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APPLICATION OF DEA METHOD TO THE EVALUATION OF THE EFFICIENCY OF POLISH OPEN PENSION FUNDS IN THE YEARS 2004-2006

Abstract

The subject of this analysis is Open Pension Funds (OPF) in the period from 2004 to 2006. The purpose of this analysis is the measurement of technical efficiency of OPF. What was applied in this evaluation was the Data Envelopment Analysis (DEA). Data related to the volume of inputs (number of members, operating costs per capita) and the size of outputs (net assets, result of investments, accounting unit's values) allowed to construct basic models of DEA (output oriented); the CCR model (constant returns to scale), BCC model (variable returns to scale) and NIRS model (non-increasing returns to scale). In order to evaluate changes in efficiency of each OPF (in the years 2004-2006) the distances of Shephard being the basis for the Malmquist indexes were calculated.

Keywords

Open Pension Funds, efficiency evaluation, data envelopment analysis (DEA), Malmquist indexes.

INTRODUCTION

This paper compares presently existing Open Pension Funds (OPF) with relation to their efficiency. What is applied to evaluate this efficiency is DEA (Data Envelopment Analysis), the method whose objects of analysis are defined as Decision Making Units (in short referred to as DMU). In this work the role of DMU is performed by particular OPFs. The inputs of OPF are determined by the number of members and operating costs per capita of a given OPF, while its returns are determined by its net assets, result of investments and accounting unit's values.

Presently about 15 OPFs operate in the market. They are as follows (next to their names the names used in this paper are presented):

AIG OPF (AIG), Allianz Polska OPF (Allianz), Bankowy OPF (Bankowy), Commercial Union OPF BPH CU WBK (Commercial), OPF „Dom" (DOM), OPF Ergo-Hestia (Ergo), Generali OPF; former name Zurich OPF (Generali), ING Nationale Nederlanden Polska OPF (NNeder), Nordea OPF; former name Sampo (Nordea), Pekao OPF (PeKaO), OPF Pocztylion (Pocztylion), OPF Polstat (Polsat), OPF PZU „Złota Jesień" (PZU), OPF Skarbiec-Emerytura (Skarbiec) and Winterthur OPF; former Credit Suisse (Winterthur).

Tables 1 and 2 presents some selected data concerning the funds mentioned above (number of members and operating costs per capita of a given OPF, net assets, result of investments and accounting unit's values) in the years 2004-06 (the first quarters).

The presented data can lead to the conclusion that the market is definitely dominated by four funds, i.e. AIG, Commercial, Nationale Nederlanden and PZU Złota Jesień. These funds comprise 71% of the market measured by the share of net assets (65% measured by the share of members).

The specific character of the institutions represented by OPF imposes special care for proper evaluation of these funds as the objective of OPF is gathering funds and then investing them with the purpose of payment to members of funds when they achieve their pension age. Therefore finding effective methods of monitoring and evaluation related to activity of a particular OPF seems to be of great importance. In her hitherto existing works, where she used multi-criteria methods and forecasts of the rankings, the author mostly focuses on the rankings of OPFs [6; 7; 8]. This paper though concentrates on the evaluation of OPF efficiency. The method applied here (i.e. DEA) was used for the first time in 1978 by Charnes, Cooper and Rhodes [2]. In Polish literature this method is mostly known from the works related to banks' efficiency evaluation [4; 10].

When using the DEA models for the evaluation of OPF efficiency, treated here as Decision Making Units, definitions of all the factors influencing OPF efficiency should be an important stage of this analysis which later are to be transferred onto defined outlays and returns (effects).

Table 1

Selected features of OPF (inputs) in the years 2004-06 (fixed prices 2004)

No.	*OPF*	Number of participants [thousand persons]			Operating costs per capita [PLN (2004) / person]		
		2004	2005	2006	2004	2005	2006
1.	AIG	969,01	960,33	998,50	6,59	9,01	8,53
2.	Allianz	250,89	272,77	295,32	8,08	9,54	11,30
3.	Bankowy	401,77	406,38	435,73	5,82	7,74	9,86
4.	Commercial	2579,83	2557,20	2572,65	8,36	9,83	12,35
5.	DOM	239,38	227,22	257,15	6,59	6,99	8,06
6.	Ergo	402,78	351,56	375,33	3,97	6,15	8,16
7.	Generali	387,25	391,78	434,95	6,48	8,99	10,29
8.	NNeder	2046,00	2122,51	2277,09	8,08	10,67	12,28
9.	Nordea	539,87	587,43	647,13	4,73	5,84	7,51
10.	PeKaO	292,12	237,18	242,57	4,42	6,86	8,73
11.	Pocztylion	457,38	362,20	356,84	3,77	6,46	8,31
12.	Polsat	261,27	225,58	249,78	3,19	3,82	4,63
13.	PZU	1902,44	1773,89	1846,40	5,55	7,61	9,47
14.	Skarbiec	601,27	491,57	453,55	4,44	6,30	8,14
15.	Winterthur	380,60	407,67	484,60	5,19	7,76	10,40

