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AFFINITY SET AND ITS APPLICATIONS*

Abstract

Affinity has a long history related to the social behavior of human, especially, the formation of social groups or social networks. Affinity has two meanings. The first is a natural liking for or attraction to a person, thing, idea, etc. The second defines affinity as a close relationship between people or things that have similar appearances, qualities, structures, properties, or features. Affinity here is simply defined as the distance/closeness between any two objects: the distance measurement could be geometric or abstract, or any type a decision maker prefers. A new forecasting method without historical memory, based on game theory and affinity set is originally proposed. The prediction performance of this new model is compared with the simple regression model for their performances on decision of buying in or selling out stocks in a dynamic market. Interestingly the qualitative model (affinity model) performs better than the quantitative model (simple regression model). Possible affinity set applications are provided as well in order to encourage readers to develop affinity models for actual applications.

Keywords

Affinity, forecasting, decision, distance.

INTRODUCTION

Affinity forms the basis for many aspects of social behavior, especially, the formation and evolution of groups or networks [6, 7, 12]. Affinity has two meanings. The first is a natural liking for or attraction to a person, thing, idea, etc. This kind of affinity is called *direct affinity* in this paper. The second defines affinity as a close relationship between people or things that have similar appearances, qualities, structures, properties, or features. This paper

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calls it *indirect affinity*. Two difficulties arise when dealing with affinity. First, affinity is, by definition, a vague and imprecise concept. Indeed, it is very difficult to precisely evaluate an affinity like friendship; it can be approximately described by linguistic terms like strong or weak. The second is that affinity often, if not always, varies with time. For example, a student may have strong affinity with the college he is studying now, but the affinity becomes weak after he graduates.

So far as we know, in literatures, there is no theory dealing with affinity as a vague and time-dependent concept, and little scholarly awareness that such a simple affinity idea could be developed for valuable models in management sciences. This paper originally proposes a theoretical framework for the affinity concept, different from fuzzy sets [13] and fuzzy relations [3]. Fuzzy set theory is the best tool for representing vague and imprecise concepts so far; however, the affinity set proposed here is not merely a fuzzy set because assuming any type of membership function here is unnecessary: the affinity concept is more general than the fuzzy concept. In the affinity set theory, allowing a decision maker uses his subjective perception of distance to from a set is possible, interesting, and innovative. Therefore, this work simply defines affinity as the closeness/distance between two objects [2]: the distance measurement could be geometric or abstract, and affinity could play various roles in a decision problem depending on decision maker achievement. Actually, the distance/closeness concept is more strongly related to Topology [4] rather than Fuzzy sets [5, 13]; however, these topology abstractions are translated/simplified into useful modeling concepts and procedures here.

The paper is organized as follows. Section 1 introduces the affinity set and related notions, formalizing indirect affinity and discussing its application. A qualitative forecasting method based on affinity set and game theory is newly presented, different from the traditional quantitative forecasting models because the historical trend is no longer necessary. The performance of this new model is compared with the simple regression model to show its value. Section 2 formalizes direct affinity. Last section concludes the paper.

1. AFFINITY SET AND INDIRECT AFFINITY

This paper refers to this type of affinity, which could be mediated by some intermediums as *indirect affinity*. Mathematically, indirect affinity can be understood as a relation between elements of a set, the subjects, with an object or medium, the relation is the affinity itself. The traditional crisp relations cannot be used to model indirect affinity for the following two reasons. First, affinity is, by definition, a vague and imprecise concept. The second is that affinity often, if not always, varies with time, for example the affinity between a student and his studying college may become stronger or weaker or have ups and downs over time.

1.1. Affinity set and affinity

We start by presenting the meaning we give to the primitive notion of *set.* Since the objective in this section is to formalize affinity time-dependence between an element and a set, our meaning should encompass the variability of shape or content of a set.

Definition 1

By affinity set we mean any object (real or abstract) that creates affinity between objects.

Example 1

A religion is an affinity set, for it creates affinity between people that makes them live a certain way of life.

Example 2

A famous artist or scientist or singer or sportsman or sportswoman is an affinity set for he or she creates affinity between people who appreciate him or her.

From the above examples we deduce that our set notion is wider than the traditional set notion and the fuzzy set notation. Let us now give a formal definition of affinity between a subject *e* and an affinity set.

