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AN INTERACTIVE APPROACH DETERMINING THE INDIFFERENCE THRESHOLDS IN PROMETHEE II

Abstract

The implementation of most multi-criteria decision aid methods requires fixing of certain parameters in order to model the decision-maker's preferences. The fixing of these parameter values must be naturally done with the decision-maker's collaboration. The parameter determination constitutes an important task, which is generally quite delicate and difficult to accomplish, for the decision-maker. In fact, the information provided at this level is inevitably subjective and partial. In this paper, we intend to determine the values of the indifference thresholds associated to usual and quasi criterion in PROMETHEE, by exploiting the information provided by the decision-maker and by using mathematical programming.

Keywords

Multi-criteria analysis, Integer programming, Preference disaggregation, PROMETHEE, Indifference thresholds.

Introduction

At the time of a multi-criteria decision aid activity, the basic preoccupation concerns the manner in which the decision will be taken in a given context. However, it can also be pertinent to pose the problem inversely: supposing that a decision has been taken, is it possible to find the rational bases allowing to explain or to justify the decision taken? Or is it possible to explain the decision-maker's preference model which leads precisely to the same decision or at least to a very "similar" decision? The philosophy of the preference disaggregation approach in the framework of a multi-criteria analysis is to determine preference modelling elements from preferential structures provided by the decision-maker, taking into account the method used for multi--criteria aggregation.

The implementation of PROMETHEE method requires fixing relative importance coefficients of the criteria, preference thresholds and indifference thresholds. In fact, we proposed an approach inferring relative importance coefficients of the criteria from preference relations provided by the decision maker [Frikha et al., 2010].

In this paper, we consider the problem of the preference disaggregation, inferring, from preference relations provided by the decision-maker, the indifference threshold values. We will focus our interest only on usual criteria and quasi-criteria. In a subsequent work, we intend to extend our results to the general case, with the other criteria's functions, requiring preference and indifference thresholds simultaneously. The organization of the paper is as follows: a brief presentation of the disaggregation approaches constitutes section 2. Section 3 is dedicated to the description of the PROMETHEE method. In section 4, we will describe the model proposed to determine the indifference threshold values. This model, including an interactive aspect, is based on mathematical programming of the goal-programming type. A fictitious numerical example is the object of section 5 and finally Section 6 contains a brief conclusion.

1. Preference disaggregation methods

Several disaggregation approaches have been developed to infer the ELECTRE method's parameters. Indeed, a first trial of ELECTRE III parameter determination from a given ranking has been presented by Richard [1981] without eventually leading to satisfactory results. Then Kiss et al. [1994] developed an interactive system called ELECCALC that determines indirectly the ELECTRE II method's parameters from decision-makers' answers to questions concerning their global preferences. In the same context of preference disaggregation methods allowing to determine certain of ELECTRE parameters' values on the basis of information provided by the decision-maker, Mousseau contributed to the development of several works. Indeed, Mousseau and Slowinski [1998] proposed a global inference approach that deduces ELECTRE TRI's parameters simultaneously from assignment examples. Continuing the same idea, Mousseau et al. [2001] proposed a partial inference approach that consists in inferring only the criteria's relative importance coefficients and the cut levels in order to deduce some trivial relations from valued ranking relations. Ngo The and Mousseau [2002] presented also an inference procedure that determines the limits of categories of ELECTRE TRI method from assignment examples provided by the decision-maker. Finally, Dias and Mousseau [2006] proposed a mathematical program to deduce the veto threshold values of ELECTRE III from ranking examples.

