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Taicir Loukil Moalla**

A NEW MULTICRITERIA DECISION SUPPORT TOOL BASED ON FUZZY SWARA AND TOPSIS

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Abstract

The problem of selecting 3PL (Third Party Logistics) suppliers is a major issue in the management of the supply chain and the improvement of the production management of a manufacturing company. A 3PL supplier can be defined as a company that provides contract logistics services to a manufacturer, supplier or main user of a product or service. It is called a third party because the logistics provider does not own the products but participates in the supply chain between the manufacturer and the user of a given product. In actual cases, several decision-makers intervene in the selection of 3PL suppliers and each one has his own points of view and wishes to take into account criteria which are not generally the same for all the decision-makers. Furthermore, the criteria have different weights. In this study, we propose a method to solve this problem. It consists of a combination of the fuzzy Stepwise Weight Assessment Ratio Analysis (SWARA) method with the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). The objective is to optimize the decision-making process and have another, more dynamic model and satisfy the needs of the decision-maker. Fuzzy SWARA is one of the new methods being used for ranking evaluation criteria based on decision makers' expected degree of importance to determine the weights of evaluation criteria (Selçuk, 2019).

This method can be used to facilitate estimation of decision makers' preferences regarding the meaning of attributes in the weight determination process. TOPSIS is a multi-criteria method for identifying solutions from a finite set of alternatives (Behzadian et al., 2012). To the

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best of our knowledge, this combination has not been developed in the literature, especially in the third-party logistic problems. The proposed model will be implemented to solve a 3PL problem of a company selling steel products.

Keywords: multicriteria decision support, 3PL suppliers, Fuzzy SWARA, TOPSIS.

1 Introduction

In a strong competitive environment, companies are constantly looking to improve their competitiveness. This is based as much on the quality of the products or services as on the costs and deadlines of their delivery. It is essential to be able to quickly respond to changes in the market. This remarkable dynamic of the economic context seems to require an ever-greater capacity for adaptation and responsiveness from the actors of an organization. Indeed, the accelerated scalability of markets has a direct impact on the necessary responsiveness of companies. The company's adaptation and responsiveness depend on its ability to interact effectively with all stakeholders. It is therefore a matter of breaking down cultural, organizational, functional and technological barriers within companies (Zouggari and Benyoucef, 2011).

Today, decision-making processes interest manufacturers due to their importance for achieving the desired level of competitiveness and the overall goals of implementing process innovations (Garcia et al., 2020). Organizational studies and process analysis constantly show companies' needs for solutions to organize their activities around a workspace and improve their competitiveness in a strong competitive context (Batarlienė and Jarašūnienė, 2017). In this context, outsourcing part of the work of one or more services of a company leads to calling on an external intermediary who carries out the given 3PL activity. Many definitions have been proposed for 3PL. It is defined as a professional logistics company that makes a profit while taking over all or part of the logistics of a company's supply chain (Lambert and Cooper, 2000). The delegated function remains under the control of the company that owns the service, but the function is fulfilled by the partner who undertakes to carry out the work and to respect the principles of the master company. The chosen partner must be competent and trustworthy because he owns part of the commercial activity. This delegation can have significant repercussions on the functioning and image of the company. By outsourcing its logistics to a 3PL provider, a company can focus on its core business and competence, save transportation costs, and gain flexibility in many aspects, such as supply chain flexibility, operations flexibility in logistics management, and flexibility with warehouse space and labour (Yanhui, Hao and Ying, 2018).

Selection and assessment of 3PL suppliers remains the most critical activity in the supply chain due to its important role and ease of chain operation (Hosang, 2017). Decisions are complicated by the fact that different criteria should be taken into account in the decision-making process. This attracts many researchers; many approaches have been proposed in the literature.

In recent years, many researchers have focused on multiple-criteria decision--making (MCDM) models for making complex decisions under several criteria. In fact, this concepts is often used in cases where a specific problem involves several different attributes, including simultaneously, quantitative and qualitative ones, such as cost, degree of importance, capacity, and lifetime (Seyedkolaei and Seno, 2021).

Therefore, our objective is to present a theoretical study and a case study dealing with the selection and evaluation of a group of suppliers, based on several criteria such as Cost, Delivery, Availability, Service, etc.

The remainder of this paper is organized as follows: Section 2 presents a literature review of main research papers dealing with this problem and describes a comparative study of the main existing methods. Section 3 presents the proposed method for solving the 3PL supplier choice problem with a solution of a practical case. Section 4 contains conclusions.

2 Literature review

The problem of choosing 3PL suppliers is one of the strategic decisions that have a significant impact on the company's performance. With the evolutions of the manufacturing systems, this decision becomes more and more critical. Different decision support approaches have been proposed in the literature for the problem of choosing 3PL providers. We classify these approaches according to their techniques: Artificial Intelligence, Methods based on total cost, Mathematical programming models, Linear weighting models and Outranking methods.

Table 1 summarizes these main approaches with the advantages and disadvantages presented in some related papers. It presents a classification into four categories of the approaches used for decision making in the problem of selecting supplier groups. For each category, the different techniques used to solve the selection problem have been presented under a finite number of important criteria. The main advantages and disadvantages of each approach have been listed to recognize their strengths and weaknesses.

Table 1: Advantages and disadvantages of different methods

Methods	Avantages	Disadvantages	Authors
Linear weighting methods	− Quick and easy to use − Take into account subjective criteria − Inexpensive implementation	− Depend on human judgments − No possibility to introduce constraints in the model	Jain, Wadhwa and Deshmukh (2007); Kahraman, Ufuk and Ziya (2003); Bozdag, Kahraman and Ruan (2003); Mafakheri, Breton and Ghoniem (2011); Nilay and Demirel (2012); Devendra and Ravi (2014); Şengül et al. (2015)
Mathematical programming	− The criteria do not have a formal common dimension − Offers several solutions − Possibility to introduce constraints in the model	− Takes into account the difficulty related to subjective criteria − Does not offer an optimal solution − Difficult to analyse the results	Ghodsypour and O’Brien (2001); Kumar (2014); Karsak and Dursun (2014)
Cost-based methods	− Help identify the structure of all costs − Allow to negotiate cost values with suppliers − Very flexible	− Access to data on costs sometimes limited − Expression of certain costs in difficult monetary terms	Jellouli and Benabdallah (2021); Hyunjun et al. (2021)
Artificial intelligence	− Offers a flexible knowledge base − Takes into account qualitative factors	− Collecting knowledge on suppliers and accessing expertise is long and difficult	Lin (2009); Zhang et al. (2020)
Outranking methods	− The model can be based on both qualitative and quantitative information − The criteria are not fully compensatory	− A large number of technical parameters is required	Chen and Zeshui (2015); Molla, Giri and Biswas (2021)

Ghodsypour and o’Brien (2001) presented an approach based on mixed non-linear programming (mono- and multi-objective cases) to solve the problem of supplier choice. Their approach takes into account the limitations of the budgets of the different customers, logistics costs, prices, etc. A numerical example is presented to show the effectiveness of the approach. Kahraman, Ufuk and Ziya (2003) presented an approach based on the fuzzy AHP method for the problem of selecting the location of entities in a supply chain. Similarly, Bozdag, Kahraman and Ruan (2003) implemented fuzzy AHP to choose the best manufacturing system. Decision makers usually find it more convenient to express interval judgments than fixed value judgments, due to the fuzzy nature of the comparison process (Bozdag, Kahraman and Ruan, 2003). Kumar (2014) proposed an approach based on the GP (Goal Programming) method in a fuzzy environment. The authors seek to optimize three main criteria: minimize the overall cost, minimize the rejections of requests made and minimize the number of late deliveries. The set is subject to various constraints related to customer requests, supplier capabilities, budgets allocated to suppliers, etc.

Yan, Chaudhry and Chaudhry (2003) present an analysis of an effective approach to 3PL service provider evaluation, focusing on operational efficiency. An intelligent vendor report management system consisting of customer report management, vendor estimation and product coding systems to select vendors during the new product development process is proposed by Choy, Lee and Lo (2003). The authors note that the complexity of the problem is based on the number of criteria and sub-criteria used in an international dimension of the problem. Jain, Wadhwa and Deshmukh (2007) present a state of the art dedicated to the methods used to solve the problem of supplier choice. They list all the methods used and list the advantages and disadvantages of each. The authors propose a method based on “Association Rules Mining Algorithms” with fuzziness, to have more flexibility in the evaluation of suppliers and decision-making. They justify the choice of fuzzy logic by the nature of the decision-making information’s used, which has a qualitative and quantitative form. The authors use a database with certain information specific to each provider in relation to the selection criteria. On a numerical example, the authors show the effectiveness of the developed method and insist that rules can be exploited via a database to provide decision makers with a more flexible evaluation of potential suppliers. Tanonkou, Benyoucef and Xie (2007) deal with a stochastic distribution network design problem where 3PL provider selection, distribution center location and demand area assignment decisions are processed simultaneously. The goal is to solve a complex optimization problem that brings together three levels of decisions: (i) choice of locations of distribution centers, (ii) selection of suppliers to ensure supplies (in one type of product) and finally (iii) assignment of demand areas to distribution.

Jain and Benyoucef (2008) deal with a problem of selecting 3PL suppliers in textile industry. The problem is to choose a number of suppliers, the modes of transport to be used and the storage policy to be adopted by the single distribution center of the chain. They present a simulation-based optimization approach using multicriteria genetic algorithms to solve this problem. Lin (2009) proposed a method for selecting suppliers by considering the effects of interdependence among the selection criteria (price, quality, delivery and technique), as well as achieving optimal order allocation among the chosen suppliers. The proposed method incorporates, accordingly, two steps: (i) combination of Analytic Network Process (ANP) with fuzzy Preference Programming (PP) in a more powerful fuzzy ANP (FANP) to select suppliers, (ii) application of multipurpose linear programming (MOLP) to determine the order assignment among the chosen suppliers. Mafakheri, Breton and Ghoniem (2011) proposed a two-stage dynamic multi-criteria programming approach for the problem of supplier choice and order allocation. In the first phase, the AHP method is used to address the multicriteria decision for the ranking of suppliers. In the second step, the order allocation model is proposed. It aims to maximize a service function for the company as well as to minimize all the supply chain costs. Nilay and Demirel (2012) used another method of group choice: the VIKOR method to solve multiple criteria decision-making problems with contradictory and non-commensurable criteria. This method is used for the choice of insurance companies by investors in Turkey. It is applied to determine the best feasible solution according to the chosen criteria.

Devendra and Ravi (2014) proposed an integer linear programming model to simultaneously determine the timing of supply, lot size, suppliers, and carriers. They proposed a model based on the GP to solve a problem of multiple choice; indeed, the intention of the model is to determine the timings (moments), the size of batch to be procured and the supplier and the carrier to be selected in each replenishment period.

Karsak and Dursun (2014) proposed a group decision-making method based on DEA and QFD. This methodology identifies the characteristics that the purchased products should possess to meet the needs of the business and then it attempts to establish the relevant vendor’s evaluation criteria.

Kumar (2014) proposed a new model consisting of two complementary methods: AHP and FGP (Fuzzy GP) to provide support for identification and classification suppliers, based on the preferences of a group of decision makers. He proposed a hybridization of two methods to solve the problem. The first is based on fuzzy AHP with the method of geometric means to prioritize and aggregate the preferences of a group decision makers. In the second, the obtained priorities are integrated with GP (Goal Programming) for the discriminant analysis to provide solution.

Şengül et al. (2015) proposed a model based on the TOPSIS Soft method for the analysis a renewable energy supply systems in Turkey. Chen and Zeshui (2015) presented a new approach, called the HF-ELECTRE II approach, which combines the idea of HFS (Hesitant Fuzzy Sets) with the ELECTRE II method to effectively aggregate different opinions of group members.

Figure 1 presents the taxonomy of the selection of supplier problem and its related approaches. The supplier selection problem is summarized in the following diagram which describes the main sub-problems and the different methods used to solve them.

Abbreviations used

TOPSIS: Technique for Order of Preference by Similarity to Ideal Solution; **FST:** Fuzzy Set Theory; **AHP:** Analytic Hierarchy Process; **QFD:** Quality Function Deployment; **GP:** Goal Programming; **DEA:** Data Envelopment Analysis; **ABC:** Activity-Based Costing; **TCO:** Total Cost of Ownership; **ANN:** Artificial Neural Network; **CBR**: Case-Based Reasoning; **RBR:** Rule-Based Reasoning.

Description of the taxonomy of the decision problem presented in Figure 1.

Selection of suppliers, divided into two sub-problems: Number of Suppliers and Choice of Suppliers.

We start with the first sub-problem: Number of Suppliers, which has several criteria, such as Characteristics of the company and Strategic plan, and a basic objective, which is the choice of suppliers. The second sub-problem, choice of Suppliers, can be solved under several criteria, such as Cost, Delivery, Service and Quality, using several types of methods, such as Outranking methods (PROMETHEE, ELECTRE), Linear weighting models (TOPSIS, FST, AHP), Mathematical programming models (QFD, DEA, GP), Artificial Intelligence (ANN, CBR, RBR) and Methods based on total cost (ABC, TCO).

In the previous section, we described some existing approaches that describe the decision problem. In this section, we provide a comparison between the different methods used for solving decision problems.

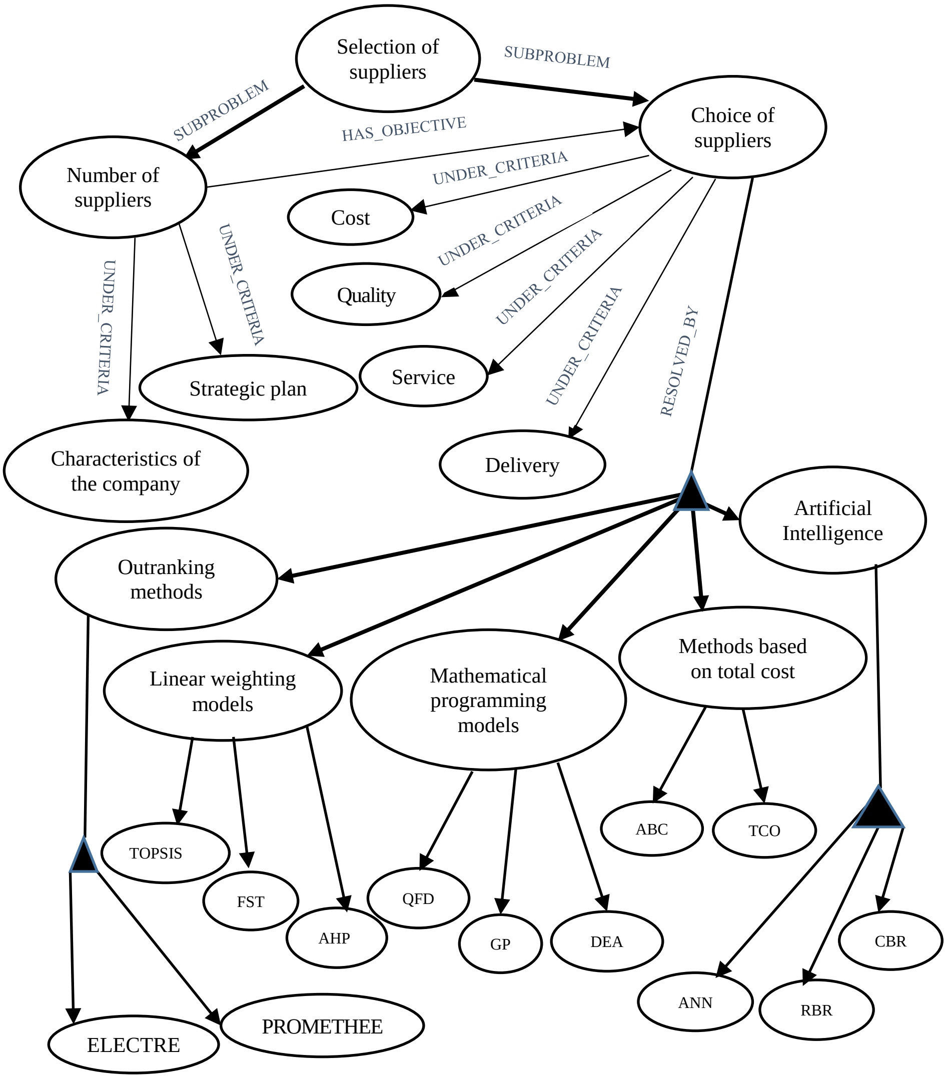

Figure 1: Selection of suppliers

3 The proposed model

We propose a new model called Fuzzy SWARA–TOPSIS. Figure 2 describes the problem. The decision-making process is described as follows: the selection of *m* suppliers (A1, A2, …, Am) taking into account the opinions of the decision-maker under several criteria (C1, C2, …, Cn).

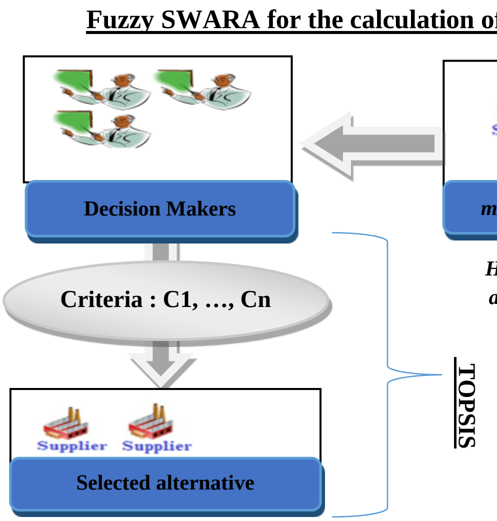

Figure 2: Description of the problem

We notice the complexity and difficulty of the analysis of the results obtained for the majority of the proposed methods. Our goal is to propose a more efficient decision support tool capable of solving the problem in a shorter time. For this reason, we propose the following model which consists of a hybrid method based on Fuzzy SWARA and TOPSIS to solve the decision problem.

3.1 Fuzzy SWARA

The SWARA method is one of the new methods used to evaluate criteria weights. Its main feature is its ability to estimate the preferences of decision makers regarding the meaning of attributes in the weight determination proces (Kersuliene, Zavadskas and Turskis, 2010).

The main reason for using Fuzzy SWARA is that it considers the expected importance of the assessment criteria identified by the experts. In addition, it is used to determine the weights of the criteria in a decision-making process in a fuzzy environment.

The steps of this method are as follows:

Step 1: Sort the evaluation criteria from maximum preference to minimum, considering the goal of decision making.

Step 2: The process is started from the second factor where the experts allocate a score between zero and one to the factor j in relation to the previous criterion $(j-1)$. This process is then applied to each factor. This ratio represents the comparative importance of $\hat{S}_j$ (Kersuliene, Zavadskas and Turskis, 2010). The values are shown in Table 3.

Step 3: Calculation of the values of the coefficient $\hat{e}_j$:

$$\hat{e}_j = \begin{cases} 1, j = 1 \\ \hat{S}_j + 1, \ j > 1 \end{cases} \tag{1}$$

Step 4: Recalculation of fuzzy weights $\hat{g}_j$:

$$\hat{g}_j = \begin{cases} 1, \ j = 1 \\ \frac{\hat{g}_{j-1}}{\hat{e}_j}, \ j > 1 \end{cases} \tag{2}$$

Step 5: Calculation of weight of fuzzy criteria $\hat{w}_j$:

$$\hat{w}_j = \frac{\hat{g}_j}{\sum_{k=1}^{n} \hat{g}_k} \tag{3}$$

where $w_j = (l, m, u)$ is the fuzzy relative importance weight of the *j*th criterion and *n* is the number of criteria.

These fuzzy weights are converted into crisp weights (w_j) by the following equation:

$$w_j = \frac{w_j^l + w_j^m + w_j^u}{3} \tag{4}$$

Moreover, let $A_1 = (l_1, m_1, u_1)$ and $B_1 = (l_2, m_2, u_2)$.

The basic arithmetic operations on triangular fuzzy numbers (TFNs) can be defined as follows:

$$A_1 + B_1 = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$$
$$A_1 - B_1 = (l_1 - l_2, m_1 - m_2, u_1 - u_2)$$
$$A_1 * B_1 = (l_1 * l_2, m_1 * m_2, u_1 * u_2)$$
$$A_1 / B_1 = (l_1 / u_2, m_1 / m_2, u_1 / l_2)$$

3.2 The TOPSIS method

TOPSIS is a method developed to classify solutions from a finite set of alternatives (Behzadian et al., 2012). The basic principle is that the best alternative should have the shortest distance from the positive ideal solution and the furthest distance from the negative ideal solution.

TOPSIS makes it possible to use the idea of a compromise solution to classify the alternatives. In addition, it helps the decision maker to establish the ranking order of the alternatives by deriving compromise indices based on the distances of the alternatives between the positive ideal solution and the negative ideal solution.

The TOPSIS method procedure can be expressed as a series of steps for *m* alternatives and *n* criteria.

Step 1: Construct the decision matrix and determine the criteria weights. The normalized decision matrix $B = (b_{ij})_{m\times n}$ is computed as follows:

$$b_{ij} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}}, \text{ for } i = 1,2,\dots,m\,; j = 1,2,\dots,n \quad (5)$$

Step 2: Calculate the normalized decision matrix. The weighted normalized decision matrix $C = (C_{ij})_{m\times n}$ is computed as follows:

$$C_{ij} = w_j * b_{ij} \;\; for\; i = 1,2,\dots,m\,; j = 1,2,\dots,n \quad (6)$$

where the weight vector of criteria is W = (w_1, w_2, …, w_n), with $\sum_{j=1}^{n} w_j = 1$.

Step 3: Determine the positive ideal solutions and negative ideal solutions:

$$P^{+} = c_1^{+}, c_2^{+}, \dots c_n^{+} = \{(Max_{j\in \mathrm{I}} c_{ij}).\,(Min_{j\in \mathrm{J}}\, c_{ij})\} \quad (7)$$

$$P^{-} = c_1^{-}, c_2^{-}, \dots c_n^{-} = \{(Min_{j\in \mathrm{I}}\, c_{ij}).\,(Max_{j\in \mathrm{J}}\, c_{ij})\} \quad (8)$$

where C^{+} is a benefit criterion, C^{-} is a cost criterion, I is the set of benefit criteria, and J is the set of cost criteria.

Step 4: Compute separation measures based on the *n*-dimensional Euclidean distance. The separation measure of the alternative A_i from P^{+} is computed as follows:

$$d_j^{+} = \sum_{j=1}^{n} \left| c_{ij} - c_j^{+} \right|, for\; i = 1,2,\dots,m \quad (9)$$

Similarly, the separation measure from $P^{-}$is computed as follows:

$$d_j^{-} = \sum_{j=1}^{n} \left| c_{ij} - c_j^{-} \right|, for\; i = 1,2,\dots,m \quad (10)$$

Step 5: Compute relative closeness coefficient to the ideal solutions. For an alternative A_i, the relative closeness coefficient is defined as follows:

$$R_i = \frac{d_i^{-}}{d_i^{-} + d_i^{+}} \quad (11)$$

Step 6: Rank the alternatives. The smaller the value of relative closeness coefficient, the better the rank of the alternative.

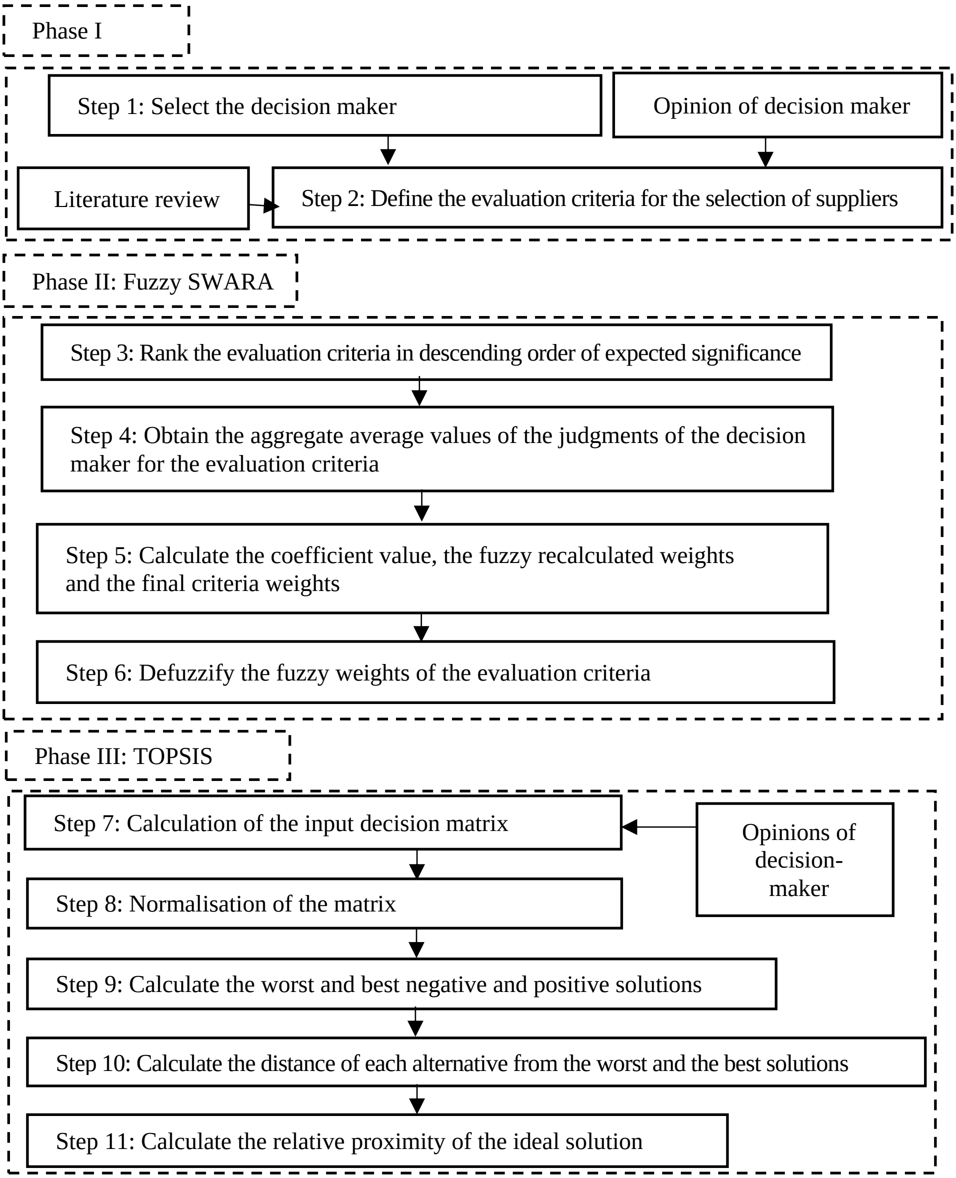

Figure 3: Algorithm of the proposed model

Figure 3 shows the details of the proposed model. First, in phase I, we identify the decision maker and obtain his opinion to define the criteria to be used in the selection of 3PL suppliers based on the literature. Then, in phase II, we apply the steps of Fuzzy SWARA to determine the criteria weights and finally, in phase III, we apply TOPSIS to select 3PL suppliers.

3.3 A case study: distribution of steel products

The new Fuzzy SWARA–TOPSIS model can be applied to a wide range of problems. The case study concerns the distribution of steel products located in Sousse, Tunisia. The company has been one of the main suppliers of steel products. It operates with a staff of approximately 100 people, employed in various divisions. The problem faced by this company is the choice of suppliers from among several.

The potential candidates are: SFAX METAL, SOQUIBAT, SOTIC, PROSID and EPPM.

Our objective is to rank and select suppliers by priority according to well--defined criteria.

The experts listed the criteria according to their expected level of importance.

Table 2: Criteria

Criteria	Designation	Maximize or minimize the value of the criterion (Max/Min)
Availability	AV	Max (AV)
Delivery	DE	Max (DE)
Quality	QU	Max (QU)
Service	SCE	Max (SCE)
Cost	C	Min (C)

From the second criterion, the (j − 1)th criterion is compared to the jth criterion using the values from Table 3. In this comparison, decision-maker use linguistic values expressing $\hat{S}_j$ which is the first step of Fuzzy SWARA. The decision maker prioritizes criteria according to their importance.

Table 3: Linguistic Values by Chang (1996)

Linguistic scale	Triangular Fuzzy Number (TFN)
Much Less Important	(0.222, 0.25, 0.286)
Less Less Important	(0.286, 0.333, 0.40)
Less Important	(0.4, 0.5, 0.667)
Moderately Important	(0.667, 1, 1.5)
Equally Important	(1, 1, 1)

The results of Fuzzy SWARA are shown in the table below.

Table 4: Fuzzy SWARA Results

Criteria	$\hat{S}_j$			$\hat{e}_j$			$\hat{g}_j$			$\hat{w}_j$			**w**
AV				1	1	1	1	1	1	0.38	0.833	0.882	0.422
C	0.667	1	1.5	1.667	2	2.5	0.4	0.5	0.6	0.152	0.417	0.529	0.221
DE	0.4	0.5	0.667	1.4	1.5	1.667	0.24	0.333	0.428	0.091	0.278	0.378	0.150
SCE	0.286	0.333	0.4	1.286	1.333	1.4	0.171	0.250	0.333	0.065	0.208	0.294	0.114
QU	0.222	0.25	0.286	1.222	1.25	1.286	0.133	0.2	0.273	0.051	0.167	0.241	0.092

The decision maker listed the criteria according to their importance level obtained. Then it assigned the $\hat{S}_j$ value to compare criteria (Step 2). Using equation 3, fuzzy weights ($\hat{w}_j$) are converted into crisp weights. The results of fuzzy SWARA are in Table 4. Next step of the proposed model is to use TOPSIS to make the final selection of suppliers by using the crisp weights.

In this paper, we consider TOPSIS to solve the decision problem. We gave a score from the interval [0, 10] for each supplier *i* compared to criterion *j*. The basic data for the decision are listed in Tables 5 and 6.

Table 5: Criteria weights

	AV	**C**	**DE**	**SCE**	**QU**
W	0.422	0.221	0.150	0.114	0.092

Table 6: Decision Matrix

	AV	**QU**	**SCE**	**DE**	**C**
SFAX METAL	8	6	6	6	8
SOQUIBAT	9	5	5	7	7
PROSID	7	6	6	7	8
SOTIC	9	5	6	6	8
EPPM	7	6	7	8	7

Table 7: Weighted Normalized decision matrix

	AV	**QU**	**SCE**	**DE**	**C**
SFAX METAL	0.186	0.106	0.066	0.044	0.043
SOQUIBAT	0.211	0.088	0.056	0.052	0.038
PROSID	0.165	0.106	0.066	0.052	0.043
SOTIC	0.211	0.088	0.066	0.044	0.043
EPPM	0.165	0.106	0.078	0.059	0.038

Table 8: Positive and Negative ideal solutions

P^+	0.211	0.106	0.078	0.059	0.038
P^-	0.165	0.088	0.056	0.044	0.043

The consecutive steps in the supplier selection problem are explained below.
Step 1: Normalize the alternatives (results in Table 7).
Step 2 + 3: Calculate the weighted normalized decision matrix. The positive and negative ideal solutions are given in Table 8.

Table 9: Separation measures

d^+	0.032	0.029	0.048	0.027	0.046
d^-	0.029	0.047	0.022	0.047	0.033

Table 10: Relative Closeness coefficients to the ideal solutions

SFAX METAL	SOQUIBAT	PROSID	SOTIC	EPPM
0.475	0.618	0.314	0.635	0.418

Step 4: Calculate the separation measures. The separations of each alternative from the positive and negative ideal solutions are given in Table 9.
Step 5: Calculate the relative closeness degrees. The results are given in Table 10.

Discussion

The methodology proposed for the classification of suppliers depends on the number of suppliers, decision makers and evaluation criteria. Our application consists in arranging and selecting suppliers of steel products in Tunisia on the basis of criteria (C, AV, …). Criteria weights were obtained by fuzzy SWARA. According to the results of this method, the most important criterion was AV, followed by C, DE, SCE, and QU. After this process, the selection of suppliers of STEEL Products were obtained by TOPSIS. The best supplier turned out to be “PROSID”, followed by EPPM, SFAX METAL, SOQUIBAT and SOTIC. To the best of our knowledge, a combination of Fuzzy SWARA and TOPSIS has not been developed and we didn’t find papers related to such a combination in the literature. This research fills this gap. In this study, the proposed model will be used for the first time.

4 Conclusions

In this paper, we presented a literature review on 3PL supplier selection problem and the different methods used to solve it. We proposed a new approach based on Fuzzy SWARA and TOPSIS methods. Within this approach, the ratings of suppliers with respect to each criterion are expressed with linguistic variables. Fuzzy SWARA is used for the calculation of criteria weights and TOPSIS for the classification of suppliers. The advantages of the proposed model are as follows: (1) it considers the relationship among various criteria and fuzzy situation for ranking suppliers; (2) it minimizes the end customer's level of dissatisfaction using demand and capacity limiting. Future studies may like to include such practices in the selection criteria to further enhance the accuracy of supplier selection and may consider fuzzy data in the evaluation process, for example. Fuzzy TOPSIS and our method can be developed as a group decision making problems.

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MULTIPLE CRITERIA DECISION MAKING

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AN EXTENSION OF THE CODAS METHOD BASED ON INTERVAL ROUGH NUMBERS FOR MULTI-CRITERIA GROUP DECISION MAKING

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Abstract

This study aims to develop a new Interval Rough COmbinative Distance-based Assessment (IR CODAS) method for handling multiple criteria group decision making problems using linguistic terms. A single decision maker is unable to express his opinions or preferences on multiple criteria decisions, while a Multi-Criteria Group Decision Making MCGDM process ensures successful outcomes when handling greater imprecision and vagueness information. A real-life case study of risk assessment is investigated using our proposed IR-CODAS method to test and validate its application; a sensitivity analysis is also performed.

Keywords: Interval Rough Numbers, group decision making, IR-CODAS method, risk assessment.

1 Introduction

The decision making process is characterized by uncertainty and subjectivity; decision makers (DMs) are often faced with a dilemma while assigning a decision to certain criteria and they evaluate the alternatives in different uncertain decision making situations. Indeed, uncertainties are generally handled using the application of Rough Set Theory (RST), especially Interval Rough Numbers IRNs.

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RST has been successfully applied in a good number of MCDM studies. For instance, Song and Cao (2017) presented a rough approach based on DEMATEL to assess the interaction between requirements of Product-Service System (PSS). A rough Technique for Ordering Preference by Similarity to Ideal Solution (TOPSIS) approach is also proposed by Song et al. (2014) to improve the effectiveness of failure mode and effect analysis technique.

Some researchers have studied IRNs. For instance, Lu, Huang and He (2011) developed a fuzzy linear programming method, based on rough intervals, to generate simultaneous water allocation strategies in agricultural irrigation systems. In turn, to solve the multi-objective hub location and hub network design problem, Niakan, Vahdani and Mohammadi (2015) used a hybrid solution, based on inexact programming, interval-valued fuzzy programming and rough interval programming.

Regarding the hybridization of extensions of rough sets, a number of approaches have been proposed, such as the hybrid DEMATEL-ANP-MAIRCA model where Pamucar et al. (2017) developed a new approach for dealing with uncertainty based on IRNs. In addition, Pamucar, Petrovic and Cirovic (2018) modified the BWM (Best-Worst Method) and MABAC (Multi-Attributive Border Approximation area Comparison) methods by integrating fuzzy rough numbers per interval. To process the uncertainty contained in group decision making, Pamucar, Edmundas and Zavadskas (2018) integrated IRNs within the MABAC and AHP methods for rating university web pages. Also, the Normalized Weighted Geometric Bonferroni Mean (NWGBM) operator of the IRNs is used by Pamucar, Božanić et al. (2018) and is applied to the DEMATEL and COPRAS model to solve the problem of selecting an optimal direction for making a temporary military route. Moreover, Pamucar, Chatterjee and Zavadskas (2019) integrated IRNs into the Best Worst Method (BWM) and Weighted Aggregated Sum Product Assessment (WASPAS) method along Multi--Attributive Border Approximation area Comparison (MABAC) to evaluate third-party logistics (3PL) providers. As the Internet of Things (IoT) technology has rapidly developed, Kao, Nawata and Huang (2019) proposed a novel Hybrid method BR-DEMATEL that integrates Bayesian theory, interval rough number, and DEMATEL for Systemic Factor Evaluation-based Technological Innovation System (TIS) for the Sustainability of IoT in the Manufacturing Industry. We can see that many researchers have studied the combination of interval rough theory and Multi-Criteria Decision Making (MCDM) methods for different decision making problems which shows the importance of using interval rough MCDM approaches.

The MCDM tackles four types of problems: ranking, sorting, choice and description. In recent years, a new ranking MCDM method has been proposed, namely COmbinative Distance-based Assessment (CODAS), developed by Keshavarz Ghorabaee et al. (2016). The ranking of alternatives is determined using two measures: The main and primary measure uses the Euclidean distance of alternatives from the Negative Ideal Solution, while the secondary measure is the Taxicab distance

Lately, CODAS has been successfully applied in Group Decision Making (GDM) in various fields. For instance, Keshavarz Ghorabaee et al. (2017) solved group decision problems using a combination of trapezoidal fuzzy numbers and the CODAS method for market segment evaluation. Moreover, Yeni and Özçelik (2019) presented the Interval-Valued Atanassov Intuitionistic Fuzzy CODAS (IVAIF-CODAS) method and applied it to a personnel selection problem. To handle uncertainty, Pamucar et al. (2018) employed integrated MCDM framework using Linguistic Neutrosophic Numbers (LNN) and the CODAS method to select the optimal Power-Generation Technology (PGT). Furthermore, Roy et al. (2019) presented an extension of the CODAS approach using Interval--Valued Intuitionistic Fuzzy Sets (IVIFS) to select the best sustainable material for the automotive instrument panel. Based on 2-tuple Linguistic Pythagorean Fuzzy Sets (2TLPFSs), He et al. (2020) developed a novel CODAS model. Remadi and Frikha (2020) developed new methodologies in group decision making where triangular intuitionistic fuzzy numbers (TIFNs) are integrated into the CODAS method to solve the green supplier selection problem. In turn, Wang et al. (2020) presented the 2-tuple linguistic neutrosophic CODAS model. CODAS has also been expanded by Lan et al. (2021) to solve multiple attribute group decision making (MAGDM) issues with Interval-valued bipolar uncertain linguistic numbers (IVBULNs) on the basis of two kinds of distance measures and aggregating operators for risk assessment of mergers and acquisitions of Chinese enterprises.

Furthermore, in real-life problems, complex decision making situations with multiple and often conflicting objectives occur. In addition, the CODAS method is a new evaluation tool and has been proved to be efficient in dealing with MCDM problems. It has a systematic and simple computation procedure. Moreover, it can be assumed that a single decision maker is unable to express their opinions or preferences regarding multiple criteria decisions. On the other hand, in many situations, the DMs are unable to provide precise values and their information is vague and cannot be evaluated exactly in numerical values. This implies that Multi-Criteria Group Decision Making can be beneficial for selecting the optimal solution. Indeed, due to a greater imprecision and vagueness of Group Decision Makers information, we suggest integrating rough set theory into CODAS. As mentioned above, a DMs' information cannot be evaluated exactly in numerical values for risk evaluation are usually uncertain, we choose to treat subjectivity and uncertainty in a group MCDM process through IRNs. We can see that although there exist papers that use IRNs in ranking methods and the aggregation operators, there has been no study on developing the CODAS method to solve multicriteria group decision making problems with IRNs. Therefore, in this paper we will approach Multi-Criteria Group Decision Making (MCGDM) problems to expand the CODAS method within Interval Rough Numbers to deal with imprecision and to develop a novel MCGDM method.

The structure of the rest of this paper is organized as follows. In Section 2, a general overview of the rough set approach as well as some fundamental concepts of Interval Rough Numbers will be presented. In Section 3, we will describe the proposed method based on IR-CODAS. In Section 4, the suggested approach will be applied to a case study of risk evaluation and a sensitivity analysis of the proposed IR-CODAS method will be performed. Finally, conclusions and suggestions will be presented.

2 Preliminaries

2.1 Rough set theory

RST is a mathematical formalism proposed in 1982 by Zdzisław Pawlak to support decision making processes. It generalizes classical set theory. A rough set is an important mathematical tool for dealing with imprecise, inconsistent and incomplete information and knowledge. This concept was introduced by Pawlak (1982).

The basic notions of RST are as follows: Indistinguishable relation on the set of actions (the objects of the decision), lower and upper approximation of a subset or of a partition of U, dependence and reduction of attributes from the set of attributes and decision rules identified with the decision classes.

For algorithmic reasons, the information about the objects is provided in the form of a data table, composed of a set of actions (alternatives) A (in rows) described by a set of attributes (criteria) R (in column). Each cell in this table indicates an assessment (quantitative or qualitative) of the object in that row using the attribute of the corresponding column. Formally, the data table can be defined by an information system S expressed by the 4-tuple $S = \{U, R, V, f\}$, $R = C \cup D$, where U is a finite non-empty set of objects (called the universe), R is a finite nonempty set of attributes, the subsets C and D are called condition attribute set and decision attribute set, respectively. V = $\bigcup_{a\in R} V_a$ where V_a is the set of values of attribute a and card(V_a) > 1, and $f: R \rightarrow V$ is an information or a description (Zhang, Xie and Wang, 2016).

