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Part I

Multiple Criteria Decision Aid: Advances in Theory and Applications

Papers presented during The Third International Conference of the Tunisian Operational Research Society (TORS '18)

From the Guest Editors

This special issue entitled Multiple Criteria Decision Aid: Advances in Theory and Applications offers a selection of papers presented and discussed at the "Third International Conference of the Tunisian Operational Research Society (TORS '18) held in Sousse (Tunisia) on 7th-9th April 2018". It was also open to the MCDA community at large. We would like to thank the Editor-in-Chief, Professor Tadeusz Trzaskalik, University of Economics in Katowice, for his support. We also thank the authors for choosing this issue to submit their papers to, and the referees for their rigorous reviews and their comments which improved the quality of papers.

This special issue presents theoretical research results and interesting applications to real-world Tunisian cases reflecting the utility of using the multicriteria approaches. The topics addressed cover, in particular, recent developments and applications of MCDA to Operations Research and Decision Aid Sciences.

Ben Moallem et al. address the problem of *Risk Prioritization Using the Analytic Hierarchy Process (AHP) in a Tunisian Healthcare Department: A Real-world Case Study*. A qualitative study based on a preliminary analysis is carried out, which allows to identify the needs, requirements and expectations of the respondents in charge of risk management of medical activities in the Obstetrics and Gynecology Department of the Academic Hospitals of Sfax. To determine the prioritization of objectives to be achieved by risk management the Analytic Hierarchy Process method is used.

Daoud Ben Amor and Moalla address the problem of *Hierarchical Structuring for the Olive Trees Irrigation Problem in Tunisia*. AHP and Shannon's entropy are hybridized to select the best alternative for water irrigation of olive trees. First, the AHP method is used to determine the priorities of all criteria of different hierarchical levels and alternatives, and to establish a classification of choice of water alternatives according to four experts. Second, since the data provided by the experts are contradictory and uncertain, the Shannon probabilistic entropy method is used. Thus, all the expert rankings are aggregated and a unique result is found.

Ghram and Moalla propose *A New Procedure of Criteria Weight Determination within the ARAS Method*. They suggest a weighting method based on mathematical programming that indirectly involves the DM's preferences within the ARAS method. Based on the DM's preferences on certain pairs of alternatives and on the criteria weights, a mathematical program is formulated and applied to the ARAS method and solved by LINGO software. A case study in rainwater management in urban areas is discussed.

Jridi, Jerbi and Kamoun introduce a new approach to solve the *Menu Planning with a Dynamic Goal Programming Approach*. Some of the studies and approaches used in the Menu Planning problem are reviewed and a Dynamic Goal Programming formulation for solving the MP problem is proposed and applied for Hemo-Dialysis (HD) patients.

This special issue shows a strong relationship between theoretical and methodological developments in MCDA. It also shows the potential offered by MCDA to solve real-world case problems. Therefore, we recommend this issue to the MCDA community. We hope that the researchers will find this collection of papers useful from both methodological and application perspectives.

*Taicir Moalla Loukil**
*Mansour Eddaly***

Prof. Taicir Moalla Loukil has received her State doctorate from the Faculty of Economics and Management of Sfax, Tunisia in 2001. She is President of the Tunisian Operational Research Society. She led the department of development and studies at the Office des ports Aériens de Tunisie prior to joining the University of Sfax. Her research activities include decision aid, combinatorial optimization, multicriteria optimization, and scheduling and logistics problems. She acted as a guest editor of a special issue on "Developments in Multiple Objective Programming and Goal Programming" of International Transactions in Operations Research (ITOR). She has authored or co-authored more than 50 papers published in scholarly journals as well as book chapters, and supervised 20 doctoral theses and over 30 master theses.

Dr. Mansour Eddaly is an assistant professor at the Higher Institute of Business Administration at the University of Gafsa. He obtained his Ph.D. degree with high mention in 2013. His main research area is the development of approaches to solve the scheduling problems. The proposed approaches include exact methods, such as branch and bound, or mathematical programming based approaches and approximation methods, based mainly on the estimation of distribution algorithms and genetic algorithms. This topic was the subject of many papers published in international journals and of many talks at international conferences.

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RISK PRIORITIZATION USING THE ANALYTIC HIERARCHY PROCESS (AHP) IN A TUNISIAN HEALTHCARE DEPARTMENT: A REAL-WORLD CASE STUDY

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Abstract

Nowadays, the Tunisian hospital environment is a complex organization in which the safety of the patient is of primary concern to the authorities. In our study we focus on the Obstetrics and Gynecology Department of CHU of Sfax. It should be noted that no risk management study dealt with the hospital logistic chain in this institution. Hence, the purpose of this paper is to develop a strategy targeting the control of risks related to the patient care activities. The proposed approach consists of two phases. First, a qualitative survey, based on 20 semi-structured interviews, is carried out to identify the problems related to care and logistic activities of the Obstetrics and Gynecology Department in CHU Sfax. Second, the assessment of the identified risks in the hospital context is a multicriteria decision problem. To perform the evaluation of the 12 objectives depending on the identified risks, we have chosen the AHP (Analytic Hierarchy Process) for its simplicity and flexibility.

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The results of this study revealed a complexity of coordination between basic and peripheral services due to several factors. Moreover, the interviewees highlighted the importance of developing a risk management strategy in the Obstetrics and Gynecology Department of Sfax. Finally, we proposed to apply our research in other services of the hospital to control other kinds of risk.

Keywords: prioritization, AHP, semi-structured interviews, obstetrics department.

1 Introduction

Promotion of safety and quality of care has become a priority for health facilities. The hospital systems in Tunisia have not attached importance to such topics despite of their many difficulties. Therefore, a literature review has been conducted using research papers by different authors in order to solve each problem encountered in the health care establishment.

After in-depth research, we found several papers dealing with the problem of risk management in the hospital environment, and in particular, with patient care. In this context, Razurel et al. (2015) have created a map of the risks to patients associated with medical treatment (PECM) in order to implement an action plan of risk reduction. To carry out this a priori risk analysis, the Preliminary Risk Analysis (APR) method was implemented by a multidisciplinary working group. The realization of this risks map allowed to distinguish 148 scenarios, 35 of which with unacceptable criticality.

Most scenarios concern generic problems: Communication (27%), Human Factor (20%), Organizational Management (16%), and Technical and Environmental Safety / Infrastructure (15%). In addition, 54 initial risk control actions were proposed and the levels of effort to implement them were evaluated. (Weber et al., 2015) led a multicentre study to map the risks associated with medical treatment for dependent elderly people in Alsace. It was conducted in 2014 on a representative sample of 23 Alsatian schools with a self-assessment questionnaire composed of 198 items completed by each institution during multidisciplinary meetings. The results showed that the regional percentages of risk management from 63% to 85%. As a result, 30 vulnerabilities were identified. An analysis of them resulted in a list of 13 possible improvement actions. In addition, the study determined difficulties related to the absence of appropriate political risk management, reflecting in particular the lack of according between the institution staff and doctors.

Moreover, Cridelich (2012) has evaluated a new method of risk analysis specific to the management of the chemotherapy patient at the University Hospital Center (CHU) of Nice. First, 53 types of failures were identified using the Failure Modes, Effects, and Criticality Analysis (FMEA) method. Then, due to the limitations of FMEA, the author chose to use a method called Functional

Resonance Analysis Method (FRAM). Since this method integrates human and organizational factors which are adopted in the chemotherapy circuit to be evaluated via current methodologies of risk management. Using the collected data, this method allowed to well providing the risk, activity, and cost axes. In order to control the risk management in the operating rooms of Sahloul University Hospital in Sousse, a hub of hospital activity (Ben Kahla-Touil, 2012) compared the available risk management methods and chose to adapt the FMECA (Failure Modes, Effects and Criticalities) in the operating rooms. Then, the author proposed a decision support system for risk management called GRAMA (Risk Management through a MultiAgent approach) to lead the stake-holders in the operating rooms towards the best decisions for the purposes of minimizing the risks. Finally, a simulation based on the proposed approach was implemented at Sahloul Hospital. Also, Cordina-Duverger (2015) has studied the various hormonal and anthropometric risk factors for women with breast cancer. The approach is based on data from a case-control study conducted on the general population in France. This was to compare women using hormonal treatments, the weights at different periods of life, various reproductive and medical characteristics, using data obtained during the interviews. On the one hand, the results showed that the carcinogenic effects of hormonal treatments were due to synthetic progestin. On the other hand, an absence of a deleterious natural progesterone effect on the breast cancer risk was noted. Veyrier et al. (2016) have dealt with risk management of the patients' medical treatment (PECM) when the hospital insures the responsibility while getting home. The researchers have chosen AMDEC as the best method. Indeed, with each employee's feedback, in the hospital, they were able to formalize a new PECM (medication management) which was optimized, secure, and controlled when the patients were home on pass. The implementation of a nurse / patient traceability of medical intake and information allowed to fulfill the patient's needs. Renet et al. (2016) have confirmed that the care pathway of cancer patients is complex and brings about several difficulties. The objective of the study was to identify and quantify the risks induced by oral anticancer drugs. Based on the proposed care model, AMDEC was used to analyze the risks. In addition, the results showed that 80% of the identified risks were related to a lack of training and / or information for patients and / or health professionals. Depending on the multiplicity and the specificity of cancer, the care pathway depends on the type of cancer. So that the modeling of the course of care proposed in this study could serve as a basis for defining a specific path for each kind of cancer.

Nolin et al. (2016) conducted a study to help improve the prevention of cytotoxic risk in the pulmonology department, in order to protect the health of the exposed staff. A preliminary study in the pulmonology department with

30 non-medical agents, as well as another study of the various departments and the completion of a semi-directive questionnaire was carried out. The results highlighted insufficient consideration of direct and / or indirect exposure to cytotoxic agents in professional practices. These were explained by insufficient training regarding the risks and by outdated equipment.

Each author has chosen to study a definite type of risk and has fixed an objective to study or a risk to focus on, as Razurel et al. (2015) who chose to manage the risk associated with patient care. Since our study deals with an unknown background in the gynecology department of Sfax, we use a literature review as a source of inspiration and we choose to perform a preliminary exploratory research to understand the context.

2 Related research

2.1 Qualitative study: Identification of needs, objectives and stakeholder expectations related to a risk management strategy

The main purpose of a preliminary analysis related to the care of patients is to develop a deep understanding of the topic. This will prevent us from spending too much time, effort or money. Nevertheless, a multitude of data collection techniques is required to define our scope and identify the risks that can be generated within this service. Qualitative research is particularly appropriate when the observed factors are difficult to measure objectively (Aubin-Auger et al., 2009). According to Roche (2009) the objective of a qualitative study is to better understand and get closer to the goal in order to shed light on several elements to conduct a qualitative study properly, several techniques are available:

- **Individual interviews.**
- **Group interviews.**
- **Projective techniques**.

Although there are other techniques, individual interviews (non-directive and semi-directive) are usually chosen, which seems the most appropriate. The purpose of the individual interviews is to gather as much information as possible from the respondents. The number of respondents can be between 10 and 100, with interviews lasting 1 to 2 hours. These interviews were of two types: non--directive and semi-directive. The non-directive interviews give the respondent an opportunity to express himself/herself without specific themes to discuss without any particular "canvas", with each respondent expressing himself or herself on the same subject. Consequently, an analysis of such an interview will obviously be very complex. For this reason, the semi-directive interviews seem the most appropriate for our study. It aims to guide the respondent through a pre-established interview guide whose main objective is to remember that all the topics on the guide interview will be addressed and get as much useful information as possible. In many cases, we have chosen semi-structured interviews, which seem easier (or less complicated!) to implement.

The results of the analysis, identified by semi-structured interviews, are structured in the form of corrective actions or alternatives. The decision maker has to decide which action should be considered first. Therefore, we deal here with a Multi-Criteria Decision Making (MCDM) problem in healthcare evaluation.

2.2 The MCDM Problem

According to Thokala et al. (2016), health care decisions are complex and involve trade-offs between multiple conflicting objectives. This has recently been identified as one of the most important issues in health system research. Using structured, explicit approaches to decisions involving multiple criteria can improve the quality of decision making a set of techniques, known as multiple criteria decision making (MCDM), are useful for this purpose. MCDM methods are widely used in other sectors, and recently there has been an increase in health care applications. In 2014, ISPOR (the International Society for Pharmaco--economics and Outcomes Research) was charged with establishing a common definition for MCDM in health care decision making and developing good practice guidelines for using MCDM to aid health care decision making (Thokala et al., 2016). This shows the need for a scientific development of MCDM to support priority setting, which has recently been identified as one of the most important issues in health system research. Baltussen & Niessen (2006) have introduced various approaches to MCDM useful to prioritize health interventions, confirmed that MCDM should allow a trade-off between various criteria, and should establish the relative importance of criteria in a way that allows a rank ordering of a comprehensive set of interventions. In this paper, we deal with an obstetrics-gynecology department where the main challenge is that the resources are limited, making it impossible to provide each action with every effective intervention they might need or want at the same time. By summery, the purpose is to determinate the importance or urgency of actions that are necessary to preserve the welfare of patient or worker, and the establishment of actions or alternatives in order of their relative importance.

MCDM comprises a broad set of methodological approaches from operations research now being used increasingly in the health care sector, and it uses a structured and logical approach to model complex decision-making problems. Since its development, AHP has been one of the most widely used MCDM because of its simplicity and flexibility (Didem & Durmus, 2018).

AHP is a useful approach for evaluating complex multiple criteria alternatives involving subjective judgment. This tool is based on a comparative judgment of the alternatives and criteria which are not equally important, that explains the use of influences to reflect the importance of each purpose. In this context, Ammar et al. (2014) mentioned that AHP (Analytic Hierarchy Process) is an aggregation multi-criteria method developed by Tomas Saaty (1980). It is an effective tool to support complex decision making. In addition, AHP is “a theory of measurement through pairwise comparisons and relies on the judgments of experts to derive priority scales” (Saaty, 2008). It is one of the more popular MCDM methods and has many advantages as well as disadvantages. One of its advantages is its ease of use. Its use of pairwise comparisons allows decision makers to weigh coefficients and compare alternatives with relative ease. It is scalable, and can easily adjust in size to accommodate decision making problems due to its hierarchical structures (Velasquez & Hester, 2013). Moreover, this method follows the decision-maker in the methodology for his problem formulation and allows to evaluate the importance of parameters.

3 The adopted methodology

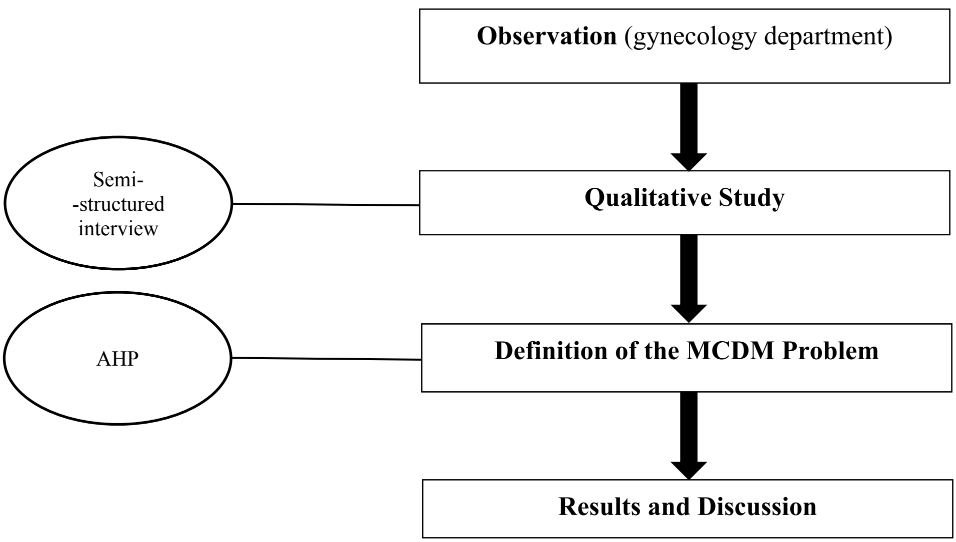

Figure 1: The adopted methodology

3.1 Observation

We visited the gynecology department and noticed the potential seriousness of adverse events associated with patient care. The presence of several types of incidents that may even put at risk the health care department together with the absence of a risk management policy, may create a number of problems, including:

- Miscommunication between departments can bring about dangerous situations.
- Misdistribution of tasks can cause incidents that put the patients' lives under risk.
- The hospital system includes a large number of activities. Communication between sectors seems difficult and the presence of several risks cannot be avoided.
- The awareness of the staff, patients, and visitors about the risks is still limited. In this regard, the managers of Sfax's Gynecology Obstetric are aware of negative effects caused by the absence of studies addressing the risks associated with the patient's medical treatment. So it is necessary to find a radical solution eliminating these failures.
- In view of the enormous flow of activities, due to insufficient human resources or equipment, the personnel is sometimes unable to take action.

The gynecology department faces several difficulties. This is why a literature review was conducted in order to solve each problem found by them in the health care establishment. Due to the lack or absence of studies in this department we have used information provided from literature and our visit. Our target is to perform a risk analysis, determine and prioritize several potential risks that can cause malfunction of the Obstetrics and Gynecology Department of Sfax.

3.2 Qualitative study

A semi-structured interview was selected for this study as a qualitative method, in the Obstetrics and Gynecology Department, for several reasons: on the one hand, this tool allows new ideas to be brought up during the interview based on what the interviewee says. On the other hand, it allows to obtain the required qualitative results and provide an appropriate balance in data collection and subsequent analysis.

20 interviews were conducted, with persons of various levels of knowledge and experience working at the hospital (nursing and administrative staff), belonging to different departments (gynecology, hygiene, supply, underwear, pharmacy, etc.). The interviews were intended to guide the response of the respondents around various themes previously defined by the interviewers and recorded in an interview guide.

In this paper, we proceeded as follows:

- **Sample selection**
- **Pre-test and validation of the interview guide**
- **Conducting interviews**
- **Analysis interviews**

A. Sample selection

We took into account different views on the risks that occurred in the Obstetrics and Gynecology Department, expressed by 20 respondents. We divide the sample in two different groups:

- Choice of respondents belonging to external departments (pharmacy, supply, underwear, hygiene).
- Selection of respondents working in the Gynecology and Obstetrics Department (resident doctors, anesthetists, instrumentalists, supervisors, nurses, midwives, workers, etc.).

B. Pre-test and validation of the interview guide

A "test interview" is necessary with a gynecologist and a midwife to decide what questions to ask them. Their opinions and reactions, and the changes they proposed, were taken into account in the creation of a final interview guide to be applied in the Obstetrics and Gynecology Department of Hedi Chaker Academic Hospital in Sfax.

C. Interviews

The purpose of an individual interview is to gather as much information as possible from the interviewees. The number of interviews can be between 10 and 100, with a duration from 1 to 2 hours. This sample makes it possible to foresee the time spent on the interviews and the cost generated by such a study, in either money or time (Roche, 2009). All interviews were conducted face-to-face during a period of three months. There are 20 interviews, and the following table provides information from the interviews.

Table 1: Interview information

<table>
<tr><th>Number</th><th>Stakeholders</th><th>Position</th><th>Duration (min)</th></tr>
<tr><td>1</td><td rowspan="4">Doctors</td><td rowspan="3">Resident</td><td>70</td></tr>
<tr><td>2</td><td>72</td></tr>
<tr><td>3</td><td>69</td></tr>
<tr><td>4</td><td>Assistant</td><td>77</td></tr>
<tr><td>6</td><td rowspan="3">Supervisors</td><td>Supervisor of internal gynecology</td><td>76</td></tr>
<tr><td>7</td><td>Supervisor of the postpartum Department</td><td>80</td></tr>
<tr><td>8</td><td>Supervisor of the Internal Gynecology Department Day Hospital</td><td>82</td></tr>
<tr><td>9</td><td rowspan="2">Nurses</td><td rowspan="2">Internal Nurse in Gynecology Department</td><td>58</td></tr>
<tr><td>10</td><td>66</td></tr>
<tr><td>11</td><td rowspan="2">Midwives</td><td>Teacher Trainer at the delivery room</td><td>67</td></tr>
<tr><td>12</td><td>Midwife in operating room</td><td>61</td></tr>
<tr><td>13</td><td>Anesthetists</td><td>Anesthetist in the operating room</td><td>63</td></tr>
<tr><td>14</td><td rowspan="3">The instrumentalists</td><td rowspan="2">Instrumentalist in the operating room</td><td>75</td></tr>
<tr><td>15</td><td>82</td></tr>
<tr><td>16</td><td>Internal Instrumentalist in Gynecology Department</td><td>55</td></tr>
<tr><td>17</td><td>Pharmacy</td><td></td><td>91</td></tr>
<tr><td>18</td><td>Administration</td><td>Manager of Supply Department</td><td>66</td></tr>
<tr><td>19</td><td>Underwear</td><td>Underwear Manager</td><td>59</td></tr>
<tr><td>20</td><td>Hygiene</td><td>Head of the Hygiene Department</td><td>74</td></tr>
</table>

D. Interviews analysis

Before the analysis, we present an interview guide composed of six themes, and the questions proposed in it focused on:

First, the risks related to the daily activity of the hospital, as identified by professionals. Second, the impact of the implementation of a risk management strategy in health care institutions, its objectives, by whom it should be managed and who are the stakeholders who can contribute to its success.

The interview guide is composed of six topics:

Topic 1: Need for Risk Management in hospital systems

- What kind of activities are you performing?
- Are there documents in which the problems that have occurred during certain operations were recorded? What is the recorded information?
- Before the medical interventions, do you prepare scenarios to facilitate interventions in unforeseen situations?
- In case of a medical intervention, do you inform the patient about the potential risks? If so, what are the impacts of this information on the patient?
- What is the frequency of white operations or staff preparations for unforeseen problems?

- Are there scenarios already prepared by the different stakeholders (even subcontractors) to intervene in the event of an incident?
- How to sensitize the staff to critical situations and various potential hazards?
- Do you consider it useful to develop a strategy aimed at controlling the risks that can be generated when you carry out your professional activity?

While it is possible to take notes about the respondents' answers, we recorded a voice clip (with permission) in order to receive appropriately the correct information.

All the respondents are unaware of the scenarios already prepared within the hospital to intervene in the event of an accident.

* 80% of the respondents (medical / paramedical committee) confirmed the existence of scenarios at the universities participating in university workshops, while 20% said that they manage the situations in time according to the experienced problem.
* ack of codified risk awareness within the hospital (codified awareness is only taught throughout the academic path for the paramedical and medical committee), and staff is verbally informed at the beginning until it becomes a routine, according to all respondents.
* 95% of the respondents emphasized the existence of non-codified corrective measures. In the event of an incident they manage the situations in time or they follow the hierarchy (recourse to the manager) if need arises.
* 95% of the respondents stated the absence of documents in which the problems that occurred during certain operations were recorded.
* 90% of the respondents found that the development of the risk management strategy is very useful.
* Some problems, such as the lack of qualified personnel, equipment and the intervention of other services or organizations, are definitely the main causes of the malfunction of the Department, according to 90% of the respondents, 25% of which added the problem of poor information flow between the internal and external stakeholders.
* 10% of the respondents underline the existence of financial and procedural constraints that prevent them from intervening.

Topic 2: Objectives, expectations and requirements for the development of a risk management strategy in hospital systems

- What are the potential goals of developing a risk management strategy in the hospital systems?
- What are the different dimensions that need to be taken into account when developing this strategy?
- Do you have any requirements or recommendations that you want to include in the proposal for a risk management strategy in the hospital systems?

From our semi-directive interviews, we can state that risk management aims to:

* Organize continual procedure reminders against incidents for the entire hospital committee, according to 70% of the respondents.
* Sensitize all the stakeholders through ongoing training, according to 50% of respondents.
* Develop job profiles that determine the specific task for each stakeholder (who does what and how?), according to 45% of the respondents.
* Improve the cooperation between the different departments, according to 40% of the respondents.
* Develop recall procedures for the recommendations made by the medical committees and forensic medicine experts, according to 30% of the respondents.
* Provide a dependent central sterilization department, according to 20% of the respondents.
* Have data traceability for the staff as well as the patients, according to 20% of the respondents.
* Have codified corrective measures relating to each incident, as reported by 20% of the respondents.
* Provide comfort and safety conditions for the staff and improve the environmental quality for the patient, according to 15% of the respondents.

Topic 3: Responsibilities at the development level of a risk management strategy in hospital systems

- What are the different stakeholders that need to participate in the development of a risk management strategy in HS (hospital systems)?
- Are there regulatory ways to be taken into account when developing a risk management strategy in HS?
- Who is the stakeholder capable of leading the development of this strategy?

All respondents stated that they are training to participate in the development of a specific strategy to manage risks

* 55% of the respondents suggest that the management of the Department should designate a management specialist to cooperate with the medical, paramedical and administrative committee; 15% of respondents said that this strategy should be led by the administration, and 30% of respondents emphasized that this strategy should be headed by Head of Department.
* 45% of the respondents underline the need for a codified, approved and updated procedure for each risk situation, for instance: if we do this, what should we do after ... why and when? etc.
* 30% of the respondents want to have a check list of the operational linen at the beginning and at the end of each operation.
* 15% of the respondents want to obtain an approach that ensures the quality and safety of the Department.

Topic 4: Environment and interaction with other departments at the level of development of a risk management strategy in hospital systems

- Are there other departments / organizations that can influence the activities you are performing?
- How do you manage the risks created by disruptions from other department(s) and / or organization(s)?
- Who is the stakeholder with whom you have most problems?

* 90% of the respondents indicated that they are in coordination with all external departments.

* Around 60% of the respondents pointed out the difficulties with the supply department and the pharmacy department, especially in terms of limited availability of single-use clothes, and wish to move towards a policy of supplying this type of clothes to avoid the risk of infections, as well as to adapt the operational linen budget to the need of the department (especially surgical gowns) with the need to sensitize all the stakeholders to this policy.

* 35% of the respondents report the problems with the Hygiene Department. These respondents disregard regular visits to this department, which they find fundamental, in order to reduce the frequency of the infection risk.

* 30% of the respondents experience difficulties with the Underwear Department.

* Multipurpose clothes are often poorly maintained according to 25% of the respondents, while the other 5% want the Underwear Department to work in the afternoon.

* 15% of the respondents notice that the Biomedical Department can influence their progress within the service, they even offer regular maintenance of equipment.

* 10% of the respondents mentioned the existence of coordination problems with the Maintenance Department.

Topic 5: Potential effects of developing a risk management strategy in hospital systems

- What are the potential impacts of developing a risk management strategy in the hospital systems on the health system and the quality service?
- How can we successfully implement a risk management strategy in the hospital systems?

* Cover the lack of human and material resources, according to 6% of the respondents.

* Create a motivating atmosphere that helps to reduce the risk and master own tasks, consequently improving the quality of care, according to 55% of the respondents.

* Sensitize and raise awareness of the staff towards the hazards and mainly towards the risks of infections from several sources, as emphasized by 50% of the respondents.
* Ensure a perfect and timely sharing of information with rapid cooperation between stakeholders, claimed 35% of respondents.
* Define a very clear risk management process for each incident, as well as corrective actions to be taken, said 10% of the respondents.

Topic 6: Do you have any additional information?

* 30% of the respondents said that the circuit of multipurpose operational linen is slow and expensive (personal expenses, energy-consuming equipment).
* 10% of doctors found themselves dealing with tasks they are not supposed to do.
* 10% of the respondents are of the opinion that setting up a quality / safety approach is essential to control the risks.
* We must see more awareness of women's psychology, said 10% of respondents.

With multidisciplinary support, we could formulate 12 alternatives that will be the objectives in this case and four criteria that are presented below (Table 2):

Table 2: Alternatives and criteria

	Criteria			
Alternatives	**Security**	**Awareness**	**Comfort**	**Communication**
A1	Create attractive signs to remind of security measures.			
A2	Provide mandatory postgraduate training controlled by an independent organization.			
A3	Develop procedures for the staff dedicated to medical and care activities.			
A4	Reduce the number of delayed surgical procedures in order to reduce the number of risks to the health of patients.			
A5	Define a policy for a single-use linen while taking into account budgetary, social and health aspects.			
A6	Establish a communication procedure with the patient.			
A7	Establish the job profile for each category of health professionals.			
A8	Trace incidents that have already occurred and take steps to control their causes.			
A9	Implement a risk policy within the respective areas of responsibility across the institution that allows an institution to consider external, internal, financial and other risks which could put the organization at risk.			
A10	Develop new methods to improve staff and equipment management (staff allocation, equipment allocation, etc.).			
A11	Provide comfort and safety for the staff and improve the quality of the environment for the patient.			
A12	Improve cooperation and information sharing between the departments to reduce daily problems, mainly with pharmacy / supply and hygiene.			

When risk analysis is done in an appropriate way, it leads to a series of recommendations that must be made to eliminate or reduce the risks. Which risk has the most impact? What are the priorities of Maternity Management? It is logical that the most serious risks, which have the highest impact, are considered first. Next, it is necessary to determine the impact on each objective. These selection decisions were made by the head of department.

4 Prioritization hierarchy at the strategy level

These objectives do not have equal importance, which explains the use of the influences in order to impact the importance of each aim. As Ammar et al. (2014) stated, the AHP (Analytic Hierarchy Process) method is a multicriteria aggregation method developed by Saaty (1980). It is an effective tool for dealing with complex decision making. Moreover, it is the multicriteria analysis best subject to responses because it guides the decision maker towards the methodology of formulation of his problem and it proposes method of evaluation of the importance of the parameters. Saaty (2001) suggested the following steps when applying AHP to study multicriteria problems. First, hierarchy, metrics and contributory factors are defined. In general, this hierarchy contains three levels: first, the focus or the goal, second, the objective/criteria for achieving the goal, and finally the evaluation criteria for deciding the objective. Step 4 consists in estimating the relative priorities (weights) of the decision criteria. We construct a set of pairwise comparison matrices for each of the lower levels with one matrix for each element in the level immediately above by using the relative AHP scale measurement shown in Table 3.

4.1 Decision hierarchy

The first step in an AHP analysis is to build a hierarchy for the decisions. This is also called decision modeling and consists in building a hierarchy to analyze the decision**.** The main objective must also be identified in this level. In our case, the goal is to choose the most important action that should be considered from among several potential alternatives. All criteria that might influence the decision are already mentioned in the previous section (P.11).

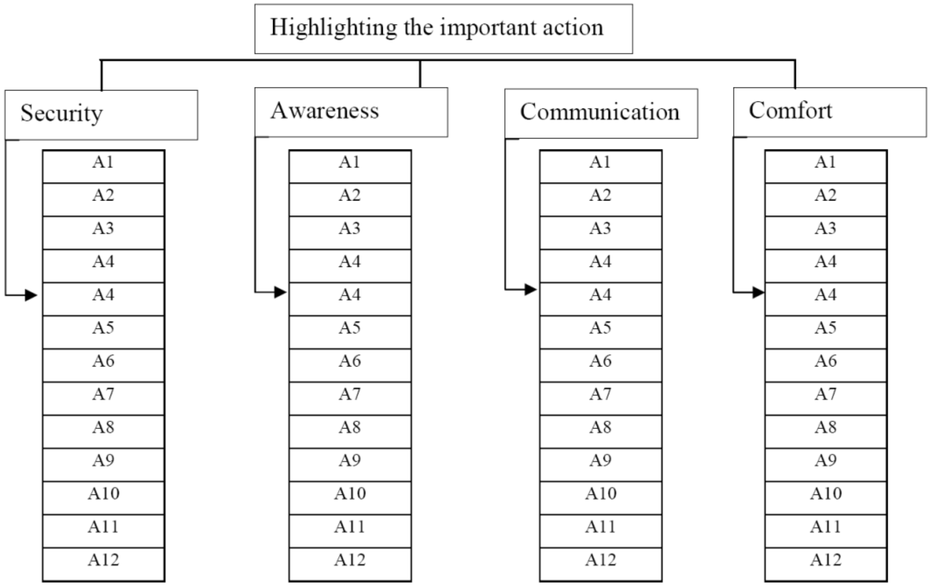

Figure 2: Decision hierarchy

4.2 Pairwise comparison matrix of criteria

Since not all the criteria have the same importance, the second step in the AHP process is to derive the relative priorities (weights) for the criteria. The relative importance between two criteria is measured on a numerical scale from 1 to 9, as shown in Table 3.

We recall that the importance of the criteria of our study was made according to the order of importance established by the decision maker.

Table 3: Saaty's pairwise comparison scale

Verbal judgment	**Numeric value**
Extremely important	9
	8
Very strongly more important	7
	6
Strongly more important	5
	4
Moderately more important	3
	2
Equally important	1

Table 4: Random consistency

N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RIC	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

The matrix is filled using the formula

$$a_{ji} = \frac{1}{a_{ij}}.$$

We define a coherence index (CI) as follows:
CI = (λ_{max} - N-1) / N, where N is the number of the elements being compared (the higher the CI, the more inconsistent the judgments, and vice versa).

A coherence ratio is defined as the ratio of the calculated consistency index and the random inconsistency coefficient (RIC) of a matrix of the same dimension. The consistency ratio is given by the following formula:

$$CR = CI/ RIC \times 100$$

CR it must be less than 10% to make consistent judgments,
where RIC is a random inconsistency coefficient that represents the average of the indices calculated at each calculation for various N (size of the square matrix).

Table 5: Pairwise comparison matrix of criteria

Criteria	Security	Awareness	Comfort	Communication	Priority Vector
Security	1	2	5.00	6	49.60%
awareness	0.50	1	5	4	31.19%
Comfort	0.2	0.20	1	0.25	6.36%
communication	0.16	0.25	4.00	1	12.85%
Sum	1.8	3.45	15	11.25	100.00%

$$\lambda_{max} = 4.095, \text{ we have } W1 = \begin{Bmatrix} 0.4960 \\ 0.3119 \\ 0.0636 \\ 0.1285 \end{Bmatrix}$$

According to the results in Table 5, it is clear that we attach greatest importance to the security criterion (0.4960), followed by awareness (0.3119) and communication (0.1285). The comfort factor has the minimum weight (0.0636).

4.3 Pairwise comparison matrix of criteria with respect to each criterion

In this step we have chosen to focus on the security criterion (Table 6). The same steps are performed for each pairwise comparison with respect to awareness, communication and comfort.

Table 6: Pairwise comparison matrix of alternatives with respect to the security criterion

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	Priority Vector
A1	1	0.142	0.5	0.2	0.25	0.5	0.2	0.2	0.2	3	3	0.166	2.60%
A2	7	1	7	4	4	6	0.5	6	0.5	8	8	2	16.60%
A3	2	0.142	1	0.2	0.25	0.333	0.2	3	0.33	3	3	0.166	3.70%
A4	5	0.25	5	1	3	4	0.33	4	0.33	7	7	0.5	9.80%
A5	4	0.25	4	0.33	1	4	0.33	3	0.33	6	6	0.5	7.20%
A6	2	0.166	3	0.25	0.25	1	0.25	4	0.2	5	5	0.25	5.00%
A7	5	2	5	3	3	4	1	4	0.5	8	6	2	16.20%
A8	5	0.166	0.33	0.25	0.33	0.25	0.25	1	0.2	5	5	0.25	4.20%
A9	5	2	6	3	3	5	2	5	1	9	9	3	20.60%
A10	0.33	0.125	0.33	0.142	0.166	0.20	0.125	0.2	0.111	1	1	0.125	1.30%
A11	0.33	0.125	0.33	0.142	0.166	0.2	0.166	0.2	0.111	1	1	0.125	1.30%
A12	6	0.5	6	2	2	4	0.5	4	0.3	8	8	1	11.50%
Sum	42.66	6.87	38.49	14.51	17.41	29.48	5.85	34.60	4.11	64.00	62.00	10.08	

$\lambda_{max} = 13.567$ CI = 0.096, CR = 9.62% < 10% (acceptable)

5 Model synthesis

In this step we calculate the overall priority (also called final priority) for each alternative; that is, the priorities that take into account not only our preference of alternatives for each criterion but also the fact that each criterion has a different weight. We are using all the values provided in the model. This step is called model synthesis.

Table 7: Synthesis of the model

	Criteria	Security	Awareness	Comfort	Communication	Overall priorities
	Criteria Weights	0.4960	0.3119	0.036	0.1285	
Actions	Action A1	0.026	0.172	0.132	0.165	0.09230455
	Action A2	0.166	0.104	0.012	0.104	0.1285696
	Action A3	0.037	0.169	0.043	0.171	0.0945846
	Action A4	0.098	0.099	0.055	0.055	0.0885336
	Action A5	0.072	0.076	0.171	0.033	0.0698129
	Action A6	0.050	0.043	0.162	0.177	0.0667882
	Action A7	0.162	0.157	0.132	0.132	0.1510343
	Action A8	0.042	0.037	0.022	0.022	0.0359913
	Action A9	0.206	0.211	0.173	0.187	0.1982444
	Action A10	0.013	0.013	0.102	0.020	0.0167447
	Action A11	0.013	0.014	0.198	0.012	0.0194846
	Action A12	0.115	0.121	0.056	0.202	

Once the above steps have been completed, it is possible to make a decision. This constitutes the last step in our AHP analysis. For this, it is necessary to compare the overall priorities obtained and whether the differences are large enough to allow for a clear choice. To give the importance (or weight) of each criterion (security, awareness, comfort and communication), action 9 is the most preferable one (with the overall priority = 0.1982444).

6 Discussion

The department stakeholders emphasized the importance of integrating an institutional risk management policy and implementing it. It is obvious that the needs and the objectives identified during the semi-structured interviews must be set up in the Department to cover all the activities. But the priorities will be influenced by the weights given to the criteria. It is useful to perform a sensitivity analysis to see how the final results would change if the weights of the criteria changed. This process allows us to understand the robustness of our original decision and what are the drivers (which criteria influenced the original results). This is an important part of the process and, in general, no final decision should be made without performing a sensitivity analysis. Note that in our example, criterion A9 (Implementation of risk policy within their respective areas of responsibility across the institution, that allows an institution to consider external, internal, financial and other risks which could threaten the organization) has a great importance (priority 19.824%). The questions that we can ask at this stage are: What would be the best objective if we changed the importance of the criteria? What if we gave the same importance to all the criteria? And what if we gave more importance, for example, to A7 (Establish the job profile for each category of health professionals)? Calculations show that even if we change the weights, the results remain the same, with high importance of criterion A9. Although the adopted methodology in this study has been quite useful in prioritizing different risks, it is not without some limitations. A major limitation is that the rating scale used in the AHP analysis is conceptual, uses a discrete scale of 1 to 9 which cannot handle uncertainty and the presence of the ambiguity in deciding the priorities of different attributes. There are also risks of bias while making pairwise comparisons of different factors. Therefore, one should be careful in assigning a relative score to different factors. This study can be further extended by considering a Fuzzy AHP approach or ANP so as to revise this model after considering some other factors in judgment expressions.

7 Conclusion

The objective of this research was to carry out a qualitative study based on a preliminary analysis in order to identify the needs, requirements and expectations of the respondents regarding risk management of medical activities in the Obstetrics and Gynecology Department of the Academic Hospitals of Sfax. Moreover, we propose to determine the prioritization of the objectives to be achieved by the risk management using the AHP method since it is an effective tool to deal with complex decision-making. It is also the best multicriteria analysis method because it follows the decision maker in the methodology to formulate his problem and in particular because it proposes a method of evaluation of the important parameters.

For this, we contacted the stakeholders of the Obstetrics and Gynecology Department in the Hédi Chaker Academic Hospital of Sfax, their risk management needs and their objectives through a qualitative study.

We analyzed the obtained findings in order to identify the objectives to be taken into account in risk management, to determine the relevance of each objective and, finally, to establish the coherence of the judgments of these objectives.

Our purpose is to provide the decision-maker with tools for decision aid to assure a continuous improvement of performance. Therefore, a framework will be allowed to be explored by a multidisciplinary team in the future. In future research, we propose to apply our results in other departments of the hospital to control other types of risks.

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HIERARCHICAL STRUCTURING FOR THE OLIVE TREES IRRIGATION PROBLEM IN TUNISIA

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Abstract

The problem of choosing the best type of water for the irrigation of olive trees is one of the decisions that have a crucial impact on the water resource management. To solve this problem, we propose a multi-expert approach, implying several quantitative and qualitative criteria and combining the AHP method and Shannon's entropy probability method. First, we use the AHP method to calculate all criteria weights for the various hierarchical levels as well as weights of the alternatives. Using the results obtained, we rank the types of water according to four experts. However, the data supplied by the experts are contradictory. We therefore combine these results according to the experts' importance. We used Shannon's entropy to determine the importance degree of each expert, to aggregate the results. The proposed approach showed that using well water was selected as the best for irrigation. Reuse of treated wastewater was classified as second, followed by desalinated brackish water and, next, by desalinated seawater.

Keywords: olive trees irrigation, multicriteria decision aid, multi-expert, AHP, incertitude, Shannon's Entropy.

1 Introduction

Water is a primary need for all living beings. It is essential for any socio-economic development. It is an important factor for the development of the agricultural, industrial, touristic and vital sectors. However, irrigation is the main water consumer in the agricultural sector. The main objective is to promote

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a stable agricultural activity when rainfall does not cover the needs of cultivated plants. Currently, the olive sector is a strategic sector in Tunisia at the economic, social, cultural and environmental levels, and the region of Sfax is one of the main olive growing regions in Tunisia. The area of olive cultivation in this region is estimated at about 340 700 ha currently representing about 19.5% of the national olive areas which cover 1.74 million ha. The olive population is about 7 million trees, representing 8.5% of the national olive population of 78 million trees. According to the Regional Commissions for Agricultural Development of Tunisia (2015)[1], Sfax is also the leading region in olive oil production, since it has contributed about 23.19% of the national production over the last decade (2006-2015) with an average production estimated at 22 674 tons. Accordingly, the olive sector is a major agricultural activity in our country. Good quality water resources in agriculture contribute to the agricultural development. In this context, the central objective is to find the best water type for the irrigation of olive trees on the basis of a study of the vegetative, productive, technological, financial, environmental and sanitary criteria related to the fruits. To achieve this goal, we propose an approach based on a multicriteria decision aid model which implies several quantitative and qualitative criteria. First, we implemented the AHP method to determine the priorities of each water type according to each expert. However, the results obtained from the AHP method appear contradictory. In order to aggregate them, Shannon's entropy is used to calculate each expert's weight. These two methods use the opinion of several experts about the choice of the best type of water for the irrigation of olive trees in the Sfax region.

AHP is a technique that facilitates complex multi-criteria decision-making, using a systematic, rational and transparent process. In addition, the AHP method helps to capture subjective and objective evaluation measures while providing a useful mechanism for verifying the consistency of the assessment measures and alternatives (Saaty, 1990; Frikha et al., 2015). In our study, we chose to work with the AHP method because our problem is hierarchically structured; it includes fifty five criteria, subcriteria and four alternatives. In addition, several experts have been contacted, which means the existence of several decision matrices. AHP, which incorporates several criteria, is proposed to determine the weights of a dataset provided by different experts. Finally, it must be verified that the information provided by the decision-makers is consistent and does not contain uncertainty.

According to our study, since the data provided are uncertain, imprecise, imperfect and conflicting, the weights of criteria and alternatives deduced from AHP are also uncertain. Consequently, we obtain judgments in the form of subjective probability distributions, which raises the question of how to combine

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[1] http://www.semide.tn/CRDA.htm

the information of several experts to obtain a better specific result. Sandri et al. (1995) have argued that uncertainty models play a crucial role in the assessment of expertise since no one can state his judgment or advice with absolute certainty. In addition, we cannot claim that the information provided has the same importance: it depends on the reliability of the expert. Hence the aggregation of information is based on the experts' weights. In conclusion, to reduce conflict, manage imperfection and calculate the experts' weights, Shannon's entropy method is used (Shannon, 1948).

2 A literature review

Several papers have dealt with the issue of water management. For instance, Domènecha et al. (2013) proposed two economic models based on growth and decay, such as (NAIADE) and (C-K-Y-L). A social multicriteria evaluation was carried out to explore the feasibility of both models. NAIADE is a new approach to improve evaluation and decision making. This method aims at evaluating each alternative with respect to each criterion and allows a ranking of the alternatives, while the C-K-Y-L approach is based on a pairwise comparison between the alternatives according to the criteria. The main objectives of these multicriteria assessments are: to compare four unconventional water sources (desalinated seawater, regenerated water, rainwater and greywater) in order to gain knowledge of their actual and perceived social-environmental performance, to find solutions to reduce water consumption, to test the feasibility and the desirability of the water supply for different alternatives and to highlight the opportunities and barriers to social and voluntary action for decay. In this paper, there are no qualitative data. Moreover, in a context of a decreasing use of water, there is a lack of reliability of a water supply system.

The multicriteria method used (NAIADE) does not supply criteria weights. Haring et al. (2016) used the AHP multicriteria decision-making method for better water management in agriculture in the Huang-Huai-hay river basin. The assessment system of irrigation water management is based on five indices or criteria, such as the technology index, the engineering index, the management index, the environmental index and the economic index.

The AHP method has been improved to calculate the weight of each index in the water management assessment indexing system for irrigation. Irrigation water management levels were obtained using the Gray correlation method and the overall fuzzy assessment method to improve the level of water management in agriculture. In this paper, the method used can give contradictory, uncertain and conflicting results. Similarly, Ben Brahim et al. (2014) used a compromise program to improve irrigation practices based on the use of recycled water and to determine if farmers would be willing to pay more for water if irrigation programs were improved and the factors influencing their decision analyzed. Their study examines a binary logistic regression analysis to meet these objectives and develops a compromise programming model based on a multi-objective technique. Compromise programming belongs to a group of multicultural analysis methods called distance methods. This technique identifies the closest solutions to the ideal through distance measurement. In this paper, farmers and policy makers used only recycled water for irrigation regardless of other types of water. Slobodan et al. (2008) proposed the Pareto optimum for the decision concerning water resources. This approach captures the uncertainty associated with weight assignment, provides decisions with a wide range of solutions to select the best one and to demonstrate the utility of the method used. A situation is said to be a Pareto optimum if it is impossible to improve the result for one actor without risk of damage to another one. The authors used ideal positive and negative solutions (TOPSIS) and a set of weights attributed to the objective functions in the form of triangular fuzzy numbers. The solution to this problem is obtained by transforming each objective function into a set of three objective functions to demonstrate the utility of the used method. Nunes et al. (2017) proposed a SWAT model to study the impact of climate and socioeconomic changes on the availability of water. This model is a tool for soil and water assessment. It is used to quantify and predict the impact of land management practices on water, sediments, and yields of agricultural chemicals.

The results obtained by the authors imply that the availability of water is resistant to climate change and that the issue of a future decrease in water availability could be solved by a supply and demand strategy. PROMETHEE is a multi-criteria overseeing method that has been applied by Abu Taleb et al. (1995).

The purpose of using this method is to minimize the extraction of groundwater that ensures quality and quantity, to obtain a high probability of cost recovery, to maximize water supply (new development projects, reuse of wastewater and others) and to promote water conservation and efficiency. A scientific analysis was developed by Lu et al. (2016), who showed the influence of the dynamic change of the ground for every period of growth of the cultures and the irrigation of the water regenerated on the yield and the quality of fruits and vegetables with regard to the irrigation drip by the subterranean waters on a ground tests of soil. They also showed that the irrigation by drip favors an increase of the yield of the tomato and allows to obtain a higher rate of water preservation. The papers listed (Domènecha and al., 2013; Sun et al., 2017; Ben Brahim et al., 2014; Slobodan et al., 2008; Nunes et al., 2017; Lu et al., 2016) focused on the reuse of treated wastewater. No paper, however, combines the four water alternatives to solve the irrigation problem, namely: the reuse of treated wastewater, the desalination of brackish water, the desalination of marine waters and the use of well water. Finally, optimization methods and multicriteria methods have not been used, so far, in the literature to solve our problem. Instead, most researchers have used qualitative scientific analysis.

3 The hierarchical structure of the problem

The choice of the best water type for olive irrigation in the region of Sfax based on the collected information from experts and researchers in the field of olive growing becomes a major challenge for the management of water resources. The determination of the best water alternative is based on several criteria: environmental (C1), production (C2), pomological (C3), physico-chemical (C4), social (C5), technological (C6) and financial (C7). Each of these criteria is divided into subcriteria of several levels. In addition, the different types of water for the irrigation of olive trees – the alternatives of our problem – are: reused treated wastewater (AL1), desalinated marine water (AL2), desalinated brackish water (AL3), and well water (AL4) (Figure 1).

