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STABILITY OF MULTICRITERIA RANKINGS – A COMPARISON

Abstract

Set of weights describing decision maker's preferences plays a crucial role in construction process of multicriteria rankings, because even small changes of preferences may lead to different results. Moreover, the stability of rankings is important due to the fact that weights are only an approximation of decision maker's preferences. Most popular approaches to rankings sensitivity usually focus on the case in which no changes occur after the modification of eights or parameters. Such analysis does not give an insight into the way ranking is changed when the given weight intervals are exceeded. Spearman's rank correlation coefficient can provide information on that case, but it does not inform directly about the scales of shifts inside the ranking. Stability analysis of investment funds rankings constructed by three multicriteria outranking methods (PROMETHEE, WSA and TOPSIS) is presented. Similarity of rankings is assessed on the basis of Spearman's correlation coefficient and chosen stability definitions. Simulation includes different weight sets describing decision maker's preferences concerning such criteria as parameters of distribution of return rates, purchase and management costs and customers' convenience.

Keywords

Multicriteria rankings, PROMETHEE method, WSA method, TOPSIS method.

Introduction

In outranking approach a decision-maker's preferences are usually introduced to the analysis in form of parameters and, in most cases, weighting vectors. They are subject to change in time, hence such an assessment involves constructing new rankings after each change. The process is time-consuming and computationally demanding. It can be also bothersome to a decision maker, because of the necessity of interviewing in order to set up the new relative importance of criteria. Moreover, usually we do not know which changes of preferences will result in a change of hierarchy i.e. in which cases a new ranking should be constructed.

It should be also considered that weighting vectors used to construct rankings are only an approximation of the decision-maker's preferences and do not reflect them exactly. Therefore it is necessary to perform a stability and sensitivity analysis.

Stability can be defined in many ways. When assessing volatility of a ranking resulting from the changes of weights or parameters, we usually focus on the case in which no shifts occur i.e. ranking is preserved after modification of the weighting vector. After conducting such an analysis it is still unknown which changes in ranking can occur when the weights exceed the given boundaries [21]. Spearman's rank correlation coefficient [19] can give some insight into that problem, but it does not inform directly on the scales of shifts inside the ranking.

The case in which ranking is preserved can be generalized by defining *k*-stability [10]. It is assumed that maintenance of the strict order of alternatives with highest evaluations is the issue of greatest importance for the decision--maker. However, there can exist problems in the case of which the decision--maker can be interested in selecting first n alternatives (i.e. alternatives that have the highest evaluations in a given ranking), but the order in this group does not play a crucial role (e.g. selecting 5 best alternatives for a further assessment by other methods). The definition proposed in [10] does not inform about the way the ranking is going to change when the change actually occurs. It informs only that some alternatives ranked among *k* objects with highest evaluations will change their positions. The scale and character of the change remain unknown.

We considered the stability problem by introducing the definition of stability of order *s*, according to which a ranking R_B is called stable of order *s* if $\max_i (d_i) = s$, where $d_i = d_{i,B} - d_{i,Z}$ is the difference of ranks of alternative *i* in the initial ranking R_B and ranking R_Z constructed using modified weights.

Stability of rankings of open-end investment funds constructed by three different multicriteria methods was analyzed on the basis of the above--mentioned definition. The results obtained indicate that the stability of rankings depends on the method applied, however this finding can result from the strong assumptions about decision-makers' preferences [18]. We intend to verify this hypothesis by simulation of the ranking stability with different assumptions and applying different weighting vectors.

The paper is organized as follows. In section 1 three multicriteria methods used to construct rankings are described. In section 2 decision criteria are proposed. The final section is focused on the presentation of simulation results and comments.

1. Multicriteria rankings

Multicriteria rankings are applied to problems in which a decision-maker is to rank the set A consisting of N alternatives a_i, $i=(1,...,N)$ taking into consideration the values acquired by criteria $f_1,...f_K$.

Three methods considered in the study represent different approaches towards solving multicriteria problems. We discuss them in detail in the following section.