Definition 1: Indiscernible relation (Zhang, Xie and Wang, 2016)

Indiscernibility arises when it is not possible to distinguish between elements of the same set. Given a subset of the attribute set B ⊆ R, an indiscernible relation ind(B) on the universe U can be defined as follows:

$$ind(B) = \{(x, y)|\ (x, y) \in U^2, \forall_{b\in B}(b(x) = b(y))\} \tag{1}$$

Definition 2: Upper and lower approximation sets (Zhang, Xie and Wang, 2016)

Given an information system S = ⟨U; R; V; f⟩, for a subset X ⊆ U, its lower and upper approximation sets are defined, respectively, by:

$$\overline{apr}(X) = \bigcup_{E_i\cap A\neq\emptyset} E_i = \{x \in U|[x] \cap X\} \neq \emptyset \tag{2}$$

$$\underline{apr}(X) = \bigcup_{E_i\subseteq A} E_i = \{x \in U|[x] \subseteq X\} \tag{3}$$

where $[x]$ denotes the equivalence class of x. The upper approximation $\overline{apr}(X)$ is the union of all elementary sets which have a nonempty intersection with A, while the lower approximation $\underline{apr}(X)$ is the union of all elementary sets which are subsets of A. In other words, the lower approximation contains the objects definitively belonging to the set, while the upper approximation contains the objects that can belong to the set. In fact, $\overline{apr}(X)$ is the largest compound set containing X, while $\underline{apr}(X)$ is the least compound set containing X.

For all the subsets X, $Y \subseteq$ U, the upper and lower approximations $\overline{apr}(X)$ and $\underline{\mathrm{apr}}(X)$ satisfy the following properties (Pawlak, 1982):

(P1) $\underline{apr}(X) \subseteq X \subseteq \overline{apr}(X)$,

(P2) $\underline{apr}(\emptyset) = \overline{apr}(\emptyset) = \emptyset$,

(P3) $\underline{apr}(U) = \overline{apr}(U) = U$,

(P4) $\underline{apr}(X \cap Y) = \underline{apr}(X) \cap \underline{apr}(Y)$,

(P5) $\overline{apr}(X \cap Y) \subseteq \overline{apr}(X) \cap \overline{apr}(Y)$,

(P6) $\underline{apr}(X \cup Y) \supseteq \underline{apr}(X) \cup \underline{apr}(Y)$,

(P7) $\overline{apr}(X \cup Y) = \overline{apr}(X) \cup \overline{apr}(Y)$,

(P8) $\underline{apr}(X) = (\overline{apr}(X^c))^c$; $\overline{apr}(X) = (\underline{apr}(X^c))^c$,

(P9) $\underline{apr}(X) = \underline{apr}(\underline{apr}(X)) = \overline{apr}(\underline{apr}(X))$,

(P10) $\overline{apr}(X) = \overline{apr}(\overline{apr}(X)), = \underline{apr}(\overline{apr}(X))$,

where X^c = U – X denotes the complement of A.

The property (P1) says that the two operators determine a range in which the given set falls. The properties (P2) and (P3) are the conditions that the operators must satisfy at the two extreme points: Ø, or the minimum element and U, or the maximum element. The properties (P4)-(P7) describe weak distributivity and distributivity of the operators $\overline{apr}$ and $\underline{apr}$. The property (P8) states that the operator pair is double. Properties (P9) and (P10) state that the result of a double application of the new operators is identical to that of a single application. It is important to note that these properties are not independent.

The universe can be divided into three disjoint regions: the positive $POS(X)$, the bounded $BRN(X)$ and the negative $NEG(X)$ regions of X which are constructed from the equivalence classes:

$$POS(X) = \underline{apr}(X) \tag{4}$$

$$BRN(X) = \overline{apr}(X) - \underline{apr}(X) \tag{5}$$

$$NEG(X) = U - \overline{apr}(X) \tag{6}$$

If $x \in POS(X)$, then x belongs to the target set X.

If $x \in BRN(X)$, then x does not belong to the target set X.

If $x \in NEG(X)$, it cannot be determined whether x belongs to the target set X or not.

Definition 3: Definable sets (Zhang et al., 2016)

The empty set and the union of elementary sets are called compound or definable sets. Given an information system S = {U, R, V, f}, for any target subset $X \subseteq U$ and attribute subset $B \subseteq R$, if and only if $\underline{apr}(X) = \overline{apr}(X)$ (i.e. the bounded region BRN(X) = $\emptyset$), then X is called a definable set with respect to B.

Definition 4: Rough Sets (Zhang, Xie and Wang, 2016)

Given an information system S = {U, R, V, f}, for any target subset $X \subseteq U$ and attribute subset $B \subseteq R$, if and only if $\overline{apr}(X) \neq \underline{apr}(X)$ (i.e. the bounded region BRN(X) ≠ $\emptyset$), then X is called a rough set with respect to B, defined by $\left[\underline{apr}(X), \overline{apr}(X)\right]$.

2.2 Interval rough numbers

Suppose we have: a set of k classes representing the preferences of the decision maker DM, $P = (J_1, J_2, ..., J_k)$, which satisfies the condition $J_1 < J_2 <, \ldots, < J_k$ and another set of z classes that also represent the DM's preferences defined in the universe U, $P^* = (I_1, I_2, \ldots, I_z)$. Suppose that all the objects recorded in an information table are defined in U and are linked to the preferences of the DM. In P*, each class of objects is represented by an interval $I_j = \{I_{ij}, I_{sj}\}$, provided that $I_{ij} \leq I_{sj}$ $(1 \leq j \leq m)$ and $I_{ij} \leq I_{sj} \in P$, such that I_{ij} is the lower interval bound, while I_{sj} is the upper interval bound of the j^{th} object class. Suppose U is the universe and let Y be an arbitrary element of U. If the upper and lower bounds of the object class are sorted, so that $I_{i1}^* < I_{i2}^* <\ldots < I_{ih}^*$ and $I_{s1}^* < I_{s2}^* <\ldots < I_{sk}^*$ $(1 \leq h, k \leq m)$, then two new sets containing the lower object class $P_i^* = (I_{i1}^*, I_{i2}^*, \ldots, I_{ih}^*)$ and the upper objects class $P_s^* = (I_{s1}^*, I_{s2}^*, \ldots, I_{sk}^*)$ are defined. Then, for any class of objects $I_{ij}^* \in P$ with $(1 \leq j \leq h)$ and $I_{sj}^* \in P$ with $(1 \leq j \leq k)$, the lower approximations of I_{ij}^* and I_{sj}^* are defined as follows (Wang and Tang; 2011):

$$\underline{Apr}(I_{ij}^*) = \cup\, Y \in U/P_i^*(Y) \leq I_{ij}^* \tag{7}$$

$$\underline{Apr}(I_{sj}^*) = \bigcup Y \in U/P_s^*(Y) \leq I_{sj}^* \tag{8}$$

The upper approximations of I_{ij}^* and I_{sj}^* are defined by the following equations:

$$\overline{Apr}(I_{ij}^*) = \bigcup Y \in U/P_i^*(Y) \leq I_{ij}^* \tag{9}$$

$$\overline{Apr}(I_{sj}^*) = \bigcup Y \in U/P_s^*(Y) \leq I_{sj}^* \tag{10}$$

So both the lower class I_{ij}^* and the upper class I_{sj}^* are defined by their lower limits $\underline{Lim}(I_{ij}^*)$ and $\underline{Lim}(I_{sj}^*)$ and their upper limits $\overline{Lim}(I_{ij}^*)$ and $\overline{Lim}(I_{sj}^*)$:

$$\underline{Lim}(I_{ij}^*) = \frac{1}{M_I} \sum P_i^*(Y) | Y \in \underline{Apr}(I_{ij}^*) \tag{11}$$

$$\underline{Lim}(I_{sj}^*) = \frac{1}{M_I^*} \sum P_s^*(Y) | Y \in \underline{Apr}(I_{sj}^*) \tag{12}$$

where M_I and M_I^* are the sum of the objects in the lower approximation of the object classes I_{ij}^* and I_{sj}^*, respectively. The upper limits $\overline{Lim}(I_{ij}^*)$ and $\overline{Lim}(I_{sj}^*)$ are defined by:

$$\overline{Lim}(I_{ij}^*) = \frac{1}{M_S} \sum P_i^*(Y) | Y \in \overline{Apr}(I_{ij}^*) \tag{13}$$

$$\overline{Lim}(I_{sj}^*) = \frac{1}{M_S^*} \sum P_s^*(Y) | Y \in \overline{Apr}(I_{sj}^*) \tag{14}$$

where M_S et M_S^* are the sum of the objects in the upper approximation of the object classes I_{ij}^* and I_{sj}^*, respectively.

For the lower class, the rough boundary interval of I_{ij}^* is an interval between its lower and upper limits, denoted by $BR(I_{ij}^*)$, while for the upper class, the rough boundary interval of I_{sj}^* is $BR(I_{sj}^*)$:

$$BR(I_{ij}^*) = \overline{Lim}(I_{ij}^*) - \underline{Lim}(I_{ij}^*) \tag{15}$$

$$BR(I_{sj}^*) = \overline{Lim}(I_{sj}^*) - \underline{Lim}(I_{sj}^*) \tag{16}$$

Then, the uncertain class of objects I_{ij}^* and I_{sj}^* can be defined using their lower and upper limits:

$$R(I_{ij}^*) = [\underline{Lim}(I_{ij}^*), \overline{Lim}(I_{ij}^*)] \tag{17}$$

$$R(I_{sj}^*) = [\underline{Lim}(I_{sj}^*), \overline{Lim}(I_{sj}^*)] \tag{18}$$

As we can see, each class of objects is defined by its lower and upper limits that represent the interval rough number, which is defined as:

$$IR(I_j^*) = [R(I_{ij}^*), R(I_{sj}^*)] \tag{19}$$

Definition 5: The distance between two IRNs (Wang et al., 2016):

Let $A_1 = ([a_1,b_1)[c_1,d_1])$ and $A_2 = ([a_2,b_2)[c_2,d_2])$ be two IRNs. The distance between them can be defined as:

$$d(A_1, A_2) = \frac{|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2|}{4} \tag{20}$$

This satisfies the properties of distance measures, which are: $d(A_1, A_2) \geq 0$ and $d(A_1, A_2) = d(A_2, A_1)$.

Definition 6: Arithmetic Operations of IRNs (Wang et al., 2016):

Let $A_1 = ([a_1,b_1)[c_1,d_1])$ and $A_2 = ([a_2,b_2)[c_2,d_2])$ be two IRNs. We define:

$$A_1 + A_2 = ([a_1, b_1)[c_1, d_1]) + ([a_2, b_2)[c_2, d_2]) = \\ = ([a_1 + a_2, b_1 + b_2][c_1 + c_2, d_1 + d_2]) \tag{21}$$

$$A_1 - A_2 = ([a_1, b_1)[c_1, d_1]) - ([a_2, b_2)[c_2, d_2]) = \\ = ([a_1 - a_2, b_1 - b_2][c_1 - c_2, d_1 - d_2]) \tag{22}$$

$$A_1 \times A_2 = ([a_1, b_1)[c_1, d_1]) \times ([a_2, b_2)[c_2, d_2]) = \\ = ([a_1 \times a_2, b_1 \times b_2][c_1 \times c_2, d_1 \times d_2]) \tag{23}$$

$$\frac{A_1}{A_2} = \frac{([a_1, b_1)[c_1, d_1])}{([a_2, b_2)[c_2, d_2])} = \left(\left[\frac{a_1}{a_2}, \frac{b_1}{b_2}\right]\left[\frac{c_1}{c_2}, \frac{d_1}{d_2}\right]\right) \tag{24}$$

$$k \times A_1 = k \times ([a_1, b_1)[c_1, d_1]) = \\ = \begin{cases} ([k \times a_1, k \times b_1)[k \times c_1, k \times d_1]) & if\ k > 0 \\ ([k \times b_1, k \times a_1)[k \times d_1, k \times c_1]) & if\ k < 0 \end{cases} \tag{25}$$

3 The IR-CODAS method

IR-CODAS is our proposed approach integrating IRNs into the CODAS multicriteria method. It allows modeling imprecision and fuzziness of the information provided.

As presented in Figure 1, IR-CODAS consists of the following steps:

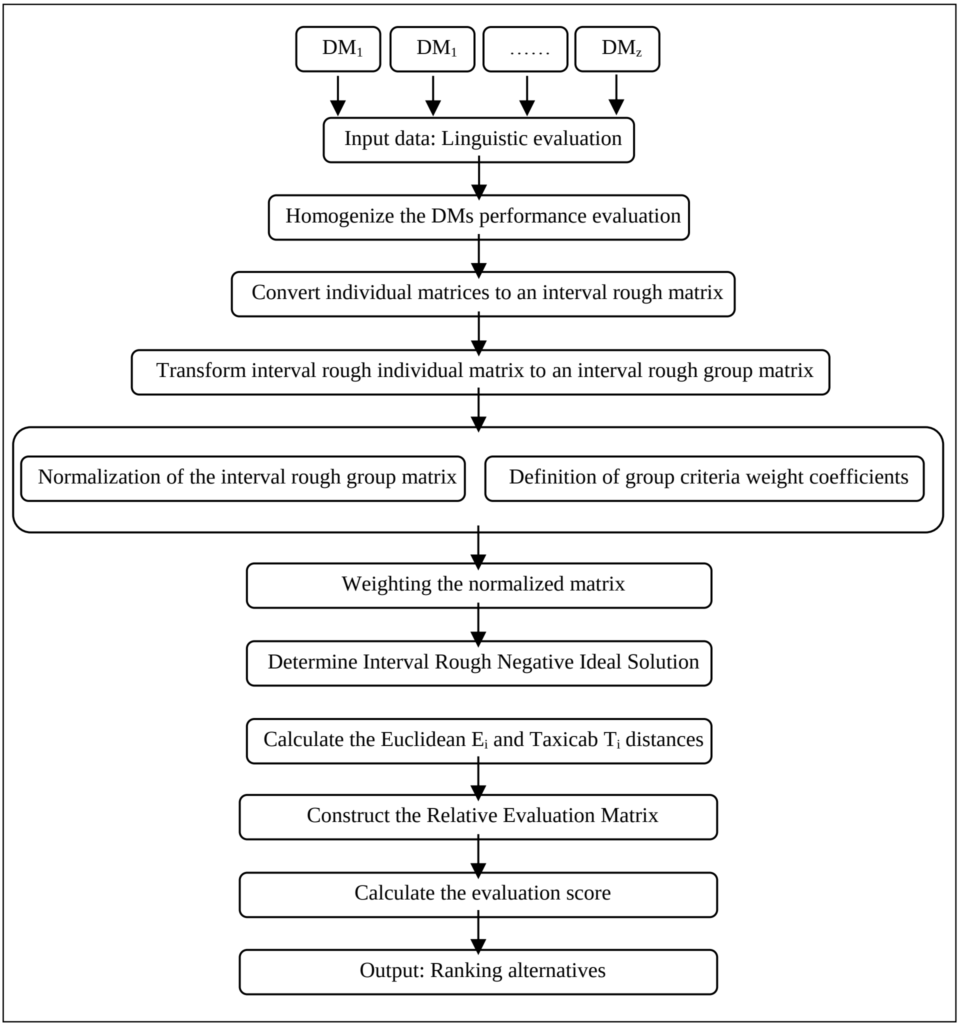

Figure 1: The structure of the proposed IR-CODAS method for an MCGDM problem

Step 1: Define a multi-criteria decision making model that consists of m alternatives $A_i (i = 1, 2, \ldots, m)$, n criteria C_j $(j = 1, 2, \ldots, n)$ and a team of k DMs, who evaluate alternatives according to all criteria. Every p^{th} DM presents his evaluation in the following matrix:

$$X^k = \left[x_{ij}^p; x_{ij}^{p*}\right]_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \ldots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} x_{11}^p; x_{11}^{p*} & x_{12}^p; x_{12}^{p*} & \ldots & x_{1n}^p; x_{1n}^{p*} \\ x_{21}^p; x_{21}^{p*} & x_{22}^p; x_{22}^{p*} & \ldots & x_{2n}^p; x_{2n}^{p*} \\ \ldots & \ldots & \ldots & \ldots \\ x_{m1}^p; x_{m1}^{p*} & x_{m2}^p; x_{m2}^{p*} & \ldots & x_{mn}^p; x_{mn}^{p*} \end{bmatrix}_{m\times n} \end{array} \tag{26}$$

where x_{ij}^p and x_{ij}^{p*} are the linguistic variables of the p^{th} DM ($p \in \{1, 2, \ldots, z\}$) for the i^{th} alternative ($i \in \{1, 2, \ldots, m\}$) according to j^{th} criterion ($j \in \{1, 2, \ldots, n\}$). Thus, matrices $X^1, X^2, \ldots\ldots, X^p, \ldots, X^k$ are obtained using performance rating for m alternatives on n criteria provided by p DMs.

Step 2: Homogenize the performance evaluations of the DMs. For each DM, matrix X^k is determined by DMs' evaluations and qualitative criterion evaluates alternatives using the following linguistic expressions provided by the group of DMs, taking into account the type of criteria (benefit or cost). As in Stevic et al. (2017), we use linguistic terms where the value of each pair x_{ij}^p is converted to an integer, as shown in Table 1.

Table 1: Linguistic scale for evaluating the alternatives

Linguistic terms	Benefit Criteria (Max)	Cost Criteria (Min)
Very Poor (VP)	1	9
Poor (P)	3	7
Medium (M)	5	5
Good (G)	7	3
Very Good (VG)	9	1

Step 3: Using equations (1-12) we convert the individual matrices to an interval rough matrix $Z^p = \left[IR\left(x_{ij}^p\right)\right]_{m\times n}$ $\forall$ $p = 1, \ldots, z$:

$$Z^p = \begin{array}{c} \\ A_1 \\ A_2 \\ \ldots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} IR\left(x_{11}^p\right) & IR\left(x_{12}^p\right) & \ldots & IR\left(x_{1n}^p\right) \\ IR\left(x_{21}^p\right) & IR\left(x_{22}^p\right) & \ldots & IR\left(x_{2n}^p\right) \\ \ldots & \ldots & \ldots & \ldots \\ IR\left(x_{m1}^p\right) & IR\left(x_{m2}^p\right) & \ldots & IR\left(x_{mn}^p\right) \end{bmatrix}_{m\times n} \end{array} \tag{27}$$

Step 4: Transform the individual interval rough matrix Z^p to a group interval rough matrix $Z = \left[IRG\left(x_{ij}\right)\right]_{m\times n}$ $\forall\, i = 1, \ldots, m$ and $\forall\, j = 1, \ldots, n$:

$$IRG(x_{ij}) = \frac{1}{z}\sum_{p=1}^{k} IR(x_{ij}^{p}) \tag{28}$$

where z is the total number of DMs.

Step 5: Normalize the elements of the group interval rough matrix Z using equation (29):

$$IR(t_{ij}) = \left(\left[t_{ij}^{i}, t_{ij}^{s}\right], \left[\, t_{ij}^{\prime i}, t_{ij}^{\prime s}\right]\right) = \\ = \begin{cases} \left(\left[\dfrac{x_{ij}^{i}}{\max\limits_{i} x_{ij}^{\prime s}}, \dfrac{x_{ij}^{s}}{\max\limits_{i} x_{ij}^{\prime i}}\right], \left[\dfrac{x_{ij}^{\prime i}}{\max\limits_{i} x_{ij}^{s}}, \dfrac{x_{ij}^{\prime s}}{\max\limits_{i} x_{ij}^{i}}\right]\right) & if \;\; \mathrm{j} \in N_b \\ \left(\left[\dfrac{\min\limits_{i} x_{ij}^{i}}{x_{ij}^{\prime s}}, \dfrac{\min\limits_{i} x_{ij}^{s}}{x_{ij}^{\prime i}}\right], \left[\dfrac{\min\limits_{i} x_{ij}^{\prime i}}{x_{ij}^{s}}, \dfrac{\min\limits_{i} x_{ij}^{\prime s}}{x_{ij}^{i}}\right]\right) & if \;\; \mathrm{j} \in N_c \end{cases} \tag{29}$$

where N_b and N_c are the sets of profit and cost criteria, respectively. In addition, $\min_i x_{ij}$ and $\max_i x_{ij}$ are the minimum and maximum values of the bounded approximate interval of the criteria, respectively.

The elements $IR\left(t_{ij}\right)$ of the normalized matrix (N) are:

$$N = \left[IR(t_{ij})\right]_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \ldots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} IR(t_{11}) & IR(t_{12}) & \ldots & IR(t_{1n}) \\ IR(t_{21}) & IR(t_{22}) & \ldots & IR(t_{2n}) \\ \ldots & \ldots & \ldots & \ldots \\ IR(t_{m1}) & IR(t_{m2}) & \ldots & IR(t_{mn}) \end{bmatrix}_{m\times n} \end{array} \tag{30}$$

Step 6: Definition of group criteria weight coefficients:

$$w_j = \frac{1}{z}\sum_{p=1}^{k} w_j^{p} \tag{31}$$

where w_j^{p} is the importance of j^{th} criterion ($j \in \{1, 2, \ldots, n\}$) provided by the p^{th} DM ($p \in \{1, 2, \ldots, z\}$).

Step 7: Weighting the previous normalized group interval rough matrix R by multiplying the obtained matrix with weighted values of the criteria:

$$IR(r_{ij}) = w_j \times IR\left(t_{ij}\right) = \left(\left[r_{\mathrm{ij}}^{i}, r_{\mathrm{ij}}^{s}\right], \left[r_{ij}^{\prime i}, r_{\mathrm{ij}}^{\prime s}\right]\right)_{m\times n} = \\ = \left[w_j t_{ij}^{i}, w_j t_{ij}^{s}\right], \left[w_j\, t_{ij}^{\prime i}, w_j t_{ij}^{\prime s}\right] \tag{32}$$

where w_j is the importance of j^{th} criterion.

We obtain the following weighted normalized group interval rough matrix:

$$R = \begin{bmatrix} ([r_{11}^i, r_{11}^s],[r_{11}'^i, r_{11}'^s]) & ([r_{12}^i, r_{12}^s],[r_{12}'^i, r_{12}'^s]) & \dots & ([r_{1n}^i, r_{1n}^s],[r_{1n}'^i, r_{1n}'^s]) \\ ([r_{21}^i, r_{21}^s],[r_{21}'^i, r_{21}'^s]) & ([r_{22}^i, r_{22}^s],[r_{22}'^i, r_{22}'^s]) & \dots & ([r_{2n}^i, r_{2n}^s],[r_{2n}'^i, r_{2n}'^s]) \\ \dots & \dots & \dots & \dots \\ ([r_{m1}^i, r_{m1}^s],[r_{m1}'^i, r_{m1}'^s]) & ([r_{m2}^i, r_{m2}^s],[r_{m2}'^i, r_{m2}'^s]) & \dots & ([r_{mn}^i, r_{mn}^s],[r_{mn}'^i, r_{mn}'^s]) \end{bmatrix}_{m\times n} \quad (33)$$

Step 8: Determine the Interval Rough negative ideal solution $IR(NIS_j)$ ($j \in \{1, 2, \dots, n\}$):

$$IR(NIS_j) = [NIS_j]_{1\times m} \quad (34)$$

$$NIS_j = \min_i IR(r_{ij}) = [NIS_j^i, NIS_j^s][NIS_j'^i, NIS_j'^s] = \\ = \left[\min_i r_{\mathrm{ij}}^i, \min_i r_{\mathrm{ij}}^s\right]\left[\min_i r_{\mathrm{ij}}'^i, \min_i r_{\mathrm{ij}}'^s\right] \quad (35)$$

Step 9: Calculate the Euclidean E_i and Taxicab T_i distances of alternatives i ($i \in \{1, \dots, m\}$) from the $IR(NIS_j)$ as follows:

$$E_i = \sqrt{\frac{\sum_{j=1}^m [(r_{ij}^i - NIS_j^i)^2 + (r_{ij}^s - NIS_j^s)^2 + (r_{ij}'^i - NIS_j'^i)^2 + (r_{ij}'^s - NIS_j'^s)^2]}{4}} \quad (36)$$

$$T_i = \frac{\sum_{j=1}^m \left[\left|r_{ij}^i - NIS_j^i\right| + \left|r_{ij}^s - NIS_j^s\right| + \left|r_{ij}^i - NIS_j^i\right| + \left|r_{ij}^i - NIS_j^i\right|\right]}{4} \quad (37)$$

The Euclidean and Taxicab distances are converted from IRNs to crisp numbers.

Step 10: Construct the Relative Evaluation Matrix *Re*:

$$Re = [h_{ik}]_{n\times n} \quad (38)$$

$$h_{ik} = (E_i - E_k) + \left(\psi(E_i - E_k) \times (T_i - T_k)\right) \quad (39)$$

where $k \in \{1, 2, \dots, n\}$ and ψ is a threshold function to determine the equality of the Euclidean distances of two alternatives, defined as follows:

$$\psi(E_i - E_k) = \begin{cases} 1 \ \ if \ \ |E_i - E_k| \geq \tau \\ 0 \ \ if \ \ |E_i - E_k| < \tau \end{cases} \quad (40)$$

In this function, τ is the threshold parameter that can be set by the decision maker. It is suggested to set this parameter at a value between 0.01 and 0.05.

Step 11: Calculate the evaluation score H_i of each alternative i ($i \in \{1, 2, \ldots, m\}$):

$$H_i = \sum_{k=1}^{n} h_{ik} \tag{41}$$

Step 12: Rank the alternatives according to the decreasing values of evaluation score H_i. The alternative with the highest evaluation score is the most desirable alternative.

4 Application of the IR-CODAS model for risk assessment

The Sfax “Hannibal” gas processing plant produces natural gas, diesel fuel, hydrogen sulfide, sulfuric acid, potassium hydrate, etc. The Sulfox unit of the Hannibal British Gas industry is focused on energy recovery, specifically the transfer of hydrogen sulfide gas H_2S to sulfuric acid H_2SO_4.

The gas treatment process generates several risks. Thus, the need to assess the risks and to know the most important of them in order to take the necessary precautions is essential to prevent them. For this reason, we test the applicability of the proposed IR-CODAS model under uncertain environment for MCGDM to the risk assessment problem. After a preliminary screening, we established that there are five types of risks of H_2S gas emissions into the atmosphere: Explosion (A_1), Fire (A_2), Leak (A_3), Respiratory fatigue (A_4) and Dysfunction of control devices (A_5). These risks are the alternatives of our model and they are evaluated by a committee of three decision makers (DM) according to four criteria: Security (C_1), Frequency of exposure (C_2), Degree of severity (C_3) and Environmental impact (C_4), where C_1 is a benefit criterion and the others are cost criteria.

Step 1: After the DMs’ evaluation of criteria, the study consider four criteria that are evaluated by a linguistic scale in three matrices (Table 2).

Table 2: Linguistic Assessing Matrix by three DMs

	DM_1				DM_2				DM_3			
	C_1	**C_2**	**C_3**	**C_4**	**C_1**	**C_2**	**C_3**	**C_4**	**C_1**	**C_2**	**C_3**	**C_4**
A_1	G, VG	P, VP	VG, VG	G, G	M, G	VP, VP	G, VG	VG, G	G, G	P, M	VG, VG	VG, VG
A_2	G, VG	P, VP	VG, VG	G, VG	VG, VG	VP, VP	G, G	M, G	G, VG	P, P	M, VG	G,VG
A_3	G, VG	G, M	G, VG	M, G	G,VG	G, P	G, VG	M, G	VG, VG	G, VG	M, VG	P, M
A_4	M, G	G, M	M, G	VP, VP	G , G	M, M	G, G	VP, VP	M ,VG	G, M	M, G	VP, P
A_5	M, G	VG, M	M, G	P, M	G, VG	VG, M	G, VG	P, P	M, G	VG, G	M, VG	VP, P

Step 2: Using Table 1, we transformed the linguistic input values, which are recorded in Table 2, into integer data shown in Table 3.

Table 3: Evaluation of alternatives by three DMs

	DM_1				DM_2				DM_3			
	C_1	**C_2**	**C_3**	**C_4**	**C_1**	**C_2**	**C_3**	**C_4**	**C_1**	**C_2**	**C_3**	**C_4**
A_1	7, 9	7, 9	1, 1	3, 3	5, 7	9, 9	1, 3	1, 3	7, 7	5, 7	1, 1	1, 1
A_2	7, 9	7, 9	1, 1	1, 3	9, 9	9, 9	3, 3	3, 5	7, 9	7, 7	1, 5	1, 3
A_3	7, 9	3, 5	1, 3	3, 5	7, 9	3, 7	1, 3	1, 5	9, 9	1, 3	1, 5	5, 7
A_4	5, 7	3, 5	3, 5	9, 9	7, 7	5, 5	3, 3	9, 9	5, 9	5, 7	3, 5	7, 9
A_5	5, 7	1, 5	3, 5	5, 7	7, 9	3, 5	1, 3	7, 7	5, 7	1, 3	1, 1	7, 9

Step 3: According to Table 3 and Equations (1-12), we convert the individual matrices to an interval rough matrix.

As an example of calculating the evaluation for the position A_4-C_1, we select the object classes x_{41}^{p} and x_{ij}^{p*}. Each class contains three elements:

$$x_{41}^{p} = \{5; 7; 5\}$$
$$x_{41}^{p*} = \{7; 7; 9\}$$

By applying expressions (7-14), we form rough sequences for each object class.

For the first object class we get:

$\underline{Lim}(5) = 5$ $\overline{Lim}(5) = \frac{1}{3}(5 + 7 + 5) = 5.67$ RN(5) = [5; 5.67]

$\underline{Lim}(7) = \frac{1}{3}(5 + 7 + 5) = 5.67$ $\overline{Lim}(7) = 7$ RN(7) = [5.67; 7]

For the second object class we get:

$\underline{Lim}(7) = 7$ $\overline{Lim}(7) = \frac{1}{3}(7 + 7 + 9) = 7.67$ RN(7) = [7; 7.67]

$\underline{Lim}(9) = \frac{1}{3}(7 + 7 + 9) = 7.67$ $\overline{Lim}(9) = 9$ RN(9) = [7.67; 9]

On the basis of rough sequences, we obtain for each DM the following interval rough numbers:

IRN(DM_1) = [5; 5.67] [7; 7.67],
IRN(DM_2) = [5.67; 7] [7; 7.67],
IRN(DM_3) = [5; 5.67] [7.67; 9].

In our case study, the evaluation of alternatives by three decision makers has been performed using interval rough numbers as shown in Table 4.

Table 4: Initial interval rough matrix for three DMs

DM$_1$				
	C$_1$	**C$_2$**	**C$_3$**	**C$_4$**
A$_1$	[6.33; 7][7.66; 9]	[6; 8][8.33; 9]	[1; 1][1; 1.67]	[1.67; 3][2.33; 3]
A$_2$	[7; 7.67][9; 9]	[7; 7.67][8.33; 9]	[1; 1.67][1; 2.33]	[1; 1.67][3; 3.67]
A$_3$	[7; 7.66][9; 9]	[2.33; 3][5; 7]	[1; 1][3; 3.67]	[2; 3][5; 5]
A$_4$	[5; 5.67][7; 7.66]	[3; 4.33][5; 5.66]	[3; 3][4.33; 5]	[8.33; 9][9; 9]
A$_5$	[5; 5.67][7; 7.66]	[1; 1.67][4.33; 5]	[1.67; 3][3; 5]	[5; 6.33][7; 7.67]
DM$_2$				
A$_1$	[5; 6.33][7; 7.66]	[7; 9][8.33; 9]	[1; 1][1.67; 3]	[1; 1.67][2.33; 3]
A$_2$	[7.67; 9][9; 9]	[7.67; 9][8.33; 9]	[1.67; 3][2.33; 3]	[1.67; 3][3.67; 5]
A$_3$	[7; 7.66][9; 9]	[2.33; 3][5; 7]	[1; 1][3; 3.67]	[1; 3.67][5; 5]
A$_4$	[5.67; 7][7; 7.67]	[4.33; 5][5; 5.66]	[3; 3][3; 4.33]	[8.33; 9][9; 9]
A$_5$	[5.67; 7][7.66; 9]	[1.67; 3][4.33; 5]	[1; 1.67][2; 4]	[6.33; 7][7; 7.67]
DM$_3$				
A$_1$	[6.33; 7][7; 7.66]	[5.7][7; 8.33]	[1; 1][1; 1.67]	[1; 1.67][1; 2.33]
A$_2$	[7.67; 9][9; 9]	[7; 7.67][7; 8.33]	[1; 1][3.67; 5]	[1; 1.67][3; 3.67]
A$_3$	[7.66; 9][9; 9]	[1; 2.33][3; 5]	[5; 6.33][9; 9]	[3; 4.33][5; 7]
A$_4$	[5; 5.67][7.67; 9]	[4.33; 5][5.66; 7]	[3; 3][4.33; 5]	[7; 8.33][9; 9]
A$_5$	[5; 5.67][7; 7.66]	[1; 1.67][3; 4.33]	[1; 1.67][1; 3]	[6.33; 7][7.67; 9]

Step 4: In this step, the DMs' individual evaluations can be fused into the group assessing matrix with IRNs using Equation (28). So, for the sequence x_{41} we obtain:

$$IRG(x_{41}^{i}) = \frac{x_{41}^{i1} + x_{41}^{i2} + x_{41}^{i3}}{z} = \frac{5 + 5.67 + 5}{3} = 5.22$$

$$IRG(x_{41}^{s}) = \frac{x_{41}^{s1} + x_{41}^{s2} + x_{41}^{s3}}{z} = \frac{5.67 + 7 + 5.67}{3} = 6.11$$

$$IRG(x_{41}'^{i}) = \frac{x_{41}'^{i1} + x_{41}'^{i2} + x_{41}'^{i3}}{z} = \frac{7 + 7 + 7.67}{3} = 7.22$$

$$IRG(x_{41}'^{s}) = \frac{x_{41}'^{s1} + x_{41}'^{s2} + x_{41}'^{s3}}{z} = \frac{7.67 + 7.67 + 9}{3} = 8.11$$

Then $IRG(x_{41}) = [5.22;\ 6.11][7.22;\ 8.11]$.

For our case, using Table 4 and Equation (28), we transform individual interval rough matrix to a group interval rough matrix shown in Table 5.

Table 5: Interval Rough Group Matrix

GIR	**C1**	**C2**	**C3**	**C4**
A1	[5.89; 6.78][7.22; 8.11]	[6; 8][7.89; 8.78]	[1; 1][1.22; 2.11]	[1.22; 2.11][1.89; 2.78]
A2	[7.45; 8.56][9; 9]	[7.22; 8.11][7.89; 8.78]	[1.22; 2.11][1.89; 2.78]	[1.22; 2.11][3.22; 4.11]
A3	[7.22; 8.11][9; 9]	[1.89; 2.78][4; 6]	[1; 1][3.22; 4.11]	[2.22; 4.56][5; 5]
A4	[5.22; 6.11][7.22; 8.1]	[3.89; 4.78][5.22; 6.11]	[3; 3][3.89; 4.78]	[7.89; 8.78][9; 9]
A5	[5.22; 6.11][7.22; 8.11]	[1.22; 2.11][3.89; 4.78]	[1.22; 2.11][2; 4]	[5.89; 6.78][7.22; 8.11]

Step 5: Equation (29) is applied to normalize the Interval Rough Group Matrix (Table 5) and we obtain the results listed in Table 6.

An example of calculating a normalized matrix for the cost criteria C_2:

$$IRG(x_{42}) = \left[\frac{\min_i x_{i2}^{i}}{x_{42}^{\prime s}}; \frac{\min_i x_{i2}^{s}}{x_{42}^{\prime i}}\right]\left[\frac{\min_i x_{i2}^{\prime i}}{x_{42}^{s}}; \frac{\min_i x_{i2}^{\prime s}}{x_{42}^{i}}\right] = \left[\frac{1.22}{6.11}; \frac{2.11}{5.22}\right]\left[\frac{3.89}{4.78}; \frac{4.78}{3.89}\right]$$
$$= [0.2; 0.4][0.81; 1.23]$$

An example of calculating a normalized matrix for the benefit criteria C_1:

$$IRG(x_{31}) = \left[\frac{x_{31}^{i}}{\max_i x_{i1}^{\prime s}}; \frac{x_{31}^{s}}{\max_i x_{i1}^{\prime i}}\right]\left[\frac{x_{31}^{\prime i}}{\max_i x_{i1}^{s}}; \frac{x_{31}^{\prime s}}{\max_i x_{i1}^{i}}\right]$$
$$= \left[\frac{7.22}{9}; \frac{8.11}{9}\right]\left[\frac{9}{8.56}; \frac{9}{7.45}\right] = [0.8; 0.9][1.05; 1.2]$$

Table 6: Normalized Interval Rough Matrix

	C1	**C2**	**C3**	**C4**
A1	[0,65; 0.75][0.84; 1.09]	[0,14; 0.27][0.49; 0.8]	[0,47; 0.82][1.22; 2.11]	[0,44; 1.12][0.9; 2.28]
A2	[0,83; 0.95][1.05; 1.21]	[0,14; 0.27][0.48; 0.66]	[0,36; 0.53][0.58; 1.73]	[0,3; 0.66][0.9; 2.28]
A3	[0,8; 0.9][1.05; 1.21]	[0,2; 0.53][1.4; 2.53]	[0,21; 0.26][0.41; 0.7]	[0,14; 0.23][0.22; 0.35]
A4	[0,58; 0.68][0.84; 1.09]	[0,2; 0.4][0.81; 1.23]	[0,13; 0.26][0.41; 0.7]	[0,14; 0.23][0.22; 0.35]
A5	[0.58; 0.68][0.84; 1.09]	[0,26; 0.54][1.84; 3.92]	[0,25; 0.5][0.58; 1.59]	[0,15; 0.29][0.28; 0.47]

Step 6: The relative importance weights of the four criteria provided by the DMs are assumed to be crisp numbers which are presented in Table 7. Then we define the group criteria weight coefficient using Equation (31).

Table 7: The relative importance weights of the four criteria by the three DMs

Criterion	DM_1	DM_2	DM_3
C_1	0,48	0,4	0,3
C_2	0,01	0,1	0,2
C_3	0,47	0,3	0,2
C_4	0,04	0,2	0,3

Step 7: Weighting the previous normalized group interval rough matrix (Table 6) by Equation (32).

Step 8 and Step 9: After normalizing and calculating the weighted normalized matrix, we determine the *IR(NIS)*, the Euclidean and Taxicab distances of alternatives given in Table 8.

Table 8: Weighted Normalized Group Interval Rough Matrix

	C_1	C_2	C_3	C_4	E_i	T_i
A_1	[0,26; 0,3][0,33; 0,43]	[0.01; 0.03][0.05; 0.08]	[0.15; 0.27][0.4;0.68]	[0.08; 0.2][0.16; 0.41]	0,42	0,44
A_2	[0,33; 0,38][0,41; 0,48]	[0.01; 0.03][0.05; 0.07]	[0.12; 0.17][0.19;0.56]	[0.05; 0.12][0.16; 0.41]	0,36	0,33
A_3	[0,32; 0,36][0,41; 0,48]	[0.02; 0.05][0.14; 0.26]	[0.07; 0.08][0.132; 0.23]	[0.02; 0.04][0.04; 0.06]	0,12	0,13
A_4	[0,23; 0,27][0,33; 0,43]	[0.02; 0.04][0.08; 0.13]	[0.04; 0.08][0.13; 0.23]	[0.02; 0.04][0.04; 0.06]	0,03	0,03
A_5	[0,23; 0,27][0,33; 0,43]	[0.03; 0.06][0.19; 0.4]	[0.08; 0.16][0.19; 0.51]	[0.03; 0.05][0.05; 0.08]	0,25	0,25
IR(NIS)	[0,23; 0,27][0,33; 0,43]	[0.01; 0.03][0.05; 0.07]	[0.04; 0.08][0.13; 0.23]	[0.02; 0.04][0.04; 0.06]		

Step 10: Construct the Relative Evaluation Matrix Re by using Table 8 and Equations (38-40) with the threshold parameter τ set to 0.03 (Table 9).

Table 9: Relative Evaluation matrix

	A_1	A_2	A_3	A_4	A_5	H_i	Rank
A_1	0,00	0,17	0,61	0,80	0,36	1,94	1
A_2	−0,17	0,00	0,44	0,63	0,19	1,08	2
A_3	−0,61	−0,44	0,00	0,19	−0,25	−1,10	4
A_4	−0,80	−0,63	−0,19	0,00	−0,44	−2,06	5
A_5	−0,36	−0,19	0,25	0,44	0,00	0,13	3

Step 11: We compute the value of the evaluation score of each alternative using Table 9 and Equation (41):

$$H_1 = 1.94;\ H_2 = 1.08;\ H_3 = -1.10;\ H_4 = -2.06;\ H_5 = 0.13$$

Step 12: We rank the alternatives in decreasing order. Evidently, the order is A_1-A_2-A_5-A_3-A_4 and from the above findings it follows that A_1 is the most dangerous risk among the five alternatives in this case study.

The traditional crisp CODAS method evaluates alternatives using crisp numbers. Indeed, crisp values as input data are insufficient to model real-life situations and complex concepts with multiple and often conflicting objectives which frequently occur in multicriteria decision aid. For instance, in risks assessment some criteria are considered very important and the way of indicating their importance needs to be more flexible. The linguistic term "very good" can be preferably expressed an IRN rather than a single crisp number. However, in this paper, we use IRNs to assess the risks, since DMs can flexibly express their opinions using linguistic terms.

On the other hand, the proposed distance-based IR-CODAS method used two types of distance in evaluation process: Euclidean distance and Taxicab distance which helps to increase the precision of ranking results in group decision making process (which is accompanied by a great amount of uncertainty and subjectivity). An interval structure can be used to synthesize the decision rules provided by the DMs. Thus, in this study, we have introduced the theory of rough sets, an approach based on IRNs for representing uncertainty in group decision making. So, IR-CODAS transforms individual linguistic matrices into interval rough matrices with different size of interval to capture preference uncertainty of the DMs.

Furthermore, a sensitivity analysis is performed to determine the effect of the different threshold parameters on the rankings. According to step 10, the Relative Evaluation Matrix *Re* depends on the threshold parameter τ that denotes the degree of closeness of the Euclidean distances of two alternatives.