Our approach is divided into two parts. The first one deals with multicriteria analysis. It will consist in an overview of the evaluation criteria and subcriteria as well as the alternatives to solve our problem. The second part handles the probabilistic analysis with multiple criteria used in the cultuvation of olive trees. These two parts deal with the opinion of several experts cultivation about the choice of the best water type for olive tree irrigation in the Sfax region.

3.1 Criteria

The choice of the best water is based on several criteria, namely: environmental (C1), production (C2), pomological (C3), physico-chemical (C4), social (C5), technological (C6) and financial (C7); each of them will be divided into more specific subcriteria (there are fifty-five criteria and subcriteria and four alternatives). These criteria generate subcriteria which are divided into subsubcriteria. Accordingly, these different levels of criteria will be represented in the form of a hierarchical structure. They were chosen after an exhaustive review of the literature on sustainable development specific to the olive sector in Tunisia. We also used discussions with researchers from an olive tree institute and with multidisciplinary researchers.

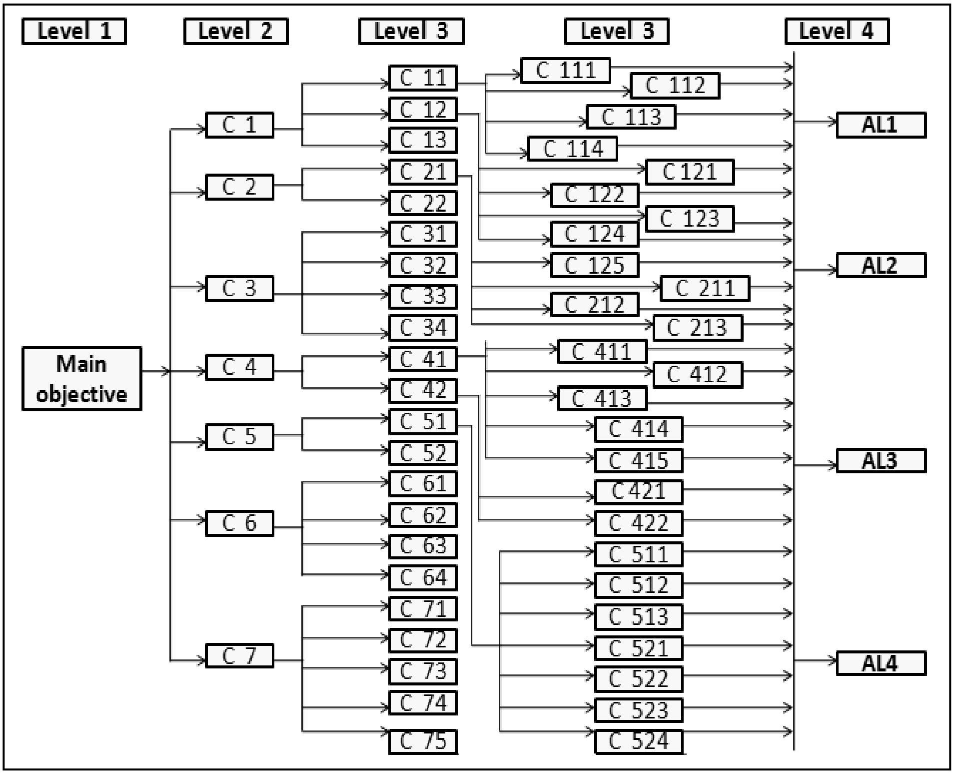

Figure 1: Hierarchical structure for choosing the best water type for olive tree irrigation

The environmental criterion **(C1)** breaks down into three subcriteria at level 3 (soil fertility (C11), salinization of irrigated soil (C12), effect on groundwater (C13)) (Figure 2). Specifically, (C11) is the ability of the land to ensure, on a regular and repeated basis, the growth of crops (Bedbabis et al., 2015) which depends on various soil components involved in the supply of plants in water and nutrients. In addition, the soil is a living vegetative cover, which facilitates the water cycle. This criterion is taken into consideration to improve the quality of the soil, its fertility and health status for the protection of the environment in the case of irrigation by different types of water. The subcriterion “soil fertility” is composed of several subsubcriteria at level 4 (preservation of the physical properties of the soil (C111) (Bedbabis et al., 2014; Ben Rouina, 2011), texture (C112), depth (C113), salinity (C114) (Bedbabis et al., 2010; Ben Ahmed et al., 2009).

The quality of water used in irrigation is a first-order factor in soil salinization. Therefore, the salinization of the irrigated soil (C12) must be minimized as long as salt has a negative effect on the physical and chemical properties of soil and water table. The effects of irrigation water on the ground are judged through the total concentration of this water in soluble salts and by the water content of absorbable sodium (Leone et al., 2007). This subcriterion includes several subcriteria at the fourth level (availability of water sources (C121), mode of irrigation (C122) (Bedbabis et al., 2015), socioeconomic factors (C123), effect of the irrigation of plants (C124), effect of the irrigation on the physico-chemical properties (C125)).

Finally, the effect on groundwater (C13) is the third subcriterion. The irrigation mode has a direct influence on the risk of contamination. Underground or gravity irrigation can affect the quality of groundwater and surface water. Direct contamination may occur during the maintenance of the irrigation system. Sprinkler irrigation creates contaminating aerosols that can be transported over long distances, while gravity-fed and flood irrigation exposes workers to high health risks, especially when land use is unprotected against soil salinization (Peasey et al., 2000).

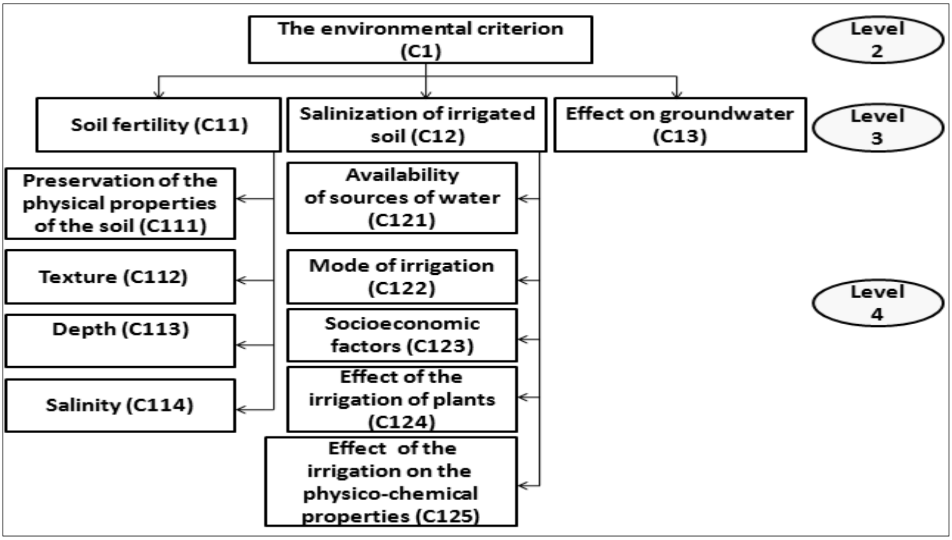

Figure 2: Decomposition of the environmental criterion into subcriteria

The production criterion (Wiesman et al., 2004) (C2) (Figure 3) splits into vegetative growth (C21) and oil quality (C22). The first subcriterion (C21) depends on several factors, such as light, water supply, mineral elements and the load of olives. In our problem and for this type of criteria, we aim at finding the best water alternative to improve production. The improvement in production is mainly due to good vegetative growth. Criterion (C21) splits into three subcriteria (number of flower clusters / linear meter per shoot (C211), number of flower buds / linear meter (C212), number of fruit tied / linear meter of the shoot

(C213)). The goal of (C22) (oil quality) is to find the impact of irrigation through this type of water on yield (Clodoveo, 2012). The higher the production of olives, the larger the increase of the amount of olive oil.

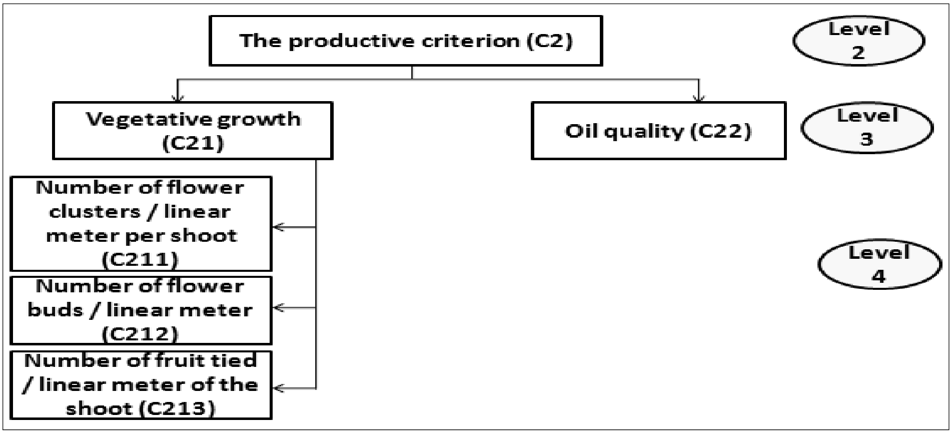

Figure 3: Decomposition of the production criterion into subcriteria

As for Criterion (**C3**) (Figure 4), there are four subcriteria at the third level (production (C31), average fruit weight (C32), pulp / core ratio (C33), fat content (C34)). Pomological criterion is used for the characterization of olive varieties. It allows to classify the varieties according to their yield in olive oil. The objective of (C31) is to obtain higher and more consistent levels of olive production while minimizing the cost of exploiting water resources (Gucci et al., 2007). In addition, (C32) is a pomological criterion for olives that must be calculated because this indicator is very important for the characterization of oil varieties olives, given its impact on the fat content and, consequently, on the oil yield (Fourati et al., 2003). In general, the average value of (C33) depends on the variety and type of water used in irrigation. Irrigation of olive trees with water of good quality leads to an improvement in the consistency of the fruit pulp, which has a direct impact on their commercial value because this consistency is an important quality criterion for olives. High water availability in the soil during the growing season increases the production, the fruit size, the pulp-core ratio and the oil content of the olives expressed as a percentage of dry weight. The fat content (C34) is a criterion of great economic importance, as the ultimate goal of olive cultivation is the production of oil. This criterion can be determined by various methods such as nuclear magnetic resonance.

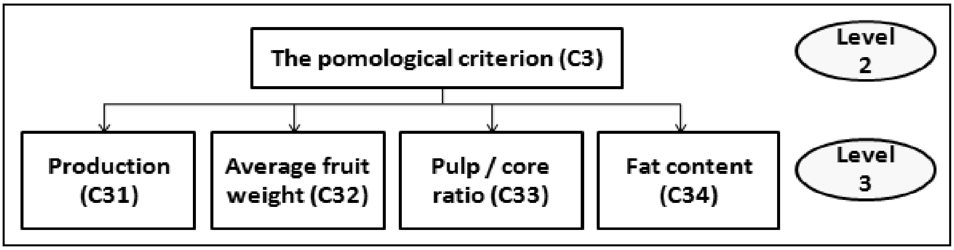

Figure 4: Decomposition of the pomological criterion into subcriteria

The physicochemical criterion (**C4**) is the fourth one considered in our problem (Figure 5). It is divided into two subcriteria, namely quality (C41) and purity (C42). These are also subdivided into subcriteria. The concept of ‘quality’, especially for virgin olive oil, must be defined and a judicial control of the respect of commercial indices and authenticity must be established (Gharsallaoui et al., 2011; Bedbabis et al., 2016). The criteria of olive oil quality are: acidity (C411), peroxide value (C412), ultraviolet absorbance (C413), chlorophyll quantity (C414) and polyphenol content (C415). The purity criterion is also divided into two subcriteria at the fourth level (oil quality (C421), acidic component (C422)). The objective of the purity criterion (C42) is to find the impact of irrigation by this type of water on olive oil. Commercially speaking, the taste has a very important effect on the quality, which is measured by organoleptic evaluation (C421) (Bedbabis et al., 2015; Bourazanis, 2016). Thus, certain characteristic defects are prohibitive for the marketing of olive oil. The most important are the olive oils obtained from olives stored in bad conditions, the olive oils appear mold if the olives are long stored even under the right conditions (mold) and the olive oils poorly preserved (rancidity). The quality of irrigation water has a direct influence on the acidic component of olive oil (C422). Indeed, olive oil consists of several types of acidic components, the most important of which is oleic acid. (It is an excellent energy food, a basic ingredient of the Mediterranean cuisine.)

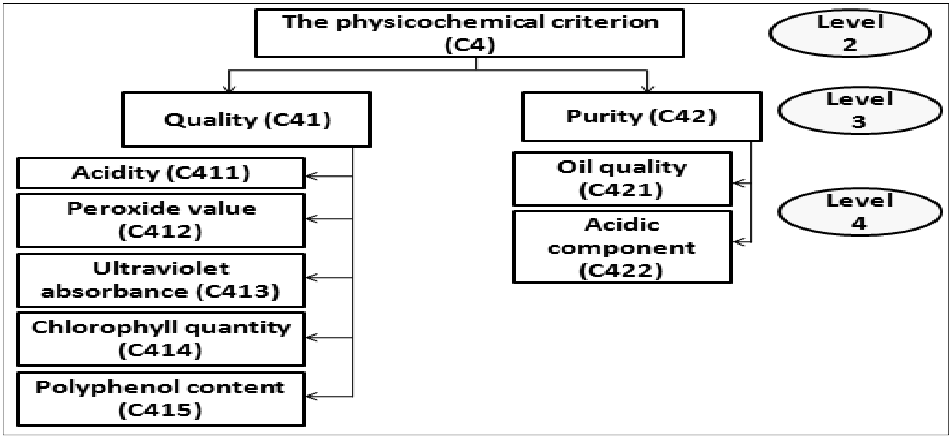

Figure 5: Decomposition of the physicochemical criterion into subcriteria

The choice of the best water is also based on a social criterion **(C5)** which is divided into two subcriteria (Figure 6) (Health Risk (C51) and Water Quality (C52)). The choice of water and irrigation method of olive trees is very important for good quality of oil. In particular, we are here concerned with the sanitary quality of the olive tree and the soil in terms of bacteria. This criterion (C51) also splits into three subcriteria at level four (tree (C511), soil (C512) and irrigation mode (C513)). Irrigation water has an influence on the sanitary quality of the olive tree (C511) (Bedbabis et al., 2015). In addition, an increase in salinity causes toxic effects which appear much more easily when the salts are brought directly into the leaves during irrigation. In addition, the irrigation mode influences soil contamination and clogging (C512) (Bedbabis et al., 2015; Petousi et al., 2015). Indeed, Azzouzi et al., (2015), found that the use of treated wastewater for 20 years is not recommended because it generates a high level of organic contaminants in the soil. (C513) has a direct influence on the risk of contamination. In 2006, the World Health Organization (WHO) recommendations have predicted risk levels, depending on the irrigation technique and crop types (WHO, 2016). As for water quality (C52), it is divided into four subcriteria at the fourth level (guarantee of the safety of the farmers (C521), no deterioration of the soil quality (C522), physicochemical characteristics of the soil (C523) and bacteriological aspects (C524) (Khabou et al., 2009). The quality of water used for irrigation is an essential parameter for crop yield, maintaining soil productivity and protecting the environment. Thus, the physical and chemical properties of the soil, such as its structure (aggregate stability) and permeability are very sensitive to the type of potentially exchangeable ions present in irrigation water (C523) (Ayoub-Tebini H., 1981). Water chosen for the irrigation of olive trees must be of good quality so as not to cause the deterioration of soil quality

(C522). Indeed, the degradation of cultivated soils depends very much on the type of water. In addition, poor water quality is a serious threat to the viability and safety of agricultural products from intensive farming systems (C524) (Asano, 1998; Materon, 2003).The safety of the operators must also be guaranteed (C521). The choice of the best category of water for the irrigation of olive trees stimulates the production and the quality of the oil which guarantees the safety of the farmers.

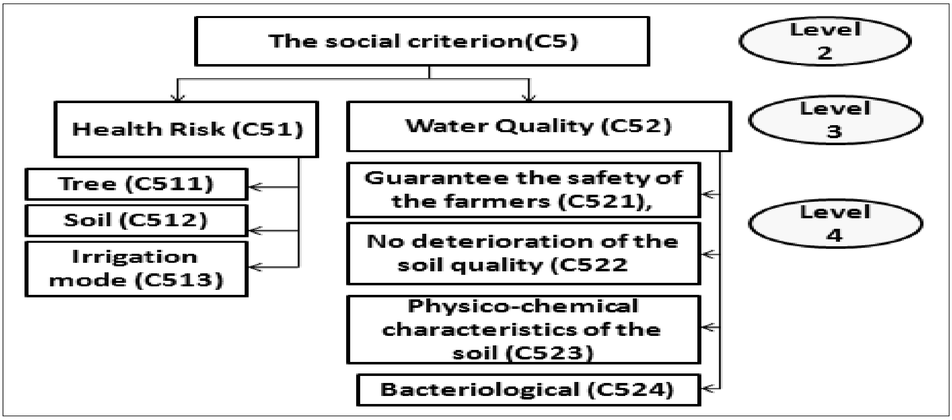

Figure 6: Decomposition of the social criterion into subcriteria

Next, we take also into account the technological criterion (**C6**) (Figure 7) which is divided into several subcriteria (irrigation technique (C61), the time required for irrigation (C62), simplicity (C63) and processing reliability (C64)). Most farmers use traditional water-intensive techniques, such as gravity irrigation, which generate significant losses through soil evaporation and deep percolation (Zin El-Abedin et al., 2018). Today, irrigation systems are diversified. Among the most effective are full coverage, drip and sprinkling. (C62) is the amount of time needed to complete the installation of an unconventional water supply system. Inadequate or poorly designed irrigation systems can spread pathogens and pollutants in crops. The objective of (C63) is to apply the most reliable irrigation technique and especially the simplest and least time-consuming. Drip irrigation is considered to be the simplest such technique in agriculture. Finally, reliability (C64) includes skills and knowledge required from farmers and workers, land ownership, and land and water rights.

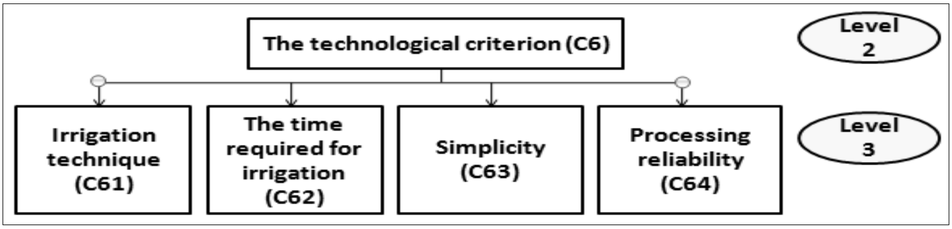

Figure 7: Decomposition of the technological criterion into subcriteria

We consider also the financial criterion (**C7**) (Figure 8). It is divided into five subcriteria (cost of irrigation (C71), water intake (C72), electrical input (C73), cost of water transfer for irrigation (C74) and amount of used water (C75)). The objective of (C71) consists in evaluating the economic efficiency of irrigation, whether it is cost-effective. This cost assessment will determine whether the selected water is the least expensive or not and will lead to significant economic gains. For (C72), access to a reliable supply of water is often the main constraint of irrigation. Furthermore, water has a fundamental role in the life of olive trees. In addition, drought directly influences plant growth and yield in arid and semi-arid regions. The use of unconventional water is the major solution for irrigation. But the farmers refuse to use it because they believe that this water is worthless. In economics, energy efficiency (C73) consists in reducing energy consumption, with an equal service level. This is the case of agriculture, installation of equipment or materials for irrigation, which facilitates the distribution of water for the farmer. The objective of (C74) is to choose the most efficient type of water with the minimum cost of transfer (Berbel**,** 2018). Irrigation water requirements depend on water requirements of the crops and the water they naturally have. In fact, the objective of (C75) is to choose the most efficient water alternative while minimizing the amount used for irrigation of olive trees. Excessive irrigation leads to costly waste, which can lead to a deterioration of the quality of the olives and results in fertilizers placed deep in soil (Nielsen**,** 2018).

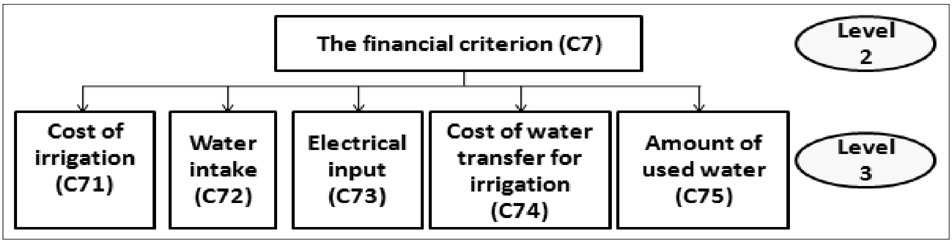

Figure 8: Decomposition of the financial criterion into subcriteria

3.2 Alternatives

The alternatives, which are the different types of water possible to use to irrigate the olive cultivation in Sfax were fixed after meetings with scientists of an institute of the olive tree. The different types of water for olive irrigation available in the Sfax region of Sfax are:

– **Reused treated wastewater (AL1)** (Bedbabis et al., 2015; Brahim-Neji et al., 2014; Bourazanis et al., 2016; Valdes-Abellan et al., 2017; Makram et al., 2012) Wastewater is a very important alternative in the context of the overall management of water resources in agriculture. Reuse of wastewater in agriculture contributes to the conservation of freshwater and energy, which improves the quality of life. Finally, reuse of wastewater in agriculture can be a way of protecting the environment and especially a mean of recycling the nutrients contained in the soil;
– **Desalinated seawater (AL2)** (Ghassemi et al., 2013; B. Rjula et al., 2010) results from a process that produces fresh water from brackish or salty water. Desalinated seawater is a resource rarely used for irrigation because of its cost. Desalination of seawater is a reliable technique which is also less expensive than the recycling waste water;
– **Desalinated brackish water (AL3)** (Valdes-Abellan et al., 2017; Wiesman et al., 2004) refers to all saline waters with less salinity than seawater. Desalination of brackish water is a solution to avoid the risk of salinity. This use will normally be for human consumption or for industrial, agricultural, activities, and so on;
– **Well waters (AL4)** (Singh, 2018; 2016; 2014; Hamamouch et al., 2017; Chen, 2018; Autovino et al., 2018). Wells are soil-based structures that extract, economically and efficiently, groundwater from an aquifer. There are three main types of wells: dug wells, dark wells and drilled wells.

4 The proposed model for choosing the best water alternative for olive trees irrigation

4.1 AHP method for ranking water alternatives

Multicriteria decision aid methods are methods for aggregating multiple criteria to choose one or more actions or solutions. In this methodological framework, we use the AHP method (Sun et al., 2016; Frikha et al., 2015) which is a powerful and flexible tool in decision-making. It is a multicriteria aggregation process developed by Saaty (1990), which makes it possible to break down a complex problem into a hierarchical system, in which binary combinations are established at each level of the hierarchy. The method begins with the definition of the main objective to be achieved or the decision to be made about determining the best type of water for irrigation of olive trees. This main goal breaks down into a hierarchical structure of evaluation criteria and subcriteria. In the last hierarchical level, we find the types of water to be evaluated (the alternatives). The AHP method consists of the following steps:

- Break the problem into a hierarchical structure (Figure 1).
- Perform binary combinations level by level: This involves pairwise comparison of the relative importance of all the elements from the same level of the hierarchy with the element from the higher level. Each expert is asked to provide matrices of pairwise comparisons of all the criteria, and of all the subcriteria corresponding to the criterion of the higher level, and so on, until reaching the matrices of comparisons of the types of water corresponding to each sub criterion.
- Determine the priorities: Three operations are necessary to calculate the priorities: add the columns of the matrix, normalize the matrix and calculate the average of the rows. We determine the weights of all criteria and subcriteria as well as the weights of water types for the irrigation of olive trees, according to each subcriterion, and that according to each of the contacted experts.
- Synthesis of the priorities: Once the priorities for all the criteria in the hierarchy have been determined, the weight of each alternative with respect to all the criteria and subcriteria are calculated and a ranking of all types of water is obtained. We thus obtain the main eigenvector of the $n \times m$ reciprocal matrix.
- Check the consistency of judgments: The AHP method validates the reliability of the results by calculating a consistency index. This index will allow us to detect significant inconsistencies in the data provided.

  The Coherence Index is calculated as follows:

$$IC = \frac{(\lambda max - n)}{(n - 1)} \tag{1}$$

where $\boldsymbol{\lambda_{max}}$ is the maximum eigenvalue, n is the size of the matrix, IC is the Coherence Index which represents the level of reliability of the judgments provided.

The Coherence Ratio (CR) is calculated as:

$$RC = \frac{\boldsymbol{IC}}{\boldsymbol{CIA}} \tag{2}$$

where CIA is a random index developed by Saaty.

Using the consistency ratio, we compare the actual reliability with theoretical reliability. If RC ≤ 0.1 (10%), the matrix is regarded as sufficiently coherent. When this value exceeds 10%, the assessments may require revisions.

To obtain a reliable result, several experts are contacted (Table 1). Reliance on a single expert can lead to unreliable and uncertain solutions, as expert knowledge of a single expert is often regarded as the best or the only source of information. In addition, the experts often share the same education and the same literature and visit the same conferences which will have a similar influence on their quantification of uncertain knowledge (Hofer, 1986). Therefore, it is mandatory to conduct an expert opinion poll when expert judgment is an important basis for quantification.

Table 1: Distribution of the sample of experts

Expertise field	Expert	Farmer	Responsible of Regional Commissariat for Agricultural Development	Researcher	Responsible of the Olive Tree Institute of Sfax	Responsible of the Agricultural Development Delegation of Sfax
Expert 1	*	*			*	
Expert 2			*			
Expert 3				*	*	
Expert 4		*				*

A questionnaire has been proposed to determine the experts' opinion. It must be carried out on an individual basis. It consists of two main parts:

- ✓ The first part is simple and consists in identifying and characterizing the respondent's situation, including his area of expertise.
- ✓ The second part deals with the objectives to be evaluated and the alternatives of the study. According to the opinion and the experience of the respondent, the comparative evaluation consists in pairwise comparisons of the importance of one criterion at each level of the hierarchy. The comparative evaluation is performed using Saaty's fundamental scale (Saaty, 1990).

We will present an explanatory example of the calculations for a single expert (Expert 3) and a single level given the large number of calculations (Tables 2-6, Figure 9).

Table 2: Pairwise Comparison Matrix of Criterion C1 for Expert 3

	C11	C12	C13
C11	1	1	0,5
C12	1	1	1
C13	2	1	1
Sum	4	3	2,5

Table 3: Determination of subcriteria weights

	C11	C12	C13	Sum	Weights
C11	0,25	0,333333333	0,2	0,783333333	0,261111111
C12	0,25	0,333333333	0,4	0,983333333	0,327777778
C13	0,5	0,333333333	0,4	1,233333333	0,411111111
Sum	1	1	1		1

Table 4: Verification of subcriteria judgment consistency

	C11	C12	C13
priority	0,261111111	0,327777778	0,411111111
C11	1	1	0,5
C12	1	1	1
C13	2	1	1

	C11	C12	C13	Sum	Sum/weight
C11	0,261111111	0,327777778	0,205555556	0,794444444	3,042553191
C12	0,261111111	0,327777778	0,411111111	1	3,050847458
C13	0,522222222	0,327777778	0,411111111	1,261111111	3,067567568
Sum	1,044444444	0,983333333	1,027777778		9,160968217
λmax					3,053656072
IC					0.026828036
RC					4.625523468

RC < 10%. Hence the judgments are consistent.

Each expert is asked to compare, pairwise, the types of water used for irrigation, denoted ALT_i i = 1,…, 4 from the fifth level of the hierarchy with respect to the criteria and subcriteria of the fourth level. (The results of weight calculations according to Expert 3 are shown in Tables 5 and 6 and Figure 9).

Table 5: The alternative weights for Expert 3

	Weights	AL1	AL2	AL3	AL4
C111	0,450043706	0,294157925	0,074249709	0,436873543	0,436873543
C112	0,117438811	0,223513911	0,076711811	0,123120592	0,576653685
C113	0,190646853	0,105916593	0,16154583	0,253647215	0,478890363
C114	0,241870629	0,29805452	0,048147717	0,377574557	0,276223206
C121	0,043882347	0,29805452	0,048147717	0,377574557	0,276223206
C122	0,554795892	0,29805452	0,048147717	0,276223206	0,276223206
C123	0,174955527	0,29805452	0,048147717	0,377574557	0,276223206
C124	0,128520914	0,633333333	0,066666667	0,066666667	0,233333333
C125	0,09784532	0,625	0,125	0,125	0,125
C13	0,411111111	0,051699819	0,185999095	0,087599731	0,674701355
C211	0,128501401	0,585714286	0,053968254	0,053968254	0,306349206

Table 5 cont.

C212	0,276610644	0,585714286	0,053968254	0,053968254	0,306349206
C213	0,594887955	0,412443439	0,053968254	0,053968254	0,306349206
C22	0,875	0,669026807	0,105099068	0,088432401	0,137441725
C31	0,272457651	0,669380843	0,142322272	0,071466171	0,116830714
C32	0,497278107	0,634259259	0,141137566	0,10542328	0,119179894
C33	0,168712378	0,616946559	0,159250009	0,098316218	0,125487214
C34	0,061551864	0,616946559	0,159250009	0,098316218	0,125487214
C411	0,205401715	0,57486631	0,069136746	0,069136746	0,286860199
C412	0,10978003	0,57486631	0,069136746	0,069136746	0,286860199
C413	0,17584118	0,57486631	0,069136746	0,069136746	0,286860199
C414	0,281755373	0,57486631	0,069136746	0,069136746	0,286860199
C415	0,227221702	0,57486631	0,069136746	0,069136746	0,286860199
C421	0,25	0,375	0,125	0,125	0,375
C422	0,75	0,375	0,125	0,125	0,375
C511	0,128501401	0,051784822	0,175668821	0,15589816	0,616648197
C512	0,276610644	0,038919414	0,117673993	0,421703297	0,421703297
C513	0,594887955	0,06223344	0,109931996	0,104118043	0,723716521
C521	0,354249354	0,042261905	0,12797619	0,12797619	0,506448413
C522	0,245306495	0,048065489	0,204895922	0,084664244	0,662374346
C523	0,068292068	0,039479576	0,310106113	0,263319901	0,38709441
C524	0,10992986	0,043030039	0,104486861	0,852483101	0,342445491
C61	0,178075397	0,667468046	0,155260412	0,11691592	0,060355621
C62	0,104662698	0,208474419	0,058027252	0,071468112	0,662030216
C63	0,104662698	0,667468046	0,155260412	0,11691592	0,060355621
C64	0,612599206	0,667468046	0,155260412	0,11691592	0,060355621
C71	0,089357579	0,187156094	0,059690355	0,063596605	0,689556946
C72	0,164650529	0,29805452	0,048147717	0,377574557	0,276223206
C73	0,219951875	0,29805452	0,048147717	0,377574557	0,276223206
C74	0,120851922	0,193877278	0,144732757	0,0444426	0,616947365
C75	0,405188095	0,411342593	0,162268519	0,190046296	0,236342593

Table 6: Alternative weights for Expert 3

Expert 3	w_{AL1}	w_{AL2}	w_{AL3}	w_{AL4}	**Sum**
w_j	4.33256669	1.206038011	1.785311004	3.658094211	10.98200992
$w_{j\ normalized}$	0.394514913	0.19819425	0.162566872	0.3309879	1

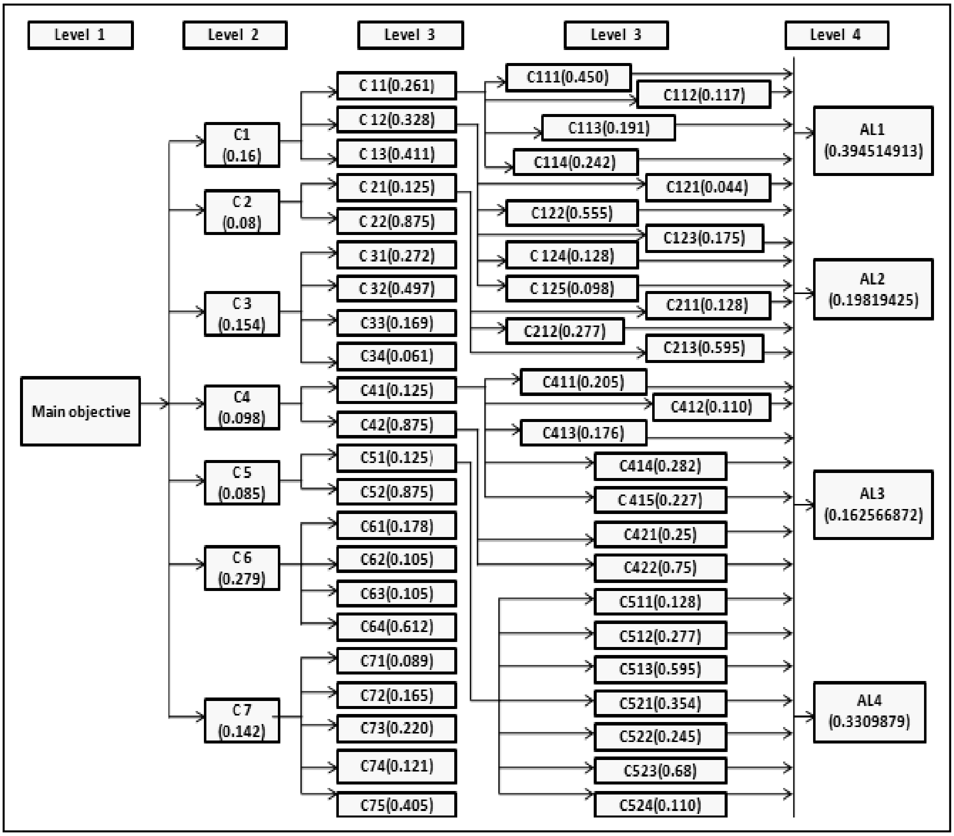

Figure 9: Presentation of the weights of the criteria, subcriteria and alternatives for Expert 3

Nevertheless, since several experts have been contacted and provided different information, the implementation of their information in the AHP method brings about different and even contradictory results. In order to reduce the contradiction and ambiguity, we should combine all the obtained results. The combination must take into account the degree of importance of each expert. For that, Shannon's entropy must be used to calculate the experts' weights.

We calculate the weights of all criteria, subcriteria, as well as the weights of alternatives with respect to each criterion. Then, we multiply the sum of each criterion weight by the alternative one, according to this criterion. Thus we obtain a vector that indicates the impact of the criterion i on each alternative. This vector represents the main eigenvector of the $m \times n$ reciprocal matrix. The results obtained for each expert are shown in Table 7.

Table 7: Weights of irrigation alternatives according to different experts

Experts	w_j	AL1	AL2	AL3	AL4
Expert 1	w_j	3. 478792956	1.343721133	1.511164155	4.666321756
	$w_{jnormalized}$	0.316253905	0.122156467	0.13737856	0.424211069
Expert 2	w_j	2.6289787795	1.642643392	1.612734043	4.315285859
	$w_{j\,normalized}$	0.257752063	0.161049121	0.158116729	0.423082087
Expert 3	w_j	4.33256669	1.206038011	1.785311004	3.658094211
	$w_{j\,normalized}$	0.394514913	0.19819425	0.162566872	0.3309879
Expert 4	w_j	2.567468293	1.42787949	1.822872115	5.392475158
	$w_{j\,normalized}$	0.229019546	0.127367615	0.162601168	0.48101167

Using these results, we can rank alternatives, according to different experts.

From the results of Expert 1, we find that AL4 ≻ AL1 ≻ AL3 ≻ AL2. The use of well water (AL4) is the best choice with the weight of 0.424. It is followed by the reuse of treated wastewater (AL1) (0.316) which is followed by the use of desalinated brackish water (AL3) (0.137) and, finally, by the use of desalinated marine waters (AL2) (0.122). For this expert, the result of the analysis suggests that the use of well water is preferred because it has the highest coefficient of importance (Figure 10).

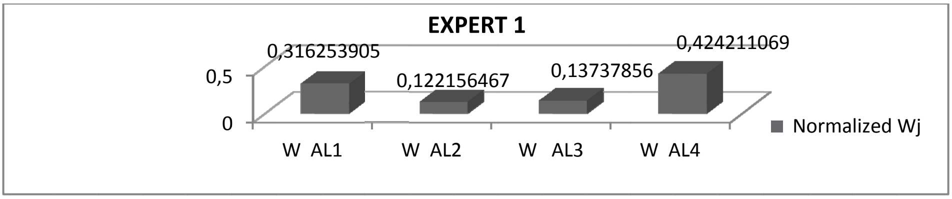

Figure 10: Ranking of water types according to the judgments of Expert 1

From the judgments provided by Expert 2, we obtain the following ranking:

AL4 ≻ AL1 ≻ AL2 ≻ AL3. According to the results obtained from the second expert, the preference of AL4 and AL1 is similar to the first one. Thus, the use of well water and the reuse of treated wastewater are the best types of water for the irrigation of olive trees. The use of desalinated marine waters is preferred to the use of desalinated brackish water according to the judgments provided by Expert 2, which is unlike the rank of Expert 1. We also note that the weights of the two best alternatives are much larger than those of the two others (Figure 11).

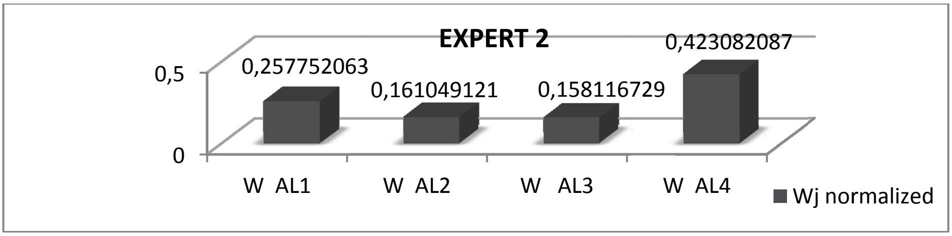

Figure 11: Ranking of water types according to the judgments of Expert 2

Applying the AHP method to the data provided by Expert 3, we obtain the following results:

AL1 ≻ AL4 ≻ AL3 ≻ AL2. According to Expert 3 the best type of water for irrigation is different as compared to the first two experts. Namely, the reuse of treated wastewater turns out to be the best choice with the weight of 39.45%. The use of desalinated brackish or marine waters are the least preferred by all experts (Figure 12).

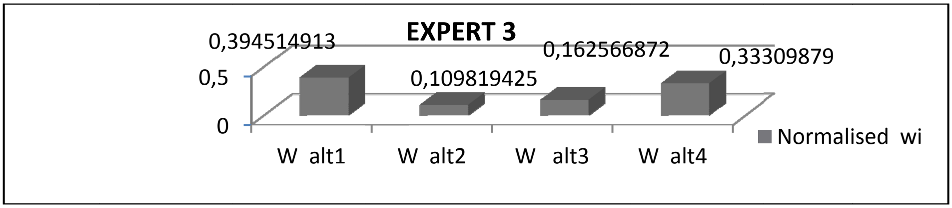

Figure 12: Ranking of water types according to the judgments of Expert 3

The ranking obtained on the basis of the pairwise comparisons provided by Expert 4 is the same as that obtained by Expert 1. However, the use of well water is by far preferred over the other types of water for the irrigation of olive trees with a coefficient of importance of almost 50%, against the other 50% divided among the other three types of water (Figure 13).

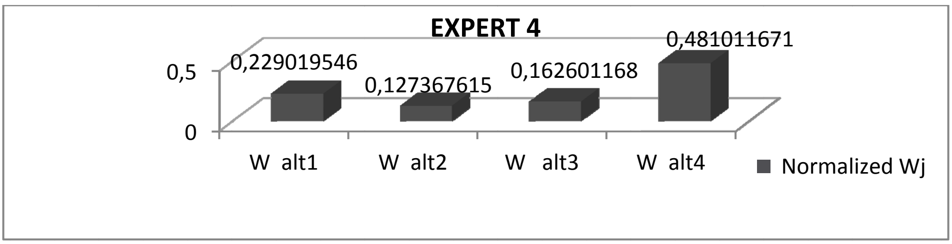

Figure 13: Ranking of water types according to the judgments of Expert 4

4.2 Shannon's entropy for aggregating the experts' rankings

The data provided by the different experts are contradictory and uncertain. In other words, judgments provided by experts are often imprecise, incomplete, uncertain and therefore unreliable due to the inherently limited precision of human evaluations. In this context, unreliability is not synonymous with a total lack of reliability, but it implies partial reliability. In order to cope with heterogeneity, Sandri et al. (1995) have argued that uncertainty models play a crucial role in assessing expertise, since no one can provide absolute certainty of his judgment or advice.

According to our study, since the data provided are uncertain, imprecise, imperfect and conflicting, the weights of the criteria and alternatives determined by the AHP method are also uncertain. These weights are assumed to be subjective probability distributions. This raises the question of how to combine the information from several experts to obtain a better specific result. We cannot regard all the information provided as having the same importance; it must depend on the reliability of the expert. Hence, the aggregation of information should be weighted according to the importance of each expert. In conclusion, to reduce conflict and manage imperfection, we use Shannon's entropy (Shannon, 1948) in order to determine the experts' weights and combine judgments. It is a mathematical function that corresponds to the quantity of information contained or delivered from an informed source, and has the properties of a suitable measure of uncertainty in a random experiment. The more different the information emitted by the source, the larger the entropy (or uncertainty about what the source emits).

4.2.1 Determination of the uncertainty (H_n) of the experts

Shannon's entropy ($\boldsymbol{H_n}$) can serve as a very convenient measure of uncertainty and information that corresponds to a finite probability space or a random experiment. This function has the properties of a suitable measure of uncertainty in a random experiment. We calculate the amount of uncertainty ($\boldsymbol{H_n}$) provided by each expert *i*.

$$H_i = H_i(P_1.....P_n) = -\sum_{j=1}^{n} W_{ij}\, Log_n\left(W_{ij}\right) \qquad (3)$$

where W_{ij} is the weight of the alternative *j* according to expert *i*, *i* = 1, ..., *m* and *j* = 1, …, *n*.

$$W_{ij} \geq 0 \text{ and} \sum_{n=1}^{n} W_{ij} \geq 1$$

Shannon's entropy is a decreasing function because the higher $\boldsymbol{H_i}$, the less informative the expert is and the more uncertainty his opinion contains. Therefore, $\boldsymbol{H_i}$ is a function to be minimized. It is then necessary to normalize $\boldsymbol{H_i}$ to find the weights $\boldsymbol{w_i}$ of expert *i, i* = 1, …, *m*.

In our case, alternative weights are assumed to be probabilities. We use Shannon's entropy method to reduce conflict and manage imperfection. This method is based on the theory of probabilities that allows to solve a problem with uncertain data. Since experts do not have the same degree of reliability and the same level of importance, we must determine their weights. The information derived from the data provided by the experts are the weights of the standardized alternatives presented in Table 8.

Table 8: Weights of water types according to each expert

	E1	E2	E3	E4
AL1	0,31625391	0,25775206	0,39451491	0,22901955
AL2	0,12215647	0,16104912	0,10981942	0,12736761
AL3	0,13737856	0,15811673	0,16256687	0,16260117
AL4	0,42421107	0,42308209	0,33309879	0,48101167

These weights are used in a probability distribution. In this case, we can determine the amount of information or uncertainty of each expert using Shannon's entropy. The results are summarized in Table 9.

Table 9: Amount of uncertainty provided by the experts

	H_1	H_2	H_3	H_4	Sum
H_i	-0,54606995	-0,56419057	-0,55200145	-0,54174727	-2,20400925

We must then normalize the uncertainty quantities of each expert. The obtained data are then summarized in Table 10.

Table 10: The standard uncertainty amount provided by the experts

	H_1	H_2	H_3	H_4
$H_{i\ normalized}$	0,2477621	0,25598376	0,25045333	0,24580082

4.2.2 Determination of the experts' weights

When aggregating the opinions of the experts, we cannot regard them as equally important and their judgments, as having the same importance. Indeed, these experts have different degrees of reliability. The more reliable the expert is, the more important his judgment will be. Therefore, to be able to aggregate the opinions of all the experts, we must calculate their weights, which express their coefficients of relative importance.

Since $\boldsymbol{H_i}$ expresses the amount of uncertainty, the higher it is, the more unreliable the expert is and the less his judgments will be considered. This function is then decreasing with the weight of the experts. We obtain:

$$\boldsymbol{w_i} = 1 - \boldsymbol{H_{i\,normalized}} \quad (4)$$

Given that $\boldsymbol{H_i}$ expresses the amount of uncertainty, it follows that the higher the uncertainty, the less reliable the expert is and the lower the weight will be. The weight will then be a decreasing function of the amount of uncertainty (Equation 4).

The weights are summarized in Table 11.

Table 11: Determination of the experts' weights

	H_1	H_2	H_3	H_4
W_i	0,7522379	0,74401624	0,74954667	0,75419918

4.2.3 Aggregation of the experts' opinions

To be able to classify the different types of water for the irrigation of olive trees, according to all the experts, we must aggregate all the weights of each alternative determined by the AHP method while considering the degree of reliability of each expert. This aggregation is based on the weighted average method. For each type of olive trees irrigation water, we calculate the priority $\boldsymbol{W'_j}$.

$$\boldsymbol{W'_j} = \sum_{i=1}^{m} \boldsymbol{w_i w_{ij}} \ \forall j = 1, \ldots, n \quad (5)$$

The results are shown in Table 12.

Table 12: Weights of water types

	E1	E2	E3	E4	W_j'
W_i	0,7522379	0,74401624	0,74954667	0,75419918	
AL1	0,31625391	0,25775206	0,39451491	0,22901955	0,7522379
AL2	0,12215647	0,16104912	0,10981942	0,12736761	0,74401624
AL3	0,13737856	0,25775206	0,16256687	0,16260117	0,74954667
AL4	0,42421107	0,42308209	0,33309879	0,48101167	0,75419918

On the basis of the determined weight values $\boldsymbol{W'_j}$, we rank the alternatives in a descending order of importance to obtain an outranking graph. The best alternative is the one with the highest $\boldsymbol{W'_j}$, and so on. The alternatives are ranked according to the weights from Table 12.

$$W_j'(E4) > W_j'(E1) > W_j'(E3) > W_j'(E2)$$

Then the ranking of the alternatives is:

AL4 ≻AL1 ≻ AL3 ≻ AL2

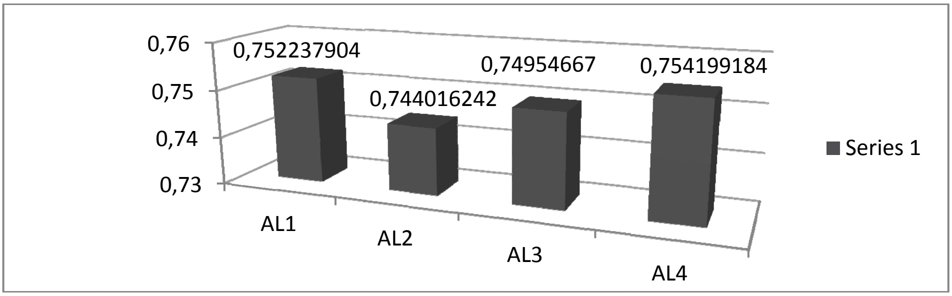

Figure 14: Ranking of alternatives according to the judgments of all experts

From the results provided by Shannon's entropy (Figure 5), we obtain that well water proves to be the best type of water for the irrigation of olive trees in the Sfax region. This alternative minimizes the impact on the environment (contamination) and demonstrates a commitment to public health. Additionally, well water offers an opportunity to enjoy free and healthy water. Hence, well water irrigation can increase olive productivity at the lowest cost. On the other hand, it is obvious that the seawater desalination alternative is considered as the worst technique, mainly because it requires a lot of time and money. So it can have environmental impact. Reuse of treated wastewater is ranked second, followed by desalination of brackish water. In fact, it has the major advantage of providing an alternative resource at a lower cost to limit water shortage, preserve natural resources and contribute to the integrated water management.