In the same context of preference disaggregation, Jacquet-Lagreze [1979] had proposed an approach to construct an additive value model that consists in assessing indirectly the model's parameters on the basis of preference holistic information. This approach is mathematically integrated in the UTA method by Jacquet-Lagrèze and Siskos [1982] through a disaggregation model of ordinal regression type, based on the formulation of linear programming. The preference disaggregation methods also appear in other versions of UTA. Indeed, the UTADIS method [Utilité Additive Discriminante] of Devaud et al. [1980] is an ordinal regression method based on the preference disaggregation approach. Given a predefined action ranking in classes, the objective of UTADIS is to estimate the additive utility function and utility thresholds that assign actions in their original classes with a minimum of ranking errors. The method UTA II, developed by Siskos [1980] is another version of the UTA method. This preference disaggregation approach is useful to assess the additive utility model. Greco et al. [2008] developed the UTAGMS method, which allows the determination of all additive value functions compatible with the preference information provided by the decision maker [a set of pairwise comparisons on a subset of alternatives, called reference alternatives]. Besides, Figueira et al. [2009] developed the UTAGRIP allowing constructing a set of additive value functions compatible with preference information composed of comparisons of reference action pairs. Moreover, Bous et al. [2010] proposed a new method called ACUTA based on the computation of the analytic centre of a polyhedron, for the selection of additive value functions that are compatible with holistic assessments of preferences provided by the decision maker. In the same context of determining additive value functions, Köksalan and Özpeynirci [2009] developed an approach that estimates an additive utility function. In fact, the decision maker is invited to assign some reference alternatives into categories during the interactive process. Else, Greco et al. (2010) proposed a model that aims at assigning actions evaluated on multiple criteria to *p* pre-defined and ordered classes. In this work, the decision maker supplies a set of assignment examples on a subset of actions, called reference actions. This information is used to determine a set of general additive value functions compatible with these assignment examples.

In the framework of multi-criteria decision aid under uncertainty, Siskos [1983] developed a stochastic ordinal regression method from UTA (stochastic UTA).

The disaggregation approaches are also applicable in a specific multi--objective optimization field, mainly in the field of the linear programming with multiple objective functions. For example, in the classic methods of Geoffrion et al. [1972] and Zionts and Wallenius [1976], the weights of the objective linear combinations are inferred locally from the judgments provided by the decision-maker at each iteration of these methods. Stewart [1987] proposed a procedure of action pruning using the UTA method, whereas Jacquet-Lagrèze et al. [1987] developed a method similar to UTA to estimate the utility function of multi-objective systems for the linear programming systems. Siskos and Despotis [1989] proposed an interactive method called ADELAIS that uses UTA in an iterative way in order to optimize an additive value function in the feasible region defined on the basis of satisfaction levels determined during each iteration. Tangian [2001] proposed a disaggregation technique to calculate quadratic multi-objective functions.

2. The PROMETHEE method

The PROMETHEE method (Preference Ranking Organization METHod for Enrichment Evaluation) [Brans and Vincke, 1985] is based on the principle of pairwise action comparison according to each criterion. It consists in defining a preference function P^k_{ij}, allowing the modeling of the decision-maker's preferences with respect to each criterion k.

When the decision-maker compares two alternatives x_i and x_j, P^k_{ij} represents the degree of preference for x_i, considering only the criterion k. The preference function's value varies between 0 and 1 and is defined separately for every criterion by:

$$P^k_{ij} = \begin{cases} 0 & \text{if} \quad d^k_{ij} \leq 0 \\ P^k_{ij} & \text{if} \quad d^k_{ij} > 0 \end{cases} \tag{1}$$

For the usual-criterion:

$$P^k_{ij} = \begin{cases} 0 & \text{if} \quad d^k_{ij} \leq 0 \quad \text{(Indifference)} \\ 1 & \text{if} \quad d^k_{ij} > 0 \quad \text{(Strict preference)} \end{cases}$$

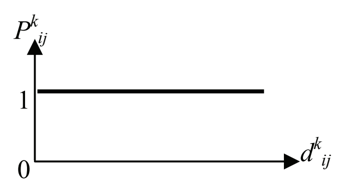

Figure 1. The preference function for the usual-criterion

In this case, there is an indifference between x_i and x_j only if $g_k(x_i) = g_k(x_j)$. As soon as these values are different, there is a strict preference for one of the alternatives. There is no parameter to determine.

For the quasi-criterion:

$$P^k_{ij} = \begin{cases} 0 & \text{if} \quad d^k_{ij} \le q_k \quad \text{(Indifference)} \\ 1 & \text{if} \quad d^k_{ij} > q_k \quad \text{(Strict preference)} \end{cases}$$

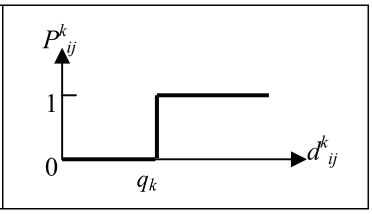

Figure 2. The preference function for the quasi-criterion

In this case, there is an indifference between x_i and x_j as a long as the slack between $g_k(x_i)$ and $g_k(x_j)$ does not exceed the indifference threshold q_k. Beyond this, the preference becomes strict. We find the usual-criterion if q_k=0.

