Lech Kruś

COMPUTER-BASED SUPPORT OF MULITCRITERIA COOPERATIVE DECISIONS – SOME PROBLEMS AND IDEAS

Abstract

The paper deals with a class of cooperation problems related to the decision situations in which several parties consider participation in a joint enterprise. Typical questions arise: When is cooperation beneficial? What should be the fair engagement of the parties in the enterprise and the fair allocation of benefits among them? Problems related to construction of a computer-based system supporting the decisions analysis made by the parties are discussed.

To construct the system, first a substantive model describing the decision situation has to be built. The model includes specifications of decision variables, exogenous variables, output quantities, criteria of the parties, constraints defining the set of admissible decisions, mathematical relations enabling derivation of the output quantities, and the criteria as dependent on given decision and exogenous variables. It is assumed that each party has given its own set of criteria, in general different, and has independent preferences over the criteria.

The sovereignty of the decision makers representing the parties is assumed, i.e. the decision makers are fully responsible for the decisions they make. The computer-based system is only a tool aiding the analysis of decision situations and a tool facilitating the consensus seeking.

The bargaining game model extended to the multicriteria case is used to describe the cooperation problems. Solution concepts formulated for the classical bargaining problem, such as the egalitarian solution and those proposed by Nash, Raiffa-Kalai-Smorodisky, and Imai are considered. Extensions of the solutions to the multicriteria bargaining problem are presented. Properties of solutions are discussed. The solution concepts can be used to derive mediation proposals generated in the system and presented to the parties for analysis in iterative procedures.

The application area includes, among others, the analysis of cooperation in the case of innovative activities, education systems, and cost allocation problems.

Keywords

Cooperative decisions, multicriteria analysis, mediation support, computer-based systems.

INTRODUCTION

The paper deals with cooperation problems related to the decision situations in which several parties consider participation in a joint enterprise. Questions arise: When the cooperation is beneficial? What should be the fair engagement of the parties in the enterprise and the fair allocation of benefits among them? In the paper problems related to the construction of computer--based systems supporting the decisions analysis made by the parties are discussed. Such systems can be built with the application of the control theory methods, the mathematical modeling technics, the optimization procedures, and the modern advanced information technology.

It is assumed that each party has a given set of criteria, in general different, and independent preferences over the criteria. Sovereignty of the decision makers representing the parties is assumed, i.e. the decision makers are fully responsible for the decisions they made. The computer-based system is only a tool aiding the analysis of the decision situations and facilitating the consensus sought.

In practice, cooperation problems are solved through a negotiation process. Before the negotiations each party should derive its Best Alternative to Negotiated Agreement (abbreviated further as BATNA) – a concept introduced by Fisher and Ury [1]. In the negotiations a party can compare the analyzed proposals to the derived BATNA and can evaluate its possible benefit from the cooperation.

The cooperation situations are modeled in the game theory: as the so--called bargaining problem for two and more players. The classical axiomatic theory of bargaining has been developed by Nash [23], Raiffa [27], Kalai and Smorodinsky [3], Roth [29], Thomson [31], and many others. The classical bargaining problem in the case of two and many issues is formulated theoretically in terms of utilities as a pair (S, d). Two parties (players) can reach any of the payoffs from the agreement set S, if they agree. The disagreement point d defines the payoffs of the players in the case when they do not reach such an agreement. It is derived on the basis of the BATNA concept; in particular it can be the status quo point.

A solution of the bargaining problem is considered as a method of chosing a point from the set S, accepted by rational players. Different solution concepts are proposed under different sets of axioms (assumed properties describing the feeling of fairness) the solution should fulfill. The argumentation for acceptation of the solution concept by the players is the following: If rational players agree on a selected set of axioms-principles and accept them as fair, why they should not accept the solution concept which fulfills the axioms?

