Kazimierz Zaraś

Hamdjatou Kane

Maciej Nowak

PRODUCTION PLANNING AND CONTROL: AN APPROACH BASED ON ROUGH SETS

Abstract

This paper deals with the problem of production process control in a job shop where the work flow is controlled by Kanban cards. Production may proceed differently depending on the lot size, number of Kanban cards used, and the decision rule for choosing the job waiting to be processed. The problem that arises consists in deciding which rule should be used, how many Kanbans should be allocated for each operation, and what lot size should be applied. Thus, the choice of the best triplet constitutes a multicriteria problem. We propose to solve the multicriteria problem by using Rough Set Approach. Taking into account operator's choices we use the dominance-based rough set approach to induce the decision rules, which can be applied to choose the best triplet from a large number of alternatives. This paper deals with the problem of production process control in a job shop where the work flow is controlled by Kaban cards. Production may proceed differently according to a lot size, number of Kanban cards used, and the decision rule for choosing the waiting job to process. The problem that arises consists in deciding which rule should be used, how many Kanbans should be allocated for each operation, and what lot size should be applied. Thus, the choice of the best triplet constitutes a multicriteria problem. We propose to solve the multicriteria problem by using Rough Set Approach. Based on the choice of the operator and using the dominance-based rough set approach we will be able to induce the decision rules, which can be applied to choose the best triplet from a large number of alternatives.

Keywords

Job shop, Kanban, multicriteria analysis, rough sets, stochastic dominance.

INTRODUCTION

This study assumes that Just-in-Time (JIT) approach is used for scheduling the production system. The work flow is controlled by Kanban cards. This technique is mainly used in a classic mass production environment with few product variations and levelled demand. Gravel and Price [4; 3] have shown how this approach can be adapted to job-shop environment. A more recent variation of this problem is known as POLCA (Paired, Over-lapping, Loops of Cards with Authorisation); it is applicable in a job-shop environment where each job can be unique [8; 9].

Production may proceed differently depending on the lot size, number of Kanban cards used, and the decision rule for choosing the job waiting to be processed. The problem that arises consists in deciding which rule should be used, how many Kanbans should be allocated for each operation, and what lot size should be applied. In general, smaller lot sizes reduce work-in-progress, but also increase the number of machine set-ups. Increasing the number of allocated Kanbans improves machine utilisation, but may also increase average work-in-progress level. Finally, the performance of a scheduling rule depends on the performance measure used. Thus, the choice of the best triplet involving the Kanban lot size, the decision rule, and the number of Kanbans constitutes a multicriteria problem. Gravel et al. [5] considered a similar problem and used Electre method [13] to model outranking relations. They assumed that completion time of each operation is known and simulated each product separately to evaluate performance of the shop under various conditions (various products, various production environments). In their study, they assumed that the decision maker (DM) is risk-averse. Nowak et al. [11] proposed a modified approach for this problem. They assumed that the DM is risk-prone and several products are processed simultaneously in the shop. The probability distribution of the operation's completion times was determined by series of simulations for each decision alternative to analyse the performance of a shop. This paper deals with solving the problem of production process control as a multicriteria problem such as in [2 ;11] but by using the Rough Set approach. By application of the Rough Set approach we don't need the explicit information about criterion weights as it is necessary to have for preference modelling with the ELECTRE method. In practice we know that criterion weights determination is not the easy task. In the Rough Set approach the DM shows us how he does his job by ordering the alternatives from the efficient set; implicit weights are given by ranking the alternatives.

This paper is structured as follows: the problem is formulated as a multicriteria problem in Section 1. Section 2 presents the rough set approach to choose the control production parameters. In Section 3, we give a job shop production example.

1. PRODUCTION PROCESS CONTROL AS A MULTICRITERIA PROBLEM

In this paper, three performance criteria are considered: makespan; average work-in-progress level; number of set-ups. The first criterion is very important, since short execution times increase effective capacity of the shop and improve the service level. The average work-in-progress level reflects the effectiveness of the firm in reducing investment in semi-finished work. Finally, the number of machine set-ups indicates the number of times the operators have to adapt to a different operation. All tree criteria will be minimized.

