Grzegorz Koloch

Tomasz Kuszewski

Tomasz Szapiro

ON STABILITY OF EDUCATIONAL RANKINGS

Abstract

The paper deals with the problem of decreasing the imprecision in multicriteria evaluation of objects. The evaluations of educational institutions presented in form of rankings are investigated. In this context the role of ranking and sources of vagueness are discussed. The general, mathematically based concept of stability of ranking is introduced and used to describe the stability properties of an educational ranking.

Keywords

Educational ranking, multicriteria evaluation, imprecision, stability.

INTRODUCTION

We consider an educational system as a set of elements (actors – households, constitutions, etc.) and relations (norms, property rights, dependence, etc). An educational system can be viewed as a complex of interrelated input-output subsystems. Each subsystem describes an actor on educational market. Inputs consist of information reflecting evaluations of tangible and intangible factors building actors' utility. This information influences costs (present or expected) and profits. We consider situations in which the actors face decision problems which can be framed as problems of choice. This view assumes that a subject is identified and that she or he can identify a set of actions (here, for simplicity, a finite set). The task is to identify the most suitable action in the given case (with suitability to be defined). Thus, we assume that each subject is able to identify a model enabling a comparison of actions and a selection of the final one. This model will be further called a *preference model*.

In the case of individuals, their choices can be explained using the concept of human capital [7]. Human capital influences individual utility which can be approximated by the private rate of return on investment in education. In this sample situation the task of the decision maker can be solved in two phases: first – the construction of a model of preference, and second – the use of this model to identify the solution. In a more mathematical setting this would mean the identification of the preference ρ in the product $X \times X$ (where X is the set of actions to be undertaken) and finding the maximal element of this set[1].

Other actors can be described similarly (although they consider different sets of options and preferences). The complexity of the system is related to the fact that the decisions of actors are interdependent. This interrelation can be illustrated by the following example. Educational demand (reflected in past households' decisions on types of studies) influence university decisions on design of educational. These offers in turn constrain the decision process of households in phase of description of households' options.

Knauff and Szapiro [10] consider three internal actors in an educational system: university management, candidates (households), and government. In the present consideration, we use more general setting and add also firms although in many educational processes firms are represented in models as external, exogenous subjects. Their objectives are summed up in Table 1 which extends the presentation introduced by Knauff and Szapiro [10].

Table 1 illustrates the overall use of ranking in an educational system from the macroeconomic point of view. In a microeconomic setting, the widespread use of rankings is even more convincing. The question arises: Is there a possibility to create a common methodology for rankings and to use this to optimize educational market decisions? Knauff and Szapiro [10] advocate negative answer to this question and recommend the use of a computer-based interactive decision supporting system assisting decision makers as a tool in flexible structuring of selection problems and manipulation with evaluation criteria following individual preference of different users of the system. In this paper we take a different perspective.

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[1] For definitions see e.g. [15, 24].

Table 1

Structure of goals of actors in general setting of educational system

Actor	Government	Households	Universities	Firms
Goal	Socially optimal outcomes of education	Optimal choice of the economic service	Efficient administration	Labor and intellectual assets
Tasks	– to provide funding – to allocate it – to inform – to participate in control	– to elicit preferences – to collect the data – to process information	– to include market orientation into managerial operation – to perform the comparative analyses	– co-funding – manifesting preferences
Tool	Ranking of educational institutions	Ranking of relevant universities	Ranking of competitors	Ranking of relevant universities
Criteria	– to reflect social preferences – to create a scheme for allocation of public funds addressed for universities	– to involve utility of a household – to aggregate the date – clarity of criteria and results	– to serve as reference in curricula design – to serve as reference in pricing programmes – to identify competitive (external) threats	– alumni (recruited staff) competence – innovations – training – expertize

We consider a situation where the use of a formalized, computer-supported procedure is not possible. In such a situation groups of experts are gathered (e.g. family in the case of households, ranking councils or committees in press rankings, etc., senate's or rector's committees at universities). They commonly agree on the sets of actions as well as on evaluation criteria, and individually evaluate actions with respect to the chosen criteria and then agree on the final evaluation which results in a choice. This process of compromising final evaluation can be formalized as an algorithm with use of weights and of maximization of weighted evaluations; this decision rule is used in the choice of the final decision. Even in this scenario the process of weight definition remains subject to non-algorithmic agreement. In the case of allocation of public funds such algorithms are designed by means of an extensive set of numerical indicators of effectiveness of educational institutions; see also [18] for more theoretical perspective. The values of an indicator is assigned to an object during a process which involves precise definitions of given indicators. However, values assigned to objects are frequently softened by averaging subjective scales or by introducing exceptions.

Algorithmic decision procedures can be used as a regulation tool. In the 1990s in Poland, an increase of the number of doctoral programmes offered by universities was a desirable target of the state policy which was achieved by including the number of Ph.D. students in the algorithm used to allocate state funding. The quick reaction of the market to this offer was also due to expectation of high return on education at this level. Not surprisingly, the number of Ph.D. skyrocketed and new programmes based on purely private funding were set up. The dynamics of this process depends heavily on the information processed by households, university managing teams, and regulating institutions.

In real-life situations, as in the example above, the processed information is incomplete in the sense that interrelated actors compare their options without knowledge of the preferences of the others. According to bounded rationality principles, see e.g. [16, 17], they simplify their description of the problem. One of the simplification strategies is related to the intuitive evaluation of the likelihood of occurrence of others choices. Such simplified evaluations are neither measured nor analytically evaluated.

The mechanisms described above lead to imprecision and may result in decisions not leading to an intended effect and thus resulting in ineffective allocation. In this paper we present a tool which can assist decision makers in the description of the range of consequences of imprecise evaluations.

The paper is organized as follows. In Section 1 typical educational rankings of importance for education are described as a background for Section 2, where we refine the remarks on subjectivity, uncertainty, and imprecision outlined above to justify the approach and the model presented in Section 3. Section 4 is focused on possible applications of the methodology introduced. The paper is concluded with remarks and references.

1. RANKINGS IN EDUCATION – THE RATIONALE AND SELECTED SCENARIOS

The relation of education to economic growth was a subject of interest of empirical economy as well as of theoretical studies. These analyses also take into account managerial perspective; for examples and reviews [13, 14, 19, 20]. Macroeconomic studies do not give clear explanation of interdependence of empirical values of variables which describe economic systems and economic growth. On the other hand, microeconomic approaches based on measurements of individual returns on educational investment do not take into account the public return on educational investment or the problem of availability of education services [5, 6, 12]. This raises the issue of public co-financing of individual educational services [4]. Public intervention in educational market influences its mechanism and requires a cautious evaluation of its impact. This turns us back to the first scenario: The public education funding allocation scenario (abbreviated further as the PEFA Scenario).

In the PEFA Scenario, we deal with the situation described partly in Introduction. In this scenario exists an administrator responsible for allocation of public funds to educational institutions according to a procedure based on an algorithm worked out by a group of experts and approved by political and social decision makers. The procedure consists of the following steps:

Step 1. The algorithm begins with the identification of a set of objects subject to evaluation and funding – this usually results from formal, legal regulations.

