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AN ANP-BASED FRAMEWORK FOR REVENUE MANAGEMENT

Abstract

Revenue management (RM) is the process of understanding, anticipating and influencing consumer behavior in order to maximize revenue. The challenge is to sell the right resources to the right customer at the right time for the right price through the right channel. Network revenue management models attempt to maximize revenue when customers buy bundles of multiple resources. An Analytic Network Process (ANP)-based framework for RM problems structuring and combining specific methods is presented. RM addresses three basic categories of demand-management decisions: price, quantity, and structural decisions. Specific models are used to model and to solve basic RM decisions. Combinations of the solutions are given by sub-networks in an ANP model.

Keywords

Revenue management, multicriteria decisions, price decisions, quantity decisions, structural decisions, Analytic network process, Dynamic Network Process.

Introduction

Revenue management (RM) is the process of understanding, anticipating and influencing consumer behavior in order to maximize revenue or profits from fixed, perishable resources. The RM area encompasses all work related to operational pricing and demand management. This includes traditional problems in the field, such as capacity allocation, overbooking and dynamic pricing, as well as newer areas, such as oligopoly models, negotiated pricing and auctions. Recent years have seen great successes of revenue management, notably in the airline, hotel, and car rental business. Currently, an increasing number of industries is exploring possibilities of adopting similar concepts [see Talluri, van Ryzin, 2004]. What is new about RM is not the demand--management decisions themselves but rather how these decisions are made. The true innovation of RM lies in the method of decision making.

Revenue Management is to sell the right product, to the right customer at the right time, for the right price through the right channel by maximizing revenue. RM is the art and science of predicting real-time customer demand and optimizing the price and availability of products according to the demand. RM addresses three basic categories of demand-management decisions:

– structural,
– price, and
– quantity decisions.

Network revenue management models attempt to maximize revenue when customers buy bundles of multiple resources. The dependence among the resources in such cases is created by customer demand.

For the basic specific problems many appropriate methods have been proposed [see Talluri, van Ryzin, 2004]. An Analytic Network Process (ANP)-based framework for RM problems structuring and combining specific methods is presented in this paper. Combinations of the solutions are given by subnetworks in an ANP model. RM problems are complex dynamic problems. The DNP (Dynamic Network Process) method was used for dynamic extensions.

1. ANP structure of the problem

The Analytic Hierarchy Process (AHP) is the method for setting priorities [Saaty, 1996]. A priority scale based on reference is the AHP way to standardize non-unique scales in order to combine multiple performance measures. The AHP derives ratio scale priorities by making paired comparisons of elements on a common hierarchy level by using a 1 to 9 scale of absolute numbers. The absolute number from the scale is an approximation to the ratio w_j / w_k and then it is possible to derive values of w_j and w_k as weights, i.e. measures of relative importance. The AHP method uses the general model for synthesis of the performance measures in the hierarchical structure:

$$u_i = \sum_{j=1}^{n} v_j w_{ji} ,$$

where u_i are global weights of alternatives, v_j are weights of criteria, and w_{ji} are weights of alternatives by individual criteria.

The Analytic Network Process (ANP) is the method [Saaty, 2001] that makes it possible to deal systematically with all kinds of dependence and feedback in the performance system. The well-known AHP theory is a special case of the Analytic Network Process that can be very useful for incorporating linkages in the system.

The structure of the ANP model is described by clusters of elements connected by their dependence on one another. A cluster groups elements that share a set of attributes. At least one element in each of these clusters is connected to some element in another cluster. These connections indicate the flow of influence between the elements.

The challenge in RM is to sell:

– the right resources,
– to the right customer,
– at the right time,
– for the right price,
– through the right channel.

There are two possibilities for a decision: to accept or to reject a request for a product. The clusters in an RM problem can consist of resources, customers, time, prices, channels, and decisions (see Figure 1).

