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AN APPLICATION OF INTERACTIVE MULTIPLE CRITERIA TECHNIC IN LABOR PLANNING*

Abstract

The aim of each production or service system is to deliver final products or services in right quantities, on time and at appropriate cost. Decisions affecting such system are usually grouped in three categories: strategic, tactical, and operational. While the strategic planning decisions are mostly focused on the development of resources satisfying the external requirements, the tactical ones are concerned with the utilization of these resources. Finally, the operational decisions deal with day-to-day operational and scheduling problems and require disaggregation of the information generated on higher levels.

Labor planing, considered in this paper, is concerned with determining staffing policies that deal with employment stability and work schedules. A staffing plan is a managerial statement of time-phased staff sizes and labor-related capacities, which takes into consideration customers' requirements and machine-limited capacities. Such plan has to balance conflicting objectives involving customer service, work-force stability, cost, and profit.

In the paper, a multicriteria decision aiding procedure is proposed for labor planning problems. Simulation technic is employed for evaluating decision alternatives with respect to criteria. Demand forecasts and calendar constraints are taken into account in the simulation model. Uncertainties related to employees' accessibility are also considered. In the second phase, an interactive multiple criteria procedure is used for selecting the final solution of the problem.

A numerical example is presented to illustrate the applicability of the proposed technic.

Keywords

Labor planning, simulation, multiple criteria.

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INTRODUCTION

The aim of each production or service system is to deliver final products or services in right quantities, on time and at appropriate cost. Decisions affecting such system are usually grouped in three categories: strategic, tactical, and operational. While strategic decisions are mostly focused on the development of resources to satisfy the external requirements, tactical ones are concerned with the utilization of these resources [2]. Finally, operational decisions deal with day to day operational and scheduling problems and require disaggregation of the information generated on higher levels.

Labor planning, considered in this paper, is concerned with determining staffing policies that deal with employment stability and work schedules [3]. A staffing plan is a managerial statement of time-phased staff size and labor-related capacities, which takes into consideration customer requirements and machine-limited capacities. Such plan has to balance conflicting objectives involving customer service, work-force stability, cost, and profit.

Various technic are employed for solving labor planning problems. Linear programming and dynamic programming are used most often. However, these approaches are based on strong assumptions that often are not satisfied. Employees' attainability varies due to planned and unexpected absences. Work-force requirements are not stable as well. Often, considerable fluctuations can be noticed even in short-term. In accounts or payroll departments, for example, work-force requirements are usually higher in the early part of the month than in the latter one.

In the paper a staffing planning problem is considered. It consists in determining the number of full-time and part-time employees for a department in which work-force requirements fluctuate in a month. Three criteria are considered: yearly labor costs, number of overtime hours worked by all employees during the whole year, and the work-force utilization rate. While the first and the second criteria are to be minimized, the last one is to be maximized. The solving procedure is based on simulation and interactive multiple criteria technic. First, simulation experiments are performed in order to evaluate alternatives with respect to criteria. It is assumed that distributions of work--force requirements for each week of the month are available. For each alternative a distribution of work-force capacity is generated taking into account the probability, that employees are show up for work. The simulation model considers calendar constraints resulting from national holidays and Polish Labor

Code rules. As a result distributional evaluations are obtained for each alternative and each criterion. In the second phase, interactive technic INSDECM-II is employed for solving a multiple criteria problem.

The paper is organized as follows. The decision problem is presented in section 1. Section 2 deals with the simulation model of the problem. In Section 3 stochastic dominance rules are briefly presented. These rules are employed in the interactive multiple criteria procedure INSDECM-II described in Section 4. Section 5 provides a numerical example. The last section gives conclusions.

1. LABOR PLANNING IN ACCOUNTS DEPARTMENTS

Each economic organization has to prepare a variety of plans. Demand forecasts prepared by a manufacturer or service organization can address short-, medium- and long-time problems. Long-range forecasts deal with capacity and strategic issues. Problems, such as facility location and expansion, new product development, research funding, and investment over a period of several years have to be solved on this level.

Labor planning is a part of medium-range planning, which starts once long-term decisions have been made. Problems concerned with matching the productivity to fluctuating demand have to be solved on this level. Production and staffing medium-range plans link organization's strategic goals with the master production schedule and work-force-schedule. A planning horizon for such plans is usually one year, although it can differ in various situations.

