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APPLICATION OF AN AHP-TYPE METHOD AT PORTFOLIO MANAGEMENT

Abstract

The article deals with an application of the methodology of Analytic Hierarchy process (AHP) and also with its newly developed modification named FVK to portfolio management. The method AHP was published already in 1980s whereas FVK is a newly created tool expanding application possibilities of the AHP. Both methods are based on the definitions of decision criteria and variants in a logical hierarchy. First, we decompose the decision problem analytically from the upper to the lowest level and then we perform a synthesis by evaluating the decision variants and eliciting the best one. Here, we apply this multi-criteria methodology to the problem of a portfolio manager making a decision when selecting the best possible instrument on the financial market. Using a case study we demonstrate how appropriate application of the above mentioned methods could show a clear way for finding a satisfactory solution of this problem.

Keywords

Multi-criteria decision making; Analytic hierarchy process; Portfolio management.

Introduction

In this paper we propose an application of the Analytic hierarchy process (AHP) to a partial task in portfolio management. Any portfolio manager who works in an asset management company deals with the problem of converting his portfolio into cash. There is a need to decide, which instrument available on the market is the best one to invest in. Of course, there are specific areas, e.g. law, contract, internal requirements (criteria), which make the problem difficult. This problem can be viewed as a multicriteria decision making (MCDM) problem and Analytic Hierarchy Process (AHP) seems to be a suitable method for solving it.

Apart from the classical portfolio selection based on the Markowitz theory, see for example [1], there are studies applying AHP to portfolio mix, see e.g. [10]. This paper represents an appropriate approach to e.g. a pension fund portfolio. This may be also a problem of pair-wise comparison, see e.g. [11]. Another application of AHP close to our approach can be found in [12]. In this paper we show that AHP results could be comparable to those obtained by mean-variance optimization. In fact our specific approach can be viewed as a development of the idea given in [10].

In [4] a new MCDM method is presented which, in some sense, is an extension of AHP allowing for using triangular fuzzy inputs and feedback between criteria. The result of our work should answer the question: whether the software tool FVK created in [4] can be used as an alternative to the well known SW Expert Choice (EC) for solving our portfolio problem.

Let us start with the basic characteristics of a decision making (DM) model. Any DM model should satisfy the following characteristics:

– should be easy to compose,
– should be intuitive (it is not always the case),
– should be flexible in all elements,
– should comply with common sense,
– should include instructions for compromise,
– should be comprehensible.

It is important that even a poor problem design (its mathematical model) brings useful insight into a detail of the problem. The logic of MCDM is based on the goal identification, elements incorporated and influencing the output. In the next stage we shall deal with the time horizon, scenarios and limiting factors, see [7].

Some studies on analytical thinking led to the development of such models in 1970s, see e.g. [5] and the references therein. It was at that time that the method for DM support called the AHP was developed. The author Thomas L. Saaty – an American professor – and his co-workers and successors found many applications for the method. For example, in everyday life (e.g. a new car purchase, a choice of carrier, and so on) or in decision making problems in society or institutions (general elections, marketing strategies, political decisions, project selection etc.) For more information, see [2] or ([5], [6]).

Since its inception, the AHP has become one of the most widely used tools for MCDM. The procedures of the AHP involve the following steps, see ([5], [6]):

- Define the problem, objectives and outcomes.
- Decompose the problem into a hierarchical structure with decision elements (criteria, detailed criteria and alternatives).
- Apply the pair-wise comparison method resulting in pair-wise comparison matrices.
- Apply the principal eigenvalue method to estimate the relative weights of the decision elements.
- Check the consistency of pair-wise comparison matrices to ensure that the judgments of decision makers are consistent.
- Aggregate the relative weights of decision elements to obtain an overall rating for the alternatives.

