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RANKING BY THE ROUGH APPROXIMATION OF A PREFERENCE RELATION FOR AN EXTRUSION PROCESS – A ROBUSTNESS STUDY

Abstract

In this paper an extrusion process example is studied to illustrate a new methodology in the field of decision engineering, which is based on the rough set approach. Rough sets are used to aid the Decision Maker in choosing the best point in the Pareto's region, which is the zone of the non-dominated solutions. The rough set approach allows us to make a rough approximation of a preference relation on a sample of experimental points chosen from the Pareto set. These approximations are done to induce the decision rules, which can be afterwards applied on whole sets of the potential points. To give the final recommendation the concept of technical robustness is suggested.

Keywords

Rough sets, decision rules, multi-criteria problem, robustness, genetic algorithms.

Introduction

In applications of the rough set approach to the model of the Decision Maker (DM) preferences in the multi-criteria problem, the starting point is his global evaluation of the subset chosen actions as examples. So, after getting the preferential information in terms of exemplary comparisons, it would be natural to build the preference model in the form of "if..., then..." decision rules.

The consequence of the ambiguity of these evaluations is that some rules are non-deterministic, i.e., they cannot be univocally described by means of "granules" of the representation of preferential information. A formal framework for dealing with the granularity of information; called Rough Set theory has been given by Pawlak [9, 7]. The Rough Sets theory assumes a representation of the information in a table whose rows correspond to objects and its columns to attributes. If a description of objects by a set of attributes is supplied by the DM, these attributes are called decisions, and the remaining ones conditions, and all together these form the decision table. This decision table is a particularly appropriate form for the description of decision sorting problems (see [8]). As was shown by Greco, Matarazzo and Slowinski [4], a direct application of the Rough Sets approach to multi-criteria decision analysis is not possible when the ordinal properties of the criteria have to be taken into account. In this paper, and following this way of reasoning, a decision table is when the objects are pairs of actions considered. For each pair of actions, the partial evaluations of the preferences with respect to each attribute are given. The attributes taken from the multi-criteria problem can be called conditions. For a chosen subset of pairs of actions, a global preference relation from the total order is supposed to be given by the DM or expert. In this table, a global preference relation will be noted as the decision attribute. In the original theory of Rough Sets the rough approximations are defined using an indiscernibility relation on the pairwise comparison table. In this paper following the Greco, Matarazzo and Slowinski [6] approach, an indiscernibility relation was substituted by a dominance relation.

This paper is structured as follows. The problem under study is formulated in Section 1. After a brief presentation of the industrial application, this section deals with the determination of the Pareto set, which is a prerequisite to obtain all possible actions. The aim is to eliminate all points, which are not optimal in the multi-criteria problem. The remaining optimal points are determined with an evolutionary approach by adapting a genetic algorithm. The Decision Maker has to choose the best action in this zone, so, in Section 2, a multi-criteria method based on the theory of rough sets is developed to support him in his choice. A few experimental points are required in this method and it is supposed that the DM is able to express his preferences in relation to these points. From these preferences the method of rough sets allows us to build the set of rules and they are applied on whole sets of points in the Pareto zone to determine the total ranking of the points in Section 3. In Section 4, the robustness of the proposed solution is discussed. The solution has to take into account a possible evolution of the input parameters and a possible change in the decision parameters. A robustness study is proposed for the application of an extrusion process, which is a didactic example. So, the results can be easily represented in two dimensions.

1. Determination of the Pareto set by an evolutionary approach

1.1. Description of the industrial process

Let us consider an industrial application concerning the problem of food granulation for cattle. The goal is to propose the best recommendation for the working conditions of an extrusion process. A pulverulent product is converted into granules due to the conjugated effects of heat, moisture and pressure. The industrialist would like to simultaneously minimise three characteristics: moisture, the friability of his product and the energy consumption of the process. Two discretized input parameters are taken into account in this study: the flour temperature T (between 35°C and 75°C) and the drawplate profile D (between 2 cm and 6 cm). All three attributes are described by functions represented in Figure 1.