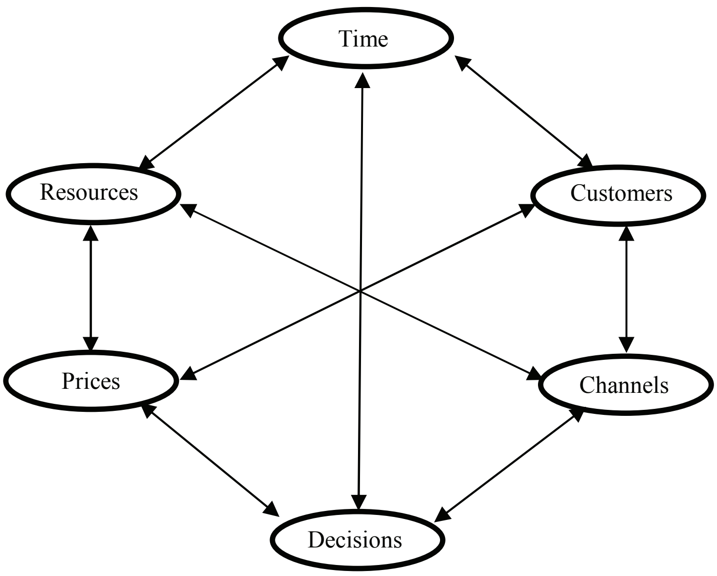

Figure 1. Clusters and connections in an RM problem

Pairwise comparisons are inputs for preference elicitation in revenue problems. A supermatrix is a matrix of all elements by all elements. The weights from the pairwise comparisons are placed in the appropriate column of the supermatrix. The sum of each column corresponds to the number of comparison sets. The weights in the column corresponding to the cluster are multiplied by the weight of the cluster. Each column of the weighted supermatrix sums to one and the matrix is column stochastic. Its powers can stabilize after some iterations to a limited supermatrix. The columns of each block of the matrix are identical in many cases, though not always, and we can read off them the global priority of units.

2. Sub-networks

The basic ANP model is completed by specific sub-networks. The sub--networks are used to model important features of the RM problems. The most important features in our ANP-based framework for revenue management are captured in sub-networks:
– time dependent resources,
– products,
– network revenue management,
– price-quantity-structure network.

Time dependent resources

A specific sub-network models time-dependent amounts of resources. The time-dependent amount of resources is given by previous decisions. The sub-network connects clusters: time, resources and decisions.

Products

A product is a sub-collection of available resources. An (m, n) matrix $A = [a_{ij}]$ is defined such that a_{ij} represents the amount of resource i used to produce one unit of product j. Every column j of A represents a different product and the collection $M = \{A_{.1}, \dots, A_{.n}\}$ is the menu of products offered by the seller.

Network revenue management

The quantity-based revenue management of multiple resources is referred to as network revenue management. This class of problems arises for example in airline, hotel, and railway management. In the airline case, the problem consists in managing capacities of a set of connecting flights across a network, a so called hub-and-spoke network (see Figure 2). In the hotel case, the problem is managing room capacity on consecutive days when customers stay multiple nights.

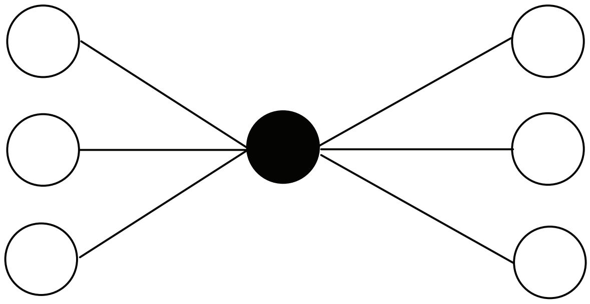

Figure 2. Hub-and-spoke network

Network revenue management models attempt to maximize some reward function when customers buy bundles of multiple resources. The Inter-dependence of resources, commonly referred to as network effects, creates difficulties in solving the problem. The classical technique of approaching this problem has been to use a deterministic LP solution to derive policies for the network capacity problem. Initial success with this method has triggered considerable research on possible reformulations and extensions, and this method has become widely used in many industrial applications. A significant limitation of the applicability of these classical models is the assumption of independent demand. In response to this, interest has arisen in recent years to incorporate customer choice into these models, further increasing their complexity. This development drives current efforts to design powerful and practical heuristics that still can manage problems of practical scope.