Labor plan should provide following information:

– how many employees are needed for each department,
– should they be full-time or part-time workers,
– what the salaries should be, etc.

In the paper labor planning in accounts and payroll departments is considered. Work-force requirements in such divisions are usually higher in the early part of the month due to fixed pay day or insurance and tax forms preparation dead-lines. In order to meet requirements both full-time and part-time employees can be hired. Overtime can also be used to satisfy work-force requirements that cannot be completed in regular time. However, overtime is expensive. According to Polish Labor Code, 50 percent bonus has to be paid if overtime work is done on working day, while 100 percent bonus is to be paid for working on Saturdays, Sundays and holidays. Additionally, the number of overtime hours worked by an employee is limited to 150 per year. Moreover, in many cases workers do not want to work a lot of overtime for extended period. Finally, increased utilization of overtime may lead to decreased productivity due to employees' tiredness. If work-force requirements fluctuations are considerable, employees' working hours may not be fully utilized in some periods. Such situation is inconvenient, as it results in the labor costs increase. It is also unfavorable from psychological point of view. Balancing various objectives in order to arrive at an acceptable staffing plan involves consideration of various decision alternatives.

The decision problem considered in this paper consists in determining the number of full-time and part-time employees. Decision alternatives are evaluated with respect to three criteria:

X_1 – yearly labor costs,

X_2 – total number of overtime hours worked by all employees in the department during the year,

X_3 – work-force utilization rate measured by the contribution of regular hours effectively worked in the total number of regular hours worked by employees.

In order to solve the problem, alternatives have to be evaluated with respect to attributes. Simulation technic is an efficient and flexible tool for doing this.

2. SIMULATION MODEL FOR LABOR PLANNING

One of the most important elements of simulation modeling is identifying appropriate probability distributions for input data. Usually, this requires analyzing empirical or historical data and fitting these data to probability distributions. Sometimes, however, such data are not available and an appropriate distribution has to be selected according to the decision maker's judgment. Once the simulation model is built, verified, and validated, it can be used for generating probability distributions of output variables.

In our problem, distributions of work-force requirements in successive weeks of each month and distribution of employees' accessibility have to be identified. This requires analyzing historical data (e.g. the number of documents processed in previous periods), as well as the information about planned and unplanned absences of employees. Based on these data probability distributions of input data can be determined.

Let us assume following notation:

t – the number of the week in the year,
d_t – work-force requirements in week t,
r_t – regular time work-force capacity in week t,
w_t – regular wage per hour,
b_t – overtime bonus (%),
o_t – number of overtime hours worked by employees in week t,
u_t – number of regular-time hours that where not effectively utilized in week t,
c_t – labor cost in week t,
C – labor cost per year,
R – the number of regular-time hours worked by all employees in the year,
O – the number of overtime hours worked by all employees in the year,
U – the number of regular-time hours that were not effectively utilized

Simulation experiment is performed as follows:

1. $t = 1, X_1 = 0, X_2 = 0, X_3 = 0$.
2. Determine the work-force requirements d_t:
   - draw a random number,
   - use inverse transformation method to determine work-force requirements in week t.
3. Determine regular time work-force capacity for each working day in the week:
   - draw a random number,
   - use inverse transformation method to determine the work-force capacity for the day (number of regular hours that may be worked by all employees in this day);

   Sum daily capacities in successive days to achieve weekly capacity r_t.
4. If $r_t > d_t$ – determine the number of hours that are not effectively utilized:
$$u_t := r_t - d_t$$
5. If $d_t > r_t$ – determine the number of overtime hours worked:
$$o_t := d_t - r_t$$
6. Calculate the labor cost:
$$c_t := r_t w_t + o_t w_t \frac{b_t}{100}$$
7. $C := C + c_t$, $R := R + r_t$, $O := O + o_t$, $U := U + u_t$.
8. If $t = 52$ – go to 9, else assume $t := t + 1$ and go to 2.

9. Calculate values of criteria:

$$X_1 = C, X_2 = O, X_3 = (R - U) / R$$

Each simulation model can be classified as one of two types: terminating and non-terminating [7]. For a terminating simulation there is a natural end point that determines the length of a run. A non-terminating simulation does not have a natural end point. As the aim of our study is to analyze department's activities during a year-long period, terminating simulation is employed. In order to obtain accurate results, simulation runs should be repeated. Such experiments have to be performed for each alternative. As a result, distributional evaluations of alternatives with respect to criteria are obtained.