In the paper the cooperative game model is used to describe the cooperation problems in the case of multicriteria payoffs of players. Solution concepts for the classical bargaining problem, like the egalitarian solution and those proposed by Nash and Raiffa, Kalai, and Smorodisky are considered. The solution concepts extended to the multicriteria case can be used to derive mediation proposals generated in the system and presented to the parties for analysis in iterative procedures. The presented mediation support with use of the computer-based system has been inspired by the single negotiation procedure frequently applied in international negotiations (see [28]).

The application area includes among others analysis of cooperation in the case of innovative activities, educational systems and cost allocation problems. The references include selected papers dealing with computer-based support in negotiations [2, 5-9, 11-18], related to the multicriteria decision analysis [10-13, 16, 18, 25, 26, 34-36], to the utility function approach [4, 19--22, 29, 30, 32, 33] and to the game theory, as mentioned above.

1. THE IDEA OF A COMPUTER-BASED SYSTEM

The proposed system includes a model representation, modules supporting unilateral analysis made by decision makers (DMs), a module generating mediation proposals, as well as modules including an optimization solver, respective data bases, procedures enabling interactive sessions realizing the mediation procedure, and a graphical interface.

The model describing the decision situation of the parties is the basis for decision analysis and is constructed by a system analyst with use of the gathered information according to the rules of system sciences. It includes the specification of decision variables, exogenous variables, output quantities, criteria, and model relations. The model parameters are identified on the basis of the collected data and should be verified as well as validated. Therefore modules containing respective data base, model editor, procedures for estimation of model parameters, and for model verification are included in the system.

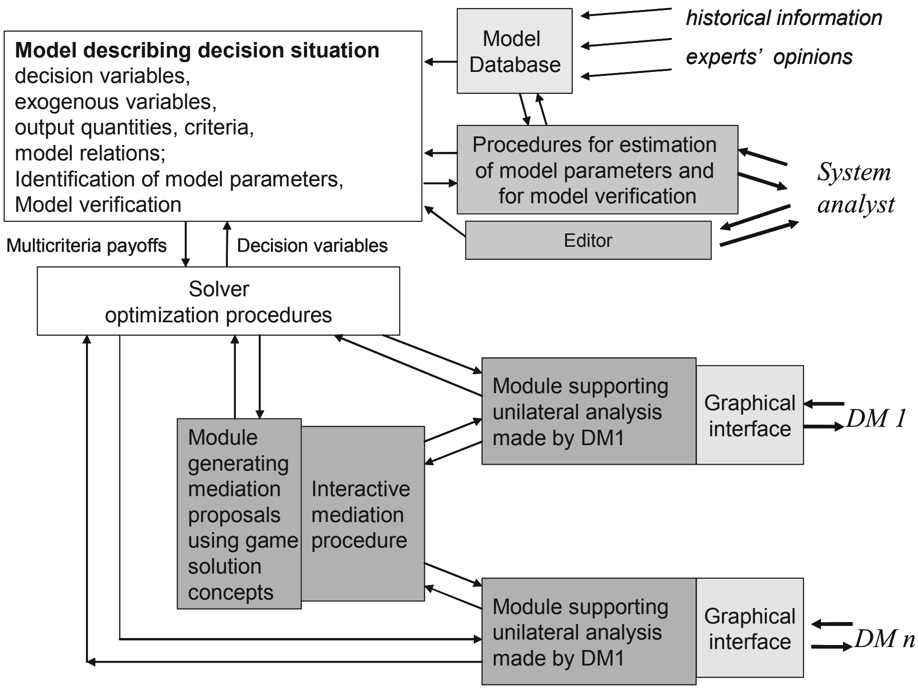

Fig. 1. General idea of a computer-based system supporting cooperative decisions

The module supporting unilateral analysis enables each DM to obtain independently information about possible multicriteria payoffs for the assumed scenario and to look for the preferred option. The analysis is made interactively.