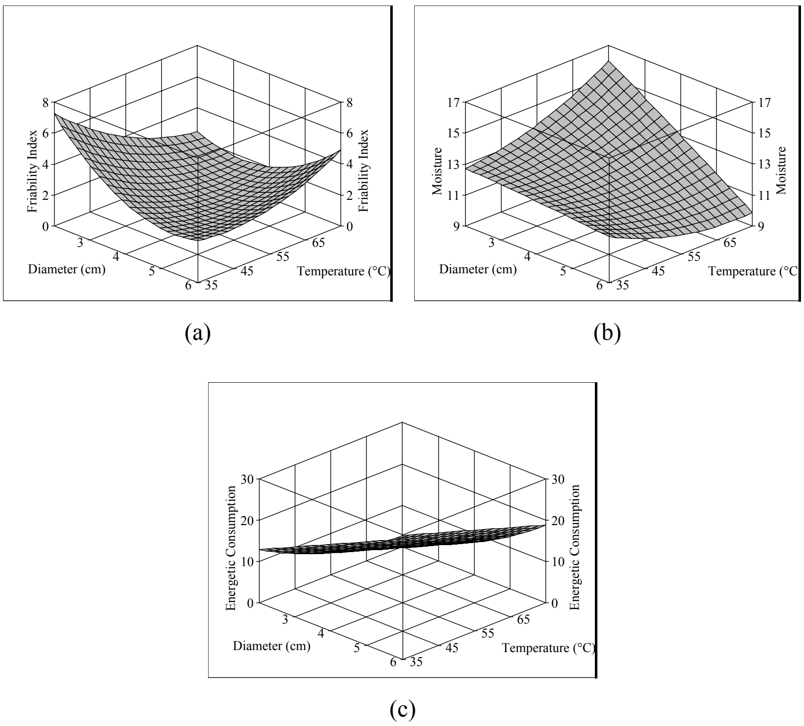

Figure 1. Evaluations of friability index, moisture and energy consumption vs temperature and drawplate profile

These evolutions are derived from Courcoux, Qannari, Melcion and Morat [1]. The modelling of these attributes depends on the information at our disposal and they are generally incomplete or vague. In this paper, discussed application is limited to the deterministic case. In the case of stochastic or determinist evaluations the modelling can be done using the Stochastic Dominances (see [12, 13, 14]).

1.2. Multiobjective optimisation algorithm

In a real-world problem, like our extrusion process, an optimal working condition does not generally exist due to the multi-criteria aspect. In the case of the multi-criteria analysis of a process, all possible points have to be determined first. These points, which are called Pareto-efficient solutions, are in fact a continuous set, which is not always possible to determine analytically. The concept of the Pareto dominance is used which implies that a point dominates another if it is better for all criteria. So, the set of the non--dominated points forms the Pareto set.

A multiobjective optimisation algorithm is developed to obtain the Pareto domain sampled by a set of points. The aim of this part is not to immediately find the preferred solution but to exclude all conditions which are not interesting, i.e. not optimal in a multi-criteria sense. An extension of the traditional genetic algorithm is then used to deal with discretized data by introducing the dominance concept. Genetic algorithms are an optimisation method which is inspired by an analogy using the evolution of populations [3]. The approach consists, after a random initialisation of points, of an evaluation of them, a selection of the best points and a recombination of these ones until a convergence of the algorithm occurs. Some thousands of points can randomly initialise the algorithm. The evaluation of each point is determined by the calculation of a dominance function which counts how many times a point is dominated by the others [2]. The non-dominated points are kept and are recombined to replace the dominated points. The procedure evaluation – recombination is applied until all points are non-dominated. This method allows us to obtain a set of points, which corresponds to Pareto efficient solutions and to also discretize a continuous problem with these points. The Pareto set is represented in Figure 2 by 5000 grey points, which define precisely the continuous multi-criteria optimal zone. This obtained domain is the prerequisite before the preference analysis of the process.

