One can look at the GTP as a special kind of the Generalized Minimum Cost Flow Problem or as a generalization of the ordinary Transportation Problem. The generalized flows, as well as some solution methods, can be found e.g. in Ahuja et al. (1993). The generalized flows were also studied by Glover et al. (1972), Goldberg et al. (1988), and Wayne (2002), among others. The particular case of the GTP was studied in particular by Balas (1966), Balas and Ivanescu (1964), and Lourie (1964). Anholcer

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and Kawa (2012) considered the two-stage GTP and its application in the supply chain in which complaints are involved.

The transportation of perishable goods is not the only application of generalized flows. In Ahuja et al. (1993) several others have been discussed. In particular, they may be used in the modeling of conversions of physical entities in financial, mineral and energy networks or machine loading. Nagurney et al. (2013) discuss, in turn, the application of generalized flows in the modeling of selected kinds of logistic chains, in particular in the distribution process of medical materials, food, pharmaceuticals and clothes.

Very often (also in the above mentioned papers) it is assumed that the demand is fixed. In fact, it is usually impossible to predict *a priori* the exact values of demand. However, in many cases it is possible to estimate its probability distribution.

The Stochastic Generalized Transportation Problem (SGTP) is the generalized version of the GTP, where one assumes that the values of demand are given as random variables. At least two approaches can be applied to transform this kind of problem into an equivalent, deterministic form. One could assume that the probability of satisfying the demand constraints has to be not less than some fixed value. This, together with the demand distribution, allows to transform the constraints (and hence the problem) into a deterministic form. However, in the case of transportation problems, another approach is more common. In this approach we remove the demand constraints and use them to introduce a new cost function, including the expected extra cost, increasing with the discrepancy between the actual value of the demand and the size of delivery. This approach has been used in such classic papers as Williams (1963), Cooper and LeBlanc (1977), but also in more recent ones, such as Holmberg and Jörnsten (1984), Holmberg (1995), Qi (1985, 1987) and Anholcer (2012, 2015). It is also worth mentioning that this approach is related to the classical Newsvendor Problem which has been known at least from the moment of the publication of Edgeworth (1888), and then analyzed and generalized by numerous authors, see e.g. Khouja et al. (1996), Şen and Zhang (1999), Chen and Chuang (2000), Yang et al. (2007), Goto (2013) (in fact, the Newsvendor Problem can be considered as an instance of the Stochastic Transportation Problem with one source and one destination).

A more general version of the Nonlinear Transportation Problem (where any convex costs at the destination points are applicable) was discussed by Anholcer (2005, 2008a, 2008b), Sikora (1993) and Sikora et al. (1991), among others. In those papers the Equalization Method was considered and it was proved to be convergent in Anholcer (2005, 2008a). The convergence of the general versions for the Nonlinear and Stochastic GTP was also proved by Anholcer (2012, 2015).

In all the above papers only the expected costs were taken under consideration. It can be useful, however, to involve also the risk, measured by variance.

This makes the problem bi-criterial. The problem of stochastic programming involving both expected cost and variance has been recently studied by Li et al. (2014) who transformed this problem into a quasi-linear form and applied it to the Transportation Problem. A version of the bi-criteria SGTP, this time with expected cost and time criteria, has been studied by Anholcer (2013). Also Nagurney et al. (2013) studied the generalized flows where two criteria (expected cost and risk) were involved (the authors assumed that the risk can be represented by a function convex with respect to the flow, which is, however, not always true; see below). Bi- and Multi-criteria Transportation Problems were discussed also e.g. by Aneja and Nair (1979), Gupta and Gupta (1983), Shi (1995), Li (2000), Basu and Acharya (2002), Khurana and Arora (2011), Kesavarz and Khorram (2011) and Kumar et al. (2012). The (linear) Generalized Transportation Problem in the multi-criteria version was studied by Gen et al. (1999), among others.

In this paper we present a method for finding efficient solutions of the Bi-criteria Stochastic Generalized Transportation Problem with two criteria: expected cost and variance. In Section 2 the problem is formulated. In Sections 3 and 4 the algorithm, together with its theoretical justification, is presented. Section 5 contains an illustrative example. The results of computational experiments are presented in Section 6. Section 7 contains final remarks.

2 Problem formulation

In the Generalized Transportation Problem, the goal is to minimize the transportation costs of a uniform good delivered from m supply points to n destination points. The amount of the transported good changes during the transportation process. More precisely, the amount delivered to demand point j from supply point i is equal to $r_{ij}x_{ij}$, where x_{ij} is the amount of the good that leaves supply point i and r_{ij} is the *reduction ratio*. The unit transportation costs c_{ij} are constant, the demand b_j of every demand point j has to be satisfied and the supply a_i of any supply point i cannot be exceeded. The model looks as follows:

$$\min\left\{f(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}\right\},$$

s. t.

$$\sum_{i=1}^{m} r_{ij}x_{ij} = b_j, j = 1, \dots, n,$$

$$\sum_{j=1}^{n} x_{ij} \le a_i, i = 1, \dots, m,$$

$$x_{ij} \ge 0, i = 1, \dots, m, j = 1, \dots, n.$$

In the Stochastic GTP (SGTP), the demands b_j are independent continuous random variables X_j with density functions φ_j. We will assume that for every $j = 1, \dots, n$ and for every $x > 0$,

$$\varphi_j(x) > 0.$$

The unit surplus cost $s_j^{(1)}$ and the unit shortage cost $s_j^{(2)}$ are defined for every destination point j. This implies that the expected extra cost at the destination j is equal to:

$$f_j(x_j) = s_j^{(1)} \int_0^{x_j} (x_j - t)\varphi_j(t)dt + s_j^{(2)} \int_{x_j}^{\infty} (t - x_j)\varphi_j(t)dt.$$

Using elementary transformations and integrating by parts, we obtain that:

$$f_j(x_j) = s_j^{(2)} \int_0^{\infty} (t - x_j)\varphi_j(t)dt + \left(s_j^{(1)} + s_j^{(2)}\right) \int_0^{x_j} (x_j - t)\varphi_j(t)dt =$$

$$= s_j^{(2)} \left(\int_0^{\infty} t\varphi_j(t)dt - x_j \int_0^{\infty} \varphi_j(t)dt\right) +$$

$$+ \left(s_j^{(1)} + s_j^{(2)}\right)\left(\left[(x_j - t)\Phi_j(t)\right]_0^{x_j} + \int_0^{x_j} \Phi_j(t)dt\right) =$$

$$= s_j^{(2)}\left(E(X_j) - x_j\right) + \left(s_j^{(1)} + s_j^{(2)}\right) \int_0^{x_j} \Phi_j(t)dt,$$

where Φ_j is the cumulative distribution function of the demand at destination j (the last equality uses the fact that $\Phi_j(0) = 0$).

Finally, the SGTP takes the form:

$$\min\left\{f(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{j=1}^{n} f_j(x_j)\right\},$$

s. t.

$$\sum_{i=1}^{m} r_{ij}x_{ij} = x_j, j = 1, \dots, n,$$

$$\sum_{j=1}^{n} x_{ij} \le a_i, i = 1, \dots, m,$$

$$x_{ij} \ge 0, i = 1, \dots, m, j = 1, \dots, n.$$

The first derivative of the expected cost function has the form:

$$f_j'(x_j) = -s_j^{(2)} + \left(s_j^{(1)} + s_j^{(2)}\right)\Phi_j(t),$$

while the second derivative is equal to:

$$f_j''(x_j) = \left(s_j^{(1)} + s_j^{(2)}\right)\varphi_j(t).$$

This means that each function f_j is twice differentiable and strictly convex on the interval where $\varphi_j(t) > 0$. This allows to use the corresponding version of the Equalization Method (Anholcer, 2012 and 2015) to solve this problem.

Of course it may happen that a Decision Maker considers the transportation costs, shortage costs and surplus costs as not equally important. In such a situation one could use three criteria instead of one, or even when using one objective, one could still introduce weights, reflecting the Decision Maker's preferences. However, this would not change the structure or the general form of the resulting weighting problem, discussed in Section 3 (Observation 1).

The second criterion of interest is variance. The formula for variance for destination j is:

$$g_j(x_j) = p_j(x_j) - q_j(x_j),$$

where:

$$p_j(x_j) = \left(s_j^{(1)}\right)^2 \int_0^{x_j} (x_j - t)^2 \varphi_j(t)dt + \left(s_j^{(2)}\right)^2 \int_{x_j}^{\infty} (t - x_j)^2 \varphi_j(t)dt$$

and:

$$q_j(x_j) = \left(f_j(x_j)\right)^2.$$

One can see that:

$$p_j'(x_j) = 2\left(s_j^{(1)}\right)^2 \int_0^{x_j} (x_j - t)\varphi_j(t)dt + 2\left(s_j^{(2)}\right)^2 \int_{x_j}^{\infty} (x_j - t)\varphi_j(t)dt$$

and:

$$p_j''(x_j) = 2\left(s_j^{(1)}\right)^2 \int_0^{x_j} \varphi_j(t)dt + 2\left(s_j^{(2)}\right)^2 \int_{x_j}^{\infty} \varphi_j(t)dt.$$

Moreover:

$$q_j'(x_j) = 2f_j(x_j)f_j'(x_j)$$

and:

$$q_j''(x_j) = 2\left[f_j'(x_j)\right]^2 + 2f_j(x_j)f_j''(x_j).$$

This means that each of the functions $g_j(x_j)$ is a twice differentiable DC-function. Namely, it is the difference of two convex functions, which are strictly convex if $\varphi_j(x_j) > 0$. However, in general, the functions g_j do not need to be convex.

As the demands are independent random variables, the variance of total extra cost is equal to the sum of the variances at the destination points. Thus the bi-criteria problem (BSGTP) takes the form:

$$\min\left\{f(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{j=1}^{n} f_j(x_j)\right\},$$

$$\min\left\{g(x) = \sum_{j=1}^{n} g_j(x_j)\right\},$$

s. t.
$$\sum_{i=1}^{m} r_{ij}x_{ij} = x_j, j = 1, \dots, n,$$
$$\sum_{j=1}^{n} x_{ij} \le a_i, i = 1, \dots, m,$$
$$x_{ij} \ge 0, i = 1, \dots, m, j = 1, \dots, n.$$

Usually the two objective functions have different minima. Our goal is to find a solution method that finds the efficient (Pareto-optimal) solutions.

3 Algorithm – the main idea

Let S denote the set of all feasible solutions of the BSGTP. The problem may be rewritten as:
$$\min f(x),$$
$$\min g(x),$$
s. t.
$$x \in S.$$
The following observation is a corollary from the well-known result about the efficiency of the solution to the weighting problem (see e.g. Miettinen, 1998, p. 78, Theorem 3.1.2).

Observation 1
If $x^\star$ is, for some $\lambda > 0$, an optimal solution to the problem:
$$\min h(x) = f(x) + \lambda g(x)$$
s. t.
$$x \in S,$$
then it is a Pareto-optimal solution of the BSGTP.

Minimizing $h(x)$ on S always leads to an efficient solution. The problem obtains then the form of a GTP with a nonlinear objective function. The function $h(x)$ is not necessarily convex, but it is a separable function in which each summand is a DC-function. Thus one can use a branch-and-bound method to determine an exact solution. We will discuss such a method in the next section.

4 Algorithm – the details

The method that we are going to present uses the ideas discussed by Falk and Soland (1969), as well as by Holmberg and Tuy (1999). Assume that the variable x_j is bounded from below and from above: $l_j \le x_j \le u_j$. Since the function $q_j(x_j)$ is convex, we have:

$$q_j(x_j) \le r_j(x_j; l_j, u_j)$$

for $l_j \le x_j \le u_j$, where:

$$r_j(x_j; l_j, u_j) = q_j(l_j) + \frac{x_j - l_j}{u_j - l_j}\left(q_j(u_j) - q_j(l_j)\right)$$

is a linear function such that:

$$r_j(l_j; l_j, u_j) = q_j(l_j)$$

and:

$$r_j(u_j; l_j, u_j) = q_j(u_j).$$

This means that for each index j we have:

$$g_j(x_j) = p_j(x_j) - q_j(x_j) \ge p_j(x_j) - r_j(x_j; l_j, u_j) = g_j^\star(x_j; l_j; u_j).$$

One can see that $g_j^\star(x_j; l_j; u_j)$ is a lower estimate of $g_j(x_j)$ on the interval $[l_j, u_j]$. Let l be the vector of the lower bounds and u the vector of the upper bounds. Let:

$$h^\star(x; l; u) = \sum_{j=1}^{n} \left(f_j(x_j) + \lambda g_j^\star(x_j; l_j; u_j)\right).$$

Of course, $h^\star(x; l; u)$ is a lower estimate of $h(x)$ on the generalized rectangle defined by the inequalities $l \le x \le u$. This means that the new problem:

$\min h^\star(x; l; u)$

s. t.

$x \in S,$

has the form of an SGTP and can be solved using the Equalization Method (see Anholcer, 2012 and 2015). Note that no additional constraints are introduced, so the set of feasible solutions does not change.

The rule of branching is as follows. After solving the problem with function $h^\star(x; l; u)$, we check whether the solution is satisfactory for some predefined accuracy level ε. If it is not, we choose j for which the difference $r_j(x_j; l_j; u_j) - q_j(x_j)$ is the largest and define two child problems by setting $l_j \coloneqq x_j$ and $u_j \coloneqq x_j$, respectively, for the new problems (recall that we do not change the set of feasible solutions; those values are used only to find the formula of the lower estimate function).

Finally, we can write the algorithm as follows (U_h and U_x denote the upper bound on the optimal value of the objective and the point at which this value is reached, respectively; for a given node v of the solution tree, $L(v)$ and $P(v)$ denote the lower bound on the optimal value of the objective and the corresponding convex problem).

Algorithm 1: The Branch and Bound Method for BSGTP
Input: initial problem, the value of $\lambda > 0$, accuracy level ε.
Output: Pareto-optimal solution $x^\star$.

1. *Initial solution*. Let the initial bounds for each x_j be:
$$l_j = 0, u_j = \sum_{i=1}^{m} r_{ij} a_i.$$
Solve (using the Equalization Method) the corresponding problem $P(v_0)$:
$\min h^\star(x; l; u)$
s. t.
$$x \in S.$$
Assume that the obtained optimum is $x^\star$. Set $U_x = x^\star$, $U_h = h(x^\star)$, and $L(v_0) = h^\star(x^\star; l; u)$, where v_0 is the root of the solution tree.
Go to step 2.
2. *Checking the optimality*. Find an active node $v^\star$, for which $L(v)$ has the minimum value. If:
$$|U_h - L(v^\star)| < \varepsilon,$$
then STOP. The solution U_x is satisfactory. Otherwise go to step 3.
3. *Branching and bounding*. Consider the problem $P(v^\star)$. Let $j^\star$ be an index j for which the difference $r_j(x_j; l_j; u_j) - q_j(x_j)$ is the largest. Remove the node $v^\star$ from the set of active nodes. Add two new active nodes v' and v'' and define the corresponding convex problems. To obtain $P(v')$, set $u_{j^\star} = x^\star_{j^\star}$ in $P(v^\star)$. To obtain $P(v'')$, set $l_{j^\star} = x^\star_{j^\star}$ in $P(v^\star)$. Let us denote the new bounding vectors by l', u', l'', u'', respectively.
Solve $P(v')$ and $P(v'')$ using the Equalization Method. Assume that the obtained optima are x' and x'', respectively. If $U_h > h(x')$, then set $U_x = x'$ and $U_h = h(x')$. Set $L(v') = h^\star(x'; l'; u')$. If $U_h > h(x'')$, then set $U_x = x''$ and $U_h = h(x'')$. Set $L(v'') = h^\star(x''; l''; u'')$.
Close all the active nodes v for which $L(v) > U_h - \varepsilon$.
Go back to step 2.

5 Illustrative example

Let us analyze a simple example that illustrates the algorithm. Assume that there are two supply points with the supply equal to $a_1 = a_2 = 15$ and three destinations, with uniform demand distribution given by the density functions:
$$\varphi_1(x_1) = \begin{cases} \frac{1}{10}, x \in [0,10], \\ 0, x \notin [0,10], \end{cases}$$

$$\varphi_2(x_2) = \begin{cases} \frac{1}{12}, x \in [0,12], \\ 0, x \notin [0,12], \end{cases}$$

$$\varphi_3(x_3) = \begin{cases} \frac{1}{14}, x \in [0,14], \\ 0, x \notin [0,14]. \end{cases}$$

The unit transportation costs c_{ij}, the reduction ratios r_{ij}, the surplus costs $s_j^{(1)}$ and the shortage costs $s_j^{(2)}$ are given in the Table 1.

Table 1: Problem parameters

Parameter	***j* = 1**	***j* = 2**	***j* = 3**	**Parameter**	***j* = 1**	***j* = 2**	***j* = 3**
c_{1j}	5	3	2	r_{1j}	0.92	0.95	0.93
c_{2j}	2	1	4	r_{2j}	0.91	0.87	0.92
$s_j^{(1)}$	1	4	5	$s_j^{(2)}$	4	6	10

Assume that we are interested in finding the solution for $\lambda = 0.5$ and $\varepsilon = 0.01$. The functions of expected costs are given by:

$$f_1(x_1) = \begin{cases} \frac{1}{4}x^2 - 4x + 20, x \in [0,10], \\ x - 5, x > 10, \end{cases}$$

$$f_2(x_2) = \begin{cases} \frac{5}{12}x^2 - 6x + 36, x \in [0,12], \\ 4x - 24, x > 12, \end{cases}$$

$$f_3(x_3) = \begin{cases} \frac{15}{28}x^2 - 10x + 70, x \in [0,14], \\ 5x - 35, x > 14. \end{cases}$$

The functions p_j have the form:

$$p_1(x_1) = \begin{cases} -\frac{1}{2}x^3 + 16x^2 - 160x + \frac{1600}{3}, x \in [0,10], \\ x^2 - 10x + \frac{100}{3}, x > 10, \end{cases}$$

$$p_2(x_2) = \begin{cases} -\frac{5}{9}x^3 + 36x^2 - 432x + 1728, x \in [0,12], \\ 16x^2 - 192x + 768, x > 12, \end{cases}$$

$$p_3(x_3) = \begin{cases} -\frac{25}{14}x^3 + 100x^2 - 1400x + \frac{19600}{3}, x \in [0,14], \\ 25x^2 - 350x + \frac{4900}{3}, x > 14. \end{cases}$$

The first bounds on the variables (corresponding to the node v_0) are defined by $0 \leq x_1 \leq 27.45$, $0 \leq x_2 \leq 27.3$ and $0 \leq x_3 \leq 27.75$. The respective linear estimates of q_j are equal to:

$$r_1(x_1) = 400 + \frac{x_1 - 0}{27.45 - 0}(504.0025 - 400)$$
$$r_2(x_2) = 1296 + \frac{x_2 - 0}{27.3 - 0}(7259.04 - 1296)$$
$$r_3(x_3) = 4900 + \frac{x_3 - 0}{27.75 - 0}(10764.0625 - 4900)$$

The solution of the problem $P(v_0)$ is as follows:

Table 2: Solution

Value	$j = 1$	$j = 2$	$j = 3$	**Value**	$j = 1$	$j = 2$	$j = 3$
x_{1j}	0.00	3.02	11.98	$p_j(x_j)$	74.03	628.64	878.24
x_{2j}	5.40	9.60	0.00	$q_j(x_j)$	40.67	446.96	628.89
x_j	4.92	11.22	11.14	$r_j(x_j)$	418.63	3747.27	7253.62
$f_j(x_j)$	6.38	21.14	25.08	$r_j(x_j) - q_j(x_j)$	377.96	3300.30	6624.73

The objectives of the initial problem and of the convex problem are $h(x^\star) = 338.215$ and $h^\star(x^\star) = -4813.279$. This means that $L(v_0) = -4813.279$ and $U_h = U(v_0) = 338.215$. Since v_0 is the only (active) node and $|U_h - L(v_0)| > \varepsilon$, we perform branching with respect to the variable x_3 (the maximum difference $r_j(x_j) - q_j(x_j)$ is $r_3(x_3) - q_3(x_3)$). Since $x_3 = 11.138$, the new nodes v_1 and v_2 will correspond to the additional constraints $x_3 \leq 11.138$ and $x_3 \geq 11.138$, respectively. After defining the functions $r_j(x_j)$ and solving the new problems, we obtain $L(v_1) = -2289.611$, $U(v_1) = 448.384$, $L(v_2) = -1995.718$ and $U(v_2) = 489.812$. $U(v_1) > U_h$ $U(v_2) > U_h$, so U_h does not change (and U_x remains the optimal solution of $P(v_0)$). Now the two active nodes are v_1 and v_2. The function L is minimized at v_1 and $|U_h - L(v_1)| > \varepsilon$, so we perform branching and continue in this way. At some moment we obtain $U(v_8) = 224.145$, which means that starting from this moment $U_h = 224.145$ and U_x becomes the optimal solution of $P(v_8)$. After a few more iterations, after branching at v_{13}, we obtain, in particular, that at v_{22} we have $L(v_{22}) = 259.418$, which means that $L(v_{22}) > U_h - \varepsilon$ and we close node v_{22}. The details for the first 51 nodes have been collected in Table 3 below. In each row, the label of node v_j is followed by the label of the parent node; two child nodes, order of branching, branching variable and its value (if the branching was performed at v_j); the values of both objectives: $U(v_j)$ and $L(v_j)$; and the actual value of U_h. At the stage presented in the table, 25 nodes are still active (A), four have been closed (C), and the branching has been performed at the other nodes.

Table 3: Beginning of the algorithm

Node (v)	**Parent node**	**Child nodes**	**Checking order**	**Branching variable**	**Branching value**	**U(v)**	**L(v)**	**Uh**
1	2	3	4	5	6	7	8	9
v0	none (root)	v1, v2	1	x3	11.138	338.215	– 4813.279	338.215
v1	v0	v3, v4	2	x2	12.393	448.384	– 2289.611	338.215
v2	v0	v5, v6	3	x2	9.393	489.812	– 1995.718	338.215
v3	v1	v7, v8	4	x3	6.089	367.325	– 1080.794	338.215
v4	v1	v9, v10	5	x3	5.672	541.545	– 936.929	338.215
v5	v2	v11, v12	6	x3	18.044	445.815	– 892.492	338.215
v6	v2	v13, v14	7	x3	15.495	558.560	– 587.665	338.215
v7	v3	v17, v18	9	x2	6.072	580.604	– 357.557	224.145
v8	v3	v15, v16	8	x2	6.072	224.145	– 447.747	224.145
v9	v4	v25, v26	13	x2	18.395	762.404	– 194.699	224.145
v10	v4	v23, v24	12	x2	16.601	387.967	– 222.306	224.145
v11	v5	v19, v20	10	x2	5.100	375.752	– 271.061	224.145
v12	v5	v31, v32	16	x2	3.641	523.392	– 128.550	224.145
v13	v6	v21, v22	11	x2	14.436	411.793	– 249.963	224.145
v14	v6	A	A	A	A	551.582	110.603	224.145
v15	v8	v29, v30	15	x1	7.933	281.030	– 134.819	224.145
v16	v8	v27, v28	14	x1	7.933	240.024	– 134.905	224.145
v17	v7	v39, v40	20	x3	4.163	637.550	– 44.627	224.145
v18	v7	v37, v38	19	x3	4.161	590.147	– 49.242	224.145
v19	v11	v35, v36	18	x1	8.462	429.345	– 67.279	224.145
v20	v11	v33, v34	17	x1	6.805	345.258	– 85.043	224.145
v21	v13	v49, v50	25	x1	5.181	344.501	56.284	224.145
v22	v13	C	C	C	C	445.887	259.418	224.145
v23	v10	v41, v42	21	x1	5.625	321.107	– 3.570	224.145
v24	v10	A	A	A	A	445.700	115.497	224.145
v25	v9	A	A	A	A	708.643	120.357	224.145
v26	v9	A	A	A	A	819.072	186.977	224.145
v27	v16	v43, v44	22	x3	8.893	245.454	9.312	224.145
v28	v16	A	A	A	A	246.101	57.107	224.145
v29	v15	v45, v46	23	x2	3.950	296.393	19.732	224.145
v30	v15	v47, v48	24	x2	3.884	303.556	55.917	224.145
v31	v12	A	A	A	A	583.376	65.850	224.145
v32	v12	A	A	A	A	499.660	102.411	224.145
v33	v20	A	A	A	A	368.272	80.426	224.145
v34	v20	A	A	A	A	340.735	115.675	224.145
v35	v19	A	A	A	A	441.503	76.627	224.145
v36	v19	A	A	A	A	440.998	126.633	224.145
v37	v18	C	C	C	C	721.613	259.574	224.145

Table 3 cont.

1	2	3	4	5	6	7	8	9
v38	v18	A	A	A	A	325.146	77.547	224.145
v39	v17	C	C	C	C	768.302	263.785	224.145
v40	v17	A	A	A	A	372.663	82.115	224.145
v41	v23	A	A	A	A	342.190	160.389	224.145
v42	v23	A	A	A	A	319.241	190.339	224.145
v43	v27	A	A	A	A	255.496	87.144	224.145
v44	v27	A	A	A	A	245.696	81.619	224.145
v45	v29	A	A	A	A	336.275	126.426	224.145
v46	v29	A	A	A	A	241.175	85.048	224.145
v47	v30	A	A	A	A	343.076	163.929	224.145
v48	v30	A	A	A	A	251.624	125.184	224.145
v49	v21	A	A	A	A	404.270	214.555	224.145
v50	v21	C	C	C	C	329.764	246.344	224.145

6 Computational experiments

Test problems were randomly generated and solved with the proposed method. Two types of demand distributions were considered: uniform $U(0,\ u)$ and exponential $Exp(\lambda)$, where u and λ were chosen uniformly at random from the intervals $[15,\ 20)$ and $[0.5,\ 0.6)$, respectively. In both cases unit transportation costs were chosen from the interval $[5,\ 10)$, surplus costs from the interval $[1,\ 2)$, shortage costs from the interval $[5,\ 10)$, reduction ratios from the interval $[0.8,\ 0.9)$ and the supply from each source point from the interval $[10,\ 20)$. The algorithm was implemented in Java SE and run on a personal computer with Intel(R) Core(TM) i7-2670 QM CPU @2.20 GHz. For both types of distributions, 100 randomly generated problems of four sizes were solved: $(m, n) = (10, 10)$, (10, 20), (10, 50) and (20, 50), that is, 800 test problems in total. The running times in seconds (average, standard deviation, minimum and maximum) are presented in Table 4:

Table 4: Running times in seconds

Problem type	**$U(0, u)$ 10×10**	**$U(0, u)$ 10×20**	**$U(0, u)$ 10×50**	**$U(0, u)$ 20×50**	**$Exp(\lambda)$ 10×10**	**$Exp(\lambda)$ 10×20**	**$Exp(\lambda)$ 10×50**	**$Exp(\lambda)$ 20×50**
AVG	0.16	0.95	159.79	2316.95	2.35	8.10	1039.79	5445.41
ST DEV	0.45	2.72	181.66	1255.08	10.82	45.34	1183.32	3449.44
MIN	0.02	0.12	16.40	289.35	0.12	0.52	96.99	718.73
MAX	6.64	37.41	1538.15	10128.26	207.66	1204.13	9029.58	29031.47

As we can see, the algorithm can be regarded as fast: the running times are less than a second or a few seconds in the case of the smaller problems and about one hour in the case of the bigger problems (up to 1000 variables). However, one needs to remember that the branch and bound methods are super-polynomial, which means that the solution times may grow very rapidly with the increasing size of the problem.

7 Final remarks

The algorithm presented above allows to find the Pareto-optimal solutions of the Bi-criteria Stochastic Generalized Transportation Problem. In this type of problem we assume that one of the criteria is the sum of the transportation cost and the expected total extra cost of all the deliveries. The second criterion is the risk measured by the variance of the expected extra cost. The resulting problem, which allows to find the efficient solutions, is a non-convex optimization problem that can be solved with a branch-and-bound method described in the paper. The subproblems solved in the nodes of the solution tree are of SGTP form and therefore can be solved using the Equalization Method. The numerical evidence shows that the presented algorithm allows to solve problems of average size in a reasonable time.

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MULTIPLE CRITERIA DECISION MAKING
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BICRITERIA OPTIMIZATION IN THE NEWSVENDOR PROBLEM WITH EXPONENTIALLY DISTRIBUTED DEMAND[1]

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Abstract

In this paper exponential distribution is implemented as a demand distribution in newsvendor model with two different and conflicting goals. The first goal is the standard objective of maximization of the expected profit. The second one is to maximize the probability of exceeding the expected profit, called survival probability. Using exponential distribution as the demand distribution allows us to obtain the exact solutions. Also for this distribution we can study the monotonicity of survival probability with respect to various model parameters analytically. Additional results are obtained when various sets of the parameters are considered. Finally, the bicriteria index which combines these conflicting objectives is optimized which gives the compromise solution. Moreover, in order to illustrate theoretical results, we present numerical examples and graphs of auxiliary functions.

Keywords: stochastic demand, newsvendor problem, bicriteria optimization.

1 Introduction

There is a great variety of stochastic models in the inventory theory. We refer to the papers of Plewa (2010), Prusa and Hruska (2011), Zipkin (2000) and the references therein. The fundamental inventory stochastic model is the newsvendor problem denoted by NVP. A survey of this topic has been given recently by Quin et al. (2011). In the basic model, the aim is to determine the order quantity which

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maximizes the expected profit. Some authors applied alternate or multiple criteria (Choi, 2012; Gaspars-Wieloch, 2015, 2016; Rubio-Herrero, 2015; Ye and Sun; 2016, Kamburowski, 2014). For instance, instead of the maximization of the expected profit, the maximization of the probability of exceeding the target profit can be used. This is called the survival probability and the corresponding objective is called satisficing or aspiration-level objective. The aspiration-level objective in NVP was first discussed by Kabak and Shiff (1978). Since then the problem was widely studied by Lau (1980) and Li et al. (1991). Recently the NVP has been extended by introducing the bicriteria decision problem. In the extended model the newsvendor incorporates two conflicting goals into the objective function. The first goal is the classic maximization of the expected profit and the second one is the satisficing-level objective. The only decision variable is the order quantity needed to satisfy uncertain demand. Parlar and Weng (2003) consider a bicriteria NVP with a moving target which is the expected profit. In this case two conflicting goals are taken into account together since there is no solution which maximizes both constraints simultaneously. The bicriteria index combines both results by assigning appropriate weights which are numbers between 0 and 1 which sum up to 1. Parlar and Weng (2003) obtained the approximate result which is then applied to the case of normally distributed demand. Arcelus et al. (2012) continued this research for uniform distribution which allows to derive precise analytic results.

It should be noted here that both normal and uniform distributions belong to the class of maximum entropy probability distributions. This class is widely used in practice and in many papers these distributions are applied to model the unknown random demand (see for instance Eren and Maglaras, 2006). The classical entropy maximizing distributions are listed by Perakis and Roels (2008). For more details, we refer also to Eren and Maglaras (2015) and Lim and Shantikumar (2007). The normal distribution is the maximum-entropy distribution on the whole real line with fixed mean and variance. On the other hand, the uniform distribution is a good choice if we only know that the demand has positive mean and support on a finite interval. Yet another distribution which approximates the unknown demand well is the exponential distribution. This is the maximum entropy distribution in the class of continuous distributions with fixed finite mean and support on the positive half-axis $[0, \infty)$ (Andersen, 1970; Harrenoes, 2001).

However, when the coefficient of variation of the demand is large, then using the normal distribution leads to excessive orders and large financial losses may occur, as it was observed by Gallego et al. (2007). For this reason, for products with a large coefficient of variation, they recommended to use another classes of distributions including the exponential distribution.

Another argument to study exponentially distributed demand is to make the model simpler and mathematically tractable. This distribution belongs to the gamma and Weibull family of distributions, which are relatively easy to work with and they often provide good approximation to the actual demand distribution when data are highly variable. The exponential distribution is used in practice to represent interarrival times of customers to a system (times between two independent events) that occur at a constant rate, as well as the time to failure of a piece of equipment. One more feature of the exponential distribution is that its failure rate is constant. More reasonable customer demand distributions such as uniform, normal, gamma and Weibull distributions (Lariviere, 2006) belong to the class of distributions with increasing failure rate.

All the above mentioned arguments justify the use of the exponential distribution as distribution of the demand in the bicriteria newsvendor problem. The exponentially distributed demand with maximization of the probability of exceeding the target profit was studied by Li et al. (1991). The difference between our paper and theirs is that they consider a constant profit goal and a two-product newsvendor, and they do not obtain so many analytical results as we do.

We use the known notions defined in the above mentioned papers but the use of the exponential distribution allows us to obtain precise results and to investigate the obtained solutions more in detail. We can study analytically the monotonicity of the survival probability with varying parameters of the model. The mathematical computations are almost elementary, but we get some additional results for specific combinations of these parameters. It is worth noting here that for the general case an analogous analysis cannot be performed because the equations involved are cubic.

The rest of the paper is organized as follows. Section 2 provides the basic notation and formulation of the single criterion models and the bicriteria newsvendor problem. Next, in Section 3 we study analytically the variability of survival probability with respect to the model parameters. We also present example graphs of the considered functions to illustrate the nature of the solutions. Moreover, we provide a numerical example to illustrate the key properties of the elements of the bicriteria problem. In Section 4 we combine both objectives in one measure called bicriteria index. The solutions can be obtained numerically as well, which is illustrated by a numerical example. The last section concludes the paper.

2 Definition of the bicriteria newsvendor problem

In this section we recall the bicriteria problem in the newsvendor model and derive the optimality conditions for exponentially distributed demand. First we consider the model with the expected profit maximization as the objective. We recall the known results and apply them in the case when the demand is exponentially distributed.

In the newsvendor model we consider a retailer who wants to acquire Q units of a given product to satisfy exponentially distributed demand. First we introduce the following notation. Define:

- $p > 0$ to be the selling price for unit (unit revenue);
- $c > 0$ to be the purchasing cost per unit;
- $s > 0$ to be the unit shortage costs;
- v to be the unit salvage value (unit price of disposing any excess inventory);
- $f(.)$ and $F(.)$ to be the probability density function and the cumulative distribution function of the demand with mean μ.

The standard assumption is $v < c < p$.

In our case the demand is exponentially distributed with the density: $f(x) = \lambda e^{-\lambda x}, x > 0,$ and the cumulative distribution function $F(x) = 1 - e^{-\lambda x}$, $x > 0$, where $\lambda > 0$ is the parameter of this distribution. Then the mean demand is $\mu = \frac{1}{\lambda}$. Define $\pi(Q, x)$ to be the retailer's profit function given by:

$$\pi(Q,x) = \begin{cases} px + v(Q-x) - cQ, if\ x \le Q \\ pQ - s(x-Q) - cQ, if\ x > Q, \end{cases}$$

where Q is the order quantity and x is the realized demand. Then the expected profit $E(Q)$ is given by:

$$E(Q) = (p-c)\mu - (c-v)(Q-\mu) - (p+s-v)\int_Q^\infty (x-Q)f(x)dx,$$

(Acelus, 2012), which in the exponential case simplifies to:

$$E(Q) = \frac{1}{\lambda}(p-v) - (c-v)Q - \frac{1}{\lambda}(p+s-v)e^{-\lambda Q}.$$

Note that $E(0) = -s/\lambda$ and $E(\infty) = -\infty$. In the expected profit newsvendor model the aim is to maximize $E(Q)$. Thus in this case the condition:

$$E'(Q) = -(c-v) + (p+s-v)e^{-\lambda Q} = 0$$

determines the order quantity maximizing the expected profit, which is given by:

$$Q_E^* = \frac{1}{\lambda}\ln\frac{p+s-v}{c-v}. \tag{1}$$

The feasibility of this solution is proved by the fact that for the second derivative we have:

$$E''(Q) = -\lambda(p+s-v)e^{-\lambda Q} < 0,$$

which implies that the function $E(Q)$ is concave. The shape of the function of $E(Q)$ for the parameters $(\lambda, v, c, p, s) = (0.003,15,16,30,50)$ is shown in Figure 1 below.

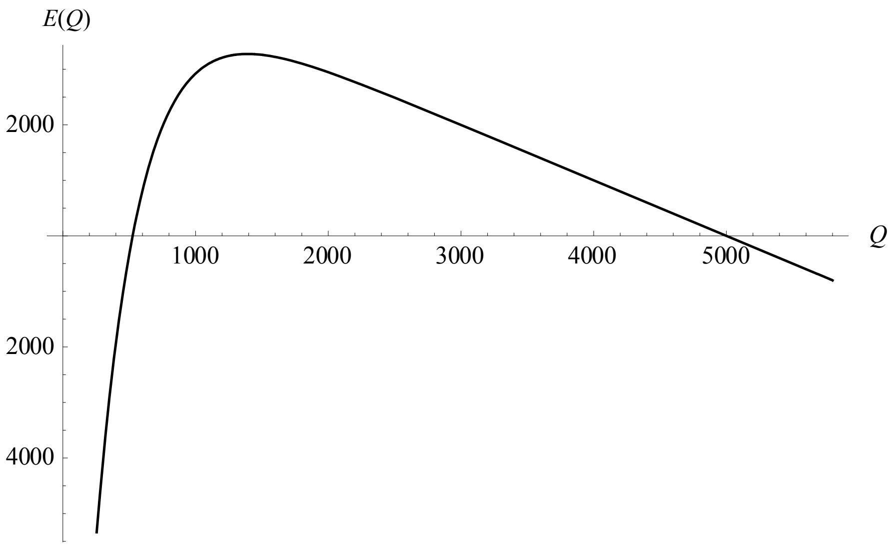

Figure 1. Expected profit function for $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$

In the other approach to the newsvendor model the probability of exceeding the expected profit is maximized instead of the expected profit itself. Let $H(Q)$ be the probability of this event, namely:

$$H(Q) = P\big(\pi(Q) \geq E(Q)\big),$$

which is called the survival probability. Its optimization with respect to Q in the exponential case will be performed in the next section. Let Q_H^* be the optimal order quantity which maximizes $H(Q)$. In the bicriteria newsvendor model both conditions mentioned above are considered together, although these objectives are conflicting with each other. Hence a new measure should be introduced which treats both constraints simultaneously. For this purpose the bicriteria index $Y(Q)$ is defined as:

$$Y(Q) = \frac{w}{E^*}E(Q) + \frac{1-w}{H^*}H(Q).$$

Here $E^* = E(Q_E^*)$ and $H^* = H(Q_H^*)$. Note also that both E^* and H^* are constants in the bicriteria function. The weight $0 \leq w \leq 1$ measures the relative importance of $E(Q)$ and $H(Q)$. If w increases, the risk-aversion decreases and $w = 1$ reflects risk-neutrality. Our aim is to find the order quantity which maximizes the bicriteria index which can be considered as a compromise solution to the bicriteria problem.

3 Optimization of the survival probability for the exponential distribution

Next we give the results for the satisficing-level objective which involves the maximization of survival probability also in the case of exponentially distributed demand. From Parlar and Weng (2003) we know that the survival probability $H(Q)$ can be written as:

$$H(Q) = \int_{D_1(Q)}^{D_2(Q)} f(x)dx,$$

where for exponentially distributed demand with parameter λ the limit functions $D_1(Q)$ and $D_2(Q)$ are given by $D_1(Q) = \max\{0, k(Q)\}$ with:

$$k(Q) = \frac{(c-v)Q + E(Q)}{p-v} = \frac{1}{\lambda}\left(1 - \frac{p+s-v}{p-v}e^{-\lambda Q}\right)$$

and:

$$D_2(Q) = \frac{(p+s-c)Q - E(Q)}{s} = \frac{1}{s}\left((p+s-v)Q - \frac{p-v}{\lambda} + \frac{p+s-v}{\lambda}e^{-\lambda Q}\right).$$

To calculate the survival probability it is necessary to analyse the behaviour of the limit functions which is done in the next subsection.

3.1 The analysis of the limit functions

First we recall some properties of the limit functions such as their monotonicity or their zeroes. The expressions presented below are easily obtained from Parlar and Weng (2003), but we need them for the exponential distribution in the following study.

Note that:

$$k(0) = -\frac{s}{\lambda(p-v)} < 0$$

and:

$$k'(Q) = \frac{p+s-v}{p-v}e^{-\lambda Q} > 0.$$

Moreover, for the second derivative of the function k we have:

$$k''(Q) = -\lambda\frac{p+s-v}{p-v}e^{-\lambda Q} < 0,$$

which implies that $D_1(Q)$ is concave and increasing. Let Q_0 be such that $k(Q_0) = 0$. Then:

$$Q_0 = \frac{1}{\lambda}\ln\frac{p+s-v}{p-v},$$

which implies that $D_1(Q)$ is equal to 0 in the interval $(0, Q_0)$. Moreover, the lower limit function tends to $\frac{1}{\lambda}$ as $Q \to \infty$.

Next for the upper limit we have:

$$D_2(0) = \frac{1}{\lambda},$$
$$D_2'(Q) = \frac{p+s-v}{s}\left(1 - e^{-\lambda Q}\right) > 0,$$
$$D_2''(Q) = \lambda \frac{p+s-v}{s} e^{-\lambda Q} > 0$$

and the upper limit $D_2(Q)$ tends to infinity as $Q \to \infty$. Therefore, the upper limit function is a convex increasing function of Q. Taking into account that:

$$D_2'(Q) - D_1'(Q) = 0$$

for $Q = Q_0$, we infer that, rather surprisingly, on the interval $[Q_0, \infty)$ the difference $D_2(Q) - D_1(Q)$ is minimized also at Q_0.

Examples of graphs of the limit functions $D_1(Q)$ and $D_2(Q)$ are presented below.

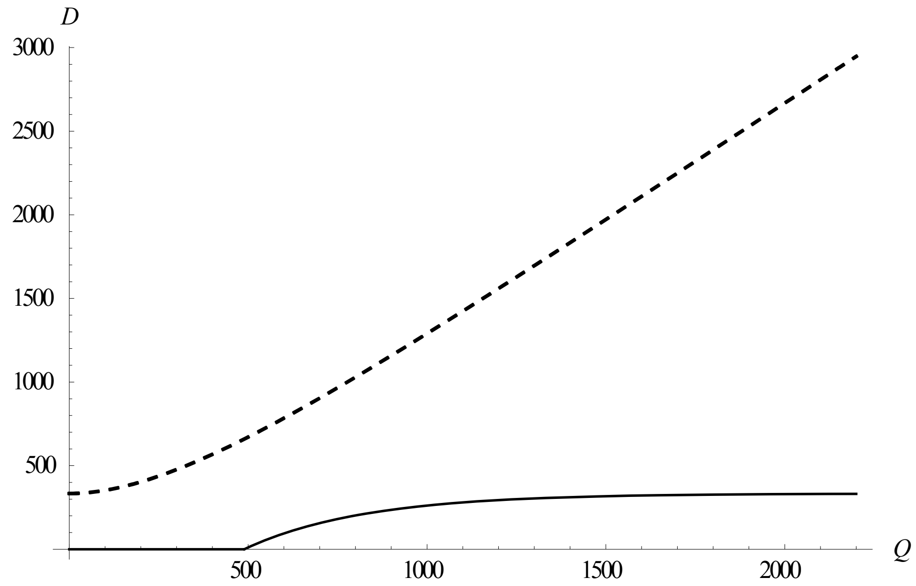

Figure 2. Limit functions $D_1(Q)$ and $D_2(Q)$ for $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$

From the expressions for the limit functions we observe in Figure 2 that the graph of the upper limit always lies under the graph of the lower one. Moreover, the minimum distance between these limits occurs for the point identical with a zero of $k(Q)$, which is the function corresponding to the lower limit.

Using these facts in the next subsection we solve the problem of optimization of the survival probability function $H(Q)$.

3.2 Optimization of survival probability $H(Q)$

First we investigate the variability of the survival probability function when the demand is exponentially distributed. It is known that $H(0) = 1 - e^{-1}$ and $H(\infty) = e^{-1}$. We stress the fact that while for uniformly distributed demand the minimum distance between the limit functions corresponds to the minimum probability $H(Q)$, this is not the case for the normal or the exponentially distributed demand. However, for the exponential distribution under some conditions on the parameters of the model, the order quantity which minimizes the distance between the limit functions is simultaneously the quantity which optimizes the survival probability function $H(Q)$. The following theorem provides satisfactory conditions which assure the existence of the maximum of $H(Q)$.

Theorem 1
If the demand distribution in the NVP is an exponential distribution with parameter λ, then the following statements hold.

a. If for some parameters p, s, v and some $\lambda > 0$, we have:
$$a(Q) < b(Q), for\ Q > Q_0, \tag{2}$$
where $a(Q) = e^{-\lambda(D_2(Q) - D_1(Q))}$ and $b(Q) = \frac{s}{p-v}\frac{e^{-\lambda Q}}{1-e^{-\lambda Q}}$, then condition (1) is satisfied for any $\lambda > 0$.
b. If (2) holds, then the survival probability function $H(Q)$ attains the maximum value at:
$$Q_H^* = \frac{1}{\lambda}\ln\frac{p+s-v}{p-v} \tag{3}$$
and the maximal survival probability is given by the formula:
$$H^* = 1 - \left(\frac{p-v}{p-v+s}\right)^{\frac{p-v+s}{s}}$$

Proof of Theorem 1. For simplicity let $a = p - v$. Then the expressions for the limit functions simplify to:
$$k(Q) = \frac{1}{\lambda}\left(1 - \frac{a+s}{a}e^{-\lambda Q}\right)$$
and:
$$D_2(Q) = \frac{1}{s}\left((a+s)Q - \frac{a}{\lambda} + \frac{a+s}{\lambda}e^{-\lambda Q}\right).$$
Note that $a > 0$ and both the limit functions and the survival probability do not depend directly on the parameters p and v but only on their difference. Then we calculate:
$$H'(Q) = e^{-\lambda Q\left(\frac{a+s}{s}+1\right)}e^{-\frac{(a+s)^2}{as}e^{-\lambda Q}} - \frac{s}{a}\frac{e^{-\lambda Q}}{1-e^{-\lambda Q}}$$

and we infer that the sign of the derivative does not depend on the parameter of the exponential distribution which proves (a).

Moreover, the variability of the function $H(Q)$ is as follows. First, it is increasing on the interval $(0, Q_0)$ since the lower limit $D_1(Q)$ is equal to 0 on $(0, Q_0)$ and the upper limit $D_2(Q)$ is increasing on this interval. It suffices to note that condition (2) is equivalent to the statement that $H(Q)$ is decreasing on (Q_0, ∞). Combining these facts, we get statements (b) and (c) of the theorem.

We illustrate the results of Theorem 1 with example graphs. In Figure 3 we present the graph of $H(Q)$ for the case when the vector of the model parameters is $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$, and then the survival probability is decreasing for all $Q > Q_0$. To illustrate the possibility of non-monotonicity of the survival probability for $Q > Q_0$, consider the following set of the model parameters $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 1)$. The graph of $H(Q)$ for this case is shown in Figure 4.

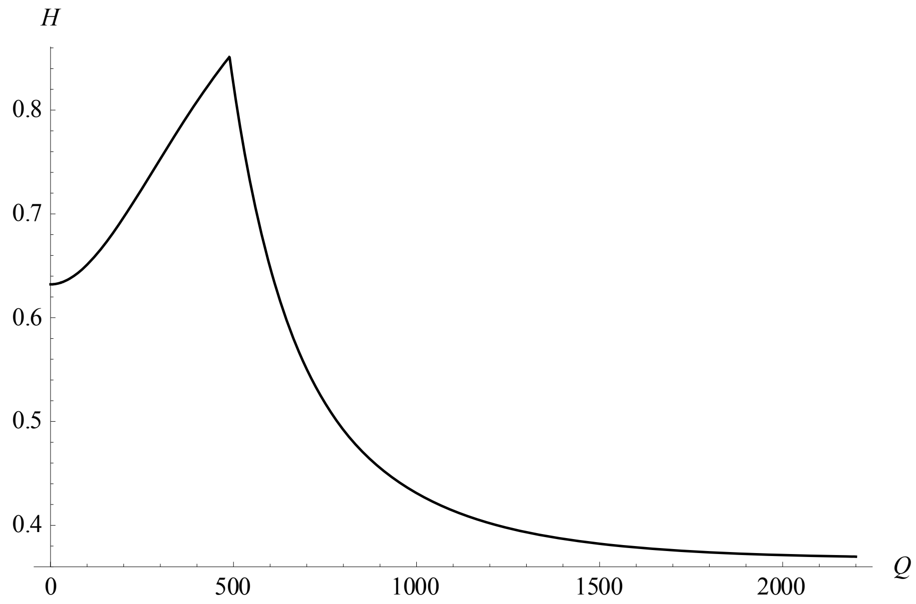

Figure 3. Survival probability for $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$

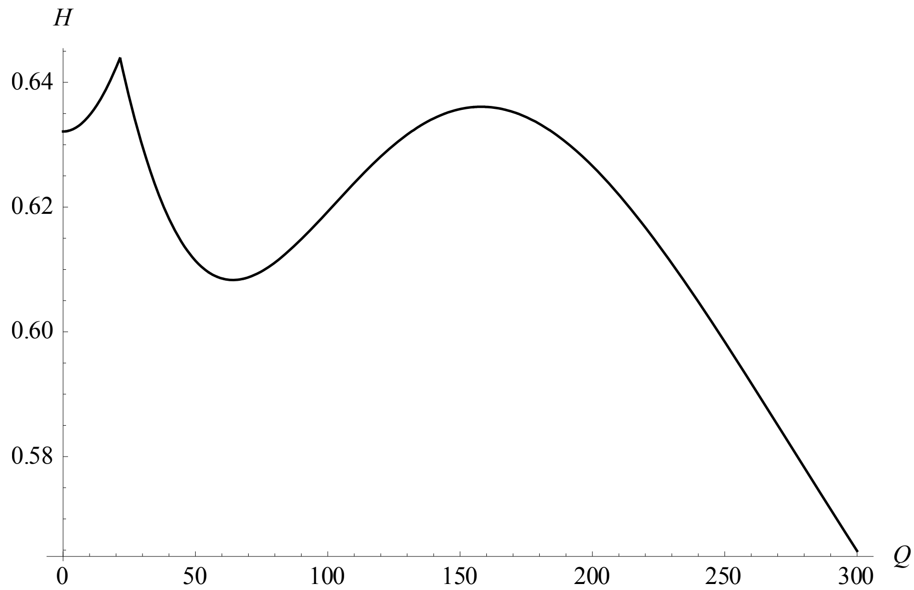

Figure 4. Survival probability for $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 1)$

We conclude this subsection with a short discussion of Theorem 1.