Definition 2

Let e *and* A *be a subject and an affinity set, respectively. Let I be a subset of the time axis* $[0,+\infty[$. *The affinity between e and* A *is represented by a function.*

The value $M_A^e(t)$ expresses the degree of affinity between the subject *e* and the affinity set A at time *t*. When $M_A^e(t) = 1$ this means that affinity of *e* with affinity set A is complete or at maximum level at time *t*; *but it doesn't mean that e belongs to* A, unless the considered affinity is belongingness. When

$M_A^e(t) = 0$ this means that e has no affinity with A at time t. When $0 < M_A^e(t) < 1$, this means that e has partial affinity with A at time t. Here we emphasize the fact that the notion of affinity is more general than the notion of membership or belongingness. The later is just a particular case of the former.

Definition 3

The universal set, denoted by U, is the affinity set representing the fundamental principle of existence. We have:

$$M_U^e(\,.\,): [0,+\infty) \to [0,1]$$
$$t \to M_U^e(t)$$

and $M_U^e(t) = 1$, for all existing objects at time t and for all times t.

In other words the affinity set defined by the affinity „existence" has complete affinity with all previously existing objects, that exist in the present, and that will exist in the future. In general, in real world situations, some traditional referential set V, such as that when an object e is not in V, $M_A^e(t) = 0$ for all t in $I \subset [0,+\infty[$, can be determined. In order to make the notion of affinity set operational and for practical reasons, in the remainder of the paper, instead of dealing with the universal set U, we will deal only with affinity sets defined on a traditional referential set V. Thus, in the remainder of the paper when we refer to an affinity set, we assume that sets V and I are given.

Definition 4

Let A be an affinity set. Then the function defining A is:

$$F_A(.,.): V \times I \to [0,1]$$
$$(e, t) \to F_A(e, t) = M_A^e(t) \tag{1}$$

An element in real situations often belongs to a set at some time and not at other times. Such behavior can be represented using the affinity set notion. The behavior of affinity set A over time can also be investigated through its function $F_A(.,.)$.

Interpretation 1

1) For a fixed element *e* in V, the function (1) defining the affinity set A reduces to the fuzzy set describing degree variation of affinity of the element *e* over time.
2) For a fixed time *t*, the function (1) reduces to a fuzzy set defined on V that describes the affinity between elements V and affinity set A at time *t*. Roughly speaking it describes the shape or "content" of affinity set A at time *t*.
3) In addition to 1) and 2), we can't say/validate affinity set as a special fuzzy set, unless we can prove that any affinity set A is included in a fuzzy set B and vice versa.
4) Any distance/closeness could be normalized to [0, 1], however, such a normalization process is not necessarily fuzzy.

The maximum affinity $M_A^e(t)=1$ may not be reached at any time in real-world problems. In order to consider various situations we introduce the following definition.

Definition 5

Let A be an affinity set and $k \in [0,1]$. We say that an element *e* is in the *t-k*-Core of the affinity set A at time *t*, denoted by *t-k*-Core(A), if $M_A^e(t) \geq k$, that is:

$$\text{t-k-Core(A)} = \{e \mid M_A^e(t) \geq k\}$$

when $k = 1$, *t-k*-Core(A) is simply called the core of A at time *t*, denoted by *t*-Core(A).

Definition 6

An observation period is defined as the period (continuous or discrete) analyzing the behavior of an element *e* of V with respect to an affinity set A (an illustration is given in Figure 1 below).

