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MCDM WITH RELATIVE IMPORTANCE OF CRITERIA: APPLICATION TO CONFIGURATION DESIGN OF VEHICLES*

Abstract

In this article, a preference model of relative importance of criteria is presented and applied to a vehicle configuration design problem formulated as a multi-objective program. The model is based on convex cones and extends the classical notion of Pareto optimality. Preferences quantifies as allowable tradeoffs are elicited from the decision maker and used for the construction of a new preference. In effect, the Pareto set is reduced which facilitates the process of choosing a final preferred solution.

Configuration design of mechanical systems corresponds to finding the placement of a set of components such that performance criteria are optimized while satisfying design constraints. The presented configuration design problem involves designing a midsize truck for optimum vehicle dynamic behavior, survivability, and maintainability in the presence of decision maker's preferences that are included *a posteriori*. A set of Pareto solutions is first generated with a multi-objective genetic algorithm, and the Pareto solutions are screened according to the preferences quantified as allowable tradeoffs. The model extracts preferred designs from the Pareto set producing a short list of "strong" or "privileged" designs, which is a useful feature when preferences are unknown.

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Keywords

Multi-criteria decision-making, preferences, Pareto optimality, relative importance, tradeoffs, Pareto set, configuration design, vehicle design.

Introduction

In the operations research as well as engineering literature on multi-criteria decision- making (MCDM), relative importance of criteria has predominantly been modeled with lexicographic ordering or weights applied to criteria with a higher weight denoting greater importance. Among many others who worked with weights, Podinovskii seems to have mastered this technique in a series of papers [22, 23, 24]. However, it has been observed that weights do not always effectively model relative importance of criteria [16] and methods that relax or even eliminate weights have also been proposed. Weight-free optimization of the most important criterion in the presence of additional constraints generated by the other criteria was proposed by Ignizio [8], Papalambros and Wilde [20], and others. Yakowitz and Lane [31] proposed a method based on a set of weights for every criterion rather than a unique weight. Noghin [18] and Noghin and Tolstykh [19] used weights to construct a vector representing the relative importance and augmenting the Pareto concept of optimality.

Theoretical foundations of the preference model used in this paper were developed by Hunt [6] and Hunt and Wiecek [7]. The approach uses tradeoffs rather than weights to augment Pareto optimality. Tradeoffs play a fundamental role in MCDM providing a direct link between the criteria and attainable outcomes. Kaliszewski and Michalowski [9, 10] developed a methodology to generate Pareto solutions with a priori-selected bound on tradeoffs. They also proposed psychologically stable solutions defined to be the solutions that attain satisfactory criterion values and have acceptable tradeoffs [11].

Our work takes a similar approach by allowing the decision maker (DM) to set a priori bounds on tradeoffs and model relative importance of criteria using these bounds. The approach augments Pareto optimality. In particular, not only are criteria expected to improve (or remain unchanged) but, additionally, a group of relatively more important criteria are allowed to improve at the expense of decaying relatively less important criteria. The augmentation reduces the Pareto set and therefore facilitates the choice of a single final solution. While the model is theoretically grounded in convex analysis, it is numerically easily available in the form of a matrix, which is used to premultiply the vector of criterion functions. The model is applicable to decision--making situations under the following axioms: (1) the DM recognizes relative importance of criteria; (2) the Pareto set is large; (3) a small subset of preferred solutions is determined in agreement with the assumed relative importance from which a final preferred solution will be selected.

This paper illustrates the applicability of the relative importance model to a vehicle configuration design problem which has been selected due to inherent presence of multiple criteria and the ongoing interest in solving complex design problems within the engineering community. In contrast to other approaches in engineering, the designer is not automatically led to a unique final design but rather he/she retains the right to choose and exercises this right within a subset of preferred designs. The model extracts preferred designs from the Pareto set producing a short list of "strong" or "privileged" designs, which is a useful feature when preferences are unknown.

The remainder of the paper includes five sections. In Section 1, we briefly review the conventional MCDM framework governed by the concept of Pareto optimality. In Section 2, we develop a model of relative importance of criteria which is based on the convex-cone approach to multi-objective programming originally proposed by Yu [32]. Sections 3 and 4 present our application. The vehicle configuration design problem is described in Section 3. In Section 4, a design methodology developed by incorporating the preference model into the configuration design problem is described and applied to the design of the FMTV truck to enhance vehicle dynamics performance, survivability, and maintainability. Significance of the method in engineering design is also discussed. Finally, Section 6 concludes the paper.

1. Preliminaries

In MCDM we typically consider the following conventional multi-objective program (MOP)

$$\begin{array}{ll} \text{minimize} & \boldsymbol{f}(\boldsymbol{x}) = [f_1(\boldsymbol{x}),\ldots,f_m(\boldsymbol{x})]^{\mathrm{T}} \\ \text{subject to} & \boldsymbol{x} \text{ in } S \subseteq R^n \end{array} \tag{1}$$

where $S \subseteq R^n$ is the set of feasible solutions in the decision (solution, design) space R^n and each criterion function $f_i(\boldsymbol{x})$, $i = 1,\ldots,m$, is real-valued. The image

$\boldsymbol{y} = f(\boldsymbol{x}) \in R^m$ of a solution $\boldsymbol{x} \in S$ is called an outcome and R^m is referred to as the objective (outcome, performance) space. The set of all attainable outcomes is defined as $Y = \{\boldsymbol{y} \in R^m : \boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}), \boldsymbol{x} \in S\}$. In the first *optimization* stage of MCDM, problem (1) is solved for Pareto solutions according to the notion of Pareto-optimality. A feasible solution $\boldsymbol{x}^1$ is said to be *Pareto* if there exists no other feasible solution $\boldsymbol{x}^2$ such that $f_i(\boldsymbol{x}^2) \leq f_i(\boldsymbol{x}^1)$ for all $i = 1,\ldots,m$, and $f_j(\boldsymbol{x}^2) < f_j(\boldsymbol{x}^1)$ for at least one j. Note that if such a solution $\boldsymbol{x}^2$ exists, then outcome $f(\boldsymbol{x}^2)$ is said to dominate outcome $f(\boldsymbol{x}^1)$. The image of a Pareto solution is called a *Pareto* outcome. In other words, an attainable outcome $\boldsymbol{y}^1$ is said to be Pareto if there exists no other attainable outcome $\boldsymbol{y}^2$ and a non-zero $\boldsymbol{d} \in R^m$, $\boldsymbol{d} \leq \boldsymbol{0}$ such that $\boldsymbol{y}^2 = \boldsymbol{y}^1 + \boldsymbol{d}$ [32]. Let $E(S, f, \text{Par})$ and $N(Y, \text{Par})$ denote the set of all Pareto solutions and Pareto outcomes of (1), respectively. In the second *decision-making* stage of MCDM, a preferred solution is selected from among Pareto solutions based on preferences elicited from the DM. This preferred solution is the final result of the MCDM process.

According to Pareto optimality used in problem (1), which is a minimization problem, during the search for Pareto outcomes one examines those directions in the outcome space, $\boldsymbol{d} \in R^m$, along which at least one criterion value decreases (improves) while the others remain unchanged, i.e., $\boldsymbol{d} \leq \boldsymbol{0}$. In this context, we define *decay* and *improvement* of criteria, and the notion of *preferred direction* in the outcome space.

