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DOMAIN SPECIFIC KEY FEATURE EXTRACTION USING KNOWLEDGE GRAPH MINING

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Abstract

In the field of text mining, many novel feature extraction approaches have been propounded. The following research paper is based on a novel feature extraction algorithm. In this paper, to formulate this approach, a weighted graph mining has been used to ensure the effectiveness of the feature extraction and computational efficiency; only the most effective graphs representing the maximum number of triangles based on a predefined relational criterion have been considered. The proposed novel technique is an amalgamation of the relation between words surrounding an aspect of the product and the lexicon-based connection among those words, which creates a relational triangle. A maximum number of a triangle covering an element has been accounted as a prime feature. The proposed algorithm performs more than three times better than TF-IDF within a limited set of data in analysis based on domain-specific data.

Keywords: feature extraction, natural language processing, product review, text processing, knowledge graph.

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1 Introduction

Online consumer reviews consist of indefinite statements based on a specific product (Park, Kim, 2008). Open-ended comments exhibit reviewer's judgment of a product based on negative and positive polarity (Willemsen et al., 2011). This type of open-ended textual content is cognate with customer satisfaction. Customer satisfaction is a metric to quantify the degree to which a customer will react based on a subject or the aspects of the subjects. Here the subject is a product, and aspects are the features. Knowledge discovery (Feldman, Dagan, 1995) from textual corpus refers to the process of bringing-out thought--provoking patterns or knowledge from unorganized text documents. In this case, it is the review data. Now, gaining knowledge from a vast database can be challenging (Houari, Rhanoui, Asri, 2015). Therefore, the primary intent is to get the most talked-about features or aspects.

Here we propose a novel method using a graph-based approach to extract key features from product review data. To generate a language pattern, POS (Parts of Speech) tagging is imperative. As post-POS tagging, we can extract features that are nouns. After doing POS tagging, we need to extract features that are nouns. Nouns are to be considered the main feature we are looking for in the central database. As per Oxford (www 8), "a word (other than a pronoun) used to identify any of a class of people, places, or things (common noun), or to name a particular one of these to know as a noun". We can consider a noun as our main feature, which we should look for as our main subject or aspect of the subjects. For example, consider the statement, '*The battery life of this camera is too short*'. As we can observe, '*battery*' and '*camera*' are those two entities, we can consider these words as a subject (or aspect), and the user's review is based on these two subjects (or aspects) that are '*camera*' and '*battery*'; also, this corroborates that identifying domain product features that are talked about by customers by using the manually tagged POS belongs to nouns (Htay, Lynn, 2013). Hence, a noun can be considered a subject or an aspect. Our whole idea is based on pivoting the noun as the main feature.

Selecting a noun as a central feature can be considered as a bias. While developing this algorithm, we are incorporating this bias from our expertise in this particular domain. Pre-mentioned is a kind of inductive bias. The word '*bias*' suggests an awareness of the predetermined notion instead of the neutral evaluation of reality (Campolo et al., 2018). In this sense, the world around us is biased. Most machine learning techniques have a predisposition towards this projection of bias. This type of bias is historical bias. It is often explored by comparing the relation between features or aspects of the elongated domain prowess. Zhao et al. (2017) show that if we compare the label '*cooking*' in a particular data set, it co-occurs inequitably in women more than men. Since most machine learning approaches rely on correlations, such biases may proliferate to learned models or classifiers.

Similarly, we can assume that when reviewing product review data for mobile phones, words like '*battery*' and '*camera*' co-occur with the mobile phone. Also, let's consider some other POS such as Adjective, Adverb, and Verb. We have developed this algorithm considering mobile review data from an e-commerce platform. As per Merriam-Webster (www 1), "An adjective is a word belonging to one of the major form classes in any of numerous languages and typically serving as a modifier of a noun to denote a quality of the thing named, to indicate its quantity or extent, or to specify a thing as distinct from something else", for instance, '*Camera is good*'. Here '*good*' is an adjective which denotes the quality of the thing named which is nothing but the camera. It is a noun. Also, as per Merriam-Webster (www 1) "a word that characteristically is the grammatical centre of a predicate and expresses an act, occurrence, or mode of being, that in various languages is inflected for agreement with the subject, for tense, for voice, for mood, or aspect, and that typically has full descriptive meaning and characterizing quality but is sometimes nearly devoid of these especially when used as an auxiliary or linking verb", for instance, '*Camera hangs a lot*'. Here '*hang*' is a verb that expresses an act, occurrence, or mode of being, that in various languages is inflected for agreement with the subject '*camera*'. Now, most reviews will have both positive and negative comments (Safrin et al., 2017). Also, it has been observed that in the case of a vast data set, people tend to use synonyms to describe product features or use the exact words. For instance, '*The camera quality is awesome*' and '*The camera is super*'. Here, '*awesome*' and '*super*' bear lexically similar meaning and are associated with a noun subject which is here '*camera*'. It conveys a positive sentiment. Similarly, in '*Camera is worst*' the word '*worst*' and '*super*' again have lexical antonym property that carries with the subject noun '*camera*'. During our research work, we have found out that if we build a graphical model considering all available nouns, adjectives, verbs, and adverbs as vertices and create an edge between each pair of vertices based on some relations, then we will develop relational triangles and the most significant number of triangles will contribute to the most talked about feature. If we can create a dictionary based on words with lexically similar meaning and associate it with our maximum number of triangles, we will likely encounter the most talked-about features. In this case, we are considering a weighted knowledge-based graph. Here we have also tried to use the *n*-gram model. The main goal of the *n*-gram model is to predict the context from the target word; the model transposes the contexts and targets and attempts to predict each context word from its target word. The main objective becomes to predict the context. We can consider a forward and backward window like the *n*-gram concept for this surrounding the target word, which is to be used for context prediction. The contexts words are nothing but the noun; in our case, we are more interested in the context words, which are nouns, surrounded by adjectives, adverbs, and verbs. So, we can say that a noun is our target word. The backward and forward windows must have the same size. Now we are focusing on finding the relation between adjective, adverb, and verbs. We have observed that since the main corpus is about review feedback, it must be associated with words that convey positive or negative polarity. We tried to bind these words with their synonym and antonym properties. For this, we have built a dictionary and trained our model with it. The output is a triangle, from where we can consider the feature that people have talked about the most. The most significant number of triangles associated with a noun is the most talked-about feature. In the following section, we will describe our approach in more detail.

2 The objective of the study and the novelty of the work

Feature extraction in Natural Language Processing (NLP) using graph theory is a new research field. Many research workers have proffered countless ideas. Hitherto the associated work (Markov, Last, Kandel, 2007; Wang, Do, Lin, 2005) has given special attention to the collocation of words and their recurrence as graphs instead of the sentence's linguistic interpretation. One research paper (Sidorov et al., 2013) has propounded linguistic information and word order in a graph for text classification; unfortunately, the result was limited to minimal texts of between 8 to 13 tokens. Shi et al. (2017) have proposed an idea to extract key phrases using knowledge graphs. They emphasized the latent relationship between two key terms (nouns and named entities) without instigating many random noises. As per them, sizeable experiments over real data show that the proposed conviction outperforms the state-of-the-art methods, including the graph-based co-occurrence methods and statistic-based clustering methods. There are two types of keyphrase extraction, supervised and unsupervised. The majority of the supervised methods accentuate key phrase extraction as a binary classification task (Hult, 2003a; 2003b; Jiang, Hu, Li, 2009; Turney, 2002; Witten et al., 1999) and evaluate some other features, such as term frequency-inverse document frequency (TF-IDF) and the position of the first occurrence of a phrase, as the inputs of a Naive Bayes classifier (Russell, Norvig, 2003). As per Shi et al. (2017, p. 1), "This is extremely expensive and time-consuming in domain-specific scenarios. To reduce manpower, investigating comparative unsupervised methods is highly desired. Thus, we focus on studying unsupervised methods to extract key phrases from a single input document (e.g., news and article)". In our proposed algorithm, we have amalgamated the concept of the knowledge graph and the term frequency based on a context of target words (noun), which is formed by an *n*-gram model. After that, we have attempted to create a relational triangle surrounding the target word. The maximum weighted triangle considers a target word (noun) which is the most talked-about feature with our proposed algorithm. The surplus words with low or no semantic meaning must be filtered out. Such words are known as stop words (Jaideepsinh, Jatinderkumar, 2016). While building a feature extraction algorithm apart from the default stop word, we need to remove some stop words manually. We are doing a feature extraction from mobile review data extracted from Amazon for a particular mobile phone from a specific company. Our objective is to find out the most negatively reviewed features. So, in this case, company names like '*Samsung*', '*Apple*' can all be considered stop words, since we are looking for the product features. We are not looking for the company that has created it. We are focused on evaluating the product. Manual removal of stop words is an uphill task; also, it can contribute to the degradation of the feature extraction model. With the proposed model, dependency on the stop word is somewhat eliminated. In their paper, Stuart Rose et al. (2010) proposed a key feature extraction algorithm, RAKE (Rapid Automatic Key Feature Extraction). Its input consists of a stop word list, a set of phrase delimiters, and word delimiters. It uses stop words and phrase delimiters to segregate the input text into candidate keywords, which are sequences of content words in the text. Co-occurrences of words within these candidate keywords identify word co-occurrence. It helps us to generate the score for candidate keywords. RAKE is a well known and widely used feature extraction algorithm, which tends to give compound words or phrases as key features that are not helpful while looking for particular words. When we apply it in mobile review data from an e-commerce website, we get compound outcomes of the type '*used camera*' or '*automatically camera close*' or '*good battery life*'. These phrases or compound words are not very helpful when we want to know about a specific feature or aspect. The same problem persists with another well-known algorithm YAKE (Yet Another Keyword Extractor) (Campos et al., 2020). Our proposed algorithm has overcome this challenge. It does not generate a compound word or phrase, but provides a single word as a critical feature. We can also consider TF-IDF (Term Frequency-Inverse Document Frequency) with Bag of Words

(BOW) for key feature extraction from text. TF-IDF is a product of the word frequency (Term Frequency) and of the measure how common or rare is that particular word in all the documents (IDF). The problem with this algorithm is that it does not capture semantics; hence, to extract a topic's features can be a tedious task. Our algorithm has tried to overcome this problem by extracting the probable features considering the semantics. This is the reason we have incorporated a concept of 'sentiN-gram', which is a fixed-sized forward and back window pivoting the probable key features (noun).

3 Literature review

Feature extraction from colossal data is a crucial task, and it is one of the parts of Natural Language Processing. Sammons et al. (2016) showed that implementing a machine-learning algorithm is unequivocal while extracting key features where the programmatic approach hinders the essence of key feature extraction. For decades, constructing a pattern recognition has required careful engineering and considerable domain expertise to design a feature extractor that transformed the raw data into a suitable internal representation or a feature vector in which the learning subsystem, often a classifier, could detect or classify patterns in the input (LeCun, Bengio, Hinton, 2015). Multiple graph-based approaches have been proposed in the field of Information Retrieval (I.R.); we have gone through some of them in our research. One of the papers (Devika, Subramaniyaswamy, 2019) dealt with a semantic graph-based keyword extraction model using a powerful social data ranking method. The authors used numeric graph metrics to associate the nodes' weight in the semantic graph. After that, they applied page ranking algorithm to arrange the nodes, which provided the most influential nodes. In another approach, the researchers provided a graph-based keyword extraction model using collective node weight (Biswas, Bordoloi, Shreya, 2018). They attempted to determine the importance of keywords by collectively taking various influencing parameters. This is one of the states of art in the field of knowledge graph.

A Knowledge Graph (K.G.) is a systematic representation of facts, consisting of entities and their relationships. Entities are real-world objects or abstract concepts. Relationships depict the relation between individual entities within a boundary. Semantic definitions of entities and their associations constitute types and properties with a comprehensible meaning (www 2). To learn unambiguous linguistic and semantic word relationships from highly distributed vector representations, a Knowledge Graph model provides an excellent result. In this paper, the researchers discussed using a knowledge graph to identify concept prerequisites (Manrique, Pereira, Mariño, 2019). They proposed a four--step approach consisting of building a knowledge graph to find probable candidate concepts; create potential pictures; formulate a model to evaluate possible ideas, and validate the idea using ground truth concepts from different domains. In another paper, researchers proposed a global level relation extractor model using knowledge graph embeddings for document-level inputs (Kim et al., 2020). This model creates a local-level knowledge graph from the input document, which will predict the global level relation from an extensive record. The synchronization between these two levels has been achieved during training. During our literature review, we have seen that the use of knowledge graphs is very pertinent for feature extraction (Zhao, Pan, Yang, 2020; Xu et al., 2020; Wang et al., 2018; Jia et al., 2018; K-CAP ’19, 2019). Using a knowledge graph gives a graphical semantic view of a topic and associated aspects of the subject. This is the reason we have incorporated the concept of knowledge graph in the formation of this algorithm. Bonatti et al. (2018) stated that “Human and Social Factors in Knowledge Graphs” provided more concrete insights as it could build on both academic and industrial research results, projects, and practical experiences. Knowledge graphs capture relevant domain knowledge, and with machine learning algorithms, we can train our model to find out a specific pattern within that particular domain. This concept of knowledge graph is the driving force behind our algorithm.

4 Methodology

As we have said earlier, our algorithm is based on graph theory, knowledge graph, and *n*-gram model; also, we have integrated sentiment analysis. Sentiment analysis is a nexus of methods, techniques, and tools to identify and obtain personal information, such as opinion from natural language (Liu, 2009). Conventionally, sentiment analysis accentuates opinion polarity, i.e., whether someone has conveyed positive, neutral, or negative views towards something (Dave, Lawrence, Pennock, 2003). The quintessence of sentiment analysis has typically been a product or a service whose review has been made public on the internet (www 3). Hence, our primary focus is to extract the features based on the reviewer’s sentiment in our research paper. We will now give the basics of Graphs, Knowledge Graph, and *n*-gram. A graph is denoted as $G = <V, E_i>$, where E_i can be defined as the set of vertices (nodes) V, and the interactions among pairs of nodes called links (edges) E. “A graph associated with each edge E (also called arc) is an ordered pair. Edge E is then directed from vertex U to vertex V, and an arrowhead on edge shows the direction. A graph is undirected if the end vertices of all the edges are unordered (i.e., edges have no direction)" (www 4; www 5). A Knowledge Graph (K.G.) is a multi-relational graph composed of entities (nodes) and relations (different types of edges). Each edge is represented as a triple of the form (head entity, relation, tail entity), also called a fact, indicating that two entities are connected by a specific relation, e.g., (Alfred Hitchcock, director of, Psycho). Although effective in representing structured data, the underlying symbolic nature of such triples usually makes K.G.s hard to manipulate (Wang et al., 2017, p. 5). A linguistic model can take a list of words and attempt to predict the word that follows them. It outputs a probability score for all the words it knows. The *n*-gram model is a linguistic model. *N*-gram means a sequence of *N* words. The definition of *n*-gram is an unambiguous definition. For instance, 'good camera' is a 2-gram, 'Display is not good' is a 4-gram. While building an NLP model with the help of *n*-gram, we can assume that it will have a pretty good idea of the 'probability' of a word's occurrence after a specific word or before a specific word. Below is our training database with seven reviews given by a customer.

Table 1: Sample dataset to explain the notion of a Senti-*n*-gram

i)	Camera quality is average
ii)	Camera quality not good
iii)	I like the camera quality
iv)	Not satisfied with camera quality
v)	The camera quality is excellent
vi)	Camera quality is average
vii)	Camera quality is also good

From this, we can see that after the word '*camera*', only the word '*quality*' occurs, which this is expected, because our central database is based on product review data. So, the term '*quality*' has a special place while providing a product review to calculate the probability of the sequence, and we have:

$$\frac{|(W_1 W_2)|}{|(W_2)|}\,. \tag{1}$$

Here we calculate the probability of the word W_1 occurring after the word W_2; as stated earlier, the following algorithm adds the sentiment analysis concept. Consider the above database with seven review data from Table 1, where we can see the customers' feelings about the camera. So, all the sentences associated with '*camera*' must contain a word that describes a positive, negative or neutral sentiment. In Table 1, we can see that sentences (iii), (v), and (vii) all convey a positive sentiment, and this is due to the words: '*like*', '*excellent*', '*good*' which occur next to '*camera*'. Here we can consider '*camera*' as a fixed element and assume an *n*-gram model before and after the '*camera*' is the same size. Within this *n*-gram, we can look for the words which convey positive and negative polarity. As we can see from our example, if we consider a window or an *n*-gram of 3 before and after the word '*camera*', we will undoubtedly find a word that conveys a '*camera*' sentiment. We call this model "Senti N-Gram". We can also see the terms such as '*camera*' and words that bear sentiment polarity value belong to a particular part of speech (such as a noun, adjective, verb, adverb). The following study is based on 200 data points. After doing the tokenization, the distribution of parts-of-speech has been observed. We have also used the Penn Treebank tag set for Parts of Speech (POS) tagging. A tag set is a set of part-of-speech tags used to label the parts of speech and other grammatical categories (case, tense, etc.) of each word token in a central text corpus. Below is the list of Penn Treebank tag sets (www 6).

Table 2: Penn Treebank tag set for Parts of Speech (POS) tagging

Tag	Description
CC	Coordinating conjunction
CD	Cardinal number
DT	Determiner
EX	Existential there
FW	Foreign word
IN	Preposition or subordinating conjunction
JJ	Adjective
JJR	Adjective, comparative
JJS	Adjective, superlative
LS	List item marker
MD	Modal
NN	Noun, singular or mass
NNS	Noun, plural
NNP	Proper noun, singular
NNPS	Proper noun, plural
PDT	Predeterminer
POS	Possessive ending
PRP	Personal pronoun

Tag	Description
PRP$	Possessive pronoun
RB	Adverb
RBR	Adverb, comparative
RBS	Adverb, superlative
RP	Particle
SYM	Symbol
TO	to
UH	Interjection
VB	Verb, base form
VBD	Verb, past tense
VBG	Verb, gerund or present participle
VBN	Verb, past participle
VBP	Verb, non3rd person singular present
VBZ	Verb, 3rd person singular present
WDT	Whdeterminer
WP	Whpronoun
WP$	Possessive whpronoun
WRB	Whadverb

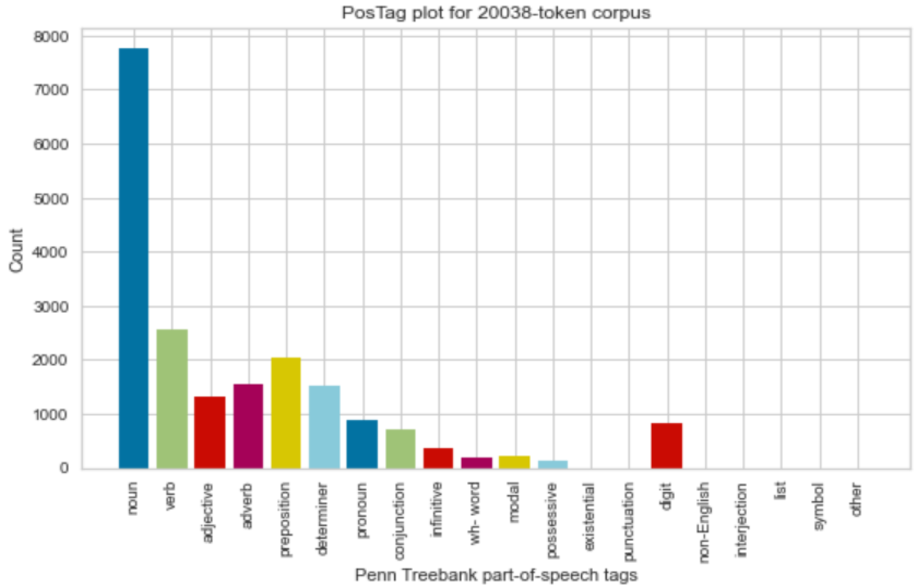

Figure 1: POS distribution of sample data

From this, we can see that Noun, Verb, Adjective, and Adverb have higher density in our main corpus.

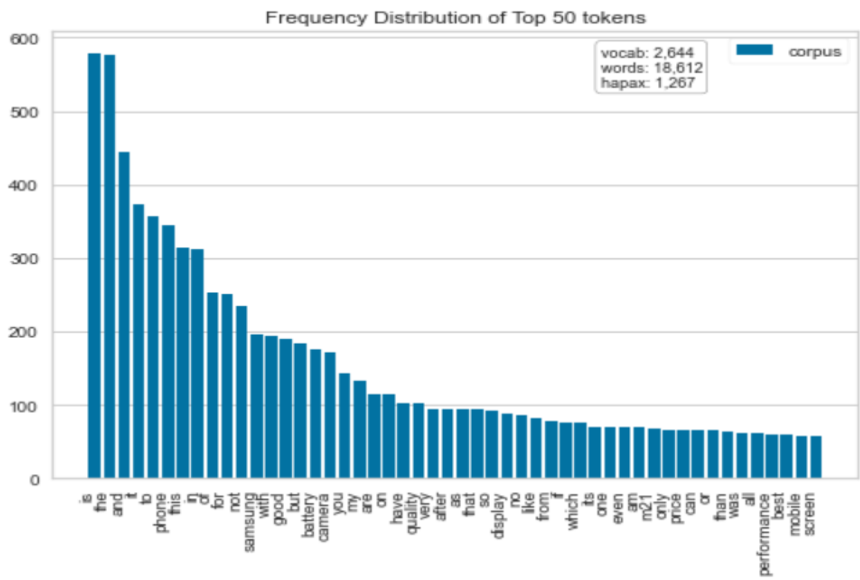

Figure 2: Frequency distribution of the top 50-word token

In the section below, we have described our proposed algorithm.

1. *From the main corpus of feedback data, do sentence tokenization.*
2. *Do word tokenization.*
3. *Do POS tagging.*
4. *Select nouns within a sentence.*
5. *Using nouns as target elements within a sentence, an n-gram model assumes a fixed window size in the target word's forward and backward direction.*
6. *By the above, pick up the forward and backward neighbour words for a fixed window size.*
7. *Consider all nouns, adverbs, adjectives, and verbs' as nodes or vertices of a graph.*
8. *Assume there is no direct relationship between the nouns (as we can consider these nouns as aspects or features of an entity; for instance, when we are looking for phone review data, camera, fingerprint, all these are aspects of the feature of a phone). They are independent. Hence there are no connections or edges between nouns (for instance, camera and fingerprint has no common edge between them).*
9. *We consider nouns as features to connect to the adverbs, adjectives, and verbs. Because the primary database is based on a product's feedback, adverbs, adjectives, and verbs bear a semantic context to nouns based on the review (subject entity). So, we can consider edges between nouns (Subject) and {adverbs, adjectives, verbs}(Description of Subject). This can be regarded as a 'knowledge graph'.*
10. *The main corpus is based on feedback data to bear positive, negative, and neutral polarity words. From this, we can say that it will bear synonymic and antonymic meaning among the words because different reviewers use different expressions, such as 'good camera', 'best camera', 'bad camera', 'worst camera'. Here 'good' and 'best' are synonyms that bear a positive sentiment; also, 'bad' and 'worst' are synonyms that carry a negative opinion, 'good' and 'best' are antonyms of 'bad' and 'worst'. Considering a product feature will generate positive and negative sentiment, so the spread of these words (adverbs, adjectives, and verbs) will be higher − higher possibilities of getting synonyms and antonyms.*
11. *We can relate two words (adverbs, adjectives, and verbs) based on synonyms and antonyms properties.*
12. *From 9 and 11, we can get a triangular relation (triangular graph).*
13. *The more triangles we can form with those words will be the most talked--about features.*

14. *An edge from a noun vertex to {adverbs, adjectives, verbs} will bear a weight similar to a number of occurrences of a particular noun and the adjacent {adverbs, adjectives, verbs} based on n-gram.*
15. *If W1 and W2 are the co-occurrences of that particular neighbour word within the window frame of a pivot word (Noun), then:*

$$\lambda = \max (W1, W2). \tag{2}$$

16. *Weight between any two nodes among Adverbs, Adjectives, and Verbs based on synonym and antonym property will always be 1.*
17. *We can define feature Strength as follows:*

$$(\text{Feature Strength})_i = \sum \lambda.\Delta. \tag{3}$$

Δ is the total number of triangles formed on the basis of the dictionary. A list has been created, based on some reoccurred words (adjective or adverbs or verbs) common in any review data to develop this dictionary. For this reason, an analysis has been done on review data from a different domain (such as Hotel review, Movie review, Product review), and the following dictionary has been created.

```
wordDict = {'good' : {'syn': ['good','impressive','attractive','fantastic','nice','gorgeous','great','outstanding','loved',
                              'better','amazing','best','great','satisfied', 'positive', 'liked', 'competitive','improved'],
           'any': ['bad','worse','pathetic','poor','average','wrong', 'scary', 'disgusting']},
 'suffering': {'syn': ['suffering','trouble','blaming', 'disappointed'], 'any' : ['troubleshooting']},
 'reasonable': {'syn': ['reasonable','economical','accepting','free'], 'any':['costly']},
 'clear': {'syn':['clear'], 'any': ['blurred']},
 'slow': {'syn': ['slow','running', 'low'],'any': ['stable']},
 'tough':{'syn':['tough','hard','strongest','strong'], 'any':['solved','resolved','easy']},
 'powerful': {'syn': ['powerful','huge']},
 'recommended': {'syn':['recommended','supported', 'natural', 'original'], 'any':['defective']}
}
```

Context dictionary based on Review data considering Synonym ('syn') and Antonym ('any').

Creating this dictionary aims to generate a lexicon-based database that will hold contextual meaning, both positive and negative, from the perspective of review data; for instance: 'good' can be associated with 'satisfied' or 'improved'. This relation is based on the synonym property; all of this bears positive sentiment. Similarly, 'disgusting' has an antonymic relationship with 'good'. A sentence can have multiple pivot words. Next, we find sentiment polarity of the neighbour words using a lexicon-based sentiment analyzer such as VADER. As we have said, each adjective, noun, verb, or adverb can be considered as a node inside a graph. Each node can be tagged as follows:

Word	POS	Sentiment Score

```
['macromode', 'NN', 0.0],
['camera', 'NN', 0.0],
['shoot', 'NN', -0.34],
['blurred', 'VBN', 0.0],
['images', 'NNS', 0.0],
['heating', 'NN', 0.0],
['issue', 'NN', 0.0],
['and', 'CC', 0.0],
['camera', 'NN', 0.0],
['quality', 'NN', 0.0],
```

Figure 3: Node structure of the graph with an example from the main corpus

All available nouns in a text cannot be considered as a feature. We must look for the nouns which occur the most in the entire database. We draw an edge from the noun (Probable feature) to the other words with POS adjectives, adverbs, and verbs on the basis of their occurrence in the previously mentioned window. It can be considered the edge’s weight for multiple word occurrences concerning the feature noun based on the neighbour window.

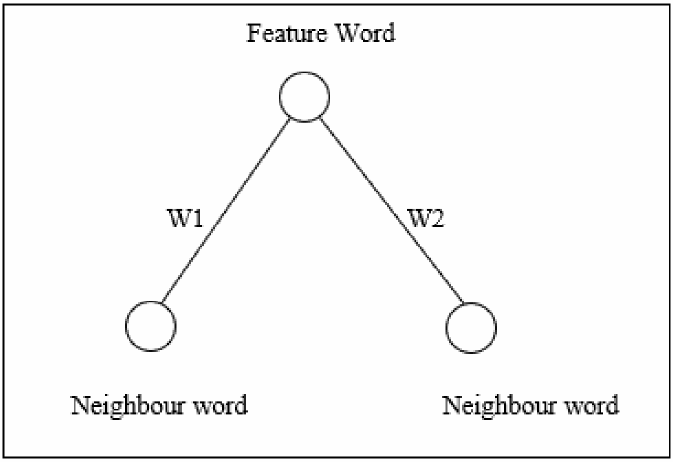

Figure 4: Initial graph structure

W1 and W2 are the co-occurrences of that particular neighbour word within the window frame of a pivot word considering all sentences within the database.

Example:

Sentence 1: ‘*Back camera pretty good, but front camera low light output is low*’.

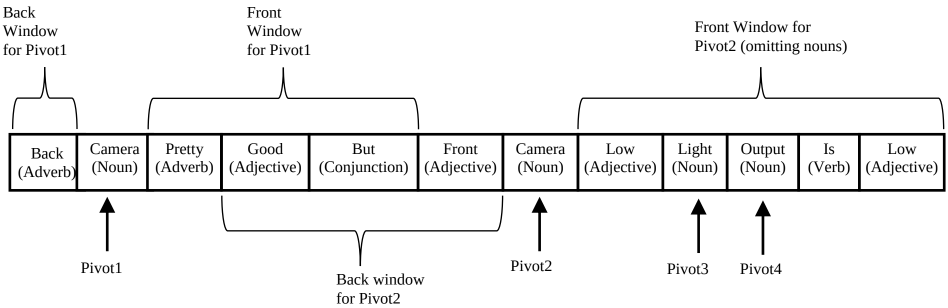

Figure 5: A review sentence structure with pivots element Noun

We consider a window of 4 from the left and right of the pivot word.

Pivot	Left of Pivot			Right of Pivot		
camera	back			pretty	good	but
camera	**good**	but	**front**	**low**	light	output
light	**front**	camera	**low**	output	is	**low**
output	camera	**low**	light	is	**low**	

Figure 6: Pivot word and sentiN-gram

Sentence 2: '*Nice rear camera and nice selfie camera but front camera struggles at night*'.

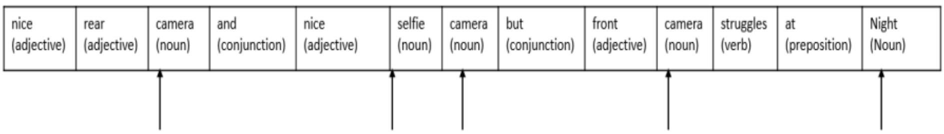

Figure 7: Pivot Nouns

Here we consider a window of 4 from the left and right of the pivot word.

We consider any noun as a feature of the product. In this case, the product is a mobile phone. We have the following nouns: [camera, light, output, selfie, night] from the two sentences above.

Pivot	Left of Pivot			Right of Pivot		
camera	**nice**	**rear**		and	nice	selfie
selfie	camera	and	**nice**	camera	but	**front**
camera	and	**nice**	selfie	camera	but	**front**
camera	camera	but	**front**	struggles	at	night
night	camera	struggles	at			

Figure 8: Pivot word and sentiN-Gram

To develop the proposed algorithm, we have used the programming language Python 3.8 on Windows 10 Home (64 bit) and different libraries to collect and extract the features. Some of the libraries used are Pandas, VADER, TextBlob, NumPy, NLTK, Spacy, Gensim, Scikit-learn, etc. The hardware used was an Intel i5 processor 2.40 GHz with 4 GB RAM.

