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TRIANGULAR NORMS IN DISCRETE DYNAMIC PROGRAMMING

INTRODUCTION

Decision problems with conflicting objectives and multiple stages can be considered as multi-objective dynamic programming problems. A survey has been presented by Li and Haimes [4], more recently by Trzaskalik [9]. Another way of generalization single-criterion dynamic programming models is to consider outcomes in partially ordered criteria set. Mitten [6] described a method for solving a variety of multistage decisions in which the real value objective function is replaced by preference relation. Sobel [7] extended Mitten's result to infinitive horizon for deterministic and stochastic problems. Preference order dynamic programming was described by Steinberg and Parks [8]. Henig [3] defined a general sequential model with returns in partially ordered set. It is shown that Bellman's principle of optimality [1] is valid with respect to maximal returns and leads to an algorithm to approximate these returns. Application of fuzzy logic to control started with the work written by Bellman and Zadeh [2]. Many contemporary approaches in this field are presented in Kacprzyk [5].

The present paper is devoted to investigate how triangular norms can be applied in the discrete dynamic programming and is the continuation of our previous papers. Basic backward procedure was formulated in Trzaskalik and Sitarz [11] and the forward procedure was worked out in Trzaskalik and Sitarz [12]. In the next paper [13] we considered dynamic programming with outcomes in fuzzy ordered structures. Fuzzy numbers and triangular norms were considered there. Some other examples of ordered structures and products of ordered structures were given in Trzaskalik and Sitarz [10; 14].

The paper consists of 4 sections. In Section 1 basic notation, backward and forward procedure are reminded. In Chapter 2 a wide extension of the idea of applying triangular norms to create ordered structures and 15 exemplary ordered structures based on triangular norms are given. Numerical analysis for these structures is performed in Section 3. Some concluding remarks are given in Section 4.

1. DYNAMIC PROGRAMMING IN PARTIALLY ORDERED STRUCTURE

Discrete dynamic process P which consists of T periods is considered. Let us assume that for t = 1,…,T:

Y_t is the set of all feasible state variables at the beginning of period t,
Y_{T+1} is the set of all states at the end of the process,
$X_t(y_t)$ is the set of all feasible decision variables for period t and state $y_t \in Y_t$.

We assume that all above sets are finite. Now let us define:

$D_t = \{d_t = (y_t , x_t): y_t \in Y_t, x_t \in X_t(y_t)\}$ – the set of all period realizations in period t,

$\Omega_t: D_t \to Y_{t+1}$ – transformations.

Process P is given if sets $Y_1, \ldots, Y_{T+1}, X_1(y_1), \ldots X_T(y_T)$ and transformations $\Omega_1, \ldots, \Omega_T$ are identified.

Let us denote:

$D = \{d = (d_1, \ldots, d_T): \forall_{t \in \{1,\ldots,T\}}\ y_{t+1} = \Omega_t(y_t, x_t)\ \ and\ \ x_T \in X_T(y_T)\}$ – the set of all process realizations

$D_t(y_t) = \{(y_t, x_t): x_t \in X_t(y_t)\}$ – the set of all realizations in period t which begin at y_t.

$d(y_t) = (y_t\, x_t, \ldots, y_T, x_T)$ – the backward partial realization which begins at y_t.

$D(y_t) = \{d(y_t): d \in D\}$ – the set of all backward partial realizations, which begin at y_t.

$D(Y_t) = \{D(y_t): y_t \in Y_t\}$ – the set of all backward partial realizations for Y_t.

$d'(y_t) = (y_1, x_1, \ldots, y_{t-1}, x_{t-1})$ – the forward partial realization which ends at $y_t = \Omega_t(y_{t-1}, x_{t-1})$.

$D'(y_t) = \{d'(y_t): d \in D\}$ – the set of all forward partial realizations, which end at y_t.

$D'(Y_t) = \{D'(y_t): y_t \in Y_t\}$ – the set of all forward partial realizations for Y_t.