1.1. SAW

The SAW method (Simple Additive Weighting) [9] allow for comparison of criteria by normalising evaluations. The alternatives evaluated are ranked according to the decreasing value of benefit function:

$$u_j = \sum_{i=1}^{K} w_i e_{ij}$$

where:

w_i – the weight associated with criterion f_i,

e_{ij} – the standardized evaluation of alternative a_j for criterion f_i.

The method of normalisation is not specified; usually the following formulas are used:

$e_{ij} = \dfrac{f_i(a_j)}{\max\limits_{j} f_i(a_j)}$ for maximized criteria (the higher evaluation the better),

$e_{ij} = \dfrac{f_i(a_j)}{\min\limits_{j} f_i(a_j)}$ for minimized criteria (higher evaluation of the criterion means also worse evaluation).

Or, respectively:

$$e_{ij} = \frac{f_i(a_j) - \min\limits_{j} f_i(a_j)}{\max\limits_{j} f_i(a_j) - \min\limits_{j} f_i(a_j)}$$

$$e_{ij} = \frac{\max\limits_{j} f_i(a_j) - f_i(a_j)}{\max\limits_{j} f_i(a_j) - \min\limits_{j} f_i(a_j)}$$

Let us notice that using the SAW method it is possible to compensate lower evaluations on some criteria by higher evaluations on others. In some cases, e.g. investment problems, this feature can be seen as beneficial, as it is possible to compensate costs (e.g. management fees) by higher returns. The method is also intuitive for the decision-maker.

1.2. TOPSIS

In the case of the TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution) [7] a ranking is constructed on the basis of the distance from two reference points: the ideal and the negative ideal solutions of the multicriteria problem. The reference points are defined as follow:

$$F^* = \left(f_1^*, f_2^*, \ldots, f_K^* \right),$$

where $f_i^* = \max_{a \in A} f_i(a)$

$$F^- = \left(f_1^-, f_2^-, \ldots, f_K^- \right),$$

where $f_i^- = \min_{a \in A} f_i(a)$.

The alternatives are ranked according to the decreasing value of the function:

$$D_p(a) = \frac{d_p^-(a)}{d_p(a) + d_p^-(a)}$$

where:

$$d_p(a) = \left(\sum_{i=1}^{K} w_i^p \left(f_i - f_i(a) \right)^p \right)^{\frac{1}{p}}$$

$$d_p^-(a) = \left(\sum_{i=1}^{K} w_i^p \left(f_i(a) - f_i^- \right)^p \right)^{\frac{1}{p}}$$

The idea of the TOPSIS method is intuitive and comprehensible for the decision-maker, as in the case of the SAW method. This method can be also regarded as more flexible than SAW, due to the fact that decision-maker's preferences can be introduced not only in the form of weights but also in the form of the parameters *p* used to calculate the distance measure.

It should be noted that it is indirectly assumed that approaching the ideal solution and moving away from the negative ideal one are of equal importance for a decision-maker.

1.3. PROMETHEE II

The PROMETHEE methods ([1], [2]) seem to be the most flexible and easy to adjust to the preferences of the decision-maker. However, similarities of rankings constructed by applying the SAW and PROMETHEE II methods were reported in previous studies ([6], [18]).

Criteria are normalised by applying the so-called generalized criteria (see [5]), which admit values between 0 and 1. We used the Gaussian criterion (*), in which case the preference function does not involve determining other parameters and making additional assumptions concerning decision-maker's preferences.

$$P_i(d_i(\cdot)) = \begin{cases} 0 & d_i(\cdot) \leq 0 \\ \\ 1 - \exp\left(-\dfrac{d_i(\cdot)^2}{2\sigma_{i^2}}\right) & d_i(\cdot) > 0 \end{cases} \tag{*}$$

where:

$\displaystyle\mathop{\forall}_{a_1, a_2 \in A} d_i(a_1, a_2) = f_i(a_1) - f_i(a_2)$ is the difference between the evaluations of the pair of alternatives a_1, a_2 on criterion f_i,

σ_i – standard deviation of evaluation of criterion f_i.

