Marek Chmielewski

Ignacy Kaliszewski

MULTIPLE CRITERIA DECISION MAKING IN FROZEN DECISION PROCESSES

Abstract

We consider a decision process of choosing an algorithm for selecting the most preferred variant. We consider the case when an algorithm has to be chosen and not changed afterwards before all feasible variants are known, as it happens e.g. in public tenders.

The fact that the chosen algorithm cannot be changed is the cause of potential regret the decision maker can resent when confronted with the selected variant.

We show how some formal tools of interactive Multiple Criteria Decision Making can be employed to confine decision maker's regret.

Keywords

Interactive multiple criteria decision making, nonautonomous processes.

INTRODUCTION

Decision processes for selecting the most preferred variant can be differentiated with respect to rights hold by the involved parties. In *autonomous processes* the sole actor of the decision process is the *decision maker* (DM). This means that the DM can carry out a decision process in a fully sovereign

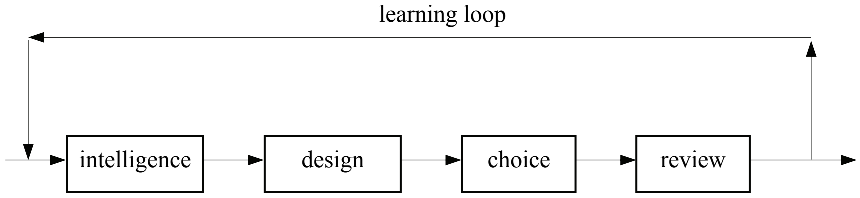

Fig. 1 Four phases of decision making process

manner. In contrast to autonomous processes, in *nonautonomous processes* manners of carrying out decision processes are restricted by rights to the process other parties may hold.

A widely adopted consensus is that every decision process consists of four phases usually closed in a feedback learning loop, namely the phase of *intelligence*, *design* (modeling), *choice*, and *review* [5] (Figure 1). With regard to the definition given above, a decision process is autonomous if in each phase the DM is sovereign in his decisions. The DM enjoys the maximal sovereignty in *interactive decision making*, i.e. when in the third phase the DM selects the most preferred variant interacting with a model, directing himself only by his own preferences.

A decision process ceases to be autonomous when the DM's sovereignty is restricted. A form of restriction can be, for example, the necessity to negotiate with the involved parties the manner in which the process is carried out or just to explain and give grounds for the manner adopted.

Below we focus on nonautonomous processes in which the only restriction is that the DM is bound to chose an unequivocal *selection algorithm* for selecting the most preferred variant and to made this algorithm public without any possibility to modify it in the future. We call such a decision process a *frozen process*.

In frozen processes consequences of impertinent choice of selection algorithm are irreversible, where pertinent choice is understood as follows: *a selection algorithm is chosen pertinently if the most preferred variant selected by this algorithm is that, which would be selected as the most preferred also in the case of an autonomous decision process*. Impertinent choice of a selection algorithm is a source of DM's (posterior) *regret*[1].

In general, chances to choose a pertinent algorithm are small. This is the case of e.g. public tenders, where parties involved are a tender calling entity, bidders, supervisory bodies, and to some extent, the whole society with its monitoring institutions (state agencies, media). Though in such cases consequences of impertinent choice of selection algorithm are mainly borne by a tender calling entity, they clearly also impact, explicitly or implicitly, other parties involved.

Further examples of frozen decision processes are public valuation procedures of individuals or institutions such as open competitions or rankings of universities.

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[1] "Regret is a negative, cognitively based emotion that we experience when realizing or imagining that our present situation would have been better, had we decided differently" [6].

The issue whether a decision process is frozen or not is context dependent. For example, banks are autonomous in credibility assessments of their clients. However, a credit officer cannot alter a credibility assessment procedure at his discretion, hence from his standpoint the assessment process is frozen.

Our aim is to provide the DM with means (methodologies and tools) to assess consequences of decisions, similar to means available for him in the case of autonomous processes. But as in the case of autonomous processes the main objective is to select the most preferred variant, in the case of frozen processes the main objective is to choose an algorithm which reflects DM's implicit and/or explicit, and usually partial preferences with respect to that variant. In other words, the objective is to minimize DM's posterior regret understood here as DM's *emotion resulting from comparing his implicit preferences for the most preferred variant and the most preferred variant selected by the chosen algorithm*.