First, the answer to the question whether condition (2) is satisfied or not does not depend on the parameter λ of the demand distribution. The value of maximal probability H^* does not depend on λ either.

Second, if condition (2) is satisfied, then comparing equations (1) and (3) we see that the order quantity Q_H^* optimizing the survival probability is strictly smaller than Q_E^* which optimizes the expected profit.

Third, in the special case when $p - v = s$, we get the optimal solution $Q_H^* = \frac{1}{\lambda}\ln 2$ and the maximal survival probability $H^* = 0.75$, so these quantities do not depend on the model parameters. Indeed, in this case the derivative $H'(Q)$ is negative for any $Q > Q_0$. To prove this statement, in the expression for $H(Q)$ we substitute $e^{-\lambda Q} = z$, where $0 < z < \frac{1}{2}$, and define an auxiliary function:

$$g(z) = H\left(-\frac{1}{\lambda}\log z\right) = e^{-2z-1} - z^2 e^{1-2z}.$$

The function $g(z)$ is increasing which is shown in Figure 5. Moreover $-\frac{1}{\lambda}\log z$ is a decreasing function of z which implies that the survival probability $H(Q)$ is decreasing.

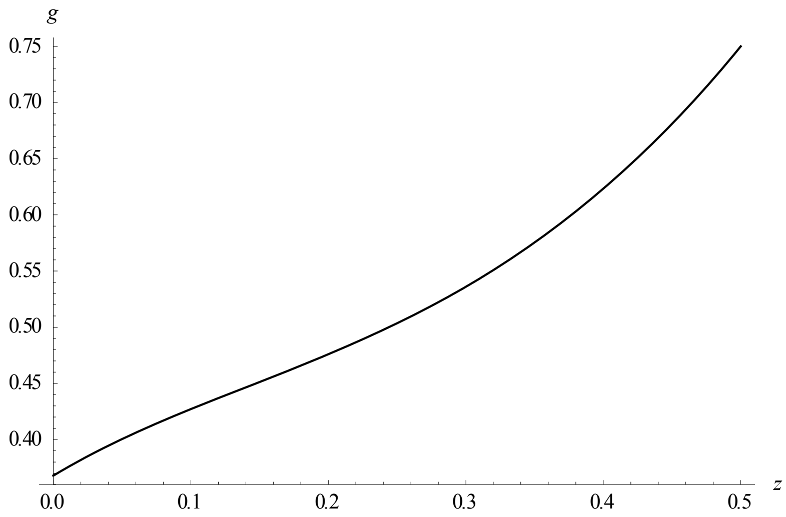

Figure 5. Auxiliary function $g(z)$

In the next subsection we investigate the monotonicity of the survival probability with respect to changes in the values of parameters.

3.3 Sensitivity analysis

The optimal order quantity in the expected profit model is different from the optimal order quantity in the aspiration-level model. In this section we study the sensitivity of the survival probability and the order quantity maximizing it with respect to the changes of the selling price, salvage value and shortage cost. For general distributions this appears to be a rather challenging problem, but for exponential distributions a full analytical study can be performed. The results are presented in the following theorem.

Theorem 2

Let A be the set of triples (p, s, v) which satisfy condition (2). For the exponentially distributed demand and any parameters (p, s, v) belonging to the set A the following statements hold.

a. Q_H^* is a decreasing function of selling price p, an increasing function of shortage cost s, and an increasing function of salvage value v.
b. H^* is a decreasing function of p, an increasing function of s and an increasing function of v.
c. Q_E^* increases if p increases, increases as s increases, increases if v increases and decreases as c increases.
d. E^* is an increasing function of p, a decreasing function of s, an increasing function of v and a decreasing function of c.

Proof of Theorem 1. For simplicity we write the expressions in terms of $a = p - v$. Then:

$$H^* = 1 - \left(\frac{a}{a+s}\right)^{\frac{a+s}{s}}$$

and:

$$Q_H^* = \frac{1}{\lambda}\ln\frac{a+s}{a}.$$

Let $x = \frac{s}{a+s}$ with the values from the interval $(0,1)$. The function $H(x) = 1 -$ $- (1-x)1x$ is increasing as x increases from 0 to 1, which proves claim (b). Statements (a), (c) and (d) are straightforward.

Now, we illustrate Theorem 2 with a numerical example. The values of the optimal solution in the classic newsvendor model and the aspiration-level model are calculated separately, taking into account the varying parameters v, c, p and s, one at a time. The parameter of the exponential distribution which modelled the random demand is assumed to be $\lambda = 0.003$. We solve the problem for $v = 11, 14, 15$, $c = 16, 17, 18$, $p = 25, 30, 35$ and $s = 20, 50, 80$. The base data values are $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$.

Table 1: Sensitivity analysis for varying parameters v, c, p and s

Parameter	$\boldsymbol{Q_E^*}$	$\boldsymbol{Q_H^*}$	$\boldsymbol{H^*}$	$\boldsymbol{E^*}$
$v = 11$	874.89	429.889	0.831	292.219
14	1165.503	472.355	0.846	2335.662
15	1391.462	488.779	0.851	3275.204
$c = 16$	1391.462	488.779	0.851	3275.204
17	1160.413	488.779	0.851	2012.507
18	1025.258	488.779	0.851	924.225
$p = 25$	1364.782	597.253	0.884	1635.218
30	1391.462	488.779	0.851	3275.204
35	1416.165	417.588	0.827	4917.168
$s = 20$	1185.116	282.433	0.851	3481.551
50	1391.462	488.779	0.918	3275.204
80	1517.959	615.276	0.943	3148.708

From Table 1 we conclude that the order quantity maximizing the survival probability increases from 429.9 to 488.8 as the salvage value increases from 11 to 15, which confirms statement (a) of Theorem 2. Next, as the unit shortage cost increases from 20 to 80, the order quantity with satisficing-level objective also increases, from 282.4 to 615.3. But if the selling price increases from 25 to 35 this order quantity decreases from 597.2 to 416.6. A similar analysis can be performed for the remaining quantities.

4 Optimal bicriteria index

In this section we give the solution to the optimization of the bicriteria index as well as some numerical examples for various values of weight w. Since the function $Y(Q)$ is continuous on the interval (Q_H^*, Q_E^*), it attains its maximum value. The derivative of the bicriteria index is equal to:

$$Y'(Q) = \frac{w}{E^*} E'(Q) + \frac{(1-w)}{H^*} H'(Q).$$

In order to optimize $Y(Q)$ it suffices to find Q such that $Y'(Q) = 0$ and then to prove that $Y''(Q) < 0$ for all $Q > Q_H^*$. If this is the case, then we get a unique Q_Y^* which maximizes the bicriteria index and satisfies the inequality $Q_H^* \le Q_Y^* \le Q_E^*$; we write $Y^* = Y(Q_Y^*)$. Note that if the second derivative satisfies $H''(Q) > 0$, then the second derivative $Y''(Q)$ is negative for weights w such that:

$$w > \frac{E^* H''(Q)}{E^* H''(Q) - H^* E''(Q)}$$

for all $Q > Q_H^*$.

In the following subsection a numerical example is given using the same base values of the parameters as in the previous example.

4.1 Sensitivity analysis

This subsection is dedicated to show the results of a numerical example. Note that the value of Q_Y^* is found here numerically. We examine the sensitivity of the optimal solution with respect to weight w. The base values are also $(\lambda, v, c, p, s) = (0.003, 15, 16, 30, 50)$. For these parameters to ensure the negativity of $Y''(Q)$ the weight w has to be greater than 0.5. For $w \le 0.5$ we take $Q_Y^* = Q_H^*$.

Table 2: Sensitivity analysis for various values of weight w

w	Q_E^*	Q_H^*	Q_Y^*	Y^*
0.0	1391.462	488.779	488.779 = Q_H^*	1.0
0.1	1391.462	488.779	488.779 = Q_H^*	0.885
0.2	1391.462	488.779	488.779 = Q_H^*	0.77
0.3	1391.462	488.779	488.779 = Q_H^*	0.665
0.4	1391.462	488.779	488.779 = Q_H^*	0.54
0.5	1391.462	488.779	488.779	0.425
0.6	1391.462	488.779	1339.517	0.783
0.7	1391.462	488.779	1359.011	0.837
0.8	1391.462	488.779	1372.915	0.891
0.9	1391.462	488.779	1383.344	0.946
1.0	1391.462	488.779	1391.462 = Q_E^*	1.0

We observe that as weight w increases, both the optimal order quantity maximizing the bicriteria index and the bicriteria index itself increase. In this case greater values of w correspond to the customer who is less risk-averse and therefore the expected profit model has an increasing influence on the bicriteria model. Hence the optimal value Q_Y^* is closer to the optimal order quantity Q_E^* of the expected profit model.

5 Conclusions

The paper is devoted to the bicriteria optimization in the newsvendor problem. One has to find the optimal order quantity which fulfils two goals. One objective is the classic optimization of the expected profit while the second one deals with the maximization of the probability of exceeding the expected profit. The assumed criteria are conflicting and there do not exist any solutions which optimize both criteria simultaneously.

We solve the bicriteria newsvendor problem with the exponential distribution as the distribution of the random demand. This distribution is widely used in many areas and in some situations it approximates the stochastic demand very well. The motivations for using this kind of distribution are explained in the introduction. The advantage of modelling the demand by an exponentially distributed random variable is the possibility of analytic derivation of exact solutions to the problem under some weak assumptions on the parameters of the model. In the paper of Arcelus et al. (2012) uniform distribution is studied which allows to find precise solutions of the optimization problem. The authors use the notions introduced by Parlar and Weng (2003) for the general distribution. They suggest to consider the problem with other demand distributions, which increases our knowledge about the bicriteria problem. Note that the solution of the bicriteria newsvendor problem presented in Parlar and Weng (2003) gives only an approximated optimal order quantity of the aspiration-level objective. In our case the order quantity maximizing the probability considered is given explicitly. Additionally, we derive the monotonicity of the solution with respect to the parameters of the model analytically. Even though the mathematics used is basic, we get some interesting results with various sets of the model parameters. It appears that the existence of a solution does not depend on the parameter of the demand distribution. To illustrate the general problem, the graphs of the expected profit, the probability of exceeding the expected profit and the limit functions are presented. Moreover, the numerical examples concerning the sensitivity of the model parameters are also given. The values of the optimal order quantities maximizing the bicriteria index are obtained numerically with Mathematica software.

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MULTIPLE CRITERIA DECISION MAKING
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EVALUATING PUBLIC BENEFIT ORGANIZATIONS IN POLAND WITH THE EVAMIX METHOD FOR MIXED DATA

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Abstract

The third sector and public benefit organizations (PBOs) play a significant role in the Polish economy. Although the third sector can boast of a long history in Poland, an intensive development of these entities has been observed since 1989. According to the current law, organizations with the public benefit status enjoy numerous benefits. This entails the need to adequately assess their activities, especially when taking into consideration the fact that they are not profit-oriented.

The aim of this paper is to propose a new assessment method for evaluating PBOs. The recommended approach is based on multi-criteria decision aiding (MCDA). The procedure proposed employs the EVAMIX technique for mixed evaluations – a hybrid of the EVAMIX method and the EVAMIX method with stochastic dominance (SD) rules. An illustrative example uses eleven PBOs from Lodz Voivodeship operating in the field of 'Ecology, animals and heritage protection'.

Keywords: Public Benefit Organization (PBO), Non-governmental Organization (NGO), annual performance report, financial report, decision analysis, MCDA, mixed data, EVAMIX method for mixed evaluations.

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1 Introduction

In modern democratic countries the broadly defined economic activity may be divided into three sectors. The first sector includes public administration, the second comprises profit-oriented institutions and organizations, and the third sector consists of non-governmental organizations (NGOs).

The third sector in Poland has a long tradition, but it has been developing more intensively since 1989. While certain regulations concerning NGOs had existed prior to that date, it was only after the collapse of communism that the unhindered development of the third sector organizations was possible (Borowiecki, Dziura, eds., 2014). According to the data provided by Poland's Main Statistical Office (GUS), there were about 27,400 registered non-profit organizations in 1997, about 67,500 in 2005 and about 100,700 in 2014 (GUS, 2009; GUS, 2016).

A special type of the third sector entity – the public benefit organization – was created in 2003 by the Polish legislators. Since these organizations are of great importance to society, they are granted many benefits, and the main benefit is the right to collect funds originating from 1% of personal income tax paid. Polish taxpayers have the right to donate part of their income tax liability to support a public benefit organization of their choice (Piechota, 2015). This benefit is the main way of supporting public benefit organizations. In 2005 PBOs received 42 million PLN from the 1% of the personal income tax paid, in 2009 it was 380 million PLN, and in 2014 the amount raised was 509 million PLN (which is twelve times as much as in 2005). The number of taxpayers who decided to donate 1% of their tax has also increased in the analysed period: in 2005 it was 0.7 million, in 2009 – 7.3 million and in 2014 – 12 million of taxpayers (GUS, 2015).

Taking into account the increasing role of the third sector organizations and public benefit organizations in Poland, it is very important to present and assess the effects of their work. In this paper we propose a tool for assessing the performance of public benefit organizations. These are the so-called non-profit organizations. Our tool is based on multi-criteria decision aiding, namely on the EVAMIX method with stochastic dominance rules (Górecka, 2010; 2012) since a number of factors need to be considered to properly evaluate such entities, and values of performance measures are not necessarily given deterministically. On the one hand, we hope that our tool will complement the existing literature. The problem of assessing non-governmental organizations has already been analysed also by e.g., Waniak-Michalak (2010), Waniak-Michalak and Zarzycka (2012; 2013; 2015), and Dyczkowski (2015b). On the other hand, our tool may possibly have some impact in practice: it could facilitate the decision to donate 1% as well as create reliability of and trust in those entities, which also depends on the transparency of activities evaluated by an adequate assessment method. Furthermore, many public benefit organizations obtain funds from local self-governments or central administration. These institutions could use the proposed method in the process of selecting candidates (organizations) to be delegated certain tasks financed with government grants and subventions.

The paper contains an introduction, three sections, and a conclusion. In the second section we present information on the public benefit status in Poland. Section three presents the proposed evaluation procedure for PBOs including a description of the EVAMIX method for mixed evaluations. In section four a case study and results of applying the MCDA approach are presented.

2 Public Benefit Organizations in Poland

The collapse of the communist system and the shift to the market economy in 1989 started a new period for the third sector in Poland. At the beginning non-profit organizations focused on the social and economic consequences of the transformation such as diminishing public social welfare provisions, unemployment and poverty. One should also emphasise that Polish NGOs did not affect significantly the political, social and economic reforms which were being implemented then (Leś, 1994 after: Leś et al., 2016).

Since 2003 work on a special status for non-profit organizations – public benefit organizations (PBOs) – have been conducted. This was related to the introduction of the Act of law of April 24th, 2003 on Public Benefit and Volunteer Work. This Act includes two important definitions: of the non-governmental organization and of the public benefit activity. According to the Act, non-governmental organizations are corporate and non-corporate entities, which are not part of the public finance sector and which do not operate for profit, including foundations and associations with the exception of political parties, trade unions and organizations of employers, professional self-governing authorities, and foundations formed by political parties (Act of law…, art. 3).

Non-governmental organizations are allowed to perform a public benefit activity which is understood as an activity that is focused on the benefit of society in the field of public tasks. The legislation indicates 37 areas of public activity, for example, social assistance, charity work, preserving national traditions, ecology, animal protection, protection of natural heritage, etc. (Act of law…, art. 3.1, 4.1).

Individual non-governmental organizations acting for public benefit may apply for the public benefit status. An appropriate entry to the National Court Register is needed in order to obtain this status. Moreover, according to the 2010 amendment of the Act, the organization has to submit evidence of its operations for public benefit for at least two years, before it applies for the public benefit status (Żak, 2012).

Entities obtaining the public benefit status gain also certain benefits regulated by the Act on Public Benefit and Volunteer Work. Those benefits facilitate the organization's activity which is generally accepted by society since the organization's activity is focused on the benefit of society. The most important benefits are the following (Act of law…, art. 24, 26, 27; Żak, 2012):

- tax exemptions as regards corporate income tax, property tax, tax on civil law transactions, stamp duty, court fees, as regards public benefit work performed by this organization,
- the right to use property owned by the State Treasury or by local self-government units, on preferential terms,
- free of charge promotion in public media: time in public radio and television to inform the general public of their activities,
- the right to receive 1% of the personal income tax, which may be used solely for public benefit work.

On the one hand, public benefit organizations are granted certain benefits, which, however, necessitate the need of transparency in those entities. From the moment of obtaining the public benefit status, organizations are obliged to fulfil reporting standards indicated by the law. Public benefit organizations must (Act of law…, art. 23): 1) prepare an annual financial report, 2) prepare an annual performance report, 3) make their financial and performance reports publicly available, 4) publish the accepted reports (financial and performance) on the website of the office of the minister competent for social security by July 15 (or 15 days after it is approved).

Information presented in annual reports is one of the most important bases for the assessment of public benefit organizations. As for the annual financial report, the relevant legislation is included in the Accounting Act of 29th September 1994. The Annual Financial Statement consists of the balance sheet (assets and funds of the public benefit organization), the income statement (the difference between the obtained income and expenses), the introduction to the financial statement and additional information.

Public benefit organizations also have to prepare an annual performance report. The law in force (since 2013) says that if the income of a public benefit organization does not exceed 100,000 PLN, this entity may prepare a simplified annual performance report. It should contain basic data on the organization, the type of its public benefit and business activities, its income and expense, number of its employees and their salaries, the number of its members and volunteers, income received from 1% of personal income tax and the way it was spent, administrative costs, other benefits that the entity made use of, and tasks commissioned by public bodies. The unabridged annual performance report includes additional information on the organization, such as its statutory goals and their realization in the reported year, more detailed information on the income gained and expenses, public tenders realized, and financial statement audits. The annual performance report form is set out in the Regulation of the Minister of Labour and Social Policy of February 12^{th} 2013 on the specimen of the annual performance report and the annual simplified performance report for public benefit organizations.

3 The proposed procedure for evaluating PBOs

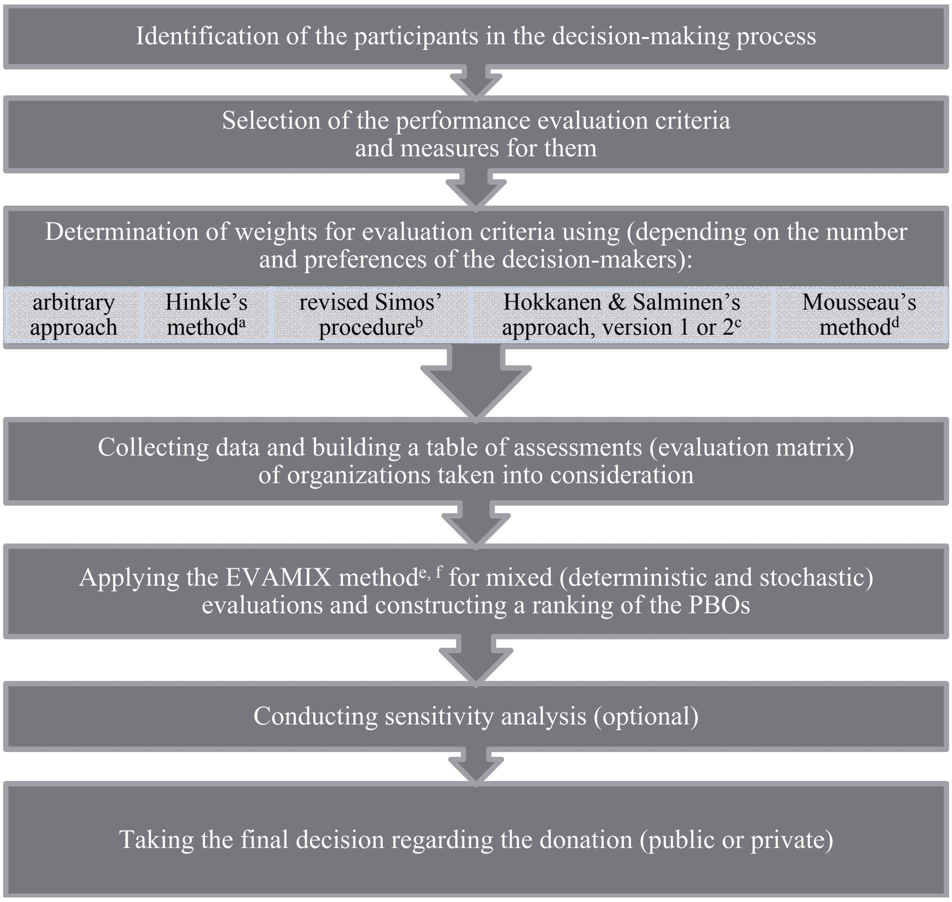
Responding to the need to develop a system for assessment and ranking public benefit organizations, for instance to help donors decide where to give their money or to determine the best and the worst entities for public co-financing, a procedure presented in Figure 1 has been proposed. In the process of developing it, advantages and disadvantages of various MCDA techniques (see Górecka 2010; 2011; 2013) have been taken into account as well as the fact that data used for evaluation will be partly qualitative and partly quantitative, and, additionally, at least some performances of alternatives (PBOs) will be evaluated in a probabilistic way.

The case of mixed data is not frequently considered in the literature and MCDA methods accepting different types of evaluations (e.g. deterministic, stochastic and fuzzy ones) are rather rare and not very well known. One multicriteria model that can be applied in such a situation is called NAIADE (see Munda, 1995; Munda et al., 1995); another one is called PAMSSEM (see Martel et al., 1997; Guitouni et al., 1999). Mixed evaluations were also considered by Zaras (2004) and Ben Amor et al. (2007). In the procedure proposed here the EVAMIX method for mixed evaluations is employed, which is a hybrid of the EVAMIX method (see Voogd, 1982; 1983) and the EVAMIX method with stochastic dominance rules (see Górecka, 2010; 2012).

In the EVAMIX method, proposed by H. Voogd (1982), the qualitative and quantitative data are distinguished and the final appraisal score of a given alternative is the result of a combination of the evaluations calculated separately for the qualitative and quantitative criteria.

In this paper it is assumed that the performances of alternatives (PBOs) are given in a deterministic and stochastic way, and that the decision-maker(s) are risk-averse. Thus, if the evaluations are stochastic, we will use FSD/SSD[1] (see Quirk, Saposnik, 1962; Hadar, Russel, 1969) and AFSD/ASSD rules (see Leshno, Levy, 2002) for modelling preferences with respect to criteria measured on a cardinal scale, and OFSD/OSSD (see Spector et al., 1996) and OAFSD/OASSD rules (see Górecka, 2009; 2014) in the case of criteria measured on an ordinal scale.

[1] If a decision-maker has also a decreasing absolute risk aversion, then the TSD rule (see Whitmore, 1970) should be additionally applied.



a) see Hinkle (1965); Rogers, Bruen (1998); b) see Figueira, Roy (2002); c) see Hokkanen, Salminen (1994; 1997); d) see Mousseau (1995); e) see Voogd (1982; 1983); f) see Górecka (2010; 2012).

Figure 1. Procedure recommended for evaluating PBOs

We assume that two situations can be considered when preferences are modelled with respect to a single criterion: preference in the wide sense and non-preference (see Roy, 1990; cf. Nowak, 2004; 2005):

- alternative a_i is preferred (strictly or weakly) to alternative a_j:

$$a_i \succ a_j \Leftrightarrow F_k^i \; SD \; F_k^j \quad \text{and} \quad \mu_k(a_i) - \mu_k(a_j) > 0, \tag{1}$$

- alternative a_j is preferred (strictly or weakly) to alternative a_i:

$$a_j \succ a_i \Leftrightarrow F_k^j \; SD \; F_k^i \quad \text{and} \quad \mu_k(a_j) - \mu_k(a_i) > 0, \tag{2}$$

- non-preference – otherwise,

and *SD* denotes stochastic dominance relation (FSD/SSD/AFSD/ASSD or OFSD/OSSD/OAFSD/OASSD).

The procedure of ordering alternatives (PBOs) consists of the following steps:

1. Determination of the qualitative dominance measures for the ordinal criteria:

$$\alpha_{ij} = \left[\sum_{k \in O} \left\{ w_k \varphi_k (a_i, a_j) \right\}^c \right]^{\frac{1}{c}}, \quad c = 1,3,5..., \tag{3}$$

where:

c – an arbitrary scaling parameter, for which any positive odd value may be chosen; the higher the value of the parameter is, the weaker the influence of the deviations between the evaluations for the less important criteria;

O – a set of qualitative (ordinal) criteria[2];

$$\varphi_k (a_i, a_j) = \begin{cases} 1 \ \text{ if } \ f_k(a_i) - f_k(a_j) > 0, \\ -1 \ \text{ if } \ f_k(a_j) - f_k(a_i) > 0, \\ 0 \ \text{ otherwise,} \end{cases} \tag{4}$$

for deterministic evaluations;

$f_k (a_i)$ – performance of alternative a_i on criterion f_k,

$$\varphi_k (a_i, a_j) = \begin{cases} 1 \ \text{ if } \ F_k^i \ SD \ F_k^j \ \ and \ \ \mu_k(a_i) - \mu_k(a_j) > 0, \\ -1 \ \text{ if } \ F_k^j \ SD \ F_k^i \ \ and \ \ \mu_k(a_j) - \mu_k(a_i) > 0, \\ 0 \ \text{ otherwise,} \end{cases} \tag{5}$$

for stochastic evaluations;

F_k^i – distribution of the evaluations of alternative a_i with respect to criterion f_k;

SD – stochastic dominance relation;

$\mu_k (a_i)$ – average performance (expected value of the evaluation distribution) of alternative a_i on criterion f_k.

2. Calculation of the quantitative dominance measures for the cardinal criteria:

$$\gamma_{ij} = \left[\sum_{k \in Q} \left\{ w_k \left(v_k(a_i) - v_k(a_j) \right) \right\}^c \right]^{\frac{1}{c}}, \quad c = 1,3,5..., \tag{6}$$

for deterministic evaluations;

where:

Q – a set of quantitative (cardinal) criteria[3],

$v_k (a_i)$ – standardised performance of alternative a_i on criterion f_k (expressed on a scale from 0 to 1);

[2] It is assumed that all the criteria are maximized.
[3] It is assumed that all the criteria are maximized.

$$\gamma_{ij} = \begin{cases} \left[\sum_{k \in Q} \left\{ w_k \left(\eta_k(a_i) - \eta_k(a_j) \right) \right\}^c \right]^{\frac{1}{c}}, & c = 1,3,5..., \text{ if } F_k^i \ SD \ F_k^j, \\ 0 \quad \text{otherwise,} \end{cases} \tag{7}$$

for stochastic evaluations;

where:

$\eta_k(a_i)$ – average standardised performance (expected value of the standardised evaluation distribution) of alternative a_i on criterion f_k;

F_k^i – distribution function representing standardised evaluations of alternative a_i with respect to criterion f_k and SD denotes stochastic dominance relation.

3. Standardisation of the dominance measures as follows:

$$\delta_{ij} = \alpha_{ij} \left(\sum_{i=1}^{m} \sum_{j=1}^{m} \left| \alpha_{ij} \right| \right)^{-1}, \tag{8}$$

$$\sigma_{ij} = \gamma_{ij} \left(\sum_{i=1}^{m} \sum_{j=1}^{m} \left| \gamma_{ij} \right| \right)^{-1}. \tag{9}$$

4. Calculation of the overall dominance measure q_{ij} for each pair of alternatives:

$$q_{ij} = w_O \delta_{ij} + w_Q \sigma_{ij}, \tag{10}$$

where:

w_O – the sum of weights of qualitative criteria,

w_Q – the sum of weights of quantitative criteria.

5. Determination of the final appraisal score u_i for each alternative:

$$u_i = \frac{1}{m} \sum_{j=1}^{m} q_{ij}. \tag{11}$$

6. Ranking of the alternatives (PBOs) according to the descending order of the final appraisal scores.

4 Illustrative example

The present study shows an application of the recommended procedure to appraising and ranking of eleven public benefit organizations from Lodz Voivodeship operating in the field of ‘Ecology, animals and heritage protection’.

Factors which a responsible charitable giver, social investor or public authority should, in our opinion, consider when selecting PBOs to support, as well as measures for them, have been identified on the basis of the literature review and the present authors’ own ideas. They are presented in Table 1.

Table 1: PBOs performance assessment factors

No.	Criterion (min/max/value of); (earlier studies)	Measure – calculation formula
f_1	Average amount of aid per beneficiary (max)	cost of unpaid statutory activities/number of beneficiaries
f_2	Average revenue generated by people involved in the organization's activities (max)	total revenue/number of people involved in the PBO's activities
f_3	Labour cost in relation to total revenue (min)	gross salaries/total revenue
f_4	Change in revenue (max); (a)	(total revenue in current year – total revenue in previous year)/total revenue in previous year
f_5	Financial stability ratio (value of 73); (b), (c)	cash and other short-term investments (in previous year)*365/total cost (in current year)
f_6	Private revenue concentration ratio (% of private financing) (max); (b), (c)	(1% of PIT + income from private sources including individual and institutional donations)/total revenue
f_7	Administrative costs ratio (% of administrative costs) (value of 6,5%); (b), (c), (d), (e)	administrative cost/total cost
f_8	Activity scope (value of 36); (b), (c)	number of beneficiaries/number of people involved in the organization's activities
f_9	Alternative labour costs (max); (b), (c)	(number of volunteers*gross salaries)/employees
f_{10}	Organization's age (max); (e)	the number of days the organization has PBO status
f_{11}	Statutory goals and activities or projects (max); (c)	do annual statements of the organization or its promotion materials define precisely statutory goals and activities or projects undertaken to achieve those objectives? (appraisal of the DM on scale 0-3)
f_{12}	Effects of activities (max) (c)	do annual statements of the organization or its promotion materials disclose accurately effects of activities undertaken by the organization in the recent period? (appraisal of the DM using scale 0-3)
f_{13}	Beneficiaries of activities (max); (c)	do annual statements of the organization or its promotion materials characterise thoroughly beneficiaries of activities conducted by the organization in the recent period? (appraisal of the DM using scale 0-3)
f_{14}	Organization's image (max); (c)	does the web-site of the organization help to create a positive image of the PBO? (appraisal of the DM on scale 0-3)

a) (www 1); b) Dyczkowski (2015a); c) Dyczkowski (2015b); d) Frumkin and Kim (2001); e) Trussel and Parsons (2008).

Source: Dyczkowski (2015a; 2015b); Waniak-Michalak (2010); Waniak-Michalak and Zarzycka (2012), own elaboration.

The analysis has been carried out on the basis of the official and publicly available annual reports (from 2014) of the organizations considered and on information from their websites. Criteria f_{11} through f_{14} have been assessed by the present authors (denoted by DM_1 and DM_2 in Table 3), who played the roles of potential givers. They have also determined weights for the evaluation criteria (arbitrarily, reaching compromise). The model of preferences for the decision-making problem as well as measurement data are presented in Table 2 and 3, while Table 4 provides the results obtained by applying the EVAMIX technique for mixed evaluations, together with a brief description of the PBOs examined.

Table 2: Model of preferences and input data – part 1

f_k	f_1 [max]	f_2 [max]	f_3 [min]	f_4 [max]	f_5 [goal: 73]	f_6 [max]	f_7 [goal: 0.065]	f_8 [goal: 36]	f_9 [max]	f_{10} [max]
w_k	0.1286	0.1238	0.0762	0.0429	0.0571	0.0667	0.0167	0.0452	0.0238	0.0333
a_i	Evaluation of alternative a_i (PBO) on criterion f_k									
A	90.98	26694.42	0.0457	0.3024	41.41	0.8865	0.2815	204.26	14319.54	4117
B	89.44	29855.98	0.4663	0.4078	106.32	0.0322	0.0625	231.50	19640.25	4171
C	572.95	21367.02	0.0000	-0.0718	14.04	0.0000	0.0007	35.56	0.00	4318
D	0.00	37448.55	0.3462	-0.0788	66.73	0.2625	0.7341	4243.43	82577.78	4151
E	71.95	30175.27	0.0597	0.1709	30.30	0.0676	0.9723	11.76	0.00	3456
F	6.93	58578.88	0.1074	-0.0311	66.67	0.9255	0.3605	3386.49	30640.66	3110
G	104.97	8645.69	0.0000	-0.0928	0.00	0.9943	0.0000	84.58	277200.00	2794
H	37.98	40155.69	0.0000	1.1761	54.74	0.4237	0.1478	1100.92	132396.92	120
I	670.70	39624.44	0.0217	-0.5438	0.00	0.1287	0.0000	48.94	0.00	519
J	1039.83	217163.46	0.0740	0.4368	12.32	0.9998	0.0000	192.00	3350.00	1450
K	1003.37	22893.19	0.0687	0.1520	76.48	0.8096	0.0715	17.65	30319.50	1427

Table 3: Model of preferences and input data – part 2

f_k	f_{11} (max)			f_{12} (max)			f_{13} (max)			f_{14} (max)		
w_k	0.0762			0.1095			0.0952			0.1048		
a_i	Evaluation of alternative a_i (PBO) on criterion f_k											
	DM_1	DM_2	$\mu_k(a_i)$	DM_1	DM_2	$\mu_k(a_i)$	DM_1	DM_2	$\mu_k(a_i)$	DM_1	DM_2	$\mu_k(a_i)$
A	3	3	3	3	2	2.5	3	3	3	3	3	3
B	3	3	3	3	3	3	3	3	3	3	3	3
C	2	2	2	1	2	1.5	1	3	2	1	2	1.5
D	2	2	2	2	2	2	2	3	2.5	2	2	2
E	3	2	2.5	2	2	2	2	1	1.5	3	2	2.5
F	3	2	2.5	3	3	3	3	3	3	3	2	2.5
G	2	2	2	1	2	1.5	2	2	2	2	3	2.5
H	3	3	3	3	3	3	3	3	3	2	1	1.5
I	2	3	2.5	2	2	2	1	1	1	2	2	2
J	2	3	2.5	2	2	2	1	1	1	1	2	1.5
K	2	2	2	2	2	2	1	1	1	2	1	1.5

Table 4: Ranking of PBOs

No.	PBO	Description of the organization	Appraisal score
1	**2**	**3**	**4**
1	**J**	The foundation has the status of PBO since 2012. Its aim is to promote kindness to animals and to prevent or suppress cruelty to and suffering among animals. It takes care of old, crippled, homeless, sick, injured or mentally ill animals including dogs and horses. It runs a sanctuary, a hospice and a home hospital	**0.0090**
2	**A**	An independent, self-financing NGO, created in February 1990, operating on the territory of Lodz Voivodeship. It has the status of PBO since 2005. It is open to cooperation with local authorities for the protection of animals. It is a non-profit organization that supports itself with donations and funds originating from 1% of personal income tax paid. The organization operates on a voluntary basis. Its aim is to promote kindness to animals, and to rescue, rehabilitate and rehome neglected and unwanted animals	**0.0046**
3	**K**	The society, based in Glowno, was founded in 2009. It has the status of PBO since 2012. It runs a sanctuary for stray and abandoned animals, takes care of animals staying there, arranges adoptions, and makes every effort to restore the animals' trust in humans. The organization promotes compassionate treatment of animals and helps people on low income to feed and care for their animals	**0.0034**
4	**F**	The foundation was registered in October 2006 on the initiative of volunteers helping in the animal shelter in Lodz. It is a non-profit organization with PBO status (since 2007) that supports all activities against animal homelessness by promoting the adoption, castration and sterilization. It conducts educational activities, promotes the practice of microchipping and registration of animals, and undertakes interventions for abused animals. From September 2013 the organization has run an adoption centre for animals of various kinds	**0.0022**
5	**H**	The foundation is an organization with PBO status, based in Lodz. Its aim is to provide care and protection for abandoned and maltreated animals, mainly by putting them in shelters. It supports all activities against animal homelessness and helps other organizations that deal with this problem	**0.0019**
6	**B**	A non-profit organization with PBO status (since 2004) and a mission to educate people about local environmental issues, and to expand their capacity to act for a more sustainable Poland. Its members believe that it is important not only to work directly in conservation and welfare, but to instil in people a love for their surroundings and their fellow inhabitants. The organization was founded in 1993, and was registered as an association in February 1997. The association is based in Lodz, Warsaw and Cracow	**-0.0012**
7	**G**	A non-profit foundation that operates on a voluntary basis. It has the status of PBO since 2008. It helps to solve any kind of problem which deals with cats, especially those that are chased from backyards and wandering, hungry or sick. The organization takes care of them by stroking, nursing or providing medicines, taking animals to the clinic and looking for homes for them	**-0.0020**
8	**I**	An organization with PBO status founded in September 2008. Its aim is to help all homeless animals, especially those which are in the Lodz shelter. The association helps people on low income towards the cost of feeding, treating, microchipping, spaying or neutering their animals. It educates people and promotes kindness to animals	**-0.0031**

Table 4 cont.

1	2	3	4
9	**C**	An organization registered in October 2002. The aim of the society is rescue, rehabilitation and re-homing of stray and unwanted animals, and the protection of animals of all kinds in need, including provision of veterinary treatment. It promotes environmental protection and the compassionate treatment of animals, and educates people in their care for animals. The organization has the PBO status since 2004 and it does not operate a business	**-0.0037**
10	**E**	The organization, based in Belchatow, was re-established in 1991 on the initiative of its pre-war members. It has the PBO status since 2006. Its aim is education in the spirit of patriotism and character formation of young people through paramilitary discipline and organization of their free time. The association organizes, among other things, environmental and ecological excursions	**-0.0040**
11	**D**	It is a PBO registered in January 2005. It is based on the territory of Lodz Voivodeship in Zgierz. The foundation runs sanctuary for abandoned animals, providing care, shelter, nourishment, veterinary treatment and re-homing for them	**-0.0071**

The ranking of public benefit organizations we have obtained shows that the best entity for donation, taking into account its effectiveness and reputation, is organization J. Organizations A, K, F and H also turned out to be quite good solutions since the values of their appraisal scores are positive. In turn, PBOs B, G, I, C and E do not seem appropriate entities for supporting by the decision-makers examined as the values of appraisal scores determined for them are negative. The worst organization for subsidising is organization D.

5 Summary

Taking the increasing role of public benefit organizations into consideration we have proposed a procedure for assessing their performance. The tool is based on the outranking MCDA technique intended for mixed evaluations, namely the EVAMIX method for mixed data, which is a hybrid of the EVAMIX method and the EVAMIX method with stochastic dominances. It can help donors to make smart and confident giving decisions. Moreover, it can be used by the authorities (self-governments or central administration) to choose organizations which should be responsible for certain tasks financed with public resources. Finally, it can help non-profit organizations to control their operations more effectively and to verify their own attractiveness as fundraisers.

The procedure discussed can be used for the evaluation of public service organizations all over the world. In the not-too-distant future we are going to apply it for charities operating in countries of the Commonwealth, for example Canada or Australia. However, we should keep in mind that measures used in the analysis should be tailored to each country's specific circumstances.

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SELECTION OF ACADEMIC CONFERENCES BASED ON ANALYTICAL NETWORK PROCESSES

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Abstract

Academic conferences are platforms established by scientists to provide broad access to their research. For this reason, it is important to have influential researchers presenting plenary talks and for the scientific community in that field to submit their work. Various organizations and academic institutions organize hundreds of academic conferences a year. Academics have to select conferences to attend, since it is not possible to participate in every conference. Conference selection takes into account such factors as: the registration fee, subject of the conference and its appropriateness, conference language and the deadline for submission. We consider the specific criteria that academics use to choose conferences and effective decision-making in this field. In this study, we use an approach based on analytic network processes (ANPs) to appropriately choose a conference based on multiple criteria.

Keywords: Analytic Network Process (ANP), multicriteria decision making, selection of an academic conference.

1 Introduction

Academic conferences are events that present the work of academics and students (in the form of papers and posters). Their contribution is very important to academic study, due to the review process at the acceptance stage and feed-

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back when work is presented. Also, such conferences are important opportunities to work on specific issues and enable researchers in the same field to meet each other.

Academic conferences are organized for various purposes. Such meetings may be on specific topics, designed to be instructive and/or a forum for academics and students to interact in their research and learn new things.

Conferences are categorized as national or international. At such meetings, papers are presented, short or long-term training seminars and public meetings are held, as well as working groups being organized. The process of accepting papers for a conference and how academics select conferences have been studied in the literature. Yüncü and Kozak (2010) developed a scale based on the criteria which Turkish academics use to select a conference. They surveyed 1100 academics in Turkey from a large number of universities and the data from this survey were analyzed with the help of software packages. Factor analysis and confirmatory factor analysis were performed to determine the attractiveness of a congress or conference. The location of a congress was a very significant factor according to this study, the extent of the recreational opportunities available in the area were identified as having the greatest impact. At the same time, the variety of accommodation and accessibility of the location were also included, among other measures affecting preferences.

Kozak and Yüncü (2011) also conducted a study on the characteristics of conventions preferred by academics. The appropriate criteria are determined by a survey which included 40 factors, such as registration fee, the cost of accommodation, dates of the conference and possible contribution to career development. Confirmatory factor analysis and *t*-tests were used to analyze these data. According to the analysis of the data obtained, the following are the most significant factors when choosing a conference; opportunities for recreation, location of the congress and overall cost. Acar and Ünsal (2013) aimed to identify the factors influencing the choice of scientific and academic conferences and willingness to take part in e-congresses. Their research was based on a pilot study and interviews with a total of 150 academics from 4 faculties. The factors included were the subject of the congress, the prestige of the institution organizing the congress, the consistency between the theme of a congress and the research interests of an academic, as well as the natural and cultural charm of the conference location. Statistical analysis was performed to describe the demographics of the faculty members surveyed and the factors that influence their preferences.

The following studies have also appeared in the literature. Go and Zhang (1997) classified the factors that influence the location of a conference. Based on regression analysis, Chacko and Fenich (2000) conducted a study of selecting the location of a congress. Ngamsom and Beck (2000) analyzed participation in international conferences and examined what influences the participants' motivation. Kim and Kim (2003) determined the significance of quality of service, accessibility etc. for selecting the site of a convention. Crouch and Louviere (2004) have developed various approaches and evaluated various alternatives for selecting the location of a congress. They also talked about the importance of competition from similar conferences. Chen (2006) used AHP to address the problem of congress selection. Lee and Back (2007) evaluated the factors influencing the submission process. Severt et al. (2007) explored a variety of factors determining participation in congresses and identified criteria like the appropriateness of conferences, training, opportunities for recreation, etc. Arslan et al. (2013) studied the level of academic support to students using a regression analysis based on data from Turkey. Dimitrios et al. (2014) investigated the best ways of increasing effectiveness when organizing conferences.

In this study, we aim to derive a rule for selecting conferences at national and international level according to the preferences of academics.

This study consists of 4 sections. In the second part, Analytic Network Processes are briefly described, together with a review of the literature. In the third part, we implement a decision rule, whose steps are described in detail. In the fourth part, we present the results of the study.

2 Analytic Network Processes

Analytic Network Processes (ANP) were developed by Saaty in 1980 as an extended version of analytic hierarchy processes. The ANP method derives a network describing the interaction of internally and externally dependent factors affecting a process. In this way, complex relationships that cannot be modeled using hierarchical structures can be modeled by ANP to aid in taking more effective decisions.

The ANP algorithm basically consists of 4 phases (Karabacak, 2012):

1. Determination of the interaction between the target and criteria.
2. Pairwise comparisons between the criteria and calculation of the eigenvalues of the corresponding matrix.
3. Forming super matrices.
4. Sorting and identifying the best alternative.

The application of the Analytic Network Process algorithm is illustrated in Figure 1.

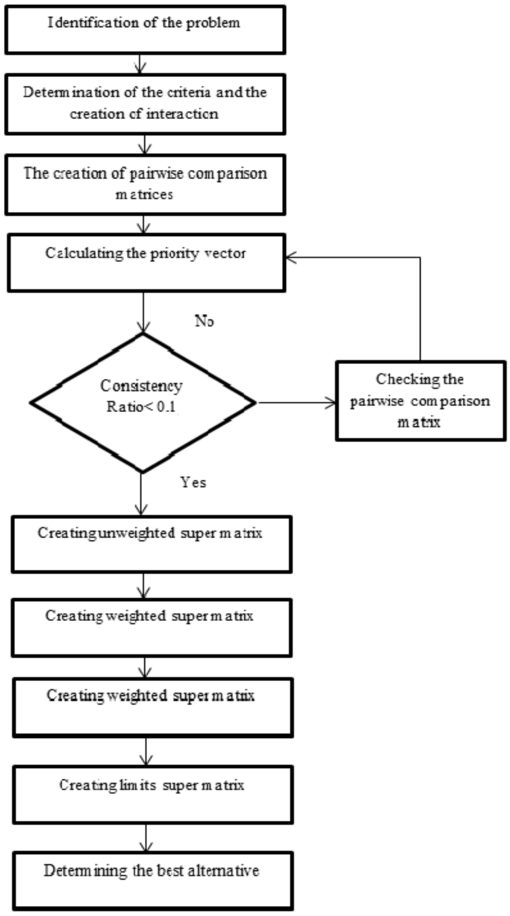

Figure 1. Algorithm for implementing ANP

The generated super-matrix shows the relationship between any pair of factors in the system. After the form of this matrix has been calculated, we derive the ranks of the alternatives and significance of the criteria, as shown in the flow chart above. Thus, the significant criteria and the ranking of alternatives are determined simultaneously.

The following studies have applied the ANP method: Lee and Kim (2000) and (2001), for example, chose a project for an information system using integrated ANP and goal programming. Meade and Presley (2002) used the ANP method for selecting research and development projects. Ravi et al. (2008) used ANP and goal programming methods for selecting reverse logistics projects. Büyüközkan and Öztürkcan (2010) evaluated six sigma projects using the ANP and Dematel methods. Cheng and Li (2005) used the ANP method for project selection. Tripathy and Biswal (2007) used 0-1 goal programming methods for project selection. Wey and Wu (2007) used ANP and 0-1 integer goal programming methods for selecting projects in transportation systems. Begičević et al. (2010) used the ANP method for selecting projects at higher education institutions. Bağ et al. (2012) used ANP and goal programming for scheduling nurse care. Tavana et al. (2013) used such algorithms for selecting high-tech projects in NASA applications. El-Abbasy et al. (2013) used the ANP method for selecting projects for highway construction. Ivanovic et al. (2013) selected road transport projects using the ANP method. Görgülü et al. (2013) assessed investment projects using the ANP and TOPSIS methods. Macura et al. (2013) selected transportation projects for reconstructing a pedestrian street in a Balkan city using the ANP method. Wang et al. (2013) integrated ANP with a fuzzy DELPHI method for project selection. Ortiz et al. (2015) made an application based on the ANP and DEMATEL-ANP methods for selecting a six sigma project in health care. Grady et al. (2015) used the ANP method for selecting international development projects. Hamurcu et al. (2015) combined ANP and goal programming for shift scheduling. Tuzkaya and Yolver (2015) applied ANP to selecting research and development projects. Jeng and Huang (2015) combined the ANP and DEMATEL methods to select national research projects. Tavana et al. (2015) use TOPSIS and integer programming to select a project. Hamurcu et al. (2016a) integrated ANP with goal programming to select monorail projects in Ankara. At the same time, Özder and Eren (2016) used multicriteria decision making and goal programming to select a supplier. Hamurcu and Eren (2016b) used ANP-TOPSIS methods to select the route for a monorail and in another study (2016c) selected appropriate monorail technology using ANP.