The values of the function $P_i()$ are used to determine the aggregated preference indices:

$$\begin{cases} \pi(a_1, a_2) = \displaystyle\sum_{i=1}^{K} P_i(a_1, a_2) w_i \\ \\ \pi(a_2, a_1) = \displaystyle\sum_{i=1}^{K} P_i(a_2, a_1) w_i \end{cases}$$

The value of the index $\pi(a_1, a_2)$ expresses the degree to which the alternative a_1 is preferred to alternative a_2. For a given alternative $a^* \in A$ the positive and negative outranking flows are defined respectively as follow:

$$\varphi^+(a^*) = \frac{1}{N-1} \sum_{a \in A}^{K} \pi(a, a^*)$$

and

$$\varphi^-(a^*) = \frac{1}{N-1} \sum_{a \in A}^{K} \pi(a^*, a)$$

The positive outranking flow for an alternative a^* is interpreted as the degree to which a^* outranks other alternatives. The greater value it acquires, the better alternative a^* is in comparison with others. The high value of the negative outranking flow expresses, in turn, the weakness of the alternative examined.

The net outranking flow (**) is defined as the balance of the positive and negative outranking flows and is the basis of the final ranking. A positive value of this flow means that the given alternative is in the group of dominating and dominated objects.

$$\varphi(a) = \varphi^+(a) - \varphi^-(a) \tag{**}$$

The alternatives are sorted according to the decreasing net flow values.

2. Criteria

In the literature it is suggested to evaluate funds assuming that their results are affected by such indicators as historical means of returns, risk, participation costs, size of the fund and minimum value of the initial investment ([3], [11], [12], [15]). Investors are assumed to be interested in the quality of services [4] and the manager's reputation [13]. Funds can be classified [14] on the basis of the i.a. rates of return, Sharpe ratio ([16], [17]), Treynor ratio [20] and VaR measure based on the quantiles [8].

We consider a set consisting of 47 open-end stock funds operating on Polish market. The data reflect their performance in the period from January to July 2008.

In the set of criteria we included nine indicators associated with the financial results of funds, costs connected with management of the assets and customer's convenience.

Three groups of criteria are listed below.

1. Criteria associated with the distribution of the return rates that inform about the expected return rate and risk. In this group we took into consideration the following measures: *expected value, standard deviation, skewness, kurtosis* and *0.05 percentile of the return rate*.

2. Criteria associated to fees and minimum values of inputs required by the fund. From the decision-maker's point of view these criteria can be treated as obstacles and costs. Minimum values of inputs mean that the decision-maker is unable to invest individually chosen amount of capital at time he/she chooses. Even though the fund may offer shares it is impossible for the decision-maker to purchase them or invest only the amount of capital he/she intends to invest. Criteria from this group include: *management fee, minimum required value of the first input, minimum required value of the next input.*

3. *Number of methods of placing orders* – this criterion describing different facilities for the decision-maker. The funds analysed offered different methods of placing orders: directly in the customer service point, by transfer, phone, fax or Internet.

3. Empirical analysis

In the empirical analysis we applied 14 scenarios presented in the Table 1. The decision-maker's preferences were described by weighting vectors. In the first scenario S1 we assumed no preference among the criteria (i.e. equal weight for each criterion). The following scenarios are based on equal (S2) and differentiated (S3-S14) weights for the groups of criteria and individual criteria as well.

Table 1

Values of weights describing decision-maker's preferences in analysis scenarios

		Expected value	Standard deviation	Skew-ness	Kur-tosis	Per-centile 0.05	Minimum required value of the first input	Minimum required value of the next input	Manage-ment fee	Number of methods of placing orders
		C1	C2	C3	C4	C5	C6	C7	C8	C9
		Max	Min	Max	Min	Max	Min	Min	Min	Max
Values of weights in scenarios	S1	1	1	1	1	1	1	1	1	1
	S2	0.067	0.067	0.067	0.067	0.067	0.11	0.11	0.11	0.33
	S3	2	2	2	2	2	1	1	1	1
	S4	2	2	2	2	2	2	2	2	1
	S5	1	1	1	1	1	1	1	1	2
	S6	1	1	1	1	1	2	2	2	2
	S7	1	1	1	1	2	2	2	2	1
	S8	2	2	2	2	2	1	1	1	2
	S9	3	3	3	3	3	1	1	1	2
	S10	3	3	3	3	3	2	2	2	1
	S11	1	1	1	1	1	2	2	2	3
	S12	1	1	1	1	1	3	3	3	2
	S13	2	2	2	2	2	3	3	3	1
	S14	2	2	2	2	2	1	1	1	3

After constructing rankings by applying weights listed above we assessed similarity of the results by calculating Spearman's rank correlation coefficient. The majority of the coefficients calculated acquire high values which indicate a considerable similarity between rankings constructed by applying different methods. A similarity between the PROMETHEE II and SAW methods is noticeable. Minimum and maximum values of correlation coefficients for rankings constructed by using the same weights vectors are presented in Table 2.