Source: Quarterly Bulletin KNUiFE, www.knuife.gov.pl

Table 2

Selected features of OPF (outputs) in the years 2004-06 (fixed prices 2004)

No.	*OPF*	Net assets [mln PLN (2004)]			Result of investments [mln PLN (2004)]			Accounting unit's values [PLN (2004)]		
		2004	2005	2006	2004	2005	2006	2004	2005	2006
1.	AIG	4282,39	5539,83	7400,97	32,14	36,12	45,04	17,16	18,63	21,09
2.	Allianz	1351,89	1725,17	2224,12	12,25	15,09	12,53	17,73	18,71	20,58
3.	Bankowy	1551,96	2059,37	2778,36	9,20	12,30	15,92	18,51	19,74	21,76
4.	Commercial	14071,95	17988,79	23724,58	130,13	140,62	146,72	18,39	19,88	22,35
5.	DOM	829,36	1041,70	1379,16	3,80	6,10	6,40	18,78	20,42	23,22
6.	Ergo	1028,01	1414,13	2072,57	8,05	10,63	12,92	18,25	19,72	22,05
7.	Generali	1663,26	2193,51	3007,24	13,18	15,32	16,15	18,53	20,09	22,72
8.	NNeder	11195,24	14717,85	20193,51	92,47	126,30	148,85	19,50	21,18	24,02
9.	Nordea	1571,26	2297,20	3207,21	13,71	18,33	15,32	19,11	20,44	22,71
10.	PeKaO	809,82	1048,96	1396,89	6,78	7,66	7,22	16,94	18,54	20,86
11.	Pocztylion	1052,66	1350,26	1796,58	7,10	10,56	9,78	16,99	18,27	20,96
12.	Polsat	484,98	571,25	776,96	3,85	4,92	3,30	19,87	21,06	24,07
13.	PZU	6983,78	8989,54	11992,08	50,69	61,85	64,67	18,45	19,93	22,28
14.	Skarbiec	1752,04	2021,80	2430,55	10,44	12,87	15,18	17,11	18,43	20,85
15.	Winterthur	1327,84	2141,12	3409,91	9,77	14,29	18,47	17,86	19,58	21,85

Source: Quarterly Bulletin KNUiFE, www.knuife.gov.pl

1. METODOLOGY OF THE OPF EFFICIENCY EVALUATION

To evaluate the efficiency of each OPF in a selected moment in time (t) the DEA method was used and to evaluate changes in returns of each OPF between two moments in time (t and t+1) the Malmquist productivity indexes were applied.

1.1. Effectiveness evaluation – DEA method

The DEA method allows to evaluate efficiency solely on the basis of data on values of inputs and outputs. It does not require any knowledge of function form defining the relation between the two categories.

The guidelines of the DEA model are as follows: there are n objects operating in a given branch, each of them makes use of m varied inputs in order to obtain s different outputs(effects). Additionally, it is assumed that the value of inputs and outputs (effects) is either bigger or equals zero; however, there is at least one input and one output bigger than zero.

The efficiency of an economic subject o is defined as the relation between the sum of weighted inputs and the sum of weighted outputs.

$$\theta = h_o(\mathbf{u}, \mathbf{v}) = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \tag{1}$$

where:
y_{ro} – r-th output of the object o,
x_{io} – i-th input of the object o,
u_r – weight defining the importance of the output r-th,
v_i – weight defining the importance of the input i-th,
s – number of outputs for the object o,
m – number of inputs for the object o.

Thus outputs and inputs are reduced to single values of a “synthetic output” (sum of weighted outputs) and a “synthetic input” (sum of weighted inputs) and their relation is the function of purpose that should be maximized. In the numerator of the expression (1) there is a “complete output” of the object o while the denominator includes a “complete input” of this object.

The DEA method does not require the knowledge of the weights u and v as for each object o the weights maximizing its efficiency h_o are searched for. The process of seeking maximizing value h_o would, however, lead to achieving incomplete solution and therefore what should be done is the introduction of additional restrictions (2) and (3) thanks to which it is possible to find the best completed solution. According to the restriction (2) for each object the quotient of the "complete output" and "complete input" is to be smaller or equal to 1. While the restriction (3) is a classic boundary restriction.