3 A case study

In this study, selection criteria and the corresponding decision rule were derived to choose international and national conferences, using the ANP method. This process is implemented using a computer application. The first problem is to choose an appropriate conference from a set of six, based on their subjects, lengths and costs. In total 21 factors were chosen based on the literature review. The factors affecting the choice of the favored conferences were determined. Table 1 gives a description of the conferences available.

The conferences available are classified in Table 1 according to the subject and length of the conference and the registration fee. The subject of the conference is split into the following categories: general or very general, specialist and sub-branches. For example, natural science conferences are classified as very general, engineering as general, industrial engineering as specialist and the scheduling of projects in industrial engineering as a sub-field. Academics made their choice by selecting the most appropriate conference according to their criteria from among these alternatives. The factors and sub-factors considered in this choice are presented in Table 2.

Table 1: Conferences available

Conference	Subject of the conference	Length of the conference (days)	Registration fee (TL)	Location of the conference	Conference language
A	General	2	300	İzmir	English/Turkish
B	Very General	2	750	İstanbul	English
C	Subfields	4	525	Antalya	English/Turkish
D	Very General	3	1200	Spain	English
E	General	3	800	Poland	English
F	Specialist	3	150	Aydın	Turkish

Table 2: Factors and sub-factors

Factor	Sub-Factor
Cost	Registration Fee
	Accommodation
	Transportation
Time	Travel Time
	Length of the Conference
	Submission Date
	Intensity of Conference
Conference	Subject of the Conference
	The Prestige of the Conference
	Conference Language
	Reputations of Main Speakers
	The Location of the Conference
	Academic Contribution of the Conference
	Social Programs
	Transport Facilities and Accessibility of Conference Venue
	Relevance
Country/City	Image of the Country/City
	Culinary Culture
	Safety
	Visa Facilitation
	Accommodation Facilities

In this study, choosing a conference was based on 4 factors and 21 sub-factors. The factors were split into cost, time, city/country and the conference itself. The cost sub-factors include all the components of costs incurred as a result of participating in a conference. These sub-factors are the registration fee, accommodation and transport costs.

Travel time, length of the conference, submission date and intensity (climate) of the conference are the sub-factors based on time. Travel time and length of the conference should not be long. The intensity (climate) of the conference is particularly affected by social activities and the submission date affects the ability of the participants to plan. City/country was another factor. It is important that the location is safe and has a wide range of accommodation. Culinary culture and the city/country's image are also among the relevant sub-factors.

The conferences themselves are described by several sub-factors, such as the conference location, transport facilities and accessibility. In addition, the subject of the conference, prestige of the conference and of its plenary speakers, language, relevance, social programs and academic contributions are also sub-factors. The conference should be located in an attractive city or region, easily accessible and close to major centers. Also, the social program is important, e.g. tours of the city or its natural surroundings. Participation in conferences is key in advancing one's academic career. Hence, it is important to select a conference that provides the maximum benefit to academics. Thus, the relevance of the conference, the conference language and reputation of the conference and its main speakers are important.

The literature and previous studies were used to determine the criteria that need to be satisfied in order to achieve the objectives of the application. Also, the views of scholars who participated in conferences and presented their academic work were taken into account. The ANP method was applied based on the factors and sub-factors defined above in order to measure the overall attractiveness of a conference and choose an appropriate conference. A network structure describing the interactions between these factors was derived. Sub-factors involving costs are associated with the following factors: Country/City, Conference and Time. Transportation costs are associated with the conference location, ease of obtaining a visa to visit the host country and the date of conference. Likewise, the views expressed by academic staff indicated an association between the subject of a conference and its academic importance. This network structure was derived using the SUPER DECISION program and is illustrated in Figure 2.

The structure of the ANP network, including the factors, sub-factors and alternatives, is shown in Figure 2. In addition, the mutual interactions between the factors are highlighted. This network is based on data from questionnaires answered by academics attending conferences. The 2nd stage of the ANP method involves calculating pairwise comparison matrices for the factors, sub-factors, alternatives and the characteristics of the alternatives. The matrices given by these binary comparisons were calculated based on Saaty's approach (1980) using a 1-9 scale and then their overall consistency was calculated. These comparisons were found to be consistent. The significance of the factors is assessed by pairwise comparisons based on the experience and knowledge of the decision makers. These pairwise comparisons are used to define super matrices representing the relationship between pairs of factors in the system. Figure 3 shows the weights of the selection criteria obtained using pairwise comparison matrices.

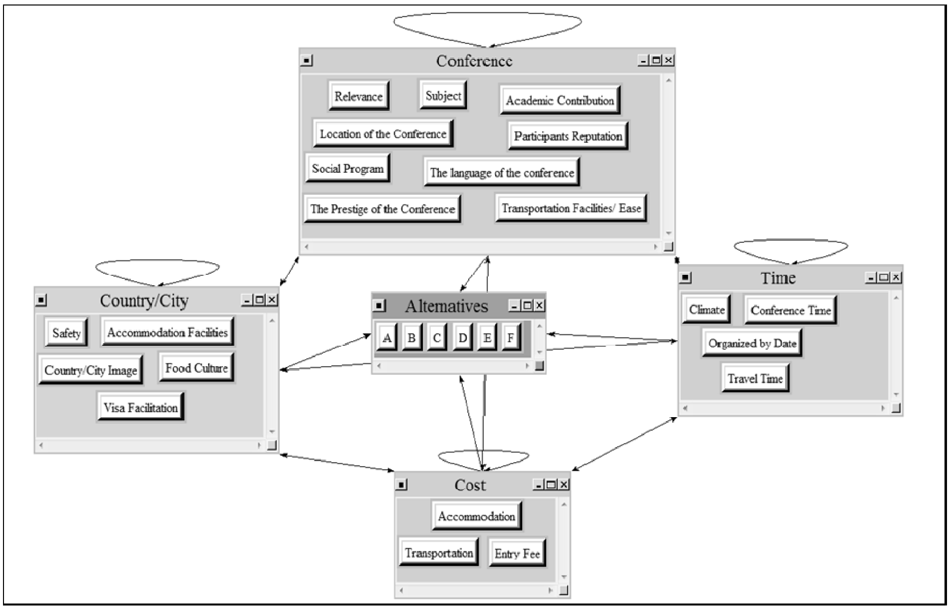

Figure 2. Structure of the ANP Network

The selection criteria are grouped into four separate groups: cost, time, country/city and the conference itself. According to these results, the subject of conference is the most important factor with a normalized weight of 0.117879. Travel costs are the second most important factor with a normalized weight of 0.107280. Academics rank the following factors most highly: “the subject of the conference”, “transport costs”, “image of country/city” and “academic contribution”. These criteria are considered to be very important when selecting one of six alternative conferences, which are evaluated as shown in Figure 4.

A network structure was derived to describe the relationship between these most significant factors and the remaining factors. By pairwise comparison of the results obtained, we obtain the following ranking of the alternatives: alternative F is in first place with value 0.218243, followed by alternative A with value 0.192537, alternative C with value 0.184145, alternative E with value 0.148414, alternative D with value 0.144356 and finally alternative B with value 0.112305.

No Icon	Academic Contribution	0.22399	0.089983
No Icon	Location of the Conference	0.09214	0.037015
No Icon	Participants Reputation	0.02189	0.008795
No Icon	Social Program	0.02818	0.011319
No Icon	Subject	0.29344	0.117879
No Icon	The Prestige of the Conference	0.04525	0.018179
No Icon	Transportation Facilities/ Ease	0.18855	0.075745
No Icon	Relevance	0.09843	0.039542
No Icon	The language of the conference	0.00812	0.003262
No Icon	Accommodation	0.35328	0.071516
No Icon	Entry Fee	0.11678	0.023641
No Icon	Transportation	0.52994	0.107280
No Icon	Accommodation Facilities	0.14206	0.027627
No Icon	Country/City Image	0.48483	0.094289
No Icon	Food Culture	0.01552	0.003018
No Icon	Safety	0.29547	0.057462
No Icon	Visa Facilitation	0.06213	0.012082
No Icon	Climate	0.34994	0.036275
No Icon	Conference Time	0.34543	0.035807
No Icon	Organized by Date	0.17890	0.018545
No Icon	Travel Time	0.12573	0.013033

Figure 3. Weights of the sub-factors (normalized values in black)

Name	Graphic	Ideals	Normals	Raw
A		0.882216	0.192537	0.018812
B		0.514586	0.112305	0.010973
C		0.843764	0.184145	0.017992
D		0.661447	0.144356	0.014104
E		0.680043	0.148414	0.014501
F		1.000000	0.218243	0.021323

Figure 4. Attractiveness of the alternatives

4 Results

Academic staff present the results of their research at conferences. Conferences are a platform for sharing knowledge and experience, thus providing significant benefits to those participating. Key experts in a field come together to present their work and interact with each other. There are many factors that influence the choice of a conference.

This study conducted research on the factors that influence the choice by academics of which conference to attend. The factors used in the study were chosen according to a literature review and questionnaires directed to academics. These factors were analyzed using the ANP method, which is then used to model a multicriteria decision problem. The most important factors in choosing a conference were the subject of the conference and travel costs.

Based on the answers given by academics, according to our analysis, there exist both internal and external interactions between the factors considered. These results were used to determine the most effective criteria for selecting a conference.

In this study, we have determined the criteria for selecting an appropriate conference for academics. A set of conferences were assessed using this approach. According to our analysis, academics look primarily at the subject of the conference and then other factors, such as cost, are taken into consideration.

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Sławomir Jarek*

REMOVING INCONSISTENCY IN PAIRWISE COMPARISON MATRIX IN THE AHP

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Abstract

The Analytic Hierarchy Process (AHP) allows to create a final ranking for a discrete set of decision variants on the basis of an earlier pairwise comparison of all the criteria and all the decision variants within each criterion. The properties of the obtained ranking depend on the quality of pairwise comparisons; this quality can be evaluated on the basis of consistency measured by means of certain measures. The paper discusses a mathematical model which is the foundation of the AHP and a starting point for a new method which allows to significantly reduce – and even eliminate – the inconsistency of pairwise comparisons measured by the consistency index. The proposed method allows to reduce the consistency index well below the threshold of 0.1.

Keywords: AHP, pairwise comparison, inconsistent pairwise comparison matrices.

1 Introduction

One of the stages of analysis of discrete multicriteria problems can be pairwise comparison. This process requires that the decision maker indicate, on a defined scale and for each pair of objects, the object which is evaluated higher or else that he/she state that they are evaluated identically. However, even for a small number of criteria the number of pairwise comparisons can be fairly large. This, in turn, may cause difficulties with expressing consistent evaluations by the decision maker. This may lead to determining an inconsistent matrix of pairwise comparisons which will therefore lack the assumed properties.

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A well-known method which heavily uses pairwise comparison is the Analytic Hierarchy Process (AHP). An essential obstacle in the application of the AHP is the above mentioned possibility of the occurrence of an inconsistent matrix of pairwise comparisons. Attempts to propose methods reducing the inconsistency of this matrix were made previously. In some papers it was suggested that the AHP itself is incorrectly constructed which leads to difficulties in proper analysis of the decision maker's preferences. One of these papers is Bana e Costa & Vansnick (2008), whose authors state: "we consider that the EM [Eigenvalue Methods] has a serious fundamental weakness that makes the use of AHP as a decision support tool very problematic".

This statement is based on their analysis of the AHP in which an essential role is played by the largest eigenvalue and the corresponding eigenvector of the pairwise comparison matrix. The authors introduced the notion of the Condition of Order Preservation (COP), which was supposed to be used to prove the weakness of the EM, including the AHP. Unfortunately, the authors, in a suggestive example, investigated a pairwise comparison matrix which, on the one hand, does not preserve the COP, and, on the other hand, was regarded in the AHP as consistent, with $c_r = 5\%$ – a value not exceeding the 10% threshold proposed by the author of the method. This example shows very well the problems encountered when analyzing an inconsistent pairwise comparison matrix, even if the degree of inconsistency is small. One can regard the specific values used in Bana e Costa & Vansnick (2008) as revealing the problematic definition of the consistency index and, at the same time, as underscoring the importance of the pairwise comparison matrix. In the present paper, an alternative method of reducing the inconsistency is proposed, which avoids the problems described above (Bana e Costa & Vansnick, 2008).

An interesting proposal of eliminating inconsistency is in the paper Benitez et al. (2011a) in the chapter "Fast computation of the consistent matrix closest to a reciprocal matrix", which describes, in the Matlab language, a function which allows to reduce the inconsistency of the pairwise comparison matrix. This method, however, is not based directly on the EM. Of extreme interest is the mathematical formula from this chapter, since it is similar to the relationship (2), derived in the present paper from the EM. Vector $\boldsymbol{w}$, given by Benitez et al. (2011a), is not based on the eigenvector of the eigenmatrix, but is determined numerically in the above mentioned function. In the proposal described further in the paper, the pairwise comparison matrix will be modified using values based on the eigenvector of the original matrix.

In the paper Zeshui (2004) a variable introducing small perturbations was added to the pairwise comparison matrix. This matrix is corrected using the values of the arithmetic or geometric weighted mean. To improve the pairwise comparison evaluations, the matrix elements with the largest values of the perturbation variables are corrected.

In the papers Saaty (2008, p. 15-16) and Saaty (2003, p. 88-90) three methods of modification of the pairwise comparison matrix have been proposed, which allow to reduce the inconsistency index. In these methods Saaty suggests to determine those elements in the matrix which influence the excessive value of the inconsistency index most. Next, new values are proposed and presented to the decision maker for his/her approval.

To correct an inconsistent pairwise comparison matrix, the paper Ergu et al. (2011) defines an algorithm based on the values of a new matrix containing a certain measure of inaccuracy of the evaluations contained in the original matrix. The authors propose a new method which allows to correct selected evaluations on the basis of the values of the measure proposed.

Another approach, proposed in the paper Siraj et al. (2012), consists in defining a certain heuristics which allows to improve the decision maker's evaluations. This heuristics is based on the ordinal consistency (transitivity) analysis. In this proposal, the relationships between the elements compared are expressed in form of a directed graph, with edges expressing direction and intensity of the decision maker's preferences. By investigating this graph it is possible to determine the number of violations of priority and, on this basis, to correct the values of the pairwise comparison matrix.

The authors of the paper Benitez et al. (2011b) propose to apply a linearization which is supposed to lead to the determination of a consistent pairwise comparison matrix whose distance from the original matrix is small. For this purpose, they define a certain measure based on Frobenius' norm. The paper contains a function in the MatLab language which allows to determine a corrected pairwise comparison matrix.

Among the existing methods of correcting inconsistent evaluations in the pairwise comparison matrix, none is based to a large extent on the eigenvector corresponding to the largest eigenvalue of the original matrix. The present paper attempts to fill this gap.

The purpose of the present paper is to propose a new method of reducing the inconsistency of the pairwise comparison matrix, which is measured with the consistency index c_r. The proposal is based on selected numerical properties of the AHP, which will be described in the next subsection of the paper. Additionally, a new scale is proposed, for the comparison of those elements which differ from each other only slightly. The proposal is based on Saaty's original scale, which introduces two different values (namely 1 and 1.1) for identical objects.

2 Basic properties of the pairwise comparison matrix in the AHP

An essential role in the AHP is played by the scale used for pairwise comparisons. Saaty (2008, p. 257) proposed two versions of the scale, described in Table 1. The first one is used for objects which are clearly different and uses values from 1 to 9. The other one is used for only slightly different objects, for which most evaluations would concentrate between 1 and 2. In this situation Saaty suggested to use values from the interval 1.1-1.9. Unfortunately, undistinguishable objects obtain different values on the two scales, namely 1 and 1.1. The reciprocals of these two values are also different, namely 1 and $\frac{1}{1.1}$, respectively. To solve this problem, in the present paper we use a different form of the second scale, with values smaller by 0.1 as compared with those in the paper Saaty (2008, p. 257), that is, from the interval 1.0-1.8. Thanks to this, identical objects are evaluated as 1, and the reciprocal of this value is also equal to 1.

Table 1: Saaty's Fundamental Scale of Absolute Numbers

Intensity of Importance	**Definition of Importance**	**Explanation**
1	Equal	Both activities contribute equally to the objective
2	Weak or slight	Intermediate importance between 1 and 3
3	Moderate	Experience and judgment slightly favor activity *i* over *j*
4	Moderate plus	Intermediate importance between 3 and 5
5	Strong	Experience and judgment strongly favor activity *i* over *j*
6	Strong plus	Intermediate importance between 5 and 7
7	Very strong or demonstrated	Activity *i* is favored very strongly over *j*; its dominance demonstrated in practice
8	Very, very strong	Intermediate importance between 7 and 9
9	Extreme	The evidence favoring activity *i* over *j* is of the highest possible order of affirmation
1.1-1.9	When all compared activities are very close: a decimal is added to 1 to show their difference as appropriate*	A better alternative way of assigning small decimals is to compare two close activities with other widely contrasting ones, favoring the larger one a little over the smaller one when using the 1-9 values
Reciprocals of above	If activity *i* has one of the above nonzero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*	A logical assumption

* Because of different properties of the first degree and its reciprocal in both scales, it is justified to use the range of degrees from the interval 1.0-1.8.

Source: Saaty (2008, p. 257).

In the consecutive subsections of the paper we propose a method supporting the process of correcting inconsistent evaluations of the decision maker. This proposal is based on numerical properties of the AHP, which will be described in the consecutive sections of the paper.

2.1 Analysis of the pairwise comparison matrix in the AHP

The AHP method uses pairwise comparisons of the individual criteria and decision variants. The results of the comparisons are saved in an *n* by *n* square matrix, which has ones on the main diagonal, and in which the symmetrical elements are mutually reciprocal. The number of those comparisons is a quadratic function of the number of the elements. The number of the necessary comparisons is expressed by the following formula:

$$\binom{n}{2} = \frac{n!}{2!\,(n-2)!} = \frac{n(n-1)}{2} = \frac{n^2-n}{2} \tag{1}$$

Comparison of two objects results in a consistent pairwise comparison matrix, since only one of the following three cases occurs:

- both objects are identical,
- the first one is evaluated higher than the second one, or
- the second one is evaluated higher than the first one.

Inconsistency of evaluations can occur already in the case of three objects. If the first object is evaluated higher than the second one, and the second one higher than the third one, then the third object cannot be evaluated higher than the first one. If this condition is not satisfied, we obtain an inconsistent pairwise comparison matrix. When investigating the random index described below, we have to generate random pairwise comparison matrices. In simulation experiments with 3×3 matrices, consistent matrices have been obtained in about 20% of cases. For larger matrices, the probability of drawing a consistent pairwise comparison matrix was extremely low. One can observe, therefore, that as the size of the pairwise comparison matrix increases, the problem with the inconsistency of evaluations can grow, too.

A certain inconsistency level was in a sense assumed in the AHP, since the decision maker's evaluations are expressed on a 9-degree scale. This number results from the natural limit of information processing by humans, described by the "seven plus or minus two" rule in Miller (1956).

In the AHP we aim at ordering the discrete decision variants, taking into account a certain hierarchy of criteria. For this purpose, a certain ranking is created, expressed by means of weight coefficients, contained in vector $\boldsymbol{w}$. This vector is normalized, hence the sum of its components is equal to 1.

To describe the AHP, as it was done in Saaty (2008), let us assume that at the beginning the values of vector $\boldsymbol{w}$ are known. An example is the problem of ordering companies with respect to their trade turnover volume. Knowing the values of $\boldsymbol{w}$, we can analytically create the pairwise comparison matrix by dividing the appropriate components of vector $\boldsymbol{w}$. If the turnover volumes are equal, then the quotient of the turnover volumes of the pair of businesses is 1. If the turnover of the first company is greater than that of the second one, the value of this quotient is larger than 1. Otherwise, it is smaller than 1. The results can be written in the form of a pairwise comparison matrix $\boldsymbol{W}$, as in (2) below:

$$\boldsymbol{W} = \boldsymbol{w} \cdot \frac{1}{\boldsymbol{w}^T} = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \frac{w_1}{w_3} & & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \frac{w_2}{w_3} & \cdots & \frac{w_2}{w_n} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & \frac{w_3}{w_3} & & \frac{w_3}{w_n} \\ & \vdots & & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \frac{w_n}{w_3} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \tag{2}$$

From the process of constructing $\boldsymbol{W}$ it follows that its main diagonal consists of ones only, and the symmetric elements are mutually reciprocal: $w_{ij} = \frac{1}{w_{ji}}$. On the basis of (2) we can state that the order of $\boldsymbol{W}$ is exactly 1. Therefore, this matrix has only one non-zero eigenvalue. Additionally, on the basis of calculations in (3), we can see that $\boldsymbol{w}$ is an eigenvector of $\boldsymbol{W}$:

$$\begin{aligned} \boldsymbol{W} \cdot \boldsymbol{w} = \boldsymbol{w} \cdot \frac{1}{\boldsymbol{w}^T} \cdot \boldsymbol{w} &= \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \frac{w_1}{w_3} & & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \frac{w_2}{w_3} & \cdots & \frac{w_2}{w_n} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & \frac{w_3}{w_3} & & \frac{w_3}{w_n} \\ & \vdots & & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \frac{w_n}{w_3} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} \\ &= \begin{bmatrix} w_1 + w_1 + w_1 + \cdots + w_1 \\ w_2 + w_2 + w_2 + \cdots + w_2 \\ w_3 + w_3 + w_3 + \cdots + w_3 \\ \vdots \\ w_n + w_n + w_n + \cdots + w_n \end{bmatrix} = \boldsymbol{w} \cdot n \end{aligned} \tag{3}$$

Usually we do not know the components of vector $\boldsymbol{w}$, only the values of matrix $\boldsymbol{W}$. To determine the values of $\boldsymbol{w}$ we analyze the eigenproblem of the form $\boldsymbol{W} \cdot \boldsymbol{w} = n \cdot \boldsymbol{w}$, where n is the eigenvalue corresponding to eigenvector $\boldsymbol{w}$. This relationship is described by formula (3).

From the relationship (3) one can conclude that the order of matrix $\boldsymbol{W}$ is exactly 1. Moreover, from this it follows that all the eigenvalues of $\boldsymbol{W}$, except one, are equal to 0. Since the main diagonal of matrix $\boldsymbol{W}$ contains only 1s, its trace is: $tr(\boldsymbol{W}) = n$. On the other hand, the trace of $\boldsymbol{W}$ is the sum of its eigenvalues, $tr(\boldsymbol{W}) = \sum_i \lambda_i$, and therefore the largest eigenvalue of $\boldsymbol{W}$ is equal to n, and the remaining ones are equal to 0. The problem of constructing the scale vector $\boldsymbol{w}$ is therefore reduced to determining the eigenvector corresponding to the largest eigenvalue of the pairwise comparison matrix $\boldsymbol{W}$.

To determine the eigenvector $\boldsymbol{w}$ it is convenient to use von Mises's exponential method. For a pairwise comparison matrix this method converges, since the difference between the two largest eigenvalues is significantly greater than 0, since $n - 0 > 0$.

It is convenient to start the calculations with the assumptions that the initial vector consists of 1s only: $\boldsymbol{w}_{(0)}^T = [1 \quad 1 \quad 1 \quad \cdots \quad 1]$. We obtain the consecutive approximations of the sought eigenvector from the formula: $\boldsymbol{w}_{(k+1)} = \boldsymbol{W} \cdot \boldsymbol{w}_{(k)}$.

Saaty proposed to normalize matrix $\boldsymbol{W}$ prior to the application of the exponential method, so that the sum of the elements in each column is equal to 1. In a sense this is consistent with the exponential method, since in the consecutive iterations of this method it is necessary to normalize the obtained approximations of the eigenvector. This operation is supposed to prevent a sudden growth of the components of vector $\boldsymbol{w}$. Calculations in (4) show the method of determining the sum in each column of matrix $\boldsymbol{W}$. At the same time, we assume that the sum of the components of vector $\boldsymbol{w}$ is equal to $\boldsymbol{s_w} = \sum_i w_i$:

$$\boldsymbol{s_k} = \boldsymbol{e}^T \cdot \boldsymbol{W} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}^{\boldsymbol{T}} \cdot \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \frac{w_1}{w_3} & & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \frac{w_2}{w_3} & \cdots & \frac{w_2}{w_n} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & \frac{w_3}{w_3} & & \frac{w_3}{w_n} \\ & \vdots & & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \frac{w_n}{w_3} & \cdots & \frac{w_n}{w_n} \end{bmatrix} = \begin{bmatrix} \frac{s_w}{w_1} \\ \frac{s_w}{w_2} \\ \frac{s_w}{w_3} \\ \vdots \\ \frac{s_w}{w_n} \end{bmatrix}^T \tag{4}$$

By dividing the columns of matrix $\boldsymbol{W}$ by the sums $\boldsymbol{s_k}$ we obtain the normalized matrix $\boldsymbol{W}$ whose structure is shown in (5):

$$\boldsymbol{W_N} = \boldsymbol{W} \cdot diag\left(\frac{1}{\boldsymbol{s_k}}\right) = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \frac{w_1}{w_3} & & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \frac{w_2}{w_3} & \cdots & \frac{w_2}{w_n} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & \frac{w_3}{w_3} & & \frac{w_3}{w_n} \\ & \vdots & & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \frac{w_n}{w_3} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \cdot diag\begin{pmatrix} \frac{w_1}{s_w} \\ \frac{w_2}{s_w} \\ \frac{w_3}{s_w} \\ \vdots \\ \frac{w_n}{s_w} \end{pmatrix} = \\ = \begin{bmatrix} \frac{w_1}{s_w} & \frac{w_1}{s_w} & \frac{w_1}{s_w} & & \frac{w_1}{s_w} \\ \frac{w_2}{s_w} & \frac{w_2}{s_w} & \frac{w_2}{s_w} & \cdots & \frac{w_2}{s_w} \\ \frac{w_3}{s_w} & \frac{w_3}{s_w} & \frac{w_3}{s_w} & & \frac{w_3}{s_w} \\ \vdots & & & \ddots & \vdots \\ \frac{w_n}{s_w} & \frac{w_n}{s_w} & \frac{w_n}{s_w} & \cdots & \frac{w_n}{s_w} \end{bmatrix} \tag{5}$$

By performing only one iteration of the exponential method, we obtain the result shown in (6):

$$\boldsymbol{w_{(1)}} = \boldsymbol{W_N} \cdot \boldsymbol{w_{(0)}} = \begin{bmatrix} \frac{w_1}{s_w} & \frac{w_1}{s_w} & \frac{w_1}{s_w} & & \frac{w_1}{s_w} \\ \frac{w_2}{s_w} & \frac{w_2}{s_w} & \frac{w_2}{s_w} & \cdots & \frac{w_2}{s_w} \\ \frac{w_3}{s_w} & \frac{w_3}{s_w} & \frac{w_3}{s_w} & & \frac{w_3}{s_w} \\ & \vdots & & \ddots & \vdots \\ \frac{w_n}{s_w} & \frac{w_n}{s_w} & \frac{w_n}{s_w} & \cdots & \frac{w_n}{s_w} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n \cdot w_1}{s_w} \\ \frac{n \cdot w_2}{s_w} \\ \frac{n \cdot w_3}{s_w} \\ \vdots \\ \frac{n \cdot w_n}{s_w} \end{bmatrix} \tag{6}$$

Moreover, it is easy to see that when we divide the resulting vector $\boldsymbol{w_{(1)}}$ by *n*, we obtain the normalized scale vector $\boldsymbol{w_n}$ since $\sum_i w_{n_i} = 1$. The relevant calculations are in formula (7):

$$\boldsymbol{w_n} = \frac{\boldsymbol{w_1}}{n} = \begin{bmatrix} \frac{w_1}{s_w} \\ \frac{w_2}{s_w} \\ \frac{w_3}{s_w} \\ \vdots \\ \frac{w_n}{s_w} \end{bmatrix} = \frac{1}{s_w} \cdot \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} = \frac{1}{s_w} \cdot \boldsymbol{w} \tag{7}$$

As mentioned before, in general we do not know the scale vector $\boldsymbol{w}$, only the pairwise comparison matrix $\boldsymbol{W}$. However, by performing the calculations shown in formulas (4) through (7), we can determine the scale vector $\boldsymbol{w}$ on the basis of matrix $\boldsymbol{W}$.

2.2 The occurrence of inconsistency in the AHP

The method of determining the scale vector $\boldsymbol{w}$ described in the previous subsection is correct as long as the order of the pairwise comparison matrix $\boldsymbol{W}$ is equal to 1. This is because the pairwise comparisons led to a consistent matrix $\boldsymbol{W}$. Unfortunately, in general, matrix $\boldsymbol{W}$ is not always consistent and therefore it is necessary to find out by how much the eigenvalue obtained exceeds n. In the case of a consistent matrix, the relationships in (8) and (9) are true. On the basis of their construction it is possible to determine the extent to which the maximal eigenvalue differs from the theoretical quantity n:

$$\boldsymbol{W} \cdot \boldsymbol{w}_n = n \cdot \boldsymbol{w}_n \tag{8}$$

$$\boldsymbol{W} \cdot \boldsymbol{w}_n = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \frac{w_1}{w_3} & & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \frac{w_2}{w_3} & \cdots & \frac{w_2}{w_n} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & \frac{w_3}{w_3} & & \frac{w_3}{w_n} \\ & \vdots & & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \frac{w_n}{w_3} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \cdot \begin{bmatrix} \frac{w_1}{s_w} \\ \frac{w_2}{s_w} \\ \frac{w_3}{s_w} \\ \vdots \\ \frac{w_n}{s_w} \end{bmatrix} = \begin{bmatrix} \frac{n \cdot w_1}{s_w} \\ \frac{n \cdot w_2}{s_w} \\ \frac{n \cdot w_3}{s_w} \\ \vdots \\ \frac{n \cdot w_n}{s_w} \end{bmatrix} \tag{9}$$

For this purpose, we divide the obtained vector (the right-hand side of (9)) by the consecutive components of $\boldsymbol{w_n}$. In the case of a consistent matrix we obtain vector $[n \quad n \quad n \quad \cdots \quad n]^T$, for which the average of the elements λ_{max} is n. In general, this average can have another value, and therefore we determine the consistency index $c_i = \frac{\lambda_{max}}{n-1}$. This index is the arithmetic mean of the eigenvalues of matrix $\boldsymbol{W}$, calculated omitting the largest eigenvalue. If the pairwise comparison matrix is consistent, then $c_i = 0$. Since this index depends on the size of matrix $\boldsymbol{W}$, Saaty proposed to correct the value of c_i by a certain random index which takes into account the size of the matrix under discussion. The consistency index $c_r = \frac{c_i}{r_i}$, where r_i is a certain random index, allows to check if matrix $\boldsymbol{W}$ is inconsistent. We assume that the pairwise comparison matrix is consistent if $c_r < 10\%$.

3 A proposal to eliminate the inconsistency in pairwise comparisons

An essential obstacle in applying the AHP are frequently occurring problems with inconsistency of pairwise comparisons. In many problems, especially those related to large-size matrices, the value of the consistency index c_r significantly exceeds the acceptable threshold of 10%. To obtain a consistent matrix, we have to correct the results of pairwise comparisons. Since matrix $\boldsymbol{W}$ reflects the decision maker's preferences, it is justified to allow him/her to participate in the correction of its contents. This approach requires additional activity from the decision maker. The proposed method analyzes the pairwise comparison matrix and points out the elements to be corrected to the decision maker. Moreover, the method suggests to him/her the values of the evaluations of the elements being corrected.

The proposed algorithm for eliminating inconsistency consists of the following steps:

1. Determine the scale vector $\boldsymbol{w_n}$ using the AHP method and check the consistency index c_r.
2. If $c_r < 0.1$, end the calculations, otherwise go to the next step.
3. Determine the new pairwise comparison matrix $\boldsymbol{W}$ from formula (11).
4. On the basis of matrix $\boldsymbol{W_s}$ and Saaty's scale determine the new proposals of pairwise comparisons.
5. Ask the decision maker to accept the proposed pairwise comparisons or to present the new evaluations of pairwise comparisons from matrix $\boldsymbol{W}$ (in particular, those values which differ most from the proposal).
6. If the decision maker accepts the new comparisons, end the calculations, otherwise go to Step 3.

4 Examples of applications

In the next two subsections we present examples illustrating applications of the proposed algorithm. The first example describes a problem in which the decision maker supplied exceptionally inconsistent evaluations of the individual variants, revealing in the consecutive iterations that according to his/her preferences, the variants compared differ only slightly from each other. During this process a transition from the classic Saaty scale 1-9 to the scale 1.0-1.8 is effected; this scale is proposed in the present paper. The next example deals with a problem described in Saaty (2003, p. 88).

4.1 The problem of an inconsistent pairwise comparison matrix

Let us consider three decision variants: a, b and c, which were evaluated by the decision maker as follows: $a > b$, $b > c$ and $c > a$ (sic!). The pairwise comparison matrix reflecting these preferences is shown in (10):

$$\boldsymbol{W} = \begin{bmatrix} 1 & 9 & \frac{1}{9} \\ \frac{1}{9} & 1 & 9 \\ \boldsymbol{9} & \frac{1}{9} & 1 \end{bmatrix} \tag{10}$$

Using the AHP we obtain a scale vector of the form $\boldsymbol{w_n} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}^{\boldsymbol{T}}$ and $c_r = 6.84 \gg 0{,}1$ (for $r_i = 0.52$). In our calculations we used the fact that the sums of the elements in the consecutive rows and columns were identical. Since c_r indicates that matrix $\boldsymbol{W}$ is strongly inconsistent, we propose the corrected matrix $\boldsymbol{W_s}$ to the decision maker. Our proposal consists in reconstructing the pairwise comparison matrix on the basis of $\boldsymbol{w_n}$, according to formula (11), which in turn is based on the relationship described in (2):

$$\boldsymbol{W_s} = \boldsymbol{w_n} \cdot \left(\frac{\boldsymbol{1}}{\boldsymbol{w_n}}\right)^{\boldsymbol{T}} \tag{11}$$

By performing the calculations we obtain the corrected pairwise comparison matrix, for which $c_r = 0$:

$$\boldsymbol{W_s} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \begin{bmatrix} 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{12}$$

It is easy to see that this proposal consists in assuming that all three variants are equivalent: $a = b = c$.

We assume that the decision maker, knowing the new matrix $\boldsymbol{W_s}$ modifies his/her evaluations and expresses them in a new matrix, shown in (13):

$$\boldsymbol{W} = \begin{bmatrix} 1 & 5 & \frac{1}{2} \\ \frac{1}{5} & 1 & 5 \\ 2 & \frac{1}{5} & 1 \end{bmatrix} \tag{13}$$

For this pairwise comparison matrix, the scale vector is $\boldsymbol{w_n} = [0.4 \quad 0.33 \quad 0.27]^{\boldsymbol{T}}$ and the consistency index is $c_r = 1.72 \gg 0.1$. Unfortunately, we have again obtained an inconsistent matrix $\boldsymbol{W}$. The corrected pairwise comparison matrix, shown in (14), has been determined from formula (11):

$$\boldsymbol{W_s} = \begin{bmatrix} 1 & 1.204 & 1.474 \\ 0.830 & 1 & 1.224 \\ 0.678 & 0.817 & 1 \end{bmatrix} \tag{14}$$

Using the 1.0-1.8 scale, we obtain the matrix shown in (15), which we present to the decision maker for evaluation. The consistency index of this matrix is $c_r = 0$:

$$\boldsymbol{W} = \begin{bmatrix} 1 & \mathbf{1.2} & \mathbf{1.5} \\ 0.833 & 1 & \mathbf{1.2} \\ 0.667 & 0.833 & 1 \end{bmatrix} \tag{15}$$

The decision maker finds that the proposed matrix correctly reflects his/her preferences.

4.2 An example from Saaty's paper

The next example is related to the problem of buying a house, with eight criteria taken into account. The decision maker expressed his/her preferences in the form of a pairwise comparison matrix, shown in Table 2.

Table 2: Pairwise comparison matrix $\boldsymbol{W}$ for the problem of buying a single family home for the given criteria

	Size	Trans.	Nbrhd	Age	Yard	Modern	Cond.	Finance
Size		5	3	7	6	6		
Trans.				5	3	3		
Nbrhd.		3		6	3	4	6	
Age								
Yard				3				
Modern				4	2			
Cond.	3	5		7	5	5		
Finance	4	7	5	8	6	6	2	

* $\lambda_{max} = 9.618$, $c_i = 0.231$, $r_i = 1.4$, $c_r = 0.165$.

Source: Saaty (2003, p. 88).

Using the AHP we conclude that the matrix in Table 2 is not consistent. From formula (11) we determine the corrected matrix, shown in Table 3.

Using Saaty's scale for the matrix from Table 3, we obtain a new pairwise comparison matrix, shown in Table 4. We assume that the decision maker accepts the proposed corrections. The consistency index decreased from 23.1% to 1%.

Table 3: Pairwise comparison matrix reconstructed from (11)

	Size	Trans.	Nbrhd	Age	Yard	Modern	Cond.	Finance	w_i
Size	1.000	2.639	1.025	9.227	4.947	3.922	0.977	0.558	1.000
Trans.	0.379	1.000	0.388	3.497	1.875	1.486	0.370	0.212	0.379
Nbrhd.	0.976	2.575	1.000	9.003	4.827	3.827	0.953	0.545	0.976
Age	0.108	0.286	0.111	1.000	0.536	0.425	0.106	0.061	0.108
Yard	0.202	0.533	0.207	1.865	1.000	0.793	0.197	0.113	0.202
Modern	0.255	0.673	0.261	2.352	1.261	1.000	0.249	0.142	0.255
Cond.	1.024	2.701	1.049	9.444	5.063	4.015	1.000	0.572	1.024
Finance	1.791	4.726	1.835	16.524	8.859	7.025	1.750	1.000	1.791

Source: Author's own calculations.

Table 4: A correct pairwise comparison matrix, based on Table 3 and after the application of Saaty's scale

	Size	Trans.	Nbrhd	Age	Yard	Modern	Cond.	Finance	w_i
Size		3	1	9	5	4			0.151
Trans.				3	2	1			0.052
Nbrhd.		3		9	5	4			0.151
Age									0.019
Yard				2					0.032
Modern				2	1				0.038
Cond.	1	3	1	9	5	4			0.151
Finance	2	5	2	9	9	7	2		0.259

* $\lambda_{max} = 8.068$, $c_i = 0.010$, $r_i = 1.4$, $c_r = 0.007$.

Source: Author's own calculations.

Analyzing the data from Table 4 we can see that three categories are regarded by the decision maker as equivalent. Table 5 shows the matrix corrected according to Saaty's proposal. For this matrix the consistency index is equal to 8.1% and is significantly higher than that for the matrix from Table 4.

Table 5: The corrected pairwise comparison matrix $\boldsymbol{W}$ for the problem of buying a family home

	Size	Trans.	Nbrhd	Age	Yard	Modern	Cond.	Finance	w_i
Size		5	3	7	6	6			0.175
Trans.				5	3	3			0.062
Nbrhd.		3		6	3	4			0.103
Age									0.019
Yard				3					0.034
Modern				4	2				0.041
Cond.	3	5	2	7	5	5			0.221
Finance	4	7	5	8	6	6	2		0.345

* $\lambda_{max} = 8.811$, $c_r = 0.083$.

Source: Saaty (2003, p. 90).

5 Summary

In this paper the author presented an iterative method of eliminating inconsistency of pairwise comparison matrices. The proposal allows to determine a consistent matrix in a single iteration. By applying the assumed scale of pairwise comparison evaluations we determine the corrected pairwise comparison matrix and present it to the decision maker for acceptance. If the decision maker does not accept the proposed changes, he/she can add necessary corrections of the pairwise comparison matrix, on the basis of the corrections proposed. This process, in which the decision maker plays an active role, lasts until a consistent matrix $\boldsymbol{W}$ is obtained. The proposed method facilitates finding out consistent preferences of the decision maker, especially in large-size problems.

This proposal removes one of the obstacles encountered by users of the AHP in complex problems. Another obstacle is the determination of random indices r_i for matrices of sizes larger than 30. For smaller matrix sizes, these indices are published, but unfortunately various authors give various lists of values for them. Another research direction will be related to the investigation of random indices used in research on consistency of pairwise comparison matrices and on a new construction of the consistency index.

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AN IMPACT OF NEGOTIATION PROFILES ON THE ACCURACY OF NEGOTIATION OFFER SCORING SYSTEMS – EXPERIMENTAL STUDY

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Abstract

In this paper an impact of the party's negotiation profile on the misperception of the preferential information provided to the negotiating parties is studied. In particular, the problems with determining an adequate and preferentially correct negotiation offer scoring system is analyzed, when the parties are supported in their decision analyses by means of the SAW technique. In the analyses we use the negotiation data from bilateral negotiation experiments conducted by means of the Inspire negotiation support system. To determine the negotiators' profiles the Thomas-Kilmann Conflict Mode Instrument was used, which allows to describe their general negotiation approach using two dimensions of assertiveness and cooperativeness. The accuracy of scoring systems was defined as the extent to which the negotiator's individual scoring system (agent's system) is concordant to the preferential information provided by the negotiator's superior (principal's system) in the form of verbal and graphical descriptions, and measured by means of ordinal and cardinal accuracy indexes.

Keywords: negotiation profile, Thomas-Kilmann Conflict Mode Instrument, SAW, SMART, negotiation offer scoring system, preferences, electronic negotiation.

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1 Introduction

Various methods and techniques of operations research, and in particular – of multiple criteria decision aiding (MCDA), play an important role from the viewpoint of measuring the negotiation outcome, its quality and efficiency (Wachowicz, 2010). They are used in prenegotiation phase to help negotiators to elicit their preferences for different resolution levels of negotiation issues and determine the quantitative negotiation offer scoring system, which allows to assign numerical score to each feasible negotiation offer defined within the negotiation template and can be used throughout the whole negotiation process to support the negotiator decisions (Raiffa et al., 2002). In particular, such systems help to measure the scales of concessions, visualize the negotiation progress and conduct the arbitration analysis aimed at finding a fair and balanced negotiation agreement.

Determining the accurate scoring systems that represent the parties' preferences adequately is important from the viewpoint of providing the negotiators with reliable decision support. If the scoring system is inaccurate, the negotiator may falsely interpret the moves of their counterpart, e.g. misinterpret concession as a reverse-concession (or *vice versa*); misevaluate the profitability of alternative offers submitted by both parties during the actual negotiation phase and, finally, accept a contract that does not yield the expected and aspired profits. It is even more important, when the principal-agent context is embodied in the negotiation problem (Spremann, 1987), in which the agents negotiate in the name of their principals, and should be able to prove that their strategies and negotiated contracts are concordant with the principals' requirements and expectations. In this case determining an accurate scoring system, that reflect the preferences of the principal correctly, is of special focus. The agent, having a scoring system discordant with the principal's preference system, may negotiate in good will a contract he will consider to be good (best). Yet, the same contract will be evaluated as poor by the principal, whose preferences were not adequately represented by the agent's scoring system. Thus, it is a key issue to provide the negotiating agents with easy to use and technically accurate decision support tools that would help them to build reliable scoring systems.

Various formal decision support models are implemented in the negotiation support systems (NSS) used in business, research and training, such as OpenNexus (http://en.opennexus.pl/), Inspire (Kersten and Noronha, 1999) or Negoisst (Schoop et al., 2003). In vast majority of situations, it is the simple additive weighting (SAW) method (Churchman and Ackoff, 1954), or its discrete version called SMART (Edwards and Barron, 1994), which are used in the decision support models in negotiation mainly for their simplicity and low cognitive demand.

In discrete negotiation problems they require assigning rating points to each element of the negotiation template assuming that more preferable issues and options obtain higher ratings. Hence, any negotiation offer can be easily evaluated by adding up the ratings of options that comprise this offer. The higher the rating, the better the offer. Naturally, applying SAW or SMART to negotiation support requires the acceptance of the fundamental assumptions that the preferences are additive and preferentially independent (Keeney and Raiffa, 1976).

Even though SAW seems easy, cognitively low-demanding and technically uncomplicated, some of its drawbacks have been recently empirically discovered. For instance, it has been observed (Roszkowska and Wachowicz, 2014) that a majority (57%) of decision makers, when given a choice of the method for defining their preferences, express them qualitatively using linguistic or descriptive labels. If quantitative scores are used, they are usually of ordinal nature. This may suggest that the negotiators may make mistakes when asked to express their preferences by means of cardinal ratings instead of more intuitive qualitative judgements. This seems to be confirmed by initial analyses conducted by us in the negotiation support context (Wachowicz et al., 2015) that reveal significant problems with determining adequate scoring systems. The question, still unanswered, is which factors influence the negotiator's ability to construct the scoring systems precisely, i.e. according to the preferential information provided by their principals.

In this paper we focus on analyzing the prenegotiation process of building a negotiation offer scoring system by means of SAW by the negotiators of various negotiation profiles[1]. These profiles are determined by means of Thomas-Kilmann Conflict Mode Instrument (Kilmann and Thomas, 1977) and describe the negotiator's assertiveness and cooperativeness by means of five different conflict modes: collaborating, competing, compromising, avoiding and accommodating. In our research we analyze a dataset of electronic negotiation experiments conducted in the Inspire system, with a predefined multi-issue bilateral business negotiation case. We study whether the ability of the negotiators to transform correctly the preferential information included in the case description (provided by the principal) into a system of ratings depends on their negotiation profile described by means of five characteristics related to the conflict modes mentioned above. Inspired by earlier research by Vetschera (Vetschera, 2007), we use a negotiation case with precise graphical information about the principal's preferences. It makes possible to measure the scale of potential inaccuracy in determining the negotiation offer scoring systems by means of two separate

[1] This paper is an extension of the conference paper presented during the International Conference on Group Decision and Negotiation 2015, Warsaw (Kersten et al., 2015).

measures of accuracy: an ordinal accuracy and a cardinal accuracy measures. The former one is more general and focuses on measuring the correctness of rank order of issues and options defined in the negotiation template. The latter one refers to measuring the cardinal differences in ratings assigned to options and issues by the negotiators (agents) with the reference ratings that reflect the principal's preferences adequately.

The paper consists of three more sections. In section 2 we describe briefly the experimental setup, i.e. the bilateral negotiation experiment conducted in the Inspire system, the notions of measuring the ordinal and cardinal accuracy of scoring systems determined by the negotiators in our experiment, as well as the Thomas-Kilmann Conflict Mode Instrument (TKI) used to determine the negotiators' profiles. In section 3 we analyze the experimental results and present the key findings regarding the structures of profiles and their impact on the scoring systems' accuracy. In section 4 we present the final conclusions as well as suggest some directions for future research.

2 Experiment setup

2.1 Negotiation case

For the purpose of this paper the bilateral negotiation experiment was organized in the Inspire negotiation support system (Kersten and Noronha, 1999). In this experiment 350 students from Poland, Austria, China, Taiwan, Great Britain, Ukraine and Canada took part. The negotiation case we used in the experiment described in details a bilateral problem of signing a new contract between the entertaining agency (WorldMusic) and the musician (Ms. Sonata). The participants were asked to play the roles of agents of the agency (Mosico) and the musician (Fado) and negotiate for them the best possible contracts. The negotiation problem was defined by means of four issues, for which the feasible resolution levels were predefined in a form of a discrete negotiation template (Table 1).

Table 1: The Mosico-Fado negotiation template

Issues	Salient options
No of new songs	11; 12; 13; 14 or 15
Royalties (%)	1.5; 2; 2.5 or 3%
Contract signing bonus ($)	$125,000; $150,000; $200,000
No of promotional concerts	5; 6; 7 or 8

The negotiators were provided with private information about the goals, expectations and priorities of the principals they represented. The private information on each principal's preferences was accessible only to the agent that represented this principal. This information was both verbal and graphical, the latter one used to illustrate the differences in strength of preferences among issues and options. The graphical preference information was presented in the form of circles (Figure 1).

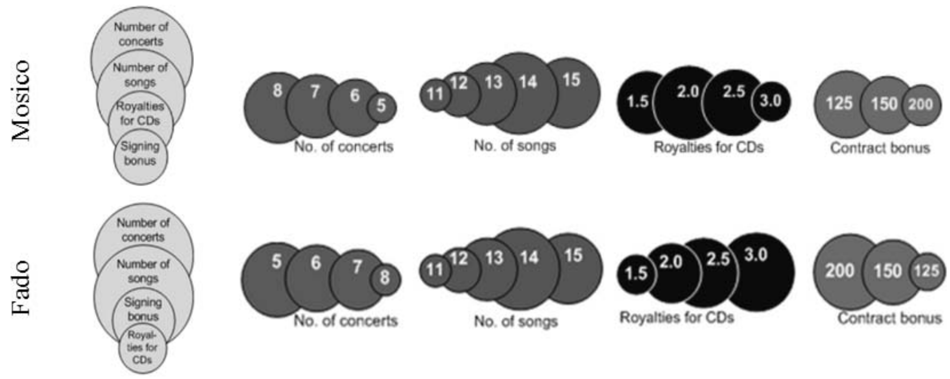


Figure 1. Graphical visualization of preferences for the Mosico-Fado case

As shown in Figure 1, the agents of Fado may learn, for instance, that the issues of number of concerts their principal (Ms. Sonata) would have to perform and number of new songs she would have to write within the contract signed with WorldMusic are the two most important issues. Both agents also knew that the resolution level "14 songs" is the best option for the principal they represent, and is somewhat more important than options: 13 and 15. The latter two are nearly equally preferred, yet 15 seems slightly better than 13.

