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MULTICRITERIAL EXAMINATION TIMETABLING WITH UNCERTAIN INFORMATION

Abstract

We consider examination timetabling at a university. This problem has been widely treated in the literature (e.g. [1]. [8], [9]); however, we propose a new approach, which belongs to the family of robust approaches. The main obvious assumption is that two examination sessions sharing at least one student cannot be scheduled at the same time. This scheduling problem will be stated as a graph coloring problem. The stability of the solution scheduled is desirable in the sense that it remains valid also when, unexpectedly, some additional students want to take the exams, for example those who failed in earlier examination sessions. This stability is defined as the robustness of examination scheduling. In [6], [10] a probabilistic robustness measure has been proposed. We propose a fuzzy approach, similarly as in [3]. We consider three different schedule robustness measures: mean value of the fuzzy number of examination conflicts considered in [3], and two new measures, put forward in this paper: the cardinality of the fuzzy number of session conflicts and the possibility that the fuzzy number of session conflicts is 0. We also consider a multicriterial approach with the minimization of the examination session days and the maximization of schedule robustness.

Keywords

Scheduling, timetabling, fuzzy number, coloring graph, robustness.

Introduction

In the paper we will use the following notions.

A fuzzy number (set) in space $\Re$, denoted by a capital letter with a tilde (e.g. $\widetilde{A}$), is defined as a set of pairs $\{(\mu_A(x), x)\}$, where $x \in \Re$ and $0 \le \mu_A(x) \le 1$ is a membership function describing to what degree $\widetilde{A}$ equals x [7], [11].

We will use discrete normal fuzzy numbers (e. g. fuzzy numbers defined on $N \cup \{0\} \subset \Re$ for which $\mu_A(x) = 1$ at least for one x). For discrete fuzzy numbers we will use the notation

$$\widetilde{A}_i = x_1 / \mu_{A_i}(x_1) + x_2 \mu_{A_i}(x_2) + \ldots + x_n \mu_{A_i}(x_n)$$

where $\{x_1, .., x_n\}$ are those numbers from $N \cup \{0\} \subset \Re$ for which the membership function takes a non-zero value.

Addition of two fuzzy numbers $\widetilde{A}$, $\widetilde{B}$ can be defined as follows:

$$\mu_{A+B}(z) = \sup_{z=x+y} \min(\mu_A(x), \mu_B(y)) \tag{1}$$

and scalar multiplication of a fuzzy number and scalar addition to a fuzzy number as:

$$\mu_{rA}(z) = \sup_{z=rx} \mu_A(rx) \tag{2}$$

$$\mu_{r+A}(z) = \sup_{z=r+x} \mu_A(rx).$$

According to these definitions, if $r = 0$ then $r\widetilde{A}$ is a crisp number equal to 0, $r + \widetilde{A} = \widetilde{A}$, and if $r = 1$ then $r\widetilde{A} = \widetilde{A}$.

The membership function $\mu_A(x)$ of the fuzzy number $\widetilde{A} = \sum_{i=1}^{n} \widetilde{A}_i$ of the sum of *n* normal discrete fuzzy numbers $\widetilde{A}_i = 0/\mu_{A_i}(0) + 1/\mu_{A_i}(1)$ takes on the form [5]:

$$\widetilde{A} = \sum_{i=1}^{n} \widetilde{A}_i = \sum_{k=0}^{n} k / \mu_A(k) = \sum_{k=0}^{n} k / \min(\lambda_k, \Lambda_k) \tag{3}$$

where:

$\lambda_0, \lambda_1, \ldots, \lambda_n$ – $\mu_{A_i}(0)$ are sorted in a non-decreasing way, with the number 1 at the end of the sequence,

$\Lambda_0, \Lambda_1, \ldots, \Lambda_n$ – $\mu_{A_i}(1)$ are sorted in a non-increasing way, with the number 1 at the beginning of the sequence.