$(W, \preceq, \circ)$ – ordered structure with binary relation $\leq$ and binary operator $\circ$ fulfilling following conditions:

$$\forall_{a \in W} \; a \leq a \tag{1}$$

$$\forall_{a,b \in W} \; a \leq b \wedge b \leq a \Rightarrow a = b \tag{2}$$

$$\forall_{a,b,c \in W} \; a \leq b \wedge b \leq c \Rightarrow a \leq c \tag{3}$$

$$\forall_{a,b,c \in W} \; a \circ (b \circ c) = (a \circ b) \circ c \tag{4}$$

$$\forall_{a,b,c \in W} \; a \leq b \;\Rightarrow\; a \circ c \leq b \circ c \wedge \; c \circ a \leq c \circ b \tag{5}$$

Relation < is defined as follows:

$$a < b \Leftrightarrow \; a \leq b \wedge a \neq b \tag{6}$$

Applying relation $\preceq$ we define for each finite subset $A \subset W$ the set of maximal elements:

$$max(A) \;= \{a^* \in A: \sim\exists_{a \in A} \;\; a^* < a\} \tag{7}$$

For $t = 1,\ldots,T$ let $(W, \preceq, \circ_t)$ be a sequence of ordered structures and $f_t:D_t \rightarrow W$ – a sequence of period criteria functions. Applying period criteria functions f_t, we define functions $F_t:D(Y_t) \rightarrow W$ in the following way:

$$F_T = f_T \tag{8}$$

$$F_t = f_t \;\circ_t\; F_{t+1} \qquad\qquad t = T\text{-}1,\ldots,1 \tag{9}$$

Functions $G_t:D'(Y_{t+1}) \rightarrow W$ are built as follows:

$$G_1 = f_1 \tag{10}$$

$$G_t = G_{t-1} \;\circ_{t-1}\; f_t \qquad\qquad t = 2,\ldots,T \tag{11}$$

According to (11) we obtain:

$$F_1 = G_T \tag{12}$$

Let $F: D \rightarrow W$ be the function defined in one of the following ways:

$$F = F_1 \tag{13}$$

$$F = G_T \tag{14}$$

F is called the multiperiod criteria function. Discrete dynamic decision process (P, F) is given if the discrete dynamic process P and the multiperiod criteria function F are defined.

Realization $d^* \in D$ is efficient, iff:

$$F(d^*) \in max\ F(D) \tag{15}$$

Our problem is to find the set of all maximal values of the process, i.e. set max F(D).

Theorem 1

Decision dynamic process *(P, F)* is given. For all $t=T-1,\ldots,1$ and all $y_t \in Y_t$ holds:

$$max\ \{F_t(D(y_t)\} = max\ \{ f_t(d_t) \circ_t max(F_{t+1}(d(\Omega_t(d_t))): d_t \in D_t(y_t)\} \tag{16}$$

$$max\{F(D)\} = max\ \{maxF_1(d(y_1)): y_1 \in Y_1\} \tag{17}$$

Proof. Trzaskalik and Sitarz (11).

Theorem 1 yields backward iterative computational method.

Backward Procedure

Step B$_1$. Compute $max\{F_T(D(y_T)\}$ for all states $y_T \in Y_T$.

Step B$_t$ (for t = T-1, …, 1). Compute $max\{F_t(D(y_t)\}$ for all states $y_t \in Y_t$ applying (16).

Step B$_{T+1}$. Compute $max\{F(D)\}$ applying (17).

Theorem 2

Decision dynamic process *(P, F)* is given. For all $t=2,\ldots,T$ and all $y_t \in Y_t$ holds:

$$max\ \{G_t(D'(y_{t+1})\} = max\ \{ max\ G_{t-1}(d'(y_t)) \circ_t f_t(y_t, x_t):\ \Omega_t(y_t, x_t) = y_{t+1}\} \tag{18}$$

$$max\{F(D)\} = max\ \{max\ G_T(d'(y_{T+1})): y_{T+1} \in Y_{T+1}\} \tag{19}$$

Proof. Trzaskalik and Sitarz [11].

Theorem 2 yields forward iterative computational method.

Forward Procedure

Step F$_1$. Compute $max\{G(D(y_2)\}$ for all states $y_2 \in Y_2$.

Step F$_t$. (for t = 3, …, T). Compute $max\ \{G_t(D'(y_{t+1})\}$ for all states $y_t \in Y_t$ applying (18).

Step F$_{T+1}$. Compute $max\{F(D)\}$ applying (19).

2. FUZZY ORDERED STRUCTURES

As examples of fuzzy ordered structures we will consider triangular norms and products of triangular norms.