1. Description of AHP

Here, we consider a three-level hierarchical decision system: On the first level we consider a decision goal G; on the second level, n independent evaluation criteria: C1, C2, ..., Cn are considered such that $\sum_{i=1}^{n} w(C_i) = 1$, where $w(C_i)$ is a positive real number – the weight, usually interpreted as a relative importance of the criterion C_i subject to the goal G. On the third level, m alternatives (variants) of the decision outcomes $V_1, V_2, ..., V_m$ are considered; again $\sum_{r=1}^{m} w(V_r, C_i) = 1$, where $w(V_r, C_i)$ is a non negative number – the weight of the alternative V_r subject to the criterion C_i, $i = 1, 2, ..., n$. It is advantageous to put the above mentioned weights into a matrix form.

Let $\mathbf{W}_1$ be the $n\times1$ matrix (weighing vector of the criteria), i.e. $W_1 = \begin{bmatrix} w(C_1) \\ \vdots \\ w(C_n) \end{bmatrix}$, and $\mathbf{W_3}$ be $m\times n$ matrix:

$$W_3 = \begin{bmatrix} w(C_1,V_1) & \cdots & w(C_n,V_1) \\ \vdots & \cdots & \vdots \\ w(C_1,V_m) & \cdots & w(C_n,V_m) \end{bmatrix} \tag{1}$$

The columns of this matrix are evaluations of alternatives according to the given criteria. Moreover, in matrix $\mathbf{W}_3$ the sums of columns are assumed to be equal to one (this property is called stochasticity, for more details see [5]). The following matrix product

$$\mathbf{Z} = \mathbf{W}_3\mathbf{W}_1 \tag{2}$$

is an $m\times1$ matrix – the resulting vector of weights of the alternatives – – expressing the relative importance of the alternatives. From formula (2) we get the weights in the following way

$$z_j = \sum_{i=1}^{n} w(C_i)w(C_i,V_j)\,,\ j=1,2,\ldots,m. \tag{3}$$

The weights $w(C_i)$, and $w(C_i,V_j)$ will be denoted in the following text simply as w_k; they are obtained from the pair-wise comparison matrix. An element of the pair-wise comparison matrix serves as a relative evaluation element from the given hierarchy level to a given element from the dominant level. Each pair of elements is evaluated on a specific scale, see below. A starting point for the calculation of weights is a pair-wise comparison matrix $\boldsymbol{S} = \{s_{ij}\}$. The value s_{ij} expresses the relative importance of elements x_i to element x_j, with respect to the superior element, in other words the ratio of w_i and w_j:

$$s_{ij} = \frac{w_i}{w_j}\,,\ i,j = 1,2,...,m. \tag{4}$$

As the weights w_k are not known in advance, (it is our goal to find them), we use for their determination additional information about the numbers s_{ij}, from the basic scale {1,2,...,9}, i.e.

$$s_{ij} \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}. \tag{5}$$

It follows from (4) that the pair-wise comparison matrix $\boldsymbol{S}$ is reciprocal, which means that

$$s_{ij} = \frac{1}{s_{ji}}\,. \tag{6}$$

In AHP, the vector $\boldsymbol{w}$ of weights w_k is calculated by a specific method based on the principal eigenvector of the pair-wise comparison matrix $\boldsymbol{S} = \{s_{ij}\}$. The following equation holds:

$$\boldsymbol{S}\,\boldsymbol{w} = \lambda_{\max}\,\boldsymbol{w}\,, \tag{7}$$

where $\lambda_{\max}$ is the maximal eigenvalue of matrix $\boldsymbol{S}$.

The values (called also intensities) from 1 to 9 in the evaluation scale (5) which are used in pair-wise comparisons can be interpreted qualitatively as follows:

Pair-wise comparison of elements x_i and x_j – number scale	Intensity of relative importance of element x_i to element x_j – word scale
1	x_i and x_j are equally important
3	x_i is more important than x_j
5	x_i is strongly more important than x_j
7	x_i is very strongly more important than x_j
9	x_i is absolutely more important than x_j

The numbers 2, 4, 6, 8 and their reciprocals are used to facilitate a compromise between slightly different judgments. Some authors also use rational numbers to form ratios from the above scale values, see [3] or [4].

