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MEASURING ECO-EFFICIENCY IN A LEONTIEF INPUT-OUTPUT MODEL

1. THE MACROECONOMIC PRODUCTION FUNCTION

The economic input-output model augmented by pollution generation and its abatement as introduced by Leontief [6] has often been used to analyse environmental and economic repercussions. Pollutants are considered as undesirable outputs of industrial activities. The aggregate generation of pollution is controlled by a specific given tolerance limit, an environmental standard. Abatement activities are absorbing pollution at the expense of intermediate and primary inputs.

In the context of pursuing sustainable development a new concept termed "eco-efficiency" has surfaced in business economics and in the public discussion on environmental policy. Efficiency of production as well as environmental efficiency should simultaneously be taken into account. Eco-efficiency is characterised by production of goods and services with minimal resource use and generation of waste and other emissions of pollutants.

Production efficiency is typically determined in relation to the production possibility frontier. In a multi-input, multi-output production technology distance functions can be used to characterise the efficiency of an economy [3; 12]. Given the input vector, the output distance function considers the maximal proportional expansion of the output vector, while the input distance function considers the minimal proportional contraction of the input vector, given the output vector. Using the input-output model based on make and use tables (denoted by *V* and *U*) for *n* industries and *n* commodities[1], such distance functions derive from the following linear optimisation problems:

[1] Without loss of generality we assume quadratic make and use tables.

1. Minimise the use of inputs (capital (K), labour (L)) without altering their proportions, for a given vector of final demand $\bar{y}$:

$$\begin{aligned} &\min_{\lambda} \gamma \\ &\text{subject to} \quad [V'\text{-}U]\lambda \geq \bar{y} \\ &\qquad\qquad\quad k'\lambda - \gamma\bar{K} \leq 0 \\ &\qquad\qquad\quad l'\lambda - \gamma\bar{L} \leq 0 \\ &\qquad\qquad\quad \lambda \geq 0, \gamma \geq 0 \end{aligned} \tag{1}$$

where V' denotes the transposed make matrix.

2. Maximise the proportional expansion of final demand $\bar{y}$ for given amounts of primary factors:

$$\begin{aligned} &\max_{\lambda} \alpha \\ &\text{subject to} \quad [V'\text{-}U]\lambda - \alpha\bar{y} \geq 0 \\ &\qquad\qquad\quad k'\lambda \leq \bar{K} \\ &\qquad\qquad\quad l'\lambda \leq \bar{L} \\ &\qquad\qquad\quad \lambda \geq 0, \alpha \geq 0 \end{aligned} \tag{2}$$

where (V'-U) is the net output flow matrix, k' the row vector of capital requirements, l' the row vector of labour requirements, $\bar{K}$ and $\bar{L}$ are the given upper limits of the respective primary inputs, and λ is the column vector of intensity levels of sector production. The scalar γ describes the proportional reduction factor of primary inputs and α the proportional expansion factor of final demand. Because the input-output model exhibits constant returns to scale the input distance function is the reciprocal of the output distance function. Thus in the optimum $\gamma = 1/\alpha$.

Models of this kind have been proposed by ten Raa [12]. To measure eco--efficiency these models are extended to include undesirable outputs, i.e. pollutants, produced in the economy, and abatement activities which reduce the emissions at the cost of desirable outputs and value added. Let then the production possibility frontier of the economy be determined by the input-output model, primary inputs, pollution generation and abatement, and final demand. Models (1) and (2) are appropriately amended and discussed in the next section. The degree by which a net-output vector, for given primary inputs and environmental standards, could be extended, can be considered as measure of eco-inefficiency. Equivalently, this could be also be achieved by a reduction of primary inputs for given environmental standards and given final demand. Including abatement activities in our model we take into account that desirable outputs are strongly disposable while undesirable ones are only weakly disposable meaning that their reduction can only be achieved by a reduction of desirable outputs or an increase of primary inputs.