Knowledge Graph Representation:

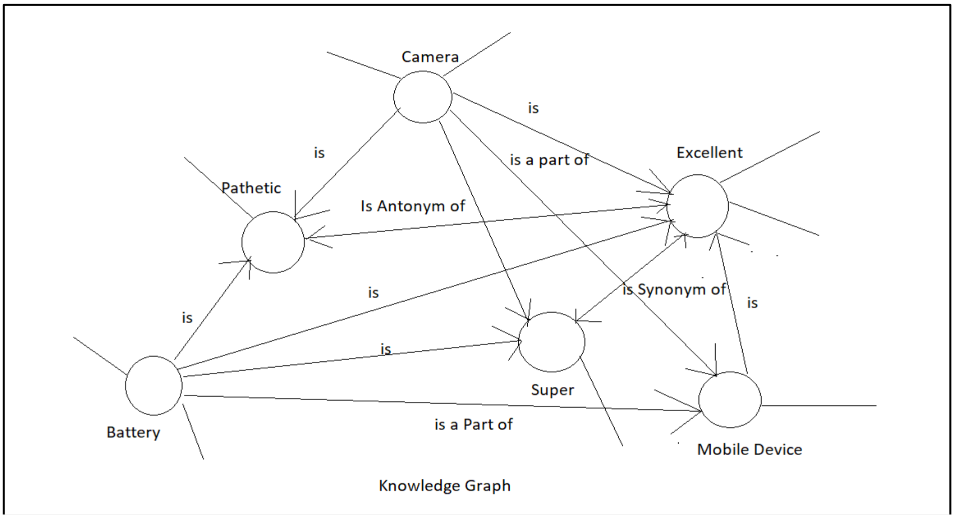

Figure 9: Relational triangle based on features (Part 1)

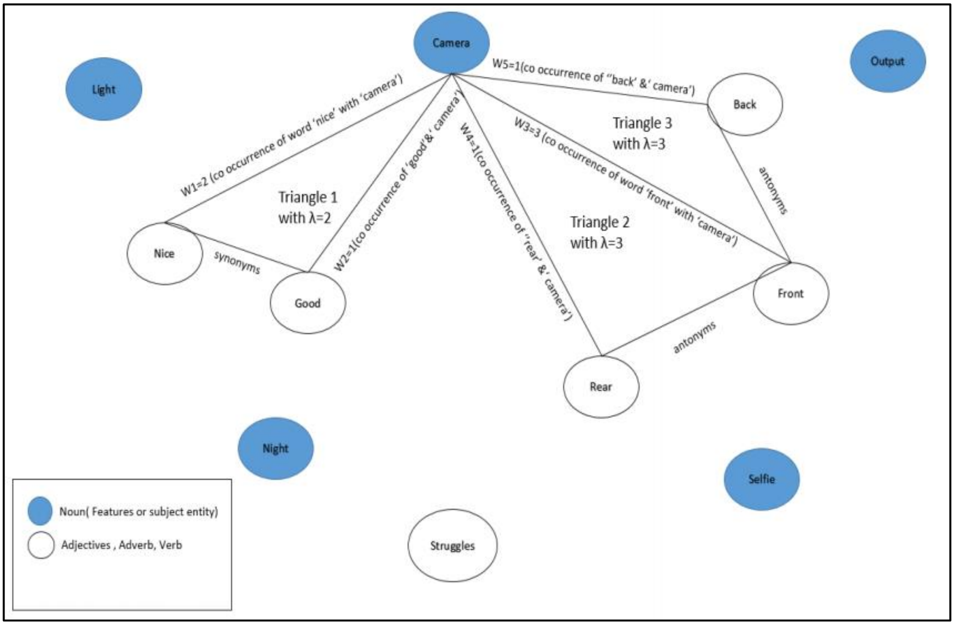

Figure 10: Relational triangle based on features (Part 2)

5 Discussion of data and result

The fundamental objective of this paper is to develop a key feature extraction algorithm. The most commonly used feature extraction algorithm in Natural Language Processing is Bag-of-Words with TF-IDF. As Ramos (2003, p. 1) said, “TF-IDF calculates values for each word in a document through an inverse proportion of the frequency of the word in a particular document to the percentage of documents the word appears in. Words with high TF-IDF numbers imply a strong relationship with the document they appear in, suggesting that the document could be of interest to the user if that word were to appear in a query”. But the major disadvantage of this method is that the most frequent TF-IDF words of a document may not make sense while extracting key features. Words like ‘this’, ‘if’, ‘the’, ‘or’, ‘what’ are most frequent; they are called Stop words. Even after the elimination of these words, content-related domain-specific words with high levels of frequency, like ‘communication’, ‘team’, ‘message’ or ‘product’, etc., occur. These words do not provide any significance to the content of each review. When we try to predict the text’s context using TF-IDF, the outcome is not productive. An alternative method using a graph-based approach has been widely used for Text Mining and Information Retrieval tasks (Vazirgiannis, Malliaros, Nikolentzos, 2018). These representations exploit concepts and techniques inherited from graph theory (e.g., node centrality and subgraph frequency) to address limitations of the classical bag-of-words representation (Aggarwal, 2018). A text can be represented as a graph in numerous ways. For instance, considering all words in a text as vertices connected by a directed edge (one-way connection). Those edges can be labeled using the relation of the words in a dependency tree. Another rendering of text can use undirected edges, for example, when representing word co-occurrences. In this way, one can capture structural and semantic information of a text, mitigate the effects of the 'curse-of-dimensionality' phenomenon, identify the most critical terms of a text, and seamlessly incorporate data from external knowledge sources (Giarelis, Kanakaris and Karacapilidis, 2020). In a recent paper, Giarelis, Kanakaris and Karacapilidis (2020, p. 1) suggested that "These approaches combine statistical tests and graph algorithms to uncover hidden correlations between terms and document classes. However, while they take into account the co-occurrences between terms to identify the most representative features of a single document (something that is not the case in traditional statistical methods), they are not able to assess the importance of a term in a corpus of documents". These problems can be obliterated if we do feature extraction from a product review data with some presumption like POS tagging and considering noun as the main feature. It also adds the logic of *n*-gram, which helps us construct a knowledge graph, as we have mentioned above. The number of relation triangles helps identify the most frequent feature and can identify the most positive or negative reviewed feature. This can be found by traversing onto the side nodes of the relevant nodes (polarity wise). The efficiency is much higher. It can remove the dependency of stop word removal altogether, which is precisely the 'Curse of dimensionality'. Here a comparative study has been done to check the effectiveness of crucial feature extraction via TF-IDF over the proposed algorithm. We have a master database of probable key features. These key features are chosen by experts who have had domain knowledge of the mobile industry for more than ten years. Below is the list of most probable features talked about by customers while providing mobile-related feedback.

```
feature_dict = {
    'camera': ['camera','selfie','front','video','photo','picture','rear','macro','image','clarity','resolution','focus',
               'photography',
               'recording','zoom'],
    'display': ['display','resolution','hz','amoled','fluid'],
    'battery': ['battery','heat', 'charge', 'charged', 'capacity'],
    'charging': ['charging','slow','charger','heating','speed','power','heat','hanging','charged','heated','hot'],
    'performance': ['performance','ram','speed','lag','hanging','hang','slow'],
    'fingerprint': ['fingerprint','finger','face','touch','rear','recognition','lock'],
    'processor':['processor','slow','ram','heating','speed','lag','hang','memory'],
    'gaming': ['gaming','ram','heating','lag','heat','hanging','hot','hang','ram','slow'],
    'sensor': ['sensor','finger','face','touch','brightness','security'],
    'sound':['sound','speaker','audio','voice','dolby','clarity','volume','atmos','recording'],
    'network':['network','speed','internet','sim','signal','voice','wifi','connectivity','heating'],
    'calling':['calling','heat','sim','voice','signal','wifi','volume','connectivity']
}
```

Figure 11: Reference keyword dataset

To compare TF-IDF's behavior and the proposed algorithm with a given data, the Jaccard Similarity Coefficient (J.C.) has been introduced. It is a statistic used to understand the similarities between sample sets. The measurement focuses on the similarity between finite sample sets and is formally defined as the size of the intersection divided by the size of the sample sets' union. Its mathematical representation is:

$$J\,(A, B) = \frac{|A| \cap |B|}{|A| \cup |B|}, \tag{4}$$

where A and B are two finite sets (A and B don't have to be the same size).

$$J\,(A, A) = 1 \text{ (Similar set)}, \tag{5}$$
$$J\,(A, B) = 0 \text{ if } |A| \cup |B| = 0. \tag{6}$$

Now consider the most frequent 300 keywords selected using the TF-IDF algorithm and the proposed algorithm and find the Jaccard Coefficient w.r.t. the reference keyword.

J (Output from TF-IDF, Reference data) = .03125
J (Output from Proposed Algorithm, Reference data) = .1134

Hence, J (Output from Proposed Algorithm, Reference data) > J (Output from TF-IDF, Reference data). We have also compared the result with the reference database after removing stop words with the most frequent 300 words. The result is:

J (Output from TF-IDF, Reference data) = .03418
J (Output from Proposed Algorithm, Reference data) = .11009

This proves that the proposed algorithm has the edge over TF-IDF. Also, after comparing the reference data with the proposed algorithm, it has been observed that it has an accuracy of 59%, while TF-IDF has an accuracy of 17%.

If we consider the 300 most frequent features extracted by TF-IDF and our algorithm, out of 63 reference features (golden features), TF-IDF has 11 similarities. Our proposed algorithm has 37 similarities; also, we have compared the behavior with a well-known recently developed algorithm YAKE (Yet Another Keyword Extraction) (Campos, 2020). We have found that if we consider a single word extracted by YAKE, then the similarities with our respective golden dataset are 29. This further proves the superiority of our proposed algorithm.

6 Conclusion & future work

When we are extracting features from ever-growing review data to check which is/are the highest affected modules, the curse of dimensionality is the biggest challenge, since even though our thinking (reviews) about a product is alike (based on sentiment polarity, Positive, Negative, and Neutral), we express it differently. So, the extraction of keywords using Natural Language Processing becomes highly provocative. But with the proposed algorithm, we have found a way where we can reduce the effect. This algorithm consists in the merging of sentiment analysis, knowledge graph, and *n*-gram, which forms a relational triangle, and the highest occurrence of the triangle leads to feature extraction. It has been shown that this algorithm has the edge over TF-IDF. It has an accuracy of 59%, while TF-IDF has an accuracy of 17%.

While proposing our algorithm, we have not considered those sentences in which words are preceded by a negation word. In the future, we will work on that and will try to tune our algorithm accordingly. Also, this particular algorithm has been attuned on product review data for most affected modules. We will therefore try to propose a generalized model. Also, we suggest a graph--based dictionary to find out the synonym and antonym relation between words.

Declaration

1. Funding: Not Applicable
2. Conflicts of interest/Competing interests: Not Applicable
3. Availability of data and material: Available
4. Code availability: Available

References

Aggarwal C.C. (2018), *Machine Learning for Text*, Springer, Cham.

Biswas S.K., Bordoloi M., Shreya J. (2018), *A Graph-based Keyword Extraction Model Using Collective Node Weight,* Expert Systems with Applications, 97, 51-59, https://doi.org/10.1016/j.eswa.2017.12.025.

Bonatti P., Decker S., Polleres A., Presutti V. (2018), *Knowledge Graphs: New Directions for Knowledge Representation on the Semantic Web* (Dagstuhl Seminar 18371), Dagstuhl Reports, 8, 29-111.

Campolo A., Sanfilippo M., Whittaker M., Crawford K. (2018), *AI Now 2017 Report,* Symposium and Workshop, January, AI Now Institute at New York University.

Campos R., Mangaravite V., Pasquali A., Jorge A., Nunes C., Jatowt A. (2020), *YAKE! Keyword Extraction from Single Documents using Multiple Local Features*, Information Sciences, 509, 257-289, DOI: 10.1016/j.ins.2019.09.013.

Dave K., Lawrence S., Pennock D.M. (2003), *Mining the Peanut Gallery: Opinion Extraction and Semantic Classification of Product Reviews*, Proceedings of the 12th International Conference on World Wide Web, 519-528.

Devika R., Subramaniyaswamy V. (2019), *A Semantic Graph-based Keyword Extraction Model Using a Ranking Method on Big Social Data*, Wireless Netw, https://doi.org/10.1007/s11276-019-02128-x.

Feldman R., Dagan I. (1995), *Knowledge Discovery in Textual Databases* (KDT), Proceedings of the First International Conference on Knowledge Discovery and Data Mining (KDD-95), Montreal, Canada, August 20-21, AAAI Press, 112-117.

Giarelis N., Kanakaris N., Karacapilidis N. (2020), *An Innovative Graph-Based Approach to Advance Feature Selection from Multiple Textual Documents*, Artificial Intelligence Applications and Innovations, 583, May 6, 96-106, DOI: 10.1007/978-3-030-49161-1_9.

Houari M., Rhanoui M., Asri B. (2015), *From Big Data to Big Knowledge: The Art of Making Big Data Alive*, 1-6, DOI: 10.1109/CloudTech.2015.7337001.

Htay S.S., Lynn K.T. (2013), *Extracting Product Features and Opinion Words Using Pattern Knowledge in Customer Reviews*, The Scientific World Journal, Vol. 2013, Article ID 394758, 5 pages, https://doi.org/10.1155/2013/394758.

Hulth A. (2003a), *Improved Automatic Keyword Extraction Given More Linguistic Knowledge*, EMNLP, 216-223.

Hulth A. (2003b), *Reducing False Positives by Expert Combination in Automatic Keyword Indexing*, RANLP, 367-376.

Jaideepsinh K., Saini J. (2016), *Stop-Word Removal Algorithm and Its Implementation for the Sanskrit Language*, International Journal of Computer Applications, 150, 15-17, DOI: 10.5120/ijca2016911462.

Jia Y., Qui Y., Shang H., Jiang R., Li A. (2018), *A Practical Approach to Constructing a Knowledge Graph for Cybersecurity*, Engineering, 4(1), 53-60, https://doi.org/10.1016/j.eng.2018.01.004.

Jiang X., Hu Y., Li H. (2009), *A Ranking Approach to Keyphrase Extraction*, SIGIR, 756-757.

K-CAP ’19 (2019), Proceedings of the 10th International Conference on Knowledge Capture, September, 131-138, https://doi.org/10.1145/3360901.3364441.

Kim K., Hur Y., Kim G., Lim H. (2020), *GREG: A Global Level Relation Extraction with Knowledge Graph Embedding*, Applied Sciences, 10, 1181.

LeCun Y., Bengio Y., Hinton G. (2015), *Deep Learning*, Nature, 521, 436-44, https://doi.org/10.1038/nature14539.

Liu B. (2009), *Handbook Chapter: Sentiment Analysis and Subjectivity. Handbook of Natural Language Processing*, Marcel Dekker, Inc., New York, NY, USA.

Manrique R., Pereira B., Mariño O. (2019), *Exploring Knowledge Graphs for the Identification of Concept Prerequisites*, Smart Learning Environments, 6, 21, https://doi.org/10.1186/s40561-019-0104-3.

Markov A., Last M., Kandel A. (2007), *Fast Categorization of Web Documents Represented by Graphs*, Advances in Web Mining and Web Usage Analysis, 4811, 56-71.

Park D.-H., Kim S. (2008), *The Effects of Consumer Knowledge on Message Processing of Electronic Word-of-mouth via Online Consumer Reviews*, Electronic Commerce Research and Applications, 7, 399-410.

Ramos J. (2003), *Using TF-IDF to Determine Word Relevance in Document Queries*, Computer Science, Proceedings of the First Instructional Conference on Machine Learning, 1-4.

Rose S., Engel D., Cramer N., Cowley W. (2010), *Automatic Keyword Extraction from Individual Documents*, DOI: 10.1002/9780470689646.ch1.

Russell S.J., Norvig P. (2003), *Artificial Intelligence − A Modern Approach: The Intelligent Agent Book*, Prentice-Hall.

SAC ’07 (2007), Proceedings of the 2007 ACM Symposium on Applied Computing, March, 807-811, https://doi.org/10.1145/1244002.1244182.

Safrin R., Sharmila K.R., Shri Subangi T.S., Vimal E.A. (2017), *Sentiment Analysis on Online Product Review*, International Research Journal of Engineering and Technology (IRJET), 4, April, 2381-2388.

Sammons M., Christodoulopoulos C., Kordjamshidi P., Khashabi D., Srikumar V., Vijayakumar P., Bokhari M., Wu X., Roth D. (2016), *Edison: Feature Extraction for NLP*, *Simplified* [in:] N. Calzolari, K. Choukri, H. Mazo, A. Moreno, T. Declerck, S. Goggi, M. Grobelnik, J. Odijk, S. Piperidis, B. Maegaard, J. Mariani (eds.), Proceedings of the 10th International Conference on Language Resources and Evaluation, LREC 2016, European Language Resources Association (ELRA), 4085-4092.

Shi W., Zheng W., Yu J.X., Cheng H., Zou L. (2017), *Keyphrase Extraction Using Knowledge Graphs*, Data Science Engineering, 2, 275288, https://doi.org/10.1007/s41019-017-0055-z.

Sidorov G., Velasquez F., Stamatatos E., Gelbukh A., Chanona-Hernández L. (2013), *Syntactic Dependency-Based N-grams as Classification Features* [in:] I. Batyrshin, M.G. Mendoza (eds.), Advances in Computational Intelligence, MICAI 2012, Lecture Notes in Computer Science, 7630, Springer, Berlin, Heidelberg, https://doi.org/10.1007/978-3-642-37798-3_1.

Turney P.D. (2002), *Learning to Extract Keyphrases from the Text*, CoRR, cs. L.G./0212013.

Vazirgiannis M., Malliaros F., Nikolentzos G. (2018), *GraphRep: Boosting Text Mining, NLP, and Information Retrieval with Graphs*, Proceedings of the 27th ACM International Conference on Information and Knowledge Management, 2295-2296.

Wang Ch., Ma X., Chen J., Chen J. (2018), *Information Extraction and Knowledge Graph Construction from Geoscience Literature*, Computers & Geosciences, 112, 112-120, https://doi.org/10.1016/j.cageo.2017.12.007.

Wang Q., Mao Z., Wang B., Guo L. (2017), *Knowledge Graph Embedding: A Survey of Approaches and Applications*, IEEE Transactions on Knowledge and Data Engineering, 29(12), December 1, 2724-2743, DOI: 10.1109/TKDE.2017.2754499.

Wang W., Do D.B., Lin X. (2005), *Term Graph Model for Text Classification*, Advanced Data Mining and Applications, 19-30.

Willemsen L.M., Neijens P.C., Bronner F., de Ridder J.A. (2011), *“Highly Recommended!” The Content Characteristics and Perceived Usefulness of Online Consumer Reviews*, Journal of Computer-Mediated Communication, 17(1), October 1, 19-38, https://doi.org/10.1111/j.1083-6101.2011.01551.x.

Witten I.H., Paynter G.W., Frank E., Gutwin C., Nevill-Manning C.G. (1999), *KEA: Practical Automatic Keyphrase Extraction*, Proceedings of the Fourth ACM Conference on Digital Libraries, 254-255.

Xu J., Kim S., Song M., Jeong M., Kim D., Kang J., Rousseau J.F., Li X., Xu W., Torvik V.I., Bu Y., Chen Ch., Ebeid I.A., Li D., Ding Y. (2020), *Building a PubMed Knowledge Graph*, Scientific Data, 7, 205, https://doi.org/10.1038/s41597-020-0543-2.

Zhao H., Pan Y., Yang F. (2020), *Research on Information Extraction of Technical Documents and Construction of Domain Knowledge Graph*, IEEE Access, 8, 168087-168098, DOI: 10.1109/ACCESS.2020.3024070.

Zhao J., Wang T., Yatskar M., Ordonez V., Chang K.W. (2017), *Men also Like Shopping: Reducing Gender Bias Amplification Using Corpus-level Constraints*, Proceedings of the 2017 Conference on Empirical Methods in Natural Language Processing, 2979-2989.

(www 1) https://www.merriam-webster.com/dictionary/adjective (accessed: 1.11.2020).

(www 2) Ji S., Pan S., Cambria E., Marttinen P., Yuar P.S. (2021), *A Survey on Knowledge Graphs: Representation, Acquisition and Applications*, IEEE Transactions on Neural Networks and Learning Systems, Xiv:2002.00388 (accessed: 8.11.2020).

(www 3) Mäntylä M.V., Graziotin D., Kuutila M. (2018), *The Evolution of Sentiment Analysis − A Review of Research Topics, Venues, and Top Cited Papers*, Computer Science Review, 27, February, 16-32, arXiv:1612.01556 [cs.CL] (accessed: 10.11.2020).

(www 4) http://web.onda.com.br/abveiga/capitulo4-ingles.pdf (accessed: 11.11.2020).

(www 5) Mutlu E.C., Oghaz T.A., Rajabi A., Garibay I., *Review on Learning and Extracting Graph Features for Link Prediction*, arXiv:1901.03425 (accessed: 11.11.2020).

(www 6) https://www.sketchengine.eu/penn-treebank-tagset/#:~:text=English%20Penn%20Treebank%20part%2Dof%2Dspeech%20Tagset&text=Atagset%20is%20a%20list%20of,(case%2C%20tense%20etc.) (accessed: 12.11.2020).

(www 7) Hellström T., Dignum V., Bensch S. (2020), *Bias in Machine Learning – What Is It Good for?* https://arxiv.org/pdf/2004.00686.pdf (accessed: 12.11.2020).

(www 8) https://www.lexico.com/definition/noun (accessed: 9.11.2020).

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IMPACT OF THE COVID-19 PANDEMIC ON THE EXPRESSION OF EMOTIONS IN SOCIAL MEDIA

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Abstract

In the age of social media, every second thousands of messages are exchanged. Analyzing those unstructured data to find out specific emotions is a challenging task. Analysis of emotions involves evaluation and classification of text into emotion classes such as Happy, Sad, Anger, Disgust, Fear, Surprise, as defined by emotion dimensional models which are described in the theory of psychology (www 1; Russell, 2005). The main goal of this paper is to cover the COVID-19 pandemic situation in India and its impact on human emotions. As people very often express their state of the mind through social media, analyzing and tracking their emotions can be very effective for government and local authorities to take required measures. We have analyzed different machine learning classification models, such as Naïve Bayes, Support Vector Machine, Random Forest Classifier, Decision Tree and Logistic Regression with 10-fold cross validation to find out top ML models for emotion classification. After tuning the Hyperparameter, we got Logistic regression as the best suited model with accuracy 77% with the given datasets. We worked on algorithm based supervised ML technique to get the expected result. Although multiple studies were conducted earlier along the same lines, none of them performed comparative study among different ML techniques or hyperparameter tuning to optimize the results. Besides, this study has been done on the dataset of the most recent COVID-19 pandemic situation, which is itself unique. We captured Twitter data for a duration of 45 days with hashtag #COVID19India OR #COVID19 and analyzed the data using Logistic Regression to find out how the emotion changed over time based on certain social factors.

Keywords: classification, COVID-19, emotion, emotion analysis, Naïve Bayes, Pandemic, Random Forest, SVM.

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1 Introduction

Emotion analysis is an advanced and fine grained version of sentiment analysis. Many surveys and papers have been published on sentiment analysis. Sentiment analysis provides the information about the positive, negative or neutral categories, but it requires more detailed and granular level analysis of the information. For example: ‘It was a bad luck! We lost the game’ vs. ‘we lost the game because of the worst decision of the umpire’; here both sentences are of negative sentiment, but the first sentence is of ‘sadness’, whereas the second sentence is of ‘anger’.

Emotion analysis fills this gap with more detailed analysis of human emotion (www 1; Russell, 2005). Emotion can be expressed through different medium, such as textual message, voice or facial expressions; this is a huge area of research work. There are multiple studies going on in each of these specific areas to decipher the human emotions and translate them to actionable forms.

Emotion analysis data can be used in multiple industry segments. Products reviews and customer feedbacks can be analyzed in different emotion categories, which helps the merchant to improve the product line or propose a suitable product to the customer to improve customer satisfaction. Social media messages can be analyzed to understand the mood and needs of the people of a particular locality or demography, which helps companies to target promotions. In the healthcare sector, emotion analysis can be helpful to diagnose the mental health of a patient through analysis of social media blogs, tweets, messages; it can also predict the sudden mood swings of people that result in suicidal attempts. In the public sector, emotion analysis can be helpful in election campaign, etc. Our study addresses the fluctuation of human emotions in the COVID-19 pandemic situation across India.

Analysis of Twitter data or the thoughts of the crowd over a long period of time presents a very clear picture and provides insightful information about the mentality of the common people. This helps the government or management to take appropriate measures in time. The COVID-19 pandemic has affected the socio-economic condition of the common people very badly and it has had a deep effect on the mental situation of the common people. This study can be performed in a specific state or area; people very often share their thoughts in social media; analyzing this textual information can be very helpful to predict forthcoming situations, so that administration can take proper measures to tackle unpredicted situations.

The rest of the paper is organized as follows: in Section 2, we discuss related work. In Section 3, we present detailed steps of our proposed approach. In Section 4, we present the analysis of the Twitter data and a visualization of the results. In Section 5, we present conclusions and recommendations.

2 Objective of the study and the novelty of the work

The main goal of this paper is to cover the COVID-19 pandemic situation in India and its impact on human emotions. As people very often express their state of the mind through social media, analyzing and tracking their emotions can be very effective for the government and local authorities to take required measures. We have analyzed different machine learning classification models, such as Naïve Bayes, Support Vector Machine, Random Forest Classifier, Decision Tree and Logistic Regression with 10-fold cross validation to find out the top ML models for emotion classification. After tuning the Hyperparameter, we got Logistic regression as the best suited model with accuracy 77% with the given datasets. We used algorithms based supervised ML technique to get the expected result. Although multiple studies were conducted earlier in the same line, none of them performed a comparative study among different ML techniques or hyperparameter tuning to optimize the results. Besides, this study has been done on the dataset of the most recent COVID-19 pandemic situation, which is itself unique.

3 Literature review

With ever increasing popularity of different social media and microblogging sites, it is of paramount interest for different researchers to analyze and construct meaningful insights from the messages. Wiebe, Wilson and Cardie (2005) worked on manual annotation of expressions of opinions and emotions in language for the corpus size of 10,000. Arora et al. (2010) worked on the automatic feature construction using subgraph mining algorithm. Gupta, Gilbert and Di Fabbrizio (2012) worked on emotion detection in email customer care using salient features and N-gram. Mohammad and Turney (2010) worked on the creation of an emotion lexicon using Amazon’s Mechanical Turk. Keshtkar and Inkpen (2010) worked on a technique to extract paraphrases for emotion terms from non-parallel corpora. Lee, Chen and Huang (2010) worked on emotion cause detection using a text-driven rule based system. Ramanand, Bhavsar and Pedanekar (2010) worked on finding actionable insights from suggestions and ‘buy’ wishes of product reviewers using a lexicon-based approach. Volkova et al. (2010) worked on an emotion analysis system, showing how an annotation task can be set up. Hanser et al. (2010) worked on enhancing news reading experiences by integrating 30 seconds long Flash animations into news article web pages depicting their content and emotional aspects. Bandhakavi et al. (2017) worked on general-purpose emotion lexicons (GPELs) that associate words with emotion categories. Gupta et al. (2020) introduced a compact and efficient way of representing text for further downstream natural language processing (NLP) tasks. Kumar, Kawahara and Kurohashi (2018) proposed a two-layered attention network based on Bidirectional Long Short-Term Memory for sentiment analysis. Hakak et al. (2017) summarized the previous studies of textual emotion analysis based on various emotional models and computational approaches used. Yanran et al. (2017) developed a high-quality multi-turn dialog dataset. Similar work was done by Hakak et al. (2019). Ahmed et al. (2020) proposed an attention-based model to detect emotion from tweets. Gupta et al. (2017) worked on a sentiment and semantics based approach for emotion detection in textual conversations.

4 Methodology

4.1 Proposed approach of emotion classification process

As shown in Figure 1, we have taken JIRA and Stack Overflow labeled dataset for training and testing purposes. We applied different preprocessing techniques to remove noise and unwanted words or characters. Then we did feature extraction using TF-IDF and N-Gram techniques. In the next step, we used five different classification algorithm with 10-fold cross validation to find out the top three algorithms, then tuned the hyperparameters to get the best algorithm for the given dataset.

In parallel to the above mentioned process, we were collecting Twitter data related to #COVID19 for 45 days. After preprocessing, we fed the Twitter data in the selected machine learning model to classify the tweets into five different emotion categories.

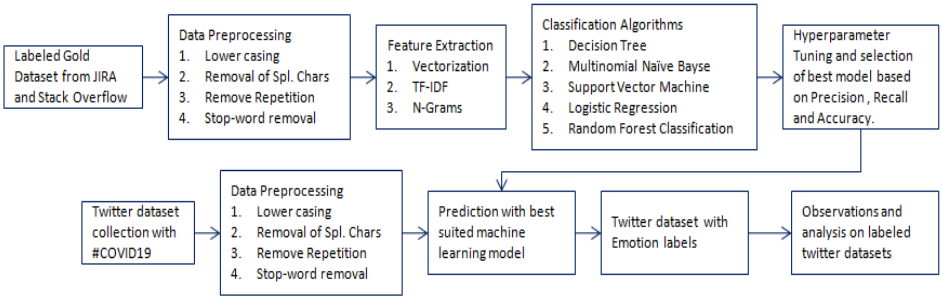

Figure 1: Proposed approach of emotion classification process

4.2 Emotion selection for pandemic situation

Emotion classification requires choosing the most appropriate model to categorize human sentiments. Researchers mostly classify emotions on the basis of the framework proposed by Shaver et al. (1987) or on Ekman’s six basic emotions (Ekman, 1971). In this study, we used two gold standards datasets annotated according to the discrete framework by Shaver et al. (1987). This framework defines a tree-structured hierarchical classification of emotions, where each level refines the granularity of the previous one, thus providing more indication about its nature. It includes six basic top-level emotions, namely: love, joy, anger, sadness, fear, and surprise. The gold standard datasets had very small leveled data for fear and surprise categories; in the COVID-19 situation the fear factor is very closely related to sadness, and surprise is quite similar to joy or happiness. So here we are broadly categorizing the emotions in four main categories: love, joy, anger, and sadness.

4.3 Training and testing dataset

Multiple research papers in related fields were consulted to get the understanding and to obtain the proven Gold standard dataset to model the system, but, unfortunately, there are very limited data sets available for the emotion types labeled. Besides, most of the datasets are crowd source data, so the degree of correctness varies depending on the person and the situations. Also on the basis of multiple feedbacks from different researchers, we have selected the labeled dataset of stack overflow and JIRA as the gold standard dataset. These datasets have been annotated according to the discrete framework by Shaver et al. (1987). It includes six basic emotions, namely: love, joy, anger, sadness, fear, and surprise. But the ‘surprise’ category was not considered very often as labeled datasets. Table 1 shows the different emotion categories of labeled data collected from Stack Overflow and JIRA. Figure 2 shows the distribution of the dataset under different emotion categories. It is observed that the majority of the datasets fall under the category of 'Love' and 'Anger'.

Table 1: Gold standard labeled Training Dataset

Dataset	Emotion Level Distribution of the Gold Dataset				
	Love	Joy	Anger	Sadness	Fear
Stack OF	1220	491	882	230	106
JIRA	166	124	324	302	0
Total	1386	615	1206	532	106

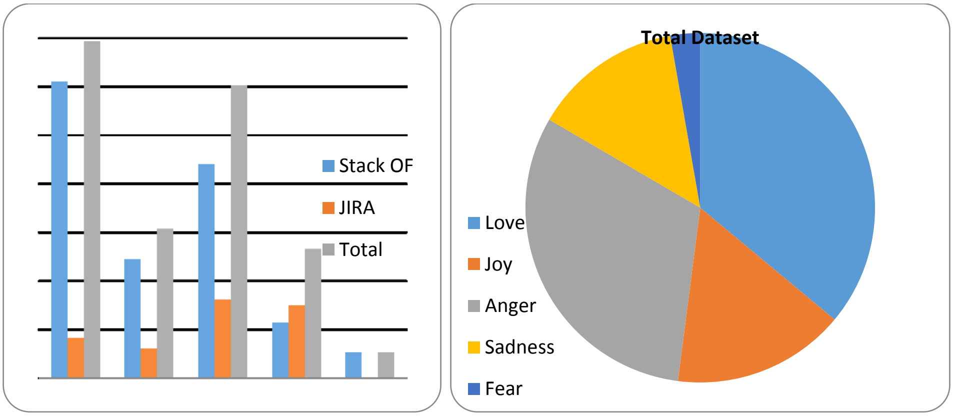

Figure 2: Distribution of the dataset under different emotion category

The distribution of the emotion categories is not uniform. So there might be biasness towards the majority of the dataset. There are five categories of emotions: Love, Joy, Anger, Sadness and Fear.

4.4 Social Media Data Collection for emotion analysis

We collected Twitter (www 2) data for a duration of 45 days starting from 15th Oct, 2020 to 30th Nov, 2020. We used tweepy API with geolocation as the central location of India and radius of 1000 km to collect Twitter data with the hashtag #COVID19India OR #COVID19. As India is one of the countries most affected by COVID-19, we collected data from all over India to get a sense how emotions are changing over a period of 45 days. We collected 2000 tweets every day and ignored all re-tweets; in total, we collected a corpus of 80 000 tweets for emotion classification. We only considered tweets written in English and we used tweet_mode as extended to collect tweets with a limit of 280 characters.

4.5 Data preprocessing

Figure 3 shows the steps for data preprocessing.

- Lowercasing: Entire text has been converted to lower case letter.
- Remove special chars: Special characters such as @,’,http ,^ etc. have been replaced by spaces.
- Remove repetitions: Repetition of the words have been removed.
- Transform short negation form: Short negation forms, such as can’t, have been replaced by cannot.
- Remove stop words: Stop words have been removed using Python NLTK library.

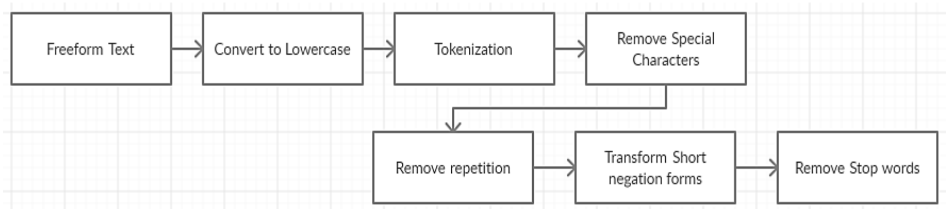

Figure 3: Flow diagram − data preprocessing steps

4.6 Feature extraction

We removed all the ‘English’ stop words and used wordnet lemmatization before proceeding with tokenization. We used CountVectorizer for vector representation of the sentences with word frequency as integer number. It built a vocabulary with the top features ordered by term frequency across the corpus. We used TF-IDF and n-gram for feature extraction. For n-gram, we tried unigram, bi-gram, and tri-gram to get the best accuracy. It was found that the combination of unigram and bi-gram provided the best result.

4.7 Classification techniques and comparative study

We did a comparative study of the five different machine learning classification techniques with default parameter setting and with 10-fold cross validations. As shown in Table 2, the results of different ML techniques were compared on the basis of precision, recall, F1-score and accuracy. The experimental results captured in Table 2 show that the top three ML models with high accuracy are SVM, LR, and RFC. These three ML models have been tuned to get better results; details of the hyperparameter tuning are given in the next section.

Table 2: Comparative study of different machine learning models

Emotions	Decision Tree			MNB			SVM			Logistic Regression			RFC		
	P	R	F1	P	R	F1	P	R	F1	P	R	F1	P	R	F1
anger	0.8061	0.7336	0.7681	0.7381	0.9066	0.8137	0.6957	0.9412	0.8001	0.7065	0.9412	0.8071	0.7905	0.8616	0.8245
fear	0.3667	0.3793	0.3729	0	0	0	0.5001	0.0691	0.1212	0	0	0	0.4615	0.2069	0.2857
joy	0.3191	0.4111	0.3593	0.5	0.0274	0.0519	0.5001	0.1849	0.2701	0.6545	0.2466	0.3582	0.405	0.3356	0.3571
love	0.7421	0.7191	0.7304	0.6435	0.9635	0.7717	0.7221	0.8315	0.7728	0.7273	0.8539	0.7855	0.7321	0.8062	0.7674
sadness	0.7647	0.7324	0.7482	0.9545	0.4437	0.6058	0.9223	0.6691	0.7755	0.9029	0.6549	0.7592	0.8264	0.7042	0.7605
Accuracy			**0.6684**			**0.6985**			**0.7193**			**0.7328**			**0.7183**
micro avg	0.5997	0.5951	0.5958	0.5672	0.4682	0.4486	0.6681	0.5391	0.5479	0.5982	0.5393	0.5421	0.6431	0.5829	0.6011
wt. avg	0.6891	0.6684	0.6773	0.6766	0.6985	0.6273	0.7033	0.7193	0.6854	0.7141	0.7328	0.6996	0.7058	0.7183	0.7082

4.8 Hyperparameter tuning

Each of the identified models can be tuned with different parameters as follows:

Logistic Regression: This ML model has been tuned with the parameters (www 3) from Table 3.