3. STOCHASTIC DOMINANCE RULES

The methodology used in this paper combines two methods that are frequently used for modeling the choice among uncertain outcomes: mean-risk approach and stochastic dominance. The former is based on two criteria: one measuring expected outcome and another one representing variability of outcomes; the latter one uses stochastic dominance rules. Two groups of stochastic dominance relations can be considered. The first group includes FSD, SSD, and TSD, which means first, second, and third degree stochastic dominance respectively (see Appendix for definitions). These rules can be applied for modeling risk-averse preferences. The FSD rule is for increasing utility function, i.e. for $u(x)$ such that $u'(x) > 0$ for all x. The SSD rule can be applied for concave increasing utility function: $u'(x) > 0$ and $u''(x) \leq 0$. Finally, the TSD rule is for decreasing absolute risk aversion (DARA) utility function, i.e. for a function with $u'(x) > 0$, $u''(x) \leq 0$, $u'''(x) \geq 0$ and $u'''(x)\cdot u'(x) \geq [u''(x)]^2$. The second group includes FSD and three types of inverse stochastic dominance: SISD, TISD1, TISD2 – second degree inverse stochastic dominance and third degree inverse stochastic dominance of the first and the second types. These rules can be applied for modeling risk-seeking preferences. The SISD rule is limited to a convex utility function: $u' > 0$ and $u'' \geq 0$, while TISD1 and TISD2 rules can be used in the case of increasing absolute risk aversion (INARA) utility function, i.e. function with $u'(x) > 0$, $u''(x) \geq 0$, $u'''(x) \geq 0$ and $u'''(x)\cdot u'(x) \leq [u''(x)]^2$ (TISD1) or $u'(x) > 0$, $u''(x) \geq 0$ $u'''(x) \leq 0$ (TISD2).

In the methodology presented here both approaches are employed. The decision maker defines his/her requirements by specifying minimal or maximal values of criteria measuring either expected outcome (mean) or variability of outcomes (standard deviation, standard semideviation, lower/upper standard semideviation from a target value, lower/upper mean semideviation from a target value, probability of below-target/over-target returns). However, SD relations between distributional evaluations are also analyzed in order to detect unclear situations.

4. INTERACTIVE PROCEDURE INSDECM-II

The procedure presented in this study is a modified version of INSDECM technic proposed in [6]. It also exploits some ideas used in the approach proposed in [5]. The first procedure is based on the interactive multiple criteria goal programming approach [8], the latter exploits the main ideas of the STEM technic [1].

INSDECM-II combines concepts that are used in multiple criteria goal programming and STEM method. In each iteration the ideal solution is generated. The elements of the ideal solution are the best values of mean for each criterion, which are individually attainable within the set of alternatives. Next, a candidate alternative is generated. It is the one that is closest to the ideal solution according to the minimax rule. Additionally, a potency matrix is generated. It is composed of the best and the worst values of mean with respect to all criteria. The candidate alternative and potency matrix are presented. If the decision maker is not satisfied with the data available, he/she may specify the kind of additional information that should be provided. By looking at the data, the decision maker may decide whether the proposal is satisfactory. If the answer is YES, the procedure ends, otherwise the decision maker is asked for defining additional requirements. It is assumed that such requirements specify minimal or maximal values of a specified distribution parameter. The consistency of the requirement formulated by the decision maker with stochastic dominance rules is analyzed. It is assumed that the requirement is not consistent with stochastic dominance rules if following conditions are simultaneously fulfilled:

- the evaluation of a_i with respect to criterion X_k does not satisfy the requirement,
- the evaluation of a_j with respect to criterion X_k satisfies the requirement,
- the evaluation of a_i with respect to X_k dominates corresponding evaluation of a_j under stochastic dominance rules.

The pair for which inconsistency occurs is presented and the decision maker is asked to confirm or relax the requirement. If the requirement is confirmed, the assumptions on the stochastic dominance rules that should be fulfilled are revised. As the decision maker confirms that he/she finds the distribution dominated according to stochastic dominance rules as better, it is assumed that this rule should not be used for modeling decision maker's preferences.