The set of alternatives includes all triplets (the lot size, the number of kanban cards and the decision rule). The set of attributes includes all criteria (makespan, average stock and the number of set-ups). Performances of each alternative with respect to the attributes are evaluated by distribution functions. The knowledge base used for the construction of these functions was obtained by using a simulation model of the process where several products are manufactured simultaneously such as in Nowak [12].

The decision situation considered here may be conceived as a problem (**A**, **X**, **E**) where **A** is a finite set of alternatives (triplets), $i = 1, 2, ..., m$; **X** is a finite set of attributes (criteria) X_k, $k = 1, 2, ..., n$; and **E** is a set of evaluations of triplets with respect to the criteria:

$$\mathbf{E} = \begin{bmatrix} X_{11} & \cdots & X_{1k} & \cdots & X_{1n} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ X_{i1} & \cdots & X_{ik} & \cdots & X_{in} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ X_{m1} & \cdots & X_{mk} & \cdots & X_{mn} \end{bmatrix}$$

We assume that the attributes are probabilistically independent and also satisfy the independence conditions which allows us to use additive utility function.

Our approach consists in building global preferences on the set of parameter triplets by first comparing their distributional evaluations in relation to each criterion to model the partial preferences and then by aggregating them into global preferences. With respect to each criterion the preferences are modelled by using the Stochastic Dominances [18; 17; 10]. The comparison of alternatives can be conducted by means of First Degree Stochastic Dominance (FSD), Second Degree Inverse Stochastic Dominance (SISD) [1] and Third Degree Inverse Stochastic Dominance (TISD1 and TISD2). The FSD is defined, if the difference between two cumulated distributions is non-positive for all x, and for at least one x this difference is strictly negative. The Second Inverse Stochastic Dominance (SISD) is defined, if the difference between two integrals from right to left on two cumulated distributions is non-positive for all x, and for at least one x this difference is strictly negative. The Third Inverse Stochastic Dominance (TISD2) is defined, if the difference between two double integrals from right to left on two cumulated distributions is non-positive for all x, and for at least one x this difference is strictly negative. Generally, if one of the inverse stochastic dominances is verified, it has been proven for increasing convex class of utility functions that the expected utility of distributional evaluation which dominates is greater or equal to the expected utility of distributional evaluation which is dominated.

Let F_{ik} and F_{jk} be cumulative distribution functions:

(1) F_{ik} FSD F_{jk} if and only if $F_{ik} \neq F_{jk}$ and $F_{ik} \leq F_{jk}$ for all $x \in [c, d]$.

(2) F_{ik} SISD F_{jk} if and only if $F_{ik} \neq F_{jk}$ and $\int_x^d F_{ik}(x_k)\,dx_k \leq \int_x^d F_{jk}(x_k)\,dx_k$

for all $x \in [c, d]$.

(3) F_{ik} TISD2 F_{jk} if and only if $F_{ik} \neq F_{jk}$ and $\int_x^d \int_x^d F_{ik}(y_k)\,dx_k\,dy_k \leq \int_x^d \int_x^d F_{jk}(y_k)\,dx_k\,dy_k$

for all x $\in [c, d]$.

(4) F_{ik} TISD1 F_{jk} if and only if $F_{ik} \neq F_{jk}$ and $\int_c^x \int_x^d F_{ik}(y_k)\,dx_k\,dy_k \leq \int_c^x \int_x^d F_{jk}(y_k)\,dx_k\,dy_k$

for all x $\in [c, d]$.

where $[c, d]$ is the interval of definition of two random variables X_{ik}, X_{jk}.

2. THE ROUGH SET APPROACH

The rough set approach is based on the Rough Set theory developed by Pawlak (1991), Pawlak and Slowinski (1994) and Greco, Matarazzo and Slowinski (1999). This theory was proposed in this paper for ranking a large number of parameter triplets from the efficient set. The Rough Set Theory relies on a tabular representation of the preferential information expressed by the DM.

These preferences are expressed using the following procedure. First, a small number (4-7) of parameter triplets chosen from different parts of efficient set are presented , and the DM is asked to order triplets from the most preferred to the least preferred.

Second, this ranking represents the DM's preferences, which are noted in a decision table with respect to the decisional attributes.