2.1. Triangular norms

Function T : [0, 1] × [0, 1] → [0, 1] is called t-norm iff:

$$T(a, b) = T(b, a) \tag{20}$$

$$T(a, T(b, c)) = T(T(a, b), c) \tag{21}$$

$$a \le a' \wedge b \le b' \Rightarrow T(a, b) \le T(a', b') \tag{22}$$

$$T(a, 1) = a \tag{23}$$

We denote:

$$[0,1]^n = [0, 1] \times \ldots \times [0, 1]$$

Each t-norm T may be extended to the function of n-arguments $a^1,\ldots,a^n$:

$$T^n : [0, 1]^n \to [0,1]$$

as follows:

$$T^2(a^1, a^2) = T(a^1, a^2) \tag{24}$$

$$T^i(a^1, a^2, \ldots, a^i) = T(T^{i-1}(a^1, a^2, \ldots, a^{i-1}), a^i) \text{ for } i = 2,\ldots,n \tag{25}$$

In the further considerations we will omit index n.

Let us consider the structure $([0,1]^n, \le, T)$. It is easy to show, that conditions (4) and (5) are fulfilled. Applying the definition of t-norms (20) – (23) we see, that:

$$\forall_{a,b,c\in[0,1]} \; T(a,T(b,c)) = T(T(a,b),c) \tag{26}$$

$$\forall_{a,b,c\in[0,1]} \; a \le b \Rightarrow T(a,c) \le T(b,c) \wedge T(c,a) \le T(c,b) \tag{27}$$

It means, that such a triple constitute ordered structure.

2.2. Product of triangular norms

We denote:

$$a = [a_1, \ldots, a_m],\; b = [a_1, \ldots, a_m]$$

and define relation $\le^m$ as a product of standard relations $\le$, i.e:

$$a \le^m b \Leftrightarrow \forall_{i=1,\ldots,m}\; a_i \le b_i \tag{28}$$

Let T be the set of triangular norms. Function $T = T_1 \times ... \times T_m$ is defined as the product of triangular norms $T_1,...,T_m \in T$, iff:

$$\forall_{a,b \in [0,1]^m} \; T(a,b) = [T_1(a_1,b_1),...,T_m(a_m, b_m)] \tag{29}$$

Applying formulas (24) and (25), each product of triangular norms T may be extended to the function of n-arguments $a^1 = [a_1^{\,1},...,a_m^{\,1}]$, ..., $a^n = [a_1^{\,n},...,a_m^{\,n}]$:

$$T^n : [0, 1]^{m \times n} \to [0,1]^m$$

Again in the further considerations we will omit index n.

Let us consider the structure $([0,1]^{m \times n}, \leq^m, T)$. It is easy to show, that conditions (4) and (5) are fulfilled. It means, that such a triple constitutes ordered structure.

2.3. Examples of ordered structures applying triangular norms

We will consider the following t-norms:

$$T_1(a, b) = \max\{a+b-1, 0\}, \quad (\text{Łukasiewicz}) \tag{30}$$

$$T_2(a,b) = \begin{cases} \min\{a,b\}, & \textit{if } \max\{a,b\} = 1 \\ 0 & \textit{otherwise} \end{cases}, \quad (\textit{weak}) \tag{31}$$

$$T_3(a, b) = a \cdot b, \quad (\textit{probablistic}) \tag{32}$$

$$T_4(a, b) = \min\{a, b\}, \quad (\textit{minimum}) \tag{33}$$

and define following ordered structures:

Structure S_1:	$([0, 1], \leq, T_1)$	
Structure S_2:	$([0, 1], \leq, T_2)$	
Structure S_3:	$([0, 1], \leq, T_3)$	
Structure S_4:	$([0, 1], \leq, T_4)$	
Structure S_{12}:	$([0, 1], \leq^2, T_{12})$	$T_{12} = T_1 \times T_2$
Structure S_{13}:	$([0, 1], \leq^2, T_{13})$	$T_{13} = T_1 \times T_3$
Structure S_{14}:	$([0, 1], \leq^2, T_{14})$	$T_{14} = T_1 \times T_4$
Structure S_{23}:	$([0, 1], \leq^2, T_{23})$	$T_{23} = T_2 \times T_3$
Structure S_{24}:	$([0, 1], \leq^2, T_{24})$	$T_{24} = T_2 \times T_4$

