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MULTIOBJECTIVE COMBINATORIAL AUCTIONS[*]

Abstract

Auctions are important market mechanisms for the allocation of goods and services. Combinatorial auctions are those auctions in which bidders can place bids on combinations of items. The advantage of combinatorial auctions is that the bidder can more fully express his preferences. This is particular important when items are complements. Allowing bidders more fully to express preferences often leads to improved economic efficiency and greater auction revenues. A typical combinatorial auction problem is the so called winner determination problem. The problem illustrates the possibility to formulate combinatorial auctions as mathematical programming problems as well as the complexity of combinatorial auctions. Auctions with complex bid structures are called multiobjective auctions, since they address multiple objectives in the negotiation space. Multiobjective optimization can be helpful for detailed analysis of combinatorial auctions. Buyers can specify weights and aspiration levels that express their desired values on the attributes of the items to be purchased. Interactive methods for multiobjective optimization are proposed for analysis of combinatorial auctions and for negotiation process.

Keywords

Combinatorial auctions, preference elicitation, multiobjective optimization, negotiation, interactive methods, Dynamic Network Process.

INTRODUCTION

Auctions are important market mechanisms for the allocation of goods and services. They are preferred often to other common processes because they are open, quite fair, easy to understand by participants, and lead to economically efficient outcomes. Many modern markets are organized as auctions. Design of auctions is a multidisciplinary effort made of contributions from economics,

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operations research, informatics, and other disciplines. Combinatorial auctions are those auctions in which bidders can place bids on combinations of items called bundles. The advantage of combinatorial auctions is that the bidder can more fully express his preferences. This is particularly important when items are complements. The auction designer also derives value from combinatorial auctions. Allowing bidders more fully to express preferences often leads to improved economic efficiency and greater auction revenues. However, alongside their advantages, combinatorial auctions raise a host of questions and challenges [see 5; 6].

Auction theory has attracted tremendous interest from both the economic side as well as the Internet industry. An auction is a competitive mechanism to allocate resources to buyers based on predefined rules. These rules define the bidding process, how the winner is determined, and the final agreement. In electronic commerce transactions, software agents that negotiate on behalf of buyers and sellers conduct auctions. The popularity of auctions and the requirements of e-business have led to growing interest in the development of complex trading models [1; 2; 9].

Classification of auctions is based on some specific characteristics as:

1. The numbers of sellers and buyers.
2. The number of items.
3. Traded items (indivisible, divisible, pure commodities, structured commodities).
4. Participants' roles in auctions (one-sided, multilateral auctions).
5. Preferences of the participants.
6. The form of the private information participants have about preference.
7. Objectives of auctions (optimization, allocation rules, pricing rules).
8. Evaluating criteria.
9. Complexity of bids (simply, related bids).
10. Organization of auctions (single-round, multi-round, sequential, parallel, price schemes).

The problem, called the winner determination problem, has received considerable attention in the literature. The problem is formulated as: Given a set of bids in a combinatorial auction, find an allocation of items to bidders that maximizes the seller's revenue. It introduced many important ideas, such as the mathematical programming formulation of the winner determination problem, the connection between the winner determination problem and the set packing problem as well as the issue of complexity.

Iterative combinatorial auctions with multiple objectives are proposed in the paper as complex trading models. A solution procedure is presented.

1. WINNER DETERMINATION PROBLEM

Many types of combinatorial auctions can be formulated as mathematical programming problems. From among different types of combinatorial auctions we present an auction of indivisible items with one seller and several buyers. Let us suppose that one seller offers a set G of m items, $j = 1, 2, \ldots, m$, to n potential buyers. Items are available in single units. A bid made by buyer i, $i = 1, 2, \ldots, n$, is defined as:

$$B_i = \{S, v_i(S)\}$$

where:

$S \subseteq M$ is a combination of items,
$v_i(S)$ is the valuation or offered price by buyer i for the combination of items S.

The objective is to maximize the revenue of the seller given the bids made by buyers. Constraints are imposed such that no single item is allocated to more than one buyer and that no buyer obtains more than one combination.

