Sahnoun Imen

Jean Marc Martel

Chabchoub Habib

PREDICTION OF BANKRUPTCY BASED ON THE MATHEMATICAL PROGRAMMING

Abstract

The concern about the prediction of bankruptcy is not recent. Since the 1960s, several methods have been developed. The most popular is certainly the Multiple Discriminant Analysis. In this paper, we are developing a method based on Mathematical Programming. The global performance of every firm is evaluated by a multicriteria index and the firms are classified into three categories: the non-failed firms, failed firms and those about which we are uncertain. The minimization of the latter constitutes the main objective of our mathematical program.

Keywords

Mathematical programming, multicriteria index, classification, predicting bankruptcy.

Introduction

The problem of classification and segregation of different firms into disconnected groups is frequently used in decision-making. The problem of binary classification, which is set down in this context, is a problem in which the number of groups is limited to two. This type of classification is appropriate to numerous problems such as the differentiation between a good and a bad customer or between failed and non-failed firms. Zopounidis and Doumpos [39] showed that discrimination problems are very common in several fields of finance, including predicting bankruptcy, credit granting, corporate mergers and acquisitions, country risk evaluation, venture capital investments, portfolio selection and management, etc.

"The definition of business failure varies across different studies depending on purposes and scopes of studies" Gu [18]. But generally, bankruptcy refers to a situation in which the firm has negative net worth as well as insufficient liquidity to meet current liabilities. "For more than 30 years, researchers from all over the world work on the problem of business failure prediction. The problem of predicting bankruptcy timely and correctly, is of great importance for financial institutions" Tsakonas et al. [36].

Multivariate Discriminant Analysis (MDA) remains undoubtedly the best-known model and the most often used in this field. It allows us to formulate the problem mathematically, to analyze the set of variables simultaneously, to reduce the dimension of the space of analysis from n (the number of independent variables) to g-1 (where g is the number of groups) and to assign the new firms to the groups defined. It defines a synthetic variable Z capable of summarizing the information included in the set of variables. Z is a linear combination of n ratios which separates the two types of firms (non-failed firms and failed firms). Z is written as follows:

$$Z = a_0 + a_1R_1 + a_2R_2 + ... + a_nR_n$$

Z_1 and Z_2 are gravity centers of the two classes. The MDA looks for Z_1 and Z_2 which are the most distant possible and such that the uncertainty zones be the narrowest in the sample of reference and validation.

Altman is the first who had used the MDA to predict bankruptcy, but later on, several works attempted to adapt this analysis to their environments such as: Collongues [6], Conan and Holder [7] and the case of the Banque of France [4], etc.

Despite the success achieved by the MDA, the method has been criticized in numerous studies such as: Joy and Tollefson [20], Scott [32], Malecot [23, 24, 25] and Eisenbeis [11, 12]. The latter raised problems concerning the applicability of the MDA.

Eisenbeis [11] summarized the difficulties with the MDA in seven points:

1. Violation of the assumption of multivariate normal distribution of the variables.
2. Use of linear instead of quadratic discriminant functions when group dispersions are unequal.
3. Unsuitable interpretation of the role of independent variables.
4. Reduction in dimensionality.
5. Group definition.
6. Inappropriate choice of a priori probabilities and costs of misclassification.
7. Problems in estimating classification error rates.

Similar difficulties have been pointed out by Malecot [23], but he tried to classify them according to their origin: statistical limit (the absence of the normality of variables, the unequal group dispersions, etc. and the limit of the transposition. Whereas Joy and Tollefson [20] revealed other difficulties related to the form of the discriminant function, the cross-validation and sampling.

The other popular functional form used by bankruptcy researchers is the logit model and the probit model. Laitinen & Laitinen [22] proposed a combined use of logistic regression and the Taylor series expansion. A comparison of the logit model and the quadratic interval logit model is described in Tseng and Lin [37]. Doganay et al. [9] proposed an integrated early warning system by combining multiple regression, discriminant analysis, logit and probit.