Definition 1.1. For MOP (1), criterion *decay* occurs when the value of a criterion function increases and criterion *improvement* occurs when the value of a criterion function decreases.

Definition 1.2. For MOP (1) with Pareto optimality, a direction $\boldsymbol{d} \in R^m$ along which criterion values improve or remain unchanged is referred to as *preferred direction.*

In other words, for (1), the preferred directions $\boldsymbol{d} \in R^m$ yield the inequalities

$$d_i \leq 0, \text{ for all } \mathrm{i} = 1,\ldots,\mathrm{m} \tag{2}$$

or, in an equivalent matrix notation:

$$\boldsymbol{Id} \leqq 0 \tag{3}$$

where $\boldsymbol{I}$ denotes the $m \times m$ identity matrix. Alternatively, the collection of all preferred directions forms the set

$$R_{\leqq}^{m} = \{\boldsymbol{d} \in R^m \text{: } \boldsymbol{Id} \leqq \boldsymbol{0}\} \tag{4}$$

known as the *Pareto cone of preferred directions* or *Pareto preference cone* where the matrix ***I*** is referred to as the *Pareto preference matrix* [29]. Figure 1 depicts the Pareto preference cone for $m = 2$. Using this perspective, the Pareto sets are often denoted as $E(S, f, R_{\leqq}^{m})$ and $N(Y, R_{\leqq}^{m})$.

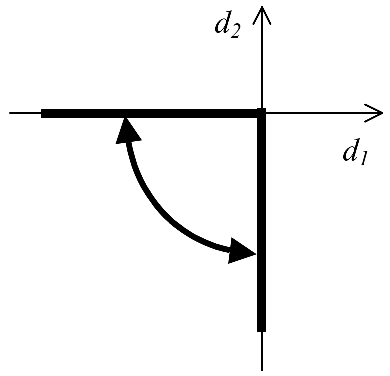

Figure 1. Pareto preference cone in R^2

2. Model of relative importance of criteria

In this section, we present an approach to modeling relative importance of criteria and give its simple description in order to lay the groundwork for the application presented in the subsequent section. For the detailed development and complete derivations of the model, we refer the reader to Hunt [6].

2.1. Relative importance preference cone

We assume that every DM follows the Pareto optimality implying that every direction in the Pareto cone is a preferred direction. However, we also assume that the indices of all criteria $\{1,\ldots,m\}$ are divided into two groups: the set of indices M corresponding to a relatively more important group of criteria that are not allowed to decay and the set of indices L corresponding to a relatively less important group of criteria that are allowed to decay. Even though the criteria represented by L are allowed to decay, if they also improve or remain unchanged then we consider ourselves fortunate.

We introduce the concept of an *allowable tradeoff* to quantify decay and improvement between two criteria.

Definition 2.1. An *allowable tradeoff* between criteria i and j, $i, j \in \{1,\ldots,m\}$, $i \neq j$, denoted a_{ij}, is the largest amount of decay in criterion i considered allowable to the DM to gain one unit of improvement in criterion j. Also, $a_{ij} \geq 0$ for all i and j, $i \neq j$.

The values of the allowable tradeoffs a_{ij} depend on the DM's preferences. If $a_{ij} = 0$ for all $i, j = 1,\ldots,m$, $i \neq j$, then the DM follows only the notion of Pareto optimality and only applies the Pareto preference cone. If the DM is willing to allow tradeoffs between criteria (i.e., there exists at least one nonzero allowable tradeoff), then additional preferred directions are appended to the Pareto cone to construct a new preference cone. An experienced DM may have previous knowledge of and experience with the decision problem to guide the assignment of allowable tradeoff values but, in general, assigning specific values to a_{ij} may be difficult. In Section 4 we show how this issue may be resolved.

The sets L and M are constructed such that $L \cup M = \{1,\ldots,m\}$ and $L \cap M = \varnothing$. The DM is required to define an allowable tradeoff a_{ij} for every pair of criteria i and j, $i,j \in \{1,\ldots,m\}$, $i \neq j$, such that $i \in L$ and $j \in M$. As assumed above, all directions in the Pareto preference cone are always preferred to the DM and are always contained in a new preference cone. However, to reflect the relative importance of criteria, other directions also become preferred and are appended to the Pareto cone to yield this new cone.

Definition 2.2. Consider (1) and let L and M be the set of indices of relatively less important and relatively more important criteria, respectively, defined by the DM. Define a set of preferred directions as

$$W = \{\boldsymbol{d} \in R^m : d_i \leq 0 \text{ for all } i \in M \text{ and } -d_k \geq \sum_{i \in M} a_{ki} d_i \text{ for each } k \in L\}$$

Note that the components d_i, $i \in M$, of directions $\boldsymbol{d} \in$ W are only allowed to be non-positive because the corresponding criteria are considered relatively more important and are never allowed to decay. The components d_k, $k \in L$, of directions $\boldsymbol{d} \in$ W are allowed to be positive to represent decay in the corresponding criteria. However, these components are also allowed to be non--positive to represent improvement or no change in the corresponding criteria, which is an attractive occurrence in an effort to minimize the criteria as prescribed in (1). Definition 2.2 also reveals that the total amount of decay allowed for each criterion indexed by $k \in L$ is bounded from below by the value of the expression:

$$\sum_{i \in M} a_{ki} d_i \leq 0 \tag{5}$$

which is always non-positive and equal to a cumulative allowable decay based upon the allowable tradeoff values and the total amount of improvement obtained in the criteria in M. If $d_k \leq 0$ for some $k \in L$, representing improvement or no change in criterion k, then $-d_k \geq 0$ and the inequality

$$-d_k \geq \sum_{i \in M} a_{ki} d_i \tag{6}$$

always holds. However, if $d_k > 0$ for some $k \in L$, representing decay in criterion k, then $-d_k < 0$ and we require that this decay be bounded by the desired expression so that inequality (6) holds again.

Definition 2.3 Consider (1) and let L and M be the sets of indices of relatively less important and relatively more important criteria, respectively, defined by the DM. The *relative importance preference cone* is defined as:

$$C^A = R^m_{\leqq} \cup W \tag{7}$$

In [6] it is shown that cone (7) can be represented in the inequality form:

$$C^A = \{\boldsymbol{d} \in R^m : \boldsymbol{Ad} \leqq \boldsymbol{0}\} \tag{8}$$

where A is an $m \times m$ relative importance preference matrix with entries Aij in row i and column j defined as:

$$\begin{aligned} &A_{ii} = 1 && \text{for all } i \in \{1,...,m\} \\ &A_{ij} = a_{ij} && \text{for all } i \in L \text{ and } j \in M \\ &A_{ij} = 0 && \text{otherwise.} \end{aligned} \tag{9}$$

The structure of $\boldsymbol{A}$ depends on the definition of the sets of criterion indices L and M. For example, for a three-criteria problem, let $L = \{2\}$ and $M = \{1, 3\}$. Then $\boldsymbol{A}$ has the following form:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

To observe the relationship between the structure of this matrix and the structure of the cone, we analyze the system of linear inequalities from the cone representation in (8). The first and last inequalities in (8), corresponding to the first and last rows of matrix $\boldsymbol{A}$, enforce that the first and third criterion may only improve or remain unchanged which is required since $M = \{1, 3\}$. The second inequality in (8), corresponding to the second row of matrix $\boldsymbol{A}$, enforces that the second criterion is allowed to decay since $L = \{2\}$

and the decay is controlled by the lower bound implied by this inequality. The structure of the cone implied by this preference can be easily presented graphically when considering two criteria at a time. For the criteria 1 and 2, the model reduces to

$$d_1 \leq 0$$

$$a_{21}d_1 + d_2 \leq 0$$

which is depicted in Figure 2.