The basic model of the network revenue management problem can be formulated as follows [see Talluri, van Ryzin, 2004]: The network has m resources which can be used to provide n products. We define the incidence matrix $A = [a_{ij}]$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, where

$a_{ij} = 1$, if resource i is used by product j, and
$a_{ij} = 0$, otherwise.

The j-th column of A, denoted a_j, is the incidence vector for product j. The notation $i \quad a_j$ indicates that resource i is used by product j.

The state of the network is described by a vector $x = (x_1, x_2, \dots, x_m)$ of resource capacities. If product j is sold, the state of the network changes to $x - a_j$.

Time is discrete, there are T periods and the index t represents the current time, $t = 1, 2, \dots, T$. We assume that within each time period t at most one request for a product can arrive.

Price-quantity-structure network

RM addresses three basic categories of demand-management decisions:

1. Price decisions:
   - How to set posted prices.
   - How to price across product categories.
   - How to price over time.
   - How to markdown over the product lifetime.
2. Quantity decisions:
   - Whether to accept or reject an offer to buy.
   - How to allocate output or capacity to different segments, products and channels.
   - When to withhold a product from market and sale it at later points in time.
3. Structural decisions:
   - How to bundle products.
   - Which selling format to use.
   - Which segmentation or differentiation mechanisms to use.
   - Which terms of trade to offer.

The price-quantity-structure network is given by interdependences of the three very important factors. The solutions of three basic categories of demand--management decisions are solved by basic methods described in next paragraphs. Interdependencies are modeled and analyzed in the ANP sub--network.

3. Price decisions

The basic pricing model of the network revenue management problem is formulated as a stochastic dynamic programming problem whose exact solution is computationally intractable. Most approximation methods are based on one of two basic approaches: to use a simplified network model or to decompose the network problem into a collection of single-resource problems.

The Deterministic Linear Programming (DLP) method is a popular one in practice. The DLP method is based on a wrong assumption that demand is deterministic and static. Approximation methods based on extensions of the basic approaches are proposed.

The revenue management general model [Bitran, Caldentey, 2003] provides a global view of the different elements and their interrelations:

– Supply.
– Product.
– Information.
– Demand.

A seller has a fixed amount of initial capacity that is used to satisfy a price-sensitive demand during a certain selling period $[0, T]$. This initial capacity is modeled by an m-dimensional vector of m resources. Capacity can be interpreted for example as rooms in a hotel, available seats for a specific origin-destination flight on a given day, etc. Capacity is essentially given and the seller is committed exclusively to finding the best way to sell it. From a pricing perspective, two important attributes of the available capacity are its degree of flexibility and its perishability. Flexibility measures the ability to produce and offer different products using the initial capacity C_0. Perishability relates to the lack of ability to preserve capacity over time. As time progresses and resources are consumed, capacity decreases. The available capacity at time t is denoted by $C_t = (c_1(t), \dots, c_m(t))$.

The knowledge of the system and its evolution over time is crucial to any dynamic pricing policy. Given an initial capacity C_0, a product menu M, and a demand and price processes, the observed history H_t of the selling process is defined as the set of all relevant information available up to t. This history should include at least the observed demand process and available capacity, and it can also include some additional information such as demand forecasts.

The set of potential customers is divided into different segments, each one having its own set of attributes. A d-dimensional stochastic process is defined as $N(t, H_t) = (N_1(t, H_t), \dots, N_d(t, H_t))$ where $N_j(t, H_t)$ is the cumulative potential demand up to time t from segment j given the available information H_t. An (n, d) matrix $B(P) = [b_{ij}]$ is defined where b_{ij} represents the units of product $i \in M$ requested by a customer in segment $j = 1, \dots, d$. The demand depends on the pricing policy $P = \{p_t, t \in [0, T]\}$ where $p_t(i, H_t)$ is the price of product $i \in M$ at time t given a current history H_t. The effective cumulative demand process in $[0, t]$ at the product level is defined as the n-dimensional vector $D(t, P, H) = B(P)N(t, H_t)$. The set of all admissible pricing policies, those that satisfy all the relevant constraints, is denoted by Π. The seller has the ability to partially serve demand if it is profitable to do so. An n-dimensional vector $S(t)$ that represents the cumulative sales up to time t is defined.