Structure S_{34}:	$([0, 1], \leq^2, T_{34})$	$T_{34} = T_3 \times T_4$
Structure S_{123}:	$([0, 1], \leq^3, T_{123})$	$T_{123} = T_1 \times T_2 \times T_3$
Structure S_{124}:	$([0, 1], \leq^3, T_{124})$	$T_{124} = T_1 \times T_2 \times T_4$
Structure S_{134}:	$([0, 1], \leq^3, T_{134})$	$T_{134} = T_1 \times T_3 \times T_4$
Structure S_{234}:	$([0, 1], \leq^3, T_{234})$	$T_{234} = T_2 \times T_3 \times T_4$
Structure S_{1234}:	$([0, 1], \leq^4, T_{1234})$	$T_{1234} = T_1 \times T_2 \times T_3 \times T_4$

These structures will be used in numerical analysis, performed below.

3. NUMERICAL ILLUSTRATIONS

We consider a dynamic process which consists of 3 periods [$T=3$]. We have:

$$Y_t = \{0,1\}, \quad for \quad t \in \{1, 2, 3, 4\}$$
$$X_t(y_t) = Y_{t+1} \quad for \quad t \in \{1, 2, 3\} \quad and \quad y_t \in Y_t$$
$$\Omega_t(y_t, x_t) = x_t \quad for \quad y_t \in Y_t \quad and \quad x_t \in X_t(y_t)$$

The sets of period realizations for t = 1,2,3 are as follows:

$$D_t = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$$

The values of period realizations are listed below:

$f_1(0, 0) = 0,35$	$f_2(0, 0) = 0,5$	$f_3(0, 0) = 0,5$
$f_1(0, 1) = 0,93$	$f_2(0, 1) = 1$	$f_3(0, 1) = 0,65$
$f_1(1, 0) = 0,3$	$f_2(1, 0) = 0,8$	$f_3(1, 0) = 0,6$
$f_1(1, 1) = 0,85$	$f_2(1, 1) = 0,82$	$f_3(1, 1) = 0,61$

The structure of the process and the values of period criteria are shown in Figure 1.

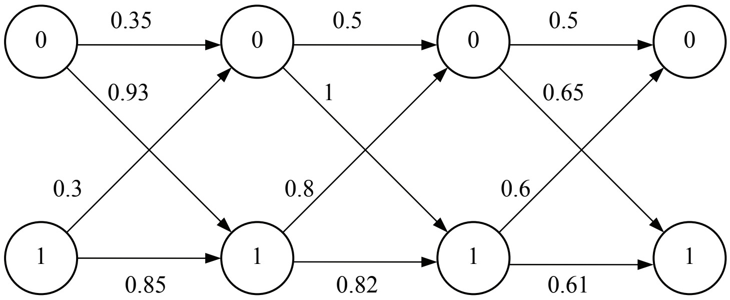

Fig. 1. The graph of the process

The sets of all realizations in period t which begin at y_t are as follows:

$$D_t(0) = \{(0, 0), (0, 1)\}$$

$$D_t(1) = \{(1, 0), (1, 1)\}$$

The sets of backward partial realizations are as follows:

$$D(y_3 = 0) = \{(0,0), (0,1)\}$$

$$D(y_3 = 1) = \{(1,0), (1,1)\}$$

$$D(y_2 = 0) = \{((0,0), (0,0));\quad ((0,0), (0,1));\quad ((0,1), (1,0));\quad ((0,1), (1,1))\}$$

$$D(y_2 = 1) = \{((1,0), (0,0));\quad ((1,0), (0,1));\quad ((1,1), (1,0));\quad ((1,1), (1,1))\}$$

$$\begin{aligned} D(y_1 = 0) = \ & \{((0,0), (0,0), (0,0)); && ((0,0), (0,0), (0,1)); \\ & ((0,0), (0,1), (1,0)); && ((0,0), (0,1), (1,1); \\ & ((0,1), (1,0), (0,0)); && ((0,1), (1,0), (0,1); \\ & ((0,1), (1,1), (1,0)); && ((0,1), (1,1),(1,1))\} \end{aligned}$$

$$\begin{aligned} D(y_1 = 1) = \ & \{((1,0), (0,0), (0,0)); && ((1,0), (0,0), (0,1)); \\ & ((1,0), (0,1), (1,0)); && ((1,0), (0,1), (1,1); \\ & ((1,1), (1,0), (0,0)); && ((1,1), (1,0), (0,1); \\ & ((1,1), (1,1), (1,0)); && ((1,1), (1,1),(1,1))\} \end{aligned}$$