Because the efficiency indicators derived from the distance function approach are based on optimisation without the possibility of altering the proportions of net outputs or primary inputs in the respective models, they do not imply necessarily Pareto-Koopmans efficiency. For example, it might still be possible to reduce a specific primary input without reducing any of the net-outputs if its slack variable is positive (see [7]). In this case a change in the proportion of primary inputs might be required to achieve efficiency. In view of the sectoral disaggregation of the economy section three proposes, therefore, a new "slack" – based measure of eco-efficiency which goes beyond the traditional proportional approaches to the changes in final demand or primary inputs. Thus, we are able to take into account the changes in the structure of final demand and the composition of primary inputs.

In section four we show the relationship of the macroeconomic efficiency model of section two to that of Data Envelopment Analysis (DEA), which is a widely used method for efficiency measurement. We derive the production possibility frontier from a multi-objective problem subject to the input-output model. Different DEA models can be constructed which provide eco-efficiency measures comparable to the proportional measures derived in the previous section.

Section five provides a demonstration of the viability of all methods discussed by applying them to Austrian input-output data and National Accounts Matrices including Environmental Accounts (NAMEA) data.

2. MEASURING ECO-EFFICIENCY OF AN ECONOMY

The augmented Leontief model known from the literature (e.g. [6; 7]) extends the economic system by pollution generation and abatement activities. We define emission-matrix $W = \{w_{hj}\}$ containing the amounts of pollutants h = 1,...,q generated by sector j = 1,...,n, emission-matrix $W_a = \{w_{hi}\}$ stating the emissions of pollutants h = 1,...,q generated by abatement activity i = 1,...,q, and matrix $U_a = \{u_{jh}{}^a\}$ containing the inputs of commodity j = 1,...,n into the technological process that removes pollutant h = 1,...q. The q-dimensional row vectors of capital and labour inputs into abatement activities are denoted k'_a and l'_a. Diagonal matrix $\hat{A}$ represents pollution eliminated and λ_a the abatement activity vector. With $\bar{y}$ the (n x 1) final demand vector as above and $\bar{w}$ vector of net generation of pollutants which remain untreated (i.e. pollution standards) we can apply the idea of distance function to this partitioned system. When treating the undesirable outputs like inputs, the proportional measure of eco-efficiency can be derived from the following model formulations:

1. Minimise the use of primary factors for a given level of final demand and tolerated pollution:

$$\begin{aligned} &\min_{\lambda} \gamma \\ &\text{subject to} \quad (V'-U)\,\lambda - U_a\lambda_a \geq \bar{y} \\ &\qquad\qquad W\lambda - (\hat{A} - W_a)\lambda_a \leq \bar{w} \\ &\qquad\qquad k'\,\lambda + k_a'\,\lambda_a - \bar{K}\gamma \leq 0 \\ &\qquad\qquad l'\,\lambda + l_a'\,\lambda_a \leq \bar{L}\gamma \leq 0 \\ &\qquad\qquad \lambda \geq 0;\ \lambda_a \geq 0;\ \gamma \geq 0 \end{aligned} \tag{3}$$

2. Maximise expansion of final demand for given levels of tolerated pollution and primary factors:

$$\begin{aligned} &\max_{\lambda} \alpha \\ &\text{subject to} \quad (V'-U)\,\lambda - U_a\lambda_a - \alpha\bar{y} \geq 0 \\ &\qquad\qquad W\lambda - (\hat{A} - W_a)\lambda_a \leq \bar{w} \\ &\qquad\qquad k'\,\lambda + k_a'\,\lambda_a \leq \bar{K} \\ &\qquad\qquad l'\,\lambda + l_a'\,\lambda_a \leq \bar{L} \\ &\qquad\qquad \lambda \geq 0;\ \lambda_a \geq 0;\ \alpha \geq 0 \end{aligned} \tag{4}$$

We note that due to the presence of the pollution subsystem representing undesirable outputs, the optimal values of α and γ are no longer the reciprocal of each other. However, by treating these undesirable outputs like inputs in the model, i.e. by changing the problem formulation into a proportional reduction of primary inputs and undesirable outputs for given final demand, the reciprocal property of the distance function can be re-established.

3. A SLACK-BASED MEASURE OF ECO-EFFICIENCY

To avoid the limitation of efficiency indicators assuming unchanged proportions of outputs or inputs and taking into account the similarity of undesirable outputs and inputs we turn now to a slack based measure of eco--efficiency.