Table 3: Logistic Regression Hyperparameters setup

Logistic Regression Hyperparameters setup		
Hyperparameters	**Test Value sets**	**Selected value set**
C	1, 2.78, 7.74	2.78
max_iter	1000, 10000, 50000	50000
penalty	l1, l2	l1
norm	l1, l2	l2
use_idf	True, False	TRUE
ngram_range	(1, 1), (1, 2), (2, 2)	(1,2)

Support Vector Machine: This ML model has been tuned with the parameters (www 4) from Table 4.

Table 4: SVM Hyperparameters setup

SVM Hyperparameters setup		
Hyperparameters	**Test value sets**	**Selected value set**
C	1, 10, 100, 1000	1
degree	2, 3, 4, 5	2
gamma	1, 0.1, 0.01, 0.001, 0.0001	1
kernel	rbf','linear'	linear
norm	l1, l2	l2
use_idf	True, False	FALSE
ngram_range	(1, 1), (1, 2), (2, 2)	(1,2)

RFC: This ML model has been tuned with the parameters (www 5) from Table 5.

Table 5: RFC Hyperparameters setup

RFC Hyperparameters setup		
Hyperparameters	**Test Value sets**	**Selected value set**
max_depth	10, 25, 50, 100	25
min_samples_leaf	2, 5	2
min_samples_split	2, 3, 5, 10	5
n_estimators	10, 100, 200, 500	500
norm	l1, l2	l2
use_idf	True, False	FALSE
ngram_range	(1, 1), (1, 2), (2, 2)	(1,1)

4.9 The best selected ML model

Table 6 shows the comparative study of the three ML models after hyperparameters tuning was performed for each of them. The comparison was based on precision, recall, F1-score and accuracy. It was found that the logistic regression has the highest accuracy score of 77%; the accuracy score has been improved by 3.6% after parameter tuning.

Table 6: Comparative study of three ML models after hyperparameter tuning

Emotions	RFC			SVM			Logistic Regression		
	P	R	F1	P	R	F1	P	R	F1
anger	0.7071	0.9273	0.8024	0.7479	0.9239	0.8266	0.7946	0.9101	0.8484
fear	0	0	0	0.5001	0.0691	0.1212	0.6154	0.2759	0.3811
joy	0.7143	0.1712	0.2762	0.6338	0.3082	0.4147	0.6588	0.3836	0.4848
love	0.7035	0.9129	0.7946	0.7238	0.8539	0.7835	0.7476	0.8736	0.8057
sadness	0.9651	0.5845	0.7281	0.9001	0.6972	0.7857	0.8718	0.7183	0.7876
Accuracy			**0.7287**			**0.7453**			**0.7692**
macro avg	0.6181	0.5192	0.5203	0.7011	0.5704	0.5864	0.7376	0.6323	0.6615
wt. avg	0.7236	0.7287	0.6845	0.7366	0.7453	0.7209	0.7626	0.7692	0.7544

5 Classification of Twitter data and visualization of results

5.1 Testing results and analysis of the classified emotions with dates

We collected Twitter data for a duration of 45 days with #COVID19India OR #COVID19, collecting a set of 2000 tweets per day. In the current pandemic situation, with the unavailability of proper medications so far and the number of cases not yet under control, people are mostly angry and frustrated. In addition, job loss due to a long lockdown period, and the overall social economic condition in India is very bad. Still, news of the vaccines against COVID-19, gradual removal of lockdown, and economic recovery raised some hope and created a positive mindset among the people. This complete picture has been reflected in our analysis of the data captured in a span of more than one month. Close to 80% of the tweets showing anger and frustration related to the COVID-19 situation, there are certain small ups and downs in the graph but over all people are angry towards the COVID-19 condition.

5.2 Visualization of results

In Figure 4, we show a comparison of different emotions: anger, fear, joy, love and sadness, over a period of 45 days and their fluctuations with time and different social-economic factors.

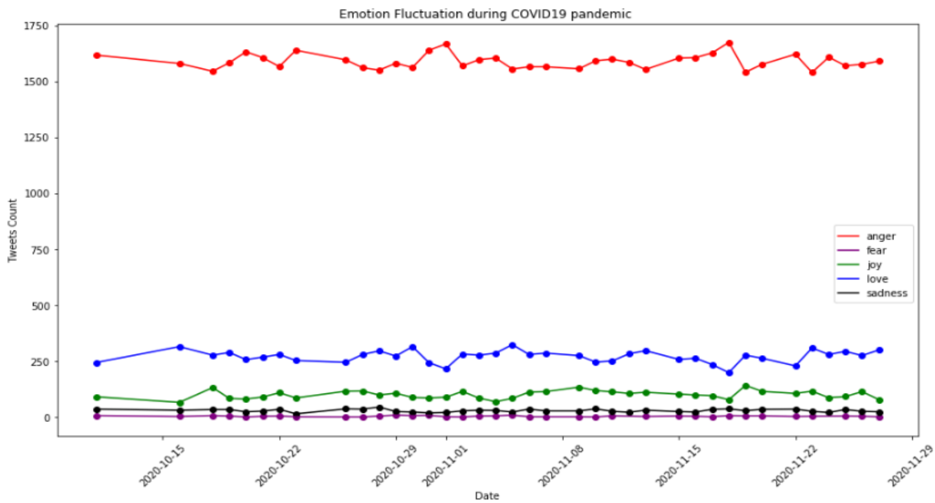

Figure 4: Emotion fluctuation during the COVID-19 pandemic

From the above plot it is evident that the ‘anger’ factor has dominated other emotions. In Figure 5, we have plotted the ‘anger’ emotion separately to get more detailed information about its trends. It can be seen that the anger factor is gradually decreasing of late.

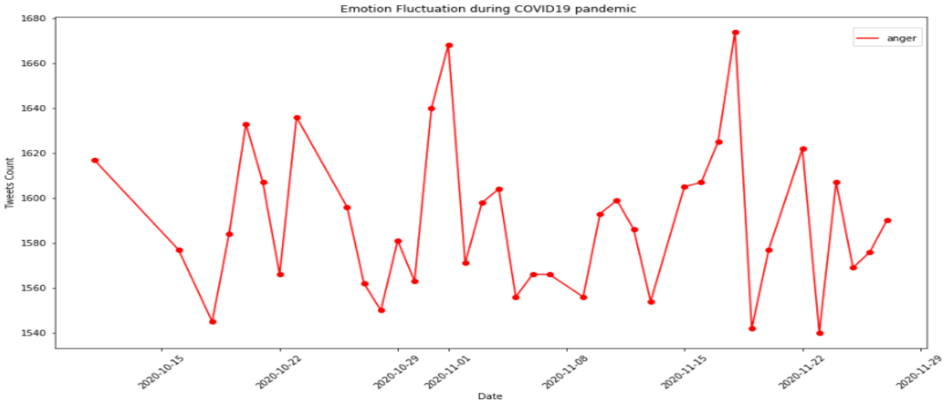

Figure 5: Variation of ‘Anger’ during the COVID-19 pandemic

6 Conclusion and recommendations

In this paper, we have done a comparative study of Decision Tree (DT), Multinomial Naive Bayes (MNB), Support Vector Machine (SVM), Random Forest Classifier (RFC) and Logistic Regression (LR) machine learning classification techniques. On the basis of precision, recall, F-score and accuracy, we selected the SVM, RFC and LR techniques for further performance optimization. Then we tuned the required parameter of the SVM, RFC and LR techniques and found that LR is the best technique with an accuracy of 77% for the given datasets. We collected tweets related to COVID-19 across India for a period of 45 days, and using the machine learning technique identified we classified the tweets into five different emotion categories. It has been observed that the ‘anger’ factor dominated the other emotions in COVID-19 related tweets. The selection technique of the ML model and its implementation can be used in a small local area to deeply analyze the mood swings of people in that area. Also, we discussed the application of this technique in other industry segments or for socio-political purposes. In future, this system can be extended to automatically identify other emotion categories in a more granular way and apart from depending on the small set of labeled dataset from crowd sourcing. This research study can be extended to decipher not only the contextual meaning of the messages, but also to perform psychological analysis of the users.

Declaration

1. Funding: Not Applicable
2. Conflicts of interest/Competing interests: Not Applicable
3. Availability of data and material: Available
4. Code availability: Available

References

Ahmed S., Reyadh A., Sithil F., Shah F., Shaafi A. (2020), *An Attention-based Approach to Detect Emotion from Tweets*, 182-187, DOI: 10.1109/IC2IE50715.2020.9274600.

Arora S., Mayfield E., Penstein-Rosé C., Nyberg E. (2010), *Sentiment Classification Using Automatically Extracted Subgraph Features*, Proceedings of the NAACL HLT 2010 Workshop on Computational Approaches to Analysis and Generation of Emotion in Text, Association for Computational Linguistics, Los Angeles, 131-139.

Bandhakavi A., Wiratunga N., Massie S., Deepak P. (2017), *Lexicon Generation for Emotion Detection from Text*, IEEE Intelligent Systems, 32, 102-108, DOI: 10.1109/MIS.2017.22.

Chang Y.-C., Yeh W.-C., Hsing Y.-C., Wang C.-A. (2019), *Refined Distributed Emotion Vector Representation for Social Media Sentiment Analysis*, PloS ONE, 14(10), 1-22.

Ekman P. (1971), *Universals and Cultural Differences in Facial Expressions of Emotion*, Nebraska Symposium on Motivation, 19.

Gupta N., Gilbert M., Di Fabbrizio G. (2012), *Emotion Detection in Email Customer Care*, Computational Intelligence, 29, 10-16, DOI: 10.1111/j.1467-8640.2012.00454.x.

Gupta S., Kanchinadam T., Conathan D., Fung G. (2020), *Task-Optimized Word Embeddings for Text Classification Representations*, Frontiers in Applied Mathematics and Statistics, 5, 67, DOI: 10.3389/fams.2019.00067.

Gupta U., Chatterjee A., Srikanth R., Agrawal P. (2017), *A Sentiment-and-Semantics-Based Approach for Emotion Detection in Textual Conversations*, https://arxiv.org/abs/1707.06996.

Hakak N., Kirmani M., Mohd M., Ahmed M., Mohsin M. (2019), *Automatic Emotion Classifier: Proceedings of ICACIE 2017, Volume 1*, Progress in Advanced Computing and Intelligent Engineering, 565-572, DOI: 10.1007/978-981-13-1708-8_52.

Hakak N., Mohd M., Kirmani M., Mohd M. (2017), *Emotion Analysis: A Survey*, 2017 International Conference on Computer, Communications and Electronics (Comptelix), 397-402, DOI: 10.1109/COMPTELIX.2017.8004002.

Hanser E., McKevitt P., Lunney T., Condell J. (2010), *NewsViz: Emotional Visualization of News Stories*, Proceedings of the NAACL HLT 2010 Workshop on Computational Approaches to Analysis and Generation of Emotion in Text, 125-130.

Keshtkar F., Inkpen D. (2010), *A Corpus-based Method for Extracting Paraphrases of Emotion Terms*, Workshop on Computational Approaches to Analysis and Generation of Emotion in Text, 35-44.

Kumar A., Kawahara D., Kurohashi S. (2018), *Knowledge-enriched Two-layered Attention Network for Sentiment Analysis*, Proceedings of the 2018 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Vol. 2 (Short Papers), 253-258.

Lee S., Chen Y., Huang C.-R. (2010), *A Text-driven Rule-based System for Emotion Cause Detection*, Proceedings of the NAACL HLT 2010 Workshop on Computational Approaches to Analysis and Generation of Emotion in Text, 45-53.

Li Y., Su H., Shen X., Li W., Cao Z., Niu S. (2017), *DailyDialog: A Manually Labelled Multi-turn Dialogue Dataset*, https://arxiv.org.pdf/1710.03957.pdf.

Mohammad S., Turney P. (2010), *Emotions Evoked by Common Words and Phrases: Using Mechanical Turk to Create an Emotion Lexicon*, Proceedings of the NAACL HLT 2010 Workshop on Computational Approaches to Analysis and Generation of Emotion in Text.

Ovesdotter Alm C., Roth D., Sproat R. (2005), *Emotions from Text: Machine Learning for Text-based Emotion Prediction*, Proceedings of the Conference on Human Language Technology and Empirical Methods in Natural Language Processing, 579-586.

Pearl L., Steyvers M. (2010), *Identifying Emotions, Intentions, and Attitudes in Text Using a Game with a Purpose*, Proceedings of the NAACL HLT 2010 Workshop on Computational Approaches to Analysis and Generation of Emotion in Text, Association for Computational Linguistics, 71-79.

Potamias R., Siolas G., Stafylopatis A. (2020), *A Transformer-based Approach to Irony and Sarcasm Detection*, Neural Computing and Applications, 32, DOI: 10.1007/s00521-020-05102-3.

Ramanand J., Bhavsar K., Pedanekar N. (2010), *Wishful Thinking: Finding Suggestions and 'Buy' Wishes from Product Reviews*, Proceedings of the NAACL HLT 2010 Workshop on Active Learning for Natural Language Processing, 54-61.

Russell J.A. (2005), *Emotion in Human Consciousness Is Built in Core Affect*, Journal of Consciousness Studies, 12, 8-10, 26-42.

Shaver P., Schwartz J., Kirson D., O'Connor C. (1987), *Emotion Knowledge: Further Exploration of a Prototype Approach*, Journal of Personality and Social Psychology, 52(6), 1061-1086.

Strapparava C., Valitutti A. (2004), *WordNet-Affect: An Affective Extension of WordNet*, Proceedings of the Fourth International Conference on Language Resources and Evaluation (LREC'04), 1083-1086.

Volkova E., Mohler B., Meurers D., Gerdemann D., Bülthoff H., Inkpen D., Strapparava C. (2010), *Emotional Perception of Fairy Tales: Achieving Agreement in Emotion Annotation of Text*, Proceedings of the NAACL HLT 2010 Workshop on Computational Approaches to Analysis and Generation of Emotion in Text, 98-106.

Wiebe J., Wilson T., Cardie C. (2005), *Annotating Expressions of Opinions and Emotions in Language*, Language Resources and Evaluation, 39(2-3), 165-210.

(www 1) *Types Emotions and Their Effect on Human Behavior*, https://www.verywellmind.com/an-overview-of-the-types-of-emotions-4163976 (accessed: 5.04.2021).

(www 2) Information collected from social networking site, www.twitter.com (accessed: 5.10.2020).

(www 3) Qiao F., *Logistic Regression Model Tuning with Scikit-learn*, https://towardsdatascience.com/(accessed: 8.01.2019).

(www 4) Dawson C., *SVM Parameter Tuning*, https://towardsdatascience.com/ (accessed: 31.05.2020).

(www 5) Koehrsen W., *Hyperparameter Tuning the Random Forest in Python*, https://towardsdatascience.com/ (accessed: 10.01.2018).

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WAVELET DECOMPOSITION APPROACH FOR UNDERSTANDING TIME-VARYING RELATIONSHIP OF FINANCIAL SECTOR VARIABLES: A STUDY OF THE INDIAN STOCK MARKET

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Abstract

In this paper, we study the effect of overall stock market sentiment in India on sectoral indices and on individual stock prices in terms of co-movement, dependence and volatility transmission along with the magnitude and persistence of the effects. The study uses wavelet decomposition framework for breaking down different financial time series into time-varying components. Quantile Regression, Wavelet Multiple Correlation and Cross-Correlation analysis, and Diebold-Yilmaz spillover analysis are then applied to investigate the nature of dependence, association, and spillover dynamics. For further focus, we have considered different time periods separately to identify the effect of market phases. Interesting results are obtained with respect to persistence of shocks, both across and within time periods. These have implications with respect to understanding market behavior and also perception of sectors and stocks.

Keywords: Wavelet Decomposition, SENSEX, Quantile Regression, Wavelet Multiple Correlation, Wavelet Multiple Cross Correlation, Diebold-Yilmaz Spillover.

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1 Introduction

Examination of behavioral patterns, causal nexus and volatility contagion in global stock markets has gained considerable attention among researchers. Proper understanding of such relationships is crucial as they have implications in investments, portfolio formation, financial flows and policy making (Glensk, Ganczarek-Gamrot and Trzpiot, 2013; Juszczuk, Kaliszewski and Miroforidis, 2017; Ghosh and Datta Chaudhuri, 2018; Ghosh and Datta Chaudhuri, 2019). The global financial crisis of 2008-09 showed that in today's globalized world, disruptions in one part of the globe can easily spread to other parts. It is thus important to study the inherent characteristics of market sentiment and its influence on other assets for effective risk management. The present study attempts to analyze the effect of overall stock market sentiment in India on sectoral indices and also on individual stock price movements in terms of co-movement, dependence and volatility transmission along with the magnitude and persistence of the effects. The analysis is made for four distinct time periods in the Indian stock market, namely: pre-crisis (2007-08), crisis (2008-09), bearish period (2015) and a bullish phase (2016). SENSEX returns have been chosen to represent overall market sentiment. Sectoral returns are represented by returns from the MID CAP Index and SMALL CAP Index. Further, daily stock returns of six different companies, namely: Bharat Forge Limited (BF), Cipla Limited (CP), Godrej Consumer Products Limited (GD), Biocon Limited (BC), Tata Consultancy Services Limited (TCS) and Glenmark Pharmaceutical Limited (GM) belonging to separate industrial sectors are analyzed to evaluate their co-movements with market sentiment and also the spillover dynamics during the four periods. The analysis can easily be extended to understand stock price movements of other companies.

Beta of Capital Asset Pricing Model (CAPM) has garnered a lot of interest in the literature. While some have looked at the time-varying properties of Beta, others have focused on "Good Beta, Bad Beta". No paper on stock price prediction has been able to ignore Beta in their model specification and analysis. Stocks have been characterized by Beta, and portfolio formation and realignment has been based on its magnitude. Although myriads of research work have attempted to model the interplay stock returns with market returns, the persistence of shocks cannot be explained by the CAPM model.

Modeling volatility has received strong traction in the literature which has resulted in ARCH, GARCH, DCC-GARCH, etc. framework-driven work to analyze the same (Datta Chaudhuri and Ghosh, 2015, 2016; Ghosh, Sanyal and Jana, 2021). As Beta is covariance divided by variance, researchers have applied this class of models, as there is volatility in the time series of returns. The purpose of this paper is to measure not the extent of market-wide shocks on individual stock returns or sectoral returns, but the persistence of the shocks. For this, we go granular and try to identify associations between market returns, individual stock returns, and sectoral returns in the short run, medium term and long term. Measurements of the extent of spillover of shocks and the duration of the shocks are the focus of the paper. We are not focusing on whether stocks are aggressive, neutral, or defensive; we are interested to know whether the responsiveness of their returns to market shocks is temporary or prolonged. In terms of length of persistence of shocks, our quest is also to understand whether market-wide shocks affect different sectors differently. If so, then this helps in portfolio formation.

The major contributions of the present study can be segregated into three broad strands. It first attempts to contribute to the literature by analyzing the dynamic interplay in terms of inspecting the duration of the same at the short, medium, and long run duration at granular level. The interplay has been decoded through careful investigation of dependence structure, co-movement dynamics, and volatility contagion effects. The methodological frameworks resort to quantile regression, wavelet driven correlation, and Diebold-Yilmaz spillover analysis in conjunction with wavelet decomposition to capture dependence, co-movement, and spillover relationship, respectively. Second, the research aims to expound the interplay in four specific regimes linked explicitly to the global financial crisis. Unlike the majority of previous work dedicated to examining interrelationships in the pre-crisis, crisis, and post-crisis phases at aggregate level, the present study further bifurcates the post-crisis phase into downswing and upswing phases for drawing inferences. The regime-driven exploration in association with wavelet-based time-frequency modeling delivers key insights and rationalizes the positioning of the paper with respect to past literature. The third contribution of the study lies in the selection of financial variables in Indian context. Again, dominance of large sector stocks while analyzing market sentiment, transmission of volatility, shocks is apparent in the literature. However, there is a dearth of research in assessment of mid- and small-cap sectors. The underlying work systematically fills the gap through incorporating both these sectors, alongside company-specific stocks, in the research agenda.

Performance of sectors of an economy reflects the demand pattern of the consumers, both national and global. This manifests itself in the performance of the companies in the different sectors, and gets reflected in their share prices. The business scenario is dynamic; companies of various sizes are trying to survive and grow. Companies of various sizes can be classified by their market capitalization and we have large cap, mid cap and small cap companies. Our focus in this paper is also on whether overall market-wide shocks affect companies of various sizes differently. Companies whose share prices take a beating, may find it hard in the short run to raise resources from the market, which in turn can adversely affect their growth prospects. Thus it is important to measure the extent of spillover of the shocks and also the duration of the shocks across sectors and companies.

The majority of financial time series during the said time periods have been observed to exhibit high degree of nonlinear and nonparametric behavior. Nonparametric behavior of the said series is a reflection of the violation of the normal distribution. Standard econometric and statistical approaches often fail to derive any significant insights from such data. Discrete Wavelet Transformation (DWT) is a nonparametric tool which is capable of modelling any high volatile and nonlinear time series in combined time and scale domain. It decomposes the original time series into a set of linear and nonlinear granular components accounting for different time horizons in an effective way. We use Maximal Overlap Discrete Wavelet Transformation (MODWT) algorithm, which is a variant of the DWT technique, and possesses several advantages over the latter in the decomposition process.

In this paper, we decompose each time series, and go granular for better understanding of the relationships within time phases. The following methods are used, on the decomposed series in different contexts, to analyze the time--varying pattern of the relationships in the Indian stock market.

1. Quantile Regression is applied in conjunction with wavelet decomposition to understand the dependence of MID CAP index returns and SMALL CAP index returns on market sentiment measured in terms of SENSEX returns.
2. Time-varying nature of association between SENSEX returns and individual company stock returns are analyzed through a separate framework called Wavelet Multiple Correlation (WMC) and Wavelet Multiple Cross Correlation (WMCC).
3. Volatility spillover of market sentiment on sectoral returns and company stock returns are explored through the Diebold-Yilmaz volatility spillover index on decomposed time components to understand time-varying characteristics of the same.

It may be noted that there exists a body of literature that explores the dynamics of co-integration and interrelationship of financial markets during the pre-crisis and crisis periods. However, the majority of these studies are either based on data of developed economies or confined to frameworks similar to CAPM. We have not come across granular wavelet-based research models to decode time-varying behavioral patterns of stock markets of developing economies during such time periods. The present paper attempts to fill this gap.

The rest of the paper is organized as follows. Section 2 is a discussion of the literature related to our research objectives and models. Section 3 presents the data profile and key statistical properties of the same. The research methodology is explained in Section 4, and Section 5 discusses the overall findings. Section 6 concludes the paper.

2 Literature review

As the objective of this paper is to examine the time-varying interaction among the financial sector variables, this section briefly outlines some recent work in financial modeling using different methodological frameworks. A careful and thorough review of cognate literature assists in identifying the trend and prevailing gaps. It should be noted that the effects of global financial crisis on markets have been studied by Gao and Mei (2019); Nikkinen, Piljak and Rothovius (2020); Bessler et al. (2021); Yamani (2021). Nevertheless, these studies mostly relied upon orthodox statistical and econometric modeling which limit the scope of finding a dynamic evolutionary pattern. To overcome the challenge, the present study resorts to wavelet-driven modeling. We have restricted the review to wavelet-driven frameworks for modeling temporal relationship specifically, as the primary objective of the present paper is to explicate the dynamics in varying timescales.

Ghosh and Datta Chaudhuri (2016) applied wavelet decomposition framework for studying interplay among historic and implied volatility indices of Indian and US markets. The results revealed time-varying nature of bidirectional and unidirectional causal structure which were effectively utilized for forecasting individual financial time series using machine learning models.

Jammazi et al. (2017) explored time-varying nature of causality structure between oil price change and stock returns of six oil-importing countries using wavelet analysis and dynamic causality test for assessing dependence in multiresolution framework. The overall outcome suggested the existence of strong causal links in the short run during the global financial crisis.

Liu et al. (2017) studied the volatility spillover phenomenon between oil and stock markets employing time-varying granular model comprising wavelet decomposition and GARCH-BEKK tool. Findings indicated that spillovers between oil and S&P 500 index reduced in long run, while a completely opposite behavior was prevalent between the MICEX index and oil market during the entire time span.

Das et al. (2018) assessed the nexus of Asian gold spot markets of China, India, Indonesia, South Korea, Thailand and Vietnam through Wavelet Coherence (WC), Wavelet Multiple Correlation (WMC) and Wavelet Multiple Cross Correlation (WMCC) models. Strong positive co-movement structure was found and Thailand emerged as the potential market leader.

Das et al. (2018) applied Wavelet Local Correlation (WLC) to examine the nature of contagion, interrelationship, and changes in association among emerging stock markets after the global financial crisis. The findings implied the presence of weaker contagion among Latin American emerging markets, significant contagion between European and Middle East markets, and insignificant long-run association between them post crisis.

Das and Kannadhasan (2018) explored the dependence of Bitcoin price on different macroeconomic factors, stock index, economic policy uncertainty, implied volatility, gold and crude oil price, using wavelet-based multi-scale approach. Findings revealed nonexistence of dependence of Bitcoin price on those factors in short run; however, significant dependence was present in the medium to long run.

Polanco-Martiez et al. (2018) carried out an exploratory research to determine the interplay between EU peripheral stock indices and the S&P Europe 350 index during pre-crisis and crisis periods through wavelet-based rolling correlation and nonlinear Granger causality test. The findings suggested the existence of stronger association and higher causal interaction in the crisis period than in the pre-crisis period.

Ghosh and Datta Chaudhuri (2019) examined pairwise association and causality between four different pairs of financial assets, namely: CBOEVIX (Implied volatility of US market) and INDIAVIX (Implied volatility of Indian market), Rupee-Dollar exchange rate and Crude oil return, DJIA return and IT sectoral return of India, and SENSEX and METAL sector return of India though MODWT-based granular Dynamic Conditional Correlation (DCC) – GARCH, Asymmetric Dynamic Conditional Correlation (ADCC) – GARCH models and the Diks-Panchenko test. The results indicated the existence of significant association at different timespan and asymmetric causal interaction. They also investigated volatility contagion dynamics of eight assets together using MODWT-based Diebold-Yilmaz spillover analysis and found the dominance of CBOEVIX in the long run.

Mishra et al. (2019) used wavelet-driven quantile regression framework to decode the interrelationship between the global crude oil price fluctuations and the Dow Jones Islamic stock index. Their findings indicated that short oil price fluctuation had positive influence on stock index, while the impact turned out to be adverse in the long run.

Mo et al. (2019) explored the influence of crude oil prices on the economic growth of the BRICS countries using wavelet-driven quantile-on-quantile tests. Positive effects on economic growth emerged, though heterogeneity across countries, periods, and quantiles were observed.

Dai et al. (2020) examined the dynamic dependence and risk contagion among oil, gold, and US foreign exchange markets in multiscale manner through deploying wavelet and time-varying vine-copula and vine-copula quantile regression approaches. Their results implied that the US foreign exchange market possessed stronger influence in the short run, while gold markets, in the medium run. Asymmetry in the risk contagion effect was also observed.

Ghosh, Sanyal and Jana (2021) resorted to continuous wavelet transformation-driven wavelet coherence analysis to model the dynamic dependence structure of financial and energy markets. The presence of long and medium run co-movement among the assets was discovered. They further combined MODWT and DCC-GARCH in an integrated framework to successfully estimate hedge ratios across the time scales.

Goodell and Goutte (2020) explored the co-movement of global COVID-19 mortality and Bitcoin price using wavelet coherence. The results indicated a strong negative association prevailing between the underlying variables.

Jiang and Yoon (2020) used wavelet multiscale decomposition and wavelet coherence analysis to inspect interaction of oil and stock markets of oil-importing and oil-exporting nations. The outcome revealed that stock markets of oil-exporting nations were relatively more affected by oil markets than those of the oil-importing nations.

Sharif, Aloui and Yarovaya (2020) used wavelet coherence and combined framework of discrete wavelet transformation and Granger causality to evaluate the dynamic interplay among daily COVID-19 infections in the US, US stock market (DJIA), geopolitical risk (GPR) in the US, economic political uncertainty (EPU) in the US, and WTI crude oil price. The overall findings suggested the presence of asymmetric association and a stronger penetration of the impact of COVID-19 infections on GPR and EPU of the US.

Thus it is amply apparent that wavelet-driven approaches have been extremely effective in modeling different financial time series data and in mining association among heterogeneous assets in time-varying manner. There is clear trend using these tools in discovering critical interaction structure at challenging times for practical implications including strategic management and policy formulations. These methodologies can delve quite effectively into financial markets with a high degree of volatility and nonlinearity at granular level. It should be noted that wavelet-driven frameworks have emerged to be extremely successful in predictive modeling of financial markets as well (Ghosh and Datta Chaudhuri, 2017; Ghosh, Sanyal and Jana, 2018; Ghosh and Datta Chauduri, 2019; Ghosh, Jana and Sanyal, 2019). Therefore resorting to wavelet--driven frameworks to accomplish the current research endeavors is truly justified. On the other hand, in terms of gap analysis and positioning the underlying work, the aforesaid review provides sufficient information and scope. It is amply evident that despite the prevalence of a substantial amount of research dedicated to wavelet-driven dynamic evaluation of interaction patterns, the scantiness of research concentrated on the Indian context specifically is apparent. Empirical studies to categorically evaluate the nature of association at the outset of black swan events such as the global financial crisis have largely been made on aggregate series which fall short of decoding short, medium, and long run patterns distinctly. Moreover, post-crisis assessment has not penetrated to a deeper level, to the best of our knowledge. The current paper effectively addresses the existing gaps through a proper selection of methodological frameworks, variables and regimes. Recently, the examination of the behavioral difference in reactions of stock markets to the global financial crisis and the COVID-19 pandemic has steadily garnered attention (Chang, McAleer and Wang, 2020; Choi, 2021). Nonetheless, the said analysis is beyond the scope of the current research.

3 Data profile

In order to analyze the dynamics of the Indian stock market, two different set of modeling exercises have been conducted during the pre-crisis, crisis and post--crisis periods.

In the first experiment, we have considered daily returns of SENSEX, BSE MID CAP INDEX (MID CAP) and BSE SMALL CAP INDEX (SMALL CAP) for understanding of the nature of behavior and nexus of sectoral indices in the Indian stock market. The study attempts to capture the influence of market sentiment on returns of MID CAP and SMALL CAP indices, assess the volatility dynamics, and measure the extent of spillovers.

The other experiment analyzes the co-movement of SENSEX return with selected Indian company stock returns, namely: Bharat Forge (BF), Cipla (CP), Godrej Consumer Products (GD), Biocon (BC), Tata Consultancy Service (TCS) and Glenmark (GM).

Daily observations on the variables from January 2007 to January 2008 have been considered for pre-crisis analysis; crisis period analysis has been performed on daily observations from February 2008 to March 2009; and daily data for two separate time periods, March 2015 to February 2016 (representing downswing) and March 2016 to November 2016 (representing upswing) have been considered.

We have estimated the key statistical properties of our dataset across different phases using well-known statistical tests. Tables 1-8 summarize the results.

Table 1: Test of normality and stationarity of SENSEX and sectors during Pre-Crisis

Series	Jarque-Bera Test	Shapiro-Wilk Test	Terasvirta Test	ADF Test
SENSEX	41.32***	0.9705***	9.39***	−18.97***
MID CAP	36.69***	0.9138***	10.34***	−16.17***
SMALL CAP	37.59***	0.9203***	10.08***	−16.98***

*** Significant at 1% level of significance.

Table 2: Test of normality and stationarity of SENSEX and sectors during Crisis

Series	Jarque-Bera Test	Shapiro-Wilk Test	Terasvirta Test	ADF Test
SENSEX	48.76***	0.9640***	10.85***	−17.58***
MID CAP	45.72***	0.9189***	11.32***	−15.84***
SMALL CAP	51.33***	0.9239***	10.57***	−16.31***

*** Significant at 1% level of significance.

Table 3: Test of normality and stationarity of SENSEX and sectors during Downswing (Post-Crisis)

Series	Jarque-Bera Test	Shapiro-Wilk Test	Terasvirta Test	ADF Test
SENSEX	39.15***	0.9146***	9.58***	−18.56***
MID CAP	34.43***	0.8977***	10.67***	−16.73***
SMALL CAP	36.82***	0.8860***	9.92***	−17.05***

*** Significant at 1% level of significance.

Table 4: Test of normality and stationarity of SENSEX and sectors during Upswing (Post-Crisis)

Series	Jarque-Bera Test	Shapiro-Wilk Test	Terasvirta Test	ADF Test
SENSEX	49.07***	0.9258***	10.17***	−18.17***
MID CAP	47.24***	0.9406***	10.89***	−16.59***
SMALL CAP	43.35***	0.9188***	10.37***	−17.36***

*** Significant at 1% level of significance.

Significant Jarque-Bera and Shapiro-Wilk test statistics across all distinct time phases imply that none of the financial variables abide by normal distribution. Likewise significant Terasvirta test statistics signify a clear presence of nonlinear behavior in daily returns of three assets. Lastly, the Augmented Dickey Fuller (ADF) test indicates that all the series exhibit stationary behavior.