The general revenue management problem is to find the solution to the following optimal control problem:

$$\sup_{P,S} E_N\left[\int_0^T p_t dS(t)\right]$$

subject to

$$C_t = C_0 - AS(t) \geq 0 \text{ for all } t \in [0; T],$$
$$0 \leq S(t) \leq D(t, P, H_t) \text{ for all } t \in [0; T],$$
$$P \in \Pi, \text{ and } S(t) \in H_t.$$

Deterministic models assume that the seller has perfect information about the demand process. They are easy to analyze and provide a good approximation for the more realistic yet complicated stochastic models. Deterministic solutions are in some cases asymptotically optimal for the stochastic demand problem [Cooper, 2002].

The simplest deterministic model considers the case of a monopolist selling a single product to a price sensitive demand during a period $[0, T]$. The initial inventory is C_0, demand is deterministic with time dependent and price sensitive intensity $\mu(p, t)$. The instantaneous revenue function $r(p, t) = p\,\mu(p, t)$ is assumed to be concave as in most real situations. The general revenue management problem can be written in this case as follows:

$$\max_P \int_0^T p_t \mu(p_t, t) dt$$

subject to

$$\int_0^T \mu(p_t, t) dt \leq C_0 .$$

This is a standard problem in calculus of variations. The optimality condition is given by

$$p_t^* = \lambda - \frac{\mu(p_t^*, t)}{\mu_p(p_t^*, t)},$$

where λ is the Lagrangian multiplier for the constraint, μ_p is the partial derivative of μ with respect to the price.

4. Quantity decisions

Demand in time period t is modeled as the realization of a single random vector $r(t) = (r_1(t), r_2(t), \dots, r_n(t))$. If $r_j(t) = r_j > 0$, this indicates that a request for product j occurred and that its associated revenue is r_j. If $r_j(t) = 0$, this indicates that no request for product j occurred. A realization $r(t) = 0$ (all components equal to zero) indicates that no request from any product occurred at time t. The assumption that at most one arrival occurs in each time period means that at most one component of $r(t)$ can be positive. The sequence $r(t)$, $t = 1, 2, \dots, T$, is assumed to be independent with known joint distributions in each time period t. When revenues associated with product j are fixed, we will denote these by r_j and the revenue vector $r = (r_1, r_2, \dots, r_n)$.

Given the current time t, the current remaining capacity x and the current request $r(t)$, the decision is to accept or not to accept the current request. We define the decision vector $u(\mathrm{t}) = (u_1(t), u_2(t), \dots, u_n(t))$ where

$u_j(t) = 1$, if a request for product j in time period t is accepted, and
$u_j(t) = 0$, otherwise.

The components of the decision vector $u(\mathrm{t})$ are functions of the remaining capacity components of vector x and the components of the revenue vector r, $u(\mathrm{t}) = u(t, x, r)$. The decision vector $u(t)$ is restricted to the set

$$U(x) = \{u \in \{0,1\}^n,\ Au \le x\}.$$

The maximum expected revenue, given remaining capacity x in time period t, is denoted by $V_t(x)$. Then $V_t(x)$ must satisfy the Bellman equation:

$$V_t(x) = E\left[\max_{u \in U(x)} \{r(t)^T u(t,x,r) + V_{t+1}(x - Au)\}\right] \tag{1}$$

with the boundary condition

$$V_{T+1}(x) = 0, \forall x.$$

A decision u^* is optimal if and only if it satisfies:

$u_j(t, x, r_j) = 1$, if $r_j \ge V_{t+1}(x) - V_{t+1}(x - a_j)$, $a_j \le x$,

$u_j(t, x, r_j) = 0$, otherwise.

This reflects the intuitive notion that revenue r_j for product j is accepted only when it exceeds the opportunity cost of the reduction in resource capacities required to satisfy the request.