To define the criteria preference function, it is necessary to determine the indifference threshold values (q). The indifference threshold (q) corresponds to the maximum value of d^k_{ij} below which the decision-maker is indifferent between the two alternatives x_i and x_j according to the considered criterion.

The preference modelling, at the time of the decision process, requires for each alternative x_i, the use of the preference indexes C_{ij}, the outgoing flow ϕ_i^+, the incoming flow ϕ_i^- and the net flow ϕ_i.

Therefore, it is necessary to calculate for every alternative x_i:

– The preference index C_{ij} which represents the degree of preference for x_i with regard to x_j over all the criteria simultaneously.

$$C_{ij} = \sum_{k=1}^{n} w_k P^k_{ij} \tag{2}$$

Where w_k is the relative importance coefficient (RIC) given to each criterion k with $w_k \ge 0$ and $\Sigma w_k = 1$, the greater the RIC, the more important the criterion.

– The outgoing flow ϕ_i^+ which represents the dominance of a with regard to other alternatives.

$$\phi_i^+ = \sum_{j=1,\, i \ne j}^{r} C_{ij} \tag{3}$$

– The incoming flow ϕ_i^- which represents the weakness of a with regard to other alternatives.

$$\phi_i^- = \sum_{i=1,\, j \neq i}^{r} C_{ji} \tag{4}$$

– The net flow ϕ_i which is the difference between the outgoing and the incoming flows.

$$\phi_i = \phi_i^+ - \phi_i^- \tag{5}$$

3. Determination of PROMETHEE's indifference thresholds

Within the framework of the PROMETHEE method, the decision-maker is invited to provide directly the analyser with information concerning alternatives, criteria, the assessment of each alternative according to each criterion, the nature of each criterion function, as well as all parameters' values required for the implementation of the method. The quantitative information that he must provide (relative importance coefficients, preference and indifference thresholds) is not always easy to put in evidence. Besides, many other factors such as the order in which criteria are presented to the decision--maker, the moment at which he is interrogated or the type of the alternative assessed, can lead to considerable variation of parameter values. Consequently, the parameters' values provided directly by the decision maker are subjective and not very reliable. In what follows, we propose to deduce some of these parameters from global information given by the decision-maker.

We suppose that the criteria relative importance coefficients (r.i.c) w_k are known. Criterion functions can only take the form of the usual-criterion or the quasi-criterion. The decision matrix (which is composed of alternatives, criteria as well as the assessment of alternatives according to each criterion) is known and the decision-maker is invited to provide us with p preference relations on some alternatives; relations of the type: alternative x_i is preferred ($\succ$) to alternative x_j. Our objective is to determine the indifference thresholds q_k associated to each criterion k through the resolution of the first mixed integer linear program. When q_k takes the value zero, the k^{th} criterion is a usual one.

Program 1:

$$Minimize\ Z = \sum_{m=1}^{p} S_m^- \tag{6}$$

Subject to

$$\left[\sum_{\substack{i=1\\ i\neq l}}^{r}\sum_{k=1}^{n} w_k P_{il}^k - \sum_{\substack{l=1\\ l\neq i}}^{r}\sum_{k=1}^{n} w_k P_{li}^k\right] - \left[\sum_{\substack{j=1\\ j\neq l}}^{r}\sum_{k=1}^{n} w_k P_{jl}^k - \sum_{\substack{l=1\\ l\neq j}}^{r}\sum_{k=1}^{n} w_k P_{lj}^k\right] + S_m^- - S_m^+ = 0 \quad \forall\ m = 1,\dots, p \tag{7}$$

$$P_{ij}^k = 0 \qquad \forall\ d_{ij}^k \leq 0 \quad \forall\ i \neq j,\ i = 1,\dots,r \text{ and } j = 1,\dots,r,\ \forall\ k = 1,\dots,n \tag{8}$$

$$P_{ij}^k \leq P_{hl}^k \qquad \forall\ d_{ij}^k \leq d_{hl}^k \quad \forall\ i \neq j,\ i = 1,\dots,r \text{ and } j = 1,\dots,r\ ;\ \forall\ k = 1,\dots,n\ ;\ \forall\ h \neq l,\ h = 1,\dots,r \text{ and } l = 1,\dots,r \tag{9}$$

$$P_{ij}^k + P_{ji}^k \leq 1 \qquad \forall\ i \neq j,\ i = 1,\dots,r \text{ and } j = 1,\dots,r\ ,\ \forall\ k = 1,\dots,n \tag{10}$$

$$P_{ij}^k \in \{0,1\} \qquad \forall\ i \neq j,\ i = 1,\dots,r \text{ and } j = 1,\dots,r,\ \forall\ k = 1,\dots,n \tag{11}$$

$$S_m^+ \geq \alpha_m \qquad \forall\ m = 1,\dots, p \tag{12}$$

$$S_m^-, S_m^+ \geq 0 \qquad \forall\ m = 1,\dots, p \tag{13}$$

where:

P^k_{ij} is the preference index of x_i on x_j according to the criterion k,
d^k_{ij} is the difference between of assessment of x_i and x_j according to the criterion k,

where

$d_{ij}^k = g_k(x_i) - g_k(x_j)$ if the criterion k is to be maximised,
$d_{ij}^k = g_k(x_j) - g_k(x_i)$ if the criterion k is to be minimised,

p is the preference relation number provided by the decision-maker,
n is the criteria number,
r is the alternative number.

The objective function (6) consists in minimizing the sum of the negative deviations. In this paper, we regard the p preference relations expressed by the decision-maker ($x_i \succ x_j$) as goals to be achieved. In fact, in PROMETHEE method, the "goal" of having x_i preferred to x_j ($x_i \succ x_j$) means that $\phi_i > \phi_j$

then $\phi_i - \phi_j > 0$. We can transform these inequalities into equalities by introducing two slack variables which represent the deviations between the achievements and the decision-maker's preferences (goals).

Let's note by S^+_m the positive deviation in case of objective exceeding and by S^-_m the negative deviation in the opposite case.

Therefore, $(x_i \phi x_j)$ means that $\phi_i - \phi_j + S_m^- - S_m^+ = 0$ with $S_m^+ \geq 0$ and $S_m^- \geq 0$.

In order to reach the goal $(x_i \phi x_j)$, it is necessary that $\phi_i > \phi_j$. We can transform this inequality into equality, by subtracting a positive deviation S^+_m then $\phi_i^+ - \phi_j^- - S_m^+ = 0$ with $S_m^+ > 0$ and $S_m^- = 0$.

In order to satisfy all the preferences expressed by the decision-maker, we must minimize all the negative deviations, which must be ideally null. The objective function will be, therefore, the minimization of the negative deviation sum (6), then the risk encountered is that at the optimality, all the positive and negative deviations are null. In this case, $\phi_i - \phi_j + S_m^- - S_m^+ = 0$ becomes $\phi_i - \phi_j = 0$ and the decision-maker will be indifferent between the alternatives i and j, which is in contradiction with the preference relations provided.

However, $(x_i \phi x_j)$ means that $\phi_i - \phi_j > 0$. We must therefore have at least a small difference between ϕ_i and ϕ_j. In order to have $\phi_i > \phi_j$, and to satisfy the equality $\phi_i - \phi_j + S_m^- - S_m^+ = 0$ with S_m^- taking its minimal value (we prefer that $S_m^- = 0$), we must have $S_m^+ > 0$. For this reason, we introduce in the program constraints of the type $S_m^+ \geq \alpha_m \quad \forall\, m = 1, \ldots, p$ (12) fixing a minimum threshold α_m to each positive deviation S^+_m in order to prevent it from being null. Now, the question is how to choose these thresholds?

We start with fixing an arbitrary threshold α_m to each S^+_m and we solve the mathematical program. At this level, we are interested in positive deviation values S^+_m only.

If the positive deviation values found are much larger than threshold values fixed in the constraints ($S_m^+ > \alpha_m$), then thresholds are well fixed.

However, if the positive deviation values found are equal to the threshold values fixed in the constraints ($S_m^+ = \alpha_m$), then there exists a risk that S^+_m would have another value smaller than α_m, but this cannot happen because of the constraint $S_m^+ \geq \alpha_m$. Hence, it took the minimum, which is equal to α_m. In this case, we decrease the threshold's value α_m and we solve the program again. We verify if the positive deviation values found are much greater than threshold values, and so forth… If the mathematical program does not have a solution, we must reduce the α_m's and we solve it again. In fact, when many constraints are satisfied, the program may have no solution. When α_m (the alternatives preference degree) are reduced, the program may have solution(s).

Concerning the other program constraints, the constraint (7) is related to preference relations provided by the decision-maker. In fact, the relation ($x_i \phi x_j$), expressed by the equality $\phi_i - \phi_j + S_m^- - S_m^+ = 0$, leads us to write the 7th constraint.