Let B be a finite subset of parameter triplets which are considered by the DM as the basis for exemplary pairwise comparisons. In addition, let C be the set of attributes (condition attributes) describing the parameter triplet, and D, the decision attribute. The decision table is defined as the 4-tuple: $T = (H, C \cup D, V_C \cup V_D, g)$ where $H \subseteq B \times B$ is a finite set of pairs of parameter triplets, $C \cup D$ is the union of two subsets of attributes, called condition and decision attributes, $V_C \cup V_D$ is the union of the domains of these attributes respectively, and g: $H \times (C \cup D) \rightarrow V_C \cup D$ is a total function where $V_C = \cup V_k$.

This function is such that:

(1) $g[(a_i, a_j), k] = 1$, if f_{ik} $SD_k f_{jk}$ is verified $\forall X_k \in C$, and $\forall (a_i, a_j) \in H$;

(2) $g[(a_i, a_j), k] = 0$, if f_{ik} not $SD_k f_{jk}$ is verified $\forall X_k \in C$, and $\forall (a_i, a_j) \in H$; and $g[(a_i, a_j), k] \in V_k, \forall X_k \in C$, and $\forall (a_i, a_j) \in H$ and $g[(a_i, a_j), D] \in V_D, \forall (a_i, a_j) \in H$.

In our decision table $g[(a_i, a_j), D]$ can also have two values on $H \subseteq B \times B$:

(1) $g[(a_i, a_j), D] = P$, if a_i is preferred to a_j $\forall (a_i, a_j) \in H_P$,

(2) $g[(a_i, a_j), D] = N$, if a_i is not preferred to a_j $\forall (a_i, a_j) \in H_N$.

These two values will be expressed with respect to the decisional attribute. The subset H_P expresses the preferences and H_N expresses non-preferences.

In general, the decision table can be presented as in Table 1.

Table 1

Decision table

		X1	X2	...	Xm	D
HP	(a_i, a_j)	$g[(a_i, a_j), 1]$	$g[(a_i, a_j), 2]$	...	$g[(a_i, a_j), m]$	$g[(a_i, a_j), D] = P$
	.	.	.	...	.	.
	.	.	.	...	.	.
HN	.	.	.	...	.	.
	.	.	.	...	.	.
	(a_s, a_t)	$g[(a_s, a_t), 1]$	$g[(a_s, a_t), 2]$	...	$g[(a_s, a_t), m]$	$g[(a_i, a_j), D] = N$

In the decision table, with respect to each conditional attribute, the pair-wise evaluation of each ranked parameter triplet provides decision rules. In our approach, we suggest the approximation of the global preference relation P by the Multiattribute Stochastic Dominance for reduced number of attribute MSD_R. This dominance can be defined as follows:

Definition 1 [15; 14]

a_i MSD_R a_j if and only if f_{ik} SD_k f_{jk} for all $X_k \in R \subseteq X$ (1) (1)

The MSD_R is the particular case of the MSD dominance defined for given a_i, a_j $\in$ A by [6] as follows:

Definition 2 [6]

a_i MSD a_j if and only if f_{ik} SD_k f_{jk} for all $X_k \in X$ (2) (2)

In the Rough set theory, the approximation of the global preference relation P by MSD_R can be done by lower and upper approximations. According to Greco et al. [5], the lower approximation can be defined as follows:

$$Q_*(P) = \bigcup_{R \subseteq X} \{(MSD_R \cap H) \subseteq P\} \tag{3}$$

The application of the lower approximation allows us to induce the following kind of decision rules:

Rule: If a_j MSD_{R_3} a_j then a_j P a_j

The upper approximation (4) may contain the Multiattribute Stochastic Dominances for reduced number of attributes which leads to the conclusion for preference or non preference. These dominances usually introduce uncertainty in the induction of the decision rules and are referred to as the boundary region (5) which added to lower approximation give us an upper approximation of the preferences. According to Greco et al. [5], the upper approximation can be defined as follows:

$$Q^*(P) = \bigcap_{R \subseteq X} \{(MSD_R \cap H) \supseteq P\} \tag{4}$$

$$BN_Q(P) = Q^*(P) - Q_*(P) \tag{5}$$

The decision rules from upper approximation of the preference P are formulated as follow:

Rule: If a_i MSD_{R_2} a_j then a_i P a_j or a_i N a_j

These uncertain rules obtained from upper approximation must be discarded to eliminate inconsistencies. The decision rules obtained from the lower approximation of the global preference P are kept and they are used to rank all parameter triplets belonging to the efficient set.