Let us assume the following notation:

K$_1$ – the set of indices of criteria, that are defined in such a way that the larger values are preferred to smaller ones,

K$_2$ – the set of indices of criteria, that are defined in such a way that the smaller values are preferred to larger ones,

$\mathbf{A}^l$ – set of alternatives considered in iteration l,

$\mathbf{I}^l$ – set of indexes i, such that $a_i \in \mathbf{A}^l$,

μ_{ik} – mean of the distributional evaluation of alternative a_i in relation to attribute k,

$\mathbf{P}_1^l$ – potency matrix:

$$\mathbf{P}_1^l = \begin{bmatrix} \underline{\mu}_1^l \cdots & \underline{\mu}_k^l & \cdots & \underline{\mu}_m^l \\ \overline{\mu}_1^l \cdots & \overline{\mu}_k^l & \cdots & \overline{\mu}_m^l \end{bmatrix}$$

where: $\overline{\mu}_k^l = \begin{cases} \max\limits_{i \in \mathbf{I}^l}\{\mu_{ik}\} & \text{for } k \in \mathbf{K}_1 \\ \min\limits_{i \in \mathbf{I}^l}\{\mu_{ik}\} & \text{for } k \in \mathbf{K}_2 \end{cases}$ $\quad \underline{\mu}_k^l = \begin{cases} \min\limits_{i \in \mathbf{I}^l}\{\mu_{ik}\} & \text{for } k \in \mathbf{K}_1 \\ \max\limits_{i \in \mathbf{I}^l}\{\mu_{ik}\} & \text{for } k \in \mathbf{K}_2 \end{cases}$

Q – number of distribution parameters chosen by the decision maker for presentation in conversational phase of the procedure,

Q$_1$ – the set of indices of parameters, that are defined in such a way, that the larger values are preferred to smaller ones,

Q$_2$ – the set of indices of parameters, that are defined in such a way, that the smaller values are preferred to larger ones,

v_{ip} – value of p-th parameter for alternative a_i, $i = 1, \ldots, \mathbf{I}^l$, $p = 1, \ldots, Q$,

$\mathbf{P}_2^l$ – additional potency matrix for attribute k in iteration l

$$\mathbf{P}_2^l = \begin{bmatrix} \underline{v}_1^l \cdots & \underline{v}_q^l & \cdots & \underline{v}_Q^l \\ \overline{v}_1^l \cdots & \overline{v}_q^l & \cdots & \overline{v}_Q^l \end{bmatrix}$$

where: $\overline{v}_k^{\,l} = \begin{cases} \max\limits_{i \in \mathbf{I}^l} \{v_{iq}\} & \text{for } q \in \mathbf{Q}_1 \\ \min\limits_{i \in \mathbf{I}^l} \{\mu_{iq}\} & \text{for } q \in \mathbf{Q}_2 \end{cases}$ $\underline{v}_k^{\,l} = \begin{cases} \min\limits_{i \in \mathbf{I}^l} \{v_{iq}\} & \text{for } q \in \mathbf{Q}_1 \\ \max\limits_{i \in \mathbf{I}^l} \{v_{iq}\} & \text{for } q \in \mathbf{Q}_2 \end{cases}$

We will assume, that Stochastic Dominance (SD) rule is fulfilled if following relation is identified:

$$\mathbf{X}_{jk} \succ_{\mathrm{SD}} \mathbf{X}_{ik} \Leftrightarrow \big((\mathbf{X}_{jk} \succ_{\mathrm{FSD}} \mathbf{X}_{ik} \vee \mathbf{X}_{jk} \succ_{\mathrm{SSD}} \mathbf{X}_{ik} \vee \mathbf{X}_{jk} \succ_{\mathrm{TSD}} \mathbf{X}_{ik}) \wedge k \in \mathbf{K}_1 \big) \vee$$
$$\big((\mathbf{X}_{jk} \succ_{\mathrm{FSD}} \mathbf{X}_{ik} \vee \mathbf{X}_{jk} \succ_{\mathrm{SISD}} \mathbf{X}_{ik} \vee \mathbf{X}_{jk} \succ_{\mathrm{TISD1}} \mathbf{X}_{ik} \vee \mathbf{X}_{jk} \succ_{\mathrm{TISD2}} \mathbf{X}_{ik}) \wedge k \in \mathbf{K}_2 \big)$$

The operation of the procedure is as follows:

I. Initial phase:

1. Calculate means of distributional evaluations of alternatives with respect to attributes μ_{ik}, $i = 1, ..., n$, $k = 1, ..., m$.
2. $\mathbf{A}^l = \mathbf{A}$.