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \quad for \quad j = 1,2,...,n \tag{2}$$

$$\begin{aligned} u_r \geq 0 \quad dla \quad r = 1,2,...,s \\ v_i \geq 0 \quad dla \quad i = 1,2,...,m \end{aligned} \tag{3}$$

Thus evaluating the efficiency related to the object *o* means solving a problem of quotient programming with the maximization function of purpose (1) and restrictions (2)-(3).

Model (1)-(3) can be transformed into a linear form by applying the transformation method of Charnes-Cooper and by making use of simple methods of linear optimization [7].

Due to the values of the transferred model (1)-(3) it is convenient to solve this problem that is dual to a particular one.

What is an advantage of a DEA method is the fact that it does not require the knowledge of function relation between expenditures and outputs. Efficiency curve is estimated on the grounds of empirical data on values of inputs and outputs (effects) in the form of segments of linear curve and thus highly recommended everywhere where it is impossible to fix the objective function relation between inputs and outputs (effects) or by finding corresponding weights.

Some economic objects for which the optimal value of the function of purpose (1) is placed in the curve of efficiency are efficient ($1/\theta = 1$), while the ones whose value lies below the curve of efficiency ($1/\theta < 1$) are as a rule inefficient, and their inefficiency amounts to ($1-\theta = 1$).

In this paper there were applied the following, outputs oriented DEA models:[*]: a model with constant returns to scale (CCR[**]), model with variable returns to scale (BCC[***]) and model with non increasing returns to scale (NIRS[****]). Each model should be solved *n* times separately for each economic object.

[*] The models presented here are in compliance with the optimization theory of dual models. In literature DEA are though called primary models. Such a reverse convention is commonly encountered.
[**] CCR; after **C**harnes, **C**ooper and **R**hodes, see [2, p. 3].
[***] BCC; after **B**anker, **C**harnes and **C**ooper, see [2, p. 23-47].
[****] NIRS; **N**on **I**ncreasing **R**etuns to **S**cale model.

The CCR model

$$
\begin{aligned}
&\theta^* = \max \theta \\
&\sum_{j=1}^{n} x_{ij}\lambda_j \le x_{io} \qquad for \quad i = 1,2,\dots,m \\
&\sum_{j=1}^{n} y_{rj}\lambda_j \ge \theta\, y_{ro} \qquad for \quad r = 1,2,\dots,s \\
&\lambda_j \ge 0 \qquad for \quad j = 1,2,\dots,n
\end{aligned}
\tag{4}
$$

The solution of the problem (4) consists in finding a maximum value θ which allows to maximize outputs in such a way so as not to exceed the inputs[*]. The efficiency ($1/\theta^*$) calculated on the grounds of the CCR model is called a *technical efficiency (e_crs)*. If $1/\theta^* = 1$ object *o* is efficient while if $1/\theta^* < 1$ object *o* is non-efficient.

The BCC model

$$
\begin{aligned}
&\theta^* = \max \theta \\
&\sum_{j=1}^{n} x_{ij}\lambda_j \le x_{io} \qquad for \quad i = 1,2,\dots,m \\
&\sum_{j=1}^{n} y_{rj}\lambda_j \ge \theta\, y_{ro} \qquad for \quad r = 1,2,\dots,s \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0 \qquad for \quad j = 1,2,\dots,n
\end{aligned}
\tag{5}
$$

The measure of efficiency ($1/\theta^*$) calculated on the grounds of the BCC model is marked as *e_vrs*. It is so-called *pure technical efficiency* (*e_vrs*) which defines how many more outputs (effects) could be achieved with the same volume of inputs.

The NIRS model

$$
\begin{aligned}
&\theta^* = \max \theta \\
&\sum_{j=1}^{n} x_{ij}\lambda_j \le x_{io} \qquad for \quad i = 1,2,\dots,m \\
&\sum_{j=1}^{n} y_{rj}\lambda_j \ge \theta\, y_{ro} \qquad for \quad r = 1,2,\dots,s \\
&\sum_{j=1}^{n} \lambda_j \le 1 \\
&\lambda_j \ge 0 \qquad for \quad j = 1,2,\dots,n
\end{aligned}
\tag{6}
$$

[*] Contrary to the outlays oriented model whose purpose is to minimize inputs while maintaining constant level of outputs.