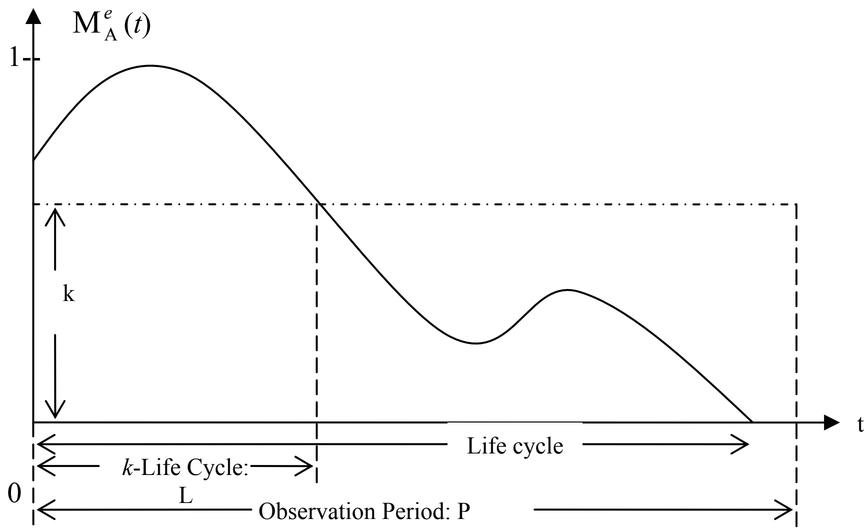

Fig. 1. Illustration of the affinity between an element *e* and an affinity set A over an observation period P

Definition 7

Let A be an affinity set and $k \in [0,1]$. A subset T (discrete or continuous) of I is said to be the *k*-life cycle of an element *e* with respect to A if:

$$M_A^e(t) \geq k, \text{ for all } t \in T \text{ and } M_A^e(t) < k, \text{ elsewhere in I}$$

In other words, the period T is the *k*-life cycle of *e* with respect to A if *e* is in the *t*-*k*-Core(A) for all *t* in T. It is the time period that element *e* keeps its affinity at least equal to *k* in I. The period of time $T_C = \{t \mid M_A^e(t) > 0, t \in I\}$ is called life cycle of the element *e* with respect to the affinity set A.

1.2. Indirect affinity

Indirect affinity occurs when affinity between subjects takes place via a medium. This section gives a formal definition of indirect affinity. The notion of harmony between objects with respect to an affinity set is also formalized.

Definition 8

Let A be an affinity set and $k \in [0,1]$. Let D be a subset of V. A k-indirect affinity degree with respect to A, at time t, between the elements of D exists, if they all belong to the t-k-Core(A), that is $D \subset t - k - Core$ (A). A k-indirect affinity degree with respect to A, during an observation period T, between the elements of D exists, if $D \subset t - k - Core$ (A) at any time t in T.

Definition 9

Let A and D be an affinity set and a subset of V, respectively. Harmony exists at time t between the elements of D with respect to A, if they all belong to t-Core(A) at time t, that is, $D \subset t - Core$ (A). In other words, harmony between the elements of D with respect to A is reached at time t when the maximum indirect affinity degree between them is $k = 1$ at this time. Harmony exists during the observation period of time T, with respect to A, between the elements of D, if there is harmony with respect to A between them at any time t in T. This definition expresses the fact that harmony is the highest level of affinity.

1.3. Operations on affinity sets

This section defines basic affinity set operations. The following definitions 10-14, assume that A and B are two given affinity sets defined on I and V.

Definition 10

We say that A and B are equal at time t if $M_A^e(t) = M_B^e(t)$, for all e in V. Then we write A = B at time t. If A and B are considered in an observation period T, then A = B during this period if $M_A^e(t) = M_B^e(t)$, for all e in V and all t in T.

Definition 11

We say that A is contained in B at time t if $M_A^e(t) \leq M_B^e(t)$, for all e in V. Then we write $A \subset B$ at time t. In the case that A and B are considered in an observation period T, then $A \subset B$ during this period if $M_A^e(t) \leq M_B^e(t)$, for all e in V and all t in T.

Definition 12

The union of A and B at time t, denoted by $A \cup B$, is defined by the function $F_{A\cup B}(t, e) = M^{e}_{A\cup B}(t) = \text{Max}\{M^{e}_{A}(t), M^{e}_{B}(t)\}$, for all e in V. In the case that A and B are considered in an observation period T, then during this period, $A \cup B$ is defined by the function $F_{A\cup B}(t, e) = M^{e}_{A\cup B}(t) = \text{Max}\{M^{e}_{A}(t), M^{e}_{B}(t)\}$, for all e in V and all t in T.

Definition 13

The intersection of affinity sets A and B at time t, denoted by $A \cap B$, is defined by the function $F_{A\cap B}(t, e) = M^{e}_{A\cap B}(t) = \text{Min}\{M^{e}_{A}(t), M^{e}_{B}(t)\}$, for all e in V. In the case that A and B are considered in an observation period T, then during this period, $A \cap B$ is defined by the function $F_{A\cap B}(t, e) = M^{e}_{A\cap B}(t) = \text{Min}\{M^{e}_{A}(t), M^{e}_{B}(t)\}$, for all e in V and all t in T.