2.2 Building the offer scoring system with SAW

Having both the negotiation template and the principal's preferences defined, as shown in Table 1 and Figure 1, respectively, the experiment's participants were supposed to determine their individual negotiation offer scoring system in the prenegotiation phase. There are many methods and techniques that can be used to determine such a system if the negotiation template consists of many issues. The problem of scoring the template is similar to an individual discrete multiple criteria decision making problem, hence a range of MCDA approaches can be of use here. A literature review reveals a few examples of using various MCDA techniques to support negotiators in rating the negotiation template, such as: UTA (Jarke et al., 1987), AHP (Mustajoki and Hamalainen, 2000) or TOPSIS

(Roszkowska and Wachowicz, 2015). Yet, the most popular technique that is applied in negotiation support systems used commonly for negotiation training, teaching or real-world problem solving is the one that derives from the multi-attribute utility and multi-attribute value theories (von Neumann and Morgenstern, 1944; Keeney and Raiffa, 1976). Formally defined as SAW, i.e. simple additive weighting (Churchman and Ackoff, 1954), it was later modified and adjusted to various decision contexts, e.g. to elicit preferences in discrete decision making problems as SMAR – simple multiple attribute rating technique (Edwards and Barron, 1994).

When applied to scoring the negotiation template, like in Inspire (Kersten and Noronha, 1999) or NegoCalc (Wachowicz, 2008) systems, SAW consists of two straightforward steps: (1) defining the issue weights (issue ratings); and (2) defining preferences for options within each issue (option ratings). Let us denote by m the number of negotiation issues and by X_j ($j = 1, \dots, m$) the sets of salient and feasible options for issue j identified within the negotiation template under consideration. The SAW-based process of building the negotiation offer scoring systems, as implemented in the Inspire system, consists of the following steps:

- **Step 1.** Assigning the weights to each of the issues in the form of cardinal ratings so that:

$$\sum_j u_j = 100. \tag{1}$$

- **Step 2.** Assigning the ratings u_{jk} to each option $x_{jk} \in X_j$ within each negotiation issue j so that:

$$u_{jk} \in \langle 0; u_j \rangle, \tag{2}$$

and the most preferred (best) option receives the maximum score resulting from the issue weight (i.e. u_j), while the worst one, the rating equal to 0.

The SAW-based scoring system obtained by means of the above algorithm can be used to evaluate each feasible negotiation offer that can be constructed out of the salient options defined within the negotiation template. The global rating $u(A)$ of the offer A under consideration is determined as the result of additive aggregation of the ratings assigned to each option that comprise this offer, i.e.:

$$u(A) = \sum_{j=1}^{m} \sum_{k=1}^{|X_j|} z_{jk}(A) \cdot u_{jk}, \tag{3}$$

where $z_{jk}(A)$ is a binary multiplier denoting if the option x_{jk} comprises an offer A (1) or not (0).

According to the rating rules described by the SAW-based rating procedure (steps 1 and 2), the best offer within the template, i.e. the one that consists of the most preferred options, will be scored with 100 rating points, while the worst offer – with the score of 0.

2.3 Measuring the accuracy of scoring systems

For the purpose of this experiment two notions of scoring system accuracy were used, both measuring the concordance of the agent's individually built scoring system with the principal's preferential information provided within the case description as private information (see Figure 1) (Roszkowska and Wachowicz, 2015). These were the *ordinal accuracy* and the *cardinal accuracy* measures.

Ordinal accuracy

Ordinal accuracy of the agent's scoring system measures the extent to which the rank order of the preferences defined by this agent for issues and options is concordant with the rank order of the principal's preferences. More precisely, if we consider n alternatives $A_1, A_2, \dots, A_n$ (that represent various options or issues in the negotiation template) ordered according to non-increasing preferences of the principal, the notion of perfect ordinal accuracy requires that cardinal ratings $u(A_i)$ assigned by the agent to each alternative i fulfill the following condition:

$$u(A_1) \geq u(A_2) \geq \cdots \geq u(A_n). \tag{4}$$

Measuring the ordinal accuracy of the whole scoring systems requires different groups of elements of the negotiation template to be considered separately. While building the negotiation offer scoring system by means of the SAW method the agent assigns the scores within a two-step procedure: (1) the issue weights are declared (a ranking of m issues is defined by means of cardinal scores); (2) m series of options are evaluated (one series for each negotiation issue $j = 1, \dots, m$). Consequently, the agent assigns scores to $m + 1$ series of alternatives, i.e. $m + 1$ rank orders defined by the principal's preferences need to be reflected by means of cardinal scores. In our experiment there are five rankings (series of alternatives) that describe the principal's preferences for the complete negotiation template, one for issue weights and four others describing the structure of preferences within each issue: number of concerts, number of songs, royalties and contract signing bonus (see Figure 1).

Consequently, the *ordinal accuracy index* of the agent's negotiation offer scoring system will be defined as a ratio of the number of series of ratings defined by ith agent that are concordant with the rank order of preferences defined by the principal (n_i^{con}) to the total number of series of ratings that the agent needed to define within the negotiation template. In our experiment $m + 1 = 5$, hence the *ordinal accuracy index* of ith agent's scoring system is defined in the following form:

$$OA_i = \frac{n_i^{\text{con}}}{5}. \tag{5}$$

If the agent's individual scoring system represents correctly all the possible rankings resulting from the principal's preferential information, the ordinal accuracy index is equal to 1. If none of the rankings is represented correctly by the ratings assigned by the agent, then $OA_i = 0$.

It seems clear that the ordinal accuracy of the agent's scoring system may be also measured in a more detailed way, e.g. by means of the Kendall or Spearman rank correlation indexes. Yet, in our study we are interested in the most general perception of quality of scoring systems, which summarizes its accuracy at the level of complete rankings.

Cardinal accuracy

The notion of cardinal accuracy of the agent's scoring system is introduced to measure not only if the scores assigned by the agent to the issues and options reflect adequately the order of the principal's preferences, but also the strength of these preferences. To build such a measure a kind of reference rating system needs to be determined, for which we assume that it reflects the principal's preferences precisely, and according to the verbal and graphical preference information provided to the agents. Hence, such a reference rating system for the principal's preferences defines precisely the cardinal ratings describing the issue weights (u_j^{ref}, for $j = 1, \dots, m$), as well as the ratings of options (feasible resolution levels) defined for each negotiation issue (u_{jk}^{ref}, for $j = 1, \dots, m$; and $k = 1, .., N_j$, where N_j is the number of options defined for jth issue).

Having the reference scoring system defined, the *cardinal inaccuracy* of ith agent's scoring system may be measured in the following way:

$$CI_i = \sum_j \left| u_j^{\text{ref}} - u_j^i \right| + \sum_j \sum_{k=1,..,N_j} u_j^{\text{ref}} \cdot \left| \bar{u}_{jk}^{\text{ref}} - \bar{u}_{jk}^i \right|, \qquad (6)$$

where:

u_j^i – the rating assigned to jth issue by ith agent;

$\bar{u}_{jk}^{\text{ref}}$ – normalized rating of kth option of jth issue in the reference scoring system;

$\bar{u}_{jk}^i$ – normalized rating of kth option of jth issue in the scoring system of ith agent.

The cardinal inaccuracy index is a non-standardized measure. If all agent's ratings are the same as the reference ratings determined on the basis of the principal's preferential information, the cardinal inaccuracy index is equal to 0. The bigger discrepancy between the principal's and the agent's ratings, the bigger the CI_i value.

Please note that measuring the cardinal inaccuracy by means of formula (6) allows to avoid double counting of errors made by the agents when determining their individual scoring system. According to the two-step SAW-based procedure of defining the preferences in the Inspire system (for details see: Kersten and Noronha, 1999; Wachowicz, 2010), the issue rating assigned by the agent in step 1 is a reference value for assigning the option ratings within this issue. The most preferred option of one issue should obtain the rating equal to the weight (rating) of this issue. Hence, any mistake made by the agent at the issue level would be copied to the option level and counted twice, as the difference $u_j^{\text{ref}} - u_j^i$ and then as the difference $u_{jk}^{\text{ref}} - u_{jk}^i$ (where k represents the most preferred option). If the normalization of option ratings is applied, the double-counting of the mistakes at the issue level can be avoided. Yet, the normalized differences in option rating between the agent's and the principal's scoring systems need to be multiplied by the issue weights (u_j^{ref}), to be comparable with the differences determined first at the issue level.

To illustrate the problem of double counting of inaccuracies let us consider the ratings assigned by the Fado agent to the options of issue "Number of promotional concerts" and compare them with the reference ratings by the principal (Ms. Sonata herself). Both the reference and normalized ratings of the agent and the principal are shown in Table 2.

Table 2: Principal's and agent's ratings for options of „Number of concerts"

Option	**Principal's ratings (u_{jk}^{ref})**	**Normalized principal's ratings ($\bar{u}_{jk}^{\text{ref}}$)**	**Agent's ratings (u_{jk}^i)**	**Normalized agent's ratings ($\bar{u}_{jk}^i$)**	**Normalized difference ($\lvert\bar{u}_{jk}^{\text{ref}} - \bar{u}_{jk}^i\rvert$)**	**Final inaccuracy ($u_j^{\text{ref}} \cdot \lvert\bar{u}_{jk}^{\text{ref}} - \bar{u}_{jk}^i\rvert$)**
5	32	1.00	17	1.00	0.00	0.00
6	25	0.78	10	0.58	0.20	6.40
7	21	0.66	5	0.29	0.37	11.84
8	0	0.00	0	0.00	0.00	0.00

From the agent's ratings displayed in Table 2 we read that he underestimated the rating (weight) of the whole issue. Instead of scoring the issue importance at the level of 32 rating points (the principal's reference rating), he assigned to it 17 rating points only. This issue-level inaccuracy will be accumulated within the first summand of formula (3), i.e. $\left|u_{\text{concerts}}^{\text{ref}} - u_{\text{concerts}}^i\right| = 32 - 17 = 15$. However, from the viewpoint of option-level accuracy, the best option (5 concerts) was correctly recognized by the agent, and he assigned to it the highest possible rating resulting within all options of this issue. He cannot be penalized for assigning 17 rating points to the option "5 concerts" instead of 32, since the specificity of the SAW algorithm does not allow him to operate with 32 rating points, if in step 1 the pool of 17 rating points was used to indicate the importance of this issue. Accumulating the non-normalized differences $u_{jk}^{\text{ref}} - u_{jk}^i$ would result in counting the previous mistake one more time here. However, the normalized differences $\left|\bar{u}_{jk}^{\mathrm{ref}} - \bar{u}_{jk}^{i}\right|$ would not indicate any problem at the option-level in such a situation. Yet, if the inaccuracies in ratings appear for the remaining options, the normalized differences would allow us to capture the scale of the problem. For instance, the option "6 concerts" assures 78% of rating points (25 out of 32) assigned by the principal to the option of "5 concerts". When we look at the agent's rating we will find that his evaluation of "6 concerts" is inaccurate. He assigned 10 rating points to this option, which amounts to 58% of the value of the best option (17 rating points were assigned to "5 concerts"). This is a difference of $0.78 - 0.58 = 0.22$ percentage points and should be included in the global value of scoring system inaccuracy. Yet, the cardinal inaccuracy index is measured in rating points, thus this percentage-based inaccuracy must be recalculated using the reference value of the rating assigned to this issue, and hence it will be equal to $u_{\mathrm{concerts}}^{\mathrm{ref}} \cdot \left|\bar{u}_{\mathrm{concerts},6}^{\mathrm{ref}} - \bar{u}_{\mathrm{concerts},6}^{i}\right| = 32 \cdot$ $\cdot \left|0.78 - 0.58\right| = 6.4$ rating points.

One technical issue needs also to be raised while determining the principal's reference scoring system in Inspire experiments. Since the graphical preferential information was provided by means of circles, there is a question of how to measure the circle sizes that would reflect the final rating values of issues and options. These can be determined either by measuring the circles' radiuses, or the circles' areas. The reference scoring systems determined by measuring radiuses and areas are shown in Table 3. There is no objective rationale that we could use to support our choice of radiuses or areas, hence in this paper we will measure the inaccuracies with respect to two references scoring systems and obtain two cardinal inaccuracy indexes for each agent: radius-based cardinal inaccuracy index (CIR), and area-based cardinal inaccuracy index (CIA).

Table 3: The reference scoring systems for Fado and Mosico determined by measuring radiuses and areas

Party	**Reference rates**															
	No. of concerts				**No. of songs**					**Royalties for CDs**				**Contract bonus**		
	5	6	7	8	11	12	13	14	15	1.5	2.0	2.5	3.0	125	150	200
Radius-based reference ratings																
Mosico	0	21	26	32	0	7	16	28	21	13	23	16	0	17	10	0
Fado	32	25	21	0	0	8	20	32	24	0	7	12	16	0	15	20
Area-based reference ratings																
Mosico	0	22	30	39	0	5	15	30	20	10	20	13	0	11	6	0
Fado	38	27	22	0	0	6	20	38	26	0	4	7	9	0	10	15

2.4 Thomas-Kilmann Conflict Mode Instrument

The Thomas-Kilmann Conflict Mode Instrument (Kilmann and Thomas, 1977) is a questionnaire-based psychometric test that has been widely used for analyzing the conflict attitudes of people in various contexts and problems (Rahim, 1983; Hignite et al., 2002). It consists of 30 questions regarding the surveyed person's attitude toward conflict and conflict solving. Each question consists of two statements, each describing the examples of different behavior in conflict and related to one of the five conflict modes: competing, collaborating, compromising, avoiding, and accommodating. All five modes are positioned in two-dimensional space described by the intensity of two personal characteristics that play a key role in conflict: assertiveness and cooperativeness, as shown in Figure 2.

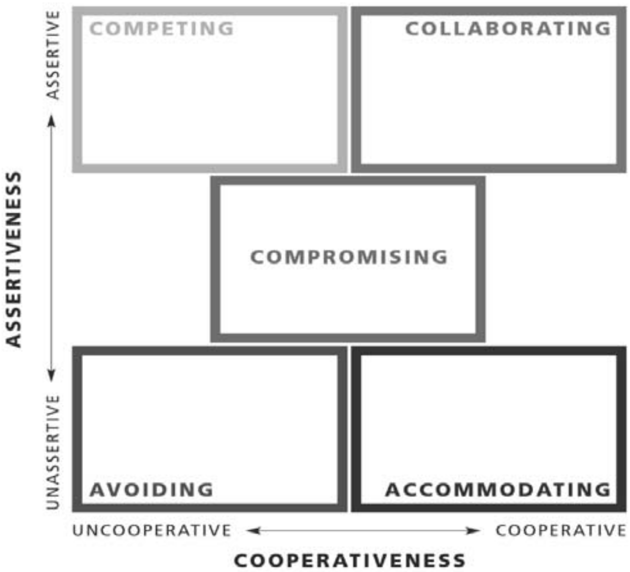


Figure 2. Thomas-Kilmann conflict modes

Source: (www 1).

The competing mode represents a high concern for self, low concern for others; collaborating – high concern for self and others, compromising – moderate concern for self and for others; accommodating – low concern for self and high concern for others, and avoiding – low concern for self and low concern for others.

Within each of 30 questions the responder chooses one of these two statements that describes their behavior better. The intensity of each mode for the responder is determined in the form of raw scores, as a total number of sentences corresponding to this mode chosen by the responder in the TKI test. Since each of these modes can be evaluated on a 0-12 scale (there are in total 12 sentences in the TKI test corresponding to each mode), the results may be represented as the percentage rates of the maximal possible scores. For instance, if the negotiator's raw scores are the following: competing − 6, collaborating − 11, compromising – 4, avoiding − 4, and accommodating – 5, we find collaborating as a leading mode, with very high intensity at the level of 92%. We would also call this person to be little compromising and avoiding at the level of 33%. The TKI results are, however, also interpreted in a relative way by comparing the responder's answers to the typical results obtained by other responders of similar profession or background that form a norm sample (Thomas et al., 2008). Such an interpretation is shown in Figure 3, where the same raw scores were compared to the reference group of 8,000 people preselected to ensure representative numbers of people by organizational level and race/ethnicity.

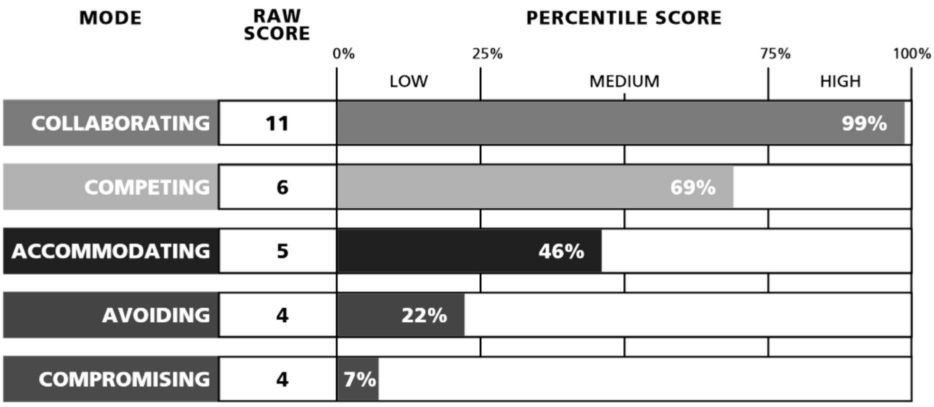


Figure 3. TKI percentile score for a preselected norm sample (reference group)

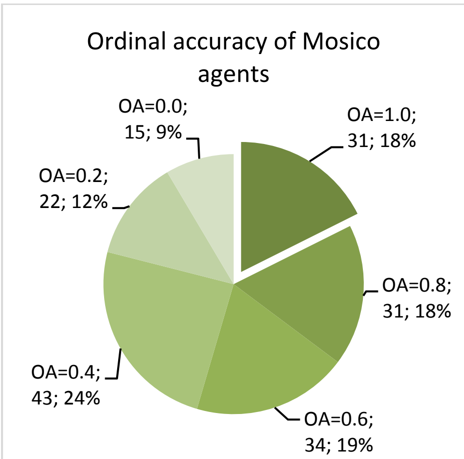
Source: (www 1).

In our experiment, however, the negotiation profile of each participant will be described by means of a non-relative numerical description of conflict modes represented in the form of raw scores, since − to the best of our knowledge − there is no experimental research that defines the reference percentiles for the intensities of the conflict modes for the international bachelor and master students.

3 Results

3.1 General findings on the inaccuracy of scoring systems depending on the agent's role

The results of the analysis of the scale of the inaccuracy in defining the scoring systems by the agents playing different roles confirm our earlier findings from the pilot studies (Roszkowska and Wachowicz, 2014; Kersten et al., 2015; Roszkowska and Wachowicz, 2015). Out of 176 representatives of WorldMusic (Mosico agents) only 31 (18%) were able to build the scoring systems that were fully concordant with the principal's (WorldMusic) structure of preferences, i.e. for which $OA = 1$. The percentage of fully accurate Fado agents (representatives of Ms. Sonata) was a little higher and equal to 22% (38 out of 174). Yet, the fraction test does not allow to reject the hypothesis on equal proportions of fully accurate Mosico and Fado agents ($p = 0.243$). The structures of ordinal accuracy indexes for representatives of both negotiation parties are shown in Figure 4.



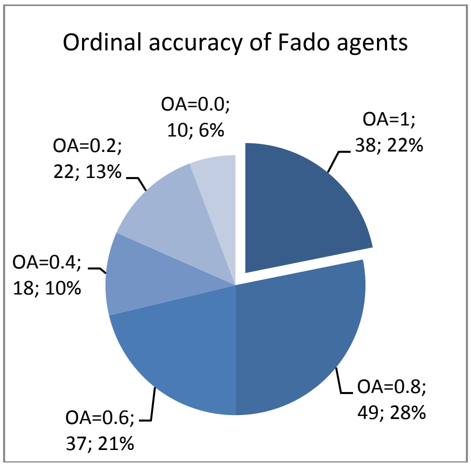


Figure 4. Structures of ordinal accuracy for Mosico and Fado agents

Despite the fact that the percentages of Mosico and Fado agents with $OA = 1$ do not differ significantly, the charts suggest the global difference in structure accuracy between the negotiation roles. Indeed, the chi-squared test determined for the contingency table representing the frequencies illustrated in Figure 4 allows to reject the hypothesis on equal structure of accuracy indexes for Mosico and Fado agents at $p = 0.007$. Hence, analyzing the charts we may conclude that Fado representatives were in general more accurate than Mosico agents.

The next issue we investigated while analyzing the general inaccuracy of agents' negotiation offer scoring systems, were the potential links between the ordinal accuracy (OA) and cardinal inaccuracy indexes (CIR and CIA). In other words, we aimed at discovering whether there is any relationship between the number of mistakes made at the ordinal level and the cardinal scale of these mistakes. To measure this relationship, the Pearson correlation coefficient was used (see Table 4).

Table 4: Relationship among OA, CIR and CIA indexes for Mosico and Fado agents

Pearson correlation coefficients	**Mosico**			**Fado**		
	CIR	CIA	OA	CIR	CIA	OA
CIR	1	.959**	-.708**	1	.924**	-.643**
CIA	.959**	1	-.602**	.924**	1	-.602**
OA	-.708**	-.602**	1	-.643**	-.602**	1

** Correlation significant at 0.01 (two-tailed).

The results shown in Table 4 confirm the existence of a very strong relationship between both cardinal inaccuracy indexes. This suggests that we could use only one of these indexes in our further analyses to make it more clear and the results easier to interpret. A strong negative relationship is also observed between OA and each of the cardinal inaccuracy indexes. The results are statistically significant and similar for both negotiation parties.

Knowing the strong relationship between CIR, CIA and OA and deriving from previous findings on different structures of ordinal accuracy for Mosico and Fado agents we may analyze the differences in average values of accuracy indexes for both roles at ordinal and cardinal levels. The results are shown in Table 5.

Table 5: Average accuracy and inaccuracy indexes for Mosico and Fado agents

Agents	**Indexes**		
	CIR	CIA	OA
Mosico	92.4	83.1	0.556
Fado	64.6	74.5	0.638
Significance (p)	0.000	0.089	0.012

The indexes' values confirm the previous findings on different structures of ordinal accuracy for the negotiation parties. The inaccuracy indexes are larger, while the ordinal accuracy index is smaller, for Mosico than for Fado agents. All the differences are significant at the level no worse than 0.089. Thus the question arises, what could influence such a difference in scoring systems accuracy between the negotiation roles in our case. To answer this question we analyzed first the detailed structures of accuracy in the representation of the principal's rank order for issues' weights and options shown in Table 6.

Table 6: Numbers of Mosico and Fado agents with accurate ratings for the subsequent elements of negotiation template

Agent	**Number (%) of agents with ratings concordant with the principal's rank order for:**				
	(1) options of no. of concerts	(2) options of no. of songs	(3) options of royalties	(4) options of contract signing bonus	(5) issue weights
Mosico	133 (76%)	51 (29%)	65 (37%)	120 (68%)	120 (68%)
Fado	135 (77%)	98 (56%)	125 (72%)	137 (79%)	60 (34%)

Cells in grey correspond to the elements of the negotiation template, for which the principal's preferences were non-monotonous.

As can be seen from Table 6, the frequencies of accurate ratings are not homogenous across all elements of the negotiation table and negotiation roles. Some elements of the negotiation template seemed to make bigger problems for agents with assigning concordant ratings than others. For three elements of the negotiation template the principals had defined non-monotonous preferences, i.e.: for options of "No. of concerts" (both principals), and for options of "Royalties" (WorldMusic) − marked in grey in Table 6. For these three elements the percentage of agents with correct ratings is lower (29-56%) than for all the remaining elements of template (68-79%), but one (Fado issue weights). These differences are statistically significant at $p = 0.000$, which was confirmed by the Cochran n-fraction test for dependent samples ($Q = 162.93$ for Mosico agents with 5 samples: elements 1–5; $Q = 42.49$ for Fado agents with 4 samples: elements 1-4). Simultaneously, the Cochran tests determined for both agents separately, and for elements described by monotonous preferences only (for Mosico – elements: 1, 4 and 5; for Fado – elements: 1, 3 and 4) did not allow to reject the H_0 hypothesis on equal fractions ($p = 0.115$ and $p = 0.084$ respectively). This allows us to confirm the following hypothesis: *The accuracy in defining the scoring system for a selected element of the negotiation template depends on the structure of preferences defined by the principal for this element, and is higher when the principal's preferences are monotonous, and lower for non-monotonous ones.*

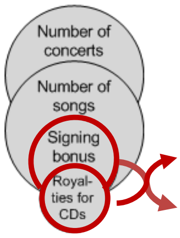
Yet, there is still a difference in rating accuracy between Fado and Mosico agents for the same element of the negotiation template, i.e. options of number of songs. The structures of the principal's preferences defined for this element by both agents are the same, but the agents' accuracies differ significantly. Thus, it seems there must be also other factors that influence this accuracy, related to the general psychological or demographical profile of the negotiators, that still need to be discovered and studied.

There is another element in the negotiation template for which the fraction of accurate ratings is also very low, i.e. the series of issue weights defined by Fado agents. Only 60 out of 174 (35%) of agents were able to assign the ratings that correctly represented Ms. Sonata's preferences. This is the lowest fraction for Fado agents and the McNemar test confirms the significance of differences in fractions of correct ratings between issue weights and other elements of the negotiation template ($p = 0.000$). However, it is not an issue of non-monotonous preferences that could play a role here. For this element of the template an order of issues described in preferential information by the principal was different than an order used in step 1 of the SAW-based preference elicitation procedure. More precisely, in preferential information the issues were described in the following order: "No. of concerts", "No. of songs", "Contract signing bonus", "Royalties" (as shown in Figure 1), while in the preference elicitation process the two least important issues were reversed in order, i.e. "Royalties" came before "Contract signing bonus" in the list (see Figure 5).

Importance of the four issues:

- You asked Ms. Sonata to think aloud the importance of issues. She said that this is quite easy, every issue is important to her. But, she added, she really does not want to have too many **promotional concerts**, so it is very important for her that she has as few concerts as possible.
- Ms. Sonata says that she must write as many **new songs** as she can, because this is her only way to enrich her fans. This issue of new songs is equally important to the first issue, promotional concerts.

…



Issue	Rating
Number of promotional concerts (per year)	38
Number of new songs	38
Royalties for the CDs (% of revenue)	10
Contract signing bonus ($)	14

Preferencial information SAW-based rating procedure

Figure 5. Differences in issue lists – preferential information vs. SAW-based rating procedure

It appears that such a technical change in listing the template elements subjected to evaluation had a stronger impact on rating accuracy than non-monotonous preferences. Here, however, it is not a specificity of preferences, but rather the agents' oversight that made the potential difficulties in accurate mapping of the principal's preferences. This allows us to formulate a general hypothesis for future research, the confirmation of which requires a deeper and more detailed analysis, that *there could be some user-specific heuristics and perception errors that affect agents' accuracy in defining their individual negotiation offer scoring systems*.

Summarizing the above results, we find that the problem of determining an accurate scoring system by agents is quite common, and hence decided to find whether the conflict/negotiation profile can differentiate among the agents with respect to the scale of scoring system inaccuracy.

3.2 Comparing profiles of Fado and Mosico agents

In the next stage our analysis was focused on the comparison of the differences in profiles of our negotiators depending on the role they were playing (Mosico or Fado). Because of the size of our research sample we could not consider different profiles taking into account the full range of potential results for each conflict mode (a 0-12 scale). We grouped the results into three classes depending on the intensity of each conflict mode: (1) low – raw scores from 0 to 4; (2) medium – raw scores from 5 to 8; and (3) high – raw scores from 9 to 12. Figures 6 and 7 show the structures of conflict modes for Mosico and Fado agents, respectively.

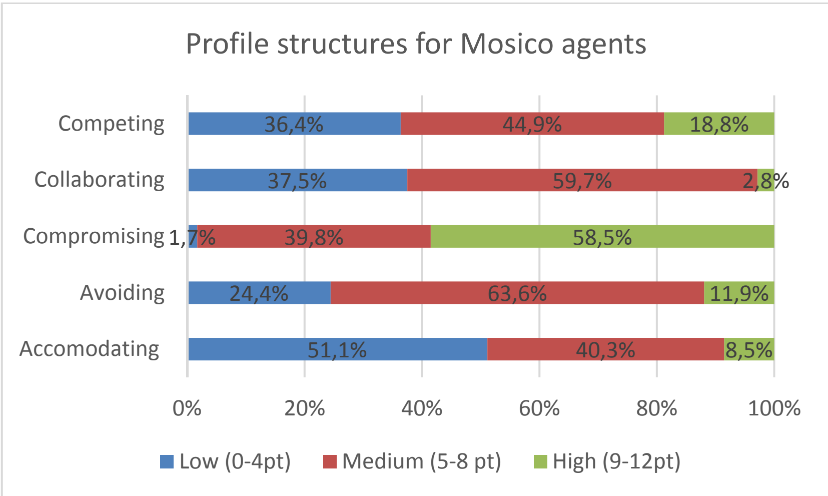


Figure 6. The levels of conflict modes intensities for Mosico agents

Comparing the structures of profiles of Mosico and Fado agents we find that for both parties the compromising mode was the most intensive. For more than 97% of agents this mode was medium or high. The fraction test confirms that the structure of compromising mode is similar for both agents ($\chi^2 = 3.48$, for two degrees of freedom, $p = 0.175$) and significantly different from the structures of other conflict modes.

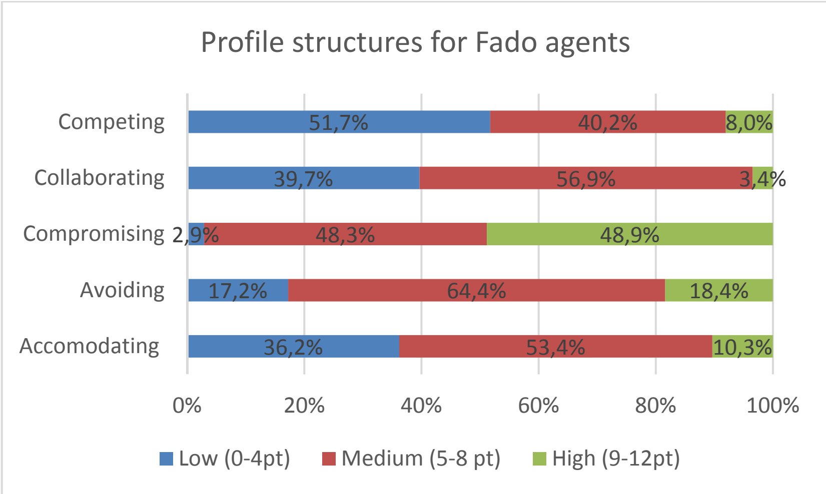


Figure 7. The levels of conflict modes intensities for Fado agents

These observations are confirmed by the analysis of the profiles conducted at the level of average profile values for both agents. The Kolmogorov-Smirnov test confirms that the distribution of intensity of each conflict mode for each agent separately is normal ($p < 0.005$), hence the parametric t-Student test can be applied to measure the significance in differences of average mode values within the profiles. The average profiles are shown in Figure 8.

As can be seen from Figure 8, Fado agents are lower in competing and compromising modes, and higher in avoiding and accommodating modes than Mosico agents. The differences for all these four modes are significant for $p < 0.038$. There is only one mode in both profiles that refers to the collaborating behavior, which does not differentiate significantly between the agents. The t-Student test does not allow to reject the hypothesis on equal average scores for the collaborating mode for Mosico and Fado agents at the level $p = 0.388$.

It is worth noting that competing and compromising modes are the ones that indicate the negotiator's medium and high assertiveness, while avoiding and accommodating ones are characteristic to unassertive decision makers (see Figure 2). Hence, we decided to perform the confirmatory factor analysis using the raw scores of conflict modes to find whether it is possible to determine a single factor describing the assertiveness level across the whole sample. As the result we obtained the following table of loadings that allow us to interpret the factor as the "unassertiveness level" (see Table 7).

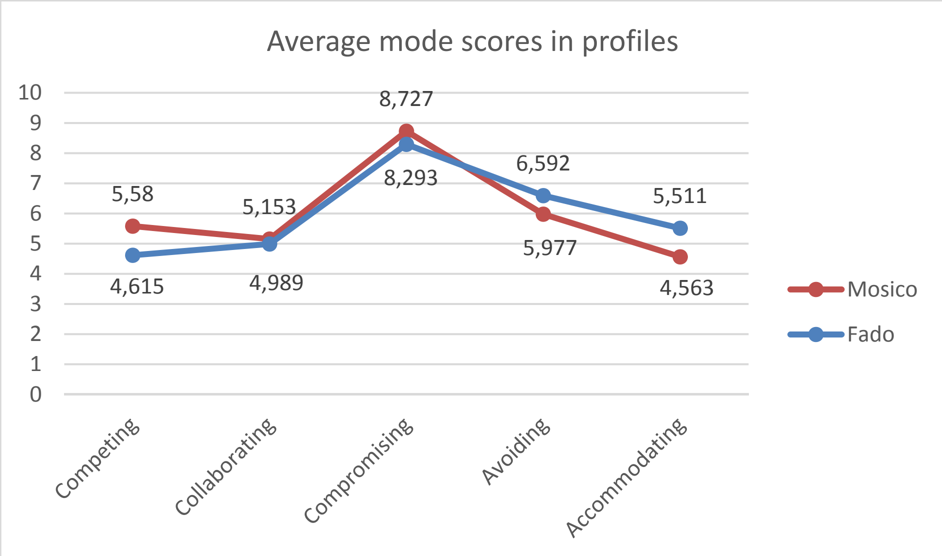


Figure 8. Average negotiation profiles for Mosico and Fado agents

Table 7: Modes' loadings to a single factor

Modes	**Factor 1** "unassertiveness"
Competing	-.837
Collaborating	-.470
Compromising	–
Avoiding	.573
Accommodating	.701

In Table 7 only the loadings greater than 0.3 are shown. They confirm our interpretation of the average profiles from

Figure 8 and the potential impact of selected modes on the intensity of unassertive behavior of agents. In our experiment high competing and collaborating scores load negatively to the unassertiveness level (yet, the collaborating mode loads distinctly less than competing), while high avoiding and accommodating ones load in a positive way.

Using the regression approach we determined the factor values for all agents in our experiment and then calculated the average unassertiveness level for Mosico and Fado agents, which are equal to -0.201 and 0.203, respectively. They seem to be good aggregates of the profiles presented in Figure 8, since Mosico agents were higher in competing and compromising, and lower in avoiding and accommodating, which we interpreted as more assertive behavior. For Fado agents the relation between modes and the unassertive factor is converse. The difference between average unassertiveness levels for Mosico and Fado agents is significant for $p = 0.000$.

On the basis of the average profile analysis conducted in this section, which leads us to the conclusion that the agents differ significantly in the structure of negotiation profiles, and general ordinal accuracy of scoring systems analyzed in section 3.1, that confirmed differences in structures of OA indexes for the agents, we may expect that the negotiation profiles (or at least some of their elements or aggregates, such as the level of unassertiveness) influence the general ordinal accuracy of scoring systems determined by the agents.

3.3 Conflict modes, profiles and scoring systems accuracy

In order to find the direct links between the intensities of conflict modes and the accuracy of scoring systems we determined the Pearson correlation coefficient among the raw scores of modes, unassertiveness level, OA, CIR and CIA separately for each negotiation role. The results are shown in Table 8.

Table 8: Pearson coefficients for accuracy indexes and conflict modes

Conflict mode	Mosico			Fado		
	CIR	CIA	OA	CIR	CIA	OA
Competing	.014	-.009	-.022	.123	.121	-.007
Collaborating	.023	.018	-.053	-.085	-.081	.045
Compromising	-.209**	-.193*	.134	-.105	-.127	-.039
Avoiding	.038	.044	-.011	-.094	-.093	.078
Accommodating	.099	.132	-.030	.078	.094	-.058
Factor 1: unassertiveness	.018	.039	.022	-.047	-.042	-.011

* Correlation significant at 0.05 (two-tailed).
** Correlation significant at 0.01 (two-tailed).

The results reveal no correlation or very weak correlation between selected conflict modes of agents and the selected accuracy measure of the scoring systems they built. The highest relationship was observed for the raw scores of the compromising mode and cardinal inaccuracy indexes only for Mosico agents. This relationship is weak, yet statistically significant at $p = 0.01$ for CIR and $p = 0.05$ for CIA. More precisely, we could conclude that being more likely to compromise results in higher inaccuracy in the scoring system determined by Mosico agents. Unfortunately, such a relationship is not confirmed for Fado agents. Consequently, we are not able to build any convincing regression model that would explain the relationship between the series of conflict modes and the final concordance of the agent's scoring system with the preferential system declared by the principal. Hence, we may hypothesize that there may be some other demographical or sociological characteristics that may affect the negotiators' ability to determine an accurate scoring system. Unfortunately, the experimental data we gathered do not allow us to perform such an in-depth analysis of the demographical profiles of the experiment participants.

Being unable to find the direct links between raw scores of a mode and accuracy levels we performed the cluster analysis for the whole negotiation profiles described by means of a series of all conflict modes in a profile for our further analysis. We applied the k-means clustering procedure, for which an optimal number of clusters was determined as a result, with an authorial approach for measuring the classification quality. We aimed at determining the smallest possible number of homogenous clusters with respect to the negotiation profiles of our agents. Hence, the clustering evaluation procedure required analyzing various classifications obtained for consecutive numbers of classes (starting from the smallest possible, i.e. two) and determining for each classification results the Kruskal-Wallis test to measure, if the distribution of mode raw scores within each conflict mode is significantly different within each of the clusters (for $p < 0.05$). The first classification that meets such requirements was "optimal". For both agents the optimal number of clusters was five. The clustering results for Mosico and Fado agents are shown in Tables 9 and 10, respectively.

Table 9: Average profiles and accuracy indexes for Mosico clusters

Cluster	Average profile					OA	CIR	CIA
	Competing	Collaborating	Compromising	Avoiding	Accommodating			
Cl1 (N = 40)	8.300	4.400	7.700	7.000	2.600	0.595	80.884	72.290
Cl2 (N = 40	5.500	3.900	8.800	5.100	6.700	0.550	102.521	92.873
Cl3 (N = 44)	2.114	5.045	8.886	7.818	6.136	0.477	104.008	95.588
Cl4 (N = 19)	9.211	7.684	7.526	2.474	3.105	0.474	97.701	83.103
Cl5 (N = 33)	4.909	6.273	10.364	5.364	3.091	0.667	75.376	67.687
K-W sign.	.000	.000	.000	.000	.000	.056	.032	.087
All Mosicos	5.58	5.15	8.73	5.98	4.56	.556	92.365	83.097

K-W sing. – p value for the Kruskal-Wallis test.

Table 10: Average profiles and accuracy indexes for Fado clusters

Cluster	Average profile					OA	CIR	CIA
	Competing	Collaborating	Compromising	Avoiding	Accommodating			
Cl1 (N = 11)	11.091	5.364	5.909	4.545	3.091	0.618	74.394	82.101
Cl2 (N = 42	4.286	4.643	9.119	8.333	3.619	0.629	55.634	63.603
Cl3 (N = 53)	1.660	5.000	8.094	7.189	8.057	0.619	65.415	76.909
Cl4 (N = 43)	5.302	5.698	9.395	4.465	5.140	0.651	67.576	77.583
Cl5 (N = 25)	7.400	4.160	6.480	6.960	5.000	0.680	68.595	79.054
K-W sign	.000	.003	.000	.000	.000	.876	.954	.858
All Fados	4.61	4.99	8.29	6.59	5.51	.638	64.613	74.500

K-W sing. – p value for the Kruskal-Wallis test.

From section 3.2 we know that for the average Mosico and Fado profiles a higher accuracy of scoring systems was observed for Fado agents (see last rows in Tables 9 and 10). Now we try to find whether the accuracy varies for different clusters of profiles within each group of agents. The Kruskal-Wallis test confirms the differences in accuracy levels for Mosico agents to be significantly different across all clusters for $p < 0.087$. Yet, the differences in accuracy do not differ significantly for Fado agents ($p > 0.858$). What is more, it is not easy to identify the cluster of negotiators with highest scoring system accuracy within each group of agents. For Mosico Cl5 seems to have the highest OA index and the lowest CIA and CIR values. Yet, the Mann-Whitney test does not confirm these values to differ significantly from the ones determined for Cl1 ($p < 0.288$), that seems to be the second best. The situation is even worse in the case of Fado agents. Here there is no single cluster that outperforms others with respect to OA, CIA and CIR simultaneously. Since the differences in OA values are really insignificant, we may try to identify the cluster with best accuracy using CIA and CIR values only. Using this rationale Cl2 will be considered as best. Yet, the situation is similar to the one we encounter in the case of Mosico agents, when we compare Cl2 of Fado agents with the second best accurate cluster, i.e. Cl3. The Mann-Whitney test will not allow to consider these two clusters as significantly different with respect to CIA and CIR values. Hence, our comparison of the most accurate clusters of Mosico and Fado agents presented below is only illustrative.

Comparing the average profiles of Mosico Cl5 and Fado Cl2 we find that no general conclusion may be drawn regarding a single profile that is most likely to generate the most accurate scoring systems, yet some regularities may be indicated for further investigation (see Figure 9).

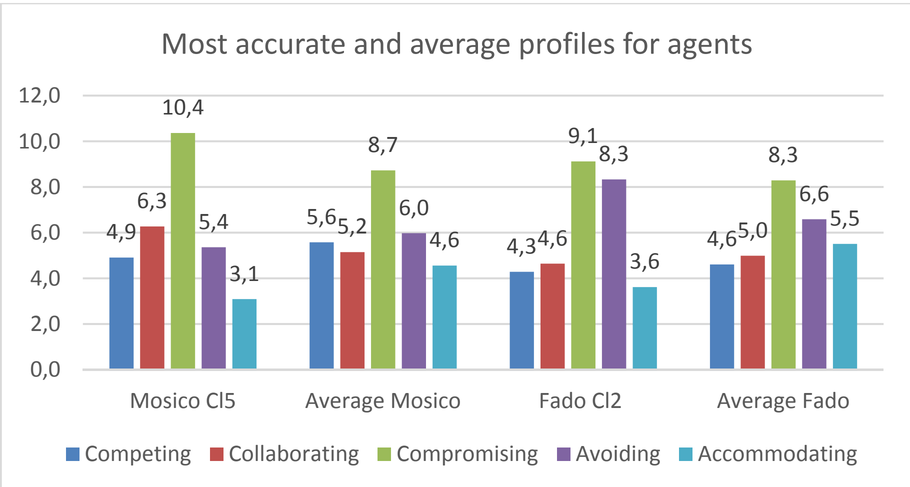


Figure 9. Comparison of most accurate (Mosico Cl5, Fado Cl2) and average profiles

The most accurate profiles of both agents are characterized by the competing level slightly lower than the average within each role and second lowest in each group. No univocal conclusions may be drawn with regard to the collaborating mode. The most accurate profile of Mosico agents is characterized by second highest raw rate of collaboration across all Mosico agents (higher than average). The situation is opposite for Fado agents. The profile of the most accurate Fados is characterized by second lowest collaborating mode, lower than average within the Fado group. Both accurate Mosico and Fado profiles have a very high level of compromising, which for Mosico is the highest across all profiles, and for Fado, second highest. The most accurate Mosico and Fado profiles differ entirely with regard to the level of the avoiding mode. They are, however, second lowest with respect to the accommodating mode, significantly lower than the average for each of the roles.

3.4 Clustering the agents with respect to accuracy indexes

In the last stage of our analysis we changed the perspective used previously in analyzing the relationship between the negotiation profiles and scoring system accuracy. We decided to conduct a more general analysis using the whole dataset without the distinction between the roles and the accuracy measures introduced. Therefore we decided to build a single inaccuracy measure (SIM) that would combine all three indexes: OA, CIA and CIR. We used exploratory factor analysis with regression-based aggregation of factors to determine the potential number of factors and loadings values with an eigenvalue threshold equal to 1 as a discriminant value for the final factor number and the varimax rotation. This analysis proved that the factor model is best fitted for only one factor and allows to explain 84% of the variance measured by three variables considered in the analysis. The loading values of OA, CIA and CIR calculated by means of the principal component method are equal to: -0.846; 0.953 and 0.952, respectively. Hence, the higher the SIM value the bigger inaccuracy of the scoring system under consideration.

Having determined the SIM values for the scoring systems of all experiment participants we identified three classes of participants that differ significantly with respect to SIM values using two-step cluster analysis and Bayesian Information Criterion. For each cluster we calculated the average profiles and SIM values (see Table 11).

Table 11: Average profiles and accuracy for three clusters of negotiators

Negotiators	Average mode values in profiles					Average assertiveness level	Average *SIM* value
	competing	collaborat-ing	compromis-ing	avoiding	accommo-dating		
Highly accurate (N = 188)	4.99	5.16	8.54	6.35	4.96	.003	-.720
Medium accurate (N = 105)	5.01	4.94	8.79	6.33	4.92	.034	.303
Little accurate (N = 57)	5.61	5.02	7.91	5.98	5.47	-.077	1.816
K-W significance	.440	.537	.050	.511	.255	.888	.000

As can be seen from Table 11, there are again no significant differences among most of conflict modes for the profiles described as highly, medium and little accurate. The only mode for which the difference can be considered as significant at p = 0.05 is compromising. Yet, it is difficult to draw unambiguous conclusions out of the average values of this mode. It seems that highly compromising negotiators (average raw score of 7.91) are on average less accurate than others. However, the highest intensity of the compromising mode (8.79) does not lead to the most accurate scoring system. It is a medium level of 8.54 that describes the negotiators of highest accuracy in defining the negotiation offer scoring systems. This confirms in some way the previous findings for individual agents (see Figure 8), where Fado agents, being more accurate than Mosico ones, were less compromising, but still at the average level above 8.00. Similarly, there are no significant differences between the clusters with respect to the general assertiveness levels.

4 Summary and conclusions

In this paper we tried to analyze the scale of inaccuracy in defining the scoring systems by the negotiator and its potential links with their negotiation profile, describing the negotiators' attitude and behavior in conflict situations. In our analyses, we used the dataset of bilateral electronic negotiations conducted in the Inspire system, for which a predefined negotiation problem was defined (the Mosico-Fado case). Within the negotiation problem applied, the agent-principal context was embodied, and the preferences of the principal were clearly described both verbally and graphically. Despite such a detailed preferential information, the students that played the roles of Mosico and Fado agents appeared to be relatively inaccurate in defining their scoring systems. Less than one third of all agents built their scoring systems in complete concordance with the principal's preferences (i.e. with $OA = 1$). We observed, however, that the accuracy differed with respect to the agents' roles. Fado agents (the buyers) were on average more accurate than Mosico agents (the sellers). The difference in accuracy seemed to be linked to the structure of the principal's preferences, i.e. non-monotonous preferences made bigger problems for agents to handle them accurately. The effect of heuristic thinking (fast thinking, not paying attention to differences in issue lists) has also affected the ordinal accuracy of assigning the issue weights. What is interesting, the average profiles of both agents also differed significantly. Fado agents, being more accurate in building their scoring system, were also less assertive than Mosicos, i.e. they had lower levels of competing and compromising modes and higher levels of avoiding and accommodating behaviors.

Unfortunately, the in-depth analyses of both the whole dataset and the agent's subsamples did not lead us to any further binding conclusions. The correlations among accuracy indexes and conflict modes appeared to be very weak; hence, it was impossible to build any regression model that would be able to describe the relationship between the negotiators' profiles and their accuracy at the satisfying level of determination and significance. Even though we succeeded in clustering the agents into classes of significantly different profiles, we were unable to prove that these classes differ significantly with respect to the scoring system accuracy, no matter which notion of accuracy was used. A converse approach that amounted to clustering the agents with respect to a single inaccuracy measure did not lead to a better explanation of the problem. It allowed only to formulate a conclusion on the desired level of compromising mode required to determine the most accurate scoring systems. The negotiators with intermediate level of compromising behavior were also the most likely to build the most accurate scoring systems. This general conclusion was also confirmed partially by correlation analysis, where for the Mosico party the compromising mode was the only one that was significantly (yet, weakly) correlated with the selected accuracy measure.