2. Application to Portfolio Management – Case Study

The main task of a portfolio manager is asset allocation, that is, the selection of new assets for a new investment. Moreover, the portfolio manager has to make predictions about the price development of each asset class and, consequently, sell some of his positions and make new investments. The trickiest part of his work is to close some losing positions. It may happen when the loss reaches a specified value, which is not bearable for the owner of the portfolio any more. This is called realization of Stop-Losses. By the word "trickiest" we mean the effect given by cutting off any recovery possibility of the price.

Nevertheless, the main motivation for portfolio management is a possibility of its diversification. Financial instruments are divided into several categories, i.g. cash, bonds, equities and others. The prices movements at asset allocation could take different directions, or, they do not have the same drift, which is reflected by correlation. There are other possible diversification styles: we distinguish credit, geographic, currency and other diversification styles depending on different characteristics of the issuer, see e.g. [1].

Since in portfolio management it is necessary to make daily decisions concerning substitution of matured instruments for some new allocations, it may be useful to apply the AHP. Here, we illustrate the application of this MCDM technique on the following practical problem.

In Table 1 we consider four instruments, which are available for sale on the financial market:

Table 1

Financial instruments

ISIN	Name	1. volatility	2. rating	3. duration	4. liquidity
CZ0002000219	Ceskomoravska Hypotecni Bank	0,03	A	0,8491	low
XS0212596240	Deutsche Bank AG	0,05	AA	0,0381	good
XS0215579946	Tesco PLC	0,08	A	1,0991	worse
CZ0001000863	Czech Republic Government Bond	0,01	A	0,4916	the best

Source: Authors.

Table 1 contains preselected instruments (bonds), considered by a portfolio manager for his investment activity. For all financial instruments we consider some characteristics – evaluation criteria. In particular, we consider 4 evaluation criteria: Crit1 – volatility, Crit2 – rating, Crit3 – duration and Crit4 – liquidity.

Volatility is one of the most popular characteristic of a financial instrument. Sometimes it is considered as a risk. We can simply say: the more volatile the price of some instrument, the higher the risk of loss. Some conservative models consider equity of volatility at the level of 30%. Bond prices have lower volatility which is given by the fact that the investment in such an instrument is not risky, of course, from the point of view of volatility. A usual expected volatility level of bonds is between 0% and 10%. Moreover, the bonds are in fact the right to get back the money invested – nominal value plus the coupon, which is usually paid through the life of the bond.

Here, we use a well known historical approach for volatility calculation. First, we calculate the changes of asset returns by the formula:

$$R_{i,t} = P_{i,t} \frac{-P_{i,t\text{-}1}}{P_{i,t\text{-}1}} = \frac{P_{i,t}}{P_{i,t\text{-}1}} - 1.$$

Next, the expected value of returns is calculated by the following formula:

$$E(R_i) = \frac{1}{N}\sum_{t=1}^{N} R_{i,t} .$$

The sample variance of returns is calculated as follows:

$$\sigma_i^2 = \frac{1}{N-1} \cdot \sum_{t=1}^{N} \left[R_{i,t} - E(R_i)\right]^2 ,$$

and the sample standard deviation of returns is calculated as:

$$\sigma_i = \sqrt{\sigma_i^2}$$

This is considered as the volatility (risk). Here, historical prices are used; however, there exist elaborated models for volatility prediction, e.g. Vasicek´s model, EWMA model or GARCH models, see [9].

The second criterion is *rating* of a given issuer or issue. Here, we use the rating format given by Moody's scale in a simplified form without increasing signs (+) and decreasing signs (-). A higher number of A-symbols indicates more positive information about the credit profile of issuer. On the lower levels of the scale, instead of symbols A, symbols B and C can be used, but issuers or issues rated under BBB are considered as speculative investments.