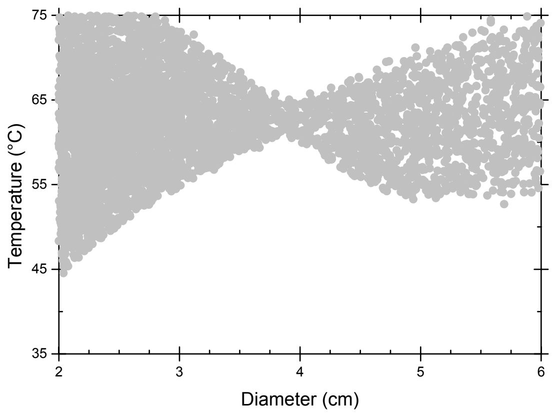

Figure 2. Pareto set of the extrusion process for temperature and drawplate profile parameters

2. Rough sets preference analysis

2.1. Algorithmic design

The basis for rough sets preference analysis is the building of a decision table for a sample of experimental points B chosen from the optimal Pareto's zone. Let X be a set of the output process attributes (in our example: moisture, the friability index and the energy consumption) describing the performance of points. Let C be the set of condition attributes describing the pairs of points, and D the decision attribute. The decision table is defined as 4-tuple, ($T = < H, C \cup D, V_C \cup V_D, g>$), where $H \subseteq BxB$ is a finite set of pairs of points, $C \cup D$ is the union of two sets of attributes, called condition and decision attributes, $V_C \cup V_D$ is the union of the values of the functions which are defined as follows [5]: g: $H \times (C \cup D) \rightarrow V_C \cup V_D$ is a total function where $V_C = \cup V_k$ (criterion k). This function is such that:

$$g[(a_i, a_j), k] = \begin{cases} 1, & \text{if } \; X_k(a_i) < X_k(a_j) \\ 0, & \text{if } \; X_k(a_i) \geq X_k(a_j) \end{cases} \tag{1}$$

$$(\textit{if criterion } k \textit{ is to minimise}) \; \forall C_k \in C \; \textit{ and } \; \forall (a_i, a_j) \in H$$

where a_i is the one point of the temperature-profile parameter combination, i, j = 1, ..., n, and $X_k(a_i)$ is the value of the performance of the point a_i in relation to the attribute $X_k \in X$ where k=1,..., m (in our example m = 3) and C_k is the condition attribute associated with the criterion k. The decision table can also take only two values $g[(a_i, a_j), D]$ on $H \subseteq B \times B$:

$$g[(a_i, a_j), D] = \begin{cases} P, & \textit{if } a_i \textit{ is preferred to } a_j; \\ N, & \textit{if } a_i \textit{ is not preferred to } a_j \end{cases} \tag{2}$$

Then, we suppose that the DM is able to express his preferences on the small number of points. The appeal of this approach is that the DMs are typically more confident exercising their evaluations than explaining them. In general, the decision table can be presented as follows in

Table 1

General presentation of the decision table

		X_1	X_2	…..	X_m	**D**
	.	.	.	…..	.	.
H_P	(a_i, a_j)	$g[(a_i, a_j),1]$	$g[(a_i, a_j),2]$	…..	$g[(a_i, a_j),m]$	$g[(a_i, a_j),D]=P$
	.	.	.	…..	.	.
	.	.	.	…..	.	.
H_N	(a_s, a_t)	$g[(a_s, a_t),1]$	$g[(a_s, a_t),2]$	…..	$g[(a_s, a_t),m]$	$g[(a_s, a_t),D]=N$
	.	.	.	…..	.	.

Let $Q \subseteq C$ be a subset of condition attributes. In relation to the subset Q of C attributes (see Zaras (2001)) we can define the Multi-attribute Stochastic Dominance noted MSD_Q for a reduced number of attributes which can be defined as follows.

Definition 1

$$\text{ai MSDQ aj if and only if } g[(ai, aj),k] = 1 \text{ for all } Ck \in Q \tag{3}$$

where $Q \subseteq C\{1, ..., m\}$

The problem is how to identify the subsets Q from which we could approximate global preferences? To answer this question we use the idea of the approximation, which was taken from the rough sets of Greco et al. [6] where an indiscernibility relation was substituted by a dominance relation.