The system generates also mediation proposals. The mediation proposals are derived with use of selected solution concepts of the theory of cooperative games and on the basis of the preferences expressed by the DMs. The mediation proposals are generated and presented to the DMs within a special mediation procedure.

Optimization technics are used in the system: in the procedures of multicriteria analysis, in the modules supporting individual unilateral analysis, and in the module generating mediation proposals to calculate game solution concepts. The respective optimization procedures are included in the solver module.

2. MODEL

To describe the cooperation situation an extension of the classical bargaining problem is considered in the case of n decision makers (DMs) further called players. For each DM (player), $i = 1,...,n$, we define:

A vector of decision variables $x_i \in \mathbf{R}^{k_i}$, where k_i is the number of variables of the player i,

– a vector of criteria (to be maximized) $y_i \in \mathbf{R}^{m_i}$, where m_i is the number of criteria of the player i.

A mathematical model describing the decision situation is given with:

– a set of admissible decisions $X^0 \subset R^K$, where $R^K = R^{k_1} \times ... \times R^{k_n}$ is the space of decisions of all the players,

– a space of payoffs of all the players $R^M = R^{m_1} \times ... \times R^{m_n}$, it is the Cartesian product of the multicriteria spaces of the players' payoff,

– a function $F : X^0 \to R^M$ defining vectors of the players' payoffs for given values of decision variables. If the function F is continuous and the set X^0 is compact, then the set of attainable payoffs $Y^0 = F(X^0)$ is also compact.

Let each player have his own reservation point $d_i \in R^{m_i}$ assumed in his multicriteria space on the basis of the BATNA concept. Then the Multi-criteria Bargaining Problem (MBP) can be defined by the disagreement point $d = (d_1, ...d_n) \in R^M$ and the agreement set S consisting of the points of the set $Y^0 \subset R^M$ dominating the point d. Each point of the agreement set can be reached if all the players agree, i.e the problem consists in the selection of the point from the set S, which could be accepted by all the players.

Remarks to the problem formulation:

1. Each DM (player) has his own set of criteria, in general different.
2. A set of attainable payoffs is considered in the space R^M which is the Cartesian product of individual multicriteria spaces of the players.
3. The set of attainable payoffs $Y^0 \subset R^M$ is in general not given explicitly.
4. The multicriteria payoffs of each player can be derived by means of a computer-based system for the given values of the decision variables of all the players, using model relations.

An example of the multicriteria bargaining problem is presented in Figure 2 in the case of two players. Player 1 has criteria y_{11} and y_{12}, player 2 has only one criterion y_{21}. In the three-dimensional space of criteria an agreement set S and a disagreement point d are shown.

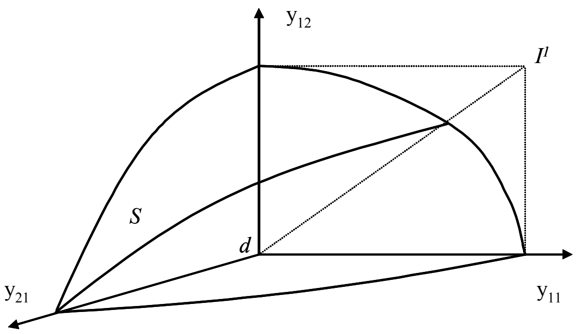

Fig. 2. An example of the multicriteria bargaining problem

The disagreement point d is based on BATNA of each player. In general case the derivation of the disagreement point may also require additional multicriteria analysis performed by each player. The agreement set S is defined by model relations, and in general is not known explicitly. The ideal point in the criteria space of the player 1 is also shown denoted by I^1.

3. UNILATERAL ANALYSIS

Each player starts from unilateral interactive multicriteria analysis of the problem. During the analysis he can obtain information about possible outcomes for different assumptions about his preferences. He has also to make assumptions about the counterplayers' outcomes or counterplayers' preferences. The analysis can be made by applying the reference point approach developed by Wierzbicki [31-33] with use of the order approximation functions. According to the approach, each player assumes reference points in the space of his criteria and the system generates the respective outcomes which are Pareto optimal in the set S. For some number of reference points assumed by a player, a characterization of the Pareto frontier of the set S can be obtained.