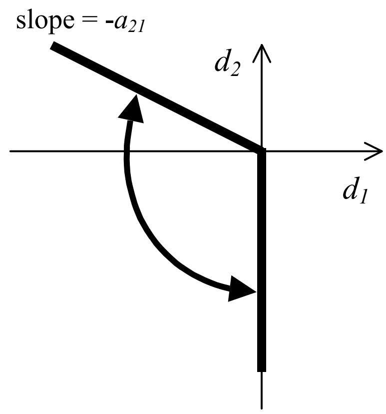

Figure 2. Relative importance preference cone in R^2

2.2. Integration of relative importance with conventional MOPs

In order to integrate the preference model given by (7-9) with the MOP given by (1) we make use of the following results from the literature. In multi-objective programming with general cones representing preferences rather than the Pareto cone, the solution sets are typically referred to as the *efficient set* $E(S, f, C)$ and the *nondominated set* $N(Y, C)$ for a cone C in R^m.

Theorem 1 [26]: Let C_1 and C_2 be cones in R^m. If $C_1 \subseteq C_2$ then $N(Y, C_2) \subseteq N(Y, C_1)$.

Theorem 2: Let C be a convex and pointed cone in R^m represented by $C = \{\boldsymbol{d} \in R^m\text{: } \boldsymbol{Ad} \leqq \boldsymbol{0}\}$. Then

(i) [18]: $\mathrm{E}(\mathrm{S}, \boldsymbol{f}, \mathrm{C}) = \mathrm{E}(\mathrm{S}, \boldsymbol{Af}, R_{\leqq}{}^{m})$;

(ii) [7]: $\boldsymbol{A}[\mathrm{N}(\mathrm{Y}, \mathrm{C})] = \mathrm{N}(\boldsymbol{A}[\mathrm{Y}], R_{\leqq}{}^{m})$.

Based on Theorem 1, we observe that since $R_{\leqq}^{m} \subseteq C^{A}$, then $N(Y, C^{A}) \subseteq N(Y, R_{\leqq}^{m})$ and the Pareto set of (1) is reduced to a subset of efficient solutions with respect to the cone C^{A}. Theorem 2 requires that the cone C be pointed. In [6] it is shown that cone C^{A} given by (8-9) is always pointed. In part (i), this theorem reveals that the efficient set with respect to the cone C^{A} is equal to the Pareto set of an MOP related to (1) and formulated as:

$$\begin{array}{ll} \min & \boldsymbol{g}(\boldsymbol{x}) = [g_1(\boldsymbol{x}),\ldots,g_m(\boldsymbol{x})]^{\mathrm{T}} = \boldsymbol{A}\,[f_1(\boldsymbol{x}),\ldots,f_m(\boldsymbol{x})]^{\mathrm{T}} \\ \text{s.t.} & \boldsymbol{x} \in S \end{array} \tag{11}$$

where $g_i(\boldsymbol{x}) = \boldsymbol{A}_i f_i(\boldsymbol{x})$ and $\boldsymbol{A}_i$ is the i-th row of matrix $\boldsymbol{A}$, $i = 1,\ldots,m$.

In view of this results we propose two methods to implement the preference model and find $E(S, f, CA)$. Let a matrix $\boldsymbol{A}$ be given.

(1) In the first one-step method, we find the set $E(S, \boldsymbol{Af}, R_{\leqq}^{m})$, i.e., solve (11) for its Pareto solutions. To do so, we may apply any suitable method from the literature.

(2) In the other two-step method, we first find the Pareto set $\mathrm{E}(S, \boldsymbol{f}, R_{\leqq}^{m})$ again applying any suitable method. In the next step, we find the Pareto set $E(E(S, \boldsymbol{f}, R_{\leqq}^{m}), \boldsymbol{Af}, R_{\leqq}^{m})$, i.e., we solve an MOP:

$$\begin{array}{ll} \min & \boldsymbol{A}\,[f_1(\boldsymbol{x}),\ldots,f_m(\boldsymbol{x})]^{\mathrm{T}} \\ \text{s.t.} & \boldsymbol{x} \in E(S, \boldsymbol{f}, R_{\leqq}^{m}) \end{array} \tag{12}$$

To accomplish this, we screen the set $E(S, \boldsymbol{f}, R_{\leqq}^{m})$ for the solutions that fulfill the preference quantified by $\boldsymbol{A}$.

In general, both methods are applicable to MCDM problems. The one--step method is especially suitable for decision-making situations in which a priori preferences are known and the resulting solutions are immediately sought. On the other hand, the two-step method is flexible and allows the DM to explore relationships between the preferences given a priori and possibly changed later and the resulting changes in the reduction of the Pareto set. Since DM's preferences are not always known in engineering design problems, we apply the two-step method to the vehicle configuration design problem presented in this paper. We first generate the Pareto set and then identify $E(S, \boldsymbol{f}, C^{A})$ and $N(Y, C^{A})$ of the design problem. For brevity, in the remaining sections we refer to these solutions as *preferred solutions* (*decisions, designs*) and to their images as *preferred outcomes*.

3. Vehicle configuration design

Configuration design of mechanical systems consists of finding optimal locations and orientations of a set of mechanical components such that design constraints are satisfied and performance criteria are met or exceeded. In the literature, it is sometimes referred to as packing, packaging, and layout optimization. Due to the complexity of such problems, algorithms that guarantee to find optimal design solutions are computationally prohibitive. Thus, researchers have directed their effort toward generating acceptable solutions in a reasonable amount of time. Recent surveys of configuration design methods are available in [1, 2].

As the demand on performance of mechanical systems increases, the number of criteria to be included in a configuration design problem is generally large leading to multi-objective programs of type (1). To deal with the multi-criteria aspect of configuration design problems, the most popular approach is to aggregate the criteria into a single objective function with a weighted sum. Drawbacks of this approach have been highlighted in several publications [4, 12, 16]. In particular, to generate the efficient set, the execution of the algorithm must be repeated with different values of the weights without a clear control over the exploration of the design space and objective space.

Other research efforts focused on developing techniques that can generate the efficient set (or its good approximation) with a single execution of the algorithm. Miao et al. [14, 15] developed an approach based on a rank-based genetic algorithm (GA), NSGA-II [5]. They applied this algorithm to the configuration design problem of the US Army family of medium tactical vehicles (FMTV) with three conflicting criteria: vehicle dynamics, survivability, and maintainability. An approximation of the Pareto set was successfully generated with high diversity in both the solution (design) space and the objective space, which is the result of an efficient and thorough exploration of the feasible space.