The slack based measure of eco-efficiency is constructed in analogy to a measure proposed by Cooper, Seiford, and Tone [2] for data envelopment analysis. The following goal programming model can be formulated when one treats the undesirable outputs of pollutants just like inputs in a conventional definition of efficiency. We formulate it as a minimisation problem of a scalar which is unit invariant and monotone, subject to the constraints of the augmented Leontief model and the relations restricting primary input use [7]:

$$\min \left\{ \rho = \frac{1 - \frac{1}{q+2}\left(\frac{S_K}{\overline{K}} + \frac{S_L}{\overline{L}} + \sum_{h=1}^{q} \frac{S_{wh}}{\overline{w}_h} \right)}{1 + \frac{1}{n}\sum_{j=1}^{n} \frac{S_{yj}}{\overline{y}}} \right\} \tag{5}$$

subject to

$$\begin{aligned} (V'-U)\,\lambda^* - U_a\lambda^*_a - S_y &= \overline{y} \\ W\lambda^* - (\hat{A} - W_a)\lambda^*_a + S_w &= \overline{w} \\ k'\,\lambda^* + k_a'\,\lambda^*_a + S_K &= \overline{K} \\ l'\,\lambda^* + l_a'\,\lambda^*_a + S_L &= \overline{L} \\ \lambda^* \geq 0;\ \lambda^*_a \geq 0;\ S_y \geq 0; S_w \geq 0; S_K \geq 0; S_L &\geq 0 \end{aligned} \tag{6}$$

where we use n sectors and q pollutants. We denote by λ^*, λ^*_a the intensity vectors, by S_y and S_w the vectors of slack variables of sector outputs and pollutants. S_K and S_L are capital and labour slacks. $\overline{y}, \overline{w}, \overline{K}, \overline{L}$ are given values.

Evidently $0 \leq \rho \leq 1$ since slacks cannot exceed the values on the right hand side. Re-defining $s_j = tS_j$ (j = y, w, L, K), and $\lambda = t\,\lambda^*$, $\lambda_a = t\,\lambda^*_a$ this fractional program can be linearised to yield the equivalent problem:

$$\min \left\{ \tau = t - \frac{1}{q+2}\left(\frac{s_K}{\overline{K}} + \frac{s_L}{\overline{L}} + \sum_{h=1}^{q} \frac{s_{wh}}{\overline{w}_h} \right) \right\} \tag{7}$$

$$\text{subject to} \quad (V'-U)\lambda - U_a\lambda_a - s_y = t\overline{y}$$
$$W\lambda - (\hat{A} - W_a)\lambda_a + s_w = t\overline{w}$$
$$k'\lambda + k_a'\lambda_a + s_K = t\overline{K}$$
$$l'\lambda + l_a'\lambda_a + s_L = t\overline{L} \tag{8}$$
$$t + \frac{1}{n}\sum_{j=1}^{n}\frac{s_{yj}}{\overline{y}} = 1$$
$$\lambda \geq 0; \lambda_a \geq 0; s_y \geq 0; s_w \geq 0; s_K \geq 0; s_L \geq 0$$

Some numerical calculations with data of the Austrian economy will provide an opportunity to compare the different eco-efficiency measures and give an impression about their usefulness.

In the next section we return to the radial efficiency measures of section two and show that these models are closely related to data envelopment analysis models which are now widely used for efficiency measurement.

4. RELATIONS TO DATA ENVELOPMENT ANALYSIS

The most obvious relationship of the models considered so far with the standard model of DEA can be found by contemplating system (1). For simplicity of exposition we use the model without pollution and abatement activities. The extension to include the environmental components is relatively straightforward and can be found in the Appendix. The idea of DEA is to use data on m inputs and n outputs of N decision making units using the same technology to derive the efficiency frontier by the "best" producing units. The efficiency frontier is defined by the data envelope of all units considered. The envelope form of the DEA model minimises the efficiency score θ, a radial contraction of the input vector of a particular decision making unit, while remaining in the feasible input set:

$$\min_{\mu} \theta$$
$$\text{subject to} \quad Q\mu \geq q^0$$
$$Z\mu - \theta z^0 \leq 0 \tag{9}$$
$$\mu \geq 0, \theta \geq 0$$

with q^0 the (n x 1) output vector and z^0 the (m x 1) input vector of the unit "0" whose efficiency is to be investigated, Q the (n x N) matrix containing the output vectors of all N units, Z the (m x N) matrix containing the input vectors of all units, μ a (N x 1) vector of coefficients to be determined, and θ a scalar. The radial contraction of the input vector z^0 generates a projection point ($Q\mu$, $Z\mu$) on the surface of the technology set spanned by the efficient subset of the N units. The projected point is a linear combination of the observed data points of those efficient units.