For each pair of alternatives $(a_i, a_j) \in H$ in the decision table we can identify Multi-Attribute Dominance MSD_Q and complementary Multi-Attribute Non-dominance not MSD_{C-Q} where C-Q is the set difference and where this second dominance can be defined as follows:

Definition 2

ai nonMSDC-Q aj , if and only if g[(ai,aj),k] = 0 for all Ck $\in$ C –Q (4)

These dominances are similar to the P-positive dominance and P-negative dominance suggested by Greco et al. [6] for ordinal data.
According to them these dominances satisfy the following property:

Property 1

If $(a_i, a_j) \in MSD_Q$ (not MSD_{C-Q}) then $(a_i, a_j) \in MSD_R$ (not MSD_{C-R})
for each $R \subseteq Q \subseteq C$ (5)

In this approach we propose to approximate the P-global preference relation by the MSD_Q relation. Usually this approximation in the rough set methodology is done by $Q_*(P)$-lower and $Q^*(P)$-upper approximations. According to Greco et al. [6] these approximations can be defined as follows:

$$Q_*(P) = \bigcup_{Q \subseteq C} \left\{ \left(MSD_Q \cap H \right) \subseteq P \right\}$$
$$Q^*(P) = \bigcap_{Q \subseteq C} \left\{ \left(MSD_Q \cap H \right) \supseteq P \right\} \quad (6)$$

The Q-boundary (doubtful region) of a set of preferences P is defined as follows:
The set $BN_Q(P)$ contains the MSD_Q which introduces uncertainty in the deduction of the decision rules using the subset of attributes Q.

$$BNQ(P) = Q^*(P) - Q^*(P) \quad (7)$$

If $BN_Q(P) \neq \varnothing$, then P is a rough set. Taking into account property 1, we obtain a certain number of the MSD_Q, which verifies the condition of the lower approximation.
Analogously, we can approximate a non-preference denoted by the letter N in the decision table by the Multi-Attribute non-dominance for a reduced number of attributes not MSD_Q:

$$Q_*(N) = \bigcup_{Q \subseteq C} \left\{ \left(not \;\; MSD_Q \cap H \right) \subseteq N \right\}$$
$$Q^*(N) = \bigcap_{Q \subseteq C} \left\{ \left(not \;\; MSD_Q \cap H \right) \supseteq N \right\} \tag{8}$$

We can induce a generalised description of the preferential information contained in a given decision table in terms of decision rules.

2.2. Application

We apply this algorithmic design in the case of our granulation process with the following discretized input parameters: temperature T (between 35 C^o and 75 C^o) and drawplate profile D (between 2 cm and 6 cm) values on the one hand, and the three output attributes: X_1: the friability index, X_2: the moisture of the granules and X_3: the energy consumption on the other hand. The criteria role has been attributed to minimise all three output attributes. To illustrate the application of the Rough Set approach, we began by showing in Table 2 the subset of five points chosen by a human expert and their evaluations in relation to each of the three attributes.

Table 2

Evaluations of the five chosen actions for three attributes

Actions	X_1	X_2	X_3
a_1	2.0495	13.361	7.8915
a_2	2.0925	14.998	5.4695
a_3	3.3122	14.096	4.7477
a_4	2.9819	10.128	20.871
a_5	4.3177	10.225	17.916

Figure 3 shows us the localisation of these five points in relation to the Pareto domain which was determined analytically from the equations (13).