Once the Pareto set has been approximated, it is then the responsibility of designers and DMs to choose a Pareto solution (design) that is acceptable based on their preferences. Since many methods have been proposed in engineering to facilitate the designers' task of eliciting preferences and incorporating them into the design problem, we refer to more recent or effective efforts. Messac [13] introduced physical programming, in which designers express preferences by specifying ranges of desirability to each criterion. Narayanan and Azarm [17] proposed an interactive method to iteratively narrow down the Pareto solution set to a size small enough for DMs to easily select the final solution. At each iteration, preferences are captured by ranking a limited set of solutions. The advantage of such an interactive method is that DMs can view a few sample points from the efficient set before zooming into a region of interest. Wan and Krishnamurty [28] recognized the possibility of establishing preference inconsistencies in utility theory and proposed a preference learning process based on a dynamic interactive procedure to help generate a set of acceptable outcomes for the problem and a set of rules for selection of a reduced set of outcomes. Finally, preference methods based on ranking of criteria and their integration in genetic algorithms have been reported by Cvetkovic and Parmee [3] and Park and Koh [21] among others.

The present paper builds upon the work of Miao et al. [15] that allows us for the incorporation of the preference model of relative importance of criteria into their configuration design methodology.

3.1. Vehicle and components

The FMTV is a midsize truck widely used in the US Army for transportation of equipment and personnel. A recent study focused on the redesign of the FMTV to accommodate the integration of a fuel cell auxiliary power unit (APU) [15]. The purpose of the APU is to provide power to non-propulsive applications such as cabin cooling, radio, and other electronic devices. While the integration of the APU increases the overall power efficiency of the vehicle, additional components of significant size must be installed on the vehicle. These components, shown in Figure 3, are the main fuel cell unit (APU), reformer, accumulator, reservoir, and pump. The redesign problem is then to find the optimum placement of these components as well as that of the already present components, i.e. engine, transmission, fuel tank, and axles – on the chassis.

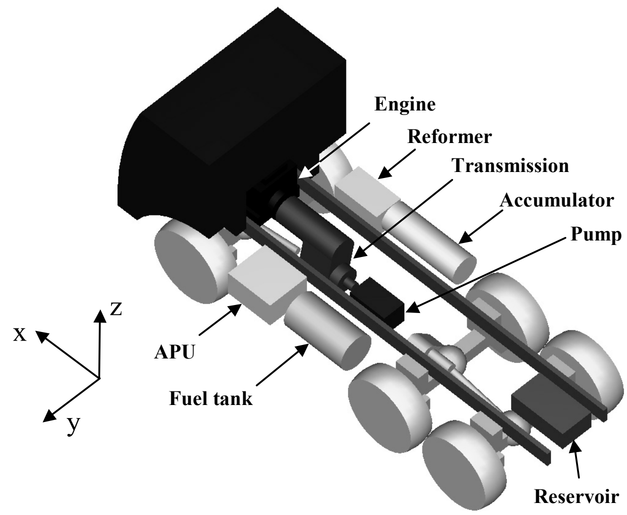

Figure 3. FMTV medium tactical vehicle and its components

Table 1

Predefined packaging sequence of vehicle components

	Component	Relative to	Degree of Freedom[a]
1	First Axle	Ground	x
2	Second Axle	First Axle	–
3	Third Axle	Second Axle	–
4	Chassis	Ground	z
5	Engine	Ground	x, z
6	Transmission	Engine	x′
7	Pump	Transmission	x′
8	Fuel tanks	Ground	x, y, z
9	Accumulator	Ground	x, y, z
10	Reservoir	Ground	x, y, z
11	APU[b]	Ground	x, y, z
12	Reformer	Ground	x, y, z

[a] x, y, z are longitudinal, transversal, and vertical directions, respectively and x′ is coordinate with respect to engine.
[b] auxiliary power unit.

A predefined packaging sequence, presented in Table 1, is used to ensure coherence of the vehicle configuration as well as to define 21 design variables for the optimization problem. Each object is allowed to translate according to the degree of freedom listed in Table 1. The location of each object is known by the location of its reference point, which is generally the center of gravity of the object. Object rotations are not considered in this problem. Note that allowing rotations is possible but would significantly increase complexity without adding value to this research effort.

3.2. Design criteria and constraints

Three conflicting design criteria are identified for this problem: vehicle dynamics performance, survivability, and maintainability. In addition, constraints are defined on the vehicle ground clearance, functional arrangements, and overlap between objects. A detailed description of the criteria and constraints briefly described below is available in [15].

The *vehicle dynamics index*, to be minimized, is an overall performance index that quantifies the dynamic behavior of the vehicle in a standard maneuver. More specifically, for this type of vehicle the index characterizes the risk of rollover and loss of control during a J-turn maneuver. An eight--degree-of-freedom vehicle dynamics model is used to numerically simulate the behavior of the vehicle [33]. In order to capture the complexity of vehicle dynamics, eight performance indices are computed during a numerical simulation. The overall vehicle dynamics index is then calculated as a weighted sum of the eight performance indices. Note that tradeoffs between these eight performance indices are not captured since the above weights are defined prior to the execution of the algorithm.

Survivability is a criterion that quantifies the ability of the vehicle to survive attacks from bullets and explosives. Since the vehicle components have different survivability priorities, a set of weights is defined and the overall vehicle survivability is calculated as the weighted sum of the survivability values of all components. The survivability of an object is defined as the level of protection provided by other objects along selected directions (bottom, rear and both sides). Each direction is associated with a weight corresponding to its importance in the overall survivability of the vehicle. The survivability of the vehicle must be maximized. However, a survivability index inversely proportional to the survivability of the vehicle is defined and minimized.

Maintainability of an object is defined as the number of objects that must be removed before direct removal of this object along selected directions. This quantification is very similar to the survivability described earlier except that it is based on the number of overlapping objects rather than the projected overlap area. To account for various maintenance priorities, an integer weight is defined for each object and the overall maintainability is defined as the weighted sum of the maintainability of all objects. The maintainability of the vehicle must be maximized. However, a maintainability index inversely proportional to the maintainability of the vehicle is defined and minimized.

Since minimizing the dynamic performance index has the tendency to lower the center of gravity of the vehicle, a *ground clearance constraint* is defined. Ground clearance comprises three terms: front, rear, and between axles. If an object is lower than the chassis, the lowest point of this object is identified and is used to calculate the angle defined with the lowest point of the wheels. The minimum value among the three angles is used as the over all ground clearance of the vehicle.

Several *functional constraints* are defined in the definition of the design variables. For instance, the transmission must be aligned with the engine output in the x-direction. This requirement is enforced by defining the design variable as the location of the transmission in the x-direction relative to that of the engine (refer to Table 1).

Finally, a *zero-overlap constraint* is imposed to ensure that none of the components collide and the configuration is spatially acceptable. Overlap is calculated as the sum of all interference volumes between every pair of vehicle components.