The outcomes characterizing the Pareto frontier in the case of the i-th player are derived by:

$$\max_{x \in X_0} [s\ (y_i(x), y_i^*)] \tag{1}$$

where:

y* is a reference point assumed by the player in the space R_{m_i},

$y_i(x)$ defines the vector of criteria of the i-th player, which are dependent on the vector x of decision variables, by the model relations,

s(y,y*) is the order approximating achievement function.

The function:

$$s(y_i, y_i^*)= \min_{1\leq j\leq mi} [a_j(y_{ji} - y^*_{ji}) + a_{m_i+1}\sum_{i=1}^{m_i} a_j(y_{ji} - y^*_{ji})] \qquad (2)$$

is an example of the achievement function suitable in this case, where $y_i^* \in R^{m_i}$ is a reference point, a_j, $1 \leq j \leq m_i$, are scaling coefficients, and $a_{m_i+1} > 0$ is a small parameter.

The assumed reference points and the obtained Pareto outcomes are stored in a database, so that a characterization of the Pareto frontier can be made and analyzed by the player.

Figure 3 presents the results of an unilateral, interactive analysis made by the player 1 in his criteria space for two different assumptions about the second player's outcomes: 1st – for the counterplayer's outcomes assumed on the level of *d*, and 2nd – for the conterplayer's aspirations assumed by the player 1.

Using the reference point approach the player can generate a number of such characterizations of the Pareto frontier. At the end, the player is asked to indicate the preferred outcome.

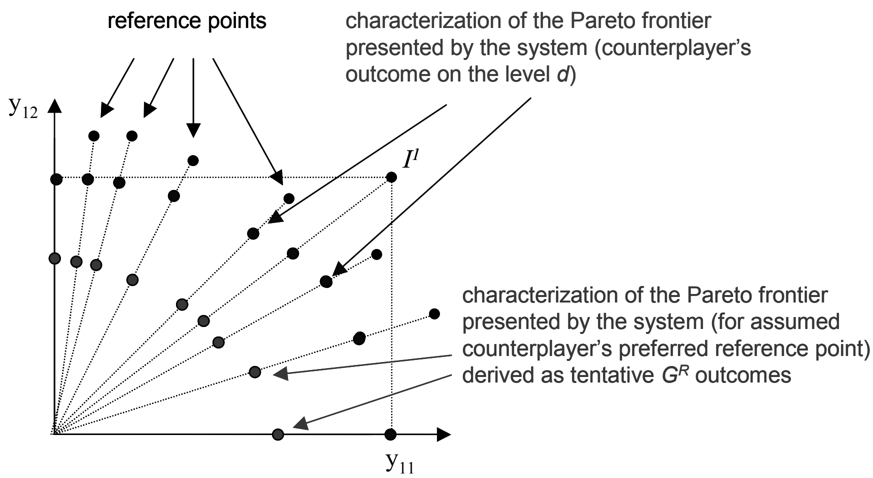

Fig. 3. Characterizations of the Pareto frontier obtained during unilateral analysis

The unilateral analysis is made by each player. Information about the indicated preferred outcomes of all players are collected.

4. SUPPORTING MEDIATION PROCESS

A procedure supporting mediation process has been proposed as inspiration of the Single Negotiation Text (SNT) procedure frequently applied in international negotiations. The SNT procedure formulated by Roger Fisher and described among others by Raiffa [28], is applied to solve crisis situations which appear in hard positional negotiations. According to the procedure, the opponents should not discuss the tasks independently nor formulate and consider counterproposals. They obtain and analyze, in consecutive rounds, proposals prepared by the mediator. In each round they work on the same text. On the basis of their opinions and suggestions, the mediator prepares an improved proposal to be analyzed in the next round.