5 Conclusion and future research

Tunisia has limited water resources distributed over time and space. In this context, better allocation and valuation of irrigation water are required. In Tunisia, olive growing is of paramount importance in our agriculture in social and economic terms. In fact, irrigation of olive trees is an effective management tool against the hazards of rainfall. Therefore the selection of the best alternative water for the irrigation of olive trees is essential to establish an effective management system. This assessment must be based on the collection of a large amount of information, obtained from several experts. In this context, we hybridized two methods: AHP and Shannon's entropy. Firstly, we used the AHP method to determine the priorities of all criteria of different hierarchical levels and alternatives, and a classification of choice of water alternatives according to each of the four experts is determined. Secondly, we used Shannon's probabilistic entropy method, since the data provided by the experts are contradictory and uncertain and therefore unreliable. Thus, we determined the importance of each expert using Shannon's entropy in order to be able to aggregate all the rankings by the experts and then determine a unique result. The proposed approach has shown that well water irrigation is the best water alternative. Among the most promising prospects, it would be interesting to analyze and measure the uncertainty of the results obtained by the AHP method in a simulation model. It is necessary to increase the use of unconventional waters for treating wastewater. It is a solution that seems efficient in the immediate or short term. But it is still insufficient considering the limitations of their use. As for desalination of seawater, it is a solution that could be serious and radical, but the cost of a cubic meter of this type of water still represents a major constraint. Finally, we must consider the desalination of seawater to solve the problem of lack of water resources in the region in the long term.

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A NEW PROCEDURE OF CRITERIA WEIGHT DETERMINATION WITHIN THE ARAS METHOD

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Abstract

In most of multicriteria aggregation methods, we need to elicit parameters that are generally determined directly by the decision-maker (DM). Direct assigning of parameters and criteria weights presents a crucial and difficult step in the decision-making process. However, this kind of information is too subjective and may affects the reliability of the results. To overcome this issue, we suggest a weighting method based on mathematical programming to incorporate the DM's preferences indirectly within the ARAS method.

Keywords: MCDA, preference disaggregation, ARAS, criteria weights.

1 Introduction

Multiple criteria decision analysis (MCDA) is a general framework for supporting complex decision-making situations with multiple and often conflicting objectives. Commonly, the multicriteria methods require setting criteria weights in order to be implemented. Therefore, the problem of criteria weight determination has gained the interest of many researchers during the past decades. There are two ways of weight elicitation: 'a priori weights' that are determined directly by the experts and 'a posteriori weights' obtained from the data. This paper adopts the 'a posteriori approach'. Hence, we focus on reducing the subjectivity and the unreliability of weight values when they are directly determined by the DM

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without excluding him from the decision making process. Thus, we propose a new procedure of preference disaggregation in order to elicit criteria weights in the ARAS method. This approach is based on preference relations provided by the decision maker, as well as on comparisons between differences of criteria weights. Our weight elicitation method is based on solving a linear program which takes into account the DM's preferences.

Our paper consists of six sections. Section 2 will give a brief survey of the state of the art of selected weighting methods; selected preference disaggregation approaches will be described. In Section 3, the different steps of the ARAS method will be presented. In section 4, we will develop a criteria determination approach based on the ARAS method. In section 5, a case study will be presented to discuss the feasibility of the proposed model. In section 6, we present conclusions and perspectives for future research.

2 A review of the literature

Chiang (2009) noted that "one of the most difficult tasks in multiple criteria decision analysis (MCDA) is determining the weights of individual criteria so that all alternatives can be compared based on the aggregate performance of all criteria". For this reason, many methods have been developed to objectively determine the values of criteria weight. For instance, Figueira and Roy (2001) proposed a version of the Simos method which takes into account a new kind of information supplied by the DM and changed some computing rules. In addition, a new software package based on the revised Simos' procedure has been implemented. In addition, Chiang (2009) proposed a measure of the relative distance, which involved the calculation of the relative position of an alternative between the anti-ideal and the ideal for ranking to seek the shortest absolute distance between an alternative and the ideal one. The author showed that the relative distance produces consistent rankings for any set of weights, regardless of how they are determined. Thus, this method is suitable for cases where no prior information can be used for determining the weights. Furthermore, Rezaei (2009) proposed a new method called BWM (Best-Worst Method). First, the DM gives the best and the worst criterion. Then, pairwise comparisons are conducted between each of these two criteria (best and worst) and the remaining ones. After that, a maximin problem is formulated and solved to determine the weights of different criteria. In the same context, Roszkowska (2013) presented a comparative overview on several rank ordering weight methods that convert the ordinal ranking of a number of criteria into numerical weights. Also, Siskos and Tsotsolas (2015) proposed a set of complementary robustness analysis rules and measures integrated in a robust Simos method for the elicitation of the criteria weights. The goal was to aid the DM and the analysts to gain insight on the whole set of weighting solutions, to select a single set of criteria weights and to apply robust rules based on multiple sets of acceptable weights.

Approaches to preference disaggregation

In the aggregation paradigm, the aggregation model is known a priori, whereas the global preference is unknown. On the other hand, the philosophy of the disaggregation involves the inference of preference models from the given global preferences.

The development of preference disaggregation methods was initiated in 1978. In the disaggregation-aggregation approach, iterative interactive procedures are used to be aggregated later to a value system (Siskos, 1980; Jacquet-Lagrèze and Siskos, 1982, 2001; Siskos and Yannacopoulos, 1985; Siskos et al., 1993). The first developed preference disaggregation method was the UTA method proposed by Jacquet-Lagrèze and Siskos (1982). The purpose of this method is to infer additive value functions from a given ranking through linear programming. Besides, Mousseau and Slowinski (1998) developed a global inference approach to determine ELECTRE III's parameters. In the same way, Lourenço and Costa (2004) developed a disaggregation approach for the determination of weight coefficients as well as a category of reference profiles of ELECTRE III. Furthermore, Dias and Mousseau (2006) developed a mathematical program to determine the veto thresholds of the ELECTRE III method. Nevertheless, Corrente et al. (2014) opted for the Robust Ordinal Regression (ROR) to determine the different values of ELECTRE parameters. On the other hand, Frikha et al. (2018) determined the ELECTRE I parameters based on the outranking relations given by the DM. In addition, Mousseau et al. (2001) solved a linear program to infer criteria weights in the ELECTRE III method. They used a pure maxmin and a standard additive objective function. In the same context, Kadzinski et al. (2017) developed a disaggregation approach to elicit the parameters of the ELECTRE III-C method. Indeed, Frikha et al. (2010) determined the relative importance of the criteria of the PROMETHEE method based on some preference relations and other information provided by the DM. Also, Frikha et al. (2011a) developed an interactive disaggregation approach to infer the indifference thresholds of the PROMETHEE II method based on some preference relations. Later, Frikha et al. (2011b) proposed an approach to elicit both preference and indifference thresholds of the PROMETHEE method. Moreover, Frikha et al. (2017) developed a mathematical programming model to determine the relative importance of the criteria as well as the preference and the indifference thresholds in the PROMETHEE method. Disaggregation methods in multi-criteria decision analysis use linear programming, in particular goal programming, in eliciting preference aggregation models (Siskos, 1983). For instance, Charnes et al. (1955) proposed a linear model by disaggregating pairwise comparisons and given measures. Greco et al. (2010) used robust ordinal regression to describe an interactive multiobjective optimization methodology called NEMO. Likewise, Kadziński et al. (2013) used ROR to establish the rank of the alternatives. Furthermore, Corazza et al. (2015) determined the parameter values of the MUlticriteria RAnking MEthod (MURAME), while Valkenhoef and Tervonen (2016) considered the elicitation of incomplete preference information for the additive utility model in terms of linear constraints on the weights using holistic pairwise comparisons given by the DM. Likewise, De Almeida et al. (2016) used partial holistic information to determine criteria weights based on Multi-Attribute Value Theory (MAVT). Furthermore, Kadziński et al. (2017) developed a set of interactive evolutionary multiple objective optimization (MOO) methods, called NEMO-GROUP.

In this paper, we propose a new approach to elicit criteria weights of the ARAS method.

3 The ARAS method

The ARAS (*Additive Ratio ASsessment*) method proposed by Zavadskas and Turskis (2010) as a ranking method. Its purpose is to select the best alternative among others. It has been applied in several fields such as technology, construction, investments, etc., to validate the selection of a decision alternative.

The steps of the ARAS method are:

Step 1

The first stage of ARAS is to create the decision-making preference matrix consisting of *m* alternatives and *n* criteria.

Let

x_{ij} be the performance value of the alternative *i* according to the criterion *j*;

m be the number of alternatives and

n be the number of criteria.

$$X=\begin{bmatrix} x_{01} & \dots x_{0j} & \cdots x_{0n} \\ \vdots & \ddots \; \vdots & \ddots \; \vdots \\ x_{i1} & \dots x_{ij} & \dots x_{in} \\ \vdots & \ddots \; \vdots & \ddots \; \vdots \\ x_{m1} & \cdots x_{mj} & \dots x_{mn} \end{bmatrix} \quad i=0,\dots,m\;;\; j=1,\dots,n$$

Step 2

The second stage in most of MCDM methods is the normalization of the decision matrix. The purpose of any normalization technique is to unify incommensurable criteria measures so that all the performances can be compared. In the literature, two normalization ways are suggested:

The criteria whose preferable values are maxima, are normalized as follows:

$$\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}} \tag{1}$$

The criteria whose preferable values are minima, are normalized as follows:

$$x_{ij} = \frac{1}{x_{ij^*}}; \tag{2}$$

$$\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}} \tag{3}$$

where

$\bar{x}_{ij}$ are the normalized values of the normalized decision matrix $\bar{X}$ and

$x_{ij}{*}$ is the optimal value of the criterion j.

x_{0j} is the initial value of the minimized criterion j.

If the optimal value of criterion j is unknown, then x_{0j} = max x_{ij}, if max x_{ij} is preferable and

x_{0j} = min $x_{ij}{*}$, if min $x_{ij}{*}$ is preferable.

Thus, the general form of the normalized decision matrix $\bar{X}$ is:

$$\bar{X} = \begin{bmatrix} \bar{x}_{01} & \dots \bar{x}_{0j} & \cdots \bar{x}_{0n} \\ \vdots & \ddots \vdots & \ddots \vdots \\ \bar{x}_{i1} & \cdots \bar{x}_{ij} & \cdots \bar{x}_{in} \\ \vdots & \ddots \vdots & \ddots \vdots \\ \bar{x}_{m1} & \cdots \bar{x}_{mj} & \cdots \bar{x}_{mn} \end{bmatrix} i = 0, \dots, m; j = 1, \dots, n$$

Step 3

The third stage consists in creating the weighted-normalized matrix $\hat{X}$.

The weighted-normalized values of all the criteria are calculated as follows:

$$\hat{x}_{ij} = \bar{x}_{ij}\, w_j \; ; i = 0,\dots,m; j = 1,\dots,n \tag{4}$$

where

$\bar{x}_{ij}$ is the normalized evaluation value of the alternative i according to the criterion j;

w_j is the weight of the criterion j and

$\sum_{j=1}^{n} w_j = 1$ (criteria weights must be normalized)

The weighted normalized matrix is:

$$\hat{X}=\begin{bmatrix} \hat{x}_{01} & \ldots \hat{x}_{0j} & \cdots \hat{x}_{0n} \\ \vdots & \ddots \ \vdots & \ddots \ \vdots \ s \\ \hat{x}_{i1} & \cdots \hat{x}_{ij} & \cdots \hat{x}_{in} \\ \vdots & \ddots \ \vdots & \ddots \ \vdots \\ \hat{x}_{m1} & \cdots \hat{x}_{mj} & \cdots \hat{x}_{mn} \end{bmatrix} i=0,\ldots,m\ ;\ j=1,\ldots,n$$

Step 4

The objective of this step is to determine the values of the optimality function, denoted by S_i, such that

$$S_i = \sum_{j=1}^{n} \hat{x}_{ij}\ ;\ i = 0,\ldots,m \tag{5}$$

Step 5

In ARAS, the value K_i of the utility function determines the relative efficiency of a feasible alternative a_i . It can be calculated as follows:

$$K_i = \frac{S_i}{S_0}\ ;\ i = 0,\ldots,m \tag{6}$$

where S_0 is the optimal value (i.e., the maximum value of S_i) and the calculated values K_i are in the interval [0,1].

Step 6

The last step of the ARAS method consists in ranking, in an increasing order, the values K_i of the utility function. As a result, we obtain the rank of all the alternatives and therefore also the best one.

Thus, we choose to change the normalization formula of ARAS to a more convenient one (normalization by the minimum-maximum) because the linear normalization technique is not symmetric. Actually, the normalized values of the alternative are lower for the benefit criteria and higher for the cost criteria (Vafaei et al., 2015).

The minimum-maximum normalization technique can be described as follows:

In the case of maximization criteria, we replace the formula ($\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}}$) by

$$\bar{x}_{ij} = \frac{x_{ij} - \min(x_{ij})}{\max(x_{ij}) - \min(x_{ij})} \tag{7}$$

In the case of minimization criteria, we use:

$$\bar{x}_{ij} = \frac{\max(x_{ij}) - x_{ij}}{\max(x_{ij}) - \min(x_{ij})} \tag{8}$$

Thus, we propose a new procedure of preference disaggregation to elicit the criteria weights of ARAS.

4 The proposed model for the determination of ARAS criteria weights

In most procedures, ranking is a necessary first step for eliciting accurate weights. Usually, criteria weights are obtained from the rank order of each criterion. Thus, ARAS has a serious flaw: the criteria weights are too subjective since they are provided directly by the DM. Therefore, we propose a mathematical programming model that aims to determine the criteria weights objectively, but without excluding the DM. For that purpose, the decision maker is asked to provide pairwise comparisons of alternatives and criteria weights. The provided information is integrated into the following program.

Program 1

$$\max \sum_{i=1}^{p} g_i \tag{9}$$

Subject to

$$\sum_{j=1}^{n} w_j \bar{x}_{Bj} - \sum_{j=1}^{n} w_j \bar{x}_{Qj} - g_i \geq 0 \; \forall \, B, Q \in A; \forall \, i=1,\dots,p \tag{10}$$

$$w_k - w_l \geq w_r - w_{l},, \; k, l, r, v \in [1,\dots,n] \tag{11}$$

$$w_k \geq w_l, \quad k, l \in [1,\dots,n] \tag{12}$$

$$g_i \geq \frac{1}{2^{(p-1)}} \; \forall \, i=1,\dots,p \tag{13}$$

$$w_j \geq e \; \forall \, j=1,\dots,n \tag{14}$$

$$\sum_{j=1}^{n} w_j = 1 \tag{15}$$

Let:
A: be the set of alternatives;
p: be the number of relations between pairwise preferences among alternative preferences provided by the decision-maker;
w_j be the weight of the j^{th}criterion;
e be a threshold.

Within ARAS, alternative B is preferable over alternative Q ($B \succ Q$) if $K_B \geq K_Q$. The degree of preference of B over Q (g_i) is the difference between the two utility degrees with respect to all the criteria, that is, K_B- K_Q = g_i for every preference relation i provided by the DM.

In order to ensure strict preference and to avoid the relationship of indifference between two alternatives, we have to maximize the sum of slack variables g_i given in Equation (9).

In addition, in ARAS, all alternatives are ranked according to the decreasing order of the values of their utility degrees. As we said before, alternative B is preferable to Q is equivalent to: the utility degree of B is greater than that of Q.

$$\text{Then, } K_B \geq K_Q \tag{16}$$

$$\text{Consequently, } \frac{S_B}{S_0} \geq \frac{S_Q}{S_0} \quad (17)$$

where S_0 is the best value.

$$\sum_{j=1}^{n} \widehat{x_{Bj}} \geq \sum_{j=1}^{n} \widehat{x_{Qj}} \quad (18)$$

where $\widehat{x_{Bj}}$ and $\widehat{x_{Qj}}$ are the normalized-weighted values of all the criteria

$$\sum_{j=1}^{n} w_j \, \bar{x}_{Bj} \geq \sum_{j=1}^{n} w_j \, \bar{x}_{Qj} \quad (19)$$

where $\bar{x}_{Bj}$ and $\bar{x}_{Qj}$ are the normalized values of the decision matrix.

Then, the preference relations expressed by the DM are modeled in the mathematical program as $\sum_{j=1}^{n} w_j \, \bar{x}_{Bj} - \sum_{j=1}^{n} w_j \, \bar{x}_{Qj} - g_i \geq 0 \; \forall$ B, $Q \in A$; $\forall \, i=1,\ldots,p$ Equation (10).

In addition to the preference relations, the DM should provide two other pieces of information. The first one concerns the comparisons of the differences of adjacent weights written as:
$w_k - w_l \geq w_r - w_v$ Equation (11). Therefore, the gap between the importance of criteria *k* and *l* is more important than that between *r* and *v*.

The second piece of information concerns a partial pre-order on criteria weights. The DM is asked to supply pairwise comparisons of criteria weights in the form $w_k \geq w_l \; \forall \, k \in [1,\ldots,n]$; $\forall \, l \in [1,\ldots,n]$ Equation (12). The number of partial pre-order constraints must not exceed (n-1).

In order to guarantee the preference between the pairs of preferences provided by the DM and to avoid the situation of indifference, we impose the condition that all slack variables (g_i) are strictly positive. Consequently, we have to set a minimum threshold for each g_i according to each preference relation. It is evident that the threshold value is strongly dependent on the number of preference relationships, hence it can be equal to $\frac{1}{2(p-1)}$. Thus, we introduce the constraint $g_i \geq \frac{1}{2(p-1)} \forall \, i=1,\ldots,p$ Equation (13).

The constraint (14) is related to a threshold of the weight values. Indeed, in the constraints of the weight determination, we should take into account the condition that all criteria weights should be strictly positive ($w_j > 0$) in order to prevent any criterion from being null and therefore ignored. Since mathematical programming deals with weak inequalities and not with strict inequalities, we should set a small positive threshold *e* associated with each importance coefficient w_j. Depending on the value of *e*, the criterion may be meaningless. The value of *e* is dependent on the number of criteria. Then, we should add the constraint $w_j \geq e \; \forall \, j= 1,\ldots,n$ to the mathematical program.

Moreover, we should take into account that all criteria weights are normalized. This means that the sum of all the weights is equal to 1. For example, if we have *n* criteria, then $\sum_{j=1}^{n} w_j = 1$ Equation (15).

Our approach is iterative and interactive. In the iterative process of determining ARAS criteria weights, the DM is free to add or to remove information whenever needed. The additional information consists in adding or even removing one or more preference relations. Each additional information and each preference relation will be modeled in the mathematical program as constraints. In real-world decision problems, the decision-makers have difficulty in providing reliable information due to time constraints and their cognitive limitations. Therefore, the preferences of the decision makers are not necessarily stable: they can evolve over time and can even contain conflicting and inconsistent information. The role of an interactive tool is to help the DM to understand his preferences and their representation in a specific aggregation method. Inconsistencies occur when the DM's preferences cannot be obtained from the aggregation method used.

5 An illustrative example

Rainwater source control is usually considered as an alternative solution of water evacuation by sanitation networks. The alternatives (infiltration and retention basin, porous pavements with tank structure, infiltration wells, draining trenches, berms, storage roofs and buried pools) are subject to pollution and floods caused by rainwater in urban areas. Therefore, water managers face many obstacles related to the diversity of management techniques of a source of rainwater. Decision support tools are therefore required to guide the water managers in the choice of the best alternative. Therefore, multiple criteria methods are needed to develop such decision support (Martin and Legret, 2005).

A storm water Best Management Practice (BMP) is a practice that is suitable for reducing the volume of overflow and treating pollutants in storm water runoff. Therefore, the alternatives represent the eight types of Best Management Practice (BMP).

- **A1:** Wet pond (retention basin): "A retention basin or wet pond is a storm water control structure with a permanent pool of water into which storm runoff is directed. Runoff from each storm is retained, allowing suspended sediment particles and associated pollutants to settle out. Water in the basin infiltrates or is displaced by runoff from a subsequent storm" (Kathryn et al., 2011).
- **A2:** Dry pond (detention basin): "A detention basin or dry pond is a structure into which storm water runoff is directed, held for a period of time (detained), and slowly released to a surface water body. A dry pond is not designed to permanently contain water. It can help to improve water quality by allowing suspended solids to settle over a period of time. The temporary storage of storm runoff water also decreases downstream peak flow rates which can reduce potential flooding" (Kathryn et al., 2011).

- **A3:** Buried pool: "Hidden basins but remaining accessible, intended to store underground rainwater" (Iowa Drainage Law Manual).
- **A4:** Berm: "A horizontal strip or shelf built on or cut into an embankment to break the continuity of a long slope, usually to reduce erosion or increase the size of the embankment" (Iowa Drainage Law Manual).
- **A5:** Porous pavement with tank structure: "Porous, permeable or pervious pavement includes several methods and materials that allow water and air to move through the pavement and into the underlying soil. Some examples of permeable pavement include specially designed and constructed concrete, asphalt, paving stones or bricks. Permeable pavement sometimes includes an underlying reservoir for additional water storage" (Kathryn et al., 2011).
- **A6:** Draining trenches (storm sewer): "A natural or artificial waterway where a stream of water flows periodically or continuously or forms a connecting link between bodies of water. Also a conduit such as a pipe conveys water" (Iowa Drainage Law Manual).
- **A7:** Storage roofs: "waterproofing coating installed on the roofs of buildings protected by grave land designed to temporarily retain rainwater" (Iowa Drainage Law Manual).
- **A8:** Infiltration wells: "an infiltration basin is a shallow impoundment designed to infiltrate storm water runoff into the soil. Infiltration basins do not release water except by infiltration, evaporation, or emergency overflow" (Kathryn et al., 2011).

These alternatives are evaluated according to eight criteria which are:

- **C1:** pollution retention (to be maximized)
- **C2:** probability of dysfunction (to be minimized)
- **C3:** need for and frequency of maintenance operations (to be minimized)
- **C4:** impact on groundwater quality (to be minimized)
- **C5:** level of approval (to be maximized)
- **C6:** contribution to development policies (to be maximized)
- **C7:** equity stake (to be maximized)
- **C8:** maintenance costs (to be minimized)

The criteria: pollution retention (C1), need for and frequency of maintenance operations (C2), impact on groundwater quality (C4), level of approval (C5) and contribution to development policies (C6) have been evaluated on the basis of the analysis of the results of a satisfaction survey on the use of alternative techniques in rain water sanitation. They are evaluated on a scale of 1 to 5 or 1 to 3.

The criterion probability of dysfunction (C2) is evaluated in %, according to a bibliographic study on different alternative techniques.

The criteria equity stake (C7) and maintenance costs (C8) are valued numerically, in € and € / year, respectively.

The DM provides the following decision matrix (Table 1).

Table 1: Decision matrix

Criteria / Alternatives	**C1 max**	**C2 min**	**C3 min**	**C4 min**	**C5 max**	**C6 max**	**C7 max**	**C8 min**
A1	4	20	3	2	5	3	38	32
A2	4	20	3	2	5	3	54	32
A3	4	20	2	2	1	1	370	32
A4	4	40	3	2	3,5	3	13	30
A5	4	60	2	2	2,5	2	54	4,5
A6	4	60	2	2	2,5	2	39	1,2
A7	1	40	2	5	1	2	0	2
A8	4	60	2	1	1	1	4	2

The normalization of the decision matrix is based on equations 7 and 8.

We get the normalized values and hence the normalized decision matrix (Table 2).

Table 2: Normalized decision matrix

Criteria / Alternatives	**C1 max**	**C2 min**	**C3 min**	**C4 min**	**C5 max**	**C6 max**	**C7 max**	**C8 min**
A1	1	1	0	0,75	1	1	0,103	0
A2	1	1	0	0,75	1	1	0,146	0
A3	1	1	1	0,75	0	0	1	0
A4	1	0,5	0	0,75	0,625	1	0,035	0,065
A5	1	0	1	0,75	0,375	0,5	0,146	0,893
A6	1	0	1	0,75	0,375	0,5	0,105	1
A7	0	0,5	1	0	0	0,5	0	0,974
A8	1	0	1	1	0	0	0,011	0,974

Thus, the manager of the civil engineering department gave the following pairwise preference relations among the alternatives:

$A_8 \succ A_4$;

$A_3 \succ A_7$;

$A_6 \succ A_8$;

$A_2 \succ A_5$;

$A_1 \succ A_3$;

He also gave some comparisons between differences of criteria weights:

$w_5 - w_6 \geq w_1 - w_4$

$w_3 - w_2 \geq w_7 - w_8$

The gap between criteria 5 and 6 is more important than that between criteria 1 and 4 (Equation 7).

Moreover, some pairwise comparisons among criteria weights are given:

$w_1 \geq w_5$

$w_4 \geq w_3$

The information provided is incorporated into the following mathematical program (program 2).

Program 2

$\max \sum_{i=1}^{5} g_i$

$\sum_{j=1}^{8} w_j \bar{x}_{A_8 j} - \sum_{j=1}^{8} w_j \bar{x}_{A_4 j} - g_1 \geq 0$

$\sum_{j=1}^{8} w_j \bar{x}_{A_3 j} - \sum_{j=1}^{8} w_j \bar{x}_{A_7 j} - g_2 \geq 0$

$\sum_{j=1}^{8} w_j \bar{x}_{A_6 j} - \sum_{j=1}^{8} w_j \bar{x}_{A_8 j} - g_3 \geq 0$

$\sum_{j=1}^{8} w_j \bar{x}_{A_2 j} - \sum_{j=1}^{8} w_j \bar{x}_{A_5 j} - g_4 \geq 0$

$\sum_{j=1}^{8} w_j \bar{x}_{A_1 j} - \sum_{j=1}^{8} w_j \bar{x}_{A_3 j} - g_5 \geq 0$

$w_5 - w_6 \geq w_1 - w_4$

$w_3 - w_2 \geq w_7 - w_8$

$w_1 \geq w_5$

$w_4 \geq w_3$

$g_i \geq 0.0625\ \forall\ i=1,\ldots,5$

$w_j \geq 0.05\ \forall\ j=1,\ldots,8$

$$\sum_{j=1}^{8} w_j = 1$$

We choose to solve the proposed model using the LINGO commercial software package. As a result, by solving this mathematical program, we obtain the following criteria weights:

$w_1 = 0.123$

$w_2 = 0.255$

$w_3 = 0.065$

$w_4 = 0.079$

$w_5 = 0.123$

$w_6 = 0.05$

$w_7 = 0.05$

$w_8 = 0.256$

Sensitivity analysis is crucial at this stage. It investigates how the uncertainty in the output of a mathematical model can be divided into different sources of uncertainty in its inputs. It is also known as the what-if analysis. In this sensitivity analysis, we will study the effect of different normalization forms on criteria weights. These forms are as follows.

The minimum-maximum normalization technique:

In the case of benefit criteria: $\bar{x}_{ij} = \frac{x_{ij} - \min(x_{ij})}{\max(x_{ij}) - \min(x_{ij})}$

In the case of cost criteria: $\bar{x}_{ij} = \frac{\max(x_{ij}) - x_{ij}}{\max(x_{ij}) - \min(x_{ij})}$

The normalization technique by the maximum:

In the case of benefit criteria: $\bar{x}_{ij} = \frac{x_{ij}}{\max x_{ij}}$

In the case of cost criteria: $\bar{x}_{ij} = 1 - \frac{x_{ij}}{\max x_{ij}}$

The linear normalization technique:

In the case of benefit criteria: $\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}$

In the case of cost criteria: $\bar{x}_{ij} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{m} \frac{1}{x_{ij}}}$

The vector normalization technique:

In the case of benefit criteria: $\bar{x}_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} {x_{ij}}^2}}$

In the case of cost criteria: $\bar{x}_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} {x_{ij}}^2}}$

Once the decision making matrix is normalized, we solve the mathematical model using the LINGO software package to get the criteria weights (Table 3).

Table 3: Weights obtained using each normalization form

Normalization form	**min-max**	**max**	**linear**	**vector**
w_1	0,123	0,151	0.148	0.259
w_2	0,255	0,205	0.05	0.05
w_3	0,065	0,05	0.05	0.05
w_4	0,079	0,067	0.356	0.05
w_5	0,123	0,151	0.148	0.259
w_6	0,05	0,05	0.05	0.05
w_7	0,05	0,084	0	0.073
w_8	0,256	0,242	0.198	0.208

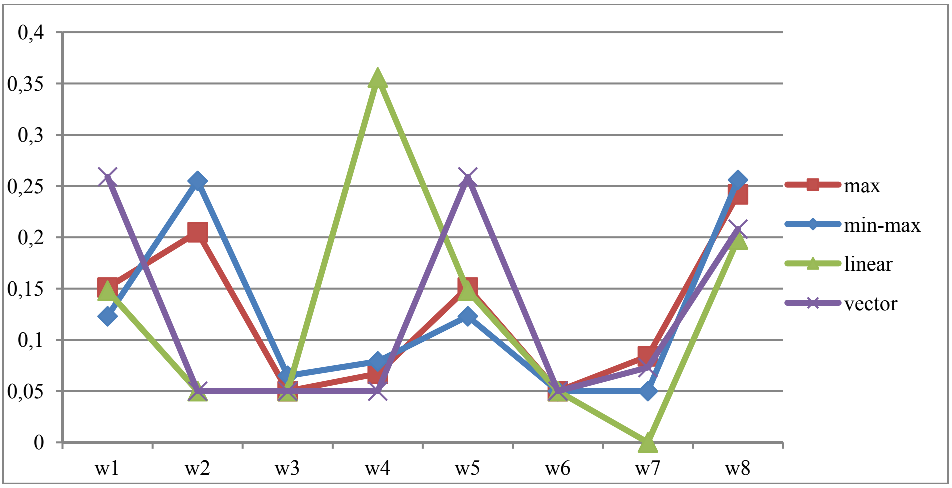


Figure 1: The curves of each normalization technique

As can be seen, the curves vary in different ways. However, there is a significant disparity in the variation of each curve. The fluctuation differs from one curve to another. As a consequence, we can conclude that this approach is sensitive to a change in the normalization technique.

The next step consists in building the weighted-normalized decision matrix in which we calculate the values of the optimality function (S_t), and the utility degree (K_t) to obtain a ranking of all the alternatives (Table 4).

Table 4: The weighted normalized decision matrix and solution

Criteria / Alternatives	C1 max	C2 min	C3 min	C4 min	C5 max	C6 max	C7 max	C8 min	S_t	K_t	Rank
A1	0,123	0,255	0	0,059	0,123	0,05	0,005	0	0,615	0,997	2
A2	0,123	0,255	0	0,059	0,123	0,05	0,007	0	0,617*	1	1
A3	0,123	0,255	0,065	0,059	0	0	0,05	0	0,552	0,894	5
A4	0,123	0,1275	0	0,059	0,077	0,05	0,002	0,017	0,455	0,737	8
A5	0,123	0	0,065	0,059	0,046	0,025	0,007	0,229	0,554	0,897	4
A6	0,123	0	0,065	0,059	0,046	0,025	0,005	0,256	0,579	0,939	3
A7	0	0,1275	0,065	0	0	0,025	0	0,249	0,467	0,756	7
A8	0,123	0	0,065	0,079	0	0	0,00055	0,249	0,517	0,837	6

* S_0 = 0.617 (the greater value).

The final ranking of the alternatives is: A2 ≻A1 ≻ A6 ≻ A5 ≻ A3 ≻ A8 ≻ A7 ≻ A4.

This means that A2 (dry pond / detention basin) is the best alternative for retaining excess rainwater since it reduces peak rate of runoff and alleviates flooding. It is also regarded as cost effective. A dry pond can be designed to improve water quality. A detention basin has the advantage that the space surrounding the pond can be landscaped to enhance the beauty of the place and provide a habitat for the inhabitants.

On the other hand, berms are considered to be the worst alternative for retaining excess rainwater because they require a lot of space. Unless fill is available nearby, the cost of transporting it to the site may be prohibitive.

The obtained results are different from those found in the paper Martin and Legret (2005). The authors used the ELECTRE III method to classify the different BMPs according to three strategies (planning, urban development and environment protection) in France. The following figure shows the resulting outranking relations.

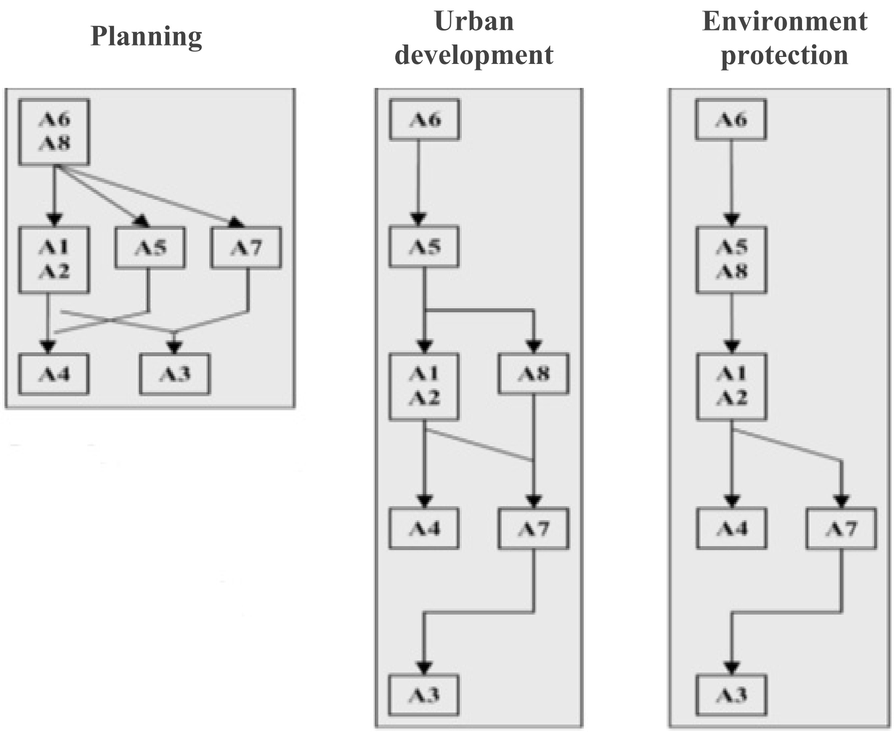
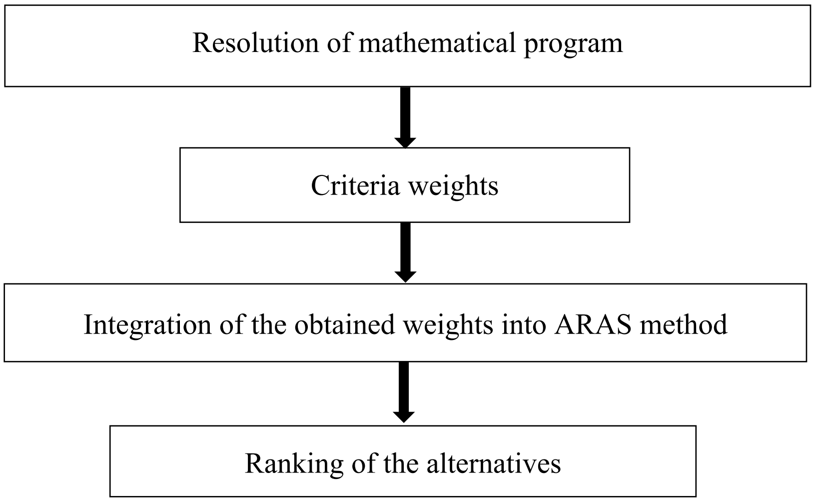


Figure 2: The outranking graphs (Martin and Legret, 2005)

The difference in the results is due to the fact that multi-criteria methods do not give the same output. In fact, the choice of a multi-criteria method is itself considered to be a multi-criteria problem. Indeed in MCDM, there is no optimal solution, rather a satisfying one (unlike in the exact methods). For instance, ELECTRE and ARAS cannot give the same result. Also, the preference relations obtained from the DM do not contradict the final rankings founded in Martin and Legret (2005), apart from two constraints ($A_2 \succ A_5$ and $A_3 \succ A_7$) which give the proposed method more consistency and reliability.

In the final analysis, the proposed model can be summarized by the following algorithm.



6 Conclusion and perspectives

In this paper, we have proposed an approach to criteria weight determination for the ARAS method. In most multicriteria aggregation problems, the DM determines directly the weight values using his own intuition. However, this information is too subjective which makes the results unreliable. To overcome this flaw, we suggested a weighting method that involves the DM indirectly in the decision-making process. The DM was asked to provide pairwise preferences among alternatives and criteria weights. On the basis of his preferences, we formulated a mathematical program using the ARAS method and solved it with the LINGO software package. Having obtained the weight values, we ranked the alternatives from the best to the worst. Finally, a case study in rainwater management in urban areas was given in order to implement the model. The main contribution of this paper is that the DM is not directly involved in the elicitation of weights, which reduces the subjectivity of the results. The proposed method can be applied to several real-world case studies. However, the proposed mathematical program is valid only for the ARAS method. It does not accept any threshold, either. In future research, we will consider eliciting criteria weights in a hierarchical structure of criteria.

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MENU PLANNING WITH A DYNAMIC GOAL PROGRAMMING APPROACH

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Abstract

Dynamic Goal Programming (DGP) represents an extension of Goal Programming (GP). It is characterized by the importance of time factor in relation to its variables. As a complex decision making problem, Menu Planning Problem (MPP) requires the development of methodologies which are able to combine different and conflicting goals incorporating the dynamic characteristics. The article reviews some of the studies and approaches used in MPP. It deals with the Standard GP model of MPP. It provides a DGP formulation for solving the MPP. An MPP for the hemodialysis (HD) patient is an application that best exemplifies the proposed dynamic formulation.

Keywords: Goal Programming, Menu Planning Problem, Standard, Static/Dynamic Programming.

1 Introduction

The present paper is reconsiders the MPP with the DGP approach. Dynamic Programming (DP) is characterized by regarding the target values as a function of time. A target value appears on the accumulated value of the objective for each period of time within the planning period. This allows the Decision Maker (DM) to control the behavior of the objectives during the whole planning period, rather than only their final values. The achievement of goals at different periods in the day is

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restricted by DP. The proposed model can be simply adapted to plan the diet/menu of individuals in different health conditions. It can also analyze several other issues if the problem is dynamic in nature with respect to certain characteristics and constraints.

Various optimization approaches have been applied to solve the MPP, including linear programming (Smith, 1959, 1974; Bassi, 1976; Foytik, 1981; Silberberg, 1985; Westrich et al., 1998; Colavita and D`Orsi, 1990; Fletcher et al., 1994), integer programming (Balinfy, 1964; Leung et al., 1995), multistage multiple-choice programming algorithm (Balinfy, 1975), mixed integer programming (Armstrong and Sinha, 1974), bi-criteria mathematical programming (Benson and Morin, 1987), mixed integer linear programming (Sklan and Dariel, 1993; Valdez-Peña and Martĩnez-Alfaro, 2003) and GP (McCann-Rugg et al., 1983).

MPP is a scheduling problem whose objective is to find an optimal combination of meals that satisfy individual nutritional, structural and other requirements during a period of time. In MPP, multiple conflicting and diversified objectives are simultaneously taken into account, which is characteristic for typical Multi--Objective Decision Making (MODM) problems. These can be effectively solved by the GP approach. Thus the obtained solution represents the best compromise that can be achieved by the decision maker. The GP model is a distance function that tends to minimize unwanted positive and negative deviations from the achievement and aspiration levels.

To the best of our knowledge, little work has been undertaken on the solution of MPP by the GP approach. Indeed, applications of the GP approach to MPP differ from one research study to another. McCann-Rugg et al. (1983) used the GP approach interactively with the dietician who determined the availability of foods and their preference aspiration level. They aimed to compare the results of manual planning and of the GP approach of various dieticians. Ferguson et al. (2006) combined the use of linear programming and GP, seeking to improve complementary nutrition practices of young children to guarantee good conditions of their growth and health. Pasic et al. (2012) built a GP nutrition optimization model that intended to meet daily nutritional needs for women and men, thereby successfully overcoming budget constraint. Gerdessen and Vries (2015) studied the impact of the achievement functions in designing diet models based on GP. Their research enables the DM to use either a MinSum function or a MinMax function or a compromise between them.

In practice, the resolution of all healthcare problems and especially those related to nutrition should not be limited to the classical and static frame, but rather requires a dynamic one that considers the evolution of the decision making process over time. For example, in an everyday situation, if an individual had a dangerous health condition (cardiovascular, diabetic or end stage renal disease, etc.), he/she would have to choose among different meals available in order to satisfy their daily nutritional requirements. If he/she decides to eat a dish to gain more energy or protein, she/he will risk a simultaneous increase of potassium and sodium, taking into account the nutritional gain from the previously-eaten dishes. The decision made at each period must take into account its effects not only on the next period, but also on all subsequent periods. A dynamic problem can be divided into a number of stages (periods) or sub-problems, with an optimal decision required at each stage. DP is similar to a sequence of interrelated decisions, in which a decision made at each stage influences the decision to be taken in what follows.

It is quite natural to rely upon dynamic characteristic of MPP in which any feasible solution provides a vector of meals satisfying nutritional, structural and other requirements. DP, a technique based on the optimality principle, was developed by Richard Bellman in the early 1950s. He stated that "an optimal policy has the property that, whatever the initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision". DP leads to optimal solutions, not only of the entire problem, but also of each of its sub-problems. For example, if we need to select projects for a 10-year program, DP gives the optimal solutions of the projects for the entire 10-year period as well as the optimal solution for any period of less than 10 years.

DGP represents an extension of classical GP in a context that assigns much importance to the dependence of its variables on time. To the best of our knowledge, although no research has investigated the use of the DGP approach to solve MPP, there are some work which has explored DGP. Trzaskalik (1997) discussed different aspects of the GP approach to multiple objectives DP. He described four approaches, namely: dynamic goal approach, dynamic hierarchical goal approach, dynamic period goal approach and dynamic hierarchical period approach. Trzaskalik (2003) applied period target values to hierarchical goal dynamic programming. A period backward approach is applied and a fixed single hierarchy of criteria is used. The proposed approach aimed to realize for the DM the possibility of interactive modeling the period backward fixed single hierarchy target goal structure of the final solution. Caballero et al. (1998) argued that most of the DGP approaches used goal values on the final value of their objective functions and developed a Lexicographic DGP (LDGP) algorithm using dynamic target values. In addition to the final values of the corresponding functions, they controlled their evolution along the planning periods. Pal and Moitra (2003) described the way of using preemptive priority-based GP to solve a class of Fuzzy Programming (FP) problems, with a set of linear and/or non-linear fuzzy goal objectives with the characteristics of DP. Hamalainen and Mantysaari (2002) developed a DGP approach, in which dynamic aspects arose from three factors; the house acts as heat storage, the price of electricity varies over time and the outdoor temperature changes during the day. Based on an

LDGP approach, Nha et al. (2013) developed a novel robust design optimization procedure that aims to implement time series based on multi-responses, unlike static responses implemented in the conventional experimental design formats and frameworks.

The remainder of the paper is organized into five sections. Section 2 presents the standard classical GP model of MP. The proposed DGP model is discussed in Section 3. Section 4 illustrates the dynamic approach through a specific example in the context of HD patient nutrition. Finally, section 5 concludes and outlines directions for future research.

2 Standard formulation of the MP model with Static GP

GP is an important method for MODM approaches. The GP model is a well--known approach for solving multi-objective programming problems which allows the DM to take into account several conflicting objectives simultaneously. Thus, the obtained solution represents the best compromise achievable. In general, the objective function of the GP model is a distance function that minimizes the unwanted positive and negative deviation. The standard and static GP model of MPP can be formulated as follows:

$$\text{Minimize} \sum_{i=1}^{N} (\delta_i^+ + \delta_i^-) \tag{1}$$

so that

$$\sum_{l=1}^{L} \sum_{j \in J_l} a_{ilj} x_{ljk} + \delta_i^- - \delta_i^+ = g_i \, \forall i = 1, \dots, N, k = 1, \dots, 7 \tag{2}$$

$$x_{ljk} \geq 0 \; with \; k = 1, \dots, 7; \; l = 1, \dots, L \; and \; j \in J_l \tag{3}$$

$$\delta_i^+ \geq 0, \delta_i^- \geq 0 \; for i = 1, \dots, N \tag{4}$$

where

- i is the set of nutrients, i = energy, protein, potassium, sodium… N
- l is the type of recipe, j, $l = 1, \dots, L$ (breakfast, snacks, lunch and dinner),
- J_l is the set of the j^{th} recipes of type l to be recommended,
- k is the k^{th}day in the week, $k = 1, \dots ,7$,
- g_i is the i^{th} nutrient requirement per day,
- a_{ilj} is a coefficient indicating the quantity of i^{th} nutrient provided in 100 g in j^{th} recipe of type l,
- x_{ljk} is the quantity of j^{th} recipe of type l to be recommended in day k
- δ_i^-, δ_i^+ are negative and positive deviations from goal g_i.

3 Standard formulation of the MP model with Dynamic GP

The term "programming" is used in DP as a synonym of "optimization" and means "planning". It is basically a step-by-step search method used in optimization problems, whose solutions may be viewed as the result of a sequence of decisions (Bhowmik, 2010). As any other optimization models, in formulating the DGP model for solving MPP, we define the problem variables, determine the objective function and specify the constraints. In particular, in the process of formulating a DP model, a recursive relationship is developed, based on the principle of optimality, which keeps recurring as we move backward stage by stage.

The aim of this section is to apply DP to MPP. To this end, let us consider the following DGP model:

$$\text{Minimize} \sum_{i=1}^{N} \sum_{t \in t_k} \delta_{it}^{+} + \delta_{it}^{-} \tag{5}$$

so that

$$\delta_{it-1}^{-} - \delta_{it-1}^{+} + \delta_{it}^{-} - \delta_{it}^{+} + \sum_{l=1}^{L} \sum_{j \in J_l} a_{ilj} x_{ljt} = g_{it}$$
$$\forall\, i = 1, \ldots, N;\; k = 1, \ldots, 7 \;and\; t \in t_k \tag{6}$$

$$\sum\nolimits_{j \in J_l} \sum\nolimits_{t \in t_k} y_{ljt} = 1 \;\forall\, t \in t_k \;and\; l = 1, \ldots, 5 \tag{7}$$

$$\delta_{it}^{+} \geq 0, \delta_{it}^{-} \geq 0 \;for\; i = 1, \ldots, N \;and\; t \in t_k \tag{8}$$

$$x_{ljt} \geq 0 \;with\; k = 1, \ldots, 7;\; l = 1, \ldots, L;\; j \in J_l and\; t \in t_k \tag{9}$$

where

- i is the set of nutrients, i = energy, protein, potassium, sodium… N
- l is the type of recipe, $l = 1, \ldots, L$ (breakfast, snacks, lunch, and dinner),
- J_l is the set of j^{th} recipes of type l to be recommended,
- k is the k^{th}day in the week, $k = 1, \ldots, 7$,
- t_k is the period t in day k, $t = 1, \ldots, T \in t_k$and $k = 1, \ldots, 7$, which are the time slots used in DP,
- g_{it} is the i^{th} nutrient requirement (goal) per period t,
- a_{ilj} is a coefficient indicating the quantity of i^{th} nutrient provided in 100 grams from j^{th} recipe of type l,

- x_{ljt} is the quantity of j^{th}recipe of type l to be recommended in the period t of day k,
- y_{ljt} is a binary variable to decide whether the recipe j of type l is included or not in period t of day k,
$\begin{cases} y_{ljt} = 1 \text{ If the recipe is included} \\ \qquad\qquad else\ 0 \end{cases}$
- $\delta_{it}^{-}, \delta_{it}^{+}$ are the negative and positive deviations from i^{th} nutrient goal in period t.

The sixth constraint above defines the following recursive relationship between the solutions of the sub-problems: It identifies the optimal solution for period t when the optimal solution given in the period $t-1$ is taken into account.