From (3) it follows that

$$\mu_A(0) = \min_{k=0,1,\dots,n}(\lambda_k) = \lambda_0 \tag{4}$$

Now we assume that the normal fuzzy numbers $\widetilde{A}_i = 0/\mu_{A_i}(0) + 1/\mu_{A_i}(1)$ are ordered in the following way:

$$\begin{cases} \mu_{A_1}(0) \le \mu_{A_2}(0) \ldots \le \mu_{An}(0), \\ \mu_{A_1}(1) \mu_{A_2}(1) \ldots \ge \mu_{An}(1), \end{cases}$$

where for n_1 fuzzy numbers $\mu_{A_i}(0)<1$, for n_2 fuzzy numbers $\mu_{A_i}(1) < 1$ and for $n - n_1 - n_2$ fuzzy numbers $\mu_{A_i}(0) = \mu_{A_i}(1) = 1$. In this case the fuzzy number $\widetilde{A} = \sum_{i=1}^{n} \widetilde{A}_i$ can be expressed as:

$$\widetilde{A} = \sum_{i=1}^{n} \widetilde{A}_i = \sum_{i=0}^{n_1 - 1} i/\mu_{A_{i+1}}(0) + \sum_{n_1}^{n-n_2-1} i/1 + \sum_{i=n-n_2}^{n} i/\mu_{A_i}(1) \tag{5}$$

In the literature there are many definitions of the cardinality of a fuzzy set. We will use the following one [7]: the cardinality of a fuzzy set $A = x_1/\mu_A(x_1) + x_2/\mu_A(x_2) + \ldots + x_n/\mu_A(x_n)$, denoted by $Card(A)$, is the real number equal to

$$Card(A) = \sum_{i=1}^{n} \mu_A(x_i). \tag{6}$$

The cardinality of the fuzzy set defined by (3) takes the form:

$$\begin{aligned} Card(A) &= \sum_{i=1}^{n} \mu_A(x_i) = \sum_{i=1}^{n_1} \mu_{A_i}(0) + (n - n_1 - n_2 + 1) \cdot 1 + \sum_{i=n-n_2+1}^{n} \mu_{A_i}(1) = \\ &= \sum_{i=1}^{n} \mu_{A_i}(0) + \sum_{i=1}^{n} \mu_{A_i}(1) - (n-1) = 1 + \sum_{i=1}^{n} \left(\mu_{A_i}(0) + \mu_{A_i}(1) - 1 \right). \end{aligned} \tag{7}$$

The mean value of a normal fuzzy set $A = 0/\mu_A(0) + 1/\mu_A(1)$ equals [2], [5]:

$$E(A) = \frac{1}{2}\left(1 + \mu_A(1) - \mu_A(0)\right) \tag{8}$$

By the linearity of the mean, the mean of the weighted sum of fuzzy sets equals:

$$E\left(\sum_{i=1}^{n} w_i A_i\right) = \sum_{i=1}^{n} w_i E(A_i) \tag{9}$$

1. Robust schedules

We consider examination timetabling at a university. We assume that there are so called "standard students" and "non standard students". Standard students are these who study according to the basic study program and non standard ones are those who repeat examinations or have an individual study program. Two examinations sharing at least one standard student cannot be scheduled on the same day. Taking into account examination conflicts, we construct the graph $G = (V, E)$, whose vertices $V = \{1,2,...,m\}$ represent the examinations *E1, E2,…, Em* and whose edges (i, j) are included in the edge set *E* if the examinations *Ei* and *Ej* share at least one standard student.

In order to schedule examinations in no more than *C* days, a *C*-colouring problem can be stated. We can construct an examination schedule by solving the following binary linear model:

$$\begin{aligned} &\sum_{c=1}^{C} x_{ic} = 1 \quad for \quad i \in V, \\ &x_{ic} + x_{jc} \le 1 \quad for \quad (i, j) \in E, \quad c = 1,2,...,C, \\ &x_{ij} = 0,1. \end{aligned} \tag{10}$$

Moreover, the stability of a schedule found by solving the problem (10) is desirable in the sense that it remains valid also when non standard students examination conflicts are taken into account. Let $G' = (V, E')$ be the complementary graph set, where the set of edges $E' = (V \times V) \setminus (E \cup I)$ (where: $I = \{(i, j) \in G : i = j\}$) represent all non standard students examination conflicts. In [10] the authors consider a the validity of a schedule found by solving the problem (10) taking as a measure the probability that such solution remains valid after random complementary edges from E' have been added to the edge set *E*. The validity of the solution of the problem (10) taking into account the non standard students examination conflicts can be defined as the robustness of this solution. In [3] the mean of the non standard students examination conflicts (a fuzzy number) was assumed to be the robustness measure of (10).

Further we assume that for each edge (i, j) in *E'* the fuzzy number

$$\widetilde{A}_{(i,j)} = 0/\mu_{A_{(i,j)}}(0) + 1/\mu_{A_{(i,j)}}(1)$$

determining whether students who take the exams *Ei*, *Ej* will turn up, is known. This fuzzy number is 1 if the examinations *Ei* and *Ej* have at least one non standard student in common and 0 if they have no non-standard students in common. Let $\widetilde{A}$ be the fuzzy number determining the number of non standard students examination conflicts.