The proposed procedure consists of a sequence of rounds $t = 1,2,\ldots T$. The rules of the procedure can be listed as follows:

- in each round each player, supported by the computer-based system, performs an interactive unilateral analysis in his criteria space and indicates a required improvement direction of his outcome according to his preferences among the criteria,
- the computer-based system generates the consecutive mediation proposals on the basis of the improvement directions indicated by all players,
- each player analyzes the proposals and introduces the required improvements of his outcome and the system generates a new improved mediation proposal.

The consecutive mediation proposal d^t is generated in the round t on the basis of the players' indications, according to the scheme:

$$d^0 = d$$

$$d^t = d^{t-1} + \alpha^t \cdot [G^t - d^{t-1}], \text{ for } t = 1, 2, \ldots, T \tag{3}$$

where $\alpha^t = \min\{\alpha^{t_1}, \ldots, \alpha^{t_n}\}$, α^{t_i} is the so-called confidence coefficient assumed by the player i in the round t, $0 < \varepsilon < \alpha^{ti} < 1$, G^t is the game solution calculated in the round t, for example the Raiffa solution, generalized in multicriteria case.

In the Cartesian product of the multicriteria spaces of the players' payoffs a point, which is a composition of the preferred outcomes indicated by the players after the unilateral analysis is found. This point denoted by U^R in Figure 4 is called the relative utopia point. It relates to the aspirations of the players. In fact it is derived according to the players' preferences expressed after the unilateral analysis. In general it is different from the ideal point defined by the maximal values of criteria in the set S.

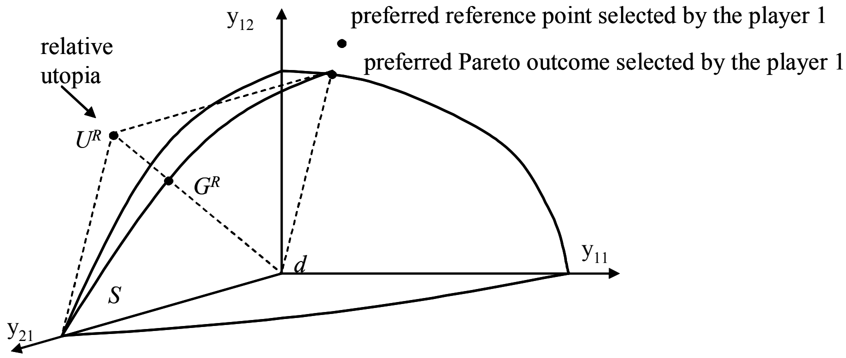

Fig. 4. Relative utopia and generalized Raiffa solution

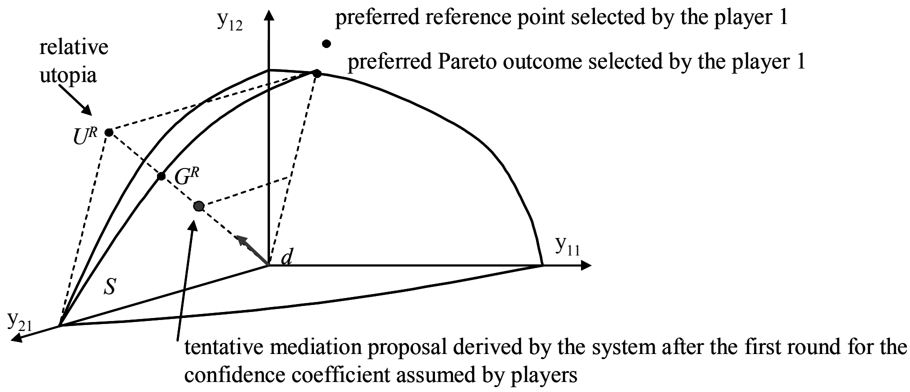

Fig. 5. Tentative mediation proposal

The generalized Raiffa solution G^R is the maximal point in S, located on the line linking the disagreement point d and the relative utopia U^R. If the confidence coefficient is relatively small (less then 1), then each player can limit the increase of the payoffs of all the players in the given round as it is presented in Figure 5. A tentative mediation proposal is derived according to the formula (3).