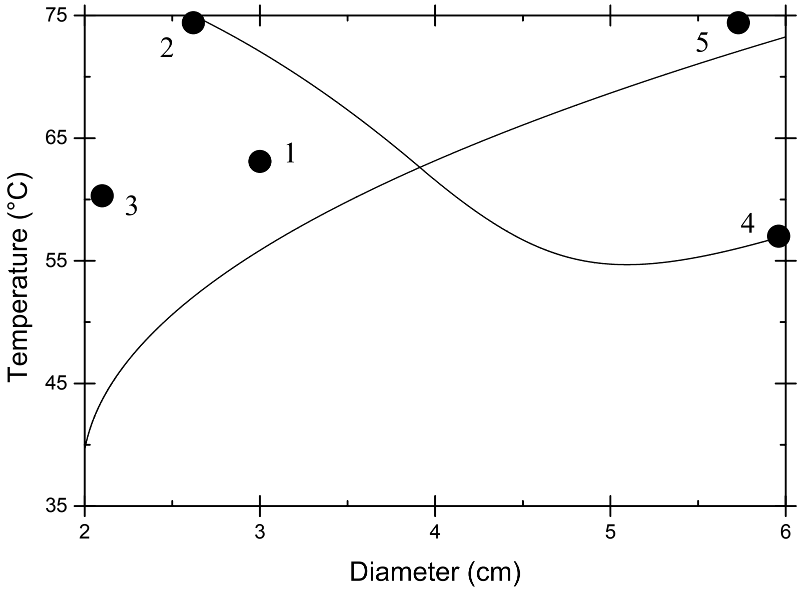

Figure 3. The subset of five points chosen by a human expert and the Pareto domain which was determined analytically

These points have been ordered by the DM from the best to the worst. Then, we can add the decision attribute D to Table 3 which makes a dichotomic partition of the set of pair points according to the values $V_D = P$, which means preference and $V_D = N$, which means non preference.

Table 3

Decision table for the five chosen points

	Pairs	X_1	X_2	X_3	D
H_P	(a_1, a_2)	1	1	0	**P**
	(a_1, a_3)	1	1	0	P
	(a_1, a_4)	1	0	1	P
	(a_1, a_5)	1	0	1	P
	(a_2, a_3)	1	0	0	P
	(a_2, a_4)	1	0	1	P
	(a_2, a_5)	1	0	1	P
	(a_3, a_4)	0	0	1	P
	(a_3, a_5)	1	0	1	P
	(a_4, a_5)	1	1	0	P

Table 3 contd.

	Pairs	X_1	X_2	X_3	D
H_N	(a_2, a_1)	0	0	1	**N**
	(a_3, a_1)	0	0	1	N
	(a_4, a_1)	0	1	0	N
	(a_5, a_1)	0	1	0	N
	(a_3, a_2)	0	1	1	N
	(a_4, a_2)	0	1	0	N
	(a_5, a_2)	0	1	0	N
	(a_4, a_3)	1	1	0	N
	(a_5, a_3)	0	1	0	N
	(a_5, a_4)	0	0	1	N

According to definition (6), from Table 4 we obtain one dominance relation which intersects with five pairs of points and which verifies the condition of the lower approximation.

Table 4

Dominance which intersects with five pairs of points in the decision table

MMD_Q /H	(a_1, a_4)	(a_1, a_5)	(a_2, a_4)	(a_2, a_5)	(a_3, a_5)
Q = {X_1, X_3}	x	x	x	x	x

From this we can induce the minimal decision rule, which can be formulated as follows:

Rule 1

$$\text{If } ai \text{ MMDQ } aj \text{ then } ai \text{ P } aj \text{ , with } Q = \{X1, X3\} \qquad (9)$$

The meaning of this rule is that if the point a_i in the Pareto's zone is better that the point a_j with respect to friability index and energy consumption, then the point a_i is preferred to point a_j. In the same way we can do the approximation of non-preference N by the Multi-Attribute non-dominance not MMD_Q. According to the definition (9) from Table 5, we obtain one non-dominance relation which intersects with five pairs of points and which verifies the condition of lower approximation.

Table 5

Non-dominance which intersects with five pairs of points in the decision table

Not MMD_Q /H	(a_4, a_1)	(a_5, a_1)	(a_4, a_2)	(a_5, a_2)	(a_5, a_3)
Q = {X_1, X_3}	x	x	x	x	x

Based on the same principle as in the case of preference choosing, we can induce the rule as follows:

Rule 2

If ai not MMDQ aj then ai N aj , with Q = {X1, X3} (10)

The last step of the suggested methodology is to apply the rules to order the whole set of points.

3. Ranking deduction from rough approximation

3.1. Algorithmic design

The overall binary preference relation noted (P) is identified if Rule 1 is fulfilled between points. If the second rule is fulfilled, the overall non--preference is identified which is noted (N). In general, it is possible to induce decision rules being propositions of the following type:

1. D++ decision rule, which is a proposition of the type:
   if ai MMDQ aj then ai P aj.
2. D-- decision rule, which is a proposition of the type: (11)
   if ai notMMDQ aj then aj N ai.