All parameter triplets in the efficient set are compared two by two to determine if they satisfy a decision rule. If the comparison of two parameter triplets with the procedure used to define the set of rules leads to a decision rule, the score associated with the first parameter triplet is incremented by one and the score for the second parameter triplet is decremented by one. Following the comparisons of all parameter triplets in the efficient set, all parameter triplets are ranked in decreasing order of score.

3. APPLICATION

We consider a company which produces sport equipment. The firm manufacturers 12 different products which are processed simultaneously in the shop. The production process of each product includes a number of operations performed on different machines (see Table 2). The number and type of operations are different for each product. Parts may return to the same machining centre in the process. 24 devices are installed in the work centre: 6 machines of type M_1, 6 machines of M_2 type, 4 machines of M_3 type, 4 machines of M_4 type, 2 machines of M_5 type and 2 machines of M_6 type.

The production planning and control are organized according to the "Just-in-Time" rules. Production orders are broken into small Kanban lots treated individually. The firm uses Kanban cards to control the work flow. Each operation has its Kanban. One or more Kanbans may be used for each operation. Before starting his work, an operator has to choose one of the waiting operations. Scheduling rules are often used to determine the order in which operations should be processed on workstations. Thus, the worker is able to decide which job queuing at the station needs to be processed first. In our study, eight decision rules are considered: (1) The first come – first served (FCFS) rule; (2) The shortest processing time (SPT) rule; (3) The same job as previously (SJP) rule; (4) The shortest next queue (SNQ) rule; (5) The minimal total time of the rest operations on the path (MTP) rule, (6) The maximal number of Kanbans awaiting processing at the workstation (MKW) rule; (7) The maximal total number of Kanbans awaiting processing at all workstations on the path (MKP) rule; (8) The priority ratio (PR) rule.

Table 2

he number of operations and units of products

Products	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	W_{11}	W_{12}
Number of operations	21	20	22	20	23	21	23	19	25	21	14	40
Number of units	120	120	120	120	120	120	60	60	60	60	60	60

The FCFS rule gives priority to a job that has been queuing at the station for the longest time. The SPT rule chooses the job with shortest planned processing time. The SJP rule assumes that the job which is the same as previously processed on the station should be chosen. The SNQ rule gives priority to the job for which the queue at the next station is the shortest. The MPT rule chooses the operation for which the total time for the remaining operations that have to be completed is minimal. The MKW rule assumes that the job with the greatest number of Kanbans waiting at the workstation should be performed first. The MKP rule is similar, but it considers all operations that have to be performed to complete the processing. The rule selects the operation, for which the total number of Kanbans for all operations on the path is maximal. Finally PR rule is based on the following ratio (6):

$$z_i = \begin{cases} 0 & \text{for } i \in K \\ l_i - \max\limits_{k \in E_i}\{l_k\} & \text{for } l_i > \max\limits_{k \in E_i}\{l_k\} \text{ for } i \in A \\ L & \text{otherwise} \end{cases} \tag{6}$$

Where:
A: the set of operations waiting for processing at the workstation i,
K: the set of final operations,
l_i: the number of unavailable Kanbans for operation i,
E_i: the set of operations which produce components, that are used together with the component produced by operation i in the next stage of the process.

Four values of lot size are considered: 5, 10, 15, and 20, while the number of Kanbans is assumed to be between 2 and 5 and eight scheduling rules. Thus, 128 triplets of parameters are considered. Three criteria are used for evaluating performance of the alternatives: makespan (measured in seconds); average work-in-progress level (measured by the average number of jobs queuing at stations); number of set-ups (the whole number of set-ups done on all stations).

The solution to the problem is as follows: (a) simulation of the production of selected products for each triplet of parameters; (b) construction of distribution functions for each triplet with respect to each attribute; (c) identification of stochastic dominances between triplets of parameters in relation to each attribute; (d) ranking of parameter triplets according to decision rules.