3.3. Problem formulation

The configuration design problem is formulated mathematically as follows:

$$\begin{array}{ll} \text{minimize} & [VDI(\boldsymbol{x}), SI(\boldsymbol{x}), MI(\boldsymbol{x})] \\ \text{subject to} & \boldsymbol{x} \in X \\ & GC(\boldsymbol{x}) \geq \alpha \\ & Overlap(\boldsymbol{x}) = 0 \end{array} \tag{13}$$

where $\boldsymbol{x}$ is the vector of 21 design variables (degrees of freedom) listed in Table 1; X is the set of feasible values for the design variables determined by their lower and upper bounds; $VDI(\boldsymbol{x})$, $SI(\boldsymbol{x})$, $MI(\boldsymbol{x})$ are the vehicle dynamics, survivability, and maintainability indices, respectively; $GC(\boldsymbol{x})$ and α are the ground clearance and its lower bound, respectively; and $Overlap(\boldsymbol{x})$ is the total overlap defined as the sum of all overlap volumes between pairs of objects

4. Implementation with relative importance of criteria

We now apply the developed preference model to problem (13). We first discuss the numerical solvability of this problem and then present the results for the case of known allowable tradeoffs and the case of unknown allowable tradeoffs.

4.1. Computational considerations

A rank-based genetic algorithm, also referred to as multi-objective GA (MOGA) [5], is used to find an approximation of the Pareto set of (13). The algorithm was implemented in a C++ computer code based on Galib [27] and reported in [15]. The individuals that are Pareto when considering the entire population have a rank equal to 1. The individuals that are Pareto when considering the population without the individuals of rank 1 have a rank equal to 2, etc. By minimizing the rank, the population evolves towards an *observed* Pareto set, which is an approximation of the *true* Pareto set for the problem.

Since the stochastic nature of a GA can lead to possibly large differences between results of different runs, MOGA was executed several times and in this paper we report the results of two different runs to illustrate the issues related to non-repeatability.

Approximation of the true nondominated set. Each of the two executions of MOGA generates a set of Pareto designs of (14): 238 and 317 designs, respectively, whose outcomes are shown as dots in Figures 4 and 5. These two sets of Pareto outcomes correspond to two approximations of the *true* Pareto set.

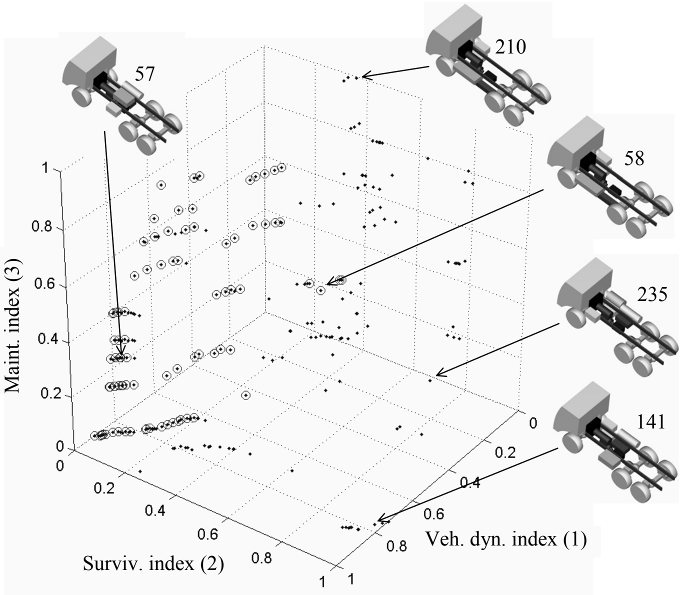

Figure 4. Pareto outcomes (dots) and preferred outcomes (circles) in case 1 of run 1: $L = \{1,3\}$, $M = \{2\}$, $a_{12} = 0.6$, $a_{32} = 0.4$

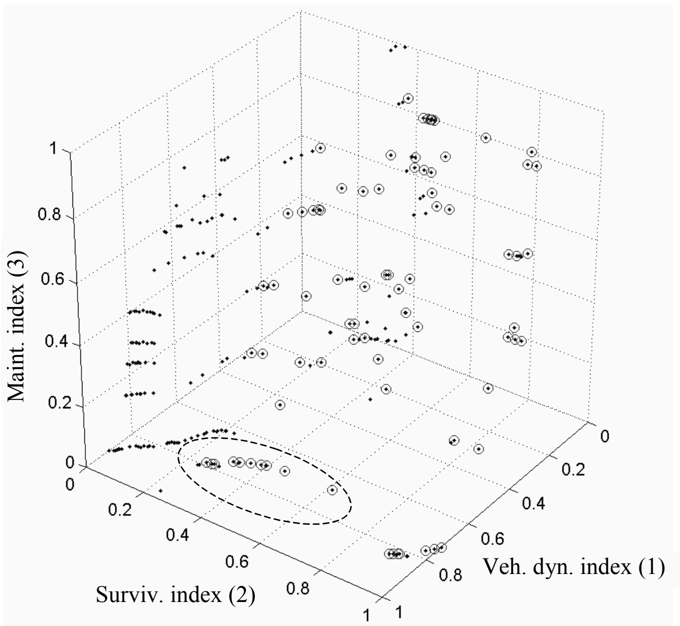

Figure 5. Pareto outcomes (dots) and preferred outcomes (circles) for case 2 of run 1: $L = \{2\}$, $M = \{1,3\}$, $a_{21} = 0.6$, $a_{23} = 0.4$

As seen in Figures 4 and 5, the two Pareto sets are significantly different. While it is virtually impossible to determine the goodness of each approximation compared to the true Pareto set, it is possible to compare them to each other. To do so, the two sets are combined and screened for overall Pareto outcomes. From the first run, 47 percent of the outcomes are Pareto when combining both runs, compared to 86 percent for the second run. This means that the second set of outcomes is a better approximation. Note that in both cases MOGA was terminated using the same convergence criteria, which is not enough to ensure the same quality of approximation of the true Pareto set. Note that a formal study of the goodness of these Pareto sets might be performed using metrics such as those proposed by Sayin [25] and Wu and Azarm [30] among others.

Normalization of criteria. After the two executions of the algorithm, all criterion values are normalized with respect to the minimum and maximum values of each set so that all normalized criterion values fall between 0 and 1. The normalization is not required but allowed by the definition of allowable tradeoffs. In fact, it is up to the DM to relate either to the normalized or the -non-normalized values of the criteria when selecting allowable tradeoff values. For example, using mass (measured in *kilograms*) and cost (measured, for instance, in *dollars*) as two conflicting criteria, there is a difference between dealing with *kilograms* rather than *units of mass* and *dollars* rather than *units of cost*. Some DMs may prefer to consider allowing decay of *x kilograms* to gain one *dollar*. Others may prefer to normalize mass and cost and consider allowing decay of *y units of mass* to gain one *unit of cost*. While the two statements are theoretically equivalent, their cognitive meaning may differ depending on the DM's experience and understanding of the problem.