Basic approximation approach

The equation (1) cannot be solved exactly for most networks of realistic size. Solutions are based on approximations of various types. There are two important criteria when judging network approximation methods: accuracy and speed. Among the most useful information provided by an approximation method are estimates of bid prices [see Talluri, van Ryzin, 2004].

Given an approximation method M that yields an estimate of the value function $V_t^M(x)$ we can approximate the displacement cost of accepting product j by gradient of the value function approximation. The bid prices are then defined by:

$$\pi_i^M(t, x) = \frac{\partial}{\partial x_i} V_t^M(x) .$$

We introduce Deterministic Linear Programming (DLP) method. The approach is to use a simplified network model, for example formulating the problem as a static mathematical program.

The DLP method uses the approximation:

$$V_t^{LP}(x) = \max r^T y \tag{2}$$

subject to

$Ay \leq x$

$0 \leq y \leq E[D]$

where $D = (D_1, D_2, \dots, D_n)$ is the vector of demand over the periods t, $t+1, \dots, T$, for product $j, j = 1, 2, \dots, n$, and $r = (r_1, r_2, \dots, r_n)$ is the vector of revenues associated with the n products. The decision vector $y = (y_1, y_2, \dots, y_n)$ represents partitioned allocation of capacity for each of the n products. The approximation effectively treats demand as if it were deterministic and equal to its mean $E[D]$.

The optimal dual variables, π^{LP}, associated with the constraints $Ay \leq x$, are used as bid prices. The DLP was among the first models analyzed for network RM [see Talluri, van Ryzin, 2004]. The main advantage of the DLP model is that it is computationally very efficient to solve. Due to its simplicity and speed, it is a popular one in practice. The weakness of the DLP approximation is that it considers the mean demand only and ignores all other distributional information. The performance of the DLP method depends on the type of network, the order in which fare products arrive and the frequency of re-optimization.

5. Structural decisions

One of structural decisions is how to bundle products. We will show on this example how to use models of combinatorial auctions. Auctions are important market mechanisms for the allocation of goods and services. Combinatorial auctions are those auctions in which bidders can place bids on combinations of items, so called bundles. The advantage of combinatorial auctions is that the bidder can more adequately express his preferences. This is particularly important when items are complements. The auction designer also derives value from combinatorial auctions. Allowing bidders more adequately to express preferences often leads to improved economic efficiency and greater auction revenues. However, alongside their advantages, combinatorial auctions raise a host of questions and challenges [see Cramton et al. (eds.), 2006; de Vries and Vohra, 2003].

The problem, called the winner determination problem, has received considerable attention in the literature. The problem is formulated as follows: Given a set of bids in a combinatorial auction, find an allocation of items to bidders that maximizes the seller's revenue. It introduced many important ideas, such as the mathematical programming formulation of the winner determination problem, the connection between the winner determination problem and the set-packing problem as well as the issue of complexity.

Winner determination problem

Many types of combinatorial auctions can be formulated as mathematical programming problems. From among different types of combinatorial auctions we present an auction of indivisible items with one seller and several buyers. Let us suppose that one seller offers a set G of m items, $j = 1, 2, \dots, m$, to n potential buyers. Items are available in single units. A bid made by buyer i, $i = 1, 2, \dots, n$, is defined as:

$$B_i = \{S, v_i(S)\},$$

where

$S \subseteq M$, is a combination of items,
$v_i(S)$, is the valuation or offered price by buyer i for the combination of items S.

The objective is to maximize the revenue of the seller given the bids made by buyers. The constraints ensure that no single item is allocated to more than one buyer and that no buyer obtains more than one combination.

Problem formulation

Bivalent variables are introduced for model formulation:
$x_i(S)$ is a bivalent variable specifying if the combination S is assigned to buyer i ($x_i(S) = 1$).