Definition 14

B is said to be the complement of A at time t if it is defined by the following function $F_{B}(t, e) = M^{e}_{B}(t) = 1 - M^{e}_{A}(t)$, for all e in V. In the case that A an B are considered in an observation period T, then during this period, B is defined by the function $F_{B}(t, e) = M^{e}_{B}(t) = 1 - M^{e}_{A}(t)$, for all e in V and all t in T.

1.4. Application of forecasting

The affinity set's potential applications are valuable in analyzing, evaluating, forecasting (predicting) the time-dependent behaviors: for example, evolving an uncertain dynamic system in a human society. In addition, predicting the demand curve with high fluctuations is also possible by an affinity set. We will give a simple example of how the affinity set can be applied in forecasting real-world problems later. In fact, any time series method can be used to predict any element e behavior in V with respect to an affinity set A based on past data, if it is possible to define affinity set A. This paper proposes a new forecasting method based on affinity set and game theory. Assume that an affinity set A and a universe V are given and some data are available at some past periods $t_1, t_2, ..., t_n$ on the behavior of elements e in V with respect to affinity set A as described in the following matrix [1]:

$$\mathbf{D} = \begin{array}{c} \\ \mathrm{A} \\ \overline{\mathrm{A}} \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & \ldots & t_n \end{array} \\ \begin{pmatrix} a_{11} & \ldots & a_{1n} \\ a_{21} & \ldots & a_{2n} \end{pmatrix} \end{array}$$

Here we can follow the similar concept in [1], $t_1, t_2, \ldots, t_n$ are regarded as the multiple attributes of the decision problem, and $\overline{\mathrm{A}}$ and A are two alternatives of this problem. But we will define a new method, which is different from [1] to resolve this affinity game. Where $\overline{\mathrm{A}}$ is the affinity set complementary to A (see Definition 14), entry a_{1j} is the affinity degree of element *e* with respect to affinity set A at the period t_j and $a_{2j} = 1 - a_{1j}$ is the affinity degree of element *e* with respect to affinity set $\overline{\mathrm{A}}$ at the same period. Here a decision maker wants to forecast element *e* behavior at the next period t_{n+1}. Interestingly we can look at the situation as a game between the decision maker and Nature. The decision maker faces an uncertain situation represented by future element *e* behavior. One way to handle the situation is to adopt the maximum decision making under uncertainty principle [3] by considering the situation as a game against Nature [1]. Thus, matrix D can be considered as a matrix game between the decision maker and Nature, where the decision maker is the maximizing player who chooses between A and $\overline{\mathrm{A}}$ and Nature is the minimizing player who chooses the time periods.

Definition 15

A pair of strategies (i_0, j_0) *where* $i_0 \in \{1,2\}$ *and* $j_0 \in \{1,..,n\}$ *are said to be the Nash equilibrium* [9] *of the matrix game* D *if*:

$$a_{ij_0} \le a_{i_0 j_0} \le a_{i_0 j}, \text{ for all } i \in \{1,2\} \text{ and } j \in \{1,..,n\} \qquad (2)$$

Assume that the game has a Nash equilibrium (i_0, j_0). In terms of affinity, this equilibrium can be interpreted as follows. If $i_0 = 1$, the decision maker will favor element *e* affinity with affinity set A rather than affinity with $\overline{\mathrm{A}}$, with affinity degree $a_{i_0 j_0}$. The decision maker in case $i_0 = 2$ will favor element *e* affinity with $\overline{\mathrm{A}}$ rather than with A, with affinity degree $a_{i_0 j_0}$. It may happen that matrix D has no Nash equilibrium in pure strategies, then the two players have to use mixed strategies. A mixed strategy for Nature is a probability distribution over the set of its pure strategies, that is, it is a vector $y = (y_1, y_2, \ldots, y_n)$ such that:

$$\sum_{j=1}^{n} y_j = 1 \text{ and } y_j \geq 0,\ j = \overline{1,n}$$

Similarly, a mixed strategy for the decision maker is a vector $x = (x_1, x_2)$ such that:

$$x_1 + x_2 = 1 \text{ and } x_i \geq 0,\ i = 1,2$$

Player payoffs become expected payoffs. Decision maker payoff is $x^T \mathrm{D} y$ and that of Nature is $-x^T \mathrm{D} y$. Any matrix game always includes a Nash equilibrium in mixed strategies [9]. A Nash equilibrium in mixed strategies is defined by:

$$x^T \mathbf{D} y^* \leq x^{*T} \mathbf{D} y^* \leq x^{*T} \mathbf{D} y$$

for all mixed strategies x and y. The mixed strategy x^* of the decision maker can be interpreted as follows. The decision maker will favor A with weight x_1 and $\overline{\mathrm{A}}$ with weight x_2. He can also use these two evaluations to rank sets A and $\overline{\mathrm{A}}$ from his point of view. The expected affinity degree of element e in the period t_{n+1} with each of the affinity sets can be defined as follows:

$$\mathrm{M}_{\mathrm{A}}^{e}\ (t_{n+1}) = \sum_{1}^{n} a_{1j} y_j^* \text{ and } \mathrm{M}_{\overline{\mathrm{A}}}^{e}\ (t_{n+1}) = \sum_{1}^{n} a_{2j} y_j^*$$

respectively. The mixed strategy y^* of Nature can be interpreted as the weights Nature assigns to the periods in order to minimize expected decision maker affinity. Let us illustrate our approach by examples.

Example 3. Decision of buy in/sale out/hold

Today, we are aware that the stock price in a market is quite unstable; in other words, the stock price curve is highly fluctuating for a company. Now we collect the actual data of Taiwan TGV Company for twenty-two periods (from October 1, 2007 to October 22, 2007) from Taiwan Stock Market [8]. Assume that a decision maker wants to predict if he can buy in or sell out his stocks in the market by updating his information and using the affinity game. The first seventeen data are used as the training base, then we predict the remaining five data. Please note that if we want to predict the eighteen data by affinity model, then the previous seventeen data will be all included in an affinity game, and if predicting the nineteen data then the previous eighteen data will be included, etc. Assume, for simplicity, that by experience he classifies his decisions into only "Buy in", "Sell out" and "Hold". These two possible states can be considered as two affinity complementary sets A (Buy in) and $\overline{\mathrm{A}}$ (Sell out), respectively. His decision will be the element *e*. And if the affinity degree of *e* to A, and that to $\overline{\mathrm{A}}$ are identical, then he chooses the "Hold" state. The price data of twenty-two dates in October 2007 are collected as in Figure 2. Assume this decision maker has recorded the affinity degrees of stock price with respect to affinity set A by the following function:

$$c_{1t} = (\frac{p_t}{p_{t-1}}), t = 2,3,...,n \quad (3)$$

and

if $c_{1t} < 1$ then $a_{1t} = 1 - c_{1t}$;
if $c_{1t} > 1$ then $a_{2t} = c_{1t} - 1$;
if $c_{1t} = 1$ then $a_{1t} = a_{2t} = 0.5$, which is a "Hold" state: no buying in and no selling out.
Here $a_{1t} = 1 - a_{2t}$ is also assumed.

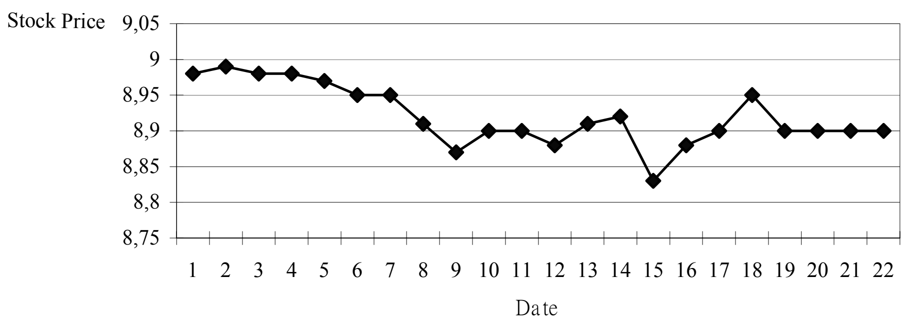

Fig. 2. Actual data of stock price (in Taiwanese Dollars per stock)

Source: [8].