Step 2. The experts work out a clear understanding of educational system goals and of an implied understanding of educational effectiveness. In the next steps they construct elements of an operational procedure to evaluate the effectiveness of the system.

Step 3. The experts define a set of variables describing objects that can be measured and used to build effectiveness indicators (criteria).

Step 4. The experts define scales to be used in measurement.

Step 5. The experts recommend administrative routines to be used in the evaluation of the objects.

Step 6. Measurement of variable values.

Step 7. Data processing.

Step 8. Implementation.

In the next section this procedure will be discussed from the point of view of its reliability.

Algorithms evaluating the effectiveness of educational institutions take into account the evaluation of their academic record. The second scenario considered here – the scenario of evaluation of individual academic records (EIAR Scenario) – describes the evaluation procedure of individual academic records authored by university employees. This evaluation is based on rating articles and other research reports published in research periodicals. Rating systems used in evaluation of research use the procedure which is described below. Again, the procedure is to be worked out by a group of experts and, as previously, is to be approved by political and social decision makers. The procedure consists of the following steps:

Step 1. The experts identify research periodicals which are taken into consideration when publications are evaluated.
Step 2. The experts design a rating system for research periodicals.
Step 3. The experts define scales for rating classes.
Step 4. The experts use a system to classify periodicals in groups.

Steps 5-8 are analogous to those in the PEFA Scenario. Again, we postpone the discussion of reliability of this scenario to the next section.

Let us consider the third procedure in the rating research periodicals scenario (RRP scenario) which is crucial for EIAR Scenario and therefore also for PEFA Scenario. The rating of research periodicals is based on evaluation of their impact on the progress of the field of research. In different countries, rating classes are defined using diverse methods. They are, however, only different solution of the same problem – the problem of measurement of impact of periodicals. Usually, the number of citations is used in the measurement. In order to exclude bias, the number of citations is transformed, e.g. self citations are excluded. Another important measure relates the significance of a periodical to the number of rejection of submitted articles. Yet another important measure is related to the degree of rigor in internal procedures of acceptance of papers (e.g. blind refereeing, participation of local authors, competence of supervising committees). RRP Scenario is based on a procedure whose Steps 1-4 involve expert compromises on goals identification and method of measurement of achievement of these goals, while its Steps 5-8 follow the previous procedures and are to be approved by political and social decision makers (in this case, at the local level).

In education, other scenarios involving rankings are used. In particular, an extensive research literature deals with rankings of economics departments. Use of rankings is important for managerial reasons: they help to attract young researchers and to retain mature ones and are an important hint in solving problem of funding allocation. Rankings results build the reputation of university departments and institutes. An interesting survey of this literature was presented by Kalaitzidakis et al. [8, 9].

Iterative procedures aimed at reduction of bias impact in rankings of periodicals have been presented in research literature. The rankings' methodology in this area was originated by seminar work of Liebowitz and Palmer [11]. The procedure of evaluation in educational systems depends heavily on the system of evaluation of research outlined in Figure 2. The figure illustrates two facts. Firstly, it shows the lack of stimuli to refresh the initial pool of reference journals. Secondly, it reveals a concentration of talented researchers in best departments. These departments prove to attract researchers with the best performance measure (which is constructed using the defined pool of journals). Thus, departments are not encouraged to take risk to refresh this pool. Other departments are not sufficiently influential to do this.

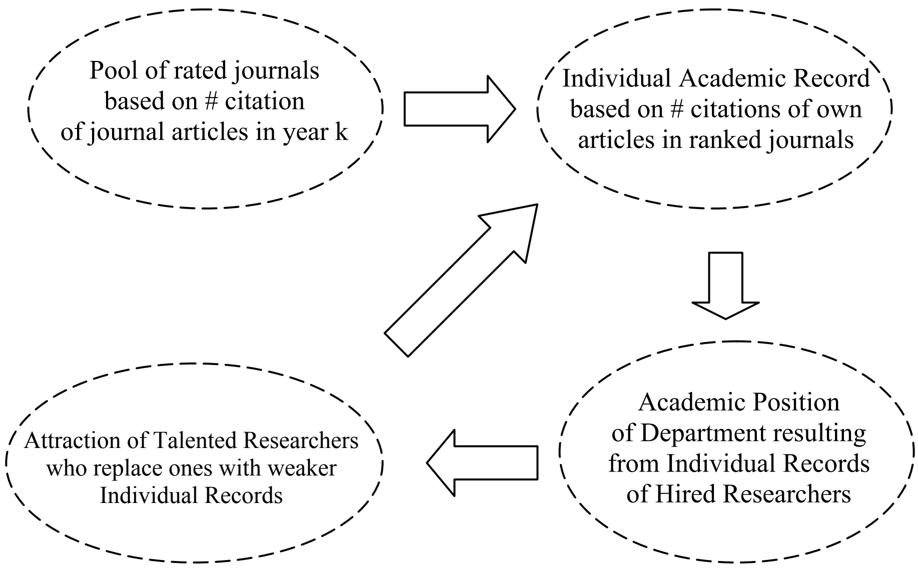

Fig. 1. Stability of traditional evaluations system

Adams et al. [1] investigate scientific influence using citations based on the data collected by the Institute for Scientific Information. They consider the top 110 US research university activities in nearly all branches of science in 1981-1999 (2.4 million papers and 18.8 million citations). Their results confirm "[…] that top institutions are more often cited by peer institutions then lower-ranked institutions are cited by their peers […]".

One of recent application of this approach together with a survey of results was presented by Amir and Knauff [2]. They rank economics departments worldwide based on the placement of their Ph.D. graduates at top--level economics departments instead of measuring the total productivity of the departments. Thus, they introduce a future orientation to the procedure of ranking of economics departments. Their results are obviously different from the earlier approaches and thus refresh results of previous rankings. However, they also show concentration of research.

As mentioned earlier, evaluation of research is an extremely important part of the evaluation of educational institutions. However, iterative analytical procedures are not used directly in such evaluations. Their results spread informally, but they form background knowledge for evaluations by experts.

A methodology (based on evaluation methods of performance in sports) of evaluation of efficiency of educational units was recently presented by Avery et al. [3]. The authors construct a ranking of US undergraduate programmes based on information about preferences of their best students. This evaluation does not use admission rate as a measure of attractiveness, since this could be manipulated by a college. Instead, the authors use independent college data, e.g. tuition discounts, alumni preferences, and use statistical inference in evaluation.

It is believed that any evaluation of teaching – programmes and methods – involves students' evaluation of their satisfaction. Weinberg et al. [2, 5] present a model used to identify the determinants of the evaluations (grades, learning measures). The model shows a weak awareness of learning effects and resulting bias.