Table 10: Difference of Euclidean distance

	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	$\mathbf{A_4}$	$\mathbf{A_5}$
$\mathbf{A_1}$	0,00	0,06	0,30	0,39	0,17
$\mathbf{A_2}$	−0,06	0,00	0,23	0,32	0,11
$\mathbf{A_3}$	−0,30	−0,23	0,00	0,09	−0,12
$\mathbf{A_4}$	−0,39	−0,32	−0,09	0,00	−0,21
$\mathbf{A_5}$	−0,17	−0,11	0,12	0,21	0,00

From the absolute value of the difference of Euclidean distance given in Table 10, it can be seen that all differences exceed 0.05. Hence, the evaluation score H_i of each alternative is the same. Even if we increase the value of τ and disregard the condition $0.01 \leq \tau \leq 0.05$, Table 11 shows that there are no changes in the rankings despite the differences in the threshold function values.

Table 11: Evaluation score H_i and ranking results with different values of τ

Alternatives	τ = 0,03		τ = 0,07		τ = 0,1	
	$\boldsymbol{H_i}$	**Rank**	$\boldsymbol{H_i}$	**Rank**	$\boldsymbol{H_i}$	**Rank**
$\mathbf{A_1}$	1,94	1	1,84	1	1,84	1
$\mathbf{A_2}$	1,08	2	1,08	2	1,08	2
$\mathbf{A_3}$	−1,10	4	−1,10	4	−1,20	4
$\mathbf{A_4}$	−2,06	5	−2,06	5	−1,96	5
$\mathbf{A_5}$	0,13	3	0,13	3	0,13	3

However, the weight coefficients of the evaluation criteria have a great influence on the results. Hence, we compute the final ranking of the alternatives by replacing the group procedure by the individual procedure, i.e. we omit step 6 and keep the importance coefficients provided by each DM; at the end, the DMs' individual evaluations scores H_i^p can be fused into the collective evaluation score GH_i for each alternative. The final ranking orders of alternatives is shown in Table 12.

Table 12: Individual and group evaluations score matrix

Alternatives	H_i^1	H_i^2	H_i^3	GH_i	Rank
$\mathbf{A_1}$	0,26	1,19	0,23	0,56	3
$\mathbf{A_2}$	0,90	0,92	−0,10	0,58	2
$\mathbf{A_3}$	0,31	−0,30	0,28	0,10	4
$\mathbf{A_4}$	−0,94	−2,70	−3,17	−2,27	5
$\mathbf{A_5}$	−0,54	0,88	2,75	1,03	1

As can be seen, dysfunction of control devices risk (A_5) is the most dangerous. Clearly, changes in the procedure of calculating criteria weights leads to a change in the ranks of individual alternatives, which confirms that the model is sensitive to changes in weight coefficients. Compared to the previous results, we can notice that the first ranked alternatives (A_1, A_2 and A_5) are the most important and it is necessary to take essential precautions to prevent them.

5 Conclusion

The CODAS method is a simple and easily applicable multi-criteria decision making method. To handle uncertainty, it is impossible to provide data with crisp numbers in an adequate way. Therefore, we propose to develop a subjective model using linguistic evaluation. Since the group decision making process proceeds in an uncertain environment, this assessment is complex. Thus, our proposed approach IR-CODAS refers to the integration of the interval rough numbers into the CODAS methods to solve group decision making problems under uncertainty.

The applicability of the proposed model is validated through a real-life case study of the gas processing industry in Sfax. Namely, our IR-CODAS approach was applied to select the most important risks in order to take the necessary precautions to prevent them. A sensitivity analysis was conducted, confirming the validity of the final results. We changed the threshold parameters values which do not influence the ranking of alternatives. Furthermore, we choose to test the final ranking using the individual procedure of each DM.

Future research intends to develop a preference disaggregation approach deducing criteria weight values and threshold parameters from the information provided by the DMs. As well, we aim to integrate interval rough numbers into other methods and develop new MCDM methods.

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DESIGNING SUSTAINABLE SUPPLY CHAINS

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Abstract

A supply chain is a complex and dynamic supply and demand network of agents, activities, resources, technology and information involved in moving products or services from supplier to customer. The suitability of supply chains can be measured by multiple criteria, such as environmental, social, economic, and others. Finding an equilibrium between the interests of members of a sustainable supply chain is a very important problem.

The main objective of the paper is to analyze the design of sustainable supply chains and to create a comprehensive model and solution methods for designing sustainable supply chains. Multiple criteria analysis and game theory is a natural choice to effectively analyze and model decision making in such multiple agent situation with multiple criteria where the outcome depends on the choice made by every agent. Multiple criteria analysis is useful for assessing sustainability of supply chains. The De Novo approach focusses on designing optimal systems. Game theory has become a useful instrument in the analysis of supply chains with multiple agents. Games are used for behavior modeling of supply chains; they focus on the allocation of resources, capacities, costs, revenues and profits. The co-opetition concept combines the advantages of both competition and cooperation into new dynamics, which can be used to not only generate more profits, but also to change the nature of the business environment for the benefit of users.

Keywords: supply chain, sustainability, multiple criteria, De Novo approach, game theory, biform games.

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1 Introduction

Supply chain management is a philosophy that provides the tools and techniques enabling organizations to develop strategic focus and achieve sustainable competitive advantage (Simchi-Levi et al., 2008). This philosophy presents management with a new focus and way of thinking about the existence and workings of the organization in a wider business environment. Supply chain management is now seen as a governing element in strategy and as an effective way of creating value for customers.

The evolution of supply chain management recognized that a business process consists of several decentralized firms and that decisions of these different units impact each other's performance, and thus the performance of the whole supply chain. Each unit attempts to optimize his own preference. Behavior that is locally efficient can be inefficient from a global point of view. Sustainability in supply chain management has become a highly relevant topic for researchers and practitioners (Brandenburg et al., 2014; Carter and Rogers, 2008; Seuring, 2013). The objective of supply chain sustainability is to create, protect and grow long-term environmental, social and economic value for all stakeholders involved in bringing products and services to market.

The main objective of the paper is to analyze the design of sustainable supply chains and to create a comprehensive model and solution methods for designing sustainable supply chains. Multicriteria analysis and game theory tools are a natural choice for modeling and effective analysis of decision making in a situation with multiple criteria and multiple agents, where the outcome depends on the choice of each agent. Multiple criteria analysis is useful for assessing sustainability of supply chains. Game theory has become a useful instrument in the analysis of supply chains with multiple agents, often with conflicting objectives.

Standard multiple criteria approaches focus on valuation of already given systems. The De Novo approach focusses on designing optimal systems (Zelený, 2010). The approach is based on reformulation of the problem by given prices of resources and the given budget. Searching for a better portfolio of resources leads to a continual reconfiguration and reshaping of systems boundaries. The De Novo approach was adapted for supply chain design. Current business conditions are changing rapidly. New products are evolving faster. Technological innovations bring improvements to the criteria and a better utilization of available resources. This dynamics must be included in the new models. These changes can lead beyond trade-off-free solutions.

The search for equilibrium in supply chains is a very important problem. Games are used for behavior modeling of supply chains; they focus on allocation of resources, capacities, costs, revenues and profits (Kreps, 1991; Cachon and Netessine, 2004). There are numerous opportunities to create hybrid models that combine competitive and cooperative behavior. The co-opetition concept combines the advantages of both competition and cooperation into new dynamics, which can be used to not only generate more profits, but also to change the nature of the business environment for the benefit of users (Brandenburger and Nalebuff, 2011). Searching for relationships with complementors (competitors whose products add value to other agents) brings ever new opportunities that bring added values. The co-opetition is based on the biform game theory (Okura and Carfi, 2014). Biform games combine non-cooperative and cooperative approaches of the traditional game theory and are promising for modeling behavior of the agents in supply chains (Brandenburger and Stuart, 2007). The authors propose to divide the biform games into so-called sequential and simultaneous shapes. The proposed procedure captures these concepts; it is flexible and open to other concepts and procedures for designing sustainable supply chains.

2 Sustainable supply chain

A supply chain is a complex and dynamic supply and demand network of agents, activities, resources, technology, and information involved in moving a product or service from the initial supplier to the ultimate customer (Tayur, Ganeshan and Magazine, eds., 2012; Snyder and Shen, 2011; Harrison, Lee and Neale, 2003). A supply chain consists of several decentralized firms; decisions of these different units impact each other's performance, and thus the performance of the whole supply chain.

A supply chain is defined as a network system that consists of clusters with:

- suppliers,
- manufacturers,
- distributors,
- retailers,
- customers,

where:

- material,
- financial,
- information,
- decision

flows connect participants in both directions. Decision flows are sequences of decisions among agents (see Fiala, 2005).

Supply chain management can be divided into four phases:

- design,
- control,
- performance evaluation,
- performance improvement.

These phases are repeated during the dynamic evolution of the environment and the supply chain. The design phase of supply chains plays an important role in supply chain management. This paper focuses on modeling this design phase.

The proposed approach promotes sustainability of supply chains through the following instruments:

- multiple criteria,
- De Novo optimization,
- technology development,
- biform games,
- the concept of co-opetition.

Sustainability of supply chains is evaluated by multiple criteria:

- environmental,
- social,
- economic,
- and others.

The model contains not only three basic aspects; other criteria can be used (technological, legal, etc.). Two models were used for multiple criteria evaluation of sustainable supply chains. Multi-objective linear programming (MOLP) is a model of optimizing a given system by multiple objectives (Steuer, 1986). Multi-objective De Novo linear programming (MODNLP) is a problem for designing an optimal system by reshaping the feasible set (Zelený, 2010). This approach seeks to find a trade-off-free solution and uses only the necessary resources for this solution, limited only by budget. The technological innovations included in the model bring improvements to the desired criteria and a better utilization of available resources.

The proposed biform game models provide suitable tools for finding an equilibrium in the agent-system by combining non-cooperative and cooperative approaches. The inclusion of the concept of co-opetition enriches the model with other aspects, including considering the influence of other agents such as competitors and complementors (Min, Feiqi and Sai, 2008). The search for equilibrium in a sustainable supply chain is based on a negotiation approach. Information exchange by negotiations reduces inefficiencies and material flows and leads to reduced environmental pollution and costs.

3 Multiple criteria analysis

The first component of the proposed procedure is multiple criteria analysis (Greco, Figueira and Ehrgott, 2016). A standard approach can be used to optimize the given system and the De Novo approach to design an optimal system. Both procedures will be described. The advantages of the De Novo approach will be explained.

3.1 Optimizing given systems

In MOLP problems, it is usually impossible to optimize all objectives together in a given system. Trade-off means that one cannot increase the level of satisfaction for an objective without decreasing it for another objective. Multi--objective linear programming (MOLP) problem can be described as follows:

$$\begin{aligned} \text{“Max” } \mathbf{z} &= \mathbf{Cx} \\ \text{s.t.} \quad \mathbf{Ax} &\leq \mathbf{b},\ \mathbf{x} \geq \mathbf{0} \end{aligned} \tag{1}$$

where **C** is a $(\bar{k}, \bar{n})$ matrix of objective coefficients, **A** is an $(\bar{m}, \bar{n})$ matrix of structural coefficients, **b** is an $\bar{m}$-vector of known resource restrictions, **x** is an $\bar{n}$-vector of decision variables. The “Max” operator is used for vector optimization. For multi-objective programming problems, the concept of efficient solutions is used (e.g. Steuer, 1986). A compromise solution is selected from the set of efficient solutions. Many methods are proposed for solving the problem. Most of the methods are based on trade-offs between objective values.

Multiple criteria supply chain model

In the next part, a multiple criteria supply chain design problem is formulated. The mathematical program determines the ideal locations for each facility and allocates the activity at each facility so that the multiple objectives are taken into account and the constraints of meeting the customer demand and the facility capacity are satisfied. The presented model of a supply chain consists of four layers with m suppliers: $S_1, S_2, \dots, S_m$, n potential producers: $P_1, P_2, \dots, P_n$, p potential distributors: $D_1, D_2, \dots, D_p$ and r customers: $C_1, C_2, \dots, C_r$.

The following notation is used:

a_i = annual supply capacity of supplier i, b_j = annual potential capacity of producer j,

w_k = annual potential capacity of distributor k, d_l = annual demand of customer l,

f_j^P = fixed cost of potential producer j, f_k^D = fixed cost of potential distributor k,

c_{ij}^S = unit transportation cost from S_i to P_j, c_{jk}^P = unit transportation cost from P_j to D_k,
c_{kl}^D = unit transportation cost from D_k to C_l, e_{ij}^S = unit pollution from S_i to P_j,
e_{jk}^P = unit pollution from P_j to D_k, e_{kl}^D = unit environmental pollution from D_k to C_l,
x_{ij}^S = number of units transported from S_i to P_j, x_{jk}^P = number of units transported from P_j to D_k, x_{kl}^D = number of units transported from D_k to C_l,
y_j^P = binary variable for build-up of the fixed capacity of producer j,
y_k^D = binary variable for build-up of the fixed capacity of distributor k.

With the above notations, the problem can be formulated as follows:

The model has two objectives: The first one expresses minimizing total costs; the second one expresses minimizing total environmental pollution.

Minimize two objectives:

$$z_1 = \sum_{j=1}^{n} f_j^P y_j^P + \sum_{k=1}^{p} f_k^D y_k^D + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^S x_{ij}^S + \sum_{j=1}^{n}\sum_{k=1}^{p} c_{jk}^P x_{jk}^P + \sum_{k=1}^{p}\sum_{l=1}^{r} c_{kl}^D x_{kl}^D$$

$$z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij}^S x_{ij}^S + \sum_{j=1}^{n}\sum_{k=1}^{p} e_{jk}^P x_{jk}^P + \sum_{k=1}^{p}\sum_{l=1}^{r} e_{kl}^D x_{kl}^D$$

subject to the following constraints:
the amount sent from the supplier to producers cannot exceed the supplier capacity:

$$\sum_{j=1}^{n} x_{ij}^S \le a_i, \quad i = 1, 2, ..., m$$

the amount produced by the producer cannot exceed the producer capacity:

$$\sum_{k=1}^{p} x_{jk}^P \le b_j y_j^P, \quad j = 1, 2, \dots, n$$

the amount shipped from the distributor should not exceed the distributor capacity:

$$\sum_{l=1}^{r} x_{kl}^D \le w_k y_k^D, \quad k = 1, 2, \dots, p$$

the amount shipped to the customer must equal the customer demand:

$$\sum_{k=1}^{p} x_{kl}^{D} = d_l\,,\ l\ =\ 1,2,\dots,r$$

the amount shipped out of producers cannot exceed the number of units received from suppliers:

$$\sum_{i=1}^{m} x_{ij}^{S} - \sum_{k=1}^{p} x_{jk}^{P} \geq 0,\ \ j\ =\ 1,2,\dots,n$$

the amount shipped out of distributors cannot exceed the quantity received from producers:

$$\sum_{j=1}^{n} x_{jk}^{P} - \sum_{l=1}^{r} x_{kl}^{D} \geq 0\,,\ k\ =\ 1,2,\dots,p$$

binary and non-negativity constraints:

$y_j^P, y_k^D \in \{0,1\}$,

$x_{ij}^S, x_{jk}^P, x_{kl}^D \geq 0,\ i = 1,2,\dots,m,\ j = 1,2,\dots,n,\ k =\ 1,2,\dots,p,\ l = 1,2,\dots,r$

The formulated model is a multi-objective linear programming problem (MOLP). The problem can be solved using MOLP methods.

3.2 Designing optimal systems

By using given prices of resources and the given budget the MOLP problem (1) is reformulated into the following MODNLP problem (2):

$$\begin{aligned} &\text{“Max”} \quad \mathbf{z} = \mathbf{Cx} \\ &\text{s.t.}\ \ \mathbf{Ax} - \mathbf{b} \leq \mathbf{0},\ \mathbf{pb} \leq B,\ \mathbf{x} \geq \mathbf{0} \end{aligned} \tag{2}$$

where $\mathbf{b}$ is an $\overline{m}$-vector of unknown resource restrictions, $\mathbf{p}$ is an $\overline{m}$-vector of resource prices, and B is the given total available budget.

From (2) follows that:

$$\mathbf{pAx}\ \leq\ \mathbf{pb}\ \leq B$$

Defining an n-vector of unit costs $\mathbf{v} = \mathbf{pA}$, we can rewrite the problem (2) as:

$$\begin{aligned} &\text{“Max”} \quad \mathbf{z} = \mathbf{Cx} \\ &\text{s.t.}\ \ \mathbf{vx} \leq B,\ \ \mathbf{x}\ \geq \boldsymbol{0} \end{aligned} \tag{3}$$

Solving single objective problems:

$$\text{Max}\ \ z^{i}\ = \mathbf{c}^{i}\mathbf{x},\quad i = 1,2,\dots,\overline{k}$$

$$\text{s.t.} \quad \mathbf{vx} \leq B, \mathbf{x} \geq 0 \tag{4}$$

$\mathbf{z}^*$ is a $\bar{k}$-vector of objective values for the ideal system, concerning budget B, where the elements of the vector are values z^i obtained by solving the set of problems (4).

The problems (4) are continuous "knapsack" problems, with the solutions:

$$x_j^i = \begin{cases} 0, j \neq j_i \\ B/v_{j_i}, j = j_i \end{cases} \text{, where } j_i \in \left\{ j \in (1, \ldots, n) \middle| \max_j (c_j^i / v_j) \right\}$$

The meta-optimum problem can be formulated as follows:

$$\text{Min} \quad f = \mathbf{vx}$$

$$\text{s.t.} \quad \mathbf{Cx} \geq \mathbf{z}^*, \mathbf{x} \geq \mathbf{0} \tag{5}$$

Solving the problem (5) provides the solution: $\mathbf{x}^*$, $B^* = \mathbf{vx}^*$, $\mathbf{b}^* = \mathbf{Ax}^*$.

The value B^* identifies the minimum budget to achieve $\mathbf{z}^*$ through solutions $\mathbf{x}^*$ and $\mathbf{b}^*$, with the given budget level $B \leq B^*$. The optimum-path ratio for achieving the best performance for a given budget B is defined as:

$$r_1 = \frac{B}{B^*}$$

The optimum-path ratio provides an effective and fast tool for the efficient optimal redesign of large-scale linear systems. The optimal system design for the budget B:

$$\mathbf{x} = r_1 \mathbf{x}^*, \mathbf{b} = r_1 \mathbf{b}^*, \mathbf{z} = r_1 \mathbf{z}^*$$

Multi-objective De Novo supply chain model

The De Novo approach can be useful in the design of the multi-criteria supply chain. Only a partial relaxation of constraints is adopted. Producer and distributor capacities are relaxed. Unit costs for capacity build-up are computed:

$p_j^P = \frac{f_j^P}{b_j}$ = cost of the unit capacity of potential producer j,

$p_k^D = \frac{f_k^D}{w_k}$ = cost of the unit capacity of potential distributor k.

Variables for build-up capacities are introduced:

u_j^P = variable for the flexible capacity of producer j,

u_k^D = variable for the flexible capacity of producer k.

The constraints for non-exceeding the producer and distributor fixed capacities are replaced by the flexible capacity constraints and the budget constraint:

$$\sum_{k=1}^{p} x_{jk} - u_j^P \leq 0\,,\; j = 1, 2, \ldots, n$$

$$\sum_{l=1}^{r} x_{kl} - u_k^D \leq 0\,,\; k = 1, 2, \ldots, p$$

$$\sum_{j=1}^{n} p_j^P u_j^P + \sum_{k=1}^{p} p_k^D u_k^D \leq B$$

The multi-objective optimization can be then seen as a dynamic process. Technological innovations bring improvements to the objectives and the better utilization of available resources. The technological innovation matrix $T = (t_{ij})$ is introduced. The elements in the structural matrix A should be reduced by technological progress.

The problem (2) is reformulated into the innovation MODNLP problem (6):

$$\text{“Max”} \quad \mathbf{z} = \mathbf{Cx}$$
$$\text{s.t.} \quad \mathbf{TAx} - \mathbf{b} \leq \mathbf{0},\; \mathbf{pb} \leq B,\; \mathbf{x} \geq \mathbf{0} \tag{6}$$

The De Novo approach provides a better solution with respect to multiple objectives and also with lower budget thanks to flexible capacity constraints. The capacity of supply chain members has been optimized as regards flows in the supply chain and budget.

3.3 An illustrative example

The De Novo approach was tested on a case study. A supply chain is proposed with three potential suppliers, three potential manufacturers, three potential distributors, and three customers. The chain is evaluated according to two criteria: the first one aimed at minimizing total costs and the second one, at minimizing overall environmental pollution.

Inputs for the model are as follows:
Capacities $a_i = 100$, $i = 1, 2, 3$; $b_j = 100$, $j = 1, 2, 3$;
$w_k = 100$, $k = 1, 2, 3$; $d_l = 50$, $l = 1, 2, 3$.
Fixed costs $f_1^P = 110$, $f_2^P = 100$, $f_3^P = 120$, $f_1^D = 120$,
$f_2^D = 110$, $f_3^D = 150$.

Unit transportation costs and unit pollution are shown in Table 1 and Table 2.

Table 1: Unit transportation costs

c_{ij}^S	1	2	3	c_{jk}^P	1	2	3	c_{kl}^D	1	2	3
1	5	10	6	1	7	5	9	1	8	3	10
2	8	9	7	2	6	8	4	2	6	5	4
3	3	6	8	3	5	7	9	3	7	3	5

Source: Authors.

Table 2: Unit pollution

e_{ij}^S	1	2	3	e_{jk}^P	1	2	3	e_{kl}^D	1	2	3
1	4	3	8	1	8	7	9	1	8	6	2
2	8	9	2	2	6	8	4	2	8	9	8
3	7	6	8	3	4	7	9	3	5	3	5

Source: Authors.

This model was solved by different approaches. The first two approaches minimize each criterion separately. The compromise solution is calculated by the traditional STEM interactive approach for multi-criteria problems using the De Novo approach. The following are non-zero values of the variables that express the number of units of the product shipped between each supply chain layer.

The following values are given for each problem-solving approach:

Min z_1: $x_{13}^S = 50$, $x_{31}^S = 100$, $x_{12}^P = 100$, $x_{31}^P = 50$, $x_{12}^D = 50$, $x_{21}^D = 50$, $x_{23}^D = 50$.
Min z_2: $x_{12}^S = 100$, $x_{23}^S = 50$, $x_{23}^P = 100$, $x_{31}^P = 50$, $x_{13}^D = 50$, $x_{31}^D = 50$, $x_{32}^D = 50$.
STEM: $x_{11}^S = 58.13$, $x_{23}^S = 91.87$, $x_{12}^P = 58.13$, $x_{31}^P = 91.87$, $x_{12}^D = 46.87$, $x_{13}^D =$ $= 45$, $x_{21}^D = 50$, $x_{22}^D = 3.12$, $x_{23}^D = 50$.
De Novo: $x_{23}^S = 62.86$, $x_{32}^S = 87.14$, $x_{21}^P = 10$, $x_{23}^P = 77.14$, $x_{31}^P = 62.86$, $x_{12}^D =$ $= 50$, $x_{13}^D = 22.86$, $x_{31}^D = 50$, $x_{33}^D = 27.14$.

Criteria values z_1, z_2 and budget B are compared according to these solutions. The De Novo solution is better in all values than the STEM solution. The De Novo approach provides better solutions with respect to both criteria and also with a lower budget due to flexible capacity constraints. The capacities of supply chain members have been optimized for flows in the supply chain and budget. The comparison of results is shown in Table 3.

Table 3: Comparison of solution results

	Min z_1	Min z_2	STEM	De Novo
z_1	2460	3490	3070	3000
z_2	3100	1800	2030	2000
B	460	490	460	365.71

Source: Authors.

4 Equilibrium searching by biform games

The second component of the proposed procedure is the search for equilibrium (Myerson, 1997). Most supply chains are composed of independent agents with individual interests and preferences. Biform games are used for searching for an equilibrium in sustainable supply chains. A biform game is a combination of non-cooperative and cooperative games for searching for an equilibrium. The authors propose to divide biform games into sequential and simultaneous forms.

4.1 Sequential biform games

A sequential biform game (Fiala, 2016a) is a two-stage game: in the first stage, players choose their strategies in a non-cooperative way, thus forming the second stage of the game, in which the players cooperate. First, suppliers make initial proposals and take decisions. This stage is analyzed using a non--cooperative game theory approach. The players search for the Nash equilibrium by solving the next problem.

An n-player non-cooperative game in the normal form is a collection

$$\{N = \{1,2,\dots,n\};\ X_1, X_2, \dots, X_n;\ \pi_1(x_1, x_2, \dots, x_n), \dots, \pi_n(x_1, x_2, \dots, x_n)\ \} \quad (7)$$

where N is a set of n players; X_i, $i = 1, 2, \dots, n$, is a set of strategies for player i; $\pi_i(x_1, x_2, \dots, x_n)$, $i = 1, 2, \dots, n$, is a pay-off function for player i, defined on a Cartesian product of n sets X_i, $i = 1, 2, \dots, n$.

Decisions of players other than player i are summarized by the vector:

$$\mathbf{x}_{-i} = (x_1, \dots, x_{i-1}, x_{i+1} \dots, x_n) \quad (8)$$

A vector of decisions $(x_1^0, x_2^0, \dots, x_n^0)$ is the Nash equilibrium of the game if:

$$x_i^0\left(\mathbf{x}_{-i}^0\right) = \text{argmax}_{x_i}\, \pi_i(x_i, \mathbf{x}_{-i}) \forall i = 1,2,\dots,n \quad (9)$$

The Nash equilibrium is a set of decisions from which no player can improve the value of his pay-off function by unilaterally deviating from it.

Next, players negotiate among themselves. In this stage, a cooperative game theory is applied to characterize the outcome of negotiation among the players over how to distribute the total surplus. Each player's share of the total surplus is the product of its added value and its relative negotiation power. Distribution of the total surplus to players can be given by Shapley values (14).

The cooperative game theory looks at the set of possible outcomes, studies what the players can achieve, what coalitions they will form, how the coalitions that do form divide the outcome, and whether the outcomes are stable and robust.

The maximal combined output is achieved by solving the following problem:

$$\mathbf{x^0} = \text{argmax}_{\mathbf{x}} \sum_{i=1}^{n} \pi_i(x_i) \tag{10}$$

When modeling cooperative games it is advantageous to switch from the normal form to the characteristic function form. The characteristic function of the game with the set N of n players is a function $v(S)$ that is defined for all subsets $S \subseteq N$ (i.e. for all coalitions) and which assigns to each subset S a value $v(S)$ with the following characteristics:

$$v(\emptyset) = 0,\ v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \tag{11}$$

where S_1, S_2 are disjoint subsets of N. The pair (N, v) is called a cooperative game of n players in the characteristic function form.

Allocation mechanisms are based on different approaches, such as Shapley values, contracts, auctions, negotiations, etc. A particular allocation policy, introduced by Shapley (1953), has been shown to possess the best properties in terms of balance and fairness (Mahjoub and Hennet, 2014). The so called Shapley vector is defined as:

$$\mathbf{h} = (h_1, h_2, \ldots, h_n) \tag{12}$$

where the individual components (Shapley values) indicate the mean marginal contribution of i-th player to all coalitions, of which she/he may be a member. Player contribution to the coalition S is calculated by the formula:

$$v(S) - v(S - \{i\}) \tag{13}$$

The Shapley value for the i-th player is calculated as a weighted sum of marginal contributions according to the formula:

$$h_i = \sum_S \left\{ \frac{(|S|-1)\,!\,(n-|S|)\,!}{n!} . [v(S) - v(S - \{i\})] \right\} \tag{14}$$

where the number of coalition members is denoted by $|S|$ and the summation runs over all coalitions $i \in S$.

4.2 Simultaneous biform games

The simultaneous biform game is a one-stage model where combinations of concepts for cooperative and non-cooperative games are applied. The combinations will be changed according to situations in problems. At this stage, multi-round negotiations take place. The first problem is a classification of situations. The situations are affected by:

- which players can cooperate,
- to what scope they can cooperate.

If all players can cooperate fully, a standard cooperative model (10) can be used with subsequent distribution of the result according to the Shapley values (14). If no one can cooperate even in a partial context, a standard non--cooperative model (9) is used.

The general simultaneous biform games are based on a negotiation process with multiple criteria (see Fiala, 1999). The negotiation concept is based on the assumption that each negotiating subject decides under pressure of objective context. The scope of cooperation is dynamic and changes over time. The effects of pressures are reflected in restrictive conditions.

Negotiation model

Suppose we have n negotiation participants. Denote by X the decision space for the negotiating process. The elements of this space are decisions $\mathbf{x} \in X$, which are vectors whose components represent the parameters of the decision. A consensus decision $\mathbf{x}^*$ should be chosen from the decision space X. The traditional game concepts assume a fixed structure and fixed sets of strategies. The sets of strategies are assumed to be dynamic $X_i(t)$, for players $i = 1, 2, \ldots, n$, depending on discrete time periods $t = 0, 1, 2, \ldots, T$. A dynamic evaluations of strategies will be also considered.

Each participant evaluates decisions using multiple criteria and compares the decisions with the target values. Multiple criteria analysis from the first component of the proposed procedure is applied. The analysis is based on the De Novo approach. The criteria are in the form of criteria functions, and all participants want to optimize their values. Each participant in negotiations may have a different number of criteria. Denote by $\mathbf{f}^1(\mathbf{x})$, $\mathbf{f}^2(\mathbf{x})$, …, $\mathbf{f}^n(\mathbf{x})$ the vector criteria functions that transform decision $\mathbf{x}$ into the vectors of target values $\mathbf{y}^1$, $\mathbf{y}^2$, …, $\mathbf{y}^n$ of the target spaces of participants Y^1, Y^2, …, Y^n. However, the participant tries to not reveal his interests and his strategy to all players. One's own negotiations and exchanges of information between the participants occur in the decision space.

The negotiation process can be represented by dynamic models. Individual time moments correspond to rounds of negotiation, in which the current joint problem representation shows the degree of consensus or conflict between the parties in the negotiations. The development of problem representations can be described as a search for consensus through the exchange of information between the participants. The negotiation process takes place at discrete time points $t = 0, 1, 2, \ldots, T$. At time T the process is completed by finding a trajectory to time horizon T. The negotiation process can be modeled as a gradual change over time of the negotiation space, which is a subset of the decision space containing acceptable decisions of participants in the negotiation time until a single-element negotiation space is reached.

For each participant, a set of acceptable decisions is formulated, which is a set of decisions that are permissible and acceptable in terms of the required aspiration levels of criteria functions. The aspiration levels $\mathbf{b}^i(t)$, $i = 1, 2, \dots, n$, $t = 0, 1, 2, \dots, T$, of criteria functions represent opportunities for added values. At the beginning of the negotiations it has the form:

$$X_i(0) = \{\mathbf{x}; \mathbf{x} \in X, \mathbf{f}^i(\mathbf{x}) \le \mathbf{b}^i(0)\},\ i = 1, 2, \dots, n \tag{15}$$

Then the negotiation space is defined at the beginning of the negotiations as an intersection of sets of acceptable decisions of all participants in the negotiations:

$$X_0(0) = \bigcap_{i=1}^{r} X_i(0) \tag{16}$$

If the negotiation space $X_0(0)$ is a single-element set, the negotiation problem is trivial. This element is the consensus. The negotiation problem becomes interesting when the negotiation space is empty or contains more than one element. In the former case, the participants have to reduce some or all of the aspiration levels of criteria functions, but the participants are involved more in the reduction of certain criteria and less in the reduction of others. In the latter case, each element of the negotiation space is acceptable to all participants, but different elements are evaluated differently, because they meet the criteria of the participants on different levels. Further negotiations are conducted at time points $t = 1, 2, \dots, T$, and should lead to a consensus decision, to achieve the single--element negotiation space $X_0(t)$.

5 Conclusion

This paper proposes and discusses a procedure for designing sustainable supply chains. This procedure takes into account multiple agents in the system and multiple evaluation criteria to solve the design problem. The procedure is flexible enough: it is, in general, open to other types of criteria and other types of agents. The De Novo approach is applied to the multiple-criteria supply chain design problem and provides a better solution than traditional approaches applied on fixed constraints. The approach is not oriented towards the optimization of some criteria, but seeks a trade-off-free solution by reformulating resource constraints only limited by the budget. The resources are saved by drawing only in the amount necessary to reach a balanced solution.

The multi-criteria approach is applied to the search for equilibrium for interested agents using biform game procedures. Biform games combine cooperative and non-cooperative approaches of game theory. The authors propose to divide biform games into sequential and simultaneous forms and to use a negotiation model for simultaneous games. The concept of co-opetition brings other aspects into design of sustainable supply chains, including other agents, such as competitors and complementors.

The procedure is open to be complemented by other concepts and approaches: for example, allocation mechanisms can be based on different approaches, such as Shapley values, contracts (Fiala, 2016a), auctions (Fiala, 2016b), and negotiations (Fiala, 1999). A combination of these concepts and approaches can be a powerful instrument for designing supply chains. The complex structure of the model can be captured using graph theory in a system consisting of an environment in which agents (nodes) create interactions (edges) and flows directed to meet the global demand. Some future research trends of sustainable supply chain management have been suggested. The proposed procedure tries to capture, at least partially, some of these trends.

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A PROMETHEE II-BELIEF APPROACH FOR MULTI-CRITERIA DECISION-MAKING PROBLEMS WITH INCOMPLETE INFORMATION

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Abstract

Multi-criteria decision aid methods consider decision problems in which many alternatives are evaluated on several criteria. These methods are used to deal with perfect information. However, in practice, it is obvious that this information requirement is too strict. In fact, the imperfect data provided by more or less reliable decision makers usually affect decision results, since any decision is closely linked to the quality and availability of information. In this paper, a PROMETHEE II-BELIEF approach is proposed to help multi-criteria decisions based on incomplete information. This approach solves problems with incomplete decision matrix and unknown weights within PROMETHEE II method. On the basis of belief function theory, our approach first determines the distributions of belief masses based on PROMETHEE II's net flows, and then calculates weights. Subsequently, it aggregates the distribution masses associated with each criterion using Murphy's modified combination rule in order to infer a global belief structure. The final alternative ranking is obtained via pignistic probability transformation. A case study of a real-world application concerning the location of a treatment center of waste from healthcare activities with infectious risk in the center of Tunisia is studied to illustrate the detailed process of the PROMETHEE II-BELIEF approach.

Keywords: multiple criteria aid, incomplete information, PROMETHEE II method, belief function theory.

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1 Introduction

Multi-criteria decision making deals with choosing, ranking or sorting alternatives on the basis of qualitative or quantitative criteria and preference judgments expressed by the decision maker. The literature presents many methods in decision analysis which come up with a satisfying decision. However, in order to be implemented, these methods assume that perfect information is available. That means the evaluations of the alternatives on the criteria as well as the preference parameters are known as exact numbers. Nevertheless, in practice, missing evaluations or imprecise information can occur. The information provided by the decision maker is usually imperfect because of its subjectivity. In fact, subjective information provided directly by the decision maker can hardly be applied successfully. The information imperfection includes the aspects of inconsistency, imprecision, incompleteness and uncertainty.

In this study, we will focus on the incompleteness of information. The incomplete information results from limited precision of human assessments which reduces its effectiveness in many applications. It may alter the final decision in practical situations, thus resulting in a gap between theory and practice. This gap is due to the fact that decision maker's preferences are not yet structured enough in his mind to allow the application of the decision making methods. For example, the decision maker cannot supply exact estimations of some parameters or he is not willing or able to define a stable preference structure or his complete evaluations of the consequences in the way required by the method. This inability might be due to his indisposition or his fear to decide exactly (Weber, 1987). Kim and Ahn (1999) claim that the possible reasons of the incompleteness of information provided by the decision maker are: (1) a decision should be made under time pressure and lack of data, (2) many criteria are intangible, as they reflect social and environmental impacts, (3) the decision maker has limited attention and information processing capability (Kahneman, Slovic and Tversky, 1982; Park et al., 1996), and (4) all group members do not have the same expertise in the given field (Ramanathan and Ganesh, 1994).

In this paper, we consider the PROMETHEE II method in which some evaluations of alternatives with respect to each criterion in the decision matrix are missing and criteria weights are unknown. In order to model the information incompleteness, the approach developed in this paper incorporates belief function theory with PROMETHEE II, the well known multi-criteria aggregation method. PROMETHEE II (Brans and Vincke, 1985; Brans, Vincke and Mareschal, 1986) is designed to solve complex problems involving multiple criteria. It presents several advantages and is considered a simple and clear method. It can also manage quantitative and qualitative criteria simultaneously and can solve the problem of incommensurability of measurement units (Sen et al., 2015). However, the standard PROMETHEE has many drawbacks. One of them is that it is time-consuming due to the high number of comparisons to be performed before any ranking can be evaluated; the number of comparisons rises quickly with the number of alternatives and criteria (Tscheikner-Gratl et al., 2017). Moreover, there is no allowance for ignorance with respect to types of alternatives and available criteria. It is also difficult for the decision maker to fix the shape of the criteria function and of parameter values (criteria weights, preference and indifference thresholds) (Sen et al., 2015). Indeed, the information concerning the parameter values provided by the decision maker is subjective and not very reliable, since it is based only on his experience, his intuition and his psychological state (Moalla Frikha, Chabchoub and Martel, 2011). Furthermore, the standard PROMETHEE II deals with perfect information only and not with incomplete, uncertain or imprecise one.

In order to alleviate these difficulties, we propose in this paper an approach integrating belief function theory into the PROMETHEE II method. Belief function theory (Dempster, 1967; Shafer, 1976) is a useful tool for representing and managing imperfect knowledge; it provides a suitable framework for dealing with incomplete data. In addition, it presents the advantage of combining distribution masses and taking decisions.

Incompleteness of information in the PROMETHEE II method can appear at different levels. Indeed, PROMETHEE II relies not only on the evaluations of alternatives with respect to criteria, but also on preference parameters, such as preference functions' thresholds and criteria weights. In our approach we focus on the incomplete decision matrix and unknown weights. The incompleteness of the decision matrix derives either from the decision maker's ignorance of some evaluations from the beginning or from the combination of alternatives having similar evaluations for a given criterion.

The paper is organized as follows: A literature review related to this research is presented in Section 2. Section 3 is devoted to a description of the PROMETHEE II method. A brief presentation of belief function theory is defined in Section 4. Section 5 develops the proposed PROMETHEE-BELIEF approach. In section 6, some examples are introduced to illustrate our approach and compare it with standard PROMETHEE. A case study of a real-world application related to management of waste from healthcare activities with infectious risk is included in Section 7. Section 8 concludes the paper and outline further research.

2 A literature review

In the literature, several studies were carried out to help decisions based on imperfect information. For instance, Tacnet proposed ER-MCDA (Tacnet, 2009; Tacnet, Batton-Hubert and Dezert, 2009; Tacnet, Batton-Hubert and Dezert, 2010) to handle imprecise and uncertain information through a combination of Analytic Hierarchy Process (AHP) and belief function theory. In addition, Dezert et al. (2010) introduced imprecise evaluations of subsets and new discounting techniques. Thereafter, Tacnet and Dezert (2011) developed the COWA-ER for decision making under uncertainty to take into account imperfect evaluations of alternatives and unknown beliefs about groups of possible scenarios. Moreover, Hyde, Mayer and Colby (2003) proposed generalized criterion functions incorporated in PROMETHEE in order to take the uncertainty in the criteria performance values into consideration. Likewise, Pelissari et al. (2019) proposed a new method for sorting decision-making problems capable of dealing with multiple imperfect data and with criteria weight elicitation. Also, Ennaceur, Eloudei and Lefèvre (2012) extended the AHP method to an uncertain environment, where the uncertainty is represented through the Transferable Belief Model (TBM) in both the criterion and the alternative levels. Furthermore, Abdennadher, Boujelben and Ben Amor (2013) were interested in the PROMETHEE method where the alternatives are evaluated on a set of ordinal criteria and where the evaluations can be uncertain and imprecise. In addition, Ennaceur, Eloudei and Lefèvre (2014; 2016) and Altieri et al. (2017) proposed an extension of the belief Analytic Hierarchy Process (AHP) method based on the belief function where information is uncertain and imprecise. Later, Chen et al. (2017) suggested a novel method, based on the Dempster-Shafer evidence theory and Analytic Hierarchy Process, to handle the dependence in human reliability analysis. Their model can deal with uncertainty in an analyst's judgment and reduce the subjectivity in the evaluation process. Furthermore, Dezert, Han and Tacnet (2017) integrated belief functions into TOPSIS and proposed Imp-BF-TOPSIS to deal with imprecise score values (intervals of real numbers). Besides uncertainty and imprecision, various papers discussed incompleteness, the third aspect of imperfection. Several methods solve this type of problems using two steps procedures. In the first step, they proceed by completing the missing values in the decision matrix through applying a learning process (Morad, Svrcek and McKay, 2000; Hong, Tseng and Wang, 2002; Fortes et al., 2006) or heuristic rules (Raymond, 1986; Kaufman, 1988; Quinten and Raaijmakers, 1999; Quinlan, 1993), or by removing the alternatives or criteria with incomplete information from the problem. In the last case, the problem structure becomes distorted. The second step consists in applying the multi-criteria method to solve the problem. All these methods present disadvantages. Heuristic methods lack scientific foundations, since they calculate the missing value through replacing it by the mean of all known values or by the most frequent value under the criterion. Learning methods present also the drawback of complexity in their application to incomplete decision matrix.