1.1. Problem formulation

Let $x_i(S)$ be a bivalent variable specifying if the combination S is assigned to buyer i ($x_i(S) = 1$). The winner determination problem can be formulated as follows

$$\sum_{i=1}^{n} \sum_{S \subseteq M} v_i(S)\, x_i(S) \rightarrow \max$$

subject to:

$$\sum_{S \subseteq M} x_i(S) \leqq 1, \quad \forall i,\ i = 1, 2, \ldots, n$$

$$\sum_{i=1}^{n} \sum_{S \subseteq M} x_i(S) \leqq 1, \quad \forall j \in M$$

$$x_i(S) \in \{0, 1\}, \quad \forall S \subseteq M, \quad \forall i,\ i = 1, 2, \ldots, n$$

The objective function expresses the revenue. The first constraint ensures that no bidder receives more than one combination of items. The second constraint ensures that overlapping sets of items are never assigned.

The winner determination problem, i.e. determination of the items that each bidder wins, is not difficult in the case of non-combinatorial auctions. It would take *O(nm)* time where *n* is the number of bidders and *m* is the number of items. But in the case of combinatorial auctions, the winner determination problem is much more complex.

1.2. Complexity of the problem

Complexity is a fundamental question in combinatorial auction design. There are some types of complexity:

- computational complexity,
- valuation complexity,
- strategic complexity,
- communication complexity.

Computational Complexity covers such questions as: How much computation is required to compute an outcome given the bid information of the bidders. This is an extremely important question because winner determination problem is an NP-complete optimization problem. The winner determination problem turns out to be an instance of a weighted set packing problem. The weighted set packing problem is a problem of finding a disjoint collection of weighted subsets of a larger set with maximal total weight. Weighted set packing is a classical NP-complete problem.

Valuation complexity deals with such questions as: How much computation is required to provide preference information within a mechanism? Estimating every possible bundle of items requires exponential space and hence exponential time. Bidders need to determine valuations for 2^m -1 possible bundles.

Strategic complexity concerns such questions as: Which of the 2^m -1 bundles to bid on? What is the best strategy for bidding? Must bidders model the behavior of other bidders and solve problems to compute an optimal strategy? For instance, in a sealed bid combinatorial procurement scenario, sellers will need to take not only their valuation of the bundles into consideration, but also the bidding behavior of their competitors. This requires sophisticated bidding logic.

Communication complexity concerns such questions as: How much communication should be exchanged between bidders and auctioneer until an equilibrium price is reached and the mechanism computes an outcome. The amount of communication between the bidders and the auctioneer can become quite high. For instance, in an iterative combinatorial auction, where individual valuations are revealed progressively in an iterative manner, the communication costs could be high if the auction were conducted in a distributed manner over space and/or time. The problem of communication complexity can be addressed through the design of careful bidding languages that provide expressive, but concise bids.

2. MULTIDIMENSIONAL AUCTIONS

Multidimensional auctions are examples of generalization of auctions. These auctions can be classified as:

– multiunit auction,
– multiitem auction,
– multiobjective auction,
– multiround auction.

Multiunit auctions contain multiple units of items and makes possible volume discount auctions. In multiitem auctions one can place bids on combinations of items; such auctions are called combinatorial auctions. In combinatorial auctions multiple objectives can be defined, for instance, as:

– revenue maximization – the seller should extract the highest possible price,
– efficiency – the buyers with the highest valuation get the goods,
– collusion possibility.

Auctions with complex bid structures are also called multiobjective auctions, since they address multiple attributes of the items (quality, quantity, price) in the negotiation space. Multiobjective optimization can be helpful for detailed analysis of combinatorial auctions.

In the iterative approach, there are multiple rounds of bidding and allocation and the problem is solved in an iterative and incremental way. Iterative combinatorial auctions are attractive to bidders because they learn about their rivals' valuations through the bidding process, which could help them to adjust their own bids.

Combinations of multidimensional characteristics are possible. We suggest to use an iterative process for multiobjective combinatorial auctions. Multiobjective combinatorial auctions require several key components to solve the process:

– preference elicitation model,
– multiobjective optimization model,
– negotiation model.