The NIRS model differs from the BCC model by a „less strict” condition concerning the factors of linear combination (λ_j).The measure of efficiency ($1/\theta^*$) calculated on the grounds of the NIRS model is denoted by *e_nirs*. It includes some information on the types of returns to scale, i.e. it answers the question whether an economic object (*o*) functions within increasing or decreasing returns of scale.

When solving the CCR model what is received is the information on complete technical efficiency of a given economic object, while the BCC model provides some information on pure technical efficiency, i.e. the one which considers variable returns to scale. If there is a considerable difference between the calculated values of efficiency in case of constant and variable returns of scale, then by comparing the two measures one can assume the existence of the returns of scale in a given group of objects. The measure of return to scale efficiency is defined in (7).

$$e\_s\_vrs = \frac{e\_crs}{e\_vrs} \quad (7)$$

The efficiency of scale (*e_s_vrs*),connected with scale (volume) of production informs, how many fewer inputs could be used if the volume of outputs were optimal. The efficiency of scale calculated in this way tells us nothing though about types of returns to scale, i.e. whether an object functions within increasing or decreasing returns to scale. Only when there is no statistically considerable difference between complete technical efficiency and pure technical efficiency it can be supposed that a given decision making unit is efficient as far as scale of engaged productivity factors are concerned. However, if the compared volumes are different we receive no answer regarding the productivity scale a given decision making unit operates in. In other words if $0 < e\_crs < e\_vrs < 1$ the obtained measure is smaller than 1 and a decision making unit is inefficient in respect to scale of the engaged productivity factors, however, the region a given decision making unit operates in is unknown. To define this aspect another measure of the scale efficiency is applied (8):

$$e\_s\_nirs = \frac{e\_crs}{e\_nirs} \quad (8)$$

Comparing the efficiency measure obtained in the model CCR with the efficiency measure obtained in the NIRS model allows to define the types of returns to scale (*e_s_nirs*). And thus if $e\_s\_nirs = 1$ the decision making unit operates within the region of increasing returns to scale, but if $e\_s\_nirs < 1$ the decision making unit operates in the region of decreasing returns to scale. In other words if $e\_crs = e\_nirs$ the object is in the region of increasing returns to scale, but if $e\_crs < e\_nirs$, the object is in the region of decreasing returns to scale.

1.2. Evaluation of efficiency changes in time – the Malmquist index

To evaluate the efficiency changes in time what is used is the index grounded to a large extent on Farrell's efficiency that was used for the first time by Malmquist [5]. He suggests that the levels of efficiency $F^t(x^t, y^t)$ and $F^{t+1}(x^{t+1} y^{t+1})$ should be compared in two different moments in time t and t+1. In this paper the function of distance by Shephard [3] $D^{\bullet}(x^{\bullet}, y^{\bullet})$ was used in place of the levels of efficiency $F^{\bullet}(x^{\bullet} y^{\bullet})$. The Malmquist index for the year t assumes the form of (9), and for the year t+1 it is (10). Index (9) compares efficiency of the period t+1 to the efficiency of the period t by using as a point of reference the technology of the t period. While index (10) compares efficiency of the period t+1to the efficiency of the period t by using as a point of reference the technology of the t+1 period.

$$M^t\left(x^t, y^t, x^{t+1}, y^{t+1}\right)=\frac{D^t\left(y^{t+1}, x^{t+1}\right)}{D^t\left(y^t, x^t\right)} \tag{9}$$

$$M^{t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right)=\frac{D^{t+1}\left(y^{t+1}, x^{t+1}\right)}{D^{t+1}\left(y^t, x^t\right)} \tag{10}$$

Measures $D^{\bullet}(x^{\bullet}, y^{\bullet})$ are technical effectivenesses $1/\theta^*$ obtained from solving the CCR model (4).The parameters of the left-hand sides of first two limits (*LHS*) and parameters of right-hand side limits (*RHS*) are changed in compliance with the rules presented below in the table 3.

Table 3

Rules for constructing the CCR model (4) in defining volumes $D^{\bullet}(x^{\bullet}, y^{\bullet})$

	$D^{\bullet}(x^{\bullet}, y^{\bullet})$	*LHS* (technology)	*RHS* (evaluated object)
1.	$D^t(x^t, y^t)$	From period t	From period t
2.	$D^{t+1}(x^{t+1} y^{t+1})$	From period t+1	From period t+1
3.	$D^t(x^{t+1} y^{t+1})$	From period t	From period t+1
4.	$D^{t+1}(x^t, y^t)$	From period t+1	From period t

In practice the formula (11) of the Malmquist index is applied which is the geometrical* of both indexes (9) and (10).

$$M^{t,t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right)=\sqrt{\frac{D^t\left(y^{t+1}, x^{t+1}\right)}{D^t\left(y^t, x^t\right)}\cdot\frac{D^{t+1}\left(y^{t+1}, x^{t+1}\right)}{D^{t+1}\left(y^t, x^t\right)}} \tag{11}$$

* Suggested by R. Färe, S. Grasskopf, B. Lindgren, P. Ross in [2, chapter 13].