The final set of decision rules is the set of minimal decision rules. According to Greco et al. [6] a D_{++} decision rule [D_{+-} decision rule] a_i MMD_Q a_j $\rightarrow a_i$ P a_j

[if ai not MMDQ aj $\rightarrow$ ai N aj] will be called *minimal* if there is not any other rule (12)

ai MMDR aj $\rightarrow$ ai P aj [if ai not MMDR aj $\rightarrow$ ai N aj] such that R$\subseteq$ Q.

For each point a_i let

$SC^{++}(a_i)$ = card({ $a_j \in$ A: there is at least one D_{++} $-$ decision rule stating that a_i P a_j }),

$SC^{+-}(a_i)$ = card({ $a_j \in A$: there is at least one D_{++} – decision rule stating that a_j P a_i }),
$SC^{-+}(a_i)$ = card({ $a_j \in A$: there is at least one D_{--} – decision rule stating that a_j N a_i }),
$SC^{--}(a_i)$ = card({ $a_j \in A$: there is at least one D_{--} – decision rule stating that a_i Na_j }).
To each $a_i \in A$ we assign a score, called *Net Flow Score* (see [6]),
$SNF(a_i) = SC^{++}(a_i) – SC^{+-}(a_i)+ SC^{-+}(a_i)- SC^{--}(a_i)$.

The final recommendation is the best point established by ranking 5000 points in decreasing order of the value of the $SNF(a_i)$ on A. The rough sets approach gives us a clear recommendation: the best combination of two input parameters is temperature 74.85 C^o and drawplate profile diameter 2.80 cm. This combination of input parameters assumes the following values of three output attributes, the friability index = 1.89, the moisture of granules = 14.84, and the energetic consumption = 6.02. Figure 4 shows the results plotted *quintile* by *quintile* and the best recommendation (black diamond) for a better understanding.

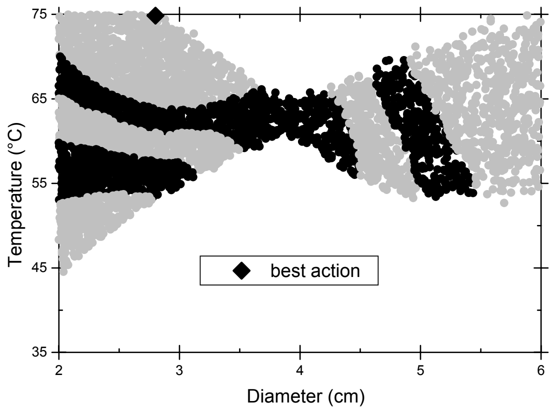

Figure 4. Total ranking of the Pareto set presented *quintile* by *quintile*

The first 1000 actions are represented in grey around the best action, the second in black, etc. From this figure, we can notice that the best point is on the border of the Pareto's zone. Then, this remark leads us to study the robustness of this best recommendation.

4. A robustness study

Robustness is connected to the fact that the decision-aid methods often contain parameters whose values have to be chosen (more or less arbitrary) by the user. In our example we have two input parameters: temperature T (between 35°C and 75°C) and drawplate profile D (between 2 cm and 6 cm). For different instances of parameter values the zone Pareto represents the feasible part of the result obtained by application of the genetic algorithm taking into account the three criteria for minimisation: the friability index, the moisture of granules and the energy consumption. The case of the robust solution was discussed by Roy and Bouyssou [10] and Roy [11]. Roy [11] suggests three kinds of assertions in order to establish recommendations: *a perfectly robust* where the domain of the result for all parameter values is well known, *an approximately robust* where a perfectly robust conclusion is not necessarily identified and *a pseudo-robust* conclusion which is a more or less formal statement concerning all parts of the result for all parameter values. In our example, the points in the Pareto's zone are approximately robust because of the variation of the working conditions of the process, which can put them out of the optimal zone.

5.1. Technical robustness

Therefore, we propose the concept of technical robustness defined as stability of the result when technical parameters have small variation. In our example, a point on the border of the Pareto set could be considered as robust where no variation of the working conditions of the process is observed. In the real world of the industrial process this condition is rarely verified. This is why we suggest the hypothesis that the robustness of the points in the Pareto's zone increases with the distance of the point from the boundary. We call this concept the technical robustness because this analysis is made before the rough set approximation of the preference, which allows us to do the final recommendation. So, another criterion may be defined to take into account the robustness of the solution. Maximise the distance between a point and the border can be considered as a supplementary objective when the Pareto's zone is already found. This concept allows us to decrease the impact of a possible instability of the system.