Many other methods dealing with multi-criteria problems with incomplete information were developed on scientific basis. Weber (1987) presented an overview of existing methods which are particularly suitable for handling incomplete information. Thereafter, several approaches have been developed to make multi-criteria group decision under incomplete data (Kim and Ahn, 1997; 1999; Kim, Choi and Kim, 1999; Ju, 2014). Also, Hua, Gong and Xu (2008) proposed the DS-AHP approach for the multi-criteria decision making problems with incomplete decision matrix. This approach first identifies all possible focal elements with the incomplete decision matrix, and then calculates the basic probability assignment of each focal element. Next, the belief interval of each decision alternative is evaluated using belief function theory. Subsequently, preference relations are determined by comparing belief intervals. Moreover, Ren and Lutzen (2017) developed a novel multi-criteria decision-making method that combines Dempster-Shafer theory and a trapezoidal fuzzy Analytic Hierarchy Process for alternative energy source selection under incomplete information conditions. Likewise, Haseli, Sheikh and Shib (2020) proposed the Base-Criterion Method, which is capable to find lost comparisons in the worst terms of the incomplete pairwise comparison matrix between base-criterion and other criteria. Moreover, Fan and Deer (2005) developed a new approach to determine the parameter ρ using belief function theory under incomplete information. In order to calculate the expected utility, an evaluation about the value of the parameter ρ must be known. The authors assume that in the case of absence of evidence available about this value, the decision maker must provide partial information about it. This incomplete information is introduced into the developed model to solve the decision-making problem. Furthermore, Ben Amor and Mareschal (2012) proposed an approach to solve decision problems under incomplete decision matrix, using different models of the imperfection representation, that is, probabilities, fuzzy logic and possibility theory. In addition, Ahn (2015) presented a method dealing with incomplete attribute weights using extreme points.

For multi-criteria decision problems with incomplete information, the majority of papers dealt with the AHP method and integrated belief function theory (Beynon, Curry and Morgan, 2000; Hua, Gong and Xu, 2008; Wang, 2006; Hsu and Wang, 2011; Huang et al., 2014; Ju, 2014). However, in our paper we are interested in incorporating belief function theory into the PROMETHEE method.

3 The PROMETHEE approach

PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) (Brans and Vincke, 1985) is a multi-criteria decision making method for ranking alternatives evaluated on several conflicting criteria according to a decision maker's preferences. It is characterized by its simplicity and clearness (Brans, Vincke and Mareschal, 1986). Therefore, it has been applied in various area including environment, management, hydrology, business, finance, logistics and transportation, energy, manufacturing, and other fields (Behzadian et al., 2010). PROMETHEE is based on the principle of pairwise comparison of alternatives with respect to each criterion. The PROMETHEE methods involve five steps:

Step 1: Calculation of the performance differences from the decision matrix: The performance difference between each pair of alternatives a_i and a_j with respect to each criterion k is calculated as follows:

$$d_{ij}^{k} = \begin{cases} g_k(a_i) - g_k(a_j) & \text{for a criterion to maximize} \\ g_k(a_j) - g_k(a_i) & \text{for a criterion to minimize} \end{cases} \tag{1}$$

where $g_k(a_i)$ and $g_k(a_j)$ show the performance of the alternatives a_i and a_j, respectively, with regard to criterion k.

Step 2: A preference function P_{ij}^{k} has to be associated with each criterion to model the decision-maker's preferences with respect to each criterion k. When the decision maker compares two alternatives a_i and a_j, P_{ij}^{k} represents the intensity of preference for a_i over a_j, considering only the criterion k. The preference function's value varies between 0 and 1 and is assessed differently according to the criterion shape. The authors of PROMETHEE proposed six shapes of criteria functions that seem to cover most of the needs occurring in the real world (usual criterion, quasi criterion, criterion with linear preference, level criterion, criterion with linear preference and indifference area and Gaussian criterion). Depending on the manner in which the DM's preference increases with the difference between assessments of alternatives a_i and a_j with respect to the criterion k, the DM fixes the form of P_{ij}^{k} and the associated parameters for every criterion.

To define the preference function, it is necessary to fix the values of indifference thresholds (q_k), preference thresholds (p_k) and inflexion point of the Gaussian curve (Gaussian threshold's σ_k).

Step 3: Calculation of the aggregated preference index π_{ij}: For each pair of alternatives, an aggregated preference index is calculated as follows:

$$\pi_{ij} = \sum_{k=1}^{n} w_k P_{ij}^{k} \tag{2}$$

where w_k is the relative importance coefficient given to each criterion k with $w_k \geq 0$ and $\sum_{k=1}^{n} w_k = 1$. The greater w_k, the more important the criterion.

The aggregated preference index represents the degree of preference for a_i over a_j with respect to all the criteria simultaneously.

Step 4: Calculation of outranking flows: For each alternative, when compared with ($m - 1$) other alternatives, a positive, a negative and a net flow are calculated as:

$$\phi_i^{+} = \frac{1}{m-1} \sum_{j \neq i} \pi_{ij} \tag{3}$$

$$\phi_i^{-} = \frac{1}{m-1} \sum_{j \neq i} \pi_{ji} \tag{4}$$

where ϕ_i^{+} and ϕ_i^{-} denote the positive and negative flows, respectively, for alternative a_i. A positive flow indicates the strength of a_i with regard to other alternatives. Similarly, a negative flow indicates the weakness of a_i with regard to other alternatives.

The net outranking flow ϕ_i is the difference between the outgoing and the incoming flows. It is obtained as follows:

$$\phi_i = \phi_i^{+} - \phi_i^{-} \tag{5}$$

Step 5: Alternative ranking

The solution of a particular decision problem depends on accepting or not the incomparability. If we accept it, we choose PROMETHEE I; otherwise, PROMETHEE II. PROMETHEE I generally leads to a ranking of the alternatives by a partial pre-order, since it accepts the incomparability, whereas PROMETHEE II leads to a ranking of alternatives by a total pre-order, as it does not accept the incomparability. According to PROMETHEE II, all the alternatives are ranked from the best to the worst one. In fact, ϕ_i can be positive or negative. The larger ϕ_i, the more x_i outranks the other alternatives, and the less it is outranked. Thus:

- a_i outranks a_j if and only if $\phi_i > \phi_j$ and
- a_i is indifferent to a_j if and only if $\phi_i = \phi_j$

Instead of calculating the aggregated preference index and multi-criteria flows (steps 3 and 4), we can simply calculate uni-criteria flows, that means the positive, negative and net flows for each alternative with respect to each criterion separately. These flows are as follows:

$$\phi_i^{k+} = \frac{1}{m-1}\sum_{j\neq i} P_{ij}^k \quad \forall \ k = 1,\dots,n \tag{6}$$

$$\phi_i^{k-} = \frac{1}{m-1}\sum_{j\neq i} P_{ji}^k \quad \forall \ k = 1,\dots,n \tag{7}$$

$$\phi_i^k = \phi_i^{k+} - \phi_i^{k-} \tag{8}$$

The multi-criteria net flow is obtained as the weighted sum of mono-criteria net flows:

$$\phi_i = \sum_{k=1}^{n} \phi_i^k w_k \tag{9}$$

Alternative are ranked in decreasing order of their net flow values (step 5).

4 Belief function theory

The belief function theory (Dempster, 1967; Shafer, 1976) is a general framework for modeling uncertainty and imprecision when the available information is imperfect. It is an interesting tool for information fusion and decision making using, combination and decision rules, respectively.

A belief function model is defined by a finite and exhaustive set Θ called the *frame of discernment* of the problem under consideration. The set of all subsets of Θ is called the *power set of* Θ and denoted by 2^Θ.

A *Basic Probability Assignment function* (BPA) is a mapping m: $2^\Theta \rightarrow [0, 1]$. It assigns to every subset $A \subseteq \Theta$ a number $m(A)$, called the mass of A, which represents the degree of belief attributed exactly to A, and to none of its subsets. This function must satisfy the following conditions: $m(\emptyset) = 0$, and $\sum\{m(A) / A \subseteq \Theta\} = 1$.

When $m(A) > 0$, A is called a *focal element* of m. The set of focal elements of m is denoted $\mathfrak{J}$ and the pair $(\mathfrak{J}, m)$ is called the *body of evidence*.

A BPA can be represented equivalently by its associated belief and plausibility functions. A *belief function* is a mapping Bel: $2^{\Theta} \to [0, 1]$, defined as:

$$Bel(A) = \sum_{B \subseteq A} m(B) \quad \forall \ A \subseteq \Theta \tag{10}$$

$Bel(A)$ measures the total belief (credibility) completely committed to $A \subseteq \Theta$.

A *plausibility function* is a mapping Pl: $2^{\Theta} \to [0, 1]$, defined as:

$$Pl(A) = \sum_{B \cap A \neq \varnothing} m(B) \quad \forall \ A \subseteq \Theta \tag{11}$$

$Pl(A)$ can be regarded as the maximum amount of belief that could be given to A.

In belief function theory, combination is an operation that plays a fundamental role. The BPAs generated by several distinct sources are combined to yield a global BPA that synthesizes the data provided by the different sources.

Let m_i and m_j denote two BPAs obtained from two distinct sources i and j in the same frame of discernment Θ. According to Dempster's rule of combination (Shafer, 1976), we have:

$$m(A) = \frac{1}{1-K} \sum_{B \cap C = A} m_i(B) \times m_j(C) \ \forall \ A \subseteq \Theta, \text{ where } K = \sum_{B \cap C = \varnothing} m_i(B) \times m_j(C) \tag{12}$$

Dempster's combination rule verifies some interesting properties (commutative and associative). However, in some situations, this rule cannot be used. When there are large conflicts between bodies of evidence, counterintuitive behavior will emerge (Zadeh, 1979). Other rules have been proposed to deal with the inconvenience of the loss of majority opinion. For instance, Murphy (2000) has, proposed, for the first time, the average rule where the belief mass of a subset $A \subseteq \Theta$ provided by independent sources are averaged to determine the global belief mass on A. Suppose there are n information sources providing n BPAs m_i for all $i = 1, \ldots, n$. The average rule is defined as:

$$m(A) = \frac{1}{n} \sum_{i=1}^{n} m_i(A) \quad \forall \ A \in \Theta \text{ and } A \neq \varnothing \tag{13}$$

This rule does not, however, offer convergence toward certainty. In fact, it is not always adequate to yield reasonable results, particularly when the evidence has a high degree of conflict. For this reason, Murphy has proposed another combination rule based on the idea of incorporating the average operation into Dempster's rule of combination. In this rule, all information sources are equally important, and, therefore, have the same weight ($1/n$) in the sum of evidence (equation 13). Murphy's averaging rule is recommended in cases when the objective is to preserve the opinion of the majority when one source contradicts several other, consistent sources.

In belief function theory, several decision rules are possible; they are, most of the times, applied to the choice of one hypothesis from among many. In order to obtain a decision without ambiguity, we should choose the hypothesis whose credibility is superior to the plausibility of any other hypothesis. Nonetheless, credibility and plausibility functions may generate, in several situations, different ranking of a single hypothesis. To overcome this inconvenience, other decision rules have been developed, based on either credibility or plausibility, such as the maximum of credibility and the maximum of plausibility. The first rule selects the hypothesis with the maximum of total belief and the second chooses the least contradicted hypothesis. However, the maximum of credibility has the drawback of not being used when focal elements are sets of hypothesis. Furthermore, Smets (2002) claims that the maximum of plausibility decision rule is subject to counterexamples. For that purpose, he transforms belief functions into a *pignistic probability function BetP* to make decisions. This transformation consists of distributing each mass $m(A)$ equally among the statements that compose $A \subseteq \Theta$. Formally, $BetP$ is defined as:

$$BetP(A) = \sum_{B \subseteq A} m(B) \frac{|A \cap B|}{|B|} \quad \forall \; A \subseteq \Theta \tag{14}$$

where $|B|$ is the cardinality of B. $BetP(A)$ can be viewed as a betting commitment to A and represents the total mass value that A can carry. The decision rule used consists in choosing the hypothesis with the maximum of pignistic probability.

$$\theta_{likely} = Arg \underset{\theta_i \in \Theta}{Max} \left(BetP\left(\theta_i\right)\right) \tag{15}$$

5 The PROMETHEE II-BELIEF approach

Let $P = \{A, C, g, f\}$ be a multi-criteria decision problem where $A = \{a_1, a_2, \ldots, a_m\}$ is a non-empty finite set of alternatives a_i and $C = \{c_1, c_2, \ldots, c_n\}$ is a non--empty finite set of decision criteria c_k. For each criterion c_k, $f : A \times C \to G$, $f(a_i, c_k) \in G$, where $g_k(a_i)$, an element of G, is called the evaluation of alternative i with respect to criterion k. If there is at least one criterion c_k and $g_k(a_i)$ which includes missing values, then problem P is called a multi-criteria decision making problem with incomplete information. Missing values in the decision matrix are denoted by asterisks, as are the unknown weights w_k associated with criterion k. For each criterion, the decision maker must provide at least two distinct evaluations of alternatives.

In the proposed PROMETHEE II-BELIEF, we first determine the focal elements, and then the distributions of belief masses using PROMETHEE's net flows. Thereafter, weights are calculated and the obtained distribution masses are combined using Murphy's modified combination rule in order to infer a global belief structure. The ranking of alternatives is based on pignistic probabilities.

5.1 Determination of focal elements from the incomplete decision matrix

Let $A = \{a_1, a_2, \ldots, a_m\}$ be the set of decision alternatives, equivalent to the frame of discernment, and A_h ($h = 1, \ldots, 2^m$) be a subset of A. Each subset $A_h \subseteq A$ such that $g_k(A_h) > 0$ is called a focal element, which can be defined from the decision matrix as follows:

Definition 1. For all a_i and $a_j \in A$ with $a_i \neq a_j$, if $g_k(a_i) = g_k(a_j)$, then a_i and a_j belong to the same focal element. Hence, both alternatives a_i and a_j are regrouped under one focal element.

Hence, from the decision matrix G we generate a new decision matrix $G' = g'_k(A_h)$, where $g'_k(A_h)$ is the evaluation of the subsets of alternatives A_h with respect to each criterion c_k ($k = 1, \ldots, n$; $h = 1, \ldots, p$; $p \leq 2^m$). The matrix G' does not contain all the 2^m subsets, but only the focal elements A_h. This step can considerably reduce the number of comparisons within the PROMETHEE II method.

5.2 Determination of belief mass distributions

Using the decision matrix G', we apply the PROMETHEE II method, which consists in comparing not only decision alternatives, but also subsets of alternatives (focal elements) A_h and A_l with $h \neq l$, pairwise with respect to each criterion k. We calculate the preference function values and then the uni-criteria positive, negative and net flows of each subset A_h for each criterion separately (equations 6-8).

In the PROMETHEE II method, net flows can be positive or negative. In order to transform net flows into belief mass distributions, and since the belief mass $m_k(A_h^k)$ must be positive, we first calculate an exponential function $\lambda^{\phi_h^k}$ for each net flow ϕ_h^k for $h = 1, \ldots, p$; $p \leq 2^m$; $k = 1, \ldots, n$. The base of the exponential function is λ.

If λ is positive and smaller than 1, $\lambda^{\phi_h^k}$ is a decreasing function. However, the belief mass distribution based on $\lambda^{\phi_h^k}$ must be an increasing function of the net flow. Hence, the base should be greater than 1. The bigger the base λ, the faster the exponential function shrinks for low values of net flow.

Next, the values of $\lambda^{\phi_h^k}$ must be normalized in order to obtain belief mass distributions for all the focal elements with respect to each criterion.

Definition 2. For all focal elements $A_h^k \in 2^A$, the belief mass distribution associated with each criterion *k* is defined as:

$$m_k(A_h^k) = \frac{\lambda^{\phi_h^k}}{\sum_{h=1}^{p} \lambda^{\phi_h^k}} \quad \forall\ k = 1,\ldots,n \text{ and } \forall\ h = 1,\ldots,p\ ;\ p \leq 2^m \tag{16}$$

The normalization rule (equation 16) guarantees that $m_k(A_h^k) \in [0, 1]$ and $\sum\left(m_k(A_h^k) / A_h^k \subseteq \mathcal{A}\right) = 1$.

Since the belief mass distribution $m_k(A_h^k)$ is an increasing function of the net flow, the higher the net flow, the greater the mass and the more preferred the given subset. The belief mass measures the strength of the subset A_h^k compared to all other subsets with respect to each criterion *k*. The more important the belief mass, the more focal element A_h^k dominates the others.

5.3 Criteria weight determination

To solve the problem of data subjectivity while providing precise weight values, we propose to determine criteria weights based on belief mass distributions deduced from mono-criteria net flows. The new method relies on the difference between two pignistic probabilities associated with two belief masses.

Formally, let us consider $BetP_k$ and $BetP_{k'}$, pignistic probabilities derived from belief masses associated respectively with criteria *k* and *k'*. The distance between betting commitments $BetP_k$ and $BetP_{k'}$ (Liu, 2006) is denoted by *difBetP* and defined as:

$$DifBetP(k,k') = \underset{A_h \subseteq \mathfrak{I}}{Max}\left(\left|BetP_k(A_h) - BetP_{k'}(A_h)\right|\right) \tag{17}$$

Definition 3. Assume *n* pignistic probabilities generated from *n* criteria. We define the similarity degree between two criteria *k* and *k'* as:

$$Sim(k,k') = 1 - DifBeP(k,k') \tag{18}$$

Definition 4. The importance degree *Imp* of each criterion is defined as:

$$Imp(k) = \sum_{\substack{k=1 \\ k \neq k'}}^{n} Sim(k, k') \quad \forall \ k = 1, \ldots, n \tag{19}$$

The more similar to other criteria criterion *k* is, the more important it is.

Definition 5. The normalized weight vector w_k of all criteria $k = 1, \ldots, n$ is defined as:

$$w_k = \frac{Imp(k)}{\sum_{k=1}^{n} Imp(k)} \tag{20}$$

This transformation ensures that weight values are normalized so that $0 \leq w_k \leq 1$ and $\sum_{k=1}^{n} w_k = 1$. Obviously, the higher the importance degree of a criterion, the higher its weight.

5.4 Combination of belief mass distribution

In the PROMETHEE II method, all mono-criteria net flows must be aggregated using weight values in order to get multi-criteria net flows. Similarly, in our proposed method, the aggregation operation involves combining belief mass distributions derived from mono-criteria net flows. The combination allows for extracting a global belief mass structure which is equivalent to a multi-criteria net flow.

To combine all deduced belief mass distributions, Murphy's rule is used. This rule considers all criteria as equally important, and therefore as having the same weight (1/*n*), which is not always reasonable in real-life cases since some criteria may be more important than others. Hence, criteria weights must be taken into consideration. We propose to modify Murphy's combination rule using the obtained objective weight values instead of equal weights. Thereafter, a modified Murphy's combination rule is applied to compute weighted average mass $\overline{m}$ as follows:

$$\overline{m}(A_h) = \sum_{k=1}^{n} \left(w_k \times m_k(A_h) \right) \ \forall A_h \subseteq \mathcal{A} \tag{21}$$

The weighted average mass must be combined $n - 1$ times using Dempster's rule to obtain an overall belief mass *m*:

$$m(A_h) = \underbrace{\overline{m}(A_h) \oplus \cdots \oplus \overline{m}(A_h)}_{(n-1)\text{ times}} \quad \forall A_h \subseteq \mathcal{A} \tag{22}$$

Our proposed aggregation approach presents many advantages. Since in multi-criteria problems criteria are conflicting and the determination of the best compromise solution is required, our proposed aggregation approach solves the problem of conflicting criteria by preserving the evaluation of the majority of criteria. In addition, it presents the advantage of considering criteria weights to calculate the overall belief mass distribution for each focal element. Finally, the global information obtained from modified Murphy's rule will be used for alternative ranking.

5.5 Alternative ranking

To rank alternatives from the best to the worst, we must first transform the obtained overall belief mass into pignistic probability *BetP* using pignistic transformation, defined as:

$$BetP(a_i) = \sum_{a_i \in A_h} \frac{m(A_h)}{|A_h|} \quad \forall \;\; a_i \in \mathcal{A} \tag{23}$$

This transformation allows to have a global evaluation for each alternative. The ranking of alternatives is performed in decreasing order of pignistic probabilities.

6 Experimental settings

In this section, a few examples are introduced to illustrate our approach and to compare it with the standard PROMETHEE II. We consider the case of a complete decision matrix with equal evaluations of some alternatives, the particular cases where an alternative is dominated, cases where an alternative is dominant, and, finally, the case of an incomplete decision matrix.

6.1 Complete decision matrix with some equal alternative evaluations

In order to compare our proposed approach PROMETHEE II-BELIEF with the standard PROMETHEE II, we consider a multi-criteria illustrative example where the elements of the decision matrix are known. The decision matrix contains alternatives with the same evaluations according with respect to some criteria. The decision matrix G is shown in Table 1.

Table 1: Decision matrix G

	C_1	C_2	C_3	C_4	C_5
A	20	15	4	$\underline{3}$	$\underline{3}$
B	52	20	3	4	$\underline{3}$
C	14	35	2	2	4
D	5	63	$\underline{1}$	1	2
E	8	10	$\underline{1}$	$\underline{3}$	1

The shapes of the criteria as well as their associated parameters (indifference thresholds q_k, preference thresholds p_k and Gaussian threshold σ_k) are:

- C_1 is a Gaussian criterion with $\sigma_1 = 10$
- C_2 is a level criterion with $q_2 = 5$ and $p_2 = 10$
- C_3 is a criterion with linear preference and indifference area with $q_3 = 0.5$ and $p_3 = 1.5$
- C_4 is a level criterion with $q_4 = 1$ and $p_4 = 1.5$
- C_5 is a quasi criterion with $q_5 = 1$

PROMETHEE II-BELIEF

In order to apply PROMETHEE II-BELIEF, decision matrix G' evaluating focal elements, belief mass distributions as well as criteria weight values are presented in appendix A (Table A.1-A.3).

The obtained weight values are introduced to modified Murphy's rule to combine all belief mass distributions. Thereafter, we transform the obtained overall belief mass into pignistic probabilities *BetP* using pignistic transformation. The results are as follows:

$BetP\ (A) = 0.4527$; $BetP\ (B) = 0.2752$; $BetP\ (C) = 0.1115$; $BetP\ (D) = 0.0286$; $BetP\ (E) = 0.1320$

The alternatives are ranked in decreasing order of their pignistic probabilities. We obtain the following ranking: $A \succ B \succ E \succ C \succ D$.

Standard PROMETHEE II

We integrate the weight values obtained from PROMETHEE II-BELIEF into the standard PROMETHEE II. The following net flows are then obtained:

$$\phi_A = 0.4590;\ \phi_B = 0.2141;\ \phi_C = -0.1556;\ \phi_D = -0.6460;\ \phi_E = 0.1285$$

According to the net flows values, we rank the alternatives from the best to the worst. We obtain the following ranking: $A \succ B \succ E \succ C \succ D$

6.2 Particular case: Dominant alternative

In this example (Table 2), we assume that alternative *B* is dominant over all criteria.

Table 2: Decision matrix with a dominant alternative

	C_1	C_2	C_3	C_4	C_5
A	20	15	3	3	5
B	**4**	**8**	**4**	**4**	**5**
C	14	35	2	2	4
D	5	63	1	1	2
E	8	10	1	3	1

PROMETHEE II-BELIEF

Calculations based on PROMETHEE II's net flows, are presented in appendix B (Table B.1-B.3). The overall belief mass is transformed into pignistic probabilities *BetP*, which are:

BetP (*A*) = 0.2037; *BetP* (*B*) = 0.6580; *BetP* (*C*) = 0.0582; *BetP* (*D*) = 0.0121; *BetP* (*E*) = 0.0680

The alternatives are ranked according to the decreasing order of their pignistic probabilities. The obtained ranking is: $B \succ A \succ E \succ C \succ D$

Standard PROMETHEE II

Using the obtained weights, PROMETHEE II's net flows are calculated:

$$\phi_A = 0.2965;\ \phi_B = 0.6874;\ \phi_C = -0.2257;\ \phi_D = -0.7826;\ \phi_E = 0.0244$$

We obtain the following ranking of the alternatives: $B \succ A \succ E \succ C \succ D$.

The results confirm the assumption of the particular case and show that alternative *B* is dominant, either for the PROMETHEE II-BELIEF approach or the standard PROMETHEE II. In fact, both *BetP*(*B*) and ϕ_B are far greater than other alternative values.

6.3 Particular case: Dominated alternative

In this example, we present a particular case where alternative *D* is dominated by all other alternatives. It has high evaluations with respect to the criteria to be minimized and low evaluations with respect to the criteria to be maximized. The assessments are presented in Table 3.

Table 3: Decision matrix with a dominated alternative

	C_1	C_2	C_3	C_4	C_5
A	20	15	4	3	3
B	52	20	3	4	3
C	14	35	2	2	4
D	**55**	**63**	**1**	**1**	**1**
E	8	10	1	3	2

PROMETHEE II-BELIEF

Calculations using the PROMETHEE II-BELIEF approach are given in appendix C (Table C.1-C.3).

Pignistic probabilities *BetP* are:

BetP (*A*) = 0.4667; *BetP* (*B*) = 0.2443; *BetP* (*C*) = 0.1202; *BetP* (*D*) = 0.0062; *BetP* (*E*) = 0.1626

The ranking of the alternatives is: $A \succ B \succ E \succ C \succ D$

Standard PROMETHEE II

Applying the standard PROMETHEE II, we obtain the following net flows:

$$\phi_A = 0.5406;\ \phi_B = 0.2539;\ \phi_C = -0.0869;\ \phi_D = -0.9939;\ \phi_E = 0.2863$$

The ranking of the alternatives is: $A \succ E \succ B \succ C \succ D$

We can clearly see that alternative *D* is dominated by all other alternatives, either for our approach or for the standard PROMETHEE II. In fact, *BetP*(*D*) has a negligible value close to 0, while ϕ_D is very low.

In all cases, we obtain the same ranking of alternatives using either PROMETHEE II-BELIEF or the standard PROMETHEE II. Nevertheless, our proposed approach presents three advantages over the standard PROMETHEE II. First, it reduces considerably the number of pairwise comparisons by regrouping alternatives with the same evaluation under the same focal element. Second, PROMETHEE II-BELIEF allows to determine objective criteria weights on the basis of scientific foundations, hence reducing subjectivity. Third, this approach is capable of solving multi-criteria decision problems with an incomplete decision matrix, which is not feasible using the standard PROMETHEE II.

6.4 Incomplete decision matrix

In this example, we omit some alternative evaluations with respect to some criteria in order to obtain the following incomplete decision matrix (Table 4):

Table 4: Incomplete decision matrix

	C_1	C_2	C_3	C_4	C_5
A	20	15	*	3	4
B	52	*	3	*	3
C	14	35	2	2	*
D	5	63	4	1	2
E	*	10	1	4	1

PROMETHEE II-BELIEF

Appendix D illustrates the details of the calculations of belief masses and criteria weights for this example (Table D.1-D.3).

Pignistic probabilities *BetP* associated with each alternative are:

$BetP\ (A) = 0.4012$; $BetP\ (B) = 0.0206$; $BetP\ (C) = 0.0735$; $BetP\ (D) = 0.3447$; $BetP\ (E) = 0.1600$

The ranking of the alternatives is: $A \succ D \succ E \succ C \succ B$

Standard PROMETHEE II

The standard PROMETHEE II cannot be applied when the decision matrix is incomplete.

7 Real-world applications

Since the management of waste from healthcare activities is of particular interest worldwide and, more specifically, in our Tunisian context, we focus in this paper on waste from healthcare activities with infectious risk. This interest originates from the fast development that recorded the structure of public and private health care in Tunisia and which has been accompanied by a corresponding increase in the number of patients treated, as well as the quantities of waste generated within health facilities. Because infectious waste is suspected to contain pathogens (bacteria, viruses, parasites or fungus) in sufficient concentration or quantity to cause disease in susceptible hosts, it has various irreversible impacts on public health and deleterious effects on the environment. Therefore, improvement in waste management was considered one of the most important concerns of the national system for the management of hazardous waste in Tunisia. An efficient management of hazardous waste consists in optimizing the location of undesirable facilities (the treatment, recycling and/or destruction plant). Location of undesirable plants is a complex process, because it combines social, environmental, political and technical objectives.

In this paper, the potential of the PROMETHEE-BELIEF approach is illustrated by a real-life case study. The Ministry of Environment considers the problem of choosing the best site for installing a new waste treatment center for healthcare activities with infectious risk in the center of Tunisia. The potential sites are nine Tunisian cities: Sousse (a_1: industrial area of Kalaa Kebira), Monastir (a_2: industrial area of Jemmel), Mahdia (a_3: industrial area of Ksour essef), Sfax (a_4: industrial area of Hencha), Gabes (a_5: industrial area of south Gabes), Kairouan (a_6: Industrial area of Hajeb Layoun), Sidi Bouzid (a_7: Industrial area of Sidi Bouzid Ouest), Kasserine (a_8: Industrial area of North of Kasserine) and Gafsa (a_9: Industrial area of South of Gafsa) (see Figure 1). These decision alternatives are evaluated with respect to five criteria.

Figure 1: Map of Tunisia

- C_1 (Installation cost in millions of TND): It includes land acquisition costs, construction and civil engineering costs, labor and administrative costs and operating costs. The Ministry of Environment intends to choose the site that minimizes this cost. Therefore, installation cost is selected in this paper as a criterion to be minimized.
- C_2 (Population in the vicinity of the waste treatment center): Infectious waste should always be assumed to potentially contain a variety of pathogenic microorganisms. This is because the presence or absence of pathogens cannot be determined at the time a waste item is produced and discarded into a container. Pathogens in infectious waste that is not well managed may enter

the human body through several routes. For this reason, the waste center presents risk to residents. Thus, it should be located in an area that is scarcely populated. Consequently, C_2 is a criterion to be minimized.

- C_3 (Quantity of waste of healthcare activities with infectious risk collected from all hospitals of the city, expressed in tons per year): The more waste healthcare facilities of the city generate, the greater the need to create a new center. Hence, quantity of waste is a criterion to be maximized.
- C_4 (Number of existing centers): The fewer treatment centers the city has, the greater the need to create new centers. Accordingly, C_4 is a criterion to be minimized.
- C_5 (Proximity to urban areas): Since treatment centers operate on waste from infectious healthcare activities, they present pollution drawbacks and risks to inhabitants and environment. Therefore, the new center should be located far from the city center, where the population is concentrated and soil and groundwater are intensively used. Alternative sites are evaluated with respect to this criterion on a 5-level scale. Level 1 and level 5 indicate that the potential waste treatment center is close and far from the city center, respectively. So, the fifth criterion is a criterion to be maximized.

The evaluations of alternative sites with respect to each criterion are presented in the following decision matrix (Table 5). In the industrial area of Hajeb Layoun in Kairouan city, the persons responsible for the project in the ministry of environment have not yet decided which location to choose because they have three proposals. Therefore, the installation cost in Kairouan remains unknown. Moreover, we fail to have the exact waste quantity since some hospitals, either in Monastir or in Gabes, present defaulting information systems. In addition, the population in the industrial area of Sidi Bouzid Ouest, as well as in the industrial area of North of Kasserine, is difficult to assess due to lack of information in municipalities of these cities. Subsequently, some evaluations are missing and the decision matrix is incomplete (asterisks denote the missing values).

Table 5: Incomplete decision matrix

City	Installation cost	Population	Waste quantity	Existing centers	Proximity to urban areas
Sousse	1350	51 196	386	2	3
Monastir	1200	55 272	*	0	4
Mahdia	1 200	48 799	208	0	3
Sfax	1 300	47 170	460	3	5
Gabes	1 350	61 699	*	0	1
Kairouan	*	35 403	113	1	5
Sidi Bouzid	850	*	90	0	2
Kasserine	850	*	104	0	1
Gafsa	900	90 742	92	2	2

In order to apply PROMETHEE II-BELIEF, we transform the decision matrix *G* (evaluating alternatives) to another decision matrix *G'* (evaluating focal elements). We regroup the alternatives with the same evaluation with respect to a criterion under a focal element. The focal element decision matrix is presented in Table 6.

Table 6: Decision matrix of focal elements for the waste treatment center location problem

City	Installation cost	Population	Waste quantity	Existing centers	Proximity to urban areas
Sousse	*	51 196	386	*	*
Monastir	*	55 272	*	*	4
Mahdia	*	48 799	208	*	*
Sfax	1 300	47 170	460	3	*
Gabes	*	61 699	*	*	*
Kairouan	*	35 403	113	1	*
Sidi Bouzid	*	*	90	*	*
Kasserine	*	*	104	*	*
Gafsa	900	90 742	92	*	*
Sousse ∪ Gabes	1350	*	*	*	*
Monastir ∪ Mahdia	1200	*	*	*	*
Sidi Bouzid ∪ Kasserine	850	*	*	*	*
Sousse ∪ Gafsa	*	*	*	2	*
Monastir ∪ Mahdia ∪ Gabes ∪ Sidi Bouzid ∪ Kasserine	*	*	*	0	*
Sousse ∪ Mahdia	*	*	*	*	3
Sfax ∪ Kairouan	*	*	*	*	5
Gabes ∪ Kasserine	*	*	*	*	1
Sidi Bouzid ∪ Gafsa	*	*	*	*	2

The criteria shapes and their associated parameters are defined as:

- Installation cost is a level criterion with $q_1 = 100$ and $p_1 = 300$.
- Population is a criterion with linear preference and indifference area with $q_2 =$ $= 10\ 000$ and $p_2 = 20\ 000$.
- Waste quantity is a level criterion with $q_3 = 20$ and $p_3 = 100$.
- Existing centers is a usual criterion.
- Proximity to urban areas is a quasi criterion with $q_5 = 3$.

We apply the PROMETHEE II method based on *G'*, then we determine the uni-criteria net flows associated with each focal element with respect to each criterion, and then we apply equation 16 to determine the belief mass distribution for each focal element with respect to each criterion. We assign the value 2 to the parameter λ because it must be greater than 1 and should not have a high value. These belief masses are given in Table 7.

Table 7: Belief mass distributions for the waste treatment center location problem

City	Installation cost	Population	Waste quantity	Existing centers	Proximity to urban areas
Sousse	0	0.0571	0.3171	0	0
Monastir	0	0.0416	0	0	0.1818
Mahdia	0	0.0796	0.0396	0	0
Sfax	0.1710	0.0997	0.6341	0.0118	0
Gabes	0	0.0238	0	0	0
Kairouan	0	0.6976	0.0050	0.1882	0
Sidi Bouzid	0	0	0.0012	0	0
Kasserine	0	0	0.0018	0	0
Gafsa	0.3879	0.0006	0.0012	0	0
Sousse ∪ Gabes	0.0121	0	0	0	0
Monastir ∪ Mahdia	0.0334	0	0	0	0
Sidi Bouzid ∪ Kasserine	0.5486	0	0	0	0
Sousse ∪ Gafsa	0	0	0	0.0471	0
Monastir ∪ Mahdia ∪ Gabes ∪ Sidi Bouzid ∪ Kasserine	0	0	0	0.7529	0
Sousse ∪ Mahdia	0	0	0	0	0.1818
Sfax ∪ Kairouan	0	0	0	0	0.3637
Gabes ∪ Kasserine	0	0	0	0	0.0909
Sidi Bouzid ∪ Gafsa	0	0	0	0	0.1818

Subsequently, criteria weight values are calculated using equations 17-20 (Table 8).

Table 8: Criteria weights for the waste treatment center location problem

	w_1	w_2	w_3	w_4	w_5
w_k	0.1730	0.1873	0.2032	0.1812	0.2552

We see that the results are reasonable, since the criterion “proximity to urban areas” is far more important than the others. In fact, individuals close to infectious healthcare waste are at a potentially threatening risk, because pathogens, chemical organic and inorganic products, acid gases from stack emissions, fugitive emissions or ash are sources of contamination of air and soil. Similarly, water can be contaminated by pathogens and chemical products; hence there is a risk to the environment and groundwater. For these reasons, the criterion “distance between the waste treatment center and the urban area (the most populated zone in the city)” should be assigned high importance. The least important criterion is the cost, because material advantages should be neglected compared to citizens’ health and environment interest. In other words, human and environmental capital is far more important than financial capital. All other criteria weights have intermediate values.

The obtained weight values are used in modified Murphy's rule in order to combine all belief mass distributions. Thereafter, we transform the obtained overall belief mass into pignistic probabilities *BetP* using pignistic transformation. The results are as follows:

BetP(Sousse) = 0.0084; *BetP*(Monastir) = 0.0519; *BetP*(Mahdia) = 0.0793; *BetP*(Sfax) = 0.1843; *BetP*(Gabes) = 0.0126; *BetP*(Kairouan) = 0.2392; *BetP*(Sidi Bouzid) = 0.2729; *BetP*(Kasserine) = 0.1458; *BetP*(Gafsa) = 0.0057.

The different sites are ranked in the decreasing order of their pignistic probabilities. The ranking obtained is:

Sidi Bouzid $\succ$ Kairouan $\succ$ Sfax $\succ$ Kasserine $\succ$ Mahdia $\succ$ Monastir $\succ$ Gabes $\succ$ Sousse $\succ$ Gafsa.

This ranking is presented to the Ministry of Environment to help the persons in charge to choose the best site (or more sites, according to their needs and their resources) for the creation of a new waste treatment center of healthcare activities with infectious risk in the center of Tunisia.

8 Conclusion and further research

In multi-criteria decision problems, information provided by the decision maker may be incomplete for various reasons. In the literature, there are only few papers solving multi-criteria decision making problems with incomplete information. Most of them discuss the AHP method. The PROMETHEE II-BELIEF approach proposed in this paper is a novel, flexible and systematic method for solving multi-criteria decision problems with incomplete information. It incorporates belief function theory into the PROMETHEE II method in order to take into account the incompleteness of information in the decision matrix as well as in criteria weight values.

Different from most of current methods, the PROMETHEE II-BELIEF approach offers the possibility of solving the problem directly on the basis of its incomplete decision matrix. In addition, it has advantages over the standard PROMETHEE II, on the objective determination way of criteria, weights instead of using values provided directly in a subjective manner. Besides, through the determination of focal elements, this approach allows to reduce considerably the number of pairwise comparisons of alternatives with respect to each criterion.

Incomplete information related to preference function thresholds is an interesting topic and can be further investigated. Another possible line of research is extending this method to the context of group decision making. Further extensions include developing approaches for an incomplete decision matrix with fuzzy and uncertain values. A promising research area is the development of a decision support system to automate the problem solving because calculations increase multiplicatively as the number of alternatives and criteria increases.

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Appendices

Appendix A: Complete decision matrix with some equal evaluations of alternatives

Table A.1: Focal elements decision matrix *G*' of case 1

	C_1	C_2	C_3	C_4	C_5
A	20	15	4	*	*
B	52	20	3	4	*
C	14	35	2	2	4
D	5	63	*	1	2
E	8	10	*	*	1
$A \cup B$	*	*	*	*	3
$A \cup E$	*	*	*	3	*
$D \cup E$	*	*	1	*	*

Table A.2: Belief mass distribution of case 1

	m_1	m_2	m_3	m_4	m_5
A	0.1767	0.2590	0.4053	0	0
B	0.0942	0.2375	0.2866	0.3717	0
C	0.2111	0.1295	0.1805	0.1858	0.3717
D	0.2684	0.0916	0	0.1475	0.1858
E	0.2496	0.2824	0	0	0.1475
$A \cup B$	0	0	0	0	0.2950
$A \cup E$	0	0	0	0.2950	0
$D \cup E$	0	0	0.1276	0	0

Table A.3: Criteria weights of case 1

	w_1	w_2	w_3	w_4	w_5
w_k	0.1987	0.2045	0.1880	0.2102	0.1986

Appendix B: Dominant alternative

Table B.1: Focal elements decision matrix *G*' of case 2

	C_1	C_2	C_3	C_4	C_5
A	20	15	3	*	*
B	4	8	4	4	*
C	14	35	2	2	4
D	5	63	*	1	2
E	8	10	*	*	1
$A \cup B$	*	*	*	*	5
$A \cup E$	*	*	*	3	*
$D \cup E$	*	*	1	*	*

Table B.2: Belief mass distribution of case 2

	m_1	m_2	m_3	m_4	m_5
A	0.1366	0.2375	0.2866	0	0
B	0.2408	0.2824	0.4053	0.3717	0
C	0.1725	0.1295	0.1805	0.1858	0.3579
D	0.2346	0.0916	0	0.1475	0.1421
E	0.2155	0.2590	0	0	0.1421
$A\cup B$	0	0	0	0	0.3579
$A\cup E$	0	0	0	0.2950	0
$D\cup E$	0	0	0.1276	0	0

Table B.3: Criteria weights of case 2

	w_1	w_2	w_3	w_4	w_5
w_k	0.2020	0.2020	0.1916	0.2123	0.1921

Appendix C: Dominated alternative

Table C.1: Focal elements decision matrix G' of case 3

	C_1	C_2	C_3	C_4	C_5
A	20	15	4	*	*
B	52	20	3	4	*
C	14	35	2	2	4
D	55	63	*	1	1
E	8	10	*	*	2
$A\cup B$	*	*	*	*	3
$A\cup E$	*	*	*	3	*
$D\cup E$	*	*	1	*	*

Table C.2: Belief mass distribution of case 3

	m_1	m_2	m_3	m_4	m_5
A	0.2304	0.2590	0.4053	0	0
B	0.1101	0.2375	0.2866	0.3717	0
C	0.2594	0.1295	0.1805	0.1858	0.3717
D	0.1083	0.0916	0	0.1475	0.1475
E	0.2918	0.2824	0	0	0.1858
$A\cup B$	0	0	0	0	0.2950
$A\cup E$	0	0	0	0.2950	0
$D\cup E$	0	0	0.1276	0	0

Table C.3: Criteria weights of case 3

	w_1	w_2	w_3	w_4	w_5
w_k	0.2047	0.2082	0.1873	0.2026	0.1972

Appendix D: Incomplete information

Table D.1: Focal elements decision matrix *G*’ of case 4

	C_1	C_2	C_3	C_4	C_5
A	20	15	*	3	4
B	52	*	3	*	3
C	14	35	2	2	*
D	5	63	4	1	2
E	*	10	1	4	1

Table D.2: Belief mass distribution of case 4

	m_1	m_2	m_3	m_4	m_5
A	0.2449	0.3317	0	0.2865	0.3407
B	0.1420	0	0.2818	0	0.2865
C	0.2754	0.1972	0.1992	0.2026	0
D	0.3377	0.1394	0.3654	0.1703	0.2026
E	0	0.3317	0.1536	0.3407	0.1703

Table D.3: Criteria weights of case 4

	w_1	w_2	w_3	w_4	w_5
w_k	0.1945	0.2103	0.1837	0.2147	0.1968

MULTIPLE CRITERIA DECISION MAKING

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A NEW APPROACH FOR CRITERIA WEIGHT ELICITATION OF THE ARAS-H METHOD

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Abstract

Criteria weight inference is a crucial step for most of multi-criteria methods. However, criteria weights are often determined directly by the decision-maker (DM) which makes the results unreliable. Therefore, to overcome the imprecise weighting, we suggest the use of the preference programming technique. Instead of obtaining criteria weights directly from the DM, we infer them in a more objective manner to avoid the subjectivity and the unreliability of the results. Our aim is to elicit the ARAS-H criteria weights at each level of the hierarchy tree via mathematical programming, taking into account the DM's preferences. To put it differently, starting from preference information provided by the DM, we proceed to model our constraints. The ARAS-H method is an extension of the classical ARAS method for the case of hierarchically structured criteria. We adopt a bottom-up approach in order to elicit ARAS-H criteria weights, that is, we start by determining the elementary criteria weights (i.e. the criteria at the lowest level of the hierarchy tree). The solution of the linear programs is obtained using LINGO software. The main contribution of our criteria weight elicitation procedure is in overcoming imprecise weighting without excluding the DM from the decision making process.