4.2. Identification of preferred designs with known preferences

Assume that the preferences are known, that is the sets L and M, and allowable tradeoffs a_{ij} can be defined unambiguously. Two cases are considered for illustration purposes. In the first case, vehicle dynamics and maintainability are the more important criteria and the DM is willing to allow 0.6 and 0.4 units of decay in survivability to gain one unit of improvement in vehicle dynamics and in maintainability, respectively, i.e., $L = \{2\}$ and $M = \{1, 3\}$, with $a_{21} = 0.6$ and $a_{23} = 0.4$. Then, the preference matrix $\boldsymbol{A}$ is written:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0.6 & 1 & 0.4 \\ 0 & 0 & 1 \end{bmatrix}$$

and according to (12), the new criteria to be minimized are $g_1 = VDI(\boldsymbol{x})$, $g_2 = 0.6VDI(\boldsymbol{x}) + SI(\boldsymbol{x}) + 0.4MI(\boldsymbol{x})$, and $g_3 = MI(\boldsymbol{x})$. The screening process described in Section 3 is then performed on the two Pareto sets to extract two sets of preferred outcomes. Table 2 lists significant results and Figures 4 and 5 show the Pareto outcomes and the preferred outcomes as dots and circles, respectively.

Table 2

Identification of preferred outcomes with known preferences

Run[a]	Case[b]	M^c	N_P[d]	VDI[e]			SI			MI			N_I[f]
				Min	Mean	Max	Min	Mean	Max	Min	Mean	Max	
1	–	0	238	0	0.50	1	0	0.30	1	0	0.33	1	-
	1	{2}	86	0.09	0.61	0.97	0	0.11	0.42	0.06	0.32	0.74	126
	2	{1,3}	78	0	0.30	0.83	0.12	0.50	1	0	0.36	0.84	210
2	–	0	317	0	0.36	1	0	0.27	1	0	0.38	1	-
	1	{2}	100	0.06	0.36	0.89	0	0.09	0.26	0	0.44	0.78	163
	2	{1,3}	96	0	0.26	0.86	0.12	0.37	1	0	0.86	0.33	301

[a] Execution of MOGA.
[b] Preference case.
[c] Set of more important criteria (Note: M = 0 means no preferences). In case 1, $a_{12} = 0.6$, $a_{32} = 0.4$. In case 2, $a_{21} = 0.6$, $a_{23} = 0.4$.
[d] Number of preferred outcomes (Number of Pareto nondominated outcomes if M = 0).
[e] Min, mean, and max values of veh. dyn. index in the set of preferred outcomes.
[f] In case 1, number of Pareto outcomes with SI < Max^{SI}. In case 2, number of Pareto outcomes with VDI < Max^{VDI} and MI < Max^{MI}.

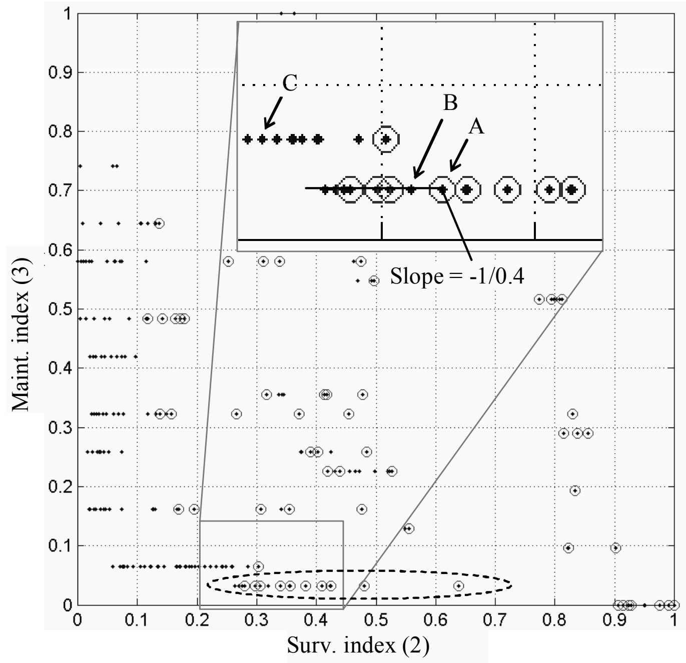

Figure 6. Projection of entire Pareto set of run 1 on two-dimensional space (SI, MI), Pareto outcomes (dots), preferred outcomes (circles), $L = \{2\}$, $M = \{1,3\}$, $a_{21} = 0.6$, $a_{23} = 0.4$

In order to further explain how preferred outcomes are identified, the distribution of Pareto outcomes in the three-dimensional objective space must be closely examined by considering the two-dimensional projections. Figure 6 shows the projection of the Pareto set of run 1 on the two-dimensional space (SI, MI). As explained in Section 3.2, the maintainability criterion assumes integer values. Therefore the normalized maintainability index can only assume a finite number of equally spaced values between 0 and 1. In this context, note the special structure of the projected Pareto set in Figure 6 showing groups of outcomes aligned horizontally. The Pareto outcomes inside the dashed oval in Figure 5 are also highlighted in Figure 6 by a dashed oval. Since they are the only outcomes with MI = 0.0323, they can be isolated from the other outcomes and projected on the two-dimensional space (VDI, SI) shown in Figure 7. Consider the outcome labeled "A" in Figure 7 and place the two-dimensional cone defined by $a_{21} = 0.6$ at "A". One can realize that there is no other outcome inside the cone. Therefore, "A" is a preferred outcome in the two-dimensional space (VDI, SI). Similarly, by placing the cone defined by $a_{23} = 0.4$ at "A" in Figure 6, one can verify that there is no other outcome inside the cone. Therefore, "A" is a preferred outcome in the two-dimensional space (SI, MI). Since "A" is preferred in both (VDI, SI) and (SI, MI), it is a preferred outcome for the overall three-dimensional problem. Comparatively, the outcome labeled "B" in Figure 7 is dominated by outcome "A", therefore "B" is not a preferred outcome. Note that the outcome labeled "C" in Figure 6 is not dominated by any outcome in the two- dimensional space (SI, MI). However, it is dominated in the space (VDI, SI) but not shown in Figure 7 since it lies in a plane defined by a higher MI value.

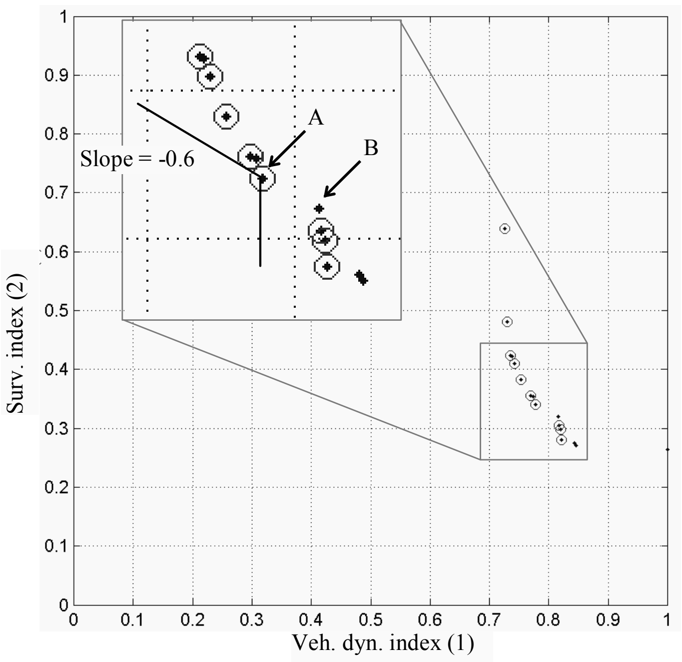

Figure 7. Projection on two-dimensional space (VDI, SI) of Pareto outcomes (dots) and preferred outcomes (circles) highlighted by dashed oval in Figure 6, $L = \{2\}$, $M = \{1,3\}$, $a_{21} = 0.6$, $a_{23} = 0.4$

In the second preference case, survivability is the more important criterion and the decision maker is willing to allow 0.6 and 0.4 units of decay in vehicle dynamics and in maintainability, respectively, to gain one unit of improvement in survivability, i.e., $L = \{1,3\}$ and $M = \{2\}$, with $a_{12} = 0.6$ and $a_{32} = 0.4$.