The preference elicitation model is used to let the buyer express his preferences. The preferences are modeled by a combination of the dynamic version of Analytic Network Process and aspiration levels. The multiobjective optimization model selects the best offer for the buyer. For analysis of iterative combinatorial auctions we propose to use interactive methods for multiobjective optimization. The negotiation model helps to find a consensus by auctions. Auctions have emerged as a particularly interesting tool for negotiations. Combinatorial auctions provide a mechanism for negotiation between buyers and sellers. Various concepts of negotiation models can be used for modeling combinatorial auctions.

3. PREFERENCE ELICITATION

The key feature that makes combinatorial auctions most appealing is the ability for bidders to express complex preferences over bundles of items, involving complementarity and substitutability. Items are complements when a set of items has greater utility than the sum of the utilities for the individual items. Items are substitutes when a set of items has less utility than the sum of the utilities for the individual items.

Two items *A* and *B* are complementary, if the following holds:

$$v(\{A, B\}) > v(\{A\}) + v(\{B\})$$

Two items *A* and *B* are substitute, if the following holds:

$$v(\{A, B\}) < v(\{A\}) + v(\{B\})$$

Different elicitation algorithms may require different means of representing the information obtained by bidders. Sandholm and Boutilier [15] describe a general method for representing an incompletely specified valuation functions. A constraint network is a labeled directed graph consisting of one node for each bundle b representing the elicitor's knowledge of the preferences of a bidder. A directed edge (a, b) indicates that bundle a is preferred to bundle b. Figure 1 represents an example of a constraint network for bundles of three items (A,B,C).

The constraint network representation is conceptually useful and can be represented explicitly for use in various elicitation algorithms. But its explicit representation is generally tractable only for small problems, since it contains 2^m nodes. For preference elicitation of bundles in a constraint network Analytic Network Process can be used.

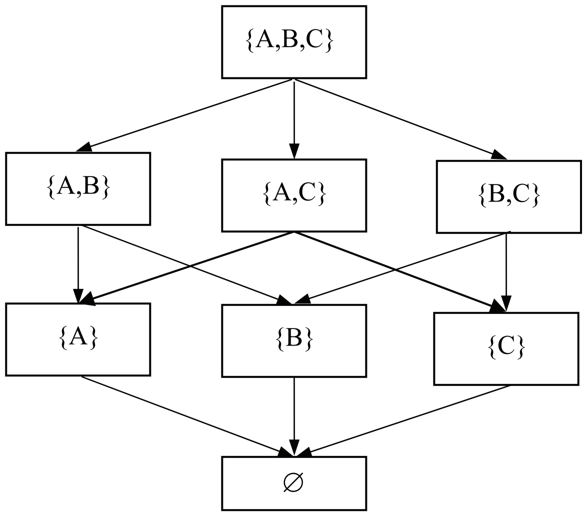

Fig. 1. Constraint network

The Analytic Hierarchy Process (AHP) is the method for setting priorities [11]. A priority scale based on reference is the AHP way to standardize nonunique scales in order to combine multiple performance measures. The AHP derives ratio scale priorities by making paired comparisons of elements on a common hierarchy level by using a 1 to 9 scale of absolute numbers. The absolute number from the scale is an approximation to the ratio w_j/w_k; it is then possible to derive values of w_j and w_k. The AHP method uses the general model for synthesis of the performance measures in the hierarchical structure:

$$u_i = \sum_{j=1}^{n} v_j w_{jk}$$

The Analytic Network Process (ANP) is the method [12] that makes it possible to deal systematically with all kinds of dependence and feedback in the performance system. The well-known AHP theory is a special case of the Analytic Network Process that can be very useful for incorporating connections in the system.

The structure of the ANP model is described by clusters of elements connected by their dependence on one another. A cluster groups elements share a set of attributes. At least one element in each of the clusters is connected to some element in another cluster. The connections indicate the flow of influence between the elements (see Figure 2).