4 An illustrative example: A hemodialysis patient diet

To illustrate the application of the DGP model for solving MPP, a specific group of patients with chronic illness was chosen. A non-diabetic HD patient with the level of Glomerular Filtration Rate (GFR) < 15 ml/min, with age less than 60 years, Ideal Body Weight (IBW) = 70kg and a Body Mass Index (BMI) between 22 and 25. The nutritional requirements for HD patients are based on the daily intake as presented in the table below:

Table 1: Recommended daily intake of nutrients for a clinically stable HD patient

Nutrients	Daily Requirements
Energy	35 Kcal/ Kg IBW
Protein	1,2g/ Kg IBW
Sodium	80 mmol
Potassium	1 mmol/ Kg IBW

We consider a Database (DB) of 66 different Tunisian recipes classified into five different types: breakfast, morning snack, lunch, afternoon snack and dinner. The DB could be enlarged to include more ingredients and recipes and help in calculating the nutritional values of all the recipes. The recipes are listed in the following table:

Table 2: Recipes and their nutritional components

Recipes /Nutrients per 100 g	Energy (kcal)	Protein (g)	Potassium (mmol)	Sodium (mmol)	Type of recipe
Barquette tuna	147.68	11.36	121.16	216.5	2 and 4
Borghol with meat	778.26	32.1	777.9	115.85	3 and 5
Lemon cake	1151.32	18.84	79.95	1015.22	1 and 2
Four quarts cake	729.18	10.61	78.68	393.6	1,2 and 4
Cannelloni with ricotta	305.13	26.2	1016.68	365.15	5
Cannelloni with spinach and ricotta	824.49	30.82	449.74	710.67	3 and 5
Chakchouka with peppers	516.72	4.71	114.12	49.74	3,5 and 2
Coca Cola	93.6	0	0	8.68	4
Chicken couscous	1169.96	35.65	902.33	151.1	3 and 5
Couscous with turkey	555.98	34.49	829.38	99.24	3 and 5
Couscous with fish	757.57	23.72	530.56	471.35	3 and 5
Fondant potatoes	596.59	11.55	110.84	398.25	4 and 1
Chocolate cake	794.1	14.33	349.15	548.68	1,2 and 4
Peach juice	19.5	0.45	95	0.6	1 and 2
Pear juice	58	0.38	119	1	1,2 and 4
Apple juice	43	0.3	75	2	1,2 and 4
Orange juice	46	0.7	169	0	1,2 and 4
Orange juice, peach and banana	147.8	4.04	349.4	55.2	2 and 4
Macaroni with chicken	932.66	43.5	1953.32	235.29	3 and 5
Mini blown escalope	253.76	5.3	71.86	437.84	2
Ojjatuna	193.67	9.75	113.02	121.62	3 and 5
Fruit paste	107	0.5	45	0.5	4 and 2
Chicken rice	968.72	35.13	542.29	121.74	3 and 5
Summer salad	93.09	0.08	19.54	4.3	3 and 5
Salad ommekhourya	205.23	0.66	161.93	39.16	3 and 5
Salt samsa	253.76	5.3	71.86	437.84	4
Grenadine syrup	79.8	0	8.4	12.9	4
Sorbet granite	92	0.5	100	8	4
Bird tongues soup	292.26	16.98	380.06	71.69	3 and 5
Spinach and ricotta tajine	305.13	26.2	1016.68	365.15	3
Tea	0.5	0	18.5	5.5	4
Coffee	2	0.07	24	5.3	4
Flavored yogurt	101	4.84	215.09	64.54	4
Fruit yogurt	113	3.5	206	55	2 and 4

To solve the MP of the HD patient problem, we used AMPL (A Modeling Language for Mathematical Programming) which applies optimization solvers such as CPLEX. AMPL is a modern modeling environment which contains an advanced architecture providing much flexibility as compared to other modeling systems. We used it for the following purposes: reading a model, analyzing data, solving/optimizing the model using CPLEX, and generating the results of the optimization.

Suppose that the day is divided into five periods (T = 5). Accordingly, the MPP will consist of five sub-problems and in each stage only one decision must be taken. The DGP model of the HD patient diet problem can be formulated as follows:

The objective function:

$$\text{Minimize} \sum_{i=1}^{4} \sum_{t \in t_k} \delta_{it}^{+} + \delta_{it}^{-} \tag{10}$$

so that

$$\delta_{it-1}^{-} - \delta_{it-1}^{+} + \delta_{it}^{-} - \delta_{it}^{+} + \sum_{l=1}^{5} \sum_{j \in J_l} a_{ilj} x_{ljt} = g_{it}$$
$$\forall\, i = 1, \dots, 4;\ k = 1, \dots, 7\ and\ t \in t_k \tag{11}$$

$$100 y_{ljt} \leq x_{ljt} \leq 200 y_{ljt}\ \forall\, t \in t_k;\ l = 1, \dots, 5\ and\ j \in J_l \tag{12}$$

$$\sum_{j \in J_l} \sum_{t \in t_k} y_{ljt} = 1\ \ \forall\, k = 1, \dots, 7\ and\ l = 1, \dots, 5 \tag{13}$$

$$y_{ljt} \in \{0, 1\}\ \forall\, l = 1, \dots, 5;\ j \in J_l\ and\ t \in t_k \tag{14}$$

$$\delta_{i0}^{+} = \delta_{i0}^{-} = 0 \tag{15}$$

$$\delta_{it}^{+} \geq 0, \delta_{it}^{-} \geq 0\ for\ i = 1, \dots, 4\ and\ t \in t_k \tag{16}$$

$$x_{ljt} \geq 0\ with\ l = 1, \dots, 5, ; j \in J_l and\ t \in t_k \tag{17}$$

where:

- i is the set of nutrients, i is energy, protein, potassium or sodium,
- l is the type of recipe, l = 1 (breakfast), 2 (morning snack), 3 (lunch), 4 (afternoon snack), 5 (dinner),
- j_l is the set of the j^{th} recipes of type l to be recommended, $l = 1, \dots, 5$,
- k is the k^{th}day in the week, $k = 1, \dots, 7$,
- t_k is the period t in day k , $t = 1, \dots, T \in t_k$ and k = 1, ..., 7, which are the time slots used in DP,
- g_{it} is the i^{th} nutrient requirement per period t of day k,

- a_{ilj} is a coefficient indicating the quantity of i^{th}nutrient provided in 100 g of j^{th} recipe of type l,
- x_{ljt} is the quantity of j^{th} recipe of type l to be recommended in period t of day k,
- y_{ljt} is a binary variable to decide whether recipe j of type l is included or not in period t of day k,
  $\begin{cases} y_{ljt} = 1 \text{ If the recipe is included} \\ \qquad else\ 0 \end{cases}$
- δ_{it}^{-},δ_{it}^{+} are negative and positive deviations from i^{th} nutrient goal in period t of day k.

The formulation of the MPP of an HD patient using DGP is expressed by the objective function in equation (10) subject to constraints from equations (11) to (17). The objective of the model is to minimize the positive and negative deviations over all periods in each day of one week. Moreover, goals have to be satisfied for the four nutrients (protein, energy, sodium and potassium). In each period of the day, the patient can have various products but one recipe from each type (breakfast, morning snack, lunch, afternoon snack, and dinner) must be chosen as defined in equation (11). It follows from constraint (12) that the quantity of each recipe included in each period must be between 100 and 200 grams. Constraint (13) implies that the binary variable y_{ljt} is used to decide whether the recipe j with type l is included in each period t of the day. The initial state of the positive and negative deviations included constraint (15) is zero. Non-negativity constraints are described by (16) and (17).

In MPP, we are faced with the incommensurability problem when objectives are expressed in different measurement units (Kcal, mmol, g, etc.). Several studies have explicitly treated this problem; worth noting here is the methodology of Kettani et al. (2004). They have indicated that a commonly used method of performing the normalization is to convert the deviations to a Euclidean distance which normalizes the positive and negative deviation variables. It is realized through assigning a set of weight coefficients to the deviations of the objective function, with the importance factor and the normalization constant (factor) mixed and aggregated together as a weight coefficient. The importance factor should be equal to 1 because all goals are supposed to be of equal importance (implicit weighting is appropriate only if the goals are of extreme importance). The normalization constant is used to allow the conversion from one scale to an equivalent one. A normalization procedure is the process of scaling a vector so that each row vector of the decision matrix is divided by its norm. This can be carried out for any norm. The normalization procedure is used to reduce the impact of large-valued features specified on a different scale (mmol, g, Kcal …) and to allow small-valued features to equally contribute to the optimization of an objective function. The advantages of the normalization factor $\frac{1}{\|a_i\|}$ are numerous. First, all criteria are measured in dimensionless units, which facilitates comparisons between the attributes. Second, the relative proportions of a_i components remain unchanged because their normalization consists in dividing them by the same constant. Third, the choice of the scale for a given objective, from among equivalent scales (ratio level), does not affect the global measure of distance due to the property: $b\|a_i\| = \|ba_i\|$.

To formulate the Normalized GP (NGP) model, we have as an objective function:

$$\text{Minimize} \sum_{i=1}^{n} \frac{1}{\|a_i\|}(\delta_i^+ + \delta_i^-) \tag{18}$$

where:

$$\|a_i\| = \sqrt{\sum_{i=1}^{n} x_i^2}$$

and x is the norm of a vector.

In order to solve the problem given above, we used an ACCESS DB with the 66 recipes presented previously. The data used to build this DB was extracted from the official DB of the Tunisian Institute of Nutrition. An optimization environment with AMPL for solving the relevant optimization problem has been established.

The proposed GP model was implemented with AMPL, and computational tests were run on a system with an Intel® Core™ i5-5200U CPU with base frequency 2.20GHz, 4GB RAM and a 64-bit operating system. The model was verified and validated in accordance with many instructions from diet experts specializing in HD patients. All the guidelines to make a balanced MP model were followed.

Different recipes for the week were obtained, and the results of the DGP model showed that the best dishes from the 66 proposed are those shown in Table 3.

Table 3: Computational results

Recipe Number	**Recipe type**	**Period of the day**	**Day 1**
1	*2*	*3*	*4*
7	Breakfast	1	100 g
34	Morning Snack	2	100 g
16	Lunch	3	194 g
21	Afternoon Snack	4	100 g
41	Dinner	5	100 g

Table 3 cont.

1	*2*	*3*	*4*
Recipe Number	**Recipe type**	**Period of the day**	**Day 2**
1	Breakfast	1	100 g
34	Morning Snack	2	100 g
5	Lunch	3	100 g
21	Afternoon Snack	4	100 g
42	Dinner	5	100 g
Recipe Number	**Recipe type**	**Period of the day**	**Day 3**
2	Breakfast	1	100 g
34	Morning Snack	2	100 g
5	Lunch	3	173 g
21	Afternoon Snack	4	100 g
51	Dinner	5	100 g
Recipe Number	**Recipe type**	**Period of the day**	**Day 4**
1	Breakfast	1	100 g
22	Morning Snack	2	100 g
6	Lunch	3	100 g
35	Afternoon Snack	4	100 g
44	Dinner	5	100 g
Recipe Number	**Recipe type**	**Period of the day**	**Day 5**
2	Breakfast	1	100 g
22	Morning Snack	2	100 g
11	Lunch	3	100 g
20	Afternoon Snack	4	125 g
37	Dinner	5	100 g
Recipe Number	**Recipe type**	**Period of the day**	**Day 6**
1	Breakfast	1	100 g
34	Morning Snack	2	100 g
12	Lunch	3	100 g
20	Afternoon Snack	4	125 g
46	Dinner	5	100 g
Recipe Number	**Recipe type**	**Period of the day**	**Day 7**
2	Breakfast	1	100 g
34	Morning Snack	2	100 g
13	Lunch	3	100 g
10	Afternoon Snack	4	123 g
39	Dinner	5	100 g

By choosing these different dishes, the patient guarantees that all his/her requirements in energy, protein, sodium and potassium are satisfied.

Applying the DGP entails taking into consideration its most important features. In other words, the MPP has to be divided into a number of sub-problems or periods t, and an optimal decision must be taken in each period regarding the correlation between these decisions.

In so doing, the best dish is scheduled in each period of the day and the daily menu consists of the chosen dishes. Decisions are interrelated in the sense that a decision taken to eat an amount of food in any period t is influenced by the quantity eaten previously (in the period $t-1$) and so on to the amount of food to be eaten next (in the period $t+1$). Due to the interrelation of the decisions, the findings of the DGP show that the amount of the chosen recipes is around 200 grams in period 3 (lunch) of the first and the third days and is superior to 100 grams in period 4 (afternoon snack) of the fifth, sixth and seventh days. In this case, the best dishes are chosen with different amounts to satisfy the main constraint of the MPP related to nutritional requirements.

We assume that the smallest unit of each dish is 100 grams. The DGP tends to simultaneously take 100 grams from each of the recipe type and take a long step in one of the recipes to complete the solution which satisfies the nutritional requirements of the day. Hence the program can give multiple solutions for each day. Moreover, swapping the daily menus between any two days of the week is possible without loss of optimality. Our future research will include a cost function which can reduce the number of multiple solutions. In addition, we have no under- or over-achievement in a real-world case which satisfies all goals related to the four nutrient requirements. Positive and negative deviations are zero in the latest periods of each day of the week for all nutrients ($\delta_{i5}^{-} = \delta_{i5}^{+} = 0$ for $\forall$ $k = 1, \ldots, 7$).

While an experienced dietician needs from a couple of minutes to a number of hours to plan manually a daily menu for an HD patient, a computer needs less than a second (0.041 second) to solve the problem and display the results of planning a weekly menu divided into five periods per day thanks to using the DGP model. For both static and dynamic models, menus are displayed for a week. In a nutshell, the longer the period and the less redundant the meals between days and periods of the day, the more obvious the importance of the DGP.

5 Conclusion

In this paper, we have presented the classical GP approach in MPP and we have underscored the importance of DGP as a better alternative. We have also presented an illustrative example focusing on a critical health condition, which is that of a patient undergoing HD. Our research has clearly shown that the proposed approach can be implemented even if in more complex and sensitive situations. It has been demonstrated that the MPP is modeled as dynamic problem and the solutions describes states that occur over time.

Based on the promising results presented in this paper, it will be interesting to assign weights to all periods of the day. Fuzzy logic can be used in further research, providing healthier intake of nutrients through food suggestion and nutritional analysis. The cost is one of the most important objectives in any MPP. In future research, we can consider the cost as a decision criterion even though the cost of a dish represents a secondary problem for patients undergoing HD or suffering from any other chronic illness.

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Part II

Regularly contributed papers

MULTIPLE CRITERIA DECISION MAKING

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GOAL SETTING IN THE NEWSVENDOR PROBLEM WITH UNIFORMLY DISTRIBUTED DEMAND

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Abstract

In the paper we introduce the newsvendor problem with a satisficing-level objective, which is defined as maximization of the probability of exceeding the moving target. This target is defined as the expected profit, multiplied by a positive constant. The constant is chosen by the management and it indicates whether the low or the high goal should be achieved. We obtain closed form solutions of this newsvendor model with uniformly distributed demand. Additionally, we consider a bicriteria problem with the satisficing--level and the classical objective.

Keywords: inventory control, newsvendor problem, bicriteria.

1 Introduction

The newsvendor problem is one of the main stochastic inventory models (Arrow et al., 1951; Khouja, 1999; Muller, 2011; Stevenson, 2009). In the classical newsvendor problem one has to determine the order quantity which maximizes the expected profit. Several authors have also introduced many relaxing assumptions to the basic inventory newsvendor problem. For a review of various kinds of newsvendor models we refer to Qin et al. (2011) and the references therein.

Sometimes companies, instead of maximizing the expected profit, make decisions based on profit targets (or goals). The profit goal can be chosen by external forces such as market conditions or by internal ones according to the budget level. For that reason another choice of newsvendor objective involves the maximization of the probability of exceeding a prespecified target profit – this is called the satisficing-level objective. The use of this objective assumes

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risk aversion; it is a more descriptive measure for the company's decision making (cf. Dechow and Skinner, 2000). This objective provides more information on how companies make decisions. The literature on management behaviour of firms indicates that meeting various profit goals is an important issue for their accounting. This subject is treated in Kabak and Shiff (1978); Lau (1980); He and Khouja (2011). The first researchers, who considered the satisficing-level objective were Kabak and Shiff (1978). Next, (Lau, 1980), developed a mathematical discussion of achieving optimal solutions under the assumption of different demand distribution rates. Recently, He and Khouja (2011) have studied the satisficing objective in the form of the maximal expected profit, but with a fixed profit target. For more information on the satisficing-level newsvendor we refer to Shi and Guo (2012).

It appears that the fixed-profit goal is sometimes specified arbitrarily. The main problem is that the profit goal does not depend on the order quantity. Hence, a more appropriate objective is introduced, namely the maximization of the probability of exceeding the expected profit. The expected profit is a moving target, since it depends on the order quantity; the probability of exceeding this goal is called survival probability. The survival probability approach is studied first in Parlar and Weng (2003) and then in Arcelus et al. (2012), Bieniek (2016; 2017). More precisely, in Parlar and Weng (2003) the problems: with the classical objective and an objective with survival probability are considered simultaneously. Their approximate result is then applied to the case of normally distributed demand. Arcelus et al. (2012) continued this research for uniform distribution, which allowed to derive precise analytic results. Recently, the present author (Bieniek, 2016; 2017) has studied the satisficing-level newsvendor for exponentially distributed demand. Bicriteria optimization discussed in the papers listed is a branch of multicriteria decision making (cf. Stevenson, 2009).

Goal-setting theory has certain psychological aspects. This issue is comprehensively described by Locke and Latham (2013) and Cyert and March (1963). It has been proven that goals affect performance and also direct attention and effort toward goal-relevant activities. High goals lead to greater effort than low goals. Faced with more difficult goals, one can work more intensely. Finally, higher goal levels result in higher performance, but they do not lead to a higher satisfaction. Goals can be a standard tool for judging satisfaction. A person trying to attain a goal will not be satisfied unless he/she attains it. Not reaching one's goal creates increasing dissatisfaction. There is a paradox that people setting difficult goals are the least satisfied ones. This is because people with high goals produce more because they are dissatisfied with less (cf. Locke and Latham, 2013).

Here we consider the survival probability, which is defined as the probability of exceeding the expected profit, multiplied by a positive constant β with values from the interval $(0,1]$. This constant is assigned by the management and it is based on the company's strategy. We use a moving goal profit taking into account the strategy of the firm. The bigger the constant β, the more difficult the goal to be achieved by the decision maker. A company is doing well if it achieves "almost" the expected profit. Note that under the assumption of a positive expected profit, it is obvious that the probability of exceeding a lower profit target is greater than the probability of exceeding a higher one. It appears that when the profit target constant β is greater than 1, a solution to the satisficing--level model may be trivial. This problem needs additional assumptions but it is beyond the scope of our paper. For that reason we limit our study to the case when $\beta \in (0,1]$, because then a solution is non-trivial for all values of the order quantity. Another reason is that achieving almost the expected profit is regarded by the management as sufficiently good and the probability, that the expected profit will be exceeded is usually very small.

We also study the bicriteria newsvendor problem, which takes into account two objectives simultaneously. One of them is the maximization of the survival probability and the second one is the classical objective of the expected profit maximization. We propose a solution to the problem similar to that presented in Arcelus et al. (2012), since in our paper customer demand is also uniformly distributed, but with the profit target involving β. All results are precise and they are given in terms of that constant. Finally, we present numerical results and graphs for various values of β.

2 Satisficing-level newsvendor with uniform distribution

First we introduce the basic notation used throughout the paper. We use the notation from Arcelus et al. (2012), since we continue the problem studied in that paper. Let $p > 0$ be the unit revenue, $c > 0$ be the unit purchase cost, $s > 0$ be the unit shortage cost and $v \in R$ be the unit salvage value. The standard assumption is that $v < c < p$. The demand is a uniformly distributed random variable X on the interval $[A, B]$, with a known density function $f(x) = 1/(B - A)$. The order quantity Q is the only decision variable in the newsvendor model.

If the realized value of the demand is x, then the profit is given by

$$\pi(Q) = \begin{cases} px + v(Q - x) - cQ, & \text{if } x \leq Q, \\ pQ - s(x - Q) - cQ, & \text{if } x > Q. \end{cases}$$

Note that the profit is random since it depends on the random demand X. Let the one-period random profit be denoted by $\pi(X, Q)$. Then the expected profit function $E(Q) = E[\pi(X, Q)]$ for uniformly distributed demand is given by

$$E(Q) = (p - v)(A + B)/2 + (v - c)Q - (p + s - v)\int_Q^\infty (x - Q)f(x)dx$$

(cf. Parlar and Weng, 2003).

The aim is to determine the optimal quantity Q which depends on the adopted optimality criterion. In the classical solution to this problem, the quantity Q which maximizes the expected profit is selected. Note that although $E(0) = -s\mu$ and $E(\infty) = -\infty$, we assume that the maximal expected profit is positive. The order quantity maximizing the expected profit for uniform distribution is equal to

$$Q_E^* = \frac{p+s-c}{p+s-v}(B - A) + A \quad (1)$$

(cf. Arcelus et al., 2012). An alternative optimality criterion, proposed by Parlar and Weng (2003), is to maximize the probability $P[\pi(\mathrm{X}, Q) \geq E(Q)]$ of exceeding the expected profit. For this problem they give an approximate solution. They also suggest to consider the survival probability in the form $P[\pi(\mathrm{X}, Q) \geq \beta E(Q)]$, where β is a positive constant. However, they state that for $\beta > 1$ some limitation on the order quantity should be imposed, which ensures that

$$\beta E(Q) \leq \pi_{\max}(Q), \quad (2)$$

where $\pi_{\max}(Q) = (p - c)Q$. For $\beta > 1$ inequality (2) does not have to be satisfied. In this case it can happen that $\beta E(Q) > \pi_{\max}(Q)$, which implies $P[\pi(\mathrm{X}, Q) \geq \beta E(Q)] = 0$. Since we want to solve the given satisficing-level problem in general, without any conditions on Q, we study the case when $0 < \beta \leq 1$. This ensures that (2) is satisfied and the optimal order quantity can take any value from the set of all possible Q without limitations. On the one hand, we use the factor β which gives flexibility to the problem and on the other hand, we provide precise solutions, which is possible for uniformly distributed demand.

From Parlar and Weng (2003) we know that the survival probability $H(Q,\beta) = P(\pi(X,Q) \geq \beta E(Q))$ can be written in the form

$$H(Q,\beta) = \int_{D_1(Q,\beta)}^{D_2(Q,\beta)} f(x)dx,$$

where the integral limits $D_1(Q,\beta)$ and $D_2(Q,\beta)$ are functions of the order quantity Q and β. Determining the variability of the limit functions is crucial to the optimization of the survival probability. First, note that for uniform distribution $D_1(Q,\beta) = \max(A, \xi_A(Q,\beta))$, where $\xi_A(Q,\beta)$ is given by

$$\xi_A(Q,\beta) = \frac{\beta E(Q) + (c - v)Q}{p - v}$$

and $D_2(Q,\beta) = \min(\xi_B(Q,\beta), B)$, where $\xi_B(Q,\beta)$ is defined by

$$\xi_B(Q,\beta) = \frac{(p + s - c)Q - \beta E(Q)}{s}.$$

Now let Q_A and Q_B be the zeros of the limit functions defined by the equations

$$D_1(Q_A, \beta) = A \text{ and } D_2(Q_B, \beta) = B.$$

Solving these quadratic equations with respect to the order quantity Q we get the expressions for Q_A and Q_B. In computations the formula for $E(Q)$ with uniformly distributed demand is used, given by

$$E(Q) = \frac{(p-v)(A+B)}{2} + (v-c)Q - \frac{(p+s-v)(B-Q)^2}{\mathrm{B}-\mathrm{A}}.$$

Lemma 1

For $0 < \beta \leq 1$

$$Q_A = B - \frac{B-A}{\beta(p+s-v)}\left((c-v)(\beta-1) + \sqrt{\alpha}\right), \tag{3}$$

and

$$Q_B = B - \frac{B-A}{\beta(p+s-v)}\left(p+s-c+\beta(c-v) - \sqrt{\gamma}\right), \tag{4}$$

where

$$\alpha = (c-v)^2(\beta-1)^2 + \frac{\beta(p+s-v)}{B-A}\left((p-v)(\beta B-(2-\beta)A) + 2B(c-v)(1-\beta)\right), \tag{5}$$

and

$$\gamma = (p+s-c+\beta(c-v))^2 - \frac{\beta(p+s-v)}{B-A}\left(2B\big(p-c+\beta(c-v)\big) - \beta(p-v)(A+B)\right). \tag{6}$$

We obtain the following conclusions concerning the shape of the limit functions. For $0 < \beta \leq 1$ the function $D_1(Q, \beta)$ is constant and equal to A on $(A, Q_A]$, and it is increasing on (Q_A, B). The function $D_2(Q, \beta)$ is increasing on (A, Q_B), and then constant and equal to B on $[Q_B, B)$.

Now we have to analyze the variability of the difference between $D_1(Q, \beta)$ and $D_2(Q, \beta)$. Since condition (2) has to be satisfied, we have $D_2(Q, \beta) - D_1(Q, \beta) = \frac{p+s-v}{s(p-v)}\left((p-c)Q - \beta E(Q)\right) \geq 0$. In some cases the minimum distance between $D_2(Q, \beta) - D_1(Q, \beta)$ exists for some $Q = Q_M$. Minimizing the difference between $D_1(Q, \beta)$ and $D_2(Q, \beta)$ we get the following lemma.

Lemma 2

Let $0 < \beta \leq 1$. If

$$s + \left(1 - \frac{1}{\beta}\right)(p-c) > 0 \tag{7}$$

then the difference $D_2(Q, \beta) - D_1(Q, \beta)$ is minimized at the unique point Q_M given by

$$Q_M = A + \frac{B-A}{p+s-v}\left(s + \left(1 - \frac{1}{\beta}\right)(p-c)\right). \tag{8}$$

Otherwise, if

$$s + \left(1 - \frac{1}{\beta}\right)(p - c) \leq 0$$

then $D_2(Q,\beta) - D_1(Q,\beta)$ is an increasing function of Q for all $A \leq Q \leq B$.

Proof

Since

$$D_2'(Q,\beta) - D_1'(Q,\beta) = \frac{p+s-v}{s(p-v)}[(1-\beta)(p-c) - \beta s + \beta(p+s-v)F(Q)],$$

then from the equality $D_2'(Q_M, \beta) - D_1'(Q_M, \beta) = 0$ we get (8). Moreover, the second derivative $D_2''(Q,\beta) - D_1''(Q,\beta) = \frac{\beta(p+s-v)^2}{s(p-v)}$ is positive for all $Q \geq 0$. Therefore, the difference $D_2(Q,\beta) - D_1(Q,\beta)$ is a convex function of Q and it attains its minimum value at Q_M. The existence of Q_M follows from the constraint (7), which ends the proof.

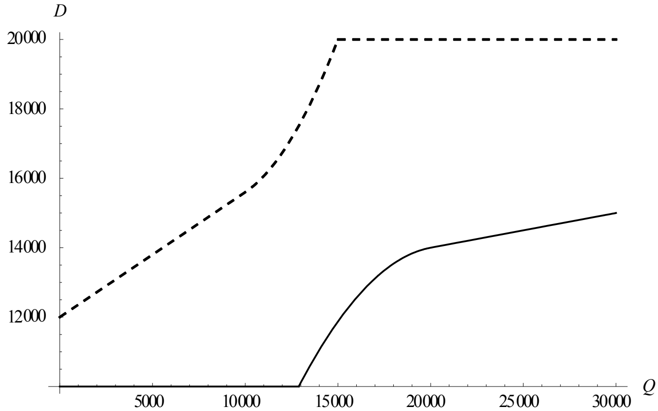

Figure 1: Limit functions D_1 (solid) and D_2 (dashed) for $\beta = 0.8$

Examples of graphs of functions D_1 and D_2 are presented in Figure 1. It should be emphasized here that if the demand is uniformly distributed then the minimum distance between the limit functions translates to the minimum probability $H(Q,\beta)$. Hence the survival probability attains the local minimum at the point Q_M if such a minimum exists. In the following theorem we study the monotonicity of $H(Q,\beta)$ when $0 < \beta \leq 1$. The results of Arcelus et al. (2011) for $\beta = 1$ can be obtained from the Theorem 1.

Theorem 1
If $0 < \beta \le 1$ and Q_M, as defined by (8), exists then $H(Q,\beta)$ is increasing with respect to Q on (A, Q_A), decreasing on (Q_A, Q_M), increasing on (Q_M, Q_B), and finally decreasing on (Q_B, B) and

$$H(Q_A,\beta) = \frac{(p+s-c)\beta + c - v - \sqrt{\alpha}}{\beta s},$$
$$H(Q_B,\beta) = \frac{p+s-c+\beta(c-v) - \sqrt{\gamma}}{\beta(p-v)},$$

where Q_A and Q_B are defined by (3) and (4) and α and γ are given by (5) and (6), respectively. Then $H(Q,\beta)$ attains its maximum value Q_H^* at Q_A or Q_B and its local minimum at Q_M. If Q_M does not exist then $H(Q,\beta)$ is increasing on (A, Q_B) and decreasing on (Q_B, B), so it attains its maximum value at Q_B.

The proof of Theorem 1 follows directly from Lemma 1. Examples of graphs of the survival probability with constant $\beta = 0.8; 0.9; 1.0$ are presented in Figure 2.

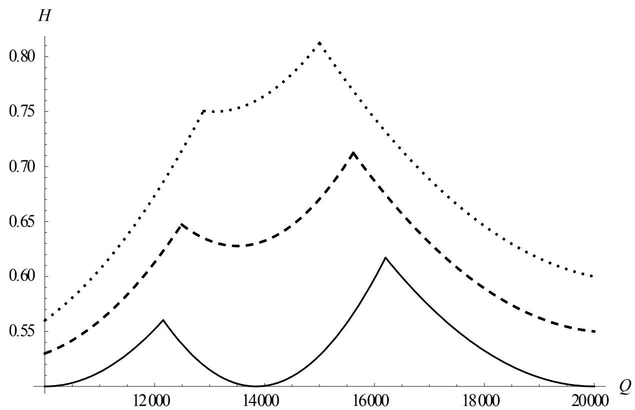

Figure 2: $H(Q,\beta)$ for uniform distribution and the model parameters $(A,B,v,c,p,s) =$ $(10000,20000,10,30,50,25)$ with $\beta = 0.8$ (dotted); $\beta = 0.9$ (dashed); $\beta = 1.0$ (solid)

3 Bicriteria problem

In the next lemma we give inequalities for Q_E^* as defined by (1), Q_A and Q_B, which are used for solving the bicriteria problem.

Lemma 3
The order quantity Q_E^* satisfies the inequalities

$$Q_A < Q_E^* < Q_B, \quad \text{if} \quad p + s - \sqrt{\gamma} < c,$$

or

$$Q_A < Q_E^* = Q_B, \quad \text{if} \quad p + s - \sqrt{\gamma} = c,$$

or

$$Q_A < Q_B < Q_E^*, \quad \text{if} \quad p + s - \sqrt{\gamma} > c.$$

Now we recall the so-called bicriteria index, i.e. a measure which combines the classical newsvendor and the satisficing models. Let $E^* = E(Q_E^*)$ and $H^*(\beta) = H(Q_H^*, \beta)$. Then the bicriteria problem is to find the order quantity Q_Y^*, which maximizes the bicriteria index $Y(Q,\beta)$ with the non-negative weight $w \in [0,1]$ defined by

$$Y(Q,\beta) = wE(Q)/E^* + (1-w)H(Q,\beta)/H^*(\beta).$$

This is a kind of a vector optimization problem with defined weights. Here the model is transformed into a scalar optimization problem. The constants E^* and H^* normalize the weighted objective function since the values of two objectives can generally be very different. For $w = 0$ the problem reduces to maximizing the survival probability and for $w = 1$ it reduces to maximizing the expected profit. For detailed discussion on this subject see Chankong and Haimes (1983), Osyczka (1984).

The constant β influences the bicriteria index since it determines Q_H^*. There are several other methods for finding a compromise solution in multiple criteria problems.

In the light of Lemma (3) we have four cases, which give the position of the order quantity Q_Y^*:

Case 1: $H(Q_B,\beta) > H(Q_A,\beta)$ and $Q_E^* > Q_B$
Case 2: $H(Q_B,\beta) > H(Q_A,\beta)$ and $Q_E^* < Q_B$
Case 3: $H(Q_B,\beta) < H(Q_A,\beta)$ and $Q_E^* > Q_B$
Case 4: $H(Q_B,\beta) < H(Q_A,\beta)$ and $Q_E^* < Q_B$.

The solution in each case for $\beta = 1$ reduces to those given in Arcelus et al. (2012). In Case (1) we get the following theorem.

Theorem 2
If $H(Q_B,\beta) > H(Q_A,\beta)$ and $Q_E^* > Q_B$ then $Q_B \le Q_Y^* \le Q_E^*$ with

$$F(Q_Y^*) = 1 - \frac{c-v}{(p+s-v)Z}\left(\frac{w}{E^*} - \frac{(\beta-1)(w-1)}{(B-A)(p-v)H^*(\beta)}\right), \quad (9)$$

where $X_\beta = w/E^* - \beta(1-w)/[(B-A)(p-v)H^*(\beta)]$ and

$$w > \frac{\beta E^*}{\beta E^* + (B-A)(p-v)H^*(\beta)}. \quad (10)$$

Proof
First we show that $Y(Q,\beta)$ is decreasing for $Q > Q_E^*$ and increasing for $Q < Q_B$. Note that $E'(Q) = (p+s-v)(1-F) - (c-v)$ and $H'(Q,\beta) = -\frac{c-v+\beta E'(Q)}{(B-A)(p-v)}$. Moreover, $Y'(Q,\beta) = X_\beta E'(Q) - (1-w)(c-v)/[(B-A)(p-v)H^*(\beta)]$, where X_β is defined in the theorem. Then $Y'(Q,\beta)|_{Q_E^*} < 0$ since $E'(Q)|_{Q_E^*} = 0$, which implies that $Y'(Q,\beta) < 0$ for $Q > Q_E^*$. Furthermore, both $H(Q,\beta)$ and

$E(Q)$ are increasing on (Q_M, Q_B) and therefore so is $Y(Q, \beta)$. The optimality is proved by:

$$Y'(Q_Y^*, \beta) = 0, \text{which implies that equality (9) holds,}$$

and $Y''(Q, \beta) < 0 \; if \; X_\beta > 0$, which implies that condition (10) holds.

The feasibility is implied from

$0 \le F(Q_Y^*) \le 1$ if $X_\beta > 0$, which implies that the condition (10) holds and completes the proof.

In the next theorem Case (4) is considered.

Theorem 3

If $H(Q_B, \beta) < H(Q_A, \beta)$ and $Q_E^* < Q_B$ then $Q_E^* \le Q_Y^* \le Q_B$ with

$$Q_Y^* = F^{-1}\left\{\frac{p+s-c}{p+s-v} + \frac{(1-w)(p-c)}{(B-A)s(p-v)X_\beta H^*(\beta)}\right\},$$

where $X_\beta = w/E^* - \frac{\beta(1-w)(p+s-v)}{(B-A)s(p-v)H^*}$ and

$$w > \frac{1/s + 1/(p-v)}{(H^*(\beta)(B-A))/(\beta E^*(p+s-v)) + 1/s + 1/(p-v)}.$$

Note that Theorem 2 is applicable to Case (3) and Theorem 3 is applicable to Case (2); the optimal solution Q_Y^* satisfies

$\min\{Q_E^*, Q_B\} \le Q_Y^* \le \max\{Q_E^*, Q_B\}$.(11)

The solution to the bicriteria problem for $w = 1$ is the same as to the expected profit maximization problem. Additionally, there exists $w_r \in [0,1)$ such that the solution to the bicriteria problem is equal to Q_Y^* for some $w > w_r$, and it is the same as the solution to the probability maximization model Q_H^* for $0 \le w < w_r$. Tables 1 and 2 below present numerical examples for Case (1). We use the same values of parameters as in [1], but additionally the constant β is involved. Note that the above expression for $\beta = 1$ reduces to the results known from Arcelus et al. (2012). We present them here to complete the overview of the problem. A numerical example is given below. Let $Y^*(\beta) = Y(Q_Y^*, \beta)$.

Table 1: Retailer policies – Case (1): $H(Q_B, \beta) > H(Q_A, \beta)$ and $Q_E^* > Q_B$ with $(A, B, v, c, p, s) = (10000,20000,10,30,50,15)$

Q_E^* =16364		E^* =236364	
β	0.8	0.9	1
$Q_A(\beta)$	12333	11866	11472
$Q_B(\beta)$	13435	14368	15222
$H(Q_A(\beta), \beta)$	0.855	0.68	0.54
$H(Q_B(\beta), \beta)$	0.9	0.77	0.66

Table 2: Retailer policies – Case (1): Bicriteria solution, parameters the same as in Table 1

β	0.8		0.9		1.0	
w	Q_Y^*	$Y^*(\beta)$	Q_Y^*	$Y^*(\beta)$	Q_Y^*	$Y^*(\beta)$
1.0	Q_E^*	1.0	Q_E^*	1.0	Q_E^*	1.0
0.9	16083	0.979	16030	0.985	15960	0.99
0.8	15679	0.961	15526	0.973	15309	0.98
0.7	15048	0.947	14679	0.968	Q_B	0.989
0.6	13926	0.941	Q_B	0.972	Q_B	0.991
0.5	Q_B	0.95	Q_B	0.977	Q_B	0.992
0.4	Q_B	0.96	Q_B	0.981	Q_B	0.994
0.3	Q_B	0.97	Q_B	0.986	Q_B	0.995
0.2	Q_B	0.98	Q_B	0.991	Q_B	0.997
0.1	Q_B	0.99	Q_B	0.995	Q_B	0.998
0.0	Q_B	1.0	Q_B	1.0	Q_B	1.0

Let us analyse Case (1). From Table 1 we see that if constant β increases from 0.8 to 1.0 then the maximal survival probability $H(Q_B, \beta)$ decreases from 0.9 to 0.66, but the optimal order quantity Q_B increases from 13435 to 15222. Moreover, the order quantity Q_A decreases from 12333 to 11474. Summarizing, for greater values of β the values of Q_A increase but the values of Q_B decrease. We also see that the probability of achieving a target profit greater than 80% of the expected profit is significantly greater than the probability for $\beta = 1$ (about 27%). Because of this, one should considered setting a goal slightly lower but one that is much more likely to be achieved.

Next, in Table 2 we see that for given β the compromise solution Q_Y^* increases from Q_B to Q_E^* as the weight w increases. Note that if we assume that $w > w_r = 0.5$ and that condition (11) is satisfied, we have $Q_Y^* = Q_B$ for $\beta = 0.8$. If $\beta = 0.9$ and $w \leq 0.6$ then $Q_Y^* = Q_B$. Finally, for $w \leq 0.7$ and $\beta = 1.0$ we get also $Q_Y^* = Q_B$.

4 Conclusions

In this research note we extend the results of Arcelus et al. (2012) concerning the solution to the bicriteria newsvendor optimization problem with uniformly distributed demand. The authors of the cited paper studied both the classical and the satisficing-level objectives simultaneously. We modify the satisficing-level objective by introducing the target profit as the expected profit multiplied by a positive constant with values from the interval (0,1]. This constant is fixed by the company management; the larger the constant is, the more difficult task for the staff is required. We limit our considerations to the interval (0,1], because setting this constant greater than one requires additional assumptions on the order quantity. Finally, we investigate the bicriteria newsvendor problem in the numerical example for various values of this constant.

We emphasize here that for the general distributions in the satisficing-level problem only bounds on the optimal order quantity can be obtained (cf. Parlar and Weng, 2003). Because of that, we use uniformly distributed demand, which substantially simplifies the expressions obtained and allows to obtain precise solutions. After the introduction of the constant to the goal profit, the derivations are not automatically transformed from the results of Arcelus et al. (2012). The constant used in the goal profit substantially changes the solutions. The model developed here can be viewed, as a tool to assist the management in determining the target level.

In future research one can investigate the problem with a high goal and the constant greater than one. Additionally, other methods, which provide precise solutions to the satisficing-level problem for any demand distribution should be found and methods other than bicriteria decision making can be proposed. Moreover, a new measure of satisfaction using the survival probability studied here can be created. In our paper the satisfiction is defined in terms of goal setting theory as the satisfaction of attaining the goal. Only two states are therefore possible: being satisfied or not. One can probably consider measuring satisfaction using a continuous measure based on our paper.

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APPLICATION OF THE GENERALISED DISTANCE MEASURE TO LOCATION SELECTION DURING ORDER-PICKING

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Abstract

When a shared storage system is used, the selection of locations from which products should be picked becomes a significant decision problem. Every storage location can be described using several criteria, such as: storage time, distance from the I/O point, degree of demand satisfaction, the number of other products to be picked near the analysed location, or others. Based on such criteria, a synthetic variable can be created to rank all these locations; the highest-ranking one is selected. Such a ranking is created using the Generalised Distance Measure (GDM); the selected locations and the picker's route based on them are compared to the results obtained using the Taxonomic Measure of Location's Attractiveness (TMAL). Both route length and picking time are compared. Also, the influence of the system of criteria weights within each method on the route length and the picking time is analysed using simulation methods.

Keywords: order-picking, Generalised Distance Measure, Taxonomic Measure of Location's Attractiveness, multiple-criteria decision making, simulation analysis.

1 Introduction

Order-picking is the most time- and cost-consuming activity in warehouse management, for both manual and automated systems (De Koster et al., 2007). Therefore, there is still room for improvement in this area, which can be done in three ways, by optimising storage assignment, orders batching, or routing methods. Every area uses different methods of improvement. Storage assignment can be improved, for example, by implementing class-based storage; orders batching, by reducing order picking time; and routing methods, by adjusting the method of travelling to the warehouse type.

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There are two main methods of storing goods in a warehouse: dedicated storage and shared storage (Bartholdi and Heckman, 2016). Dedicated storage consists in storing each product in one location containing one product only. It enables the picker (to a certain degree, depending on the number of SKUs[1] stored in the warehouse) to easily remember storage locations and makes order picking relatively fast and efficient. Its main drawback is its inefficiency (the storing space is utilised, on the average, in about 50% (Bartholdi and Heckman, 2016). Shared storage, on the other hand, consists in storing each product in any one of many locations, with many products stored in each location (Bartholdi and Heckman, 2016). This storage method greatly improves the utilisation of the storing space, but results in the products being scattered among various locations, often very distant from each other. Also, locations of all products change continually, which makes it impossible for the picker to remember them. It is then necessary to use a warehouse management system.

When a shared storage system is used, the products ordered can be picked from many locations. The question arises: Which location should be selected to pick the given product? The problem remains pretty much unsolved in the literature. Bartholdi and Heckman (2016) mention that during order-picking, the picker can select the most convenient location (to reduce labour) or the least-filled locations (which is more labour-intensive, but frees storage space for future replenishment orders). Gudehus and Kotzab (2012) specified several take-out strategies for a product which can be accessed from more than one location:

- FIFO – units are picked according to their arrival time to the warehouse.
- Priority of partial units – locations with the lowest content of the product are accessed first, even if it increases labour.
- Quantity adjustment (the opposite to the previous one) – the picker retrieves the product from the locations containing the entire requested quantity, even if it generates low amounts of products at these locations.
- Taking the access unit – if the amount of the product at the given location exceeds or is equal to the quantity requested, the entire unit is taken after the excess quantity is put aside.

There are thus several criteria relevant to the strategy of location selection during order picking. From the above-listed take-out strategies, we can think of at least two of them: storage time and the amount of product at a given location. However, other criteria can be also taken into account. To improve the picker's travel time, we can select locations close to the I/O point[2]. Also, if there are

[1] SKU (*Stock Keeping Unit*) – the smallest physical unit of a product.

[2] The I/O (input/output) point is the location from where the picker starts picking the products ordered and collects picked products.

many products in the order, we can select those storage locations which are close to each other, so that the picker can complete the order without too much travelling around the warehouse. Moreover, in a high-storage warehouse the storage level is also an important criterion. The products should be picked from low levels first, since the picker can reach them directly from the floor. To reach locations on higher levels, he/she must use a ladder or forklift; therefore, products from those levels should be picked next.

All these criteria can be considered separately, partially or in their entirety. If the decision maker intends to consider some, or all of them, then his/her decision will be based on a multiple-criteria approach. There are many methods that support multiple-criteria decision making. In general, we deal with multiple-objective mathematical programming problems, or multiple-criteria evaluation problems (Trzaskalik, 2015). In the former, alternatives are not explicitly known: there may be even an infinite number of them. Such problems are solved by means of a mathematical decision model. In the latter, alternatives are known; they are described by multiple criteria and the best one is selected by ordering them. For the location selection problem, multiple-criteria evaluation methods can be applied. The decision maker knows all the alternatives in the problem: in this case they are the storage locations of products ordered. Some of the many multiple-criteria decision analysis methods are: AHP, ANP, ELECTRE, SAW, COPRAS, TOPSIS. The best ones are those that allow the decision maker to make many decisions in a short time. They should be easily implemented in software and should require minimum attention from the decision maker, therefore such methods as SAW (Podvezko, 2011) or TOPSIS (Hwang and Yoon, 1981) are the most obvious choice. They both can rank the decision alternatives (SAW does it without creating the so-called "pattern" object, while TOPSIS is based on the distances from both "pattern" and "anti-pattern").

The present author, in his previous papers, designed a simple multiple-criteria decision-making technique, based on the Synthetic Measure of Development (Hellwig, 1968) and called the Taxonomic Measure of Location's Attractiveness (Polish abbreviation TMAL) (Dmytrów, 2015). All the above-mentioned methods are based on Euclidean distances, which can be used only for criteria measured on an interval or a ratio scale. However, some criteria can be measured on a weaker, ordinal scale, for which Euclidean distances cannot be used. In this case, we can use the Generalised Distance Measure (GDM), proposed by Walesiak (2000). Although GDM was not meant as a multiple-criteria decision-making technique, but rather as a measure for the calculation of the distance matrix in object classification, or as a synthetic measure of development in methods of linear ordering. In the latter, its application is similar to the Synthetic

Measure of Development, used in the TMAL method. The goal of the present paper is to compare GDM with TMAL as multiple-criteria decision-making techniques for the selection of locations in order-picking.

2 Analytical methods applied

2.1 Specification of decision criteria and applied systems of weights

As mentioned before, to select the pick location of a product, various strategies can be used. In this paper, three criteria are applied:
x_1 – distance from the I/O point,
x_2 – degree of demand satisfaction,
x_3 – number of other products picked in the neighbourhood of the location analysed.

The first criterion is measured in contractual units, that is, shelf width. It is measured on a ratio scale and has negative impact.

The degree of demand satisfaction has positive impact. It is measured on a ratio scale and is calculated from the following formula:

$$x_2 = \begin{cases} \frac{l}{z}, & \text{if} \quad z > l \\ 1 & \text{if} \quad l \geq z \end{cases}, \tag{1}$$

where l – number of units of the product picked from the location analysed and z – demand for the picked product.

The third criterion – the number of other products picked in the neighbourhood of the location analysed – has positive impact. It is measured on a ratio scale and is a numerical and discrete variable. It should be mentioned here that the notion of a neighbourhood depends on the warehouse type. In a high-storage warehouse, this can be the rack. In a typical low-storage warehouse, this can be the racks within an aisle (which will be assumed here).

The criteria used to create the synthetic variable to classify the alternatives should be weighed. There are many methods to weigh the decision criteria, which can be classified as statistical and formal, and expert. Statistical methods can be based on the variability of criteria: The higher the share of variability of the given criterion in the total variability, the higher weight should the criterion have (Kukuła, 2000). Another statistical and formal method is based on the Shannon entropy (Lotfi and Fallahnejad, 2010). Among expert methods is AHP, in which experts specify their preferences by comparing the criteria pairwise (Trzaskalik, 2015). The weights can also be specified purely subjectively, with the decision-maker deciding the importance of each criterion. In our case, seven combinations of weights have been analysed (see table 1).

Table 1: Analysed combinations of weights

Combinations of weights	x_1	x_2	x_3
C1	0.333	0.333	0.333
C2	0.5	0.25	0.25
C3	0.25	0.5	0.25
C4	0.25	0.25	0.5
C5	0.4	0.4	0.2
C6	0.4	0.2	0.4
C7	0.2	0.4	0.4

Source: Author's own elaboration.

Combination C1, in which every criterion has the same weight, is the reference. In combinations C2, C3 and C4 one criterion is twice as important as the other two and therefore its impact on the final decision is the same as the total impact of the remaining two criteria. In combinations C5, C6 and C7 two criteria are twice as important as the remaining one. The combinations of weights have been selected so as to analyse how well the algorithm performs in each situation and whether making one or two criteria more important than the other(s) will improve the system's performance.

2.2 Construction of TMAL and GDM

The construction of both TMAL and GDM consist of several steps, repeated for each product ordered. The steps for TMAL are as follows:

- The distance from the I/O point (x_1) is changed into a criterion with positive impact by calculating its inverse.
- The values of all criteria are normalised. We use quotient inversion:

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}, \tag{2}$$

  where x_{ij} – value of j-th criterion in i-th alternative (location). Many other normalisation formulas are possible (Walesiak, 2016). This formula was selected so as to preserve the differences in mean values and variability.
- The maximum normalised values form the so-called "perfect alternative" or the pattern.
- Euclidean distances between the pattern and each location are calculated.
- Mean weighed distances from the pattern for all combinations of weights are calculated.