Table 5: Test of normality and stationarity of SENSEX and six company stocks during Pre-Crisis

Series	Jarque-Bera Test	Shapiro-Wilk Test	Terasvirta Test	ADF Test
SENSEX (SN)	41.32***	0.9640***	9.39***	−18.97***
BHARAT FORGE (BF)	44.58***	0.9709***	9.16***	−20.13***
CIPLA (CP)	43.02***	0.9678***	9.28***	−19.46***
GODREJ (GD)	51.16***	0.9711***	11.37***	−21.37***
BIOCON (BC)	47.80***	0.9728***	10.61***	−18.84***
TCS (TC)	52.40***	0.9719***	10.15***	−20.44***
GLENMARK (GM)	40.39***	0.9621***	9.21***	−18.50***

*** Significant at 1% level of significance.

Table 6: Test of normality and stationarity of SENSEX and six company stocks during Crisis

Series	Jarque-Bera Test	Shapiro-Wilk Test	Terasvirta Test	ADF Test
SENSEX (SN)	48.76***	0.9724***	10.85***	−17.58***
BHARAT FORGE (BF)	49.34***	0.9733***	11.16***	−19.21***
CIPLA (CP)	46.77***	0.9716***	10.27***	−17.32***
GODREJ (GD)	49.94***	0.9611***	10.06***	−18.17***
BIOCON (BC)	50.13***	0.9745***	12.13***	−19.66***
TCS (TC)	48.72***	0.9732***	11.48***	−17.43***
GLENMARK (GM)	45.60***	0.9603***	10.75***	−16.95***

*** Significant at 1% level of significance.

Table 7: Test of normality and stationarity of SENSEX and six company stocks during Downswing (Post-Crisis)

Series	Jarque-Bera Test	Shapiro-Wilk Test	Terasvirta Test	ADF Test
SENSEX (SN)	39.15***	0.9718***	9.58***	−18.56***
BHARAT FORGE (BF)	42.23***	0.9723***	9.92***	−20.31***
CIPLA (CP)	41.79***	0.9706***	9.46***	−19.46***
GODREJ (GD)	43.47***	0.9729***	10.07***	−21.54***
BIOCON (BC)	42.28***	0.9648***	10.19***	−20.83***
TCS (TC)	44.34***	0.9701**	11.10***	−22.42***
GLENMARK (GM)	38.75***	0.9613***	9.32***	−18.11***

*** Significant at 1% level of significance.

Table 8: Test of normality and stationarity of SENSEX and six company stocks during Upswing (Post-Crisis)

Series	Jarque-Bera Test	Shapiro-Wilk Test	Terasvirta Test	ADF Test
SENSEX (SN)	49.07***	0.9713***	10.17***	−18.17***
BHARAT FORGE (BF)	51.16***	0.9706***	11.58***	−19.10***
CIPLA (CP)	47.82***	0.9643***	9.75***	−17.88***
GODREJ (GD)	53.49***	0.9739***	12.29***	−20.38***
BIOCON (BC)	48.18***	0.9685***	10.53***	−18.85***
TCS (TC)	46.74***	0.9661***	10.01***	−17.76***
GLENMARK (GM)	45.11***	0.9624***	9.89***	−17.68***

*** Significant at 1% level of significance.

Similar to findings of market sentiment and sectoral indices, daily returns of all six company stocks are found to be nonparametric (as the underlying series barely exhibits any sign of normal distribution), nonlinear and stationary in nature across all time phases. As the key endeavor of this research is to identify the time-varying characteristics, i.e. persistence, dependence, and volatility, wavelet-based time series decomposition approach has been used. Wavelet decomposition framework can simultaneously carry out inspection of behavioral characteristics of financial data in both different time and scales. It can also effectively model time series exhibiting high degree of non-parametric, nonlinear, and nonstationary behavior with great accuracy. Hence the deployment of wavelet-based examination frameworks in this paper is well justified as it serves the research purpose and is appropriate for dealing with financial data under consideration.

4 Methodology

This section outlines the detailed operational processes of the methodological frameworks used sequentially. As stated, we have deployed quantile regression, wavelet correlation, and Diebold-Yilmaz spillover analysis in conjunction with wavelet decomposition to accomplish research goals. The rationale behind the use of different research tools lies in the process of analyzing the dependence structure, co-movement, and volatility spillover separately. Unlike the plethora of previous research, we have not restricted the investigation to one particular form of interrelationship.

Wavelet Decomposition: Through wavelet transformation, the original series is expressed as a set of superimposed wavelets or orthogonal components. It generates father ($\varphi(t)$) and mother ($\psi(t)$) wavelets in scale-wise manner by translating and dilating the original function ($f(t)$). The mother wavelet is a square integrable function that generates a family of daughter wavelets by scaling and translating operations. Mathematically, it can be expressed as:

$$\psi_{s,u}(t) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-u}{s}\right) \tag{1}$$

The parameter s is a scaling or dilation factor, controlling the length of the wavelet and u is a location parameter that specifies where the wavelet is centered.

The mother wavelets account for the high frequency or detailed parts on each scale by noting the amount of stretching of the wavelet, while the father wavelet essentially represents the low-frequency component of the time series. The daughter wavelets are generated from the finite length high oscillating mother wavelet through scaling and translation operations. Unlike the mother wavelet, the father wavelet is a single elementary waveform which largely determines the quality of transformation. We have used MODWT for decomposition process utilizing the Haar filter. The present study considers six levels of decomposition for executing time-varying inspection. Table 9 explains the time mapping.

Table 9: Time interpretation of different frequencies

Scale 1	D_1	2~4 days (Intraweek Scale)
Scale 2	D_2	4~8 days (Weekly Scale)
Scale 3	D_3	8~16 days (Fortnightly Scale)
Scale 4	D_4	16~32 days (Monthly Scale)
Scale 5	D_5	32~64 days (Monthly to Quarterly Scale)
Scale 6	D_6	64~128 days (Quarterly to Biannual Scale)

Quantile Regression: Generally, to establish a relationship between an independent variable and a set of variables, a model with linear specification is formulated which is conditional upon its mean. Thus, the results essentially focus upon the mean value relationship. However, the relationship at different levels of conditional distribution of the dependent variable remains unexplored. To model the complex dependence of time series, Quantile Regression (QR) technique is applied on time-varying wavelet decomposition. The QR technique was introduced by Koenker and Bassett (1978), Mensi et al. (2014) and Nusair and Al-Khasawneh (2018) have applied these techniques.

Let Y be the dependent variable, which is assumed to be dependent on X linearly. Thus, the τ^{th} conditional quantile of a function may be specified as follows:

$$Q_Y(\tau|X) = inf(b|G_Y(b|X) \geq \tau) = \sum_k \beta_k(\tau)X_k \qquad (2)$$

where the conditional distribution function of $(Y|X)$ is represented by $G_Y(b|X)$; b is an actual realization of X.

The relationship of dependence between vector X and the τ^{th} conditional quantile of function is determined by $\beta(\tau)$. The dependence is conditional if the exogenous variables are added to X, while the dependence is unconditional if the exogenous variables are excluded from X. The complete dependence structure of Y is determined by $\beta(\tau)$ for $\tau \in [0,1]$. There can be three possible dependence structures between Y and vector of independent variables X: (a) where the value of $\beta(\tau)$ for different values of is similar (dissimilar), the relationship is symmetric (asymmetric) for lower and higher quantiles, (b) where the value of $\beta(\tau)$ does not change, the relationship is constant, and (c) where the values of $\beta(\tau)$ increases (decreases) with the values of τ, the relationship is monotonically increasing (decreasing).

For a given τ, the coefficients of $\beta(\tau)$ are estimated by minimization of the weighted absolute deviations between Y and X:

$$\hat{\beta}(\tau) = \arg min \sum_{t-1}^{T} \left(\tau - 1_{Y_t < \sum_k \beta_k(\tau) X_k}\right) |Y_t - \sum_k \beta_k(\tau) X_k| \quad (3)$$

where T denotes the temporal information whilst the usual indicator function is denoted by $1_{Y_t < \sum_k \beta_k(\tau) X_k}$. The solution to this problem is arrived by using a linear programming algorithm (Koenker and D'Orey, 1987). To obtain the standard error for the estimated coefficients, the pair-bootstrapping procedure proposed by Buchinsky (1995) is used. The asymptotically valid standard errors under heteroskedasticity and misspecifications of the QR function are obtained by this pair-bootstrapping procedure.

Quantile Regression of wavelet-decomposed values of the variables allows to model the interaction dynamics between the variables across different timescales and also at different market conditions. This model has been applied by Das et al. (2018); Karlsson et al. (2017); Mensi et al. (2014).

Wavelet Multiple Correlation and Cross Correlation: To overcome certain shortcomings of traditional pairwise correlation and cross correlation, Fernández-Macho (2012) introduced WMC and WMCC techniques. These use maximal overlap discrete wavelet transformation (MODWT) on a multivariate stochastic process $X_t = (x_{1t}, x_{2t}, \dots, x_{nt})$ to generate wavelet coefficients $W_{ijt} = \left(w_{1jt}, w_{2jt}, \dots, w_{njt}\right)$ at respective scales (λ_j). The WMC $(\varphi_x(\lambda_j))$ represents a set of multi-scale correlation determined from the stochastic process. The square root of regression coefficient of determination associated with the respective scales (λ_j) is calculated in linear combination of wavelet coefficients having maximum coefficient of determination. The coefficient of regression of a variable (z_i) on a regressor set $(z_k, k \neq i)$ is estimated as:

$$R^2 = 1 - 1/\rho^{ii} \quad (4)$$

where ρ^{ii} denotes the i^{th} diagonal element of the inverse of the complete correlation matrix (Fernández-Macho, 2012). WMC is calculated as:

$$\varphi_x(\lambda_j) = \sqrt{1 - \frac{1}{\max diag\ P_j^{-1}}} \quad (5)$$

where P_j represents the correlation matrix defined on W_{jt} and max diag (.) operator chooses the largest element in the diagonal of the argument. The coefficient of determination linked to regression of z_i on the remaining variables can be expressed as square of correlation between actual observations (z_i) and estimated values $(\hat{z}_i)$. Hence alternatively, the WMC can also be expressed as:

$$\varphi_x(\lambda_j) = \frac{Cov(w_{ijt}, \widehat{w}_{ijt})}{\sqrt{Var(w_{ijt})Var(\widehat{w}_{ijt})}} \tag{6}$$

The wavelet covariance and variance are computed as:

$$Cov(w_{ijt}, \widehat{w}_{ijt}) = \bar{\gamma}_j = \frac{1}{T_j}\sum_{t=L_j^{-1}}^{T-1} w_{ijt}\widehat{w}_{ijt} \tag{7}$$

$$Var(w_{ijt}) = \bar{\delta}_j^2 = \frac{1}{T_j}\sum_{t=:j^{-1}}^{T-1} w_{ijt}^2 \tag{8}$$

$$Var(\widehat{w}_{ijt}) = \bar{\varepsilon}_j^2 = \frac{1}{T_j}\sum_{t=:j^{-1}}^{T-1} \widehat{w}_{ijt}^2 \tag{9}$$

Maximization of the coefficient of determination is driven by w_{ij} on set of regressors $\{w_{kj}, k \neq i\}$. The fitted values of regression are represented by $\widehat{w}_{ij}$. The number of the affected wavelet coefficients due to the boundary of the wavelet filter having length L and scale λ_j is given by:

$$L_j = (2^j - 1)(L - 1) + 1 \tag{10}$$

while the number of unaltered coefficients is governed by the following equation:

$$\tilde{T} = T - L_j + 1 \tag{11}$$

The WMCC can be computed by putting a lag (τ) between the actual and estimated figures of the criterion construct at each scale (λ_j). Mathematically, the WMCC is formulated as:

$$\varphi_x\tau(\lambda_j) = Cor(w_{ijt}, \widehat{w}_{ijt+\tau}) = \frac{Cor(w_{ijt}, \widehat{w}_{ijt+\tau})}{\sqrt{Var(w_{ijt})Var(\widehat{w}_{ijt+\tau})}} \tag{12}$$

Confidence intervals can be constructed assuming that $X = (X_1, \ldots, X_T)$ represents a realization of equation 16 in terms of a multivariate Gaussian stochastic process. The wavelet coefficients are generated by applying J^{th} order MODWT to individual time series components.

$$\widetilde{W}_j = [\widetilde{W}_{j0}, \ldots, W_{j,T-1}] = \left[(\widetilde{w}_{1j0}, \ldots, \widetilde{w}_{nj0}), \ldots, (\widetilde{w}_{1j,\frac{T}{2^j}-1})\right], j = 1, \ldots, J \tag{13}$$

Finally, the confidence interval of the coefficient of correlation is calculated as:

$$CI_{1-\alpha}\left(\varphi_x(\lambda_j)\right) = tanh\left[\tilde{z}_j \pm \emptyset_{1-\alpha/2}^{-1} / \sqrt{\frac{T}{2^j} - 1}\right] \tag{14}$$

where $\emptyset_q^{-1}$ represents 100q% point of the standard normal distribution.

WMC and WMCC techniques have been used to understand the association between the returns of SENSEX and six company stocks during the pre-crisis, crisis and post-crisis periods.

Diebold-Yilmz Spillover (2009): The N-dimensional VAR(p) model is formulated as:

$$Y_t = \phi_1 Y_{t-1} + \cdots + \phi_p Y_{t-p} + \epsilon_t \tag{15}$$

where ϵ_t is i.i.d error component and the coefficients $(\phi_1 \dots \phi_p)$ represent the magnitude and direction of interrelationships between the target and the respective independent constructs. The system can also be expressed as a moving average representation:

$$Y_t = \epsilon_t + A_1\epsilon_{t-1} + A_2\epsilon_{t-2} + \cdots \tag{16}$$

where A_i denote the respective coefficients.

The contained information of coefficient matrices linked to spillover can be formulated with H-step ahead forecast error variance decompositions using equation 17:

$$Y_{t+H} - P(Y_{t+H}|Y_t, Y_{t-1}) = \epsilon_t + A_1\epsilon_{t+H-1} + A_2\epsilon_{t+H-2} + \cdots + A_{H-1}\epsilon_{t+1} \tag{17}$$

where $P(Y_{t+H}|Y_t, Y_{t-1})$ represents the *H*-step ahead forecast at time *t*.

If Σ_ϵ represents the covariance matrix of ϵ and $A_0 \coloneqq I_N$, then the covariance matrix of forecast's error can be calculated as:

$$\Sigma_{\epsilon,H} = \sum_{h=0}^{H-1} A_h \Sigma_\epsilon A_h^{Tr} \tag{18}$$

Diebold and Yilmaz invented the spillover index (SOI) to measure spillovers in terms of contribution of shocks from one variable to another in forecast's error variance using equation 19:

$$SOI = 100 \times \frac{1}{N}\sum_{i=1}^{N} \frac{\sum_{i\neq j}\sum_{h=0}^{H-1}(A_h L)_{ij}^2}{\sum_{h=0}^{H-1}\left(A_h \Sigma_\epsilon A_h^T\right)_{ii}} = 100 \times \left(1 - \frac{1}{N}\sum_{i=1}^{N} \frac{\sum_{h}^{H-1}(A_h L)_{ii}^2}{\sum_{h=0}^{H-1}\left(A_h \Sigma_\epsilon A_h^T\right)_{ii}}\right) \tag{19}$$

where N denotes the number of variables under consideration, L is the lower--triangular Cholesky factor of Σ_ϵ.

We have used this spillover analysis technique to measure the extent of spillovers from SENSEX returns to returns from other chosen assets across the time horizons.

Performing granular Diebold-Yilmaz spillover analysis in conjunction with wavelet decomposition assists in effective critical evaluation of the dynamics of spillover transmission in terms of magnitude and duration. Using the said framework effort has been made to capture short, medium and long run volatility contagion patterns during the pre-crisis, crisis and post-crisis periods.

5 Results and analysis

5.1 Dependence analysis

Quantile regression (QR) has been performed on the decomposed time series to understand the dynamic dependence of MID CAP and SMALL CAP sectors on market sentiment on four quantiles (0.05, 0.1, 0.9, 0.95). The three lower quantiles, i.e. 0.05 and 0.1, represent the bearish market state while the upper quantiles, 0.9 and 0.95, denote the bullish phase.

In order to test the robustness of our approach, our data set spans four periods, namely: a bullish phase prior to the 2008 world financial crisis, a bearish phase during the crisis, a bullish phase during 2015 and a bearish phase in 2016. While the wavelet method decomposed the aggregate time series data into period-wise movements, ranging from one week to six months, the quantile regression method identified the upswings and downswings within each of the four periods. The following two-way tables for each of the periods summarize the results.

Table 10: Dependence structure during Pre-Crisis

	Downswing		Upswing	
MID CAP SMALL CAP	.05	.1	.9	.95
D1	.76 .69	.78 .73	.62 .59	.60 .51
D3	.68 .58	.69 .56	.70 .65	.59 .43
D6	.79 .79	.77 .75	.53 .31	.43 .19

We can observe from Table 10 that the effects of SENSEX returns on MID CAP returns are much stronger during downswings than in upswings in the very short run, i.e. two to four days, and also within a fortnight. This effect gets stronger as time progresses beyond three months. This is also true of the effect of SENSEX returns on SMALL CAP returns. However, during the upswing, the effect wanes after a fortnight. The effect on SMALL CAP returns falls drastically. The results indicate that in a downswing, that is with bad news, SMALL CAP and MID CAP stock returns take a beating immediately. During an upswing, however, they do not move with market sentiment after a couple of months. So there is an asymmetry in effects of bad news and good news on such stock returns.

Table 11: Dependence structure during Crisis

	Downswing				Upswing			
	.05		.1		.9		.95	
SMALL CAP \ MID CAP	MID CAP	SMALL CAP	MID CAP	SMALL CAP	MID CAP	SMALL CAP	MID CAP	SMALL CAP
D1	.67	.57	1.43	.55	.67	.57	.74	.60
D3	.80	.78	.78	.75	.84	.76	.81	.73
D6	1.20	1.31	1.22	1.22	1.26	1.26	1.28	1.25

Table 11 summarizes the extent and duration of the effect of market sentiment on the MID CAP and SMALL CAP stocks during the 2008 financial crisis. It can be observed that first, the long term effects were much more severe than those in the pre-crisis phase. Second, the long term effects were stronger than the short term effects within this period. Third, these stocks did not respond to any market corrections within the crisis period. Fourth, there is no significant difference in the effects of market shocks on these two classes of stocks.

Table 12: Dependence structure during the Post-Crisis Bearish phase

	Downswing				Upswing			
	.05		.1		.9		.95	
SMALL CAP \ MID CAP	MID CAP	SMALL CAP	MID CAP	SMALL CAP	MID CAP	SMALL CAP	MID CAP	SMALL CAP
D1	1.03	1.01	.98	1.04	1	1	1.1	1.02
D3	1.07	1.12	1.14	1.15	1.02	1	1	.98
D6	1.28	1.54	1.13	1.29	1.19	1.49	1.26	1.60

In this post-crisis bearish phase during 2015, the effect of market sentiment has been quite severe in both the MID CAP and SMALL CAP segments, over a significant time period, as reported in Table 12. The magnitudes are quite high, and in the short run, the effect on the SMALL CAP segment has been relatively more severe than on the MID CAP stocks. In the upswing phase, in this overall bearish market sentiment, both sets of stocks responded positively, and the long term effects were stronger sensing a market reversal.

Table 13: Dependence structure during the Post-Crisis Bullish phase

	Downswing		Upswing	
MID CAP / SMALL CAP	.05	.1	.9	.95
D1 (MID CAP)	.92	.93	.77	.95
D1 (SMALL CAP)	1.12	.76	.85	1.04
D3 (MID CAP)	.83	.83	.91	.93
D3 (SMALL CAP)	.73	.76	.89	1
D6 (MID CAP)	.92	.93	.98	.95
D6 (SMALL CAP)	1.01	1.07	.78	.74

Table 13 suggests that even in a bullish phase, during downward correction, the SMALL CAP stocks get affected more than the MID CAP stocks. Furthermore, between upswing and downswing, there is an asymmetry in effects for the SMALL CAP stocks. The returns from such stocks are not as sensitive to positive news as they are to negative news. The results indicate that for the MID CAP stocks, the effects of bad news or good news were similar across time spans.

Thus the overall findings of QR assist us in comprehending the time-varying nature of the influence of market sentiments on sectoral level stock returns during pre-crisis, crisis, and post-crisis periods.

5.2 WMC & WMCC

As explained earlier, WMC and WMCC have been applied on SENSEX returns of (SN), BF, CP, GD, BC, TC and GM to study the overall dynamic interaction in phase-wise manner. Basically, they extract the nature co-movement and the leader responsible for driving the association. The following figures present the outcome of WMC and WMCC analysis, respectively.

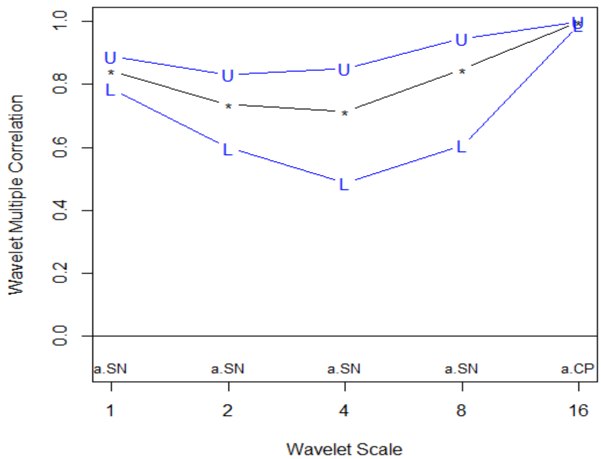

Figure 1: WMC plot of the Pre-Crisis phase

The wavelet scale 1 to 5 accounts for D1 to D5. The WMC plot reveals that at D1 and D5 scales, the correlation coefficient attains its local and global maxima values, respectively. From D1 to D3, the correlation decreases, while from D3 to D5 it increases monotonically. Hence it can be concluded that during the pre-crisis period, a high degree of association existed among the considered assets initially in the short run. The interaction gradually decayed slowly from short run to medium run; it gained momentum again from medium to long term. It can be seen that SN was leading on the D1 to D4 scale, while CP, on the D5 scale.

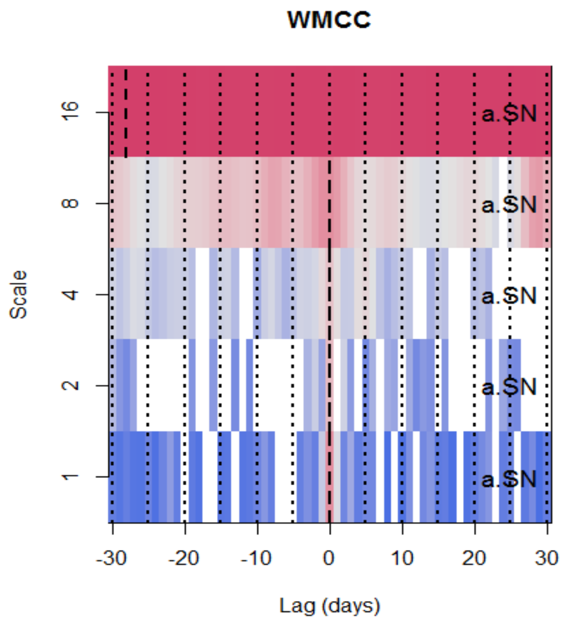

Figure 2: WMCC plot of the Pre-Crisis phase

The WMCC plots further explore the association at various lag lengths apart from point-wise estimation. The plots are generated using Daubechies ‘la8’ filter allowing a time lag of one month (30 days). The localization of the highest wavelet correlation is indicated by vertical dashed lines. Different colors are used to measure the strength of association: shaded region represents strongest association, whereas blank region implies no correlation.

The WMCC figure of the pre-crisis period suggests the existence of strong interaction localization at the largest scale, D5, which persisted for a lag of up to one month approximately. SENSEX emerged as the market leader across all the scales.

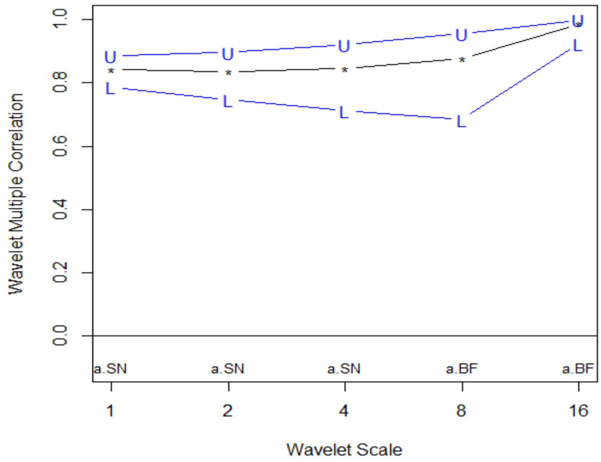

Figure 3: WMC plot of the Crisis phase

The WMC plot indicates a high degree of interaction as manifested through the correlation coefficient prevailing throughout the crisis period. At the D5 scale, i.e., in long run, it became maximum. Hence, it can be inferred that during this period, the interaction got stronger with time. SN emerged as the dominant driver of the association at the D1, D2 and D3 scales, while BF was the leader at the D4 and D5 scales. The distance between the upper and lower bands narrowed and the actual association also did not fall with time.

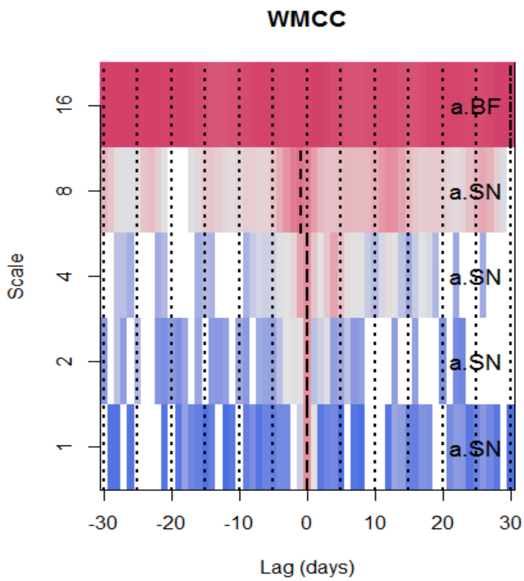

Figure 4: WMCC plot of the Crisis phase

The WMCC plot of the crisis period points out the presence of a highest degree of association mostly in the longest scale, i.e., D5, for a lag of up to nearly one month. SN emerged as the leader in the D1 to D4 scales, while BF acted as leader in the D5 scale.

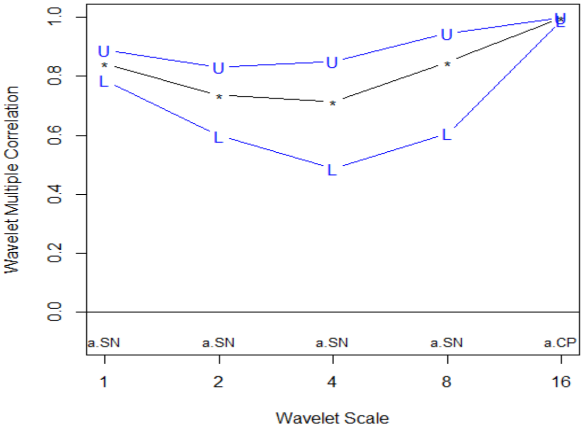

Figure 5: WMC plot of the Post-Crisis downswing phase

During the post-crisis downswing period, the WMC plot reveals that the association nexus initially marginally decreased from D1 to D3 scale. Then from D3 to D5 scale, the interaction again gradually gained momentum as a steep increase in the correlation coefficient is apparent. Hence during the post-crisis downswing phase, the seven assets quickly entered into high interaction phase which gradually faded to fortnightly scale. Again, from the fortnightly to the quarterly period the interaction gradually became intense. SN and CP emerged as the leaders at D1-D4 and D5 scales, respectively.

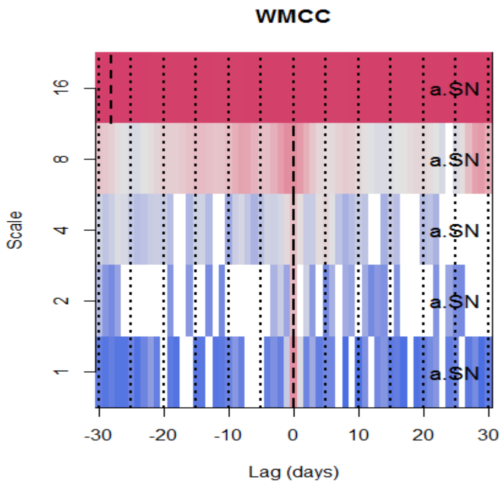

Figure 6: WMCC plot of the Post-Crisis downswing phase

Similarly to the pre-crisis and post-crisis phases, during the downswing phase of the post-crisis period, the WMCC plot reveals strong presence of a correlation among the seven financial time series across a lag period of one month in the D5 scale. SN has been found to be the leader across all time scales.

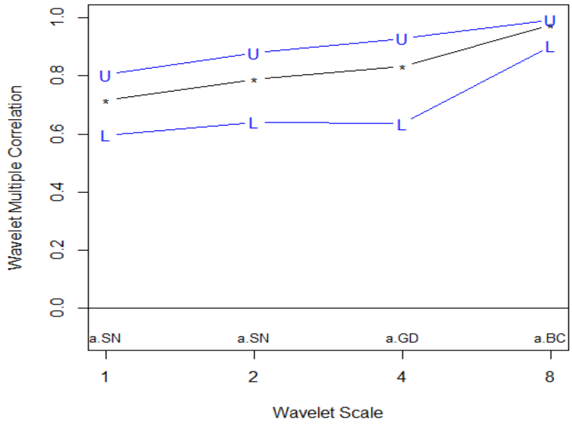

Figure 7: WMC plot of the Post-Crisis upswing phase

The plot suggests that during the post crisis upswing period, the association among the seven financial time series gradually became stronger from the D1 to D4 scale, i.e., from short to medium run duration. Two new entrants, i.e. GD and BC, are found to lead the association at the D3 and D4 scales, while SN remained the leader at the D1 and D2 scales.

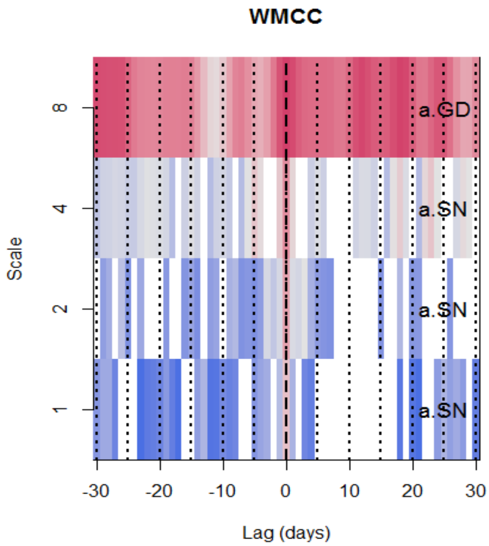

Figure 8: WMCC plot of the Post-Crisis upswing phase

During the upswing period of the post-crisis phase, the WMCC figure suggests the strongest concentration of association in the D4 scale. However, the cross correlation is unable to find the exact duration of the lag periods, as the spread of the association is not uniform and appears to be fading. GD has been found to drive cross correlation at the D4 scale, while SN leads them in the remaining scales.

Hence, it can be concluded that market sentiment had a profound association with six company stocks during different time phases. Market sentiment has also been found to be leading the association at distinct time scales. The insights generated through WMC and WMCC can be effectively exploited for time-varying portfolio diversification. In a nutshell, the following inferences can be drawn:

- In the upswing phase of the pre sub-prime crisis, markets were on the high and rising. There was herding and the association increased with time. However, in the short run there was portfolio realignment and these stocks may not have found favor. The dispersion between the upper and lower bounds becomes high, indicating portfolio alignment. But in the long run, every stock was good, implying that in the presence of herding, market participants paid scant attention to the fundamentals of stocks and bought indiscriminately, feeling that every stock would give significant positive returns.
- During the crisis phase, there was severe herding from the very beginning, and as the crisis deepened, the association increased. The dispersion was also much lower, showing that the market was sliding uniformly and fast. There was no time for portfolio alignment.
- In the downswing, there was portfolio realignment, and these stocks do not show association. However, as time progressed, some herding can be seen in the long run. Markets fell, and so did returns from these stocks. Portfolio realignment led to an increase in the dispersion, which decreased with time, as herding set in.
- In the next upswing phase, a bullish phase, the market was looking good. So all the stocks moved up together and as the upswing persisted, the association also improved. This is also evident from the fact that the dispersion was much lower than in the downswing phase.

5.3 Diebold-Yilmaz spillover analysis

One of the objectives of the paper is to understand volatility spillover of market sentiment on sectoral returns and company stock returns. We have applied Diebold-Yilmaz spillover analysis on decomposed components of the respective series of returns to measure the volatility spillovers. We report the level of spillovers at three levels, D2, D4, and D6, which act as proxies for short, medium and long run durations. The following tables summarize spillover percentage during the pre-crisis, crisis, and post-crisis phases.

Table 14: Spillover during the PRE-CRISIS phase

Scale	SMALL CAP	MID CAP	BF	CP	GD	BC	TC	GM
D2	31.59	37.88	11.23	3.85	51.86	15.74	1.16	10.14
D4	31.56	37.20	2.89	29.16	27.22	22.60	10.35	16.76
D6	46.30	28.73	6.67	14.81	7.84	2.27	0.89	11.22

It can be seen that during the pre-crisis phase, a significant amount of volatility was transmitted from SENSEX to both SMALL CAP and MID CAP in the short, medium and long runs. During the long run, the maximum amount of volatility transmission could be observed from the market to the SMALL CAP sector, while MID CAP received maximum spillover in the short run. In the short and medium time spans, volatility contagion between SENSEX and MIDCAP was more intense than between SENSEX and SMALL CAP. Among the company stocks, GD received the maximum impact from SENSEX in the short run, whereas CP was the maximum recipient in the medium run. In the long run, the amount of spillovers from market sentiment to all company stocks dwindled. This was a bullish phase, and for established companies, fundamentals became important rather than short-term volatility.