After the transformations the Malmquist index can be presented in the form of a ratio (12) dividing the Malmquist index (11) into two terms (13) and (14).

$$M^{t,t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = TE\left(x^t, y^t, x^{t+1}, y^{t+1}\right) \times TC\left(x^t, y^t, x^{t+1}, y^{t+1}\right) \quad (12)$$

The first term (13) stands for the *change in technical efficiency* which defines a relative change in efficiency of a given object between two periods *t* and *t* + 1 but without including any changes in the curve of efficiency (as efficiency is measured in respect to a curve from a proper time period either *t* or *t* + 1).

The other term (14) stands for the *technical change* (connected with technological progress), which defines relative change in technology (presented in the change in curve of efficiency), measured separately in relation to the technologies from two different periods of time, i.e. the efficiency of a given object in the period *t* is measured with respect to the technology of the period *t* + 1 and the efficiency of an object in the period *t* + 1 is measured with respect to the technology of the period *t.*

$$TE\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{D^{t+1}\left(y^{t+1}, x^{t+1}\right)}{D^t\left(y^t, x^t\right)} \quad (13)$$

$$TC\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \sqrt{\frac{D^t\left(y^{t+1}, x^{t+1}\right)}{D^{t+1}\left(y^{t+1}, x^{t+1}\right)} \cdot \frac{D^t\left(y^t, x^t\right)}{D^{t+1}\left(y^t, x^t\right)}} \quad (14)$$

The coefficient of the change in efficiency of an object is the result of the Malmquist index calculation. It is assumed that for the value of the index bigger than 1 in the period of time in question, a relative increase in efficiency took place, while the value smaller than 1 means decrease in efficiency, finally the value that equals 1 means maintaining the same level of efficiency.

1.3. Evaluation of efficiency in time - other indexes of dynamics

In the DEA analysis, apart from the Malmquist indexes, there are also used indexes of dynamics of efficiency based on the models taking into account variable returns to scale (the BCC models). To differentiate the measures of Shephard obtained by the solution of the CCR models (constant returns to scale) and BCC models (variable returns to scale) we define the distances in the following way:

- $D^{\bullet}_{CRS}\left(x^{\bullet}, y^{\bullet}\right)$ for the CCR models (4) and
- $D^{\bullet}_{VRS}\left(x^{\bullet}, y^{\bullet}\right)$ for the BCC models (5). We define and calculate these measures in the way described in 2.2. We apply here the BCC models (instead of CCR).

Here are the signaled indexes:
- *PTE* – the index of *change of clear technical efficiency* and
- *SE* – the index of *change of scale of efficiency*.

$$PTE\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{D_{VRS}^{t+1}\left(y^{t+1}, x^{t+1}\right)}{D_{VRS}^{t}\left(y^{t}, x^{t}\right)} \quad (15)$$

$$SE\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{D_{CRS}^{t+1}\left(y^{t+1}, x^{t+1}\right)}{D_{CRS}^{t}\left(y^{t}, x^{t}\right)} \Bigg/ \frac{D_{VRS}^{t+1}\left(y^{t+1}, x^{t+1}\right)}{D_{VRS}^{t}\left(y^{t}, x^{t}\right)} \quad (16)$$

3. OPF EFFICIENCY EVALUATION (2004-06)

The results of DEA calculations for the years 2004-2006 are presented in the tables 4-6. With 15 OPFs it was required the solution* 3×3×15+2x2×15=195 of linear optimization models (each year – by 15 models: CCR, BCC and NIRS and additionally by 2x2x15 of modified CCR while calculating Shephard measures).

The efficiency based on the CCR model is called a technical efficiency (*e_crs*) or in other words complete efficiency. The value of the coefficient $1/\theta^*$ (*e_crs*) is in between $\langle 0,1 \rangle$. While the value $1/\theta^* = 1$ says that a given OPF is efficient and that means complete transformation of outlays into returns. Whereas in case of all OPFs where $1/\theta^* < 1$ (efficiency index is <1) it means that they function inefficiently against the others. In other words, with no increased outlays they should improve their returns by $(1 - 1/\theta^*)\times 100\%$.