Formal approaches are rarely used in analyses of educational processes. Complexity of these processes forces researchers to use advanced methods which are hard to communicate to wider audience. According to bounded rationality principles, precise formal descriptions and algorithms are replaced by simplification strategy and group evaluations. And so we return to rankings.

In the next section subjectivity, uncertainty, and imprecision of evaluation procedures in educational environment will be discussed in the case of PEFA, EAIR, and RRP scenarios.

2. SUBJECTIVITY, UNCERTAINTY, AND IMPRECISION IN EDUCATION

Let us now clarify the terminology. First, let us recall that we assume that solving decision problems requires its structuring. Thus, one copes with three partial problems: identification of the set of actions to be undertaken, description of one's own preferences, and finally, determination and implementation of the final decision. We consider problems of choice with the finite set of options assumed to be identified. It is assumed that the solution of the second task – the description of preferences – is solved through identification of goals of the decision maker and construction of respective set of criteria. By "criteria" we mean here the methods of evaluation of options (Figure 2).

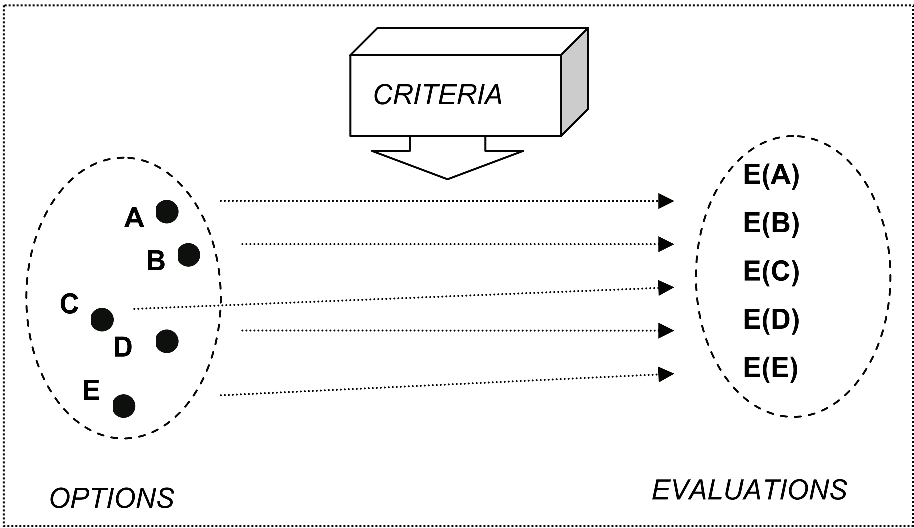

Fig. 2. The initial structure of a selection problem

In the next section we consider the situation in which evaluations are assumed to be numerical (ordered) and to measure the achievement of goals of the decision maker. In this phase of structuring, the decision space is represented by an m-dimensional (where m is the number of criteria used) vector space (of evaluations) with a quasi-order defined by single criteria values (in the weakly ordered space $(R,\geq)$). The problem boils down to determination of non-dominated evaluation; the final solutions are options with best evaluations.

Subjectivity occurs when there is no method (unique and independent on the decision maker) to define an evaluation (in numerical cases: when there is no unique definition of criteria functions or numerical scales). *Uncertainty* occurs when an environmental variable determines a set of evaluations of an option, but the determination of the proper evaluation is not possible. A problem with uncertainty is transformed into a *risk problem* when the set of environmental values is supplied with the structure of a probability space (Ω, F, P), i.e. a measure space with a measure P that satisfies the probability axioms and the probability theory is used to describe uncertainty of outcomes of the option.

Imprecision occurs when the options or their evaluations can be described numerically only as subsets (usually: intervals). A problem with imprecision is transformed into a *fuzzy problem* when imprecision is described using fuzzy set theory.

Table 2 illustrates applications of these concepts for the PEFA, EAIR, and RRP scenarios presented in Section 2.

Table 2

Subjectivity, uncertainty/risk, and imprecision/fuzziness in educational scenarios

	Subjectivity	Uncertainty/risk	Imprecision/Fuzziness
PEFA Scenario	Different expert choices of institution effectiveness criteria	Different expert valuations with respect to agreed criteria	Quantification of evaluations on qualitative scales, use of evaluation dispersion
EAIR Scenario	Different expert choices of research performance criteria	Different expert valuations with respect to agreed criteria	Quantification of evaluations on qualitative scales, use of evaluation dispersion
RRP Scenario	Different expert choices of periodical impact criteria	Different expert valuations with respect to agreed criteria	Quantification of evaluations on qualitative scales, use of evaluation dispersion

Several sources of non-deterministic factors influencing evaluations have been discussed in research literature. For example, Kalaitzidakis et al. [9] recall that rankings of periodicals are based on evaluation of past achievements while they are intended to influence future actions. Also, different periods of evaluation may result in biased comparison of rankings' results. In most rankings new periodicals and innovative research stand at lost positions.

Sample situations in Table 2 show the necessity to use rankings in scenarios outlined in the previous section and lead to the question regarding the impact of subjectivity, uncertainty/risk, and imprecision/fuzziness in educational ranking procedures. In this paper, we attempt to investigate impact of imprecision of evaluation on the final ranking; it is called the *Preference Stability Problem Analysis*.

3. SUBJECTIVITY, UNCERTAINTY, AND IMPRECISION IN RANKING DECISIONS

In this section a basic analysis of rankings is presented and then the concept of rankings stability is introduced.

Let us consider the set **O** called in the sequel the set of *objects*. Every element $\mathbf{o} \in \mathbf{O}$ is called an *object*. Let us consider m objects $\mathbf{o}_i \in \mathbf{O}$, i = 1,..., m.

Let us also consider a mapping $\mathbf{c} : \mathbf{O} \to \mathbf{R}^{n+}$ which assigns a vector $\mathbf{c}(\mathbf{o})$, $\mathbf{c}(\mathbf{o}) = (c_1(\mathbf{o}), c_2(\mathbf{o}),..., c_n(\mathbf{o}))$, to each object $\mathbf{o} \in \mathbf{O}$. $\mathbf{R}^{n+}$ denotes a space of vectors with non-negative coordinates. Each vector $\mathbf{c}(\mathbf{o}) = (c_1, c_2,..., c_n)$ is called a *vector of characteristics* of an object **o**. If $\mathbf{c}(\mathbf{o}) = \mathbf{x}$, we also say that **x** is (represents) an object **o**. Therefore two objects with the same vector of characteristics are considered the same object. Let $(\mathbf{R}^n, < , >)$ be a Euclidean space with a standard scalar product $< , >, < , > : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$.

Definition 3.1

Let $\mathbf{A} = \{\mathbf{a} \geq \mathbf{0} : <\mathbf{a}, \mathbf{1}> = 1\}$, where **1** is a unit vector in R^n. The set **A** is called a *set of weights*. The vectors $\mathbf{a} \in \mathbf{A}$ are called *weighting vectors*.

Weighting vectors have non-negative coordinates which sum up to 1.