Every modification in the preference relations' information provided by the decision-maker will induce modifications at the level of preference functions' values, variables of the program. The modifications of the P^k_{ij}'s values can result either in changes in the indifference threshold values, or in its maintenance at its initial value (because different preference function matrices can give the same value of q_k).

As for the constraint (8) of the program, it expresses the cases where the preference function is null. The preference function P^k_{ij}, whose assessment differs according to the criterion nature, is defined separately for every criterion in (1). Hence, the constraint: If $d_{ij}^k \le 0$ then $P_{ij}^k = 0$.

The constraint (9) expresses the comparison between preference functions' values, while basing on the comparison between alternative assessment differences. Three cases are presented:

– $\boldsymbol{d^k_{hl} \le q_k}$ hence $P^k_{hl}=0$, and since $d^k_{ij} \le d^k_{hl}$ then $P^k_{ij}=0$ (therefore $P^k_{ij} = P^k_{hl} = 0$).

– $\boldsymbol{d^k_{ij} > q_k}$ hence $P^k_{ij}=1$, and since $d^k_{ij} \le d^k_{hl}$ then $P^k_{hl}=1$ (therefore $P^k_{ij} = P^k_{hl} = 1$).

– $\boldsymbol{d^k_{ij} \le q_k < d^k_{hl}}$ hence $P^k_{ij}=0$ and $P^k_{hl}=1$ (therefore $P^k_{ij} < P^k_{hl}$).

From these three cases, we conclude that $P_{ij}^k \le P_{hl}^k$ if $d_{ij}^k \le d_{hl}^k$

The constraint (10) requires that the sum of the symmetrical preference functions' values not exceed 1. Indeed, the assessment difference matrix is symmetrical with regard to the diagonal, where $i = j$. It means that if $d^k_{ij} = a$, then $d^k_{ji} = -a$ ($a \in \Upsilon$). In fact, when $d^k_{ij} \le 0$ then $P^k_{ij} = 0$ and when $d^k_{ji} > 0$ then $P^k_{ji} = 0$ or $P^k_{ji} = 1$, all depends on the indifference threshold value q_k. Therefore, P^k_{ij} and P^k_{ji} cannot, both of them, take the value 1. Either one is null and the other is equal to 1 or each of them is null. Hence, $P_{ij}^k + P_{ji}^k \le 1$.

Besides, the constraint (11) indicates that the preference function in the cases of usual-criterion and quasi-criterion is a binary variable that can only take the values 0 and 1. Indeed,

$$P_{ij}^{k} = \begin{cases} 0 & \text{if} \quad d_{ij}^{k} \leq q_{k} \\ 1 & \text{if} \quad d_{ij}^{k} > q_{k} \end{cases} \tag{14}$$

The last constraint represented in the program of indifference threshold determination is (13): the constraint of no negativity ($S_m^-, S_m^+ \geq 0$) which requires that slack variables not be negative.

The solution of this program provides us with the values of the variables $P^k{}_{ij}$, $S^+{}_m$ and $S^-{}_m$.

From the values of the $P^k{}_{ij}$, $d^k{}_{ij}$ and the relation (14), and by taking into account that q_k is the indifference threshold that corresponds to the smallest assessment difference leading to conclude the strict preference, we deduce the indifference threshold values q_k.

The program can have multiple solutions. In this case, we determine all the program solutions (possible values of the $P^k{}_{ij}$), and we deduce indifference thresholds q_k associated with each solution. We assume that all indifference threshold values are integer. All these threshold values found permit to respect the preference relations provided by the decision-maker.
In addition, the threshold values found permit us to find out the nature of the criteria. Indeed, if $q_k = 0$, the criterion is usual, and if q_k is strictly positive, we have the quasi-criterion.

After having found out all the possible solutions of the thresholds q_k, and in the framework of an interactive approach, we ask the decision-maker to provide information concerning intervals for the indifference thresholds q_k. Among solutions, we look for the one or ones that belong to the intervals.

– If none of the solutions belong to the interval, we ask the decision-maker to change the q_k's intervals.
– If a solution is found, we communicate it to the decision-maker.
– If more than one solution are found in an interval, we ask the decision--maker to reduce the k^{th} interval, or we give him solutions (whose number is reduced), and ask him to choose one of them.