Let us now consider two measures (criteria) of the robustness of the session schedule for exams taken by non-standard students. One of them will be the possibility that the fuzzy number of non standard students examination conflicts $\widetilde{A}$ is 0 – $\mu_A(0)$. We can construct the best robust solution solving the following binary linear programming model. According to (3) and (4) the model takes the following form:

$$m_o \to \max$$

subject to:

$$\begin{aligned}
&\sum_{c=1}^{C} x_{ic} = 1 \quad \text{for} \quad i \in V, \\
&x_{ic} + x_{jc} \le 1 \quad \text{for} \quad (i, j) \in E, \quad c = 1,2,...,C, \\
&z_c \le \sum_{i=1}^{n} x_{ic} \le M z_c \quad \text{for} \quad c = 1,2,...,C, \\
&2y_{ij} \le x_{ic} + x_{jc} \le 1 + y_{ij} \quad \text{for} \quad (i, j) \in E', \quad c = 1,2,...,C, \\
&m_0 \le 1 - y_{ij} + \mu_{A_{(i,j)}}(0) y_{ij} \quad \text{for} \quad (i, j) \in E', \\
&m_0 \le 1
\end{aligned} \tag{11}$$

$x_{ic} = 0,1$, for i = 1,2,…,n, c = 1,2,…,C,

$y_{ij} = 0,1$ $(i, j) \in E'$,

$z_c = 0,1$ for c = 1,2,…,C.

$m_o \in \Re^+$,

M – a big number

where:

$$x_{ic}=\begin{cases}1 & if \quad vertex \quad i \quad is \quad colored \quad c,\\ 0 & if \quad vertex \quad i \quad isn't \quad colored \quad c,\end{cases}$$

$$y_{ij}=\begin{cases}1 & if \quad vertices \quad i,j \quad are \quad equally \quad colored,\\ 0 & if \quad vertices \quad i,j \quad aren't \quad equally \quad colored,\end{cases}$$

$$z_c=\begin{cases}1 & if \quad color \quad c \quad is \quad used,\\ 0 & if \quad color \quad c \quad isn't \quad used,\end{cases}$$

In the model the first and the second conditions ensure that the vertices connected with an arc are colored with different colors. The third condition expresses the requirement that if a color is used, then at least one vertex has to be colored with it (if the day c has been taken into account in the session schedule, then at least one exam has to be scheduled on that day. The fourth condition sets y_{ij} equal to 1 if the exams E_i and E_j for non standard students conflict for the given schedule. The fifth and the sixth condition together with the maximization of m_o define $m_0=\mu_A(0)=\min_{k=1,\dots,n}(\lambda_k)$.

Another robustness measure of the solution of (10) is the cardinality of the fuzzy set of non-standard students examination conflicts. Taking into account (7), the 0-1 linear model determining the most robust solution with this criterion takes on the following form:

$$1+\sum_{(i,j)\in E'}\left(\mu_{A_{(i,j)}}(1)+\mu_{A_{(i,j)}}(0)-1\right)y_{ij}\rightarrow \min$$

subject to:

$$
\begin{aligned}
&\sum_{c=1}^{C} x_{ic} = 1 \quad \text{for} \quad i \in V, \\
&x_{ic} + x_{jc} \le 1 \quad \text{for} \quad (i,j) \in E, \quad c = 1,2,...,C, \\
&z_c \le \sum_{i=1}^{n} x_{ic} \le M z_c \quad \text{for} \quad c = 1,2,...,C, \\
&2y_{ij} \le x_{ic} + x_{jc} \le 1 + y_{ij} \quad \text{for} \quad (i,j) \in E', \quad c = 1,2,...,C, \\
&x_{ic} = 0,1 \quad , \quad \text{for } i = 1,2,\ldots,n, \ c = 1,2,\ldots,C, \\
&y_{ij} = 0,1 \qquad (i,j) \in E', \\
&z_c = 0,1 \qquad \text{for } c = 1,2,\ldots,C.
\end{aligned}
\tag{12}
$$