The third criterion is *duration*. The bond price function $f(x)$ is approximated by the *Taylor's expansion*. The first member of this expansion is called the duration, i.e.:

$$f(x + \Delta x) = f(x) + f^{'}(x) \cdot \Delta x \, .$$

The price after certain time is calculated with help of the first member of Taylor's expansion in the following way:

$$P_1 = P(y + \Delta y) = P(y) + P'(y) \cdot \Delta y$$

where y is a yield to maturity, P is a price at the beginning of time period and P_1 is a price of bond after the change of interest rates.

Modifying the equation by subtraction and division of the starting price P we get:

$$P_1 \frac{-P}{P} = \frac{\Delta P}{P} = \frac{1}{P} \cdot \frac{dP}{dy} \Delta y,$$

The right side of the equation

$$\frac{1}{P} \cdot \frac{dP}{dy} = \frac{1}{P} \cdot \sum -t \cdot CF_t (1+y)^{-t-1} = -MD$$

is called the modified duration, where CF_t is the expected cash flow an owner of the bond will receive till the maturity of the bond. The negative sign of MD is a reflection of the reverse relationship between the yield curve represented here by y and the price of the bond.

The modified duration is expressed by the Macauloy's duration as follows:

$$D = \frac{\frac{dP}{P}}{\frac{dy}{(1+y)}} = \frac{1}{P} \cdot \sum t \cdot CF_t (1+y)^{-t} ,$$

and, consequently, we obtain:

$$MD = \frac{1}{1+y} D .$$

The above formulae show that the results reflect the cash flows weighted by time. The bonds, which do not pay coupons, have the duration equal to their time to maturity. Portfolio managers usually use the second expressed duration, which is a MD with the positive sign, because they consider this number as an average time to maturity of their portfolio. The MD is the parameter of a portfolio, which is usually requested by contract and must be watched after.

The fourth criterion is *liquidity*. Here, the empirical approach is used: In Table 1 the relative evaluation is carried out by pair-wise comparison.

2.1. Solving the Problem by AHP and Expert Choice

Now, we shall solve the problem by the special SW tool named Expert Choice (EC), see [13], based on the AHP theory. The original data of our problem are given in Table 1. For evaluating the liquidity criterion which is given in ordinal expressions as well as the other qualitative criterion rating we use pair-wise comparison on the Saaty's scale mentioned earlier in Section 1. We proceed simirarily for evaluating relative importance of all individual criteria. Table 2 shows the pair-wise comparison matrix of the criteria importance given by a portfolio manager.

Table 2

Pair-wise comparison matrix of importance of the individual criteria

Criteria	**Crit 1**	**Crit 2**	**Crit 3**	**Crit 4**	
Crit 1	1	1/3	1/2	1/2	**- volatility**
Crit 2	3	1	3	2	**- rating**
Crit 3	2	1/3	1	2	**- duration**
Crit 4	2	1/2	1/2	1	**- liquidity**

Source: Authors.

Table 3 contains the weights of criteria calculated by the well known eigenvector method mentioned earlier, see Eq. (7). It is clear that rating and duration are the most important criteria.

Table 3

Relative importance of the criteria obtained by pair-wise comparison

Criteria	Weights
Volatility	0,079
Rating	0,526
Duration	0,246
Liquidity	0,149

Source: Authors.

Table 4 shows the pair-wise comparison matrix of liquidity.

The values of the other criteria are calculated explicitly from the original data in Table 1.

Table 4

Pair-wise comparison matrix of Liquidity

Zn=	Var 1	Var 2	Var 3	Var 4	
Var 1	1	1/5	1/3	1/4	**- Ceskomoravska hypotecni banka**
Var 2	5	1	2	1/2	**- Deutsche Bank AG**
Var 3	3	1/2	1	1/2	**- Tesco PLC**
Var 4	4	2	2	1	**- Czech Republic Government bond**

Source: Authors.

Table 5 shows the result of calculation of each variant and criterion in the final, normalized form, i.e. the sum of all numbers in each column is equal to 1.