What can be concluded from the calculations included in the table 4 is the fact that in 2004 the Allianz, Commercial, DOM, Nationale Nederlanden, Polsat and the PZU funds represented the optimal (model) efficiency. In the year 2004 the Generali could have increased its returns by 4,3% and the Bankowy by 7,3%. As far as the aspect of efficiency is concerned other funds functioned much worse. The increase of their efficiency in relation to the outlays incurred should have amounted from 9,7% (in case of Wintherthur) to 25,1% (in case of Skarbiec). In the years 2005-06 the Allianz, Commercial, DOM, Nationale Nederlanden and Polsat represented also optimal efficiency. The remaining funds were marked with greater inefficiency. The worst results were achieved by Nordea whose increase in returns should have amounted to almost 25,5% in the first quarter of 2006 (in 2005 by 23,3%).). PZU (the third one as far as the value of its assets is concerned) should have increased its returns by 12,9% (in 2005)

* To solve the CCR models, BCC and NIRS Solver tools of Excel were used.

and by 13,1% (in 2006) at the same assets. At this point it should be also mentioned that in the years 2003-2006 Polsat achieved the highest rate of return (60%) right before the DOM Fund (59,4%).

As a result of the BCC model solution there was achieved a so-called pure technical efficiency (*e_vrs*). The relation of two efficiencies (*e_s_vrs*), i.e. complete one to pure one allows to evaluate whether a given unit is within the scope of the returns to scale. The index *e_s_vrs*=1 says about constant benefits of scale. When the relation between the complete technical efficiency and pure technical efficiency is<1 the funds are within the variable efficiency of scale. As far as the researched OPFs are concerned constant benefits of scale were noted in case of Allianz, Commercial, DOM, Nationale Nederlanden, PeKaO, Polsat and PZU in 2004-06.

To decide whether in case of variable returns to scale the type is either increasing or decreasing the NIRS model may be applied. For the Allianz, Commercial, DOM, Nationale Nederlanden, Polsat and the PZU in the years 2004-06 (PZU only in 2004) funds the index of efficiency type *e_s_nirs* calculated as the relation of complete efficiency to the efficiency calculated by the use of NIRS model equals one. It means that the funds mentioned above operated in the period in question in the area of growing returns to scale. The remaining funds operated in the area of decreasing returns to scale in the years 2004-06 (PZU in 2005-06).

To compare the changes in efficiency of OPF in time the Malmquist productivity index was applied. The comparison was carried out for some successive years of the period 2004-06. The determination of Malmquist index per each OPF required prior calculation of so-called Shaphard measurements (compare table 5).

When comparing so-called Malmquist indexes (the relation of returns to outlays) in different periods of time ($M_{t,t+1}$) we can see the increase in efficiency most of all pension funds (compare table 6). Considering the modified Malmquist index being a geometrical average of the index for the year *t* and *t*+1, we can see that the increase in efficiency most of all results from the changes in the so-called technological efficiency ($TC_{t,t+1}$) that takes into consideration the change of the curve of efficiency location. The technical efficiency, measuring the change in relative efficiency in the periods 2004-05 and 2005-06 ($TE_{t,t+1}$) is constans or decreased in the majority of funds.

Table 4

Measures of OPF efficiency in the years 2004-06 obtained by means of DEA

l.p.	*OPF*	*e_crs*			*e_vrs*			*e_nirs*			*e_s_vrs*			*e_s_nirs*		
		2004	2005	2006	2004	2005	2006	2004	2005	2006	2004	2005	2006	2004	2005	2006
1.	AIG	0,865	0,865	0,889	0,883	0,892	0,898	0,883	0,892	0,898	0,980	0,970	0,990	0,980	0,970	0,990
2.	Allianz	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**
3.	Bankowy	0,928	0,897	0,890	0,964	0,952	0,930	0,964	0,952	0,930	0,963	0,942	0,957	0,963	0,942	0,957
4.	Commercial	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**
5.	DOM	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**
6.	Ergo	0,831	0,864	0,917	0,919	0,936	0,935	0,919	0,936	0,935	0,904	0,923	0,981	0,904	0,923	0,981
7.	Generali	0,957	0,935	0,943	0,985	0,981	0,979	0,985	0,981	0,979	0,972	0,953	0,963	0,972	0,953	0,963
8.	NNeder	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**
9.	Nordea	0,801	0,767	0,745	0,963	0,970	0,943	0,963	0,970	0,943	0,832	0,791	0,790	0,832	0,791	0,790
10.	PeKaO	0,890	0,938	**1,000**	0,913	0,973	**1,000**	0,890	0,938	**1,000**	0,975	0,964	**1,000**	**1,000**	**1,000**	**1,000**
11.	Pocztylion	0,791	0,788	0,833	0,856	0,867	0,884	0,856	0,867	0,884	0,924	0,909	0,942	0,924	0,909	0,942
12.	Polsat	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**	**1,000**
13.	PZU	**1,000**	0,871	0,869	**1,000**	0,955	0,927	**1,000**	0,955	0,927	**1,000**	0,912	0,937	**1,000**	0,912	0,937
14.	Skarbiec	0,749	0,779	0,818	0,864	0,874	0,870	0,864	0,874	0,870	0,867	0,891	0,940	0,867	0,891	0,940
15.	Winterthur	0,903	0,914	0,922	0,925	0,950	0,949	0,925	0,950	0,949	0,976	0,962	0,972	0,976	0,962	0,972