II. Iteration l:

3. Identify candidate alternative a_i:

   $a_i := \arg\min\limits_{j \in \mathbf{I}^l} \{ d_{jk}^{\,l} \}$

   where $d_{jk}^{\,l}$ is calculated as follows:

$$d_{jk}^{\,l} = \max_{k=1,\ldots,m} \left\{ w_k^l \left| \overline{\mu}_k^{\,l} - \mu_{jk} \right| \right\} \qquad w_k^l = \frac{1}{r_k^l} \left[\sum_{i=1}^{m} \frac{1}{r_i^l} \right]^{-1} \qquad r_k^l = \left| \overline{\mu}_k^{\,l} - \underline{\mu}_k^{\,l} \right|$$

   In the case of a tie choose any a_i minimizing the value of $d_{jk}^{\,l}$.

4. Present the data to the decision maker:
   - means of distributional evaluations of the candidate alternative a_i: μ_{ik}, $k = 1, ..., m$,
   - potency matrix $\mathbf{P}_1^l$.
5. Ask the decision maker whether he/she is satisfied with the data that are presented. If the answer is YES – go to 7.
6. Ask the decision maker to specify parameters of distributional evaluations to be presented; calculate distribution parameters v_{ip} for i such that $a_i \in \mathbf{A}^l$, $p = 1, \ldots, Q$; calculate additional potency matrix $\mathbf{P}_2^l$; present additional potency matrix to the decision maker.

7. Ask the decision maker whether he/she is satisfied with the candidate alternative. If the answer is YES – the final solution is alternative a_i – go to 17, else – go to 8.
8. Ask the decision maker to specify an additional requirement.
9. Generate Al+1 the set of alternatives satisfying the requirement specified by the decision maker.
10. Calculate potency matrices $\mathbf{P}_1^{l+1}$ and $\mathbf{P}_2^{l+1}$; present matrices $\mathbf{P}_1^{l}$, $\mathbf{P}_2^{l}$, $\mathbf{P}_1^{l+1}$ and $\mathbf{P}_2^{l+1}$ to the decision maker; ask the decision maker whether he/she accepts the move from $\mathbf{P}_1^{l}$ and $\mathbf{P}_2^{l}$ to $\mathbf{P}_1^{l+1}$ and $\mathbf{P}_2^{l+1}$. If the answer is NO, then go to 4, else go to 11.
11. For each pair (a_j, a_i) such that $a_j \in \mathbf{A}^l \setminus \mathbf{A}^{l+1}$ and $a_i \in \mathbf{A}^{l+1}$ identify SD relation between $\mathbf{X}_{jk}$ and $\mathbf{X}_{ik}$. Generate the set of inconsistencies:

$$\mathbf{N}^l = \left\{ \left(a_j, a_i\right), a_j \in \mathbf{A}^l \setminus \mathbf{A}^{l+1}, a_i \in \mathbf{A}^{l+1}, \mathbf{X}_{jk} \succ_{\mathrm{SD}} \mathbf{X}_{ik} \right\}$$

12. If $\mathbf{N}^l = \varnothing$, then assume $l = l + 1$; go to 3, else go to 13.
13. Choose the first pair $\left(a_j, a_i\right) \in \mathbf{N}^l$; calculate:

    $\Pr(X_{ik} \le s_r)$, $\Pr(X_{jk} \le s_r)$

    where:

    $$s_r = \min\left(\alpha_i, \alpha_j\right) + r\frac{\max\left(\beta_i, \beta_j\right) - \min\left(\alpha_i, \alpha_j\right)}{R} \quad \text{for } r = 0, 1, \ldots, R$$

    α_i, β_i – lower and upper bound for evaluations of X_{ik},

    α_j, β_j – lower and upper bound for evaluations of X_{jk},

    R – number of observations. Initially R can be set to 10, the decision maker can increase (decrease) the value of R if he/she finds the data to be not enough detailed (too detailed).

    Present the data to the decision maker pointing that a_j is to be rejected, while a_i is to be accepted. Ask the decision maker what is his/her decision – propose the decision maker:

    a) accept a_i and reject a_j,
    b) accept both a_j and a_i,
    c) reject both a_j and a_i.

    If the decision maker's decision is (a), go to 11, if the decision is (b), go to 15, otherwise go to 16.
14. $\mathbf{N}^l = \mathbf{N}^l \setminus \left\{\left(a_j, a_i\right)\right\}$; go to 12.