Table 5

Weights of criteria and weights of variants

Variant	Volatility	Rating	Duration	Liquidity
V1	0,201	0,200	0,039	0,088
V2	0,121	0,400	0,864	0,197
V3	0,075	0,200	0,030	0,231
V4	0,603	0,200	0,067	0,484

Source: Authors.

Table 6 shows the result of the final synthesis calculated as weighting average (3) using both the calculation method called the Distributive mode and the calculation method called the Ideal mode. In the Distributive mode, all values of each criterion (i.e. in each column) are normalized, i.e. divided by the sum of the values of the respective criterion, see Table 5, whereas in the Ideal mode, all values of each criterion (i.e. in each column) are divided by the maximal value of the respective criterion, i.e. the highest value of each criterion is then equal to 1. In both modes the resulting ranking of the variants is identical. For more details, see [5].

Table 6

Final synthesis by AHP

Distributive mode	Weights	Rank	Ideal mode	Weights	Rank
V1	0,144	4	V1	0,161	4
V2	0,462	1	V2	0,416	1
V3	0,153	3	V3	0,173	3
V4	0,242	2	V4	0,250	2

Source: Authors.

Summarizing the results in Table 6, we can see a clear dominance of variant V2 over all other variants. Variant V4, which is ranked as the second best, has significantly lower weight. The weights of V1 and V3 are very similar to each other, and significantly lower than V4. Consequently, the best choice from the given variants is V2, hence available cash should be invested into variant V2.

2.2. Solving the Problem by FVK

In this part we solve the same problem as in section 2.1 by an alternative method. The AHP method was published as early as in 1980s, and now it is considered a "classical" methodology; on the other hand, FVK is a newly created tool expanding application possibilities of the AHP. The acronym FVK stands for Fuzzy Multicriteria Method (in Czech language). Here, we compare and discuss the results obtained by both methods.

When comparing the AHP and FVK we find out some significant differences:

- In FVK the vector of weights w_k is calculated from the pair-wise comparison matrix $\boldsymbol{S} = \{s_{ij}\}$ by the geometric mean as follows:

$$w_k = \frac{\left(\prod_{j=1}^{n} s_{kj}\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} s_{ij}\right)^{1/n}} \tag{8}$$

- FVK reduces some disadvantages of the principal eigenvector method used in AHP (see [5]),
- FVK allows for reflecting criteria interdependency, which is not considered in the classical AHP.
- FVK enables to use fuzzy evaluations, specifically by triangular fuzzy numbers (i.e. triangular membership functions). Hence, FVK is convenient in situations, where the decision maker has vague information for evaluation (here we will not use this feature).

All results presented below have been calculated by a software tool named FVK. This software application has been created as an add-on for MS Excel 2003 within the GACR project No. 402060431, see [3].

Table 7 shows the criteria weights calculated by (8); they are calculated from pair-wise comparison matrix in Table 2. As compared with Table 3 the weights in Table 7 are different; however, the order of the importance of the criteria is the same.

Table 7

Weights of criteria by FVK

Criteria	Weights
Volatility	0,119065
Rating	0,456456
Duration	0,238131
Liquidity	0,186347

Source: Authors.

Table 8 shows the final weights of the variants and the ranking according to FVK. Again, the best variant is V2; however, the variants on the third and the fourth place interchanged their positions.

Table 8

Final synthesis by FVK

Zn=	Weights	Rank
Var 1	0,176003	3
Var 2	0,396322	1
Var 3	0,127531	4
Var 4	0,300144	2

Source: Authors.

In the AHP we assume that the decision criteria are mutually independent. In practice, it is, however, not the case. Generally, the criteria are frequently interdependent, one criterion directly or indirectly influences the other one, e.g. rating strongly influences liquidity etc. On the other hand, FVK enables also to reflect influences between the criteria, which enables a deeper analysis of convenient alternatives. The influences (interdependences) between the criteria are evaluated also by pair-wise comparison,

The values in the pair-wise comparison matrix evaluating the influences between Crit 1 and other criteria (see Table 9) can be interpreted as follows: Crit 2 influences Crit 1 two times (2) more than Crit 3. Crit 2 influences Crit 1 four times (4) more than Crit 4. Crit 3 influences Crit 1 three times (3) more than Crit 4, etc.