Keywords: Multiple Criteria Decision Aiding, preference disaggregation, ARAS-H, criteria weights, mathematical programming.

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1 Introduction

Multiple criteria decision analysis (MCDA) is a general framework for supporting complex decision-making situations with multiple and often conflicting objectives (Belton and Stewart, 2002; Greco, Figueira and Ehrgott, 2016; Ishizaka and Nemery, 2013). Most of multi-criteria methods require fixing criteria weights in order to be implemented. Indeed, the problem of criteria weight determination has gained the interest of many researchers during the past decades. In fact, there are two types of weight elicitation: 'a priori weights', determined directly by experts, and 'a posteriori weights', obtained from the data (Jacquet-Lagreze and Siskos, 2001). This paper adopted the 'a posteriori approach'.

We are therefore interested in reducing the subjectivity and the unreliability of weight values provided directly by the DM without excluding him from the decision making process. The paper is divided into five sections. In section 2, we present a brief state of the art survey of some weighting methods that deal with hierarchical structure of criteria. In section 3, we develop the criteria weight determination approach of the ARAS-H method. In section 4, an empirical example is presented to discuss the feasibility of the proposed model. In section 5, we present conclusions and our main perspectives.

2 A review of the literature

Very few authors have worked on criteria weight elicitation within hierarchical methods. To start with, Corrente, Greco and Słowiński (2016) proposed a generalization of the SRF (Simos-Roy-Figuiera) method (Figueira and Roy, 2002) to deal with weight elicitation in hierarchical structure of criteria. In the SRF method, the DM ranks the criteria from the least important to the most important with the possibility of ex-aequo between them. Then, he is asked to put some blank cards between two successive subsets of criteria to increase the difference of importance between the criteria in these two subsets. Finally, he defines the ratio z of the importance of the most important subset of criteria to that of the least important one. Moreover, Corrente et al. (2017) developed the imprecise SRF to deal with imprecise preference information given by the DM on the number of blank cards and on the ratio z. Therefore, the imprecise SRF method helps the DM to obtain the weights of criteria and sub-criteria on the basis of incompletely determined preference information. As a consequence of considering imprecise preference information in SRF, there is an infinity of compatible vectors of weights satisfying the constraints translating this preference information. Furthermore, Salo and Hämäläinen (1992) developed a preference programming method called Preference Assessment by Imprecise

Ratio Statements (PAIRS) in which the preference judgments are given as linear constraints on the weight ratios of the criteria and attribute-wise values of the alternatives. In addition, Keeney and Raiffa (1993) used Multi--Attribute Value Theory (MAVT) to elicit criteria weights. The attributes are grouped under more general upper level criteria and the weighting is carried out separately on each branch of the value tree. Thus, on each branch, the DM gives local weights to the criteria, which describe the relative importance of their consequence range under the ascending next level criterion. The overall weight of each attribute is calculated by multiplying its local weight by the local weights of all the ascending upper level criteria. On each branch of the value tree, the sum of the local weights is normalized to one. Consequently, the overall weight of each criterion is the sum of the overall weights of all its next level sub-criteria, and the sum of the overall weights of all the attributes will also be one. In what follows, we present an illustrative table of the criteria weights elicitation methods when dealing with a hierarchical structure.

Table 1: A review of methods of criteria weight determination

References	Criteria weight elicitation techniques	Methods
Corrente et al. (2017)	SRF	ELECTRE-III-H
Corrente, Greco and Słowiński (2016)	Extension of the SRF	ELECTRE Tri-H
Del Vasto-Terrientes et al. (2015a)	Simos	ELECTRE-III-H
Del Vasto-Terrientes et al. (2015b)	Simos	ELECTRE-III-H
Del Vasto-Terrientes et al. (2016a)	Simos	ELECTRE-TRI-B-H
Del Vasto-Terrientes et al. (2016b)	Subjective	ELECTRE-III-H

As can been seen in Table 1, the major studies used either the Simos' procedure or the SRF technique in order to elicit the hierarchical ELECTRE and PROMETHEE methods. However, both these methods have been criticized for their subjectivity. The Simos' method is based on an unrealistic assumption (lack of essential information) and leads to the process criteria having the same importance (i.e., the same weight) in a not robust way (Schärlig, 1996). Also, the Simos' and the SRF methods are considered to be subjective weighting ones. To overcome the imprecise weighting, we suggest preference programming which takes into account the DM's preferences. In an earlier paper, we suggested a criteria weight determination procedure for the ARAS method (Zavadskas and Turskis, 2010) through mathematical programming (Ghram and Frikha, 2018). Likewise, we proceed to develop a set of mathematical programs, which takes into account the DM's preferences, to elicit ARAS-H criteria weights at each node of the hierarchy tree. In fact, the ARAS-H method is an extension of the classical ARAS method in the case of hierarchically structured criteria (Ghram and Frikha, 2019; Ghram and Frikha, 2021).

3 The proposed model for ARAS-H criteria weight inference

The aggregation paradigm states that the aggregation model is known a priori, whereas the global preference is unknown. On the other hand, the philosophy of disaggregation involves the inference of preference models from the given global preferences. The development of preference disaggregation methods was initiated in 1978. In the disaggregation-aggregation approach, iterative interactive procedures are used to be aggregated later to a value system (Siskos, 1980). The first developed preference disaggregation methodology was the UTA (Jacquet-Lagreze and Siskos, 1982). The purpose of this method is to infer additive value functions from a given ranking through linear programming so that these functions are as consistent as possible with the global decision-maker's preferences. Thus, we adopt the preference disaggregation methodology in order to elicit criteria weights of the ARAS-H method. Our weight elicitation procedure is based on the solution of linear programs which take into account the DM's preferences. Consequently, the DM has to introduce some preference information which report his value system. Thus, this approach is based on preference relations provided by the decision maker, as well as on comparisons between differences of criteria weights and some weight partial pre-orders. By involving the DM in the weight elicitation process, we allow the DM to express his preference information not only comprehensively, but also partially, by considering preference information with respect to a sub-criterion at an intermediate level of the hierarchy. Thus, the DM can obtain results not only with respect to the comprehensive view, but also at the intermediate levels of the hierarchy. The process of weight elicitation is considered to be a set of mathematical programs. Their number depends on the number of the levels in the hierarchy. Henceforth, we adopt a bottom-up approach to elicit ARAS-H criteria weights. We start with the last level l. The aim is to obtain all elementary criteria weights from preference relations given by the DM on some pairs of alternatives according to intermediate criteria of the upward level. This process is generated until we reach the root criterion.

3.1 Determination of the elementary criteria weights

For each sub-criterion $G_{(r,\ n(r))}$, the DM is asked to give preference relations between some pairs of alternatives. Also, he is asked to provide some comparisons between differences of elementary criteria weights and certain elementary weight partial pre-orders. Those preference relations are modeled in Program 1. The solution of the following mathematical program will provide all elementary criteria weights.

Program 1:

The following notations have been introduced by Corrente, Greco and Slowiński (2012).

Let:

A be the set of alternatives;

EL: the set of indices of all elementary sub-criteria;

$n(r)$: the number of sub-criteria of G_r in the subsequent level;

$G_r \in G$, with $r = (i_1, \ldots, i_h) \in I_G$, denote a sub-criterion of the first level criterion G_i at level h;

$G_{(r,\, n(r))}$: the direct sub-criteria of G_r.

We define:

p to be the number of relations between a pair of alternative preferences provided by the DM;

z: a threshold;

w_j : the weight of the elementary criterion j;

$\bar{x}_{Dj} / \bar{x}_{Fj}$ are the normalized performance values of the alternatives D and F, respectively, according to the elementary criterion j.

Thus, Program 1 can be written in the form:

$$\text{Max} \sum_{i=1}^{p} e_i \tag{1}$$

Subject to:

$$\sum_{j \in EL} w_j \bar{x}_{Dj} - \sum_{j \in EL} w_j \bar{x}_{Fj} - e_i \geq 0 \quad \forall D, F \in A; \forall i = 1, \ldots, p \tag{2}$$

$$w_k - w_s \geq w_r - w_v \;\; \forall k,s,r,v \in EL \tag{3}$$

$$w_k \geq w_l \quad \forall k,l \in EL \tag{4}$$

$$e_i \geq \frac{1}{2^{(p-1)}} \quad \forall i = 1, \ldots, p \tag{5}$$

$$w_j \geq z \quad \forall j \in EL\,; \; z \geq 0 \tag{6}$$

$$\sum_{j \in \mathrm{EL}} w_j = 1 \tag{7}$$

The objective function (eq. 1) expresses the maximization the sum of slack variables between a pair of alternatives as expressed by the DM. This slack variable insures a strict preference between two alternatives.

The first constraint concerns the degree of preference e_i, which measures the intensity of preference of an alternative over the other ones and is calculated as the difference between their utility degrees according to the intermediate sub--criterion $G_{(r,\, n(r))}$. In other words, in the ARAS-H method, all alternatives are ranked according to a decreasing order of their utility degree values. For instance, an alternative D is preferable to F is equivalent to: the utility degree of D is greater than that of F on intermediate sub-criterion $G_{(r,\, n(r))}$.

Then, $K_{G_{(r,n\,(r))}}(D) \geq K_{G_{(r,n\,(r))}}(F)$.

Consequently, $\frac{S_D}{S_0} \geq \frac{S_F}{S_0}$ on intermediate sub-criterion $G_{(r,\,n\,(r))}$, where S_0 is the best value.

Equally, $\sum_{j\,\in\,\mathrm{EL}} \hat{x}_{Dj} \ \geq\ \sum_{j\,\in\,\mathrm{EL}} \hat{x}_{Fj}$,
where $\widehat{x_{Dj}}$ and $\widehat{x_{Fj}}$ are the weighted normalized values of all elementary criteria.

Signify, $\sum_{j\,\in\,\mathrm{EL}} w_j\, \bar{x}_{Dj} \geq \sum_{j\,\in\,\mathrm{EL}} w_j\, \bar{x}_{Fj}$,
with $\bar{x}_{Dj}$ and $\bar{x}_{Fj}$ being the normalized values of the decision making matrix.

Next, the preference relations expressed by the DM are modeled in the mathematical program as: $\sum_{j\,\in\,\mathrm{EL}} w_j\, \bar{x}_{Dj} - \sum_{j\,\in\,\mathrm{EL}} w_j\, \bar{x}_{Fj} - e_i \geq 0 \ \ \forall\ D, F \in A$; $\forall\ i = 1, \ldots, p$ (eq. 2).

Besides the preference relations, the DM must provide two other information types. The first one concerns comparisons of the differences of adjacent weights presented as: $w_k - w_s \geq w_r - w_v$ (eq. 3). Therefore, the gap between the importance of elementary criteria k and l is more important than that between r and v. The second information type concerns a partial pre-order on elementary criteria weights. Nevertheless, the DM is invited to supply some comparisons between some pairs of criteria weights of the form $w_k \geq w_l \ \ \forall\, k,l \in EL$ (eq. 4). The number of partial pre-order constraints must not exceed $(n - 1)$. In order to guarantee the preference between the pairs of preferences provided by the DM and to avoid the indifference, we must impose that all slack variables (e_i) are strictly positive. Consequently, we have to fix a minimum threshold associated with each e_i related to each preference relation. It is evident that the threshold value is strongly dependent on the number of preference relationships. As an illustration, the threshold value is fixed to be $\frac{1}{2(p-1)}$. We introduce the constraint $e_i \ \geq\ \frac{1}{2(p-1)}\, \forall\ i=1, \ldots, p$ (eq. 5) into the mathematical Program 1. The constraint (eq. 6) concerns the threshold of the weight values. Indeed, in the constraints of the weight determination mathematical program, we must take into account the requirement that all criteria weights must be strictly positive ($w_j > 0$) in order to restrict any criterion from being null and therefore ignored. Since mathematical programming deals with large inequalities and not strict inequalities, we must fix a small positive threshold z associated with each importance coefficient w_j. Therefore, we must add to the mathematical program the constraint $w_j \geq z\ \forall\ j \in EL$ (eq. 6). In addition, we must take into consideration that criteria weights are normalized. It means that the sum of weights of elementary criteria must be equal to 1 ($\sum_{j\,\in\,\mathrm{EL}} w_j = 1$; eq. 7).

3.2 Determination of the intermediate sub-criteria weights

We note that if the number of the levels in the hierarchy exceeds three (l > 3), Program 2 is used. It is repeated until we reach the first level of intermediate criteria. Consequently, the DM is asked to give the same information as in Program 1, but this time according to the first-level intermediate criteria G_i. Those preference relations are included in Program 2. Hence, the solution of Program 2 gives the weights of the intermediate criteria at level h.

Program 2:

Corrente, Greco and Słowiński (2012) have introduced the following notations:

I_G: the set of indices of the particular criteria representing the positions of the criteria in the hierarchy;

$G_r \in G$, with $r = (i_1, \ldots, i_h) \in I_G$: a sub-criterion of the first-level criterion G_i at level h;

$LB(G_r)$: the set of indices of sub-criteria of the second-last level descending from criterion/sub-criterion G_r.

Thus, Program 2 can be written in the form:

$$\text{Max} \sum_{q=p}^{t} e_q \tag{8}$$

Subject to:

$$\sum_{j \in \text{LB}(G_r)} w_j K_j(D) - \sum_{j \in \text{LB}(G_r)} w_j K_j(F) - e_q \geq 0 \ \forall\, D,F \in A;\ q = p, \ldots, t \tag{9}$$

$$w_k - w_s \geq w_r - w_v \ \ \forall\, k,s,r,v \in \text{LB}(G_r) \tag{10}$$

$$w_k \geq w_l \ \ \forall\, k,l \in \text{LB}(G_r) \tag{11}$$

$$e_q \geq \frac{1}{2^{(p-1)}} \ \ \forall\, q = p, \ldots, t \tag{12}$$

$$w_j \geq z \ \ \forall\, j \in LB(G_r);\ z \geq 0 \tag{13}$$

$$\sum_{j \in \text{LB}(G_r)} w_j = 1 \tag{14}$$

Likewise, we have to maximize in the objective function (eq. 8) of Program 2, the sum of slack variables e_q, to insure the strict preference and to avoid indifference between two alternatives.

The constraint (eq. 9) concerns the degree of preference e_q, which measures the intensity of preference of an alternative over the other ones and is calculated as the difference between their utility degrees according to the first-level criterion G_{ih}. In the ARAS-H method, the statement that an alternative D is preferable over alternative F ($D \succ F$) on the first-level criterion G_{ih} is expressed by $K_{G_{ih}}(D) \geq K_{G_{ih}}(F)$. Therefore, $\sum_{j \in \text{LB}(G_r)} w_j K_j(D) \geq \sum_{j \in \text{LB}(G_r)} w_j K_j(F)$.

In addition to the preference relations, the DM must provide some comparisons of the differences of adjacent weights in the form: $w_k - w_s \geq w_r - w_v$ (eq. 10). Therefore, the gap between the importance of intermediate criteria k and l is more important than that between r and v. Also, the DM is asked to give a partial pre-order on intermediate criteria weights. Furthermore, the DM is invited to supply some comparisons between some pairs of criteria weights of the form $w_k \geq w_l$ $\forall$ k,l $\in$ LB (G_r) (eq. 11). However, the number of partial pre-order constraints must not exceed $(n - 1)$.

In order to guarantee the preference between the pairs of preferences provided by the DM and to avoid the indifference, strictly positive slack variables (e_q) are imposed. Consequently, we have to fix a minimum threshold associated with each e_q, related to each preference relation equals to $\frac{1}{2^{(p-1)}}$. Thus, $e_q \geq \frac{1}{2^{(p-1)}} \forall q = 1, \ldots, p$ (eq. 12).

The constraint (17) concerns the threshold of the weight values. Indeed, we must take into account the fact that all criteria weights must be strictly positive ($w_j > 0$) in order to prevent any criterion from being null and therefore ignored. Since mathematical programming deals with large inequalities and not strict inequalities, we must fix a small positive threshold z associated with each importance coefficient w_j. Next, we must add to the mathematical program the constraint $w_j \geq z \; \forall \, j \in LB \, (G_r)$ (eq. 13). In addition, we must take into account that criteria weights are normalized, that is, the sum of weights of intermediate criteria descending from G_{ih} must be equal to 1 ($\sum_{j \in LB \, (G_r)} w_j = 1;$ eq. 14).

3.3 Determination of the first-level intermediate criteria weights

The DM is asked to give a preference relation between a pair of alternatives according to the *root criterion*. He is also asked to provide some comparisons between differences of the first-level intermediate criteria weights and some first-level intermediate weight partial pre-orders. The solution of Program 3 gives the weights of the first-level intermediate criteria.

Program 3:

Corrente, Greco and Słowiński (2012) have defined the following notations:

m: the number of the first-level criteria (root criteria) $G_1 \ldots G_m$;

I_G: the set of indices of particular criteria representing the position of the criteria in the hierarchy.

Hence, Program 3 can be written in the form:

$$\text{Max } e_{t+1} \tag{15}$$

Subject to:

$$\sum_{j \in I_G} w_j K_j(D) - \sum_{j \in I_G} w_j K_j(F) - e_{t+1} \geq 0 \;\; \forall D,F \in A \tag{16}$$

$$w_k - w_s \geq w_r - w_v \;\; \forall k,s,r,v \in I_G \tag{17}$$

$$w_k \geq w_l \;\; \forall k,n \in I_G \tag{18}$$

$$e_{t+1} \geq \frac{1}{2^t} \tag{19}$$

$$w_j \geq z \; \forall j \in I_G; z \geq 0 \tag{20}$$

$$\sum_{j \in I_G} w_j = 1 \text{ for the root criterion } G_m. \tag{21}$$

In order to insure strict preference and to avoid indifference between two alternatives, we have to maximize in the objective function of Program 3, the slack variable e_{t+1} (eq. 15). As we said before, in the ARAS-H method, the statement that an alternative D is preferable than alternative F ($D \succ F$) on the root criterion G_m is expressed by $K_{G_m}(D) \geq K_{G_m}(F)$, that is, $\sum_{j \in I_G} w_j K_j(D) \geq$ $\geq \sum_{j \in I_G} w_j K_j(F)$. In fact, e_{t+1} presents the degree of preference of D over F and it is interpreted as the difference between the two utility degrees according to the root criterion G_m (eq. 16). As an illustration, $K_{I_G}(D) - K_{I_G}(F) = e_{t+1}$ for the preference relation (t + 1) provided by the DM. In addition to the preference relation, the DM must provide two other information types. The first one concerns comparisons of the differences of adjacent weights in the form: $w_k - w_s \geq w_r - w_v$ (eq. 17). Therefore, the gap between the importance of the first-level intermediate criteria k and l is more important than that between r and v. The second information type concerns a partial pre-order on the first-level intermediate criteria weights. Nevertheless, the DM is invited to supply some comparisons between some pairs of criteria weights of the form $w_k \geq w_l$ $\forall k$, $n \in I_G$ (eq. 18). The number of partial pre-order constraints must not exceed (n − 1). In order to guarantee the preference between the pairs of preferences provided by the DM and to avoid the indifference, we must impose that the slack variable (e_{t+1}) be strictly positive. Consequently, we have to fix a minimum threshold associated with the slack variable (e_{t+1}). Thus, we introduce the constraint $e_{t+1} \geq \frac{1}{2^t}$ (eq. 19).

The constraint (20) concerns the threshold of the weight values. Surely, all criteria weights must be strictly positive ($w_j > 0$) in order to prevent any criterion from being null and therefore ignored. Since mathematical programming deals with large inequalities and not strict inequalities, we must fix a small positive threshold z associated with each importance coefficient w_j. Henceforth, we must add to the mathematical program the constraint $w_j \geq z$ $\forall j \in$ $\in I_G$. In addition, we must take into consideration that criteria weights are normalized. It means that the sum of weights of the first-level intermediate criteria descending from the root criterion G_m must be equal to 1. For instance, if we have m root criteria, then $\sum_{j \in I_G} w_j = 1$ (eq. 21).

Our approach is iterative interactive. Within the iterative process of determining ARAS-H criteria weights, the DM can add or remove information whenever he is not satisfied with the given result. The additional information consists in adding or withdrawing one or more preference relations. Thus, each preference relation is modeled in a mathematical program as a constraint. Certainly, in real-world decision problems, decision-makers have difficulties with providing reliable information due to time restriction and their cognitive limitations. The preferences of decision makers are therefore not necessarily stable: they can contain inconsistent and conflicting data. The role of an interactive tool is to help the DM to understand his preferences and their representation in a multi-criteria aggregation method. Inconsistencies appear when DM's preferences cannot be guaranteed by the aggregation method used.

4 An illustrative example

The aim of this example is to present websites designed to promote tourist destination brands. Websites have become crucial tools for communicating destination brands and for selling a variety of tourism services and related products (Fernández-Cavia and Huertas-Roig, 2010). Since the problem is considered to be a complex one, the DM organized the set of criteria into a hierarchical structure as expressed in Figure 1. Thus, ten tourist websites in different regions in the world (Andalusia, Catalonia, Barcelona, Madrid, Santiago de Compostela, Rias Baixas, Stockholm, Wales, Rome and Switzerland) are evaluated, according to a set of hierarchically structured criteria. The following dataset comes from a Spanish research project entitled "Online Communication for Destination Brands. Development of an Integrated Assessment Tool: Websites, Mobile Applications and Social Media (CODETUR)" completed in 2012, whose main objective was to identify a website evaluation framework to help expert managers to enhance and optimize online communication of their brands. In this section, we discuss the analysis of this dataset with the criteria weight elicitation procedure of the ARAS-H method.

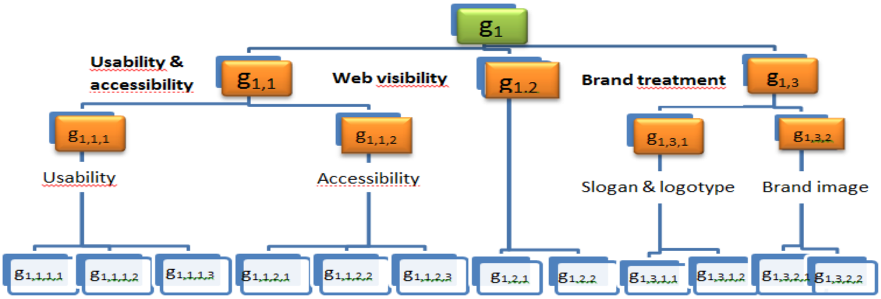

Figure 1: Criteria hierarchy tree

Table 2: Normalised decision matrix

	$g_{1,1,1,1}$	$g_{1,1,1,2}$	$g_{1,1,1,3}$	$g_{1,1,2,1}$	$g_{1,1,2,2}$	$g_{1,1,2,3}$	$g_{1,2,1}$	$g_{1,2,2}$	$g_{1,3,1,1}$	$g_{1,3,1,2}$	$g_{1,3,2,1}$	$g_{1,3,2,2}$
Andalusia	1	1	0.6	0	0.125	0.16	0.58	0.42	1	0.5	1	0.43
Catalonia	0.48	0	0	0.67	0.375	0.387	0	0	0	1	0.4	0
Barcelona	1	0.43	0.8	0.67	0.125	1	0.61	0.86	0	0.9	1	1
Madrid	0.66	1	0.6	0.33	0.5	0.613	0.64	0.44	0	0	0	0.43
Santiago	1	0.71	0.2	0.33	0	0.32	1	0.31	1	0.25	0.6	0.43
Rias Baixas	0	0	0	0.67	0.125	0.16	0.19	0	1	0	0	0
Stockholm	1	1	1	1	0.125	0.387	0.61	0.75	1	0.4	0.2	0.57
Wales	0.66	0.57	0	1	0.375	0	0.89	0.22	1	0.5	0.6	0
Rome	1	1	1	1	0.125	0.16	0.61	0.22	0	0	0.4	0.43
Switzerland	0.52	1	1	1	0.125	0.16	0.75	1	1	0.75	1	1

The DM is asked to give his preference relations at each node of the hierarchy.

On Usability ($g_{1,1,1}$): the DM prefers Madrid over Wales

Mad $\succ$ Wales means,

$K_{(1,1,1)}(\text{Mad}) - K_{(1,1,1)}(\text{Wales}) - e_1 \geq 0$. Then,

$(w_{(1,1,1,1)}\,\bar{x}_{(1,1,1,1)}(\text{Mad}) + w_{(1,1,1,2)}\,\bar{x}_{(1,1,1,2)}(\text{Mad}) + w_{(1,1,1,3)}\,\bar{x}_{(1,1,1,3)}(\text{Mad})) -$
$- ((w_{(1,1,1,1)}\,\bar{x}_{(1,1,1,1)}(\text{Wales}) + w_{(1,1,1,2)}\,\bar{x}_{(1,1,1,2)}(\text{Wales}) + w_{(1,1,1,3)}\,\bar{x}_{(1,1,1,3)}(\text{Wales})) - e_1 \geq 0$

On Accessibility ($g_{1,1,2}$): the DM prefers Stockholm over Rias

Stock $\succ$ Rias is equivalent to:

$K_{(1,1,2)}(\text{Stock}) - K_{(1,1,2)}(\text{Rias}) - e_2 \geq 0$. In other words,

$(w_{(1,1,2,1)}\,\bar{x}_{(1,1,2,1)}(\text{Stock}) + w_{(1,1,2,2)}\,\bar{x}_{(1,1,2,2)}(\text{Stock}) + w_{(1,1,2,3)}\,\bar{x}_{(1,1,2,3)}(\text{Stock})) -$
$- ((w_{(1,1,2,1)}\,\bar{x}_{(1,1,2,1)}(\text{Rias}) + w_{(1,1,2,2)}\,\bar{x}_{(1,1,2,2)}(\text{Rias}) + w_{(1,1,2,3)}\,\bar{x}_{(1,1,2,3)}(\text{Rias})) - e_2 \geq 0$

On Web visibility ($g_{1,2}$): the DM prefers Switzerland over Andalusia

Switz $\succ$ Anda means,

$K_{(1,2)}(\text{Switz}) - K_{(1,2)}(\text{Anda}) - e_3 \geq 0$. Consequently,

($w_{(1,2,1)}\ \bar{x}_{(1,2,1)}$ (Switz) + $w_{(1,2,2)}\ \bar{x}_{(1,2,2)}$ (Switz)) – ($w_{(1,2,1)}\ \bar{x}_{(1,2,1)}$ (Anda) + $w_{(1,2,2)}$ $\bar{x}_{(1,2,2)}$(Anda)) – $e_3 \geq 0$

On Slogon & Logotype ($g_{1,3,1}$): the DM prefers Santiago over Rome

Sant ≻ Rome Thus,

$K_{(1,3,1)}$ (Sant) – $K_{(1,3,1)}$ (Rome) – $e_4 \geq 0$. In other terms,

($w_{(1,3,1,1)}\ \bar{x}_{(1,3,1,1)}$ (Sant) + $w_{(1,3,1,2)}\ \bar{x}_{(1,3,1,2)}$ (Sant)) – ($w_{(1,3,1,1)}\ \bar{x}_{(1,3,1,1)}$ (Rome) + + $w_{(1,3,1,2)}\ \bar{x}_{(1,3,1,2)}$ (Rome)) – $e_4 \geq 0$

On Brand Image ($g_{1,3,2}$): the DM prefers Barcelona over Wales

Barc ≻ Wales means that:

$K_{(1,3,2)}$ (Barc) – $K_{(1,3,2)}$ (Wales) – $e_5 \geq 0$. Hence,

($w_{(1,3,2,1)}\ \bar{x}_{(1,3,2,1)}$ (Barc) + $w_{(1,3,2,2)}\ \bar{x}_{(1,3,2,2)}$ (Barc)) – ($w_{(1,3,2,1)}\ \bar{x}_{(1,3,2,1)}$ (Wales) + + $w_{(1,3,2,2)}\ \bar{x}_{(1,3,2,2)}$ (Wales)) – $e_5 \geq 0$

Hence, the DM provides other information types concerning the thresholds of both weights and slack variables, in addition to comparisons between differences of elementary criteria weights and some elementary criteria weights partial pre-orders.

We use the LINGO software for the solution of the three mathematical programs.

Program 1:

Max $\sum_{i=1}^{5} e_i$

Subject to:

$w_{1112} \times 0.43 + w_{1113} \times 0.6 - e_1 \geq 0$

$w_{1121} \times 0.33 + w_{1123} \times 0.23 - e_2 \geq 0$

$w_{121} \times 0.17 + w_{122} \times 0.58 - e_3 \geq 0$

$w_{1311} + w_{1312} \times 0.25 - e_4 \geq 0$

$w_{1321} \times 0.4 + w_{1322} - e_5 \geq 0$

$w_{1321} - w_{1311} \geq w_{1312} - w_{1322}$

$w_{1123} - w_{1121} \geq w_{1122} - w_{1112}$

$w_{122} - w_{1112} \geq w_{1111} - w_{121}$

$w_{1111} \geq w_{1121}$

$w_{1122} \geq w_{1311}$

$w_{1321} \geq w_{1113}$

$w_{1312} \geq w_{1322}$

$w_{1111} + w_{1112} + w_{1113} + w_{1121} + w_{1122} + w_{1123} + w_{121} + w_{122} + w_{1311} + w_{1312} + w_{1321} +$ $+ w_{1322} = 1$

$e_i \geq 0.0625 \quad i = 1 \ldots 5$

$w_j \geq 0.015$ $j \in$ {(1,1,1,1), (1,1,1,2), (1,1,1,3), (1,1,2,1), (1,1,2,2), (1,1,2,3), (1,2,1), (1,2,2), (1,3,1,1), (1,3,1,2), (1,3,2,1), (1,3,2,2)}

The solution of Program 1 provided the elementary criteria weights.

```
W1112        0.3902952E-01
W1113        0.7619551E-01
W1121        0.1500000E-01
W1123        0.2502174
 W121        0.1500000E-01
 W122        0.1033621
W1311        0.1500000E-01
W1312        0.1900000
W1321        0.7619551E-01
W1322        0.1900000
W1122        0.1500000E-01
W1111        0.1500000E-01
```

After determining all elementary criteria weights, we proceed to the construction of the weighted-normalized decision matrix (see Table 3 in Appendix).

Thus, the obtained utility degrees values K_i will be used in constraints of type:

$\sum_{j\in \mathrm{LB}\,(G_r)} w_j\, K_j\,(B) - \sum_{j\in \mathrm{LB}\,(G_r)} w_j\, K_j(Q) - e_i \geq 0$; $\forall$ B, $Q \in A$;$\forall\, i = p, \ldots, t$ (see Table 4 as an example in Appendix).

On Usability and Accessibility ($g_{1,1}$): the DM prefers Stockholm over Catalonia Stock ≻ Cata means,

$K_{(1,1)}$ (Stock) – $K_{(1,1)}$ (Cata) – $e_6 \geq 0$. Hence,

($w_{(1,1,1)}\, K_{(1,1,1)}$ (Stock) + $w_{(1,1,2)}\, K_{(1,1,2)}$ (Stock)) – ($w_{(1,1,1)}\, K_{(1,1,1)}$ (Cata) + $w_{(1,1,2)}\, K_{(1,1,2)}$ (Cata)) – $e_6 \geq 0$

On Brand treatment ($g_{1,3}$): the DM prefers Andalusia over Santiago

Anda ≻ Sant is equivalent to,

$K_{(1,3)}$ (Anda) – $K_{(1,3)}$ (Sant) – $e_7 \geq 0$. Thus,

($w_{(1,3,1)}\, K_{(1,3,1)}$ (Anda) + $w_{(1,3,2)}\, K_{(1,3,2)}$ (Anda)) – ($w_{(1,3,1)}\, K_{(1,3,1)}$ (Sant) + $w_{(1,3,2)}\, K_{(1,3,2)}$ (Sant)) – $e_7 \geq 0$

Similarly, the DM provides other information types concerning the thresholds of both weights and slack variables, in addition to comparisons between differences of intermediate criteria weights and some intermediate criteria weights partial pre-orders. Thus, Program 2 can be written in the form:

Program 2:

Max $\sum_{q=6}^{7} e_q$

$w_{111} \times 0.856 - w_{112} \times 0.27 - e_6 \geq 0$

$w_{131} \times 0.217 + w_{132} \times 0.2 - e_7 \geq 0$

$w_{111} \geq w_{131}$

$w_{132} \geq w_{112}$
$w_{132} - w_{111} \geq w_{112} - w_{131}$
$w_{132} - w_{131} \geq w_{111} - w_{112}$
$w_{111} + w_{112} + w_{131} + w_{132} = 1$
$e_q \geq 0.0625 \;\; \forall\, q = 6, 7$
$w_j \geq 0.015 \; j \in \{(1,1,1), (1,1,2), (1,3,1), (1,3,2)\}$

The solution of Program 2 gave us the weights of $g_{1,1,1}$, $g_{1,1,2}$, $g_{1,3,1}$ and $g_{1,3,2}$.

```
W111          0.4850000
W112          0.1500000E-01
W131          0.1500000E-01
W132          0.4850000
```

The final step in the weight elicitation process is to calculate the values of the utility degree K_i with respect to the first-level sub-criteria using the previously obtained weights (see Table in Appendix as an example).
On the root criterion g_1: the DM prefers Rome over Madrid
Rome ≻ Mad means,
K_1 (Rome) – K_1 (Mad) – $e_8 \geq 0$. Therefore,
($w_{(1,1)} K_{(1,1)}$ (Rome) + $w_{(1,2)} K_{(1,2)}$ (Rome) + $w_{(1,3)} K_{(1,3)}$ (Rome)) –
($w_{(1,1)} K_{(1,1)}$ (Mad) + $w_{(1,2)} K_{(1,2)}$ (Mad) + $w_{(1,3)} K_{(1,3)}$ (Mad)) – $e_8 \geq 0$

In the same way, the DM provide us with other information type concerning the thresholds of both weights and slack variables in addition to comparisons between differences of first-level intermediate criteria weights and some first--level intermediate criteria weight partial pre-orders. Therefore, Program 3 can be written in the form:

Program 3:
Max e_8
$w_{11} \times 0.0778 - w_{12} \times 0.2099 + w_{13} \times 0.097 - e_8 \geq 0$
$w_{11} - w_{13} \geq w_{13} - w_{12}$
$w_{11} \geq w_{13}$
$w_{11} + w_{12} + w_{13} = 1$
$e_8 \geq 0.008$
$w_j \geq 0.015 \; j \in \{(1,1), (1,2), (1,3)\}$

The solution of Program 3 gave us the weights of the first-level intermediate criteria to construct the complete pre-order (Figure 2) from ranking the utility degrees K_i obtained in Table 7 (see Appendix).

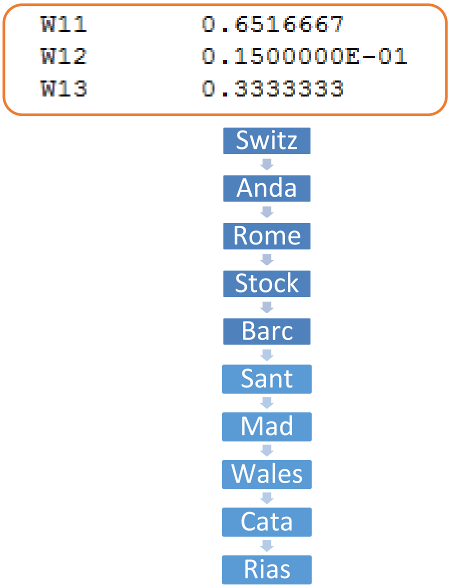

Figure 2: The complete pre-order

As we can notice, Switzerland outranks all the other alternatives. It is considered to be the best web tourist destination brand, whereas Rias Baixas is considered to be the worst.

In the final analysis, the proposed model can be summarized in the following Figure 3.

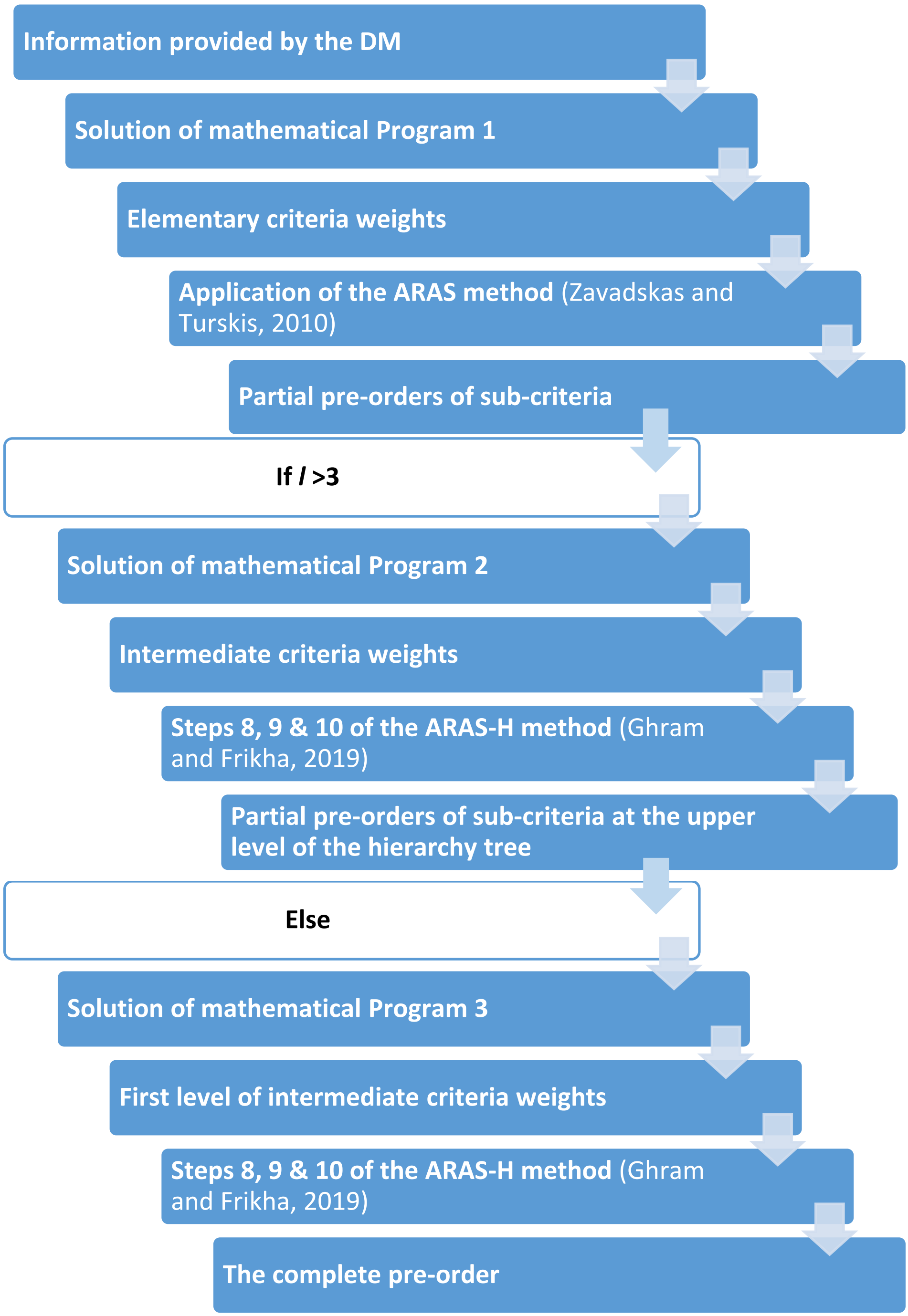

Figure 3: A flow chart of the proposed approach

5 Conclusion and perspectives

In this paper, we developed a criteria weight elicitation procedure for the ARAS-H method. Its aim is to overcome the imprecise weighting encountered in most of multi-criteria aggregation problems, in which the DM determines directly the weight values using his own intuition. However, the direct weight elicitation is too subjective, which makes the results unreliable. To overcome this issue, we suggested a weighting method based on preference programming which takes into account the DM's preferences. Therefore, the DM is involved indirectly in the decision-making process by expressing his preference relations on some pairs of alternatives, some comparisons between differences of criteria weights and some weight partial pre-orders. A set of mathematical programs were developed and solved by the LINGO software package in order to elicit ARAS-H criteria weights at each level of the hierarchy tree. Therefore, the DM can express his preference information not only in a comprehensive way, but also in a partial way, that is, considering preference information with respect to each criterion in the hierarchy tree. Thus, he can analyze the obtained rankings according to each criterion apart from detecting the main anomalies of the given problem. An illustrative example was presented at the end of the paper to showcase the feasibility of the proposed approach by ranking the tourist destination brands across Europe. The main contribution of this paper is that the DM is not involved directly in the weight elicitation, which reduces the subjectivity of the results. Thus, he interacts partially through preferential information. Nevertheless, the proposed model is valid only for the ARAS-H method. For future research, we consider developing the ARAS-H method in the context of a fuzzy environment and to elicit criteria weights of the fuzzy ARAS-H method.