15. $\mathbf{A}^{l+1} = \mathbf{A}^{l+1} \cup \{a_j\}$, $\mathbf{N}^l = \mathbf{N}^l \setminus \{(a_j, a_i)\}$; go to 12.
16. $\mathbf{A}^{l+1} = \mathbf{A}^{l+1} \setminus \{a_i\}$, $\mathbf{N}^l = \mathbf{N}^l \setminus \{(a_j, a_i)\}$; go to 12.
17. End of the procedure.

III. Comments:

Step 6: The decision maker may specify various distribution parameters to be presented during the conversational phase of the procedure. Some examples are: standard deviation, lower/upper standard semideviation from a target value ψ, lower/upper mean semideviation from a target value ψ, probability of getting the outcome not exceeding a target value ψ, probability of getting the outcome not less than a target value ψ.

Step 8: The decision maker defines additional requirements by specifying minimal or maximal acceptable values of distribution parameters. Thus, the decision maker's additional requirements may be formulated as follows:

– mean not less/not greater than a specified target value ψ,
– standard deviation not greater than a specified value ξ,
– lower standard semideviation from a target value ψ not greater/not less than a specified value ξ,
– upper standard semideviation from a target value ψ not less/not greater than a specified value ξ,
– lower mean semideviation from a target value ψ not greater/not less than a specified value ξ,
– upper mean semideviation from a target value ψ not less/not greater than a specified value ξ,
– probability of getting outcome not exceeding a specified target value ψ not greater/not less than a specified value α,
– probability of getting outcome not less than a specified target value ψ not less/not greater than a specified value α.

5. ILLUSTRATIVE EXAMPLE

To illustrate the procedure let us consider a labor planning problem in a payroll department. Twenty alternative staffing plans are considered (Table 1).

Table 1

The set of alternatives **A**

Alternative	Number of employees			
	Full-time	Part-time (6/8)	Part-time (4/8)	Part-time (2/8)
a_1	4	0	0	0
a_2	3	1	0	0
a_3	3	0	1	0
a_4	3	0	0	1
a_5	3	2	0	0
a_6	3	0	2	0
a_7	3	0	0	2
$a8$	3	1	1	0
a_9	3	1	0	1
a_{10}	3	0	1	1
a_{11}	2	3	0	0
a_{12}	2	0	3	0
a_{13}	2	0	0	3
a_{14}	2	2	1	0
a_{15}	2	2	0	1
a_{16}	2	1	2	0
a_{17}	2	1	0	2
a_{18}	2	0	2	1
a_{19}	2	0	1	2
a_{20}	2	1	1	1

Three criteria are used for evaluating the performances of alternative plans:

X_1 – yearly labor costs,

X_2 – total number of overtime hours worked by all employees in the department during the year,

X_3 – work-force utilization rate measured by the contribution of regular hours effectively worked to the total number of regular hours worked by employees.

Based on past experience distributions of work-force requirements for each week of the month have been estimated (Table 2).

Table 2

Distributions of work-force requirements

Week 1 and 5		Week 2	
Work-force requirements (hours per day)	Probability	Work-force requirements (hours per day)	Probability
96	0,30	88	0,30
108	0,40	100	0,40
120	0,20	112	0,20
132	0,10	124	0,10
Week 3		**Weeks 4**	
Work-force requirements (hours per day)	Probability	Work-force requirements (hours per day)	Probability
80	0,30	88	0,30
92	0,40	100	0,40
104	0,20	112	0,20
116	0,10	124	0,10

For each alternative distribution of daily work-force's capacity has been estimated. It was assumed that the probability of an employee's absence is equal to 0,15. Table 3 presents the distribution for alternative a_1.

Table 3

Distribution of a daily work-force capacity
(man-hours)

Capacity (hours)	Probability
0	0,0005
8	0,0115
16	0,0975
24	0,3685
32	0,5220

Simulation has been applied for generating distributional evaluations of alternatives. Table 4 presents results of simulation experiments.