To obtain better evaluation of the situation of the pension fund market the efficiency *e_crs* was compared with the yield index. To do so the shown in picture 1 was divided into four parts by means of straight lines corresponding to the average level of efficiency and average level of yield. Thus the created areas are called as follows: “stars”, “sleeping ones”, “question marks” and “poor dogs”. The correlogram in this form is called a x BCG matrix*. The pension funds that are within the area of „stars” belong to well-managed funds (very good financial strategy and level of productivity). “The stars” of the year 2006 are Nationale Nederlanden, Commercial, DOM, Generali, PeKaO, and Allianz. “The sleeping ones” are Pocztylion, PZU, AIG and Bankowy, which despite the high yield could use their potentials better (too little efficiency). “The question marks” (Ergo, Skarbiec, Nordea) have quite a chance to increase their efficiency at the proper strategy of development being applied. “The poor dogs” are in the worst situation as their chances to improve the yield despite great efficiency are slight. In 2006 Winterthur and Polsat faced the same situation.

* The name *BCG matrix* comes from the name of the firm *Boston Consulting Group*, which was the first one to propose such a division of a correlogram at the evaluation of economic subjects. Quote from [1].

Table 5

Changes in OPF efficiency 2004-06 – distance measures according to Shephard

l.p.	OPF	$D_t(x_t,y_t)$		$D_t(x_{t+1},y_{t+1})$		$D_{t+1}(x_{t+1},y_{t+1})$		$D_{t+1}(x_t,y_t)$	
		2005/04	2006/05	2005/04	2006/05	2005/04	2006/05	2005/04	2006/05
1.	AIG	0,865	0,865	1,057	1,093	0,865	0,889	0,714	0,716
2.	Allianz	1,000	1,000	1,170	1,171	1,000	1,000	0,975	1,179
3.	Bankowy	0,928	0,897	1,013	1,012	0,897	0,890	0,802	0,801
4.	Commercial	1,000	1,000	1,287	1,311	1,000	1,000	1,088	1,181
5.	DOM	1,000	1,000	1,172	1,073	1,000	1,000	0,864	1,022
6.	Ergo	0,831	0,864	0,983	0,988	0,864	0,917	0,882	0,870
7.	Generali	0,957	0,935	1,033	1,073	0,935	0,943	0,842	0,904
8.	NNeder	1,000	1,000	1,269	1,263	1,000	1,000	0,935	0,990
9.	Nordea	0,801	0,767	0,911	0,871	0,767	0,745	0,822	0,813
10.	PeKaO	0,890	0,938	1,045	1,072	0,938	1,000	0,771	1,005
11.	Pocztylion	0,791	0,788	0,892	0,921	0,788	0,833	0,873	0,805
12.	Polsat	1,000	1,000	1,227	1,062	1,000	1,000	1,130	1,094
13.	PZU	1,000	0,871	0,947	0,949	0,871	0,869	1,019	0,920
14.	Skarbiec	0,749	0,779	0,909	0,923	0,779	0,818	0,803	0,684
15.	Winterthur	0,903	0,914	1,030	1,063	0,914	0,922	0,777	0,850