The winner determination problem can be formulated as follows:

$$\sum_{i=1}^{n} \sum_{S \subseteq M} v_i(S)\, x_i(S) \rightarrow \max$$

subject to:

$$\sum_{S \subseteq M} x_i(S) \leqq 1, \quad \forall\, i, \; i = 1, 2, \dots, n,$$

$$\sum_{i=1}^{n} \sum_{S \subseteq M} x_i(S) \leqq 1, \quad \forall\, j \in \mathrm{M},$$

$$x_i(S) \in \{0, 1\}, \; \forall \; S \subseteq M, \; \forall\, i, \; i = 1, 2, \dots, n.$$

The objective function expresses the revenue. The first constraint ensures that no bidder receives more than one combination of items. The second constraint ensures that overlapping sets of items are never assigned.

6. Dynamic Network Process

RM problems are complex dynamic problems. The ANP is static but for the RM problem, time dependent decision making is very important. The DNP (Dynamic Network Process) method was introduced [Saaty, 2003]. There are two ways to study dynamic decisions: structural, by including scenarios, and functional, by explicitly involving time in the judgment process. For the functional dynamics there are analytic or numerical solutions. The basic idea of the numerical approach is to obtain the time dependent principal eigenvector by simulation.

The Dynamic Network Process seems to be an appropriate instrument for analyzing dynamic networks [Fiala, 2006]. The method is appropriate also for the specific features of RM problems. The method computes time dependent weights for decisions and combinations of decisions.

We used the ANP software package Super Decisions developed by Creative Decisions Foundation (CDF) for some experiments in testing the possibilities of the expression and evaluation of the dynamic RM models [see Figure 3].

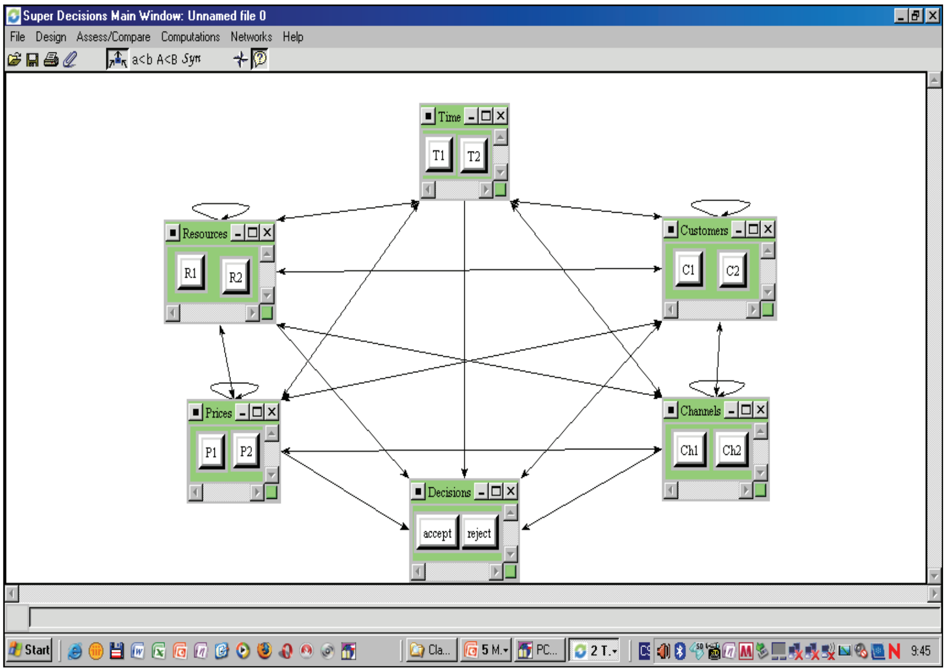

Figure 3. Super Decisions – RM model

Conclusions

RM problems are important subjects of intensive economic research. A possible flexible ANP/DNP framework is presented. Analytic Network Process methodology is useful for structuring the RM problem and for combining specific models. Sub-networks are used for sophisticated analyses of RM processes. Specific models are used to model and to solve basic RM decisions (price, quantity, structure). Approximations, heuristics, or iterative approaches are used for solving the specific models. Dynamic Network Process is an appropriate approach for explicitly involving time in the RM processes. The combination of such approaches can give more complex views on RM problem.

Acknowledgements

The research project was supported by Grant No. P402/10/0197 „Revenue management – models and analyses" from the Grant Agency of the Czech Republic.

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