In frozen decision processes the range of information about variants can vary between two extremes, from full information (all variants are known before a selection algorithm is chosen) to lack of any information (no variant is known before a selection algorithm is chosen). In general, DM's regret should be minimal in the former case. In all other cases the DM can make use of *hypothetical variants*. Hypothetical variants convey DM' his expectations about and, at the same time, his expertise on possible variants. Creating and evaluating hypothetical variants is the main (and practically only) tool to confine DM's regret.

In this paper we analyze frozen decision processes. Our aim is to identify the extent of freedom the DM possesses when choosing selection algorithm, as well as to identify his ability to minimize his regret. We also aim at establishing a methodology for supporting the DM in choosing selection algorithms.

We adopt a simplifying assumption that the only party which in a frozen decision process can resent regret is the DM.

The outline of the paper is as follows. In Section 1 we introduce necessary definitions and notation. In Section 2 we present those elements of Multiple Criteria Decision Making [4, 1. 3], which we use in Section 3 to propose how to support choice of selection algorithms in the case of frozen decision processes. Next, we present conclusions and we point to further possible directions of research.

1. DEFINITIONS AND NOTATION

Let x denote a decision variant, X – a space of decision variants, X_0 – a finite subset of feasible variants, and $X_0 \subseteq X$. Then the Multiple Criteria Decision Making (MCDM) problem is formulated as:

$$\begin{aligned} &\text{“}max\text{”} f(x) \\ &x \in X_0 \subseteq X \end{aligned} \tag{1}$$

where $f : X \to R^k,\ f = f(f_1, \ldots, f_k);\ f_i : X \to R,\ i = 1, \ldots, k,\ k \geq 2$, are objective (criteria) functions; “max” denotes the operator of deriving all efficient variants in X_0 according to the definition of efficiency given below.

In MCDM to compare variants x one makes use of their *outcomes* $f(x)$. Relations between outcomes in space R^k induce relations between variants in space X.

Below we make use of the following notation: $y = f(x)$ and $Z = f(X_0)$. Outcome $\overline{y} \in Z$ is called efficient if $y_i \geq \overline{y}_i,\ i = 1, \ldots, k,\ y \in Z$ implies $y = \overline{y}$. Variant $\bar{x} \in X_0$ is called efficient if $\bar{y} = f(\bar{x})$ is efficient.

Observe that in frozen processes the DM deals with X_0 and Z, which may contain hypothetical variants and hypothetical outcomes.

2. THE PROPOSED APPROACH

As a vehicle for supporting the DM in choosing selection algorithm in the case of frozen decision processes we employ interactive MCDM methods, which on the base of model (1) enable the DM to select the most preferred outcome (variant).

In interactive MCDM at each iteration the DM evaluates at least one pair of outcomes and establishes preference $\succ$ between them. The selection of the most preferred outcome is made possible by assuming that DM's preferences are consistent with his *implicit value function*.

Since outcomes are vectors of numbers, it is reasonable to assume that DM's preference relation $\succ$ is a partial order. Since there is no reason to exclude the case that two or more variants have the same outcome, the relation induced in the set of feasible variants X_0 by relation $\succ$ is in general a quasi-partial order.

Consistency between the implicit value function and preference relation $\succ$ (we assume, by analogy to interactive MCDM, that consistency holds), entails implication:

$$y \succ y' \ \Rightarrow \ v(y) > v(y') \tag{2}$$

which establishes a set of conditions on the DM's implicit value function.

We employ here *weighted linear scalarizing functions*, widely used in MCDM, namely:

$$\sum_i \lambda_i y_i \tag{3}$$

where $\lambda_i \geq 0, \ i = 1, ..., k$. For these functions condition (2) reduces to:

$$y \succ y' \ \Rightarrow \ \sum_i \lambda_i y_i > \sum_i \lambda_i y'_i \tag{4}$$

Evaluating h pairs of outcomes results in a system of inequalities:

$$\sum_i \lambda_i y_i^h > \sum_i \lambda_i y_i'^h, \ h = 1, ... \tag{5}$$

where y^h and y'^h are elements of pair h.

It is easy to show that if $y' \succ y''$ and $y''' \succ y''$, $y' \neq y'' \neq y'''$ and $y'_i \geq y'''_i$, $i = 1, ..., k$ (i.e. y' *dominates* y'''), then inequality $\sum_i \lambda_i y'''_i > \sum_i \lambda_i y''_i$ is redundant. Hence, it is suffcient to evaluate only pairs of efficient outcomes. This entails the following observation.

Lemma 2.1

The most preferred outcome selected consistently to implication (2), where $v(\cdot)$ are weighted linear scalarizing functions, is efficient.