Table 6

Changes in OPF efficiency 2004-06 – Malmquist and other indexes

l.p.	OPF	$M_{t,t+1}$		$TE_{t,t+1}$		$TC_{t,t+1}$		$PTE_{t,t+1}$		$SE_{t,t+1}$	
		2005/04	*2006/05*	*2005/04*	*2006/05*	*2005/04*	*2006/05*	*2005/04*	*2006/05*	*2005/04*	*2006/05*
1.	AIG	**1,217**	**1,253**	1,000	1,028	1,217	1,219	1,010	1,007	0,990	1,021
2.	Allianz	**1,095**	**0,997**	1,000	1,000	1,095	0,997	1,000	1,000	1,000	1,000
3.	Bankowy	**1,105**	**1,119**	0,967	0,992	1,143	1,128	0,988	0,977	0,978	1,016
4.	Commercial	**1,088**	**1,054**	1,000	1,000	1,088	1,054	1,000	1,000	1,000	1,000
5.	DOM	**1,165**	**1,025**	1,000	1,000	1,165	1,025	1,000	1,000	1,000	1,000
6.	Ergo	**1,076**	**1,097**	1,040	1,061	1,035	1,034	1,018	0,999	1,021	1,062
7.	Generali	**1,095**	**1,095**	0,977	1,009	1,121	1,085	0,996	0,998	0,981	1,011
8.	NNeder	**1,165**	**1,129**	1,000	1,000	1,165	1,129	1,000	1,000	1,000	1,000
9.	Nordea	**1,031**	**1,020**	0,958	0,971	1,076	1,050	1,007	0,972	0,951	0,999
10.	PeKaO	**1,195**	**1,066**	1,054	1,066	1,134	1,000	1,066	1,028	0,989	1,037
11.	Pocztylion	**1,009**	**1,099**	0,996	1,057	1,013	1,040	1,013	1,020	0,983	1,036
12.	Polsat	**1,042**	**0,985**	1,000	1,000	1,042	0,985	1,000	1,000	1,000	1,000
13.	PZU	**0,900**	**1,015**	0,871	0,998	1,033	1,017	0,955	0,971	0,912	1,028
14.	Skarbiec	**1,085**	**1,191**	1,040	1,050	1,043	1,134	1,012	0,995	1,028	1,055
15.	Winterthur	**1,158**	**1,123**	1,012	1,009	1,144	1,113	1,027	0,999	0,986	1,010

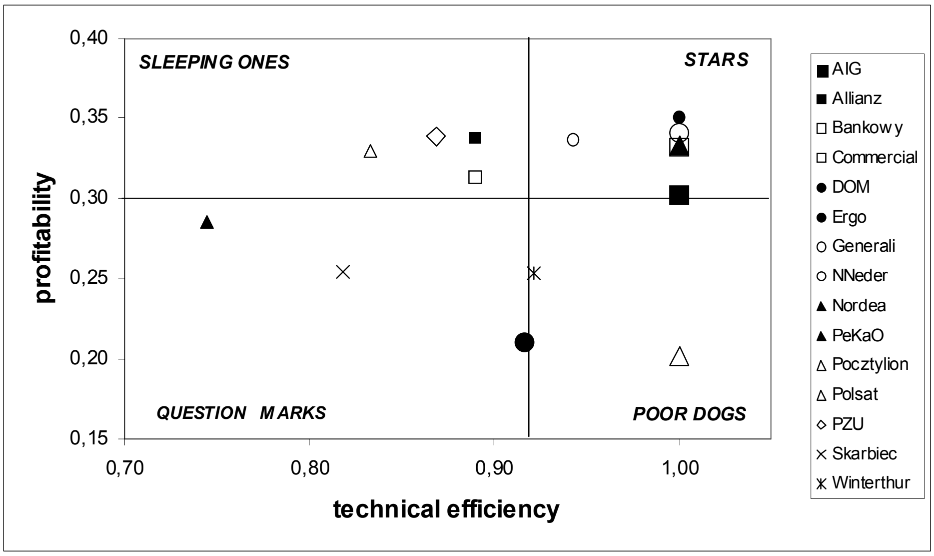

Fig. 1. OPF in the year 2006 – profitability and efficiency (BCG matrix)

SUMMARY

The findings can be summarized as follows:

1. In the period of 2004-06 almost all the funds were inefficient in case of assumed both constant and variable returns to scale (exluding Allianz, Commercial, Dom, Nationale Nederlanden, Polsat founds).
2. In the period of 2004-06 almost all the funds operated within the area of increasing returns to scale (exluding Allianz, Commercial, Dom, Nationale Nederlanden, Polsat founds).
3. In the years 2004-06 the efficiency of almost all pension funds increased period by period (growth in efficiency by 1,5-25,3%).
4. The executed research on efficiency and its changes in time confirms that two funds remain the leaders in the market of pension funds and they are: Commercial Union and Nationale Nederlanden.

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