First, a series of one hundred simulations had been done for each triplet to build distributional evaluations with respect to each criterion. In our case, the set of alternatives includes 128 triplets but a certain number of triplets were rejected because of the constraints involved by the DM such that makespan cannot be longer than 85 hours by week; the average number of jobs waiting for all operations no more than 4600 by week; and the number of set-ups for all machines no more than 4000 by week. It was also assumed that the probability of reaching unsatisfactory attribute value should not exceed 0,05. The result of this verification was that 71 triplets were rejected. Next we started to identify types of stochastic dominance between alternatives with respect to attributes. According to prospect theory [7], we assumed that the decision-maker is risk-prone and so we used FSD, SISD, TISD1 and TISD2 (as defined in section 2) to explain relations between alternatives. Tables 3, 4 and 5 show the relations between selected alternative pairs explained by stochastic dominance with respect to the attribute X_1 (makespan). These dominances can be used to determine the multi-attribute stochastic dominance (MSD). By verification of the multi-attribute dominance rule (see definition 1, in Section 3) on the remaining subset of 57 alternatives, we obtained 44 efficient triplets as shown in Table 6.

Table 3

Stochastic dominance for attribute X_1 (makespan)

X_1	5_3_2	5_3_3	5_4_4	10_3_3	10_3_4	10_4_2	10_4_3	10_6_2	10_6_3
5_3_2	X	FSD	SISD	FSD	FSD	SISD	FSD	SISD	TISD1
5_3_3	X	X	X	FSD	FSD	X	SISD	X	X
5_4_4	X	TISD1	X	FSD	FSD	FSD	FSD	X	X
10_3_3	X	X	X	X	TISD1	X	X	X	X
10_3_4	X	X	X	X	X	X	X	X	X
10_4_2	X	TISD1	X	FSD	FSD	X	FSD	X	X
10_4_3	X	X	X	FSD	FSD	X	X	X	X
10_6_2	X	FSD	SISD	FSD	FSD	SISD	FSD	X	X
10_6_3	X	FSD	SISD	FSD	FSD	FSD	FSD	TSD	X

Table 4

Stochastic dominance for attribute X_2 (number of set-ups)

X_2	5_3_2	5_3_3	5_4_4	10_3_3	10_3_4	10_4_2	10_4_3	10_6_2	10_6_3
5_3_2	X	X	FSD	X	X	X	X	X	X
5_3_3	FSD	X	FSD	X	X	X	X	FSD	FSD
5_4_4	X	X	X	X	X	X	X	X	X
10_3_3	FSD	FSD	FSD	X	X	FSD	FSD	FSD	FSD
10_3_4	FSD	FSD	FSD	FSD	X	FSD	FSD	FSD	FSD
10_4_2	FSD	TISD1	FSD	X	X	X	X	FSD	FSD
10_4_3	FSD	FSD	FSD	X	X	FSD	X	FSD	FSD
10_6_2	FSD	X	FSD	X	X	X	X	X	X
10_6_3	FSD	X	FSD	X	X	X	X	TISD1	X

Table 5

Stochastic dominance for attribute X_3 (average stock)

X3	5_3_2	5_3_3	5_4_4	10_3_3	10_3_4	10_4_2	10_4_3	10_6_2	10_6_3
5_3_2	X	FSD	FSD	FSD	FSD	FSD	FSD	FSD	FSD
5_3_3	X	X	FSD	FSD	FSD	FSD	FSD	X	FSD
5_4_4	X	X	X	FSD	FSD	FSD	FSD	X	X
10_3_3	X	X	X	X	FSD	X	FSD	X	X
10_3_4	X	X	X	X	X	X	X	X	X
10_4_2	X	X	X	FSD	FSD	X	FSD	X	X
10_4_3	X	X	X	X	SSD	X	X	X	X
10_6_2	X	FSD	FSD	FSD	FSD	FSD	FSD	X	FSD
10_6_3	X	X	FSD	FSD	FSD	FSD	FSD	X	X