Table 15: Spillover during the CRISIS phase

Scale	SMALL CAP	MID CAP	BF	CP	GD	BC	TC	GM
D2	27.26	43.05	3.88	1.86	34.45	7.11	2.45	9.41
D4	60.65	62.16	0.84	2.85	41.69	16.89	5.71	2.28
D6	30.93	35.92	0.26	29.36	20.28	5.43	18.23	6.37

In the crisis phase, large spillovers can be observed and both the MID CAP and SMALL CAP indices received maximum spillover in the medium run. SMALL CAP received comparatively minimum spillover in the short run. That is, once the global financial crisis hit the markets, the MID CAP sector suffered the immediate brunt; then it intensified and spilled over to the SMALL CAP sector. It can be concluded that contagion was stronger in the medium run between market sentiments and sectoral indices. Considering the individual companies, only GD was affected, and to some extent CP.

Table 16: Spillover during the POST-CRISIS downswing phase (2015)

Scale	SMALL CAP	MID CAP	BF	CP	GD	BC	TC	GM
D2	41.75	22.35	1.54	0.44	49.93	29.08	0.02	15.72
D4	13.43	25.92	3.25	12.15	24.84	51.27	1.08	2.91
D6	49.24	61.47	12.70	72.41	1.14	0.10	10.51	0.29

In the post-crisis downswing phase the SMALL CAP sector received the highest spillover in the short run and also in the long run. The effect intensified for the MID CAP sector in the long run. In the case of individual companies, GD and BC were affected by the market in the short and medium run, and CP was most affected in the long run.

Table 17: Spillover during the POST-CRISIS upswing phase (2016)

Scale	SMALL CAP	MID CAP	BF	CP	GD	BC	TC	GM
D2	26.33	26.39	4.85	4.32	78.84	3.06	1.13	7.18
D4	35.97	58.04	0.35	38.26	1.70	4.54	8.21	1.25
D6	54.04	49.18	21.97	0.34	19.73	6.40	2.05	17.80

During the post-crisis upswing phase, spillovers received by SMALL CAP and MID CAP gradually increased from the short run to the long run. Among company stocks, GD and CP once again emerged as the top recipients of spillovers in the short run and medium run, respectively, while BF received maximum spillover in the long run.

It can be concluded that we have successfully captured and interpreted the interplay of financial markets represented by SMALL CAP, MID CAP, market sentiment and six company stocks through proper deployment of research methodologies. The present paper constitutes a significant contribution to existing research. The overall findings indicate that the relationships do not follow a uniform pattern across different timescales (Das et al., 2018; Ghosh and Datta Chaudhuri, 2019).

6 Conclusion

The performance of different sectors of an economy reflects the demand pattern of the consumers, both national and global. This manifests itself in the performance of the companies in the different sectors, and consequently gets reflected in their share prices. Furthermore, there are companies of different sizes which can be classified by their market capitalization, namely large cap, mid cap and small cap companies. Following the principles of microeconomics, the size of a company depends on the prevailing competitive position and economies of scale. Our focus in this paper is to examine to what extent overall market-wide shocks affect the different sectors and also companies of various sizes. The study also considers these effects over different time periods to understand whether the intensity and duration of the effect of shocks undergo any change.

The analysis is important as companies, whose share prices get affected by shocks, may find it hard in the short run to raise resources from the market, which in turn can adversely affect their growth prospects. Thus, the study attempts to measure the extent of spillover of the shocks and also the duration of the shocks across sectors and companies over different time periods.

In this paper, we study the effect of overall stock market sentiment in India on sectoral indices and on individual stock prices in terms of co-movement, dependence and volatility transmission along with the magnitude and persistence of the effects. The study uses a wavelet decomposition framework for breaking down different financial time series into time-varying components. Quantile Regression, Wavelet Multiple Correlation and Cross-Correlation analysis, and Diebold-Yilmaz spillover analysis are then applied to investigate the nature of dependence, association, and spillover dynamics. For further focus, we have considered different time periods separately to identify the effect of market phases. Interesting results are obtained with respect to persistence of shocks, both across time periods and within time periods. These have implications for the understanding of market behavior and also perception of sectors and stocks.

The contribution of the present study can be segregated into three broad strands. First, it analyzes the dynamic interplay among the variables at a granular level to understand the effects in the short, medium, and long run. The interplay has been decoded through careful investigation of dependence structure, co-movement dynamics, and volatility contagion effects. The methodological frameworks include quantile regression, wavelet driven correlation, and Diebold-Yilmaz spillover analysis in conjunction with wavelet decomposition to capture dependence, co-movement, and spillover relationship, respectively. Second, the paper analyzes four different time periods. Unlike the majority of previous studies dedicated to examining interrelationships in the pre-crisis, crisis, and post-crisis phases at aggregate level, the present paper further splits the post-crisis phase into downswing and upswing phases for drawing inferences. The regime-driven exploration in association with wavelet-based time-frequency modeling delivers key insights and rationalizes the positioning of the paper with respect to past literature. Third, the paper looks at companies of different sizes and also sectoral indices in terms of market capitalization.

The paper brings out the time-varying nature of association, dependence and volatility spillover dynamics of Indian stock market using wavelet-based frameworks. The results reveal the nature of interaction during the global financial crisis, pre-crisis and post-crisis phases. It has been observed that market sentiment has played a pivotal role in influencing both sectoral level stock indices and company level stock returns during the specified time periods. However, the magnitude of the impact has not been homogenous. The key insights generated from our granular level analysis are as follows:

- In terms of dependence, during the pre-crisis period, a comparatively stronger dependence of the MID CAP and SMALL CAP sectors on market sentiment has been detected in the bearish phase than in the bullish phase.
- During the crisis phase, the long run dependence was stronger than the short and medium run in both bearish and bullish market states.
- In the post-crisis downswing period, the dependence structure between market sentiment and SMALL CAP has been noticed to be relatively stronger than market sentiment and the MID CAP counterpart. A similar scenario prevailed during the post crisis upswing phase as well.
- In the context of association between market sentiment and six company stocks, our framework reveals that during the pre-crisis period there existed considerable opportunities for portfolio diversification.
- In the sub-prime crisis phase the association became extremely strong between market sentiment and stocks of the chosen companies, which implied very little opportunity for portfolio diversification.
- During the post-crisis downswing phase, evidence of herding behavior has been discovered in the long run time scale. The association bond became stronger during the subsequent upswing phase.
- As per the wavelet based spillover analysis, strong volatility spillover from market sentiment to the MID CAP sector was noticed in the short and medium duration during the pre-crisis bullish period. SMALL CAP was marginally less sensitive to shocks as compared to MID CAP in the short and medium run. Among the companies, GD and CP were the top recipients of volatility shocks. A similar phenomenon was noticed in the crisis phase.
- During both post-crisis downswing and upswing phases SMALL CAP and MID CAP received significant volatility spillover from market sentiment. Apart from CP and GD, BF and BC received a high degree of volatility.

The granular frameworks have successfully decoded the interaction dynamics of the Indian stock market in phase-wise time-varying manner in the bullish and bearish market states explicitly. The insights can be effectively exploited for practical applications including portfolio design and alignment and policy formations in a dynamic manner. The scope of the present study is restricted to six chosen companies. The framework can easily be extended to other Indian companies and other sectoral indices for modeling dynamic dependence structure. In the future, the impact of the COVID-19 pandemic on the Indian stock market in the form of association, dependence and spillover will be investigated.

References

Bessler W., Beyenbach J., Rapp M.S., Vendrasco M. (2021), *The Global Financial Crisis and Stock Market Migrations: An Analysis of Family and Non-family Firms in Germany*, International Review of Financial Analysis, 74, 101692, 1057-5219.

Buchinsky M. (1995), *Estimating the Asymptotic Covariance Matrix for Quantile Regression Models a Monte Carlo Study*, Journal of Econometrics, 68, 303-338.

Chang C.L., McAleer M., Wang Y.A. (2020), *Herding Behaviour in Energy Stock Markets during the Global Financial Crisis, SARS, and Ongoing COVID-19**, Renewable and Sustainable Energy Reviews, 134, 110349, p. 1-15.

Choi S.Y. (2021), *Analysis of Stock Market Efficiency during Crisis Periods in the US Stock Market: Differences between the Global Financial Crisis and COVID-19 Pandemic*, Physica A: Statistical Mechanics and Its Applications, 574, 125988, p. 1-20.

Dai X., Wang Q., Zha D., Zhou D. (2020), *Multi-scale Dependence Structure and Risk Contagion between Oil, Gold, and US Exchange Rate: A Wavelet-based Vine-copula Approach*, Energy Economics, 88, 104774, p. 1-58.

Das D., Kannadhasan M. (2018), *Do Global Factors Impact Bitcoin Prices? Evidence from Wavelet Approach*, Journal of Economic Research, 23, 227-264.

Das D., Kannadhasan M., Al-Yahyee K.H., Yoon S.M. (2018), *A Wavelet Analysis of Co-Movements in Asian Gold Markets*, Physica A: Statistical Mechanics and Its Applications, 492, 192-206.

Datta Chaudhuri T., Ghosh I. (2015), *Using Clustering Method to Understand Indian Stock Market Volatility*, arXiv:1604.05015.

Datta Chaudhuri T., Ghosh I. (2016), *Artificial Neural Network and Time Series Modeling Based Approach to Forecasting the Exchange Rate in a Multivariate Framework*, arXiv:1607.02093.

Diebold F.X., Yilmaz K. (2009), *Measuring Financial Asset Return and Volatility Spillovers, with Application to Global Equity Markets*, The Economic Journal, 119, 158-171.

Fernández-Macho J. (2012), *Wavelet Multiple Correlation and Cross-correlation: A Multiscale Analysis of Eurozone Stock Markets*, Physica A: Statistical Mechanics and Its Applications, 391, 1097-1104.

Gao H.L., Mei D.C. (2019), *The Correlation Structure in the International Stock Markets during Global Financial Crisis*, Physica A: Statistical Mechanics and Its Applications, 534, 122056, p. 1-8.

Ghosh I., Datta Chaudhuri T. (2016), *Understanding and Forecasting Stock Market Volatility through Wavelet Decomposition, Statistical Learning and Econometric Methods*, SSRN Electronic Journal, DOI: 10.2139/ssrn.2930876.

Ghosh I., Datta Chaudhuri T. (2017), *Fractal Investigation and Maximal Overlap Discrete Wavelet Transformation (MODWT)-based Machine Learning Framework for Forecasting Exchange Rates*, Studies in Microeconomics, 5, 105-131.

Ghosh I., Datta Chaudhuri T. (2018), *Stock Market Portfolio Construction: A Four-stage Model Based on Fractal Analysis*, South Asian Journal of Management, 25, 117-149.

Ghosh I., Datta Chaudhuri T. (2019), *A Wavelet Approach towards Examining Dynamic Association, Causality and Spillovers*, International Journal of Data and Network Science, 3, 23-36.

Ghosh I., Jana R.K., Sanyal M.K. (2019), *Analysis of Temporal Pattern, Causal Interaction and Predictive Modeling of Financial Markets Using Nonlinear Dynamics, Econometric Models and Machine Learning Algorithms*, Applied Soft Computing, 82, 105553, p. 1-17.

Ghosh I., Sanyal M.K., Jana R.K. (2018), *Fractal Inspection and Machine Learning-based Predictive Modelling Framework for Financial Markets*, Arabian Journal for Science and Engineering, 43, 4273-4287.

Ghosh I., Sanyal M.K., Jana R.K. (2021), *Co-movement and Dynamic Correlation of Financial and Energy Markets: An Integrated Framework of Nonlinear Dynamics, Wavelet Analysis and DCC-GARCH*, Computational Economics, 57, 503-527.

Glensk B., Ganczarek-Gamrot A., Trzpiot G. (2013), *Portfolio Analysis on Polish Power Exchange and European Energy Exchange*, Multiple Criteria Decision Making, 8, 18-30.

Goodell J.W., Goutte S. (2020), *Co-movement of COVID-19 and Bitcoin: Evidence from Wavelet Coherence Analysis*, Finance Research Letters, https://doi.org/10.1016/j.frl.2020.101625.

Jammazi R., Ferrer R., Jareno F., Shahzad J. (2017), *Time-varying Causality between Crude Oil and Stock Markets: What Can We Learn from a Multiscale Perspective?* International Review of Economics & Finance, 49, 453-483.

Jiang Z., Yoon S.M. (2020), *Dynamic Co-movement between Oil and Stock Markets in Oil-importing and Oil-exporting Countries: Two Types of Wavelet Analysis*, Energy Economics, 90, 104835, p. 1-33.

Juszczuk P., Kaliszewski I., Miroforidis J. (2017), *Trade-Off Guided Search for Approximate Pareto Optimal Portfolios*, Multiple Criteria Decision Making, 12, 49-59.

Karlsson H.K., Karlsson P., Månsson K., Sjölander P. (2017), *Wavelet Quantile Analysis of Asymmetric Pricing on the Swedish Power Market*, Empirica, 44, 249-260.

Koenker R., Bassett G. (1978), *Regression Quantiles*, Econometrica Journal of the Econometric Society, 46, 33-50.

Koenker R.W., D'Orey V. (1987), *Algorithm AS 229: Computing Regression Quantiles*, Journal of the Royal Statistical Society. Series C (Applied Statistics), 36, 383-393.

Liu X., An H., Huang S., Wen S. (2017), *The Evolution of Spillover Effects between Oil and Stock Markets across Multi-scales Using Wavelet Based GARCH-BEKK model*, Physica A: Statistical Mechanics and Its Applications, 465, 374-383.

Mensi W., Hammoudeh S., Reboredo J.C., Nguyen D.K. (2014), *Do Global Factors Impact BRICS Stock Markets? A Quantile Regression Approach*, Emerging Markets Review, 19, 1-17.

Mishra S., Sharif A., Khuntia S., Meo M.S., Abdul S., Khan R. (2019), *Does Oil Prices Impede Islamic Stock Indices? Fresh Insights from Wavelet-based Quantile-on-Quantile Approach*, Resources Policy, 62, 292-304.

Mo B., Chen C., Nie H., Jiang Y. (2019), *Visiting Effects of Crude Oil Price on Economic Growth in BRICS Countries: Fresh Evidence from Wavelet-based Quantile-on-quantile Tests*, Energy, 178, 234-251.

Nikkinen J., Piljak V., Rothovius T. (2020), *Impact of the 2008-2009 Financial Crisis on the External and Internal Linkages of European Frontier Stock Markets*, Global Finance Journal, 100481, p. 1-9.

Nusair S.A., Al-Khasawneh J.A. (2018), *Oil Price Shocks and Stock Market Returns of the GCC Countries: Empirical Evidence from Quantile Regression Analysis*, Economic Change and Restructuring, 51, 339-372.

Polanco-Martinez J.M., Fernandez-Macho J., Neumann M.B., Faria S.H. (2018), *A Pre-crisis vs. Crisis Analysis of Peripheral EU Stock Markets by Means of Wavelet Transform and a Nonlinear Causality Test*, Physica A: Statistical Mechanics and Its Applications, 490, 1211-1227.

Sharif A., Aloui C., Yarovaya L. (2020), *COVID-19 Pandemic, Oil Prices, Stock Market, Geopolitical Risk and Policy Uncertainty Nexus in the US Economy: Fresh Evidence from the Wavelet Based Approach*, International Review of Financial Analysis, 70, 101496, p. 1-9.

Yamani E. (2021), *Foreign Exchange Market Efficiency and the Global Financial Crisis: Fundamental versus Technical Information*, The Quarterly Review of Economics and Finance, 79, 74-89.

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EXTENDED CHOICE FUNCTIONALS – A CARDINAL FRAMEWORK FOR THE ANALYSIS OF CHOICE UNDER RISK

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Abstract

We propose a framework that extends the one developed by Professor Amartya Sen (with Arrowian roots), for the analysis of choice under risk by an individual, hereafter referred to as a decision maker. The framework is based on the decision maker's state dependent numerical evaluations − referred to as utility, worth, or pay-off − of the alternatives. We provide several examples to illustrate meaningful possibilities in the model proposed here. The expected utility choice functional assigns to each given state--dependent data profile (i.e., a pair consisting of a profile of state-dependent evaluation functions and a probability distribution over states of nature) the non-empty set of alternatives obtained by maximizing expected utility. A significant result in this paper, which illustrates the workability of our frameworks of analysis, is an axiomatic characterization of the expected utility choice functional using purely combinatorial techniques.

Aim/Purpose: To use a minor extension of the Arrow-Sen model of social choice theory to study individual decision making/aiding under risk and with state dependent evaluation functions.

Methodology: Combinatorics (theory of finite sets).

Findings: Plausible decision-aids for decision making under uncertainty with state dependent evaluation functions.

Research Implications: Exactly same model and results apply for the study of "weighted" multi-criteria decision making/aiding with state dependent evaluation functions.

Contribution: Apart from useful decision-aids for managerial decision making under risk and operations research, we provide an axiomatic characterization of the expected utility choice functional.

Keywords: risk, state-dependent evaluation, extended choice functionals.

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1 Introduction

Here we propose a framework for the analysis of choice under risk by an individual, hereafter referred to as a decision maker. The framework is based on the decision maker's state dependent numerical evaluations − referred to as utility, worth, or pay-off − of the alternatives. This framework is an extension of a model described in Sen (1970). A framework for the analysis of choice under risk, when the state-dependent preferences of the decision maker are expressed through rankings of alternatives, is motivated in Lahiri (2019b) and the framework in its entirety is discussed in Lahiri (2020a). Related axiomatic analysis, when the decision maker believes that all states of nature are equiprobable, is available (2019c) and a concrete analysis concerning the existence of "preferred with probability at least half" winners and when beliefs can be represented by any probability distribution is described in Lahiri (2020b). The problem of choosing one or more alternatives from a given set of alternatives was raised and rigorously formulated for the first time in a seminal contribution on majority voting by Pattanaik (1970). For the classical theory of decision making under uncertainty in the state dependent case − which is the other and major motivation behind this paper − one may refer to Karni (1985). Karni (1985) and Sen (1970) comfortably surpass the prerequisites related to decision making that is required to understand the frameworks of analyses developed here. An informative overall perspective of decision theory can be found in Resnik (1987). In the concluding section of this paper, we discuss a representation of uncertain prospects as ordered pairs of evaluation functions and probability distributions on the set of states of nature, motivated by a similar attempt in chapter 2 of Resnik (1987).

The reasons for our interest in state-dependent preferences are precisely the same as the ones discussed in Karni (1985), i.e., it is so obviously true that it does not need justification beyond citing trivial day-to-day examples as Karni has done in his book. Hence we can comfortably move ahead with our understanding of state-dependent preferences as in Karni (1985).

The major justification for the framework and the investigation presented in this paper is that the classical theory of decision making under uncertainty that rests on the assumption of maximization of expected utility (state-dependent or not) has significant limitations. It has often failed to be consistent with observed human behaviour in situations involving risk (i.e., uncertainty with probabilistic information about all states of nature available to or plausibly attributable by the decision maker) as was shown in the seminal work of Maurice Allais, also known as the Allais paradox (see Allais, 1953).

After defining the extended choice functional, we provide several examples of choice functionals. However, in order to show that our framework of analysis is very general and a "workable" model for the purpose of axiomatic characterizations, we provide here an axiomatic characterization of the expected utility choice functional. Related results are available in Lahiri (2019a). Going beyond that is the agenda for future research.

2 The model and some examples of extended choice functionals

The concept of an extended choice functional that is developed here, is a direct consequence of the concept of a social welfare functional introduced in Sen (1970) or its choice theoretic equivalent − choice functional − discussed in Lahiri (2019c), Sen's framework has been the subject of extensive as well as intensive research, that lead to a comprehensive survey by d'Aspremont and Gevers (2002).

Consider a decision maker (DM) faced with the problem of choosing one or more alternatives from a non-empty finite set of alternatives X. Let $\Psi(X)$ denote the set of all non-empty subsets of X. For a positive integer $n \geq 3$, let $N = \{1, 2, \ldots, n\}$ denote the set of states of nature. The satisfaction from the chosen alternative is realized only after the state of nature reveals itself.

We assume the satisfaction derived from the chosen alternative is represented by a numerically measurable worth or pay-off referred to as the **evaluation of the chosen alternative**.

An **evaluation function** is a function $u: X \times N \to \mathbb{R}$ such that for each alternative $x \in X$ and state of nature $i \in N$, $u(x,i)$ is the evaluation of x, in state of nature i. Let $\mathcal{U}$ denote the set of all evaluation functions.

Given $u \in \mathcal{U}$ and $x \in X$, we will often use $u(x)$ to denote the point $(u(x,1), \ldots, u(x,n))$ in $\mathbb{R}^N$ (the n-dimensional Euclidean space).

It is easy to see that $\{u(x) | \, u \in \mathcal{U} \text{ and } x \in X\} = \mathbb{R}^N$.

An **admissible set of evaluation functions** is any non-empty subset $\mathcal{D}$ of $\mathcal{U}$.

We denote vectors in $\mathbb{R}^N$ by letters a, b, c, d, etc. and when there is need for us to be explicit about (for instance) vector a, we write it as $(a_1, \ldots, a_n)$. $\mathbb{R}^N_+$ denotes the set $\{a \in \mathbb{R}^N | \, a_i \geq 0 \text{ for all } i \in N\}$.

The DM's beliefs about the possibility of the various states of nature being realized is summarized by a probability distribution, i.e., $p \in \mathbb{R}^N_+$ such that $\sum_{i=}^{n} p_i = 1$. Let P^N denote the set of all probability distributions on N. Let π denote the **equi--probability** distribution, i.e. $\pi \in P^N$ such that $\pi_i = \frac{1}{n}$ for all $i \in N$.

Given a probability distribution p, the **set of most likely states of nature at** p is denoted by $ML(p) = \{i \in N| p_i \geq p_j \text{ for all } j \in N\} = \operatorname*{argmax}_{i \in N} p_i$.

A **feasible set of probability distributions** (about the future states of nature being realized) is a non-empty subset of P^N denoted by Q. For whatever reasons, the DM's beliefs are restricted to belong to Q.

An **extended choice functional** (ECFL) on $\mathcal{D}\times Q$ is a function F: $\mathcal{D}\times Q \rightarrow \Psi(X)$, such that for each $(u,p) \in \mathcal{D}\times Q$, the decision maker chooses an alternative from F(u,p).

Before we proceed to examples, let us introduce the concept of regret which we shall require subsequently.

Given $u \in \mathcal{D}$, $x \in X$ and $i \in N$, **the regret from choosing** x **given** u **in state of nature** i, is: $\text{regret}(x,u,i) = \max_{y \in X} u(y,i) - u(x,i)$.

Note: If $\mathcal{D} = \{u \in \mathcal{U}| \text{ for all } i \in N,\ u(.,i): X \rightarrow \{1,2, \dots, \#X\} \text{ is a one-to-one function}\}$, where #X denotes the cardinality of X, then each u could be considered to be an assignment of state-dependent rank-score of an alternative, with a higher rank-score corresponding to a better ranking.

Example 1 (Min-max Regret Choice Functional): An ECFL on $\mathcal{D}\times Q$ is said to be the **Min-max Regret Choice Functional**, denoted F^{mMR}, if for all $(u,p) \in \mathcal{D}\times Q$: $F^{mMR}(u,p) = \operatorname{argmin}_{x \in X}[\max_{i \in ML(p)} \text{regret}(x,u,i)]$.

Research on issues related to Example 1, but in an entirely different framework and from an entirely different perspective, is available in Puppe and Schlag (2009).

Example 2 (Max-min or Pessimistic rule): An ECFL on $\mathcal{D}\times Q$ is said to be the **Max-min rule**, denoted Mm, if for all $(u,p) \in \mathcal{D}\times Q$: $Mm(u,p) = \operatorname{argmax}_{x \in X}[\min_{i \in ML(p)} u(x,i)]$.

Example 3 (Max-max or Optimistic rule): An ECFL on $\mathcal{D}\times Q$ is said to be the **Max-max rule**, denoted MM, if for all $(u,p) \in \mathcal{D}\times Q$: $MM(u,p) = \operatorname{argmax}_{x \in X}[\max_{i \in ML(p)} u(x,i)]$.

Example 4 (Hurwicz's pessimism-optimism criterion): Let $\alpha \in [0,1]$. α is called the pessimism index. An ECFL on $\mathcal{D}\times Q$ is said to be the **Hurwicz** α **rule**, denoted H^α, if for all $(u,p) \in \mathcal{D}\times Q$: $H^\alpha(u,p) = \operatorname{argmax}_{x \in X}[\alpha \min_{i \in ML(p)} u(x,i) + (1-\alpha) \max_{i \in ML(p)} u(x,i)]$.

Example 5 (Pessimism-optimism regret criterion): Let $\alpha \in [0,1]$. α is called the pessimism index. An ECFL on $\mathcal{D}\times Q$ is said to be the **Regret** α **rule**, denoted Regret^α, if for all $(u,p) \in \mathcal{D}\times Q$: $\text{Regret}^\alpha(u,p) = \operatorname{argmax}_{x \in X}[(1-\alpha) \min_{i \in ML(p)} \text{regret}(x,u,i) + \alpha \max_{i \in ML(p)} \text{regret}(x,u,i)]$.

Example 6 (Expected Utility Choice Functional): An ECFL on $\mathcal{D}\times Q$ is said to be the **Expected Utility Choice Functional**, denoted F^e, if for all $(u,p)\in \mathcal{D}\times Q$: $F^e(u,p) = \operatorname{argmax}_{x\in X} \sum_{i=1}^{n} p_i u(x,i)$.

In subsequent sections we are concerned with the expected utility choice functional which given state-dependent data chooses alternatives by maximizing expected utility. The properties we invoke for our axiomatic characterization are not very unusual and seem plausible in the context of our analysis.

3 Some important properties of extended choice functionals

In this section we introduce some important axioms for extended choice functionals.

We shall be **assuming** in what follows that $\pi\in Q$.

An ECFL F on $\mathcal{D}\times Q$ is said to satisfy the **Weak Domination criterion** (WD), if for all $u\in\mathcal{D}$, $p\in Q$ and $x,y\in X$, $u(x,i) > u(y,i)$ for all $i\in N$ implies $y\notin F(u,p)$.

An ECFL F on $\mathcal{D}\times Q$ is said to satisfy **Independence of Irrelevant Alternatives** (IIA), if for all $u,v\in\mathcal{D}$, $p\in Q$ and $x,y\in X$ with $x \neq y$, the following holds: $[u(x,i) = v(x,i)$, $u(y,i) = v(y,i)$ for all $i\in N$, $x\in F(u,p)$, $y\notin F(u,p)]$ implies $[y\notin F(v,p)]$.

An ECFL F on $\mathcal{D}\times Q$ is said to satisfy **Equi-Probability Identical Evaluation** (E-PIE), if for all $u\in\mathcal{D}$, and $x,y\in X$ with $x \neq y$, the following holds: $[u(x,i) = u(y,i)$ for all $i\in N$ and $x\in F(u,\pi)]$ implies $[y\in F(u,\pi)]$.

An ECFL F on $\mathcal{D}\times Q$ is said to satisfy **Equi-Probability Anonymity** (E-PAnon), if for all $u,v\in\mathcal{D}$, $i,j\in N$ and $x\in X$: $[v(x,k) = u(x,k)$ for all $k\in N\backslash\{i,j\}$, $v(x,i) = u(x,j)$, $v(x,j) = u(x,i)]$ implies $[F(v,\pi) = F(u,\pi)$.

An ECFL F on $\mathcal{D}\times Q$ is said to satisfy **Equi-Probability Additivity** (E-PAdditivity), if for all $u,v\in\mathcal{D}$, and $a\in\mathbb{R}^N$:$[v(x) = u(x) + a$ for all $x\in X]$ implies $[F(v,\pi) = F(u,\pi)]$.

An ECFL F on $\mathcal{D}\times Q$ is said to satisfy **Evaluation Probability Conjunction** (EvPC), if for all $u,v\in\mathcal{D}$ and $p\in Q$ satisfying $v(x,i) = p_i u(x,i)$ for all $(x,i)\in X\times N$, it is the case that $F(u,p) = F(v,\pi)$.

4 The significance of Evaluation Probability Conjunction

EvPC is a fairly strong assumption, which is summarized in the following proposition whose proof is quite straightforward.

Proposition 1: An ECFL F on $\mathcal{U}\times Q$ is the expected utility choice functional on $\mathcal{U}\times Q$ if and only if the following two properties are satisfied.

(i) $F(u,\pi) = F^e(u,\pi)$ for all $u\in\mathcal{U}$;

(ii) F satisfies EvPC on $\mathcal{U}$.

Proof: It is easy to see that the expected utility choice functional on $\mathcal{U}$ satisfies (i) and (ii). Hence, suppose an ECFL F on $\mathcal{U}$ satisfies (i) and (ii) and let $(u,p)\in\mathcal{U}\times Q$.

By EvPC, $F(u,p) = F(v,\pi)$, where for all $(x,i)\in X\times N$: $v(x,i) = p_i u(x,i)$.

By (i) $F(v,\pi) = F^e(v,\pi) = \text{argmax}_{x\epsilon X} \sum_{i=1}^{n} v_i(x,i) = \text{argmax}_{x\epsilon X} \sum_{i=1}^{n} p_i u(x,i) =$ $= F^e(u,p)$.

This proves the proposition. Q.E.D.

Using Proposition 1 and the main axiomatic characterization in Lahiri (2019a), we can easily obtain an axiomatic characterization of the EUCFL on $\mathcal{U}\times Q$.

5 An axiomatic characterization of expected utility choice functional

The first lemma of this section leads to the starting point of the discussion of subjective expected utility theory due to Leonard Savage in lecture 7 of Rubinstein (2019).

A binary relation R on $\mathbb{R}^N$ whose asymmetric part is denoted P(R) and symmetric part is denoted I(R) is said to satisfy:

(i) **reflexivity (or be reflexive)** if for all $a\in\mathbb{R}^N$ it is the case that aRa holds;

(ii) **completeness (or be complete)** if for all a, $b\in\mathbb{R}^N$ it is the case that either aRb or bRa holds;

(iii) **transitivity (or be transitive)** if for all a, b, $c\in\mathbb{R}^N$: [aRb & bRc] implies [aRc];

(iv) **anonymity (or be anonymous)** if for a, $b\in\mathbb{R}^N$ and one-to-one functions (permutations) $\rho:N\to N$ on N: [$b_{\rho(i)} = a_i$ for all $i\in N$] implies [aI(R)b];

(v) **additivity** if for a, b, $c\in\mathbb{R}^N$: [aRb] implies [(a+c)R(b+c)].

Give a binary relation R on $\mathbb{R}^N$and any non-empty finite subset A of $\mathbb{R}^N$, let Best (A,R) = $\{a\in A|\ aRb \text{ for all } b\in A\}$.

Suppose F is an ECFL on $\mathcal{U}\times Q$. Define a binary relation R on $\mathbb{R}^N$ as follows: for a, $b\in\mathbb{R}^N$, aRb if and only if for some $u\in\mathcal{U}$ there exist x, $y\in X$ such that $u(x,i) =$ $= a_i$, $u(y,i) = b_i$ for all $i\in N$ and $x\in F(u,\pi)$.

Claim 1: Suppose F is an ECFL on $\mathcal{U}\times Q$ satisfying WD, IIA and E-PIE. Let $u\in\mathcal{U}$, $x\in F(u,\pi)$ and $y\in X$. Then $y\in F(u,\pi)$ if and only if u(y)I(R)u(x).

Proof: $u \in \mathcal{U}$, $x \in F(u,\pi)$ and $y \in X$ implies $u(x)Ru(y)$. Hence we have to show that for $u \in \mathcal{U}$, $x \in F(u,\pi)$ and $y \in X$, [$y \in F(u,\pi)$ if and only if $u(y)Ru(x)$].

If $y \in F(u,\pi)$, then by definition of R, we have $u(y)Ru(x)$. Hence suppose $u(y)Ru(x)$ and, towards a contradiction, suppose $y \notin F(u,\pi)$.

$u(y)Ru(x)$ implies there exist $v \in \mathcal{U}$ and z, $w \in X$ with $v(z) = u(y)$, $v(w) = u(x)$ and $z \in F(v,\pi)$.

Let $v^* \in \mathcal{U}$ with $v^*(y) = v^*(z) = u(y)$, $v^*(x) = v^*(w) = u(x)$ and for all $x' \in X \backslash \{x,y,z,w\}$ and $i \in N$, $v^*(x',i) < \min\{u(i,x), u(i,y)\}$.

By WD, $F(v^*,\pi) \subset \{x,y,z,w\}$.

If $z \notin F(v^*,\pi)$, then by E-PIE, $y \notin F(v^*,\pi)$.

Thus, $F(v^*,\pi) \subset \{x,w\}$ and by E-PIE, $F(v^*,\pi) = \{x,w\}$.

Since $v^*(z) = v(z)$, $v^*(w) = v(w)$, $w \in F(v^*,\pi)$, $z \notin F(v^*,\pi)$ and $z \in F(v,\pi)$ contradicts IIA. Thus, $z \in F(v^*,\pi)$ and by E-PIE, $y \in F(v^*,\pi)$.

Since $v^*(y) = u(y)$, $v^*(x) = u(x)$, $x \in F(u,\pi)$, $y \notin F(u,\pi)$ and $y \in F(v^*,\pi)$ contradicts IIA. Thus, $y \in F(u,\pi)$.

This proves the claim. Q.E.D.

Lemma 1: Suppose F is an ECFL on $\mathcal{U} \times Q$ satisfying WD and IIA. Then R is a weak order on $\mathbb{R}^N$, i.e., R is reflexive, complete and transitive. If, in addition, F satisfies E-PIE, then for all $u \in \mathcal{U}$: $F(u,\pi) = \{y \in X | \, u(y) \in \text{Best}(\{u(x) | \, x \in X\},R)\}$.