Setting $Q = [V'\text{-}U]$, $Z = [k', l']'$, $q^0 = \bar{y}$, and $z^0 = (\bar{K}, \bar{L})'$ the problem (9) is equivalent to problem (1) with θ = γ. But there is a significant difference in the economic interpretation of DEA model (9) and a general DEA model. While DEA uses inputs and outputs of different independent decision making units, the I-O model uses data of one country but disaggregated into interrelated sectors. One way to exploit the formal similarity of the problem consists in generating such levels of outputs of and inputs into sectors as can optimally be generated by the given input-output system. This will establish the production possibility set or the input requirement set depending on the formulation.

In essence we propose to formulate a multi-objective optimisation problem in which final demand for each commodity is maximised subject to restraints on the production of other outputs and required inputs, or each input is minimised for the given levels of final demand. Denoting by s the vector of n slack variables of the n goods and by s_K and s_L the slacks in the capital and labour input relation, the following model is solved j times for given nonnegative values of sector net-outputs and inputs to obtain the maximal value of each slack variable s_j for j = 1,..., n, K, L.

$$
\begin{aligned}
&\max s_j \\
&\text{subject to} \quad (V'-U)\,\lambda - s = \bar{y} \\
&\qquad k'\lambda + s_K = \bar{K} \\
&\qquad l'\lambda + s_L = \bar{L} \\
&\qquad s_j \geq 0 \quad \text{all} \quad j = 1,\ldots,n,K,L \\
&\qquad \lambda_i \geq 0 \quad \text{all} \quad i = 1,\ldots,n.
\end{aligned}
\tag{10}
$$

For each of the n + 2 solutions of (10) the values of the net-output column vector y* are then obtained by $y^* = \bar{y} + s$ and those of the inputs by $K^* = \bar{K} - s_K$ and $L^* = \bar{L} - s_L$. These sets of values are arranged row-wise in a pay-off matrix with the maximal (or minimal) values appearing in the main diagonal while the off-diagonal elements provide the levels of other sector net--outputs (or inputs) compatible with the individually optimised one. Thus, each column of the pay-off matrix yields an efficient solution, i.e. characterises a potential efficient point which can be generated by the economic system. In other words each column represents a ficticious decision making unit. Each of these points is constructed independently of the other points but taking account of the entire systems relations. The independence derives from the fact that each hypothetical experiment to find the maximal net output of a particular sector (or a particular minimal input) is conducted independently from that of another sector output or input, although all are using the same technology. In this way the experiments can be taken to generate data equivalent to those of hypothetical firms with different input and output characteristics, which all use the same production technique. The whole set of such efficient solutions can, therefore, establish the frontier of the production possibility set (or the input requirement set). Thus, the efficient envelope of the economy is defined by:

$$\begin{bmatrix} Q^* \\ -- \\ Z^* \end{bmatrix} = \begin{bmatrix} y^*_{11} & y^*_{12} & \cdots & y^*_{1,n+2} \\ \vdots & \vdots & & \vdots \\ y^*_{n1} & y^*_{n2} & \cdots & y^*_{n,n+2} \\ -- & -- & -- & --- \\ K_1^* & K_2^* & \cdots & K_{n+2}^* \\ L_1^* & L_2^* & \cdots & L_{n+2}^* \end{bmatrix} \tag{11}$$

This efficient frontier constitutes the standard envelope (a notion introduced by Golany and Roll [4]) for the DEA model measuring the efficiency of the economy given by the actual output and input data (q^0,z^0). For this purpose we solve the following problem (12):

$$\begin{aligned} &\min_{\mu} \; \theta \\ &\text{subject to} \quad Q^* \mu \geq q^0 \\ &\qquad\qquad\quad Z^* \mu - \theta z^0 \leq 0 \\ &\qquad\qquad\quad \mu \geq 0, \theta \geq 0 \end{aligned} \tag{12}$$

The efficiency score, the scalar θ gives us the proportion of all inputs of the economy which must be sufficient – compared to the production frontier – to achieve the given output levels. In other words (1-θ) describes the necessary reduction of all inputs of the economy to achieve the efficiency frontier. Therefore θ describes the efficiency of the economy.