Table 6

Alternatives analyzed in the last step of the procedure

Alternative	Lot-size	Scheduling rule	No. of Kanbans	Alternative	Lot-size	Scheduling rule	No. of Kanbans
1	5	SJP	2	23	15	MKW	4
2	5	SJP	3	24	15	MKW	5
3	5	SNQ	4	25	15	MKP	2
4	10	SJP	3	26	15	MKP	3
5	10	SJP	4	27	15	MKP	4
6	10	SNQ	2	28	15	MKP	5
7	10	SNQ	3	29	15	PR	4
8	10	MKW	2	30	15	PR	5
9	10	MKW	3	31	20	SPT	2
10	10	MKW	4	32	20	SPT	5
11	10	MKW	5	33	20	SNQ	2
12	10	MKP	2	34	20	MKW	2
13	10	MKP	3	35	20	MKW	3
14	10	MKP	4	36	20	MKW	4
15	10	MKP	5	37	20	MKW	5
16	10	PR	5	38	20	MKP	2
17	15	SPT	2	39	20	MKP	3
18	15	SJP	2	40	20	MKP	4
19	15	SJP	3	41	20	MKP	5
20	15	SNQ	2	42	20	PR	3
21	15	MKW	2	43	20	PR	4
22	15	MKW	3	44	20	PR	5

Next, we attempt to build a decision table (Table 7) for pairwise comparison between 5 triplets chosen to make an exercise with industrial operator. The preferences in the decision table were supposed finally to be the same as those analyzed by ELECTRE method in Nowak et al. [11] case. The evaluations with respect to the decisional attribute partition the set of pairs of triplets H into: those which express preferences and those which express non preferences. With respect to the conditional attributes, for each pair of triplets, we can identify the Multiattribute Dominances for reduced number of attributes (MSD_R).

This table shows for the first three pairs of triplets, two attribute dominances with respect to X_2 and X_3.

Table 7

Decision table

0	1	1	P
0	1	1	P
0	1	1	P
1	0	1	P
0	1	0	P
0	1	0	P
1	0	1	P
1	0	0	P
1	0	1	P
1	0	1	P
0	0	0	N
1	0	0	N
1	0	0	N
0	1	0	N
1	0	0	N
1	0	0	N
0	1	0	N
0	1	1	N
0	1	0	N
0	1	0	N

In our approach, we suggest the approximation of the global preference relation P by the Multiattribute Stochastic Dominance to reduce the number of attribute MSD_R. The application of the lower approximation (3) for twenty examples (see Table 5) of the pairwise comparison of the triplets using the software package 4eMKA2 allows us to induce the first decision rule which is based on the two attribute dominances with respect to the attributes X_1 (makespan) and X_3 (number of set-ups).

Rule 1: If a_i $MSD_{x1,x3}$ a_j then a_i P a_j

By application of the upper approximation (4) of preferences we can identify the boundary region, which contains 15 pairs of triplets out of 20 in the decision table (Table 5). Larger boundary region implies the weaker quality of approximation. This is why the quality of approximation of preference is equal to only 0.21.

$$\gamma_Q(P) = \frac{|Q_*(P)|}{|Q^*(P)|} = \frac{4}{19} = 0.21 \tag{7}$$

Rules 2 and 3 are induced from upper approximation of preferences.

Rule 2: If a_i MSD_{X_2} a_j then a_i P a_j or a_i N a_j

Rule 3: If a_i MSD_{X_1} a_j then a_i P a_j or a_i N a_j

Finally, we keep the first certain decision rule to model the overall binary preference relation. The last step of the suggested methodology is to apply this decision rule to order the entire set of forty-four triplets. The extraction of the list of pairs of triplets supporting the decision rule of 44 triplets is presented in Table 6. For each triplet a_i we have:
$SC_{++}(a_i) = card(\{ a_j \in A$: there is at least one D_{++} decision rule stating that $a_i P a_j \})$,
$SC_{+-}(a_i) = card(\{ a_j \in A$: there is at least one D_{++} decision rule stating that $a_j P a_i \})$,

If we identify the pairs of triplets with the decision rule which it corresponds to, we have one of two following situations for each triplet. The triplet a_i dominates the others or is dominated by them.