For the last few years, several theoretical studies have been devoted to the use of Mathematical Programming (MP) techniques to solve the classification problems. Several experiences and studies showed that MP can overclassify statistical techniques of discriminations. Glover, Keene and Duea [16] mention the following advantages:

– The MP methods are free from underlying parametric assumptions.
– Various objectives and more complex problems are easily accommodated.
– Misclassification costs can be easily incorporated into the model.
– Some MP methods, especially Linear Programming, lend themselves to sensitivity analyses.

The approaches of Mathematical Programming applied to the discrimination problems try to build a discriminant function or a hyperplane which separates the two groups in order to classify a new entity in the corresponding group. The hyperplane corresponds to the following equation:

$$AX = b$$

where A is the set of alternatives.

The objective is to determine the weighting vector X and a scalar b, so that we assign, as correctly as possible, the individuals of group 1 to the one side of the hyperplane and the individuals of group 2 to the other side. Therefore, the goal is to minimize the weighted sum of boundary violation.

Mathematical programming methods were employed early to solve the discriminant problem. Grinold is one of the pioneers who formulated a MP for failure prediction in 1972. More recently different clustering and discrimination methodologies have been developped, based on mathematical programming. Among these models are MSD (minimize the sum of distances) presented by Freed and Glover [14], MMD (minimize the maximum distance) proposed by Freed and Glover [15] and MIP (mixed-integer programming). Various combinations of these basic methods have been proposed in literature.

With the technological and scientific development, other discrimination models appeared and were applied to the problems of bankruptcy prediction. Among these models we can mention: the Recursive Partitioning Algorithm, rough sets [28], the expert system, the neural networks [1, 30, 35, 31, 2], the genetic algorithm [19, 28, 13] and the Multicriteria Decision methods such as ELECTRE TRI [38, 33], UTADIS [10], MINORA, PROMETHEE [5, 11] (for more details see [8]). Kumar and Ravi [21] presented a review of the work done, during the 1968-2005, in the application of statistical and intelligent techniques to solve the bankruptcy prediction problem faced by banks and firms.

By relying on the Multicriteria Index (MI) suggested by Martel et al. [27] in order to compare the performance-size of mutual funds and by taking advantage of the competitiveness and the flexibility of Mathematical Programs, we have formulated a mathematical program which integrates this index to assess the global performance of the firms. The latter allows us to predict the bankruptcy of the firms by classifying them into two categories: failed firms and non-failed firms. Martel and Khoury [26] proposed a procedure of binary classification of firms which refers to this index.

Our paper includes 5 sections. In the second section there is a brief description of the Multiciteria Index. Our MP model of bankruptcy prediction will be presented in the third section. An empirical illustration is given in the fourth section and the last one summarizes the content of the paper.

1. A Multicriteria Index (MI) of financial performance

As the Multicriteria Index (MI) of Martel et al. [27] is inspired by PROMETHEE (Preference Ranking Organisation MEthods for THe Enrichment Evaluation), we start by presenting basically this procedure as developed by Brans et al. [5]. This method classifies a set of alternatives on the basis of pairwise comparisons. It defines a preference function for every criterion.

Suppose:

$$a, a' \in A$$

where A is the set of the alternatives.

The preference function, denoted $P_j(a, a')$, expresses the decision-maker's preference for the alternative a over a' according to the criterion C_j. This function is defined as follows:

$$P_j(a, a') = \begin{cases} 0 & \text{if} \quad C_j(a) \le C_j(a') \\ H_j(d_j(a, a')) & \text{if} \quad C_j(a) > C_j(a') \end{cases}$$

where $d_j = C_j(a) - C_j(a')$.

Every criterion function $H_j(d_j(a, a'))$ (see Figure 1) takes its values between 0 and 1. In other words the decision-maker's preference varies from indifference (0) to the strict preference (1) for each criterion.

Brans et al. [5] proposed six types of functions (H_j) that correspond to different types of criteria (Usual Criterion, Quasi-Criterion, Criterion With Linear Preference, Level-Criterion, Criterion With Linear Preference and Indifference Area and Gaussian Criterion).