The vector μ provides the weighting pattern in the construction of the projection point on the efficient surface derived from a radial input contraction. It informs about the weight a particular artificial decision making unit (i.e. a particular efficient solution as described by (11)) has in the projection of the given state of the economy to the efficient frontier. All those units $i = 1,...,N$ (here $N = n + 2$) with $\mu_i > 0$ form the "peer group" defining the efficient production level for the economy under investigation.

The extension of the model to include the environment is straightforward, remembering that quantities of undesirable outputs (pollutants) are treated like inputs in view of them being minimised. A short derivation is given in the Appendix.

According to Korhonen and Luptáčik [5] four different DEA models can be constructed: The first model (A) is based on the idea of presenting all outputs as a weighted sum, but using negative weights for undesirable outputs. Here, the efficiency is measured by proportional reduction of inputs only. In model (B) undesirable outputs are taken as inputs. Efficiency is measured by a proportional simultaneous reduction of inputs and undesirable outputs. If efficiency is measured by the ratio of the weighted sum of desirable outputs minus inputs to that of undesirable outputs we obtain model (C). Model (D) is an output oriented model where efficiency is measured by proportional improvements of outputs and constitutes the reciprocal formulation of model (B). It has been shown that the eco-efficient frontier is independent of the specific model used.

Model (3) which measures efficiency only by proportional reduction of inputs can, therefore, be considered equivalent to model (A). Changing the second constraint of the input oriented model (3) to:

$$W\lambda - (\hat{A} - W_a)\lambda_a - \gamma\overline{w} \leq 0 \qquad (13)$$

i.e. treating the undesirable outputs exactly like primary inputs, implies DEA-model (B), while keeping constraint (13) but replacing the primary input constraints by:

$$\begin{aligned} k'\lambda + k_a'\lambda_a &\leq \overline{K} \\ l'\lambda + l_a'\lambda_a &\leq \overline{L} \end{aligned} \qquad (14)$$

leads to DEA model (C). Model (4) is seen to correspond to model (D) because of efficiency measured by the proportional increase of outputs.

These different model versions for eco-efficiency measurement permit decompositions of the efficiency score according to desirable outputs, undesirable outputs, and inputs.

5. AN EMPIRICAL DEMONSTRATION FOR THE AUSTRIAN ECONOMY

The following models use a highly aggregated version of the Austrian input-output table of 1995 [10] and NAMEA data [11; 13] for air and water pollution. The empirical examples are calculated for five sectors, two pollutants and two primary inputs as follows (the numbering of intensity and slack variables follows the item numbers):

Sectors:
1. Agriculture, forestry, mining (mill. ATS).
2. Industrial production (mill. ATS).
3. Electricity, gas, water, construction (mill. ATS).
4. Trade, transport and communication (mill. ATS).
5. Other public and private services (mill. ATS).

Pollutants:
6. Air pollutant (NO_x, tons per year) (Source: [11]).
7. Water pollutant (P, tons per year) (Source: [13]).

Primary Inputs:
8. Labour (total employment, 1000 persons).
9. Capital (gross capital stock, 1995, nominal, mill. ATS) (Source: [1]).

5.1. Proportional and slack based eco-efficiency measures

A first experiment employs the simple models (1) and (2) with levels of capital and labour corresponding to a 5% underutilisation of both of these inputs. As expected the proportional efficiency measure α yields $1.05, (output could be expanded by 5% proportionally) and the minimum γ equals 0.952, the reciprocal value of α. The λ values are the same for all sectors, i.e. $\lambda_i = 1.05$ for model (1) and equal to one for model (2) (i.e. the same output can be produced by a 4.76189% reduction of both inputs).