M – a big number

where:

$$
x_{ic} = \begin{cases} 1 & \textit{if vertex } i \textit{ is colored } c, \\ 0 & \textit{if vertex } i \textit{ isn't colored } c, \end{cases}
$$

$$
y_{ij} = \begin{cases} 1 & \textit{if vertices } i,j \textit{ are equally colored}, \\ 0 & \textit{if vertices } i,j \textit{ aren't equally colored}, \end{cases}
$$

$$
z_c = \begin{cases} 1 & \textit{if color } c \textit{ is used}, \\ 0 & \textit{if vertex } c \textit{ isn't used}, \end{cases}
$$

1.1. Example

The examinations for seven courses *E1*, *E2*, *E3*, *E4*, *E5*, *E6*, *E7* for a particular program at the university must be scheduled within no more than C = 5 days. The graph $G = (V, E)$ for the standard students examination conflicts is shown in Figure 1. The vertices $V = \{1, 2, 3, 4, 5, 6, 7\}$ represent the examination numbers. If the examinations *Ei* and *Ej* share at least one standard student, the edge (i, j) has been included in graph *G*. The adjacency matrix is:

$$M_G = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ & 0 & 1 & 1 & 0 & 0 & 1 \\ & & 0 & 0 & 1 & 0 & 0 \\ & & & 0 & 1 & 1 & 1 \\ & & & & 0 & 1 & 0 \\ & & & & & 0 & 0 \\ & & & & & & 0 \end{pmatrix}$$

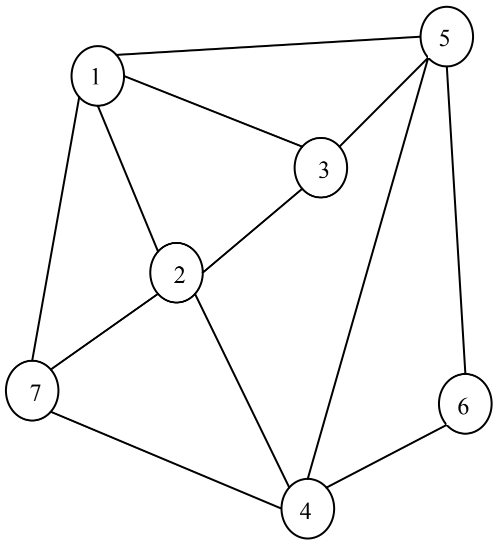

Figure 1. The graph G = (V,E) for the standard students examination conflicts

The matrix of the complementary graph $G' = (V, E')$, where a set of edges $E' = (V \times V) \setminus E \setminus \cup I$ represent all possible non standard students examination conflicts is:

$$M_{G'} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 1 & 1 & 0 \\ & & 0 & 1 & 0 & 1 & 1 \\ & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 1 \\ & & & & & 0 & 1 \\ & & & & & & 0 \end{pmatrix}$$

Using information from past years, we determine the possibility of an examination conflict for non-standard students:

$$\widetilde{A}_{G'} = \begin{pmatrix} 0 & 0 & 0 & \widetilde{A}_{(1,4)} & 0 & \widetilde{A}_{(1,6)} & 0 \\ & 0 & 0 & 0 & \widetilde{A}_{(2,5)} & \widetilde{A}_{(2,6)} & 0 \\ & & 0 & \widetilde{A}_{(3,4)} & 0 & \widetilde{A}_{(3,6)} & \widetilde{A}_{(3,7)} \\ & & & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & \widetilde{A}_{(5,7)} \\ & & & & & 0 & \widetilde{A}_{(6,7)} \\ & & & & & & 0 \end{pmatrix}$$

In the above matrix, $\widetilde{A}_{(i,j)}$ is a fuzzy number equal to 1 if the examinations Ei and Ej have at least one non standard student in common and 0 if the examinations Ei and Ej have no non-standard students in common. These fuzzy numbers take the following forms:

$\widetilde{A}_{(1,4)} = 0/1 + 1/0{,}6$ $\widetilde{A}_{(1,6)} = 0/1 + 1/0{,}3$ $\widetilde{A}_{(2,5)} = 0/0{,}6 + 1/1$

$\widetilde{A}_{(2,6)} = 0/0{,}8 + 1/1$ $\widetilde{A}_{(3,4)} = 0/1 + 1/0{,}5$ $\widetilde{A}_{(3,6)} = 0/0{,}4 + 1/1$

$\widetilde{A}_{(3,7)} = 0/1 + 1/0{,}7$ $\widetilde{A}_{(5,7)} = 0/0{,}1 + 1/1$

$\widetilde{A}_{(6,7)} = 0/0{,}1 + 1/0{,}4$.