Proof: Given a, $b \in \mathbb{R}^N$ and x, $y \in X$, let $u \in \mathcal{U}$ such that $u(x,i) = a_i$, $u(y,i) = b_i$ and $u(z,i) < \min\{a_i,b_i\}$ for all $i \in N$ and $z \in X \backslash \{x,y\}$.

Since F satisfies WD, $F(u,\pi)$ is a non-empty subset of $\{x,y\}$. Thus either aRb or bRa. Hence R is reflexive and complete.

To show that R is transitive, suppose aRb and bRc for some a, b, $c \in \mathbb{R}^N$ with $a \neq b \neq c \neq a$. Thus, there exist u, $v \in \mathcal{U}$ and x, y, $z \in X$ such that $u(x,i) = a_i$, $u(y,i) = b_i = v(y,i), c_i = v(z,i)$ for all $i \in N$, $x \in F(u,\pi)$ and $y \in F(v,\pi)$.

Let $u^* \in \mathcal{U}$ such that for all $i \in N$, $u^*(x,i) = u(x,i) = a_i$, $u^*(y,i) = u(y,i) = v(y,i) = b_i$, $u^*(z,i) = v(z,i) = c_i$ and $u^*(w,i) < \min\{a_i, b_i, c_i\}$ for all $w \in X \backslash \{x,y,z\}$.

By WD, $F(u^*,\pi)$ is a non-empty subset of $\{x,y,z\}$. Towards a contradiction suppose that $x \notin F(u^*,\pi)$.

Then by WD, $F(u^*,\pi)$ is a nonempty subset of $\{y,z\}$.

If $y \in F(u^*,\pi)$, then along with $x \notin F(u^*,\pi)$, $x \in F(u,\pi)$ and [for all $i \in N$ $u^*(x,i) = = u(x,i) = a_i$, $u^*(y,i) = u(y,i) = b_i$], we get a violation of IIA. Thus, $y \notin F(u^*,\pi)$. Thus, $F(u^*,\pi) = \{z\}$ implying $z \in F(u^*,\pi)$ and $y \notin F(u^*,\pi)$.

However, $z \in F(u^*,\pi)$ and $y \notin F(u^*,\pi)$ along with $y \in F(v, \pi)$ and [for all $i \in N$, $u^*(y,i) = v(y,i) = b_i$, $u^*(z,i) = v(z,i) = c_i$] leads to a violation of IIA.

Thus, $z \notin F(u^*,\pi)$ and so $F(u^*,\pi) = \phi$, which contradicts the definition of an ECFL.

Thus $x \in F(u^*,\pi)$ and so xRz.

Thus, R is transitive.

That $F(u,\pi) \subset \{y \in X | u(y) \in Best(\{u(x)|x \in X\},R)\}$ follows immediately from the definition of R. Now suppose that in addition to WD and IIA, F satisfies E-PIE.

Let us show that $\{y \in X | u(y) \in Best(\{u(x)|x \in X\},R)\} \subset F(u,\pi)$.

Let $y \in X$ be such that $u(y) \in Best(\{u(x)|x \in X\},R)$ and let $z \in F(u,\pi)$. Since $F(u,\pi) \subset \{y \in X| u(y) \in Best(\{u(x)|x \in X\},R)\}$, $u(z) \in Best(\{u(x)|x \in X\},R)$. Thus, $u(y)I(R)u(z)$ and since $z \in F(u,\pi)$ it follows from claim 1 that $y \in F(u,\pi)$.

Thus, $\{y \in X| u(y) \in Best(\{u(x)|x \in X\},R)\} \subset F(u,\pi)$ and hence $F(u,\pi) =$ $= \{y \in X| u(y) \in Best(\{u(x)|x \in X\},R)\}$. Q.E.D.

It is possible to follow the discussion in lecture 7 of Rubinstein (2019) with the lemma 1 as given and arrive at an axiomatic characterization of EUCFL on $\mathcal{U} \times Q$. However, then we would require using either the separating or the supporting hyperplane theorem, which we do not want to do, since we want our axiomatic characterization to be based entirely on combinatorial techniques. We do not want to use any continuity assumption and/or topological properties of finite dimensional Euclidean space to prove our axiomatic characterization. Thus, we follow the route provided below.

Lemma 2: Suppose F is an ECFL on $\mathcal{U} \times Q$ that satisfies E-PAnon. Then R satisfies anonymity.

Proof: Since any permutation can be obtained as a succession of pair-wise interchanges it is enough to establish the result for the case of a permutation ρ such that for some $i, j \in N$ with $i \neq j$, $\rho(i) = j$, $\rho(j) = i$ and $\rho(k) = k$ for all $k \in N \backslash \{i,j\}$.

Thus, let $a, b \in \mathbb{R}^N$ with $a_i = b_j$, $a_j = b_i$ and $a_k = b_k$ for all $k \in N \backslash \{i,j\}$.

Let $u \in \mathcal{U}$ and $x, y \in X$ with $u(x) = a$, $u(y) = b$ and for all $z \in X \backslash \{x,y\}$ and $k \in N$, $u(z,k) = \beta$, where β $\min\{\min\{a_k,b_k\}|k \in N\} - 1$.

By WD, $F(u,\pi) \subset \{x,y\}$.

Without loss of generality suppose $x \in F(u,\pi)$. By the definition of R, $u(x)Ru(y)$, i.e., aRb.

Now let $v \in \mathcal{U}$ with $v(z,i) = u(z,j)$, $v(z,j) = u(z,i)$ for all $z \in X$ and $v(z,k) =$ $= u(z,k)$ for all $z \in X$ and $k \in N \backslash \{i,j\}$.

By E-PAnon, $F(v,\pi) = F(u,\pi)$ and so $x \in F(v,\pi)$.

By definition of R, $v(x)Rv(y)$, i.e., bRa.

Hence aI(R)b. Q.E.D.

Lemma 3: Suppose F is an ECFL on $\mathcal{U}\times Q$ that satisfies E-PAdditivity. Then R satisfies additivity.

Proof: Let a, b, $c\in\mathbb{R}^N$ and suppose aRb. By the definition of R, there exist $u\in\mathcal{U}$, x, $y\in X$ with u(x) = a, u(y) = b and $x\in F(u,\pi)$. Let $v\in\mathcal{U}$ be such that v(z) = = u(z) + c for all $z\in X$.

By E-PAdditivity, $F(v,\pi) = F(u,\pi)$ and so $x\in F(v,\pi)$.

By the definition of R, v(x)Rv(y), i.e. (a+c)R(b+c).

Thus, R is additive. Q.E.D.

Lemma 4: Suppose F is an ECFL on $\mathcal{U}\times Q$ that satisfies WD, IIA and E-PAdditivity. Let $<a^{(0)}, a^{(1)}, \ldots, a^{(n-1)}>$ be a sequence in $\mathbb{R}^N$, such that $a^{(k)}I(R)a^{(m)}$ for all k, $m\in\{0,1, \ldots, n-1\}$. Then $na^{(0)}I(R)\sum_{k=0}^{n-1}a^{(k)}$.

Proof: By lemma 1, R is a weak order on $\mathbb{R}^N$.

Suppose $ma^{(0)}I(R)\sum_{k=0}^{m-1}a^{(k)}$ for all $1 \le m \le K$ for some $K < n$.

Now $Ka^{(0)}I(R)\sum_{k=0}^{K-1}a^{(k)}$ and additivity of R implies $(K+1)a^{(0)}I(R)(a^{(0)} + + \sum_{k=0}^{K-1}a^{(k)})$.

But $a^{(0)}I(R)a^{(K+1)}$ and additivity of R implies $(a^{(0)} + \sum_{k=0}^{K-1}a^{(k)})I(R)(\sum_{k=0}^{K-1}a^{(k)}+ a^{(K)})$.

By transitivity of R, we get $(K+1)a^{(0)}I(R)\sum_{k=0}^{K}a^{(k)}$.

By a standard induction argument we now get $na^{(0)}I(R)\sum_{k=0}^{n}a^{(k)}$. Q.E.D

Lemma 5: Suppose F is an ECFL on $\mathcal{U}\times Q$ that satisfies WD, IIA, E-PAnon and E-PAdditivity. Let $a\in\mathbb{R}^N$, ρ be the permutation on N such that ρ(j) = j+1 for all $j\in\{1, \ldots, n-1\}$, ρ(n) = 1, $a^{(0)} = a$ and for $k\in\{1, \ldots, n-1\}$, let $a_j^{(k)}= a_{\rho(j)}^{(k-1)}$ for all $j \in 1, \ldots, n$. Then $aI(R)\frac{1}{n}\sum_{k=0}^{n}a^{(k)}$, where every coordinate of $\sum_{k=0}^{n}a^{(k)} = \sum_{j=1}^{n}a_j$.

Proof: By lemma 2, $aI(R)a^{(k)}$, for all k = 0,1, …, n-1 and $(\frac{1}{n}a)I(R)(\frac{1}{n}a^{(k)})$, for all k = 0,1, …, n – 1.

The lemma now follows from lemma 4. Q.E.D.

The following proposition is the stepping stone to our main result.

Proposition 2: Suppose F is an ECFL on $\mathcal{U}\times Q$ that satisfies WD, IIA, E-PIE, E-PAnon and E-PAdditivity. Then for all $u\in\mathcal{U}$, $F(u,\pi) = F^e(u,\pi)$.

Proof: Suppose F is an ECFL on $\mathcal{U}\times Q$ that satisfies WD, IIA, E-PIE, E-PAnon and E-PAdditivity and let $u\in\mathcal{U}$. Let $x\in F(u,\pi)$ and towards a contradiction suppose there exists $y\in X$ with $\sum_{i=1}^{n}u(y,i) > \sum_{i=1}^{n}u(x,i)$. Thus $\frac{1}{n}\sum_{i=1}^{n}u(y,i) > \frac{1}{n}\sum_{i=1}^{n}u(x,i)$.

By lemma 5, u(x)I(R)a and u(y)I(R)b, where $a_k = \frac{1}{n}\sum_{i=1}^{n} u(x,i)$ and $b_k =$ $= \frac{1}{n}\sum_{i=1}^{n} u(y,i)$ for all k∈N.

By lemma 1, F(u,π) = {z∈X|u(z)Ru(w) for all w∈X}.

Thus, u(x)Ru(z) for all z∈X and by transitivity of R, aRu(z) for all z∈X. Further, by transitivity of R, aRb.

Let v∈$\mathcal{U}$ with v(x) = a, v(y) = b, and v(z) = u(z) for all z∈X\{x,y}.

Since v(y,k) = $b_k > a_k$ = v(x,k) for all k∈N, by WD, x∉F(v,π) = = {{z∈X|u(z)Ru(w) for all w∈X}.

Hence it is not the case that aRb and aRv(z) for all z∈X\{x,y}.

Since v(z) = u(z) for all z∈X\{x,y}, it is not the case that aRb and aRu(z) for all z∈X\{x,y}, leading to a contradiction.

Thus, F(u,π) ⊂ F^e(u,π). Let x∈ F(u,π) ⊂ F^e(u,π) and y∈F^e(u,π).

Since x,y∈ F^e(u,π) implies $\frac{1}{n}\sum_{i=1}^{n} u(y,i) = \frac{1}{n}\sum_{i=1}^{n} u(x,i)$, by lemma 5, u(x)I(R)u(y).

Since F(u,π) = {z∈X| u(z)Ru(w) for all w∈X}, x∈F(u,π) and u(x)I(R)u(y), by transitivity of R, y∈{z∈X| u(z)Ru(w) for all w∈X}= F(u,π).

Thus, F(u,π) = F^e(u,π). Q.E.D.

With propositions 1 and 2 in place, we can prove the main theorem of this paper.

Theorem 1: An ECFL F on $\mathcal{U}$×Q is the expected utility choice functional if and only if it satisfies WD, IIA, E-PIE, E-PAnon, E-PAdditivity.and EvPC.

Proof: It is easy to verify that the EUCFL on $\mathcal{U}$×Q satisfies the six properties. Hence let us suppose that F is an ECFL on $\mathcal{U}$×Q that satisfies the six properties and let u∈$\mathcal{U}$. Let us show that F(u,π) = F^e(u,π).

By proposition 3, F(u,π) = F^e(u,π).

The theorem now follows from Proposition 1. Q.E.D.

An examination of the procedure by which we arrived at theorem 1, suggests that in order to axiomatically characterize the expected utility choice functional, weaker assumptions would suffice.

An ECFL F on $\mathcal{D}$×Q is said to satisfy the **Equi-Probability Weak Dominatio criterion** (E-PWD), if for all u∈$\mathcal{D}$ and x,y∈X, u(x,i) > u(y,i) for all i∈N implies y∉F(u,π).

An ECFL F on $\mathcal{D}$×Q is said to satisfy **Equi-Probability Independence of Irrelevant Alternatives** (E-PIIA), if for all u,v∈$\mathcal{D}$ and x,y∈X with x ≠ y, the following holds: [u(x,i) = v(x,i), u(y,i) = v(y,i) for all i∈N, x∈F(u,π), y∉F(u,π)] implies [y∉F(v,π)].

The alternative axiomatic characterization of EUCFL on $\mathcal{U}$ based on the using the above four properties instead of their analogues used in theorem 2 is the following.

Theorem 2: An ECFL F on $\mathcal{U}\times Q$ is the expected utility choice functional if and only if it satisfies E-PWD, E-PIIA, E-PIE, E-PAnon, E-PAdditivity and EvPC.

6 Representation of uncertain prospects as an element in the domain of a choice functional

The following is based on Chapter 2 of Resnik (1987), where "states of nature" are related to "consequences".

Given a non-empty set $\mathcal{X}$, an uncertain prospect on $\mathcal{X}$ is a probability distribution p on $\mathcal{X}$ with finite support, i.e., support (p) = $\{x \in \mathcal{X} \mid p(x) > 0\}$ is a non-empty finite set. The elements of $\mathcal{X}$ are called prizes or consequences. If $\mathcal{X} = \mathbb{R}$, then the prizes are interpreted as monetary gains and losses, depending on whether the real number is positive or negative.

Let $\{p^{(1)}, \ldots, p^{(K)}\}$ for some positive integer K be a non-empty finite set of uncertain prospects.

Let $X = \{p^{(1)}, \ldots, p^{(K)}\}$ denote the set of alternatives from which the decision maker is required to choose. Note that $\bigcup_{j=1}^{K} \mathrm{support}(p^{(j)})$ is a non-empty finite subset of $\mathcal{X}$.

Let $N = \{1,2, \ldots, K\}\times\bigcup_{j=1}^{K} \mathrm{support}(p^{(j)})$ denote the set of states of nature. Clearly N is non-empty and finite.

Let $v:\{0\}\cup\mathcal{X}\rightarrow\mathbb{R}$ satisfying $v(0) = 0$ denote the utility function of the decision maker. The utility function is defined on a set consisting of consequences and the real number 0, which could belong to $\mathcal{X}$. Intuitively, $v(x) > v(0)$ means that x is a gain and $v(x) < v(0)$ means that x is a loss.

The corresponding evaluation function $u:X\times N\rightarrow\mathbb{R}$ of the decision maker is defined as follows: for all $p^{(k)}\in X$ and $(j,x)\in N$, $u(p^{(k)}, (j,x)) = v(x)$ if $k = j$ and $u(p^{(k)}, (j,x)) = v(0)$ if $k \neq j$.

The decision maker's beliefs about the occurrence of the states of nature in N is given by a probability distribution q on N such that for all $(j,x)\in N$, $q_{(j,x)} = $ $= \frac{1}{\sum_{k=1}^{K} \#(\mathrm{support}(p^{(k)}))}$, where for each $k\in\{1, \ldots, K\}$, $\#(\mathrm{support}(p^{(k)})$ is the cardinality of support $(p^{(k)})$.

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References

Allais M. (1953), *Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école Américaine*, Econometrica, 21(4), 503-546.

d'Aspremont C., Gevers L. (2002), *Social Welfare Functional and Interpersonal Comparability* [in:] K. Arrow, A. Sen, K. Suzumura (eds.), *Handbook of Social Choice and Welfare. Volume 1*, Amsterdam, Elsevier Science, 459-541.

Karni E. (1985), *Decision Making under Uncertainty: The Case of State-Dependent Preference*, Harvard University Press.

Lahiri S. (2019a), *The Utilitarian Social Choice Function. Revista Investigacion Operacional*, 40(4), 503-515.

Lahiri S. (2019b), *State Salient Decision Rules for Choice under Uncertainty*, AIMS International Journal of Management, 13(3), 101-105.

Lahiri S. (2019c), *The No-spoiler Condition for Choice Correspondences*, Multiple Criteria Decision Making, 14, 60-74.

Lahiri S. (2020a), *Extended Choice Correspondences and an Axiomatic Characterization of the Probabilistic Borda Rule*, Mathematical Methods in Economics and Finance (forthcoming in).

Lahiri S. (2020b), *Generalized Sen-Coherence and Existence of Preferred with Probability at Least Half Winners*, International Journal of Operations Research, 17(3), 93-99.

Pattanaik P.K. (1970), *Sufficient Conditions for the Existence of a Choice Set under Majority Voting*, Econometrica, 38(1), 165-170.

Puppe C., Schlag K. (2009), *Choice under Complete Uncertainty when Outcome Spaces Are State-dependent*, Theory and Decision, 66, 1-16.

Resnik M.D. (1987), *Choices: An Introduction to Decision Theory*, University of Minnesota Press, Minneapolis.

Rubinstein A. (2019), *Lecture Notes in Microeconomic Theory*, Revised Version of the First Version Published in 2005, Princeton University Press, Princeton and Oxford.

Sen A.K. (1970), *Collective Choice and Social Welfare*, Holden Day, San Francisco.

Part II
Regularly contributed papers

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COMPLEMENTARITY OF THE GRAPHICAL ANALYSIS FOR INTERACTIVE AID AND DOMINANCE-BASED ROUGH SET APPROACH APPLIED TO THE CLASSIFICATION OF NON-URBAN MUNICIPALITIES

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Abstract

Graphical analysis for interactive aid (GAIA) and the dominance-based rough set approach (DRSA) are compared as methods of explaining the solution to a multi-criteria ranking problem obtained using the preference ranking organization method for the enrichment of evaluations (PROMETHEE). The classification of 52 municipalities in Northern Quebec in terms of the socioeconomic situation is based on three attributes: home conditions, employment and demographic potential. The classification provided to the decision maker is aggregated information. To facilitate decision-making, the problem is first considered as a sorting task, in which municipalities are distributed into three categories: best (B), worst (W) or intermediate (I), based on the PROMETHEE ranking. In order to improve the position of a municipality thus categorized, the decision maker needs information that will answer the questions: What criteria are relevant to the municipality? What criteria are in conflict? What are the critical values of the criteria? We show that GAIA and DRSA provide convergent and complementary information that allow enrichment of the answers to these questions.

Keywords: management decision support, multi-criteria analysis, GAIA, dominance-based rough set approach.

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1 Introduction

The dominance-based rough set approach (DRSA) proposed by Greco, Matarazzo and Slowinski (2001) is a mathematical tool that may be used to support decision-making processes in many fields such as medicine, banking and engineering (Pawlak, 2002). It has been applied also on a larger scale to feasibility studies and risk analyses for the purpose of facilitating the prioritization and selection of sustainable economic development projects in non-urban regions in the province of Quebec (Zaras, Marin, Boudreau, 2012; Marin, Zaras, Boudreau-Trudel, 2014).

In the present study, three socioeconomic indicators were measured using Canadian census data representing 52 of the 62 municipalities in Northern Quebec[1] (www 1). These indicators were: housing conditions, employment and demographic potential. The decision maker required a synthesis of information for the ranking of the municipalities.

To solve this multi-criteria classification problem, the PROMETHEE method (preference ranking organization method for enrichment evaluations) was used. First developed in 1982 by J.P. Brans (1982), it has since undergone several refinements (Brans, Mareschal, Vincke, 1984, 1986; Brans, Vincke, 1985; Mareschal, 1986, 1988; Brans, Mareschal, 1992, 1994). In order to decide how to improve the position of a municipality, the decision maker needed answers to the following questions: What criteria are relevant to the municipality? What are the critical values of the criteria for advancing to the next category?

The PROMETHEE method was used to rank the municipalities for sorting into one of three categories: best (B), worst (W) and intermediate (I). DRSA was then used to extract the decision rules. This method plays, in this context, the role of explaining the classification obtained by the multi-criteria method. Another method that can play this role is geometrical analysis for interactive assistance or GAIA. This is an interactive geometric normalization method that allows data to be written and interpreted.

Considering the position of all the municipalities projected in relation to the GAIA plan origin appears very helpful to the decision maker. The municipalities nearest to the origin are in the intermediate category (I). Decision rules are extracted for municipalities that are above and below the origin, which makes it possible to determine the critical values for specific criteria. In the present paper, we show that the GAIA and DRSA methods provide convergent and complementary information, thus enriching the answers to the questions being asked by the decision maker.

[1] Statistics Canada does not have data for municipalities inhabited by fewer than 200 individuals.

This paper is structured as follows. The problem is formulated in Section 2. Section 3 presents the multi-criteria method PROMETHEE and how ranking is obtained using this method. In Section 4, the GAIA and DRSA methods are applied to explain the classification. Section 5 provides a comparison of the usefulness of the two explicative methods for answering the questions asked by the decision maker.

2 Formulation of the multi-criteria problem

The first stage was the ranking of the 52 municipalities in northern Quebec. Since three indicators were used, this task is a multi-criteria ranking problem. It can be represented using the AXE model, where:

A is a finite set of actions (municipalities) a_i for $i = 1, 2\ldots 52$;

X is a finite set of criteria X_k for $k = 1, 2, 3$; and

E is the set of municipality evaluations $\mathbf{e}_{i,k}$ with respect to criterion X_k.

The main aim of the multi-criteria approach is to obtain an overall preference among the set of municipalities, which is based on performance evaluated with respect to each criterion. The municipalities were evaluated using Statistics Canada census data from 2012 (www 1) (results of the 2016 census were not all available).

3 The PROMETHEE II multi-criteria method

PROMETHEE II is a complete ranking method for solving multi-criteria decision problems. All actions are compared even when comparison is difficult to perform (Mareschal, 2013). The PROMETHEE II ranking is based on the computation of preference flows, which confirm the results of the pairwise comparisons of the actions and allows all actions to be ranked from the best to the worst. To achieve this, we need to compute three different preference flows:

- Φ^+ : the positive (or leaving) flow,
- Φ^- : the negative (or entering) flow,
- Φ : the net flow.

The positive preference flow $\Phi^+(a)$ gives an overall measurement of the strengths of action a by computing how much it is preferred over the $n-1$ others. The higher the computed $\Phi^+(a)$, the better the action:

$$\Phi^+(a) = \frac{1}{n-1}\sum_{b\neq a}\pi(a,b) \tag{1}$$

where $\pi(a,b) = \sum_{j=1}^{k} P_j\,(a,b)\cdot w_j \;\forall a,b \in A$ which aggregates the preferences $(0 \leq P_j \leq 1)$ by taking into account weights attributed by the decision maker to the various criteria (w_j). For our purposes, the same weight was used for each criterion.

The negative preference flow $\Phi^{-}(a)$ provides an overall measurement of the weaknesses of action a by computing how much the $n-1$ other actions are preferred. The smaller the computed $\Phi^{-}(a)$, the better the action:

$$\Phi^{-}(a) = \frac{1}{n-1}\sum_{b \neq a} \pi(b,a) \quad (2)$$

The net preference flow $\Phi(a)$ is the difference between the positive and the negative preference flows:

$$\Phi(a) = \Phi^{+}(a) - \Phi^{-}(a) \quad (3)$$

In summary, $\Phi(a)$ considers and aggregates both the strengths and the weaknesses of the action into a single score (Mareschal, 2013). The value of $\Phi(a)$ can be either positive or negative. As is the case with $\Phi^{+}(a)$, the greater the computed $\Phi(a)$, the better the action.

The PROMETHEE II ranking is based on the net preference flow. For example, action a is preferred to action b in the PROMETHEE II ranking if and only if it is preferred to b given that net preference flow:

$$aP^{II}b \ \text{ if and only if } \ \Phi(a) > \Phi(b) \quad (4)$$

where P^{II} means "is preferred to in the PROMETHEE II ranking".

3.1. Practical example

The data is from a real-life example, the North Quebec Development Plan. These data concern one of the most important issues related to the development of employment. In our example, we considered three selected criteria from among many that were taken in the analysis of this plan: employment, condition of housing in the municipality and demographic potential.

With respect to the three criteria, we used the measurement that come from the information available on the 2006 and 2011 censuses from Statistics Canada. They are defined by Statistics Canada as follows:

Demographic potential: Number of people aged 15 and under divided by the number of people.

Employment: Number of people employed divided by the number of people aged 15 and over.

Housing conditions: Number of private dwellings in need of major repair divided by the number of private dwellings.

A multi-criteria analysis of 52 municipalities in relation to the three above criteria using the PROMETHEE II method allowed to determine the ranking of municipalities presented in Table 1.

Table 1: Ranking of actions with PROMETHEE II

Rank	Actions	$\Phi(a)$	Category	Rank	Actions	$\Phi(a)$	Category
1	26	0,6078	B	27	48	0,0131	I
2	9	0,4771	B	28	18	−0,0131	I
3	31	0,4641	B	29	15	−0,0196	I
4	11	0,4052	B	30	20	−0,0261	I
5	6	0,3333	B	31	5	−0,0327	I
5	19	0,3333	B	32	14	−0,0523	I
7	13	0,3268	B	32	37	−0,0523	I
8	3	0,3137	B	34	44	−0,0654	I
9	25	0,2810	B	35	24	−0,1046	I
10	12	0,2745	B	36	38	−0,1111	I
10	39	0,2745	B	37	41	−0,1373	I
12	32	0,2484	B	38	16	−0,2026	I
13	22	0,2353	B	39	28	−0,2157	I
14	35	0,2157	I	39	34	−0,2157	W
15	33	0,2026	I	41	30	−0,2288	W
16	4	0,1765	I	42	46	−0,2614	W
17	1	0,1699	I	43	42	−0,2810	W
18	52	0,1569	I	44	49	−0,3203	W
19	8	0,1373	I	45	40	−0,3464	W
20	27	0,1242	I	46	47	−0,3595	W
20	17	0,1242	I	47	45	−0,3725	W
22	2	0,1176	I	48	43	−0,3791	W
23	7	0,0850	I	49	51	−0,4314	W
24	23	0,0458	I	50	36	−0,5163	W
25	10	0,0392	I	51	50	−0,7124	W
26	21	0,0261	I	52	29	−0,7516	W

This ranking helps us to determine which action is better than the other, which is the best or the worst, and to rank all actions between these two extremes. Although we have at this point no explanation for the ranking, we can distribute the municipalities among the three classes, namely the best (class B, i.e. those in the top quartile, n = 13), the worst (class W, i.e. those in the bottom quartile, n = 13) and the intermediates (class I, i.e. those in the two middle quartiles).

4 Explanatory methods

4.1 GAIA

The first explanatory method that we used was derived from Visual PROMETHEE, which represents the completed ranking obtained from PROMETHEE II. Developed under the supervision of the creator of PROMETHEE, Bertrand Mareschal, this software is available online (www 2). Compared to products like PromCalc and Decision Lab, it provides more visually appealing and informative representations, including GAIA.

4.1.1 Explanation of classification with GAIA

GAIA provides decision makers with a powerful tool for the analysis of the discriminating force between criteria and between their conflictual characters (Brans, Mareschal, 2002). We show our results in Figure 1. The plan is generated from principal component analysis.

In a GAIA plan, the discriminating force between criteria is indicated in terms of axis length. In our study, the longest axis is from the criteria employment. The orientation of the axes to each other indicates the level of similarity between the criteria. The closer they are, the more similar the criteria are in terms of preference (the cosine of the angle between them will be closer to 1). On the contrary, the more they are directed in opposite positions, the more the criteria express independence of preferences (the cosine of the angle between them will approach −1). As the axes become orthogonal, the preferences become independent. The view in the first two principal components (U-V) is the best quality. In our case it groups 87.1% of the information. With regard to the positioning of the actions, the closer these are in the GAIA plan, the more similar they are for the decision maker. When actions occur in clusters, they perform similarly based on all criteria. The closer an action is to the end of the axis of a criterion, the better this action is on the basis of this criterion. For example, action 26 in Figure 1 obtains the best evaluation on the basis of the employment criterion.

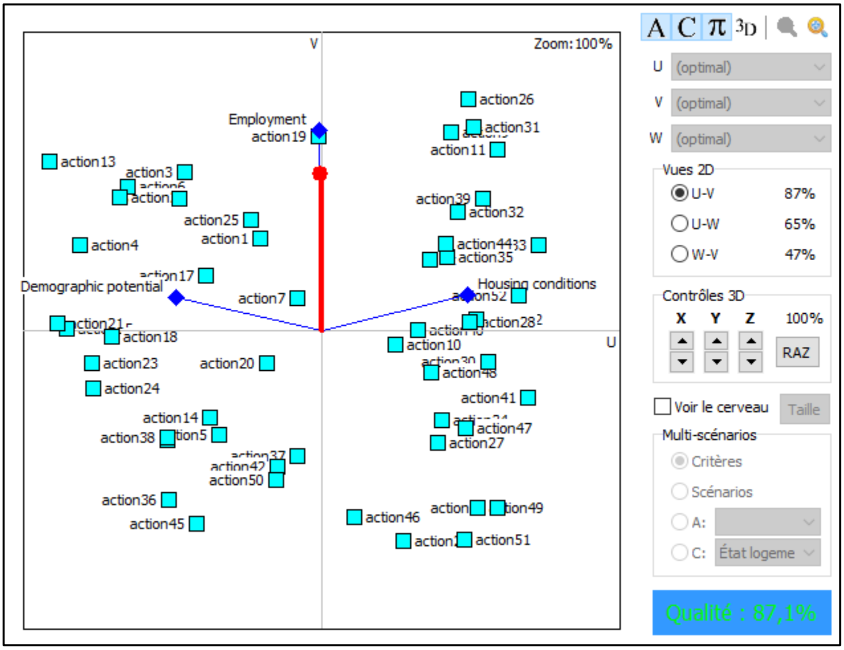

Figure 1: The GAIA plan

It is also possible to detect differences between actions by comparing their relative positions in the GAIA plan. Two actions differ if they are situated in different locations in the plan, for example, A19 and A46, as well as A28 and A21.

To obtain the explanation of the ranking provided by PROMETHEE II, we examine the decisional axis (π) on the GAIA plan, which appears in vertical axis in Figure 1. Representing the multi-criteria net flow, this axis indicates the criteria that are in agreement or not with the net flow. The reliability of this information increases with the length of the decisional axis. The best actions are thus the farthest possible in the direction of this axis. In our study, we see that the decisional axis is rather long and stuck on the axis of the employment criterion. We also see that the farthest action in this direction is action 26. This ranking thus represents the PROMETHEE II ranking presented in Table 1. In our multi-criteria problem, the most relevant criterion is employment, which is on the V axis as well. To be ranked the best, an action has to have the best score for that criterion, which was the case in our study. Complementary to the fact that the decisional axis is on the V axis, the remaining two criteria, namely demographic potential and housing conditions, are compensatory and influence the positioning of actions over the U axis. For example, action 13 scores strongly for demographic potential (39.9% versus an average of 24.14%), but below average for housing conditions (50% versus 26.3%). For this reason, the position of action 13 is far on the U axis on the GAIA plan. Another example is action 7, for which both demographic potential (30.1%) and housing conditions (24.6%) are close to the averages (respectively 24.14% and 26.3%). This action is more balanced and therefore positioned nearly midway between both criteria axes on the GAIA plan.

The GAIA plan also helps the decision maker to classify the municipalities. For the first classification step (overall ranking based on the three criteria), municipalities in class B are preferred strongly and correspond to the first 13 actions starting from the upper side of the V axis on the GAIA plan. Class W municipalities correspond to strong non-preference and the first 13 actions starting from the lower side of V axis. Municipalities in class I correspond to weak preference, weak non-preference or indifference. They are found in the middle of the V axis.

4.2 The dominance-based rough set approach

The second explanatory method that we considered is based on the *rough set theory* proposed by Pawlak (1982, 1991) and developed by Greco, Matarazzo, Slowinski (2001) and Zaras (2004). Centered on the principle of dominance, this method is called the dominance-based rough set approach (DRSA). DRSA in effect helps decision makers identify a reduced set of criteria (reducts) that provides the same quality of classification of the original set of actions as obtained using (in our case) the PROMETHEE II net flow score.

In rough set theory, the decision problem is represented as a table in which the rows correspond to actions while the columns correspond to attributes (see Table 2). In our study, the actions are the municipalities. The attributes are of two types: conditional and decisional. The conditional attributes correspond to the three criteria, namely housing conditions, demographic potential and employment. The decisional attribute is the classification of the municipality on the basis of the PROMETHEE II net flow score to one of three categories: the best – class B, the worst – class W and the intermediate − class I.

Table 2: Decision table #1

	X_1	X_2	X_3	**D**
a_1	$e[(a_1)\ 1]$	$e[(a_1)\ 2]$	$e[(a_1)\ 3]$	$e(a_1) = \{B, I, W\}$
a_2	$e[(a_2)\ 1]$	$e[(a_2)\ 2]$	$e[(a_2)\ 3]$	$e(a_2) = \{B, I, W\}$
...	...	...	...	...
a_{52}	$e[(a_{52})\ 1]$	$e[(a_{52})\ 2]$	$e[(a_{52})\ 3]$	$e(a_{52}) = \{B, I, W\}$

Then $e[(a_1)\ 1]$ is the evaluation of municipality a_1 with respect to criterion $\mathbf{X_1}$ and $e(a_1) = \{B, I, W\}$ is the classification of municipality a_1 corresponding to the appropriate class from the PROMETHEE II net flow score (B, I or W).