- TMAL is calculated as the complement to unity from the ratio of the mean weighed distance of each location from the maximum value obtained in the previous step.
- TMAL values are sorted in the descending order.
- The highest-ranking locations are selected, until the demand for each product is satisfied.

The Generalised Distance Measure (GDM) is based on the generalised correlation coefficient, using the Pearson Product-Moment Correlation Coefficient and the Kendall τ correlation coefficient (Walesiak, 2011):

$$d_{ik} = \frac{1}{2} - \frac{\sum_{j=1}^{m} w_j a_{ikj} b_{kij} + \sum_{j=1}^{m} \sum_{l=1, l \neq i,k}^{n} w_j a_{ilj} b_{klj}}{2\left[\sum_{j=1}^{m} \sum_{i=1}^{n} w_j a_{ilj}^2 \cdot \sum_{j=1}^{m} \sum_{i=1}^{n} w_j b_{klj}^2\right]^{\frac{1}{2}}}, \tag{3}$$

where:
d_{ik} – distance (similarity) measure,
$i, k, l = 1, 2, \ldots, n$ – index of the alternative (location),
$j = 1, 2, \ldots, m$ – index of the criterion,
w_j – weight of j-th criterion.

For variables measured on an interval or a ratio scale, the values of a and b are calculated from the following formulas:

$$a_{ipj} = x_{ij} - x_{pj} \text{ for } p = k, l, \qquad b_{krj} = x_{kj} - x_{rj} \text{ for } r = i, l, \tag{4}$$

where x_{ij} (x_{kj}, x_{ij}) is i-th (k-th, l-th) value of j-th criterion.

Using GDM we can calculate the distance between objects (in multivariate statistical analysis) or decision alternatives (in multiple-criteria decision-making problems). The main advantage of GDM over the most commonly used distance measures, such as Euclidean, Mahalanobis or Manhattan, is that it allows to use criteria measured on an ordinal scale. It can be used for the determination of the distance matrix in classification procedures or in linear ordering of objects (in multivariate statistical analysis) or decision alternatives (in multiple-criteria decision-making problems).

The procedure of using GDM in linear ordering is as follows (Walesiak, 2003):

- There is no need to change the criteria with negative impact into ones with positive impact.
- The values of each criterion are normalised using formula (2).
- The so-called “perfect alternative”, or the pattern, is created. For the criteria with negative impact, the pattern values are the minimum values among all the alternatives. For the criteria with positive impact, the pattern consists of the maximum values among all the alternatives.

- The distance of each alternative (location) from the pattern is calculated using formula (3), applying the substitutions given by (4).
- The values of GDM for the alternatives (locations) are sorted in the ascending order.
- The highest-ranking locations are selected, until the demand for each product is satisfied.

2.3 Assumptions of the simulation experiment

A simulation experiment has been performed, with the following assumptions:

- A simple, rectangular warehouse was assumed.
- The warehouse contained 1000 locations with one main aisle and 20 aisles between racks. Every rack contained 25 locations.
- The warehouse used chaotic storage system.
- Every order consisted of ten products.
- Every product was stored in four locations.
- The available amounts of products in each location varied from a single unit to the amount that satisfied the demand twice.
- For both TMAL and GDM and all combinations of weights, 100 orders were generated.
- For every product picked, every method and every combination of weights, a ranking of locations was created.
- The highest-ranking locations were selected until the demand was satisfied.
- Once the locations had been selected, the picker's route was determined using *s-shape* heuristics (Le-Duc, 2005).
- For each route, its length was measured, and the order-picking time was calculated.
- The order-picking time was the sum of the picker's travel and collection times. It was assumed that the time of traversing a distance unit (shelf width) was 2 seconds and the time of collecting the product from the location, 10 seconds.
- For TMAL and GDM, it was analysed, using the one-way ANOVA, whether both route lengths and picking times were significantly different.
- If the null hypothesis was to be rejected, using *post-hoc* Tukey's HSD test, pairwise comparisons were performed.
- For every combination of weights, mean route length and order-picking time obtained using TMAL and GDM were compared using the paired *z*-test for independent samples.

3 Results of the simulation analysis

3.1 Comparison of results for each combination of weights within each method

Mean route lengths, order-picking times and results of the ANOVA for TMAL are presented in Table 2.

Table 2: Mean route lengths, order-picking times (in minutes) and results of the ANOVA for TMAL

Specification	C1	C2	C3	C4	C5	C6	C7
Route length	367.88	362.92	350.28	383.64	346.98	376.10	378.42
	ANOVA $F = 9.612$, p-value $p < 0.0001$						
Order-picking time	14:38	14:33	13:55	15:15	13:49	15:05	14:56
	ANOVA $F = 8.516$, p-value $p < 0.0001$						

Source: Author's own elaboration.

The one-way ANOVA showed that both mean route lengths and order-picking times varied depending on the combination of weights. The results of *post-hoc* Tukey's test are presented in Table 3.

Table 3: Results of Tukey's test for TMAL (significant differences are marked in bold)

	C2	C3	C4	C5	C6	C7
Route length						
Tukey's criterion $T = 17.736$						
C1	4.96	17.60	15.76	**20.90**	8.22	10.54
C2		12.64	**20.72**	15.94	13.18	15.50
C3			**33.36**	3.30	**25.82**	**28.14**
C4				**36.66**	7.54	5.22
C5					**29.12**	**31.44**
C6						2.32
Order-picking time						
Tukey's criterion $T = 39.622$						
C1	5.12	**43.20**	37.52	**48.50**	27.04	18.18
C2		38.08	**42.64**	**43.38**	32.16	23.30
C3			**80.72**	5.30	**70.24**	**61.38**
C4				**86.02**	10.48	19.34
C5					**75.54**	**66.68**
C6						8.86

Source: Author's own elaboration.

For the route length, the best results (the shortest route lengths) were obtained for combination C5 (0.4; 0.4; 0.2). This means that in order to minimise the picker's route length, the decision-maker should weigh both the distance from the I/O point and the degree of demand satisfaction twice as much as the number of other products in the neighbourhood of the location analysed. The mean route length for this combination was significantly shorter than the route lengths obtained for combinations C1, C4, C6 and C7. The mean route length for combination C5 was shorter by 9.6% than that for the worst combination, that is, C4. The results obtained by the reference combination C1 were exactly in the middle.

For the order-picking time, the best results were also obtained for combination C5. The worst results (longest order-picking times) were obtained for combination C4. The mean order-picking time obtained for combination C5 was shorter by 9.4% than that for the worst combination C4. Also, the results for combination C5 were significantly better than those obtained for combinations C1, C2, C4, C6 and C7. The results obtained for the reference combination C1 were exactly in the middle, as in the case of route length.

The mean route lengths, order-picking times and the results of the ANOVA for GDM are presented in Table 4.

Table 4: Mean route lengths, order-picking times (in minutes)
and results of the ANOVA for GDM

Specification	C1	C2	C3	C4	C5	C6	C7
Route length	366.66	361.36	343.72	383.08	340.72	369.26	372.94
	ANOVA $F = 10.306$, p-value $p < 0.0001$						
Order-picking time	14:34	14:29	13:40	15:14	13:33	14:49	14:43
	ANOVA $F = 11.647$, p-value $p < 0.0001$						

Source: Author's own elaboration.

Similarly as in the case of TMAL, the one-way ANOVA for GDM showed that both mean route lengths and order-picking times varied significantly depending on the combination of weights. The results of *post-hoc* Tukey's test for the results obtained by GDM are presented in Table 5.

As in the case of TMAL, the shortest route lengths for GDM were obtained for combination C5 (0.4; 0.4; 0.2). The mean route length for this combination was significantly shorter than the route lengths obtained for combinations C1, C2, C4, C6 and C7. The mean route length for combination C5 was shorter by over 11% than that for the worst combination, that is, C4. The results obtained by the reference combination C1 for GDM were exactly in the middle, as previously.

Table 5: Results of Tukey's test for GDM (significant differences are marked in bold)

	C2	C3	C4	C5	C6	C7
Route length						
Tukey's criterion $T = 17.659$						
C1	5.30	**22.94**	16.42	**25.94**	2.60	6.28
C2		17.64	**21.72**	**20.64**	7.90	11.58
C3			**39.36**	3.00	**25.54**	**29.22**
C4				**42.36**	13.82	10.14
C5					**28.54**	**32.22**
C6						3.68
Order-picking time						
Tukey's criterion $T = 39.603$						
C1	4.90	**54.18**	**39.64**	**60.88**	14.8	9.36
C2		**49.28**	**44.54**	**55.98**	19.70	14.26
C3			**93.82**	6.70	**68.98**	**63.54**
C4				**100.52**	24.84	30.28
C5					**75.68**	**70.24**
C6						5.44

Source: Author's own elaboration.

For GDM the shortest order-picking times were obtained for combination C5. The worst results – longest order-picking times – were obtained for combination C4. The mean order-picking time obtained for combination C5 was shorter by 11% than that for the worst combination C4. Also, the results for combination C5 were significantly better than those obtained for all other combinations, except for C3. The route lengths obtained for the reference combination C1 were exactly in the middle, as previously.

3.2 Comparison of the results obtained using each method

The results of the paired z-test for independent samples for both methods are presented in Table 6.

Table 6: Mean route lengths, order-picking times (in minutes) for TMAL and GDM and the results of the paired z-test

Specification	C1	C2	C3	C4	C5	C6	C7
Route length							
TMAL	367.88	362.92	350.28	383.64	346.98	376.10	378.42
GDM	366.66	361.36	343.72	383.08	340.72	369.26	372.94
z	0.200	0.228	0.907	0.084	0.879	0.989	0.856
p**-value**	0.710	0.705	0.591	0.733	0.595	0.581	0.598

Table 6 cont.

Specification	C1	C2	C3	C4	C5	C6	C7
Order-picking time							
TMAL	14:38	14:33	13:55	15:15	13:49	15:05	14:56
GDM	14:34	14:29	13:40	15:14	13:33	14:49	14:43
z	0.271	0.234	0.932	0.105	1.033	0.990	0.891
***p*-value**	0.697	0.704	0.588	0.729	0.575	0.581	0.593

Source: Author's own elaboration.

For both route length and order-picking time, GDM performed always better than TMAL, although the differences were not statistically significant. The difference between the mean route lengths varied from less than half of a unit for combination C4 to almost seven units for combination C6. For the order-picking time, the differences varied from one second for combination C4 to sixteen seconds for combination C6. Therefore, in this case, both methods are practically equally efficient.

4 Conclusions

In this paper, GDM has been applied as a multiple-criteria decision-making technique for the selection of locations in order-picking. Although GDM is usually not regarded as a decision-making support tool, its construction enables us to use it for this purpose. This measure has been previously used in decision making, as a distance measure in other techniques, such as TOPSIS (Wachowicz, 2011). Here, it has been used as a technique to create a ranking of the alternatives. The alternatives were the locations in a warehouse to be visited by the picker to complete the orders.

In the analysed simulation example, GDM generated similar results as TMAL, which is based on the classical Hellwig's Synthetic Measure of Development. Although the results obtained by GDM were slightly better than those obtained by TMAL, the differences were not statistically significant. Within each method, seven combinations of weights were analysed. Once the locations had been selected, route lengths and order-picking times were calculated. As regards both route length and order-picking time, the best results for both methods were obtained for combination C5 (0.4; 0.4; 0.2). For both methods this combination generated significantly better results than most of the other combinations. This means that the decision-maker should attach particular importance to the distance from the I/O point and the degree of demand satisfaction. The best locations are those closest to the I/O point and with the highest degree of demand satisfaction. The number of other products picked in the neighbourhood of the location analysed is not as important as these two criteria. A comparison of the results obtained by the best method and combination of weights (GDM with C5) with the results obtained by the worst method and combination of weights (TMAL with C4) shows that both the picker's route length and the order-picking time can be shortened by about 11%. Of course, these results were obtained with the assumption that a chaotic storage system was used, hence they cannot be generalised for other storage systems, such as ABC or XYZ class-based storage systems.

Further research will include a comparison of multiple-criteria decision-making techniques for an ABC class-based storage system with within-isle and across-isle storing strategies. As the GDM method allows for using criteria measured on other than interval and ratio scales, other criteria, such as storage level in a high-storage warehouse or the presence (or absence) of complete packages at every location, will be added. Also, other methods of heuristics for the determination of the picker's route (*return*, *midpoint*, *largest gap*, or *combined*) will be analysed.

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MULTIPLE CRITERIA DECISION MAKING

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FUZZY TOPSIS METHOD FOR GROUP DECISION MAKING

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Abstract

Multiple Criteria Decision Making methods have become very popular in recent years and are frequently applied in many real-life situations. The increasing complexity of the decision problems analysed makes it less feasible to consider all the relevant aspects of the problems by a single decision maker. As a result, many real-life problems are discussed by a group of decision makers.

The aim of the paper is to present a new approach for ranking of alternatives with fuzzy data for group decision making using the TOPSIS method. In the proposed approach, all individual decision information of decision makers is taken into account in determining the ranking of alternatives and selecting the best one. The key stage of this method is the transformation of the decision matrices provided by the decision makers into matrices of alternatives. A matrix corresponding to an alternative is composed of its assessments with respect to all criteria, performed by all the decision makers. A numerical example illustrates the proposed approach.

Keywords: fuzzy numbers, TOPSIS, group decision making, aggregation fuzzy numbers.

1 Introduction

Multiple Criteria Decision Making (MCDM) methods have become very popular in recent years and are frequently applied in many real-life situations (for more information see, e.g., Behzadian et al., 2012; Abdullah and Adawiyah, 2014). One of the most popular and widely applied MCDM methods is the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) proposed by Hwang and Yoon (1981). The basic idea of this method is fairly straightforward. It uses two reference points: the so-called positive ideal solution (PIS) and negative ideal solution (NIS) as benchmarks. The chosen alternative is that

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which has both the shortest distance from the PIS and the longest distance from the NIS. The PIS is a solution that maximizes all the benefit criteria and minimizes all the cost criteria, whereas the NIS is a solution that maximizes all the cost criteria and minimizes all the benefit criteria.

The classical TOPSIS method is based on the information provided by the decision maker (DM) or expert as exact numerical values. However, in some real-life situations, the DM may not be able to precisely express the value of the ratings of alternatives with respect to criteria or else he/she uses linguistic expressions. In such situations, when evaluations are based on unquantifiable, incomplete, or unobtainable information, the DM may use other data formats, such as: interval numbers (Jahanshahloo et al., 2006a; Yue, 2011), fuzzy numbers (Chen, 2000; Jahanshaloo et al., 2006b), ordered fuzzy numbers (Roszkowska and Kacprzak, 2016; Kacprzak, 2019), hesitant fuzzy sets (Senvar et al., 2016), intuitionistic fuzzy sets (Boran, Genc et al., 2009) and other.

On the other hand, the increasing complexity of decision problems analysed makes it less feasible to consider all the relevant aspects of the problems by a single DM. Therefore, many real-life problems are considered by a group of DMs. In such situations, the individual decisions made by each DM (usually in the form of an individual decision matrix) are often aggregated to form a collective decision (also in the form of a collective decision matrix). This collective decision is the starting point for the ranking of the alternatives or the selection of the best one.

One of the most popular and often used methods of aggregation, in MCDM methods such as TOPSIS, is arithmetic mean (Chen, 2000; Wang and Chang, 2007; Roszkowska and Kacprzak, 2016). This type of aggregation of individual decisions is also used in practice, e.g., in certain sports, such as snowboard slopestyle or halfpipe. Each participant is evaluated by a group of referees (as DMs) and the average of the referees' scores is taken as the final result for each participant. On the other hand, due to this method of aggregation of individual information, some significant information of the individual decisions of DMs is not taken into consideration. As an example, consider a group of two decision makers who make assessments using the following point scale: $\{1, 2, 3, 4, 5\}$. Let us note that regardless whether their assessment of an alternative with respect to a criterion is in the form "1 and 5", "2 and 4" or "3 and 3", the aggregation results are the same and equal to "3". This means that such an averaged result does not reflect the discrepancies of the individual decisions (preferences of DMs) and that using such averaged information may lead to an incorrect final decision.

The aim of this paper is to present a new approach for ranking of alternatives with fuzzy data for group decision making using the TOPSIS method. In the proposed approach, all individual decision information of DMs is taken into account in determining the ranking of alternatives and selecting the best one. The key stage of this method is the transformation of the decision matrices provided by the decision makers into matrices of alternatives. A matrix corresponding to an alternative is composed of its assessments with respect to all criteria, performed by all the decision makers. Since all individual decision matrices are normalized beforehand with respect to the type of criterion, the positive ideal solution in this approach is a matrix composed of maximal assessments, and the negative ideal solution is a matrix composed of minimal assessments. The distances of alternatives from the PIS and the NIS, in contrast to the classic TOPSIS and to the method based on the aggregation of the individual decisions made by each DM, are the distances between matrices. Using the coefficient of relative closeness of each alternative to the positive ideal solution, a ranking of alternatives is created and the best one is indicated.

The rest of the paper is organized as follows. In Section 2 basic definitions and notations of fuzzy numbers are introduced. In Section 3 the TOPSIS method and its fuzzy extension are presented. The proposed approach and a numerical example are described in Section 4 and Section 5, respectively. Section 6 is devoted to the comparison of the proposed approach with other, similar approaches. Finally, concluding remarks are in Section 7.

2 Fuzzy numbers

In this section some definitions related to fuzzy sets and fuzzy numbers used in the paper are briefly outlined.

Definition 1. (Zadeh, 1965). Let X be a universe set. A fuzzy subset A in a universe of discourse X is characterized by a membership function $\mu_A(x)$ which associates with each element x in X a real number from the interval [0,1]. The function $\mu_A(x)$ is called the grade of membership of x in A.

Definition 2. (Dubois and Prade, 1980). The support of a fuzzy set A is the ordinary subset of X $\text{supp}A = \{x \in X: \mu_A(x) > 0\}$.

Definition 3. (Dubois and Prade, 1980). A fuzzy set A is normalized iff $\exists x \in X, \mu_A(x) = 1$.

Definition 4. (Dubois and Prade, 1980; Zimmermann, 2001). A fuzzy set A is convex iff $\forall\ x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]\ \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$.

We can now define the concept of a fuzzy number.

Definition 5. (Dubois and Prade, 1980; Zimmermann, 2001). A fuzzy number A is a convex, normalized fuzzy subset A of the real line $\mathbb{R}$ such that:

a) there exists exactly one $x_0 \in \mathbb{R}, \mu_A(x_0) = 1$ (x_0 is called the mean value of A),
b) $\mu_A(x)$ is piecewise continuous.

If fuzzy subset A of the real line $\mathbb{R}$ is convex and normalized, its membership function is piecewise continuous, and there exists more than one element $x_0 \in \mathbb{R}$, $\mu_A(x_0) = 1$ then A is called a flat fuzzy number (Dubois and Prade, 1980).

In many practical applications of fuzzy numbers, positive triangular and positive trapezoidal fuzzy numbers are used. Figure 1 shows the characteristic points of such numbers, which describe them uniquely. The positive triangular fuzzy number A is denoted by

$$A = (a_A, b_A, c_A), \tag{1}$$

where $0 \leq a_A \leq b_A \leq c_A$, and its membership function is of the form (Fig. 1a)

$$\mu_A(x) = \begin{cases} \frac{x-a_A}{b_A-a_A} & \text{for} \quad a_A \leq x \leq b_A \\ \frac{c_A-x}{c_A-b_A} & \text{for} \quad b_A \leq x \leq c_A \end{cases}, \tag{2}$$

while the positive trapezoidal fuzzy number A is denoted by

$$A = (a_A, b_A, c_A, d_A), \tag{3}$$

where $0 \leq a_A \leq b_A \leq c_A \leq d_A$, and its membership function is of the form (Fig. 1b)

$$\mu_A(x) = \begin{cases} \frac{x-a_A}{b_A-a_A} & \text{for} \quad a_A \leq x \leq b_A \\ 1 & \text{for} \quad b_A \leq x \leq c_A \\ \frac{d_A-x}{d_A-c_A} & \text{for} \quad c_A \leq x \leq d_A \end{cases}. \tag{4}$$

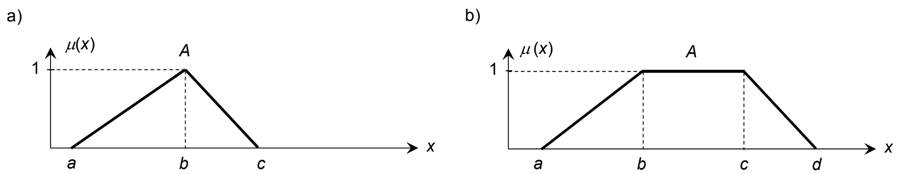

Figure 1: a) A triangular positive fuzzy number A; b) A trapezoidal positive fuzzy number A

If in a positive trapezoid fuzzy number $A = (a_A, b_A, c_A, d_A)$ we have $b_A = c_A$, then A becomes a positive triangular fuzzy number. Let $A = (a_A, b_A, c_A, d_A)$ and $B = (a_B, b_B, c_B, d_B)$ be two positive trapezoidal fuzzy numbers and let $r \in \mathbb{R}$.

Definition 6. The arithmetic operations (used later in the paper) are defined as follows

$$A + B = (a_A + a_B, b_A + b_B, c_A + c_B, d_A + d_B), \quad (5)$$

$$A \cdot B = (a_A \cdot a_B, b_A \cdot b_B, c_A \cdot c_B, d_A \cdot d_B), \quad (6)$$

$$r \cdot A = (r \cdot a_A, r \cdot b_A, r \cdot c_A, r \cdot d_A). \quad (7)$$

In some fuzzy MCDM methods, including fuzzy TOPSIS, it is necessary to measure the distance between fuzzy numbers, and to perform maximum and minimum operations on them.

Definition 7. The distance (d) calculated by the vertex method and the maximum (max) and minimum (min) operations are defined as

$$d(A, B) = \sqrt{\frac{1}{4}[(a_A - a_B)^2 + (b_A - b_B)^2 + (c_A - c_B)^2 + (d_A - d_B)^2]}, \quad (8)$$

$$\max(A, B) = (\max\{a_A, a_B\}, \max\{b_A, b_B\}, \max\{c_A, c_B\}, \max\{d_A, d_B\}), \quad (9)$$

$$\min(A, B) = (\min\{a_A, a_B\}, \min\{b_A, b_B\}, \min\{c_A, c_B\}, \min\{d_A, d_B\}). \quad (10)$$

3 The TOPSIS method

In this section the classical TOPSIS method and its fuzzy extension are presented. Let us assume that the decision maker has to choose one of m possible alternatives described by n criteria. The rating of alternative A_i $(i = 1,...,m)$ with respect to criterion C_j $(j = 1,...,n)$ is denoted by x_{ij}. The set of criteria is divided into two subsets: benefit criteria (greater value is better) denoted by B and cost criteria (lower value is better) denoted by C. Let $W = (w_1, w_2, \dots, w_n)$ be the vector of criteria weights.

The original TOPSIS method assumes that the rating x_{ij} of the alternatives with respect to the criteria, as well as the criteria weights w_j, are expressed precisely by real numbers. It consists of the following steps:

Step 1

Determination of the decision matrix X

$$X = (x_{ij}) \quad (11)$$

where $x_{ij} \in \mathbb{R}$.

Step 2

Calculation of the normalized decision matrix R using vector normalization

$$R = (r_{ij}) \quad (12)$$

where $r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{m} x_{kj}^2}}$.

Step 3

Calculation of the weighted normalized matrix V by multiplying the columns of the normalized decision matrix R by the associated weights $w_j \in \mathbb{R}$ satisfying $\sum_{j=1}^{n} w_j = 1$

$$V = (v_{ij}) \tag{13}$$

where $v_{ij} = r_{ij} \cdot w_j$.

Step 4

Determination of the positive ideal solution A^+

$$A^+ = (v_1^+, v_2^+, \dots, v_n^+) \tag{14}$$

where $v_j^+ = \{(\max_i v_{ij} \ \text{if}\, j \in B), (\min_i v_{ij} \ \text{if}\, j \in C)\}$

and the negative ideal solution A^-

$$A^- = (v_1^-, v_2^-, \dots, v_n^-) \tag{15}$$

where $v_j^- = \{(\min_i v_{ij} \ \text{if}\, j \in B), (\max_i v_{ij} \ \text{if}\, j \in C)\}$.

Step 5

Calculation of the Euclidean distances of each alternative A_i from the positive ideal solution A^+

$$d_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2} \tag{16}$$

and from the negative ideal solution A^-

$$d_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}. \tag{17}$$

Step 6

Calculation of the relative closeness of each alternative A_i to the positive ideal solution A^+

$$RC_i = \frac{d_i^-}{d_i^+ + d_i^-}. \tag{18}$$

Step 7

Ranking of the alternatives A_i according to their relative closeness to the ideal solutions A^+ (the larger the value of RC_i the better the alternative A_i). The best alternative is the one with the largest value of RC_i.

In real-life decision making problems it is usually difficult to express evaluations precisely using real numbers, due to a lack of knowledge and data or to subjective and imprecise expert judgments. In such situations, instead of exact numbers, fuzzy numbers can be used. The fuzzy TOPSIS method based on positive triangular fuzzy numbers proposed by Chen (2000) consists of the following steps:

Step 1

Define the fuzzy decision matrix X

$$X = (x_{ij}) \tag{19}$$

where $x_{ij} = (a_{x_{ij}}, b_{x_{ij}}, c_{x_{ij}})$ is a positive triangular fuzzy number.

Given a group of K decision makers, the rating of alternatives with respect to each criterion can be calculated as $x_{ij} = \frac{1}{K}\left(x_{ij}^1 + x_{ij}^2 + \cdots + x_{ij}^K\right)$, where x_{ij}^k $(k = 1,2,\dots,K)$ is the rating of alternative i with respect to criterion j provided by decision maker k.

Step 2

Construct the normalized fuzzy decision matrix R using linear normalization

$$R = \left(r_{ij}\right) \tag{20}$$

where

$$r_{ij} = \begin{cases} \left(\frac{a_{x_{ij}}}{\max_i c_{x_{ij}}}, \frac{b_{x_{ij}}}{\max_i c_{x_{ij}}}, \frac{c_{x_{ij}}}{\max_i c_{x_{ij}}}\right) & \text{if} \quad j \in B \\ \left(\frac{\min_i a_{x_{ij}}}{c_{x_{ij}}}, \frac{\min_i a_{x_{ij}}}{b_{x_{ij}}}, \frac{\min_i a_{x_{ij}}}{a_{x_{ij}}}\right) & \text{if} \quad j \in C \end{cases}. \tag{21}$$

Step 3

Construct the weighted normalized fuzzy matrix V by multiplying the columns of the normalized fuzzy decision matrix R by the associated weights $w_j \in \mathbb{R}$ satisfying $\sum_{j=1}^{n} w_j = 1$

$$V = \left(v_{ij}\right) \tag{22}$$

where $v_{ij} = r_{ij} \cdot w_j = (a_{r_{ij}}, b_{r_{ij}}, c_{r_{ij}}) \cdot w_j = (a_{r_{ij}} \cdot w_j, b_{r_{ij}} \cdot w_j, c_{r_{ij}} \cdot w_j)$.

Step 4

Determine the fuzzy positive ideal solution as follows

$$A^+ = (v_1^+, v_2^+, \dots, v_n^+) \tag{23}$$

where $v_j^+ = \max_i v_{ij}$ and the fuzzy negative ideal solution

$$A^- = (v_1^-, v_2^-, \dots, v_n^-) \tag{24}$$

where $v_j^- = \min_i v_{ij}$.

Step 5

Calculate the distances of each alternative A_i from the positive ideal solution A^+

$$d_i^+ = \sum_{j=1}^{n} dd\left(v_{ij}, v_j^+\right) \tag{25}$$

and from the negative ideal solution A^-

$$d_i^- = \sum_{j=1}^{n} dd\left(v_{ij}, v_j^-\right) \tag{26}$$

where the distance dd between two positive triangular fuzzy numbers $A = (a_A, b_A, c_A)$ and $B = (a_B, b_B, c_B)$ is equal to

$$dd(A,B) = \sqrt{\frac{1}{3}[(a_A - a_B)^2 + (b_A - b_B)^2 + (c_A - c_B)^2]}. \tag{27}$$

Step 6

Calculate the relative closeness of alternative A_i to the ideal solution A^+

$$RC_i = \frac{d_i^-}{d_i^+ + d_i^-} . \tag{28}$$

***Step** 7*
Rank the alternatives A_i and select the one with the largest value of RC_i.

4 The proposed approach

In this section the proposed approach is presented. Consider an MCDM problem for group decision making. Let $\{A_1, A_2, \dots, A_m\}$ $(m \geq 2)$ be a discrete set of m feasible alternatives, $\{C_1, C_2, \dots, C_n\}$ $(n \geq 2)$ be a finite set of criteria, $w = (w_1, w_2, \dots, w_n)$ be the vector of criteria weights, such that $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. Let $\{DM_1, DM_2, \dots, DM_K\}$ $(K \geq 2)$ be a group of decision makers.

In the process of group decision making, the DMs are asked to assess alternatives with respect to criteria. In many real-life situations, when the DMs' knowledge of the analysed subject is incomplete, or the available data are inaccurate, or when the ratings are expressed linguistically, fuzzy numbers can be used. In that case, each DM provides a decision matrix of the form

$$X^k = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{bmatrix} \end{array} \quad (29)$$

where $x_{ij}^k = \left(a_{x_{ij}^k}, b_{x_{ij}^k}, c_{x_{ij}^k}, d_{x_{ij}^k}\right)$ is a positive trapezoidal fuzzy number representing the rating of alternative A_i $(i = 1,2, \dots, m)$ with respect to criterion C_j $(j = 1,2, \dots, n)$ provided by decision maker DM_k $(k = 1,2, \dots, K)$.

The decision matrix X^k (29) can be constructed in various ways, for instance, using crisp evaluations x_{ij}^{k*}. The transformation is carried out by extending the support and kernel of the crisp evaluation to the estimated or assumed imprecision bound of evaluation. For empirical data from the range $[L, U]$, the crisp value $x_{ij}^{k*} \in [L, U]$ can be transformed into the trapezoidal fuzzy number $\left(a_{x_{ij}}, b_{x_{ij}}, c_{x_{ij}}, d_{x_{ij}}\right)$, where $a_{x_{ij}} = \max\{L, x_{ij}^{k*} - 2\sigma\}$, $b_{x_{ij}} = \max\{L, x_{ij}^{k*} - \sigma\}$, $c_{x_{ij}} = \min\{U, x_{ij}^{k*} + \sigma\}$, $d_{x_{ij}} = \min\{U, x_{ij}^{k*} + 2\sigma\}$ where σ is the assumed or estimated imprecision bound of empirical data (for more details and a numerical example, see Rudnik and Kacprzak, 2017). Another very popular way of constructing the fuzzy decision matrix X^k (29) is to use linguistic variables to evaluate the ratings of alternatives with respect to various criteria (for more details, see e.g. Bonissone and Decker,1986; Shemshadi et al., 2011; Kacprzak, 2017; Hatami-Marbini and Kangi, 2017).

Next, in order to ensure comparability of criteria, the fuzzy decision matrix X^k is normalized. The normalized fuzzy decision matrix

$$Y^k = \begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{matrix} C_1 & C_2 & \cdots & C_n \\ \left[\begin{matrix} y_{11}^k \\ y_{21}^k \\ \vdots \\ y_{m1}^k \end{matrix}\right. & \begin{matrix} y_{12}^k \\ y_{22}^k \\ \vdots \\ y_{m2}^k \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \ddots \\ \cdots \end{matrix} & \left.\begin{matrix} y_{1n}^k \\ y_{2n}^k \\ \vdots \\ y_{mn}^k \end{matrix}\right] \end{matrix} \tag{30}$$

is calculated using the following formulas

$$y_{ij}^k = \begin{cases} \left(\frac{a_{x_{ij}^k}}{\max_i d_{x_{ij}^k}}, \frac{b_{x_{ij}^k}}{\max_i d_{x_{ij}^k}}, \frac{c_{x_{ij}^k}}{\max_i d_{x_{ij}^k}}, \frac{d_{x_{ij}^k}}{\max_i d_{x_{ij}^k}}\right) & \text{if} \quad j \in B \\ \left(\frac{\min_i a_{x_{ij}^k}}{d_{x_{ij}^k}}, \frac{\min_i a_{x_{ij}^k}}{c_{x_{ij}^k}}, \frac{\min_i a_{x_{ij}^k}}{b_{x_{ij}^k}}, \frac{\min_i a_{x_{ij}^k}}{a_{x_{ij}^k}}\right) & \text{if} \quad j \in C \end{cases}. \tag{31}$$

Using the vector of criteria weights $w = (w_1, w_2, \dots, w_n)$, the weighted normalized fuzzy decision matrix is calculated for each DM

$$V^k = \begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{matrix} C_1 & C_2 & \cdots & C_n \\ \left[\begin{matrix} v_{11}^k \\ v_{21}^k \\ \vdots \\ v_{m1}^k \end{matrix}\right. & \begin{matrix} v_{12}^k \\ v_{22}^k \\ \vdots \\ v_{m2}^k \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \ddots \\ \cdots \end{matrix} & \left.\begin{matrix} v_{1n}^k \\ v_{2n}^k \\ \vdots \\ v_{mn}^k \end{matrix}\right] \end{matrix} \tag{32}$$

where $v_{ij}^k = w_j y_{ij}^k = \left(w_j a_{y_{ij}^k}, w_j b_{y_{ij}^k}, w_j c_{y_{ij}^k}, w_j d_{y_{ij}^k}\right)$. The matrices V^k form the basis for the construction of weighted normalized fuzzy decision matrices for each alternative A_i

$$W^i = \begin{matrix} \\ DM_1 \\ DM_2 \\ \vdots \\ DM_K \end{matrix} \begin{matrix} C_1 & C_2 & \cdots & C_n \\ \left[\begin{matrix} v_{i1}^1 \\ v_{i1}^2 \\ \vdots \\ v_{i1}^K \end{matrix}\right. & \begin{matrix} v_{i2}^1 \\ v_{i2}^2 \\ \vdots \\ v_{i2}^K \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \ddots \\ \cdots \end{matrix} & \left.\begin{matrix} v_{in}^1 \\ v_{in}^2 \\ \vdots \\ v_{in}^K \end{matrix}\right] \end{matrix}. \tag{33}$$

Matrices W^i constitute the basis for the construction of the ranking of the alternatives and the selection of the best one using the fuzzy TOPSIS method. The positive ideal solution A^+ is determined as follows

$$A^+ = \begin{matrix} \\ DM_1 \\ DM_2 \\ \vdots \\ DM_K \end{matrix} \begin{matrix} C_1 & C_2 & \cdots & C_n \\ \left[\begin{matrix} v_1^{1+} \\ v_1^{2+} \\ \vdots \\ v_1^{K+} \end{matrix}\right. & \begin{matrix} v_2^{1+} \\ v_2^{2+} \\ \vdots \\ v_2^{K+} \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \ddots \\ \cdots \end{matrix} & \left.\begin{matrix} v_k^{1+} \\ v_n^{2+} \\ \vdots \\ v_n^{K+} \end{matrix}\right] \end{matrix} \tag{34}$$

where $v_j^{k+} = \max_i v_{ij}^k$, and the negative ideal solution A^- is determined as follows

$$A^{-} = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ DM_1 \\ DM_2 \\ \vdots \\ DM_K \end{matrix} \begin{bmatrix} v_1^{1-} & v_2^{1-} & \cdots & v_k^{1-} \\ v_1^{2-} & v_2^{2-} & \cdots & v_n^{2-} \\ \vdots & \vdots & \ddots & \vdots \\ v_1^{K-} & v_2^{K-} & \cdots & v_n^{K-} \end{bmatrix} \quad (35)$$

where $v_j^{k-} = \min_i v_{ij}^k$. Next, the distances of each alternative A_i represented by matrix W^i from PIS

$$d_i^+ = \sum_{k=1}^{K} \sum_{j=1}^{n} d\left(v_{ij}^k, v_j^{k+}\right) \quad (36)$$

and from NIS

$$d_i^- = \sum_{k=1}^{K} \sum_{j=1}^{n} d\left(v_{ij}^k, v_j^{k-}\right) \quad (37)$$

are calculated. Using these distances, the relative closeness coefficients RC_i to PIS for each alternative A_i is calculated

$$RC_i = \frac{d_i^-}{d_i^- + d_i^+}. \quad (38)$$

According to the descending values of RC_i, all alternatives A_i are rank ordered and the best one is selected.

Remark 1

Note that if in the proposed approach we use triangular fuzzy numbers and the distance measure between two triangular fuzzy numbers (27), and if there is only one DM, i.e. $K = 1$, then the proposed approach is equivalent to the fuzzy TOPSIS method proposed by Chen (2000).

Remark 2

Note that if we take into account the form of the matrices W^i, the positive ideal solution A^+ and the negative ideal solution A^-, the proposed approach can be regarded as a simultaneous application of the fuzzy TOPSIS for each of the DMs represented by the corresponding rows of these matrices.

5 Numerical example

In this section our new approach is presented on a numerical example. Consider a fuzzy MCDM problem for group decision making, consisting of the set of three feasible alternatives $\{A_1, A_2, A_3\}$ rated with respect to the set of three benefit criteria $\{C_1, C_2, C_3\}$ by a group of three decision makers $\{DM_1, DM_2, DM_3\}$, with the vector of criteria weights $w = (0.4, 0.2, 0.4)$. The DMs have used trapezoidal fuzzy numbers to rate the alternatives with respect to the criteria and their evaluations are shown in Table 1. Using formula (31), the decision matrices are normalized and using the vector w of criteria weights, the weighted normalized fuzzy decision matrix is calculated for each DM (see Table 2). Next, these matrices are transformed into the weighted normalized fuzzy decision matrices for each alternative (see Table 3). Using these matrices, the positive ideal solution A^+ and the negative ideal solution A^- are determined (see Table 4). Finally, the distances of each alternative from the positive ideal solution d_i^+ and from the negative ideal solution d_i^+ are calculated (see Table 5). This allows to calculate the relative closeness coefficient RC_i and the rank order R of the alternatives (where $\prec$ means "inferior to"):

$$A_3 \prec A_2 \prec A_1.$$

Hence, alternative A_1 should be selected.

Table 1: Individual decision matrices provided by the decision makers

		C_1	C_2	C_3
	A_1	(72,78,84,90)	(72,77,82,87)	(85,87,89,91)
DM_1	A_2	(77,78,79,80)	(69,77,85,93)	(83,85,87,89)
	A_3	(80,85,90,95)	(59,68,77,86)	(80,82,84,86)
	A_1	(77,79,81,83)	(68,73,78,83)	(82,85,88,91)
DM_2	A_2	(93,95,97,99)	(76,79,82,85)	(65,72,79,86)
	A_3	(79,81,83,85)	(72,79,86,93)	(81,84,87,90)
	A_1	(85,86,87,88)	(76,79,82,85)	(80,86,92,98)
DM_3	A_2	(79,82,85,88)	(81,84,87,90)	(81,85,89,93)
	A_3	(77,80,83,86)	(84,86,88,90)	(81,84,87,90)

Table 2: Weighted normalised decision matrices

		C_1	C_2	C_3
	A_1	(0.3032,0.3284,0.3537,0.3789)	(0.1548,0.1656,0.1763,0.1871)	(0.3736,0.3824,0.3912,0.4000)
DM_1	A_2	(0.3242,0.3284,0.3326,0.3368)	(0.1484,0.1656,0.1828,0.2000)	(0.3648,0.3736,0.3824,0.3912)
	A_3	(0.3368,0.3579,0.3789,0.4000)	(0.1269,0.1462,0.1656,0.1849)	(0.3516,0.3604,0.3692,0.3780)
	A_1	(0.3111,0.3192,0.3273,0.3354)	(0.1462,0.1570,0.1677,0.1785)	(0.3604,0.3736,0.3868,0.4000)
DM_2	A_2	(0.3758,0.3838,0.3919,0.4000)	(0.1634,0.1699,0.1763,0.1828)	(0.2857,0.3165,0.3473,0.3780)
	A_3	(0.3192,0.3273,0.3354,0.3434)	(0.1548,0.1699,0.1849,0.2000)	(0.3560,0.3692,0.3824,0.3956)
	A_1	(0.3864,0.3909,0.3955,0.4000)	(0.1689,0.1756,0.1822,0.1889)	(0.3265,0.3510,0.3755,0.4000)
DM_3	A_2	(0.3591,0.3727,0.3864,0.4000)	(0.1800,0.1867,0.1933,0.2000)	(0.3306,0.3469,0.3633,0.3796)
	A_3	(0.3500,0.3636,0.3773,0.3909)	(0.1867,0.1911,0.1956,0.2000)	(0.3306,0.3429,0.3551,0.3673)

Table 3: Weighted normalised decision matrices for the alternatives

		C_1	C_2	C_3
	DM_1	(0.3032,0.3284,0.3537,0.3789)	(0.1548,0.1656,0.1763,0.1871)	(0.3736,0.3824,0.3912,0.4000)
A_1	DM_2	(0.3111,0.3192,0.3273,0.3354)	(0.1462,0.1570,0.1677,0.1785)	(0.3604,0.3736,0.3868,0.4000)
	DM_3	(0.3864,0.3909,0.3955,0.4000)	(0.1689,0.1756,0.1822,0.1889)	(0.3265,0.3510,0.3755,0.4000)
	DM_1	(0.3242,0.3284,0.3326,0.3368)	(0.1484,0.1656,0.1828,0.2000)	(0.3648,0.3736,0.3824,0.3912)
A_2	DM_2	(0.3758,0.3838,0.3919,0.4000)	(0.1634,0.1699,0.1763,0.1828)	(0.2857,0.3165,0.3473,0.3780)
	DM_3	(0.3591,0.3727,0.3864,0.4000)	(0.1800,0.1867,0.1933,0.2000)	(0.3306,0.3469,0.3633,0.3796)
	DM_1	(0.3368,0.3579,0.3789,0.4000)	(0.1269,0.1462,0.1656,0.1849)	(0.3516,0.3604,0.3692,0.3780)
A_3	DM_2	(0.3192,0.3273,0.3354,0.3434)	(0.1548,0.1699,0.1849,0.2000)	(0.3560,0.3692,0.3824,0.3956)
	DM_3	(0.3500,0.3636,0.3773,0.3909)	(0.1867,0.1911,0.1956,0.2000)	(0.3306,0.3429,0.3551,0.3673)

Table 4: Positive ideal solution and negative ideal solution

		C_1	C_2	C_3
A^+	DM_1	(0.3368,0.3579,0.3789,0.4000)	(0.1548,0.1656,0.1828,0.2000)	(0.3736,0.3824,0.3912,0.4000)
	DM_2	(0.3758,0.3838,0.3919,0.4000)	(0.1634,0.1699,0.1849,0.2000)	(0.3604,0.3736,0.3868,0.4000)
	DM_3	(0.3864,0.3909,0.3955,0.4000)	(0.1867,0.1911,0.1956,0.2000)	(0.3306,0.3510,0.3755,0.4000)
A^-	DM_1	(0.3032,0.3284,0.3326,0.3368)	(0.1269,0.1462,0.1656,0.1849)	(0.3516,0.3604,0.3692,0.3780)
	DM_2	(0.3111,0.3192,0.3273,0.3354)	(0.1462,0.1570,0.1677,0.1785)	(0.2857,0.3165,0.3473,0.3780)
	DM_3	(0.3500,0.3636,0.3773,0.3909)	(0.1689,0.1756,0.1822,0.1889)	(0.3265,0.3429,0.3551,0.3673)

Table 5: The distances of each alternative from the positive ideal solution d_i^+, the negative ideal solution d_i^-, the relative closeness coefficients RC_i and the ranking order R of alternatives

	d_i^+	d_i^-	RC_i	R
A_1	0.1338	0.1602	0.5448	**1**
A_2	0.1494	0.1467	0.4954	**2**
A_3	0.1523	0.1338	0.4677	**3**

6 Comparison of the proposed approach with other and similar approaches

In this section, the proposed approach is compared with other similar methods. Figures 2, 3a and 3b show the hierarchical structure of the classical TOPSIS (Hwang and Yoon, 1981), the TOPSIS for group decision making with aggregation of individual decision matrices, and the proposed approach, respectively.

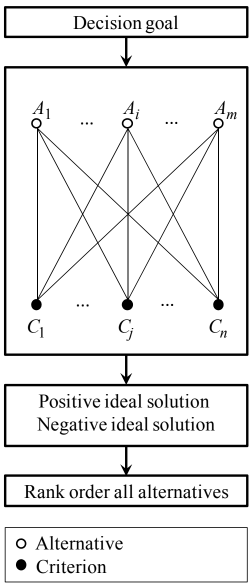

Figure 2: The hierarchical structure of the classical TOPSIS

Here the results (rankings of alternatives) using the proposed method ($\boldsymbol{PA}$) are compared with the method which aggregates the individual weighted normalized decision matrices into an aggregated collective matrix (which in TOPSIS is the starting point for the ranking of alternatives), based on data from the example in Section 5 (Table 1). For the calculation of the aggregated collective matrix $X = \left(x_{ij}\right)$ the following aggregation methods, known from the literature, are used:

- $\boldsymbol{AGG1}$ – arithmetic mean (Chen, 2000; Wang and Chang, 2007; Roszkowska and Kacprzak, 2016), defined by

$$x_{ij} = \frac{1}{K}\sum_{k=1}^{K} x_{ij}^{k} = \left(\frac{1}{K}\sum_{k=1}^{K} a_{x_{ij}^{k}}, \frac{1}{K}\sum_{k=1}^{K} b_{x_{ij}^{k}}, \frac{1}{K}\sum_{k=1}^{K} c_{x_{ij}^{k}}, \frac{1}{K}\sum_{k=1}^{K} d_{x_{ij}^{k}}\right),$$

- $\boldsymbol{AGG2}$ – geometric mean (Shih et al., 2007; Ye and Li, 2009), defined by

$$x_{ij} = \left(\prod_{k=1}^{K} x_{ij}^{k}\right)^{\frac{1}{K}} =$$
$$\left(\left(\prod_{k=1}^{K} a_{x_{ij}^{k}}\right)^{\frac{1}{K}}, \left(\prod_{k=1}^{K} b_{x_{ij}^{k}}\right)^{\frac{1}{K}}, \left(\prod_{k=1}^{K} c_{x_{ij}^{k}}\right)^{\frac{1}{K}}, \left(\prod_{k=1}^{K} d_{x_{ij}^{k}}\right)^{\frac{1}{K}}\right),$$

- $\boldsymbol{AGG3}$ – modified arithmetic mean (Shemshadi et al., 2011; Nadaban et al., 2016), defined by

$$x_{ij} = \left(\min_{k} a_{x_{ij}^{k}}, \frac{1}{K}\sum_{k=1}^{K} b_{x_{ij}^{k}}, \frac{1}{K}\sum_{k=1}^{K} c_{x_{ij}^{k}}, \max_{\mathrm{k}} d_{x_{ij}^{k}}\right),$$

- $\boldsymbol{AGG4}$ – modified geometric mean (Ding, 2011; Chang et al., 2009; Hatami--Marbini and Kangi, 2017), defined by

$$x_{ij} = \left(\min_{k} a_{x_{ij}^{k}}, \left(\prod_{k=1}^{K} b_{x_{ij}^{k}}\right)^{\frac{1}{K}}, \left(\prod_{k=1}^{K} c_{x_{ij}^{k}}\right)^{\frac{1}{K}}, \max_{k} d_{x_{ij}^{k}}\right).$$

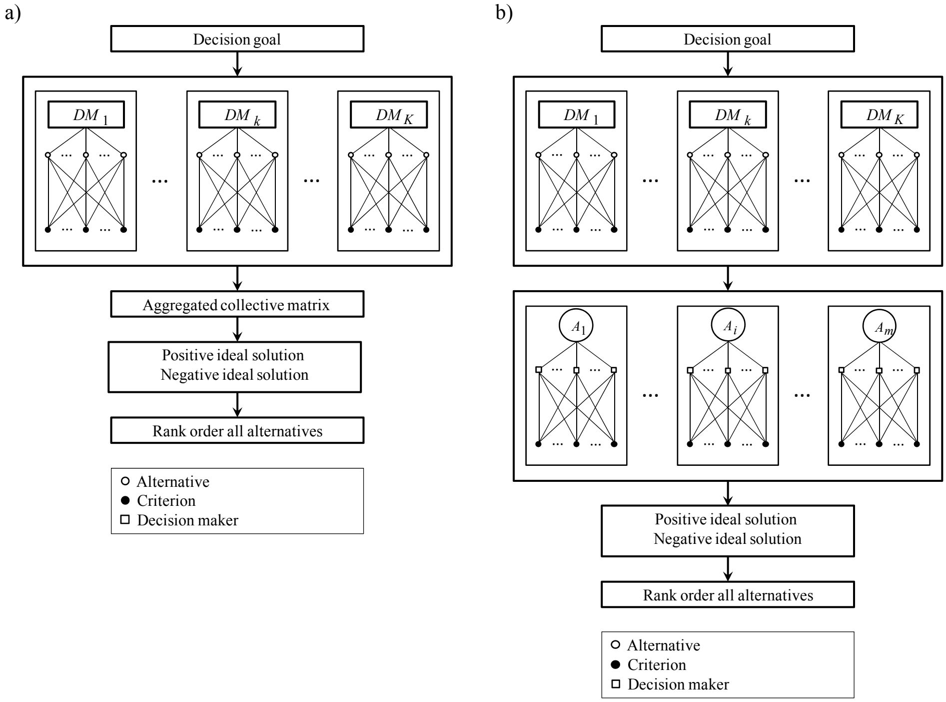

Figure 3: a) The hierarchical structure of the TOPSIS method for group decision making with aggregation of individual decision matrices, b) The hierarchical structure of the proposed TOPSIS method for group decision making

Table 6 shows the distance of each alternative A_i from the positive ideal solution d_i^+ and the negative ideal solution d_i^-, as well as the relative closeness coefficients RC_i and rank order R of the alternatives using the proposed method and different aggregation methods. The last column, denoted by J, consists of the normalized (summing up to 1) values of the relative closeness coefficients of each alternative to the ideal solution, which allows to highlight the differences between the final scores of the alternatives. Next, Table 7 and Fig. 4 show the ranking of the alternatives. Let us note that the aggregation methods using arithmetic mean and geometric mean give the same rank order of the alternatives: $A_3 \prec A_1 \prec A_2$, but different from that of the proposed approach. These methods swap the order of alternatives A_1 and A_2. This means that the final ranking order of the alternatives and the choice of the best one depend on the method used. Let us also note that the modified arithmetic mean and the modified geometric mean result in the same rank order of the alternatives $A_2 \prec A_3 \prec A_1$, which is also different from the proposed approach and from the aggregation methods using arithmetic mean and geometric mean. In these cases, alternative A_1 is the best; the results are the same as those obtained using the proposed method. Taking into account column J in Table 6 and Figure 4b, we can notice that the aggregation methods using arithmetic mean and geometric mean result in a fairly high score of alternative A_2 and a fairly low final score of alternative A_3 in comparison with the other methods analysed (which result in less diverse final scores).