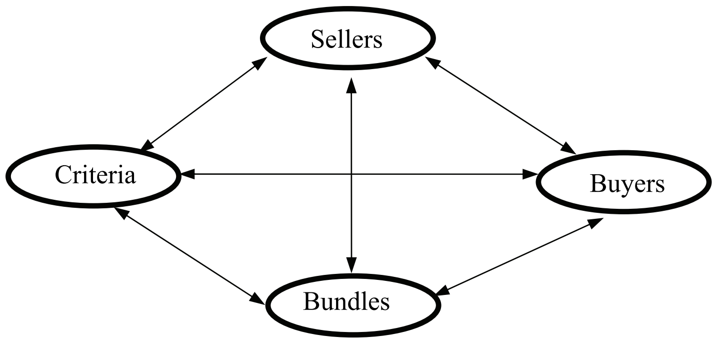

Fig. 2. Clusters and connections in multiobjective combinatorial auctions

The clusters in multiobjective combinatorial auctions can be sellers, buyers, bundles of items, and also evaluating criteria. Paired comparisons are inputs for preference elicitation in combinatorial auctions. A supermatrix is a matrix of all elements by all elements. The weights from the paired comparisons are placed in the appropriate column of the supermatrix. The sum of each column corresponds to the number of comparison sets. The weights in the column corresponding to the cluster are multiplied by the weight of the cluster. Each column of the weighted supermatrix sums to one and the matrix is column stochastic. Its powers can stabilize after some iterations to limited supermatrix. The columns of each block of the matrix are identical in many cases, though not always, and we can read the global priority of units.

Recent work has focused on the question of how to limit the amount of valuation information provided by bidders by adaptively limiting the precision of the bids that are specified.

Combinatorial auctions can be divided into auctioneer-side allocation auctions and bidder-side allocation auctions. The bidder-side allocation auctions were developed for small problems where bidders can cooperate in order to find a better allocation in each iteration without external help. In the auctioneer-side allocation auctions the auctioneer solves the winner determination problem after the bids are collected. The auctioneer provides then some kind of feedback to support the bidders in improving their bids in the next iteration. Usually the bidder's current winning bids and item prices are used as the feedback. The key challenge in the iterative combinatorial auction design is to provide information feedback to the bidders after each iteration. Assigning prices to items and/or item bundles was adopted as the most intuitive mechanism of providing feedback.

The AHP and ANP are static, but in decision analysis in the modern world it is very important to take time into account. The DHP/DNP (Dynamic Hierarchy Process/Dynamic Network Process) methods have been introduced [12]. There are two ways to study dynamic decisions: structural, by including scenarios, and functional by explicitly involving time in the judgment process. For the functional dynamics there are analytic or numerical solutions. The basic idea of the numerical approach is to obtain the time dependent principal eigenvector by simulation.

The Dynamic Network Process seems to be the appropriate instrument for analyzing dynamic network effects [7]. The method is appropriate also for the specific features of multiobjective combinatorial auctions. The method computes time dependent weights for bundles of items of weights of bidders (Figure 3).

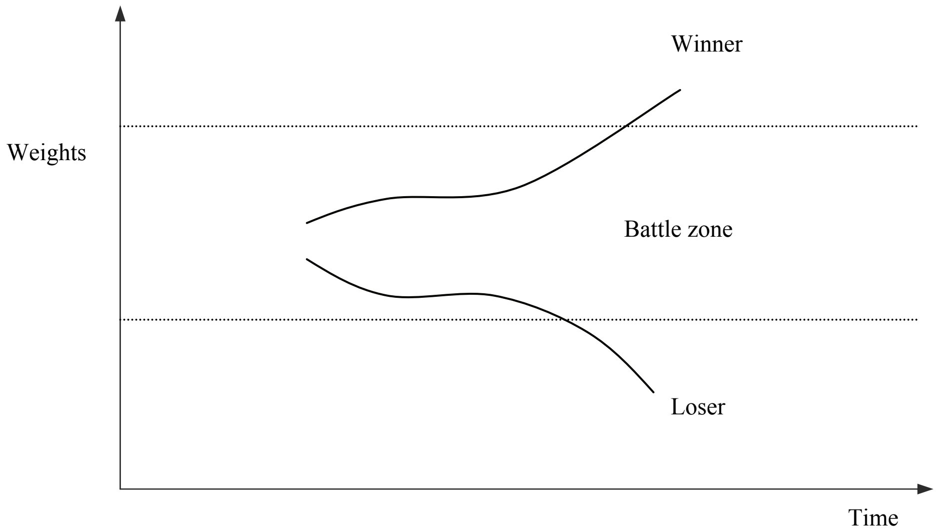

Fig. 3. Time dependent weights

In the multiobjective combinatorial auction model we take into account the auctioneer, bidders, criteria, and packages as clusters and different types of connections in the system. There are also some dependencies and feedback among elements and clusters. The dynamic version of the model is tested.