The most robust schedule according to the model (11) with the robustness criterion $\mu_A(0) \to \max$ is the following one:

Day 1: *E3*, *E4*,
Day 2: *E7*,
Day 3: *E1*, *E6*,
Day 4: E5,
Day 5: *E2*.

The fuzzy number of the number of non standard students examination conflicts takes the form: $\tilde{A} = 0/1 + 1/0{,}5 + 2/0{,}3$, Figure 2a. For this solution $\mu_A(0) = 1$, while $Card(\tilde{A}) = 1{,}8$.

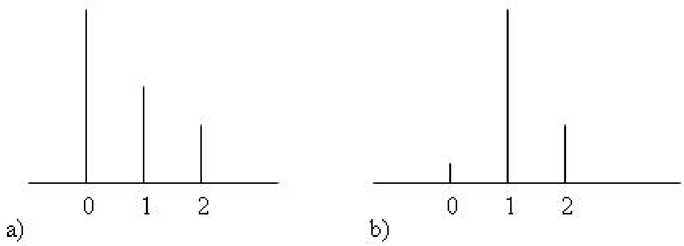

Figure 2. The form of the fuzzy number of the number of conflicts for examination schedule for one-criterial problems

And for the robustness criterion $Card(\tilde{A}) \to \min$ the session schedule according to the model (12) is as follows:

Day 1: *E5*, *E7*,
Day 2: *E3*,
Day 3: *E1*, *E6*,
Day 4: *E4*,
Day 5: *E2*,

and the fuzzy number of the number of non standard students examination conflicts takes on the form: $\tilde{A} = 0/0{,}1 + 1/1 + 2/0{,}3$, Figure 2b. For this solution we have $\mu_A(0) = 0{,}1$ and $Card(\tilde{A}) = 1{,}4$.

The schedules generated have high robustness according to one criterion and low robustness according to the other one.

2. A multicriteria approach

Let us consider a multicriteria optimization problem, where our goal is to construct a schedule with a small number of session days and at the same time one which would be as robust as possible. The following robustness criteria are assumed:

- The possibility that the non standard students examination conflicts are zero,
- The power of the fuzzy number of the non standard students examination conflicts,
- The mean value of the fuzzy number of the non standard students examination conflicts, (8) (9).

The corresponding multicriteria programming model takes on the following form

$$
\begin{aligned}
&F_1\left(x_{11},\ldots,x_{iC}, y_{ij} \in E', z_1,\ldots,z_C\right) = \sum_{c=1}^{C} z_c \to \min \\
&F_2\left(x_{11},\ldots,x_{iC}, y_{ij} \in E', z_1,\ldots,z_C\right) = m_o \to \max \\
&F_3\left(x_{11},\ldots,x_{iC}, y_{ij} \in E', z_1,\ldots,z_C\right) = 1 + \sum_{(i,j)\in E'} \left(\mu_{A(i,j)}(1) + \mu_{A(i,j)}(0) - 1\right) y_{ij} \to \min \\
&F_4\left(x_{11},\ldots,x_{iC}, y_{ij} \in E', z_1,\ldots,z_C\right) = \frac{1}{2} \sum_{(i,j)\in E'} \left(1 + \mu_{A(i,j)}(1) - \mu_{A(i,j)}(0)\right) y_{ij} \to \min
\end{aligned}
\tag{13}
$$

with the constraints of (11).

To solve (13), we suggest to use the method of objectives prioritizing.

2.1. Example

Let us consider the problem from section 2.1. We assume the following hierarchy of objectives: F_1, F_2, F_3, F_4.

We will get a schedule with the minimal number of session days by minimizing the function $F_1 = \sum_{c=1}^{C} z_c$ with the constraints of (13) with an additional one: $m_0 = 0$. This schedule is:

Day 1: *E2*, *E5*,

Day 2: *E1*, *E4*,

Day 3: *E3, E6*, *E7*.

The fuzzy number of the number of non standard students examination conflicts takes the form: $\widetilde{A} = 0/0{,}4 + 1/0{,}6 + 2/1 + 3/0{,}7 + 4/0{,}6 + 5/0{,}4$, Figure 3a. The robustness of this schedule according to the measures we consider is $F_1 = \mu_A(0) = 0{,}4$, $F_2 = Card(\widetilde{A}) = 3{,}7$, $F_2 = E(\widetilde{A}) = 1{,}95$.