Definition 3.2

A function $E : \mathbf{A}\times\mathbf{O}\to\mathbf{R}$ is said to be an *evaluation function*.

An evaluation function assigns to each object a value which depends on the weighting vector and on the object itself.

Throughout this paper, we assume that for every $\mathbf{a} \in \mathbf{A}$ and $\mathbf{o} \in \mathbf{O}$ the evaluation function is given by $E(\mathbf{a}, \mathbf{o}) = <\mathbf{a}, \mathbf{x}>$, where $\mathbf{x} = \mathbf{c}(\mathbf{o})$.

Definition 3.3

Let $\mathbf{a} \in \mathbf{A}$, $\mathbf{o}_1, \mathbf{o}_2 \in \mathbf{O}$ and let E, $E : \mathbf{A}\times\mathbf{O}\to\mathbf{R}$, be an evaluation function. Let $\mathbf{x} = \mathbf{c}(\mathbf{o}_1)$ and $\mathbf{y} = \mathbf{c}(\mathbf{o}_2)$. The relation ρ defined by the condition: $\mathbf{o}_1 \rho \mathbf{o}_2 \Leftrightarrow E(\mathbf{a}, \mathbf{o}_1) \geq E(\mathbf{a}, \mathbf{o}_2)$ is called a *preference*.

Definition 3.4

Let $\mathbf{o}_i \in \mathbf{O}$, $i = 1,...,m$, and let ρ be a preference. Given the set $\{\mathbf{o}_1, \mathbf{o}_2,\dots, \mathbf{o}_m\}$ of m objects, an ordered set $R = (\{\mathbf{o}_1, \mathbf{o}_2,\dots, \mathbf{o}_m\}, \rho)$ is said to be a *ranking* of objects $\mathbf{o}_i$, $i = 1,...,m$.

From Definition 3.3 it follows that the preference ρ is a *linear order*. A *strict* order induced by ρ will be denoted by ». Therefore $\mathbf{x}»\mathbf{y} \Leftrightarrow \mathbf{x}\rho\mathbf{y} \wedge \neg\mathbf{y}\rho\mathbf{x}$. If $\mathbf{x}»\mathbf{y}$ then $\mathbf{x}$ is preferred to $\mathbf{y}$. If $\mathbf{x}\rho\mathbf{y} \wedge \mathbf{y}\rho\mathbf{x}$, then $\mathbf{x}$ is equivalent to $\mathbf{y}$, and the notation $\mathbf{x}\approx\mathbf{y}$ is used in this case. The relation $\approx$ is called the *indifference*.

Let $\mathbf{x},\mathbf{y} \in \mathbf{R}^{n+}$ represent two objects and let $\mathbf{a} \in \mathbf{A}$ (i.e. $\mathbf{a}$ is a weighting vector). For every set $X \subset R^n$, the set $X^\perp$ is given by the following expression:

$$X^\perp = \{\mathbf{y}\in\mathbf{R}^n : \forall\mathbf{x}\in X <\mathbf{y},\mathbf{x}> = 0\}\equiv\{\mathbf{x}\}^\perp$$

Remark 3.5

The indifference is characterized by the following algebraic property:

$$\mathbf{x} \approx \mathbf{y} \Leftrightarrow \mathbf{y} - \mathbf{x} \in \{\mathbf{a}\}^\perp$$

Proof

From the definitions we have:

$$\mathbf{x} \approx \mathbf{y} \Leftrightarrow <\mathbf{a},\mathbf{y}> = <\mathbf{a},\mathbf{x}> \Leftrightarrow <\mathbf{a},\mathbf{y} - \mathbf{x}> = 0 \Leftrightarrow \mathbf{y}-\mathbf{x}\in \{\mathbf{a}\}^\perp$$

■

Remark 3.6

The indifference relation $\approx$ is an equivalence relation.

Proof

For all $\mathbf{x},\mathbf{y}\in\mathbf{R}^n$ we have the following implication:

$$<\mathbf{a},\mathbf{x}-\mathbf{x}> = 0 \Rightarrow \mathbf{x} - \mathbf{x}\in \{\mathbf{a}\}^\perp \Rightarrow \mathbf{x}\approx\mathbf{x}$$

Now, if $\mathbf{x} \approx \mathbf{y}$, then $\mathbf{y} - \mathbf{x}\in \{\mathbf{a}\}^\perp$, then $\mathbf{x} - \mathbf{y} = -(\mathbf{y} - \mathbf{x})\in \{\mathbf{a}\}^\perp$, hence $\mathbf{y} \approx \mathbf{x}$.

Finally, if $\mathbf{x} \approx \mathbf{y}$ and $\mathbf{y} \approx \mathbf{z}$, then $\mathbf{x} - \mathbf{y}\in \{\mathbf{a}\}^\perp$ and $\mathbf{y} - \mathbf{z}\in \{\mathbf{a}\}^\perp$ then $\mathbf{x} - \mathbf{z} = \mathbf{x} - \mathbf{y} + \mathbf{y} - \mathbf{z}\in \{\mathbf{a}\}^\perp$, hence $\mathbf{x} \approx \mathbf{z}$.

Since $\{\mathbf{a}\}^\perp$ is a vector space, then for every $\mathbf{x}\in R^{n+}$ the affine space $W_\mathbf{x}$,

$$W_\mathbf{x} \equiv \{\mathbf{x}+\alpha : \alpha\in \{\mathbf{a}\}^\perp\} \equiv \mathbf{x}+\{\mathbf{a}\}^\perp$$

is a layer of $\{\mathbf{a}\}^\perp$ in $\mathbf{R}^n$. Hence Remark 3.5 implies the following corollary.

Corollary 3.7

The indifference is algebraically characterized by the following condition:

$$\mathbf{x} \approx \mathbf{y} \Leftrightarrow W_{\mathbf{x}} = W_{\mathbf{y}}$$

Proof

By definition, the tangent space $W_{\mathbf{x}}$, i.e. $\{\mathbf{a}\}^{\perp}$, is the same as in case of $W_{\mathbf{y}}$. If $W_{\mathbf{x}} = W_{\mathbf{y}}$, i.e. $\mathbf{x}+\{\mathbf{a}\}^{\perp}=\mathbf{y}+\{\mathbf{a}\}^{\perp}$, then $\mathbf{x} = \mathbf{y} + \alpha$ for some $\alpha\in\{\mathbf{a}\}^{\perp}$. Hence $\mathbf{x} - \mathbf{y} \in\{\mathbf{a}\}^{\perp}$, which means, from Remark 3.5, that $\mathbf{x} \approx \mathbf{y}$. If $\mathbf{x} \approx \mathbf{y}$ then, from Remark 3.5, $\mathbf{y} - \mathbf{x} \in \{\mathbf{a}\}^{\perp}$, which means that $\mathbf{x} = \mathbf{y} + \alpha$ for some $\alpha\in\{\mathbf{a}\}^{\perp}$. Than, for every $\beta\in\{\mathbf{a}\}^{\perp}$, we have $\mathbf{x} + \beta = \mathbf{y} + \alpha + \beta \in W_{\mathbf{y}}$, since $\alpha + \beta \in \{\mathbf{a}\}^{\perp}$. Therefore $W_{\mathbf{x}}\subset W_{\mathbf{y}}$. Due to Remark 3.6 we have $\mathbf{x} \approx \mathbf{y} \Rightarrow \mathbf{y} \approx \mathbf{x}$, hence if we reverse the sequence of variables, then $W_{\mathbf{y}}\subset W_{\mathbf{x}}$.