The tentative mediation proposal derived in the round t is treated as the disagreement point d^t in the next round $t + 1$. Next, unilateral analysis is performed by each player who explores the set of points from S and dominating the point d^t. This is illustrated in Figure 6.

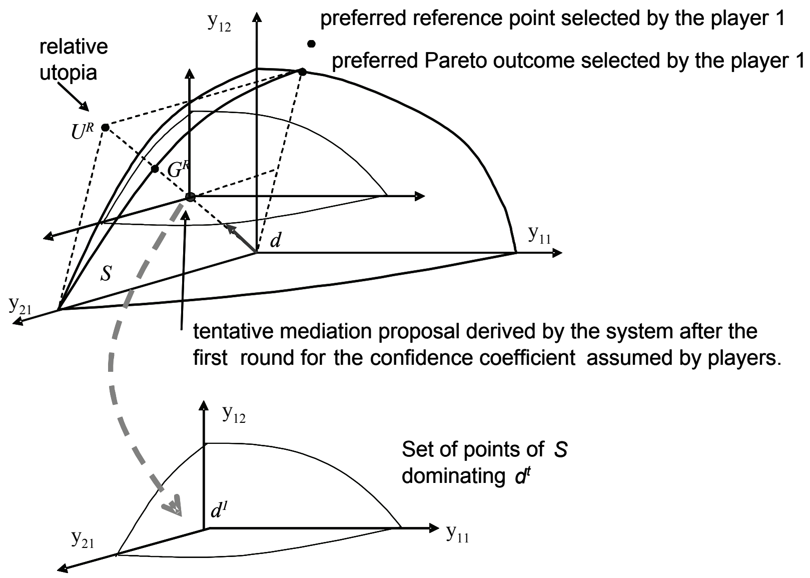

Fig. 6. The mediation proposal treated as the disagreement point in the problem analyzed in the next round

The preferred outcome selected by the player defines a direction in his multicriteria space. The directions of all players define a hyperplane in the Cartesian product of the spaces. The relative utopia and generalized Raiffa solutions lie on the hyperplane. Other theoretical solutions of the game theory lying on the hyperplane can be considered based on, for example, the egalitarian concept or the Nash cooperative solution concept.

The egalitarian solution maximizes gain of equal coordinates. It satisfies the axioms of weak Pareto optimality, symmetry, and strong monotonicity.

The Nash cooperative solution maximizes the product of the increases of the payoffs. It satisfies the axioms of Pareto optimality, symmetry, scale invariance, and independence of irrelevant alternatives [23].

The generalized Raiffa solution concept mentioned before satisfies the axioms of weak Pareto optimality, symmetry, scale invariance, and restricted monotonicity [12, 16]. The solution can be only weakly Pareto optimal even if the set S is convex. This means that the payoff of some player can be improved without decreasing the payoffs of other players. The application of the reference approach and of the achievement function of the form (2) partially solves the problem. The maximization of the function for $x \in S$ and decreasing parameter $a_{n+1} \rightarrow 0_+$ results in the lexicographic order applied to two separate terms of the function. For further discussion of the reference point method and the lexicographic ordering see [10, 26].