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Appendix

Table 3: Weighted normalized decision matrix

	$g_{1,1,1,1}$	$g_{1,1,1,2}$	$g_{1,1,1,3}$	$g_{1,1,2,1}$	$g_{1,1,2,2}$	$g_{1,1,2,3}$	$g_{1,2,1}$	$g_{1,2,2}$	$g_{1,3,1,1}$	$g_{1,3,1,2}$	$g_{1,3,2,1}$	$g_{1,3,2,2}$
Andalusia	0.039	0.076	0.009	0	0.001875	0.01648	0.0087	0.0798	0.076	0.095	0.015	0.00645
Catalonia	0.01872	0	0	0.1675	0.005625	0.0398301	0	0	0	0.19	0.006	0
Barcelona	0.039	0.03268	0.012	0.1675	0.001875	0.103	0.00915	0.1634	0	0.171	0.015	0.015
Madrid	0.02574	0.076	0.009	0.0825	0.0075	0.0631699	0.0096	0.0836	0	0	0	0.00645
Santiago	0.039	0.05396	0.003	0.0825	0	0.03296	0.015	0.0589	0.076	0.0475	0.009	0.00645
Rias Baixas	0	0	0	0.1675	0.001875	0.01648	0.00285	0	0.076	0	0	0
Stockholm	0.039	0.076	0.015	0.25	0.001875	0.0398301	0.00915	0.1425	0.076	0.076	0.003	0.00855
Wales	0.02574	0.04332	0	0.25	0.005625	0	0.01335	0.0418	0.076	0.095	0.009	0
Rome	0.039	0.076	0.015	0.25	0.001875	0.01648	0.00915	0.0418	0	0	0.006	0.00645
Switzerland	0.02028	0.076	0.015	0.25	0.001875	0.01648	0.01125	0.19	0.076	0.1425	0.015	0.015

Table 4: Optimality values and utility degrees of the alternatives according to sub-criterion «Usability»

$g_{1,1,1}$	$g_{1,1,1,1}$	$g_{1,1,1,2}$	$g_{1,1,1,3}$	S_i	K_i	Rank
Andalusia	0.039	0.076	0.009	0.124	0.953846154	2
Catalonia	0.01872	0	0	0.01872	0.144	8
Barcelona	0.039	0.03268	0.012	0.08368	0.643692308	6
Madrid	0.02574	0.076	0.009	0.11074	0.851846154	4
Santiago	0.039	0.05396	0.003	0.09596	0.738153846	5
Rias Baixas	0	0	0	0	0	9
Stockholm	0.039	0.076	0.015	0.13	1	1
Wales	0.02574	0.04332	0	0.06906	0.531230769	7
Rome	0.039	0.076	0.015	0.13	1	1
Switzerland	0.02028	0.076	0.015	0.11128	0.856	3

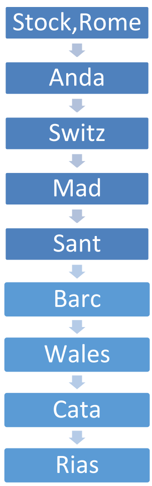

Figure 4: Partial pre-order according to sub-criterion «Usability»

Table 5: Optimality values and utility degrees of the alternatives according to sub-criterion «Accessibility»

$g_{1,1,2}$	$g_{1,1,2,1}$	$g_{1,1,2,2}$	$g_{1,1,2,3}$	S_i	K_i	Rank
Andalusia	0	0.065	0.0528	0.1178	0.230128	9
Catalonia	0.1005	0.195	0.127611	0.423111	0.826568	3
Barcelona	0.1005	0.065	0.33	0.4955	0.967983	2
Madrid	0.0495	0.26	0.202389	0.511889	1	1
Santiago	0.0495	0	0.1056	0.1551	0.302995	8
Rias Baixas	0.1005	0.065	0.0528	0.2183	0.42646	7
Stockholm	0.15	0.065	0.127611	0.342611	0.669307	5
Wales	0.15	0.195	0	0.345	0.673974	4
Rome	0.15	0.065	0.0528	0.2678	0.52316	6
Switzerland	0.15	0.065	0.0528	0.2678	0.52316	6

Table 6: Utility degrees of the alternatives according to the first-level sub-criterion «Usability & Accessibility»

$g_{1,1}$	K_i
Andalusia	0.463
Catalonia	0.081
Barcelona	0.326
Madrid	0.421
Santiago	0.364
Rias Baixas	0.009
Stockholm	0.5
Wales	0.271
Rome	0.499
Switzerland	0.429

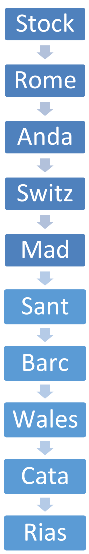

Figure 5: Partial pre-order according to the first-level sub-criterion «Usability & Accessibility»

Table 7: Utility degrees and the ranking of the alternatives according to the «Root criterion»

g_1	K_i	Rank
Andalusia	0.4197	2
Catalonia	0.0888	9
Barcelona	0.3761	5
Madrid	0.3082	7
Santiago	0.322	6
Rias Baixas	0.008	10
Stockholm	0.3902	4
Wales	0.2281	8
Rome	0.3908	3
Switzerland	0.444	1

MULTIPLE CRITERIA DECISION MAKING
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AN INTUITIONISTIC FUZZY EXTENSION OF THE CODAS-SORT METHOD

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Abstract

Currently, an important issue in multi-criteria decision-making (MCDM) problems are vagueness and lack of precision of decision-making information because of insufficient data and incapability of the decision maker to process the information. Intuitionistic fuzzy sets (IFS) are a solution to eliminate the vagueness and the uncertainty. While fuzzy sets (FS) deal with ambiguity and vagueness problem, IFSs have more advantages. Moreover, the CODAS-SORT method cannot handle the uncertainty and ambiguity of information provided by human judgments. The aim of this study is to develop an IF extension of CODAS-SORT combining this method with the IFS theory. To achieve this, we use the fuzzy weighted Euclidean distance and fuzzy weighted Hamming distance instead of the crisp distances. A case study of a supplier selection assessment is used to clarify the details of our proposed method.

Keywords: multicriteria decision aid, sorting methods, CODAS-SORT, intuitionistic fuzzy set.

1 Introduction

MCDM helps the decision maker to evaluate several conflicting criteria. In real life, most problems have multiple objectives and need an assessment of several criteria. As a result, MCDM has become a significant problem and a great deal

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of research has gone into helping the decision maker to choose the best decisions. The categorization and classification of MCDM methods are defined in different ways by the authors. According to Roy (1985), the goal of MCDM is to solve one of three types of decision-making problems: (1) identifying a single best alternative or selecting a few best alternatives (choice), (2) ranking the alternatives from the best to the worst (ranking), (3) sorting the alternatives into predefined homogeneous classes (sorting). The study and application of the first two problems has occurred in several areas, while the sorting problems are handled in few studies.

ELECTRE-Tri (Yu, 1992) is the first variant of ELECTRE for the sorting problems. After that, a few studies applied the transformation of ranking methods to deal with sorting problems, e.g., Electre Tri-C (Almeida-Dias, Figueira and Roy, 2010), ELECTRE Tri-nC (Almeida-Dias et al., 2010), ELECTRE-SORT (Ishizaka and Nemery, 2014), ELECTRE Tri-nB (Fernandez et al., 2017). ELECTRE is not the only ranking method that has been adapted to solve the sorting problem. For example, UTADIS was introduced as a sorting variant of the UTA method (Jacquet-Lagreze and Siskos, 1982). The Promethee variants in the sorting environment are the best known. Figueira, Smet and Brans (2004) developed PROMETHEE TRI, which is the first variant of PROMETHEE to solve a sorting problem. PROMSORT is a sorting methodology based on PROMETHEE (Araz and Ozkarahan, 2005). PROMSORT has two important advantages over PROMETHEE TRI. FlowSort (Nemery and Lamboray, 2008) is an variant of Promethee. Then, Ishizaka, Pearman and Nemery (2012) developed a sorting extension of AHP, namely AHPSort, while Nemery et al. (2012) developed GAIASort, an extension of GAIA. TOPSIS-Sort (Sabokbar et al., 2016) supports sorting problems with TOPSIS. MACBETHSort (Ishizaka and Gordon, 2017) is a sorting variant of MACBETH. VIKORSORT (Demir et al., 2018) is a sorting extension of VIKOR and DEASORT (Ishizaka et al., 2018) is a sorting extension of DEA.

Keshavarz Ghorabaee et al. (2016) proposed a different outranking MCDM method which addresses the ranking problem with the calculation of two distances. This advantage gives CODAS more credibility for the decision maker. The Euclidean distance of alternatives from the “negative-ideal” solution is the first measure and the Taxicab distance is the secondary measure. The most desirable alternative is the one farthest from the negative ideal solution. In this method, the Taxicab distance is used as a secondary measure when there are two incomparable alternatives according to the Euclidean distance. According to these cases, the calculation of the assessment score of the alternatives is a combination of the Euclidean and Taxicab distances. The assessment score makes it possible to rank the alternatives from the best to the worst.

For sorting, Ouhibi and Frikha (2019) introduced CODAS-SORT, a variant of CODAS. The assignment rules use two measures. The first measure is the Euclidean distance and the second one is the Taxicab distance. The difference between these two distances define the assignment rules. However, a problem of human judgments is its ambiguity but the CODAS-SORT method cannot deal with this problem. For this reason, we resort to use an IF environment.

According to Zadeh (1975), FST is an extension of classical set theory (Lemaire, 1990). In real-life conditions, the information and data collected are multiple and sometimes contradictory. For this reason, the evaluation criteria are difficult to express. To solve this problem, the concepts from the IFS theory are more appropriate for dealing with vagueness than other generalized FS models (Gautam, Abhishekh and Singh, 2016). Atanassov (1986) introduced an IFS that is an extension of the classical FST; it is characterized by a membership function and a non-membership function.

In this study, an IF extension of the CODAS-SORT method is proposed to handle the sorting problem in an uncertain environment. A case study is added to indicate the reliability of the proposed IF-CODAS-SORT method. The rest of this paper is organized as follows. In Section 2, some basic concepts and definitions of intuitionistic fuzzy sets are presented. In Section 3, an extension of the CODAS-SORT method is proposed to handle IF multi-criteria decision--making. Then the proposed IF-CODAS-SORT method is applied to a case study in Section 4. Finally, conclusions and suggestions for further research are presented.

2 Fuzzy sets and intuitionistic fuzzy sets

In this section, the basic definitions for the IFS and some IFS-based MCDM problems are reviewed.

Definition 1. Fuzzy sets (Zadeh, 1975)
FST is an extension of classical set theory. However, there is a relaxation of the concept of membership that occurs in the classical theory (Lemaire, 1990). The set X is a universe of discourse, and a fuzzy set $\tilde{a}$ is characterized by a membership function $\mu_{\tilde{a}}(x)$, for $x \in X$, which measures the degree of x belonging to $\tilde{a}$. α. $\mu_{\tilde{a}}(x)$ represents the membership of x in $\tilde{a}$.

$$\alpha = \{(x, \mu_{\tilde{\alpha}}(x)) | x \in X\} \tag{1}$$

Definition 2. Intuitionistic fuzzy set
IFS introduced by Atanassov (1986) is an extension of the classical FST, which is a suitable way to deal with vagueness.

Assuming that X is a collection of objects x and $\beta \in X$ is a fixed set, the IFS β on X is defined as (Atanassov, 1986):

$$\beta = \left\{\left(x, \mu_\beta(x), \upsilon_\beta(x)\right) \middle| x \in X\right\} \tag{2}$$

where $\mu_\beta(x): X \rightarrow [0,1]$, $x \in X \rightarrow \mu_\beta(x) \in [0,1]$ represents the degree of membership of element $x \in X$ in set β, and $\upsilon_\beta(x): X \rightarrow [0,1]$, $x \in X \rightarrow \upsilon_\beta(x) \in$ $\in [0,1]$ is the degree of non-membership of element $x \in X$ in set β.

μ_β and $\upsilon_\beta(x)$ usually satisfy $0 \leq \mu_\beta(x) + \upsilon_\beta(x) \leq 1$ for all $x \in X$. Besides the degree of membership and non-membership, an indeterminacy degree, so-called "hesitancy degree" of x to β, which is different from the numbers $\mu_\beta(x)$ and $\upsilon_\beta(x)$ and which measures the degree of indeterminacy of $x \in X$ to β is defined as:

$$\pi_\beta(x) = 1 - \mu_\beta(x) - \upsilon_\beta(x) \tag{3}$$

Accordingly, an intuitionistic fuzzy number β can be represented as $\beta = (\mu_\beta, \upsilon_\beta, \pi_\beta)$, which included the degree of membership, of non--membership, and of indeterminacy.

Definition 3. Arithmetic operations (Xu and Yager, 2006)
Let $\gamma = \left(\mu_\gamma, \upsilon_\gamma, \pi_\gamma\right)$ and $\beta = (\mu_\beta, \upsilon_\beta, \pi_\beta)$ be two intuitionistic fuzzy numbers; the arithmetic operations on these numbers are defined as follows:

Addition:

$$\begin{gathered}\gamma \oplus \beta = \left(\mu_\gamma, \upsilon_\gamma, \pi_\gamma\right) \oplus \left(\mu_\beta, \upsilon_\beta, \pi_\beta\right) = \\ = \mu_\gamma + \mu_\beta - \mu_\gamma \mu_\beta, \upsilon_\gamma \upsilon_\beta, 1 + \mu_\gamma \mu_\beta - \mu_\gamma - \mu_\beta - \upsilon_\gamma \upsilon_\beta\end{gathered} \tag{4}$$

$$\begin{gathered}\bigoplus_{j=1}^{n} \gamma_j = \bigoplus_{j=1}^{n} \left(\mu_{\gamma_j}, \upsilon_{\gamma_j}, \pi_{\gamma_j}\right) = \\ = 1 - \prod_{j=1}^{n}(1 - \mu_{\gamma_j}), \prod_{j=1}^{n} \upsilon_{\gamma_j}, \prod_{j=1}^{n}(1 - \mu_{\gamma_j}) - \prod_{j=1}^{n} \upsilon_{\gamma_j})\end{gathered} \tag{5}$$

Multiplication:

$$\begin{gathered}\gamma \otimes \beta = \left(\mu_\gamma, \upsilon_\gamma, \pi_\gamma\right) \otimes \left(\mu_\beta, \upsilon_\beta, \pi_\beta\right) = \\ = (\mu_\gamma \mu_\beta, \upsilon_\gamma + \upsilon_\beta - \upsilon_\gamma \upsilon_\beta, 1 + \upsilon_\gamma \upsilon_\beta - \mu_\gamma \mu_\beta - \upsilon_\gamma - \upsilon_\beta)\end{gathered} \tag{6}$$

$$\begin{gathered}\bigotimes_{j=1}^{n} \gamma_j = \bigotimes_{j=1}^{n} \left(\mu_{\gamma_j}, \upsilon_{\gamma_j}, \pi_{\gamma_j}\right) = \\ = \left(\prod_{j=1}^{n} \mu_{\gamma_j}, \prod_{j=1}^{n}(1 - \upsilon_{\gamma_j}), 1 - \prod_{j=1}^{n} \mu_{\gamma_j}) - \prod_{j=1}^{n}(1 - \upsilon_{\gamma_j})\right)\end{gathered} \tag{7}$$

Scale multiplication:

$$\lambda_\gamma = \left(1-\left(1-\mu_\gamma\right)^\lambda, \left(\upsilon_\gamma\right)^\lambda, \left(1-\mu_\gamma\right)^\lambda - \left(\upsilon_\gamma\right)^\lambda\right) \tag{8}$$

where λ is a crisp number.

Definition 4. Geometric distance (Szmidt and Kacprzyk, 2000)
The Hamming distance is defined as:

$$D(\gamma,\beta) = \frac{1}{2}\sum_{j=1}^{n}\left(\left|\mu_\gamma(x_j)-\mu_\beta(x_j)\right| + \left|\upsilon_\gamma(x_j)-\upsilon_\beta(x_j)\right| + \left|\pi_\gamma(x_j)-\pi_\beta(x_j)\right|\right) \tag{9}$$

The Euclidean distance is defined as:

$$D(\gamma,\beta) = \sqrt{\frac{1}{2}\sum_{j=1}^{n}\left[\left(\mu_\gamma(x_j)-\mu_\beta(x_j)\right)^2 + \left(\upsilon_\gamma(x_j)-\upsilon_\beta(x_j)\right)^2 + \left(\pi_\gamma(x_j)-\pi_\beta(x_j)\right)^2\right]} \tag{10}$$

3 The intuitionistic fuzzy CODAS-SORT method

In this section, we present an IF extension of the CODAS-SORT method to deal with sorting problem. As already declared, CODAS-SORT is a new sorting method based on CODAS. It is easy to apply and simple to deal with for DM. The use of two measures defines the assignment rules. The first measure is the Euclidean distance and the second one is the Taxicab distance. However, we cannot use the Euclidean and Taxicab distances in IF problems, because they are defined in a crisp environment. Because of that, we replaced the Taxicab distance by the Hamming distance. Since the aim of this study is to propose an IF extension of CODAS, instead of crisp distances, we use the fuzzy weighted Euclidean distance and the fuzzy weighted Hamming distance, which were introduced by Li (2007). Suppose that we have *n* alternatives and *m* criteria.

The steps of the IF-CODAS-SORT method are the following:

Step 1. Construct the IF decision matrix (D_X):

Determining the IF decision-making matrix. Assuming that there are *m* alternatives ($A_1, A_2, \ldots, A_m$) to be evaluated with respect to *n* criteria ($M_1, M_2, \ldots, M_n$):

$$D_X = \begin{array}{cccccc} & M_1 & M_2 & & M_n \\ A_1 & (\mu_{11}^X, \upsilon_{11}^X, \pi_{11}^X) & (\mu_{12}^X, \upsilon_{12}^X, \pi_{12}^X) & \cdots & (\mu_{1n}^X, \upsilon_{1n}^X, \pi_{1n}^X) \\ A_2 & (\mu_{21}^X, \upsilon_{21}^X, \pi_{21}^X) & (\mu_{22}^X, \upsilon_{22}^X, \pi_{22}^X) & \ldots & (\mu_{2n}^X, \upsilon_{2n}^X, \pi_{2n}^X) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & (\mu_{m1}^X, \upsilon_{m1}^X, \pi_{m1}^X) & (\mu_{m2}^X, \upsilon_{m2}^X, \pi_{m2}^X) & \ldots & (\mu_{mn}^X, \upsilon_{mn}^X, \pi_{mn}^X) \end{array} \tag{11}$$

where D_X is the decision-making matrix, and $\mu_{ij}^X, \upsilon_{ij}^X, \pi_{ij}^X$ are the relative performances of the i^{th} alternative with respect to the j^{th} criterion.

Step 2. Construct the IF profile matrix (D_Y):

Determining the IF decision-making matrix. Assuming that there are l profiles ($B_1, B_2, \dots, B_l$) to be evaluated with respect to n criteria $M_1, M_2, \dots, M_n$):

$$D_Y = \begin{array}{cccccc} & M_1 & M_2 & & M_n & \\ B_1 & (\mu_{11}^Y, v_{11}^Y, \pi_{11}^Y) & (\mu_{12}^Y, v_{12}^Y, \pi_{12}^Y) & \cdots & (\mu_{1n}^Y, v_{1n}^Y, \pi_{1n}^Y) \\ B_2 & (\mu_{21}^Y, v_{21}^Y, \pi_{21}^Y) & (\mu_{22}^Y, v_{22}^Y, \pi_{22}^Y) & \dots & (\mu_{2n}^Y, v_{2n}^Y, \pi_{2n}^Y) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ B_l & (\mu_{l1}^Y, v_{l1}^Y, \pi_{l1}^Y) & (\mu_{l2}^Y, v_{l2}^Y, \pi_{l2}^Y) & \dots & (\mu_{ln}^Y, v_{ln}^Y, \pi_{ln}^Y) \end{array} \quad (12)$$

where D_Y is the profiles matrix, and $\mu_{kj}^X, v_{kj}^X, \pi_{kj}^X$ are the relative performances of k^{th} profile with respect to j^{th} criterion.

Step 3. Determine the IF negative-ideal solution:

NIS_j is the anti-ideal solution of the decision matrix:

$$NIS_j = (u_j^X, v_j^X, \pi_j^X), \text{j} = 1, 2, \dots, \text{n} \quad (13)$$

$$t = arg\min_i (u_{ij}^X) \quad (14)$$

$$t = u_{tj}^X \quad (15)$$

$$u_j^X = u_{tj}^X \quad (16)$$

$$v_j^X = v_{tj}^X \quad (17)$$

$$\pi_j^X = 1 - u_{tj}^X - v_{tj}^X \quad (18)$$

MIS_j is the anti-ideal solution of the profiles matrix:

$$MIS_j = (u_j^Y, v_j^Y, \pi_j^Y), \text{j} = 1, 2, \dots, \text{n} \quad (19)$$

$$t = arg\min_k (u_{kj}^Y) \quad (20)$$

$$t = u_{kj}^Y \quad (21)$$

$$u_j^Y = u_{kj}^Y \quad (22)$$

$$v_j^Y = v_{kj}^Y \quad (23)$$

$$\pi_j^Y = 1 - u_{kj}^Y - v_{kj}^Y \quad (24)$$

Step 4. Calculate the IF weighted Euclidean distances of alternatives from the IF negative-ideal solution:

$$E_{a_i} = \sqrt{\frac{1}{2}\sum_{j=1}^{n} \left(u_{ij}^X - u_j^X\right)^2 + \left(v_{ij}^X - v_j^X\right)^2 + \left(\pi_{ij}^X - \pi_j^X\right)^2} \quad (25)$$

where E_{a_i} denotes the Euclidean distance between the action a_i and the negative-ideal solution NIS_j:

$$E_{b_k} = \sqrt{\frac{1}{2}\sum_{k=1}^{n} \left(u_{kj}^Y - u_j^Y\right)^2 + \left(v_{kj}^Y - v_j^Y\right)^2 + \left(\pi_{kj}^Y - \pi_j^Y\right)^2} \quad (26)$$

where E_{b_k} denotes the Euclidean distance between the limit b_k and the negative--ideal solution MIS_j.

Step 5. Calculate the IF weighted Hamming distances of alternatives from the IF negative-ideal solution:

$$H_{a_i} = \frac{1}{2}\sum_{j=1}^{n}\left(\left|u_{ij}^{X} - u_{j}^{X}\right| + \left|v_{ij}^{X} - v_{j}^{X}\right| + \left|\pi_{ij}^{X} - \pi_{j}^{X}\right|\right) \tag{27}$$

where H_{a_i}denotes the Hamming distance between the action a_i and the negative--ideal solution NIS_j:

$$H_{b_k} = \frac{1}{2}\sum_{j=1}^{n}\left(\left|u_{kj}^{Y} - u_{j}^{Y}\right| + \left|v_{kj}^{Y} - v_{j}^{Y}\right| + \left|\pi_{kj}^{Y} - \pi_{j}^{Y}\right|\right) \tag{28}$$

where H_{b_k} denotes the Hamming distance between the limit b_k and the negative--ideal solution MIS_j.

Step 6. Determine the relative assessment matrix:

$$R(a_i, b_k) = \left[E_{a_i} - E_{b_k}\right] + (\psi\left[E_{a_i} - E_{b_k}\right] * \left[H_{a_i} - H_{b_k}\right]) \tag{29}$$

where $k \in \{1, 2, \ldots, n\}$ and ψ denotes a threshold function to determine the equality of the Euclidean distances of two alternatives, and is defined as follows:

$$\psi(x) = \begin{cases} 1 & \text{if } |x| \geq \tau \\ 0 & \text{if } |x| < \tau \end{cases} \tag{30}$$

In this function, τ is the threshold parameter that can be set by the DM. It is suggested to fix this parameter at a value between 0.01 and 0.05.

If the difference between the Euclidean distances of two alternatives is less than τ, these two alternatives are also compared by the Hamming distance. In this study, we use τ = 0.02 for the calculations.

Step 7. Assign alternatives to categories: To assign an alternative a_i to one of the predefined categories, there are two ways that depend on the type of the available profile provided by the decision maker:

<u>Central profiles:</u>
If central profiles have been defined, the alternative a_i is assigned to the class C_k which has the smallest | $R(a_i, b_k)$|.
If | $R(a_i, b_k)$| is the smallest then $a_i \in C_k$.

<u>Limiting profiles:</u>
When the difference between the two distances is minimal, the alternative and the center of the category are very near and if the difference is negative or positive, the alternative belongs to the category that has the minimum difference. If limiting profiles have been defined and |R(a_i, b_k)| is the smallest then there are two cases:

− If $R(a_i, b_k) \geq 0$ then alternative a_i is assigned to class C_k.
− If $R(a_i, b_k) < 0$ then alternative a_i is assigned to class C_{k-1}.

4 Case study: Suplier selection

The case problem allows evaluating and assessing seven suppliers (a_1, a_2, a_3, a_4, a_5, a_6, a_7). The proposed evaluation framework was applied at a company N, a maker of perfumery, hygiene, health and cosmetic products. An expert evaluates the suppliers with respect to four criteria: Price, Product quality, Delivery and Agility. These seven suppliers are divided into three groups: Worst C_1, Moderate C_2, and Best C_3. After determining the list of alternatives, we evaluate them with regard to each criterion (Table 1).

Table 1: Performance matrix

	C_1	C_2	C_3	C_4
$\boldsymbol{a_1}$	(0.50, 0.10, 0.40)	(0.60, 0.10, 0.30)	(0.40, 0.10,0.50)	(0.70, 0.10, 0.20)
$\boldsymbol{a_2}$	(0.20, 0.40, 0.40)	(0.40, 0.20, 0.40)	(0.50, 0.10, 0.40)	(0.20, 0.30, 0.50)
$\boldsymbol{a_3}$	(0.40, 0.50, 0.10)	(0.50, 0.10, 0.40)	(0.40, 0.20, 0.40)	(0.50, 0.10, 0.40)
$\boldsymbol{a_4}$	(0.50, 0.10, 0.40)	(0.50, 0.10, 0.40)	(0.40, 0.30, 0.30)	(0.20, 0.40, 0.40)
$\boldsymbol{a_5}$	(0.50, 0.20, 0.30)	(0.60, 0.10, 0.30)	(0.50, 0.20, 0.30)	(0.70, 0.20, 0.10)
$\boldsymbol{a_6}$	(0.20, 0.20, 0.60)	(0.40, 0.20, 0.40)	(0.50, 0.10, 0.40)	(0.40, 0.30, 0.30)
$\boldsymbol{a_7}$	(0.40, 0.10, 0.50)	(0.50, 0.00, 0.50)	(0.40, 0.30, 0.30)	(0.50, 0.20, 0.30)
NIS_j	(0.20, 0.20, 0.60)	(0.40, 0.20, 0.40)	(0.40, 0.10, 0.50)	(0.20, 0.30, 0.50)

Thereafter, the decision maker is invited to provide a list of classes. We evaluate the limiting profiles with regard to each criterion (Table 2).

Table 2: Limiting profiles

	C_1	C_2	C_3	C_4
$\boldsymbol{b_1}$	(0.40, 0.20, 0.40)	(0.40, 0.20, 0.40)	(0.40, 0.25, 0.35)	(0.40, 0.25, 0.35)
$\boldsymbol{b_2}$	(0.60, 0.30, 0.10)	(0.60, 0.30, 0.10)	(0.55, 0.30, 0.15)	(0.55, 0.30, 0.15)
MIS_j	(0.40, 0.20, 0.40)	(0.40, 0.20, 0.40)	(0.40, 0.25, 0.35)	(0.40, 0.25, 0.35)

Next, we calculate the Euclidean and Hamming distances of the alternatives and limits from the negative-ideal solution (Tables 3 and 4):

Table 3: Euclidian and Hamming distances (actions)

	Distances	
Alternatives	$\boldsymbol{E_{a_i}}$	$\boldsymbol{H_{a_i}}$
$\boldsymbol{a_1}$	0.29	1
$\boldsymbol{a_2}$	0.05	0.3
$\boldsymbol{a_3}$	0.28	1
$\boldsymbol{a_4}$	0.125	0.7
$\boldsymbol{a_5}$	0.36	1.2
$\boldsymbol{a_6}$	0.05	0.3
$\boldsymbol{a_7}$	0.17	0.9

Table 4: Euclidian and Hamming distances (profiles)

	Distances	
Profiles	$\boldsymbol{E_{b_i}}$	$\boldsymbol{H_{b_i}}$
$\boldsymbol{b_1}$	0	0
$\boldsymbol{b_2}$	0.205	0.82

The construction of the relative evaluation matrix is as follows (Table 5):
First, we set τ = 0.02
Example of calculation:

$$h(a_1, b_1) = (0.29 - 0) + [(0.29 - 0) * (1 - 0)] = 0.58$$

The other relative evaluations are shown in Table 5.

Table 5: Relative evaluation matrix

	$\boldsymbol{b_1}$	$\boldsymbol{b_2}$
$\boldsymbol{a_1}$	0.58	0.1
$\boldsymbol{a_2}$	0.065	−0.075
$\boldsymbol{a_3}$	0.56	0.89
$\boldsymbol{a_4}$	0.11	−0.07
$\boldsymbol{a_5}$	0.79	0.2
$\boldsymbol{a_6}$	0.065	−0.074
$\boldsymbol{a_7}$	0.323	−0.038

The assignment of alternatives to categories is presented in Table 6.

For example:
Since $|R(a_3, b_1)|$ is the smallest, we have $R(a_3, b_1) \geq 0$, and alternative a_3 is assigned to class C_2.
Since $|R(a_4, b_2)|$ is the smallest, we have $R(a_4, b_2) < 0$, and alternative a_4 is assigned to class C_2.

Table 6: The final classification of the actions

Actions	Categories
a_1	C_3
a_2	C_2
a_3	C_2
a_4	C_2
a_5	C_3
a_6	C_2
a_7	C_2

Suppliers a_1 and a_5 are assigned to the best group, whereas a_2, a_3, a_4, a_6 and a_7 are assigned to the moderate group.

Sensitivity analysis

A sensitivity analysis is also performed in this part to demonstrate the stability of the sorting result. First, five values of τ are generated. Then we solve the problem using each of these cases. The generated values of τ are shown in Table 7 and the sorting results, in Figure 1.

Table 7: Sorting results with different values of τ

Actions	0.01 C_3	0.02 C_2	0.03 C_3	0.04	0.05
a_1	3	3	3	3	3
a_2	2	2	2	3	2
a_3	2	2	1	2	2
a_4	2	3	2	2	2
a_5	3	3	3	3	3
a_6	2	2	2	2	2
a_7	2	2	3	1	1

According to Figure 1 and Table 7, we can notice a good stability in the sorting of actions when the threshold parameter τ varies from 0.01 to 0.05. However, the modification of the τ parameter has a minor and neglected impact on the sorting of actions that can undermine the validity of the results. Consequently, we can affirm the performance of the IF-CODAS-SORT method.

As indicated by the conclusions of this analysis, we can claim that our proposed method is proficient to handle MCDM problems.

However, it may be seen from Table 7, that every one of the differences in sorting occurred between the successive classes, which confirms the consistency of the results.

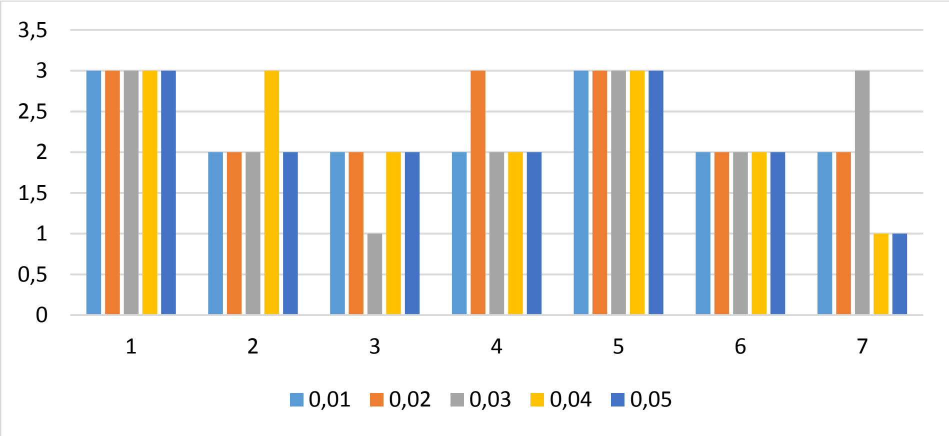

Figure 1: Sorting results with different values of τ

5 Application of the IR-CODAS model for risk assessment

The development of an intuitionistic fuzzy CODAS-SORT method is the objective of this study. Indeed, CODAS-SORT deals with the sorting MCDM problem. This method sorts the alternatives into ordered classes based on the central and limiting profiles and using exact values. Since it is difficult for decision makers to precisely express their preferences, we have developed an IF-CODAS-SORT method which uses intuitionistic fuzzy numbers to express uncertain evaluations.

An advantage of our result is that the assignment rules are based on the use of two measures. The first measure is based on the Euclidean distance. The second measure is the Hamming distance. The assignment rules are based on the difference between the two distances.

In the future, we intend to develop an IF-CODAS-SORT approach in the group decision context (IF-GD-CODAS-SORT).

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MULTIPLE CRITERIA DECISION MAKING

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THE MULTICRITERIA GROUP DECISION MAKING FLOWSORT METHOD UNDER UNCERTAINTY

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Abstract

Crisp values are insufficient to model real-life situations and imprecise ideas are frequently represented in multicriteria decision aid analysis. In fact, it is difficult to treat the evaluation criteria precisely and to fix exact preferences rating. The triangular intuitionistic fuzzy numbers succeeded to treat this kind of ambiguity in a great deal of research than other forms of fuzzy representation functions. The field of sorting issues is an active research topic in the multiple criteria decision aid (MCDA). This study extended one of the sorting methods, FLOWSORT, for solving multiple criteria group decision-making problems. This extension described the preferences rating of alternatives as linguistic terms which can be easily expressed in triangular intuitionistic fuzzy numbers. To validate our extension, an illustrative example as well as an empirical comparison with other multi-criteria decision making methods is presented.

Keywords: multicriteria group decision making, sorting problematic, intuitionistic fuzzy set, FlowSort method.

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1 Introduction

Multi-criteria decision making (MCDM) is considered an essential part of modern decision science and operational research. It is the process of finding the best compromise among the feasible alternatives. It provides a wide variety of methodologies and techniques that enable the systematic treatment of decision problems under multiple criteria. The MCDA methods can be applied to four different kinds of analyses that can be performed in order to provide significant support to decision-makers (Remadi and Frikha, 2019). These are: (1) the choice of the best alternative, (2) the ranking of the set of the alternatives from the best to the worst, (3) the description of the features of the alternatives and (4) the classification of the alternatives into predefined homogenous groups.

In this paper, we study the ordinal classification problem, also called the sorting problem. It consists in orienting a decision problem to an assignment of alternatives to one of the predefined, ordered and homogenous categories or classes. Each class is a set of alternatives with similar properties or even values for the same properties, when compared to the alternatives of from the other classes. Many methods have been proposed during the previous decades. Among these, we can mention the well-known sorting methods, the ELECTRE-TRI (Shen, Xu and Xu, 2016), the THESEUS (Fernandez and Navarro, 2011), etc. Relying on the PROMETHEE (Brans, Mareschal and Vincke, 1984) methodology, several authors proposed the PROMETHEE-TRI (Figueira, Smet and Brans, 2004), the PROMSORT (Araz and Ozkarahan, 2007) and the FlowSort (Nemery and Lamboray, 2007). In fact, the PROMETHEE is one of the best known MCDM methods, since it is easy to use, simple to process and uses fewer parameters than the other MCDM methods such as ELECTRE (Govindan and Jepsen, 2016). Figueira, Smet and Brans (2004) were pioneers in the PROMETHEE-TRI method, extending it to the sorting context, but it used incompletely ordered categories. In 2007, Araz and Ozkarahan (2007) proposed the PROMSORT method which used completely ordered categories, but the assignment of the alternatives was not independent.

Developed by Nemery and Lamboray in 2007, FlowSort (Nemery and Lamboray, 2007) was proposed for assigning actions to completely ordered categories defined by limiting profiles or central profiles. It solves the drawbacks of PROMETHEE-TRI (Figueira, Smet and Brans, 2004) and PROMSORT (Araz and Ozkarahan, 2007) and treats the problematic sorting issue for independent assignments and completely ordered categories. The evaluation of alternatives and preference parameters of FlowSort are defined as crisp values. But, in a real-world situation, decisional problems are multidimensional and ambiguous in nature.

So, it is difficult to express the evaluation criteria precisely. Many extensions of FlowSort have been developed to solve these problems. Indeed, Janssen and Nemery (2012) proposed an extension of FlowSort to the case of input data imprecision. Moreover, Campos, Mareschal and Almeida (2015) extended FlowSort to introduce a fuzzy sorting method called Fuzzy FlowSort (F-FlowSort). For a simplified FlowSort version, Assche and De Smet (2016) found the parameters of a sorting model using classification examples in the context of traditional sorting and interval sorting. Moreover, Pelissari et al. (2019) suggested a new multicriteria method, SMAA-Fuzzy-FlowSort, for sorting problems under uncertainty through applying the Stochastic Acceptability Analysis to Fuzzy FlowSort.

As stated above, the fuzzy set (FS) theory (Zadeh, 1965) has been successfully applied in a good number of studies. However, this theory is not flawless as it uses only the membership degree of an element to a fuzzy set which is between zero and one. Actually, it is necessary to define the non-membership degree of an element to a fuzzy set, because it is not necessarily equal to 1 minus the degree of membership. To overcome this limitation, the intuitionistic fuzzy set theory concept seems more suitable to deal with uncertainty than other generalized fuzzy sets forms (Zhang, Jin and Liu, 2013). Furthermore, compared to the traditional fuzzy sets, the IFS can describe the fuzzy nature of the real world more comprehensively (Wang, Han and Zhang, 2012). In fact, it provides more flexibility to treat real-life problems under an uncertain environment, because when the area of applications changes, the intuitionistic fuzzy sets are easy to modify (Zhang, Jin and Liu, 2013).

Due to the complexity of the socio-economic environment, single decision-makers are unable to express their opinions or preferences on multiple criteria. In fact, multiple criteria group decision making (MCGDM) problems constitute an important research area that has drawn the attention of many researchers. In addition, the intuitionistic fuzzy set theory was applied to solve real-life complex Multicriteria Group Decision Making problems. Park, Cho and Kwun (2011), for instance, extended the group decision-making VIKOR method to an interval-valued intuitionistic fuzzy environment, in which the information about attribute weights was partially known. In addition, Chen (2015) developed an extended TOPSIS (Chen and Hwang, 1992) method which included the comparison approach to address multiple criteria group decision-making medical problems in the interval-valued intuitionistic fuzzy set framework. In the context of sorting problem, Shen, Xu and Xu (2016) provided a new outranking sorting method for solving Multi-Criteria Group Decision Making (MCGDM) problems using Intuitionistic Fuzzy Sets (IFS). Furthermore, Lolli et al. (2015) introduced a group decision support system, named FlowSort-GDSS, for sorting failure modes into priority classes.

Thus, the first aim of our research, which is also at the heart of its originality, was to develop an extension of the FlowSort method to deal with the imprecision issue, using the IFS theory to solve MCGDM problems. It consists in aggregating the individual sorting results in a collective one and calculating the personal and the group satisfaction degrees. Shen, Xu and Xu (2016) defined the personal satisfaction degree as the mean average of the comparison of the group sorting results and the individual sorting results and the group satisfaction degree as the weighted average of the personal satisfaction degrees. If satisfaction is low, it will be necessary to recollect the input data.

In addition, human judgments including preferences are difficult to define as numerical values. Also, the linguistic terms can simplify the process of an alternative rating by decision makers (DMs). Several operations on fuzzy numbers have been used to convert linguistic terms into IF numbers in the literature; the easiest to use are Triangular intuitionistic fuzzy numbers (Gautam, Singh and Singh, 2016). And here comes our second main original contribution, which lies in our choice to describe our decisional matrix through linguistic terms which are then, converted into triangular intuitionistic fuzzy values.

The remaining of this paper is organized as follows: in the second section, we present the FlowSort method using crisp evaluations. We introduce the IFS theory notations and definitions in the third section. The fourth section is devoted to develop developing an extension of the FlowSort method based on the IFS theory to solve the Multicriteria group decision making problem. Section five includes a numerical example and a comparison of the achieved results with those of other MCDA methods. The final section provides conclusions and suggests further research issues.