■

The fact that **x** is equally preferred as **y** if and only if **x** and **y** belong to the same layer of $\{\mathbf{a}\}$ results in:

Corollary 3.8

The layers of the space $\{\mathbf{a}\}^{\perp}$ constitute classes of abstraction of the indifference relation $\approx$.

Proof

In Remark 3.6 it is noticed that $\approx$ is an equivalence. Furthermore $\mathbf{x} + \{\mathbf{a}\}^{\perp}=\{\alpha\in R^n : \alpha = \mathbf{x} + \beta$ for some $\beta\in\{\mathbf{a}\}^{\perp}\} = \{\alpha \in \mathbf{R}^n : \alpha - \mathbf{x}\in\{\mathbf{a}\}^{\perp}\}$ – is the class of abstraction of **x** with respect to relation $\approx$.

■

If $\mathbf{x}»\mathbf{y}$ then the layer $W_{\mathbf{x}}$ is said to be positioned above layer $W_{\mathbf{y}}$. Let $\cos(\mathbf{a},\mathbf{b})$ denote the cosine of the angle between the vectors **a** and **b** in $(\mathbf{R}^n, < , >)$ and let $|\mathbf{a}|$ denote a canonical norm in $(\mathbf{R}^n, < , >)$, i.e. $|\mathbf{a}| = <\mathbf{a},\mathbf{a}>^{1/2}$. Remark 3.5 and Corollary 3.7 can be rephrased as follows:

Remark 3.9

The indifference is geometrically characterized by the following condition:

$$\mathbf{x} \approx \mathbf{y} \Leftrightarrow \cos(\mathbf{a},\mathbf{y}-\mathbf{x}) = 0$$

Proof

$$\mathbf{x} \approx \mathbf{y} \Leftrightarrow <\mathbf{a},\mathbf{y} - \mathbf{x}> = 0 \Leftrightarrow |\mathbf{a}|\,|\mathbf{b}|\cos(\mathbf{a},\mathbf{y}-\mathbf{x}) = 0$$

■

In the same manner one shows that **x**»**y** ⇔ cos (**a**,**y**–**x**) < 0. Therefore W_x is positioned above W_y if and only if cos (**a**,**y**–**x**) < 0. If we assign to each layer W_x a number $\xi = \langle \mathbf{a},\alpha \rangle$ for some $\alpha \in W_x$, then higher positioned layers are associated with bigger numbers ξ. The number associated with a given layer is equal to the evaluation of objects forming this layer.

Let us consider two objects $\mathbf{x} \in W_x$ and $\mathbf{y} \in W_y$. The positioning of layers in $\mathbf{R}^n$ and therefore the positioning of objects **x** and **y** in a ranking depends on the weighting vector **a**. Let us assume that **a** is such, that **x**»**y**. The following question arises: how can the weighting vector **a** be changed (to **a**'), so that for **a**' the relation **x**»**y** still holds. First, let us notice that changes of the form: **a**'= q**a**, $q > 0$, $q \neq 1$ are not feasible, since it was assumed that $\langle \mathbf{a},\mathbf{1} \rangle = 1$ and $\langle q\mathbf{a},\mathbf{1} \rangle = q\langle \mathbf{a},\mathbf{1} \rangle \neq 1$ for $q \neq 1$. Additionally, such changes would preserve the space $\{\mathbf{a}\}^\perp$, thus $\{\mathbf{a}\}^\perp = \{\mathbf{a}'\}^\perp$, which results in the same positioning of layers of $\{\mathbf{a}\}^\perp$ and therefore the same positioning of objects as in the initial ranking.

If **x**»**y** then, due to Remark 3.9, cos(**a**,**y** – **x**) < 0. If after the change of **a** to **a'** this inequality still holds, i.e. cos(**a'**,**y**–**x**) < 0, then we still have **x**»**y**. Therefore any change of **a** to **a'** that preserves the sign of cos(**a**',**y** – **x**), i.e. keeps the angle between vectors **a**' and **y** – **x** within the interval (π/2,π] and thus preserves the relation **x** » **y**. The boundary case is cos(**a'**,**y** – **x**) = 0, i.e. **x** ≈ **y**. Assume **a**' is chosen in such a way that the boundary case applies.

Corollary 3.10

For any weighting vector **a**'' given by **a**'' = p**a** + q**a**', where $p,q \geq 0$, $p + q = 1$, $q \neq 1$, the order of **x** and **y** is preserved, i.e. **x** » **y**.

Proof

$$\cos(\mathbf{a}'',\mathbf{y}-\mathbf{x}) = (p\langle \mathbf{a},\mathbf{y}-\mathbf{x}\rangle + q\langle \mathbf{a}',\mathbf{y}-\mathbf{x}\rangle)/(|\mathbf{a}''|\,|\mathbf{y}-\mathbf{x}|) =$$
$$= p\langle \mathbf{a},\mathbf{y}-\mathbf{x}\rangle/(|\mathbf{a}''|\,|\mathbf{y}-\mathbf{x}|)<0$$

■

Corollary 3.10 states that if **a** is a current weighting vector for which **x** » **y**, and **a**' makes **x** and **y** equivalent, then for any vector of the form **a**'' the angle between the vectors **x** and **y** is in (π/2,π]. Therefore the angle between **y** – **x** and **a** or, more specifically, between **y** – **x** and $\{\mathbf{a}\}^\perp$ represents area where changes of **a** preserve ranking. This observation suggests that the angle between the vectors **y** – **x** and **a**, or between **y** – **x** and its orthogonal projection on $\{\mathbf{a}\}^\perp$ could be used as an indicator of the stability of rankings outcomes. The investigation of this issue is left for future studies.

Two other notions of rankings stability will now be introduced: the notion of k-stability and k-ε stability. In the next section a numerical procedure will be implemented which, for a given ranking, calculates the numbers *k* and ε for which the ranking is k-ε stable.

Let us recall that the positioning of objects in a ranking R depends on the chosen weighting vector $\mathbf{a}\in\mathbf{A}$. This relation will be denoted by $R_{\mathbf{a}}$.

Definition 3.11

Let $\mathbf{a}\in\mathbf{A}$ and let $R_{\mathbf{a}}$ be a ranking. The set $D_R \subseteq \mathbf{A}$ defined as: $D_{R\mathbf{a}} = \{\mathbf{d}\in\mathbf{A} : R_{\mathbf{d}} = R_{\mathbf{a}}\}$ is said to be *a stability set* of the ranking $R_{\mathbf{a}}$.