FINAL REMARKS

In the paper, a computer-based system supporting cooperative decisions is proposed and a model-based approach is applied. The model of the cooperation problem is formulated with use of ideas from the game theory. The system supports multicriteria analysis of the problem performed independently by parties with use of the reference point approach. Each player by assuming a reference point in his criteria space can use the system to generate a set of outcomes characterizing Pareto frontier of possible outcomes. It is made by solving maximization problems with specially constructed achievement functions. The system generates also Pareto optimal compromise outcomes. They are derived taking into account the information on the parties' preferences expressed in a special interactive procedure. The outcomes satisfy the axioms of cooperative solutions formulated in the theory of games generalized to the multicriteria case. They can be treated as mediation proposals aiding the players in looking for the consensus. The parties using the system can understand the nature of the cooperation problem, learn what their real preferences among the criteria are, analyze the possible outcomes, and make the final decision about cooperation consciously.

The paper continues the line of research presented in the references [11--18]. It is a part of the research including development of methods and computer experiments in the case of different cooperation problems. The research includes decision situations described by the multicriteria bargaining problems, but also by the multicriteria noncooperative games, the multicriteria cooperative games with and without side payments. In the research the utility function approach, which is an alternative to the direct multicriteria analysis, is also developed. In particular, the concepts proposed by R. Kulikowski [19-22] are applied to the support decision analysis taking into account the presence of risk. The concepts extend ideas developed in the papers [4, 24, 30, 32, 33] and are applied among others in the case of financial analysis [19, 21], analysis of innovative activities [15, 20], and analysis of education decisions [22].

REFERENCES

1. Fisher R., Ury W.: Getting to Yes. Hougton Mifflin, Boston 1981.
2. Jarke M., Jelassi M.T., Shakun M.F.: Mediator: Towards a Negotiation Support System. "European Journal of Operation Research" 1987, Vol. 31, pp. 314-334.
3. Kalai E., Smorodinsky M.: Other Solutions to Nash's Bargaining Problem. "Econometrica" 1975, Vol. 43, pp. 513-518.
4. Kahneman D., Tversky A.: Prospect Theory: An Analysis of Decision under Risk. "Econometrica" 1979, Vol. 47, No. 2.
5. Korhonen P., Moskowitz H., Wallenius J., Zionts S.: An Interactive Approach to Multiple Criteria Optimization with Multiple Decision Makers. "Naval Research Logistics Quarterly" 1986, Vol. 33, pp. 589- pp. 602.
6. Kersten G.E.: A Procedure for Negotiating Efficient and Non-Efficient Compromises. "Decision Support Systems" 1988, No. 4, pp. 167-177.
7. Kersten G., Michalowski W., Szpakowicz S., Koperczak Z.: Restructurable Representations of Negotiations. "Management Science" 1991, Vol. 37, No. 10, pp. 1269-1290.
8. Kersten G., Szapiro T.: Generalized Approach to Modeling Nego-tiations. „EJOR" 1986, 26.
9. Kersten G., Noronha S.J.: WWW-based Negotiation Support: Design, Implementation and Use. "Decision Support Systems" 1999, 25, 2
10. Kostreva M.M., Ogryczak W., Wierzbicki A.: Equitable Aggregations and Multiple Criteria Analysis. "European Journal of Operational Research" 2004, Vol. 158, pp. 362-377.
11. Kruś L.: Some Models and Procedures for Decision Support. In Bargaining. In: Multiple Criteria Decision Support. Eds P. Korhonen, A. Lewandowski, J. Wallenius. Springer Verlag, Berlin 1991.
12. Kruś L., Bronisz P.: Some New Results in Interactive Approach to Multicriteria Bargaining. In: User-Oriented Methodology and Techniques of Decision Analysis and Support. Eds J. Wessels, A.P. Wierzbicki. Springer Verlag, Berlin 1993.
13. Kruś L.: Multicriteria Decision Support in Negotiations. "Control and Cybernetics" 1996, Vol. 25, No. 6, pp. 1245-1260.
14. Kruś L., Bronisz P.: Cooperative Game Solution Concepts to a Cost Allocation Problem. "European Journal of Operational Research" 2000, Vol. 122, No. 2, pp. 258-271.