Based on the examination of Figures 4 and 5 and Table 2, several remarks can be made:

1. The set of Pareto outcomes is reduced significantly in both runs and both preference cases. In run 1, out of 238 outcomes, 86 (36%) and 78 (33%) are preferred in the first and second preference cases, respectively.
2. As expected, since the two preference cases have conflicting meanings, outcomes from different regions of the Pareto set are identified as preferred outcomes. In Figure 4, the preferred outcomes tend to have a low survivability index, which is due in part to the fact that survivability is the more important criterion. As explained in the following remark, however, this is not the only reason.
3. In Figure 5, since the more important criteria are vehicle dynamics and maintainability, one may expect the preferred outcomes to have a low vehicle dynamics index and low maintainability index. However several preferred outcomes, such as those highlighted by the dashed oval in Figure 5, have a high vehicle dynamics index. Therefore, it is not because the values of the more important criteria are large that an outcome cannot be preferred. Rather, this depends on the relative distribution of the Pareto outcomes in the three-dimensional objective space.
4. For preference case 1, not all outcomes with a low survivability index are preferred. This is due to the fact that these outcomes are dominated by at least one other outcome. Table 2 shows that in case 1 of run 1, from the set of preferred outcomes, the maximum SI value is $\mathrm{Max}^{\mathrm{SI}} = 0.42$. In the region defined by $\mathrm{SI} \leq 0.42$, there are 126 outcomes of which 86 are preferred.
5. In Figure 4, several truck configurations are provided to show the differences in the design space between regions of the outcome space. Configurations that have a high vehicle dynamics index tend to have high center of gravity by raising the height of the chassis and placing heavy components higher. For example, outcomes 57 and 141 correspond to such designs and differ by their survivability index while outcome 210 has a very low vehicle dynamic index partly because of low center of gravity at the expense of maintainability.

4.3. Sensitivity to allowable tradeoff values

The allowable tradeoff values a_{ij} have an impact on the number of preferred outcomes. Graphically, this is simply explained by the fact that as the allowable tradeoff values a_{ij} increase, the related preference cone opens up and the chance for a design to lie inside the cone increases. In order to illustrate this sensitivity, the allowable tradeoff values are varied incrementally between 0 and 3 for two cases: (L = {2}, M = {1,3}) and (L = {1,3}, M = {2}). The sensitivity is graphically represented in Figure 8 for run 1. A similar sensitivity, not shown, was found for run 2.

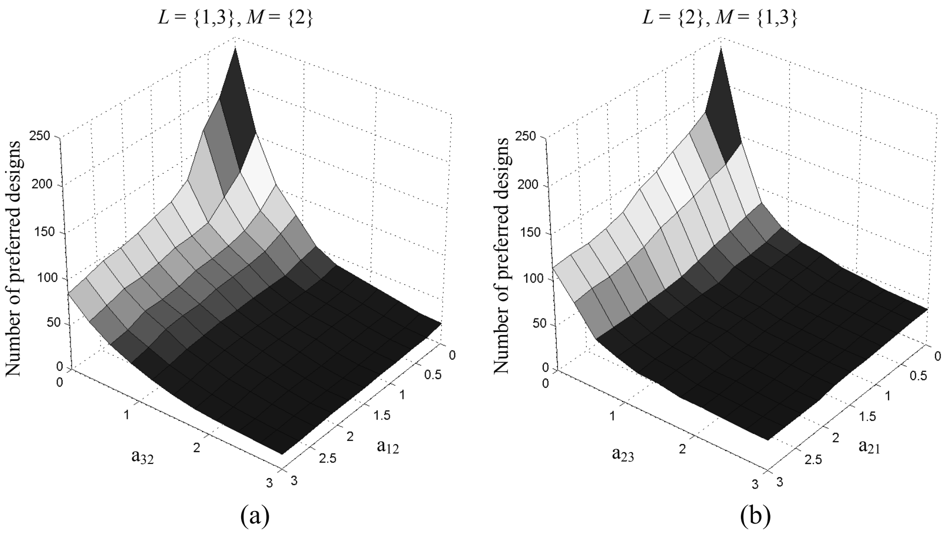

Figure 8. Effect of allowable tradeoff values on reduction of set of preferred outcomes in run 1 for two preference cases: (a) = {1,3}, *M*={2}, (b) *L* = {2}, *M* = {1,3}

Case 1 has a greater impact on the reduction than case 2 for large values of allowable tradeoffs (the set of preferred outcomes is reduced to 16 in case 1 and 33 in case 2 for all $a_{ij} = 3$). However, case 2 is more effective in reducing the set for low values of allowable tradeoffs.

One can notice that both cases show an asymmetric behavior. It appears that a_{12} does not have as much impact on the reduction of the set of preferred outcomes as a_{32} in case 1 for low values of allowable tradeoffs.

4.4. Identification of strong designs with unknown preferences

If *L*, *M*, and/or a_{ij} are not explicitly known, we introduce the term *strong design,* to describe designs that prevail for any combination of *L* and *M* and for any value of a_{ij} and propose a method to identify them. The image of a strong design in the outcome space is referred to as a *strong outcome*.

The idea is to consider multiple preference cases and vary allowable tradeoff values. In the configuration design problem, the permutation of the three criteria leads to six preference cases: ($L = \{1, 2\}$, $M = \{3\}$), ($L = \{1, 3\}$, $M = \{2\}$), ($L = \{2, 3\}$, $M = \{1\}$), ($L = \{1\}$, $M = \{2, 3\}$), ($L = \{2\}$, $M = \{1, 3\}$), and ($L = \{3\}$, $M = \{1, 2\}$). For each case, the two allowable tradeoff values a_{ij} are set to three different values: 0.6, 0.8, 1.0. The screening process identifies the preferred outcomes for each case and all outcomes are then ranked based on the number of cases for which they are preferred. The strong outcomes are then those that are preferred for the greatest number of preference cases.

Note that the two allowable tradeoff values can vary independently, which means that a large number of cases could be considered. In this paper, we selected to keep the two values equal in all cases. The results are reported in Table 3 and shown in Figures 9 and 10.