Table 6: The results obtained using the proposed method and different aggregation methods

	ALT.	d_i^+	d_i^-	RC_i	R	J
PA	A_1	0.1338	0.1602	0.5448	1	0.3613
	A_2	0.1494	0.1467	0.4954	2	0.3285
	A_3	0.1523	0.1338	0.4677	3	0.3102
$AGG1$	A_1	0.0253	0.0226	0.4721	2	0.3280
	A_2	0.0220	0.0267	0.5478	1	0.3806
	A_3	0.0318	0.0230	0.4194	3	0.2914
$AGG2$	A_1	0.0244	0.0242	0.4980	2	0.3405
	A_2	0.0230	0.0261	0.5322	1	0.3639
	A_3	0.0315	0.0240	0.4322	3	0.2955
$AGG3$	A_1	0.0319	0.0473	0.5971	1	0.3484
	A_2	0.0354	0.0431	0.5492	3	0.3204
	A_3	0.0405	0.0532	0.5677	2	0.3312
$AGG4$	A_1	0.0316	0.0475	0.6004	1	0.3498
	A_2	0.0356	0.0428	0.5460	3	0.3181
	A_3	0.0401	0.0532	0.5699	2	0.3320

Table 7: The rankings of alternatives based on the relative closeness coefficients

	RANKING - RC_i
PA	$A_3 \prec A_2 \prec A_1$
$AGG1$	$A_3 \prec A_1 \prec A_2$
$AGG2$	$A_3 \prec A_1 \prec A_2$
$AGG3$	$A_2 \prec A_3 \prec A_1$
$AGG4$	$A_2 \prec A_3 \prec A_1$

a) b)

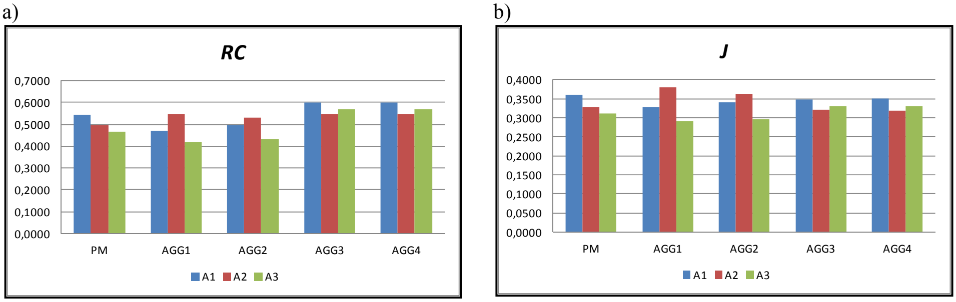

Figure 4: The rankings of alternatives based on: a) the relative closeness coefficients (RC_i), b) the normalized relative closeness coefficients (J)

7 Conclusions

In this paper an extended TOPSIS method based on fuzzy numbers for group decision making problems has been presented. Most papers in the literature aggregate the individual decision matrices provided by the DMs into a collective decision matrix which is the starting point for the ranking of alternatives or the selection of the best one, using arithmetic mean, geometric mean or their modifications. Such an averaged result does not reflect the discrepancies between the individual assessments or the preferences of the DMs. By contrast, in the proposed approach, all individual decision data of the DMs are taken into account in determining the ranking of alternatives and the selection of the best one.

The numerical example has shown that the proposed approach, as compared with other methods of aggregation of individual decision matrices of each DMs, can give a different final result, both as regards the ranking of alternatives and the selection of the best one.

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THREE WELFARE ORDERINGS THAT ARE FULLY COMPARABLE REVISITED

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Abstract

We are concerned with welfare orderings on the set of evaluation vectors. In our framework the number of agents, criteria or states of nature is fixed and an evaluation vector assigns a real valued evaluation to each criteria, agent or state of nature. Hence the space of evaluation vectors is a finite dimensional Euclidean space. In such a context we provide axiomatic characterizations of the utilitarian, maximin and leximin welfare orderings. The axiomatic characterization of the utilitarian welfare ordering is based on a quasi-linearity property. The axiomatic characterizations of the maximin and leximin welfare orderings are obtained by suitably modifying the axioms used by Barbera and Jackson (1988).

Keywords: social Welfare Orderings, Maximin, Leximin.

1 Introduction

In this paper we are concerned with axiomatic characterizations of orderings (reflexive, complete and transitive binary relations) defined on the set of finite dimensional evaluation vectors. We refer to these orderings as welfare orderings. The economic interpretation of an evaluation vector depends on the context. In the case that the context is the traditional one discussed in Amartya Sen's extension of Arrowian social welfare function (which Sen refers to as social welfare functional), then an evaluation vector is the vector of utilities obtained (or evaluations assigned) by each individual in a society to a particular social state. In the case that the context is the one about rational decision making by a single individual, there are two possible sub-cases each with its own interpretation and terminology. One is the scenario of multi-criteria or multi-attribute decision

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making. In this case each coordinate of an evaluation vector is the evaluation along a particular criterion and the evaluation vector itself is the ensemble of criterion-wise evaluations for the entire list of criteria one is concerned with. The second scenario is the one concerning decision making under complete uncertainty (more popularly known as ambiguity these days) where an agent faces the possibility of confronting in a future period exactly one of a finite set of uncertain states of nature. In this case each coordinate of an evaluation vector is the utility that one attains if a particular state of nature is realised and the evaluation vector itself is the ensemble of state dependent utilities for the entire list of states of nature exactly one of which is going to be realized at a future date. In this final scenario that we consider, the decision maker has no prior probabilities over the set of future states of nature. In each case that we have discussed so far, the problem is to obtain a welfare ordering over the set of evaluation vectors.

We consider three different welfare orderings which are very well known in welfare economics and share a common characteristic in that all three of them satisfy full comparability. Full comparability says if the evaluation vector $\mathbf{x}$ is at least as good or favourable as evaluation vector $\mathbf{y}$, then the evaluation vector $\mathbf{x}'$ should also be at least as good or favourable as the evaluation vector $\mathbf{y}'$, where the evaluation vectors $\mathbf{x}'$ and $\mathbf{y}'$ are obtained from $\mathbf{x}$ and $\mathbf{y}$ by multiplying each and every coordinate of $\mathbf{x}$ and $\mathbf{y}$ by the same positive real number (i.e. making the same change of scale along all directions) and then shifting the origin by the same amount in all directions.

The first welfare ordering that we consider is the utilitarian welfare ordering. Using well known results for numerical representation of quasi-linear preferences of a consumer as discussed in microeconomic consumer choice theory and with minimal investment in new technology on our part we are able to arrive at a completely new axiomatic characterization of the utilitarian welfare ordering. The result we establish says that a welfare ordering satisfies strict domination, continuity and quasi-linearity in every component if and only if it is a utilitarian welfare ordering. Quasi-linearity in a component means that the relation between two evaluation vectors is preserved if the evaluation at that component (or coordinate) is increased by the same real number for the two evaluation vectors. Quasi-linearity in every component means that that this property holds for every component.

The next two welfare orderings we consider are the maximin welfare ordering and the leximin welfare ordering. Our analysis of these two welfare orderings parallels the discussion of these two welfare orderings that is reported in Barbera and Jackson (1988). Unlike Barbera and Jackson (1988), in our framework the number of components (agents/criteria/states of nature) is fixed. In such a context we provide axiomatic characterizations of the maximin and leximin welfare orderings by suitably modifying the axioms used by Barbera and Jackson (1988). In the context of social welfare functionals and hence social welfare orderings, the maximin welfare ordering is best described as the “dictatorship of the least well off” and is therefore incompatible with any concept of negative liberty. On the other hand the leximin welfare ordering ranks one evaluation vector over the other, if and only if at the least rank where the two evaluation vectors disagree, the first vector has a higher evaluation than the second. Four of the seven axioms that we use are exactly those used by Barbera and Jackson (1988). Our proof of theorem 1 coincides almost word for word with the proof of theorem 1 in their paper. However, since our framework is different our results are different from their results and proofs of results merit mentioning, however close they may be to the corresponding proofs in the earlier work.

Of all the axioms we use in the characterization of maximin and leximin welfare orderings, only two are really unfamiliar to those who are acquainted with the literature on welfare orderings and therefore require some motivation. These two axioms are convexity with respect to duplicated evaluations and improvement impatience. Convexity is best explained in a two agent social welfare framework. In a two agent social welfare framework, convexity says that if an evaluation vector is preferred to a given perfectly egalitarian evaluation vector, then a third evaluation vector that is obtained from the first by replacing the evaluation of the “better off” agent by the average evaluation of the first vector is also preferred to the perfectly egalitarian evaluation vector. Hence, reasons for preferring a non-egalitarian evaluation vector to a perfectly egalitarian one are required to be quite compelling. The motivation for improvement impatience is much simpler. If there are two evaluation vectors sharing a common minimum evaluation, and there are just two different evaluation values in each evaluation vector, then the one which has fewer components getting the minimum evaluation is the preferred evaluation vector. In the context of social welfare functionals, this clearly points towards a social welfare ordering with a favourable bias towards utility distributions with fewer “least well-off” individuals.

For the broad framework and general definitions of utilitarian, maximin and leximin welfare orderings as defined in our paper one may refer to d’Aspremont (1985). We however try to adhere to the equivalent definitions of maximin and leximin that is available in Barbera and Jackson (1988). Since this paper relates to work done thirty years ago, a more recent survey of the literature such as the one by Bossert and Weymark (2004), should convince the reader that our results are original and no duplication of past effort occurs in our work.

2 The Model

Let N = {1,2,...,L} for some positive integer L, denote the set of individuals/ criteria/states of nature. An **evaluation vector** is an element of $\mathbb{R}^N$. A binary relation R on $\mathbb{R}^N$ is a subset of $\mathbb{R}^N \times \mathbb{R}^N$. If $(x,y) \in R$, then we write it as xRy. An ordering on $\mathbb{R}^N$ is a complete, reflexive and transitive binary relation on $\mathbb{R}^N$. If R is a binary relation on $\mathbb{R}^N$ then let P denote the asymmetric part and I denote the symmetric part of R. Henceforth we shall refer to binary relations on $\mathbb{R}^N$ as binary relations. Given a binary relation R and $X \subset \mathbb{R}^N \times \mathbb{R}^N$, let R|X denote $R \cap X$. R|X is called the **restriction of** R to X. We will refer to orderings on $\mathbb{R}^N$ as **welfare orderings.**

It may appear that the concept of a welfare ordering is very restrictive since we require a welfare ordering to be an ordering on $\mathbb{R}^N$. In welfare economics, we are often confronted with orderings on $\mathbb{R}^N_+$ as for instance the Nash (1950) welfare ordering. However such orderings can be harmlessly extended to all of $\mathbb{R}^N$ as the following definition reveals.

The **Nash welfare ordering** R_{Na} is defined as follows: let u be the real valued function defined on $\mathbb{R}^N$ such that for all $x \in \mathbb{R}^N_+$, $u_{Na}(x) = \prod_{i=1}^{L} x_i$ and for all $x \in \mathbb{R}^N \setminus \mathbb{R}^N_+$, $u_{Na}(x) = 0$. Then for all $x,y \in \mathbb{R}^N$, $xR_{Na}y$ if and only if $u_{Na}(x) \geq u_{Na}(y)$.

We will not dwell further on the Nash welfare ordering.

Given $x \in \mathbb{R}^N$ and $i \in N$, let x_{-i} denote the vector in $\mathbb{R}^{N \setminus \{i\}}$ such that for all $j \in N \setminus \{i\}$, the j[th] coordinate of x_{-i} is equal to the j[th] coordinate of x, i.e. x_j. The vector x can also be written as (x_i, x_{-i}).

Given, $x,y \in \mathbb{R}^N$, (a) $x \geq y$ denotes $x_i \geq y_i$ for all $i \in N$; (b) $x \leq y$ denotes $x_i \leq y_i$ for all $i \in N$; (c) $x > y$ denotes $x \geq y$ and $x \neq y$; (c) $x < y$ denotes $x \leq y$ and $x \neq y$; (d) $x >> y$ denotes $x_i > y_i$ for all $i \in N$.

Notation: Let e denote the vector in $\mathbb{R}^N_+$ all whose coordinates are equal to 1 and for $k \in \{1,\dots,N\}$, let $e^{(k)}$ denote the k[th] unit coordinate vector, i.e. the vector whose k[th] coordinate is equal to 1 and all other coordinates are equal to zero. Then given any $x \in \mathbb{R}^N$, $x = \sum_{k=1}^{L} x_k e^{(k)}$. Further, if $k \in \{1,\dots,N\}$, then x_{-k} is the vector in $\mathbb{R}^{\{1,\dots,N\} \setminus \{k\}}$ whose j[th] coordinate is x_j for $j \neq k$. We may represent x as (x_k, x_{-k}).

For $a \in \mathbb{R}$ and $x \in \mathbb{R}^N$ let $J(a,x) = \{i \in N | x_i \leq a\}$ and let #J(a,x) denote the cardinality of J(a,x).

The **utilitarian welfare ordering** R_U is defined as follows: there exists positive real numbers $\alpha_1, \alpha_2, \dots, \alpha_L$ such that for all $x,y \in \mathbb{R}^N$, xRy if and only if $\sum_{k=1}^{L} \alpha_k x_k \geq \sum_{k=1}^{L} \alpha_k y_k$.

The **maximin welfare ordering** R_{Mm} is defined as follows: $\forall x,y \in \mathbb{R}^N$, $xP_{Mm}y$ if and only if $\exists a \in \mathbb{R}$ such that $J(a,x) = \phi$ and $J(a,y) \neq \phi$.

The **leximin welfare ordering** R_{Lm} is defined as follows: $\forall x,y\in\mathbb{R}^N$, $xP_{Lm}y$ if and only if $\exists a\in\mathbb{R}$ such that $\#J(a,x) < \#J(a,y)$ and $\#J(b,x) = \#J(b,y)$ for all $b < a$.

We shall be concerned with the following axioms on welfare orderings.

A welfare ordering R is said to satisfy:

(1) **full-comparability** if for all $x^1, x^2, y^1, y^2\in\mathbb{R}^N$ satisfying $x^2_k = ax^1_k + b$, $y^2_k = ay^1_k + b$ for all $k\in N$, where a is a strictly positive real number and b is any real number, it is the case that x^1Ry^1 implies x^2Ry^2.

All the orderings discussed in this paper satisfy full-comparability.

A welfare ordering R is said to satisfy:

(2) **domination** if for all $x,y\in\mathbb{R}^N$, $x \geq y$ implies xRy and $x >> y$ implies xPy.

(3) **strict domination in the k^th component (or component k)** if for all $x,y\in\mathbb{R}^N$, $[x_j = y_j$ for all $j \neq k$ and $x_k > y_k]$ implies [x P y];

(4) **strict domination** if for all $x,y\in\mathbb{R}^N$, $x > y$ implies xPy.

(5) **continuity** if for all sequences $<x^n|n\in\mathbb{N}>$ and $<y^n|n\in\mathbb{N}>$ in $\mathbb{R}^N$ with $\lim_{n\to\infty} x^n = x\in\mathbb{R}^N$ and $\lim_{n\to\infty} y^n = y\in\mathbb{R}^N$, x^nRy^n for all $n\in\mathbb{N}$ implies xRy.

(6) **quasi-linearity in component k** if for all $x,y\in\mathbb{R}^N$, x R y implies $(x + \alpha e^{(k)})R(y + \alpha e^{(k)})$ for all $\alpha > 0$.

(7) **quasi-linearity in all components** if it satisfies quasi-linearity in coordinate k for all $k\in N$.

(8) **symmetry** if for all permutations σ on N such that $\forall x,y, x', y'\in\mathbb{R}^N$ satisfying $x'_i = x_{\sigma(i)}$ and $y'_i = y_{\sigma(i)}$ $\forall i\in N$ it is the case that xRy if and only if $x'Ry'$.

(9) **convexity with respect to duplicated evaluations** if for all $a,b,c\in\mathbb{R}$ with $a \leq b$ and $x,y,z\in\mathbb{R}^N$, $x_1 = y_1 = a$, $x_i = b$, $y_i = \frac{a+b}{2}$ for $i > 1$, $z_i = c$ for all $i\in N$, it is the case that xPz implies yPz.

(10) **strong convexity with respect to duplicated evaluations** if for all $a,b,c\in\mathbb{R}$ and $x,y,z\in\mathbb{R}^N$, $x_1 = y_1 = a$, $x_i = b$, $y_i = \frac{a+b}{2}$ for $i > 1$, $z_i = c$ for all $i\in N$, it is the case that xPz implies yPz.

(11) **improvement impatience** if for all $a,b,c\in\mathbb{R}$, with $b > a$, $c > a$, $x,y\in\mathbb{R}^N$ and $K\in\{1,...,L-1\}$: $x_i = y_i = a$ $\forall i = 1,...,K$, $y_{K+1} = a$, $x_i = b$ $\forall i\in\{K+1,...,L\}$and $y_i = c$ $\forall i\in\{K+2,...,L\}$ only if $K +2 \leq L$, implies xPy.

(12) **shuffling** if for all permutations σ, ρ on N such that $\forall x,y, x', y'\in\mathbb{R}^N$ satisfying $x'_i = x_{\sigma(i)}$ and $y'_i = y_{\rho(i)}$ $\forall i\in N$ it is the case that xRy if and only if $x'Ry'$.

(13) **ascending order separability** if for all $x,y, x', y'\in\mathbb{R}^N$ with $x_j \leq x_{j+1}$, $y_j \leq y_{j+1}$, $x'_j \leq x'_{j+1}$, $y'_j \leq y'_{j+1}$ for all $j = 1,...,L-1$ and any $i\in N$ satisfying $x_i = y_i$, $x'_i = y'_i$, $x_{-i} = x'_{-i}$, $y_{-i} = y'_{-i}$ it is the case that xRy if and only if $x'Ry'$.

(14) **separability** if for all $x,y, x', y'\in\mathbb{R}^N$ and any $i\in N$ satisfying $x_i = y_i$, $x'_i = y'_i$, $x_{-i} = x'_{-i}$, $y_{-i} = y'_{-i}$ it is the case that xRy if and only if $x'Ry'$.

Barbera and Jackson (1988) refer to something very similar to separability as the "sure thing principle". Clearly, separability implies ascending order separability, although the converse is not true. For instance R_{Mm} satisfies ascending order separability but not separability. That R_{Mm} does not satisfy separability is established in the following example.

Example 1
Let x = (4,3). y = (4,2), $x^{'}$ = (1,3), $y^{'}$ = (1,2). Thus, $xP_{Mm}y$ but $x^{'}I_{Mm}y^{'}$. Thus, $y^{'}R_{Mm}x^{'}$ but not $xR_{Mm}y$. This hold in spite of $x_1 = y_1$, $x_1^{'} = y_1^{'}$, $x_2 = x_2^{'}$, $y_2 = y_2^{'}$.

Note that shuffling implies symmetry. However, the converse is not true and shuffling is a much stronger property than symmetry. This will be shown in example 11.

Note further that both symmetry and shuffling are implied by the property known as anonymity.

A welfare ordering R is said to satisfy **anonymity** if for all x, y∈$\mathbb{R}^N$ and permutation σ on N, $y_i = x_{\sigma(i)}$ for all i∈N implies xIy.

It is also the case that our main results remain intact if we replace symmetry and shuffling by anonymity. In fact, since we are concerned with <u>orderings</u> on $\mathbb{R}^N$ anonymity and shuffling are equivalent properties.

Let $\mathfrak{J} = \{(x,y) \in \mathbb{R}^N \times \mathbb{R}^N \mid \min_{i\in N} x_i \neq \min_{i\in N} y_i\}$.

The restrictions of R_{Mm} and R_{Lm} to $\mathfrak{J}$ agree with each other.

3 Some well known preliminary results

In this section we present some well known preliminary results which immediately lead to an axiomatic characterization of the utilitarian welfare ordering.

The following two propositions along with their proofs can be found in Rubinstein (2012). Proposition 2 requires proposition 1 for its proof.

Proposition 1
Let R be a welfare ordering that satisfies continuity, domination and strict domination in component k. If R is quasi-linear in component k then there exists a function $v:\mathbb{R}_+^{N\setminus\{k\}} \to \mathbb{R}$ such that for all x,y∈$\mathbb{R}_+^N$, x R y if and only if $x_k + v(x_{-k}) \geq y_k + v(y_{-k})$.

Proposition 2
Let R be a welfare ordering that satisfies continuity, domination and strict domination. If R is quasi-linear in all components then there exists positive real numbers $\alpha_1, \alpha_2, \ldots, \alpha_L$ such that the function $u: \mathbb{R}_+^N \to \mathbb{R}$ defined by $u(x) = \sum_{k=1}^{L} \alpha_k x_k$ for x∈$\mathbb{R}_+^N$ satisfies the following property: for all x,y∈$\mathbb{R}_+^N$, xRy if and only if $u(x) \geq u(y)$.

4 Quasi-linearity and utilitarian welfare orderings

In this section we present and prove an axiomatic characterization of the utilitarian welfare ordering using quasi-linearity. Before doing so we introduce the following lemma.

Lemma 1
Let R be a welfare ordering that satisfies dominance, continuity and quasi--linearity in all components. Then for all $x,y \in \mathbb{R}^N$ and $z \in \mathbb{R}^N_+$: xRy implies (x+z)R(y+z) and xPy implies (x+z)P(y+z).

Proof
Suppose R is a welfare ordering that satisfies dominance, continuity and quasi--linearity in all components. Let $x,y \in \mathbb{R}^N$ and $z \in \mathbb{R}^N_+$.

Suppose xRy.

Let $M = \{k \in N | z_k > 0\}$. If $M = \phi$, then $z = 0$ so that (x+z)R(y+z). Hence suppose, $M \neq \phi$. Without loss of generality suppose, $M = \{1,...,K\}$ for some positive integer $K \leq L$. Thus, $x + z = x + \sum_{k=1}^{K} z_k e^{(k)}$ and $y + z = y + \sum_{k=1}^{K} z_k e^{(k)}$. By quasi-linearity, xRy implies $(x+z_1e^{(1)})R(y+z_1e^{(1)})$ and if $K > 1$, then for all $J < K$, $(x + \sum_{k=1}^{J} z_k e^{(k)})$ R $(y + \sum_{k=1}^{J} z_k e^{(k)})$ implies $(x + \sum_{k=1}^{J+1} z_k e^{(k)})$ R $(y + \sum_{k=1}^{J+1} z_k e^{(k)})$. Thus by a standard finite induction argument we get (x+z)R(y+z).

Now suppose xPy and towards a contradiction suppose (y+z)R(x+z). By quasi- linearity we have (x+z)R(y+z), so that we have (y+z)I(x+z). By continuity of R, xPy implies that there exists $\varepsilon > 0$ sufficiently small, so that we have $(x-\varepsilon e)$Py. By quasi-linearity we get $(x + z-\varepsilon e)$R(y+z). Transitivity of R along with $(x + z-\varepsilon e)$R(y+z) and (y+z)I(x+z) implies $(x + z-\varepsilon e)$R(x+z). This contradicts dominance since $x + z >> x + z-\varepsilon e$. Hence we must have, (x+z)P(y+z). Q.E.D.

Proposition 3
Let R be a welfare ordering. Then R satisfies continuity, domination, strict domination and quasi-linearity in all components if and only if it is utilitarian.

Proof
Let $R = R_U$. Then it is easily verified that it is a welfare ordering that satisfies continuity, domination, strict domination and quasi-linearity in all components. Hence suppose R is a welfare ordering that satisfies continuity, domination, strict domination and quasi-linearity in all components. Then by proposition 2, there exists positive real numbers $\alpha_1, \alpha_2, \ldots, \alpha_L$ such that for all $x,y \in \mathbb{R}^N_+$, xRy if and only if $\sum_{k=1}^{L} \alpha_k x_k \geq \sum_{k=1}^{L} \alpha_k y_k$.

Let $x,y \in \mathbb{R}^N$. Then there exists a non-negative real number b such that x + be and y + be both belong to $\mathbb{R}^N_+$. Suppose xRy. Then by quasi-linearity and lemma 1 it must be the case that (x + be)R(y+be). From the previous paragraph it follows that (x + be)R(y+be) if and only if $\sum_{k=1}^{L} \alpha_k(x_k + b) \geq \sum_{k=1}^{L} \alpha_k(y_k+b)$. However, $\sum_{k=1}^{L} \alpha_k(x_k + b) \geq \sum_{k=1}^{L} \alpha_k(y_k+b)$ if and only if $\sum_{k=1}^{L} \alpha_k x_k + b\sum_{k=1}^{L} \alpha_k \geq \sum_{k=1}^{L} \alpha_k y_k + b\sum_{k=1}^{L} \alpha_k$, while the latter holds if and only if $\sum_{k=1}^{L} \alpha_k x_k \geq \sum_{k=1}^{L} \alpha_k y_k$. Thus, xRy implies $\sum_{k=1}^{L} \alpha_k x_k \geq \sum_{k=1}^{L} \alpha_k y_k$.

Conversely suppose $x,y \in \mathbb{R}^N$ and $\sum_{k=1}^{L} \alpha_k x_k \geq \sum_{k=1}^{L} \alpha_k y_k$. Towards a contradiction suppose yPx. Now, there exists a non-negative real number b such that x + be and y + be both belong to $\mathbb{R}^N_+$. By lemma 1 we have (y+be)P(x+be). Thus, we have $\sum_{k=1}^{L} \alpha_k(y_k + b) > \sum_{k=1}^{L} \alpha_k(x_k+b)$. This leads to $\sum_{k=1}^{L} \alpha_k y_k > \sum_{k=1}^{L} \alpha_k x_k$, contradicting $\sum_{k=1}^{L} \alpha_k x_k \geq \sum_{k=1}^{L} \alpha_k y_k$. Thus we must have xRy. Thus, $R = R_U$, i.e. R is utilitarian.

5 Some preliminary results concerning maximin and leximin welfare orderings

In this section we present some preliminary results concerning maximin and leximin welfare orderings.

Lemma 2

Both R_{Mm} and R_{Lm} satisfy strong convexity with respect to duplicated evaluations and hence convexity with respect to duplicated evaluations.

Proof

Let $a,b,c \in \mathbb{R}$ and $x,y,z \in \mathbb{R}^N$ with $x_1 = y_1 = a$, $x_i = b$, $y_i = \frac{a+b}{2}$ for $i > 1$, $z_i = c$ for all $i \in N$. (a) Suppose we have $xP_{Mm}z$. Thus, $\min\{a,b\} > c$. But $\min\{a, \frac{a+b}{2}\} \geq \min\{a,b\} > c$. Thus we have $yP_{Mm}z$.

(b) Suppose we have $xP_{Lm}z$. If $xP_{Mm}z$ then by (a) we have $yP_{Mm}z$ which implies $yP_{Lm}z$.

If it is not the case that $xP_{Mm}z$, then $\min\{a,b\} = c$ and $\max\{a,b\} > c$.

Case 1: $\min\{a,b\} = a$.

Then, $b > a = c$ and so $\frac{a+b}{2} > a = c$.

Thus, $\min\{a, \frac{a+b}{2}\} = c$ and $\max\{a, \frac{a+b}{2}\} > c$ and so $yP_{Lm}z$.

Case 2: $\min\{a,b\} = b$.

Thus, $a > b = c$ and so $\frac{a+b}{2} > b = c$. Thus, $\min\{a, \frac{a+b}{2}\} > c$ and so we have $yP_{Lm}z$.

Q.E.D.

Lemma 3

The restrictions of R_{Mm} and R_{Lm} to $\mathfrak{I}$ agree with each other. Let R be a welfare ordering that satisfies symmetry, domination and convexity with respect to duplicated evaluations. Then $R|\mathfrak{I} = R_{Mm}|\mathfrak{I} = R_{Lm}|\mathfrak{I}$.

Proof

That $R_{Mm}|\mathfrak{I} = R_{Lm}|\mathfrak{I}$ follows from the respective definitions. Hence let us suppose, R is a welfare ordering that satisfies symmetry, domination and convexity with respect to duplicated evaluations. We need to show that $R|\mathfrak{I} = R_{Mm}|\mathfrak{I} = R_{Lm}|\mathfrak{I}$.

We first prove that for $a,b,c,d \in \mathbb{R}$, it is the case that $[b \geq a, c > a, d > a]$ implies xPy where $x_1 = c$, $y_1 = a$, $x_i = d$ and $y_i = b$ $\forall i > 1$. Call this statement (i).

Suppose not. Then yRx. Let $\varepsilon > 0$ be such that $c > a + \varepsilon$, $d > a + \varepsilon$.

Then by dominance $xP(a+\varepsilon)e$, where e is the unit vector in $\mathbb{R}^N$.

By transitivity of R, we have $yP(a+\varepsilon)e$.

By convexity with respect to duplicated evaluations we have $z^1P(a+\varepsilon)e$, where $z_1^1 = a$ and $z_i^1 = \frac{a+b}{2}$ $\forall i > 1$.

By convexity with respect to duplicated evaluations again we have $z^2P(a+\varepsilon)e$, $z_1^2 = a$ and $z_i^2 = \frac{1}{2}a + \frac{1}{2}(\frac{a+b}{2}) = \frac{3a+b}{4} = \frac{(2^2-1)a+b}{2^2}$ $\forall i > 1$.

On the n^{th} repetition of convexity with respect to duplicated evaluations again we have $z^nP(a+\varepsilon)e$, $z_1^n = a$ and $z_i^n = \frac{(2^n-1)a+b}{2^n}$ $\forall i > 1$.

Now, $\lim_{n\to\infty} \frac{(2^n-1)a+b}{2^n} = a$. Hence there exists $K \in \mathbb{N}$, such that $\forall n \geq K$, $a+\varepsilon > \frac{(2^n-1)a+b}{2^n}$.

Thus, $(a + \varepsilon)e \gg z^n$ $\forall$ $n \geq K$ and hence by domination $(a + \varepsilon)ePz^n$ $\forall$ $n \geq K$, leading to a contradiction.

Hence we have xPy.

Now let $(x,y) \in \mathfrak{I}$ and suppose $a = \min_{i\in N} x_i$ and $b = \min_{i\in N} y_i$. Suppose without loss of generality that $b > a$. Let $d = \frac{2b+a}{3}$ and $c = \max_{i\in N} x_i$. Thus, $b > d > a$ and $c \geq a$.

By symmetry we can suppose $x_1 = a$. Then by domination we have (a,c,c,....,c)Rx.

By (i) we have deP(a,c,c,...,c).

By domination we have bePde as well as yRbe.

Thus, yRbe, bePde, deP(a,c,c,...,c), (a,c,c,...,c)Px and transitivity of R implies yPx.

Since it is easy to verify that the restriction to $\mathfrak{I}$ of R_{Mm} agrees with the restriction to $\mathfrak{I}$ of R_{Lm} and both R_{Mm} and R_{Lm} satisfy symmetry, domination and convexity with respect to duplicated evaluations, our lemma is proved. Q.E.D.

6 An axiomatic characterization of the maximin welfare ordering and logical independence of the axioms

In this section we obtain an axiomatic characterization of the maximin welfare ordering and provide examples to show that the axioms we use are logically independent.

Proposition 4
The welfare ordering R_{Mm} is uniquely characterized by symmetry, domination, convexity with respect to duplicated evaluations and continuity.

Proof
It is easy to see that R_{Mm} satisfies symmetry and continuity. Convexity with respect to duplicated evaluations of R_{Mm} follows from lemma 2. Let us verify that R_{Mm} satisfies domination. Let $x,y\in\mathbb{R}^N$ and suppose $x \geq y$. Without loss of generality suppose $x_i \leq x_{i+1}$ for all i = 1,..., n-1. Thus, $\min_{i\in N} x_i = x_1 \geq y_1 \geq \min_{i\in N} y_i$. Thus, $xR_{Mm}y$. Further if $x >> y$, then $\min_{i\in N} x_i = x_1 > y_1 \geq \min_{i\in N} y_i$ and so $xP_{Mm}y$. Thus R_{Mm} satisfies domination.

Hence suppose R is a welfare ordering that satisfies symmetry, domination, convexity with respect to duplicated evaluations and continuity. We know from lemma 3, that $R|\mathfrak{I} = R_{Mm}|\mathfrak{I}$. Hence suppose $(x,y)\notin\mathfrak{I}$. Thus, $\min_{i\in N} x_i = \min_{i\in N} y_i$. Let $<\varepsilon^n|n\in\mathbb{N}>$ be sequence of strictly positive real numbers converging to 0. Let $<x^{n-}|n\in\mathbb{N}>$ and $<x^{n+}|n\in\mathbb{N}>$ be two sequences in $\mathbb{R}^N$ such that $\forall n\in\mathbb{N}$ and $i\in N$, $x_i^{n-} = x_i - \varepsilon^n$ and $x_i^{n-} = x_i - \varepsilon^n$. Then for all $n\in\mathbb{N}$, $\min_{i\in N} x_i^{n-} = \min_{i\in N} x_i - \varepsilon^n < \min_{i\in N} y_i < \min_{i\in N} x_i + \varepsilon^n = \min_{i\in N} x_i^{n+}$.

By lemma 3, x^{n+} Py and yPx^{n-} $\forall n\in\mathbb{N}$. Further, $\lim_{n\to\infty} x^{n+} = x = \lim_{n\to\infty} x^{n-}$. Thus by continuity, we have xRy and yRx, i.e. xIy.
Thus, $R = R_{Mm}$. Q.E.D.

Let us show that the properties we use in proposition 4 are logically independent.

Example 2
(A welfare ordering that satisfies symmetry, domination, convexity with respect to duplicated evaluations but not continuity): Let $R = R_{Lm}$. Then R satisfies symmetry, domination, convexity with respect to duplicated evaluations. But it does not satisfy continuity. Let L = 2 $x^n = (\frac{1}{n}, 1)$ and $y^n = (0,2)$ for all $n\in\mathbb{N}$. Thus,

x^nRy^n for all n∈ℕ. However, $\lim_{n\to\infty} x^n = (0,1)$, $\lim_{n\to\infty} y^n = (0,2)$ and (0,2)P(0,1). Thus R does not satisfy continuity.

Example 3

(A welfare ordering that satisfies symmetry, domination and continuity but not convexity with respected to duplicated evaluations): Let R be such that for all x,y∈$\mathbb{R}^N$, xRy if and only if $\sum_{i=1}^{n} x_i \geq \sum_{i=1}^{n} y_i$. Clearly R satisfies symmetry, domination and continuity. Let L = 3, x = (1, 2, 2), y = $(1,\frac{3}{2},\frac{3}{2})$ and z = (1.65, 1.65, 1.65). Then $\sum_{i=1}^{3} x_i = 5$, $\sum_{i=1}^{3} z_i = 4.95$ and so we have xPz. However, $\sum_{i=1}^{3} y_i = 4 < 4.95 = \sum_{i=1}^{3} z_i$ and so zPy. Thus R violates convexity with respect to duplicated evaluations.

Example 4

(A welfare ordering that satisfies symmetry, convexity with respect to duplicated evaluations and continuity but not domination): Let R be such that for all x,y∈$\mathbb{R}^N$, xRy if and only if $\max_{i\in N} x_i \leq \max_{i\in N} y_i$. It is easy to verify that R satisfies symmetry, convexity with respect to duplicated evaluations and continuity. Let a, b∈ℝ with a < b. Let e be the vector in $\mathbb{R}^N$ with all its coordinates equal to 1. Thus be >> ae, but aePbe. Thus, R violates domination.

Example 5

(A welfare ordering that satisfies domination, convexity with respect to duplicated evaluations, continuity but not symmetry): Let R be such that for all x,y∈$\mathbb{R}^N$, xRy if and only if $x_1 \geq y_1$. Clearly, R satisfies domination, convexity with respect to duplicated evaluations and symmetry. However, R does not satisfy symmetry. Let L ≥ 2, x,y∈$\mathbb{R}^N$ with $x_1 > y_1$ and $x_2 < y_2$. Let σ be the one--to-one function from N to N, such that σ(1) = 2, σ(2) = 1 and σ(i) = i for all i∈N\{1,2}. Let $x', y' \in \mathbb{R}^N$ with $x_i' = x_{\sigma(i)}$ and $y_i' = y_{\sigma(i)}$ for all i∈N. Then, xPy but $y'Px'$ contradicting symmetry.

7 An axiomatic characterization of the leximin welfare ordering and logical independence of the axioms

Now let us consider the leximin welfare ordering.

Lemma 4

R_{Lm} satisfies separability (and hence ascending order separability).

Proof

To show that R_{Lm} satisfies separability let us consider x,y, x', $y' \in \mathbb{R}^N$ and i∈N satisfying $x_i = y_i$, $x_i' = y_i'$, $x_{-i} = x_{-i}'$, $y_{-i} = y_{-i}'$ and $xR_{Lm}y$. Let ξ, η, ξ', η' be the arrangement of x,y, x', y' in ascending order. If $xI_{Lm}y$, then ξ = η. Thus when we replace x_i by x_i' and y_i by y_i' we get $\xi' = \eta'$, since $x_i = y_i$ and $x_i' = y_i'$. This is because in the ascending order arrangements the position of x_i is the same as the position of y_i and position of x_i' is same as the position of y_i'. Hence suppose, $xP_{Lm}y$. Thus there exists $a\in\mathbb{R}$ such that $\#J(a,x) < \#J(a,y)$ and $\#J(b,x) = \#J(b,y)$ for all $b < a$. Since $x_i = y_i$, $\#(J(a,x)\backslash\{x_i\}) < \#(J(a,y)\backslash\{y_i\})$ and $\#(J(b,x)\backslash\{x_i\}) = \#(J(b,y)\backslash\{y_i\})$ for all $b < a$.

Now $\forall b\in\mathbb{R}$, $x_i' \le b$ if and only if $y_i' \le b$. This is because $x_i' = y_i'$. Thus, $\forall b\in\mathbb{R}$, $x_i' \in J(b,x)\backslash\{x_i\}$ if and only if $y_i' \in J(b,y)\backslash\{y_i\}$. Thus, $\#J(a,x') < \#J(a,y',)$ and $\#J(b,x') = \#J(b,y')$ for all $b < a$. Hence, $x'P_{Lm}y'$. Q.E.D.

We are now in a position to state and prove the following proposition.

Proposition 5

R_{Lm} is uniquely characterized by shuffling, domination, convexity with respect to duplicated evaluations, improvement impatience and ascending order separability.

Proof

It is easy to see that R_{Lm} satisfies shuffling and improvement impatience. Convexity with respect to duplicated evaluations of R_{Lm} follows from lemma 2. That it satisfies ascending order separability follows from lemma 4. Let us verify that R_{Lm} satisfies domination. Let $x,y\in\mathbb{R}^N$ and suppose $x \ge y$. Without loss of generality suppose $x_i \le x_{i+1}$ for all $i = 1,..., n-1$. Let $\sigma:N\to N$ be a one-to-one function such that $y_{\sigma(i)} \le y_{\sigma(i+1)}$ for all $i = 1,..., n-1$. Now $x_1 \ge y_1 \ge y_{\sigma(1)}$ so that if $x_1 > y_1$ or $y_1 > y_{\sigma(1)}$, then $xP_{Lm}y$. Thus, $x >> y$ implies $xP_{Lm}y$. Hence suppose, $x_1 = y_1 = y_{\sigma(1)}$. If $x_i = y_{\sigma(i)}$ for all $i\in N$, then $xI_{Lm}y$ and so $xR_{Lm}y$. Hence suppose, $K = \min\{i\in N|\ x_i \ne y_{\sigma(i)}\}$. Clearly $K > 1$ and $K \le L$. Towards a contradiction suppose, $x_K < y_{\sigma(K)}$ so that $y_K = y_{\sigma(i)}$ for some $i < K$. Now, for $i \le K$, $y_i \le x_i \le x_K < y_{\sigma(K)}$. Thus, $\sigma(i)\in\{1,...,K\}$ for $i\in\{1,...,K\}$. Thus, $x_K \ge y_{\sigma(i)}$ for $i\in\{1,...,K\}$ and so $x_K \ge y_{\sigma(K)}$, leading to a contradiction. Along with $x_K \ne y_{\sigma(K)}$, $x_K \ge y_{\sigma(K)}$ implies $x_K > y_{\sigma(K)}$. Since $x_i = y_{\sigma(i)}$ for all $i\in\{1,...,K-1\}$, we get $xP_{Lm}y$.

Hence suppose R is a welfare ordering that satisfies shuffling, domination, convexity with respect to duplicated evaluations, improvement impatience and separability. Since shuffling implies symmetry, by lemma 3 we get that $R|\mathfrak{I} = R_{Lm}|\mathfrak{I}$. Hence suppose $(x,y)\notin\mathfrak{I}$. Thus, $\min_{i\in N} x_i = \min_{i\in N} y_i = a$ (say). By shuffling we may assume $x_i \le x_{i+1}$ and $y_i \le y_{i+1}$ for $i = 1,..., L-1$. Thus, $x_1 = y_1 = a$. Suppose $x_i = y_i$ $\forall i = 1,...,K$. If $K = L$, then by reflexivity of R we have xIy and so $xI_{Lm}y$. Hence suppose $K < L$.

Case 1: $K = L-1$.

Thus, $y_L \ne x_L$. Without loss of generality suppose, $x_L > y_L$.

By improvement impatience the evaluation vector $(y_L,...,y_L, x_L)$ is preferred to the evaluation vector y_Le.
By separabilty the evaluation vector $x = (x_1,...,x_{L-1}, x_L)$ is preferred to the evaluation vector $y = (y_i,...,y_{L-1},y_L)$ since $x_i = y_i$ for i = 1,..., L-1 and so in this case R agrees with R_{Lm}.
Case 2: K < L-1.
Thus, $K + 1 \leq L-1 < L$ and $y_{K+1} \neq x_{K+1}$. Without loss of generality suppose, $x_{K+1} > y_{K+1}$.
By improvement impatience the evaluation vector $(y_{K+1},...,y_{K+1}, x_{K+1},....,x_{K+1})$ is preferred to the evaluation vector $(y_{K+1},...., y_{K+1}, y_{K+1}, y_L,...., y_L)$.
In the first vector the first K co-ordinates are y_{K+1} and the remaining L-K coordinates are x_{K+1}. In the second vector the first K+ 1 coordinates are y_{K+1} and the remaining L- (K+1) coordinates are y_L.
By separability the evaluation vector $(x_1,...,x_K, x_{K+1},....,x_{K+1})$ is preferred to the evaluation vector $(y_1,...., y_K, y_{K+1}, y_L,...., y_L)$, since $x_i = y_i$ for i = 1,...,K.
Thus we can write $(x_1,...,x_K, x_{K+1},....,x_{K+1})P(y_1,...., y_K, y_{K+1}, y_L,...., y_L)$.
By dominance we have $xR(x_1,...,x_K, x_{K+1},....,x_{K+1})$ and $(y_1,...., y_K, y_{K+1}, y_L,...., y_L)Ry$.
By transitivity of R, we get xPy.
Once again P agrees with P_{Lm}.
This proves the proposition. Q.E.D.

It is worth observing that R_{Mm} does not satisfy improvement impatience. This observation is immediate from the fact that if $x,y\in\mathbb{R}^N$ satisfies the conditions in the definition of improvement impatience, then it must be the case that $xI_{Mm}y$, contrary to the requirement xPy.

Let us now show that the properties we use in proposition 5 are logically independent.

Example 6

(A welfare ordering that satisfies shuffling, domination, convexity with respect to duplicated evaluations, improvement impatience but not ascending order separability):
Let L =3 and $\mathcal{Z} = \{x\in\mathbb{R}^3|$ there exists $i,j\in\{1,2,3\}$ with $i\neq j$ and $x_i = x_j\}$. Let R be a binary relation on $\mathbb{R}^N$ such that $R|\mathcal{Z}\times\mathcal{Z} = R_{Lm}|\mathcal{Z}\times\mathcal{Z}$ and for all $(x,y)\in(\mathbb{R}^3\times\mathbb{R}^3)\backslash(\mathcal{Z}\times\mathcal{Z})$, xRy if and only if $xR_{Mm}y$. It is easy to verify that R is an ordering which satisfies shuffling, domination, convexity with respect to duplicated evaluations, improvement impatience. However, R does not satisfy separability. Let x = (2,2,3), y = (2,3,3). Thus, $x,y\in\mathcal{Z}$ and we have yPx since it is the case that $yP_{Lm}x$. Let $x' = (1,2,3)$ and $y' = (1,3,3)$. Thus, $(x', y')\in(\mathbb{R}^3\times\mathbb{R}^3)\backslash(\mathcal{Z}\times\mathcal{Z})$ and so $x' I_{Mm} y'$ implies $x'Iy'$. This happens in spite of $x_1 = y_1 = 2$, $x_1' = y_1' = 1$, $x_2 = x_2' = 2$, $y_2 = y_2' = 3$, $x_3 = x_3' = 3$, $y_3 = y_3' = 3$. Thus, R does not satisfy separability.

Example 7

(A welfare ordering that satisfies shuffling, domination, convexity with respect to duplicated evaluations, ascending order separability but not improvement impatience). Let R = R_{Mm}. Then R satisfies all the properties required in proposition 5 except for improvement impatience.

Example 8

(A welfare ordering that satisfies shuffling, domination, improvement impatience, ascending order separability but not convexity with respect to duplicated evaluations). Let L = 2 and let R be a binary relation on $\mathbb{R}^N$ such that for all $x,y \in \mathbb{R}^N$, xRy if and only if $x_1 + x_2 \geq y_1 + y_2$. Then R satisfies all the properties required in the statement of proposition 5, other than convexity with respect to duplicated evaluations. Let x = (1,7), y = (1,4), z = (3,3). Then we have a = 1, b = 7, c = 3, xPz and zPy violating convexity with respect to duplicated evaluations.