This first step will enable the decision maker to determine which criteria are the most relevant to getting a specific classification. Attribute reduction is one of major topics in the DRSA. In fact, this is not only a tool for identifying the most relevant criteria, but it also allows determination of the critical threshold value to be reached in order to upgrade the categorization.

4.2.1 The DRSA explanation of classification

To get an explanation of classification from DRSA, we extracted the rules using the 4eMK2 program (Greco, Matarazzo, Slowinski, 1999). Based on the rough set with dominance relation, 4eMK2 is another multi-criteria decision analysis method for classification problems. The rules explain the classification of actions by pointing out which condition(s) must be met in order to earn a specific classification. Our results are shown as decision rules as follows:

Rule 1. (Employment <= 32.3) => (Dec at most W); [6; 46.15%]
{29, 43, 45, 46, 49, 51}

This rule is confirmed by 6 of the 13 worst municipalities. This means that if the employment rate is at most equal to 32.3%, then the municipality can be classified as the worst.

Rule 2. (Home Conditions >= 61.5) => (Dec at most W); [2; 15.38%]
{36, 50}

This rule is confirmed by 2 of the 13 worst municipalities. This means that if the housing conditions rate is at least equal to 61.5%, then the municipality can be classified as the worst.

Rule 3. (Demographic Potential <= 10.4 & Employment <= 50.8) => (Dec at most W); [3; 23.08%]
{29, 30, 47}

This rule is confirmed by 3 of the 13 worst municipalities. This means that if the demographic potential rate is at most equal to 10.4% and employment at most equal to 50.8% then the municipality can be classified as the worst.

Rule 4. (Demographic Potential <= 8.1 => (Dec at most W); [2; 15.38%]
{29, 40}

This rule is confirmed by 2 of the 13 worst municipalities. This means that if the demographic potential rate is at most equal to 8.1%, then the municipality can be classified as the worst.

Rule 5. (Employment <= 48.0) & (Demographic Potential <= 28.9) => (Dec at most W); [8; 61.54%]
{29, 34, 36, 42, 43, 46, 49, 50, 51}

This rule is confirmed by 8 of the 13 worst municipalities. This means that if the employment rate is at most equal to 48.0% and the demographic potential rate is at most equal to 28.9% then the municipality can be classified as the worst.

Rule 6. (Employment >= 62.5) => (Dec at least B); [6; 46.15%]
{3, 9, 11, 19, 26, 31}

This rule is confirmed by 6 of the 13 best municipalities. This means that if the employment rate is at least equal to 62.5%, then the municipality can be classified as the best.

Rule 7. (Demographic Potential >= 39.9) => (Dec at least B); [1; 7.69%]
{13}

This rule is confirmed by 1 of the 13 best municipalities. This means that if the demographic potential rate is at least equal to 39.9%, then the municipality can be classified as the best.

Rule 8. (Home Conditions <= 1.07) => (Dec at most W); [1; 7.69%]
{12}

This rule is confirmed by 1 of the 13 worst municipalities. This means that if the housing conditions rate is at most equal to 1.07%, then the municipality can be classified as the worst.

Rule 9. (Demographic Potential >= 35.5) & (Employment >= 59.2) => (Dec at least B); [2; 15.38%]
{6, 22}

This rule is confirmed by 2 of the 13 best municipalities. This means that if the demographic potential rate is at least equal to 35.5% and the employment rate is at least equal to 59.2% then the municipality can be classified as the best.

Rule 10. (Employment >= 59.6) & (Home Conditions <= 25.6) => (Dec at least B); [1; 7.69%]
{25}

This rule is confirmed by 1 of the 13 best municipalities. This means that if the employment rate is at least equal to 59.6% and the housing conditions rate is at most equal to 25.6% then the municipality can be classified as the best.

Rule 11. (Employment>=59.6) & (HomeCond<=11.8) & (DemographPotenc>=28) => (Dec at least B); [6, 46.15%]
{9, 11, 26, 31, 32, 39}

This rule is confirmed by 6 of the 13 best municipalities. This means that if the employment rate is at least equal to 59.6% and the housing conditions rate is at most equal to 11.8% and the demographic potential rate is at least equal to 28% then the municipality can be classified as the best.

The municipalities which do not comply with the eleven rules mentioned above belong to intermediate class I.

The 4eMK2 software also makes it possible to identify the reducts. Reducts may be composed of single or multiple attributes, which allows us to classify the actions with the same quality. In other words, reducts of attributes lead to the same classification, but by taking into account reduced number of attributes in relation to the initial set. In our example, we have a reduct made up of two attributes: employment and demographic potential.

4.2.2 The DRSA explanation of the GAIA results

The municipalities belonging to classes B and W are presented in the GAIA plan in Figure 2. This figure shows a greater number of municipalities belonging to the intermediates class on the side of the potential demographic axis which represents the greatest uncertainty and this, perhaps, is an explanation of the reduct which is composed of two attributes: employment and demographic potential.

The rules listed in the previous section determine the critical values of the criteria which allow the sorting of municipalities on those which are the best, those which are the worst and those which are intermediate.

Decision-makers whose municipalities are in class W or I, are interested in the critical values which will allow their municipality to pass to class B. In relation to the employment criterion, as pointed by rule 6, the minimal value is 62.5%, which is confirmed by 6 of the 13 best municipalities (46.15%), circled between the first and second quadrant of the coordinates (see Figure 2). Here we can see a certain balance between two dimensions which are opposed to each other. In relation to the demographic potential criterion (rule 7), this is a value of 39.9%. In relation to the housing conditions criterion (rule 8), this is a value equal to 1.07%. It is possible to lower the critical value with respect to the employment criterion down to 59.2%, but in combination with other criteria, such as housing conditions critical value (rule 11) which should be at most 11.8%. This rule is confirmed by 6 of the 13 best municipalities (46.15%), circled in the first quadrant of the coordinates (see Figure 2). Here we can see all the six municipalities on the side of the housing conditions axis, which explain this combination. Both rules (11 and 6) are confirmed by 4 of the 13 best municipalities with the highest values in relation to the employment criterion.

From the other side, we have rule 9 which is the combination of two criteria. The critical employment criterion value equal to 59.2% and the demographic potential critical value equal to 35.5% which is confirmed by 2 of the 13 best municipalities (15.38%), circled in the second quadrant of the coordinates (see Figure 2) on the side of the demographic potential axis.

Finally, we have rule 10 made of three criteria: employment whose critical value is equal to 59.2%, housing condition whose critical value is equal to 25.6% and demographic potential whose critical value is equal to 32%. This rule is confirmed by one of the 13 best municipalities (7.69%), namely municipality 25 (see Figure 2) on the side of the demographic potential axis.

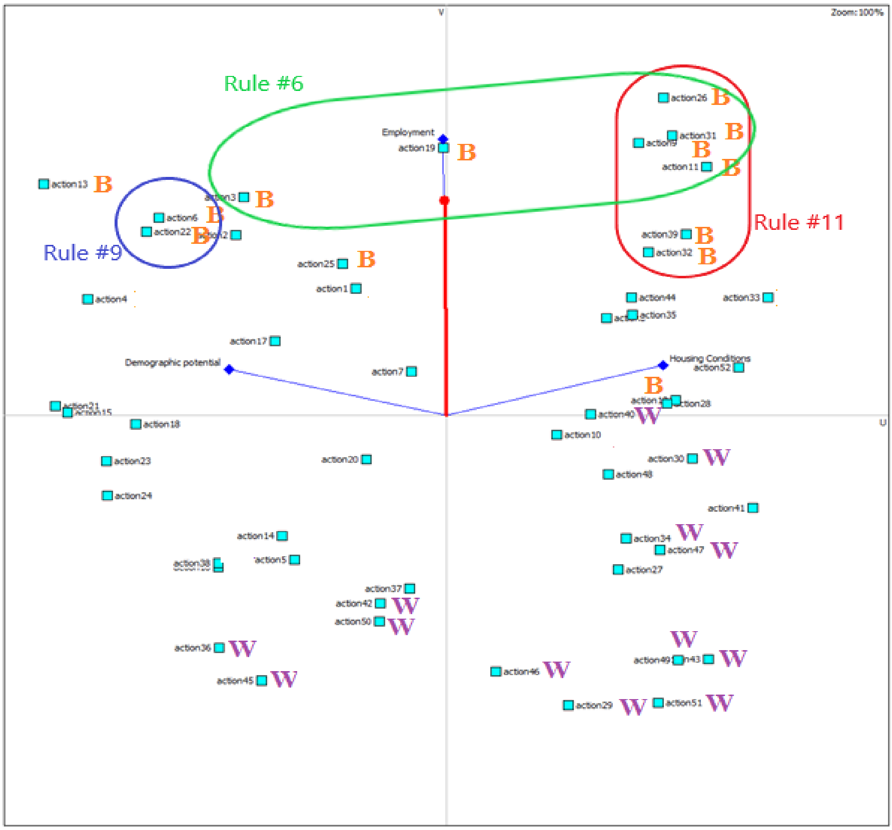

Figure 2: Municipalities belonging to classes B and W in the GAIA plan

5 Discussion

GAIA and DRSA are two methods that explain the ranking of potential actions obtained by the PROMETHEE multi-criteria method, each in its own way. The decision maker needs this explanation for a better understanding of the ranking of each action and to find a way to improve, if possible, the arrangement of the more distant actions in the classification.

The DRSA analysis of our case contains one reduct consisting of the employment rate and of the demographic potential rate. GAIA considers employment rate as similar to the housing conditions rate and the demographic potential rate when they are in balance. In the GAIA plan, it covers the decision axis marked as vertical. If the housing conditions rate and the demographic potential rate are unbalanced, then the points that represent municipalities are located in the GAIA plan above or below the center of the vertical axis, depending on which criterion is dominant. Obviously, this relationship is not linear, and the results can be seen on the GAIA plan for the municipalities, the increasing trend of the housing conditions rate being above the center of the vertical axis and the increasing trend of the demographic potential rate being below the center of vertical axis. For example, full balance is observed for belonging to the group of the best municipalities (action 19), located on the vertical axis of the employment in which the demographic potential rate is equal to 27.8% and the housing conditions rate is equal to 24.3%. The employment rate criterion (71.5% for this municipality) shows on the decision axis that this municipality has a very good performance.

6 Conclusions

We have compared GAIA and DRSA as approaches to assisting decision-making processes based on action ranking obtained using the multi-criteria method called PROMETHEE. The goal of using these approaches was to explain the ranking of 52 municipalities in northern Quebec in terms of employment statistics. The multi-criteria method provides aggregate information that is insufficient to answer the questions of the decision maker, such as how to upgrade the classification of a municipality categorized in a three-level scheme consisting of the designations 'best', 'intermediate' and 'worst', which criteria are relevant to the municipality, what critical values of a criterion determine the placement of municipality in one category or another, and so on. The application of GAIA and DRSA provides answers to these questions. GAIA allows visualization of a municipality and criteria positions on the initial main component plan, and DRSA allows determination of the critical values of the criteria. We have shown that the two methods complement each other, by explaining how three criteria can be reduced to two or by clustering the municipalities on the GAIA plan.

References

Brans J.-P. (1982), *L'ingénierie de la décision, Élaboration d'instruments d'aide à la décision. Méthode PROMETHEE* [in:] R. Nadeau, M. Landry (eds.), *L'aide à la décision: Nature, instruments et perspectives d'avenir*, Presses de l'Université Laval, Québec, Canada, 183-214.

Brans J.-P., Mareschal B. (1992), *PROMETHEE V: MCDM Problems with Additional Segmentation Constraints*, INFOR, 30, 2, 85-90.

Brans J.-P., Mareschal B. (1994), *The PROMCALC & GAIA Decision Support for MCDA*, Decision Support Systems, 12, 297-310.

Brans J.-P., Mareschal B. (2002), *PROMETHEE-GAIA une méthodologie d'aide à la décision en présence de critères multiples*, Éditions de l'Université de Bruxelles, Bruxelles.

Brans J.-P., Mareschal B., Vincke P. (1986), *How to Select and How to Rank Projects: The PROMETHEE Method*, European Journal of Operational Research, 24, 228-238.

Brans J.-P., Mareschal B., Vincke P. (1984), *PROMETHEE: A New Family of Outranking Methods in MCDM*, Operational Research, North Holland, 477-490.

Brans J.-P., Vincke P. (1985), *A Preference Ranking Organisation Method. The PROMETHEE Method for MCDM*, Management Science, 31, 6, 647-656.

Greco S., Matarazzo B., Slowinski R. (1999), *The Use of Rough Sets and Fuzzy Sets in MCDM* [in:] T. Gal, T. Hanne, T. Stewart (eds.), *Advances in Multiple Criteria Decision Making*, Kluwer Academic Publishers, Dordrecht, Boston 14.1-14.59, http://idss.cs.put.poznan.pl/site/71.html.

Greco S., Matarazzo B., Slowinski R. (2001), *Rough Sets Theory for Multi-criteria Decision Analysis*, European Journal of Operational Research, 129, 1, 1-47, https://doi.org/10.1016/S0377-2217(00)00167-3.

Mareschal B. (1986), *Stochastic Multicriteria Decision Making under Uncertainty*, European Journal of Operational Research, 26, 58-64.

Mareschal B. (1988), *Weight Stability Intervals in Multicriteria Decision Aid*, European Journal of Operational Research, 33, 54-64.

Mareschal B. (2013), *Visual PROMETHEE 1.4 Manual*, VP solutions, http://www.promethee-gaia.net/assets/vpmanual.pdf (accessed: 18 March 2019).

Marin J.-C., Zaras K., Boudreau-Trudel B. (2014), *Use of the Dominance-based Rough Set Approach as a Decision Aid Tool for the Selection of Development Projects in Northern Quebec*, Modern Economy, 5, 7, 723-741, http://dx.doi.org/10.4236/me.2014.57067.

Pawlak Z. (1982), *Rough Sets*, International Journal of Parallel Programming 11, 5, 341-356.

Pawlak Z. (1991), *Rough Sets: Theoretical Aspects of Reasoning About Data*, Kluwer Academic Publishing, Dordrecht, http://dx.doi.org/10.1007/978-94-011-3534-4.

Pawlak Z. (2002), *Rough Set Theory and Its Applications*, Journal of Telecommunications and Information Theory, 3, 7-10.

Zaras K. (2004), *Rough Approximation of a Preference Relation by a Multi-attribute Dominance for Deterministic, Stochastic and Fuzzy Decision Problems*, European Journal of Operational Research, 159, 196-206, https://doi.org/10.1016/S0377-2217(03)00391-6.

Zaras K., Marin J.-C., Boudreau-Trudel B. (2012), *Dominance Rough Set Approach as a Decision--making Method for the Selection of Sustainable Development Projects*, American Journal of Operational Research, 2, 506-508, http://dx.doi.org/10.4236/ajor.2012.24059.

(www 1) Statistics Canada (2012), Labour Force Survey, https://www150.statcan.gc.ca/n1/en/catalogue/71M0001X (accessed: 18 March 2019).

(www 2) PROMETHEE-GAIA software (2013), http://www.promethee-gaia.net/vpa.html (accessed: 18 March 2019).

MULTIPLE CRITERIA DECISION MAKING

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CREATING A NEW CULTURAL TOURISM PRODUCT RELATED TO LOCAL POST-INDUSTRIAL HERITAGE AS A MULTIPLE CRITERIA DECISION AIDING PROBLEM

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Abstract

Creating local cultural tourism products (CTPs) requires the involvement of many entities, e.g., local government units, culture and tourism institutions, non-government organisations, etc., as well as organisational, technical, financial, and human resources. When deciding on a future product venture, it is important to establish visions, goals, or the product's desired functions common to all these actors. Then, the alternative instances of such CTP need to be designed and examined from the viewpoint of these goals, which may be mutually exclusive due to the various preferences of the actors. Furthermore, despite their importance, these criteria may conflict with, e.g., the tourism policy of local authorities. The issues mentioned above make the decisions regarding creating CTPs very complex and have behavioural, group multiple criteria character. This paper aims to identify the structural elements of creating the best possible CTP promoting local post-industrial heritage in the Czeladź Commune. The specific assumptions, conditions, and criteria are defined to formulate the problem for further consideration using multiple criteria group decision-making (GDM) approaches.

Keywords: multiple criteria decision making, group decision-making, multiple criteria problems in cultural tourism, decisions on new cultural tourism products.

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1 Introduction

The development of cultural tourism, especially at the local and regional levels, has contributed to the growing importance of territorial marketing (Bagautdinova et al., 2012), promotion of a given local government unit (LGU), (Felsenstein and Fleischer, 2003; Panasiuk, 2020), and expanding the cultural and tourist offer (Bec et al., 2021). New CTPs have become an original and distinctive offer, attracting tourists from outside the LGU and the inhabitants themselves (Smith et al., 2021). However, before the new CTP is implemented, the decision to plan and design it goes through a long and procedural way of acceptance. There are many factors that affect the complexity of the problem related to creating a new CTP, e.g., the definition of the decision problem (Hammond, Keeney and Raiffa, 2002), identification of different CTP variants (Cemali, 2010), adequate selection of the set of evaluation criteria (Ginevičius and Podvezko, 2005), and analyses of entities and stakeholders, e.g., formal and informal decision-makers, experts (Crane and Ruebottom, 2011).

Economists and cultural, geographical and sociological scholars have been involved in a debate for at least two decades aiming to clarify the relation between destinations development and the use management production of the assets on which such development is based initially. In the case of cultural tourism, the issue is to "exploit wisely" the heritage for tourist use. In the variety of issues, methods, recommendations, and caveats generated by such discussion, two themes emerge as particularly productive for practical policy developments: the compatibility between the development of tourism industry and the preservation of the heritage "out of the market":

a) the compatibility between the development of tourism industry and the preservation of the heritage "out of the market";
b) the existing and potential synergies and tensions between the "global" tourism system and the "local" socio-economic development (Russo and van der Borg, 2002).

Yet another issue that may affect the complexity of the issues is the contemporary interdisciplinary character of such areas as: recreation, tourism, culture, etc. The importance of culture and heritage is becoming more and more obvious, both in local and regional development. Cultural factors are important because they directly affect economic performance and development, and therefore the competitiveness of the region. Moreover, a significant increase of the complex relations between culture, heritage, geography, tourism, economy and experience economy can be observed. Furthermore, cultural tourism offers a clear definition of each specific term, and defines the newest forms and trends in cultural tourism and experience economy (Bujdosó et al., 2015).

Given the very high complexity of the problem related to the creation of CTPs, the issue of implementing an appropriate multiple criteria behavioural group decision aiding tool arises. Theoretical and practical support for management decisions on CTPs is becoming a significant challenge for such areas as operational research, Multiple Criteria Decision-Making (MCDM) or optimisation.

The main objective of this paper is to identify the problem of creating a new CTP related to local post-industrial heritage as a MCDM problem. This requires structurisation and identification of its key elements, which allows producing grounds for further decision-making analyses from the viewpoint of any selected stakeholder of this process. The paper's contribution is twofold. First, we structure in detail the real-world problem of creating a new CTP for post-industrial heritage of the Czeladź Commune. Second, due to such a structurisation, we show what may be the elements of the decision making problem when any new CTP of post-industrial heritage is analysed; in particular, what types of criteria can be considered by the analysts and decision makers.

This problem is multidimensional, and specific relationships (exclusions, correlations, synergies) can be distinguished between individual dimensions. Therefore, several major levels of multidimensionality can be distinguished, such as:

1) multiplicity of entities and stakeholders involved (e.g., decision-makers (DM), experts, consumers) (Robson and Robson, 1996);
2) diversity of means and resources: technical, organisational, financial, etc. (Lee-Ross and Pryce, 2010; Nogueira and Pinho, 2015);
3) divergent perspectives on the mission, vision, objectives and CTP functions (Smith, 1994; Russo and van der Borg, 2002; Vucetic, 2009);
4) different CTP characteristics: e.g., attributes, features which can be considered as evaluation and finally as decision criteria (Keane, 1997; Lohmann, 2004; Mason, 2016);
5) identification of scales to measure criteria depending on their quantitative and qualitative characteristics (Davies, 2003; Weber and Taufer, 2016);
6) problems of selecting the key criteria from among the set of all identified criteria that will allow the most comprehensive evaluation of CTP alternatives (different types of criteria are presented in Section 4) (Ban, 2012; Blázquez, Molina and Esteban, 2012; Logunova et al., 2020; Nair et al., 2012; Stefano et al., 2015; Szromek and Herman, 2019);
7) diversity of base and potential of post-industrial heritage;
8) a multiplicity of categories, types and forms of a tourist product:
   a) e.g., product-thing, product-service, product-route – which can take the real, virtual or mixed form of the trail,

b) product types depending on their complexity: mono-products, systemic products (multi-products) etc.
c) different forms (instances): real product, virtual product, hybrid product etc. (Burkart and Medlik, 1981; Mason, 2016; Medlik and Middleton, 1973; Stokes, 2008; Yu and Xu, 2019).

In this paper, we focus on identifying the possibility for developing a CTP in the Czeladź Commune using the potential provided by the post-industrial cultural heritage of the town. We can understand this possibility as a set of all strengths, weaknesses, benefits, opportunities, threats (some intangible costs) and risks, which are located in the aspects listed above. The multifaceted analysis of the problem concerning the creation of a new CTP related to the promotion of post-industrial cultural heritage, presented below, was performed from the point of view of people working at the "Saturn" Museum in Czeladź.

The remainder of this paper is organised in the following way. Section 2 presents the essence of tourism products. In Section 3 diversity of stakeholders is presented. The issue related to resources as well as goals, and functions related to creation of a new CTP is presented in Section 4. The aspect of CTP criteria evaluation is described in Section 5. Diversity of the base and potential of post-industrial heritage is described in Section 6. In Section 7 we describe the multiplicity of categories, types, kinds, forms and forms of a tourist product. Finally, in Section 8, the summary and discussion are presented.

2 The essence of tourism products

There are many definitions of tourism products in the literature. Burkart and Medlik (1974, p. 45) emphasize the process of purchasing all things, their consumption and use, as well as the tourist's activity and activities related to the implementation of trips, travel or stay. Medlik and Middleton (1973, p. 32) define a tourist product in terms of area, because from the moment when tourists travel for a specific purpose, their destination has become the most important product they buy and they claim that the tourism products consists of a variety of elements which is a package that is not integral to each other and meet the needs of tourists from leaving his residence to the place of destination and back again to the place of origin. Thus, it can be concluded that there are three elements that form a tourism product, namely (Muainuddin and Hasan, 2018):

a) the attractiveness of the destination,
b) facilities of destination,
c) the ease of destinations.

Furthermore, the following six tourism product elements make up an integrated tourism package (Tresna and Nirmalasari, 2018; Yu and Xu, 2019):

a) objects and attractions,
b) services of travel agents and tour operators,
c) transportation services,
d) accommodation, restaurants, recreation and entertainment possibilities,
e) souvenir services,
f) business support.

Tourism product is a series of interrelated services produced by various companies (economic aspect), community services (social aspect) and service branches. The formulation of the components of tourism products was proposed by P. Mason (2016). They are namely:

1) attractions, which are a tourist attraction of natural, cultural and man-made origin such as festivals and performing arts;
2) accessibility, the ease of obtaining or achieving organisational goals such as tourism, travel agents;
3) amenities of the facility to obtain pleasure; in this case the tourism product can take the form of accommodation, cleanliness and hospitality, networking, the network of cooperation relating to the products offered by local, national or international organization.

Tourism product is something that can be offered to tourists to visit a tourist destination. Tourism products can be natural, cultural or community handicrafts. It is a service that can be enjoyed by tourists in a tourist destination, which is supported by tourist attractions, facilities and services, product price, or accessibility support that can facilitate travel activities. In turn, the definition presented by the World Trade Organization (WTO) in 1985 describes cultural tourism as movements of persons for essentially cultural motivations such as study tours, performing arts and cultural tours, travel to festivals and other cultural events, visits to sites and monuments, travel to study nature, folklore or art, and pilgrimages (von Rohrscheidt, 2008).

Regarding the general information about tourism products, it can be concluded that the local post-industrial CTP is a multi-component set of all goods and services enabling the consumer-tourist to sightsee post-industrial heritage, stay, rest and recreate in a geographically determined area of post-industrial heritage.

As mentioned above, the process of establishing CTPs is very complex. If we consider a specific context of such a product related to the local heritage of post-industrial areas in the Czeladź Commune, the level of complexity increases further. This is caused by the diversity of entities, decision-makers, stakeholders, the diversity of types of cultural heritage, and the lack of integrity of the previously promoted post-industrial heritage. Below we analyse the elements that comprise this multidimensionality and identify key important elements for the future decision-making problem of ‘which CTPs to choose to promote post-industrial heritage in Czeladź in the best possible way’.

3 Diversity of stakeholders

Stakeholders’ identification was based on an analysis of organisational structures, formal information provided by various entities and based on competencies and responsibilities of various human resources in institutions located in the Czeladź Commune. In general, four types of stakeholders can be distinguished from different entities located in Czeladź:

1. Stakeholders at the level of the LGU of the Czeladź Commune.
2. Stakeholders in units, entities and institutions subordinate to the Czeladź Commune.
3. Stakeholders in (non-governmental organisations) NGOs whose statutory activity is related to promoting culture, tourism, cultural tourism.
4. Stakeholders in municipal educational institutions (related to the subject of cultural tourism).

A more detailed description of the above classification is presented in Table 1.

Table 1: Planned entities and stakeholders in the process of creating a cultural tourism product related to the post-industrial cultural heritage of the Czeladź Commune

Name of the stakeholder group	**Institution/entity**	**Person/persons representing a unit, institution or entity**
1	2	3
Stakeholders at the level of the LGU	Czeladź Commune Hall	− Mayor of the Czeladź Commune and/or Deputy Mayor for Social Affairs − Head and/or Deputy Head of the City Promotion, Culture and International Cooperation Department − an employee of the City Promotion, Culture and International Cooperation Department − Head and/or Deputy Head of the City Development and External Funds Department − employee of the City Development and External Funds Department
Stakeholders in organisational units and institutions and entities subordinate to the LGU	“Culture Mine” – municipal culture institution Municipal Sports and Recreation Centre “Saturn” Museum	− Manager of the “Culture Mine” − Director of the Municipal Sports and Recreation Centre − Head of the Sport, Organisation and Coordination of Events Department

Table 1 cont.

1	2	3
	"Saturn" Museum – Department: Contemporary Art Gallery "Elektrownia" Municipal Public Library House of Culture "Odeon"	− Director of the "Saturn" Museum − Head of the History of the City and Dąbrowski Basin Mining Department − Employee of the History of the City and Dąbrowski Basin Mining Department − Manager of the Contemporary Art Gallery "Elektrownia" − Deputy Manager of the Contemporary Art Gallery "Elektrownia" − Employee of the Contemporary Art Gallery "Elektrownia" − Director of the Municipal Public Library − Manager of the House of Culture "Odeon"
Stakeholders in NGOs whose statutory activity is related to promoting the Czeladź Commune	Associations, foundations, public benefit rganisations, etc. − Educational and Cultural Association − "Razem" − "Czeladź Is Cool" Association − Public Society "Czeladź" − "The Mine of Magnificent Climates" Association	− Decision-makers in NGOs from the Czeladź Commune
Stakeholders in municipal educational institutions	Primary and secondary schools: − Primary School No. 1 − Primary School No. 2 − Primary School No. 3 − Primary School No. 4 − Primary School No. 5 − Primary School No. 7 − General Education and Technical Education Schools (secondary school)	− Headmasters (and/or their deputies) of primary and secondary schools and/or − Teachers of primary and secondary schools

4 Diversity of resources, visions, and goals

Various actors that are involved in the CTP development process have different means and resources. Their specificity may affect the definition of one's own vision and goals of CTP implementation related to the promotion of post-industrial heritage.

In order to implement a new CTP (regardless of its form: real, virtual, or combined), appropriate means and resources are necessary, such as:

a) organisational – requires an appropriate organisational structure of entities, appropriate delegation of powers to individual stakeholders and an appropriate management style;
b) technical – concerns a variety of materials, devices, equipment and infrastructure. It is difficult to imagine, for example, a thematic well-prepared post-industrial route without easy communication accessibility (e.g., public transport stops, bicycle routes, tourist routes, etc.);
c) financial – that must be secured in the budget of the Czeladź Commune (which is the main substantive provider of the new product) and the budget of the leading institution; here, a question arises regarding an "optimal" level of financial resources; decision-makers will have to deal with such a problem – what is the best ratio of the funds involved to the benefits and positive effects of a new product?
d) personnel (human resources) – that requires proper selection of people involved in the decision-making process and the executive process; here, the following issues are also important: experience in creating new ventures, competences and knowledge, managerial skills, interpersonal relations and behavioural aspects of decision-making or advisory activities.

Different goals and functions

Each stakeholder may have a different view on the vision and goals of creating a new CTP related to post-industrial heritage and the functions it needs to fulfil. Differences in views on these issues may result, among other things, from (Smith, 1994; Russo and van der Borg, 2002; Vucetic, 2009):

a) the nature of the main activity of the entity,
b) personal beliefs and concepts,
c) conflict of matters between different entities,
d) prejudices, stereotypes and lack of motivation to overcome stereotypes and habits in making decisions, as well as risk aversion of creating above-standard solutions,
e) not being aware of the existence of other alternative visions, goals or functions,
f) lack of stakeholders' awareness of alternative variants of the potential CTP.

For example, the authorities of the Czeladź Commune may strive to ensure that the new product is:

- a tool for the promotion of the Commune (primarily it plays promotional and PR functions),
- a new item in the tourist and cultural offer of the Commune.

On the other hand, the managers of the "Saturn" Museum in Czeladź may see the new product as:

- fulfilling educational functions, disseminating the history of the industry, etc.,
- a new offer in the activities of the Museum,
- a presentation of new ideas and creativity of their employees.

5 Criteria of CTP evaluation

The CTPs have many different characteristics that can be described by various attributes. Depending on, for example, the visions, goals and functions of the planned product, the properties mentioned above can be considered as criteria for assessing a given alternative of the future CTP. There may be many different objectives, evaluation criteria and related preferences of the identified stakeholders (Keane, 1997; Lohmann, 2004; Mason, 2016). The second aspect is related to the fact that the criteria may be either measurable or not, namely:

a) non-measurable evaluation criteria, such as: mission, objectives, action plan, strategy, functions, program, location;
b) measurable evaluation criteria, e.g., number of attractions, number of tourists in a given destination, estimated future volume of tourist traffic, potential tourist capacity of the region.

The research approach to eliciting preferences over such criteria should therefore take into account their dual character. Since many stakeholders may define a large number of potential criteria, the next issue is selecting the subset of key criteria that will best match the specificity and features of CTPs related to the promotion of post-industrial heritage. After analysing the problem from the viewpoint of our DMs – the management of the "Saturn" Museum – the following set of criteria was defined, as shown in Table 2.

Table 2: The list of potential CTP decision criteria

Criteria and possible sub-criteria	Meaning of the criteria/sub-criteria
1	2
C1: Development prospects: a) an action plan b) a development strategy	What activities are planned for product development? What development strategy is planned for the product?
C2: Attractiveness of the product from the point of view of tourists: a) main attractions b) USP – unique selling proposition c) objects of heritage, objects of spatial development	In what ways are the product attractive? a) main attractions related to the product b) USP – unique features that motivate tourists to take advantage of the product offer c) heritage features, land-use facilities that attract tourists to a given area and area
C3: Commercialisation of the cultural tourism product: a) bundling of services and offers b) using various distribution channels c) joint marketing strategy d) the number and names of travel agencies, sales portals offering this product, etc.	What actions are taken to commercialise the tourist product – activities related to introducing the cultural tourism product to the market?
C4: Innovation in product development: − solutions within the scope of the offer itself − management solutions − marketing solutions	What innovative solutions will be applied as regards: − the offer (e.g., offering services/products previously unknown on the market, completely new) − management (e.g., functioning under an innovative formula of partnership with other entities) − marketing (the use of innovative promotion methods, research, communication with the market, etc.)?
C5: New technologies in product development and its promotion	What new technologies will be implemented in product development and its promotion? e.g., beacons, QR codes, mobile applications, the use of web 2.0/3.0 technology, travel planners, geotagging, multimedia platforms, multimedia and/or interactive exhibitions, ICT systems, online booking/purchase, e-books, 3D exhibitions, holograms, virtual tours, etc.
C6: Economic, social and economic importance of the product for the development of the region: a) the economic potential of the product, including the ability to generate jobs b) product image − perception of the offer on the tourist market c) economic effect d) integrating the local community e) estimated future volume of tourist traffic f) product receptivity	What economic and social aspects of a cultural tourism product will be important for the development of the region?

Table 2 cont.

1	2
C7: Timeless cultural significance	Does the product show important solutions for value created by people, timeless importance for culture in the development of architecture, technology, fine arts, town planning or landscape design?
C8: Contribution to cultural tradition or civilisation	Does the product make a unique or at least exceptional contribution to a cultural tradition or civilisation – living or vanishing?
C9: A product identifies events or traditions with a universal meaning	Does the product identify events or traditions that are directionally or literally related to a thought (idea), belief, artistic or literary work, distinguished by a universal meaning?
C10: Old age criterion	How important is the time of creation related to the specific historical context of a given artefact, place, object, area, etc.?
C11: Authenticity criterion	Does the product meet preservation conditions? − the state of a given artefact, place, facility, area, etc. − the legitimacy of maintaining the existing spatial layout of a place, facility, area − susceptibility to adaptation to new functions/supplementing the infrastructure of the place, facility, area with elements necessary for making available Is the product authentic, naturally tied to the region (its natural conditions, history, culture); does it transmit a specific message about the region, tells about it?
C12: Uniqueness criterion	Does the originality of the product take into account elements such as: a) author/authors or creator/creators – significant for the development of architecture, the figure of the author/design team of a single site, facility/complex of buildings/spatial layout, area, etc. b) form – significant architectural value resulting from the formal analysis of the place, object, area: − exceptional stylistic features or their uniqueness in a given territory − type of design solution − a unique type of construction technology used − the unique nature of the building material − a solution based on a "typical project", the occurrence of which in a given place, facility, the area is incidental or has analogies in ethnically or culturally different areas c) function – a function of a place, object, area
C13: Criterion of complementarity and integrity of the object	Does the product have complementarity and integrity in contexts such as: a) spatial: − further surroundings: the importance of the place, object, area as an integral element embedded in the landscape − closer surroundings: in the context of a group of places, objects − immediate surroundings: connection of a place, object with other objects, organisation of the area, role in the functional team of places, objects

Table 2 cont.