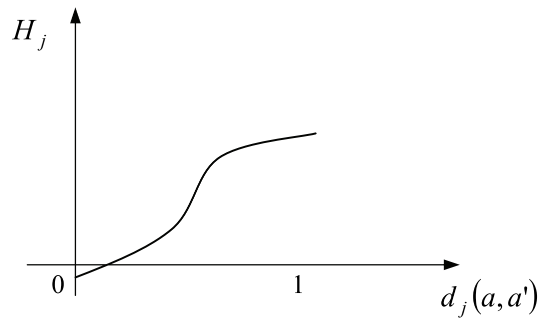

Figure 1. Criterion function

Following the definition of the criterion function, we are applying an aggregation procedure which consists of the following steps:

1. We calculate for each alternative a a global preference (the performance) index of a over a' taking into account all criteria. Therefore for every pair of alternatives (a, a'), we calculate:

$$P(a, a') = \sum_j W_j P_j(a, a')$$

where W_j is the relative importance of every criterion C_j, with $\Sigma W_j = 1$.

2. We use this index to calculate the out flows and the in flows for each alternative *a*. The out flows represents the preference (the performance) of the alternative *a* over other alternatives:

$$P^{+}(a) = \sum_{e \neq e'} P\,(a, a')$$

The in flows represent the preference (the performance) of the set of other alternatives over the alternative *a*:

$$P^{-}\,(a) = \sum_{e \neq e'} P\,(a', a)$$

3. Finally, we use the notion of net flow to evaluate the performance of the alternative *a*:

$$P(a) = P^{+}(a) - P^{-}(a)$$

Then *a* is preferred to (or is more “performing” than) *a'* if $P(a) > P(a')$.

When the number of pairwise comparison raises, Martel and al. [27] and Martel and Khoury [26] introduced the notion of ideal and anti-ideal. Thus, Martel and Khoury [27] have proposed a procedure that allows an absolute evaluation of each alternative by comparing it with two fictitious firms: one is ideal (a^*) and the other anti-ideal (a_*).

$$V\,(a^*) = \{\, C_1^* \ldots\ldots\ldots C_j^* \ldots\ldots\ldots\ldots C_m^* \}$$

where $$C_j^* = Max\left[Max\ C_j(a), \overline{C_j} + 2\sigma_j\right]$$

for a criterion to maximize

and $$C_j^* = Min\left[Min\ C_j(a), \overline{C_j} - 2\sigma_j\right]$$

for a criterion to minimize,

where $\overline{C}_j$ and σ_j are the average and the gap type of assessments according to C_j, respectively.

We suppose that each criterion is measured on a cardinal scale. We added and reduced $2\sigma_j$ since we work with a sample of an ordered population and we use a confidence interval. We could use + or $-2.5\sigma_j$ or even + or $-3\sigma_j$.

So, the ideal firm is constructed by forming the vector of the best assessments with respect to each of the criteria. However, the anti-ideal firm is the combination of the worst assessments with respect to each of the criteria.

$$V(a_*) = \{ C_{*1} \ldots\ldots\ldots C_{*j} \ldots\ldots\ldots\ldots C_{*m} \}$$

where $C_{*j} = Min\left[Min\ C_j(a), \overline{C_j} - 2\sigma_j\right]$

for a criterion to maximize,

and $C_{*j} = Max\left[Max\ C_j(a), \overline{C_j} + 2\sigma_j\right]$

for a criterion to minimize.

For every firm $a / a \in A$, Martel and Khoury [26] constructed a MI that satisfies the following inequality:

$$MI(a_*) \leq MI(a) \leq MI(a^*)$$

By relying on PROMETHEE II and the pairwise judgments between the alternatives
a^*, a and a_*, we can determine their net flow $P(a^*)$, $P(a)$ and $P(a_*)$. And we get:

$$P(a^*) = 1 + \sum W_j H_j(d^*)$$

$$P(a) = \sum W_j \left[H_j(d_*) - H_j(d^*)\right]$$

$$P(a_*) = -\left[1 + \sum W_j H_j(d_*)\right]$$

where

$P_j(a^*, a) = H_j(d^*)$, $P_j(a, a_*) = H_j(d_*)$ and $d^* = C_j(a^*)\text{-}C_j(a)$, $d_* = C_j(a)\text{-}C_j(a_*)$ for a criterion to maximize; $d^* = C_j(a)\text{-}C_j(a^*)$, $d_* = C_j(a_*)\text{-}C_j(a)$ for a criterion to minimize.