Table 2

Minimum and maximum values of correlation coefficients for rankings constructed by applying different methods in all scenarios

	PROMETHEE/TOPSIS	PROMETHEE/SAW	TOPSIS/SAW
Min r_s	0.7484	0.9744	0.7516
Max r_s	0.7978	0.9940	0.8171

The minimum values of rank correlation coefficients for rankings constructed by applying the same method in different scenarios are presented in Table 3. It can be noticed that results obtained by the TOPSIS method are most dependent on the values of weights.

Table 3

Minimum values of correlation coefficients for given methods

	PROMETHEE II	SAW	TOPSIS
Min r_s	0.8019	0.8231	0.6911

Once rankings had been constructed in each scenario the weight for each criterion was successively changed as follows:

$$w_i = \begin{cases} w_i \cdot \left(1 + \dfrac{j}{10}\right) & i = k \\ \\ w_i & i \neq k \end{cases};$$

where:

$j \in \overline{1,90}$ – number of iteration.

After each change the weights were normalized and a new ranking was constructed. In each iteration we compared the new ranking with the initial one by calculating the stability of order measure.

The results obtained in all scenarios are similar. The orders of stability of the SAW and PROMETHEE II methods are similar. Their difference average, depending on the values of weights, 2-3.5. The stability of the TOPSIS method is noticeably lower.

Figures 1-3 illustrate stability of rankings for the S13 scenario. The results are representative, because in all scenarios, the methods acted in a similar way. A considerable similarity of the PROMETHEE II and SAW methods can be noticed. Those two methods can be regarded as least sensitive to weight changes. There is also a substantial difference between them and TOPSIS. In the case of *skewness* and *kurtosis* criteria the difference mentioned is the least and the values of order of stability measure, comparable for all three methods. For the *management fee* criterion the order of stability of the TOPSIS method was, in turn, lower than for others.