1	2
	b) historical/cultural/social: − local and regional importance: significance for the history of the place, facility, area, e.g., the historical basis for the creation of the region − link to significant historical events or customs − an example of mutual cultural/national/ethnic relations Does the product have complementarity and integrity in relationships such as: a) integrity: − the importance of a place, object in a group of places, objects − the importance of the place, object in the context of the landscape or urban layout b) complementarity: − a wide typological cross-section of a given type of place, facility, area − fully preserved nature of the place, facility, area, − continuity and complementary character of the facility in the spatial structure of a place, area
C14: General infrastructure	Does the product affect the development of such infrastructure elements as: − tourist facilities − accommodation base − food and entertainment facilities − public communication accessibility − transport at the destination − tourist equipment rentals
C15: Tourist events	How does the product stimulate the tourist events in the region? In cycling, Nordic walking, hiking etc.?
C16: Cultural events	How does the product stimulate the cultural events in the region? In festivities, picnics, festivals, exhibitions, etc.?
C17: Piloting and guiding tourist trips	Does the product provide professional service by a tour leader, guide, curator or other substantively prepared persons?
C18: Image of the place	What is the image of a place? (‘image’ understood as the sum of beliefs, ideas and images a person has in relation to a given place)
C19: New experiences, emotions and impressions, new social contacts	Does the product provide knowledge of the place, attraction, value, heritage, excitement, fascination with the visited place, establishing a relationship with people who experience and feel alike?
C20: Usability criterion for tourists	How does the CTP meet the specific needs of buyers (tourists)?
C21: Criterion of consistency of elements and complexity	Are all elements of the CTP compatible with each other? Is the product comprehensive – does it constitute a wide, integrated offer; it is not a random combination of attractions, all elements of the product are consistent and well thought out?

Table 2 cont.

1	2
C22: The criterion of originality, recognisability, distinguishability	How original, recognisable, and distinguishable the product is? For instance: − the cultural tourism product should stand out from other products on the market − the product differs from the competitors' offer because it is unrivalled − the product should be recognisable in the tourism, culture and related industries and it is recognisable thanks to the name, graphic characters, slogans used to distinguish and promote it
C23: Brand strength criterion for a cultural tourism product	What is the strength of the brand related to advertising products marked with a logo, e.g., books, calendars, guides, decorative and utility ceramics, etc.
C24: Criterion of complementarity of the tourist offer	Is a given CTP itself the purpose of the visit and how it complements the local tourist offer or a place of excursion for tourists staying in the near and distant surroundings?
C25: Criterion of high quality	Is a given CTP characterised by the high quality that meets the needs of specific market segments – tourists associate the brand with quality?
C26: Criterion of promotion, creation of the region's image, region's recognition by the product, region's competitiveness	Does the product promote the area of the LGU, increasing the value of the LGU's tourist offer and building its image? Does it strengthen the competitiveness of the LGU on the regional market of tourist services?
C27: Product bidder investment stimulation criterion	Does the CTP give the feedback effect as an impulse for further action?
C28: Criterion of tourist travel motivation	Is the product a motivation to come not only for cultural tourism enthusiasts but also for visitors with less specialised interests?
C29: The criterion of inspiration and modelling	Does the product become a model and inspiration, influencing the attractiveness of other products and the environment?
C30: The criterion of a better quality of life for the inhabitants of the region, new fashion, a new trend in culture, tourism, recreation	To what extent does the product increase the standard of living of the inhabitants in the region? To what extent does the product contribute to creating a new lifestyle, becoming over time a permanent point of visit for residents together with guests from outside the region? (e.g., for a family visit)
C31: Criterion of cooperation with other entities	How does the product contribute to cooperation with business entities from other industries, reporting a constant demand for their services (electrical, telecommunications, transport, outsourcing and others)?
C32: Educational criterion	Does the product affect the quality of the educational offer (e.g. giving the possibility of conducting thematic lessons or creating educational trials)?
C33: The criterion of shaping local/regional identity	How much does the product influence the feeling of ties with the region and the feeling of pride in living there?

Source: Own elaboration based on: Abdurahman et al. (2016); Fuadillah and Murwatiningsih (2018); Logunova et al. (2020); Nair et al. (2012); Ramírez-Guerrero et al. (2020); (www 1).

The above-mentioned unified criteria form a set which characterises CTP in its general form, without indicating what cultural potentials this product relates to. The product related to the promotion of post-industrial heritage will be assessed against specific criteria, best suited to the specificity of the post-industrial heritage. The task of the stakeholders involved in the process of creating a new CTP is to select the most important, non-redundant criteria.

Depending on the stakeholder, various subset of criteria may play significant role in evaluating future instances of CTP. To select such a subset of criteria, techniques from GDM may be used that would build the ranking of the most important ones out of the list prepared individually by each stakeholder or decision maker (Morais and Almeida 2012; Morais, Costa and Almeida, 2014; Nurmi, 1981; Raiffa, Richardson and Metcalfe, 2002; Roszkowska and Wachowicz, 2015).

However, the issue of how to select and assign weights (priorities) to key criteria that fit in the post-industrial heritage CTP context is a separate issue which falls outside the scope of this paper and will be the subject of our further research.

6 Diversity of the base and potential of post-industrial heritage

The identification of post-industrial heritage was made on the basis of an analysis of Polish bibliographic resources such as: Binek-Zajda, Lazar and Szaleniec, 2016; Chmielewska et al., 2016; Domaszewski, 2000; Kurek, 2012; Lazar and Binek-Zajda, 2015. The diversity of elements comprising this heritage (which is presented in Table 3) is also an additional factor that determines the complexity of the problem of creating CTP promoting post-industrial heritage. In view of the above, stakeholders in the process of establishing CTP may face such issues as:

a) What elements of this heritage are to be taken into account?
b) How to classify (by what criteria) these elements?
c) Should not individual elements of this heritage have a specific weight or priority?
d) What is the core of this heritage?

An example of a detailed classification of the heritage base of the “Saturn” mine is presented in Table 3.

Table 3: Classification of the post-industrial heritage base of the former "Saturn" mine

Distinguishing the type of heritage base	**The name of the heritage items**
1	2
Type I: Facilities included in the "Saturn" mine	1) Assembly hall: a) guild hall b) bathhouse c) cloakroom d) administration e) meeting room 2) Electric workshop 3) Transformers and converters room 4) Building with compressors 5) Mechanical workshop 6) Power plant (power plant building, compressors room and switching station) 7) Boiler house building – with two chimneys 8) Garages 9) Shaft No. 1: a) overhang building with the hoisting tower of shaft No. 1 b) the engine room of shaft No. 1 10) Shaft No. 2: a) the superstructure of shaft No. 2 b) the building of the engine room of shaft No. 2 11) Sorting building 12) Railway siding 13) Building of the fire brigade 14) Weight 15) Non-existent, demolished buildings from the 1930s. 16) Powder Magazine (facility outside the mining plant complex) 17) Coal dump
Type II: Housing estate – workers' and clerks' housing	1) "Stara Kolonia" housing estate 2) "Nowa Kolonia" housing estate
Type III: Housing estate – public utility buildings	1) Clerks' club 2) School 3) Teachers' house 4) Workers' hotel 5) Gazebo in the Jordanowski Park
Type IV: Housing estate – management buildings	1) Mine management building 2) Director's Villa
Type V: Machine and equipment infrastructure	1) Machines and devices in the former power plant: a) Power system. "Wanda" power generator – reversible compressor b) Compressor by Belliss & Morcom c) Power system. Generator set I d) Power system. Generator set II e) Power system. Backup power generator – Brown Boveri converter

Table 3 cont.

1	2
	f) Control and measurement desk g) Gantry h) Compressor control cabinet by Belliss & Morcom i) Signalling board j) Switchgear units, exciter units k) Control and measurement cabinets l) Movable links for switchgear m) Piping parts n) Other ancillary infrastructure 2) Machines and devices in former mechanical workshops 3) Machines and devices in buildings above the shafts 4) Machines and devices in engine rooms
Type VI: Parks, gardens and estate greenery	1) The Jordanowski Park 2) The mulberry garden
Type VII: Sports fields and other sports facilities	1) Playing field in the mulberry garden (in the northern part) 2) Playing field in the mulberry garden (in the southern part) 3) Playing field in the "Stara Kolonia" housing estate (in the northern part) 4) Playing field in the "Stara Kolonia" housing estate (in the southern part) 5) Playing field in the Jordanowski Park

Source: Own elaboration based on: Binek-Zajda, Lazar and Szaleniec (2016, pp. 49-95).

As regards the former "Czeladź" mine, it is also possible to classify the types of distinguished heritage. It can be divided into several types according to the criteria given in Table 4.

Table 4: Classification of the post-industrial heritage base of the former mine „Ernest Michał" („Czeladź", „Czeladź-Milowice")

Distinguishing the type of heritage base	Object name
1	2
Type I: Housing estate – workers' and clerks' housing	1) Housing for officials − clerks' houses at Sikorski St. 2) Workers' housing 3) Complex of 4 residential buildings at the "Juliusz Lair" mining shaft 4) Buildings in Francuska St. 5) Complex of houses in Betonowa St. 6) A complex of 3 buildings servicing the "Jan Keller" mining shaft 7) Complex of 6 houses in Płocka St. 8) Construction investment in Trzeci Szyb St.
Type II: Housing estate – public buildings	1) 4 schools 2) Orphanage for children 3) Teachers' house at Kosciuszko St. 4) 2 night shelters

Table 4 cont.

1	2
	5) Hospital 6) Pharmacy 7) Canteen – a dining room for mine employees 8) Clerks' Club in Sikorski St. with a garden and a tennis court 9) Shops
Type III: Housing estate – management buildings	1) Building of the Delegate of the Administrative Council (House of the Delegate) known as "Viannay's Palace" 2) Building of the Deputy Delegate of the Administrative Council 3) Building of the main mine mechanic (house for engineers) 4) Support staff buildings 5) Building of the Head Office 6) The building at 19, 3 Kwietnia St. – house of a senior official
Type IV: Church and parishes	Church in Piaski (originally the temple functioned as Saint Angela's church, in the erection decree it was called the Church of the Seven Sorrows of the Blessed Virgin Mary, in 1939 it was changed to the Church of Our Lady of Sorrows)
Type V: Housing estate greenery	1) The original layout of buildings with green spaces between the houses in 3 Kwietnia St. and later Sikorski St. 2) Park in Piaski in the vicinity of the Head Office 3) Gardens surrounding the management's houses: a) villa of the Delegate of the "Saturn" Mining and Industry Society Board b) houses of clerical staff 4) Private gardens surrounding 4 houses for mining supervision in Francuska St. 5) Row plantings of trees along local streets 6) The Jordanowski Park in Mickiewicz St.
Type VI: Playing fields	1) The playing field of the Reserve NCO in the vicinity of the Jordanowski Park in Mickiewicz St. 2) The playing field of the Police Station of the State Police at the intersection of Kościuszko St. and 3 Kwietnia St. 3) The pitch of the "Sokół" Gymnastic Society at the intersection of Francuska St. and Nowopogońska St. 4) The playing field of the "Gwiazda" Sports Club on the green square in Betonowa St.
Type VII: Objects included in the "Ernest-Michał" mine ("Czeladź", "Czeladź--Milowice")	1) Mine railway station 2) Railway siding (connecting with the Warsaw-Vienna Iron Road) 3) Sorting building 4) Central power plant 5) Boiler room building 6) Shaft structures and equipment: a) extraction shafts b) ventilation shafts

Table 4 cont.

1	2
	7) Auxiliary objects: a) mechanical workshops b) machine halls c) forges d) electrical switching stations e) stables f) carpentry shops g) gatehouses

Source: Own elaboration based on: Lazar and Binek-Zajda (2015, pp. 20-28, 71-95).

The post-industrial heritage database presented in Tables 3 and 4 shows a huge variety of objects, elements, places and other traces. The above--mentioned differentiation of this base (according to the criteria proposed by us) may contribute to the creation of tourist sub-products. Therefore, it is possible to combine certain common elements that occur in the post-industrial heritage of two mines or create separate sub-products related only to a given coal mine. The wide and diverse base of post-industrial elements prove the great potential of this heritage in the Czeladź Commune. They give opportunities to design CTPs in the form of mono-products, system products, network products, integrated area products, and others. This gives an opportunity to create a CTPs that will comprehensively and interdisciplinarily exhibit the original values of local tourism industrial and technical facilities.

7 Multiplicity of categories, types, kinds, forms and forms of a tourist product

The CTP is not only one homogeneous product or, in other words, a mono-product. There are many classifications into categories, types or forms introduced by various researchers. In general, it can be seen that a CTP may appear in such categories as (Burkart and Medlik, 1981; Mason, 2016; Medlik and Middleton, 1973; Stokes, 2008; Yu and Xu, 2019):

1. Thing, material good or real-product (souvenirs, promotional gadgets, tourist guidebook, map, etc.).
2. Service (thematic tourist guide, accommodation and catering services, etc.).
3. Event (festivals, festivities, picnics, etc.).
4. Sets of services (themed tourist rallies, cultural trips, etc.).
5. Object or facility (monuments, buildings, architectural real-estates).
6. Route or trail (traditional, virtual, mixed, etc.).
7. Area (e.g. comprehensive cultural heritage area).

We can also identify CTPs from the perspective of the complexity of their structure and divide them into three main groups as shown in Figure 1 (Panasiuk, 2017).

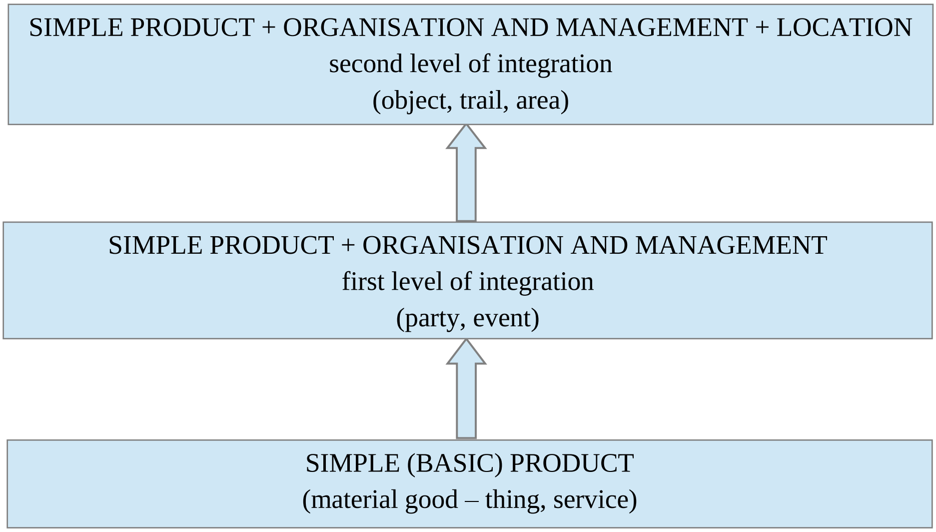

Figure 1: Tourism product/CTP pyramid

Source: Own elaboration based on: Panasiuk (2017).

Another criterion for distinguishing CTPs is their form related to their way of existence, namely:

1) Real (traditional, tangible).
2) Virtual (concerning the Internet space, computer programs and applications for mobile devices).
3) Multimedia (consists of, e.g., digital photos, figures, graphic elements, sounds, music, interactive elements, films, animations).
4) Mixed – hybrid (e.g., real-virtual or real-multimedia real-multimedia-virtual).

The CTP typology shows that the product of tourism of industrial and technical facilities can be designed as a particular portfolio of elements selected in terms of specific preferences of stakeholders. Stakeholders' preferences must address issues such as:

a) How to choose individual components (simple products)?
   - according to specific criteria?
   - according to certain subjective "rankings" by stakeholders in the process of creating a new CTP?
b) How many components should be selected?
c) Which product categories, types, and instances should be linked together to

make them clear and logical (e.g., a virtual product-area, a multimedia product-tourist service?)

The DM from whose point of view we analyse this example identified the following products (Table 5):

Table 5: The CTP portfolios related to the promotion of post-industrial cultural heritage in the Czeladź Commune

Suggested name of an alternative variant of CTP	**Combination in relation to the multidimensionality of the product**	**Description of the alternative variant of CTP**
Portfolio product A	Material good (thing) in multimedia form + virtual service + virtual route + virtual objects	Interactive and multimedia map of post-industrial attractions, on website Virtual guide service of the most important attractions Thematic virtual route through the most important post-industrial attractions Virtual sightseeing of objects (virtual walks ~~in~~ through the facilities)
Portfolio product B	Material good (thing) + event + services set + virtual route	Guidebook with map in printed form Thematic cultural event with attractions: games, plays, competitions Thematic, sports, tourist rally with elements of learning about post-industrial culture Thematic virtual route through the most important post-industrial attractions
Portfolio product C	Product-thing in multimedia form + route + questing	Interactive and multimedia map of post-industrial attractions, on website Traditional thematic route through the most important post-industrial attractions Questing of post-industrial heritage
Portfolio product D	Material good (thing) + virtual service + virtual route + event + product-area	Information boards at heritage sites and brochures related to post-industrial attractions Thematic virtual guide service regarding each of the distinguished elements of heritage – on the website Thematic virtual route through the most important post-industrial attractions Thematic tourist and cultural event with attractions: games, plays, competitions Thematic, geographically determined area of the former two coal mines, their patronage housing estates and other infrastructure sites

In the process of setting up the CTPs, the stakeholders need to face the issue of solving complex system problems concerning the creation of various types of such products, e.g., CTPs which consist of only one category listed above, one type only or any combinations of them.

8 Discussion and summary

In this paper we identified the key elements of the structure of the problem of designing new CTP based on the post-industrial heritage in the Czeladź Commune. This way we reached the goal of this paper, which was to identify the problem of creating a new CTP related to local post-industrial heritage as a MCDM problem. The proposed perspective of the identification related to the decision-making problem has several advantages, in particular:

1) Multifaceted nature takes into account the diversity of determinants of the decision-making problem.
2) Multidimensional approach adapts to the changing decision problem.
3) Multidimensional perspective of problems related to creating a new CTP should not discriminate against any decision conditions or factors.
4) The idea of the presented approach to decision-making has an open formula; since in the course of further research on other behavioural models it will turn out that one of them can be used.

Our intention is to use the structure of the problem identified in this paper in a future decision-making problem of selecting the most promising CTP at the level of the LGU – the Czeladź Commune. The direction of further research includes designing research surveys, conducting an interactive decision experiment and producing recommendations regarding alternative CTPs to be implemented.

The multidimensionality of the process of creating CTPs and the high complexity of the problem will require in future analyses the use of an appropriate integrated and hybrid methodological approach, which includes:

a) classic MCDM/Multiple Criteria Decision Aiding (MCDA) methods,
b) issues of behaviourism and psychology,
c) GDM perspective,
d) methods for aggregating individual assessments,
e) author’s innovative approach to the modification of the above-mentioned methods.

The presented multifaceted approach to the decision problem related to the plan to create a new CTP was developed from the point of view of knowledge, competence, but also subjective opinions of the employees of the “Saturn” Museum in Czeladź. If research is conducted among a group of identified stakeholders, there is a probability that the multifaceted nature of the problem will be extended by further aspects, which may increase the problem complexity even more.

Identification and characterisation of the multidimensionality of the problem is the basis for group, multiple criteria decision analysis in a behavioural context to be conducted in the future. The general scheme of proceeding in the decision analysis may take the form shown in Figure 2.

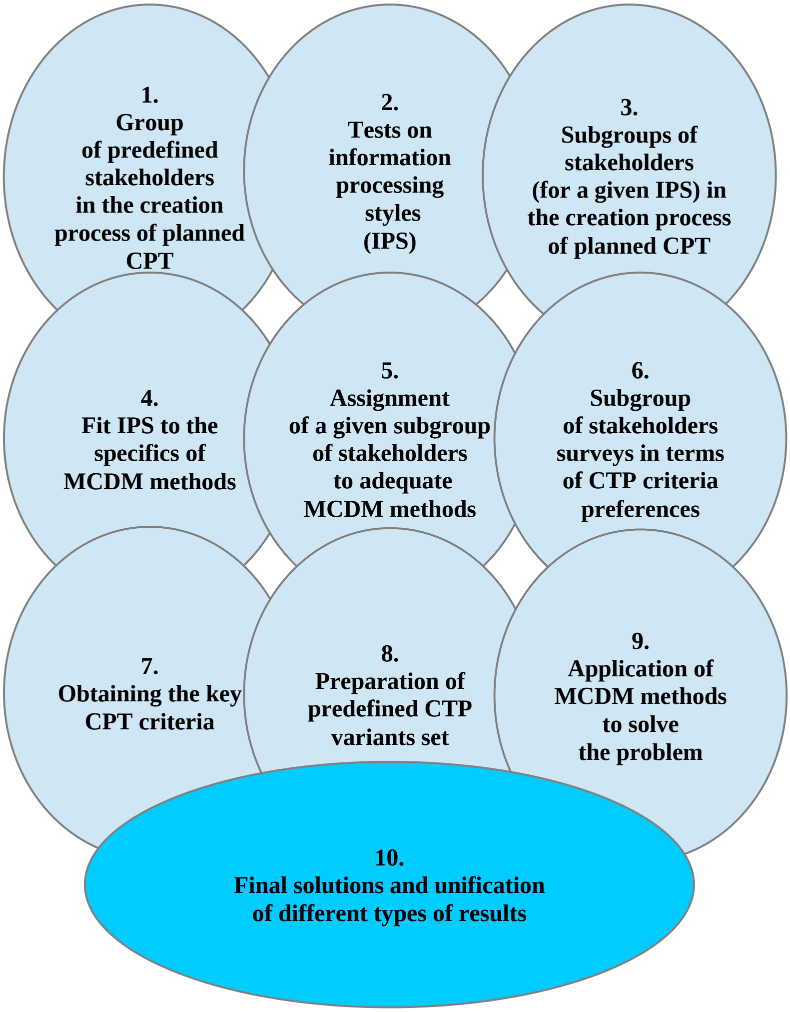

Figure 2: The general scheme of proceeding in the decision analysis

Source: Author’s own elaboration.

In the first step of the future research (to be conducted among the respondents) properly prepared tasks and psychological tests will be used. In the next step, after the respondents have solved the tasks and tests, the obtained results are analysed in terms of the dominant style of information processing. Individual respondents (characterised by a specific information processing style) will be assigned to the groups of particular decision-making styles and abilities. Then, the characteristic features of a given information processing style will be confronted with the specificity of algorithms of selected MCDA or MCDM methods, and the best possible recommendations of the method to the distinguished styles will be given. The use of this selected method by individual respondents from a given group will be one of the steps in the further stages (in the second phase) of the study.

Finally, a GDM approach will be implemented to identify the final recommendation of the CTP best matching the goals and priorities of all stakeholders and decision makers involved from the Czeladź Commune.

The structuring of the above-mentioned decision problem is based on the three initial stages of the universal PrOACT (Problem, Objectives, Alternatives, Consequences, Tradeoffs) algorithm. The theoretical assumptions of multidimensionality and multiple criteria structure create a certain conceptual framework which a group decision-maker (a group of stakeholders) can use in a practical aspect. The concepts of creating new CTPs related to a diverse cultural heritage become very complicated due to the multifaceted nature, variety of factors (entities, means, goals, functions, attributes, etc.). An additional element that affects the complexity of the CTPs creation problem are various types of relationships (cause-effect relationships, positive or negative relationships). Therefore, the decision analysis and the MCDM theory are able to support the stakeholders of such a complex decision problem by means of problem structuring tools and adequate methods of selecting the best solutions.

References

Abdurahman A.Z.A., Ali J.K., Khedif L.Y.B., Bohari Z., Ahmad J.A., Kibat S.A. (2016), *Ecotourism Product Attributes and Tourist Attractions: UiTM Undergraduate Studies*, Procedia – Social and Behavioral Sciences, 224, 360-367.

Archival collections of the “Saturn” Museum in Czeladź, Album of the Mining and Industrial Society “Saturn”, Inventory No. MS/HG/461.

Archival collections of the “Saturn” Museum in Czeladź, Design of the Mine Building, Inventory No. MS/HG/153.

Bagautdinova N., Gafurov I., Kalenskaya N., Novenkova A. (2012), *The Regional Development Strategy Based on Territorial Marketing (The Case of Russia)*, World Applied Sciences Journal, 18 (Special Issue of Economics), 179-184.

Ban O. (2012), *The Opportunity of Indirect Determination of the Importance of the Attributes of the Tourist Product in Evaluating the Consumer's Satisfaction*, International Conference on Economics, Business and Marketing Management, IPEDR 29, IACSIT Press, Singapore.

Bec A., Moyle B., Schaffer V., Timms K. (2021), *Virtual Reality and Mixed Reality for Second Chance Tourism*, Tourism Management, 83, 1-5.

Binek-Zajda A., Lazar S., Szaleniec I. (2016), *Coal Mine and Workers' Settlement "Saturn". History, Architecture, People* (in Polish), Public Society "Czeladź" and the "Saturn" Museum in Czeladź, Czeladź.

Blázquez J., Molina A., Esteban Á. (2012), *Key Quality Attributes According to the Tourist Product*, European Journal of Tourism Research, 5(2), 166-170.

Bujdosó Z., Dávid L., Tőzsér A., Kovács G., Major-Kathi V., Uakhitova G., Katona P., Vasvári M. (2015), *Basis of Heritagization and Cultural Tourism Development*, Procedia – Social and Beavioral Sciences, 188, 307-315.

Burkart A.J., Medlik S. (1974), *Tourism – Past, Present and Future*, Heinemann, London.

Cemali S. (2010), *Alternative Tourism and Touristic Product Variation Strategies: The Alakir (KUMLUCA) Valley*, International Symposium on Geography. Environment and Culture in the Mediterranean Region, http://web.deu.edu.tr/geomed2010/2007/Sari.pdf (accessed: 12.08.2021).

Chmielewska M., Lamparska M., Pytel S., Jurek K. (2016), *Patronage Housing Estates in Zagłębie. Tourist Route Project* (in Polish), Association for the Protection of Natural and Cultural Heritage "MOJE MIASTO", Będzin.

Crane A., Ruebottom T. (2011), *Stakeholder Theory and Social Identity: Rethinking Stakeholder Identification*, Journal of Business Ethics, 102(SUPPL.), 77-87.

Davies B. (2003), *The Role of Quantitative and Qualitative Research in Industrial Studies in Tourism*, International Journal of Tourism Research, 5(2), March/April, 97-111.

Domaszewski K. (2000), *From a Trip to Saturn* (in Polish), "Zeszyty Czeladzkie", No. 7, Czeladź, 8-9.

Felsenstein D., Fleischer A. (2003), *Local Festivals and Tourism Promotion: The Role of Public Assistance and Visitor Expenditure*, Journal of Travel Research, 41(4), 385-392.

Fuadillah N., Murwatiningsih M. (2018), *The Effect of Place Branding, Promotion and Tourism Product Attribute to Decision to Visit through the Destination Image*, Management Analysis Journal, 7(3), 328-339.

Ginevičius R., Podvezko V. (2005), *Generation of a Set of Evaluation Criteria*, Business: Theory and Practice, 6, 199-207.

Hammond J.S., Keeney R.L., Raiffa H. (2002), *Smart Choices: A Practical Guide to Making Better Decisions*, Broadway Books, New York.

Keane M.J. (1997), *Quality and Pricing in Tourism Destinations*, Annals of Tourism Research, 24(1), 117-130.

Kurek R. (2012), *The Beginnings and Development of Industry in Czeladź* (in Polish) [in:] J. Drabina (ed.), *History of Czeladź* (in Polish), 1, Czeladź.

Lazar S., Binek-Zajda A. (2015), *Piaski Housing Estate. History and Architecture* (in Polish), Czeladź.

Lee-Ross D., Pryce J. (2010), *Human Resources and Toursim. Skills, Culture and Industry*, Channel View Publications, Bristol-Buffalo-Toronto.

Logunova N., Kalinkina S., Lazitskaya N., Tregulova I. (2020), *Methods and Criteria for Assessing the Effectiveness of Cruise Tourism Development*, VIII International Scientific Conference Transport of Siberia, IOP Conf. Series: Materials Science and Engineering, 918.

Lohmann M. (2004), *New Demand Factors in Tourism, European Tourism Forum,* October 15, Budapest.

Mason P. (2016), *Tourism Impacts, Planning and Management*, third edition, Routledge, New York.

Medlik S., Middleton V.T.C. (1973), *Product Formulation in Tourism*, Tourism and Marketing, 13, AIEST, Berne, 173-201.

Morais D.C., Almeida A.T. (2012), *Group Decision Making on Water Resources Based on Analysis of Individual Rankings*, „Omega", 40, Oxford, 42-52.

Morais D.C., Costa A.P.C.S., Almeida A.T. (2014), *Group Decision Model for Outsourcing IT Services*, „Procedia Technology", 16, 562-568.

Muainuddin M.M.A.M., Hasan M.N. (2018), *Domestic Tourism Forecasting in Pahang: Comparison of Selected Techniques*, Global Business and Management Research: An International Journal, 10(3) (Special Issue), 930-937.

Nair M.B., Ramachandran S., Shuib A., Herman S., Nair V. (2012), *Multi-criteria Decision Making Approach for Responsible Tourism Management*, The Malaysian Forester, 75, 2, 135-145.

Nogueira S., Pinho J.C. (2015), *Stakeholder Integrated Analysis: The Specific Case of Rural Toursim in the Portuguese Peneda-Gerês National Park*, International Journal of Tourism Research, 17(4), July/August, 325-336.

Nurmi H. (1981), *Approaches to Collective Decision Making with Fuzzy Preference Relations*, Fuzzy Sets Syst., 6, 1981, 249-259.

Panasiuk A. (2017), *From Basic Tourism Products to a Comprehensive Offer of a Tourism Area*, Barometr Regionalny, 15(1), 17-24.

Panasiuk A. (2020), *Policy of Sustainable Development of Urban Tourism*, Pol. J. Sport Tourism, 27(2), 33-37.

Raiffa H., Richardson J., Metcalfe D. (2002), *Negotiation Analysis: The Science and Art of Collaborative Decision Making*, The Belknap Press of Harvard University Press, Cambridge-London.

Ramírez-Guerrero G., García-Onetti J., Chica-Ruiz J.A., Arcila-Garrido M. (2020), *Concrete as Heritage: The Social Perception from Heritage Criteria Perspective, The Eduardo Torroja's Work*, International Journal of Design & Nature and Ecodynamics, 15(6), 785-791.

Robson J., Robson I. (1996), *From Shareholders to Stakeholders: Critical Issues for Tourism Marketers*, Tourism Management, 7(17), November, 533-540.

Rohrscheidt A.M. von (2008), *Cultural Tourism – Concerning the Definition*, Turystyka Kulturowa, 1, 46-62.

Roszkowska E., Wachowicz T. (2015), *Application of Fuzzy TOPSIS to Scoring the Negotiation Offers in Ill-structured Negotiation Problems*, European Journal of Operational Research, 242, 3, 920-932.

Russo A.P., van der Borg J. (2002), *Planning Considerations for Cultural Tourism: A Case Study of Four European Cities*, Tourism Management, 23, 631-637.

Smith M.K., Pinke-Sziva I., Berezvai Z., Buczkowska-Gołąbek K. (2021), *The Changing Nature of the Cultural Tourist: Motivations, Profiles and Experiences of Cultural Tourists in Budapest*, Journal of Tourism and Cultural Change, 1-19.

Smith S.L.J. (1994), *The Toursim Product*, Annals of Tourism Research, 21(3), 582-595.

State Archives in Katowice – Mining and Industrial Society "Saturn".

State Archives in Katowice – Society of the Nameless Coal Mines "Czeladź" in Czeladź-Piaski.

Stefano N.M., Casarotto Filho N., Barichello R., Sohn A.P. (2015), *Hybrid Fuzzy Methodology for the Evaluation of Criteria and Sub-criteria of Product-service System (PSS)*, Procedia CIRP, 30, 439-444.

Stokes R. (2008), *Tourism Strategy Making: Insights to the Events Tourism Domain*, Tourism Management, 29(2), 252-262.

Szromek A.R., Herman K. (2019), *A Business Creation in Post-Industrial Tourism Objects: Case of the Industrial Monuments Route*, Sustainability, 11(5), 1-17.

Tresna P.W., Nirmalasari H. (2018), *Sustainable Competitive Advantage Strategies of Tourism Products in Pangandaran District*, Review of Integrative Business and Economics Research, 7, Supplementary Issue 3, 34-47.

Vucetic A. (2009), *Impact of Tourism Policy on Development of Selective Tourism*, https://ssrn.com/abstract=3579681 (accessed: 31.07.2021).

Weber F., Taufer B. (2016), *Assessing the Sustainability of Tourism Products – As Simple as It Gets*, International Journal of Sustainable Development and Planning, 11(3), 325-333.

Yu X., Xu H. (2019), *Cultural Heritage Elements in Tourism: A Tier Structure from a Tripartite Analytical Framework*, Journal of Destination Marketing & Management, 39-50.

(www 1) https://www.gstcouncil.org/gstc-criteria/ (accessed: 26.06.2021).