Table 1

Performance comparison of affinity model and simple regression

Model\Periods	t_{18}	t_{19}	t_{20}	t_{21}	t_{22}
Affinity Game	Hold	Hold	Hold	Hold	Hold
Simple Regression	8.85	8.84	8.84	8.83	8.82
Actual Data	8.95	8.90	8.90	8.90	8.90

Source: [8].

According to the actual data, the affinity matrix is easily computed (see Appendix). The suggested decision is summarized in Table 1, which is compared with the simple regression model (only using time t as the explainable variable). Interestingly the affinity model suggests the "Hold" state, which seems to be better than predicting the declining trend by the simple regression model. Because if a declining trend is forecasted, then the action of selling out stocks will be considered by this decision maker. However, the "Hold" state suggested by the affinity model hints the decision maker makes more profits if he keeps these stocks from the time period: t_{17}. Affinity game predicts that the stock price will remain almost stable during the analyzed time. It is clear that affinity model performs better than the simple regression model in this experiment. Of course, the function (3) could be assumed by various types, the decision maker can choose any type that he prefers. The affinity spirit is eventually, a decision maker is encouraged to try/develop any possible measurement to find/explore/analyze the *special pattern* in a time-dependent data set or input/output system. And this *special pattern* is arbitrarily defined by a decision maker, just like that distance/closeness have general definitions in Topology [4]. A *special pattern* could vary with time and space, once the decision maker catches the core of this *special pattern*, he could explain the observations or predict some useful outcomes. Actually, there is an old saying: *what you measured the world filters what you see*. Thus, various measurements for modeling are natural and should be encouraged.

1.5. Affinity set depending on time and other variables

The affinity of element e with respect to affinity set A in real-world situations often depends implicitly on other variables than time. These variables generally express condition or constraint variability that affect affinity evaluation. Studying element *e* behavior with respect to time and other variables may be practically desirable. A decision maker may even study element *e* behavior at a fixed time with respect to other variables. This section extends the affinity set definition to the case where desired variables appear explicitly. This definition makes it possible to study *e* affinity behavior over time and with respect to other variables as well.

Definition 16

Let e and A *be an element and an affinity set, respectively. Assume that the affinity of* e *with respect to* A *depends on some variable w that takes its values in a traditional set* W. *In order to make the variable w appear in the affinity definition between e and* A, *we introduce the following affinity:*

$$\mathrm{M}_{\mathrm{A}}^{e}\ (\ .\ ): \mathrm{I} \times \mathrm{W} \rightarrow [0,1]$$

$$(t,w) \rightarrow \mathrm{M}_{\mathrm{A}}^{e}\ (t,w)$$

The value $\mathrm{M}_{\mathrm{A}}^{e}(t,w)$ expresses the degree of affinity between element *e* and A at time *t* with respect to *w*.

Thus, depending on the problem at hand, the decision maker can use Definitions 2 or 16 of affinity between an element *e* and an affinity set A.

Definition 17

Let A *be an affinity set depending on a variable* $w \in \mathrm{W}$. *Then the function defining* A *is defined by:*

$$F_{\mathrm{A}}\ (.,\ .\ ,\ .): \mathrm{V}{\times}\mathrm{I}{\times}\mathrm{W} \rightarrow [0,1]$$

$$(e,\ t,w) \rightarrow F_{\mathrm{A}}\ (e,\ t,w) = \mathrm{M}_{\mathrm{A}}^{e}\ (t,w)$$

where V *is the traditional referential as in Section 1.*

2. DIRECT AFFINITY

Direct affinity is a natural liking for or attraction to a person or a thing, or an idea, etc. Direct affinity involves two elements: the affinity subjects and the affinity that takes place between them. Mathematically, direct affinity can be understood as a binary relation between elements of a set, where the elements are the subjects and the relation is affinity. The traditional crisp binary relations cannot be used to model direct affinity for the following two reasons: First, affinity is, by definition, a vague and imprecise concept. Indeed, it is very difficult to give a precise evaluation of affinity like friendship; it can be approximately described by linguistic terms like strong or weak;

the second is that affinity often, if not always, varies with time, for example friendship may become stronger or weaker or have ups and downs over time. Thus, the adequate way to model direct affinity is to use time-dependent fuzzy relations. Affinity can be considered as a particular case of the following general framework.