Table 9

Pair-wise comparison matrix (influences between volatility and other criteria)

Crit 1	Crit 2	Crit 3	Crit 4	
Crit 2	1	2	4	- rating
Crit 3	1/2	1	3	- duration
Crit 4	1/4	1/3	1	- liquidity

Source: Authors.

In Table 10 influences of Crit 2 – Rating by other criteria is presented:

Table 10

Pair-wise comparison matrix (influences between rating and other criteria)

Crit 2	Crit 1	Crit 3	Crit 4	
Crit 1	1	2	3	- volatility
Crit 3	1/2	1	1	- duration
Crit 4	1/3	1	1	- liquidity

Source: Authors.

In Table 11 influences of Crit 3 – Duration by other criteria is presented:

Table 11

Pair-wise comparison matrix (influences between duration and other criteria)

Crit 3	Crit 1	Crit 2	Crit 4	
Crit 1	1	1	1	- volatility
Crit 2	1	1	1	- rating
Crit 4	1	1	1	- liquidity

Source: Authors.

In Table 12 influences of Crit 4 – Liquidity by other criteria is presented:

Table 12

Pair-wise comparison matrix (influences between liquidity and other criteria)

Crit 4	Crit 1	Crit 2	Crit 3	
Crit 1	1	1/2	2	- volatility
Crit 2	2	1	5	- rating
Crit 3	1/2	1/5	1	- duration

Source: Authors.

In the last table Table 13 the final weights and the corresponding ranking of the variants is presented. In comparison to the previous case, the weights of the criteria are calculated by FVK, particularly by the method of geometric mean taking into account interdependences (infuences) between the criteria, see [3,4].

Table 13

Final evaluation of variants according FVK

Zn=	Weights	Rank
Var 1	0,192401	3
Var 2	0,371611	1
Var 3	0,111567	4
Var 4	0,324421	2

Source: Authors.

When comparing the results obtained by FVK with those obtained earlier by AHP we conclude: The best variant is again Variant 2 and the second-best is again Variant 4. However, Variant 1, ranked in the case of AHP as the fourth, is now located on the third place. In this particular example, from the viewpoint of the investor, who is focused on the top variants, both AHP and FVK supply equivalent results. In general, we should, however, be careful as the results obtained by these methods could be different, particularly in case of strong interdependences between criteria.

Conclusion

In this paper we tried to show that an application of MCDM methods in portfolio management may be useful. Here, we applied the classical Saaty's AHP and, at the same time, the newly developed modification of AHP named FVK extending the application power of AHP as well as reducing some of its theoretical shortages.

In the AHP we assume that the decision criteria are mutually independent; however, it is usually not the case. Generally, the criteria are interdependent: one criterion either directly or indirectly influences the other one. The new method, FVK, enables also to reflect influences between the criteria, which enables a deeper analysis of all convenient alternatives. The influences (interdependences) between the criteria are evaluated also by pair-wise comparison.

A comparison of the results obtained by FVK with those obtained earlier by AHP, in this particular application, shows that from the viewpoint of the investor, both methods give more or less equivalent results. In general, we should, however, be careful as the results obtained by these methods could differ, particularly in case of strong interdependences between criteria.

By the help of MCDM methods, the portfolio manager is able to acquire quick information (feedback) about advantages of the asset allocation into some specific product. Consequently, every specific requirement of a contract can be reflected by the methods applied. For example, liquidity evaluation could be derived from the liquidity spread. On the one hand, this approach is much more dependent on input data; on the other hand, the suggested modification could increase the objectivity of the model. Further extensions could be made by the implementation of ex-ante volatility, see [8]. Moreover, the rating inputs taken from the external rating agencies could be derived also from the rating models developed within the project BASEL II.

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