Expected value

Standard deviation

Sekwness

Kurtosis

0.05 percentile

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............  PROMETHEEII
──────────  TOPSIS
──────────  SAW
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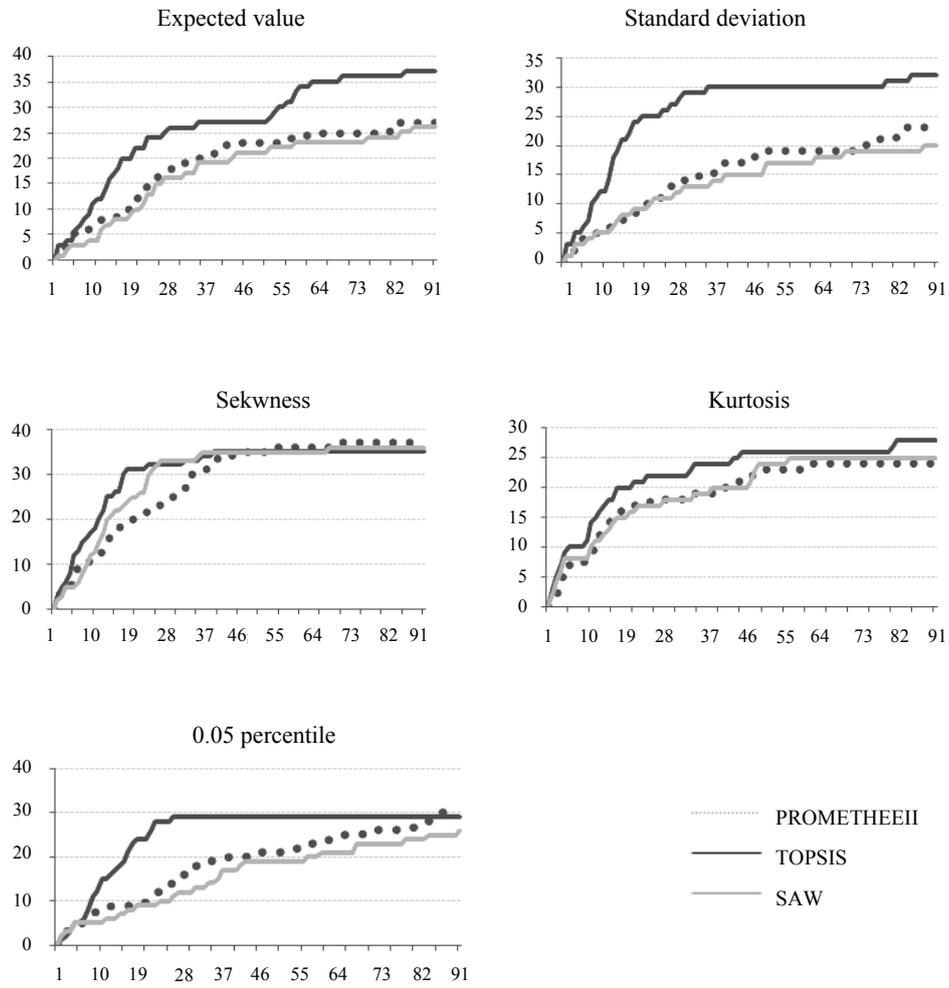
Figure 1. Order of stability in all iterations for the methods compared and the first group of criteria
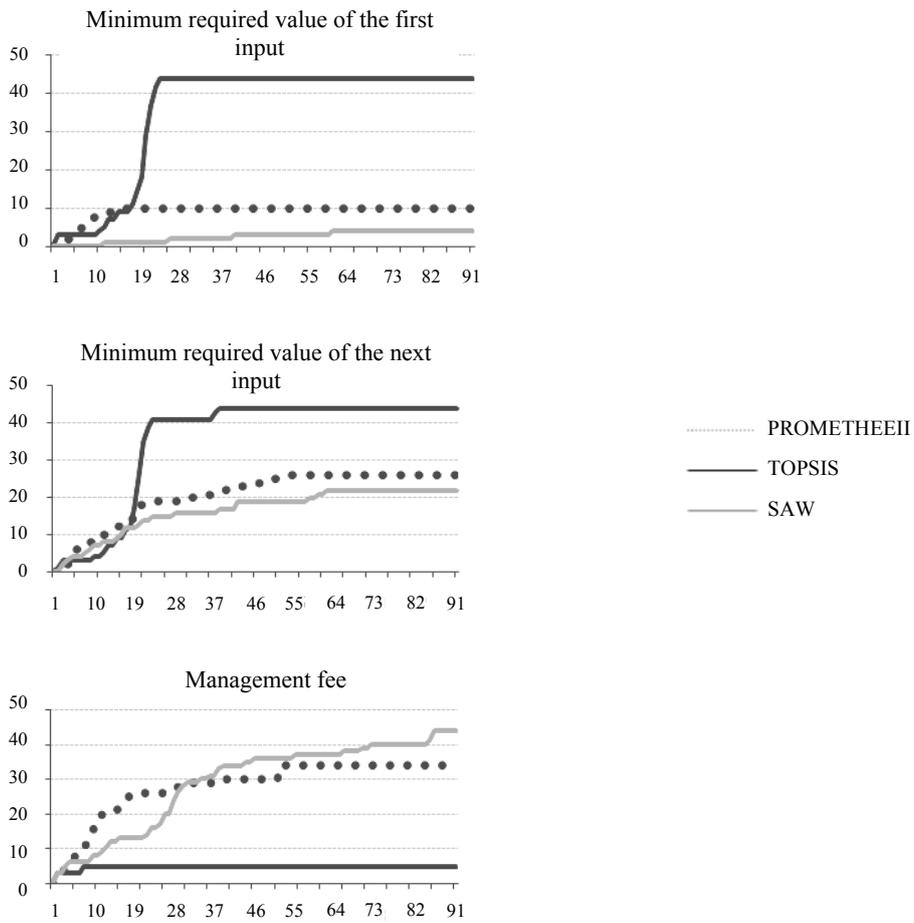
Figure 2. Order of stability in all iterations for the methods compared and the second group of criteria
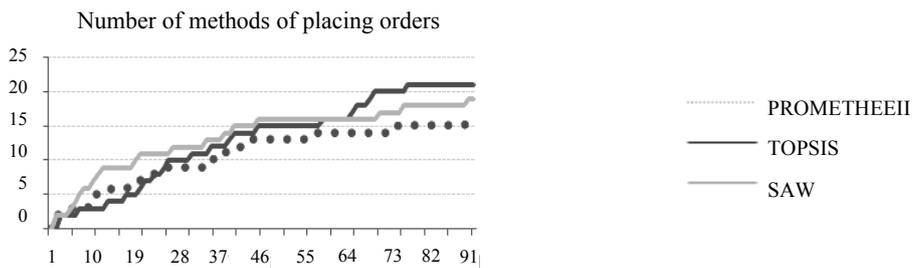
Figure 3. Order of stability in all iterations for the methods compared and the *number of methods of placing orders* criterion