The stability set of a ranking $R_{\mathbf{a}}$ consists therefore of the weighting vectors $\mathbf{d}\in\mathbf{A}$ for which the order of objects in $R_{\mathbf{d}}$ is the same as in $R_{\mathbf{a}}$. Let $R_{\mathbf{a}}|_k$ denote a ranking consisting of k objects from $R_{\mathbf{a}}$ which have highest evaluations in $R_{\mathbf{a}}$.

Definition 3.12

Let $R_{\mathbf{a}}$ be a ranking. If for every $\mathbf{d}\in\mathbf{A}$ we have $R_{\mathbf{d}}|_k = R_{\mathbf{a}}|_k$ then the ranking $R_{\mathbf{a}}$ is said to be k-*stable*.

Note that if $D_{R\mathbf{a}} = \mathbf{A}$ then the order of objects in $R_{\mathbf{a}}$ is the same regardless of which $\mathbf{a} \in \mathbf{A}$ is chosen. If, however, only the order of the first k objects in $R_{\mathbf{a}}$ does not depend on **a** then $R_{\mathbf{a}}$ is k-stable.

Definition 3.13

Let $R_{\mathbf{a}}$ be a ranking. A set $D_{R\mathbf{a}}|_k \subseteq \mathbf{A}$ defined as: $D_{R\mathbf{a}}|_k = \{\mathbf{d} \in \mathbf{A} : R_{\mathbf{d}}|_k = R_{\mathbf{a}}|_k\}$ is said to be the *k-stability set* of a $R_{\mathbf{a}}$.

The k-stability set $D_{R\mathbf{a}}|_k$ of a ranking $R_{\mathbf{a}}$ consists therefore of the weighting vectors $\mathbf{d}\in\mathbf{A}$ for which the order of the first k elements in $R_{\mathbf{d}}$ is the same as in $R_{\mathbf{a}}$.

Let $\varepsilon \geq 0$. Let $\mathbf{J}(\varepsilon) = \{\varepsilon_1, \varepsilon_2,..., \varepsilon_n\}$ denote an n-element set whose elements are equal to ε or $-\varepsilon$, i.e. $\varepsilon_j \in\{\varepsilon,-\varepsilon\}$, $j = 1,...,n$. Additionally, let us assume that the elements of $\mathbf{J}(\varepsilon)$ sum up to 0. If $\varepsilon > 0$ then such combination of elements ε_j exists for even numbers n only. Therefore for odd numbers n any element of $\mathbf{J}(\varepsilon)$ is assumed to be 0.

Let $\varepsilon \geq 0$. Let us consider the *group* $P_{\mathbf{J}(\varepsilon)}$ *of permutations* of $\mathbf{J}(\varepsilon)$, i.e. the set of all bijective functions $\sigma_i : \mathbf{J}(\varepsilon)\to\mathbf{J}(\varepsilon)$. We have $P_{\mathbf{J}(\varepsilon)} = \{\sigma_i, i = 1,2,...,n!\}$.

From now on, the set $\mathbf{J}(\varepsilon)$ and the elements of $P_{\mathbf{J}(\varepsilon)}$ will be represented as vectors, so $\sigma_i\in P_{\mathbf{J}(\varepsilon)}$ is the i-th permutation of vectors' $\mathbf{J}(\varepsilon)$ co-ordinates.

Definition 3.14

Let $\mathbf{a}\in\mathbf{A}$ and let $P_{\mathbf{J}(\varepsilon)}$ be the group of permutations of $\mathbf{J}(\varepsilon)$. A number $\varepsilon \geq 0$ satisfying $\mathbf{a}+\sigma\in\mathbf{A}$ for every $\sigma\in P_{\mathbf{J}(\varepsilon)}$ is said to be *feasible*.

Definition 3.15

Let $\varepsilon \geq 0$ be feasible and let $P_{\mathbf{J}(\varepsilon)}$ be the group of permutations of $\mathbf{J}(\varepsilon)$. Each vector $\sigma\in P_{\mathbf{J}(\varepsilon)}$ is said to be an *ε-perturbation*. Let $\mathbf{a}\in\mathbf{A}$. For every ε-perturbation $\sigma\in P_{\mathbf{J}(\varepsilon)}$ the vector $\mathbf{a'} = \mathbf{a} + \sigma$ is said to be an *ε-perturbed weighting vector* **a**.

Definition 3.16

Let $\mathbf{a}\in\mathbf{A}$ and let $R_{\mathbf{a}}$ be a ranking. Let **a**' be an ε-perturbed weighting vector **a**. If every $\mathbf{a}'\in D_{R\mathbf{a}}$ then $R_{\mathbf{a}}$ is called *ε-robust*.

Let us note that a ranking $R_{\mathbf{a}}$ is ε-robust if the ranking $R_{\mathbf{a}+\sigma}$ preserves the order of objects for all ε-perturbations $\sigma\in P_{\mathbf{J}(\varepsilon)}$.

Corollary 3.17

Let $\mathbf{a}\in A$ and let $R_{\mathbf{a}}$ be a ranking. Let $\varepsilon \geq 0$ be feasible. If $R_{\mathbf{a}}$ is ε-robust then $R_{\mathbf{a}}$ is δ-robust for all feasible $0 \leq \delta \leq \varepsilon$.