We used the alpha version of the ANP software Super Decisions developed by Creative Decisions Foundation (CDF) for some experiments for testing the possibilities of the expression and evaluation of the multi-objective combinatorial auction models (Figure 4).

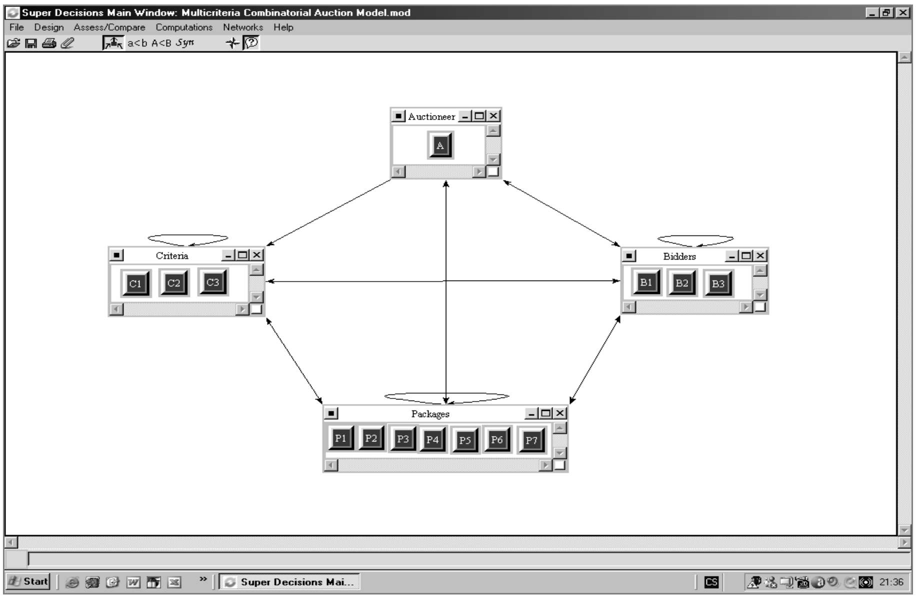

Fig. 4. Multiobjective Combinatorial Auction Model

We propose to combine the weight model of preference elicitation with multiobjective model based on aspiration levels ALOP [8]. The general formulation of an individual multiobjective decision problem is expressed as follows:

$$z(x) = (z_1(x), z_2(x), ..., z_k(x)) \to \text{“max”}$$
$$x \in X \tag{1}$$

where X is a decision space, x is a decision alternative and $z_1, z_2, ..., z_k$ are the objectives. The decision space is defined by objective restrictions and by mutual goals of all the agents in the aspiration level formulation. The decision alternative x is transformed by the objectives to objective values $z \in Z$, where Z is an objective space. Every agent has his own objectives.

It appears that people tend to satisfy given conditions rather than attempt to optimize them. That means substituting the goals of reaching specified aspiration levels for the goals of maximizing. We denote by $y^{(t)}$ aspiration levels of the objectives and by $\Delta y^{(t)}$ changes of aspiration levels in the step t. We search for alternatives such that:

$$z(x) \geq y^{(t)}$$
$$x \in X \tag{2}$$

According to heuristic information from the results of the condition (2) the agent changes the aspiration levels of objectives for step t + 1:

$$y^{(t+1)} = y^{(t)} + \Delta y^{(t)} \tag{3}$$

We can formulate the multiobjective decision problem as a state space representation. The state space corresponds to the objective space Z, where the states are the aspiration levels of the objectives $y^{(t)}$ and the operators are changes of the aspiration levels $\Delta y^{(t)}$. The start state is a vector of the initial aspiration levels and the goal state is a vector of the objective levels for the best alternative.