The only exception among the scenarios analysed was the *management fee* criterion. For weights assumed in scenarios S2, S3, S8-S10 and S14, the order of stability of the TOPSIS method was higher and comparable with the other methods (see Figure 4), but the similarity of the SAW and PROMETHEE II methods was preserved. The scenarios in which the above-mentioned changes were noticed have one feature in common – low weights for criteria from the second group (except for S2 scenario).
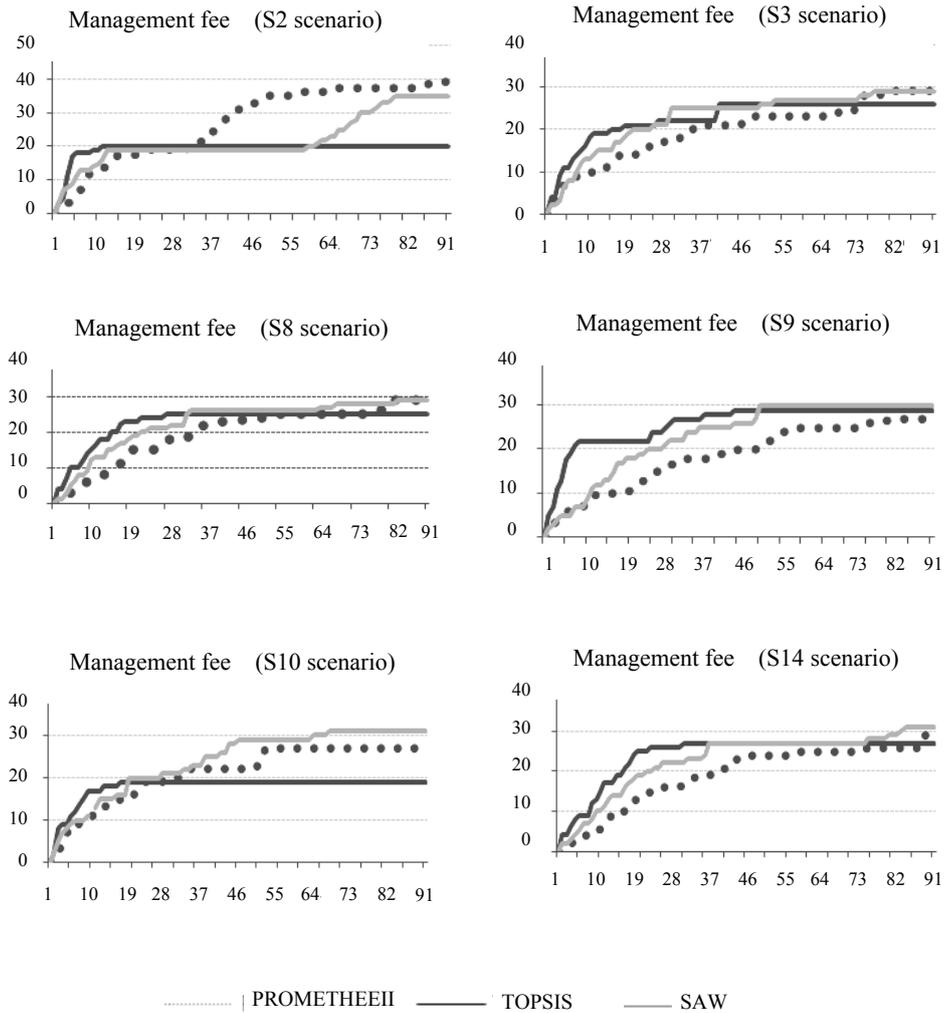