To facilitate the comparison between firms, Martel and Khoury [26] normalized the global performance $P(a)$ of every firm. The index thus obtained, called Multicriteria index (MI), varies between 0 and 1:

$$MI(a) = \frac{P(a) - P(a_*)}{P(a^*) - P(a_*)}$$

So for every firm, Martel and Khoury [26] calculated its MI in order to assign it to a predefined category. Then, they determined two thresholds MI^* and MI_* permitting to partition the set of the firms into three categories:

– "Failed firms",
– "Non-failed firms",
– "We don't know".

The rule is that if $MI\ (a) > MI^*$ the firm is assigned to the non-failed category, if $MI\ (a) < MI_*$ the firm is assigned to the failed category and if the $MI\ (a) \in [MI_*, MI^*]$ then the firm is assigned to the category “we don't know”.

Finally, Martel and Khoury [26] presented an *ad hoc* method (a sort of “trial and error”) for the determination of the coefficients (W_j) in order to minimize the numbers of misclassified firms while trying to reduce the width of the uncertainty zone which is limited by $[MI_*, MI^*]$. This method includes several iterations before obtaining the “best” classification. So it is a quite heavy process which offers no guarantee of success. It is mainly at this level that the MP can be very useful.

2. A model for predicting bankruptcy

Inspired by the work of Martel and Khoury [26] we have developed a model for predicting bankruptcy based on mathematical programming. As MI presented in the previous section generates a non-linear mathematical program whose linearization may be long and complex, we have decided to adopt a new form of normalization.

Let's suppose that:

$$x = P\left(a^*, a\right) = \sum_j W_j P_j\left(a^*, a\right)$$

$$z = P(a, a_*) = \sum W_j P_j (a, a_*)$$

For all j we have $W_j \geq 0, \sum W_j = 1$, $P_j(a^*, a) = H_j(d_j(a^*, a))$ and $P_j(a, a_*) = H_j(d_j(a, a_*))$

With $0 \leq H_j(d_j(a, a')) \leq 1$, we can deduce that $0 \leq x \leq 1$ and $0 \leq z \leq 1$,

Knowing that:

$$P(a) = P(a, a_*) - P(a^*, a) = z - x$$

we concluded that: $-1 \leq P(a) \leq 1$.

So, we can do a simple linear normalization by adding a unit and by dividing by 2. We get:

$$0 \leq 1/2(P(a) + 1) \leq 1$$

where: $P\ (a) = \sum W_j \left[H_j(d_*) - H_j(d^*)\right]$

This normalised performance will play the role of the score in our classification model. So, we will try to determine two critical values of performance: P_L (lower performance) and P_U (upper performance) which will limit the uncertainty zone $[P_L, P_U]$.

In order to minimize the number of misclassified firms and the uncertainty zone, we have been inspired by the extension of "DEA-Discriminant Analysis" (DEA-DA) developed by Sueyoshi [34]. Yet, our method can be perceived as a sort of combination of two programs: the MSD (Minimize the sum deviation) and the MMO (Minimize the Misclassified Observations), composed of two steps. The first one minimizes the uncertainty zone by minimizing the maximal deviation s. So this step can be formulated as follows:

Min s

Subject to

$$[1/2(P(a_i)+1)] - d + s \geq 0 \qquad \text{for all} \quad i \in G_1$$

$$[1/2(P(a_i)+1)] - d - s \leq 0 \qquad \text{for all} \quad i \in G_2$$

$$\sum_{j=1}^{k} W_j = 1$$

With $P(a_i) = \sum W_j \left[H_j(d_*) - H_j(d^*)\right]$

G_1: group of non-failed firms.
G_2: group of failed firms.
d: a positive critical score.
s: maximal deviation.
W_j: a vector of coefficients.