Table 3

Strong designs identified by considering six preference cases

Run 1						Run 2					
$a_{ij} = 0.6$		$a_{ij} = 0.8$		$a_{ij} = 1.0$		$a_{ij} = 0.6$		$a_{ij} = 0.8$		$a_{ij} = 1.0$	
DN	N_c	DN	N_c	DN	N_c	DN	N_c	DN	N_c	DN	N_c
206	6	206	5	206	5	250	6	250	6	250	6
159	6	159	5	221	4	260	5	315	4	298	4
121	6	121	5	182	4	256	5	298	4	257	4
116	6	116	5	89	4	172	5	260	4	228	4
104	6	89	5	238	3	315	4	257	4	148	4
89	6	104	4	236	3	300	4	228	4	130	4
54	6	54	4	…	…	298	4	174	4	128	4
221	5	…	…	159	3	257	4	172	4	111	4
…	…	221	4	54	3	240	4	148	4	86	4
182	4	182	4	121	2	237	4	130	4	75	4
236	4	236	4	116	2	228	4	…	…	…	…
238	3	238	3	104	2	…	…				

DN: Design number.
N_c: Number of preference cases for which outcome is preferred.

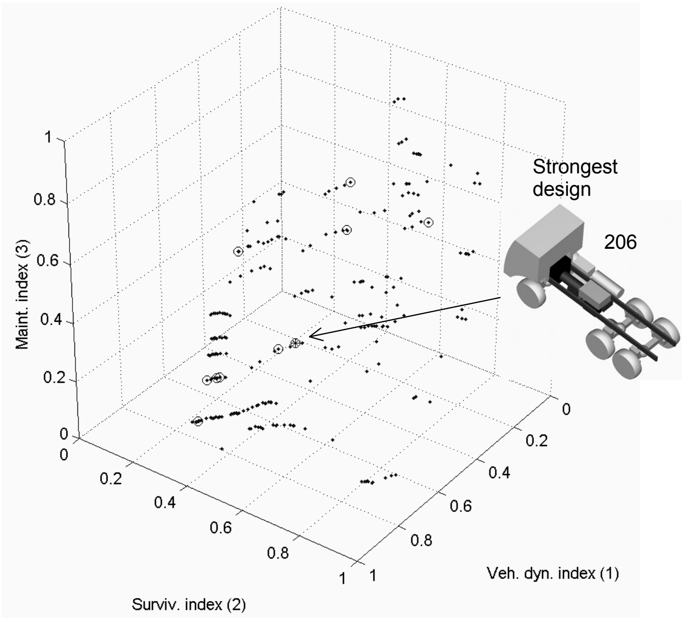

Figure 9. Strong outcomes (top ten) in run 1 identified by considering six preference cases

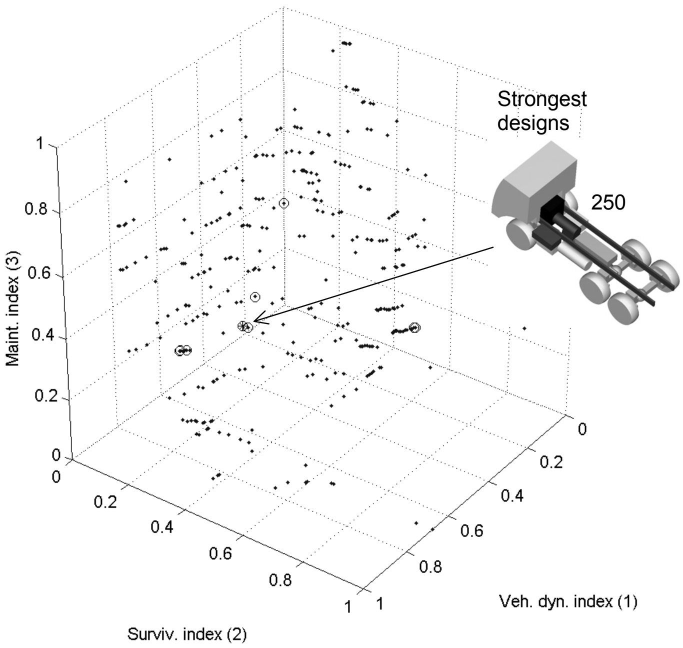

Figure 10. Strong outcomes (top ten) in run 2 identified by considering six preference cases

Based on Figures 9 and 10, it appears that the strong outcomes are not concentrated in a small region of the outcome space. On the contrary, they are evenly distributed within the Pareto set. This is due to the fact that the screening process works on the observed Pareto set rather than the true Pareto set. In particular, the observed outcomes' proximity to the true Pareto set interacting with the assumed preference may both determine whether they are extracted or not. This remark directly concerns managers and decision makers who, because of the complexity of real-world problems, only have access to approximations as opposed to the desired true outcomes of the multi-criteria optimization problem.

Table 4

Non-normalized criterion values of selected designs

Run	Outcome	VDI	SI	MI
1	Min	12.3	-18.4	-47
	206	22.9	-17.5	-42
	Max	35.0	-12.9	-16
2	Min	13.4	-20.4	-47
	250	22.5	-19.4	-41
	Max	36.1	-11.6	-10

In both runs, only one outcome is consistently at the top of the list. These are outcomes number 206 in run 1 and number 250 in run 2, whose non-normalized criterion values are listed in Table 4. Note that the two truck configurations, shown in Figures 9 and 10, are fairly different. By examining the range of variation of each criterion, one can verify that the two outcomes are similar in the outcome space while different in the design space. Several other outcomes appear at the top of the list in both cases, however, less consistently. For instance, outcomes 159 and 221 in run 1 and outcomes 257 and 228 in run 2 are considered strong designs. This discussion may represent a sample analysis a designer would perform when choosing a final configuration design for the FMTV. The two strong outcomes, 206 and 250, resulting from the a posteriori analysis with unknown preferences, can be considered candidates for the final preferred outcome. The choice between these two "privileged" outcomes may be dictated by other considerations (e.g., cost) that have not been included in multi-objective program (13).

Conclusion

A model of relative importance of criteria has been presented and applied to the configuration design of a midsize truck for optimum vehicle dynamic behavior, survivability, and maintainability. The presented application, although related to a specific design problem, demonstrates the special feature of the model that a small subset of preferred solutions is determined in agreement with the assumed relative importance from which a final preferred solution is selected. The model therefore offers an alternative to scalarizing approaches that convert MOPs into scalar optimization problems with tradeoff information available only at their optimal solutions, and to genetic algorithms that produce approximations of the true Pareto set from which the choice of a final preferred solution is difficult.

The paper also highlights the issues related to the stochastic nature of methods for finding Pareto solutions. Two sets of Pareto outcomes were first generated with a multi- objective genetic algorithm. These sets were then screened according to the new preference model. It is shown that for known preferences this model can be used to extract preferred outcomes from the Pareto set. In addition, a method has been proposed to produce a short list of "strong" solutions for the case where preferences are unknown or non-quantifiable, which generally results from DM's lack of experience and understanding of the problem or lack of consensus between multiple DMs.

Finally, the versatility of the proposed model allows preferences to be also included in the problem *a priori* to directly obtain a set of preferred solutions and thus offers promise for practical use in real-world engineering and other problem. Indeed, a majority of the issues discussed here in relation to engineering design, remain relevant for applications in business, management, and other domains of human life.

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