The decision maker has decided that he can accept four session day (but not more) if this results in higher robustness. Adding to the constraints of (13) the constraint $\sum_{c=1}^{C} z_c \le 4$ we solve our problem by taking as the optimization criterion the second objective in the hierarchy $F_2 = m_o \to \max$. We get the following schedule:

Day 1: *E1*, *E4*,

Day 2: *E3*, *E7*,

Day 3: *E2*, *E6*,

Day 4: *E5*.

The fuzzy number of the number of non standard students examination conflicts takes the form: $\widetilde{A} = 0/0{,}8 + 1/1 + 2/1 + 3/0{,}7 + 4/0{,}6$, Figure 3b. The criteria take on the following values: $F_1 = \mu_A(0) = 0{,}8$, $F_2 = Card(\widetilde{A}) = 3{,}1$, $F_2 = E(\widetilde{A}) = 1{,}25$. It is a four-day schedule whose robustness is better from the point of view of all the robustness measures considered here.

Let us now assume that the decision maker has decided that he would be satisfied with a schedule for which the possibility that there are no non standard students examination conflicts is 0.6 (not less), but which has a smaller number of non standard students examination conflicts. We check whether it is possible to minimize the objective $F_3 = 1 + \sum_{(i,j)\in E'} \left(\mu_{A_{(i,j)}}(1) + \mu_{A_{(i,j)}}(0) - 1 \right) y_{ij}$ with the constraints from (13) with the two additional ones: $\sum_{c=1}^{C} z_c \leq 4$ and $m_o \geq 0{,}6$. The solution of this problem is:

Day 1: E2, E5,

Day 2: E1, E6,

Day 3: E3, E4,

Day 4: E7.

The fuzzy number of the number of non standard students examination conflicts takes the form: $\widetilde{A} = 0/0{,}6 + 1/1 + 2/1 + 3/0{,}5 + 4/0{,}3$, Figure 3c. The values of the criteria are: $F_1 = \mu_A(0) = 0{,}6$, $F_2 = Card(\widetilde{A}) = 2{,}4$, $F_2 = E(\widetilde{A}) = 1{,}1$.

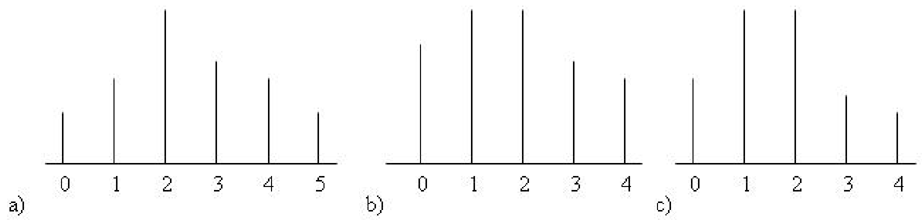

Figure 3. The form of the fuzzy number of the number of conflicts for examination schedule for multicriterial problem

In some cases it might be interesting to consider the fourth criterion in the hierarchy: $F_4 = \frac{1}{2} \sum_{(i,j)\in E'} \left(1 + \mu_{A_{(i,j)}}(1) - \mu_{A_{(i,j)}}(0) \right) y_{ij} \to \min$. For the example under consideration it did not bring anything new.

Conclusions

In the paper [10] the authors showed that the problem (11) is NP-Complete for the criterion function $F_0(y_{ij}) = \sum_{(i,j)\in E'} p_{ij} \cdot y_{ij} \to \min$ for non negative penalties p_{ij}. In our case the following inequality holds true: $0 \leq p'_{ij} = \left(\mu_{A_{(i,j)}}(1) + \mu_{A_{(i,j)}}(0) - 1 \right) \leq 1$. We can use the function

$f\left(p'_{ij}\right)=\frac{p_{ij}}{\max\left(p_{ij}\right)}$ as a polynomial mapping function which maps the problem with non negative penalties p_{ij} to the problem with penalties $0 \le p'_{ij} \le 1$. This implies that the fuzzy robust examination scheduling problem minimizing the power of the fuzzy number of the non standard students examination conflicts is NP-Complete. So, only a small-size fuzzy robust examination scheduling problem can be solved by means of binary linear programming models; for large-size problems some heuristics have to be applied to obtain appropriate solutions. To solve a probability robust examination scheduling problem a genetic algorithm has been proposed [10]. It can be also used to solve fuzzy robust examination scheduling problem. However, it seems that problems of optimal examination schedule have small dimensions and for such problems the solution of (13) is obtained very quickly.

Future research will deal with other graph problems such as assignment, map coloring, open shop problems. We will also analyze models, in which the non standard students examination conflicts are modeled as fuzzy variables.

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