Figure 4. Order of stability in all iterations for the methods compared and the *management fee* criterion

The similarity of the SAW and PROMETHEE II methods is confirmed by the evaluation of the r_s coefficients. The lowest values of the coefficients for rankings constructed on the basis of modified weights and initial rankings (depending on criteria) are presented in Table 4. Taking into consideration the results presented, it can be noticed that for most criteria the rankings are not prone to change substantially due to modifications of weights. In the case of the TOPSIS method the similarity measured in this way is the smallest.

Table 4

Minimum values of rank correlation coefficients for given methods

	PROMETHEE II	TOPSIS	SAW
Expected value	0.811	0.569	0.827
Standard deviation	0.804	0.473	0.794
Skewness	0.385	0.156	0.378
Kurtosis	0.798	0.665	0.804
Percentile 0.05	0.630	0.248	0.641
Minimum required value of the first input	0.954	0.817	0.986
Minimum required value of the next input	0.931	0.744	0.897
Management fee	0.798	0.657	0.810
Number of methods of placing orders	0.768	0.772	0.782

In the next step we assessed the stability of rankings using the approach presented in [10]. By modifying the weights in the same way as before we examined how many alternatives with highest ranks remain on their positions in the initial ranking. For the sake of simplicity, in this case we present only those modifications that occur in the initial iterations, because of the substantial changes of stability defined in this way as the number of iterations increase.

In the case when no changes in the ranking occur it can be noticed that almost all modifications lead to instability. The criteria *0.05 percentile* and *minimum required value of the first input* are the only exceptions, as in these cases in the majority of scenarios no shifts occurred in the initial iterations. In general, increasing values of these two criteria did not lead to shifts among highly classified alternatives.

With the exception of a few particular cases, similarities between the SAW and PROMETHEE II methods are noticeable, but they are not as distinct as in the case of the previously applied approach. The numbers of alternatives with highest evaluations that preserve their ranks in the chosen sample scenarios are compared in Table 5.

Number of alternatives with highest evaluations that preserve their ranks in 3 first iterations
for SAW and PROMETHEE II methods

		S5			S6			S7			S10			S12			S13		
		j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3
PROMETHEE II	C1	25	13	13	7	5	5	8	8	5	13	13	3	39	6	6	19	6	6
	C2	12	12	11	16	7	7	19	19	9	3	3	3	5	5	5	15	7	6
	C3	8	0	0	11	10	6	8	8	7	0	0	0	12	12	9	6	5	5
	C4	12	12	12	15	11	6	13	11	11	3	3	3	47	17	10	19	12	6
	C5	47	47	47	47	47	47	47	47	47	47	37	37	47	47	47	47	47	47
	C6	16	12	12	16	16	16	20	20	20	22	22	22	14	14	14	47	20	20
	C7	9	9	9	20	5	5	8	8	8	30	14	14	16	9	9	18	18	7
	C8	12	10	8	6	4	0	4	4	0	3	3	3	8	8	7	5	4	4
	C9	16	14	11	14	7	7	8	8	5	15	3	3	21	5	5	15	15	11
SAW	C1	10	10	10	15	5	5	9	6	6	14	1	1	47	7	6	20	6	6
	C2	10	10	10	10	8	5	9	9	9	1	1	1	6	6	6	7	7	5
	C3	8	0	0	11	10	10	9	9	8	0	0	0	12	12	12	6	6	5
	C4	13	13	9	15	13	13	12	12	12	1	1	1	47	6	6	7	7	7
	C5	38	38	38	47	43	43	38	17	17	47	47	23	47	44	44	47	36	36
	C6	15	15	15	15	15	14	17	17	17	47	20	20	13	13	14	16	16	16
	C7	9	9	9	13	13	9	17	17	17	19	19	16	21	7	6	7	7	7
	C8	9	8	8	8	8	4	10	6	6	1	1	1	12	10	5	5	5	5
	C9	10	10	10	10	10	5	9	7	5	14	1	1	14	13	13	16	7	7