We need to emphasize that the findings and general results we obtained from the experimental analysis are focused on the enriching of the general knowledge on the use and usefulness of the decision support tools applied in negotiation support and the potential factors that influence their use and usefulness rather than on providing any additional support directly to the negotiation parties (asymmetric negotiation support) in the negotiation process. Usually, the parties do not know each other so well or are unable to investigate the profiles of the counterparts based on public information to be able to determine the detailed negotiation profile of their counterparts and derive from them additional information on their accuracy and the potential misinterpretation of the negotiation moves and concessions made. The information about the negotiation profiles of both parties is confidential and may be accessible only by a third party, such as a negotiation support system or a mediator. These third parties can use it to model the best ways, methods and tools to support the negotiation parties in the best possible way, taking into account the negotiators' cognitive limitations, skills and expert knowledge. The last one is actually a part of our ongoing project, and the research presented in this paper was focused only on selected behavioral issues that can be studied when analyzing the general profile of the parties.

As the initial results confirmed the differences in accuracy depending on the role the participants played in our experiment, this may suggest that there are other characteristics of the negotiators that may have an impact on their accuracy in determining the scoring systems, different from the ones described in the TKI test. There may be some demographical or sociological characteristics or also background issues (such as educational level or field) that may affect the results. Therefore, in our future research we will conduct an exploratory analysis of other potential factors that could be used to describe the negotiator's profiles in a different way. We will investigate the applicability of other tests, such as Rational-Experiential Inventory (Handley et al., 2000) or Scott-Bruce (Scott and Bruce, 1995) tests that allow to measure the decision making profile of the respondents.

Acknowledgements

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A MODEL TO SUPPORT NEGOTIATIONS ON THE ELECTRICITY MARKET

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Abstract

Liberalization on the electricity market in Poland is related to the possibility of free choice of electricity supplier. On a liberalized market, suppliers have to compete to gain new customers and retain the old ones. The suppliers have to satisfy the customers' needs − which are more and more complex − and customize their approach. Therefore, negotiations of electricity sale conditions become an usual practice.

The purpose of this study is to propose a way of supporting the negotiation process of electricity sale conditions between a supplier and a customer. To solve this problem, the scoring method has been used.

Keywords: electricity, active and passive negotiation support, negotiation offer evaluation system, scoring method, SAW.

1 Introduction

Nowadays we see a progressing process of liberalization on the electricity market in Poland. According to the principle of TPA (Third Party Access), starting with 2007 every consumer of electricity in Poland can freely choose the supplier. On the other hand, suppliers of electricity have to compete with each other to gain new consumers and retain the old ones. For this reason, negotiations of conditions of offers to sell electricity become a common practice.

The purpose of this study is to present a proposal for supporting the negotiation process of conditions of electricity sales between a supplier and a consumer. A sample negotiation problem from the electricity market is presented.

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The paper consists of few main sections. First few sections are the introduction of basic concepts related to the electricity market, the negotiation problem, and the system of evaluation of negotiation offers. There's also a description of the algorithm of the method used to construct such a system, which is the scoring method.

The next sections contains the description of a sample application of the scoring method on the electricity market to the construction of an offer evaluation system. The section contains also the description of assumptions regarding the elements of the negotiation problem and the evaluation of negotiation offers. Furthermore, examples of passive and active negotiation process support are presented, together with the author's proposal of suggesting a non-dominated negotiation compromise.

Passive negotiation support means that information on the negotiation process in progress is elaborated and visualized, without suggesting any solutions. Active support, on the other hand, is related to a recommendation of negotiation compromises (Wachowicz, 2013).

The last section is a summary with conclusions and future research directions.

2 Problem formulation and a system for negotiation offer evaluation

Negotiation analysis was introduced in the 1980s, when a formal description and assumptions of a negotiation process were suggested. This description became a basis for the construction of models describing negotiation processes (Raiffa, 1982; Kopańska-Bródka, Wachowicz, 2013). Nowadays, to support the negotiation process, tools based on the following are used:

- game theory (Brams, 1990, cited in: Kopańska-Bródka, Wachowicz, 2013);
- decision-making theory (Raiffa et al., 2002, cited in: Kopańska-Bródka, Wachowicz, 2013);
- mathematical programming (Kersten et al., 1991, cited in: Kopańska-Bródka, Wachowicz, 2013).

Selected papers dealing with negotiations on electricity markets focus on the problem of automation of this process through the notion of multi-agent systems with defined negotiation strategies and bidding rules (Kaleta et al., 2009; Brazier et al., 2002).

First, we have to define three basic categories related to the electricity market and used in the present paper. An energy supplier is a business entity (an energy-trading company) which sells and buys energy on the market. In this paper, the role of the market is played by the Polish Energy Exchange (PEE)[1]. The cus-

[1] Detailed information on the PEE is available on the web page www.tge.pl.

tomer purchases energy for further resale, and is interested in purchasing a forward contract for the supply of electricity. The customer is not a direct participant in the market, and that is why he/she wants to purchase from a supplier.

The negotiation problem consists of three main elements: negotiation issues, together with the levels of their implementation, and variants of agreement. Given the set G of J negotiation issues $G = \{g_1, g_2, \ldots, g_J\}$, $j = 1, \ldots, J$, with the levels of implementation defined for each issue, we can also define the set of agreement variants $A = \{a_1, a_2, \ldots, a_I\}$, $i = 1, ..., I$. The set A consists of vectors of implementation levels of the negotiation issues from the set G: $a_i = [x_{i1}, x_{i2}, \ldots, x_{iJ}]^T$[2].

The negotiation issues in the classical decision problem represent the criteria, while the variants of agreement correspond to decision variants. The elements of a negotiation issue are the subject of discussions and are determined by both parties during the pre-negotiation stage.

The negotiation problem thus defined can undergo further evaluation. The evaluation can be related to negotiation issues, levels of their implementation, and − as a consequence − agreement variants. As a result we obtain a system of negotiation offer evaluation with a defined vector of weights of negotiation issues[3] $w = [w_1, w_2, \ldots, w_J]^T$ and a vector of evaluations of all agreement variants $V = [v(a_i)]^T$, $i = 1, \ldots, I$ (Wachowicz, 2013).

The system of negotiation offer evaluation enables each negotiator to sort out the information on the problem. In this system, the negotiators' preferences are presented explicitly as evaluations and weights. Finally, on the basis of this system both passive and active negotiation process support is made possible at the stage of actual negotiations.

Moreover, such a system allows for a quick and unique evaluation and comparison of negotiation offers; it also allows to preserve the negotiator's rational way of thinking. A negotiation offer evaluation system makes it also possible to justify the decision to suggest the next offer in response to the moves of the other party (Simoms, Tripp, 2003, after: Wachowicz, 2013).

The offer evaluation system is constructed separately by each negotiator at the pre-negotiation stage. To construct such a system we can use the scoring method.

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2 These are not the only elements of the problem indicated in the literature, cf. Wachowicz (2013).

3 Not all methods of construction of offer evaluation systems require the evaluation of how essential a given negotiation issue is. An example is the ELECTRE TRI method, see Roy (1971), after: Wachowicz (2013).

3 The scoring method

The scoring method (Trzaskalik, ed., 2014) is a modification of the Simple Additive Method (SAW) and is used to construct systems of negotiation offer evaluation. In the literature it is pointed out that other multi-criteria methods can be used, such as BIPOLAR, VIKOR, TOPSIS, AHP and their later modifications (see Keeney, Raiffa, 1976; Saaty, 1980; Hwang, Yoon, 1981; Konarzewska-Gubała, 1989; Opricovic, 1998; Wachowicz et al., 2012). The SAW method is one of the simplest and most often used multi-criteria methods. It was introduced by Churchman and Ackhoff in 1954 (Churchman, Ackhoff, 1954, after: Trzaskalik, ed., 2014). Its algorithm consists of three steps:

1. Assignment of the weights w_j [0, P] to the negotiation issues[4]. The most essential issues receive the highest weights. We also assume that $\sum_{j=1}^{J} w_j = P$.
2. The individual evaluation of k implementation levels within the negotiation issues $v_j\left(x_j^k\right) \in \left[0; w_j\right]$. The most essential levels receive the highest evaluations.
3. Calculation of the global evaluations of agreement variants based on the multi-attribute value function $v(a_i) = \sum_{j=1}^{J} v_j\left(x_j^{a_i}\right)$. The most favorable variants receive the score P, while the least favored ones, the score 0 (Wachowicz, 2013).

4 An example of a negotiation issue

A customer wants to purchase energy in the form of forward contracts for further resale. Moreover, he/she wants to actively manage the purchase price of his/her volume. That is why the customer decided to negotiate a dynamic purchase, that is, one where he/she decides when and what share of volume to purchase by submitting a purchase order to the potential supplier. The total power of contracts within this dynamic purchase is 10 MW for baseload supply (BASE–Y_15) and 4 MW for peakload supply (PEAK5–Y_15)[5].

For the purpose of this paper we make certain assumptions as to the elements of the negotiation problem and the offer evaluation system.

[4] In the scoring method, points are assigned as weights and evaluations. The value P is determined by the customer and the supplier separately. Usually, P is set to 100 which means that weights are assigned as points from the interval [0, 100]. One should note that evaluations from the range between 0 and 100 are assigned also in the SMART method.

[5] Definitions of forward contracts for baseload supply and peakload supply can be found in Kudyba (2014).

The individual negotiation issues with their weights and the implementation levels with the scores assigned do not change during the negotiation. Otherwise, the set of non-dominated compromises would change in time. If during the negotiation the negotiators decided to include a new issue into the discussion, it would mean that the offer evaluation system would have to be constructed anew.

The negotiators do not have information on weights and scores of their opponents. If a negotiator had information on the preferences of his/her opponents, he/she could influence the choices of the other party's consecutive offers. Such a situation is undesirable, since it is not in agreement with the general principles of conducting business.

It is assumed, however, that the information on weights and scores of both negotiators can be voluntarily passed on to a third party (an arbitrator) and used only to support the negotiation process.

5 Problem definition

In the pre-negotiation stage the parties agreed as to the following issues:

- maximal power of the standard product ordered in a single purchase order;
- pricing method or the method of setting the purchase price on the market at the time of purchase;
- mark-up of the supplier who fulfills the contract.

The parties agreed that one purchase order can include the purchase of both products at the same time[6]. For instance, if a 5 MW volume is selected for both products, the customer receives 5 MW of baseload power and 4 MW of peakload power. The next orders will concern baseload supply only. The customer cannot contract for more than 10 MW of baseload power and 4 MW of peakload power. It has been agreed that the power interval will be negotiated in the range from 1 MW to 10 MW. The essential options within this negotiation issue are 1.5 MW and 10 MW.

The second negotiation issue concerns the method of setting the purchase price of the volume in each purchase order, which is a qualitative issue. The price itself is of course a strictly quantitative issue, but this issue consists in the method of agreeing on the price, and not in its specific value. The parties agreed to discuss the following proposal:

- the settlement occurs according to the price of sale offer posted on the exchange quotation board at the time when the decision of purchase is made;

[6] That is, contracts denoted BASE-Y_15 and PEAK-Y_15.

- the settlement occurs according to the average price of transactions on the PEE before 12:00; the customer can submit the purchase order by 12:30 on the same day;
- the settlement occurs according to the average price of transactions on the PEE on the day preceding the submission of the order; the customer can submit the purchase order by 8:00 on the following day;
- the settlement occurs according to the average price of transactions on the PEE on the day preceding the submission of the order; but the customer can decide to submit the order during the entire day following the publication of the average transaction price.

One should note that in this negotiation issue there are no intermediate options.

The last issue to be negotiated is the supplier's profit margin. The margin suggested by the supplier is the markup on the liquidity risk related to the volatility of prices on the market for products being ordered. When a customer submits a purchase order of a volume, the supplier is not always able to buy the product immediately, and price quotations change in time. Hence the margin as a markup for the liquidity risk calculated on the basis of price volatility.

The analysis of price volatility concerned BASE–Y_15 and PEAK5–Y_15 products. To measure volatility, standard deviation of the logarithmic returns was used. The supplier assumed a uniform risk calculation as the average of markups for base and peakloads, weighted by the joint power of standard products.

Daily return volatility for the BASE–Y_15 product is 0.47%, which gives the markup of 0.79 PLN/MWh assuming the settlement rate of 169.37 PLN/MWh. Analogously, the return volatility for the PEAKS–Y_15 product is 0.64%, which gives the markup of 1.40 PLN/MWh assuming the settlement rate of 202.02 PLN/MWh. The average markup weighted with the total volume of orders is 0.88 PLN/MWh[7].

On the basis of liquidity analysis the supplier obtains information on the number of days needed to purchase the given power, depending on the settlement methods selected. The number of days taken into account in the calculation of the markup for risk depending on the settlement option selected and the volume ordered is presented in Table 1. The values of markup for 1-, 2-, 3-, and 4-day risk are 0.88, 1.25, 1.53, and 1.77 PLN/MWh, respectively.

On the basis of the calculations of each markup the supplier knows that selecting the fourth method of settlement and the maximal power of the order of 10 MW, the markup for the service equal to 1.77 PLN/MWh should protect

[7] One should note that the daily mark-up was calculated on the basis of daily price volatility, while mark-ups for longer periods will be calculated using the square root principle, described in Marcinkowska (2009).

him/her from price volatility. For the purpose of the negotiation process, these amounts have been rounded up to integers. The supplier's margin will be fixed as a value from the interval between 1.00 and 2.00 MWh with the accuracy of 0.01 PLN/MWh. The essential options in this case are 1.00, 1.50, and 2.00 MWh[8].

Table 1: Number of days for the calculation of markup for volatility risk depending on the option

***t*-day markup**	**Settlement 1**	**Settlement 2**	**Settlement 3**	**Settlement 4**
1 MW	1	2	2	3
2 MW	1	2	2	3
3 MW	1	2	2	3
4 MW	1	2	2	3
5 MW	1	2	2	3
6 MW	2	3	3	4
7 MW	2	3	3	4
8 MW	2	3	3	4
9 MW	2	3	3	4
10 MW	2	3	3	4

Source: Author's own elaboration.

6 Construction of systems of negotiation offer evaluation

After having agreed on negotiation issues and options within each issue, both parties begin to prepare their systems of negotiation offer evaluation. It is assumed that the threshold value *P* is 100 points. The evaluations assigned to the essential options are listed in Table 2[9].

Table 2: Evaluations of the individual options assigned by each party in the negotiation process

Negotiation issue	**Essential option**	**Supplier's evaluations**	**Customer's evaluations**
Maximal volume in a single order	1 MW	**40**	0
	5 MW	30	20
	10 MW	0	**30**
Settlement method	Settlement 1	**30**	0
	Settlement 2	25	10
	Settlement 3	5	30
	Settlement 4	0	**40**
Supplier's profit margin	2,00	**30**	0
	1,50	11	25
	1,00	0	**30**

Source: Author's own elaboration.

8 Evaluations of the intermediate options (between the essential options) are based on linear approximation.

9 Boldface denotes the weights assigned to each negotiation issue.

The issue of power in the order submitted: The customer assessed the importance of this issue to be 30 points, while the supplier 40 points. From the point of view of the customer a high power of the order increases the chances of flexible management of purchase cost: the higher the power, the greater the chance of purchasing the entire volume in shorter time. The rate of increase of the score for powers from 1 MW to 5 MW is greater: it is equal to 20 pts, while the increase of score for powers from 5 MW to 10 MW is equal to 10 pts. A smaller power in a single order means less problems for the supplier with the purchase on the market and that is why the supplier evaluates options with lower powers higher.

The issue of the settlement method: The most important issue for the customer (its weight is 40 pts) who evaluates highest a settlement based on the average price over the transactions from day $t-1$ (also 40 pts). In this case the customer has the opportunity to watch the prices on the next day, and therefore to check whether the current price tendency is favorable for him/her or not. The least preferred is the first method since it is tied to the current price and does not allow to forecast its further increase or decrease. For the supplier, the importance of this issue is 30 pts, and the preferred settlement method is the first one, although the second one is also acceptable. On the other hand, the third and fourth methods are evaluated much lower. What is appealing in these two methods for the customer is at the same time less so for the supplier.

The issue of the supplier's margin is just as essential for the supplier as the issue of settlement. The highest score is assigned to the highest profit margin, that is, 2.00 PLN/MWh. This score decreases to 11 pts for the margin of 1.50 PLN/MWh and is 0 pts for the margin of 1.00 PLN/MWh. For the customer the margin is as essential as the power ordered; he/she prefers most the margins from the range between 1.00 to 1.50 PLN/MWh, assigning to them scores from 30 pts to 25 pts. Further on, the score decreases gradually to 0 for the highest profit margin of 2.00 PLN/MWh.

One should note that Table 2 contains only the essential options for each negotiation issue, on the basis of which the systems of negotiation offer evaluation of both negotiators were constructed.

There are several power variants to be negotiated in the orders, of value from 1 MW to 10 MW; there are thus ten possibilities. In the settlement issue there are only four options; no intermediate variants are possible. The supplier's margin, on the other hand, is a value from the range between 1.00 and 2.00 PLN/MWh, determined with the accuracy of 0.01 PLN/MWh, and therefore there are as many as 101 implementation levels for this issue. The evaluations of all the options which are not deemed essential have been determined using linear interpolation.

The negotiation space for the problem is presented in Figure 1.

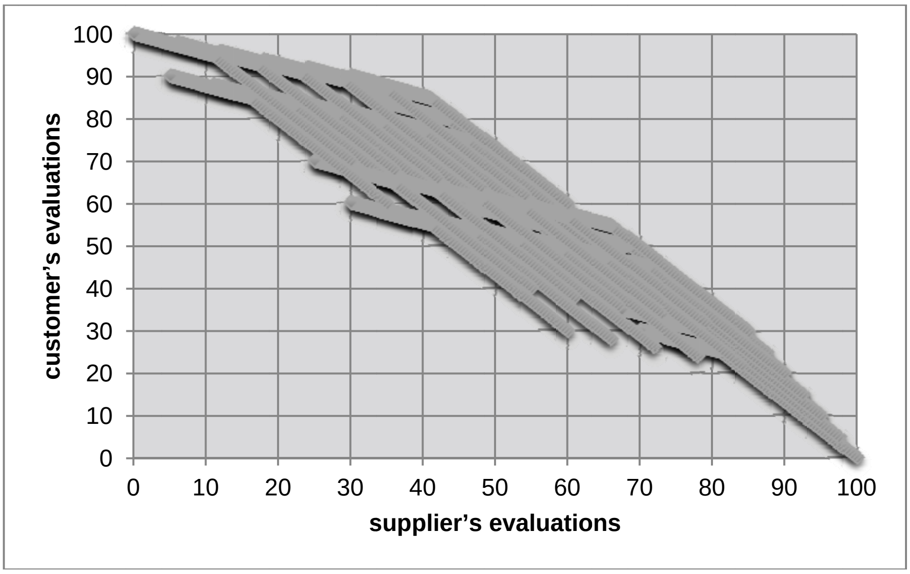

Figure 1. Negotiation space for the problem under discussion

Source: Author's own elaboration.

7 The phase of actual negotiations

In the phase of actual negotiations the supplier put forward seven offers, while the customer six. The compromise was reached in the 13th round. The first offer came from the supplier: it was one of the most favorable ones, that is, an offer of 1 MW of power with the first settlement method and the profit margin of 1.90 PLN/MWh. This offer scored 96.2 pts, according to the supplier's offer evaluation system. The customer countered with an offer evaluated at 94 pts: an order for 7 MW of power with the fourth settlement method and the profit margin of 1.00 PLN/MWh.

In the third negotiation round the supplier suggested a modification of his offer, with the profit margin lowered by 0.10 PLN/MWh, and the settlement method changed to the second one. The evaluation of his offer falls from 96.2 pts to 87.4 pts. The customer, on the other hand, evaluates the supplier's offer at 20 pts, which is an increase of 15 pts as compared to the customer's evaluation of the previous offer by the supplier. In response to that, the customer suggests an offer with the profit margin higher by 0.35 PLN/MWh as compared with his previous offer (which was an order of 7 MW with the fourth settlement method and the profit margin of 1.00 PLN/MWh). The implementation levels of the other issues remained unchanged.

The supplier gives up the profit margin and the minimal power in the order, but returns to the suggestion to use the first settlement method, and puts forward an offer of 3 MW with settlement #1 and margin 1.70. In the sixth round the customer did not make any concessions as regards the settlement method, but lowered the power and the margin down to the values of 5 MW and 1.40, respectively. These concessions, in turn, resulted in the next offer on the part of the supplier: the same power of 5 MW and a more favorable second settlement method, but the margin higher by 0.20 PLN/MWh, that is, equal to 1.90 PLN/MWh. The supplier, based on the analysis of liquidity risk, knows that such parameters of the order and settlement are safeguarded by the margin proposed.

In the eighth round the customer decided to increase the power to 7 MW and the margin, to 1.55 PLN/MWh at the expenses of the settlement method. He chose method #3, less profitable for him. In response, the supplier suggested the same volume and price algorithm, but a lower markup 1.80 PLN/MWh, which covers the potential liquidity risk. The customer counters with an offer of profit margin lower by 0.10 PLN/MWh, but insists on the third settlement method.

In the eleventh round the supplier agrees to the power and margin, but the settlement method remains a contentious issue. The customer suggest a higher margin to compensate for the settlement method and as a result, in the 13th round, the supplier − satisfied with the power at the level of 5 MW and a higher profit margin of 1.80 PLN/MWh − agrees to the third settlement method. According to his analysis, this margin will cover the liquidity risk as high as four days.

In the 13th round the parties achieved a compromise, whereby the customer will be able to order a maximum of 5 MW of power by 8 am, the price will be equal to the average over the transactions on the PEE on the previous day plus the profit margin of 1.80 PLN/MWh.

The compromise was assigned the score of 57.4 pts by the supplier. From the point of view of the customer's evaluation system, the compromise was worth 60 pts. The list of all the offers analyzed in the actual negotiation phase is presented in Table 3.

Table 3: Evaluation of each option by both parties participating in the negotiation process

Negotiation round	**Offer maker**	**Offer: power/settlement method/margin**	**Global score of the offer from the supplier's point of view**	**Global score of the offer from the customer's point of view**
1	2	3	4	5
Round 1	Supplier	1 MW/Settlement 1/1,90	96,2	5
Round 2	Customer	7 MW/Settlement 4/1,00	18	94
Round 3	Supplier	1 MW/Settlement 2/1,80	87,4	20
Round 4	Customer	7 MW/Settlement 4/1,35	25,7	90,5
Round 5	Supplier	3 MW/Settlement 1/1,70	83,6	25

Table 3 cont.

1	2	3	4	5
Round 6	Customer	5 MW/Settlement 4/1,40	38,8	86
Round 7	Supplier	5 MW/Settlement 2/1,90	81,2	35
Round 8	Customer	7 MW/Settlement 3/1,55	35,9	76,5
Round 9	Supplier	5 MW/Settlement 2/1,80	77,4	40
Round 10	Customer	5 MW/Settlement 3/1,70	53,6	65
Round 11	Supplier	5 MW/Settlement 2/1,70	73,6	45
Round 12	Customer	5 MW/Settlement 3/1,80	57,4	60
Round 13	Supplier	5 MW/Settlement 3/1,80	57,4	60

Source: Author's own elaboration.

8 Negotiation process support

Using the information from the offer evaluation system, both parties can use tools which allow for active and passive negotiation process support.

An example of passive support is the use of negotiation history plots (Wachowicz, 2013). Plots of negotiation history for the supplier and the customer in the problem analyzed are presented in Figures 2 and 3.

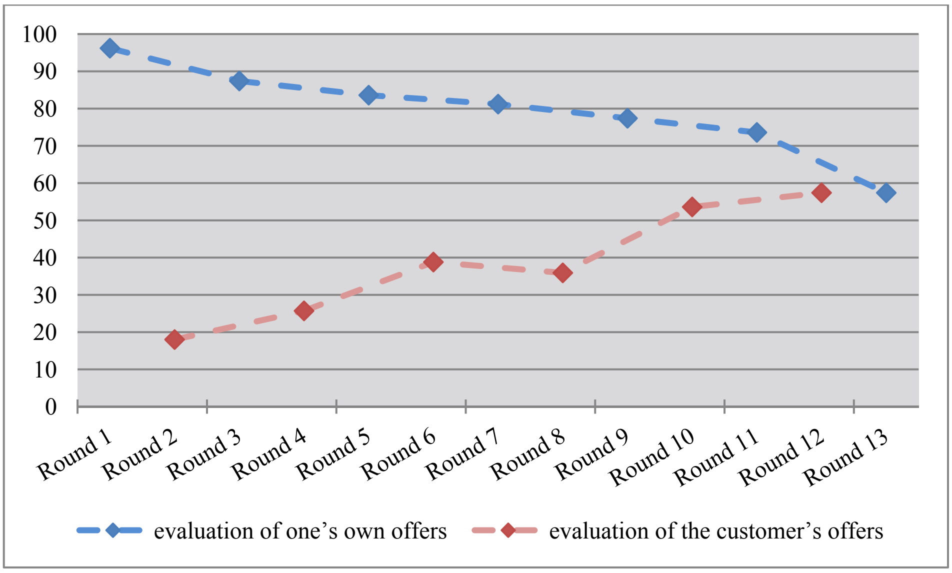

Figure 2. Plot of negotiation history from the supplier's point of view

Source: Author's own calculations.

In the negotiation history plot accessible to the supplier we can see the scores of the agreement variants suggested by the supplier and by the customer[10]. The customer's agreement variants are evaluated according to the supplier's evaluation system. Using this plot, the potential supplier can obtain information as to the customer's response to the offers suggested.

From the point of view of the supplier one can note that the dynamics of his/her compromises is relatively constant: the supplier made the greatest concessions at the beginning and the end of the negotiations. The supplier started the negotiations with an offer evaluated at 96.2 pts and ended with a compromise evaluated at 57.4 pts. The concession scale is therefore 38.8 pts. Worth noting are the supplier's evaluations assigned to the customer's offer. In round 6, the customer suggested an offer evaluated at 38.8 pts, in the next round the supplier countered with an offer evaluated at 81.2 pts (a score lower by 2.4 pts as compared with round 5). In response, in round 8, the customer came up with an offer evaluated by the supplier at 35.9 pts (that is, lower by 2.9 pts as compared with the offer from round 6). This reversal of the evaluation trend does not mean that the customer changed his/her attitude to a tougher one or to non-cooperative behavior, the more so that the same offers are evaluated at 86 and 76.5 pts in the customer's evaluation system. In reality, therefore, the customer made a concession of 9.5 pts between rounds 6 and 8.

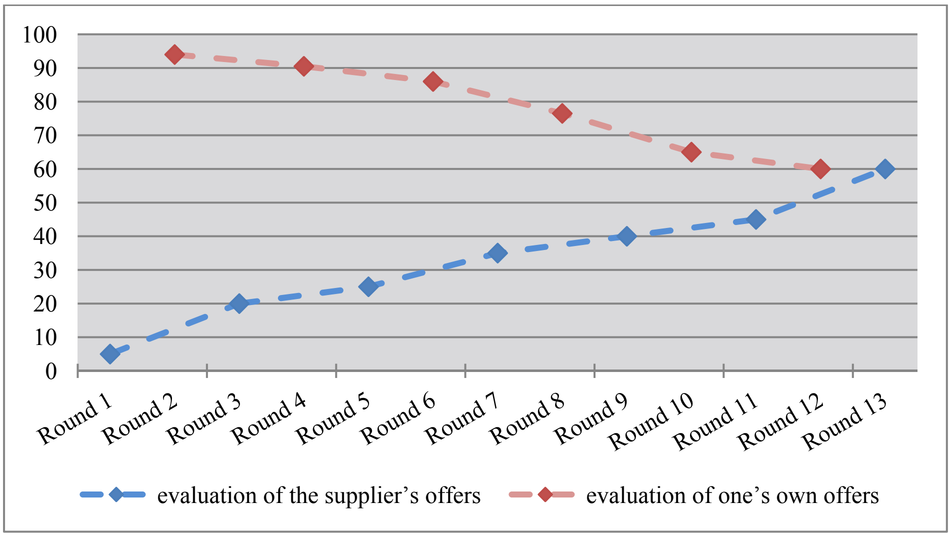

Figure 3. Plot of negotiation history from the point of view of the customer

Source: Author's own calculations.

[10] One should point out that the supplier has no access to the customer's evaluations.

The concession scale of the supplier's offers from the point of view of the customer is 55 pts. The first offer scored 5 pts, while the compromise 60 pts. The customer's range of concessions for round 6 did not exceed 5 pts. In the second half, however, the customer was more inclined to make concessions: in rounds 8 and 10 the scores of offers were lowered by 10 pts on the average.

Active support of the negotiation process consists in suggesting non-dominated negotiation compromises. If the compromise S^{sq} worked out by the parties[11] is not a non-dominated compromise, one can suggest to them a new, non-dominated solution S_i^*.

This way, the selection of the compromise suggested will exhaust the negotiation capabilities of both parties (in the negotiation parlance: the entire negotiation pie will be consumed). The negotiating parties do not have to and do not always agree to select a new, non-dominated negotiation compromise (Wachowicz, 2013).

The compromise reached by the supplier and the customer in the example analyzed is a dominated solution S^{sq} = (57.4; 60)[12]. Active support can include a suggestion to both parties to select instead a non-dominated compromise from among those situated within the domination cone[13].

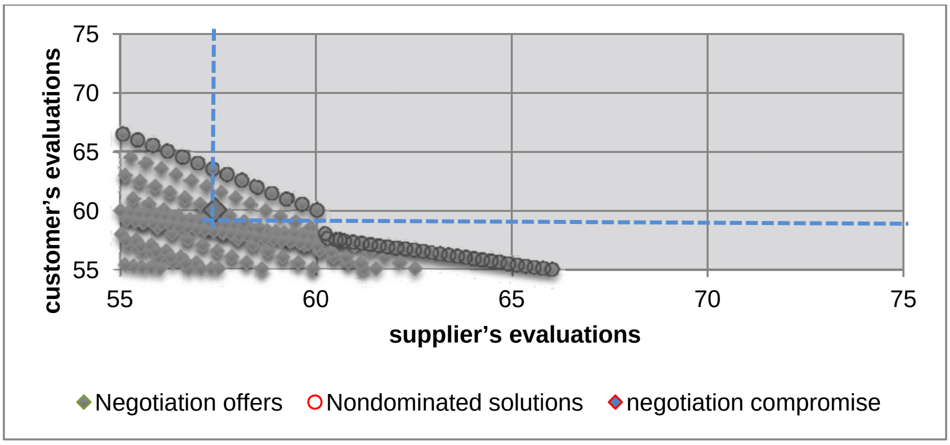

Figure 4. Compromise of the negotiators against non-dominated solutions in the negotiation space

Source: Author's own elaboration.

[11] The *status quo* point, according to game theory and Nash's arbitration scheme is the result of a game in the case when the players have not reached an agreement on their own, see Nash (1950). In negotiation analysis the possibility of using a dominated negotiation compromise was analyzed, instead of a *status quo* point (cf. Kopańska-Bródka, Wachowicz, 2013). So does also the present author.

[12] This compromise is shown in Figure 4.

[13] The domination cone is marked in Figure 4 by a dashed line.

When we deal with a concrete proposal of a dominated compromise, it is easy to identify non-dominated solutions situated within the domination cone. In our case, however, a problem appears: that of the criterion which should be used when selecting the specific non-dominated solution to be presented to the negotiating parties. Various approaches to solve this problem are suggested in the literature[14]. One can use the Euclidean or the taxicab metric which measures the distances between the compromise and the non-dominated solutions within the domination cone. When metrics are used, the recommendation concerns the selection of the solution closest to the compromise in the sense of the metric applied (Wachowicz, 2013).

Another solution proposed is Raiffa's solution of balanced increments. This solution consists in using both negotiators' potential to determine the bliss point $S^* = \left(s_1^*, s_2^*\right)$ on the basis of the status quo $S^{sq} = \left(s_1^{sq}, s_2^{sq}\right)$. In particular, as the point S^{sq} one can take the compromise reached by the parties during the actual negotiation phase.

If the criteria are maximized, S^{sq} lies below the effective limit, while S^* lies above it. The intersection of the straight line determined by these two points with the effective limit is the recommended non-dominated solution $S^{rec} = \left(s_1^{rec}, s_2^{rec}\right)$. This solution can be suggested to the negotiators to improve the joint result. The idea of determining the coordinates of the points S^* and S^{rec} using the negotiation potentials is presented in Figure 5.

Incremental analysis assumes that mixed strategies are recommended as non-dominated negotiation compromises (Kopańska-Bródka, Wachowicz, 2013; Raiffa, 1953).

When the possibility of selecting mixed negotiation compromises is excluded, the use of the effective limit can cause problems. It may be impossible to determine negotiation potentials according to Raiffa's algorithm, since a non-dominated compromise, allowing to determine the point S^* uniquely, does not always exist. This situation is shown in Figure 6.

[14] Suggesting non-dominated negotiation compromises is related to the notion of fair solution. Various approaches to this can be found in: Zeuthen (1930); Nash (1953); Brams (1990); Raiffa et al. (2002).

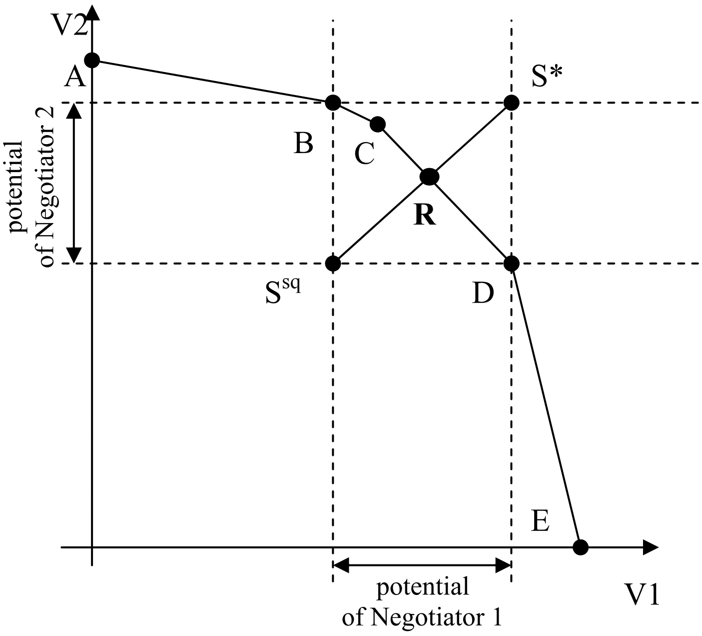

Figure 5. Balanced increment solution based on negotiation potential

Source: Kopańska-Bródka, Wachowicz (2013).

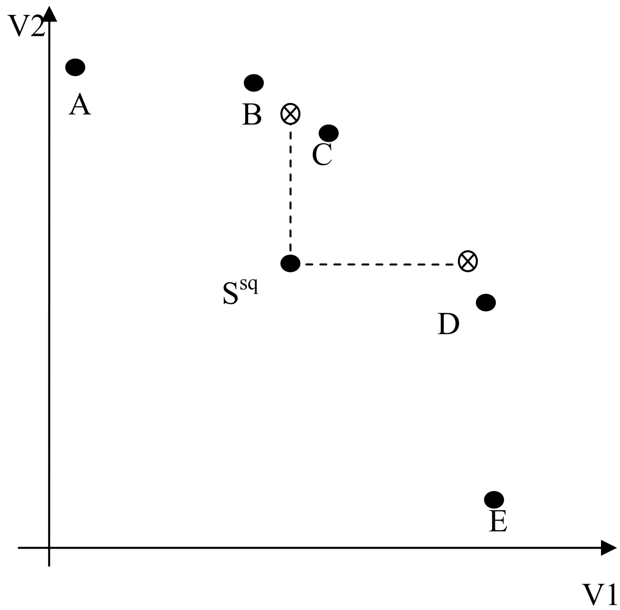

Figure 6. Balanced increment solution in the discrete case

Source: Author's own elaboration based on: Kopańska-Bródka, Wachowicz (2013).

The set of non-dominated solutions in Figure 6 consists of five points *A* through *E*. It can be seen that it is not possible to determine negotiation potentials uniquely. When determining the potential for the first negotiator (scores V1) it is not obvious to which point, *B* or *C*, the straight line from S^{sq} should be drawn. The choice of either point influences the coordinates of S^{*} and therefore the recommended compromise S^{rec}.

This problem can be solved using linear interpolation between the points analyzed. One should keep in mind, however, that the interpolated points between *B* and *C* represent mixed negotiation compromises. As a result, the recommended solution can also be a mixed compromise, which is not always acceptable for the negotiators, since it follows from the detailed characteristics of the negotiation problem. The characteristics of the criterion related to the determination of the settlement method for the purchase order excludes the possibility of recommendation of mixed negotiation compromises.

The classic version of the balanced increment solution can be applied despite these problems. Instead of selecting the determined mixed compromise one can select another pure compromise closest to the mixed one.

The effective limit of the problem analyzed is formed by the straight lines through the points listed in Table 4[15].

Table 4: Points on the effective limit

Points	**Scores**	
	supplier	**customer**
A	100	0
B	90	20
C	85	30
D	66	55
E	41	85
F	30	90
G	0	100

Source: Author's own elaboration.

The recommended point is the mixed compromise S^{rec} = (59,61(6);62,66). It lies between two non-dominated points *D* and *E*. These compromises differ only by the implementation of the settlement method. S^{rec} has been determined according to the idea of balanced increments as the intersection of the line through *D* and *E* $v^{K} = 1{,}2v^{S} + 134{,}2$ with the line perpendicular to it and passing through

[15] For a detailed algorithm calculating the effective limit using analysis of critical ratios, see Raiffa et al. (2002); Wachowicz (2013).

S^{sq} and the bliss point located above the effective limit $v^K = 1{,}2v^S - 8{,}88$. The variables v^K and v^S represent the evaluations of the customer and supplier, respectively.

While rejecting at first the possibility of accepting mixed compromises, one should note that the lines through the point S^{sq} which determine the negotiation potentials are identical with the domination cone for this point. The dominated compromise S^{sq} reached by the negotiators[16] is overlapped by the domination cone which determines the distance from all the non-dominated points included in the cone. The distance is understood here as the difference between the evaluations assigned to non-dominated solutions and those assigned to the compromise S^{sq}. Distances can be interpreted as improvements of the results of each negotiator achievable by selecting any available non-dominated solution. The interpretation of distances refers to Raiffa's potentials.

As the criterion for the recommendation of a non-dominated solution we take the quotient of the improvements of the results as close to 1 as possible. The formula for the improvement quotient (also called increment ratio) can be written as follows:

$$\frac{s_{i1}^0 - s_1^{sq}}{s_{i2}^0 - s_2^{sq}} \to 1,\ s_{i2}^0 - s_2^{sq} \neq 0 \tag{1}$$

where:

$S^{sq} = \left(s_1^{sq}, s_2^{sq}\right)$ is the dominated compromise achieved by the negotiators;

$S_i^0 = \left(s_{i1}^0, s_{i2}^0\right)$ is the ith non-dominated solution.

The denominator and numerator of the improvement quotient show how much the evaluations of both negotiators will increase if a specific suggested compromise is chosen. One can say that the increment ratio of evaluations of both parties is an indicator of the improvement of the solution. If this measure is exactly equal to 1, this means that the selection of a new non-dominated compromise will improve the evaluations of both negotiators by the same number of points. In this case one should choose that compromise for which the quotient is as close to 1 as possible. The quotient value greater than 1 means that the improvement of the compromise is more favorable for the negotiator whose evaluations are calculated in the numerator of formula (1). The quotient value smaller than 1 means that the improvement is more favorable for the negotiator whose evaluations are calculated in the denominator of this formula.

[16] Kopańska-Bródka, Wachowicz (2013) also suggest using the compromise achieved by the negotiators instead of point S^{sq}.

The improvement quotient described by formula (1) has a construction flaw: It can be calculated only if the difference of evaluations in the denominator is non-zero. The zero value would mean that the choice of a non-dominated compromise is not related to an increase in evaluation for one negotiator. Moreover, a quotient with 0 in the denominator would be undetermined. To avoid this situation, the difference of evaluations in the denominator must be positive. The construction of the index is therefore sensitive to the order of negotiators. One would have to choose that order for which this quotient can be determined[17].[2]

The ratio of evaluation increments is related to the notion of the proportion of the potential (Raiffa et al., 2002) (POP). The POP indices are calculated separately for each negotiator, taking into accounts the reservation values of each party[18]. The calculation of the improvement quotient, as opposed to POP, requires only points S^{sq} and $S^0{}_i$. The quotient itself is calculated jointly for both negotiators.

Within the cone of dominating solutions there are seven non-dominated points denoted from S_1^0 to S_7^0 with the coordinates: S_1^0 = (60; 60), S_2^0 = (59.62; 60,5), S_3^0 = (59.24; 61), S_4^0= (58.86; 61.5), S_5^0 = (58.48; 62), S_6^0 = (58.1; 62.5) and S_7^0 = (57.72; 63). The detailed results of the potential recommendation using various selection criteria are shown in Table 5. Four criteria of the selection of non-dominated compromise were analyzed: taxicab metric, Euclidean metric, Raiffa's balanced increment solution, and the notion of improvement quotient.

Table 5: Evaluations of each option by both negotiating parties

No	Evaluations supplier	Customer	Taxicab metric	Euclidean metric	Raiffa's approach	Improvement quotient
S_1^0	60	60	**2,60**	2,60	2,69	0,00
S_2^0	59,62	60,5	2,72	2,28	2,16	0,23
S_3^0	59,24	61	2,84	**2,09**	1,70	0,54
S_4^0	58,86	61,5	2,96	**2,09**	1,38	**1,03**
S_5^0	58,48	62	3,08	2,27	**1,31**	1,85
S_6^0	58,1	62,5	3,20	2,60	1,53	3,57
S_7^0	57,72	63	3,32	3,02	1,93	9,37

* The values in this column are the distances of the points S_1^0 through S_7^0 from the compromise recommended by the balanced increment solution with the coordinates (59.61(6); 62.66). The distances have been calculated using the Euclidean metric.

** The numerator contains the differences of the customer's evaluations, the denominator – those of the supplier.

Source: Author's own calculations.

[17] In future research it is planned to eliminate this flaw by applying the minimum and maximum functions.

[18] Reservation values: *cf.* Kopańska-Bródka, Wachowicz (2013).

Suggesting non-dominated compromises on the basis of the taxicab metric consists in proposing the point S_1^0 as the closest to the compromise achieved by the negotiators. The selection of the Euclidean metric, on the other hand, changes this recommendation to points S_3^0 and S_4^0 as the least removed from the compromise achieved. The point closest to the mixed compromise calculated by means of Raiffa's approach is S_5^0.

Recommending point S_4^0 agrees with the idea of the improvement quotient. Worth noting is the non-dominated compromise S_1^0, from the point of view of the notion of the improvement quotient. This point, although recommended on the basis of the Euclidean metric, is at the same time evaluated as the worst one by the improvement quotients. This is because the selection of S_1^0 is related to an improvement of the result from 57.4 to 60 pts only for the supplier. Therefore, these measures exclude a point at which the improvement will not occur for one of the parties.

9 Summary

In the era of liberalization of the energy market in Poland an individual approach to each customer, especially to an industrial one, and negotiations of the conditions of agreement will become standard. For that reason it is justified to use methods supporting negotiations in this area.

The scoring method, being a modification of one of the simplest multi-criteria method, is relatively simple. Nonetheless, the evaluations of negotiation issues and of levels of their implementation are critical for the negotiators. Their results interweave throughout the entire negotiation process and, as a result, influence the negotiation compromise.

The notion of the increment quotient proposed here is a simple and fairly intuitive measure which explicitly shows the improvement of the results of both negotiators. However, it is not always possible to calculate it because of its quotient construction. Further research will include a modification of its construction so as to eliminate the indefinite symbols in the result.

Moreover, the improvement of the result by the same number of points, that is, absolutely, does not have to mean the same improvement relatively. For that reason, in future research, a construction of this index for relative (percentage-wise) improvement will be considered.

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M U L T I P L E   C R I T E R I A   D E C I S I O N   M A K I N G

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A NEW FUZZY MEASURE FOR THE ANALYTIC HIERARCHY PROCESS WITH THE CHOQUET INTEGRAL

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Abstract

A new fuzzy measure is presented in this paper. Using the assumption that the decision maker is able to provide the pairwise additivity degree between attributes, our method uses Zimmerman's approach to solve the fuzzy multi-objective problem: a simple problem for computing fuzzy density is derived. Having done that, we use this new fuzzy measure to implement an analytic hierarchy process (AHP) with dependent attributes using the Choquet integral. Our identification procedure for fuzzy density is much easier because it reduces the resolution complexity using a linear programming problem rather than the complicated power form used traditionally.

Keywords: fuzzy measure, Choquet integral, linear programming problem, AHP.

1 Introduction

Several methods have been proposed for fuzzy measures. However, the identification of a fuzzy measure could be the most difficult step when fuzzy integrals are applied to solve MCDM problems, because $2^n - 2$ values of the fuzzy measure have to be provided by the decision-maker(s) for an MCDM problem with n criteria (Larbani, Huang, Tzeng, 2011). In earlier reviews by Grabisch (1995), Grabisch et al. (2008), Grabisch and Labreuche (2010), the identification methods are classified into three groups: semantic methods (guessing the fuzzy meas-

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ure on the basis of semantic considerations), learning methods (optimization methods) and methods combining semantic and learning methods. In addition, the main approaches (least square, minimum split and minimum variance) to fuzzy measure identification based on the Möbius transform of capacity and *k*-additivity are reviewed and their advantages and disadvantages are discussed. The least square method is historically the first approach; it can be regarded as a generalization of the classic multiple linear regression. The goal is to minimize the average quadratic distance between the overall utilities computed by means of the Choquet integral and the desired overall scores provided by the decision-maker(s). However, the objective function is not strictly convex so that the solution is not unique. The maximum split method is based on linear programming; the idea of this approach is to maximize the minimal difference between the overall utilities of objects that have been ranked by the decision-maker(s) using partial weak order. This method is quite simple; but, similarly, it does not have a unique solution. The idea of the minimum variance method is to favor the "least specific" capacity, if any, compatible with the initial preferences of the decision-maker (s). It may lead to a unique solution; however, a unique solution doesn't exist if there are "poor" initial preferences; for example, if the decision maker faces very high positive or negative interaction indices or a very uneven Shapley value.

Since computing the fuzzy density of a fuzzy measure is complicated by its power form, many scholars tried their best to simplify this problem. For example, Lee and Leekwang (1995) developed an identification method for fuzzy measures based on evolutionary algorithms. Wang and Chen (2005) used the sampling method with genetic algorithm, the complexity reduced to the data number of $O(2^n/n)$. Takahagi (2000) proposed an approach based on two types of pairwise comparison. The first one is based on the pairwise comparison values of interaction degrees between criteria. The second one is based on the pairwise comparison values of weights of criteria. Thus, the complexity of data collection can be reduced to $n(n-1)$. In addition, Corrente et al. (2016) showed that the application of the Choquet integral presenting two main problems for the necessity to determine the capacity, which is the function that assigns a weight not only to all single criteria but also to all subset of criteria, and the necessity to express on the same scale evaluations on different criteria. They adopted the recently introduced Non-Additive Robust Ordinal Regression (Greco et al., 2010) for the first problem, which takes into account all the capacities compatible with the preference information provided by the DM; with respect to the second one they build the common scale for the considered criteria using the Analytic Hierarchy Process (AHP). This permits to reduce considerably the number of pairwise comparisons usually required by the DM when applying the AHP.

The Analytic Hierarchy Process (AHP) is an effective tool for selecting the best alternative in multiple criteria decision making (MCDM) (Liou, Tzeng, 2007; Saaty, 1980; Tzeng et al., 2005; Yang et al., 2008). In multiple criteria decision making (MCDM) fuzzy measures are used to represent interactions between the attributes (Chen, 2001; Chen, Larbani, 2006; Chen, Tzeng, 2001); namely, the aspects of independence, complementarity and redundancy of attributes, which are also the challenges of the AHP (Bortot, Marques Pereira, 2013). Our method of identification is very easy to implement because the optimization problem we solve is linear and the number of its constraints is small compared with the optimization problems in methods cited above.

Our paper is organized as follows: in Section 2, the definitions of a fuzzy measure and λ-fuzzy measure are reviewed. In Section 3, the new fuzzy measure is presented and defined. In Section 4, we propose the Choquet integral AHP which uses the new fuzzy measure. In Section 5, a numerical example is used to illustrate our results. Finally, conclusions and recommendations are in Section 6.

2 Overview of the literature

In this section we will review basic definitions and concepts of fuzzy measure, Choquet integral and AHP.

2.1 Fuzzy measure

Sugeno (1974) presented a theory of fuzzy measures and fuzzy integrals in modeling the human subjective evaluation process (Ishii, Sugeno, 1985; Kambara et al., 1997).