From a theoretical point of view the borderline of the Pareto zone is represented by three non-dominated branches determined by the locus of points where isocriteria of the attributes are tangent *pairwise*:

$$D = \frac{0.005T^2 - 0.135335T + 7.52324}{-0.0105T + 0.9656} \qquad \textit{for}\ D \le 3.91\,cm \ \ \textit{and}\ \ T \ge 62.6\ {}^oC$$

$$D = \frac{4.2125 * 10^{-4} T^2 - 0.0322555T + 0.9161}{-4.175 * 10^{-4} T + 0.166196} \qquad \begin{array}{l}\textit{for}\ \ 2 \le D \le 6\,cm \\ \textit{and}\ \ 35 \le T \le 75\ {}^oC\end{array} \qquad (13)$$

$$3.0198 * 10^{-4} T^2 + (-0.04227664 + 0.00295924D)T + (0.712418 + 0.1116062D - 0.0268384D^2 = 0$$

$$\textit{for}\ D \ge 3.91\,cm\ \textit{and}\ T \le 62.6\ {}^oC$$

In Figure 3 the border of the theoretical Pareto domain is showed by the bold line with the five points chosen by the human expert. The distance between a point and the border is calculated in terms of temperature because the drawplate profile is considered stable and can have standard characteristics. For given drawplate profile D_0 two distances are calculated between the temperature T_0 and the lower border temperature $T_1(D_0)$ and the higher border temperature $T_2(D_0)$. These distances are maximised in relation to zero in such a way that:

$$d_1 = \text{Max}\ \{T_0 - T_1(D_0), 0\};\ d_2 = \text{Max}\ \{T_0 - T_2(D_0), 0\} \qquad (14)$$

The negative value of the difference between temperatures shows us that the point is out of the Pareto domain. Finally, the value of the distance which is taken to the technical robustness analysis is as follows:

$$d = \text{Min}\ \{d_1; d_2\} \qquad (15)$$

In our granulation process, the rough sets preference analysis is applied by taking into account a fourth attribute X_4: the technical robustness, which is a distance d to maximise. Table 6 shows the five points chosen by the human expert, their respective four conditional attributes and ranking (it is the same ranking as previously). So, our rules changed and another total ranking are deducted.

Table 6

Evaluations of the five chosen actions for four attributes

Actions	X_1	X_2	X_3	X_4
a_1	2.0495	13.361	7.8915	7.2253
a_2	2.0925	14.998	5.4695	0.6671
a_3	3.3122	14.096	4.7477	16.674
a_4	2.9819	10.128	20.871	0.0625
a_5	4.3177	10.225	17.916	0.0000

Then, we can add the decision attribute D to Table 7 which makes a dichotomic partition of the set of pair points.

Table 7

Decision table for the five chosen points and for four conditional attributes

	Pairs	X_1	X_2	X_3	X_4	D
H_P	(a_1, a_2)	1	1	0	**1**	**P**
	(a_1, a_3)	1	1	0	0	P
	(a_1, a_4)	1	0	1	1	P
	(a_1, a_5)	1	0	1	1	P
	(a_2, a_3)	1	0	0	0	P
	(a_2, a_4)	1	0	1	1	P
	(a_2, a_5)	1	0	1	1	P
	(a_3, a_4)	0	0	1	1	P
	(a_3, a_5)	1	0	1	1	P
	(a_4, a_5)	1	1	0	0	P
H_N	(a_2, a_1)	0	0	1	**0**	**N**
	(a_3, a_1)	0	0	1	1	N
	(a_4, a_1)	0	1	0	0	N
	(a_5, a_1)	0	1	0	0	N
	(a_3, a_2)	0	1	1	1	N
	(a_4, a_2)	0	1	0	0	N
	(a_5, a_2)	0	1	0	0	N
	(a_4, a_3)	1	1	0	0	N
	(a_5, a_3)	0	1	0	0	N
	(a_5, a_4)	0	0	1	0	N

According to definition (6), from Table 8 we obtain one dominance relation which intersects with seven pairs of points and which verifies the condition of the lower approximation.