Example 9

(A welfare ordering that satisfies shuffling, convexity with respect to duplicated evaluations, improvement impatience, ascending order separability but not domination). Let L= 2 and let R be a binary relation on $\mathbb{R}^N$ such that for all $x,y \in \mathbb{R}^N$, xPy if and only if either (i) $\min\{x_1, x_2\} < \min\{y_1, y_2\}$; or (ii) $\min\{x_1, x_2\} = \min\{y_1, y_2\}$ but $\max\{x_1, x_2\} > \max\{y_1, y_2\}$. It is easy to see that R is an ordering that satisfies shuffling, improvement impatience and ascending order separability. Let us show that R satisfies convexity with respect to duplicated evaluations. Let $a, b \in \mathbb{R}$ with $a \leq b$, x = (a,b), y = (a, $\frac{a+b}{2}$) and z = (c,c). Suppose xPz.

Case 1: $\min\{x_1, x_2\} = a$ and $a < c$.

Thus, $b \geq a$, so that $\frac{a+b}{2} \geq a$. Thus, $\min\{y_1, y_2\} = a < c$ and so yPz.

Case 2: $\min\{x_1, x_2\} = a$ and $a = c$.

Thus, $b \geq a$ and $b > c$. Hence $b > a$. Thus, $\frac{a+b}{2} > a = c$. Thus, yPz.

Thus R satisfies convexity with respect to duplicated evaluations.

However R does not satisfy domination. Let x = (1,2), y = (3,4).Since, $\min\{x_1, x_2\} < \min\{y_1, y_2\}$ we have xPy, in spite of $y >> x$. Thus R violates domination.

Example 10

(A welfare ordering that satisfies domination, convexity with respect to duplicated evaluations, improvement impatience, ascending order separability but not shuffling). Let L = 2 and let R be the lexicographic ordering on $\mathbb{R}^N$, i.e. for all $x,y \in \mathbb{R}^N$, xPy if and only if either (i) $x_1 > y_1$; or (ii) $x_1 = y_1$ and $x_2 > y_2$. It is easily verified that R satisfies domination, convexity with respect to duplicated evaluations, improvement impatience, ascending order separability. However if x = (1,2), y = (2,1), x' = (2,1) and y' = (1,2), then we have yPz and $x'Py'$ although x' and y' are obtained from x and y respectively, by interchanging the coordinates.

The above examples show that the axioms used in proposition 5 are logically independent. The next example shows that in proposition 5, we cannot replace shuffling with symmetry in order to obtain an axiomatic characterization of R_{Lm}.

Example 11

(A welfare ordering different from R_{Lm} that satisfies symmetry, domination, convexity with respect to duplicated evaluations, improvement impatience, ascending order separability but not shuffling). Let L = 2 and R be a binary relation on $\mathbb{R}^N$ such that for all $x,y \in \mathbb{R}^N$, xRy if and only if $\#\{i|x_i \geq y_i\} \geq \#\{i|y_i \geq x_i\}$. Then clearly R is an ordering and satisfies symmetry, domination, convexity with respect to duplicated evaluations, improvement impatience, ascending order separability. To show that R does not satisfy shuffling, let x = (1,3) and y = (2,1). Then clearly, xIy. However, if we let $\sigma:\{1,2\}\rightarrow\{1,2\}$ be the identity function and $\rho:\{1,2\}\rightarrow\{1,2\}$ to be such that $\rho(1) = 2$, $\rho(2) = 1$,then we get $x'P\,y'$, where $x'_i = x_{\sigma(i)}$ and $y'_i = y_{\rho(i)}$ $\forall i \in \{1,2\}$.

We already know that R_{Mm} satisfies symmetry, domination, convexity with respect to duplicated evaluations, ascending order separability, but not improvement impatience which the ordering defined in example 11 (i.e. majority rule on $\mathbb{R}^N$) satisfies. Similarly majority rule on $\mathbb{R}^N$ satisfies symmetry, domination, convexity with respect to duplicated evaluations, improvement impatience, ascending order separability but not continuity that R_{Mm} satisfies. That majority rule on $\mathbb{R}^N$ does not satisfy continuity is shown in the following example.

Example 12

Let L = 2 and R be the ordering defined in example 11. Let x = (1,0) and for $n \in \mathbb{N}$, let $y^n = (0, \frac{1}{n})$. Then, y^nIx for all $n \in \mathbb{N}$ which implies y^nRx for all $n \in \mathbb{N}$. However, $y = (0,0) = \lim_{n\rightarrow\infty}(0,\frac{1}{n}) = \lim_{n\rightarrow\infty} y^n$ and we have xPy. Thus, R is not continuous.

8 Conclusion

In this paper we obtain new axiomatic characterizations for three different welfare orderings. The interesting fact about these three welfare orderings is that they satisfy full-comparability- a desirable property that is easily established as in the surveys that we cite in this paper and a fact that we do not need to use in our axiomatic characterizations. The three welfare orderings we consider are of considerable importance in group decision theory as well as in the theory of choice in the presence of ambiguity. These orderings play a very significant role in applied multi-criteria decision making too. Hence researchers have periodically come up with new characterizations of these welfare orderings in order to understand them better and convey their importance to others whose work have an interface with group and multicriteria decision making. We hope that this paper will also serve the same purpose, and prove itself to be incerementally useful.

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APPLICATION OF MULTIOBJECTIVE DYNAMIC PROGRAMMING TO THE ALLOCATION AND RELIABILITY PROBLEM

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Abstract

The paper deals with a model of the allocation and reliability problem. This static problem, presented as a multistage decision process, can be solved using multiobjective dynamic programming. The goal of this paper is to formulate the allocation and reliability problem as a multistage decision process, to find the set of all its efficient solutions, to use the weighted sum method for multistage and single-stage criteria, as well as to perform sensitivity analysis.

Keywords: multiobjective dynamic programming, allocation and reliability problem, efficient solutions, scalarization methods.

1 Introduction

Multiple objective dynamic programming (MODP) deals with multistage decision processes, in which multiple objectives are taken into consideration. The term MODP covers models of tasks which allow to solve various problems such as: the multiple criteria knapsack problem (Klamroth, Wiecek, 2000), the problem of space heating under a time-varying price of electricity (Hämäläinen, Mäntysaari, 2002), the supplier selection-order allocation problem (Mafakheri et al., 2011), or the location-routing model for relief logistic planning under uncertainty on demand, travel time, and cost parameters (Bozorgi-Amiri, Khorsi, 2016). Those problems are usually of dynamic character. MODP methods are used to analyse multistage decision processes in which a given (usually finite) period is divided into a fixed number of stages. Dynamic programming is also often used to model appropriately formulated static problems. This is also the

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case for the mathematical economics problem discussed in this paper, namely the problem of allocation and reliability (A&R).

Multicriteria evaluation of a multistage process is performed using a vector criteria function whose multistage components are certain compositions of single-stage evaluations. These components have to be separable and monotone scalar functions (Mine, Fukushima, 1979; Mitten, 1964; Trzaskalik, 1990; Abdelaziz et al., 2018; Chen, Fu, 2005), such as additive or multiplicative compositions.

In multicriteria problems, because of the conflicting nature of the objectives, a dominating solution – that is, a solution whose all multistage components admit "best" values simultaneously – usually does not exist. As vector optimal solutions we take those solutions whose multistage evaluations are not dominated. It is not possible (in the criteria space) to improve the value of any multistage criterion without worsening the value of at least one of the remaining criteria.

The basic method of solving multicriteria problems consists in searching for non-dominated solutions (in the criteria space) and for the corresponding efficient solutions (in the decision space). This is the case also for MODP problems. Often, however, finding all non-dominated solutions is difficult calculation-wise, and the set obtained can be very large. For that reason, finding this set is of little direct help to the decision maker in making the final decision. Therefore, analogously to other multicriteria problems, various scalarization methods can be used, which allow (on the basis of additional preferences of the DM) to find a solution taking into account the DM's preferences as a single-criteria optimization problem. It is generally accepted that the solution obtained using a scalarization method should be an efficient solution.

The scalarization method used in this paper is the method of weighted sum of multistage criteria. It can be proven that to each non-negative vector of coefficients there corresponds an efficient realization (Trzaskalik, 1993). In multicriteria dynamic optimization a new possibility (as compared with static vector optimization) occurs: the DM can express his/her preferences by specifying the preferred relations between stage criteria. In the case of a bicriteria problem it is also possible to perform an effective sensitivity analysis.

As opposed to many other optimization problems, such as linear programming problems, no standard formulation of the dynamic programming model exists. Various problems are mutually related by their solving method, which uses optimality equations, constructed on the basis of the optimality principle (Bellman, 1957) and its vector counterpart (Trzaskalik, 1998). In this paper we will use the standard description of a multistage, multicriteria decision process (Trzaskalik, 1998; 2015).

The allocation problem is one of the static problems which can be solved by means of dynamic programming methods (Bellman, 1957; Nowak and Trzaskalik, 2014). The A&R problem, discussed in the present paper, can be described as follows: Given is a system consisting of T modules and a certain amount of resource R. The profits from the operation of the system and its reliability are related to the amount of the resource allocated to the individual modules. The function expressing the profits resulting from the system's operation is the sum of the profits from the operations of the individual modules, while the function expressing the reliability of the entire system is a multiplicative function. The allocation of the resource for the operation of the individual modules should be planned so as to maximize both the profits resulting from the operation of the entire system and its reliability.

The goal of the present paper is to formulate the A&R problem as a multistage decision process, to find the set of all its efficient solutions, to use the weighted sum method for multistage and single-stage criteria, and to perform sensitivity analysis.

The paper consists of five sections. In Section 2, the A&R problem is presented as a multistage decision process. A discrete problem illustrating this problem is also presented, together with the graph of this process. Section 3 shows a possible application of optimality equations and Bellman's vector optimality principle to finding the complete set of non-dominated solutions in the criteria space and of efficient solutions in the decision space. In Section 4, the problem of applying the weighted sum method is discussed and a sensitivity analysis of the problem is performed. Conclusions end the paper.

2 The A&R problem as a multistage decision process

To present the problem in question as a discrete multistage decision process, one should determine the number of stages, the sets of admissible states and decisions, the transfer function (which describes the transformations of the system in consecutive stages), and the method of evaluating the process.

The A&R problem, presented in the previous section, can be formulated as a multistage decision process as follows. The number of stages is determined by the number of modules, that is, T. The allocation of the resource is performed consecutively for the individual modules: in stage 1 we allocate resources for the operation of module 1, in stage 2 – for module 2, etc., and finally in stage T we allocate resources for the operation of module T. The process state y_t at the beginning of stage t ($t \in 1,...,T$) is the amount of the resource available after the allocation in the previous stages had been performed. The set of all admissible states at the beginning of stage t is denoted by Y_t. Decision x_t at stage t consists in the allocation of the entire remaining resource or its part for the operation of module t. The set of all admissible decisions for stage t, if at the beginning of this stage the process was in state y_t, is denoted by $X_t(y_t)$. The pair consisting of state y_t and the corresponding admissible decision x_t is the stage realization of the process, denoted by $d_t = (y_t,x_t)$.

The transformation of the system from state $y_t \in Y_t$ to state $y_{t+1} \in Y_{t+1}$, when the decision $x_t \in X_t(y_t)$ is made, is described by the transfer function of the form:

$$y_{t+1} = \Omega_t(y_t, x_t) = y_t - x_t \quad (1)$$

The sequence of admissible states and decisions of the process such that:

$$y_1 \in Y_1,\ x_1 \in X_1(y_1),\ y_2 = \Omega_1(y_1, x_1),\ \ldots,\ y_T = \Omega_t(y_{T-1}, x_{T-1}),\ x_T \in X_T(y_T) \quad (2)$$

is an admissible realization of the process, denoted by d. The set of all admissible realizations of the process is denoted by D.

The evaluation of the operation of the individual modules is described by the stage profit functions $F_t^1(y_t,x_t)$ and stage reliability functions $F_t^2(y_t,x_t)$ for t = 1,...,T. The evaluation of the operation of the entire system is described by the vector criterion function. The first component of this function describes the profits from the operation of the system; it is an additive function of the form

$$F^1(d) = \Sigma_{t=1}^{T} F^1(d_t), \quad (3)$$

while its second component describes the reliability of the system's operation, is multiplicative, and of the form

$$F^2(d) = \Pi_{t=1}^{T} F^1(d_t) \quad (4)$$

The vector criterion function which describes the operation of the system is of the form

$$F(d) = [F^1(d), F^2(d)]' \quad (5)$$

To illustrate the type of the process discussed we consider a simple system consisting of three modules. Six units of the resource are available. The profits from the individual modules and their reliability depending on the amount of the resource are shown in Table 1.

Table 1: Values of the stage criteria (dummy data)

	Module 1		**Module 2**		**Module 3**	
Amount alocated	**Profit**	**Reliability**	**Profit**	**Reliability**	**Profit**	**Reliability**
0	0	0.9	0	0.9	0	0.9
1	1.2	0.97	3	0.94	2.8	0.96
2	2	0.991	4.8	0.964	4.5	0.984
3	2.7	0.9973	5.5	0.9784	6.5	0.9936
4	3.3	0.9992	6.8	0.987	7.8	0.9974
5	3.7	0.9998	7.9	0.9922	9.0	0.999
6	4	0.9999	8.5	0.9953	10	0.9994

We will determine the sets of admissible states of this process at the beginning of the consecutive stages. The initial state is given as 6, that is, $Y_1 = \{6\}$. At the beginning of stage 2 the process can be in state 0 (if the entire remaining resource is allocated to module 1), in state 1 (if module 1 is allocated five units), or else in one of the consecutive states 2, 3, 4, 5 or 6, which are interpreted analogously to states 0 and 1.

At the beginning of stage 3 the process can be in state 0 (if the entire resource had been allocated previously to modules 1 and 2), in state 1 (if modules 1 and 2 had been allocated five units), or else in one of the remaining states 2, 3, 4, 5, 6. Since we plan to allocate the entire resource, the final state is given as 0. We obtain the following sets of admissible states:

$$Y_1 = \{6\}\ Y_2 = (0,1,2,3,4,5,6\} \qquad Y_3 = \{0,1,2,3,4,5,6\} \qquad y_4 = \{0\}$$

Now we will deal with the sets of admissible decisions for the consecutive admissible states. In the first stage, having six units at our disposal, we can either allocate no resource for the realization of module 1 allocate 1, 2, 3, 4, 5, or 6 units. Hence,

$$X_1\{6\} = \{0, 1, 2, 3, 4, 5, 6\}.$$

Analogously, we determine the sets of admissible decisions for the consecutive admissible states of the second stage. We obtain:

$X_2(0) = \{0\}$ $X_2(1) = \{0, 1\}$ $X_2(2) = \{0, 1, 2\}$ $X_2(3) = \{0, 1, 2, 3\}$
$X_2(4) = \{0, 1, 2, 3, 4\}$ $X_2(5) = \{0, 1, 2, 3, 4, 5\}$ $X_2(6) = \{0, 1, 2, 3, 4, 5, 6\}$

Since we have to use up the entire resource, and a certain amount will remain at the beginning of stage 3, we allocate this remaining amount entirely for the realization of module III. Therefore

$X_3(0) = \{0\}$ $X_3(1) = \{1\}$ $X_3(2) = \{2\}$ $X_3(3) = \{3\}$
$X_3(4) = \{4\}$ $X_3(5) = \{5\}$ $X_3(6) = \{6\}$

The following obvious condition has to be satisfied when the sets of admissible decisions for each state y_t are being constructed:

$$y_t \geq x_t$$

The values $F_t^1(y_t, x_t)$ describe the profit from the operation of module t, while $F_t^2(y_t, x_t)$ describes its reliability. Using the values from Table 1, we obtain the following values of $F_t^1(y_t, x_t)$:

$F_1^1(6, 0) = 0$ $F_1^1(6, 1) = 1,2$ $F_1^1(6, 2) = 2$ $F_1^1(6, 3) = 2,7$
$F_1^1(6, 0) = 3,3$ $F_1^1(6, 0) = 3,7$ $F_1^1(6, 0) = 4$
$F_2^1(y_2, 0) = 0$ $F_2^1(y_2, 1) = 3$ $F_2^1(y_2, 2) = 4,8$ $F_2^1(y_2, 3) = 5,5$
$F_2^1(y_2, 4) = 6,8$ $F_2^1(y_2, 5) = 7,9$ $F_2^1(y_2, 6) = 8,5$
$F_3^1(0, 0) = 0$ $F_3^1(1, 1) = 1,8$ $F_3^1(2, 2) = 4,5$ $F_3^1(3, 3) = 6,5$
$F_3^1(4, 4) = 7,8$ $F_3^1(5, 5) = 9$ $F_3^1(6, 6) = 10$

and the values of $F_2^t(y_t, x_t)$:

$F_1^2(6, 0) = 0.9$ $F_1^2(6, 1) = 0.97$ $F_1^1(6, 2) = 0.991$ $F_1^2(6,3) = 0.9973$
$F_1^2(6, 4) = 0.9992$ $F_1^2(6, 5) = 0.9998$ $F_1^2(6, 6) = 0.9999$
$F_2^2(y_2, 0) = 0.9$ $F_2^2(y_2, 1) = 0.94$ $F_2^2(y_2, 2) = 0.964$ $F_2^2(y_2,3)= 0.9784$
$F_2^2(y_2, 4) = 0.987$ $F_2^2(y_2, 5) = 0.9922$ $F_2^2(y_2, 6) = 0.9953$
$F_3^2(0. 0) = 0.9$ $F_3^2(1, 1) = 0.96$ $F_3^2(2, 2) = 0.984$ $F_3^2(3,3) = 0.9936$
$F_3^2(4, 4) = 0.9974$ $F_3^2(5, 5) = 9,999$ $F_3^2(6, 6) = 0.9994.$

By $F^1(y_1, x_1, y_2, x_2, y_3, x_3)$ we denote the profits from the operation of the system, while by $F^2 (y_1, x_1, y_2, x_2, y_3, x_3)$, its reliability. We obtain:

$$F^1(y_1, x_1, y_2, x_2, y_3, x_3) = F_1^1(y_1, x_1) + F_2^1(y_2, x_2) + F_3^1(y_3, x_3)$$
$$F^2(y_1, x_1, y_2, x_2, y_3, x_3) = F_1^1(y_1, x_1) \cdot F_2^2(y_2, x_2) \cdot F_3^2(y_3, x_3)$$

Figure 1 is a graphical representation of the process. The vertices of the graph represent the admissible states of the process, and the edges are the decisions.

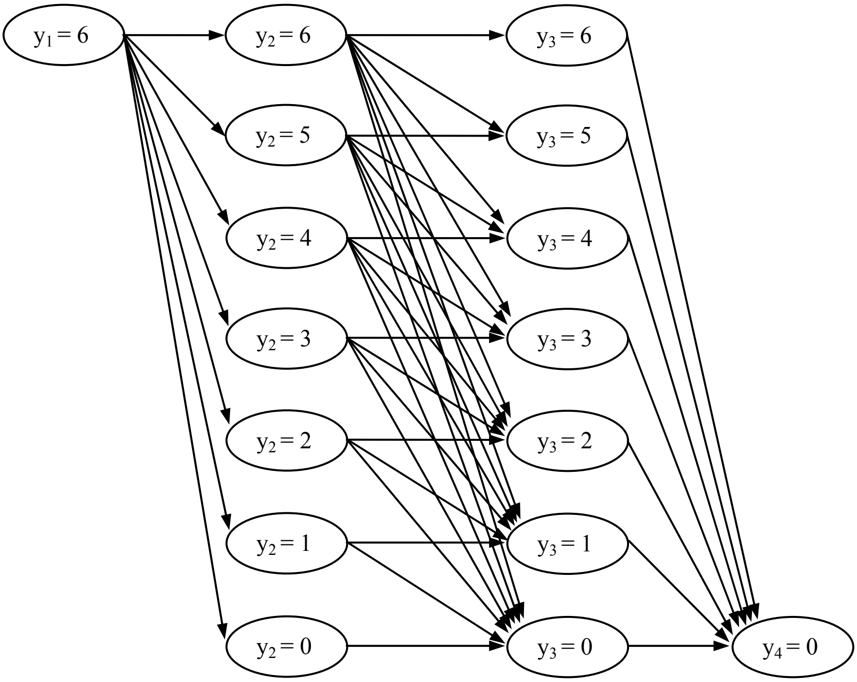

Figure 1: Graph of the process

3 The determination of the set of non-dominated evaluations and of the set of efficient realizations

Realization d' dominates realization d if

$$\forall_{k=1,\ldots,K} F^k(d') \geq F^k(d) \wedge \exists_{l=1,\ldots,K} F^l(d') > F^l(d) \qquad (6)$$

which we denote by $F(d') \geq F(d)$

Realization d* is called an efficient realization if no other realization exists whose evaluation dominates the evaluation of d*; that is, if the following condition is satisfied:

$$\sim\exists_{d' \in D} F(d') \geq F(d) \qquad (7)$$

The set of all efficient realizations is denoted by D*. The problem of vector maximization for a discrete multistage decision process consists in finding the set D* and the corresponding set F(D*) of non-dominated evaluations. We formulate this problem as follows:

$$\text{'Max'} \ \{F(d): d \in D\} \tag{8}$$

To find the sets F(D*) and D* we use the vector optimality principle, which is a modification of the optimality principle (Klötzer, 1978; Li, Haimes, 1989).

An efficient strategy has the following property: regardless of the initial state and the initial decision, the remaining decisions have to constitute a sequence of decisions efficient with respect to the state resulting from the first decision.

We formulate optimality equations, which in our case are of the form:
for t = T:

$$G_T^*(y_T) = \text{'max'} \ \{F_T(y_T, x_T): x_T \in X_T(y_T)\} \tag{9}$$

for t = T-1,…,1

$$G_t^*(y_t) = \text{'max'}\{F_t(y_t) \bullet_t G_{t+1}(\Omega_t(y_t, x_t)) : x_t \in X_t(y_t)\} \tag{10}$$

where 'max' denotes the set of non-dominated vectors of the given subset, and $\bullet_t$ denotes the stage operator which combines the evaluations in stage t.

The set $G_T^*(y)$ contains non-dominated evaluation vectors for module T. The first component of each vector in this set describes the possible profit from the operation of this module, while the second, its reliability, if y_T units of the resource can be allocated for the operation of this module. The values $G_T^*(y_T)$ are calculated consecutively for all the states $y_T \in Y_T$.

The set $G_t^*(y_t)$ contains non-dominated evaluation vectors for modules from t through T. Their first components express the possible profit from the operation of these modules, while the second, their reliability, if t units of the resource can be allocated for the operation of module t.

Detailed calculations for the numerical example are in Appendix 1. As a result, we obtain four efficient realizations which are shown in Table 2.

Table 2: Efficient realizations and non-dominated evaluation vectors

Efficient realizations	**Non-dominated evaluation vectors**
d^A = (6,0, 6,2, 4,4)	[12.6, 0.8653]
d^B = (6,1, 5,2, 3,3)	[12.5, 0.921]
d^C = (6,2, 4,2, 2,2)	[11.3, 0.94]

4 Weighted sum approach

4.1 Multistage approach

Let $z = [z^1,\dots,z^K]$ be a vector with non-negative, non-zero components, that is, $z \in R_+^K \setminus \{0\}$. For each fixed $z \in R_+^K \setminus \{0\}$, we write the scalar maximization problem in the form:

$$\mathrm{Max}\ \Sigma_{k=1}^{K} z^k F^k(d) : d \in D\}. \tag{11}$$

Let $D^0(z)$ be the set of all optimal solutions of problem (1). Using the general properties of efficient solutions in multicriteria programming, we can prove the following theorems (Trzaskalik, 1993):

Theorem 1

If for $z^0 \geq 0$ d^0 is an optimal solution of problem (11) and one of the following conditions is satisfied:

$$z^0 > 0 \tag{12}$$

$$\mathrm{card}\ D^0(z^0) = 1 \tag{13}$$

$$\mathrm{card}\ F(d^0) = 1 \tag{14}$$

then z^0 is an efficient realization of the given process, that is, $d^0 \in D^*$.

Theorem 2

The following holds:

$$\forall_{z \geq 0}\ D^0(z) \subset D^* \tag{15}$$

These theorems can be used to search for efficient solutions of our A&R problem. First let us note that in this bicriteria problem each criterion is expressed in different units. Hence, to present these criteria jointly as a weighted sum, first we have to normalize the values of the multistage criteria functions. The most convenient way of normalization of multistage criteria is to perform the transformation:

$$\Phi^k(d) = F^k(d)/F^{*k}(d) \tag{16}$$

for $k = 1,\dots, K$, $d \in D$, where

$$F^{*k} = \mathrm{Max}\ \{F^k(d), d \in D, k = 1,\dots,K\} \tag{17}$$

The results of our numerical experiment are shown in Table 3.

Table 3: Results of the calculations for $z^1 = 0.9$, $z^2 = 0$

1	**2**	**3**	**4**	**5**	**6**	**7**	**11**	**12**	**13**	**8**	**9**	**10**	**14**	**15**	**16**	**17**	**18**
1	6	0	6	0	6	6	0	0	10	0.9	0.9	0.9994	10	0.8095	0.7937	0.8611	0.8004
2	6	0	6	1	5	5	0	3	9	0.9	0.94	0.999	12	0.8452	0.9524	0.8991	0.947
3	6	0	6	2	4	4	0	4.8	7.8	0.9	0.964	0.9974	12.6	0.8653	1	0.9205	0.9921
4	6	0	6	3	3	3	0	5.5	6.5	0.9	0.9784	0.9936	12	0.8749	0.9524	0.9307	0.9502
5	6	0	6	4	2	2	0	6.8	4.5	0.9	0.987	0.984	11.3	0.8741	0.8968	0.9298	0.9001
6	6	0	6	5	1	1	0	7.9	2.8	0.9	0.9922	0.96	10.7	0.8573	0.8492	0.9119	0.8555
7	6	0	6	6	0	0	0	8.5	0	0.9	0.9953	0.9	8.5	0.8062	0.6746	0.8576	0.6929
8	6	1	5	0	5	5	1.2	0	9	0.97	0.9	0.999	10.2	0.8721	0.8095	0.9278	0.8213
9	6	1	5	1	4	4	1.2	3	7.8	0.97	0.94	0.9974	12	0.9094	0.9524	0.9674	0.9539
10	6	1	5	2	3	3	1.2	4.8	6.5	0.97	0.964	0.9936	12.5	0.9291	0.9921	0.9884	0.9917
11	6	1	5	3	2	2	1.2	5.5	4.5	0.97	0.9784	0.984	11.2	0.9339	0.8889	0.9934	0.8993
12	6	1	5	4	1	1	1.2	6.8	2.8	0.97	0.987	0.96	10.8	0.9191	0.8571	0.9777	0.8692
13	6	1	5	5	0	0	1.2	7.9	0	0.97	0.9922	0.9	9.1	0.8662	0.7222	0.9214	0.7421
14	6	2	4	0	4	4	2	0	7.8	0.991	0.9	0.9974	9.8	0.8896	0.7778	0.9463	0.7946
15	6	2	4	1	3	3	2	3	6.5	0.991	0.94	0.9936	11.5	0.9256	0.9127	0.9846	0.9199
16	6	2	4	2	2	2	2	4.8	4.5	0.991	0.964	0.984	11.3	0.94	0.8968	1	0.9071
17	6	2	4	3	1	1	2	5.5	2.8	0.991	0.9784	0.96	10.3	0.9308	0.8175	0.9902	0.8347
18	6	2	4	4	0	0	2	6.8	0	0.991	0.987	0.9	8.8	0.8803	0.6984	0.9365	0.7222
19	6	3	3	0	3	3	2.7	0	6.5	0.9973	0.9	0.9936	9.2	0.8918	0.7302	0.9487	0.752
20	6	3	3	1	2	2	2.7	3	4.5	0.9973	0.94	0.984	10.2	0.9225	0.8095	0.9813	0.8267
21	6	3	3	2	1	1	2.7	4.8	2.8	0.9973	0.964	0.96	10.3	0.9229	0.8175	0.9818	0.8339
22	6	3	3	3	0	0	2.7	5.5	0	0.9973	0.9784	0.9	8.2	0.8782	0.6508	0.9342	0.6791
23	6	4	2	0	2	2	3.3	0	4.5	0.9992	0.9	0.984	7.8	0.8849	0.619	0.9413	0.6513
24	6	4	2	1	1	1	3.3	3	2.8	0.9992	0.94	0.96	9.1	0.9017	0.7222	0.9592	0.7459
25	6	4	2	2	0	0	3.3	4.8	0	0.9992	0.964	0.9	8.1	0.8669	0.6429	0.9222	0.6708
26	6	5	1	0	1	1	3.7	0	2.8	0.9998	0.9	0.96	6.5	0.8638	0.5159	0.9189	0.5562
27	6	5	1	1	0	0	3.7	3	0	0.9998	0.94	0.9	6.7	0.8458	0.5317	0.8998	0.5685
28	6	6	0	0	0	0	4	0	0	0.9999	0.9	0.9	4	0.8099	0.3175	0.8616	0.3719

Description:

Column 1 – realization
Column 2 – state y_1
Column 3 – decision $x_1 \in X_1(y_1)$
Column 4 – state y_2
Column 5 – decision $x_2 \in X_1(y_2)$
Column 6 – state y_3
Column 7 – decision $x_1 \in X_1(y)$
Column 8 – value of $F_1^1(y_1, x_1)$
Column 9 – value of $F_2^1(y_1, x_1)$
Column 10 – value of $F_3^1(y_1, x_1)$
Column 11 –value of $F_2^2(y_1, x_1)$
Column 12 – value of $F_2^2(y_1, x_1)$
Column 13 – value of $F_2^3(y_1, x_1)$
Column 14 – value of $F^1(d)$
Column 15 – value of $F^2(d)$
Column 16 – value of $\Phi^1(d)$
Column 17 – value of $\Phi^2(d)$
Column 18 – $0.9\Phi^1(d) + 0.1\Phi^2(d)$

Thanks to the small size of the problem, we can present all the realizations of the process.

4.2 Sensitivity analysis

In the case of a bicriteria problem, we can write:

$$\text{Max } \{z^1F^1(d) + z^2F^2(d) : d \in D\} \tag{18}$$

Consider an arbitrarily fixed point $\check{z} = [\check{z}^1, \check{z}^2]' \in R_+^2$. Substituting for z the components of vector ž we obtain the problem:

$$\text{Max } \{\check{z}^1F^1(d) + \check{z}^2F^2(d) : d \in D\} \tag{19}$$

which allows to generate the efficient realization corresponding to vector ž. By $Z^+(\check{z})$ we denote the set of the points of the half-line starting at [0, 0] and passing through ž, without the point [0, 0], that is,

$$Z^+(\check{z}) = \{[z^1, z^2] : z^2 = (\check{z}^1/\check{z}^2)\cdot z^1\} \tag{20}$$

Solving problem (18) for a fixed z, we obtain efficient realizations corresponding to z. Since $z^2 = (\check{z}^1/\check{z}^2)\cdot z^1$, problem (19) can be written in the form:

$$\text{Max } \{z^1F^1(d) + (\check{z}^1/\check{z}^2)\cdot z^1F^2(d) : d \in D\} \tag{21}$$

Problem (21) is equivalent to the following problem:

$$\text{Max } \{z^1 (\check{z}^1 F^1(d) + \check{z}^2F^2(d)) : d \in D\} \tag{22}$$

which, in turn, is equivalent to (19). This means that each point of half-line Z(ž) generates the same efficient realizations. Therefore, to determine the set of efficient realizations generated by the points of a given line, it suffices to find this set for one point of the line. The most convenient to use are points satisfying the following relationship:

$$z^1 + z^2 = 1.$$

Hence it suffices to consider the problem:

$$\text{Max } \{z^1F^1(d) + z^2F^2(d) : z^1 \geq 0, z^2 \geq 0, z^1 + z^2 = 1, \ d \in D \tag{23}$$

which can be replaced by the equivalent problem:

$$\text{Max } \{\mu F^1(d) + (1 - \mu) F^2(d) : 0 \leq \mu \leq 1, \ d \in D\} \tag{24}$$

To determine the values of parameter μ for which realization d' is efficient, one has to solve the corresponding systems of inequalities.

In our problem there are three efficient realizations: d^A, d^B and d^c. The relevant systems of inequalities are of the following form:

for realization d^A:

$$\mu F^1(d^A) + (1 - \mu) F^2(d^A) \geq \mu F^1(d^B) + (1 - \mu) F^2(d^B)$$
$$\mu F^1(d^A) + (1 - \mu) F^2(d^A) \geq \mu F^1(d^C) + (1 - \mu) F^2(d^C)$$

for realization d^B:

$$\mu F^1(d^B) + (1 - \mu) F^2(d^B) \geq \mu F^1(d^A) + (1 - \mu) F^2(d^A)$$
$$\mu F^1(d^B) + (1 - \mu) F^2(d^B) \geq \mu F^1(d^C) + (1 - \mu) F^2(d^C)$$

for realization d^C:

$$\mu F^1(d^C) + (1-\mu) F^2(d^C) \geq \mu F^1(d^A) + (1-\mu) F^2(d^A)$$
$$\mu F^1(d^C) + (1-\mu) F^2(d^C) \geq \mu F^1(d^B) + (1-\mu) F^2(d^B)$$

Substituting normalized numerical values, we obtain:

for realization d^A:

$$\mu\, 1 + (1-\mu)\, 0{,}920 \geq \mu\, 0{,}992 + (1-\mu)\, 0{,}988$$
$$\mu\, 1 + (1-\mu)\, 0{,}920 \geq \mu\, 0{,}897 + (1-\mu)\, 1$$

for realization d^B:

$$\mu\, 0{,}992 + (1-\mu)\, 0{,}988 \geq \mu\, 1 + (1-\mu)\, 0{,}920$$
$$\mu\, 0{,}992 + (1-\mu)\, 0{,}988 \geq \mu\, 0{,}897 + (1-\mu)\, 1$$

for realization d^C:

$$\mu\, 0{,}897 + (1-\mu)\, 1 \geq \mu\, 1 + (1-\mu)\, 0{,}920$$
$$\mu\, 0{,}897 + (1-\mu)\, 1 \geq \mu\, 0{,}992 + (1-\mu)\, 0{,}988$$

Solving these systems of inequalities we see that:

d^A is an efficient realization for $\mu \in [0.885, 1]$,

d^B is an efficient realization for $\mu \in [0.112, 0.885]$,

d^C is an efficient realization for $\mu \in [0, 0.112]$,

Moreover, for $\mu = 0.112$ and $\mu = 0.885$ problem (24) has two optimal solutions. Hence every point of the line $Z^+(0.112, 0.888)$ allows to generate both d^B and d^C, while every point of the line $Z^+(0.885, 0.15)$ allows to generate d^A and d^B. Every point of the plane $R^+\backslash 0\}$ allows to generate an efficient realization. It can happen, however, that one of the systems of inequalities will be inconsistent, which means that there exist efficient realizations which cannot be generated using problem (18).

The solution obtained is illustrated in Figure 2.

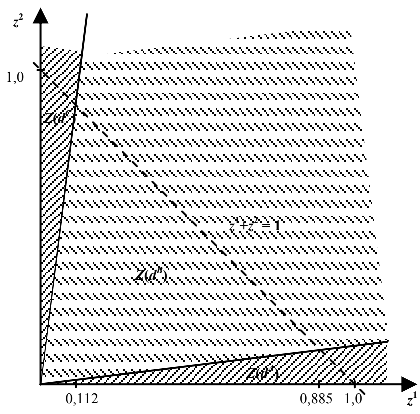

Figure 2: Graphical representation of sets $Z(d^A)$, $Z(d^B)$ and $Z(d^C)$

4.3 Stage weighted sum approach

In this subsection we consider a situation in which the DM intends to express his/her preferences as regards stage values. The preferences will be expressed as utilities, for the DM, of the normalized values of the individual stage criteria. Normalization will be performed with respect to maximal stage values for stage criteria. Hence we define new, normalized values of these criteria as follows:

$$\Phi_t^k(d_t) = F_t^k(d_t)/K \cdot F_t^k(d_t^*) \tag{25}$$

where d_t^* – arg max $\{F_t^k(d_t): d_t \in D_t\}$

We assume that the utility function is in additive form and obtain the problem:

$$\text{Max } \{\Sigma_{k=1}^K \Sigma_{t=1}^T \alpha_t^k \Phi_t^k(d): d \in D\} \tag{26}$$

in which we assume that α_t^k are non-negative and normalized, that is,

$$\forall_{t=1,\ldots,T} \Sigma_{k=1}^K \alpha_t^k = 1 \tag{27}$$

Normalization is possible for each non-negative $\{\alpha_t^k\}$; it facilitates the interpretation of the results.

Due to the form of the objective function in problem (26), we can decompose it and solve it using the standard dynamic programming method, using the functional equations

for t = T:

$$g_T(y_T) = \text{Max } \{\Sigma_{k=1}^K \alpha_T^k \Phi_T^k(y_T, x_T): x_T \in X_T(y_T)\} \tag{28}$$

for t = T-1,…,1

$$g_t(y_t) = \text{Max } \{\Sigma_{k=1}^K \alpha_t^k \Phi_t^k(y_t, x_t) + g_{t+1}(\Omega_t(y_t, x_t): x_t \in X_T(y_t)\} \tag{29}$$

Using these equations we find the optimal realization of the process.

Our discussion will be illustrated by a numerical example. We will use again the numerical data from Table 1, and stage-normalize them using formula (25). The results are shown in Table 4.

Table 4: Normalized values of stage criteria

1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	0	0.9	0.9001	0	0	0.9	0.9042	0	0	0.9	0.90054
1	1,2	0.3	0.97	0.9701	3	0.35	0.94	0.9444	2,8	0.28	0.96	0.960576
2	2	0.5	0.991	0.9911	4,8	0.56	0.964	0.9686	4,5	0.45	0.984	0.984591
3	2,7	0.675	0.9973	0.9974	5,5	0.65	0.9784	0.983	6,5	0.65	0.9936	0.994197
4	3,3	0.825	0.9992	0.9993	6,8	0.8	0.987	0.9917	7,8	0.78	0.9974	0.997999
5	3,7	0.925	0.9998	0.9999	7,9	0.93	0.9922	0.9969	9	0.9	0.999	0.9996
6	4	1	0.9999	1	8,5	1	0.9953	1	10	1	0.9994	1

The consecutive columns are as follows. Column 1 contains the amount of the resource allocated for the operation of each module. Columns 2, 6 and 10 contain profits resulting from the allocation of the given amount of the resource for the operation of modules 1, 2 and 3, respectively, while columns 3, 7, and 11 contain the values normalized using formula (25). Columns 4, 8 and 12 describe the reliability of the modules related to the amount of the resource allocated, while columns 5, 9 and 13 contain the normalized values.

We will consider three example problems:

Problem 1

The DM assumed that the profits from the operation of all the modules are less important than their reliability. This situation can be described by the following example data:

$\alpha_1^1 = 0.25$ $\alpha_1^2 = 0.75$ $\alpha_2^1 = 0.2$, $\alpha_2^2 = 0.8$ $\alpha_3^1 = 0.1$ $\alpha_3^2 = 0.9$

Problem 2

The DM assumed that the profits from the operation of all the modules are equally important as their reliability. This situation can be described by the following example data:

$\alpha_1^1 = 0.5$ $\alpha_1^2 = 0.5$ $\alpha_2^1 = 0.5$, $\alpha_2^2 = 0.5$, $\alpha_3^1 = 0.5$, $\alpha_3^2 = 0.5$

Problem 3

The DM assumed that the profits from the operation of all the modules are more important than their reliability. This situation can be described by the following example data:

$\alpha_1^1 = 0.75$ $\alpha_1^2 = 0.25$ $\alpha_2^1 = 0.8$, $\alpha_2^2 = 0.2$ $\alpha_3^1 = 0.9$ $\alpha_3^2 = 0.1$

Calculations using formulas (25) and (26) result in the solutions shown below.

Realization $d^{(10)}$ is the solution of problem 1. The value of the objective function is 0.5991.

Realization $d^{(16)}$ is the solution of problem 2. The value of the objective function is 0.7432.

Realization $d^{(21)}$ is the solution of problem 3. The value of the objective function is 0.8990.

We will compare these results with the solution of the A&R problem in its initial formulation obtained by searching for the complete set of efficient realizations. It turns out that realizations $d^{(10)}$ and $d^{(16)}$ are efficient realizations of the initial problem, while realization $d^{(21)}$, which is a solution of problem 3, is not an efficient realization. Hence, we perform efficiency testing and generate efficient realizations better that the tested one – if such realizations exist (Trzaskalik, 1990). In the case of $d^{(21)}$, realization $d^{(3)}$ is a better efficient realization.

5 Conclusions

In the paper we have presented a bicriteria A&R problem. Both multistage criteria considered – profit and reliability – are stage-wise separable and monotone, which allows to decompose the problem and to apply optimality equations to find the complete set of efficient realizations. A combination of these two criteria in one objective function, however, is not a separable scalar function, and therefore it is not possible to find optimal solutions using functional equations. In this case it is necessary to apply brute force or else approximation methods, using, for instance, genetic algorithms.

In the case of a weighted sum problem with stage values we can obtain solutions which are not efficient solutions of the initial problem. To check the efficiency of the realization obtained, we use the algorithm for checking efficiency and generating efficient realizations better than the realization tested, if such realizations exist.

In our case the weighted sum of multistage components was not separable. An open question remains: In the case of a separable function and an arbitrary choice of coefficients of stage functions, would we always obtain an efficient solution?

The problem of finding the set of non-dominated solutions in the criteria space and the corresponding set of efficient realizations has been discussed in detail in previous papers (Trzaskalik, 1990, 1998). It would be interesting to further investigate the issue of sensitivity analysis for MODP problems, since it has not been thoroughly researched so far. Another issue worth investigating in detail is that of the properties of the stage weighted sum approach.

The approach used in this paper is based on the application of a linear utility function. Another direction of research should be investigating the possibility of ordering efficient realizations from the most satisfying to the least satisfying based on determination of decision rules by means of rough sets. An example of such an application can be found in the paper by Renaud et al. (2007).

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Appendix 1

Stage 3

Assume that at the beginning of Stage 3 we have y_3 resource units available, $y_3 \in Y_3$. We find the sets:

$$G_3^*(y_3) = \text{'max'} \{F_3(y_3, x_3): x_3 \in X_3(y_3)\}$$

We calculate:

$G_3^*(0)$ = 'max' {[0, 0.9]} = {[0, 0.9]} $x_3^*(0) = \{0\}$

$G_3^*(1)$ = 'max' {[2.8, 0.96]} = {[2.8, 0.96]} $x_3^*(1) = \{1\}$

$G_3^*(2)$ = 'max' {[4.5, 0.984]} = {[4.5, 0.984]} $x_3^*(2) = \{2\}$

$G_3^*(3)$ = 'max' {[6.5, 0.9936]} = {[6.5, 0.9936]} $x_3^*(3) = \{3\}$

$G_3^*(4)$ = 'max' {[7.8, 0.9974]} = {[7.8, 0.9974]} $x_3^*(4) = \{4\}$

$G_3^*(5)$ = 'max' {[9.0, 0.999]} = {[9.0, 0.999]} $x_3^*(5) = \{5\}$

$G_3^*(6)$ = 'max' {[10, 0.9994]} = {[10, 0.9994]} $x_3^*(6) = \{6\}$

Stage 2

Assume that at the beginning of Stage 2 we have y_2 resource units available, $y_2 \in Y_2$. We find the sets:

$$G_2^*(y_2) = \text{'max'}\ F_2(y_2, x_2) + G_3^*(y_2 - x_2): x_2 \in X_3(y_2)$$

We calculate:

$G_2^*(0)$ = 'max' {[0, 0.9] $\bullet_2$ [0, 0.9]} = 'max' {[0, 0.81]} = [0, 0.81]
and $x_2^*(0) = \{0\}$

$G_2^*(1)$ = 'max' {[0, 0.9 $\bullet_2$ [2.8, 0.96] = 'max' {[2.8, 0.864] = {[2.8, 0.864]
[3, 0.94] $\bullet_2$ [0, 0.9]} [3.0, 0.846]} [3, 0.846]}
and $x_2^*(1) = \{0, 1\}$

{[0, 0.9] $\bullet_2$ [4.5, 0.984] {[4.5, 0.8856]
$G_2^*(2)$ = 'max' [3, 0.94] $\bullet_2$ [2.8, 0.96] = 'max' [5.8, 0.9024] = {[5.8, 0.9024}
[4.8, 0.964] $\bullet_2$ [0, 0.9]} [4.8, 0.8676]}
and $x_2^*(2) = \{1, 2\}$

{[0, 0.9] $\bullet_2$ [6.5, 0.9936] {[6.5, 0.8942]
[3, 0.94] $\bullet_2$ [4.5, 0.984] = 'max' [7.5, 0.9250]
$G_2^*(3)$ = 'max' [4.8, 0.964] $\bullet_2$ [2.8, 0.96] [7.6, 0.9254] = {[7.6, 0.9254]}
[5.5, 0.9784] $\bullet_2$ [0. 0.9]} [5.5, 0.8806]}
and $x_2^*(3) = \{2\}$

$$\begin{array}{llllll}
 & . \quad \{[0, 0.9] \bullet_2 [7.8, 0.9974] & & \{[7.8, 0.8977] & & \\
 & , \quad [3, 0.94] \bullet_2 [6.5, 0.9936] & & [9.5, 0.9340] & & \{[9.5, 0.9340] \\
G_2^*(4) = \text{'max'} & [4.8, 0.964] \bullet_2 [4.5, 0.984] & = \text{'max'} & [9.3, 0.9486] & = & [9.3, 0.9486]\} \\
 & [5.5, 0.9784] \bullet_2 [2.8, 0.96] & & [8.3, 0.9393] & & \\
 & [6.8, 0.987] \bullet_2 [0, 0.9]\} & & [6.8, 0.8883]\} & &
\end{array}$$

and $x_2^*(4) = \{1, 2\}$

$$\begin{array}{llllll}
 & \{[0, 0.9] \bullet_2 [9.0, 0.999] & & \{[9.0, 0.8991] & & \\
 & [3, 0.94] \bullet_2 [7.8, 0.9974] & & [10.8, 0.9376] & & \\
 & [4.8, 0.964] \bullet_2 [6.5, 0.9936] & & [11.3, 0.9578] & & \{[11.3, 0.9578] \\
G_2^*(5) = \text{'max'} & [5.5, 0.9784] \bullet_2 [4.5, 0.984] & = \text{'max'} & [10.0, 0.9627] & = & [10.0, 0.9627]\} \\
 & [6.8, 0.987] \bullet_2 [2.8, 0.96] & & [9.6, 0.9475] & & \\
 & [7.9, 0.9922] \bullet_2 [0, 0.9]\} & & [7.9, 0.8230]\} & &
\end{array}$$

and $x_2^*(5) = (2, 3)$

$$\begin{array}{llllll}
 & \{[0, 0.9] \bullet_2 [10, 0.9994] & & \{[10.0, 0.8995] & & \\
 & [3, 0.94] \bullet_2 [9.0, 0.999] & & [12.0, 0.9391] & & \\
 & [4.8, 0.964] \bullet_2 [7.8, 0.9974] & & [12.6, 0.9615] & & \{[12.6, 0.9615] \\
G_2^*(6) = \text{'max'} & [5.5, 0.9784] \bullet_2 [6.5, 0.9936] & = \text{'max'} & [12.0, 0.9721] & = & [12.0, 0.9721]\} \\
 & [6.8, 0.987] \bullet_2 [4.5, 0.984] & & [11.3, 0.9712] & & \\
 & [7.9, 0.9922] \bullet_2 [2.8, 0.96]\} & & [10.7, 0.8930]\} & &
\end{array}$$

and $x_2^*(6) = (2, 3)$

Stage 1

At the beginning of Stage 1 the process is in state $y_1 = 6$. We find:

$$G_3^*(y_3) = \text{'max'}\ \{F_3(y_3, x_3) : x_3 \in X_3(y_3)\}$$

that is,

$$\begin{array}{llllll}
 & \{[0, 0.9] \bullet_1 [12.6, 0.9615] & & \{[12.6, 0.8654] & & \\
 & [0, 0.9] \bullet_1 [12.0, 0.9721] & & [12.0, 0.8749] & & \\
 & [1.2, 0.97] \bullet_1 [11.3, 0.9578] & & [12.5, 0.9291] & & \\
 & [1.2, 0.97] \bullet_1 [10.0, 0.9627] & & [11.2, 0.9338] & & \\
 & [2, 0.991] \bullet_1 [9.5, 0.9340] & & [11.5, 0.9256] & & \\
G_1^*(6) = \text{'max'} & [2, 0.991] \bullet_1 [9.3, 0.9486] & & [11.3, 0.9401] & & \{[12.6, 0.8654] \\
 & [2.7, 0.9973] \bullet_1 [7.6, 0.9254] & = \text{'max'} & [0.3, 0.9229] & = & [12.5. 0.9291] \\
 & [3.3, 0.9992] \bullet_1 [5.8, 0.9024 & & [9.1, 0.9017] & & [11.3, 0.9401]\} \\
 & [3.7, 0.9998] \bullet_1 [2.8, 0.864] & & [6.5, 0.8638] & & \\
 & [3.7, 0.9998] \bullet_1 [3, 0.846] & & [6.7, 0.8458] & & \\
 & [4, 09999] \bullet_1 [0, 0.81]\} & & [4.0, 0.8099]\} & &
\end{array}$$

and $x_1^*(6) = (0, 1, 2)$

We obtain the following efficient realizations:

$d^A = (6,0, 6,2, 4,4)$	$F(d^A) = [12.6, 0.8654]$
$d^B = (6,1, 5,2, 3,3)$	$F(d^B) = [12.5, 0.9291]$
$d^C = (6,2, 4,2, 2,2)$	$F(d^C) = [11.3, 0.9401]$

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THE ACCURACY OF SYMMETRIC NEGOTIATION SUPPORT BASED ON SCORING SYSTEMS BUILT BY HOLISTIC APPROACH AND DIRECT RATING

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Abstract

In negotiations, the parties can be offered decision support based on formal scoring systems. These systems can be determined by means of various preference elicitation techniques and multiple criteria decision aiding (MCDA) approaches. In most situations the simplest tool is used, namely the direct rating technique (DR). In this paper we analyze to what extent the scoring system obtained by means of a mix of MARS (Measuring Attractiveness near Reference Solutions) and UTASTAR (Utilités Additives) holistic preference elicitation approaches accurately reflects the negotiator's preferences; and how much its potential inaccuracy may affect the symmetric support given to the parties. We compare the differences in the recommendation of Nash bargaining solutions offered to the parties when the bargaining analysis is determined by means of holistic and DR approaches and analyze which of them misrepresent the actual negotiation situation more. The results show that there are no significant differences when the quality of average recommendations are compared, yet the DR-based scoring system recommends the true Nash bargaining solution for more negotiation instances than the holistic one does.