Derivation of efficient outcomes is carried with the use of *scalarizing functions*, which attain their extremal values at eficient outcomes. In particular, one can make use of weighted linear scalarizing functions. For each $\lambda_i, \ i = 1, ..., k$, maximization of the corresponding linear scalarizing function yields an efficient outcome [4, 1, 3].

Each vector λ satisfying (5) and $\lambda_i \geq 0, \ i = 1, ..., k,$ defines function $\sum_i \lambda_i y_i$ preserving (in the sense of (4)) all *h* relations $y^h \succ y'^h$

The set of all such vectors (we denote this set as $\bar{\Lambda}$) defines the extent of exibility the DM has when choosing a selection algorithm consisting in maximizing a linear scalarizing function.

Clearly, $\bar{\Lambda} \subseteq \Lambda = \{\lambda \,|\, \lambda_i \geq 0,\ i = 1, ..., k\}$. Without loss of generality we assume that elements of Λ satisfy additional condition $\sum_i \lambda_i = 1$.

3. SUPPORTING CHOICE OF SELECTION ALGORITHMS

It is rational to assume that at the start of choosing a selection algorithm the DM has only some vague preferences with respect to the most preferred outcome. The aim of the algorithm selection (frozen) process is to specify those preferences and evoke more partial preferences. Recall that set X_0 can, and in some cases should, include hypothetical variants.

If selection algorithm is chosen when all variants are known, one can expect that the DM chooses always an algorithm which selects the most preferred variant and hence there is no cause for regret. However, this is not always the case, as shown in the following example.

Example 3.1

Let three variants be given which outcomes are y^1 = (2, 6), y^2 = (3, 3), y^3 = (6, 2). *Let outcome* y_2 *be the most preferred outcome. Then, by* (2):

$$\lambda_1 y_1^2 + \lambda_2 y_2^2 > \lambda_1 y_1^1 + \lambda_2 y_2^1 \quad (hence\ \lambda_1 > 3\lambda_2)$$

$$\lambda_1 y_1^2 + \lambda_2 y_2^2 > \lambda_1 y_1^3 + \lambda_2 y_2^3 \quad (hence\ 3\lambda_1 < \lambda_2)$$

But it is easy to check that this system is inconsistent, i.e. $\bar{\Lambda} = \emptyset$. *This entails that there is no function* $\sum_i \lambda_i y_i$ *such that it attains its largest value for* y^2.

Weighted linear functions allow deriving the most preferred outcomes only if they are located on the convex hull (i.e. the smallest polyhedral set containing *Z*) of *Z*. Below, for the sake of clarity of presentation, we assume that this is the case. We admit that this is an oversimplification of reality, but we do this because weighted linear functions are predominantly used in interactive MCDM (a good example of such a „standard" are methods to select a winner in public tenders). Relaxation of this assumption would require using e.g. weighted Tchebycheff functions [4, 1, 3] as scalarizing functions. This, however, falls outside of the scope of this paper.

The idea of supporting choice of selection algorithm consists in analyzing set $\bar{\Lambda}$. Recall that elements of this sets are vectors λ satisfying system of conditions (5), where y^h and y'^h are pairs of outcomes for which the DM expressed preference (in the sense of relation $\succ$).

In particular, if all outcomes are known and the DM points to the most preferred (and hence efficient) outcome y^j, then the set of conditions:

$$y^j \succ y, \text{ for each } y \in Z \setminus \{y^j\} \tag{7}$$

is to hold. Then set $\bar{\Lambda}$ contains all vectors λ, for which:

$$\sum_i \lambda_i y_i^j > \sum_i \lambda_i y_i \text{ for each } y \in Z \setminus \{y^j\} \tag{8}$$

In this case we distinguish set $\bar{\Lambda}$ as Λ^j. Hence, for each $\lambda \in \Lambda^j$ the algorithm (Algorithm Δ) defined as follows:

$$\text{“select } \arg(\max_{y \in Z} \sum_i \lambda_i y_i)\text{”}$$

selects y^j. In other words, set Λ^j is *the stability set* of outcome y^j with respect to perturbations of λ.