4. MULTIOBJECTIVE OPTIMIZATION MODEL

To find the ideal alternative we use the depth-first search method with backtracking procedure. The heuristic information is the distance between an arbitrary state and the goal state.

We propose an interactive procedure ALOP (Aspiration Levels Oriented Procedure) for multiobjective linear programming problems, where the decision space X is determined by the linear constraints:

$$X = \{x \in R^n; Ax \le b, x \ge 0\} \tag{4}$$

and $z_i = c_i x$, $i = 1, 2, \dots, k$, are linear objective functions. Then $z(x) = Cx$, where C is a coefficient matrix of objectives.

The decision alternative $x = (x_1, x_2, \dots, x_n)$ is a vector of n variables. The agent states the aspiration levels $y^{(t)}$ for the objectives values. There are three possibilities for the aspiration levels $y^{(t)}$. The problem (2) can be feasible, infeasible, or else; has a unique nondominated solution. We verify the three possibilities by solving the problem:

$$v = \sum_{i=1}^{k} w_i^+ d_i^+ \rightarrow \min \tag{5}$$

$$Cx - d^+ = y^{(t)}$$

$$x \in X, d^+ \ge 0$$

The value of the objective function in the problem (5) can be interpreted as an increase of utility.

If the following holds:

– $v > 0$, then the problem is feasible and d_i^+ are proposed changes $\Delta y^{(t)}$ of aspiration levels which achieve a nondominated solution in the next step,

– $v = 0$, then we obtained a nondominated solution,
– the problem is infeasible, then we search for the nearest solution to the aspiration levels by solving the goal programing problem:

$$v = \sum_{i=1}^{k} \frac{1}{\bar{z}_i} (d_i^+ + d_i^-) \to \min \tag{6}$$

$$Cx - d^+ + d^- = y^{(t)}$$

$$x \in X, d^+ \geq 0, d^- \geq 0$$

The solution of the problem (6) is feasible with changes of the aspiration levels $\Delta y(t) = d^+ - d^-$. For small changes of nondominated solutions the duality theory is applied. Dual variables to the objective constraints in the problem (6) are denoted by u_i, $i = 1, 2, ..., k$.

If the following holds:

$$\sum_{i=1}^{k} u_i \Delta y_i^{(t)} = 0, \tag{7}$$

then for some changes $\Delta y^{(t)}$ the value $v = 0$ is not changed and we obtained another nondominated solution. The agent can state k-1 small changes of the aspiration levels $\Delta y_i^{(t)}$, $i = 1, 2, \dots, k$, $i \neq r$, then the change of the aspiration level for criterion r is calculated from (7). The agent chooses a forward direction or backtracking. Results of the procedure ALOP are the path of tentative aspiration levels and the ideal solution.

5. NEGOTIATION MODEL

We propose a two-phase interactive approach for solving multiobjective negotiation problems:
1. Finding the ideal alternative for individual agents.
2. Finding a consensus for all the agents.

In the first phase each agent searches for the ideal alternative by the ALOP procedure. In the second phase a consensus can be obtained by the search process and the principle of cooperativeness is applied. The heuristic information for the agent is the distance between his proposal and the opponent's proposal. We assume that all the agents found their ideal alternatives. We propose an interactive procedure GROUP-ALOP for searching for a consensus.

For simplicity we assume the model with two agents:

$$\begin{aligned} & z^1(x) \rightarrow \text{“max”} \\ & z^2(x) \rightarrow \text{“max”} \\ & x \in X \end{aligned} \tag{8}$$

The agents search for a consensus on a common decision space X and change the aspiration levels of the objectives y^1, y^2. The sets of feasible alternatives for the aspiration levels y^1 and y^2 are X^1 and X^2.

$$\begin{array}{ll} z^1(x) \geq y^1 & z^2(x) \geq y^2 \\ x \in X & x \in X \end{array} \tag{9}$$