Expanding the model for pollutants and abatement we repeat the exercise with the same levels of inputs as before. Model (3) yields a minimum value of γ equal to 0.95194 with $\lambda_i = 0.999$ for $i = 1,...,5$ and $\lambda_6 = 0.99867$, $\lambda_7 = 0.90727$. We observe a slight change compared to the simple model for the intensities of the output sectors, but quite different values for the abatement intensities λ_6 and λ_7. Calculating the output oriented model (4) the maximum $\alpha = 1.04899$ is not the reciprocal value of min γ. Here again the intensities are almost the same for the outputs but different for the pollutants ($\lambda_i = 1.0494$ for $i = 1,...,5$ and $\lambda_6 = 1.137$, $\lambda_7 = 1.1007$). We observe that the efficiency measure of the extended model gives a proportional factor of expansion or reduction of outputs (respectively inputs) while intensities reveal disproportionate abatement activities.

Let us now compare these results with the slack based efficiency measure (7) under the same assumptions on primary inputs. Minimisation of τ yields a value of 0.409853 with a $t = 0.823$. All output slacks except that of sector one are zero, but pollution slacks and the capital slack are positive. The following table states the results including the optimal intensity values:

Table 1

λ1 = 1. 017 840 7	s1 = 25366.251
λ2 = 0.834 212 68	s2 = s3 = s4 = s5 = s8 = 0
λ3 = 0.832 229 32	s6 = 54206.968
λ4 = 0.831 069 16	s7 = 831.700 32
λ5 = 0.827 642 68	s9 = 75486.407
λ6 = 2. 454 075 3	t = 0.823 188 57
λ7 = 1. 995 758 5	τ = 0.409 853 52

The (inefficient) economy produces too much of agricultural output thereby generating more pollution of both kinds requiring higher abatement intensities (λ_6, λ_7). The positive pollution slacks indicate that too much undesirable outputs are generated while the capital slack shows that capital utilisation is by 0.69% only. The limiting primary factor is labour which is fully utilized.

5.2. The empirical eco-efficiency analysis with DEA

To construct the envelope as described in section 4 the nine problems (10) are solved. The resulting pay-off matrix is given below:

Table 2

Pay-off table

		y1	y2	y3	y4	y5
max	y1	71.409204	1037.514	287.531	638.251	918.239
max	y2	28.693	1217.22728	287.531	638.251	918.239
max	y3	28.693	1037.514	424.7812	638.251	918.239
max	y4	28.693	1037.514	287.531	755.2308	918.239
max	y5	28.693	1037.514	287.531	638.251	1011.103
min	Poll1	28.693	1037.514	287.531	638.251	918.239
min	Poll2	28.693	1037.514	287.531	638.251	918.239
min	K	28.693	1037.514	287.531	638.251	918.239
min	L	28.693	1037.514	287.531	638.251	918.239

		Poll1	Poll2	Capital	Labour
max	y1	65.850	1010.34	10618.188	4123.830
max	y2	65.850	1010.34	10705.312	4123.830
max	y3	65.850	1010.34	10785.810	4123.830
max	y4	65.850	1010.34	10704.571	4123.830
max	y5	65.850	1010.34	10840.940	4052.4773
min	Poll1	0	1010.34	10458.756	3962.061
min	Poll2	65.850	0	10403.477	3947.831
min	K	65.850	1010.34	10324.70548	4123.830
min	L	65.850	1010.34	10840.940	3927.553

Using this pay-off table for the same experiment as above (i.e. with 5% capital and labour surplus) the DEA model with pollutants (cf. Appendix (17)) is solved yielding a minimum θ value of 95.33 for the economy, while all other artificial units are 100% efficient as they should be from the construction of the efficiency frontier. The projection of the economy to the efficient frontier is performed by using the following coefficients:

$\mu_1 = 0.0279$, $\mu_2 = 0.2399$, $\mu_3 = 0.0871$, $\mu_4 = 0.2267$, $\mu_5 = 0.4109$,
$\mu_6 = 0.0074$, $\mu_7 = 0.0074$, $\mu_8 = \mu_9 = 0$.

The θ value indicates the inefficiency in the use of primary factors and excess pollution. In other words, both primary factors and both pollution levels should be reduced by 4.7% in order for the economy to become efficient. This is DEA-model B of Korhonen and Luptáčik [5].