Example 3.2

Given are three variants with outcomes as in Example 3.1. Set $\Lambda^1 \subseteq \bar{\Lambda}$ such that for all Set $\lambda \in \Lambda^1$ Algorithm Δ selects outcome y^1 is given by the system of inequalities:

$$\lambda_1 y_1^1 + \lambda_2 y_2^1 > \lambda_1 y_1^2 + \lambda_2 y_2^2 \quad (\textit{hence } 3\lambda_2 > \lambda_1)$$

$$\lambda_1 y_1^1 + \lambda_2 y_2^1 > \lambda_1 y_1^3 + \lambda_2 y_2^3 \quad (\textit{hence } \lambda_2 > \lambda_1)$$

On the other hand, set $\Lambda^3 \subseteq \bar{\Lambda}$ such that for all $\lambda \in \Lambda^3$ Algorithm Δ selects outcome y^3 is given by the system of inequalities:

$$\lambda_1 y_1^3 + \lambda_2 y_2^3 > \lambda_1 y_1^1 + \lambda_2 y_2^1 \ (\textit{hence } \lambda_1 > \lambda_2)$$

$$\lambda_1 y_1^3 + \lambda_2 y_2^3 > \lambda_1 y_1^2 + \lambda_2 y_2^2 \ (\textit{hence } 3\lambda_2 > \lambda_2)$$

Sets Λ^1, Λ^3 are obviously disjoint, as illustrated in Figure 2.

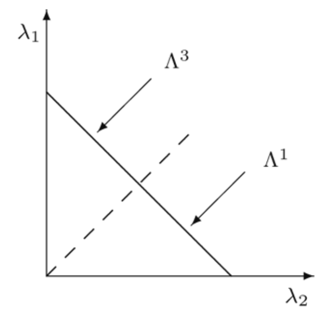

Fig. 2. Stability regions of y^1 and y^3 (Example 3.2)

If (8) does not hold, set $\bar{\Lambda}$ contains vectors λ for which Algorithm Δ can select different outcomes (and therefore to different variants) and DM's regret varies accordingly.

Example 3.3

Given are three variants with outcomes as in Example 3.1. Assume that set $\bar{\Lambda}$ defined by the following condition:

$$\lambda_1 y_1^1 + \lambda_2 y_2^1 > \lambda_1 y_1^2 + \lambda_2 y_2^2 \quad (hence\ 3\lambda_2 > \lambda_1)$$

Then for some $\lambda \in \bar{\Lambda}$ Algorithm Δ can select the following outcomes: y^1 and y^3.

In general, $cl(\bar{\Lambda}) = \bigcup_j cl(\Lambda^j),\ \Lambda_j \cap \Lambda_i = \emptyset,\ i \neq j$, where Λ^j is the set of vectors λ such that for any $\lambda \in \Lambda^j$ Algorithm Δ selects only y^j, $j = 1,...,L$, $L \leq |Z|$, and $cl(\cdot)$ denotes closure of a set. Set Λ^i determine stability regions of outcomes y^i, $i = 1, ..., L$.

Lemma 3.1

The most preferred outcome is a vertex of the convex hull of Z.

Proof

By the adopted assumption, the most preferred outcome, say outcome y^m, is located on the convex hull of *Z*. Suppose it is not a vertex. Then there exists at least one outcome y such that:

$$\sum_i \lambda_i y_i^m = \sum_i \lambda_i y_i \text{ for some } \lambda \in \Lambda^j$$

Hence, by (2), $y^m \not\succ y$, which by (8) is a contradiction.

■

From Lemma 3.1 we infer the following obvious observation.

Lemma 3.2

Partition of set $cl(\bar{\Lambda})$ *into subsets* $cl(\Lambda^i)$ *depends only on effcient vertices of the convex hull of Z.*

By the above lemma system of inequalities (5) can be confined exclusively to inequalities generated by efficient vertices of the convex hull of *Z*.

The DM can control the level of his regret imposing conditions that certain outcomes (and therefore the corresponding variants) are not selected by Algorithm Δ. This is equivalent to imposing conditions on admissible vectors λ . Namely, if he wants that y^j is not selected by Algorithm Δ he has to set weights $\lambda \in \bar{\Lambda}$ such that $\lambda \notin \Lambda^j$.

On the other hand, the DM can control the level of his regret imposing the condition that a certain outcome (and the corresponding variant) is selected by Algorithm Δ as the most preferred. Here again, this is equivalent to imposing a condition on admissible vectors λ . Namely, if he wants that y^j is selected by Algorithm Δ he has to set weights $\lambda \in \bar{\Lambda}$ such that $\lambda \in \Lambda^j$.

Identification of sets of weights for which Algorithm Δ selects given outcome y^j leads to the following question: which vector λ from set Λ^j ensures the maximum stability of the most preferred outcome y^j with respect to perturbations of decision problem parameters?