To each triplet a_i, we assign a score $NFS(a_i)$ called the *net flow score* [5] where:

$$NFS(a_i) = S_{++}(a_i) - S_{+-}(a_i).$$

In the ranking problem, the final recommendation is the total pre-order established by SNF (a_i) on the set of triplets shown in Table 9. By comparing two rankings obtained by the ELECTRE method and the Rough Set method, we find them very similar because the preferences were supposed to be the same in the decision table. Usually, the preferences are given by the DM while ranking small number of alternatives from the efficient set (Table 10).

Table 8

Pairs of alternatives supporting decision Rule 1

(5_3_2 ; 5_3_3)	(5_3_2 ; 5_4_4)	(5_3_2 ; 10_3_3)	(5_3_2 ; 10_3_4)	(5_3_2 ; 10_4_2)	(5_3_2 ; 10_4_3)
(5_3_2 ; 15_6_4)	(5_3_2 ; 15_6_5)	(5_3_2 ; 15_7_2)	(5_3_2 ; 15_7_3)	(5_3_2 ; 15_7_4)	(5_3_2 ; 15_7_5)
(5_3_2 ; 20_7_5)	(5_3_3 ; 10_3_3)	(5_3_3 ; 10_3_4)	(5_3_3 ; 10_4_3)	(5_3_3 ; 15_2_2)	(5_3_3 ; 15_3_2)
(5_3_3 ; 20_6_3)	(5_3_3 ; 20_6_4)	(5_3_3 ; 20_6_5)	(5_4_4 ; 10_3_3)	(5_4_4 ; 10_3_4)	(5_4_4 ; 10_4_2)
(5_4_4 ; 20_6_4)	(5_4_4 ; 20_6_5)	(10_3_3 ; 10_3_4)	(10_3_3 ; 15_3_2)	(10_3_3 ; 15_3_3)	(10_3_3 ; 20_2_5)
(10_4_2 ; 20_4_4)	(10_4_2 ; 20_6_2)	(10_4_2 ; 20_6_3)	(10_4_2 ; 20_6_4)	(10_4_2 ; 20_6_5)	(10_4_3 ; 10_3_4)
(10_6_2 ; 15_2_2)	(10_6_2 ; 15_3_2)	(10_6_2 ; 15_3_3)	(10_6_2 ; 15_4_2)	(10_6_2 ; 15_6_2)	(10_6_2 ; 15_6_3)
(10_6_2 ; 20_7_4)	(10_6_2 ; 20_7_5)	(10_6_3 ; 5_4_4)	(10_6_3 ; 10_3_3)	(10_6_3 ; 10_3_4)	(10_6_3 ; 10_4_2)
(10_6_3 ; 20_2_5)	(10_6_3 ; 20_4_2)	(10_6_3 ; 20_6_2)	(10_6_3 ; 20_6_3)	(10_6_3 ; 20_6_4)	(10_6_3 ; 20_6_5)
(10_6_4 ; 15_6_2)	(10_6_4 ; 15_6_3)	(10_6_4 ; 15_6_4)	(10_6_4 ; 15_6_5)	(10_6_4 ; 20_2_2)	(10_6_4 ; 20_2_5)
(10_6_5 ; 15_2_2)	(10_6_5 ; 15_3_2)	(10_6_5 ; 15_3_3)	(10_6_5 ; 15_4_2)	(10_6_5 ; 15_6_2)	(10_6_5 ; 15_6_3)
(10_7_2 ; 10_3_3)	(10_7_2 ; 10_3_4)	(10_7_2 ; 10_4_3)	(10_7_2 ; 10_7_5)	(10_7_2 ; 15_2_2)	(10_7_2 ; 15_3_2)
(10_7_2 ; 20_2_2)	(10_7_2 ; 20_2_5)	(10_7_2 ; 20_4_2)	(10_7_2 ; 20_6_2)	(10_7_2 ; 20_6_3)	(10_7_2 ; 20_6_4)
(10_7_3 ; 15_2_2)	(10_7_3 ; 15_3_2)	(10_7_3 ; 15_3_3)	(10_7_3 ; 15_4_2)	(10_7_3 ; 15_6_2)	(10_7_3 ; 15_6_3)
(10_7_3 ; 20_6_3)	(10_7_3 ; 20_6_4)	(10_7_3 ; 20_6_5)	(10_7_3 ; 20_7_2)	(10_7_3 ; 20_7_3)	(10_7_3 ; 20_7_4)