The following properties can be proved (see [9]). Solving the modified model (3) with additional constraint (13), i.e. considering the proportional reduction in both, primary inputs and undesirable outputs, the value of the efficiency score γ is exactly equal to θ of the previous DEA-model. If we calculate the output oriented model (DEA-model D in [5]) we obtain the efficiency score of 1.04899 which is exactly the reciprocal of the input oriented value θ. For given levels of primary factors and net-pollution the net output (i.e. final demand) of all sectors could be increased by 4.9% to make the economy efficient. The same efficiency score follows from model (4) as can be seen from the calculated value of α in the previous subsection.

CONCLUSIONS

The purpose of this paper was to present new alternative measures for eco-efficiency. We basically distinguished two concepts: The first was based on a linear programming input-output model which is able to provide optimal intensity levels for production and abatement activities. The other was based on the construction of a production possibility frontier by multiple objective optimisation. Then, using data envelopment analysis the eco-efficiency of an economy related to this hypothetical frontier was estimated. The results of the DEA application show the potential improvements of eco-efficiency with particular outputs, primary inputs, and undesirable outputs. An analysis of the relationships between the concepts shows an equivalence of radial efficiency measures. However, because of their different model structures useful additional insights and interpretations of the same criterion of performance can be obtained.

APPENDIX

The extension of the simple input-output-DEA model (12) to incorporate the environmental aspects can be achieved along the following lines. First, problem (10) is re-written to incorporate abatement activities and pollution generation using the notation of section 2. Combining the k given primary inputs in (k x 1) vector $\bar{z}$ we have to solve $n + q + k$ problems (n sectors, k primary inputs, q pollutants):

$$
\begin{aligned}
&\max s_i \\
&\text{subject to} \quad (V'-U)\,\lambda - U_a\lambda_a - s_1 = \bar{y} \\
&\qquad W\lambda - (\hat{A} - W_a)\lambda_a + s_2 = \bar{w} \\
&\qquad v'\,\lambda + v_a{}'\lambda_a + s_3 = \bar{z} \qquad (15)\\
&\qquad s_j \geq 0, \quad j = 1,2,3 \\
&\qquad \lambda \geq 0;\ \lambda_a \geq 0; \\
&\qquad i = 1,\ldots,n,n+1,\ldots,n+q,n+q+1,\ldots,n+q+k
\end{aligned}
$$

where s_1 and λ are (n x 1) vectors, s_2 and λ_a are (q x 1) vectors, and s_3 is a (k x 1) vector. After obtaining the n +q + k solutions the (n + q + k) x (n + q + k) pay-off matrix is constructed with the submatrices of output (Q_1^*), pollution (Q_2^*) and input values (Z^*) calculated from $y^* = \bar{y} + s_1$, $w^* = \bar{w} - s_2$ and $z^* = \bar{z} - s_3$ of all solutions. The efficient envelope is then given by:

$$
\begin{bmatrix} Q_1^* \\ -- \\ Q_2^* \\ -- \\ Z^* \\ \end{bmatrix} = \begin{bmatrix} y_1^* & y_2^* & \cdots & y_{n+q+k}^* \\ -- & -- & -- & --- \\ w_1^* & w_2^* & \cdots & w_{n+q+k}^* \\ -- & -- & -- & --- \\ z_1^* & z_2^* & \cdots & z_{n+q+k}^* \end{bmatrix} \qquad (16)
$$

to be incorporated in the extended DEA model (17) (denoting observed desirable outputs by q_1^0, undesirable ones by q_2^0 and inputs by z^0:

$$
\begin{aligned}
&\min_{\mu} \ \theta \\
&\text{subject to} \quad Q_1^*\mu \geq q_1^0 \\
&\qquad Q_2^*\mu - \theta q_2^0 \leq 0 \qquad (17)\\
&\qquad Z^*\mu - \theta z^0 \leq 0 \\
&\qquad \mu \geq 0, \theta \geq 0
\end{aligned}
$$

The weighting vector μ now has dimension N = (n + q + k). Its positive elements determine the efficient production or reference set onto which the economy with performance (q_1^0, q_2^0, z^0) is projected when it is inefficient.

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