Table 4

Results of simulation experiments

Alternative	Mean of distributional evaluation		
	Cost (PLZ)	Overtime (hours)	Work-force utilization rate (%)
a_1	122473,14	91,9	76,18%
a_2	108509,64	131,5	80,70%
a_3	95054,28	198,1	85,19%
a_4	97601,70	314,7	89,78%
a_5	121693,85	42,5	68,36%
a_6	92769,60	86,6	76,33%
a_7	95109,00	199,2	85,39%
a8	107170,28	61,7	72,15%
a_9	107630,42	86,6	76,27%
a_{10}	93626,28	129,1	80,74%
a_{11}	107119,38	60,6	72,14%
a_{12}	65306,73	193,3	85,41%
a_{13}	77149,47	731,7	97,10%
a_{14}	92780,49	84,6	76,36%
a_{15}	93556,76	125,7	80,72%
a_{16}	78733,29	124,9	80,79%
a_{17}	82582,17	304,9	89,95%
A_{18}	67694,00	306,5	89,99%
A_{19}	71629,94	482,1	94,00%
A_{20}	80156,84	194,6	85,39%

Final solution is generated as follows:

I. Iteration 1:

$\mathbf{A}^1 = \mathbf{A} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}\}$

3. Candidate alternative is identified: $a_{17.}$
4. The data are presented to the decision maker:
   - means of distributional evaluations of the candidate alternative: μ17 1 = 82582,17, μ17 2 = 304,9, μ17 3 = 89,95%,
   - potency matrix $\mathbf{P}_1^1$ (Table 5).

Table 5

Potency matrix $\mathbf{P}_1^1$

	Criterion		
	1	2	3
$\underline{\mu}_k^1$	122473,14	731,7	68,36%
$\overline{\mu}_k^1$	65306,73	42,5	97,10%

5. The decision maker is not satisfied with data presented.
6. The decision maker is interested in the probability that the number of overtime hours (criterion No. 2) is not less then 240: $Q = 1$, $v_{i\,1} = \Pr(X_{i\,2} \geq 240)$. Additional potency matrix $\mathbf{P}_2^1$ is calculated (Table 6), the data are presented to the decision maker: $v_{17\,1} = \Pr(X_{17\,1} \geq 240) = 0{,}871$.

Table 6

Potency matrix $\mathbf{P}_2^1$

	$v_{i\,1} = \Pr(X_{i\,2} \geq 240)$
$\underline{v}_1^1$	1,000
$\overline{v}_1^1$	0,000

7. The decision maker is not satisfied with the candidate alternative.
8. The decision maker specifies additional requirement:

$$\Pr(X_{i\,2} \geq 240) \leq 0{,}2$$

9. Set of alternatives satisfying the requirement specified by the decision maker is generated:

$$\mathbf{A}^2 = \{a_1, a_2, a_3, a_5, a_6, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{14}, a_{15}, a_{16}, a_{20}\}$$

10. Potency matrices $\mathbf{P}_1^2$ and $\mathbf{P}_2^2$ are calculated (Tables 7 and 8); matrices $\mathbf{P}_1^1$, $\mathbf{P}_2^1$, $\mathbf{P}_1^2$, and $\mathbf{P}_2^2$ are presented to the decision maker who accepts the move from $\mathbf{P}_1^1$ and $\mathbf{P}_2^1$ to $\mathbf{P}_1^2$ and $\mathbf{P}_2^2$.

Table 7

Potency matrix $\mathbf{P}_1^2$

	Criterion		
	1	2	3
$\underline{\mu}_k^2$	122473,14	198,1	68,36%
$\overline{\mu}_k^2$	65306,73	42,5	85,41%

Table 8

Potency matrix $\mathbf{P}_2^2$

	$v_{i\,1} = \Pr(X_{i\,1} \le 277250)$
$\underline{v}_1^2$	0,195
$\overline{v}_1^2$	0,000

11. None inconsistencies are identified.
11. As $\mathbf{N}^1 = \varnothing$, so $l = 2$.

II. Iteration 2:

3. Candidate alternative is identified: a_{16}.
4. The data are presented to the decision maker:
   - means of distributional evaluations of the candidate alternative: $\mu_{16\,1} = 78733{,}29$, $\mu_{16\,2} = 124{,}9$, $\mu_{16\,3} = 80{,}79\%$,
   - potency matrix $\mathbf{P}_1^2$ (Table 7).
5. The decision maker is satisfied with data presented.
6. The decision maker is not satisfied with the candidate alternative.
7. The decision maker specifies additional requirement:

$$\mu_{i\,3} \ge 85{,}0\%$$

9. Set of alternatives satisfying the requirement specified by the decision maker is generated:

$$\mathbf{A}^3 = \{a_3, a_{12}\}$$

10. Potency matrix $\mathbf{P}_1^3$ is generated (Table 9); matrices $\mathbf{P}_1^2$ and $\mathbf{P}_1^3$ are presented to the decision maker who accepts the move from $\mathbf{P}_1^2$ to $\mathbf{P}_1^3$.