Proof

Let $\mathbf{x},\mathbf{y}\in R_{\mathbf{a}}$ and let $\mathbf{x} \gg \mathbf{y}$. Let $\sigma\in P_{\mathbf{J}(\varepsilon)}$. If $R_{\mathbf{a}}$ is ε-robust then $\mathbf{a}' = \mathbf{a} + \sigma\in D_{R\mathbf{a}}$. Let $\mathbf{I} = \sigma/\varepsilon$. If $\mathbf{a}'\in D_{R\mathbf{a}}$ then $\mathbf{x}\gg\mathbf{y}$ holds for $R_{\mathbf{a}'}$, hence $\langle\mathbf{a}',\mathbf{y}-\mathbf{x}\rangle = \langle\mathbf{a},\mathbf{y}-\mathbf{x}\rangle + \varepsilon\langle\mathbf{I},\mathbf{y}-\mathbf{x}\rangle < 0$. Let $\phi = \mathbf{I}\delta$ for some feasible $0 \leq \delta \leq \varepsilon$. Note that $\phi\in P_{\mathbf{J}(\delta)}$. Let $\mathbf{a}'' = \mathbf{a} + \phi$. Note that $\langle\mathbf{a}'',\mathbf{y}-\mathbf{x}\rangle = \langle\mathbf{a},\mathbf{y}-\mathbf{x}\rangle + \langle\phi, \mathbf{y} - \mathbf{x}\rangle = \langle\mathbf{a},\mathbf{y}-\mathbf{x}\rangle + \delta\langle\mathbf{I},\mathbf{y}-\mathbf{x}\rangle < 0$. This is because $\langle\mathbf{a},\mathbf{y}-\mathbf{x}\rangle < 0$ and $\delta \leq \varepsilon$. If $\langle\mathbf{I},\mathbf{y}-\mathbf{x}\rangle \leq 0$ then $\langle\mathbf{a},\mathbf{y}-\mathbf{x}\rangle + \delta\langle\mathbf{I},\mathbf{y}-\mathbf{x}\rangle < 0$ for all $\delta \geq 0$. If $\langle\mathbf{I},\mathbf{y}-\mathbf{x}\rangle > 0$ then $\delta\langle\mathbf{I},\mathbf{y}-\mathbf{x}\rangle \leq \varepsilon\langle\mathbf{I},\mathbf{y}-\mathbf{x}\rangle$, hence $\langle\mathbf{a},\mathbf{y}-\mathbf{x}\rangle + \delta\langle\mathbf{I},\mathbf{y}-\mathbf{x}\rangle \leq \langle\mathbf{a},\mathbf{y}-\mathbf{x}\rangle + \varepsilon\langle\mathbf{I},\mathbf{y}-\mathbf{x}\rangle < 0$. This means that $\mathbf{x} \gg \mathbf{y}$ holds for $\mathbf{a}' = \mathbf{a} + \phi : \phi = \mathbf{I}\delta$. As noted above, $\phi\in P_{\mathbf{J}(\delta)}$. $\sigma\in P_{\mathbf{J}(\varepsilon)}$ was arbitrary and we required $\phi = \mathbf{I}\delta = \sigma(\delta/\varepsilon)$, hence if σ is any permutation of $\mathbf{J}(\varepsilon)$ then ϕ is the corresponding permutation of $\mathbf{J}(\delta)$. This means that $\mathbf{x} \gg \mathbf{y}$ holds for all $\mathbf{a}' = \mathbf{a} + \phi : \phi\in P_{\mathbf{J}(\delta)}$, that is, that $R_{\mathbf{a}}$ is δ-robust.

■

Remark 3.18

The requirement for δ to be feasible in Corollary 3.18 is not necessary.

Proof

It was assumed in Corollary 3.14 that $\varepsilon \geq 0$ is feasible and that $0 \leq \delta \leq \varepsilon$. Let $\mathbf{a} \in \mathbf{A}$, $\varepsilon \geq 0$ and $\sigma \in P_{\mathbf{J}(\varepsilon)}$. Let $\mathbf{a}' = \mathbf{a} + \sigma$. A number $\varepsilon \geq 0$ is feasible if for all $\sigma \in P_{\mathbf{J}(\varepsilon)}$ the following two conditions hold: $<\mathbf{a} + \sigma, \mathbf{1}> = 1$ and $\mathbf{a} + \sigma \geq 0$. Let us observe that the first condition holds by definition of $\mathbf{J}(\varepsilon)$. The second condition holds if $\varepsilon \leq \min(\min(a_1,...,a_n), 1 - \max(a_1,...,a_n)) = r$, where $(a_1,...,a_n) = \mathbf{a}$. If $0 \leq \delta \leq \varepsilon$ and $\mathbf{a}' = \mathbf{a} + \phi : \phi \in P_{\mathbf{J}(\delta)}$, then, the analogous first condition for δ holds by definition of $\mathbf{J}(\delta)$ and the second one follows from the fact that $\delta \leq \varepsilon \leq r$.

■

The notion of k-stability and ε-robustness leads to their following composition.

Definition 3.19

Let $\mathbf{a} \in \mathbf{A}$ and let $R_\mathbf{a}$ be a ranking. Let $\mathbf{a}'$ be an ε-perturbed weighting vector $\mathbf{a}$. If every $\mathbf{a}' \in D_R|_k$ then $R_\mathbf{a}$ is called *k-ε-stable*. Let us note that a ranking $R_\mathbf{a}$ is k-ε-stable if the ranking $R_{\mathbf{a}+\sigma}$ preserves the order of k objects with the highest evaluations for all ε-perturbations $\sigma \in P_{\mathbf{J}(\varepsilon)}$.

In this section two concepts of ranking stability have been introduced. On the basis of these concepts the stability of an educational ranking will be investigated in the following section.

4. CASE STUDY AND SIMULATION OF PREFERENCE STABILITY

In this section an empirical study of a ranking of economic and business schools (both public and private) in Poland is conducted. The ranking under consideration was published as a cover story in 2004 by the nationwide periodical "Polityka" and hundred schools were investigated. The ranking procedure was constructed by a panel of experts invited by "Polityka". The data on universities were taken from Ministry of Education and National Research Committee and from own surveys.

"Polityka's" ranking is of the same form as the rankings considered in Section 3. For the comparison of universities six aggregate criteria were taken into account: *academic position*, *academic staff potential*, *pro-student orientation*, *contacts with social and business environment including international relations*, *selectivity,* and *infrastructure*. Therefore every object

o (university) was characterized by a six-element vector **x** = **c**(**o**). For each university, appropriate values were assigned to each of the six criteria. These values constituted co-ordinates of **x**. The team of experts agreed on a weighting vector **a** whose co-ordinates reflected, as far as experts' subjective perception is concerned, the relative importance of criteria. The employed weights were: 25%, 20%, 20%, 15%, 10%, and 10%, respectively, i.e. **a** = (0.25, 0.2, 0.2, 0.15, 0.1, 0.1).

The declared mission of the "Polityka", ranking was created to assist households (and candidates) in choice of university which would closely match their expectations. "Polityka" informed that weights used to rank universities did not follow any scientific survey, but were a compromise of subjective experts' judgements. In order to enable readers to individually process the data and to modify published results, "Polityka" provided its audience not only with final results, but also with criteria, values of experts' measurements and with the weight system agreed and used by them. The readers were also encouraged to modify themselves experts' weights in order to better reflect own subjective preferences and thus to arrive to own results. "Polityka" warned however that a reader who is inexperienced in the field may find the comparison of universities difficult. The difficulty results from several reasons: many schools use similar names, their offer is difficult to evaluate, and the concepts used in criteria are not always easy to understand for the wide audience (the cognitive barrier). A user who has problems with strict numeric correction of experts' weights turns to interval preferences in order to better elicit her preference. This means that although she or he cannot strictly determine the weights, she or he can define intervals in which weights can be found. If the ranking remains the same for all weights from this interval (robustness of a ranking) then users' weighting vector is equivalent to the one used by experts. In our parlance this means that the order of objects in the ranking is *robust* with respect to *perturbations* in weights from user-defined interval.

The concept of stability presented in Section 3 allows assist users of rankings who may experience cognitive barriers in evaluations (e.g. resulting from the use of technical language in criteria description) and who attempt to elicit interval preferences. It is possible to provide the user with the answer to the question: How big the difference between his weights and the experts' weights (as described in Section 3) can become and still preserve the ordering of universities by the experts?