15. Kruś L.: A System Supporting Financial Analysis of an Innovation Project in the Case of Two Negotiating Parties. "Bulletin of Polish Academy of Sciences" 2002, Seria Technica, Vol. 50, No. 1, pp. 93-108.
16. Kruś L.: Multicriteria Decision Support in Bargaining, a Problem of Players Manipulations. In: Multiple Objective and Goal Programming. Eds T. Trzaskalik, J. Michnik. Physica Verlag, Springer, Berlin 2002.
17. Kruś L.: A Computer Based System Supporting Analysis of Cooperative Strategies. In: Artificial Intelligence and Soft Computing – ICAISC 2004. Lecture Notes in Computer Science. Eds L. Rutkowski, J. Siekmann, R. Tadeusiewicz, L. Zadeh. Springer, Berlin 2004.
18. Kruś L.: A Multicriteria Approach to Cooperation in the Case of Innovative Activity. "Control and Cybernetics" 2004, Vol. 33, No. 3.
19. Kulikowski R.: Portfolio Optimization: Two Factors Utility Approach. "Control & Cybernetics" 1998, No. 3.
20. Kulikowski R.: URS methodology – a Tool for Stimulation of Economic Growth by Innovations. "Bulletin of the Polish Academy of Sciences" 2002, Sci. Tech., Vol. 50, No. 1.
21. Kulikowski R.: On General Theory of Risk Management and Decision Support Systems. "Bulletin of the Polish Academy of Sciences" 2003, Sci. Tech., Vol. 51, No. 3.
22. Kulikowski R., Kruś L.: Support of Education Decisions. In: Group Decisions and Voting. Eds J. Kacprzyk, D. Wagner. EXIT, Warszawa 2003.
23. Nash J.: The Bargaining Problem. "Econometrica" 1950, Vol. 18, pp. 155-162.
24. Neumann J. von, Morgenstern O.: Theory of Games and Economic Behaviour. Princeton University Press, Princeton 1953.
25. Ogryczak W.: Multiple Criteria Optimization and Decisions under Risk. "Control and Cybernetics" 2002, Vol. 31, No. 4.
26. Ogryczak W.: On Nucleolar Refinement of the Reference Point Method. Paper presented at MCDM Workshop. Ustroń 2007 (submitted to the workshop proceedings).
27. Raiffa H.: Arbitration Schemes for Generalized Two-Person Games. "Annals of Mathematic Studies" 1953, No. 28, pp. 361-387.
28. Raiffa H.: The Art and Science of Negotiations. Harvard University Press, Cambridge 1982.
29. Roth A.E.: Axiomatic Models of Bargaining. Lecture Notes in Economics and Mathematic Systems, Vol. 170, Springer Verlag, Berlin 1979.
30. Savage L.J.: The Foundations of Statistics. Wiley, New York 1954.
31. Thomson W.: Two Characterization of the Raiffa Solution. "Economic Letters" 1970, Vol. 6, 225-231.
32. Tversky A., Kahneman O.: The Framing of Decisions and the Psychology of Choice. "Science" 1981, Vol. 211, 453-480.

33. Tversky A.: Utility Theory and Additivity Analysis of Risky Choices. "Experimental Psychology" 1967, Vol. 75, pp. 27-37.
34. Wierzbicki A.P.: On the Completeness and Constructiveness of Parametric Characterization to Vector Optimization Problems. "OR-Spectrum" 1986, Vol. 8, pp. 73-87.
35. Wierzbicki A.P., Kruś L., Makowski M.: The Role of Multi-Objective Optimization in Negotiation and Mediation Support. Theory and Decision. Special issue on "International Negotiation Support Systems: Theory, Methods, and Practice" 1993, 34, 2.
36. Wierzbicki A.P., Makowski M., Wessels J.: Model-Based Decision Support Methodology with Environmental Applications. Kluwer Academic Press, Dordrecht-Boston 2000.