2 The FlowSort method

The FlowSort method is an ordinal classification method based on the ranking methodology of the PROMETHEE method. We first summarized the PROMETHEE algorithm which is based on the principle of pairwise comparisons of the alternatives. It aggregates the preference information of a DM through valued preference relations (Brans, Mareschal and Vincke 1984; Brans and Mareschal, 2005). Let $A = \{a_1, a_2, \dots, a_n\}$ be a set of alternatives and $G = \{g_1, g_2, \dots, g_m\}$ be a set of criteria. A w_k weight, $k = 1, \dots, m$, for each criterion should be well-known by the DM.

The preference function $P^k(a_i, a_j)$ represents the preference intensity of a_i over a_j according to criterion g_k, for $i = 1, \dots, n$, $j = 1, \dots, n$ and $k = 1, \dots, m$: $P^K(a_i, a_j) = P[d^K(a_i, a_j)]$, where $d^K(a_i, a_j) = g_k(a_i) - g_k(a_j)$ for a criterion to maximize and $d^K(a_i, a_j) = g_k(a_j) - g_k(a_i)$ for a criterion to minimize.

Six different types of preference functions were defined by Brans and Mareschal (2005).

Therefore, we need to calculate the outgoing flow $\phi_i^+(a_i) = \frac{1}{N-1}\sum_{x\in A}(a_i, x)$ and the incoming flow $\phi_i^-(a_i) = \frac{1}{N-1}\sum_{x\in A}(x, a_i)$, for each alternative a_i. Three relations can be defined as follows:

- The preference (P):

If $\phi_i^+(a_i) > \phi_i^+(a_j)$ and $\phi_i^-(a_i) \leq \phi_i^-(a_j)$; or $\phi_i^+(a_i) = \phi_i^+(a_j)$ and $\phi_i^-(a_i) < \phi_i^-(a_j)$ or $\phi_i^+(a_i) > \phi_i^+(a_j)$ and $\phi_i^-(a_i) = \phi_i^-(a_j)$; a_iP a_j,

- The indifference (IND):

If $\phi_i^+(a_i) = \phi_i^+(a_j)$ and $\phi_i^-(a_i) = \phi_i^-(a_j)$; a_iIND a_j,

- The incomparability (INC):

If $\phi_i^+(a_i) > \phi_i^+(a_j)$ and $\phi_i^-(a_i) < \phi_i^-(a_j)$; or $\phi_i^+(a_i) > \phi_i^+(a_j)$ and $\phi_i^-(a_i) < \phi_i^-(a_j)$; a_iINCa_j.

PROMETHEE II proposed the net flow $\phi = \phi_i^+(a_i) - \phi_i^-(a_i)$ to overcome the incomparability of alternatives. Two rules can be defined as follows:

- the preference (P): a_iP a_j iff $\phi(a_i) > \phi(a_j)$;
- the indifference (IND): a_i IND a_j iff $\phi(a_i) = \phi(a_j)$.

The FlowSort was proposed to assign a set of *n* alternatives *A* to *k* ordered categories $C_1, C_2, \ldots, C_k$ evaluated according to *m* criteria *G*. Each category is defined by a set of limiting profiles $R = \{r_1, r_2, \ldots, r_{k+1}\}$ or by a set of *k* central profiles (centroids) for *k* ordered categories $\tilde{R} = \{\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_k\}$ defined by the DM. So, to avoid conflicts, we note that each category can be defined by a set of reference profiles $R^* = \{r_1^*, r_2^*, \ldots\}$ founded by Nemery and Lamboray (2007). For each alternative a_i for all $i \in \{1, 2, \ldots, n\}$, let us define a set $R_i^* = R^* \cup \{a_i\}$, where a_i is the action to be assigned.

The assignment of alternatives is deduced from their relative position with respect to the reference profiles, in terms of positive, negative and net flows. It depends on the simultaneous comparison of the alternative with all the reference profiles (Nemery and Lamboray, 2007). The positive, negative and net flows are computed as follows by using equation (1):

$$\phi_{R_i^*}^+(x) = \frac{1}{|R_i^*|-1}\sum_{y\in R_i^*}\pi(x, y), \tag{1}$$

$$\phi_{R_i^*}^-(x) = \frac{1}{|R_i^*|-1}\sum_{y\in R_i^*}\pi(y, x), \tag{2}$$

$$\phi_{R_i^*}(x) = \phi_{R_i^*}^+(x) - \phi_{R_i^*}^-(x), \tag{3}$$

where $|R_i^*|$ is the number of elements in the set R_i^*.

Three different assignment rules based on the positive, negative and the net flows are defined as follows:

$$C_{\phi^+}(a_i) = C_K \text{ if } \phi^+_{R_i^*}(r_k) > \phi^+_{R_i^*}(a_i) \geq \phi^+_{R_i^*}(r_{k+1}), \tag{4}$$

$$C_{\phi^-}(a_i) = C_k \text{ if } \phi^-_{R_i^*}(r_k) \leq \phi^-_{R_i^*}(a_i) < \phi^-_{R_i^*}(r_{k+1}), \tag{5}$$

$$C_{\phi}(a_i) = C_K \text{ if } \phi_{R_i^*}(r_k) > \phi_{R_i^*}(a_i) \geq \phi_{R_i^*}(r_{k+1}). \tag{6}$$

3 Intuitionistic fuzzy set theory

To deal with uncertainty and vagueness, fuzzy set theory (Zadeh, 1965) was used as an efficient tool, and has had a great success in innumerable fields. Let X denotes a universe of discourse. A fuzzy set A in X is defined as a set of ordered pairs:

$$A = \{<x, \mu_A(x)>|\, x \in X\},\ \mu_A(x) \in [0,1] \text{ is the degree of belongingness of x in A.} \tag{7}$$

The intuitionistic fuzzy set theory (Atanassov, 1986) is a generalization of the fuzzy set theory (Zadeh, 1965). It solves the problem that a non-membership degree is not always equal to $1 - \mu_A(x)$ in real life. The IFS theory is characterized by assigning a membership degree and a non-membership degree to each element. Let a set *X* be fixed, an intuitionistic fuzzy set (IFS) *A* in *X* is defined as follows:

$$A = \{<x, \mu_A(x), \nu_A(x)>|\, x \in X\},\ \mu_A(x),\ \nu_A(x) \in [0,1], \tag{8}$$

where $\mu_A(x)$ and $\nu_A(x)$ are defined, respectively, as the degree of membership and the degree of non-membership of the element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The fuzzy set (Zadeh, 1965) is defined by $A = \{<x, \mu_A(x)>|\, x \in X\}$ and can be defined as an IFS by $A = \{<x, \mu_A(x), 1 - \mu_A(x)>|\, x \in X\}$. For each IFS *A* in *X*, the degree of hesitancy of *x* to *A* is $\theta_A(x) = 1 - \mu_A(x) - \nu_A(x)$. If $\theta_A(x) = 0$, then *A* is reduced to a fuzzy set.

The IFS is able to describe the data which may involve uncertain information. An ill-known quantity may therefore be expressed with an intuitionistic fuzzy number (IFN). Several functions such as trapezoidal (Banerjee, 2012) triangular (Li, Nan and Zhang, 2012), interval number (Sengupta and Pal, 2009), among others, can be used to explain the intuitionistic fuzzy numbers. The simplest one is to present the membership and the non-membership functions by the triangular fuzzy numbers (TIFNs).

The TIFN (Li, Nan and Zhang, 2012) is represented by the two sets of triplets $A_{(TIFN)}= \{(a_1, a_2,\ a_3);\ (a'_1, a_2,\ a'_3)\}$, where a_2 is the mean value of the intuitionistic fuzzy numbers μ_A (x) and ν_A (x), a_1 and a_3 are, respectively, the left and the right boundaries of μ_A (x), a'_1 and a'_3 are, respectively, the left and the right boundaries of ν_A (x), and $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$. The TIFN membership and non-membership are given as follows:

$$\mu_{A_{(TIFN)}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, for\ a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, for\ a_2 \leq x \leq a_3, \\ 0\ otherwise \end{cases} \tag{9}$$

$$\nu_{A_{(TIFN)}}(x) = \begin{cases} \frac{a_2-x}{a_2-a'_1}, for\ a'_1 \leq x \leq a_2 \\ \frac{x-a_2}{a'_3-a_2}, for\ a_2 \leq x \leq a'_3. \\ 1\ otherwise \end{cases} \tag{10}$$

In many real-life situations, the information cannot be evaluated exactly in numerical values but rather in linguistic variables. The linguistic terms are words and sentences of a natural language. The Linguistic Intuitionistic Fuzzy Number (LIFN) is a special intuitionistic fuzzy number which can describe the vagueness existing in real-life decision-making more easily (Liu and Qin, 2017). The linguistic variables have some special transformations forms to IFNs. These may include the trapezoidal, triangular, and rectangular forms. The most popular kind of IFNs are triangular numbers. We opted for the Gautam, Singh and Singh (2016) transformations because of their simplicity and ease of operation. They express the linguistic variables as positive TIFNs as shown in Tables 1 and 2.

Table 1: Linguistic variables for the rating

Very Poor (VP)	<0, 0, 1; 0, 0, 2>
Poor (P)	<0, 1, 3; 0, 1 ,4>
Medium Poor (MP)	<1, 3, 5; 0.5, 3 ,5.5>
Fair (F)	<3, 5, 7; 2, 5, 8>
Medium Good (MG)	<5, 7, 9; 4.5, 7, 9.5>
Good (G)	<7, 9, 10; 6, 9, 10>
Very Good (VG)	<9, 10, 10; 8, 10, 10>

Source: Gautam, Singh and Singh (2016).

Table 2: Linguistic variables for the weight importance of each criterion

Very Low (VL)	<0, 0, 0.1; 0, 0, 0.2>
Low (L)	<0, 0.1 ,0.3; 0, 0.1, 0.4>
Medium Low (ML)	<0.1, 0.3, 0.5; 0.05, 0.3, 0.5, 0.5>
Medium (M)	<0.3, 0.5, 0.7; 0.2, 0.5, 0.8>
Medium High (MH)	<0.5 ,0.7, 0.9; 0.45, 0.7, 0.95>
High (H)	<0.7, 0.9, 1; 0.6, 0.9, 1>
Very High (VH)	<0.9, 1, 1; 0.8, 1, 1>

Source: Gautam, Singh and Singh (2016).

Let us consider $A_{(TIFN)}$= $\{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ and $B_{(TIFN)}$= $\{(b_1, b_2, b_3); (b'_1, b_2, b'_3)\}$. The operations on triangular intuitionistic fuzzy numbers are the following:

$A_{(TIFN)} + B_{(TIFN)} = \{(a_1+ b_1, a_2+ b_2, a_3+ b_3); (a'_1+ b'_1, a_2+ b_2, a'_3+ b'_3)\}$, (11)

$A_{(TIFN)} - B_{(TIFN)} = \{(a_1- b_3, a_2 - b_2, a_3- b_1); (a'_1 - b'_3, a_2 - b_2, a'_3 - b'_1)\}$, (12)

$A_{(TIFN)} * B_{(TIFN)} =\{(a_1* b_1, a_2* b_2, a_3* b_3); (a'_1* b'_1, a_2* b_2, a'_3* b'_3)\}$. (13)

Let k be a scalar number:

If $k > 0$ then $k* A_{(TIFN)}= \{(k * a_1, k * a_2, k * a_3); (k * a'_1, k * a_2, k * a'_3)\}$, (14)

If $k < 0$ then $k* A_{(TIFN)}= \{(k * a_3, k * a_2, k * a_1); (k * a'_3, k * a_2, k * a'_1)\}$. (15)

Gani and Abbas (2014) defined the defuzzification of a triangular intuitionistic number to ordinal number as follows:

$$A= \frac{(a_1+2a_2+a_3)+(a'_1+2a_2+a'_3)}{8}. \quad (16)$$

4 IFS-FlowSort for multicriteria group decision making

Our research aim is to develop an IFS FlowSort method where an ill-known quantity is expressed with an intuitionistic fuzzy number. Our proposed extension adopts linguistic values as input data to simplify the collection of data. Next, we have to transform the linguistic preference rating and the linguistic weights to triangular intuitionistic fuzzy numbers (TIFNs). In addition, our extension solves the multicriteria group decision making problems (MCGDM). It consists in aggregating the individual sorting results in a collective one and calculating the personal and the group satisfaction degrees. If there is a low satisfaction, it will be necessary to recollect the input data.

As presented in Figure 1, IFS-FlowSort for an MCGDM algorithm can be divided into four phases: (*i*) the construction of the linguistic evaluation matrix, (*ii*) the implementation of IFS-FlowSort (Remadi and Frikha, 2019) of each individual decision maker separately, (*iii*) the aggregation of the individual sorting results in a collective one, (*iv*) the satisfaction evaluation.

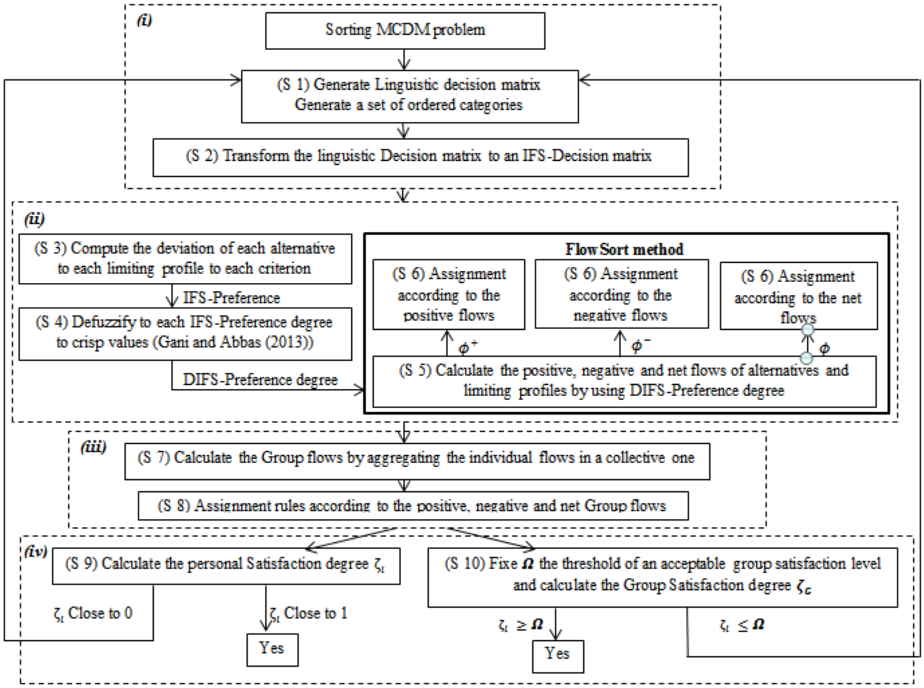

Figure 1: The procedure of the IFS-FlowSort method for an MCGDM problem

According to the definitions in Sections 2 and 3, and as presented in Figure 1, the implementation of the IFS-FlowSort method for MCGDM is as follows:

In the first phase, we have to create a linguistic evaluation matrix. To solve the sorting of the MCGDM problem, it is necessary to describe:

A = $\{a_1, a_2, \dots, a_n\}$ a set of n alternatives.

G = $\{g_1, g_2, \dots, g_m\}$ a set of m criteria evaluated by $W_g = \{w_{g1}, w_{g2}, \dots, w_{gm}\}$ criteria weights.

C = $\{c_1, c_2, \dots, c_t\}$ a set of t classes.

D = $\{d_1, d_2, \dots, d_y\}$ a set of y decision makers (DM) evaluated by $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_y\}$ DM weights. The DM weights are assumed to be crisp numbers.

$X_{(l)} = (x_{ij(l)})_{n*m}$ is the linguistic performance rating for the alternative a_i ($i = 1, 2, \dots, n$) on criterion c_j ($j = 1, 2, \dots, m$) according to the DM d_l ($l = 1, 2, \dots, y$).

The parameter values such as the criteria weights, the DM weights and the preference and the indifference degrees are assumed to be unique for all the DMs.

Then, the DMs are also invited to determine the set of ordered categories $C_1 \rhd C_2 \rhd \dots \rhd C_t$, where $C_h \rhd C_l$ for $h < l$, denote that the category C_h is preferred to the category C_l. Each category is defined by one central profile or two reference profiles. Let $R = \{r_1, r_2, \dots, r_{t+1}\}$ be the set of limiting profiles, where r_h and r_{h+1} are the upper and the lower bounds of C_h, respectively. There are t central profiles (centroids) for t ordered categories $\tilde{R} = \{\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_t\}$ defined by the DM. When there is no distinction between the set of limiting profiles and the set of centroids, there are exist the reference profiles $R^* = \{r_1^*, r_2^*, \dots\}$. Let us define the set $R_i^* =$ $= R^* \cup \{a_i\}$ where a_i is the action to be assigned (Step 1).

The second phase is to transform the linguistic performance ratings decisional matrix $X_{(l)}$, the linguistic criterion weights and the linguistic DM weights to triangular intuitionistic fuzzy numbers. Table 1 and Table 2 show the linguistic scales and the corresponding IFNs according to Gautam, Singh and Singh (2016):

$x_{ij(l)}=\{(x^1_{ij(l)}, x^2_{ij(l)}, x^3_{ij(l)}); (x'^1_{ij(l)}, x^2_{ij(l)}, x'^3_{ij(l)})\}$, $i = 1, 2, \dots, n$,
$j = 1, 2, \dots, m$, $l = 1, 2, \dots, y$. (17)

where $x^2_{ij(l)}$ is the mean value of the intuitionistic fuzzy numbers $\mu\ (x_{ij(l)})$ and $\nu\ (x_{ij(l)})$, a_1 and a_3 are, respectively, the left and the right boundaries of $\mu(x_{ij(l)})$, a'_1 and a'_3 are, respectively, the left and the right boundary of $\nu\ (x_{ij(l)})$, and $x'^1_{ij(l)} \le x^1_{ij(l)} \le x^2_{ij(l)} \le x'^3_{ij(l)} \le x^3_{ij(l)}$ (Step 2).

After that, we should construct and exploit the individual Intuitionistic Fuzzy FlowSort procedure:

The preference degrees $\pi\ (A, B)$ of each alternative A over an alternative B are computed using the arithmetic operation on triangular intuitionistic fuzzy numbers for all the alternatives A, B of R_i^* (Step 3).

$\pi\ (A, B) = \sum w_j * P_j(A, B)$, (18)

$\pi\ (A, B) = \sum w_j * P_j(f_j(A) - f_j(B))$,

where $f_j(A) = (a_1, a_2, a_3;\ a'_1, a_2, a'_3)$, $f_j(B) = (b_1, b_2, b_3;\ b'_1, b_2, b'_3)$ and $w_j = (w_1, w_2, w_3;\ w'_1, w_2, w'_3)$ are triangular intuitionistic fuzzy numbers.

$\pi\ (A, B) = \sum w_j * P_j((a_1, a_2, a_3;\ a'_1, a_2, a'_3) - (b_1, b_2, b_3;\ b'_1, b_2, b'_3))$,

$\pi\ (A, B) = \sum w_j * P_j(a_1 - b_3,\ a_2 - b_2, a_3 - b_1;\ a'_1 - b'_3,\ a_2 - b_2, a'_3 - b'_1)$,

$\pi\ (A, B) = \sum w_j * P_j(\alpha_1, \alpha_2, \alpha_3; \alpha'_1, \alpha_2, \alpha'_3)$,

where $\alpha_1 = a_1 - b_3, \alpha_2 = a_2 - b_2, \alpha_3 = a_3 - b_1, \alpha'_1 = a'_1 - b'_3, \alpha'_3 = a'_3 - b'_1$.

$\pi\ (A, B) = \sum w_j * ({\alpha_1}^{P_j}, {\alpha_2}^{P_j}, {\alpha_3}^{P_j};\ {\alpha'_1}^{P_j}, {\alpha_2}^{P_j}, {\alpha'_3}^{P_j})$,

$\pi\ (A, B) = \sum(w_{1j}{\alpha_1}^{P_j}, w_{2j}\ {\alpha_2}^{P_j}, w_{3j}{\alpha_3}^{P_j};\ w'_{1j}{\alpha'_1}^{P_j}, w_{2j}{\alpha_2}^{P_j}, w'_{3j}{\alpha'_3}^{P_j})$,

$\pi\ (A, B) = (\sum w_{1j}{\alpha_1}^{P_j}, \sum w_{2j}{\alpha_2}^{P_j}, \sum w_{3j}{\alpha_3}^{P_j}; \sum w'_{1j}{\alpha'_1}^{P_j}, \sum w_{2j}{\alpha_2}^{P_j}, \sum w'_{3j}{\alpha'_3}^{P_j})$. (19)

Then, each preference degree (A, B) should be defuzzified to transform the intuitionistic fuzzy number into a real number. We suggest the use of Gani and Abbas (2014) operator given in (12), since it is easier to use and, therefore, the use of IFS-FlowSort will be simple (Step 4):

$$\pi_d(A, B) = \frac{\left(\sum w_{1j}{\alpha_1}^{P_j} + 2*\sum w_{2j}{\alpha_2}^{P_j} + \sum w_{3j}{\alpha_3}^{P_j}\right) + \left(\sum w'_{1j}{\alpha'_1}^{P_j} + 2*\sum w_{2j}{\alpha_2}^{P_j} + \sum w'_{3j}{\alpha'_3}^{P_j}\right)}{8}. \quad (20)$$

The positive, negative and net flows of each alternative A of R_i^* are computed according to the defuzzified outranking degree (A, B) (Step 5):

$$\phi^+_{R_i^*}(A) = \frac{1}{|R_i^*|-1}\sum_{B\,\epsilon R_i^*}\pi_d(A,\ B), \tag{21}$$

$$\phi^-_{R_i^*}(A) = \frac{1}{|R_i^*|-1}\sum_{B\,\epsilon R_i^*}\pi_d(B,\ A), \tag{22}$$

$$\phi_{R_i^*}(A) = \phi^+_{R_i^*}(A) - \phi^-_{R_i^*}(A). \tag{23}$$

As in FlowSort, three different assignment rules based on the positive, negative and net flows are defined as follows (Step 6):

$$C_{\phi^+}(a_i) = C_t \text{ if } \phi^+_{R_i^*}(r_t) > \phi^+_{R_i^*}(a_i) \geq \phi^+_{R_i^*}(r_{t+1}), \tag{24}$$

$$C_{\phi^-}(a_i) = C_{\mathrm{t}} \text{ if } \phi^-_{R_i^*}(r_t) \leq \phi^-_{R_i^*}(a_i) < \phi^-_{R_i^*}(r_{t+1}), \tag{25}$$

$$C_{\phi}(a_i) = C_t \text{ if } \phi_{R_i^*}(r_t) > \phi_{R_i^*}(a_i) \geq \phi_{R_i^*}(r_{t+1}). \tag{26}$$

The third phase is the implementation of the group decision making IFS FlowSort procedure:

We calculate the positive, negative and net flows of the group of DMs by aggregating the individual flows in collective ones (Step 7):

$$\phi^{+G}_{R_i^*} = \sum_{l=1}^{y}\lambda_l * \phi^+_{R_i^*}(A), \tag{27}$$

$$\phi^{-G}_{R_i^*} = \sum_{l=1}^{y}\lambda_l * \phi^-_{R_i^*}(A), \tag{28}$$

$$\phi^{G}_{R_i^*} = \sum_{l=1}^{y}\lambda_l * \phi_{R_i^*}(A). \tag{29}$$

Afterwards, we assign alternatives according to the group flows values (Step 8):

$$C_{\phi^{+G}}(a_i) = C_t \text{ if } \phi^+_{R_i^*}(r_t) > \phi^{+G}_{R_i^*}(a_i) \geq \phi^+_{R_i^*}(r_{t+1}), \tag{30}$$

$$C_{\phi^{-G}}(a_i) = C_t \text{ if } \phi^-_{R_i^*}(r_t) \leq \phi^{-G}_{R_i^*}(a_i) < \phi^-_{R_i^*}(r_{t+1}), \tag{31}$$

$$C_{\phi^{G}}(a_i) = C_t \text{ if } \phi_{R_i^*}(r_t) > \phi^{G}_{R_i^*}(a_i) \geq \phi_{R_i^*}(r_{t+1}). \tag{32}$$

In the last phase, we have to calculate the personal and the group satisfaction degrees.

The personal satisfaction degree is the mean average of the comparison of the group sorting results and the individual sorting results (Step 9):

$$\zeta_l = \frac{\sum_{i=1}^{n}\Psi_l(A_i)}{n} \tag{33}$$

where $\Psi_l(A_i) = \begin{cases} 1 \; if \; s_l(A_i) = S(A_i) \\ 0 \qquad otherwise \end{cases}$,

where $s_l(A_i)$ is the alternative A_i sorting result of the l^{th} person, $S(A_i)$ is its group sorting result. If ζ_l is close to 1, it means that the personal satisfaction is high, while if ζ_l is close to 0, there is a low personal satisfaction, and consequently it is necessary to recollect data.

The group satisfaction degree is the weighted average of the personal satisfaction degrees (Step 10):

$$\zeta_G = \sum_{l=1}^{y}\lambda_l\zeta_l = \sum_{l=1}^{y}\lambda_l\frac{\sum_{i=1}^{n}\Psi_l(A_i)}{n}, \tag{34}$$

DMs are invited to fix $\boldsymbol{\Omega} \in [0, 1]$ as a threshold of an acceptable group satisfaction level. If $\zeta_G \geq \boldsymbol{\Omega}$, it means that there is a group agreement; else, it is also necessary to recollect data.

5 A numerical example

At this step of our research, we tested the applicability of the proposed IFS-FlowSort method for MCGDM through its application to the example of Gautam, Singh and Singh (2016). In fact, we considered an MCDM sorting problem concerning the assignment of alternatives applied to IFS-TOPSIS (Chen, 2015) to illustrate the implementation of our proposed approach.

In this decision problem, a software company desires to hire a system analyst. After a preliminary screening, four candidates $\{A_1, A_2, A_3, A_4\}$ remain for further assignment to three categories: C_1 is to be selected, C_2 is to be discussed and C_3 is to be rejected. The four potential alternatives can be evaluated by three DMs according to five criteria: g_1 (Emotional steadiness), g_2 (Oral communication skill), g_3 (Personality), g_4 (Past experience), g_5 (Self-confidence). The weights of the five criteria and the performance rating (shown in Table 3) are described using the linguistic term set w_j= {H, VH, H, VH, MH}. As already mentioned, the weights of the DMs are assumed to be a crisp number λ_l = {0.3; 0.5; 0.2}. We suppose that the indifference threshold q_j = 0 and the preference threshold p_j = 7 for $j = 1, \ldots, 5$. The limiting profiles of the criteria are given in Table 4.

Table 3: The candidates' ratings according to the three DMs

Criteria	Alternatives	Decision Makers		
		DM_1	DM_2	DM_3
g_1	A_1	MG	G	MG
	A_2	G	G	MG
	A_3	VG	G	F
	A_4	F	F	F
g_2	A_1	G	MG	F
	A_2	VG	VG	VG
	A_3	MG	G	VG
	A_4	MP	P	P
g_3	A_1	F	G	MG
	A_2	VG	VG	VG
	A_3	G	MG	VG
	A_4	MG	MP	P
g_4	A_1	VG	G	F
	A_2	VG	VG	VG
	A_3	G	VG	MG
	A_4	F	F	F
g_5	A_1	F	F	F
	A_2	VG	MG	G
	A_3	G	G	MG
	A_4	F	MP	P

Table 4: The limiting profiles

	g_1	g_2	g_3	g_4	g_5
IR_1	10	10	10	10	10
IR_2	6	6	6	6	6
IR_3	4	4	4	4	4
IR_4	0	0	0	0	0

To construct the intuitionistic fuzzy decision matrix, we transformed the linguistic performance rating (shown in Table 3) into triangular intuitionistic fuzzy number by employing equation (19) (Table 5).

The transformed intuitionistic fuzzy set weight of each criterion is the following: w_j = {<0.7, 0.9, 1; 0.6, 0.9, 1>, <0.9, 1, 1; 0.8, 1, 1>, <0.7, 0.9, 1; 0.6, 0.9, 1>, <0.9, 1, 1; 0.8, 1, 1>, <0.5, 0.7, 0.9; 0.45, 0.7, 0.95>}.

Table 5: The IFS Decision matrix

Criteria	Alternatives	Decision Makers		
		DM_1	DM_2	DM_3
g_1	A_1	<5, 7, 9; 4.5, 7, 9.5>	<7, 9, 10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>
	A_2	<7, 9, 10; 6, 9, 10>	<7, 9,10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>
	A_3	<9, 10, 10; 8, 10, 10>	<7, 9, 10; 6, 9, 10>	<3, 5, 7; 2, 5, 8>
	A_4	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>
g_2	A_1	<7, 9, 10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>	<3, 5, 7; 2, 5, 8>
	A_2	<9, 10, 10; 8, 10, 10>	<9, 10, 10; 8, 10, 10>	<9, 10, 10; 8, 10, 10>
	A_3	<5, 7, 9; 4.5, 7, 9.5>	<7, 9, 10; 6, 9,10>	<9, 10, 10; 8, 10, 10>
	A_4	<1, 3, 5; 0.5, 3, 5.5>	<0, 1, 3; 0, 1, 4>	<0, 1, 3; 0, 1, 4>
g_3	A_1	<3, 5, 7; 2, 5, 8>	<7, 9, 10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>
	A_2	<9, 10, 10; 8, 10, 10>	<9, 10, 10; 8, 10, 10>	<9, 10, 10; 8, 10, 10>
	A_3	<7, 9, 10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>	<9, 10, 10; 8, 10, 10>
	A_4	<5, 7, 9; 4.5, 7, 9.5>	<1, 3, 5; 0.5, 3, 5.5>	<0, 1, 3; 0, 1, 4>
g_4	A_1	<9, 10, 10; 8, 10,10>	<7, 9, 10; 6, 9, 10>	<3, 5, 7; 2, 5,8>
	A_2	<9, 10, 10; 8, 10,10>	<9, 10, 10; 8, 10, 10>	<9, 10, 10; 8, 10,10>
	A_3	<7, 9, 10; 6, 9, 10>	<9, 10, 10; 8, 10, 10>	<5, 7, 9; 4.5, 7, 9.5>
	A_4	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>
g_5	A_1	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>
	A_2	<9, 10, 10; 8, 10, 10>	<5, 7, 9; 4.5, 7, 9.5>	<7, 9, 10; 6, 9, 10>
	A_3	<7, 9, 10; 6, 9, 10>	<7, 9, 10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>
	A_4	<3, 5, 7; 2, 5, 8>	<1, 3, 5; 0.5, 3, 5.5>	<0, 1, 3; 0, 1,4>

We applied individual procedures to each DM evaluation. First, we computed the deviation of each pair of alternatives according to each criterion using the arithmetic IFS operations to obtain the intuitionistic fuzzy preference degrees as mentioned in Step 3. Then, we defuzzified the IF-preference degrees to crisp numbers using equation (22). Finally, we calculated the positive, negative and net flows values of each DM (see Tables 6-9).

The individual results show that, according to DM1 and DM2, the candidates A_1, A_2 and A_3 are assigned to C_1 (to be selected), but candidate A_4 is assigned to C_3 (to be rejected). As for DM3, A_1 is assigned to C_2 (to be discussed).

Table 6: The positive, negative and net flows of DM1

ϕ_{DM1}		IR_1	IR_2	IR_3	IR_4	A_i
	$\phi^+_{R_1}$	2.51	1.51	1.08	0.88	1.52
R_1	$\phi^-_{R_1}$	0.41	0.73	1.333	2.62	0.6
	ϕ_{R_1}	2.09	0.78	−0.24	−1.73	0.89
	$\phi^+_{R_2}$	2.50	1.72	1.39	0.97	2.09
R_2	$\phi^-_{R_2}$	0.089	0.59	1.29	2.61	0.093
	ϕ_{R_2}	2.41	1.13	0.10	−1.64	1.99
	$\phi^+_{R_3}$	2.50	1.58	1.23	0.96	1.77
R_3	$\phi^-_{R_3}$	0.26	0.623	1.3	2.615	0.29
	ϕ_{R_3}	2.25	0.96	−0.06	−1.66	1.48
	$\phi^+_{R_4}$	2.50	1.35	0.8	0.72	0.87
R_4	$\phi^-_{R_4}$	0.74	0.95	1.43	2.615	1.24
	ϕ_{R_4}	1.76	0.397	−0.63	−1.896	−0.37

Table 7: The positive, negative and net flows of DM2

ϕ_{DM2}		IR_1	IR_2	IR_3	IR_4	A_i
	$\phi^+_{R_1}$	2.50	1.52	1.13	0.88	1.53
R_1	$\phi^-_{R_1}$	0.37	0.68	1.31	2.615	0.47
	ϕ_{R_1}	2.13	0.84	−0.18	−1.73	1.05
	$\phi^+_{R_2}$	2.50	1.66	1.33	0.96	1.96
R_2	$\phi^-_{R_2}$	0.154	0.60	1.29	2.615	0.17
	ϕ_{R_2}	2.35	1,06	0.035	−1.65	1.79
	$\phi^+_{R_3}$	2.50	1.60	1.25	0.97	1.82
R_3	$\phi^-_{R_3}$	0.23	0.62	1.29	2.615	0.267
	ϕ_{R_3}	2.27	0.98	−0.04	−1.645	1.56
	$\phi^+_{R_4}$	2.50	1.33	0.72	0.62	0.67
R_4	$\phi^-_{R_4}$	0.86	1.16	1.59	2.61	1.73
	ϕ_{R_4}	1.64	0.17	−0.87	−1.99	−1.06

Table 8: The positive, negative and net flows of DM3

ϕ_{DM3}		IR_1	IR_2	IR_3	IR_4	A_i
	$\phi^+_{R_1}$	2.50	1.36	0.84	0.79	0.99
R_1	$\phi^-_{R_1}$	0.67	0.87	1.37	2.615	1.02
	ϕ_{R_1}	1.83	0.49	−0.53	−1.82	−0.03
	$\phi^+_{R_2}$	2.50	1.66	1.32	0.96	1.95
R_2	$\phi^-_{R_2}$	0.16	0.61	1.29	2.615	0.18
	ϕ_{R_2}	2.34	1.05	0.03	−1.65	1.77
	$\phi^+_{R_3}$	2.50	1.54	1.15	0.91	1.60
R_3	$\phi^-_{R_3}$	0.35	0.68	1.315	2.615	0.46
	ϕ_{R_3}	2.15	0.86	−0.17	−1.70	1.14
	$\phi^+_{R_4}$	2.50	1.33	0.74	0.54	0.61
R_4	$\phi^-_{R_4}$	0.83	1.14	1.58	2.615	1.67
	ϕ_{R_4}	1.67	0.19	−0.85	−2.07	−1.06

We aggregate the individual results into a collective one. The group positive, negative and net flows are presented in Table 9. The group results show that candidates A_1, A_2 and A_3 are assigned to C_1 (to be selected) and candidate A_4 is assigned to C_3 (to be rejected).

Table 9: The group of positive, negative and net flows

ϕ_G		IR_1	IR_2	IR_3	IR_4	A_i
R_1	$\phi^+_{R_1}$	2.50	1.48	1.06	0.86	1.41
	$\phi^-_{R_1}$	0.44	0.73	1.33	2.615	0.61
	ϕ_{R_1}	2.06	0.75	−0.27	−1.75	0.8
R_2	$\phi^+_{R_2}$	2.50	1.68	1.34	0.96	2.00
	$\phi^-_{R_2}$	0.13	0.60	1.29	2.615	0.15
	ϕ_{R_2}	2.36	1.08	0.05	−1.65	1.85
R_3	$\phi^+_{R_3}$	2.50	1.58	1.22	0.95	1.77
	$\phi^-_{R_3}$	0.26	0.63	1.29	2.615	0.32
	ϕ_{R_3}	2.24	0.95	−0.08	−1.66	1.45
R_4	$\phi^+_{R_4}$	2.50	1.34	0.75	0.63	0.72
	$\phi^-_{R_4}$	0.82	1.09	1.54	2.615	1.57
	ϕ_{R_4}	1.68	0.24	−0.79	−1.98	−0.85

By calculating the personal satisfaction degrees ($\zeta_1(A_i)$ = 1, $\zeta_2(A_i)$ = 1 and $\zeta_3(A_i)$ = 0.75), we can conclude that there is a full satisfaction for the DM1 and DM2 and a high satisfaction for DM3. After fixing the threshold of an acceptable group satisfaction level to $\boldsymbol{\Omega}$ = 0.9, the group satisfaction degree (ζ_G = 0.95) shows an agreement among the group of DMs.

In order to compare results, the same input data were used and applied to the FlowSort, the F-FlowSort, the PROMETHEE (Brans, Mareschal and Vincke, 1984), the TOPSIS (Chen and Hwang, 1992) and the IFS-TOPSIS (Chen, 2015) methods. As can be seen in Table 10, assignments are closely similar except for the fourth alternative, when considering the assignment based on the positive and negative flows for F-FlowSort. So, the alternative 4 can be unambiguously assigned to category 3. IFS-FlowSort can successfully correct this ambiguous assignment by using the perfect information given by the IFS values. In addition, we have found identical results when applying FlowSort. Also, some relationship can be noticed when comparing the results given by the ranking methods. In fact, the results given by PROMETHEE (Brans, Mareschal and Vincke, 1984), TOPSIS (Chen and Hwang, 1992) and IFS-TOPSIS (Chen, 2015), and by IFS-FlowSort for MCGDM are almost the same. As it can be seen in Figure 2, if we can group alternatives into three ordered categories from the best to the worst; the 1st, 2nd and 3rd alternatives are always the most preferred, so it can be logically assigned to the first category, there is no alternative that can be middle preferred and the 4th alternative is always the worst one. However, this observation cannot be generalized, since many studies are wanted in this area.

Table 10: Comparison with other sorting methods

Scenarios	FlowSort			F-FlowSort			IFS-FlowSort-GDM		
	K_{ϕ^+}	K_{ϕ^-}	K_{Φ}	K_{ϕ^+}	K_{ϕ^-}	K_{Φ}	K_{ϕ^+}	K_{ϕ^-}	K_{Φ}
A_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1
A_2	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1
A_3	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1
A_4	K_3	K_3	K_3	K_3	K_2	K_2	K_3	K_3	K_3

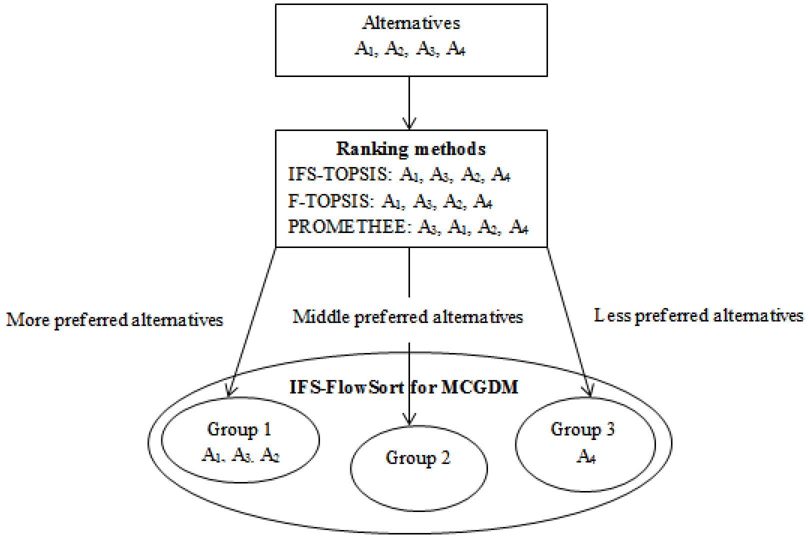

Figure 2: Comparison with ranking methods

6 Conclusion

The ordinal classification MCDM problem is one of the most important issues in management science and operational research. It is the process of structuring and sorting decision problems into ordered predefined categories when multiple conflicting criteria are deployed. The FlowSort method succeeded in solving this issue in a great deal of research. It classified items into ordered categories using the limiting and the centroid profiles based on exact values. However, the process of decision making is often prone to uncertainty and imprecision as it implies human judgement and cognitive thinking. So, the use of crisp values becomes inefficient to solve MCDM problems. The concept of intuitionistic fuzzy sets (IFS) achieves a great success to deal with the fuzziness of MCDM problems. For that reason, we introduced it and modeled IFS FlowSort. However, it is sometimes difficult for DMs to describe their opinions as intuitionistic fuzzy information. Thus, in this paper we presented preference ratings as linguistic terms and suggested transforming them into triangular intuitionistic fuzzy numbers. In addition, this study focused on a group decision making problem where a group of individuals collectively shares the responsibility for sorting a set of alternatives. In fact, we integrated the MCGDM problem by proposing the FlowSort method. To illustrate this extension, a practical example was presented and validated through a comparison with other MCDM methods As a result, we can conclude that our extension seems coherent in a sorting context and in the uncertainty logic. The proposed FlowSort method is simple to process and easy to use, especially for decision-makers who are familiar with PROMETHEE. As a future research perspective, we can modify the suggested method to solve MCGDM problems based on the input aggregation procedure.

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M U L T I P L E   C R I T E R I A   D E C I S I O N   M A K I N G

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MOLPTOL – A SOFTWARE PACKAGE FOR SENSITIVITY ANALYSIS IN MOLP

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Abstract

The paper introduces a new software package, MOLPTOL, for sensitivity analysis in multi-objective linear programming. In this application, which is available for free of charge on the web page (https:// sites.google.com/view/molptol), the tolerance approach as a measure of sensitivity is used. The motivation for creating MOLPTOL is the lack of such tools to date. MOLPTOL is novel for multi-criteria decision-making methods based on sensitivity analysis. The paper presents some new computational methods for obtaining the supremal tolerances as well.