An application which for given $k > 0$ finds maximal ε such that the considered ranking $R_{\mathbf{a}}$ is k-ε stable was implemented in the Matlab environment. This application generates, for a given weighting vector $\mathbf{a} \in \mathbf{A}$ (experts' weights), feasible ε-disturbed weighting vectors, i.e. vectors of the form: $\mathbf{a}' = \mathbf{a} + \sigma : \sigma \in P_{\mathbf{J}(\varepsilon)}$, $\varepsilon \geq 0$ – feasible, and checks if for all[2] $\sigma \in P_{\mathbf{J}(\varepsilon)}$ the order of objects in the ranking (universities) is preserved, that is if $\mathbf{a}' \in D_{R\mathbf{a}}|_k$.

A direct application of definitions from Section 3 shows that the ranking of "Polityka" is 2-ε stable for any feasible ε (this is denoted in the Table 3 by ε-robust). This result shows that for every feasible[3] change of experts' weights (according to definitions from Section 3) the sequence of the first two universities in the ranking remains the same. In this ranking the relative positioning of the first two universities does not change: the first university will always be better ranked then the second one. However, these statements are not true for *every* weighting vector.

Another use of the procedure is recommended in search of a leader in the pool of ranked objects when visible disagreement of experts on weights' definitions occurs. This situation puts credibility of weighting in question and may lead to failure of search of a leader. The 2-ε stability of ranking means that if their deviation from the weighting vector identifying the leader is feasible then the order of the first two schools is correct.

For $k = 3$ the ranking turns out to be 3-0.077 stable, which means that if the weighting vector is in the 0.077-neighborhood of experts' weights then the ranking remains the same.

In the case of $k = 4$ the ranking is not k-ε stable for any $\varepsilon \geq 0.01$. This is due to the fact that the third and the fourth schools have equal evaluations for the weighting vector assigned by the experts. In this case the result is not stable. If, however, the fourth university drops out then the ranking becomes 4-0.052 stable. The interpretation is the same as in the case of 3-0.077 stability. For $k = 5$ the ranking is still stable for $\varepsilon = 0.052$, that is it is 5-0.052 stable. For $k = 6$ we have 6-0.034 stability and for $k = 7$ we have 7-0.033 stability.

In general, stability drops as new objects arrive, but up to five objects the weights can be distorted by up to 5 pp, which in the case of a 10% weight constitutes half of this value. A summary of the results is presented in Table 3 (the left part).

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[2] In fact, not all permutations are considered, but only these, which imply different ε-perturbed vectors.

[3] Notice, that feasible ε must satisfy $\varepsilon \leq 0.1$, since 10% is the minimal weight assigned by experts (otherwise we end up with negative weights).

Apart from the evaluation of stability of the whole ranking, calculations were done for universities located in Warsaw only. Table 3 (the right part) presents the results for Warsaw schools. In this case, contrary to the overall results, the ranking is far more stable.

Table 3

k-ε stability of "Polityka" educational ranking for $k = 1,2,\ldots,7$

K	ε	k	ε
1	ε-robust (10%)	1	ε-robust (10%)
2	ε-robust (10%)	2	ε-robust (10%)
3	0.077 (7.7%)	3	ε-robust (10%)
4	0.052 (5.5%)*	4	ε-robust (10%)
5	0.052 (5.2%)	5	ε-robust (10%)
6	0.034 (3.4%)	6	ε-robust (10%)
7	0.033 (3.3%)	7	ε-robust (10%)

Results for whole ranking (left) and for Warsaw only (right).
* Fourth object dropped out.

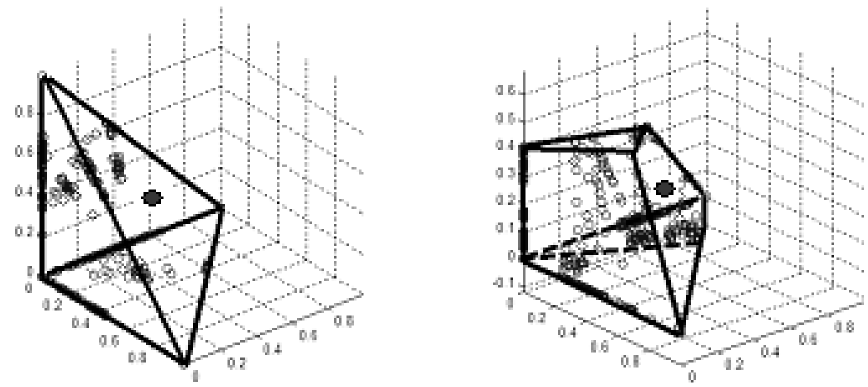

Fig. 3. Plot of ε-perturbed weighting vectors for which the order of objects in the ranking is preserved in the case of a less (left) and more (right) stable ranking (black lines are shown for visualization purposes)

Let us now present the graphical interpretation of the concept of ranking stability. Assume that for a given n-dimensional weighting vector **a**, say **a** = (1/3,1/3,1/3), the ranking is k-ε stable for some k. We take all ε-perturbed weighting vectors **a**' (for these vectors the order of objects in the considered ranking is preserved) and plot them. If $n > 3$, we also have to choose three out of n available co-ordinates and project all ε-perturbed weighting vectors **a**' on a corresponding 3-dimensional space. If $n \leq 3$, as it is in our case, we simply plot these vectors. Figure 3 presents the perturbed vectors in the case of a more (to the left) and a less (to the right) stable ranking. The red dot represents the vector **a**. One can see that in the case of a less stable ranking, the ε-perturbed weighting vectors **a**' are concentrated in greater degree than in the case of a more stable ranking. For $n > 3$, one can observe projections of ε-perturbed weighting vectors **a**' on axes and the conclusion remains the same. This observation allows us to visualize the concept of ranking stability in a simple way.

CONCLUDING REMARKS

In the paper educational rankings were discussed and mathematically described. First, the great importance of such rankings was shown: three scenarios were discussed in detail. It appears that educational decisions deal necessarily with factors which are difficult to represent in formal, mathematical way such as subjectivity, uncertainty, and imprecision. Ranking is a widely used analytical method which can tackle soft properties in a procedural way. Rankings are simple and admit easy interpretations, but they also involve a good deal of subjectivity. Thus, they can lead to different results for different users depending on the choice of weighting values.

A mathematical description of rankings allows for construction of an algorithm facilitating a search of the preferred object – an educational institution or a programme. The case study with real-life data shows that the methodology presented provides the user with meaningful information assisting him in overcoming cognitive barriers and in her preference elicitation.

Without loss of generality and for the sake of simplicity of demonstrations the argument was kept at a simplified level. However, the methodology presented here can be extended, in a natural way, in several directions. Firstly, more sophisticated mathematical models can be developed, e.g. models including continuous variables or based on set theoretical analysis of interval preferences. Secondly, the concept of 2-ε-stability can be extended to include additional questions related to management of educational institutions, e.g. what are conditions for an educational unit sufficient to remain in a group of ranking leaders? Finally, lack of space forced us to leave for future presentations the reverse problem of finding the range of *forced preference change* which would lead to ranking result compatible with university preferences.

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