Definition 2.1 (Sugeno, 1974). Let g be a set function defined on the power set $\beta(X)$ of X, and satisfying the following properties:

Property 1. Boundary conditions:
$g : \beta(X) \rightarrow [0,1]$
$g(\phi) = 0$ and $g(X) = 1$

Property 2. Monotonicity:
$\forall A, B \in \beta(X)$ if $A \subseteq B$, then $g(A) \leq g(B)$

Property 3. Continuity:
If $F_k \in \beta(X)$ for $1 \leq k < +\infty$, and the sequence $\{F_k\}$ is monotone (in the sense of inclusion), then $\text{im}_{k\rightarrow+\infty}\, g(F_k) = g(\lim_{k\rightarrow+\infty} F_k)$.
It has to be noted that if X is finite then property 3 can be omitted.

The following are three special fuzzy measures; each measure is defined by certain constraints on g.

(a) Probability measure:

$$A, B \in \beta(X) \text{ and } A \cap B = \phi \rightarrow g(A \cup B) = g(A) + g(B) \tag{1}$$

(b) F-additive measure:

$$A, B \in \beta(X) \text{ and } A \cap B = \phi \rightarrow g(A \cup B) = g(A) \vee g(B) \tag{2}$$

where $a \vee b = max\ \{a,b\}$

(c) λ-measure:

$$A, B \in \beta(X) \text{ and } A \cap B = \phi \rightarrow g(A \cup B) = g(A) + g(B) + \lambda g(A) g(B) \tag{3}$$

where $\lambda \in (-1,\infty]$.

Sugeno constructed the λ-measure as a special case of a fuzzy measure. Here, the measure is based on the parameter λ, which describes the degree of additivity. We have three important types of λ-measures.

1) if $\lambda > 0$, then $g_\lambda(A \cup B) > g_\lambda(A) + g_\lambda(B)$, the measure is superior additive, which implies multiplicative effects between A and B;
2) if $\lambda = 0$, then $g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B)$, the measure is additive;
3) if $\lambda < 0$, then $g_\lambda(A \cup B) < g_\lambda(A) + g_\lambda(B)$, the measure is subadditive, which implies substitutive effects between A and B.

If $X = \{x_1, x_2, \ldots, x_n\}$ i.e. X is finite, the fuzzy measure can be written as (Sugeno, 1974):

$$\begin{aligned} g_\lambda(\{x_1, x_2, \ldots, x_n\}) &= \sum_{i=1}^{n} g_i + \lambda \sum_{i1=1}^{n-1} \sum_{i2=i1+1}^{n} g_{i1} g_{i2} + \ldots + \lambda^{n-1} g_1 g_2 \ldots g_n = \\ &= \frac{1}{\lambda} \left| \prod_{i=1}^{n} (1 + \lambda\ g_i) - 1 \right|, \text{ for } -1 < \lambda < \infty \end{aligned} \tag{4}$$

where $g_i = g_\lambda(\{x_i\})$, $i = 1..n$, define the fuzzy density of the fuzzy measure g_λ. The power form is inspired by the utility function of Keeney and Raiffa (1976).

2.2 The Choquet integral

Consider a fuzzy measure space $(X, \beta(X), g)$ with $X = \{x_1, x_2, \ldots, x_n\}$. The Choquet integral of a function h: $X \rightarrow [0,1]$ with respect to g is defined as follows (Yang et al., 2008; Sugeno, 1974):

$$\int_X h(X) \bullet g(H) = \bigvee_{j=1}^{n} \left[h(x_j) \wedge g(H_j) \right] \tag{5}$$

where $h(x_j) \geq h(x_{j+1})$ for $1 \leq j \leq n-1$, $a \wedge b = \min\{a,b\}$ and $Hj = \{x_1, x_2, \ldots, x_j\}$, $j = 1, 2, \ldots, n$. When the Choquet integral is used to describe an MCDM problem, a value of the function h can be thought of as the performance of a particular attribute for an alternative, and g represents the decision maker's subjective evaluation of the importance of the attributes. The Choquet integral of h with respect to g gives the overall evaluation of an alternative. Furthermore, we have (Sugeno, 1974):

$$\int hdg = h(x_n)g(\mathrm{H}_n) + [h(x_{n-1}) - h(x_n)]g(\mathrm{H}_{n-1}) + \ldots + [h(x_1) - h(x_2)]g(\mathrm{H}_1) =$$
$$= h(x_n)[g(\mathrm{H}_n)\text{-}g(\mathrm{H}_{n-1})] + h(x_{n-1})[g(\mathrm{H}_{n-1})\text{-}g(\mathrm{H}_{n-2})] +\ldots +h(x_1)g(\mathrm{H}_1) \qquad (6)$$

where $\mathrm{H}_1 = \{x_1\}$, $\mathrm{H}_2 = \{x_1, x_2\},\ldots, \mathrm{H}_n = \{x_1, x_2, \ldots, x_n\} = X$.

The basic concept of equation (6) can be illustrated as shown in Figure 1.

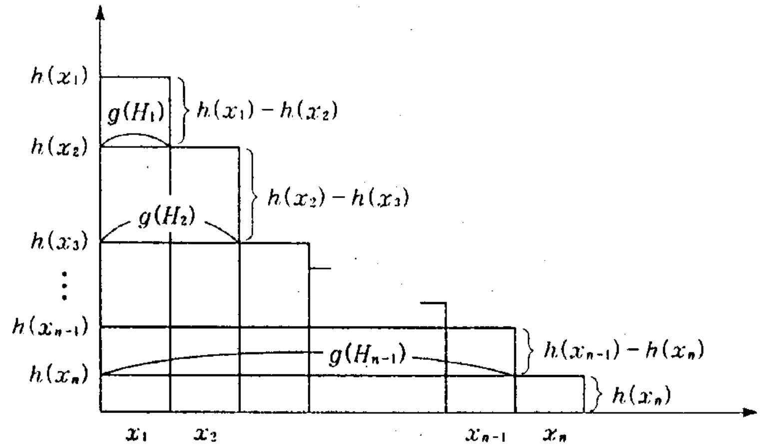

Figure 1. The Choquet integral

The Choquet integral is a powerful tool to measure the subjective human evaluation (Ishii, Sugeno, 1985; Kambara et al., 1997). The main reason for that is that it is not necessary to assume the independence between the attributes in the Choquet integral model.

2.3 The Analytic Hierarchy Process (AHP)

Saaty (1980) introduced a method of computing relative weights using a positive pairwise comparison matrix using the eigenvector method: let **P** be the positive pairwise comparison matrix with respect to n attributes:

$$\mathbf{P} = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} \cdots & \frac{w_2}{w_n} \\ & & \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} \cdots & \frac{w_n}{w_n} \end{bmatrix} \qquad (7)$$

where $\dfrac{w_a}{w_b}$ represents the relative importance of the a-th attribute over the b-th attribute, where a, $b \in \{1, 2, \ldots, n\}$. Multiplying **P** by the relative importance vector: $\mathbf{W} = (w_1, w_2, \ldots, w_n)^t$, we get the following equation:

$$\mathbf{PW} = \begin{bmatrix} \dfrac{w_1}{w_1} & \dfrac{w_1}{w_2} \cdots & \dfrac{w_1}{w_n} \\ \dfrac{w_2}{w_1} & \dfrac{w_2}{w_2} \cdots & \dfrac{w_2}{w_n} \\ \\ \dfrac{w_n}{w_1} & \dfrac{w_n}{w_2} \cdots & \dfrac{w_n}{w_n} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ w_n \end{bmatrix} \tag{8}$$

In general, the value of $\dfrac{w_a}{w_b}$ is subjectively given by the decision maker. Saaty (1980) uses the maximal eigenvalue: $EV_{\max}$ to find the solution **W** of equation (8) as shown in the following equation:

$$(\mathbf{P} - EV_{\max}\, \mathbf{I})\mathbf{W} = \mathbf{0} \tag{9}$$

where **I** is the identity matrix. A set of linear equations for $w_1, w_2, \ldots, w_n$ can be obtained from equation (9); the final values of $w_1, w_2, \ldots, w_n$ are computed using the normalization condition:

$$w_1 + w_2 + \ldots + w_n = 1 \tag{10}$$

3 A new fuzzy measure with variable additivity degree

In this section we propose a generalization of the λ-measure (Larbani et al., 2011). When the degree of additivity λ depends on the sets considered i.e.:

$$A, B \in \beta(X) \text{ and } A \cap B = \phi \rightarrow g(A \cup B) = g(A) + g(B) + \lambda_{AB}$$

where $\beta(X)$ is the set of all subsets of a set $X = \{x_1, x_2, \ldots, x_n\}$ of attributes and λ_{AB} is the additivity degree between subsets A and B.

Procedure 3.1

Step 1. Assume that the decision maker is able to provide a pairwise evaluation of the interdependence of the attributes, i.e. for each pair of different attributes i and j the decision maker is able to assign/guess their additivity degree λ_{ij} with $-1 \le \lambda_{ij} \le 1$ and $0 \le \max\lambda_{ij} \le 1$. Here λ_{ij} plays the role of the correlation coefficient in the traditional statistics. Actually, this is a simple idea if we trace it back to the traditional statistics. In the traditional statistics, a positive correlation means that the two variables are synergistic: An increasing effect of one variable leads to a similarly increasing effect of another; conversely, a negative correlation means that the two variables may be substitutive. The decision maker is encouraged to determine these λ_{ij} by his/her arbitrary perception.

Step 2. The decision maker can give only one fuzzy estimation of the value of the density g_i for each attribute x_i. Without loss of generality and for the ease of presentation, we assume that the decision maker's fuzzy estimation of each density g_i is a fuzzy value number $\widetilde{g}_i = [a_i, b_i]$, with $0 \le a_i \le b_i \le 1$. It should be noted here that the decision maker has the freedom to choose a_i, b_i according to his/her experience and knowledge about the given attribute. Here we assume $g_i = (1 - \alpha)a_i + \alpha b_i$, $0 \le \alpha \le 1$. Therefore density g_i, $i = 1, \ldots, n$ of the fuzzy measure can be determined by solving the following optimization problem:

$$
\begin{gathered}
\text{Max } (g_1, g_2, \ldots, g_n) \\
\text{such that } 0 \le g_i + g_j + \lambda_{ij} \le 1, \text{ for all } i, j \text{ with } i \ne j: \\
\sum_{1}^{n} g_i + \max_{\substack{i,j \in \{1..n\},\\ i \ne j}} \lambda_{ij} = 1 \\
g_i = (1 - \alpha)a_i + \alpha b_i,\ i = 1, \ldots, n.
\end{gathered} \tag{11}
$$

where α represents the achievement level of the fuzzy numbers $\widetilde{g}_i$, $i = 1, \ldots, n$, the larger the better. In fact, any density that satisfies the constraints of problem (11) can be taken as a feasible solution; however, we look for the maximal value of the objective function in this problem. Now the fuzzy measure can be determined on the basis of the density obtained from problem (11). When the fuzzy multi-objective approach of Zimmerman (1985) is applied, problem (11) can be reduced to problem (12):

$$
\begin{gathered}
\text{Max } \alpha \\
\text{such that } 0 \le g_i + g_j + \lambda_{ij} \le 1, \text{ for all } i, j \text{ with } i \ne j: \\
\sum_{1}^{n} g_i + \max_{\substack{i,j \in \{1..n\},\\ i \ne j}} \lambda_{ij} = 1 \\
g_i = (1-\alpha)a_i + \alpha b_i,\ i = 1, \ldots, n.
\end{gathered} \tag{12}
$$

Proposition 3.1. The set function defined by:

$$
g(A) = \sum_{i \in A} g_i + \max_{\substack{i,j \in A\\ i \ne j}} \lambda_{ij} \tag{13}
$$

for all subsets of X such that Card(A) ≥ 2 and $g(\varnothing) = 0$, is a fuzzy measure.

Proof. By construction we have $g(\varnothing) = 0$ and $g(X) = 1$. Let us now prove that given two subsets A and B of X such that $A \subset B$, we have $g(A) \le g(B)$. Since $A \subset B$, we have:

$$\sum_{i\in A} g_i \le \sum_{i\in B} g_i \text{ and } \max_{\substack{i,j\in A\\ i\neq j}} \lambda_{ij} \le \max_{\substack{i,j\in B\\ i\neq j}} \lambda_{ij}$$

Adding these two inequalities we get:

$$g(A) = \sum_{i\in A} g_i + \max_{\substack{i,j\in A\\ i\neq j}} \lambda_{ij} \le g(B) = \sum_{i\in B} g_i + \max_{\substack{i,j\in B\\ i\neq j}} \lambda_{ij}$$

Now it remains to prove that for any subset A of X we have $0 \le g(A) \le 1$. If A has one or two elements, the inequality $0 \le g(A) \le 1$ holds according to the constraints of problem (11). Assume now that A has more than three elements. Let $\{i, j\}$ be any two elements in A; then, according to the first part of this proof, we have:

$$g(A) \ge g(\{i, j\}) = g_i + g_j + \lambda_{ij} \ge 0$$

On the other hand, we have $A \subset X$, therefore $g(A) \le g(X) = 1$, hence $g(A) \le 1$. Given two subsets A and B of X such that $A \cap B = \varnothing$, we have:

$$g(A\cup B) = \sum_{i\in A\cup B} g_i + \max_{\substack{i,j\in A\cup B\\ i\neq j}} \lambda_{ij} = \sum_{i\in A} g_i + \sum_{i\in B} g_i + \max_{\substack{i,j\in A\cup B\\ i\neq j}} \lambda_{ij} =$$

$$= g(A) + g(B) + \max_{\substack{i,j\in A\cup B\\ i\neq j}} \lambda_{ij} - \Big(\max_{\substack{i,j\in A\\ i\neq j}} \lambda_{ij} + \max_{\substack{i,j\in B\\ i\neq j}} \lambda_{ij}\Big)$$

Hence A and B are independent if and only if the degree of additivity in $A \cup B$ is equal to the sum of degrees of additivity within A and within B, that is:

$$A \text{ and } B \text{ are independent} \Leftrightarrow \max_{\substack{i,j\in A\cup B\\ i\neq j}} \lambda_{ij} = \Big(\max_{\substack{i,j\in A\\ i\neq j}} \lambda_{ij} + \max_{\substack{i,j\in B\\ i\neq j}} \lambda_{ij}\Big)$$

To summarize we give a procedure for an effective implementation of our method of identification.

Procedure 3.2

Step 1. Ask the decision maker to provide/guess the pairwise degrees of additivity λ_{ij}, if attribute i and attribute j are mutually substitutive then λ_{ij} should be less than zero; while if attribute i and attribute j are mutually complementary then λ_{ij} should be larger than zero, and the fuzzy evaluations of the density are $\tilde{g}_i$, $i = 1, \ldots, n$ of attributes.

Step 2. Solve problem (12). If it has no solution then return to Step 1. The decision maker has to enlarge the interval of fuzzy evaluations $\tilde{g}_i$, $i = 1, \ldots, n$ or/and reevaluate the additivity values λ_{ij}. If (12) has a solution, go to Step 3.

Step 3. Let g_i, $i = 1, \ldots, n$ be the solution of problem (12) with the largest α, then identify the fuzzy measure by formula (13).

4 The Choquet Integral AHP of the new fuzzy measure

In this section we will compare the results from the traditional AHP (Zeleny, 1982) and the Choquet Integral AHP with the new fuzzy measure. Assume now that the traditional AHP pairwise comparison matrix **P** in (7) is given. The Choquet Integral AHP is defined by the triplet (X, **P**, g_i), where X is the set of attributes, **P** is the pairwise comparison matrix and g_i is the computed fuzzy density. Now we will show how the attribute weights are calculated in the Choquet Integral AHP. We will use the Choquet Integral to calculate the new weights of the attributes in order to take into account the interdependence of attributes. Let us assume, for the time being, that the λ-fuzzy measure g describing the interdependence of attributes is known (identification will be performed in the next section). Let w_i represent the normalized weight of attribute i from (10) and let $h(x_i) = w_i$. Let $\mathrm{H}_1 = \{x_1\}$, $\mathrm{H}_2 = \{x_1, x_2\}, \ldots$, $\mathrm{H}_n = \{x_1, x_2, \ldots, x_n\} = X$ and $h(x_1) \geq h(x_2) \geq \ldots \geq h(x_n)$. If this ordering does not hold, one can reorder the attributes accordingly. Assume for the time being that the λ-fuzzy measure g is known. Now, according to (6) we have:

$$\int hdg = h(x_n)[g(\mathrm{H}_n) - g(\mathrm{H}_{n-1})] + h(x_{n-1})[g(\mathrm{H}_{n-1}) - g(\mathrm{H}_{n-2})] + \ldots + h(x_1)g(\mathrm{H}_1)$$

Let us consider the vector $\mathbf{W}^f = (w_1^f, w_2^f, \ldots, w_n^f)$ with the following components (Chen, 2001):

$$\begin{aligned} w_1^f &= h(x_1)g(\mathrm{H}_1) \\ w_2^f &= h(x_2)\,[g(\mathrm{H}_2) - g(\mathrm{H}_1)] \\ &\vdots \\ w_n^f &= h(x_n)\,[g(\mathrm{H}_n) - g(\mathrm{H}_{n-1})] \end{aligned} \tag{14}$$

This Choquet Integral gives an aggregated evaluation of the effect of interdependence of the attributes on the weights $h(x_i) = w_i$, $i = \overline{1,n}$ given by the traditional AHP where attributes are assumed to be independent. It is then natural to write the new weight of any attribute i as $h(x_i)[g(\mathrm{H}_i) - g(\mathrm{H}_{i-1})]$ based on the Choquet Integral (6); that is, as the corresponding term in the Choquet Integral. Furthermore, the coefficient $[g(\mathrm{H}_i) - g(\mathrm{H}_{i-1})]$ can be interpreted as the effect of the interdependence of the attributes on the weight $h(x_i) = w_i$ of attribute i.

Definition 4.1. We define the vector $\mathbf{W}^{f'} = (w_1^{f'}, w_2^{f'}, \ldots, w_n^{f'})$ of weight attributes in the Choquet Integral AHP as the normalized vector of $\mathbf{W}^f$. Thus, we have:

$$w_1^{f'} + w_2^{f'} + \ldots + w_n^{f'} = 1 \tag{15}$$

where $w_i^{f'} = \dfrac{w_i^f}{\sum_{j=1}^{n} w_j^f}$, $i = 1, \ldots, n$.

Now, we will use a numerical example to show how our ideas work. We use Procedures 3.1 and 3.2 to compute each g_i, then apply equations (14)-(15) to get our modified weights.

5 A numerical example

Here we propose two applications of our new fuzzy measure. The first one consists in adjusting the weights in the traditional AHP, the second one, in finding the dependency (λ_{ij}) in a decision process.

Consider the following management problem: an enterprise always faces many negative impacts (factors), which could lead to a reduction in productivity. For example, decreased productivity may result from improper human resource management, financial management, management of technology (MOT), operations management, etc. However, since the available resources: time and money are limited when tackling the negative impacts (factors) of an enterprise, the manager wants to rank these possible causes of decreased productivity; the cause with the highest priority could be tackled first.

Table 1: Comparison matrix with respect to three causes

$\frac{w_a}{w_b}$	**Poor management of human resources (IHR)**	**Poor management of innovative technologies (IIT)**	**Poor management of manufacturing operations (IMO)**
Poor management of human resources (IHR)	1	$\frac{1}{3}$	$\frac{1}{2}$
Poor management of innovative technologies (IIT)	3	1	3
Poor management of manufacturing operations (IMO)	2	$\frac{1}{3}$	1

Assuming that a manager finds three possible causes of decrease in productivity in an enterprise, namely: poor management of human resources (IHR), poor management of innovative technologies (IIT) and poor management of manufacturing operations (IMO). The pairwise comparison matrix with respect to these three causes is shown in Table 1; the AHP is used to rank them. However, they are correlated with each other in practice. For example, IHR and IIT could happen at the same time and have an adverse effect on the enterprise – these two causes may have mutual multiplicative effects ($0 < \lambda < \infty$). Thus, the fuzzy integral AHP is applied in this example to remove this limitation of the traditional AHP. Two approaches: the traditional AHP and the Choquet integral AHP with a new fuzzy measure, are proposed and their results are compared. First, by applying the procedures from Section 2.3, we can obtain the weights of the traditional AHP: $\mathbf{W}^{T3}$= [0.16(IHR), 0.59(IIT), 0.25(IMO)]^t. Next, we assume $\lambda_{ij} = 1$ for $i = j$, $\lambda_{ij} = 0.2$ for $i \neq j$, $0.2 \leq g_1 \leq 0.3$, $0.2 \leq g_2 \leq 0.3$ and $0.3 \leq g_3 \leq 0.4$ to solve problem (12). Thus, we get $\alpha = 0.33$, $g(\mathrm{H}_1) = 0.23$, $g(\mathrm{H}_2) = 0.86$, $g(\mathrm{H}_3) = 1$; therefore, $\mathbf{W}_{T3}^{f'}$ = [0.08(IHR), 0.84(IIT), 0.08(IMO)]^t. Comparing these two models, we see that the traditional AHP doesn't have such a significant impact on IIT as does the Choquet integral AHP with a new fuzzy measure. In other words, this new model has the ability to emphasize the major cause and tends to ignore the less important causes.

The actual relationship between fuzzy densities is an interesting problem, worth exploring further. To summarize, according to our new model, when the relationship between fuzzy densities is assumed, it is possible to trace back the hidden interaction between attributes. In addition, computational difficulties are reduced because our model is the linear problem (12) instead of problem (4).

6 Conclusions and recommendations

In the well-known traditional AHP, the relative weights from Satty (1980) define the core of the problem. The AHP technique is widely and commonly used to choose the best alternative with many competing attributes; however, the interdependencies among the competing attributes are seldom considered (Bortot, Marques Pereira, 2013). Since the substitutive and multiplicative effects, i.e., additivity degrees between attributes surely influence the final decision, a process to implement the AHP accommodating such a realistic situation should be developed. We successfully propose and validate a new fuzzy measure, which is quite simple when it is compared with the traditional fuzzy measure expressed in power form. The way of finding fuzzy density is also simple in this paper.

More advanced topics should be discussed in near future, for example, what is the right/correct assumption of the relationship between fuzzy densities? An evolutionary scheme may be useful to solve this complicated problem by arranging the collected data into a training set and a validation set. Furthermore, how to implement a large scale AHP with many levels in the real world on the basis of the results of this paper? And how to use the Choquet integral AHP in machine learning, knowledge acquisition or data mining? Such problems are interesting.

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A NEW APPROACH TO THE RANK REVERSAL PHENOMENON IN MCDM WITH THE SIMUS METHOD

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Abstract

When a ranking is obtained for a set of projects, the introduction of a new project, worse than the others, may sometimes perturb the ranking. This is called rank reversal, and happens in most Multi Criteria Decision Making models. The purpose of this paper is to demonstrate that a new method, based on Linear Programming, is immune to rank reversal, which is proved by analyzing the algorithm used to solve the problem. The paper also examines a situation that produces rank reversal when two or more projects have close or identical values.

Keywords: projects, ranking, linear programming, Simplex, rank reversal, SIMUS.

1 Introduction

Given a Multiple Criteria Decision-Making (MCDM) scenario, for instance with four projects A-B-C-D, subject to several criteria and solved by any method, the result indicates preference of some projects over others and this preference/equality constitutes a ranking. For instance, in this case the ranking – obtained using any decision-making method – could be: B $\succcurlyeq$ A $\succcurlyeq$ D $\succcurlyeq$ C. The symbol '$\succcurlyeq$' means is preferred to or equal to, or precede; therefore, B is preferred to A, which is preferred to D, which is preferred to C. Rank Reversal (RR) produces changes in the ranking by altering or even reversing the order of preferences. Rank reversal was observed by Belton and Gear (1983) in the Analytic Hierarchy Process (AHP) (Saaty, 1987). Rank reversal is considered undesirable since it shows weakness in the model used for decision-making. Some authors

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suggest that a comparison between different models to determine the most appropriate and reliable one – something that has not been achieved yet – could be made by taking into account robustness and strength that is, preserving ranking stability when the original system of projects is modified by changing the number of projects. Wang and Triantaphyllou (2006) and Maleki and Zahir (2012) performed an exhaustive analysis of the occurrence of RR in different models.

Experience shows that several scenarios may alter a ranking, namely:

1. Adding a worse project.
2. Adding a better project.
3. Adding a project which is nearly or entirely identical to another one.
4. Deleting a project.

The addition of a new project E, worse than any in the ranking, can sometimes disrupt the ranking. Common sense and intuition say that if E is worse than all the others, it should go to the end of the ranking, and then the ordering should not be altered. Conversely, if E is better than all the others, it should go to the top of the ranking, but without altering the order. Neither case produces RR but placing the new project in some other, intermediate position in the ranking may do so.

For instance, the ranking above can be written as E $\succcurlyeq$ B $\succcurlyeq$ A $\succcurlyeq$ D $\succcurlyeq$ C, if E is the best project, or as B $\succcurlyeq$ A $\succcurlyeq$ D $\succcurlyeq$ C $\succcurlyeq$ E if it is the worst one, or as B $\succcurlyeq$ A $\succcurlyeq$ E $\succcurlyeq$ $\succcurlyeq$ D $\succcurlyeq$ C if it is better than D and C. Observe that the ranking preserves the ordering since it has incorporated only the preference of E over D and C. If E is identical to any other element of the original set, its inclusion will not produce RR, and therefore does not influence the ranking. This is what common sense says, but the real-life situation may be different.

There is no doubt about the necessity of determining the causes for this 'phenomenon' and diverse theories have been developed to explain it. Analysis and discussions have been going on for years and different explanations have been given. Let us start here by analyzing a new project or vector whose components are: 1) its contribution relative to the associated cost or benefit (C_j), and 2) its performance values for the set of criteria (a_{ij}).

In this paper four cases are analyzed. The literature on RR asserts that if a worse project is introduced no changes should be produced in the ranking. But how do we define a worse or a better project? This is a fundamental issue but it is not addressed here. From this author's point of view this is the nub of the question, because on what basis can we assert that a project vector is worse or better than others? Noting that a new project has a lesser cost or a larger benefit (C_j) than others is not enough; the (a_{ij}) values also play a very important if not a larger role than (C_j). Comparing the influence of its performance values (a_{ij}), however, is a more complicated issue and not an obvious one, because for a particular criterion a certain value (a_{ij}) of the new project can be better than the corresponding value of other projects, while for another criterion it could be the opposite, taking into account the action of each criterion, of course.

Assume, as usual in Linear Programming (LP), that columns represent projects and rows represent criteria. It can happen, for instance, that for criterion i_3 the performance value (a_{34}) (i.e. the performance value in the third row or criterion (3) and the fourth column or project (4)), is better than any other performance value for this row, while for criterion i_2 it is the opposite. In addition, most models use weights for criteria, and then it may happen that criterion i_3 has more influence than criterion i_2, which can produce a change in the ranking.

According to Wang and Triantaphyllou (2008), a reliable and stable method for decision-making should not produce RR, when subject to any of the three different tests:

Test number 1*:* ”An effective MCDM method should not change the indication of the best project when a non-optimal project is replaced by another worse project (given that the relative importance of each decision criterion remains unchanged”.

Test number 2: ”The rankings of projects by an effective MCDM method should follow the transitivity property”.

Test number 3: ”For the same decision problem and when using the same MCDM method, after combining the rankings of the smaller problems that an MCDM problem is decomposed into, the new overall ranking of the projects be identical to the original overall ranking of the un-decomposed problem”.

Other researchers believe that the most difficult situation appears when two projects have very close performances (or are nearly identical), or when they are identical (see Saaty, 1987; Belton and Gear, 1983). Cascales and Lamata (2012), even assert that ”It is well known that when the projects are very close the order between them can depend on the method used on their evaluation” (see also Li, 2010).

As an example in the case of a maximization criterion, the new project may have a performance that is worse than all of the others with respect to that criterion, or better, or in between. Consequently, stating that a new project vector is worse than those already existent, we mean that all performances with respect to all criteria, as well as the corresponding (C_j), must be worse than the others which in reality is possible but uncommon. Some authors (Wang and Triantaphyllou, 2008) try to analyze this issue by using random numbers in a simulation, which certainly may correspond to reality for a new project vector.

This author's opinion is that there could be situations where the existence of better performances can lead to a change of the ranking – but not to a RR. That is, if a raking shows a D > B > A > C > E > F, the introduction of a new alternative **G**, which is better than **C,** or **G > C**, means that the new ranking will be D > B > A > **G > C >** E > F. As seen, the new ranking does not show changes in the other precedence.

The objective of this paper is to demonstrate that a new model called SIMUS – Sequential Interactive Model of Urban Systems (Munier, 2011a and 2011b; Teames International, 2014), is not subject to RR. To prove this assertion it is necessary to know how this model works, and this is briefly explained in Section 2.

2 The SIMUS model

It is assumed that the reader has some knowledge of the LP technique (Kantorovich, 1939); see MIT (2016) for very clear explanation and examples, as well as Romero and Balteiro (2013). LP is taught in most undergraduate courses on MCDM, and therefore it is not explained here. Instead we provide here a detailed explanation of how SIMUS works. When LP is applied to an initial decision matrix, with the purpose of maximizing or minimizing an objective function, it uses the Simplex algorithm (Dantzig, 1963), which identifies the best solution. This is Pareto efficient, and consequently cannot be improved, that is, it is optimal. The Simplex algorithm is solved, for instance, by the 'Solver' software (FrontLineSystems, 2015), which is used in SIMUS.

As an example, consider three projects subject to five criteria as shown in the initial matrix of Figure 1, a problem that will be solved via the SIMUS model, in order to explain its functioning.

To understand this model it is necessary to take into account that for SIMUS, objective functions and criteria are equivalent, because both are linear functions, and both are subject to maximization, minimization or equalization. Consequently, in the initial matrix all criteria are at some moment used as objective functions. A thorough explanation of SIMUS with many examples can be found in TEAMES International (2004), and downloaded as free fully operational software from decisionmaking.esy.es.

SIMUS starts by using the first criterion as the objective function, by removing it from the decision matrix, and the Simplex algorithm determines the best solution or project, if such a solution exists. This preference is visualized by comparing values or scores that the algorithm assigns to each project (the higher the better). Thus, when the first objective is processed, the result is saved in a matrix called Efficient Results Matrix (ERM) and indicates that project 1 has the score of 0.57, project 2 has the score of 0.91, while project 3 has the score of 0, meaning that this last project is not a part of the solution. Consequently, according to this first objective, the best solution is project 2, although the two scores are Pareto efficient or optimal.

When the second criterion is used as objective function it appears that only project 3 is selected with the score of 1, while the lack of positive values in the other two projects indicates that these are not selected by this objective. The same procedure is followed for criteria 3, 4 and 5 and the respective scores are saved in the ERM matrix. Since criteria may have different units they have to be normalized, and then the normalized Efficient Result Matrix (ERM) is built. Any normalization system can be used, and SIMUS allows to choose from Total sum in a row, Maximum value in a row, Euclidean formula and Min-Max. Whatever the system chosen, the results or ranking are not changed.

The next stage is to add up all values in each column (SC), which gives, for instance, the score of 2.27 for project 2. Note that projects 2 and 3 satisfy three criteria each, while project 1 satisfies only two; their relation with the total number of criteria constitutes the Participation Factor (PF). That is, project 1 has a participation of 2/5 while projects 2 and 3 participate with 3/5 each. This participation is used as a weight for projects since a large number of PF means that the corresponding project satisfies more criteria. These (PF) are then normalized resulting in the Normalized Participation Factor (NPF). This ratio is obtained taking into account the number of values and the number of criteria as mentioned above, thus, for instance, for project 1 it is 2/5 = 0.4.

For each project or column, the (NPF) is then multiplied by the column sum (SC) and its product constitutes the score for that project, as can be seen in the boxed row. The higher the better, consequently, the best project is 2 followed by projects 3 and 1. This allows for building the ERM ranking as depicted. Thus, this result was obtained taking into account for each project its values for all criteria.

In the second stage SIMUS considers the values by row in the ERM matrix, that is, it analyzes for each criterion its values for all projects. Here the model finds the differences with all the other values in the same row, starting from the highest value in the first row. The result is saved in a new square matrix formed by the projects. This new matrix is called Project Dominance Matrix or PDM. The process is repeated for the same row for the next highest value and this procedure is repeated with all the values. That is, the model finds the degree by which a project dominates or outranks another.

Next, all values in a row are added; the result gauges the dominance of a project in that row. Thus, project 1 has the dominance value of 1.9. The same addition is applied to each column, and the model finds the degree by which a project is outranked by or subordinated to another. In this case, project 1 has the subordinate value of 3.2. The net difference for the same project gives the net value as a score. Thus, the score for project 1 is 1.9 – 3.2 = -1.3.

SIMUS orders them in decreasing order and constructs a ranking. Even when scores are different for the same project in ERM and PDM, their rankings coincide, that is: Ranking from ERM = Ranking from PDM.

Consequently, the same problem is solved by **two different procedures** and **the same ranking is obtained.**

Initial Matrix

	Project 1	Project 2	Project 3
Criterion 1	0.23	0.91	
Criterion 2			0.63
Criterion 3		0.50	
Criterion 4	1.18	0.56	1.18
Criterion 5	0.29	1.89	1

After running Simplex with all objectives this is the resulting efficient matrix

Efficient Results Matrix (ERM)

	Project 1	Project 2	Project 3
Criterion 1	0.57	0.91	
Criterion 2			1
Criterion 3		1	
Criterion 4	3.96		0.74
Criterion 5		1	0.53

Efficient Results Matrix (ERM) Normalized

	Project 1	Project 2	Project 3	
Criterion 1	0.38	0.62		
Criterion 2			1	
Criterion 3		1		
Criterion 4	0.84		0.16	
Criterion 5		0.66	0.34	
Sum of Column (SC)	1.22	2.27	1.50	Number of criteria
Participation Factor (PF)	2	3	3	5
Norm. Participation Factor (NPF)	0.40	0.60	0.60	
Final Result (SC x NPF)	**0.49**	**1.36**	**0.90**	

ERM Ranking **Project 2 - Project 3 - Project 1**

Project Dominance Matrix (PDM)

	Subordinated projects			Row sum of	
Dominant proj.	Project 1	Project 2	Project 3	dominant projects	Net dominance
Project 1		0.8	1.1	1.9	-1.3
Project 2	1.9		1.9	3.8	1.8
Project 3	1.3	1.2		2.5	-0.5
Column sum of subordinated projects	3.2	2.0	3.0		

PDM Ranking **Project 2 - Project 3 - Project 1**

Figure 1. Initial matrix, and ERM and PDM matrices in the SIMUS method

3 Why SIMUS does not produce rank reversal?

The simple and straight answer is: because it is based on the Simplex algorithm that does not allow it. To understand this very important algorithm, consider the following problem:

Table 1 shows the initial data of an example, which consists in selecting the best project of a renewable energy power plant using one of two sources of renewable energy: Solar energy (x_1) and Photovoltaic (x_2). Its elements are:

Table 1: Initial data

	Projects or projects				
	Solar energy x_1	Photovoltaic x_2			
Unit cost (Cj)	0.72	0.68	**Objective function Z = 0.72 x_1 + 0.68 x_2 (MIN)**		
Criteria	Project's or projects' contributions (aij) or performances		Action	Action operator	Constant value (B)
Efficiency index	0.85	0.75	MAX	≤	1
Financial index	0.78	0.98	MIN	≥	0.84
Land use index	0.92	0.65	MAX	≤	0.94
Generation index	0.99	0.60	MIN	≥	0.80

Z is the objective function minimizing the total unit cost. Its equation is $Z = 0.72, x_1 + 0.68\ x_2$.

Cj: Unit cost related to each project.

0.72: (C_1), unit cost for project x_1.

0.68: (C_2), unit cost for project x_2.

a_{ij}: Values corresponding to alternatives x_1 and x_2 for all criteria. The problem consists in determining the values of x_1 and x_2 that satisfy the objective function.

The simplex algorithm starts with this initial matrix arranged in the form of a tableau as shown in Table 2.

Table 2: First Simplex tableau

Cj			0.72	0.68	0	0	0	M	M	Ratio	
	Basic	bi	x1	x2	s1	s2	s3	A1	A2		
0	s1	1	0.85	**0.75**	1					1.18	
M	A1	0.84	0.78	0.98		-1		1		0.93	
0	s3	0.94	0.92	0.65			1			1.02	
M	A2	0.8	**0.99**	**0.6**					1	0.81	Key ro
Objective function	Zj	**1.64M**	1.77M	1.35M	0	0	0	M	M		
Index row		Cj - Zj	0.72-1.77M	0.68-1.75M	0	0	0	0	0		
			Key column								

The tableau includes artificial variables Aj for minimization (using the ≥ operator in the corresponding equation), with a very large cost value M, and slack variables sj for maximization (using the ≤ operator), to convert the inequalities to equations, and with cost values equal to '0'. At the beginning of the computation the objective function Z = 1.77M, that is (0.78M + 0.99M), which is extremely high and corresponds to artificial variables or projects A1 and A2, both of which constitute the initial solution of the problem. This is the starting point for the computation. To improve this performance the Simplex algorithm uses two indexes: The Index row (Cj-Zj) and the Key row (bi/aij), where b_i is the right hand side value for the ith criterion.

The first index selects the variable to be entered into the system to improve the solution, that is, to decrease the cost. This is obtained by selecting the most negative value in the index row (0.72-1.77M); in this case the most negative value is related to alternative or project x_1 (Solar energy). The corresponding column (shaded), is called Key column. To preserve the dimensions of the problem (this is a two dimensional problem, because we have two projects), it will be now necessary to eliminate one of the artificial projects. This is done by using the key row (shaded), which indicates that A2 must be eliminated – see Chinneck (2000), for a justification. In the next step the algorithm recalculates the complete matrix, because the basis has changed, and we get the second Simplex tableau shown in Table 3.

Table 3: Second Simplex tableau

Cj			0.72	0.68	0	0	0	M	M	Ratio	
	Basic	bi	x_1	x_2	s_1	s_2	s_3	A_1	A_2		
0	s_1	0.31		0.23	1				-0.86	0.36	
M	A_1	0.21		**0.51**		-1		1	-0.79	0.27	Key row
0	s_3	0.00		0.09			1		-0.93	-0.19	
0.72	x_1	0.81	1	0.61					1.01	-0.80	
Objective function	Zj	**0.21M**	0.72	0.51M+0.44	0	-M	0	M	0.79M		
Index row	Cj - Zj		0	0.44-0.51M	0	M	0	0	0.21M		
				Key column							

Note that project x_1 is now a unit vector, and therefore is in the basis. The objective function is now: Zj = 0.51M + 0.44, that is (0.51xM + 0.61 x 0.72), which is still a very high value, but considerably less than the first one. Therefore the cost has been reduced. The process is now repeated, i.e, the algorithm looks for the most negative Cj-Zj value and finds that it corresponds to project 2 (Photovoltaic) (0.44-0.51M). The key row index is applied again and then the artificial project A_1 is removed. The process continues until there is no more negative Cj-Zj, as shown in Table 4. As can be seen there are no more negative Cj-Zj and this indicates that the final and optimal solution has been reached with x_1 (Solar) = 0.56 and x_2 (PV) = 0.41.

Table 4: Third Simplex tableau

Cj			0.72	0.68	0	0	0	M	M
	Basic	bi	x_1	x_2	s_1	s_2	s_3	A_1	A_2
0	s_1	0.22			1	0.46		-0.46	
0.68	x_2	**0.41**		1		-1.97		1.97	-2
0	s_3	0.00				0.18	1	-0.09	
0.72	x_1	**0.56**	1			1.19		-1.19	
Objective function	Zj	**0.68**	0.72	0.68	0	-0.48	0.00	0.68	0.73
	Index row	Cj - Zj	0	0	0	0.48	0	M-0.68	M-0.73

The process has been explained in some detail to show how the Simplex always selects a better project, based on its Cj and its aij values (from Zj). In a more complicated scenario, the number of projects and criteria is irrelevant**; the Simplex will select only those projects that improve previous solutions**; consequently, it is impossible to select a project that does not satisfy this condition.

3.1 Adding a new project

Let us see now how the system reacts when a new project is introduced about which we do not know if it is better or worse than the existing projects. Of course, with the introduction of this new project, the **original problem with *n* projects has changed, and so it is a new one.** The new problem will have $n + 1$ projects, but the same rules apply. Assume that to our original problem with two projects we add a third one (x_3). If we apply the Simplex to this new problem the algorithm will perform as before when there were two projects. Consequently, if C_3-Z_3 of the new project is positive respecting to C_1-Z_1 and C_2-Z_2, this **new project will never be selected.** This is the reason why no rank reversal can be produced in SIMUS. However, if the cost of opportunity of x_3 is better than the cost of opportunity of x_1 and x_2, then the new project will be selected as the best project in the ranking. Naturally, this is not rank reversal, but the result of introducing a new project that is better than the existent ones. However, even in this last case the original order in the ranking must be preserved. A complete and thorough explanation of the Simplex Tableau is found in Kothari (2009) and in MIT (2016).

3.2 Adding an exact copy of an existing project

According to some researchers the most likely scenario for RR is when two projects (or the existing one and a new vector) are nearly or entirely identical. In this section we analyze this case and demonstrate that SIMUS is immune to this phenomenon. For instance, we can introduce a new project x_3 identical to x_1 to our solar power problem (both are shaded in Table 5).

Table 5: First Simplex tableau – introducing x_3 identical to x_1

Cj				0.72	0.68	0.72	0	0	0	0	M	Ratio	
		Basic	bi	x_1	x_2	x_3	s_1	s_2	s_3	A_1	A_2		
	0	s_1	1	0.85	0.75	0.85	1					1.18	
	M	A_1	0.84	0.78	0.98	0.78		-1			1	0.93	
	0	s_3	0.94	0.92	0.65	0.92			1			1.02	
	M	A_2	0.8	0.99	0.6	0.99				-1		0.81	Key row
Objective function		Zj	**1.64M**	1.77M	1.35M	1.77M	0	0	0	-M	M		
Index row		Cj - Zj		0.72-1.77M	0.68-1.75M	0.72-1.77M	0	0	0	M	0		
				Key column									

This demonstrates that if we have two identical vectors as projects, the system considers only one of them and rejects the other one. Consequently the ranking is preserved.

According to the rule it will be now logical to introduce x_1 or x_3 since both have the largest negative value. We can introduce either, because when transformed they will both be basic variables, but only one of them will be in the solution (see Table 6).

Table 6: Second Simplex tableau

Cj				0.72	0.68	0.72	0	0	0	0	M	Ratio	
		Basic	bi	x_1	x_2	x_3	s_1	s_2	s_3	A_1	A_2		
	0	s_1			0.23		1				-0.86	0.36	
	M	A_1			**0.51**			-1		1	-0.79	0.27	Key row
	0	s_3			0.09				1		-0.93	-0.19	
	0.72	x_1		1	0.61	1					1.01	-0.80	
Objective function		Zj	**0.21M**	0.72	0.51M+0.44	0.72	0	-M	M	M	0.79M		
Index row		Cj - Zj		0	0.44-0.51M	0	0	M	0	0	0.21M		

Continuing with the process we must select x_2 as the entering variable and criterion 2 (A_1) (shaded), as the leaving variable. The transformation values are in Table 7.

Table 7: Third Simplex tableau

Cj				0.72	0.68	0.72	0	0	0	M	M
		Basic	bi	x_1	x_2	x_3	s_1	s_2	s_3	A_1	A_2
	0	s_1					1	0.46		-0.46	
	0.68	x_2	**0.41**		1			-1.97		1.97	-2
	0	s_3						0.18	1	-0.09	
	0.72	x_1	**0.56**	1		1		1.32		-1.19	
Objective function		Zj	**0.68**					-0.48	0	0.08	1
Index row		Cj - Zj		0	0	0	0.48	0	0	M-0.68	M-0.73

The outcome has the same values as before and the same ranking.

3.3 Demonstration of absence of RR in SIMUS when more than a single project is added

Starting from an initial problem several scenarios are proposed. Note that these involve much stricter conditions as those found in the literature on RR where, in general only one scenario is examined at a time, while here we are using more than one and even mixing different scenarios.

3.4 Solving a problem with SIMUS software

Assume the initial matrix shown in Table 8 with five projects (in bold case) is given. Projects 6, 7 and 8 are added later. The system uses Euclidean normalization but, as mentioned before, any other can be used. This case is solved using SIMUS software and the result is shown on the last screen (Figure 2). Observe that SIMUS provides two solutions in its ERM and PDM matrices. The ERM solution is found in the solid black row while the PDM solution is in the solid black PDM column. Note, however, that both ERM and PDM rankings are identical.

Table 8: Initial decision matrix with five projects

Initial decision matrix Added project						**Added project**	**Added projects**	
Project 1	**Project 2**	**Project 3**	**Project 4**	**Project 5**	Project 6 **worse** than any original	Project 7 **better** than any original	Project 8 = Project 3	**Action**
6200	**6050**	**4800**	**5100**	**3800**	3600	6500	4800	**MAX**
3	**4.2**	**2.5**	**6.1**	**3.10**	2.4	6.5	2.5	**MAX**
20	**20**	**21**	**30**	**32**	35	18	21	**MIN**
4	**3**	**2.5**	**3**	**5**	2.4	5.5	2.5	**MAX**

The result is: **4 ≽ 5 ≽ 3 ≽ 2 ≽ 1.**

Efficient Results Matrix (ERM) Normalized

	Project 1	Project 2	Project 3	Project 4	Project 5	
Target 1		0.32	0.68			
Target 2				1.00		
Target 3						
Target 4					1.00	
Sum of Column (SC)	0.00	0.32	0.68	1.00	1.00	Number of targets
Participation Factor (PF)	0	1	1	1	1	4
Norm. Participation Factor (NPF)	0.00	0.25	0.25	0.25	0.25	
Final Result (SC x NPF)	0.00	0.08	0.17	0.25	0.25	

ERM Ranking Project 4 - Project 5 - Project 3 - Project 2 - Project 1

Project Dominance Matrix (PDM)

	Dominated projects					Row sum of	
Dominant proj.	Project 1	Project 2	Project 3	Project 4	Project 5	dominant projects	Net dominance
Project 1						0.0	-3.0
Project 2	0.3			0.3	0.3	0.9	-1.4
Project 3	0.7	0.4		0.7	0.7	2.4	0.4
Project 4	1.0	1.0	1.0		1.0	4.0	2.0
Project 5	1.0	1.0	1.0	1.0		4.0	2.0
Column sum of dominated projects	3.0	2.4	2.0	2.0	2.0		

PDM Ranking Project 4 - Project 5 - Project 3 - Project 2 - Project 1

Figure 2. The original problem – the final screen of SIMUS showing the results for the initial set of projects (5)

3.4.1 Adding project 6 ‘worse’ than the others

Now we add project 6, which is obviously worse than any other since its performances are lower in maximization and higher in minimization. Figure 3 shows the result, which as can be seen replicates the ranking of the situation with only five projects. Project 6 is added but with ‘0’ score’ in the ERM matrix, meaning that it is not considered.

Output: Original ranking preserved.

Efficient Results Matrix (ERM) Normalized

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6		
Target 1		0.32	0.68					
Target 2				1.00				
Target 3								
Target 4					1.00			
Sum of Column (SC)	0.00	0.32	0.68	1.00	1.00	0.00		Number of targets
Participation Factor (PF)	0	1	1	1	1	0		4
Norm. Participation Factor (NPF)	0.00	0.25	0.25	0.25	0.25	0.00		
Final Result (SC x NPF)	0.00	0.08	0.17	0.25	0.25	0.00		

ERM Ranking Project 4 - Project 5 - Project 3 - Project 2 - Project 1 - Project 6

Project Dominance Matrix (PDM)

Dominant proj.	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Row sum of dominant projects	Net dominance
	Dominated projects							
Project 1							0.0	-3.0
Project 2	0.3			0.3	0.3	0.3	1.3	-1.1
Project 3	0.7	0.4		0.7	0.7	0.7	3.1	1.1
Project 4	1.0	1.0	1.0		1.0	1.0	5.0	3.0
Project 5	1.0	1.0	1.0	1.0		1.0	5.0	3.0
Project 6							0.0	-3.0
Column sum of dominated projects	3.0	2.4	2.0	2.0	2.0	3.0		

PDM Ranking Project 4 - Project 5 - Project 3 - Project 2 - Project 1 - Project 6

Figure 3. The original problem with ‘worse’ project 6 added

3.4.2 Adding project 7 keeping project 6 and with $x_3 = x_6 = x_7$

Now we keep project 6 identical to project 3 and add project 7 also identical to 3 and 6 (see Figure 4). The result is again the same ranking: 4 $\succcurlyeq$ 5 $\succcurlyeq$ 3 $\succcurlyeq$ 2 $\succcurlyeq$ 1. Note that project 6 and 7 have ‘0’ scores.

Output: Original ranking preserved.