The application of the first step to a sample of firms allows us to classify them into three categories; one for non-failed firms, one for failed firms and one where we are not certain. The last one will be limited by $(d^* + s^*)$ and $(d^* - s^*)$ (these are the corresponding values P_U and P_L). If $s^* = 0$ there is no uncertainty zone and all the firms are correctly classified and therefore the program stops at this step. Otherwise we move to the second step to minimize the number of misclassified firms (those belonging to the uncertainty zone).

$$Min \sum_{i \in D_1 \cup D_2} X_i$$

Subject to

$$[1/2(P(a_i)+1)] - P_c(a) - P_i + M X_i = 0 \qquad \text{for all} \quad i \in D_{1m}$$

$$[1/2(P(a_i)+1)] - P_c(a) + \varepsilon - M X_i + N_i = 0 \qquad \text{for all} \quad i \in D_2$$

$$\sum_{j=1}^{k} W_j = 1$$

With $P(a_i) = \sum W_j \left[H_j(d_*) - H_j(d^*)\right]$

Where X_i is a binary variable which is equal to:

– 0 if the corresponding firm is well classified,

– 1 if the corresponding firm is misclassified.

$P_c(a)$: a critical performance.

P_i: a positive deviation.

N_i: a negative deviation.

W_j: the coefficient of the criterion C_j.

M: a large value.

ε : A quite small value.

$D_1 = \left\{ i \in G_1 / [1/2(P(a_i)+1)] \le d^* + s^* \right\}$.

$D_2 = \left\{ i \in G_2 / [1/2(P(a_i)+1)] \ge d^* - s^* \right\}$.

This last step allows us the establishments of credit to predict with less risk the probability of bankruptcy of firms belonging to the uncertainty zone.

To predict the financial situation of a new firm, we should calculate its performance *P(a)* by using the W_j generated in the first step and we compare the classification thresholds (d^*+s^*) and $(d^* - s^*)$ with $[1/2(P(a)+1)]$.

If $[1/2(P(a_i)+1)] > d^* + s^*$ then the firm a_i is non-failing,

If $[1/2(P(a_i)+1)] < d^* - s^*$ then the firm a_i is failing.

And if $(d^* - s^* \le [1/2(P(a_i)+1)] \le d^* + s^*)$ then the firm a_i belongs to the uncertainty zone and the "decision-maker" should calculate the performance of the firm a_i by using the W_j generated in the second step. Then:

if $[1/2(P(a_i)+1)] > P_c^*$ the firm a_i can be predicted as non-failing,

if $[1/2(P(a_i)+1)] < P_c^*$ the firm a_i can be predicted as failing.

3. An empirical illustration

Our sample is made up of fifty four firms involved in the Saving system (this sample has been extracted from [26]). Three of them have been eliminated because they proved to be aberrant. The remaining fifty-one firms have been divided into two subsamples. The first subsample is composed of forty four firms is used for the determination of the value of parameters of the model and the second subsample of control composed of seven firms to validate the obtained results.

First, Martel and Khoury [26] considered twelve Financial Ratios, but they kept only three which were significant following the application of the software STEPDISC of SAS. These three ratios are:

X_2: TOTAL DEBT/TOTAL ASSET.

X_4: ACTIVE SHORT-TERM/PASSIVE SHORT-TERM.

X_8: LOG 10 (TOTAL ASSET).

Where X_2 should be minimized, X_4 maximized, and X_8 maximized. So for our illustration we have relied on these three variables already found significant.

The application of the first step of our method to this sample (Annex 1), gave an s^* equal to 0.000089 thus different to zero. Hence our sample will be discriminated in three categories according to the values of s^* and d^*. Having obtained a value d^* equal to 0.500099, we can classify our sample as follows:

The firms for which $[1/2(P(a_i)+1)] > 0.500188$ are non-failing firms,

The firms for which $[1/2(P(a_i)+1)] < 0.50001$ are failing firms,

Whereas the firms for which $[1/2(P(a_i)+1)] \in [0.50001 ; 0.500188]$ have to go through the second step of our method in order to predict their financial situation.

Table 1 classifies the different firms of our sub-sample according to their performance and thus we can see the different categories: the firms which are in bold belong to the uncertain category; those which are above (1) are the non-failing firms whereas those which are below (0) are the failing firms.

Table 1

Result of the application of the first step of our method

FIRMS	CATEGORY	1/2(P(*a*)+1)
36	1	0,996960
12	1	0,995436
24	1	0,993684
50	1	0,983583
40	1	0,906997
38	1	0,852619
44	1	0,679543
20	1	0,544961
48	1	0,532709
34	1	0,529385
28	1	0,512472
52	1	0,503516

Table 1 contd.