Definition 18

Let V *and* I *be a referential set and a subset of the time axis* $[0,+\infty[$, *respectively. A time dependent fuzzy relation R such that:*

$$R_{(.,.)}(.) : \mathrm{I} \times (\mathrm{V} \times \mathrm{V}) \to [0,1] \qquad (4)$$

$$(t,(e,s)) \to R_{(e,s)}(t)$$

is called direct affinity on the referential V.

Interpretation 2

1. *For any fixed time t the relation (2) reduces to an ordinary fuzzy relation* [3]:

$$R_{(.,.)}(t) : \mathrm{V} \times \mathrm{V} \to [0,1]$$

$$(e,s) \to R_{(e,s)}(t)$$

that expresses the intensity or the degree of affinity between any couple of elements in V. *Hence affinity fuzziness between elements is taken into account in Definition 18*.

2. *For any fixed couple of elements* $(e,s) \in \mathrm{V}$, *the relation (4) reduces to a fuzzy set defined on the time-set* I*:*

$$R_{(e,s)}(.) : \mathrm{I} \to [0,1]$$

$$t \to R_{(e,s)}(t)$$

that expresses affinity evolution over time between elements e *and* s.

Thus, the time-dependent fuzzy relation (4) expresses the most important characteristic of direct affinity: Fuzziness and time-dependence.

Definition 18 can be extended to affinity between groups of elements as follows.

Definition 19

Let R be a time-dependent fuzzy relation defined on a subset of time axis I *and a referential V. Let* A *and* B *be two subsets of* V. *Then the affinity between* A *and* B *can be described by the following function:*

$$R_{(\mathrm{A},\mathrm{B})}(.):\mathrm{I}\to[0,1] \qquad (5)$$
$$t\to R_{(\mathrm{A,B})}(t)$$

where $R_{(A,B)}(.)$ *can be defined by many ways, depending on the decision maker. We propose the following four examples:*

1) $R_{(\mathrm{A},\mathrm{B})}(t) = \max\limits_{(e,s)\in\mathrm{A}\times\mathrm{B},\ e\neq s} R_{(e,s)}(t)$, for all $t\in\mathrm{I}$
2) $R_{(\mathrm{A},\mathrm{B})}(t) = \min\limits_{(e,s)\in\mathrm{A}\times\mathrm{B},\ e\neq s} R_{(e,s)}(t)$, for all $t\in\mathrm{I}$
3) $R_{(\mathrm{A},\mathrm{B})}(t) = \alpha \max\limits_{(e,s)\in\mathrm{A}\times\mathrm{B},\ e\neq s} R_{(e,s)}(t) + (1-\alpha)\min\limits_{(e,s)\in\mathrm{A}\times\mathrm{B},\ e\neq s} R_{(e,s)}(t)$, for all $t\in\mathrm{I}$, *where* α *is a number in* $[0,1]$ *that expresses the degree to which the decision maker prefers the maximum of affinity to its minimum.*
4) in the case A and B are finite $R_{(\mathrm{A},\mathrm{B})}(t) = \sum\limits_{(e,s)\in\mathrm{A}\times\mathrm{B},\ e\neq s}\lambda_{(e,s)}R_{(e,s)}(t)$, for all $t\in\mathrm{I}$, where $\lambda_{(e,s)}\geq 0$ is the weight assigned by the decision maker to the couple (e,s) for $e\neq s$ and $\sum\limits_{(e,s)\in\mathrm{A}\times\mathrm{B},\ e\neq s}\lambda_{(e,s)}=1$.

Here also for practical purpose we define the *t*-*k*-affinity.

Definition 20

Let R be a time-dependent fuzzy relation defined on a subset of time axis I and *a referential* V. *Let* $k\in[0,1]$, *and* $t\in$ I. Then:

1) *we say that a couple (e, s) has k affinity degree at time t or t-k-affinity degree if* $R_{(e,s)}(t)\geq k$,
2) *a subset D of V has t-k-affinity degree if* $R_{(D,D)}(t)\geq k$. *Thus, the t-k-affinity degree of subsets depends on how affinity is defined between groups or subsets as indicated in Definition 19,* 1)-4).