Efficient Results Matrix (ERM) Normalized

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7
Target 1		0.32	0.68				
Target 2				1.00			
Target 3							
Target 4					1.00		
Sum of Column (SC)	0.00	0.32	0.68	1.00	1.00	0.00	0.00
Participation Factor (PF)	0	1	1	1	1	0	0
Norm. Participation Factor (NPF)	0.00	0.25	0.25	0.25	0.25	0.00	0.00
Final Result (SC x NPF)	0.00	0.08	0.17	0.25	0.25	0.00	0.00

ERM Ranking Project 4 - Project 5 - Project 3 - Project 2 - Project 1 - Project 6 - Project 7

Project Dominance Matrix (PDM)

Dominant proj.	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Row sum of dominant projects	Net dominance
	Dominated projects								
Project 1								0.0	-3.0
Project 2	0.3			0.3	0.3	0.3	0.3	1.6	-0.8
Project 3	0.7	0.4		0.7	0.7	0.7	0.7	3.8	1.8
Project 4	1.0	1.0	1.0		1.0	1.0	1.0	6.0	4.0
Project 5	1.0	1.0	1.0	1.0		1.0	1.0	6.0	4.0
Project 6								0.0	-3.0
Project 7								0.0	-3.0
Column sum of dominated projects	3.0	2.4	2.0	2.0	2.0	3.0	3.0		

PDM Ranking Project 4 - Project 5 - Project 3 - Project 2 - Project 1 - Project 6 - Project 7

Figure 4. Adding projects 6 and 7 identical to project 3 simultaneously

3.4.3 Adding a new project identical to another one and simultaneously adding one regarded as the best

We add project 6 which is identical to project 3 and also add project 7 which is regarded as the best of all (see Figure 5). The result is: $4 \succcurlyeq 5 \succcurlyeq 3 \succcurlyeq 2 \succcurlyeq 1$.

Output: Original ranking preserved.

Efficient Results Matrix (ERM) Normalized

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7
Target 1		0.04	0.96				
Target 2				1.00			
Target 3							
Target 4					1.00		
Sum of Column (SC)	0.00	0.04	0.96	1.00	1.00	0.00	0.00
Participation Factor (PF)	0	1	1	1	1	0	0
Norm. Participation Factor (NPF)	0.00	0.25	0.25	0.25	0.25	0.00	0.00
Final Result (SC x NPF)	0.00	0.01	0.24	0.25	0.25	0.00	0.00

ERM Ranking Project 4 - Project 5 - Project 3 - Project 2 - Project 1 - Project 6 - Project 7

Project Dominance Matrix (PDM)

Dominant proj.	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Row sum of dominant projects	Net dominance
	Dominated projects								
Project 1								0.0	-3.0
Project 2	0.0			0.0	0.0	0.0	0.0	0.2	-2.7
Project 3	1.0	0.9		1.0	1.0	1.0	1.0	5.7	3.7
Project 4	1.0	1.0	1.0		1.0	1.0	1.0	6.0	4.0
Project 5	1.0	1.0	1.0	1.0		1.0	1.0	6.0	4.0
Project 6								0.0	-3.0
Project 7								0.0	-3.0
Column sum of dominated projects	3.0	2.9	2.0	2.0	2.0	3.0	3.0		

PDM Ranking Project 4 - Project 5 - Project 3 - Project 2 - Project 1 - Project 6 - Project 7

Figure 5. Project 6 identical to project 3 and project 7 regarded as the best

3.4.4 Deleting a project from the original portfolio

We are deleting Project 3 (see Figure 6). The result is: 4 ≽ 5 ≽ 2 ≽ 1.

Output: Original ranking preserved.

Efficient Results Matrix (ERM)

	Project 1	Project 2	Project 3	Project 4	Project 5
Target 1	0.35	1.21			
Target 2				1.22	
Target 3					
Target 4					1.63

Efficient Results Matrix (ERM) Normalized

	Project 1	Project 2	Project 3	Project 4	Project 5
Target 1	0.22	0.78			
Target 2				1.00	
Target 3					
Target 4					1.00
Sum of Column (SC)	0.22	0.78	0.00	1.00	1.00
Participation Factor (PF)	1	1	0	1	1
Norm. Participation Factor (NPF)	0.25	0.25	0.00	0.25	0.25
Final Result (SC x NPF)	0.06	0.19	0.00	0.25	0.25

ERM Ranking Project 4 - Project 5 - Project 2 - Project 1 - Project 3

Project Dominance Matrix (PDM)

	Dominated projects						
Dominant proj.	Project 1	Project 2	Project 3	Project 4	Project 5	Row sum of dominant projects	Net dominance
Project 1			0.2	0.2	0.2	0.7	-1.9
Project 2	0.6		0.8	0.8	0.8	2.9	0.9
Project 3						0.0	-3.0
Project 4	1.0	1.0	1.0		1.0	4.0	2.0
Project 5	1.0	1.0	1.0	1.0		4.0	2.0
Column sum of dominated projects	2.6	2.0	3.0	2.0	2.0		

PDM Ranking Project 4 - Project 5 - Project 2 - Project 1 - Project 3

Figure 6. Deleting project 3

4 Summary of scenarios and results

Table 9 summarizes our findings.

Table 9: Summary of results for different scenarios

		Ranking	Comments	Result referred to ranking
Original project				
	Original result taking into account only **five** projects or projects	**4 ≽ 5 ≽ 3 ≽ 2 ≽ 1** **Figure 2**		Initial ranking
Adding one project to original scenario				
Adding project 6 ‘worse’ than others	Project added has worse values in all criteria than all other projects	**4 ≽ 5 ≽ 3 ≽ 2** ≽ 1 ≽ 6 **Figure 3**		Ranking preserved

Table 9 cont.

Adding identical projects to original scenario				
Projects 6 and 7 identical to project 3	Simultaneous addition plus identity with an existing project	**4 ≽ 5 ≽ 3 ≽ 2** ≽ 1 ≽ 6 ≽ 7 **Figure 4**	Project 6 and 7 are not considered since their score is '0'	Ranking preserved
Adding an identical project and at the same time adding another one which is regarded as the best				
Project 6 identical to project 3 and project 7 regarded as the best	Simultaneous **addition** of one project identical to another existent one plus addition of another project regarded as the best	**4 ≽ 5 ≽ 3 ≽ 2** ≽ 1 ≽ 6 ≽ 7 **Figure 5**	Project 6 and 7 are not considered since their score is '0'	Ranking preserved
Projects deletion from the original scenario				
Delete project 3		**4 ≽ 5 ≽ 2 ≽ 1** ≽ 3 **Figure 6**	Project 3 is eliminated since its value is '0'	Ranking preserved

5 Conclusion

The goal of this paper is to demonstrate that when LP is used for decision-making no RR occurs. This was shown by examining the original algebraic procedure of the Simplex algorithm created by Dantzig (1963). It clearly reveals that the incorporation of a new project regarded as worse than the existing projects cannot alter the ranking because the algorithm takes into account both the contribution (cost or benefit) and the performances of the new project. To put it simply, the algorithm works by analyzing and comparing opportunity costs, minimizing or maximizing them. It is a well-known fact that RR occurs also when a project is deleted from the scenario, or when two projects are nearly or entirely identical. These two scenarios were also examined in this paper by modifying the original problem and solving each using SIMUS. Four different scenarios were considered with more than one project added at the time and also including projects with identical data. The author believes that the algebraic analysis performed and the examples proposed validate our claim.

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APPLICATION OF THE MARS METHOD TO THE EVALUATION OF GRANT APPLICATIONS AND NON-RETURNABLE INSTRUMENTS OF START-UP BUSINESS FINANCING[1]

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Abstract

This paper discusses the issue of evaluation of grant and loan proposals submitted by start-up businesses. A multi-criteria model for the evaluation of proposals for start-up business financing is proposed, based on the MARS method and taking into account three criteria: professional experience of the person planning to start a business, evaluation of the business plan, and evaluation of credit history of the applicant. Modelling of the expert's preferences was based on verbal comparisons of decision variants from the reference set consisting of solutions close to the ideal solution. The usefulness of the model has been verified using data from loan applications submitted to the Business Friendly Fund, operating in one of cooperative banks in the Podlaskie voivodeship.

Keywords: MARS, MACBETH, ZAPROS, credit application, start-up business financing, holistic approach.

1 Introduction

The development of the business sector, especially of Small and Medium-sized Enterprises (SMEs) and of micro-enterprises is an important factor affecting the financial situation of countries. According to a report of the European Commis-

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sion (www 2), the SME sector, with the exclusion of the financial branch, constituted 99.8% of all enterprises in the European Union (the EU-28 countries). For each square kilometer of the current area of the EU there are five enterprises from this sector, and almost 90 million people are employed in this sector, which constitutes 67% of the total employment. The SME sector creates 58% of the added value in the EU. According to the statistics of the World Bank (www 3), the SME sector creates 45% employment in the world and 33% of the world added value. If we take into account businesses from the "grey zone" (non-registered businesses), the numbers are even larger. The World Bank estimates that within the next 15 years this sector will contribute to the creation of 600 million work places in the entire world and points out that the main problem in the development of this sector is the lack of access to the financing of the investments. Fifty percent of businesses from SME in the world have problems with obtaining bank credit, and this number grows to 70% if we take into account non-registered businesses. These data show the very important role of the micro-enterprises and SMEs in the development of the world economy, while it is also pointed out that the main obstacle in the development of this sector is the lack of access to capital for new investments (Beck, Demirguc-Kunt, Martinez Peria, 2011).

One of measures of investment appeal of a country or region is the balance of newly opened and closing businesses. In 2014 in Poland, the number of newly created enterprises was almost twice as high as compared with 2003-2005[2], which is related, to a large extent, with the distribution of EU funds in Poland. Start-up businesses can now be financed using various types of financial instruments, such as grants or preferential loans within various assistance programs. Since a start-up is a new entity, which has no history of its operation, evaluation of its applications for funds is a difficult task. The problem of evaluation of such applications can be regarded as a weakly or non-structured multiple criteria problem, with incomplete or imprecise available preference information and with data of various types, and such that assessing the application requires expert knowledge. Several tools which can be used to solve this problem can be found among methods of multi-criteria analysis of decision problems (Figueira, Greco, Ehrgott, 2005; Roy, 1990; Trzaskalik, 2014). Among the applicable methods are: fuzzy methods, such as fuzzy TOPSIS or fuzzy SAW (Chen, Hwang, 1992) which take into account incomplete information and allow to handle data of various types; fuzzy methods based on linguistic approach (Herrera, Alonso, Chiclana, Herrera-Viedma, 2009; Herrera, Herrera-Viedma, 2000); methods us-

[2] Data from GUS BDL.

ing verbal scores, e.g., MACBETH (Bana e Costa, Vansnick, 1999), ZAPROS (Larichev, Moshkovich, 1997), or preference information given indirectly in the form of decision examples for a reference set of decision variants, such as UTA (Siskos, Grigoroudis, Matsatsinis, 2005; Jacquet-Lagrèze, Siskos, 1982; 2001), GRIP (Figueira, Greco, Słowiński, 2009; Greco, Mousseau, Słowiński, 2008), MARS (Górecka, Roszkowska, Wachowicz, 2014, 2016; Roszkowska, Wachowicz, 2015), fuzzy modeling (Jagielska, Matthews, Whitfort, 1999). Also, the theory of rough sets is used in research on risk involved in start-up business financing (Pawlak, 1982). The decision problem consisting in granting or not granting funding can be represented using a decision system in which the conditional attributes are variables from the model, and the conclusion (the system decision) is a dichotomic variable denoting a "good" client and a "bad" one (Medina, Cueto, 2013). Fuzzy concluding can be a useful tool in the assessment of risk involved in starting an individual business, where those assessing a grant or loan application have limited information on both the applicant and the microeconomic environment of the future businessperson (Konopka, 2013).

In the present study the MARS method (Górecka, Roszkowska, Wachowicz, 2014) is used to solve the problem of evaluating grant and loan applications of start-up businesses. A multi-criteria model of evaluating applications of this type is used, with three criteria: professional experience of the person planning to start the business, evaluation of the business plan for the start-up, and assessment of the applicant's credit history. The MARS (*Measuring Attractiveness near Reference Solutions*) method, which is a hybrid of the methods ZAPROS and MACBETH, is used to aggregate those criteria, and therefore to classify and evaluate the applications on the basis of verbal assessments by experts. The usefulness of the model proposed has been verified using data from loan applications from the Business-Friendly Fund in one of the largest cooperative banks in the Podlaskie voivodeship.

The paper consists of six sections. In Section 2 the problem of start-up business financing in Poland is presented. Section 3 points out the specific nature of the evaluation of the applications for start-up business financing, with particular emphasis on the assessment of risk involved in the evaluation of such applications; included is also a justification of the choice of the MARS method for the construction of a multi-criteria model of such applications. Section 4 presents the basic assumptions of the MARS method. A theoretical model of risk assessment of start-up business financing based on this method is presented in Section 5. The usefulness of this model has been verified using data from loan applications from the Business-Friendly Fund in one of the largest cooperative banks in the Podlaskie voivodeship. The last section presents conclusions.

2 Start-up business financing

The economic development of a country depends to a large extent on the business sector. According to the statistics of GUS (Central Statistical Office, Poland), the SME sector contributes ca. 48% of Poland's GNP (Raport o stanie sektora…, 2014) and employs 6.2 million of working population of Poland. From the time of Poland's accession to the EU, the number of newly created enterprises has grown and at present it is equal to ca. 400 thousand annually. This index has increased almost twice as compared with the years 2003-2005. One should stress, however, that the number of closed down businesses also increased. The phenomenon of increasing appeal of starting and conducting one's own business is related, to a large extent, to the access to capital for investments or with funding for start-up businesses. Non-returnable grants or returnable capital with preferential interest rates stimulate entrepreneurship of both small business and large enterprises. At the moment, 30% of newly created businesses in Poland close down within the first year of operation.

In the case of financing a start-up business with non-returnable grants, this index grows (in the Podlaskie voivodeship, for instance, it is ca. 50%). Two years after the financing with a non-returnable grant, 70% businesses created this way close down. This is a relatively large number, resulting both from the circumstances in which businesses operate in Poland and from the lack of professional experience which could be used in managing a business independently. A start-up business is by definition an enterprise with a high probability of failure, particularly vulnerable to various risk factors: those related to business climate and market, political and system-related, socio-demographic, and technical (De Servigny, Renault, 2004). From this discussion it follows that one should search for tools for the evaluation of grant applications and returnable instruments of start-up business funding which take into account the specific nature of creating and operating a start-up business. An apt decision as to granting funding to a business is also simply in the public interest.

3 Assessment of risk involved in financing a start-up business

Assessment of risk involved in financing a business is closely related to the assessment of the creditworthiness of the business. Creditworthiness is here understood as the legal and financial ability to take out and repay credit instruments on time (Cleary, 1999). The relationship between the credit risk involved in granting credit and creditworthiness of a business can be expressed as follows: the greater the creditworthiness of a business, the smaller the risk involved in financing the business. The assessment of creditworthiness of an existing business is based on an analysis of the current and past financial condition of the business, including its financial results, balance analysis, and cash flow analysis; analysis of the business plan of the enterprise to be financed; security analysis, as well as legal analysis of the investment. To simplify and shorten the time of the evaluation of applications, in the case of businesses already existing, credit scoring methods are used (Altman, Sabato, Wilson, 2010; Altman, Sabato, 2007; Thomas, Edelman, Crook, 2002). Risk analysis of existing businesses is a difficult problem which becomes even more difficult in the case of the evaluation of a start-up business. Commercial banks in Poland do not, by definition, grant financial assistance to businesses which have not been operating for at least 6 to 12 months. Hence typical commercial solutions for the assessment of start-up risk, such as assessment of financial condition by means of Altman's model (Altman, 1968), are lacking. Lack of available information on the history of business operations is the key factor complicating the evaluation of a credit application.

As mentioned above, the decision to finance or to refuse financing a start-up business should be based on objective, accessible information, that is, on information on the professional experience of the applicant, on the business plan of the start-up and on information from the BIK, BIG, and KRD databases. This list does not include information on financial security of the start-up business which should be, because of increased risk, a binary variable. Since the information obtained is mostly qualitative, declared by the applicant him- or herself[3], this knowledge should be regarded as incomplete and uncertain. Therefore, the assessment of start-up business financing can be regarded as a unique problem, weakly structured or non-structured, requiring expert knowledge, and based to a large extent on verbal scores in decision making (Larichev, Moshkovich, 1995; Nemery, Ishizaka, Camargo, Morel, 2012).

These assumptions justify the choice of the MARS method, which is a hybrid of the ZAPROS and MACBETH methods (Górecka, Roszkowska, Wachowicz, 2014) for the solution of this problem. In the MARS method, as in the MACBETH method (Bana e Costa, Vansnick, 1999) verbal scores are used to compare decision variants from the given reference set. Next, these scores are used to aggregate the criteria, and therefore to classify and evaluate the applications.

4 General assumptions of the MARS method

The MARS method (*Measuring Attractiveness near Reference Solutions*) (Górecka, Roszkowska, Wachowicz, 2014; 2016) is based on two methods: ZAPROS (acronym of the Russian name *Closed Procedures near Reference*

[3] The applicant should submit and sign a statement certifying, under penalty of perjury, that the information in the documents presented are factually correct.

Situations) (Larichev and Moshkovich, 1995) and MACBETH (*Measuring Attractiveness by a Categorical Based Evaluation Technique*) (Bana e Costa, Vansnick, 1999) and allows to completely rank decision variants evaluated on an interval scale. It is based on the disaggregation-aggregation paradigm (Greco, Mousseau, Słowiński, 2008), which means that a pre-order is created on the set of reference variants, and then assessment is made on the basis of this information. Next, a ranking of decision variants, defined on the entire set, is created. The order on the reference set is constructed using verbal scores on a 6-degree semantic scale; quantitative information on the characteristics of the decision variants evaluation is not used.

The following notation is used:

- $F = \{f_1, f_2, \ldots, f_n\}$ is the set of criteria,
- X_k is a finite set of verbal scores with respect to kth criterion, $k = 1, 2, \ldots, n$, where $|X_k| = n_k$,
- $X = \prod_{k=1}^{n} X_k$ is the set of all possible vectors in the n-criteria space,
- $Y \subseteq X$ is the reference set of vectors, that is, the set of vectors whose all components except one have the best values possible, and the vector whose all components have the best values possible.

The MARS procedure consists of the following stages (Górecka, Roszkowska, Wachowicz, 2014; 2016):

Stage 1. Determination of the ordering scales for all the criteria considered in the decision problem.

Stage 2. Pairwise comparison of hypothetical vectors from the set $Y \subseteq X$, whose all components except one have the best values possible, with a vector whose all components have the best values possible.

The comparison consists in the qualitative assessment of the difference in attractiveness between two vectors from the reference set using six semantic categories: d_1 – the difference in attractiveness between the vectors is "very small", d_2 – "small", d_3 – "moderate", d_4 – "large", d_5 – "very large", and d_6 – "extremely large". Pairwise comparisons are performed using the M-MACBETH program which additionally verifies the consistency of the information given by the decision maker, suggesting changes in the case of inconsistency (www 1).

Stage 3. Solution of the PL-MACBETH problem and determination of point scores from 0 to 100 for the decision variants compared.

To solve the linear programming problem PL we can use the M-MACBETH program.

Stage 4. Determination of the final scores of decision variants and their ordering with respect to the ideal variant.

The final scores of decision variants L_i for $i = 1, 2, \ldots, m$ are calculated as follows: As the score in the decision variant we take the point score p_{ik} from the

0-100 scale assigned to the options within each criterion. Next, the distance L_i from the ideal variant is calculated as follows:

$$L_i = \sum_{k=1}^{n} (100 - p_{ik}) \tag{1}$$

where p_{ik} is the point score of ith alternative with respect to kth criterion, $k = 1, 2, \ldots, n$, $i = 1, \ldots, n_k$.

The decision variants are sorted in increasing order according to their distance from the ideal variant. The best variant is that for which the final score is lowest.

The last stage is the determination of the normalized distance $L_i(norm)$.

Stage 5. Normalization of the final scores of decision variants follows the formula:

$$L_i(norm) = \frac{L_i}{\max\limits_i L_i} . \tag{2}$$

where $0 \leq L_i(norm) \leq 1$.

5 A model of risk assessment involved in start-up business financing based on the MARS method

The starting point in the construction of our model was the assumption of the criteria for the evaluation of credit applications and the determination of their scope taking into account the specific nature of granting credit to start-up businesses, as well as the possibilities of obtaining relevant information. Three criteria were taken into account in the model, related to: professional experience of the applicant, the business plan of the start-up, and the banking history of the applicant.

During the interview with the coordinator and with experts on risk who evaluate credit applications for the Business-Friendly Fund (an interview with three people), levels of criteria implementation have been determined and described verbally. Table 1 presents the set of criteria for the evaluation of credit applications, developed on the basis of the interview together with evaluation scales for each criterion.

Table 1: Description of the criteria and scales used in the model

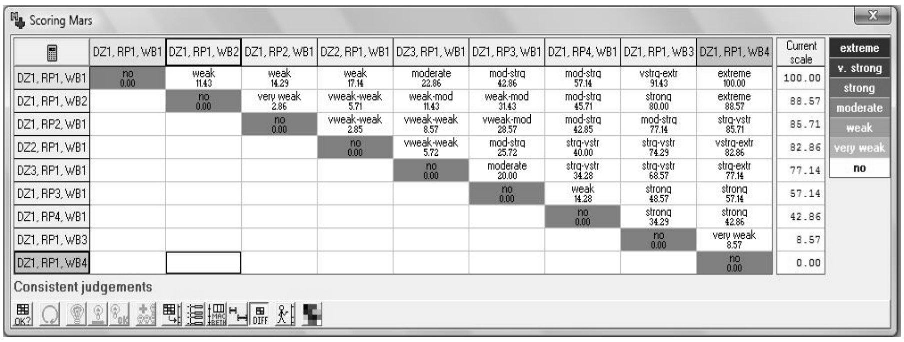
Criterion	Characteristic	Evaluation scale
f_1	Professional experience (DZ)	DZ1: Fully consistent with the business idea
		DZ2: Has at least one year of experience in the relevant industry
		DZ3: No professional experience relevant for the business idea
f_2	Feasibility of the business idea (RP)	RP1: Cautious and realistic assumptions
		RP2: Assumptions too optimistic, but realistic even in an unfavorable business climate
		RP3: Assumptions realistic in an exceptionally favorable business climate
		RP4: Unrealistic financial and business assumptions
f_3	Credit history (WB)	WB1: The applicant has credit obligations without delinquencies
		WB2: The applicant has no credit obligations or has obligations with delinquencies not exceeding 10 days
		WB3: The applicant has credit obligations with delinquencies of 10 to 30 days
		WB4: The applicant has credit obligations with delinquencies exceeding 30 days

Source: Authors' own elaboration.

The reference set X consists of nine variants: (DZ1, RP1, WP1), (DZ1, RP1, WP2), (DZ1, RP1, WP3), (DZ1, RP1, WP4), (DZ1, RP2, WP1), (DZ1, RP3, WP1), (DZ1, RP4, WP1), (DZ2, RP1, WP1), (DZ1, RP1, WP1).

In the next step, according to the MARS procedure, each expert compared decision variants from the reference set using the M-MACBETH program.

Table 2: Comparison of variants from the reference set using the M-MACBETH program, made by one expert



Scoring Mars

	DZ1, RP1, WB1	DZ1, RP1, WB2	DZ1, RP2, WB1	DZ2, RP1, WB1	DZ3, RP1, WB1	DZ1, RP3, WB1	DZ1, RP4, WB1	DZ1, RP1, WB3	DZ1, RP1, WB4	Current scale
DZ1, RP1, WB1	no 0.00	weak 11.43	weak 14.29	weak 17.14	moderate 22.86	mod-strg 42.86	mod-strg 57.14	vstrg-extr 91.43	extreme 100.00	100.00
DZ1, RP1, WB2		no 0.00	very weak 2.86	vweak-weak 5.71	weak-mod 11.43	weak-mod 31.43	mod-strg 45.71	strong 80.00	extreme 88.57	88.57
DZ1, RP2, WB1			no 0.00	vweak-weak 2.85	vweak-weak 8.57	vweak-mod 28.57	mod-strg 42.85	mod-strg 77.14	strg-vstr 85.71	85.71
DZ2, RP1, WB1				no 0.00	vweak-weak 5.72	mod-strg 25.72	strg-vstr 40.00	strg-vstr 74.29	vstrg-extr 82.86	82.86
DZ3, RP1, WB1					no 0.00	moderate 20.00	strg-vstr 34.28	strg-vstr 68.57	strg-extr 77.14	77.14
DZ1, RP3, WB1						no 0.00	weak 14.28	strong 48.57	strong 57.14	57.14
DZ1, RP4, WB1							no 0.00	strong 34.29	strong 42.86	42.86
DZ1, RP1, WB3								no 0.00	very weak 8.57	8.57
DZ1, RP1, WB4									no 0.00	0.00

extreme, v. strong, strong, moderate, weak, very weak, no

Consistent judgements

Source: Authors' own elaboration using the M-MACBETH program.

A list of point scores of the levels of implementation of criteria obtained by the experts using the M-MACBETH method is shown in Table 3.

It is worth noting that the experts agreed as to the ranking of the decision variants from the reference set, and they differed only in their scores assigned to the individual levels of implementation of decision variants and in the rankings of all decision variants obtained from them. The MARS method does not require that the decision maker directly determines the relevance of each criterion.

Table 3: Expert evaluations p_{ik} 0-100

Expert no	**Point score of the levels of implementation of decision variants**										
	DZ1	DZ2	DZ3	RP1	RP2	RP3	RP4	WB1	WB2	WB3	WB4
1	100	82,86	77,14	100	85,71	57,14	42,86	100	88,57	8,87	0
2	100	78,57	66,67	100	90,48	35,71	30,95	100	95,24	8,33	0
3	100	90,62	84,38	100	93,76	78,12	68,75	100	96,88	53,12	0
Average	100	84,02	76,06	100	89,98	56,99	47,52	100	93,56	23,44	0

Source: Authors' own elaboration based on the information obtained.

Distances L_i from the ideal variant and the normalized distances for each decision variant determined on the basis of the experts' average point score are shown in Table 4.

Table 4: Distance of each decision variant from the ideal decision variant

Variant	Criterion			Point score on the 0-100 scale			Distance	Distance	Position
	f_1	f_2	f_3	f_1	f_2	f_3	L_i	L_i(*norm*)	
1	2	3	4	5	6	7	8	9	0
W1	DZ1	RP1	WB1	100,00	100,00	100,00	0,00	0,00	1
W2	DZ1	RP1	WB2	100,00	100,00	93,56	6,44	0,04	2
W3	DZ1	RP1	WB3	100,00	100,00	23,44	76,56	0,43	24
W4	DZ1	RP1	WB4	100,00	100,00	0,00	100,00	0,57	28
W5	DZ1	RP2	WB1	100,00	89,98	100,00	10,02	0,06	3
W6	DZ1	RP2	WB2	100,00	89,98	93,56	16,45	0,09	5
W7	DZ1	RP2	WB3	100,00	89,98	23,44	86,58	0,49	26
W8	DZ1	RP2	WB4	100,00	89,98	0,00	110,02	0,62	31
W9	DZ1	RP3	WB1	100,00	56,99	100,00	43,01	0,24	13
W10	DZ1	RP3	WB2	100,00	56,99	93,56	49,45	0,28	14
W11	DZ1	RP3	WB3	100,00	56,99	23,44	119,57	0,68	34
W12	DZ1	RP3	WB4	100,00	56,99	0,00	143,01	0,81	40
W13	DZ1	RP4	WB1	100,00	47,52	100,00	52,48	0,30	15
W14	DZ1	RP4	WB2	100,00	47,52	93,56	58,92	0,33	16
W15	DZ1	RP4	WB3	100,00	47,52	23,44	129,04	0,73	37
W16	DZ1	RP4	WB4	100,00	47,52	0,00	152,48	0,86	43
W17	DZ2	RP1	WB1	84,02	100,00	100,00	15,98	0,09	4
W18	DZ2	RP1	WB2	84,02	100,00	93,56	22,42	0,13	6
W19	DZ2	RP1	WB3	84,02	100,00	23,44	92,54	0,52	27
W20	DZ2	RP1	WB4	84,02	100,00	0,00	115,98	0,66	33
W21	DZ2	RP2	WB1	84,02	89,98	100,00	26,00	0,15	8
W22	DZ2	RP2	WB2	84,02	89,98	93,56	32,44	0,18	10
W23	DZ2	RP2	WB3	84,02	89,98	23,44	102,56	0,58	30
W24	DZ2	RP2	WB4	84,02	89,98	0,00	126,00	0,71	36
W25	DZ2	RP3	WB1	84,02	56,99	100,00	58,99	0,33	17
W26	DZ2	RP3	WB2	84,02	56,99	93,56	65,43	0,37	18
W27	DZ2	RP3	WB3	84,02	56,99	23,44	135,55	0,77	39
W28	DZ2	RP3	WB4	84,02	56,99	0,00	158,99	0,90	45
W29	DZ2	RP4	WB1	84,02	47,52	100,00	68,46	0,39	20
W30	DZ2	RP4	WB2	84,02	47,52	93,56	74,90	0,42	22
W31	DZ2	RP4	WB3	84,02	47,52	23,44	145,02	0,82	42
W32	DZ2	RP4	WB4	84,02	47,52	0,00	168,46	0,95	47
W33	DZ3	RP1	WB1	76,06	100,00	100,00	23,94	0,14	7
W34	DZ3	RP1	WB2	76,06	100,00	93,56	30,37	0,17	9
W35	DZ3	RP1	WB3	76,06	100,00	23,44	100,50	0,57	29
W36	DZ3	RP1	WB4	76,06	100,00	0,00	123,94	0,70	35
W37	DZ3	RP2	WB1	76,06	89,98	100,00	33,95	0,19	11
W38	DZ3	RP2	WB2	76,06	89,98	93,56	40,39	0,23	12
W39	DZ3	RP2	WB3	76,06	89,98	23,44	110,51	0,63	32
W40	DZ3	RP2	WB4	76,06	89,98	0,00	133,95	0,76	38
W41	DZ3	RP3	WB1	76,06	56,99	100,00	66,95	0,38	19
W42	DZ3	RP3	WB2	76,06	56,99	93,56	73,38	0,42	21
W43	DZ3	RP3	WB3	76,06	56,99	23,44	143,51	0,81	41

Table 4 cont.

1	2	3	4	5	6	7	8	9	0
W44	DZ3	RP3	WB4	76,06	56,99	0,00	166,95	0,95	46
W45	DZ3	RP4	WB1	76,06	47,52	100,00	76,42	0,43	23
W46	DZ3	RP4	WB2	76,06	47,52	93,56	82,85	0,47	25
W47	DZ3	RP4	WB3	76,06	47,52	23,44	152,98	0,87	44
W48	DZ3	RP4	WB4	76,06	47,52	0,00	176,42	1,00	48

Source: Authors' own elaboration.

On the basis of point scores and an interview with the expert, the decision variants have been grouped with respect to the degree of risk involved in granting funds. The grouping is shown in Table 5. Various shades of grey denote the four groups of risk involved in start-up business financing.

Table 5: Grouping of decision variant with respect to financing risk

Position	**Variant**	**Distance**	**Position**	**Variant**	**Distance**	**Position**	**Variant**
1	W1	0,000	17	W25	0,3344	33	W20
2	W2	0,036	18	W26.	0,371	34	W11
3	W5	0,057	19	W41	0,379	35	W36
4	W17	0,091	20	W29	0,388	36	W24
5	W6	0,093	21	W42	0,416	37	W15
6	W18	0,127	22	W30	0,425	38	W40
7	W33	0,136	23	W45	0,433	39	W27
8	W21	0,147	24	W3	0,434	40	W12
9	W34	0,172	25	W46	0,470	41	W43
10	W22	0,184	26	W7	0,491	42	W31
11	W37	0,192	27	W19	0,525	43	W16
12	W38	0,229	28	W4.	0,567	44	W47
13	W9	0,244	29	W35	0,570	45	W28
14	W10	0,280	30	W23	0,581	46	W44
15	W13	0,297	31	W8	0,624	47	W32
16	W14	0,334	32	W39	0,626	48	W48

Source: Authors' own elaboration.

The assignment of decision variants to groups is as follows:

Group 1 (items 1-6 in Table 5, $L(norm) \in [0;0{,}127)$). This group contains applicants with level DZ1 professional experience; with business plan evaluated as level RB1 or RB2; and with credit history at levels WB1 or WB2. Applicants with level DZ2 professional experience, level RB1 business plan, and level WB2 credit history have also been assigned to this group. These are, therefore, applicants with **very low financing risk**.

Group 2 (items 7-16 in Table 5, $L(norm) \in [0{,}136;0{,}334)$). This group contains applicants with level DZ1 business experience, level RB3 business plan, and with credit history at level WB1 or WB2. Applicants with level DZ2 business experience, level RB2 business plan, and level WB2 credit history have also been assigned to this group. The third subgroup here consists of loan-takers with level DZ3 business experience, business plan assessed at level RB1 or RB2, and level WB1 or WB2 credit history. These applicants represent, therefore, a **low financing risk**.

Group 3 (items 17-23 in Table 5, $L(norm) \in [0{,}334;0{,}433)$). This group contains applicants with level DZ1 business experience, level RB4 business plan, and with credit history at level WB1 or WB2. A second subgroup here consists of applicants with level DZ2 business experience, level RB3 or RP4 business plan, and level WB1 or WB2 credit history. The third subgroup here consists of applicants with level DZ3 business experience, business plan evaluated at level RB3, and level WB1 or WB2 credit history. The fourth subgroup consists of applicants with level DZ3 business experience, level RB4 business plan, and level WB1 credit history. These applicants represent, therefore, a **moderate financing risk**.

Group 4 (items 24-48 in Table 5, $L(norm) \in [0{,}433;1)$). This group consists of applicants whose credit history was evaluated at WB3 or WB4 level, irrespective of the levels of implementation of the remaining evaluation criteria. Furthermore, this group contains applicants with level DZ3 business experience, level RB4 business plan, and level WB2 credit history. This group is, therefore, one with a **high financing risk**.

The next step consisted in empirical verification of the model using data on the payback quality of 64 loans taken to finance start-up businesses in the Podlaskie voivodeship between January 2013 and February 2015 (Table 6). Among the companies that obtained such a loan, seven had at least one delinquency. The model presented here indicated correctly five of them. When identifying companies which had no problems paying off their loans, the model erred once, assigning a good loan-taker to Group 4 (variant (DZ1, RB1, WB3)).

Table 6: Assigning companies to the groups and decision variants

Group 1			Group 2			Group 3			Group 4		
position	variant	number of companies	position	variant	number of companies	position	variant	number of companies	position	variant	number of companies
1	W1	8	8	W21	7	15	W13	1	24	W3	1
2	W2	8	10	W22	5	17	W25	1	25	W46	1
3	W5	18	11	W37	1	18	W26	2	30	W23	2
5	W6	4	13	W9	2	21	W42	1	34	W11	1
			14	W10	1						

Source: Authors' own elaboration based on bank data.

The structure of loan-takers is shown on Figures 1a and 1b. It is worth noting that 38 of 64 businesses (59.4%) have been qualified as belonging to Group 1, 16 companies (25%), to Group 2, five companies (7.8%), to Group 3, and five companies to Group 4.

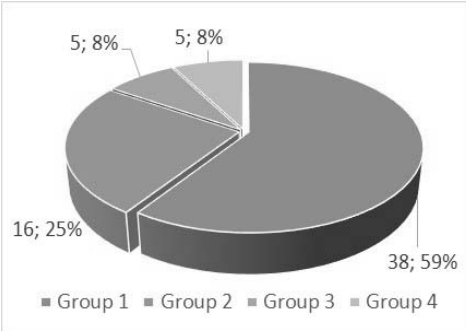


Figure 1a. Loan-takers divided into groups

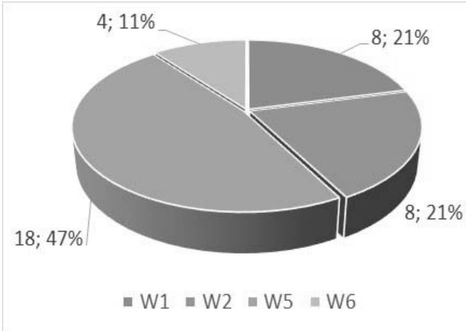


Figure 1b. Loan-takers divided into decision variants in Group 1

Source: Authors' own elaboration based on data in Table 6.

In the analysis of the number of loan-takers assigned to each decision variant, variants W5, W1, and W2 are worth noting. The loan-takers occurring most often have experience entirely consistent with their business idea, business assumptions calculated too optimistically, but feasible even in a disadvantageous economic climate; they have also credit obligations without delinquencies (variant W5). Second (eight people) are "ideal" loan-takers (variant W1) and loan-takers whose experience is entirely consistent with their business idea, who have cautious and realistic business assumptions and either have no credit obligations or have obligations with delinquencies of ten days or less (variant W2).

6 Final conclusions

Deciding whether to grant financing to a start-up business is a difficult task, primarily because of a lack of historical data on which one could base the evaluation. The problem of selecting the appropriate beneficiary for financial support in starting a business becomes complicated if we take into account the fact that preferential loans and non-returnable grants are directed mostly at unemployed, young people with modest professional experience (people up to 25 or 30 years old), handicapped people, as well as people living in rural areas – that is, at people who cannot obtain a loan from a commercial bank. On the other hand, institutions implementing European programs require that exorbitantly high indicators as regards the quality of the loan portfolio be achieved, for instance, as regards funds irretrievably lost or the number of agreements terminated. The minimization of losses can be achieved only with help of appropriate tools which allow to include expert knowledge in the evaluation of loan or grant applications. Research (Peters, 1990) indicates that when experts assess the risk, they do it not in numerical quantities but, to a large extent, using natural language. Therefore research on the inclusion of tools handling incomplete data and data in linguistic or fuzzy form, in the evaluation of applications for start-up business financing should be conducted on a larger scale.

In our paper we have presented our own proposal of using the MARS method in the evaluation and ranking of credit applications. An advantage of our approach is the possibility of taking into account expert scores expressed verbally in the evaluation of start-up business financing. This holistic approach allows, moreover, for comparing decision variants from the reference set only, and, on this basis, to evaluate the decisions in the entire set of decision variants. It is worth noting that the construction of the reference set, related to the ZAPROS procedure, is transparent and comprehensible for the expert, and pairwise comparisons of entire decision variants are natural from the point of view of the problem under discussion (a decision variant is identified with the description of the situation of a specific applicant). Another advantage of our approach is that it does not require an assessment of the relevance of the criteria (weights) of the credit application evaluations, which could constitute an additional difficulty for the expert. Further research will deal with verification of the empirical usefulness of the model proposed, as well as with identification of other methods using verbal scores, such as MACBETH, ZAPROS, methods based on holistic approach such as UTA, GRIP (Figueira, Greco, Słowiński, 2008, 2009), applications of rough sets (Pawlak, 1982; Medina, Cueto, 2013), or fuzzy reasoning (Konopka, 2013) for the evaluation of credit applications.

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A SOLVING PROCEDURE FOR THE MULTIOBJECTIVE DYNAMIC PROBLEM WITH CHANGEABLE GROUP HIERARCHY OF STAGE CRITERIA DEPENDENT ON THE STAGE OF THE PROCESS

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Abstract

We consider multiobjective, multistage discrete dynamic decision processes. In this paper we propose an interactive procedure which allows to solve the problem of optimal control of such a process in the case when the decision maker has determined a group hierarchy of stage criteria. This hierarchy is changeable and depends on the stage of the process. The proposed algorithm is illustrated by a numerical example.

Keywords: multiobjective multistage decision process, multiobjective dynamic programming, hierarchical problem, group hierarchy.

1 Introduction

The present paper is a continuation of the discussion conducted in Trzaskalik (in press). We consider decision processes consisting of a finite number of stages, determined by the decision maker. The decisions are made at the beginning of the consecutive stages and evaluated using many evaluation criteria. In the evaluation of the feasible process realizations we will use both stage criteria, which are related to the specific stages of the process, and multistage criteria, used to evaluate the overall realization of the process. Problems of this type are classified as problems of multiobjective dynamic programming. We consider the most frequently occurring situation, in which multistage criteria are sums of stage criteria.

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When formulating the issue of process realization evaluation we refer to the general notion of optimality in multiobjective problems (Steuer, 1986). We assume that the components of the vector criteria function are the consecutive multistage criteria. As vector-optimal realizations we admit those which are non-dominated (in the criteria space) or efficient (in the decision space) (Trzaskalik, 1990).

Among the varied topics dealt with currently there are many problems in which the hierarchization of the evaluation criteria is an essential element. An overview of the problems discussed has been presented in Trzaskalik (in press).

A change in importance of the criteria often influences decision making. Not infrequently, to achieve a better stage evaluation of a criterion which is important at the given stage, the decision maker is inclined to give up on the optimization of the realization of the multistage objectives. Obtaining such immediate profits can, however, have a very negative impact on the evaluation of the entire process. For that reason, in the case of criteria hierarchization, it seems justified to focus the analyses on the values of both the stage and multistage criteria.

The present paper attempts to answer the question about the method of controlling a multistage process so as to take into account at the same time both the tendency to multiobjective optimization of the entire process and the time-varying group hierarchy of stage criteria. We will discuss in detail one of many possible situations, in which the stage hierarchy varies in the consecutive stages and depends on the stage. We will present an interactive proposal of the solution of this problem, in which the decision maker actively participates in the process of finding the final realization of the process.

The present paper consists of six sections. In Section 2, we define the notation used and present the notion of vector optimization for a multiobjective decision process. Section 3 describes the idea of the group hierarchy of criteria. In Section 4 we formulate the hierarchical problem discussed in the paper and propose a solution procedure. A detailed solution of an illustrative numerical example is in Section 5. A summary completes the paper.

2 Multistage, multiobjective discrete decision process (Trzaskalik, in press)

We define $\overline{1,T}$ to be the set of all integer numbers from 1 to T and denote it as follows:

$$\overline{1,T} = \{1,\ldots,T\} \tag{1}$$

We consider a discrete decision process consisting of T stages. Let y_t be the state variable at the beginning of the stage number t, Y_t – the set of all feasible state variables for stage t, x_t – the decision variable for stage t and $X(y_t)$ – the set of all feasible decision variables for stage t and state y_t. We assume that all sets of states and decisions are finite. A stage realization is defined as follows:

$$d_t \cong (y_t, x_t) \tag{2}$$

Let D_t be the set of all stage realizations in stage t. We define $d_t(y_t)$ as the stage realization which begins in state y_t. The set of all stage realizations which begin in a given state y_t is defined as follows:

$$D_t(y_t) = \{d_t(y_t) \in D_t \colon d_t = (y_t, x_t) \wedge x_t \in X_t(y_t)\} \tag{3}$$

We assume that for $t \in \overline{1,T}$ the transformations:

$$\Omega_t \colon D_t \rightarrow Y_{t+1} \tag{4}$$

are given. A sequence of stage realizations:

$$(d_1, \ldots, d_T) = (y_1, x_1, y_2, x_2, \ldots, y_T, x_T) \tag{5}$$

is called a process realization and denoted as d, if:

$$\forall_{t \in \overline{1,T}}\, y_{t+1} = \Omega_t(y_t, x_t) \tag{6}$$

Let D be the set of all process realizations.

We assume that we consider K criteria and that for each stage t and $k \in \overline{1,K}$, stage criteria functions $F_t^k \colon D_t \rightarrow R$ are defined. For the given realization d we obtain the values:

$$\begin{array}{cccc} F_1^1(d_1) & F_1^2(d_1) & \ldots & F_1^K(d_1) \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ F_T^1(d_T) & F_T^2(d_T) & \ldots & F_T^K(d_T) \end{array}$$

F is a vector-valued criterion function for the evaluation of the entire process and its components F^k, $k \in \overline{1,K}$, are defined as follows:

$$F^k(d) = \sum_{t=1}^{T} F_k^T(d_t) \tag{7}$$

We postulate maximization of all the components of F.

Let us assume that two process realizations: $\bar{d}$, $\tilde{d}$ and vectors:

$$F(\bar{d}) \cong [F^1(\bar{d}), \ldots F^K(\bar{d})]$$
$$F(\tilde{d}) \cong [F^1(\tilde{d}), \ldots F^K(\tilde{d})]$$

are given. The relation of domination ≥ is defined as follows:

$$F(\bar{d}) \geq F(\tilde{d}) \Leftrightarrow \forall_{k \in \overline{1,K}} [F^k(\bar{d}) \geq F^k(\tilde{d})] \wedge \exists_{i \in \overline{1,K}} [F^i(\bar{d}) > F^i(\tilde{d})] \tag{8}$$

If $F(\bar{d}) \geq F(\tilde{d})$ we say that vector $F(\bar{d})$ dominates vector $F(\tilde{d})$ and realization $\bar{d}$ is better than realization $\tilde{d}$. Realization $\overset{*}{d}$ is said to be efficient if:

$$\sim \underset{\bar{d} \in D}{\exists} F(\bar{d}) \geq F(\overset{*}{d}) \tag{9}$$

Let $\overset{*}{D}$ be the set of all efficient realizations for the given criterion function F. The problem of finding $\overset{*}{D}$ is called the dynamic vector optimization problem. The set:

$$\overset{*}{D}(\bar{d}) \cong \{\overset{*}{d} \in \overset{*}{D} : F(\overset{*}{d}) \geq F(\bar{d})\} \tag{10}$$

consists of all efficient realizations which are better than $\bar{d}$. The algorithm of finding the set of all efficient realizations of the process and the algorithm of finding the set of efficient realizations better than the chosen one is described in Trzaskalik (1990).

3 Group hierarchy of criteria

The issue of hierarchization of criteria has been presented many times in the literature dealing with multiobjective decision making, in particular in papers on goal programming. This hierarchization is understood in two ways. In the first approach, the criteria are assigned weight coefficients and the importance of a criterion is reflected by the appropriate value of this coefficient: the more important the criterion, the larger the value of the weight coefficient. In the second approach, hierarchy levels are introduced. Criteria on higher levels are regarded as more important than those on lower levels; criteria on the same level are equally important for the decision maker. For criteria situated at the same hierarchy level weight coefficients can also be used (Jones, Tamiz, 2010).

When hierarchy levels are used, we can introduce a single hierarchy or a group hierarchy. In the former case, a hierarchy level contains only a single criterion. In the latter case, a hierarchy level can contain more than one criterion (Galas, Nykowski, Żółkiewski, 1987).

In a discussion of hierarchical problems with a single criteria hierarchy it is important to create an appropriate numbering of criteria. The criteria can be numbered so as to assign the number 1 to the most important criterion, the number 2 to the second-most important criterion – one that is less important than criterion number 1 but more important than all the remaining criteria, and so on. A similar method of numbering can be applied in the case of group hierarchy. Criteria from a more important group will have numbers lower than all the less important criteria; criteria from the same group are equally important. Therefore, the numbering of criteria within one group is ambiguous.

The issue of criteria hierarchization discussed above appears also when multistage decision processes are considered. In such cases, both stage criteria and multistage criteria can occur. When a hierarchy of stage criteria is established, we can hierarchize multistage criteria in the same way as described above.

A different situation occurs when the importance of stage criteria for the decision maker vary from stage to stage. This is the case of a changeable stage hierarchy. We assume that at the given stage, stage criteria have been divided into a certain number of groups, depending on their importance. Each group contains criteria which are equally important for the decision maker. Moreover, a hierarchy of stage criteria can undergo changes in the consecutive stages.

The issue of hierarchization of multistage and stage criteria was discussed before by the present author. In Trzaskalik (1997) the issue of searching for the best process realization was discussed, in the situation when a hierarchy of multistage criteria was given. Each time when the consecutive (with respect to importance) criteria were analyzed, the stage structure of the consecutive process realizations was analyzed. The changeability of hierarchies of stage criteria was discussed in other papers, too. In Trzaskalik (1995) a hierarchy dependent on the joint value of the stage criteria obtained in previous stages was discussed, while in Trzaskalik (1992), a hierarchy dependent on the current state of the process. These discussions were continued in Trzaskalik (1998a, 1998b), which dealt also with the case of group hierarchy. Changeable, weighted relevance of stage objectives was discussed in Trzaskalik (2009). In each of those cases, the process realization, which satisfies best the assumptions regarding the hierarchization of stage and multistage criteria, was compared with the set of efficient realizations.

The changeable group hierarchy of stage criteria discussed further will be illustrated by an example. We consider a 3-stage process. In stage 1, the stage criteria are F_1^1, F_1^2, F_1^3 i F_1^4, in stage 2 they are F_2^1, F_2^2, F_2^3 i F_2^4, in stage 3 they are F_3^1, F_3^2, F_3^3 i F_3^4. In the proposed notation the lower index is the stage number, while the upper index is the criterion number. We call the criteria with the same value of the upper index single-name criteria; they refer to the same aspect of the process under consideration.

An possible method of dividing the criteria could be the following. In stage 1 the decision maker divided the criteria into two groups: more important: F_1^2 and F_1^3 and less important: F_1^1 and F_1^4. In stage 2 all the criteria are equally important, therefore we have a single group of stage criteria. In stage 3 the criteria were divided into three groups. The first group contains only one, the most important, criterion $F_3^{2.}$ The second group contains the second-most important criterion F_3^1. The third group contains the two least important criteria F_3^3 and F_3^4.