Let us consider first stability of the most preferred outcome y^j with respect to perturbations of vectors λ (in the sense of value $||\lambda - \lambda'||_2$, where λ' is perturbed vector). Outcome y^j is the most stable (robust) with respect to perturbations of vector λ if such a vector is the most distant from all constraints which define set Λ^j. Such a vector can be defined in the following way.

Vector λ most distant from a constraint, which defines Λ^j as a consequence of relation $y^j \succ y^l$, can be found by solving the following optimization problem:

$$\max_{\lambda \in cl(\Lambda^j)} \sum_i \lambda_i (y_i^j - y_i^l)$$

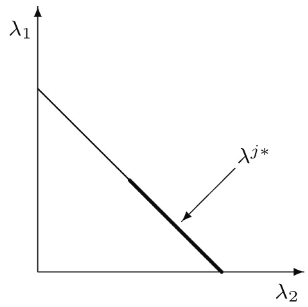

Fig. 3. Stability of an outcome in the weight space

For all constraints which define Λ^j vector λ maximizing all $\sum_i \lambda_i(y_i^j - y_i^l)$, $l = 1, ..., L$ simultaneously, can be found by solving the following optimization problem (Figure 3):

$$\begin{gathered} \max t \\ \sum_i \lambda_i y_i^j - \sum_i \lambda_i y_i^l \geq t, \ l = 1, ..., L \\ \lambda \in cl(\Lambda^j) \end{gathered} \tag{9}$$

Consider now stability of the most preferred outcome y^j with respect to variations of the remaining outcomes (in the sense of value $||y - y'||_2$ where y' is perturbed outcome). Recall that in the considered problem some variants can be hypothetical and therefore their outcomes can vary. Outcome y^j is the most stable with respect to variations of outcomes y^l, $l = 1, ..., L$, $y^l \neq y^j$, if the minimal of differences:

$$\sum_j \lambda_j y_j^i - \sum_j \lambda_j y_j^l, \ l = 1, ..., L$$

is maximal (Figure 4). Hence, vector λ satisfying this requirement can be again found by solving problem (9).

Therefore vector λ for which outcome y^j is the most stable with respect to variations of weights at the same time ensures the highest stability of y^j with respect to variations of other outcomes. As the former observation pertains to the weight space, the latter observation pertains to the outcome space.

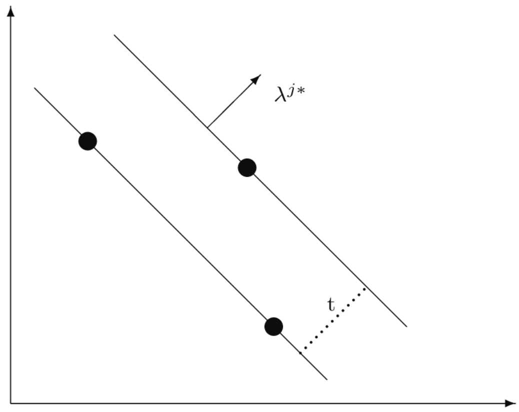

Fig. 4. Stability of an outcome in the outcome space

The interpretation in the weight space is rather straightforward. Vector λ^{j*} which solves (9) corresponds to the point in set Λ^j located in the same distance to all constraints which define this set.

More interesting is interpretation of vector λ^{j*} in outcome space. This vector in normal to the hyperplane tangent to y^j and at the same time maximizing the minimal distance to all efficient outcomes adjacent to y^j.

CONCLUDING REMARKS AND DIRECTIONS FOR FURTHER RESEARCH

Probably the most spectacular area of the above considerations can be public tenders, where public money is involved. In many occasions the variant selected is not the most preferred (in the earlier defined sense), but by the rules of public tenders (where decision processes are frozen) it is the winner. This causes regret, often formulated verbally: If I (we) had known that *such* variants were proposed, I (we) would have chosen a different selection algorithm.

Careful analysis of frozen decision process problems with the help of the technics discussed above could in many cases reduce regret. This is of particular importance when tender is organized for the first time in a field new to the DM, where a significant regret can materialize.

It is rather obvious that the process of choosing an algorithm for selection of the most preferred variant, besides considering hypothetical variants, has to also address selection of criteria. One should not regard model (1) as given and fixed, but model specification should be also an element of the process of algorithm selection. Appropriate model specification in frozen decision processes will be a subject of further research.

Another issue for further research will be also the possible use of other scalarizing functions than linear weighted functions.

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