Remark 1

Depending on information available for the time-dependent fuzzy relation describing affinity (2)-(3), direct affinity can be used to study networks (social or nonsocial). Indirect affinity can also be used to analyze, describe, forecast, and predict network behavior or its elements regarding the considered affinity. For example, with knowledge that network evolution over time follows a differential equation or a stochastic process, that is, the function

$t \to R_{(e,s)}(t)$ *is a solution of a differential equation or a stochastic process, then based on initial data one can predict network behavior at any time* $t \in I$ *regarding the considered affinity. Social network analysis* [6, 12] *is one area for direct affinity application. In addition, the direct affinity concept is valuable in developing network grouping or network controlling.*

CONCLUSIONS AND RECOMMENDATIONS

This paper proposes a basic framework for the affinity concept, allowing its investigation by fuzzy set tools and other nonfuzzy methods. Of course, fuzzy tools are not the only way to explore affinity. Readers should realize that the affinity model proposed in Example 3 is quite different from the fuzzy set and rough set [10, 11] because we don't need to assume any type of fuzzy membership function [10] or use the upper bound and lower bound to approximate a set [11]. Instead, the closeness or distance between any two objects within a time series data set is directly assumed, then it will form the basis of an affinity set. Numerous measurements of closeness/distance could exist in Example 3, but we only propose/assume one way here.

We studied two types of affinity: Indirect affinity and direct affinity. This work pointed out that indirect affinity requires a medium and introduces the affinity set for indirect affinity formalization, which actually represents the medium. The affinity of elements with respect to an affinity set is represented by a fuzzy set defined on the time axis. Then the affinity between elements (indirect affinity) is defined via their affinity to the affinity set. We have formalized direct affinity as a time-dependent fuzzy relation and present a new forecasting method based on affinity set and game theory. Finally, we indicate some potential areas for possible application of direct affinity and indirect affinity. Many issues are not fully discussed in this paper. One of them is the numerical determination of functions $t \to M_A^e(t)$ and $t \to R_{(e,s)}(t)$ that represent affinity in indirect affinity and direct affinity, respectively. Another issue is exploration of the affinity set notion. We believe that investigating affinity in social networks or engineering control using our framework is a worthwhile topic of research. We also hope that this paper will inspire and attract more researchers for investigating the affinity concept. The evolutionary algorithms will be beneficial when we try to find/explore the *special pattern* hidden in a large scale data set; for example, evolving the *special pattern* that maximizes a specified/predefined affinity.

Appendix

Actual data and affinity degree

Date	Traded Stocks	Average Price	+Up/-Down	$c_{1t} = (\frac{p_t}{p_{t-1}})$	a_{1t}	a_{2t}
07/Oct/01	3,252,380	8.98	-0.03			
07/Oct/02	1,058,000	8.99	+0.01	1.001114	0.998886	0.001114
07/Oct/03	1,660,201	8.98	-0.01	0.998888	0.001112	0.998888
07/Oct/04	1,018,000	8.98	+0.00	1	0.50	0.50
07/Oct/05	1,200,500	8.97	-0.01	0.998886	0.001114	0.998886
07/Oct/06	999,000	8.95	-0.02	0.99777	0.00223	0.99777
07/Oct/07	970,912	8.95	+0.00	1	0.50	0.50
07/Oct/08	1,177,913	8.91	-0.04	0.995531	0.004469	0.995531
07/Oct/09	1,696,000	8.87	-0.04	0.995511	0.004489	0.995511
07/Oct/10	1,687,000	8.90	+0.03	1.003382	0.996618	0.003382
07/Oct/11	781,000	8.90	+0.00	1	0.50	0.50
07/Oct/12	789,000	8.88	-0.02	0.997753	0.002247	0.997753
07/Oct/13	1,409,000	8.91	+0.03	1.003378	0.996622	0.003378
07/Oct/14	622,300	8.92	+0.01	1.001122	0.998878	0.001122
07/Oct/15	783,535	8.83	-0.09	0.98991	0.01009	0.98991
07/Oct/16	1,702,000	8.88	+0.05	1.005663	0.994337	0.005663
07/Oct/17	859,000	8.90	+0.02	1.002252	0.997748	0.002252
07/Oct/18	1,449,956	8.95	+0.05			
07/Oct/19	586,000	8.90	-0.05			
07/Oct/20	1,985,956	8.90	+0.00			
07/Oct/21	1,166,913	8.90	+0.00			
07/Oct/22	1,209,000	8.90	+0.00			

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