The assessment of the stability of the methods considered determined by the means of this approach does not allow to draw conclusions. In some cases (see Tables 6-7) the results are marked only for a given set of criteria, whereas modifications of weights associated to other criteria lead to different outcomes. The results obtained in some scenarios (see scenarios S6 and S4) suggest that in rankings constructed using the TOPSIS method more alternatives with high evaluations remain in their positions when weights are modified. In the following tables we present results for scenarios with most regularities.

Table 6

Number of alternatives with highest evaluations that preserve their ranks in first iterations
for PROMETHEE II and TOPSIS methods

| | PROMETHEE II | | | | | | | | | TOPSIS | | | | | | | | |
| | S6 | | | S9 | | | S11 | | | S4 | | | S5 | | | S6 | | |
	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3
C1	7	5	5	8	8	5	6	6	6	13	11	11	13	12	12	14	14	14
C2	16	7	7	19	19	9	14	6	6	7	7	7	15	13	8	16	8	3
C3	11	10	6	8	8	7	12	8	5	9	3	3	3	3	3	16	10	8
C4	15	11	6	13	11	11	15	15	5	13	13	13	28	13	13	16	16	16
C5	47	47	47	47	47	47	47	47	47	22	17	17	11	11	11	28	19	19
C6	16	16	16	20	20	20	16	16	16	22	17	17	11	11	11	28	28	28
C7	20	5	5	8	8	8	8	8	8	22	22	22	12	2	2	32	10	10
C8	6	4	0	4	4	0	5	5	4	8	8	8	11	4	3	16	16	16
C9	14	7	7	8	8	5	13	6	6	13	13	12	15	11	11	19	14	14

Table 7

Number of alternatives with highest evaluations that preserve their ranks in first iterations values
in chosen scenarios for SAW method

| | S3 | | | S4 | | | S5 | | | S6 | | | S7 | | |
	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3	j=1	j=2	j=3
C1	9	6	6	11	6	6	10	10	10	15	5	5	9	6	6
C2	6	6	0	6	6	6	10	10	10	10	8	5	9	9	9
C3	11	11	4	21	0	0	8	0	0	11	10	10	9	9	8
C4	6	6	6	6	6	6	13	13	9	15	13	13	12	12	12
C5	47	47	47	29	29	29	38	38	38	47	43	43	38	17	17
C6	20	20	20	16	16	16	15	15	15	15	15	14	17	17	17
C7	6	6	6	6	6	6	9	9	9	13	13	9	17	17	17
C8	16	15	0	8	8	8	9	8	8	8	8	4	10	6	6
C9	9	6	6	11	6	6	10	10	10	10	10	5	9	7	5

Concluding remarks

We have compared the stability and similarity of rankings of open-end stock funds constructed by applying different multicriteria methods based on different stability and similarity measures.

The simulation results presented in the paper indicate a considerable similarity between the SAW and PROMETHEE II methods. Similarities reported in the earlier studies turn out to be independent of the values of weights. The rankings constructed by applying those two methods are similar with regard to rank correlation coefficient and order of stability measure.

Nonetheless, the stability assessment based on the approach suggested in [10] does not lead to clear conclusions. In some of the cases considered the similarity of the SAW and PROMETHEE II methods is still noticeable. However, they lack regularities and their results are more dependent on the values associated to weights. Moreover, the higher flexibility of the TOPSIS method is not noticeable in this case. It can suggest that shifts in open-end investment funds rankings obtained when applying the TOPSIS method occur among the alternatives in middle and low positions in the ranking.

The results presented in the paper imply that the SAW and PROMETHEE II methods with the Gaussian generalized criterion lead to similar rankings and respond to the modifications of the weighting vector in a very similar way. The rankings obtained by their application are also less dependent on the values of the weights than those constructed by the TOPSIS method.

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