FIRMS	CATEGORY	1/2(P(a)+1)
54	1	0,502765
42	1	0,500368
22	1	0,500295
25	**0**	**0,500188**
18	**1**	**0,500129**
14	**1**	**0,500103**
46	**1**	**0,500089**
1	**0**	**0,500083**
21	**0**	**0,500080**
10	**1**	**0,500019**
30	**1**	**0,500010**
26	**1**	**0,500010**
16	**1**	**0,500000**
49	0	0,499991
47	0	0,499987
41	0	0,499986
15	0	0,499981
45	0	0,499961
53	0	0,499913
17	0	0,499879
51	0	0,499850
37	0	0,499850
11	0	0,499637
35	0	0,499333
29	0	0,499194
19	0	0,499167
23	0	0,499066
39	0	0,498638
9	0	0,498471
27	0	0,497531
43	0	0,424236
33	0	0,000005

The performances of the firms are calculated with the weight provided by the application of the first step of our model ($W_1 = 0.994396$; $W_2 = 0.000050$; $W_3 = 0.005554$), where W_1: the coefficient of the ratio X_2, W_2: the coefficient of the ratio X_4 and W_3: the coefficient of the ratio X_8.

By applying the second step of our method to the firms belonging to the uncertainty zone and by refering to the discriminating threshold (critical performance) $P_c(a)$ equal to 0.499318, we obtain two misclassified firms (as shown in Table 2, firms 26 and 30 are really non-failing but predicted as failing).

Table 2

Result of the application of the second step of our method

FIRMS	CATEGORY	$1/2(P(_a)+1)$
10	1	0,501369
18	1	0,500788
46	1	0,500622
14	1	0,499995
16	1	0,499318
21	0	0,498318
1	0	0,498317
26	**1**	**0,497338**
25	0	0,495333
30	**1**	**0,465743**

The coefficients provided by the application of the second step of our model are: W_1 = 0.924542; W_2 = 0.006659 and W_3 = 0.068799.

To validate the results of our model, we have applied the coefficients obtained from the first step of our model to the sub sample test made up of seven firms. These firms, their normalized performances as well as their classification are given in Table 3.

Table 3

Classification of firms of sub sample test

FIRMS	CATEGORY	1/2(P(a)+1)
8	1	0,732815
2	1	0,635040
6	1	0,610619
4	1	0,583682
7	0	0,490657
3	0	0,480655
5	0	0,305725

According to the rule of classification provided by the first step of our model the firms 8, 2, 6 and 4 have been predicted as non-failed whereas the firms 7, 3 and 5 have been predicted as failing. For this sub sample, we notice that the seven firms have been well classified and none of them belongs to the zone of uncertainty.

Conclusion

This paper presented a new linear programme to solve a multi-criteria classification problem. In contrast to other discriminating linear programmes the proposed one provided not simply a description of the alternatives, but also performing information that helps in the identification of the most and the least performed alternatives. The concepts introduced by the PROMETHEE methods were used to develop an appropriate multi-criteria index for the classification of alternatives.

"In most of the existing approaches, the classification of the alternatives is determined on the basis of their comparison to some reference profiles (fictitious alternatives) that define the boundaries of the classes. Nevertheless, the definition of these reference profiles, as class boundaries, is not always clear" [10]. Therefore we used the techniques of linear programming to define the boundaries of classes and the weight W_j of the criterion C_j.

The proposed method consists of two steps. The first seeks to determine the narrowest uncertainty zone whereas the second seeks to minimize the number of misclassified firms belonging to this zone. Thus the proposed model provides in its first step two discriminating thresholds which permitted to classify firms into three categories and in the second step classifies again the firms belonging to the uncertainty zone.

This model applied to a sample of 44 firms, 22 of which are non-failed and 22 failed, generated results which seem promising. Indeed, we have managed to classify the two groups of firms obtaining only two misclassified ones. Finally, to validate the results obtained, we have used a sub sample of control and all the firms have been perfectly classified.

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