Keywords: negotiation support, bargaining solutions, efficient frontier, direct rating, holistic preference elicitation, UTASTAR.

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1 Introduction

Decision theory offers many analytical approaches, methods and techniques to support decision makers in their individual and group decisions (Figueira et al., 2005; Anderson et al., 2018). It is also used in negotiation support, to assist the parties in finding mutually satisfying, fair and efficient compromises (Raiffa et al., 2003). Such negotiation support can be offered to the negotiators provided that they prepare themselves accurately in advance, i.e. in the prenegotiation phase. There are many different check-lists of the prenegotiation activities the parties should follow to make sure that they are prepared comprehensively (Zartman, 1989; Simons and Tripp, 2003). They focus on the problem definition, defining its structure (called the negotiation template), eliciting the negotiator's preferences and building the formal negotiation offer scoring system (Raiffa, 1982; Wachowicz, 2010). Since most of the negotiation problems involve multiple issues, the prenegotiation protocols usually implement methods and techniques from multiple criteria decision aiding (MCDA). The direct rating (DR) technique (Keeney and Raiffa, 1976) is considered to be one of the least cognitively demanding and technically least complicated MCDA approaches, and hence it is widely used in negotiation teaching, experiments, simulations and also in negotiation support systems (Raiffa, 1982; Kersten and Noronha, 1999; Schoop et al., 2003).

Using an adequate and efficient MCDA tool for preference elicitation and determination of an accurate negotiation offer scoring system is of critical importance for negotiators. Such a scoring system is used individually by the parties (asymmetric support) to analyze the profitability of the offers submitted, allows to compare the balance of the concessions made by each of the negotiators, analyzing the dynamics of the negotiation process, offering proactive support by a third party in suggesting the counteroffers as well as to analyze the negotiation process from the mutual perspective and to maximize the joint value of the contract or, if deadlocks occur, to determine the arbitration solutions for the parties (symmetric support). Inaccurate scoring systems result in misinterpretation of the negotiation process and lead to agreements that do not meet the true aspiration levels of the negotiators. The problem of determining an accurate scoring system is also very important in representative negotiations (Hanycz et al., 2008). When the agents negotiate on behalf of their principals, they must be sure that the support offered to them takes into consideration their principals' preferences precisely. Hence, the preference elicitation tool should be designed in a cognitively easy way that helps human decision makers to generate an accurate scoring system, assuming the agents (and, in general, the negotiators) are willing to declare their true preferences (Lee and Thompson, 2011). Otherwise, the negotiation contract, despite being considered satisfying on the basis of the ratings provided by the support tool or system, may happen to be weak, if not unprofitable.

Unfortunately, despite its simplicity, the DR approach is sometimes misused by negotiators. As shown in a series of representative negotiation experiments conducted in the Inspire system (Roszkowska and Wachowicz, 2014; Roszkowska and Wachowicz, 2015) the agents (who negotiated the contracts on behalf of their principals) were often unable to determine the scoring systems that were ordinally accurate and the ratings they used did not represent their principals' preferences well. Such inaccuracy also impacted significantly the quality of contracts. What is also important, later analyses did not allow to draw binding conclusions linking these inaccuracies with the motivations and goals of the agents (Kersten et al., 2017). Thus, the misuse of the DR mechanism may be also caused by low cognitive capabilities, a limited number sense or insufficient mathematical background of the negotiators. Hence, a new question arises: whether implementing alternative preference elicitation mechanisms can reduce preference misrepresentation and ensure more reliable decision support for the negotiators.

Alternative MCDA techniques, such as Analytic Hierarchical Process (AHP), even swaps or TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution), have been suggested for use in multi-issue negotiation support (Mustajoki and Hamalainen, 2000; Wachowicz, 2010; Roszkowska and Wachowicz, 2015). One group of methods that appear best predisposed for use in negotiation support are disaggregation methods (Jacquet-Lagreze and Siskos, 2001; Doumpos and Zopounidis, 2011), which aim at deriving the preferences of the decision maker from their holistic declarations of priorities using examples of potential solutions. The decision maker does not need to operate with quantitative evaluations and express their preferences qualitatively by considering which of the examples are better, and which are worse. This eliminates the negative effects of the negotiator's lack of decision making and mathematical knowledge and therefore the holistic approach is regarded as easy and intuitive (Siskos and Grigoroudis, 2010; Ghaderi et al., 2017). The holistic approach has already been proposed to support group and negotiation decision making problems, e.g., in the Mediator system, where the UTA (Utilités Additives) technique was implemented (Jarke et al., 1987); or in the MARS (Measuring Alternatives Near Reference Solutions) approach, where elements of ZAPROS (Closed Procedures near Reference Situation[1]) and MACBETH (Measuring Attractiveness by

[1] An acronym from Russian words.

a Categorical Based Evaluation Technique) were used (Górecka et al., 2016). These were, however, theoretical proposals only, and their applicability was not tested empirically. In a previous paper we showed that UTASTAR linked with certain notions of MARS can be used to determine the scoring systems that do not differ significantly in terms of accuracy from the ones determined by means of the DR technique (Kersten et al., 2017), therefore it is a potentially interesting and efficient tool in asymmetric individual support.

The goal of this paper is to analyze how the potential inaccuracy of the scoring systems determined by means of a mix of MARS and UTASTAR holistic approach may affect the symmetric negotiation support that can be offered to the negotiators by a third party such as an arbitrator or a negotiation support system. We analyze the records of the bilateral representative negotiation experiments conducted in the Inspire negotiation support system (Kersten and Noronha, 1999) and compare the same negotiation instances in which the preference information was provided by the agents on the basis of the information they received from their counterpart. Using the agents' preference declarations, the scoring systems were built in two ways: (1) by means of DR and (2) simulated using the MARS-UTASTAR approach. For both types of scoring systems Nash bargaining solutions are determined and compared to the one that would be the true recommendation if the principals negotiated themselves.

The paper consists of four next sections. Section 2 discusses the issue of negotiation support together with the importance of the scoring systems for symmetric and asymmetric negotiation support. In section 3 the problem of measuring the accuracy of the scoring systems is briefly presented in the context of individual and representative negotiations. In section 4 our experiment is discussed along with our approach, while in section 5 the results are presented. We conclude with a discussion and suggestions for future research.

2 Negotiation support

2.1 Negotiation template and the scoring system

Many researchers emphasize the importance of the prenegotiation preparation (Stein, 1989; Zartman, 1989; Simons and Tripp, 2003). It allows to gather all required information, prepare the negotiation strategy, analyze the potential solutions and assign to each of them a clear motivation line that can be used in the bargaining phase. It is also important from the viewpoint of the scope and quality of the negotiation support that can be offered to the negotiators by software systems or third parties. Both the individual (asymmetric) and mutual (symmetric) support may be offered to the negotiators if the negotiation problem is structured in the form of the so-called negotiation template (which is a detailed description of the structure of the negotiation problem) and if the parties' preferences are elicited for each element of the template (see Raiffa et al., 2003; Roszkowska et al., 2017).

To define a template, the countable sets X_i of salient options (x_i^j) are defined for each negotiation issue g_i, for $i = 1, \ldots, m$ and $j = 1, \ldots, |X_i|$. The template is defined as the set of issues and their resolution levels (options)

$$T = \left\{ \{g_i\}_{\forall i}, \{x_i^j\}_{\forall i,j} \right\}. \quad (1)$$

The negotiation offer scoring system is a system of cardinal ratings that represent the negotiator's preferences for all the elements of template T. Formally, it is represented as ~~a~~ the set of issue weights v_i and option ratings $v(x_i^j)$

$$S = \left\{ \{v_i\}_{\forall i}, \{v(x_i^j)\}_{\forall i;j} \right\}. \quad (2)$$

We will assume that the preferences are additive, therefore each feasible negotiation offer a which consists of selected salient options x_i^j can be evaluated using the scoring system S according to the following formula:

$$V(a) = \sum_{i=1}^{m} \sum_{j=1}^{|X_i|} z_i^j(a) \cdot v(x_i^j), \quad (3)$$

where $z_i^j(a)$ indicates if the jth option of the ith issue contains offer a (1) or not (0).

2.2 Using scoring systems in negotiation support

The negotiation template scoring system may be used during the whole negotiation process to support various activities of the negotiators (Young, 1991; Raiffa et al., 2003) in their individual activities, i.e. to offer an asymmetric support. In the prenegotiation preparation phase, after the scoring system has been built, the negotiator may use it for planning the concession strategy. They may be also used in the actual conduct of negotiations to visualize the negotiation progress by means of a negotiation history graph with concession paths of both parties. The subsequent offers submitted by the parties are scored according to $V(\cdot)$ of the scoring function of the negotiator and represented in the graph as separate data series. The negotiator may analyze the graph and consider if the concessions of both parties are reciprocal and which elements of the negotiation strategy should be implemented as an adequate response to the counterpart's moves. A negotiator's own concession paths show his/her true concessions when falling, and reverse concessions when rising. Conversely, the counterpart's concession paths show his/her concessions when rising and reverse concessions when falling. The scoring systems can also be used by negotiation support systems (NSS) to assist the negotiators in the construction of their offers in an actual conduct of negotiations (Kersten and Noronha, 1999; Schoop et al., 2003; Wachowicz, 2008) by implementing offer generators which find packages of various trade-offs (consisting of the options that vary as much as possible among the offers) for rating levels declared by the negotiators themselves.

The scoring systems of both negotiators may be applied to provide mutual symmetric support to suggest a fair solution in the negotiation process if the parties are unable to reach it themselves. This situation can occur when the aspirations of the parties are set extremely high and their willingness for concessions is limited. This may lead to deadlocks and impasses, for which the only solution is the intervention of a third party suggesting a fair solution (a compromise contract) designed on the basis of the scoring systems of both parties and taking into account their reservation levels declared as BATNA (Best Alternatives To Negotiated Agreements). Symmetric support may also be used when the parties negotiate their contracts themselves. The analysis is then focused on the verification of the efficiency of the negotiated agreement and on searching for the possible fair improvements.

In a symmetric negotiation all the feasible negotiation offers resulting from the template are presented in the rating spaces of the negotiating parties simultaneously. Consequently, each offer is represented as a vector of ratings, as shown in Figure 1.

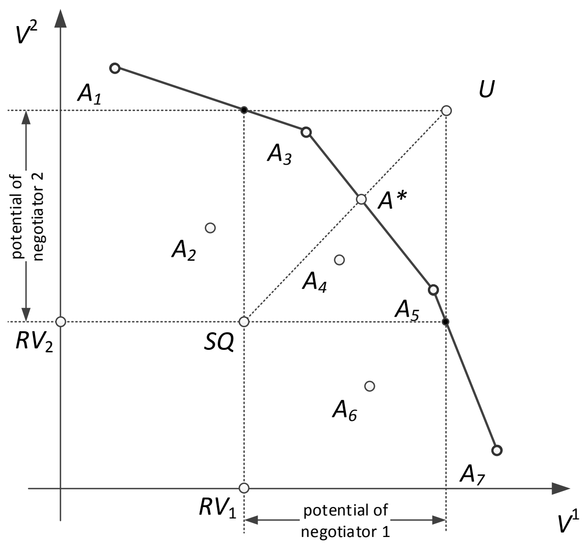

Figure 1: Symmetric analysis of the fair negotiation agreement

The status quo point (SQ) represents the reservation levels of the parties (RV_1 and RV_2, respectively), i.e., the worst outcomes that they are going to accept in these negotiations. The negotiators will not accept the contract of the values worse than their reservation levels, since they prefer to accept their BATNA (the solutions external to the negotiation process) instead. Therefore the search for a fair bargaining solution should be focused solely on the offers that outperform the SQ point and are located on the efficient frontier, i.e., are not outperformed by any other offer from the set of feasible offers.

In Figure 1 an efficient frontier consists of offers A_1, A_3, A_5, A_7. For the status quo SQ the third party may easily conclude that it is inferior and outperformed by A_3, A_5 and A_4. The first two are at the same time the efficient ones, and if the third party suggested a fair bargaining solution that left no gains at the bargaining table, only these two could be considered. Unfortunately, one of the offers (A_3) benefits more negotiator 2, while the other one (A_5) favors more negotiator 1. To identify a single and adequately balanced negotiation agreement one of the notions of fair solutions may be applied (Nash, 1950; Kalai and Smorodinsky, 1975; Gupta and Livne, 1988).

Figure 1 presents a notion of such a solution suggested by Raiffa (1953). Raiffa's idea is based on maximizing the proportion of the negotiators' potentials, which are the differences between the rating of the contract resulting from the joint reservation level (the status quo point) and the rating of the potential maximal improvement of this contract, assuming that no gains are granted to the second party. These maximal improvements are represented as an utopia point U. The intersection of the line joining SQ and U with an efficient frontier constitutes the fair Raiffa solution A^*. Note that A^* can be obtained by randomizing between A_3 and A_5. Since these two packages differ in one issue only, the randomizing amounts to finding a fair option between two neighboring salient options of these issues.

Another option is to use the notion of the Nash bargaining solution (1953). The Nash bargaining agreement is the unique solution to a two-person bargaining problem that satisfies the axioms of scale invariance, symmetry, efficiency, and independence of irrelevant alternatives. Nash proved that the solutions satisfying these axioms can be obtained by solving the following maximization problem

$$\max_{V_1(a),\, V_2(a)} \left(V_1(a) - RV_1\right) \cdot \left(V_2(a) - RV_2\right) \quad (4)$$
$$\text{s.t } V_1(a) \geq RV_1,\, V_2(a) \geq RV_2.$$

In many negotiation situations the Nash and Raiffa solutions are located close to each other (Raiffa et al., 2002).

Note that this symmetric negotiation analysis can also be implemented in the post-negotiation phase for those negotiators who negotiate their own contracts. In this case instead of using the point SQ, an actual contract is selected and subject to improvements. However, further in this paper we will analyze the problem of identifying the fair bargaining solution based on the scoring system of the agents, instead of improving the actually negotiated agreement.

3 Representative negotiations and scoring system accuracy

As shown in section 2, the scoring system offers a wide range of support possibilities. However, to ensure the reliability of the support, the scoring systems need to be accurate, i.e., they should reflect the negotiators' preferences correctly. In the preference elicitation process, each negotiator has implicitly defined their system of preferences $\mathcal{S}$ (usually in the form of non-organized and non-structured declarations and verbal descriptions) in the form of the scoring system S^N. However, during this process the negotiators' cognitive limitations related to their skills and/or the specificity of the preference elicitation technique can manifest themselves. As a result, the scoring system S^N can inaccurately represent $\mathcal{S}$. More specifically, if we assume that there is an ideal formal representation of $\mathcal{S}$ in the form of a reference scoring system S^R, then S^N can be different or discordant from S^R.

A similar problem can occur in representative negotiations, i.e., when negotiations are conducted by external negotiators (agents) on behalf of their principals. In representative negotiations the preference system $\mathcal{S}^P$of the principal, is communicated to the agent who builds the scoring system S^A reflecting the principal's preferences best[2]. As previously, it can be assumed that a theoretical formal representation of $\mathcal{S}^P$ in the form of the scoring systems S^P can be formulated, but the principal, due to his/her limited skills and formal knowledge, cannot operate with S^P directly or impart their preferences using $\mathcal{S}^P$[3].

The accuracy or concordance of S^A with respect to S^P (or S^R) may be measured in two ways, at the ordinal or cardinal level (see (Roszkowska et al., 2017)). Ordinal accuracy checks if S^A represents the same rank order of preferences as

[2] We will assume that no other incentives play a role here since the agents want to represent their principals in the best possible way, being aware that they will be evaluated on the basis of the results and their efforts during the negotiation process (Lee and Thompson, 2011).

[3] If the principal were able to define S^P, the problem would not exist for the agents, since they would only have to copy S^P into S^A.

S^P, while the cardinal accuracy measures the differences in the strength of preferences in both scoring systems. The ordinal inaccuracy index is defined by the following formula

$$OI(S^P, S^A) = |L| - \sum_{l=1}^{|L|} r_l, \quad (5)$$

where L is the set of all possible pairs of the negotiation template elements and r_l is a binary indicator describing concordance (1) or discordance (0) of the ranks resulting from the ratings for lth pair in S^P and S^A.

The cardinal inaccuracy index is defined as

$$CI(S^P, S^A) = \sum_{i=1}^{m} \sum_{j=1}^{|X_i|} \left| v^P\left(x_i^j\right) - v^A\left(x_i^j\right) \right|, \quad (6)$$

where $v^P\left(x_i^j\right)$ and $v^A\left(x_i^j\right)$ are the ratings of jth option of ith issue in S^P and S^A, respectively.

In the next sections we will try to find the difference in the accuracy of S^A as determined by means of DR and the holistic approach, and how using these approaches affects the results of the symmetric support as regards the recommendations of fair bargaining solutions

4 Organization of the negotiation experiment

4.1 Problem

We will consider the problem of representative negotiations, in which the scoring systems of the agent (S^{A1}) and their counterpart (S^{A2}) are used by the third party to suggest the efficient and fair solutions, as it was described in section 2.2. In our analyzes we will use the dataset from the bilateral negotiation experiments organized in the Inspire system (see Kersten and Noronha, 1999; Roszkowska et al., 2017).

In this negotiation the representative of a musician (Fado) negotiates a contract with the representatives of an entertainment company (Mosico). The template consists of four issues: number of promotional concerts, number of songs, royalties and contract signing bonus. For all these issues the sets of salient options are predefined. The principals provide their agents with the preference information described verbally and additionally visualized using bar graphs (see Appendix 1). Since the visualization is fairly precise the reference scoring systems of the principals (S^P) can be easily determined by measuring the bar sizes separately for the Fado and the Mosico parties. In Inspire the agents build their individual scoring systems by means of a hybrid conjoint approach (Angur et al., 1996) and the major focus is put on declaring the ratings using DR. The negotiation support offered to the parties is based on their scoring systems.

When analyzing the issues related to symmetric negotiation support and suggesting the fair bargaining solutions for the parties, one may assume that if the inaccuracy of S^{A1} and S^{A2} is not large, the negotiation spaces and efficient frontier obtained for S^{A1} and S^{A2} do not differ significantly from the ones that would be obtained from the actual preference systems of the principals, i.e., from S^{P1} and S^{P2}. Conversely, for the agents' scoring systems with high inaccuracy indexes the efficient frontiers may be totally different. This may therefore affect the final recommendation as regards the fair solution (Figure 2).

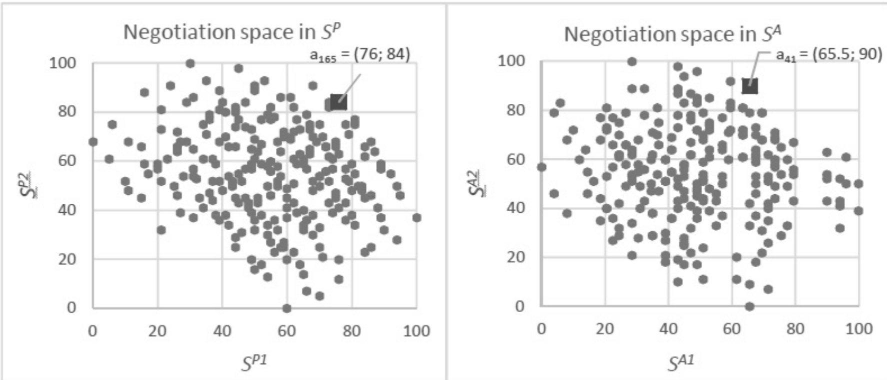

Figure 2: Negotiation space for the principals' (S^{P1} and S^{P2}) and agents' (S^{A1} and S^{A2}) scoring systems

Figure 2 shows examples of the negotiation spaces of the 240 feasible negotiation offers for the two Inspire negotiators. All feasible offers were scored separately using the principals' (S^P) and the agents' (S^A) scoring systems. S^{A1} and S^{A2} appear to be quite inaccurate, since the entire negotiation space as seen by the agents (right chart) differs significantly from what their principals see (left chart). The shapes of the efficient frontiers are also different. Finally, the fair solutions determined using the notion of the Nash bargaining solution (with $SQ = (0, 0)$) are also different. For the principals, this is another offer, a_{165}, which specifies the following contract: {7 concerts; 14 songs; 2.5% of royalties; \$200K of contract value} and results in ratings 76 and 84 for principal 1 and 2, respectively. For the agents, the Nash solution identifies as the fair solution offer a_{41}, which specifies the following contract: {5 concerts; 14 songs; 2% of royalties; \$150K of contract value} with 61.5 and 90 rating points for agent 1 and agent 2, respectively. Thus, we see that the inaccuracy of the agents' scoring systems may lead to a significant change in the recommendation of the bargaining solution. The question is: how often this happens when the holistic approach is implemented to elicit the scoring system, and how often, when the classic DR approach is used.

4.2 Method

As mentioned previously, in Inspire the scoring systems are determined individually using the conjoint hybrid approach. One of the phases of this algorithm requires the agents to provide the preference information by direct assignment of ratings. Therefore we can easily find the DR-based scoring systems (S_{DR}^{A}) and determine their accuracy (Roszkowska et al., 2017). In this paper we will also use S_{DR}^{A}'s to simulate the symmetric negotiation support and identify the recommendations of fair solutions.

The preference information provided in Inspire by the agents will also be used to simulate the scoring systems determined by means of the holistic approach. The holistic approach tested in this paper implements the modified UTASTAR algorithm (Siskos and Yannacopoulos, 1985). In UTASTAR, instead of assigning the numerical scores $v(x_i^j)$ directly, the negotiator ranks the selected offers defined in the reference set $A_R \subset A$ and this information is used to build a linear program that minimizes errors in the estimations of offers from A_R. By solving the program, the ratings of salient options $v(x_i^j)$ are determined. It is assumed that the marginal scoring functions are piece-wise linear between the neighboring salient options. Hence, for a quantitative issues any option from between x_i^j and x_i^{j+1} can be evaluated using a linear interpolation between their ratings, i.e. between $v(x_i^j)$ and $v(x_i^{j+1})$.

Taking into account the method of preference elicitation in UTASTAR, this approach seems well suited to the problem of low cognitive capabilities or decision making skills of potential negotiators. They do not need to operate with numbers while declaring their priorities, nor are they are forced to declare the importance (weights) of the negotiation issues directly. On the contrary, they define only the examples of offers that can appear on the negotiation table and are asked to rank order them. There is also no need to provide the information on strength of preferences.

One of the problems with using UTASTAR is the definition of the reference set A_R. In our earlier papers we show that depending on the informativeness of such a set, the scoring system can happen to be more or less accurate (Roszkowska et al., 2017). In this paper we will therefore use a reference set of example alternatives determined according to another MCDA holistic approach called MARS (Górecka et al., 2016). In MARS the alternatives are built on the basis of the negotiation template and a general declaration by the negotiators of the best negotiation option for each issue. Using this information the alternatives are composed that consist of the best options for all the issues but one. Such a composition of offers allows the negotiators to easily compare any two offers, since it requires to analyze a trade-off between two issues only. An example of the MARS-based reference set for the Mosico party is shown in Table 1.

Table 1: The MARS-based offers close to the ideal one for the Mosico party in the Inspire negotiation

Offer	Concerts	Songs	Royalties	Contract
1	5	14	2.0	125 000
2	6	14	2.0	125 000
3	7	14	2.0	125 000
4	8	11	2.0	125 000
5	8	12	2.0	125 000
6	8	13	2.0	125 000
7	8	14	1.5	125 000
8	8	14	2.0	125 000
9	8	14	2.0	150 000
10	8	14	2.0	200 000
11	8	14	2.5	125 000
12	8	14	3.0	125 000
13	8	15	2.0	125 000

In our experiment we will use the preference information provided by the Inspire agents for their S_{DR}^{A}'s to rank the MARS-based alternatives; the resulting rank order will be used to feed the UTASTAR linear programming model. Since the preferences specified by the Inspire principals are non-monotonous (see Appendix 1) the final LP model needs to be set up according to its extensions that handle unimodal preferences. We will implement the UTA-NM algorithm tuned to produce normalized results (Despotis and Zopounidis, 1995). By solving the final LP model we will obtain the holistic-based (MARS-UTASTAR) rating system S_{UTA}^{A}.

The systems S_{DR}^{A} and S_{UTA}^{A}will be used independently to determine the negotiation spaces for each negotiation dyad. Next, for each negotiation space the Nash arbitration procedure will be run to identify the fair bargaining solution (Nash, 1950), as described in section 2.2. The solutions determined for S_{UTA}^{A}- and S_{DR}^{A}-based spaces will be compared to the actual Nash bargaining solution in the principals' negotiation spaces (the S^{P}-based ones) and the deviations will be measured. A direct comparison of the efficaciousness of the support provided by means of DR- and UTA-based scoring systems may also be performed by comparing the average Nash products.

Note that in our experiment we focus on the problem of searching for the fair bargaining solutions independently from the actual results the agents obtained in their negotiation in Inspire. For the scoring spaces of both agents we determine the Nash arbitration recommendation simulating the situation in which the negotiators are unable to find the contract themselves. As there is no additional information about the BATNA or the reservation levels of the principals in preference description, we used $SQ = (0, 0)$. In this way we eliminate other factors that can affect the results when the improvements of the actual negotiation agreements are analyzed, such as the differences in negotiation skills, strategies and motivations that made the agents end their negotiation in a particular zone of the negotiation space for which there exists a limited number of improvements (e.g., one) which are identical regardless whether the agent's or the principal's scoring systems are considered, and – consequently – the recommendations are also identical and 100% accurate.

As mentioned before, in our analyses we use the dataset from the Inspire system's bilateral negotiation experiments conducted in a few rounds between spring 2015 and spring 2017. From the whole dataset we removed the incomplete records of those dyads which did not reach an agreement. This allowed to gather a database of 706 records of agents (353 representatives of Mosico and 353 of Fado).

5 Results

We analyzed the Inspire dataset using a special-purpose spreadsheet add-in using the VBA code that implemented the modified MARS-UTASTAR approach with the NM rescaling method. The results we obtained from our dataset indicate that the agents had a great problem with determining the accurate scoring systems using the hybrid conjoint algorithm with the DR approach. Only 23% of the Fado agents and 31% of the Mosicos were able to build a scoring system ordinally fully accurate with the preferences of their principals. The accuracy of the DR-based scoring system resulting from the conjoint hybrid algorithm and the simulated UTASTAR-based one for the whole group of experiment participants is shown in Table 2.

Table 2: Average ordinal and cardinal inaccuracy of the DR- and UTA-based scoring systems for all participants of the Inspire experiment

Agent	Ordinal inaccuracy		Cardinal inaccuracy	
	S_{DR}^{A}	S_{UTA}^{A}	S_{DR}^{A}	S_{UTA}^{A}
Mosico	3.45	2.91	73.32	108.88
Fado	3.34	2.94	67.07	86.72

Note: All differences significant for p<0.001 in the Mann-Whitney test.

Interestingly, the scoring systems determined by the modified MARS-UTASTAR algorithm appeared ordinally more accurate to S^P than the ones determined by means of the DR technique, but they were cardinally more inaccurate than the latter. The poor results of the holistic scoring systems in terms of cardinal accuracy may result from the fact, that the MARS-UTASTAR LP model was tailored for a specific situation, with an assumed (and correct) order of preferences for options. This can also be a reason for the better ordinal fit of the UTA-based scoring system whose model enforced a particular monotonicity of marginal scoring functions and only in an extreme situation could this monotonicity be violated, i.e., when a monotonically increasing marginal rating function was represented by the MARS-UTASTAR LP model as a monotonically non-decreasing one. Therefore, to eliminate the negative effects of the model setup we limited our further analysis to the subset of agents that were ordinally accurate with $OI(S^P, S^A_{DR}) = 0$, i.e., who met the requirements of monotonicity of marginal utility functions required by our UTASTAR model. This allowed us to avoid the collision of assumptions of the preference elicitation technique with the true structures of the agents' preferences.

For the dataset limited to the initially ordinally accurate agents, the scoring systems determined by means of the modified MARS-UTASTAR algorithm (S^A_{UTA}) appeared as accurate with respect to the principal's preferences (S^P) as the ones determined by means of DR in terms of ordinal accuracy. When cardinal accuracy is considered, the results are not so evident. The UTASTAR- and DR-based scoring systems seems significantly different in accuracy depending on their role (see Table 3). They differ significantly for the Mosico party, but not for the Fado one.

Table 3: Average ordinal and cardinal inaccuracy of the DR- and UTA-based scoring systems for the initially accurate agents

Agent	Ordinal inaccuracy		Cardinal inaccuracy	
	S^A_{DR}	S^A_{UTA}	S^A_{DR}	S^A_{UTA}
Mosico	0.00	0.00	37.68*	51.40*
Fado	0.00	0.01	33.26	33.58

* $p<0.001$ in the Mann-Whitney test

Within the reduced subset of Inspire's database we have selected the dyads of agents that negotiated with each other (i.e. those who were simultaneously ordinally accurate) to find the possible results of symmetric support for them. Surprisingly we found out that there were only 32 such dyads (out of 353 of all negotiating pairs). For these dyads we determined the alternatives that can be recommended as Nash bargaining solutions if the negotiation fails (for d_1 and d_2 equal to 0). The results of the symmetric support recommendations for the DR- and MARS-UTASTAR-based scoring systems are shown in Tables 4 and 5.

Table 4: The fair solution recommendations in S^A_{UTA}

Offer no.	Offer				Number of recommendations	Principal's ratings	
	concerts	songs	royalties	contract		Mosico	Fado
164	7	14	2.5	150 000	10 (31%)	81	77
165	7	14	2.5	200 000	5 (16%)	76	84
162	7	14	2.0	200 000	5 (16%)	81	75
104	6	14	2.5	150 000	3 (9%)	73	84
101	6	14	2.0	150 000	3 (9%)	78	75
225	8	14	2.5	200 000	2 (6%)	84	66
161	7	14	2.0	150 000	2 (6%)	86	68
222	8	14	2.0	200 000	1 (3%)	89	57
102	6	14	2.0	200 000	1 (3%)	73	82
Sum:					32 (100%)		

Table 5: The fair solution recommendations in S^A_{DR}

Offer no.	Offer				Number of recommendations	Principal's ratings	
	concerts	songs	royalties	contract		Mosico	Fado
165	7	14	2.5	200 000	9 (28%)	76	84
162	7	14	2.0	200 000	6 (19%)	81	75
102	6	14	2.0	200 000	4 (12%)	73	82
164	7	14	2.5	150 000	4 (12%)	81	77
101	6	14	2.0	150 000	3 (9%)	78	75
105	6	14	2.5	200 000	2 (6%)	68	91
161	7	14	2.0	150 000	2 (6%)	86	68
228	8	14	3.0	200 000	1 (3%)	69	68
104	6	14	2.5	150 000	1 (3%)	73	84
Sum:					32 (100%)		

It is worth noting that the Nash recommendation for S^P is the following offer: {7 concerts; 14 songs; 2.5% of royalties and 200 000 of contract signing bonus} (shaded in the tables). We can see that such a solution was not a common recommendation among other fair solutions suggested by the Nash procedure when the DR and MARS-UTASTAR scoring systems are used. Only in five negotiations (16%) conducted by the agents supported according to S^A_{UTA} and in nine (28%) supported by S^A_{DR} the symmetric support is the same as it would be for the principals (if S^P were used). The fraction test would confirm that these proportions are significantly different, but the sample is too small for this conclusion to be accepted as binding.

On the other hand, when the efficiency of such fair solution recommendation is considered we find out that for S_{DR}^{A} the Nash algorithm indicated the inefficient solutions, i.e., the Pareto-dominated ones, for six negotiating dyads (19% of cases). These are offers 102, 104 and 228. For the symmetric analysis based on S_{UTA}^{A} there were only four negotiating dyads that would receive inefficient recommendation (offers 102 and 104).

To find a single scalar measure of the efficiency of the symmetric support that can be offered to the parties as the consequences of their using S_{UTA}^{A} and S_{DR}^{A} we decided to determine the Nash product of the fair solution recommendation for each dyad. Note that this product is determined in the scoring spaces of their principals (S^{P}) to find out how the inaccuracy of both S_{UTA}^{A} and S_{DR}^{A} affects the true result of the key stakeholders. The results are shown in Table 6.

Table 6: The fair solution recommendations in the S_{DR}^{A}- and S_{UTA}^{A}-based negotiation spaces

S_{UTA}^{A}-based negotiation space

Offer id	No. of offers	Principal score		Nash product
		Mosico	Fado	
164	10 (31%)	81	77	6237
165	5 (16%)	76	84	6384
162	5 (16%)	81	75	6075
104	3 (9%)	73	84	6132
101	3 (9%)	78	75	5850
225	2 (6%)	84	66	5544
161	2 (6%)	86	68	5848
222	1 (3%)	89	57	5073
102	1 (3%)	73	82	5986
Average:				6077

S_{DR}^{A}-based negotiation space

Offer id	No. of offers	Principal score		Nash product
		Mosico	Fado	
165	9 (28%)	76	84	6384
162	6 (19%)	81	75	6075
102	4 (12%)	73	82	5986
164	4 (12%)	81	77	6237
101	3 (9%)	78	75	5850
105	2 (6%)	68	91	6188
161	2 (6%)	86	68	5848
228	1 (3%)	69	68	4692
104	1 (3%)	73	84	6132
Average:				6101

When we look at the average distance from the Nash fair solution observed in the negotiation spaces determined by means of each type of scoring systems, we find out that the rating products are similarly distant from the true Nash solution determined for the principals, which is equal to 6384. Interestingly, the average products for S_{DR}^{A}- and S_{UTA}^{A}-based support, equal to 6101 and 6077, respectively, are insignificantly different ($p = 0.97$ in Wilcoxon test).

6 Discussion and conclusions

As shown in the previous section, the holistic approach for generating the negotiation offer scoring system may be perceived as an interesting alternative to the classic direct rating. It is commonly perceived as being cognitively less demanding than DR, so in practical applications it should be evaluated by the negotiators highly with respect to its usefulness and ease of use. And here, in our research we were able to show that it also is capable to determine the scoring systems of the accuracy not lower that obtained in DR. Such a conclusion cannot be, naturally, derived from the results presented in Table 2, where the cardinal accuracy of the holistically determined scoring systems is much worse (significantly at $p < 0.001$) than of those determined by direct rating. However, one needs to be aware that this is partially an effect of a fixed MARS-UTASTAR LP model used in deriving the scoring systems from the rank orders of example offers. In our design we assumed that the agents are willing to represent the principal's preferences accurately, so they declare the best and worst options for each issue (which is necessary to construct the LP model) according to the principal's best and worst choices. In fact, in the experiment some of them might have made mistakes in such declarations, so the assumptions of the LP model did not fit their true declarations and hence produced a scoring system with greater inaccuracies. This is confirmed by the results presented in Table 3, where we limited the analysis to those agents who declared the preferences in ordinal accordance to their principals' preference information. Here, the general structure of the preferences fit the model's assumptions and the results show that the holistic and the DR-based scoring systems perform similarly.

There are, however, differences in performance between the groups of agents playing different roles. The scoring systems determined by DR and by the holistic approach do not differ significantly for one group of agents, i.e., the Fados; but they do for the Mosicos. In a typical decision-making situation, in which human agents use different methods to elicit their preferences there may be many reasons for such differences. The group of agents playing one role may have different decision making skills and cognitive capabilities and hence may be able to declare their preferences in an equally accurate way using DR or the holistic approach (such as the Fados in our experiment) than the other group (the Mosicos). On the other hand, there may be nuances in the structure of preferences that may cause problems in accurate disaggregation of them by the agents, which may result in better accuracy of the DR-based scoring systems than of the holistic ones. The latter may also cause problems for technical reasons, i.e., in the appropriate setup of the MARS-UTASTAR LP model. For instance, setting too few equidistant breakpoints for the determination of the marginal value function may result in false ratings for some resolution levels that are important to the agent but lie between any two neighboring breakpoints. Since in our experiment the UTA-based scoring systems were simulated using the numerical preferences defined earlier by the agents, no behavioral issues related to a cognitive limitation could have caused the differences in accuracy of the scoring systems for the Mosicos but not for the Fados. Therefore we presume that they could result from the differences in the structures of preferences between the roles that the agents had to represent, which (in the case of the Mosicos) may not fit well the structure of the LP model we used.

From the viewpoint of the symmetric support that can be offered to the parties in bilateral negotiation, the holistic approach does not seem very efficacious. It appears that based on the holistically defined scoring systems the third party's recommendation of the fair negotiation agreement differs for a vast majority of instances of negotiating agents (84%) from the one that could be offered to their principals. Note that we have studied the most optimistic situation, i.e. the one in which two parties negotiate, both having ordinally accurate scoring systems. The situation for more inaccurate agents can be even worse. A risk was also identified (12.5%), only slightly worse than the change for the best fair solution recommendation (16%), that an inefficient final contract can be suggested to the agents! When the average quality of recommendations was measured as the Nash product for the holistic approach (6077) it appeared significantly different from the "optimal" product value of 6384 resulting from the principal's recommendations ($z = -6.025$, $p < 0.001$). But again, if we compare these results with the ones obtained in a similar analysis for S_{DR}^{A} (the average Nash product equal to 6101) it appears that the differences between the recommendations do not differ significantly. Both the holistic and the direct rating approaches reveal the same level of efficaciousness (unfortunately, somewhat poor) in providing the negotiators with a reliable symmetric support. This clearly shows that while offering to the negotiators (and agents) various decision support tools one needs to make sure that the users are able to use this tool and to ensure a good quality of preference information to be provided.

A checkup mechanism should be introduced in the prenegotiation preference elicitation protocols that would analyze the reliability of the preference information provided by the negotiators and ensure additional runs of interactions if the detected accuracy is too low.

We need to emphasize that the results obtained in our experiment come from a purposely designed negotiation experiment in which the S_{UTA}^{A} were obtained in a simulation to ensure their comparability with S_{DR}^{A} that were derived from the hybrid conjoint measurement approach implemented in the Inspire system. Consequently, we did not test here the users' individual ability to generate the scoring systems in a holistic way. The situation we analyzed assumed only that some level of accuracy is feasible. Some unpublished results from our in-class prenegotiation experiments show that the negotiators may be unable to determine holistically the scoring systems of good quality when unsupported in the construction of A_R or in the declaration of certain parameters of the UTASTAR LP model. If such an additional support is offered, and the prenegotiation protocol is additionally designed to hybridize the holistic approach with the possibilities of a manual tuning of S_{UTA}^{A}, the accuracy of the support may be even higher than those obtained by means of single DR- or UTA-based approaches. This has been already proved by the initial experiments conducted in the eNego system (https://web.ue.katowice.pl/enego/).

Future research dealing with designing the prenegotiation protocol ensuring an efficient asymmetric and symmetric negotiation support should be therefore focused on identifying the determinants and characteristics of the negotiators (agents and principals) that make them prone to misinterpret the preference information and to declare it incorrectly in the preference elicitation process. Identifying the groups of agents of various cognitive capability who are able to go through the preference elicitation smoothly and correctly will allow to adjust the potential protocols and methods to reduce or extensively eliminate the potential errors, biases and heuristics.

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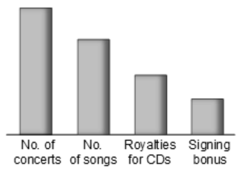
Appendix 1
Principal's preference information in the Inspire experiment

Mosico

Before meeting Fado you discussed the Agency's priorities and requirements with senior management. Senior managers could not give you very detailed information regarding the importance of the negotiated issues and options, but during a few short meetings they gave you many indications as to the relative importance of the issues and the agency's preferences. To help visualize the relationship between the issues you drew bars with their height indicating the issues' importance. You did the same for the options of each issue.

Note: The bars are only indicative of the management's preferences as you did not measure precisely the height of each bar. You drew them quickly to show to the management so that they could see whether you correctly understood their intentions.

Importance of the four issues:

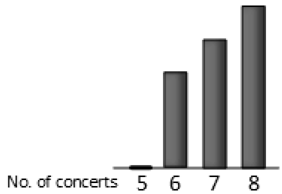


- It is clear that the most important issue is the **number of promotional concerts**. This is because successful concerts are critical to the artists' popularity and approval ratings. Without the concerts the agency cannot establish the artist in a particular market.
- The second most important issue is the **number of new songs**. Obviously the artist has to produce new songs to be recognized and accepted.
- **Royalties for CDs** are less important than the number of songs. The management considers the royalties to be a motivating factor for the artist to produce good CDs.

- The **contract signing bonus** is the least important issue. It is less important than the royalties for CDs. This is because the agency views a contract as an investment opportunity that can bring in many of millions of dollars. The bonus size is seen as a token of appreciation, but obviously within limits.
- The illustration of the issue importance is given in the figure.

1. Number of promotional concerts

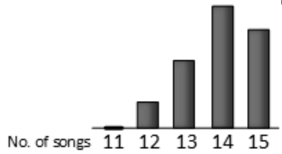
This is the most important issue for the management. The more concerts the better for WorldMusic. From your discussion with the management, it follows that:



- The most preferred option is 8 concerts.
- The difference between 7 and 8 concerts is almost the same as between 6 and 7 concerts.
- 5 concerts is significantly worse than 6.
- Less than 5 concerts cannot be accepted because it makes little sense in the entertainment business.

2. Number of new songs

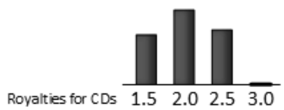
It is a long established practice that too few songs are disastrous but too many are also not profitable. The best number of songs is 14; 14 songs make two full CDs.



- 15 songs are worse than 14 because it is considered somewhat too many.
- 13 songs are a little worse than 15.
- 12 songs are worse than 13 because 13 songs allow the discarding of the worst song if necessary.
- Having 11 new songs is the worst option because only one CD can be produced.

3. Royalties for CDs

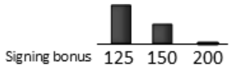
Royalties strongly depend on the artist's present standing. Typically, WorldMusic pays between 2.0% and 2.5% royalties. If the artist is very well known during contract signing, the royalties can go up to 3%. Based on the research done regarding Ms. Sonata's standing, the management considers:



- 2.0 % the best option;
- 2.5% is considered somewhat too high.
- The management prefers 2.0% much more than 1.5% because of the artist's standing. And it makes little sense to try and save a little now and loose the artist's interest in cooperating with the agency.
- The research done convinced the management that 3.0% is too much.

4. Contract signing bonus

This issue is considered the least important, although the agency does not want to be seen as throwing money away. The management's preference is to pay less rather than more.



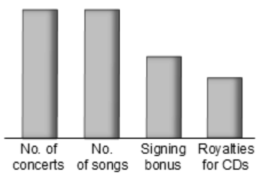
The information you obtained about the agency's top management preferences is your guide in your negotiations with Fado. It reflects WorldMusic strategic directions in the next three years and will provide guidance not only for this negotiation but also for negotiations with other artists. Therefore the ratings are quite sensitive and you were told not to discuss them with anyone.

Fado

You organized a meeting with Ms. Sonata to discuss these issues. Based on your experience, you know that artists have difficulties expressing their preferences over these issues. You used simple software to help Ms. Sonata visualize her preference on issues and options in the negotiation. During the meetings she was able to give you many indications as to the relative importance of the issues and her preferences. To show Ms. Sonata the relationship between the issues you drew bars with their height indicating the issues' relative importance. You did the same for the options of each issue.

Note: The bars are only indicative of Ms. Sonata's preferences as you did not measure precisely the height of each bar. You drew them quickly and show to Ms. Sonata so that she could see whether you correctly understood her intentions.

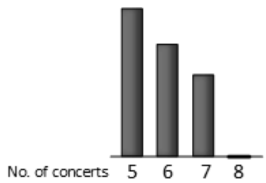
Importance of the four issues:



- You asked Ms. Sonata to think aloud the importance of issues. She said that this is quite easy, every issue is important to her. But, she added, she really does not want to have too many **promotional concerts**, so it is very important for her that she has as few concerts as possible.
- Ms. Sonata says that she must write as many **new songs** as she can, because this is her only way to enrich her fans. This issue of new songs is equally important to the first issue, promotional concerts.
- **Signing bonus** is less important than the first two issues. Although she would like to make money, she must remain true to herself; that is, write and sing songs.
- She is the least concerned with the **royalties for CDs.**
- The illustration of the issue importance is given in the figure.

1. Number of promotional concerts

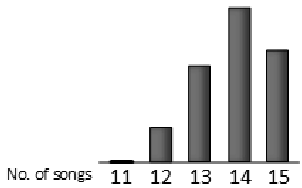
This issue is very important because Ms. Sonata would rather have no concerts at all. She understands that it is not possible so her preference is the fewer concerts the better.



- She finds that between 5 and 7 concerts, every additional concert is equally bad for her.
- But she considers giving 8 concerts a lot worse than 7.

2. Number of new songs

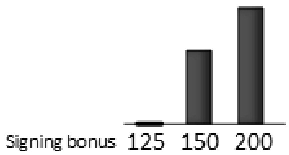
Ms. Sonata likes writing songs. After you noted that the maximum number of songs is 15 in the contract form, she was surprised.



- She said that the best for her would be writing 14 songs because she also writes poetry and short stories.
- 15 songs somewhat worse than 14, because she thinks it is a bit too many.
- Her preference for 13 is a little lower than 15.
- She added that 12 songs is barely acceptable, while 11 is not enough.

3. Contract signing bonus

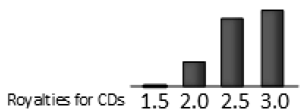
Ms. Sonata considers this issue much less important than the first two issues. This is not to say that the bonus is not important; her obvious preference is to obtain a higher bonus rather than a lower one.



She notes, however, that the difference between 125 and 150 thousand dollars is greater than between 150 and 200 thousand.

4. Royalties for CDs

This is the least important issue for Ms. Sonata but, she notes it does not mean that royalties for CDs is unimportant.



She naturally prefers higher royalties rather than lower. However, her preference for 1.5% and 2.0% are much lower than her preference for 2.5% because she thinks that receiving a very low royalty insults her musical talents. The 3.0% is obviously the best but not so different form 2.5%.

Your ratings will guide you in your negotiations with Mosico. Because they reflect Ms. Sonata's preferences and also describe her attitude towards monetary and non-monetary issues, she instructed you not to discuss them with anyone.