Denoting as $\boldsymbol{K}_t^i$ the ith most important group of criteria at stage t, and by I_t^i , the set of indices corresponding to the numbers of stage criteria in set $\boldsymbol{K}_t^i$, we can write:

$\boldsymbol{K}_1^1 = \{F_1^2, F_1^3\}$ $\quad$ $\boldsymbol{K}_1^2 = \{F_1^1, F_1^4\}$

$\boldsymbol{K}_2^1 = \{F_2^1, F_2^2, F_2^3, F_2^4\}$

$\boldsymbol{K}_3^1 = \{F_3^2\}$ $\quad$ $\boldsymbol{K}_3^2 = \{F_3^1\}$ $\quad$ $\boldsymbol{K}_3^3 = \{F_3^3, F_3^4\}$

$I_1^1 = \{2, 3\}$, $\quad$ $I_1^2 = \{1, 4\}$

$I_2^1 = \{1, 2, 3, 4\}$

$I_3^1 = \{2\}$ $\quad$ $I_3^2 = \{1\}$ $\quad$ $I_3^3 = \{3, 4\}$

At stage 1 the criteria from group $\boldsymbol{K}_1^1$, that is, F_1^2 and F_1^3, are equally important; at the same time, each of them is more important than the remaining criteria for this stage, that is, F_1^1 and F_1^4. On the other hand, criteria F_1^2 and F_1^3 are equally important and less important than both F_1^1 and F_1^4.

At stage 2 all the criteria are equally important: none is less or more important, hence all of them belong to the same group $\boldsymbol{K}_2^2$.

At stage 3 criterion F_3^2 belonging to the (1-element) group $\boldsymbol{K}_t^i$ is more important than all the remaining criteria, that is, F_3^2, F_3^3 and F_3^4. Criterion F_3^1, belonging to the second-most important (1-element) group $\boldsymbol{K}_t^i$, is less important than F_3^2 but more important than the criteria from group $\boldsymbol{K}_3^3$, that is, F_3^3 and F_3^4. Finally, criteria F_3^3 and F_3^4 are equally important.

This example shows that the number of groups into which the criteria are divided can vary from stage to stage. In particular, at some stages, all criteria can be equally important. One-element criteria groups can also occur. Also, the composition of the groups can vary from stage to stage.

Further in the paper we assume that in each process stage t (t = 1, …, T) the decision maker divided the stage criteria into i_t groups denoted $\boldsymbol{K}_t^i$, with the corresponding sets of stage criteria numbers denoted $I_t^i,\ i \in \overline{1, i_t}$. This way we obtain a division into the following groups of criteria:

for stage 1: $\boldsymbol{K}_1^1, \boldsymbol{K}_1^2, \ldots, \boldsymbol{K}_1^{i_1}$

for stage 2: $\boldsymbol{K}_2^1, \boldsymbol{K}_2^2, \ldots, \boldsymbol{K}_2^{i_2}$

……………………………..

for stage T: $\boldsymbol{K}_T^1, \boldsymbol{K}_T^2, \ldots, \boldsymbol{K}_T^{i_T}$

and sets of indices:

for stage 1: $I_1^1, I_1^2, \ldots, I_1^{i_1}$

for stage 2: $I_2^1, I_2^2, \ldots, I_2^{i_2}$

……………………………..

for stage T: $I_T^1, I_T^2, \ldots, I_T^{i_T}$

We assume that:

$$\forall_{t\in\overline{1,T}}\forall_{k,l\in\overline{1,i_t}}\, I_t^k \cap I_t^l = \varnothing$$

4 Description of the procedure

The purpose of the procedure described in this section is the selection of a process realization which, on the one hand, fulfills the decision maker's expectations as to the achievement of stage goals according to the group hierarchy described in the previous section and, on the other hand, realizes the multistage objectives in the best possible way. This procedure makes the decision maker aware of the consequences of the stage decisions which he/she makes to realize the multistage objectives. It also points out new possibilities which result from the analysis of both the stage and multistage objectives. Below we describe the consecutive stages of the procedure, comparing it with the procedure proposed in Trzaskalik (in press) for a single hierarchy of stage criteria.

Selection of the initial stage

We find the maximum value for each stage criterion F_1^j from group $\boldsymbol{K}_1^{\,1}$. We normalize the stage values for each criterion from this group. This allows to sum up the normalized values for each process state under consideration. As the initial state we propose to select the one for which the sum of normalized values is largest. If there are more than one such states, we can select any of them; the consecutive states will be considered when the procedure is repeated (if at all).

Satisfactory stage realizations

Stage realizations satisfactory with respect to the given group of multistage criteria are such process realizations for which the values of stage criteria are optimal or almost optimal in the given state. That is, their stage values are within the tolerance intervals given by the decision maker.

We solve the problem for the consecutive stages, starting with the first stage. At any given stage, we consider all the criteria groups consecutively, according to the hierarchy determined by the decision maker, starting with the group of the most important criteria.

When considering a given group of stage criteria, we take into account all stage decisions admissible for the given process state. We ask the decision maker to give a preliminary tolerance interval for the maximum values for all the stage criteria from the criteria group under consideration. As the initial set of satisfactory realizations we take those realizations for which all the stage values are within the given intervals. The cardinality of this set depends on the extent to which the decision maker is willing to give up the optimal values for the stage criteria from the given group. For that reason, if the tolerance intervals determined by the decision maker turn out to be too narrow, we suggest than he/she extends them. As a result, the decision maker agrees to lower even more the requirements as regards the criterion under consideration. On the other hand, if the cardinality of the realization set is too large, the decision maker can narrow the suggested tolerance interval, which guarantees better values for the criteria from this group in the final solution. When the decision maker accepts the tolerance interval, we obtain a set of realizations satisfactory with respect to the given group of stage criteria. This allows to consider the next most important group of stage criteria (if it exists).

Selection of the stage decision
When all the hierarchized criteria from each group of the consecutive hierarchy levels are considered in this manner, the decision maker selects the final stage decision from the last set of satisfactory stage realizations. To select this decision one can use the value of the index which characterizes the joint relative change of the value of the given realization with respect to the possible maximal changes of the individual stage criteria. A method of the construction of this index, analogous to that proposed in Trzaskalik (in press), will be presented in the detailed description of the algorithm. Once we know the stage decision we use the transfer function and determine the initial state in the next stage.

Generating a satisfactory process realization
This procedure is repeated for the consecutive stages, including the last one. The result is a satisfactory process realization which fulfills the decision maker's expectations as regards the levels of the stage criteria (according to the group hierarchy assumed). As in Trzaskalik (in press), we call this realization a satisfactory realization for short. It is added to the set of potential realizations, from among which the decision maker will make the final selection.

Testing of the efficiency of the satisfactory realization
This part of the procedure is analogous to the procedure for a single hierarchy described in Trzaskalik (in press). Using the procedure for efficiency testing we check if the generated satisfactory realization is an efficient realization. If it is not, we generate better efficient realizations and add them to the set of potential realizations, i.e., the realizations from among which the decision maker will select the final realization. Therefore, in the set of potential realizations we have a satisfactory realization and efficient realizations better than this one (if they exist).

Generating the consecutive satisfactory realizations

This procedure fragment is analogous to the one described in Trzaskalik (in press). The decision maker performs a preliminary analysis of the set of potential realizations; he/she can decide that this set suffices to make the final decision (which is to indicate one of the potential realizations as the final realization) or else may conclude that it is necessary to extend the set of potential realizations by repeating the entire procedure, taking as the initial state of the process one of the states not yet considered.

Selection of the final realization

If the decision maker does not see the need to expand the set of potential realizations, then he/she uses expert knowledge to analyze in detail (jointly with an analyst) the values of the stage and multistage criteria of all the potential realizations generated. As a result, the decision maker can select as the final decision that satisfactory realization which is at the same time an efficient one (if it exists, of course). As the final realization, the decision maker can also select a satisfactory realization which is not efficient or else an efficient realization which is not satisfactory.

Below is a detailed description of the algorithm proposed.

Algorithm

Step 1. The decision maker determines a group hierarchy of stage criteria for each stage; this hierarchy is described in detail in the previous subsection.

Step 2. Denote by D^P the set of potential realizations and set $D^P = \varnothing$.

Step 3. Consider *stage* criteria from group $\boldsymbol{K}_1{}^1$. These are criteria $F_1{}^j$ with $j \in I_1$. The set Y_1 of states for the first stage is finite. Assume that it consists of N elements which can be written as the following sequence:

$$Y_1 = \{y_1{}^{(1)}, y_1{}^{(2)}, \ldots, y_1{}^{(N)}\} \tag{11}$$

For each stage criterion $F_1{}^j$ from set $\boldsymbol{K}_1{}^1$ calculate the maximum value:

$$F_1^{j^*} = \max_{d_1 \in \boldsymbol{D}_1} F_1^j(d_1) \tag{12}$$

For all stage realizations d_1 from set D_1 calculate the normalized values:

$$f_1^j(d_1) = \frac{F_1^j(d_1)}{F_1^{j^*}} \tag{13}$$

For the consecutive stage criteria from set $\boldsymbol{K}_1{}^1$ and for the consecutive initial states for Stage 1, that is for $y_1{}^{(n)} \in Y_1$ calculate:

$$S^j\left(y_1^{(n)}\right) = \sum_{x_1 \in \boldsymbol{X}_1\left(y_1^{(n)}\right)} f_1^j\left(y_1^{(n)}, x_1\right) \tag{14}$$

For the consecutive initial states for Stage 1, sum up the normalized values for all stage criteria from set $\boldsymbol{K}_1^1$:

$$S\left(y^{(n)}\right) = \sum_{j \in \boldsymbol{I}_1^1} S^j\left(y_1^{(n)}\right) \tag{15}$$

As the initial state select state $y_1^{(p)}$, for which the sum $S(y_1^{(p)})$ is largest.
If the decision maker does not accept this proposal, ask him/her to indicate the preferred initial state.
Step 4. Set $t = 1$.
Step 5. Set $y_t = y^{(p)}$.
Step 6. Set $D_t = D_t(y^p)$.
Step 7. Set $i = 1$.
Step 8. Set $I = I_t^i$.
Step 9. If $I = \varnothing$, go to Step 13.
Step 10. Select $j \in I$, set $I = I \setminus \{j\}$.
Step 11. Find stage process realization $d_t^{j*}(y_t) \in D_t$, for which stage criterion F_t^j has its maximum value F_t^{j*}.
Step 12. Inform the decision maker what the value of F_t^j* is and ask him/her to give the value ε_t^j which determines the tolerance interval $[F_t^j* - \varepsilon_t^j, F_t*]$ for the criterion under consideration.
Step 13. Select those stage realizations from set D_t for which criterion F_t^j attains a value from the interval $[F_t^j* - \varepsilon_t^k, F_t^j*]$. Denote the set of these stage realizations by $D_t^{(j)}$. Return to Step 9.
Step 14. Find the intersection of sets $D_t^{(j)}$:

$$D_t^i = \bigcap_{j \in I_t^i} D_t^{(j)} \tag{16}$$

Step 15. Inform the decision maker about the cardinality of the set obtained and ask for approval. If the decision maker accepts this cardinality, go to Step 17.
Step 16. If the decision maker finds this cardinality too large or too small, ask him/her to repeat the analysis of set $\boldsymbol{K}_t^i$. Return to Step 8.
Step 17. Check if $i = i_t$. If so, go to Step 19.
Step 18. Set $D_t = D_t^{i,}$ and $i = i + 1$. Return to Step 8.
Step 19. Select the preferred stage realization from the reduced set D_t of realizations as described below. Check if there are dominated stage realizations in set D_t. If so, delete them. Assume that D' has cardinality P_t'. For each stage realization $d_t^{(p)} \in D_t'$ calculate the coefficient f_{pk} for the consecutive criteria, by dividing $F_t^k(d_t^{(p)})$ by the largest obtainable value of stage criterion F_t^k in $\boldsymbol{D}_t^{i_t}$. We obtain:

$$f_{pk} = \frac{F_t^k\left(d_t^{(p)}\right)}{\max\limits_{d \in D'} F^k\left(d_t\right)} \tag{17}$$

Form matrix $\boldsymbol{F} = [f_{pk}]$ of size $P' \times K$ with these values. As the stage decision, suggest to take that decision numbered p^o for which the sum of the elements of the corresponding row in matrix F is largest.

If the decision maker does not accept this suggestion, he/she should perform the selection independently, by analyzing the values of matrix $\boldsymbol{F}$.

Step 20. Check if $t = T$. If so, go to Step 23.

Step 21. Using the transfer function, determine the process state at the end of the stage. This state is at the same the initial state for the next stage. We have:

$$y_{t+1} = \Omega_t(y_t, x_t) \tag{18}$$

Step 22. Set $t = t + 1$, $y_t = y_{t+1}$, $D_t = D_t(y_t)$ and go to Step 7.

Step 23. Let d^o be the process realization obtained. Add d^o to the set D^P of potential realizations.

$$D^P = D^P \cup \{d^o\} \tag{19}$$

Step 24. Using the algorithm for efficiency testing, check if the generated realization is efficient. If not, generate the set $D^*(y^*)$ of efficient realizations better than the realization obtained.

Step 25. Add the realizations from set $D^*(y^*)$ (if any) to set D^P of potential realizations.

$$D^{P:} = D^P \cup D^*(y^*) \tag{20}$$

Step 26. Ask the decision maker to perform a preliminary analysis of set D^p. Ask the decision maker if he/she want to extend this set by repeating the procedure to obtain another satisfactory realization. If not, go to Step 28.

Step 27. Ask the decision maker to indicate as the next initial state a state not previously considered. Go to Step 3.

Step 28. The decision maker, using expert knowledge, analyzes the set of potential decision, taking into account the stage hierarchy and the value of the stage and multistage criteria. As a result, the decision maker:

a) indicates one of the potential realizations as the final realization,
b) repeats the procedure starting with Step 2, obtaining a new potential realization,
c) eliminates certain realizations obtained previously from the set of potential realizations,
d) changes the stage hierarchy and repeats the entire procedure,
e) gives up making the decision using the procedure described above.

5 Numerical example

We consider a two-stage decision process in which the transfer function is of the form:

$$y_{t+1}^{(j)} = \Omega_t(\mathrm{y}_t^{(i)}, x_t^{(j)}) = x_t^{(j)}$$

that is, the decision consists in the selection of the initial state for the next stage.

We denote stage realization d_t^{ij}, which begins in state $y_t^{(i)}$, when decision $x_t^{(j)}$ is taken, as follows:

$$d_t^{ij} = (y_t^{(i)}, x_t^{(j)})$$

The values of the stage criteria, the same in both stages, are shown in Tables 1-3.

Table 1: Values of stage criteria F_t^1 ($t = 1, 2$)

	Decision	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
State	0	420	451	433	494	462	400	455	459	438	452
	1	443	417	499	429	486	498	438	494	424	436
	2	429	490	491	434	494	484	420	480	458	482
	3	430	489	413	492	488	434	487	423	482	496
	4	414	407	418	409	460	456	454	452	419	446
	5	454	489	409	454	416	413	439	441	434	492
	6	455	462	427	483	460	437	456	493	468	436
	7	438	439	494	449	446	422	491	437	425	455
	8	490	418	449	410	429	454	439	422	434	438
	9	437	424	447	497	433	480	488	464	406	492

Table 2: Values of stage criteria F_t^2 ($t = 1, 2$)

	Decision	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
State	0	69	66	69	59	54	64	55	63	58	60
	1	59	55	63	62	53	67	61	65	62	69
	2	57	63	65	54	55	52	56	59	69	61
	3	52	67	61	61	69	59	65	51	52	69
	4	68	52	64	56	56	62	67	66	67	61
	5	53	65	69	63	68	50	50	58	64	54
	6	58	58	65	52	69	61	57	54	56	57
	7	51	56	63	58	52	53	52	60	53	62
	8	52	58	69	51	50	50	56	51	55	54
	9	60	63	60	52	51	53	69	59	63	53

Table 3: Values of stage criteria F_t^3 ($t = 1, 2$)

	Decision	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
State	0	153	162	177	180	182	189	182	178	189	157
	1	152	175	151	156	176	179	161	153	170	166
	2	186	175	168	176	174	173	175	152	188	151
	3	189	152	167	159	162	189	157	159	150	190
	4	180	177	158	176	186	158	170	172	172	180
	5	172	172	155	167	153	174	178	160	179	158
	6	162	156	172	186	180	157	155	150	172	162
	7	151	170	167	169	173	168	174	159	150	154
	8	159	158	165	154	155	171	152	185	165	162
	9	176	153	190	164	150	180	161	155	166	159

We proceed to show an application of the procedure proposed.

Step 1. The decision maker provides a group hierarchy of stage criteria for the consecutive stages. We have: $\boldsymbol{K_1}^1 = \{F_1^1, F_1^2\}$, $\boldsymbol{K_1}^2 = (F_1^3\}$, $\boldsymbol{K_2}^{\mathbf{1}} = \{F_2^3\}$, $\boldsymbol{K_2}^{\mathbf{2}} = \{F_2^1, F_2^2\}$, $I_1^1 = \{1, 2\}$, $I_1^2 = \{3\}$, $I_2^1 = \{3\}$, $I_2^2 = \{1, 2\}$.

Step 2. Set $D^P = \varnothing$.

<u>*Selection of the initial state*</u>

Step 3. The decision maker accepted the proposed selection of the initial state, presented in the description of the algorithm. The detailed calculations are shown in the appendix. As the initial state we take $y_1^{(2)}$.

<u>***Stage 1***</u>

Step 4. Set $t = 1$.

Step 5. Set $y_t = y_1^{(p)}$.

Step 6. Set $D_t = D_t(y_t^{(p)})$.

Step 7. Set $i = 1$.

<u>*First group of criteria*</u>

Step 8. Set $I = I_1^1 = \{1, 2\}$.

Step 9. We have $I \neq \varnothing$.

Step 10. Select $j = 1$, set $I = I \setminus \{1\} = 2$.

Step 11. Criterion F_1^1 has its maximum value for stage realization d_1^{24}. We have $F_1^{1*}(d^{24}) = 494$.

Step 12. The decision maker determined $\varepsilon_1^1 = 49$, hence the tolerance interval for criterion F_1^1 is [445, 494].

Step 13. The following realizations are in the interval determined by the decision maker:

$$D_1^{(1)} = \{d_1^{21}, d_1^{22}, d_1^{24}, d_1^{25}, d_1^{27}, d_1^{28}, d_1^{29}\}.$$

Step 9. We have $I \neq \varnothing$.

Step 10. Select $j = 2$, set $I = I \setminus \{2\} = \varnothing$.

Step 11. Criterion F_1^2 has its maximum value for stage realization d^{28}. We have $F_1^{2*}(d^{28}) = 69$.

Step 12. The decision maker determined $\varepsilon_1^2 = 9$, hence the tolerance interval for criterion F_1^1 is [60, 69].

Step 13. In the interval determined by the decision maker there are realizations from the set:

$$D_1^{(2)} = \{d_1^{21}, d_1^{22}, d_1^{28}, d_1^{29}\}$$

Step 9. Since $I = \varnothing$, go to Step 12.

Step 14. Find the set:

$$D_1^1 := D_1^{(1)} \cap D_1^{(2)} = \{\{d_1^{22}, d_1^{28}\ d_1^{29}\}$$

Step 15. The decision maker accepts the cardinality of set D_1^1.

Second group of criteria

Step 17. Set $i = i + 1 = 2$. We have $i = 2 \leq 2 = i_2$.

Step 18. Set $D_t = D_1^1$.

Step 8. Set $I = I_1^2 = \{3\}$.

Step 9. We have $I \neq \varnothing$.

Step 10. Select $j = 3$, set $I = \mathrm{I} \setminus \{3\} = \varnothing$.

Step 11. Criterion F_1^3 has its maximum value on set D^1 for stage realization d^{28}. We have $F_1^{3*}(d^{28}) = 188$.

Step 12. The decision maker determined $\varepsilon_1^3 = 18$, hence the tolerance interval for criterion F_1^3 is [170, 188].

Step 13. In the interval determined by the decision maker there are realizations from the set:

$$D_1^{(2)} = \{d_1^{22}, d^{28}\}$$

Step 9. We have $I = \varnothing$.

Step 14. Find:

$$D_1^2 := D_1^{(3)} = \{d_1^{22}, d_1^{28}\}$$

Step 15. The decision maker accepts the cardinality of set D_1^2.

Step 17. We have $i = 2 = i_2$.

Selection of the stage realization

Step 19. Compare the values of the stage criteria for the stage realizations from set D^2. We have:

$\mathrm{F}_1^1(\mathrm{d}^{22}) = 491$, $\mathrm{F}_1^1(\mathrm{d}^{22}) = 65$, $\mathrm{F}_1^1(\mathrm{d}^{22}) = 168$

$\mathrm{F}_1^1(\mathrm{d}^{28}) = 458$, $\mathrm{F}_1^1(\mathrm{d}^{28}) = 69$ $\mathrm{F}_1^1(\mathrm{d}^{28}) = 188$

Create the matrix:

$$\begin{bmatrix} 1 & 0{,}942 & 0{,}0897 \\ 0{,}923 & 1 & 1 \end{bmatrix}$$

The sums of the elements are: for d^{22} = 2,839, for d^{28} = 2,923. Select d^{28}.

Step 20. We have: $t = 1 < T$.

Step 21. We have: $y_2 = \Omega_1(d^{28}) = y_2^{(8)}$

Step 22. Set $t = 1 + 1 = 2$, $y_t = y_{t+1}$, $D_2 = D_2(y_2^{(8)})$ and go to Step 7.

Stage 2

First group of criteria

Step 7. Set $i = 1$.

Step 8. Set $I = I_2^1 = \{3\}$.

Step 9. We have $I \neq \varnothing$.

Step 10. Select $j = 3$, set $I = I / \{3\} = \varnothing$.

Step 11. Stage criterion F_2^3 has its maximum value for stage realization d_2^{87}. We have $F_2^{3*}(y^{(8)}) = 185$ for $x_2^7 \in X_1(y^{(2)})$.

Step 12. The decision maker determined $\varepsilon_1^1 = 15$, hence the tolerance interval for criterion F_2^3 is [170, 185].
Step 13. The following realizations are in the interval determined by the decision maker:

$$D_2^{(3)} = \{d_2^{85}, d_1^{87}\}.$$

Step 9. We have $I = \varnothing$.
Step 14. Find the set:

$$D_1^2 := D_1^{(3)} = \{\{d_1^{22}, \mathrm{d}_1^{28}\}$$

Step 15. The decision maker does not accept the cardinality of set D_1^2 and suggests that it be extended. Set $I = I_2^1 = \{3\}$ and return to Step 9.
<u>Extension of set D_1^2</u>
Step 9. We have $I \neq \varnothing$.
Step 10. Select $j = 3$, set $I = I / \{3\} = \varnothing$.
Step 11. Criterion F_1^1 has its maximum value for stage realization d_2^{87}. We have $F_2^{3*}(y^{(8)}) = 185$ for $x_2^7 \in X_1(y^{(2)})$.
Step 12. The decision maker determined $\varepsilon_1^1 = 25$, hence the tolerance interval for criterion F_2^3 is [160, 185].
Step 13. The following realizations are in the interval determined by the decision maker:

$$D_2^{(3)} = \{d_2^{82}, d_2^{85}, d_1^{87}, d_2^{88}, d_2^{89}\}$$

Step 9. We have $I = \varnothing$.
Step 14. Find the set:

$$D_2^2 := D_1^{(3)} = \{d_2^{82}, d_2^{85}, d_1^{87}, d_2^{88}, d_2^{89}\}$$

Step 15. The decision maker accept the cardinality of set D_1^2.
<u>Second group of criteria</u>
Step 17. We have $i = 1 < i_t$.
Step 18. Set $D_t = D_t^1$, $i = i + 1$.
Step 8. Set $I = I_2^2 = \{1, 2\}$.
Step 9. We have $I \neq \varnothing$.
Step 10. Select $j = 1$, set $I = I / \{1\} = \{2\}$.
Step 11. Criterion F_2^1 has its maximum value for stage realization d_2^{85}. We have $F_2^{1*}(y^{(8)}) = 454$ for $x_2^5 \in X_2(y_2^{(8)})$.
Step 12. The decision maker sets $\varepsilon_2^1 = 20$. The tolerance interval is [434, 454].
Step 13. Determine the stage realizations which fall within the given tolerance interval. We have:

$$D_2^{(1)} = \{d_2^{82}, d_2^{85}, d_2^{88}, d_2^{89}\}$$

Step 9. We have $I \neq \varnothing$.
Step 10. Select $j = 2$, set $I = I / \{2\} = \varnothing$.

Step 11. Criterion F_2^2 has its maximum value for stage realization d_2^{82}. We have $F_2^{1*}(y^{(8)}) = 69$ for $x_2^5 \in X_2(y_2^{(8)})$.
Step 12. The decision maker sets $\varepsilon_2^1 = 7$. The tolerance interval is [62, 69].
Step 13. Determine the stage realizations which fall within the given tolerance interval. We have:

$$D_2^{(2)} = \{d_2^{82}\}$$

Step 9. We have $I = \varnothing$.
Step 14. Find the intersection of sets $D_t^{(j)}$:

$$D_2^2 = D_2^{(1)} \cap D_2^{(2)} = \{d_2^{82}\}$$

Step 15. The decision maker accept the cardinality of this set.
Step 17. Set $i = 2 = i_2$.
Selection of the stage realization
Step 19. Since D_2^2 has one element only, the preferred stage realization is d_2^{82}.
Generating a satisfactory process realization
Step 20. We have $t = 2 = T$.
Step 23. Add the generated process realization $d^{282} = (d_1^{28}, d_2^{82})$ to the set of potential realizations. We have:

$$D^P = D^P \cup \{d^{282}\} = \{d^{282}\}$$

Testing the efficiency of the satisfactory realization
Step 24. Using the algorithm of efficiency testing, check that the generated realization is efficient.
Selection of the final realization
Step 26. The decision maker does not want to extend the set D^P of potential realizations.
Step 28. The decision maker indicates d^{282} as the final realization.

6 Summary

The interactive procedure proposed in this paper allows to include the decision maker into the process of solving the problem. Of fundamental importance is here the decision maker's (or the advisory team's) expert knowledge. The key theoretical aspect of the proposed procedure is the use of the algorithm for testing the efficiency of the potential realizations generated at each stage and, related to this, the possibility of generating better efficient realizations (if they exist) and of performing appropriate comparisons. Such a situation did not occur in the presented example because the potential realization generated as a result of the algorithm turned out to be efficient, but it occurred in the numerical example in Trzaskalik (in press), which can be a model for such situations. The selection of the final realization is then performed using the decision maker's expert knowledge.

Further research should take into account numerical aspects of the proposed solutions, both for single hierarchy and for group hierarchy, discussed in the present paper. For this purpose one should perform simulations with randomly generated criteria values. Taking into account the significant number of the necessary courses of action, one should discuss the possibility of determining the proposed rules of behavior for the decision maker in the situations when he/she makes decisions and of automating these decisions.

Another direction of theoretical research should deal with extending the hierarchical approach to stochastic and fuzzy decision processes.

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Appendix. Determination of the initial state

We find the largest values for each state for the first-level criteria.

Criterion F_1^1

State	0	1	2	3	4	5	6	7	8	9	max
0	420	451	433	494	462	400	455	459	438	452	494
1	443	417	499	429	486	498	438	494	424	436	499
2	429	490	491	434	494	484	420	480	458	482	494
3	430	489	413	492	488	434	487	423	482	496	496
4	414	407	418	409	460	456	454	452	419	446	460
5	454	489	409	454	416	413	439	441	434	492	492
6	455	462	427	483	460	437	456	493	468	436	493
7	438	439	494	449	446	422	491	437	425	455	494
8	490	418	449	410	429	454	439	422	434	438	490
9	437	424	447	497	433	480	488	464	406	492	497

Criterion F_1^2

State	0	1	2	3	4	5	6	7	8	9	max
0	69	66	69	59	54	64	55	63	58	60	69
1	59	55	63	62	53	67	61	65	62	69	69
2	57	63	65	54	55	52	56	59	69	61	69
3	52	67	61	61	69	59	65	51	52	69	69
4	68	52	64	56	56	62	67	66	67	61	68
5	53	65	69	63	68	50	50	58	64	54	69
6	58	58	65	52	69	61	57	54	56	57	69
7	51	56	63	58	52	53	52	60	53	62	63
8	52	58	69	51	50	50	56	51	55	54	69
9	60	63	60	52	51	53	69	59	63	53	69

We normalize the values, sum them up for the consecutive states and select the largest value.

Criterion 1	State	0	1	2	3	4	5	6	7	8	9
	0	0,841683	0,903808	0,867735	0,98998	0,925852	0,801603	0,911824	0,91984	0,877756	0,905812
	1	0,887776	0,835671	1	0,859719	0,973948	0,997996	0,877756	0,98998	0,849699	0,873747
	2	0,859719	0,981964	0,983968	0,869739	0,98998	0,96994	0,841683	0,961924	0,917836	0,965932
	3	0,861723	0,97996	0,827655	0,985972	0,977956	0,869739	0,975952	0,847695	0,965932	0,993988
	4	0,829659	0,815631	0,837675	0,819639	0,921844	0,913828	0,90982	0,905812	0,839679	0,893788
	5	0,90982	0,97996	0,819639	0,90982	0,833667	0,827655	0,87976	0,883768	0,869739	0,985972
	6	0,911824	0,925852	0,855711	0,967936	0,921844	0,875752	0,913828	0,987976	0,937876	0,873747
	7	0,877756	0,87976	0,98998	0,8998	0,893788	0,845691	0,983968	0,875752	0,851703	0,911824
	8	0,981964	0,837675	0,8998	0,821643	0,859719	0,90982	0,87976	0,845691	0,869739	0,877756
	9	0,875752	0,849699	0,895792	0,995992	0,867735	0,961924	0,977956	0,92986	0,813627	0,985972
Criterion 2	0	1	0,956522	1	0,855072	0,782609	0,927536	0,797101	0,913043	0,84058	0,869565
	1	0,855072	0,797101	0,913043	0,898551	0,768116	0,971014	0,884058	0,942029	0,898551	1
	2	0,826087	0,913043	0,942029	0,782609	0,797101	0,753623	0,811594	0,855072	1	0,884058
	3	0,753623	0,971014	0,884058	0,884058	1	0,855072	0,942029	0,73913	0,753623	1
	4	0,985507	0,753623	0,927536	0,811594	0,811594	0,898551	0,971014	0,956522	0,971014	0,884058
	5	0,768116	0,942029	1	0,913043	0,985507	0,724638	0,724638	0,84058	0,927536	0,782609
	6	0,84058	0,84058	0,942029	0,753623	1	0,884058	0,826087	0,782609	0,811594	0,826087
	7	0,73913	0,811594	0,913043	0,84058	0,753623	0,768116	0,753623	0,869565	0,768116	0,898551
	8	0,753623	0,84058	1	0,73913	0,724638	0,724638	0,811594	0,73913	0,797101	0,782609
	9	0,869565	0,913043	0,869565	0,753623	0,73913	0,768116	1	0,855072	0,913043	0,768116
Σ		17,22898	17,72911	18,36926	17,35212	17,52865	17,24931	17,67404	17,64105	17,47475	17,96419

The suggested initial state is $y_1^{(2)}$.

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ANALYSIS OF THE MULTIPLE ATTRIBUTE DECISION MAKING PROBLEM WITH INCOMPLETE INFORMATION ABOUT PREFERENCES AMONG THE CRITERIA

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Abstract

The paper presents an overview of methods used in solving Multiple Attribute Decision Making (MADM) problems in the case of incomplete information about preferences among criteria, which are defined by explicit attributes of the problems. The paper presents the following methods: dominance, maxmin, maxmax, based on game theory, ELECTRE IV and parametric approach associated with Linear Partial Information and AHP. The presented methods focus on the problem of evaluation of investment projects in a hard coal mine.

Keywords: incomplete inter-criteria information, ELECTRE IV, Linear Partial Information, AHP.

1 Introduction

In the paper we discuss Multiple Criteria Decision Making (MCDM) problems with a finite set of decision variants, that is, Multiple Attribute Decision Making (MADM) problems. MADM problems are MCDM problems with clearly defined attributes of decision variants. The criteria are defined by the attributes and the set of decision variants is mostly complete. The problems considered in the paper are discrete MCDM problems. Many methods and approaches to solve such problems have been developed. An overview of such procedures can be found in the following papers: Roy (1985); Figueira, Greco and Ergott (eds.) (2005); Tzeng, Chiang and Li (2011); Trzaskalik (ed.) (2014a, 2014b).

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In the paper Hwang and Yoon (1981) the main features of MADM problems are defined and compared with Multiple Objective Decision Making (MODM) problems.

Table 1: Characteristics of MADM vs MODM

	MADM	MODM
Criteria defined by	attributes	objectives
Objective	ill defined (implicit)	clearly defined (explicit)
Attribute	explicit	implicit
Constraint	inactive (incorporated into attributes)	active, clearly defined
Decision variant	predefined, usually a finite number	infinite number of variants
Decision problem	discrete	continuous
Interaction with DM	occasionally	mostly
Application	choice, selection, classification, evaluation	design

Source: Hwang and Yoon (1981, p. 4).

The present paper deals with methods of decision support when information about preferences among criteria is not available. Such analytical situations occur when the decision maker does not want to or cannot determine the relation among the criteria importance which usually happens at the beginning of the process of solving the problem. The goal of the paper is to present methods that do not require information about preferences among the criteria. The main objective of the present paper is to present selected methods of multiple attribute decision-making support synthetically when no information about preferences among the criteria is available. The methods focus on the actual problem of evaluation of investment projects in a hard coal mine.

The methods presented cover a selected spectrum of preferences modeling. The ELECTRE IV method (Roy and Bouyssou, 1993) is one of methods consisting in constructing a relational system of preferences of the decision maker based on the outranking relation.

The most common alternative to this approach is the AHP method (Saaty, 1980). This method allows to transform a verbal assessment into a numeric one using pairwise comparison, and therefore determines an ordering of the decision variants.

Simple methods: dominance, maxmin, maxmax (Hwang and Yoon, 1981) and methods based on game theory (Madani and Lund, 2011) usually do not require explicit aggregation of assessments of decision variants. The relations between variants result from comparison of assessments associated with each criterion.

The use of the idea of Linear Partial Information (Kofler, 1993) to solve MCDM problems (Michalska, 2011, 2012; Michalska and Pospiech, 2010, 2011;

Pospiech, 2014) involves taking into account: partial information about preferences, analysis of marginal distributions of criteria weights and determining the order of decision variants by using Wald's maxmin criterion.

The diversity of the presented approaches makes it difficult to present them clearly and comprehensively, or to compare them. Therefore the discussion in this paper focuses on the most important aspect of our topic, which is – to put it simply – the lack of requiring full information about relations between the criteria or full information about their weights.

In our paper we do not discuss the interactive approach where information about preferences in iterations is given. We assume that information about preferences among the criteria is not available: the decision maker cannot or does not want to give it. However, the situation analyzed in this paper can occur at the beginning of the interactive procedure.

2 Basic methods: Dominance, Maxmin, Maxmax and methods based on game theory

The dominance method consists in reducing the number of decision variants by removing dominated variants. The application of the Dominance method by the decision-maker shows a passive attitude or may be a preliminary part of analysis that allows to reduce a set of decision variants.

The Maxmin method requires standardization of decision variants. In this method, for each decision variant, the worst estimate for the variant is determined in terms of the criteria analyzed, and then the best one is selected among the estimates determined. The selected estimate indicates the best variant.

When using the Maxmax method the procedure is similar as in the Maxmin method. For each decision variant the best estimate is determined according to the criteria. The highest estimate indicates the best decision variant. This method assumes an optimistic approach of the decision-maker to the decision problem and the selected variant allows to reach at least one objective at the highest level possible.

The Maxmin approach was the basis for defining multi-criteria problem as two-person zero-sum game (Kofler, 1967). Later models were developed in the form of n-person games in which the player is associated with a criterion, the strategy with a decision variant, and payoffs of each player with the variant's estimate according to a given criterion. The game defined in this way may be considered as played once (Wolny, 2007) or in many moves until a stable solution is achieved (Madani and Lund, 2011). In the former case, using the general theory of equilibrium selection and risk dominance (Harsanyi and Selten, 1990), an equilibrium (in Nash's sense) is indicated that represents accordingly the best decision variant. In the latter case different equilibriums are considered (starting with Nash's equilibrium, through general meta-rationality, symmetric meta-rationality, sequential stability, limited moves stability to non-myoptic stability), and analysis of stable solutions points out the solutions to the original multiple attribute problem.

3 The ELECTRE IV method

This method belongs to the family of ELECTRE methods (ELimination Et Choix Traduisant la REalité − ELimination and Choice Expressing Reality), introduced by the so-called French school in multi-criteria decision analysis (Roy, 1990; Roy and Bouyssou, 1993), and is characterized by modeling of the decision-maker's preferences by means of constructing a relative system of his preferences based on outranking. The feature distinguishing the ELECTRE IV method is the lack of requirement of weights for the criteria analyzed: it is only assumed that none of the criteria is more important than half of them. All the methods from the ELECTRE group are based on pairwise comparison of decision variants. For each criterion the threshold values are usually defined. The thresholds are as follows: q − indifference, p − preference and veto. In the ELECTRE IV method the comparison of two variants consists in verifying whether at least one type of relation occurs: quasi-dominance, canonical dominance, pseudo-dominance, sub-dominance and veto-dominance. All the aforementioned types of dominance represent weakening premises for the occurrence of outranking – if quasi-dominance occurs, all the other ones also appear, if canonical dominance occurs, all the other ones appear except for quasi-dominance etc. On the basis of these relations two partial preorders are set using the distillation procedure. The combination of two such preorders generates the final preorder (Vallée and Zielniewicz, 1994).

4 Linear Partial Information in the AHP method and in additive methods

The analytical hierarchy process (the AHP method) has a wide range of applications. It was introduced by Saaty (1977, 1980) and has been developed since then. AHP allows to estimate decision variants according to the criteria by determining the relative weights that reflect the usability of variants for each criterion. The relative weights are determined based on the transformation of the so-called comparison matrices, which, in turn, are generated using pairwise comparison of decision variants and criteria.

One matrix is generated for each criterion and, additionally, the criteria comparison matrix. These matrices are used in constructing a partial series of decision variants according to each criterion and criteria ranking. The values of relative weights inform about the decision-maker's preferences: the higher the weight the better the variant or criterion.

Relative weights resulting from comparison decision variants generate matrix W; furthermore, relative weights of criteria form vector w. The final ranking is obtained using vector weights $w^* = W \cdot w$. However, it should be noted that the values of matrix W may be treated as standardized estimates of decision variants (similarly to vector values w as standardized criteria weights). Therefore, the aggregation of estimates is performed according to all criteria by using a weighted sum, which is a feature of additive methods.

In the problem analyzed in this paper, the components of vector w are unknown; in the AHP procedure the decision-maker does not want or cannot present his preferences in relation to criteria or he reveals them partially, e.g. in the form of linear bounds such as: 'the first criterion is at least as important as the second criterion' ($w_1 \geq w_2$), 'the second criterion is at least as important as the third and fourth ones together' ($w_2 \geq w_3+w_4$). In such a situation the application of the idea of Linear Partial Information (LPI) (Kofler, 1993) is proposed to solve the multi-criteria problem (Michalska and Pospiech, 2011). The idea of this approach consists in:

1) determining the extremal distribution of the criteria weights – the space of feasible values of weights is a simplex and each vertex of the simplex defines an extremal distribution of weights,
2) solving the problem for these distributions (a ranking of variants is established for each distribution) using the AHP method (in general, any MCDM method which allows to order variants and requires weights of criteria can be used),
3) determining the final ranking of variants on the basis of the rankings of variants and using Wald's criterion.

5 Lack of preferences and equivalence of criteria

Let us analyze the problem of the evaluation of investment projects in a hard coal mine involving longwalls. Four criteria were set for this problem (deposit size, total costs, methane hazard, rockburst hazard). The data are presented in Table 2. In the case of minimized criteria, negative estimates were adopted to obtain the same direction of optimization.

The data presented in Table 2 were subject to multi-criteria analysis as shown in the papers Sojda and Wolny (2014); Wolny (2014). It may be noted that variant a_1 dominates a_3; a_2 dominates a_3 and a_8; a_4 dominates a_5; a_6 dominates a_3, a_7 and a_9; a_8 dominates a_3. Nevertheless, the dominance method does not order the set of the variants analyzed.

Table 2: Decision variant estimates according to the criteria analyzed

Investment project – longwall	f_1 – output volume, resources estimated [thousand tons]	f_2 – total cost [PLN thousand]	f_3 – methane hazards (category of hazards)	f_4 – rockburst hazards (category of hazards)
a_1	411	-55 252	-2	-1
a_2	469	-58 251	-1	-1
a_3	297	-82 739	-3	-1
a_4	1581	-89 022	-2	-2
a_5	1092	-99 118	-2	-2
a_6	966	-78 119	-2	-1
a_7	650	-84 084	-4	-1
a_8	414	-68 300	-1	-1
a_9	737	-85 071	-4	-1

Source: Data from a mining company.

In the paper Wolny (2015) the use of the ELECTRE IV method is presented using the threshold values from Table 3. (The criterion f_4 does not differentiate much among the decision variants taking into account the threshold values: all the variants can be treated equivalently with respect to this criterion. However, this criterion was not removed from the analysis performed in the paper for two reasons. First, the differences in the assessments of variants for the indifferent variants are important for the formation of quasi- and canonic dominance relations in the ELECTRE IV method. Second, this criterion is important for the decision maker).

Table 3: Threshold values: indifference, preference and veto

	Indifference threshold	**Preference threshold**	**Veto threshold**
Criterion	$q_k[f_k(a_i)]$	$p_k[f_k(a_i)]$	$v_k[f_k(a_i)]$
f_1	10	50	1000
f_2	100	1000	50000
f_3	0	1	3
f_4	2	3	4

Source: Data obtained from the decision-maker.

For the purpose of this paper, for all methods requiring standardization of decision variants estimates, the relative weights used which resulted from the application of the AHP method as well as information about preferences related to each criterion expressed by threshold values are included in Table 3. This means that the final unification of assessments of decision variants (Table 4) are approximations of preferences expressed by thresholds.

Table 4: Standardized estimates of decision variants

Investment project – longwall	f_1 – output amount, resources estimated [thousand tons]	f_2 – total costs [PLN thousand]	f_3 – methane hazards (category of hazards)	f_4 – rockburst hazards (category of hazards)
a_1	0.019	0.164	0.126	0.125
a_2	0.019	0.158	0.238	0.125
a_3	0.017	0.107	0.007	0.125
a_4	0.730	0.094	0.126	0.063
a_5	0.124	0.018	0.126	0.063
a_6	0.028	0.116	0.126	0.125
a_7	0.022	0.104	0.006	0.125
a_8	0.019	0.137	0.238	0.125
a_9	0.023	0.102	0.006	0.125

When calculating the extreme distribution of weights (in the method using the LPI idea), it was additionally assumed that the weight of each criterion constitutes at least 20% of weight of the other criteria – in this way the significance of the analyzed criteria was defined. The following constrains should be taken account: $w_k = 0{,}2 \cdot \sum_{\substack{i=1 \\ i \neq k}}^{4} w_i , k = 1{,}2{,}3{,}4$ and $\sum_{i=1}^{4} w_i = 1$, where w_k is the weight of kth criterion. Consequently, we obtain the following extreme distributions of weights (w_1, w_2, w_3, w_4): (0.500, 0.167, 0.167, 0,167), (0.167, 0.500, 0.167, 0.167), (0.167, 0,167, 0.500, 0.167), (0.167, 0.167, 0.167, 0.500). These weights generate orderings presented in Table 5.

Table 5: Rankings resulting from extreme distributions of weights

	Extreme distributions of weights and the corresponding rankings				**MAX (pessimistic place in order)**	**Ran-king**
	(0.500,0.167, 0.167,0.167)	(0.167,0.500, 0.167,0.167)	(0.167,0.167, 0.500,0.167)	(0.167,0.167, 0.167,0.500)		
a_1	5	4	4	4	5	**4**
a_2	2	2	2	2	2	**2**
a_3	9	6	7	6	9	**7**
a_4	1	1	1	1	1	**1**
a_5	3	9	6	9	9	**7**
a_6	6	5	5	5	6	**5**
a_7	8	7	8	7	8	**6**
a_8	4	3	3	3	4	**3**
a_9	7	8	9	8	9	**7**

The orderings (rankings) obtained by different methods, as compared with the solution obtained by including different weight values of the criteria are presented in Table 6.

Table 6: Final rankings of decision variants for compared multi-criteria methods

Variant	Minmax method	Maxmax method	AHP + LPI	ELECTRE IV	Equivalent criteria	
					AHP	ELECTRE III
a_1	5	3	4	3	4	3
a_2	3	2	2	2	2	1
a_3	7	5	7	9	7	7
a_4	1	1	1	4	1	2
a_5	6	4	7	7	6	5
a_6	2	4	5	1	5	2
a_7	8	5	6	8	8	6
a_8	4	2	3	5	3	4
a_9	8	5	7	6	9	6

The rankings obtained differ, but they are a result of the transformation of the same set of information. It should be noted that all the methods analyzed, except for the methods from the ELECTRE family, are consistent in terms of optimum (according to these methods, the best variant is a_4). Differentiation is an obvious consequence of different approaches and notions that characterize the methods analyzed. The idea of using LPI in the AHP method (due to the method of standardization of estimates used here, a simple additive method is identical with it in this example) is based on the use of the Minmax method for the rankings generated by the extreme distributions of weights – from this point of view this approach is compared with a simple application of the Minmax method and the AHP method with equivalent weights of criteria. The ELECTRE IV method, in turn, uses a completely different approach, therefore the ranking obtained with it is compared with the ranking generated by the ELECTRE III method with equivalent criteria.

The values of correlation coefficients of Spearman ranks between the achieved rankings were adopted in order to examine the similarity of rankings. The data are presented in Table 7.

Table 7: Values of correlation coefficients of Spearman ranks between the examined orderings together with their critical value of significance level

	Minmax method	Maxmax method	AHP + LPI	ELECTRE IV	Equivalent criteria	
					AHP	ELECTRE III
Minmax method		**.8196** **p = .007**	**.8204** **p = .007**	**.7811** **p = .013**	**.8876** **p = .001**	**.8938** **p = .001**
Maxmax method	**.8196** **p = .007**		**.9519** **p = .000**	.5746 p = .106	**.9678** **p = .000**	**.7809** **p = .013**
AHP + LPI	**.8204** **p = .007**	**.9519** **p = .000**		.6375 p = .065	**.9363** **p = .000**	**.8230** **p = .006**
ELECTRE IV	**.7811** **p = .013**	.5746 p = .106	.6375 p = .065		.6333 p = .067	**.9252** **p = .000**
AHP	**.8876** **p = .001**	**.9678** **p = .000**	**.9363** **p = .000**	.6333 p = .067		**.8391** **p = .005**
ELECTRE III	**.8938** **p = .001**	**.7809** **p = .013**	**.8230** **p = .006**	**.9252** **p = .000**	**.8391** **p = .005**	

The results obtained indicate a strong correlation between the rankings. It should be taken into account that similar rankings are obtained when all the criteria are assigned equal weights and no information about preferences among the criteria is available (this applies to the ELECTRE methods as well as to other methods). The strong correlation of rankings obtained using the Maxmax method and the AHP method with the inclusion of the LPI idea as well as with equivalent criteria is also interesting – it may be explained by the method of standardization of estimates of decision variants based on the AHP method (the values of all estimates are non-negative and sum up to one).

6 Summary

To summarize the analysis performed, it may be stated that the egalitarian approach to criteria, consisting in assigning identical weights to them, is also a kind of approximation of decision-maker's preferences. It is consistent but not the same as in the case of the methods developed strictly to support the decision-maker in multiple attribute decisions with imperfect information about preferences among the criteria.

The methods connected with game theory were described, using Wald's criterion – from a simple Minmax method to using its idea in the AHP method without inter-criteria information or the notion of linear partial information. Next to the method related to AHP, the ELECTRE IV method was presented which does not require determination of the weights of the criteria analyzed. Further in the paper, using an example, rankings generated by various methods were compared in order to answer the question: can the solution of the problem without information about preferences among the criteria be identified with the solution of the problem with equivalent criteria?

The main conclusion of the paper follows from the analyses which show that the rankings of variants generated by the various methods are similar but not identical. However, a strong correlation of the orderings, lack of perfect information about the preferences among the criteria on the one hand, and lack of premises questioning the egalitarian approach to the criteria on the other hand, indicate a possibility of equivalent understanding of criteria in this type of problems. This approach to the criteria is not identical with treating them equivalently, but it implies similar results as in the case of equivalent criteria.

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