Table 9

Ranking of triplets according to the Rough Set approach

Triplet	S_{++}	S_{+-}	NFS	Rang	Triplet	S_{++}	S_{+-}	NFS	Rang
5_3_2	33	0	33	1	20_8_3	0	0	0	23
10_7_2	28	0	28	2	15_8_5	0	0	0	24
10_7_4	28	0	28	3	15_8_4	0	0	0	25
10_7_3	28	0	28	4	10_8_5	0	0	0	26
10_7_5	27	3	24	5	15_6_2	16	18	-2	27
10_6_2	24	1	23	6	15_6_3	15	19	-4	28
15_7_3	25	5	20	7	15_6_4	13	20	-7	29
10_6_4	21	1	20	8	15_6_5	12	21	-9	30
15_7_2	24	5	19	9	15_2_2	11	22	-11	31
10_6_3	20	1	19	10	10_4_3	5	16	-11	32
10_6_5	20	2	18	11	20_6_2	9	24	-15	33
15_7_5	23	6	17	12	20_6_4	7	26	-19	34
15_7_4	23	7	16	13	20_2_2	4	25	-21	35
5_4_4	15	5	10	14	20_6_5	6	27	-21	36
20_7_2	18	9	9	15	20_6_3	5	26	-21	37
5_3_3	18	10	8	16	15_4_2	3	25	-22	38
20_7_5	17	10	7	17	15_3_2	3	28	-25	39
10_4_2	14	7	7	18	10_3_3	4	30	-26	40
20_7_4	17	11	6	19	20_4_2	1	30	-29	41
20_7_3	17	11	6	20	10_3_4	1	33	-32	42
20_8_4	0	0	0	21	15_3_3	0	34	-34	43
20_8_5	0	0	0	22	20_2_5	0	35	-35	44

Table 10

Results obtained from Rough Set and Electre methods

Rough Set approach				Electre method			
Rank	Triplet	Rank	Triplet	Rank	Triplet	Rank	Triplet
1	5_3_2	23	20_8_3	1	10_7_2	23	5_3_3
2	10_7_2	24	15_8_5	2	10_7_3	24	10_4_2
3	10_7_4	25	15_8_4	3	5_3_2	25	10_8_5
4	10_7_3	26	10_8_5	4	10_7_4	26	15_6_2
5	10_7_5	27	15_6_2	5	15_7_2	27	15_6_3
6	10_6_2	28	15_6_3	6	15_7_3	28	15_2_2
7	15_7_3	29	15_6_4	7	10_7_5	29	15_6_4
8	10_6_4	30	15_6_5	8	15_7_5	30	15_6_5
9	15_7_2	31	15_2_2	9	15_7_4	31	20_6_2
10	10_6_3	32	10_4_3	10	20_8_4	32	5_4_4
11	10_6_5	33	20_6_2	11	20_8_5	33	20_2_2
12	15_7_5	34	20_6_4	12	10_6_2	34	20_6_4
13	15_7_4	35	20_2_2	13	20_8_3	35	20_6_5
14	5_4_4	36	20_6_5	14	10_6_4	36	20_6_3
15	20_7_2	37	20_6_3	15	20_7_2	37	10_3_3
16	5_3_3	38	15_4_2	16	10_6_3	38	10_4_3
17	20_7_5	39	15_3_2	17	10_6_5	39	15_3_2
18	10_4_2	40	10_3_3	18	20_7_3	40	15_4_2
19	20_7_4	41	20_4_2	19	15_8_5	41	15_3_3
20	20_7_3	42	10_3_4	20	20_7_4	42	10_3_4
21	20_8_4	43	15_3_3	21	20_7_5	43	20_4_2
22	20_8_5	44	20_2_5	22	15_8_4	44	20_2_5

CONCLUSION

The given procedure constitutes a dynamic decision aid in the production process control. By changing parameters and attributes, we can adapt it to other production environments.

We have used a Rough Set approach to choose the best triplet (Kanban lot size, the decision rule and the number of Kanbans). The set of decision rules induced by application of Rough Set techniques represents the preference model of the DM and can be used to order very large efficient set of triplets.

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