Table 8

Dominance which intersects with seven pairs of points in the decision table

MMD_Q/H	(a_1, a_2)	(a_1, a_4)	(a_1, a_5)	(a_2, a_4)	(a_2, a_5)	(a_3, a_5)	(a_4, a_5)
$Q = \{X_1, X_4\}$	x	x	x	x	x	x	x

From this we can induce the minimal decision rule, which can be formulated as follows:

Rule 3

$$\text{If } a_i \text{ MMD}_Q\, a_j \text{ then } a_i \text{ P } a_j \text{, with } Q = \{X_1, X_4\} \qquad (16)$$

The meaning of this rule is that if the point a_i in the Pareto's zone is better that the point a_j with respect to friability index and to the distance from the boundary, then the point a_i is preferred to point a_j. In the same way we can do the approximation of non-preference N by the Multi-Attribute non--dominance not MMD_Q. According to the definition (9) from Table 9, we obtain one non-dominance relation which intersects with seven pairs of points and which verifies the condition of lower approximation.

Table 9

Non-dominance which intersects with seven pairs of points in the decision table

MMD_Q /H	(a_2, a_1)	(a_4, a_1)	(a_5, a_1)	(a_4, a_2)	(a_5, a_2)	(a_5, a_3)	(a_4, a_5)
$Q = \{X_1, X_4\}$	x	x	x	x	x	x	x

Based on the same principle as in the case of preference choosing, we can induce the rule as follows:

Rule 4

$$\text{If } a_i \text{ not MMD}_Q\, a_j \text{ then } a_i \text{ N } a_j \text{, with } Q = \{X_1, X_4\} \qquad (17)$$

The last step of the robustness analysis is to apply the rules 3 and 4 to order once again the whole set of points.

The rough approximation then gives a new recommendation: 66.00°C as the best temperature and 2.82 cm is the best drawplate profile. This combination of input parameters assumes the following values for the four output attributes: $X_1 = 1.99$, $X_2 = 13.86$, $X_3 = 6.62$ and $X_4 = 7.53$. We can notice that this action is different when compared to the one found in the previous part.

But, the most important information from this ranking is that the best recommendation is a robust one, because a little variation of the temperature (e. g. difficulty to control the temperature in the granules) does not alter the product quality, the point stays in the Pareto set. Figure 5 shows the results of this new total ranking plotted *quintile* by *quintile* and the best recommendation (black diamond).

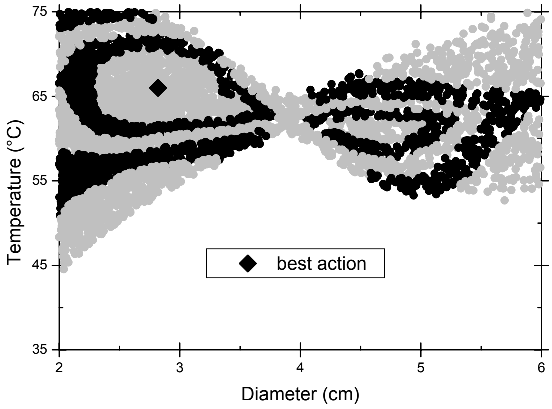

Figure 5. Total ranking of the Pareto set by taking into account the technical robustness

We can notice that the first *quintile* is represented in two parts in the middle right and the middle left of the Pareto's zone. This leads to confirm the importance of a criterion like the technical robustness.

Conclusion

The Rough Set approach has been used for the analysis of preferential information concerning multi-attribute choices in the Pareto's zone. This information is given by the DM as set of pairwise comparisons among some reference points. Taking into account these preferences, we deduce the rules. These rules represent the preference model of the DM, which can be applied to a whole set of points in the Pareto's zone.

After obtaining of the total ranking of the actions, the robustness of the results has been studied. The technical robustness has been added as another criterion to take into account a possible degradation of the working condition. So, this study has been able to propose a very good action, which can be defined as a robust one from technical points of view.

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