The consensus set S of the negotiations is the intersection of feasible sets X^1 and X^2:

$$S = X^1 \cap X^2 \tag{10}$$

When the aspiration levels change, the consensus set S is also changed. The agents search for one element consensus set S by alternating the consensus proposals. The image of partner's proposal can be taken as the aspiration levels in one's own objectives space. In searching for a consensus the distance between the proposals is heuristic information. The paths of the tentative aspiration levels can be used for the backtracking procedure. The forward directions can be directed by the proposed new aspiration levels in step $t + 1$:

$$y^1(t+1) = (1-\alpha)y^1(t) + \alpha z^1(x^2)$$
$$y^2(t+1) = (1-\beta)y^2(t) + \beta z^2(x^1)$$

where $\alpha, \beta \in \langle 0,1 \rangle$ are the coefficients of cooperativeness.

CONCLUSIONS

Combinatorial auction is an important subject of intensive economic research which promises to increase efficiency and reduces exposure to risk in an economic environment where synergy is significant. The winner determination problem is by far the most researched issue in combinatorial auctions. The problem illustrates the possibility to formulate combinatorial auctions as mathematical programming problems and also the complexity of them.

We propose to use multiobjective iterative combinatorial auctions. Multiobjective optimization can be helpful for detailed analysis of combinatorial auctions. Iterative process helps the bidders express their preferences. A possible flexible approach is presented. The approach is based on the Dynamic Network Process and Aspiration Level Oriented Procedure. The combination of such approaches can give more complex views on auctions. The iterative method is used for multiobjective optimization and also negotiation which model helps to find a consensus by auctions.

REFERENCES

1. Bellosta M., Brigui I., Kornman S., and Vanderpooten D.: A Multi-Criteria Model for Electronic Auctions. ACM Symposium on Applied Computing 2004, pp. 759--765.
2. Bichler M.: An Experimental Analysis of Multi-Attribute Auctions. “Decision Support Systems” 2000, 29, pp. 249-268.
3. Bikhchandani S. and Ostroy J.M.: The Package Assignment Model. “Journal of Economic Theory” 2002, 107(2), pp. 377-406.
4. CDF (Creative Decisions Foundation) – www.creativedecisions.net [2000].
5. Combinatorial Auctions. Eds Cramton P., Shoham Y. and Steinberg R. MIT Press, Cambridge, 2006.
6. de Vries S. and Vohra R.V.: Combinatorial Auctions: A Survey. “INFORMS Journal of Computing” 2003, 15 (1), pp. 284-309.
7. Fiala P.: An ANP/DNP Analysis of Economic Elements in Today's World Network Economy. „Journal of Systems Science and Systems Engineering” 2006, 15, pp. 131-140.
8. Fiala P.: Models of Cooperative Decision Making. In: Multiple Criteria Decision Making. Eds: T. Gal and G. Fandel. Springer, 1937.
9. Oliveira E., Fonsesca J.M., and Steiger-Garao A.: Multi-Criteria Negotiation in Multi-Agent Systems. In: 1st International Workshop of Central and Eastern Europe on Multi-Agent Systems (CEEMAS'99). St. Petersbourg 1999.
10. Rothkopf M., Pekeč A., and Harstad R.: Computationally Manageable Combinational Auctions. “Management Science” 1998, 8, pp. 1131-1147.
11. Saaty T.L.: The Analytic Hierarchy Process. RWS Publications, Pittsburgh 1996.
12. Saaty T.L.: Decision Making with Dependence and Feedback: The Analytic Network Process. RWS Publications, Pittsburgh 2001.
13. Saaty T.L.: Time Dependent Decision-Making; Dynamic Priorities in AHP/ANP: Generalizing from Points to Functions and from Real to Complex Variables. Proceedings of the 7th International Conference on the Analytic Hierarchy Process. Bali 1999, pp. 1-38.
14. Sandholm T.: Algorithm for Optimal Winner Determination in Combinatorial Auctions. “Artificial Intelligence” 2002, 135, pp. 1-54.
15. Sandholm T., Boutilier C.: Preference Elicitation in Combinatorial Auctions. In: Combinatorial Auctions. Eds P. Cramton, Y. Shoham, and R. Steinberg. MIT Press, Cambridge 2006.