The sets of all backward partial realizations are as follows:

$$D(Y_3) = D(y_3 = 0) \cup D(y_3 = 1)$$

$$D(Y_2) = D(y_2 = 0) \cup D(y_2 = 1)$$

$$D(Y_1) = D(y_1 = 0) \cup D(y_1 = 1)$$

The set of the realizations of the process can be presented as:

$$D = D(Y_1)$$

The sets of forward partial realizations are as follows:

$$D'(y_2 = 0) = \{(0,0);\ (1,0)\}$$
$$D'(y_2 = 1) = \{(0,1);\ (1,1)\}$$

$$D'(y_3 = 0) = \{((0,0),\ (0,0));\quad ((0,1),\ (1,0));\quad ((1,0),\ (0,0));\quad ((1,1),\ (1,0))\}$$
$$D'(y_3 = 1) = \{((0,0),\ (0,1));\quad ((0,1),\ (1,1));\quad ((1,0),\ (0,1));\quad ((1,1),\ (1,1))\}$$

$$\begin{aligned} D'(y_4 = 0) = \ & \{((0,0),\ (0,0),\ (0,0)); && ((0,0),\ (0,1),\ (1,0)); \\ & ((0,1),\ (1,0),\ (0,0)); && ((0,1),\ (1,1),\ (1,0)); \\ & ((1,0),\ (0,0),\ (0,0)); && ((1,0),\ (0,1),\ (1,0); \\ & ((1,1),\ (1,0),\ (0,0)); && ((1,1),\ (1,1),(1,0))\} \end{aligned}$$

$$\begin{aligned} D'(y_4 = 1) = \ & \{((0,0),\ (0,0),\ (0,1)); && ((0,0),\ (0,1),\ (1,1)); \\ & ((0,1),\ (1,0),\ (0,1)); && ((0,1),\ (1,1),\ (1,1)); \\ & ((1,0),\ (0,0),\ (0,1)); && ((1,0),\ (0,1),\ (1,1); \\ & ((1,1),\ (1,0),\ (0,1)); && ((1,1),\ (1,1),(1,1))\} \end{aligned}$$

Let us consider process realization $d = (d_1, d_2, d_3)$. According to formulas (8) and (9) we have:

$$F_3(d_3) = f_3(d_3)$$
$$F_2(d_2, d_3) = T_i(f_2(d_2), F_3(d_3))$$
$$F_1(d_1, d_2, d_3) = T_i[f_1(d_1), (F_2(d_2, d_3)]$$
$$F(d) = F_1(d_1, d_2, d_3)$$

The same result we will obtain applying formulas (10) and (11):

$$G_1(d_1) = f_1(d_1)$$
$$G_2(d_1, d_2) = T_i[F_1(d_1), f_2(d_2)]$$
$$G_3(d_1, d_2, d_3) = T_i[F_2(d_1, d_2), f_3(d_3)]$$
$$F(d) = G_3(d_1, d_2, d_3)$$

At the beginning we will consider ordered structures $S_1 – S_4$. The process is a single criterion maximization problem now and we will apply norms $T_1 – T_4$ as the operators.

Let us look at the numerical computations applying formulas (8) and (9) for subsequent norms $T_1 - T_4$ and the process realization $d = [(0, 0), (0, 0), (0, 0)]$.

For the norm T_1 we obtain:

$F_3(d_3) = F_3(0, 0) = f_3(0, 0) = 0,5$

$F_2(d_2, d_3) = T_1(f_2(0, 0), F_3(0, 0)) = T_1(0,5, 0,5) = max \{0,5 + 0,5 - 1, 0\} = 0$

$F_1(d_1, d_2, d_3) = T_1(f_1(d_1), F_2(d_2, d_3)) = T_1(0,35, 0) = max (0,35 + 0 - 1, 0\} = 0$

For the norm T_2 we obtain:

$F_3(d_3) = F_3(0, 0) = f_3(0, 0) = 0,5$

$F_2(d_2, d_3) = T_2(f_2(0, 0), F_3(0, 0)) = T_2(0,5, 0,5) = 0$

$F_1(d_1, d_2, d_3) = T_2(f_1(d_1), F_2(d_2, d_3)) = T_2(0,35, 0) = 0$

For the norm T_3 we obtain:

$F_3(d_3) = F_3(0, 0) = f_3(0, 0) = 0,35$

$F_2(d_2, d_3) = T_3(f_2(0, 0), F_3(0, 0)) = T_3(0,5, 0,5) = 0,5 \cdot 0,5 = 0,25$

$F_1(d_1, d_2, d_3) = T_3[f_1(d_1), F_2(d_2, d_3)] = T_1(0,35, 0,25) = 0,875$

For the norm T_4 we obtain:

$F_3(d_3) = F_3(0, 0) = f_3(0, 0) = 0,35$

$F_2(d_2, d_3) = T_4[f_2(0, 0), F_3(0, 0)] = T_4(0,5, 0,5) = min \{0,5, 0,5\} = 0,5$

$F_1(d_1, d_2, d_3) = T_4(f_1(d_1), F_2(d_2, d_3)) = T_4(035, 0,5) = min \{0,35, 0,5\} = 0,35$

We can continue computations for the next realization of the process. The results are gathered in Table 1.

Table 1

Values of multiperiod criterion function

d	F(d)			
	T_1	T_2	T_3	T_4
(0,0, 0,0, 0,0)	0	0	0,09	0,35
(0,0, 0,0, 0,1)	0	0	0,11	0,35

d	F(d)			
	T_1	T_2	T_3	T_4
(0,0, 0,1, 1,0)	0	0,35*	0,21	0,35
(0,0, 0,1, 1,1)	0	0,35*	0,21	0,35
(0,1, 1,0, 0,0)	0,23	0	0,37	0,50
(0,1, 1,0, 0,1)	0,33	0	0,45	0,60
(0,1, 1,1, 1,0)	0,35	0	0,46	0,60
(0,1, 1,1, 1,1)	0,36*	0	0,47*	0,61
(1,0, 0,0, 0,0)	0	0	0,08	0,30
(1,0, 0,0, 0,1)	0	0	0,10	0,30
(1,0, 0,1, 1,0)	0	0,3	0,18	0,30
(1,0, 0,1, 1,1)	0	0,30	0,18	0,30
(1,1, 1,0, 0,0)	0,15	0	0,34	0,50
(1,1, 1,0, 0,1)	0,30	0	0,44	0,65*
(1,1, 1,1, 1,0)	0,27	0	0,42	0,60
(1,1, 1,1, 1,1)	0,28	0	0,43	0,61

The best realizations in the considered ordered structures are marked in the Table 1.

Instead of inspection process, the methods described in Section 2 ensure to obtain optimal solutions. We will apply the backward method.

Step B_1

$\max \{F_3(D(y_3 = 0)\} = \max \{F_3(0,0), F_3(0,1)\} = \max \{0,5, 0,65\} = 0,65$

$\max \{F_3(D(y_3 = 1)\} = \max \{F_3(1,0), F_3(1,1)\} = \max \{0,6, 0,61\} = 0,61$

Step B_2

$\max F_2(D_2(0)) = \max T_1(f_2(0,x_2), \max F_3(D(y_3 = x_2) =$

$= \max \{T_1(f_2(0,0), \max F_3(D(0)), T_1(f_2(0,1), \max F_3(D(1))\}$

$= \max \{T_1(0,5, 0,65), T_1(1, 0,61) = \max \{0,15, 0,61\} = 0,61$

$\max F_2D_2(1) = \max T_1(f_2(1,x_2), \max F_3(D(y_3 = x_2) =$

$= \max \{T_1(f_2(1,0), \max F_3(D(0)), T_1(f_2(1,1), \max F_3(D(1))\}$

$= \max \{T_1(0,8, 0,65), T_1(0,82, 0,61) = \max \{0,45, 0,43\} = 0,45$

Step B_3

max $F_1(D(0))$ = max $T_1(f_1(0,x_2)$, max $F_2(D(y_2 = x_1)$ =

= max $\{T_1(f_1(0,0)$, max $F_2(D(0))$, $T_1(f_1(0,1)$, max $F_2(D(1))\}$

= max $\{T_1(0,35, 0,61)$, $T_1(0,93, 0,45)$ = max $\{0, 0,38\}$ = 0,38

max $F_2D(1)$ = max $T_1(f_2(1,x_2)$, max $F_3(D(y_3 = x_2)$ =

= max $\{T_1(f_1(1,0)$, max $F_2(D(0))$, $T_1(f_1(1,1)$, max $F_2(D(1))\}$

= max $\{T_1(0,3, 0,65)$, $T_1(0,85, 0,45)$ = max $\{0, 0,30\}$ = 0,30

Step B_4

max $\{F_1(D(0), F_1(D(1)\}$ = max $\{0,38, 0,30\}$ = 0.38

Now we will consider multicriteria processes based on the next ordered structures, described in Section 2.3. We will assume that we have two, three or four criteria and the value for all the period criteria are the same for a given realization. For instance, in the four criteria process and the ordered structure S_{1234} for t = 1,2,3 we have:

$$f_t(y_t, x_t) = [f_t^1(y_t, x_t), f_t^2(y_t, x_t), f_t^3(y_t, x_t), f_t^4(y_t,x_t)]$$

and for a given period realization (y_t, x_t) it holds:

$$f_t^1(y_t,x_t) = f_t^2(y_t,x_t) = f_t^3(y_t,x_t) = f_t^4(y_t,x_t)$$

We will apply forward and backward procedure for the considered process in the structure S_{1234}. The consecutive stages of computations in the forward procedure are given in Table 2, and for the backward procedure – in Table 3.

Table 2

The forward method in the structure S_{1234}

Step	*max $G_t(d(0))$*	*max $G_t(d(1))$*
F_1	[0.35, 0.35, 0.35, 0.35]	[0.93, 0.93, 0.93, 0.93]
F_2	[0.73, 0, 0.74, 0.8]	[0.75, 0, 0.76, 0.82] [0.35, ,35, 0.35, 0.35]
F_3	[0, 0.35, 0.21, 0.35] [0.35, , 0, 0.46, 0.60]	[0, 0.35, 0.21, 0.35] [0.3, 0, 0.44, 0.65] [0.36, 0, 0.47, 0.61]

Step	*max $G_t(d(0))$*	*max $G_t(d(1))$*
F_4	max F(D) [0, 0.35, 0.21, 0.35] [0.3, 0, 0.44, 0.65] [0.36, 0, 0.47, 0.61]	

Table 3

The backward method in the structure S_{1234}

Step	max $F_t(d(0))$	max $F_t(d(1))$
B_1	[0.65, 0.65, 0.65, 0.65]	[0.61, 0.61, 0.61, 0.61]
B_2	[0.61, 0,61, 0.61, 0.61]	[0.45, 0, 0.52, 0.65] [0.43, 0, 0.50, 0.61]
B_3	[0, 0.35, 0.21, 0.35] [0.36, 0, 0.47, 0.61]	[0.3, 0, 0.44, 0.65]
B_4	Max F(D) [0, 0.35, 0.21, 0.35] [0.3, 0, 0.44, 0.65] [0.36, 0, 0.47, 0.61]	

We can consider this problem as mulitcriteria one. Every realization is desribed by 4 numbers (for each T-norm). The efficient realizations obtained by using one of forward or backward method are presented in Table 2.

Table 4

The efficient realizations (x)

	Ordered structures										
	S_{12}	S_{13}	S_{14}	S_{23}	S_{24}	S_{34}	S_{123}	S_{124}	S_{134}	S_{234}	S_{1234}
(0,0, 0,0, 0,0)											
(0,0, 0,0, 0,1)											
(0,0, 0,1, 1,0)	x				x			x			
(0,0, 0,1, 1,1)	x			x	x		x	x		x	x
(0,1, 1,0, 0,0)											
(0,1, 1,0, 0,1)											

	Ordered structures										
	S_{12}	S_{13}	S_{14}	S_{23}	S_{24}	S_{34}	S_{123}	S_{124}	S_{134}	S_{234}	S_{1234}
(0,1, 1,1, 1,0)											
(0,1, 1,1, 1,1)	x	x	x	x		x	x	x	x	x	x
(1,0, 0,0, 0,0)											
(1,0, 0,0, 0,1)											
(1,0, 0,1, 1,0)											
(1,0, 0,1, 1,1)											
(1,1, 1,0, 0,0)											
(1,1, 1,0, 0,1)			x		x	x		x	x	x	x
(1,1, 1,1, 1,0)											
(1,1, 1,1, 1,1)											

CONCLUDING REMARKS

The considerations in the paper show that applying t-norms to discrete programming models seems to be easy and natural. In the numerical examples the number of efficient realizations was not large. The next step will be to consider the possibility to apply the proposed methodology to model decision makers' preferences.

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