Keywords: multi-objective linear programming, sensitivity analysis, computer software.

1 Introduction

The general idea of using sensitivity analysis in optimization aims to deal with the uncertainty and imprecise data of the considered model. Sensitivity analysis plays an important role in decision problems as well. Usually it is used in the case of perturbations of parameters which often appears in real-life problems. In this paper we consider sensitivity analysis in multiple-objective linear programming (MOLP) problems. Since many constraints and objectives are formulated in a linear way, MOLP problems are often used in practice. Here, we focus on maintaining efficiency of a given efficient solution taking into account the perturbation that can be applied simultaneously to objective functions coefficients.

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The main drawback of sensitivity analysis in MOLP is the lack of tools in multi-criteria decision making problems based on sensitivity analysis. In general, such tools should be easily applied by a decision maker. Hence, software which helps to analyze these problems would be a significant simplification of the decision process. Moreover, the computational methods to obtain the measures of sensitivity (for example, the supremal tolerances) need development. Due to the above disadvantages in this field, this paper provides new computational methods of sensitivity analysis and shows their properties. Based on these methods, MOLPTOL – a new software package for obtaining supremal tolerances in MOLP problems – is presented. Moreover, MOLPTOL can be also used as a tool in decision problems in which sensitivity analysis is important for the decision maker. In such problems the perturbation of objective functions coefficients is taken into account. The solution proposed by MOLPTOL can be easily used on the dedicated web page: https://sites.google.com/view/molptol.

The paper consists of the following sections: section 1 provides the introduction; section 2 presents related papers related to the subject of the paper; section 3 introduces the basic objects and notation; section 4 presents the theoretical background of computation methods used in MOLPTOL; section 5 shows the MOLPTOL software; section 6 describes a market model; section 7 illustrates an application of MOLPTOL in the market model; and the final section summarizes the paper.

2 Related papers

The following approaches to sensitivity analysis in MOLP are worth mentioning:

- the tolerance approach,
- the range set approach,
- the standard approach,
- the robust approach,
- the partial preference relations approach.

Let us shortly describe the above approaches by presenting the related papers.

The tolerance approach aims to find a value (tolerance) representing the perturbation that can be applied simultaneously to objective functions coefficients without affecting the efficiency of a given efficient solution. We distinguish two main forms of the tolerance approach: additive and percentage tolerances. In the case of the additive tolerance we focus on additive perturbations. The percentage tolerance approach, however, represents the relative (percentage) perturbations. This approach in linear programming comes from Wendell (1982). The use of the tolerance approach in MOLP was proposed by Hansen, Labbe and Wendell (1989). Hladik (2008a, 2008b) develops this concept in theoretical and computational ways, while Borges and Antunes (2002) present sensitivity analysis of the weights in MOLP. Applications of the tolerance approach in the transportation problem can be found in the papers by Paratane and Bit (2020), as well as Badra (2004, 2006).

The range set approach of to sensitivity analysis comes from linear programming theory for sensitivity analysis of optimal solutions, see Gass (1975) and Gal (1995). The use of this approach in MOLP was proposed by Benson (1985). The range set approach aims to find the values of parameter that can be applied to a given direction of the objective coefficients without affecting the efficiency. Methods of computing the range set in MOLP are given by Hladik et al. (2019).

The standard approach to sensitivity analysis is the extension of this method in linear programming (Gal, 1995). Initial research on using this approach to MOLP was done by Sitarz (2010, 2011). The standard approach aims to find values (a parameter set) of one selected objective function coefficient that can be applied without affecting the efficiency. Pourkarimi (2015) proposes building a ranking of all efficient faces by using stability measures based on standard sensitivity analysis.

The robust approach presented by Georgiev, Luc and Pardalos (2013) consists of analysis of the efficient solutions that remain efficient when the objective matrix is slightly perturbed by means of the Euclidean norm for the matrix of objective functions coefficients. Moreover, in that paper we find algorithms to compute the radius of robustness. In turn, Pourkarimi and Soleimani-Damaneh (2016) propose the so-called robustness order which is defined as the interiority order of the matrix of objective functions coefficients. In the paper by Goberna et al. (2015), MOLP problems with uncertainty both in the objective function and the constraints are considered.

The partial preference relations introduced by Podinovski (2012) present sensitivity analysis in the form of a parametric partial order. This approach can be applied to the sensitivity analysis by taking into account the changes of parameters of the order. Moreover, Podinovski and Potapov (2019) expand this theory by introducing parameters connected with boundaries of intervals for criteria value tradeoffs uncertainty.

3 MOLP problem and tolerance approach

In this paper, we consider the following MOLP problem:

$$VMax\ \{Cx \colon x \in X\}, \tag{1}$$

where $X = \{x \in \mathbb{R}^n: Ax \le b\} \subset \mathbb{R}^n$ is a given set, with $A \in \mathbb{R}^{m,n}$ and $b \in \mathbb{R}^m$; matrix $C \in \mathbb{R}^{k,n}$ is given by the linear objective functions $c^i x$ for $i = 1, \dots, n$. One can find a detailed description of MOLP problems in books by Steuer (1986) or Zeleny (1982). A feasible solution $x^* \in X$ is called an efficient solution to (1) if there is no $x \in X$ such that:

$$Cx^* \le Cx \ \wedge \ Cx^* \ne Cx.$$

We can check efficiency of the given feasible solution by using the following theorem, Ehrgott (2005).

Theorem 1. A feasible solution x^* is efficient if and only if the following linear program:

$$\begin{gathered} Max \ e^T w \\ Cx - Iw = Cx^* \\ x \in X \\ w \ge 0, \end{gathered}$$

where I is identity matrix and e is vector of ones, has an optimal objective function value of zero.

We consider the sensitivity in the sense of the remaining efficiency of a given feasible solution $x^* \in X$. Furthermore, we analyze the sensitivity analysis in the case of changing matrix C. Let matrix $G \in \mathbb{R}^{n,k}$ be given. We introduce a δ, G-neighbourhood of matrix $C = [c_{ij}]$ as follows:

$$O_{\delta,G}(C) = \{D = [d_{ij}] \in \mathbb{R}^{n,k}: \ |d_{ij} - c_{ij}| < \delta |g_{ij}| \ \text{ if } g_{ij} \ne 0, \ \ d_{ij} = c_{ij} \ \text{ if } g_{ij} = 0\}.$$

In this case we consider the following problem obtained from (1) by using $D \in O_{\delta,G}(C)$:

$$VMax \, \{Dx: x \in X\}. \tag{2}$$

Definition 1. A tolerance for an efficient solution x^* is any real δ such that x^* remains efficient to (2) for all $D \in O_{\delta,G}(C)$. The supremal tolerance is denoted by δ^{sup}.

We look closer at the two types of the tolerance presented above: an additive tolerance and percentage tolerance. These tolerances represent the additive perturbation and the percentage perturbation of all coefficients of matrix C.

Definition 2. An additive tolerance is a tolerance for matrix G consisting only of ones:

$$g_{ij} = 1 \ \text{ for all } i, j.$$

Definition 3. A percentage tolerance is a tolerance for matrix G consisting of $|c_{ij}|$:

$$g_{ij} = |c_{ij}| \text{ for all } i, j.$$

4 Computation methods used in MOLPTOL

Computing the supremal tolerance is based on problem (3) given by Hladik and Sitarz (2013):

$$\begin{gathered} \delta^{sup} = Min\ \delta \\ A^1(x - x^*) \leq 0 \\ C(x - x^*) + \delta G|x - x^*| \geq 0 \\ e^T G|x - x^*| = 1 \\ \delta \geq 0, \end{gathered} \tag{3}$$

where A^1 is a submatrix of A consisting only of the active constraints for x^* and e is a vector of ones. Moreover, G is a given matrix representing the method of perturbation of the coefficients of matrix C (the way of introducing matrix G was presented in section 3).

Problem (3) is NP hard, thus we are looking to improve it. In MOLPTOL, we can improve computation by using the properties of (3) and two methods: decomposition procedure and bisection procedure. The detailed descriptions of the above methods are given in the next subsections.

4.1 Decomposition procedure

Computing the supremal tolerance by using the decomposition method is based on the decomposition of problem (3) into 2^n simpler problems, according to the signs of $(x - x^*)_i$. The composition is given by a vector $z \in \{\pm 1\}^n$. For each vector z we build the following problem:

$$\begin{gathered} \delta_z = Min\ \delta \\ A^1(x - x^*) \leq 0 \\ C(x - x^*) + \delta G diag(z)(x - x^*) \geq 0 \\ diag(z)(x - x^*) \geq 0 \\ e^T G diag(z)(x - x^*) = 1 \\ \delta \geq 0, \end{gathered} \tag{4}$$

where matrix $diag(z)$ is a diagonal matrix with the coefficients z_i.

The supremal tolerance is given by the following equation:

$$\delta^{sup} = Min_{z \in \{\pm 1\}^n}\ \delta_z.$$

Problem (4), proposed by Hladik and Sitarz (2013), is not NP-hard, but it is still difficult to solve. Thus, we propose the following method which is based on solving the sequence of linear programming problems to obtain δ^{sup}.

Theorem 2. The feasible set of problem (4) with $\delta = \bar{\delta}$ is non-empty if and only if $\delta_z \leq \bar{\delta}$.

Proof. Let us assume that the feasible set of problem (4) for $\delta = \bar{\delta}$ is non-empty. In this case, there exists a pair $(\bar{\delta}, \bar{x})$ which is a feasible solution for (4). Since δ_z is the minimum of all feasible δ, we have $\delta_z \leq \bar{\delta}$.

Now, let us assume that $\delta_z \leq \bar{\delta}$. Hence, δ_z is an optimal solution for (4); it is also a feasible solution for (4) with some x_z. Moroeover, by using the inequality:

$$diag(z)(x_z - x^*) \geq 0,$$

we obtain:

$$0 \leq C(x_z - x^*) + \delta_z G diag(z)(x_z - x^*) \leq C(x_z - x^*) + \bar{\delta} G diag(z)(x_z - x^*).$$

Thus, the pair $(\bar{\delta}, x_z)$ fulfils the above condition of (4). Furthermore, the rest of the conditions of (4) are fulfilled as well. Thus $(\bar{\delta}, x_z)$ is a feasible solution for (4).

Corollary 1. The feasible set of problem (4) is empty with $\delta = \bar{\delta}$ if and only if $\delta_z > \bar{\delta}$.

We use the following theorem to check if the feasible set of problem (4) is non-empty.

Theorem 3. The feasible set for (4) with $\delta = \bar{\delta}$ is non-empty if and only if the following linear problem:

$$\begin{gathered} Min\ \ v \\ A^1(x - x^*) \leq 0 \\ -C(x - x^*) - \ \bar{\delta} G diag(z)(x - x^*) \leq 0 \\ -diag(z)(x - x^*) \leq 0 \\ e^T G diag(z)(x - x^*) + v = 1 \\ v \geq 0, \end{gathered} \tag{5}$$

has an optimal objective function value of zero.

Proof. By substituting $\delta = \bar{\delta}$ into problem (4) and introducing a new non--negative variable v, we obtain (after some operations) the following linear constraints for the feasible set of (4):

$$A^1(x-x^*) \le 0$$
$$-C(x-x^*) - \bar{\delta} G diag(z)(x-x^*) \le 0$$
$$-diag(z)(x-x^*) \le 0$$
$$e^T G diag(z)(x-x^*) + v = 1.$$

By using Proposition 6.15 from Ehrgott (2005), the above linear set of constraints is non-empty if and only if problem (5) has an optimal objective function value of zero.

Remark 1. The main property of problem (5) is that it is a linear programming problem, thus it is easy to solve.

4.2 Sets of vectors $z \in \{\pm 1\}^n$

Let Z denote the set of all vectors $z \in \{\pm 1\}^n$. The number of elements of set Z is very important: it can reduce the computation time. Thus, we focus on Z more closely. In this subsection we present methods to narrow Z. First, let us present some observations (Hladik and Sitarz, 2013):

(i) If condition $x \ge 0$ is assumed, then for i such that $x_i^* = 0$, we set $z_i = 1$ (in other words we omit the case of $z_i = -1$).

By setting $y = diag(z)(x-x^*)$ we obtain:

(ii) Let $i \in \{1, \dots, n\}$. If $\min_y\{y_i : A^1 y \le 0\} \ge 0$, then we set $z_i = 1$ (we omit $z_i = -1$).

(iii) Let $i \in \{1, \dots, n\}$. If $\max_y\{y_i : A^1 y \le 0\} \le 0$, then we set $z_i = -1$ (we omit $z_i = 1$).

Applying (i), (ii) and (iii), we obtain the initial set $Z_0 \subseteq Z$, for which:

$$\delta^{sup} = Min_{z \in Z} \ \delta_z = Min_{z \in Z_0} \ \delta_z. \tag{6}$$

Now, we proceed with a new observation used in our method which is based on problem (5) and theorems 2 and 3. By taking into account this new observation we can omit some vectors z in formula (6). Suppose that we have a set $Z_i \subseteq Z$ and:

$$Min_{z \in Z} \delta_z = Min_{z \in Z_i} \delta_z. \tag{7}$$

Moreover, let $\bar{\delta}$ be given (which is the approximate value of δ^{sup}).

Definition 4. Let $\bar{Z}_i$ be defined as follows:

$$\bar{Z}_i = \{z \in Z_i \text{: problem (5) has an optimal objective function value of zero with } z \text{ and } \bar{\delta}\}.$$

By using definition 4, we formulate the next theorem.

Theorem 4. If $\bar{Z}_i \neq \emptyset$, then:

$$\delta^{sup} \leq \bar{\delta} \text{ and } Min_{z\in Z_i}\, \delta_z = Min_{z\in \bar{Z}_i}\, \delta_z.$$

Otherwise, $\bar{\delta} \leq \delta^{sup}$.

Proof. If $\bar{Z}_i \neq \emptyset$, then there exists $\bar{z}$ such that problem (5) has an optimal objective function value of zero with $\bar{\delta}$. Thus, by using theorems 2 and 3, $\delta_{\bar{z}} \leq \bar{\delta}$. Moreover, by using the fact that:

$$\delta^{sup} = Min_{z\in\{\pm 1\}^n} \quad \delta_z,$$

we have $\delta^{sup} \leq \delta_{\bar{z}} \leq \bar{\delta}$, which means that $\delta^{sup} \leq \bar{\delta}$. Moreover, for all $z \notin \bar{Z}_i$ we have (theorem 2) $\bar{\delta} < \delta_z$, which means that $\delta_{\bar{z}} < \delta_z$, thus:

$$Min_{z\in \bar{Z}_i}\, \delta_z \leq \delta_{\bar{z}} \leq Min_{z\notin \bar{Z}_i}\, \delta_z.$$

Since $Z_i = \bar{Z}_i \cup \bar{Z}_i^{'}$ we have:

$$Min_{z\in Z_i}\, \delta_z = Min_{z\in \bar{Z}_i}\, \delta_z.$$

If $\bar{Z}_i = \emptyset$, then for all $z \in Z_i$ we have (theorem 2) $\bar{\delta} < \delta_z$, thus by using (7) we obtain $\bar{\delta} \leq \delta^{sup}$.

Remark 2. The main property of theorem 4 is the fact that it is possible to reduce set *Z* in order to find δ^{sup}. We omit vectors *z* for which problem (5) does not have an optimal objective function value of zero.

4.3 An algorithm for obtaining $\boldsymbol{\delta^{sup}}$

In this algorithm we use the bisection procedure for seeking the supremal tolerance. While the idea of bisection is taken from the optimization numerical methods, in the case of supremal tolerance, the bisection has been adopted together with theorem 4. Figure 1 presents the algorithm to obtain δ^{sup}. Let us introduce the parameters and their initial values:

i – index for steps,

Z_i – the set of vectors z considered in step i; we start with Z_0 defined in subsection 4.2,

δ^L – the left endpoint of interval; we have the initial constraint $\delta^{sup} \geq 0$; moreover, in most cases, the supremal tolerance is close to zero; thus, the initial value of δ^L is equal to zero,

δ^R – the right endpoint of interval; according to our numerical experiments, the initial value of δ^R should be taken as follows: $\delta^R = \max_{i,j}|c_{ij}|$,

$\bar{\delta}$ – the middle-point of interval; from the nature of the bisection method the initial value of $\bar{\delta}$ is equal to $(\delta^L + \delta^R)/2$,

S – number of steps; the precision of the approximate value of δ^{sup} is related to the number of steps; to have the error equal to $\varepsilon = 0.001$, we should take $S = [\log_2(\delta^R/\varepsilon)] + 1$.

Step $i \geq 0$: If $i = S$, then the approximate value of δ^{sup} is equal to $\bar{\delta}$, otherwise proceed as follows:

Check if $\bar{Z}_i \neq \emptyset$

If yes, then set:

$\delta^R = \bar{\delta}$

$\bar{\delta} = (\delta^L + \delta^R)/2$

$i = i + 1$

$Z_i = \bar{Z}_{i-1}$

and go to Step i

If not, then set:

$\delta^L = \bar{\delta}$

$\bar{\delta} = (\delta^L + \delta^R)/2$

$i = i + 1$

$Z_i = Z_{i-1}$

and go to Step i

Figure 1: Algorithm to obtain δ^{sup}

5 MOLPTOL – a short description of the software

MOLPTOL is a software package that runs on Windows systems with the .NET 4.0 platform installed. It handles the problem in the form of (1). Moreover, the non-negativity condition ($x \geq 0$) can be added by one click. The sensitivity analysis of a given vector x^* proceeds by means of the two approaches: the supremal additive tolerance and the supremal percentage tolerance. Moreover, MOLPTOL uses Express, a numerical tool (free of charge) that is a version of the Microsoft Solver Foundation library (MSF). A description of this library can be found on the web page: http://msdn.microsoft.com. The software can be used free of charge on the web page: https://sites.google.com/view/molptol.

6 A market model

In economic theory, there is a market model studied in isolation (Mas-Colell, Whinston and Green, 1995). We consider a model with N goods and M agents. The initial endowment of agent i is given by vector $e_i = (e_{1,i}, \ldots, e_{N,i})$. Let u_i denote the linear utility function of agent i. Each agent wants to maximize his utility function. The feasible allocations are the vectors

$x = (x_{1,1}, \dots, x_{N,1}, \dots \dots, x_{1,M}, \dots, x_{N,M}) \geq 0$ which for all $i \in \{1, \dots, N\}$ satisfy the following condition:

$$\sum_{j=1}^{M} x_{i,j} = \sum_{j=1}^{M} e_{i,j}$$

By using the above description, we can formulate the following MOLP problem:

$$\begin{gathered} VMax \begin{bmatrix} u_1(x_{1,1}, \dots, x_{N,1}), \\ \dots \\ u_M(x_{1,M}, \dots, x_{N,M}) \end{bmatrix} \\ \sum_{j=1}^{M} x_{i,j} = \sum_{j=1}^{M} e_{i,j}, \quad \text{for } i \in \{1, \dots, N\} \\ x_{1,1}, \dots, x_{N,1}, \dots \dots, x_{1,M}, \dots, x_{N,M} \geq 0. \end{gathered} \tag{8}$$

The efficient solutions of problem (8) are called the Pareto optimal allocations. The decision problem in the market model is to find an allocation which is Pareto optimal and satisfies additional decision maker's preferences.

7 An application of MOLPTOL in the market model

The analysis of the market model can be done by means of sensitivity analysis of the initial data, which may be imprecise and changeable, especially the coefficients of the utility functions. We look for the Pareto allocations which are the least sensitive by means of changing these coefficients. Thus, by using MOLPTOL, we check if the given allocations are Pareto optimal. Moreover, we compute the supremal tolerances for these allocations. For further analysis, we recommend the allocation with the biggest supremal tolerance. We proceed to such an analysis by using MOLPTOL in the following case scenario. We consider a model with three goods, three agents, and the following initial endowments:

$$e_1 = (2,4,2), \; e_2 = (2,2,2), \; e_3 = (6,2,2).$$

Moreover, the agents have the following utility functions:

$$u_1(x_{1,1}, x_{2,1}, x_{3,1}) = x_{1,1} + 4x_{2,1} + 5x_{3,1},$$

$$u_2(x_{1,2}, x_{2,2}, x_{3,2}) = x_{1,2} + x_{2,2},$$

$$u_3(x_{1,3}, x_{2,3}, x_{3,3}) = x_{2,3} + x_{3,3}.$$

The MOLP problem connected with the above market model takes the following form:

$$
\begin{gathered}
VMax \begin{bmatrix} x_{1,1} + 4x_{2,1} + 5x_{3,1} \\ x_{1,2} + x_{2,2} \\ x_{2,3} + x_{3,3} \end{bmatrix} \\
x_{1,1} + x_{2,1} + x_{3,1} = 10 \\
x_{1,2} + x_{2,2} + x_{3,2} = 8 \\
x_{1,3} + x_{2,3} + x_{3,3} = 6 \\
x_{1,1}, x_{2,1}, x_{3,1}, x_{1,2}, x_{2,2}, x_{3,2}, x_{1,3}, x_{2,3}, x_{3,3} \geq 0.
\end{gathered}
\tag{9}
$$

The initial parameters for MOLPTOL are as follows:

$$
C = \begin{bmatrix} 1 & 4 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix},
$$

$$
A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}, \; b = \begin{bmatrix} 10 \\ 8 \\ 6 \end{bmatrix}.
$$

Moreover, the non-negativity condition is assumed. Let us analyze the following allocations:

$$
\begin{gathered}
x^a = (2, 4, 2, 8, 2, 0, 0, 2, 4), \\
x^b = (0, 0, 6, 10, 0, 0, 0, 8, 0), \\
x^c = (5, 0, 0, 5, 0, 0, 0, 8, 6).
\end{gathered}
$$

By using MOLPTOL we obtain that:

- allocation x^a is not efficient,
- allocation x^b is efficient and the supremal percentage tolerance is equal to 5.57%,
- allocation x^c is efficient and the supremal percentage tolerance is equal to 99.98%.

Thus, for further consideration, allocation x^a is omitted (because is not efficient). Furthermore, we conclude that allocation x^b is more sensitive than allocation x^c (based on the values of the supremal percentage tolerance). From this point of view, allocation x^c is better than allocation x^b. The consideration above includes only sensitivity analysis; in an actual decision-making problem, more aspects should be taken into consideration. However, the presented analysis can help make a decision in which sensitivity analysis is important.

8 Summary

Sensitivity analysis in MOLP problems by means of the tolerance approach was considered. New computational methods to obtain the supremal tolerances were provided as well. The methods used the decomposition procedure and the bisection procedure. Based on the proposed algorithm, MOLPTOL – a new software package for obtaining supremal tolerances in MOLP problems – was presented. It can be used free of charge on the web page: https://sites.google.com/view/molptol. A market model and an application of MOLPTOL to it were presented. The application illustrated the possibilities of using MOLPTOL in decision problems in which sensitivity analysis is important for the decision maker. Further research and improvement of MOLPTOL will consist of:

- adding other sensitivity analysis methods, for instance, standard sensitivity analysis or the range set approach;
- extending the software by introducing fuzzy numbers or interval coefficients;
- taking into account other tools beside MSF, for instance, the Gurobi solver;
- finding more applications of MOLPTOL.

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A NEW APPLICATION OF THE GENERALIZED TRAVELING SALESMAN PROBLEM IN INDUSTRY 4.0 AND 5.0

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Abstract

A novel application of the generalized traveling salesman is proposed. The practical problem considered is optimization of different optimization criteria in various models of a mixed assembly workstation. Several models that give rise to interesting optimization problems are discussed.

Keywords: generalized traveling salesman problem, flexible assembly workstation.

1 Introduction

The generalized traveling salesman problem (GTSP), also known as the 'travelling politician problem', deals with 'states' that have (one or more) 'cities' and the salesman has to visit exactly one 'city' from each 'state'. In analogy with the traveling salesman problem, it is natural to consider the problem on directed graphs.

The definition of the generalized traveling salesman problem (TSP) below, based on Nobert and Laporte (1983) and Noon and Bean (1991), is as follows. Let $G = (V, E)$ be an n-node graph whose edges are associated with non-negative costs. We will assume w.l.o.g. that G is a complete graph (if

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there is no edge between two nodes, we can add an edge with an infinite or large enough cost).

We denote the cost of an edge $e = (i, j) \in E$ by $c(i, j)$. It is usual to allow different costs depending on the direction of the edge. The GTSP is called symmetric if and only if the equality $c(i, j) = c(j, i)$ holds for every two nodes $i, j \in V$.

Let $V_1, \ldots, V_p$ be a partition of V into p subsets called clusters (i.e. $V = V_1 \cup V_2 \cup ... \cup V_p$ and $V_\ell \cap V_k = \emptyset$ for all $\ell, k \in \{1, ..., p\}$, $k \neq \ell$). The GTSP asks for finding a minimum-cost tour H spanning a subset of nodes such that H meets each cluster V_i, $i \in \{1, ..., p\}$. The problem involves two related decisions: choosing a node subset $S \subseteq V$, such that $|S \cap V_k| \geq 1$, for all $k = 1, \ldots, p$ and finding a minimum cost Hamiltonian cycle in the subgraph of G induced by S. Formally,

Generalized traveling salesman problem, GTSP

Input: A graph $G = (V, E)$ with weighting function $c : E \mapsto \mathbb{R}_0$, and a partition $P_V = \{V_1, V_2, \ldots, V_m\}$, where $V_i \cap V_j = \emptyset$ for all $i \neq j$, and $\cup_{i=1}^m V_i = V$.

Question: Find a cycle in G that contains a vertex from each set V_i such that its weight is minimal.

Here, $\mathbb{R}_0$ denotes the set of non-negative real numbers.

Clearly, TSP is a special case of GTSP, where each of the clusters has exactly one element, $|V_i| = 1$. There are also several variations, for example asking for a cycle that must contain either exactly one or at least one vertex of each cluster, or allowing instances in which the sets V_i are not disjoint.

In this short note, we propose a new application of the generalized traveling salesman. The practical problem considered is to optimize various optimization criteria in various models of a mixed assembly workstation. It has motivated definition of several optimization problems that generalize the GTSP. The main contribution of this paper are definitions of the models that are, to the best of our knowledge, new in the area of application. The problem formulations may provide firm ground for future studies that will include development of heuristics and case studies on industrial applications. While, on one hand, the new applications may be of interest because they motivate further theoretical studies of related optimization problems, we believe that, on the other hand, the transfer of theoretical results directly to engineering studies and industrial applications is even more important.

The rest of the paper is organized as follows. In the next section we recall some related work. The number of sources cited is large because we wish to serve readers with both theoretical and practical expertise and interests. However, the material touches several popular research areas and therefore the section does not aim to be a comprehensive survey. Section 3 provides some details of the application that is currently a hot topic in the development of smart factories within industry 4.0 (and 5.0). The main contribution, elaboration of several models and related optimization problems is given in Section 4. In the last section we include conclusions and discuss some ideas for future work.

2 Related work

Regarding the complexity of GTSP, it is well known that the (asymmetric) generalized traveling salesman problem can be transformed into a standard asymmetric traveling salesman problem with the same number of cities, and a modified distance matrix (Noon and Bean, 1993). Therefore, the asymmetric generalized traveling salesman problem is NP-hard. More precisely, it has been proved (Sahni and Gonzalez, 1976) that assuming P$\neq$NP, no polynomial-time TSP heuristic can guarantee $A(I)/OPT(I) \leq 2^{p(N)}$ for any fixed polynomial p and all instances I. Better approximation results hold for the TSP with triangle inequality. Classical result of Christofides provides a 3/2-approximation algorithm for symmetric TSP (Christofides, 1976). It is not known whether the factor 3/2 is the best possible, however, assuming P$\neq$NP, there exists an $\varepsilon > 0$ such that no polynomial-time TSP heuristic can guarantee $A(I)/OPT(I) \leq 1 + \varepsilon$ for all instances I satisfying the triangle inequality (Arora et al., 1992). As ATSP is a generalization of TSP, it is at least as hard as TSP. Very recently, a constant factor approximation for ATSP with triangle inequality has been developed by Svensson et al. (2020).

A number of practical applications of GTSP are given in Laporte et al. (1996) and (www 2). One application is encountered in ordering a solution to the cutting stock problem in order to minimize knife changes. Another is concerned with drilling in semiconductor manufacturing, see e.g., U.S. Patent 7,054,798 (www 2). Further examples listed in Laporte et al. (1996) include the covering tour problem, material flow system design, post-box collection, stochastic vehicle routing and arc routing. GTSP is also called the 'Set TSP problem' (www 1) or Equality Generalized Traveling Salesman Problem (E-GTSP) (Helsgaun, 2015). Furthermore, it should be noted that the same or very closely related problems are sometimes studied under different names. For example, the papers Gentilini et al. (2013) and Elbassioni et al. (2009) study the TSP with neighborhoods, and Gulczynski et al. (2006) close enough TSP, both being closely related to GTSP. Here we recall a selection of papers in which various algorithms, mainly heuristics were used to solve the GTSP, as our list is not meant to be a comprehensive survey. Several approaches were considered for solving the GTSP: a branch-and-cut algorithm for symmetric GTSP is described and analyzed in Fischetti et al. (1997). In Noon and Bean (1991), a Lagrangian-based approach for asymmetric GTSP is given. Genetic algorithms were used in Snyder and Daskin (2006), and in Silberholz and Golden (2007). Gutin and Karapetyan (2009) proposed a reduction algorithm that can be used as a preprocessing that decreases the size and consequently the computation time of all solvers they consider. An efficient composite heuristic for the Symmetric GTSP is proposed in Renaud and Boctor (1998). An application of ant algorithms to GTSP is reported in Pintea et al. (2017). The asymmetric case of GTSP was also studied in Laporte et al. (1987).

As the asymmetric generalized traveling salesman problem can be transformed into a standard asymmetric traveling salesman problem with the same number of cities (Noon and Bean, 1993), any ATSP solver can be used for transformed GTSP. Furthermore, TSP is among the most studied optimization problems (www 2), and it seems that the majority if not all known heuristics were applied and tested, some of them also invented for TSP. The reservoir of ideas that may be used to solve the GTPS it thus enormous. However, while it is well known that competitive heuristics as a rule employ specific properties of the problem or even of the subset of the instances studied, we wish to recall that Occam's razor principle applies to design of heuristics as well (Žerovnik, 2015). This leads to conclusion that development of heuristics and/or approximation algorithms suited for specific variants of GTSP and/or specific domains may still be worth investigating.

3 Motivation: Flexible assembly with mobile robot

Numerous research activities in new technologies of Industry 4.0 go hand-in-hand with the research of Industry 5.0 technologies which again puts the human worker in the focus. Many tasks at smart industrial assembly workplaces require manual ergonomic workstations which must be smart, flexible and agile. Also a worker must be digitalized, his activities must be simulated in advance and optimally combined with the activities of a collaborative robot. With this regard a huge variety of workers activities should be taken into account (Nogueira et al., 2018; Borgss et al., 2019). When designing ergonomic work conditions and jobs regarding the product all possible information on products, job processes, tools, machines, tasks, limitations, etc. should be considered (Leber et al., 2018). It is of utmost importance to predict the single times required to complete individual work tasks by the worker and also by the collaborative robot. This may be very helpful when planning of necessary staff, material requirements and in prediction of productivity (Rasmussen et al., 2018; Dianat et al., 2018; de Mattos et al., 2018; Lanzottia et al., 2019).

Due to increasing competition in the global market and to meet the need for rapid changes in product variability, it is important to introduce self-configurable and smart solutions, especially in manual assembly stations and also within the entire process chain, to ensure more efficient, flexible, agile and ergonomic performance of the manual assembly process. For example, in Turk et al. (2020a), a smart assembly workstation is discussed that is self-configurable according to the anthropometry of the individual worker, the complexity of the assembly process, the product characteristics, and the product structure. See also Dianat et al. (2018) and Tornstrom et al. (2008).

In general, both ergonomic design of an assembly workstation and reliable estimation of execution time of basic manual assembly tasks (Turk et al., 2020b) may not be straightforward. Below we assume that before considering practical instances of the optimization problems, the corresponding study has been done and hence we are given the necessary data. Besides optimization of the production speed, it is worth to consider some other aspects of the production process. Therefore, the assumed available data include, in addition to production times, also some quantities corresponding to the manual assembly station itself and especially to the working conditions and consequently the satisfaction and well-being of the worker. With this regard, the working process should be structured according to ergonomic rules combined with the digitalization of the information flow and Poka-yoke approach, including the low-cost intelligent automation. In our formal models, we work with configurations of the assembly workstation. The configuration is associated with (or, defined by) its features, including:

- height adaptation and positioning of the table,
- adaptation of the buffers position to achieve primary gripping position,
- pick-by-light approach,
- digitalized product structure, which should automaticaly change with the new product or product variant,
- digitalized instructions on monitor or through augmented reality,
- setup of the chair,
- setup of the assembly nest, including its rotation and positioning abbility,
- the person working at the workstation,
- lighting with automatically adapted luminosity according to the workers needs,

- the content (material) of the boxes should be put optimally in accordance with the product structure, assembly sequence and mass of the product parts etc.

Note that if two workers may work using the same settings, there may be a difference in their speed, so we consider these as two different configurations. Of course, for different workers often also the type of setup will be different, depending on product structure, its variety and the number of product parts. Clearly, in such cases we need to model the change when only the workers are shifted without altering other settings.

4 The general model

Given a product P to be assembled, there may be a number of settings of the workplace that are feasible for this particular task. Note that by our definition, the configuration C determines both the product $P(C)$ and the worker $W(C)$ who is foreseen to work at this configuration. In other words, given a product P there may be several workers that can do the job, and for each of the workers there may be several feasible configurations. Furthermore, with each configuration C we may associate several features, for example, we can define:

- T(C), the time needed for worker $W(C)$ to complete the task related to product $P(C)$;
- R(C), the reliability of the operation performed at configuration C, which can in turn be defined as the proportion of products of poor quality, or by some other measure;
- S(C), a parameter (here called suitability) when person $W(C)$ assembles product P and the workplace is at configuration C, which can be given either as a number, a vector, or even as an element of a set, e.g. {excellent, good, poor, forbidden}; for example, 'poor' may mean that it is likely that working in this configuration for a longer period is a health hazard for the worker.

Clearly, we can define a complete graph where the vertices are the configurations and the weights are defined as follows. Given two configurations, C_i and C_j, denote the time needed to switch from C_i to C_j by $c(C_i, C_j) = c(i, j)$. Note that we may have $c(i, j) \neq c(j, i)$ hence the asymmetric version of the problem. Note that instead of time, the weights $c(i, j)$ on directed edge may have a more general meaning, the cost of operation.

Assume we need to complete the order that is a list of tuples (product, quantity), c.f. (P_1, n_1),(P_2, n_2), $\ldots (P_k, n_k)$. Given a set of available configurations, assuming that the set includes at least one feasible configuration for each of the products, the task is to define the order of production that takes minimal time (or, optimizes some other criteria).

With each product, we can associate several configurations, i.e. all C with $P(C) = P$. The set of all configurations is thus naturally partitioned into the sets that correspond to the products.

Formally, the general problem is defined as follows:

Generic Assembly workplace plan, AWP

Input: A directed graph $G = (V, A)$ with weighting functions $c : E \mapsto \mathcal{E}$, $f : V \mapsto \mathcal{V}$. An order of products and quantities $\{(P_1, n_1), (P_2, n_2), \ldots, (P_m, n_m)\}$, where each product P_i is associated with a set of configurations $V_i \neq \emptyset$. This gives the partition $\{V_1, V_2, \ldots, V_m\}$, where $V_i \cap V_j = \emptyset$ for all $i \neq j$, and $\cup_{i=1}^{m} V_i$.

Question: Find a tour in G that contains exactly one vertex from each set V_i such that the objective function is optimal.

Note that the weighting functions c and f are very general here. The edge weighting function c will in most cases map to $\mathcal{E} = \mathbb{R}_0$. The weights of vertices (configurations) may also be simply production times, i.e. $f : V \mapsto \mathbb{R}_0$. In many cases, f may model more features of the configuration, for example $f(C) = (\mathrm{T}(C), \mathrm{R}(C), \mathrm{S}(C))$.

Below we discuss and define a number of more specific problems related to more specific models. To this aim, we will have to elaborate:

- necessarily, the objective function(s) and
- additional assumptions and/or limitations on the instances.

Objective functions

Let us start with a model where we only minimize time, and consider first a rather general case. Denote by $C_{w(i)}$ the configuration that follows the configuration C_i in the tour w. Hence, in general, the production time of a product P_i depends on the configuration $C(P_i) \in V_i$ and the quantity n_i, and the objective function is thus:

$$\mathrm{T}(w) = \sum_i (\mathrm{T}(C(P_i), n_i) + f(C_i, C_{w(i)}))\,. \tag{1}$$

First simplification may be to assume that the quantities of each product are low enough so that they can be made without interruption, and consequently the time needed depends linearly on the quantities:

$$\mathrm{T}(w) = \sum_i (n_i \mathrm{T}(C(P_i)) + f(C_i, C_{w(i)}))\,. \tag{2}$$

If we add another assumption, namely that the production time does not depend on the configuration, then the first term does not depend on the tour, and we have the objective function:

$$\mathrm{T}(w) = \sum_i f(C_i, C_{w(i)}) \,. \tag{3}$$

Observe that (3) implies AWP under the last assumption is equivalent to GTSP. In other words:

Theorem 4.1 *Problem AWP is a generalization of GTSP, and is NP-hard.*

Multicriterial optimization

As already indicated before, in modelling the assembly workplace it is natural to consider other criteria besides time only. The criteria (1)-(3) may be supplemented by e.g. reliability:

$$\mathrm{R}(w) = \sum_i (\mathrm{R}(C(P_i), n_i)) \,, \tag{4}$$

or/and suitability (ergonomicity):

$$\mathrm{S}(w) = \sum_i (\mathrm{S}(C(P_i), n_i)) \,, \tag{5}$$

to obtain a multicriterial optimization problem with objective function:

$$(\mathrm{T}(w), \mathrm{R}(w), \mathrm{S}(w)) \,.$$

Stochastic optimization

Until now, we have assumed that we are given a fixed order of products and quantities $\{(P_1, n_1), (P_2, n_2), \ldots, (P_m, n_m)\}$. In modern times, industrial production is largely shifted from mass production to small, often custom designed series, and to production on demand for a known end customer. Thus it is important to consider the versions of optimization problems that are stochastic. Here we discuss the situation where we have, instead of a fixed order, a set of likely orders, or pre-orders, that are to be confirmed or altered 'just before production'. We are interested in computing an a priori plan of production that will be optimal on average. In other words, given probabilities of orders in the provisional order, we wish to plan an a priori plan of production that will have minimal expected cost. For simplicity, assume that we only wish to minimize time.

First, recall the probabilistic traveling salesman problem (PTSP) (Jaillet, 1988) that generalizes the TSP aiming to find an a priori tour that has minimal expected length. The cities that need not be visited are not skipped, while the other cities are visited in the order that is defined by the a priori solution. PTSP was among the first stochastic versions of problems in combinatorial optimization.

Similarly, AWP with general objective function (1) can be generalized to probabilistic AWP. The goal is to find an a priori tour that visits all clusters that are present in the realization of the order, and has minimal expected cost. We assume that probabilities of each pair (P_i, n_i) are known. This may be possible to estimate, for example, when we work for known customers.

Formally, the stochastic version of the problem can be defined as follows:

PROBABILISTIC ASSEMBLY WORKPLACE PLAN, PAWP

Input: A directed graph $G = (V, A)$ with weighting functions $c : E \mapsto \mathcal{E}$, $f : V \mapsto \mathcal{V}$. An order of products and quantities $\{(P_1, n_1), (P_2, n_2), \ldots, (P_m, n_m)\}$, with probabilties p_i giving the probability that the i-th order will be confirmed, and where each product P_i is associated with a set of configurations $V_i \neq \emptyset$. This gives the partition $\{V_1, V_2, \ldots, V_m\}$, where $V_i \cap V_j = \emptyset$ for all $i \neq j$, and $\cup_{i=1}^{m} V_i$.

Question: Find a tour in G that contains exactly one vertex from each set V_i such that the expected weight of the tour is mimimal.

5 Conclusions and future work

In this short note, we have provided a new application of the generalized traveling salesman. The practical problem, various models of mixed assembly workstation, has motivated definition of several optimization problems that generalize the GTSP. The contribution of this paper is the definition of the models that are novel to the best of our knowledge.

This is the first step in the research that will be continued along several avenues. On one hand, we are going to gather instances from design of particular workstations in real industrial environment thus bulding a database of realistic instances. On the other hand, we are going to study heuristics for the general optimization problem and its specific variants. The heuristics that we are going to start with is the remove and reinsert heuristics (Žerovnik, 1995; Brest and Žerovnik, 1999, 2005; Pesek et al., 2007; Zupan et al., 2016). Basically the same idea appears, under a different name, in Lahyani et al. (2017). This heuristics is very simple in its basic version, which means that it can be easily generalized and/or adapted to similar problems. In short, remove and reinsert heuristics is a multistart local search, more precisely: iterative improvement, type heuristics. After constructing an initial tour, a series of small perturbations are made that are accepted if the objective function is improved. Both the tour construction and the perturbations are based on the basic procedure that inserts a node into the existing tour that traverses the active nodes. When constructing the tour, a small subset of nodes is activated at first and an optimal tour is found. Then, the inactive nodes are activated in random order and inserted. Perturbation starts with a selection of active nodes that are unactivated, and then again reinserted in random order. For details, we refer to the previous studies.

In particular, in the past, remove and reinsert heuristics was tested both on probabilistic TSP (Žerovnik, 1995) and on asymmetric TSP (Brest and Žerovnik, 1999, 2005), and has proved very competitive.

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