Table 9

Potency matrix $\mathbf{P}_1^3$

	Criterion		
	1	2	3
$\underline{\mu}_k^3$	95054,28	198,1	85,19%
$\overline{\mu}_k^3$	65306,73	193,3	85,41%

11. The set of inconsistencies is generated.
12. As $\mathbf{N}^2 = \emptyset$, so $l = 3$.

III. Iteration 3:

3. Candidate alternative is identified: a_{12}.
4. Presentation of the data to the decision maker:
   - average evaluations of the candidate alternative a_{12}: $\mu_{12\,1} = 65306{,}73$, $\mu_{12\,2} = 193{,}3$, $\mu_{12\,3} = 85{,}41\%$,
   - potency matrix $\mathbf{P}_1^3$ (Table 9).
5. The decision maker is satisfied with data presented.
7. The decision maker is satisfied with the candidate alternative – a_{22} is the final solution
10. The end of the procedure.

As the alternative a_{12} is the final solution, so 2 full-time employees and 3 part-time employees working 4 hours per day should be hired.

CONCLUSIONS

Usually, the objective of labor planning is to minimize cost over the planning period. However, other issues may be also important. Minimizing overtime and maximizing work-force utilization rate are also analyzed when a staffing plan is prepared. As these criteria are in conflict, we are faced with a multiple criteria decision problem.

The main purpose of this paper was to give comprehensive, yet simple methodology for labor planning problems. A new methodology for determining the number of full-time and part-time employees was presented. Although this approach was applied for labor planning in a payroll department, it could be easily adapted to other organizations.

The procedure uses two approaches: stochastic dominance and interactive methodology. The former is widely used for comparing uncertain prospects, the latter is a multiple criteria technic that is probably most often used in real--world applications. These two concepts have been combined in the INSDECM--II procedure.

Appendix

Notation:

$F(x)$, $G(x)$ – cumulative distribution functions

$$F(x) = \Pr(X_F \le x)$$

$$G(x) = \Pr(X_G \le x)$$

$\overline{F}(x), \overline{G}(x)$ – decumulative distribution functions

$$\overline{F}(x) = \Pr(X_F \ge x)$$

$$\overline{G}(x) = \Pr(X_G \ge x)$$

Definition 1:

$F(x) \succ_{\text{FSD}} G(x)$ if and only if

$F(x) \ne G(x)$ and $H_1(x) = F(x) - \text{G}(x) \le 0$ for all $x \in [a,b]$

Definition 2:

$F(x) \succ_{\text{SSD}} G(x)$ if and only if

$F(x) \ne G(x)$ and $H_2(x) = \int_a^x H_1(y)dy \le 0$ for all $x \in [a,b]$

Definition 3:

$F(x) \succ_{\text{TSD}} G(x)$ if and only if

$F(x) \ne G(x)$ and $H_3(x) = \int_a^x H_2(y)dy \le 0$ for all $x \in [a,b]$

Definition 4:

$\overline{F}(x) \succ_{\text{SISD}} \overline{G}(x)$ if and only if

$\overline{F}(x) \ne \overline{G}(x)$ and $\overline{H}_2(x) = \int_x^b \overline{H}_1(y)dy \ge 0$ for all $x \in [a,b]$

where: $\overline{H}_1 = \overline{F}(x) - \overline{G}(x)$

Definition 5:

$$\overline{F}(x) \succ_{\text{TISD1}} \overline{G}(x) \text{ if and only if}$$

$$\overline{F}(x) \neq \overline{G}(x) \text{ and } \overline{H}_3(x) = \int_x^b \overline{H}_2(y)dy \geq 0 \text{ for all } x \in [a,b]$$

Definition 6:

$$\overline{F}(x) \succ_{\text{TISD2}} \overline{G}(x) \text{ if and only if}$$

$$\overline{F}(x) \neq \overline{G}(x) \text{ and } \widetilde{H}_3(x) = \int_a^x \overline{H}_2(y)dy \geq 0 \text{ for all } x \in [a,b]$$

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