After having deduced the indifference threshold values associated with each criterion, we apply the PROMETHEE method in order to get the total alternative ranking. We present them, together with the preference functions, to the decision-maker. He can then modify the alternative ranking (change the starting information on his preferences or add another preference relation in contradiction with the final alternative ranking).

In this case, the modified information will be modelled in the mixed integer linear program in order to determine the new indifference threshold values that will be presented to the decision-maker. This interactive approach of the indifference threshold determination is summarized in the following chart (Figure 3).

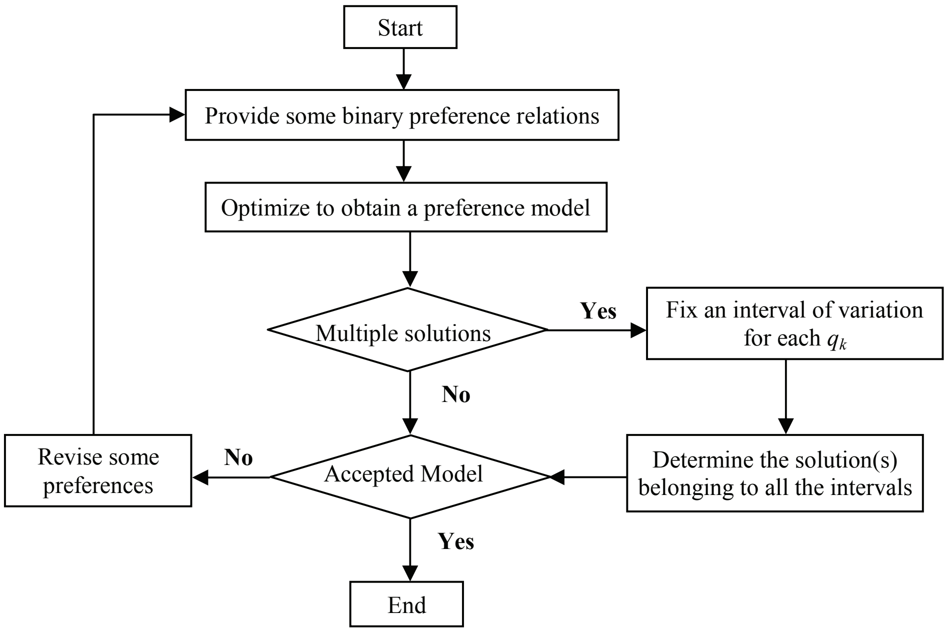

Figure 3. General diagram of the interactive approach

4. Illustrative example

We suppose a decision problem with three criteria C_1, C_2 and C_3 and six alternatives *A*, *B*, *C*, *D, E* and *F* is given. The criteria's r.i.c w_k are given, the indifference threshold q_k as well as the type of each criterion function (usual-criterion or quasi-criterion) are to be determined. The decision-maker provides the following decision matrix (Table 1).

Table 1

Decision matrix

Alternatives	Criteria		
	C_1 (Max)	C_2 (Max)	C_3 (Min)
A	6	5	28
B	4	2	25
C	5	7	35
D	6	1	27
E	6	8	30
F	5	6	26
Normalized r.i.c	0,3	0,5	0,2

The decision-maker provides the following information concerning some binary preference relations: $E\phi F$, $A\phi D$, $D\phi B$, $F\phi C$, $E\phi B$.

The assessment difference matrices d^k_{ij}, as well as the preference function matrices P^k_{ij} are represented in Table 2, using the following formula: $P^k_{ij} = 0$ if $d^k_{ij} \leq 0$.

Table 2

Assessment difference matrices and preference function matrices

For $k = 1$

d^1_{ij}	**A**	**B**	**C**	**D**	**E**	**F**
A		2	1	0	0	1
B	−2		−1	−2	−2	−1
C	−1	1		−1	−1	0
D	0	2	1		0	1
E	0	2	1	0		1
F	−1	1	0	−1	−1	

P^1_{ij}	**A**	**B**	**C**	**D**	**E**	**F**
A		P^1_{12}	P^1_{13}	0	0	P^1_{16}
B	0		0	0	0	0
C	0	P^1_{32}		0	0	0
D	0	P^1_{42}	P^1_{43}		0	P^1_{46}
E	0	P^1_{52}	P^1_{53}	0		P^1_{56}
F	0	P^1_{62}	0	0	0	

For $k = 2$

d^2_{ij}	**A**	**B**	**C**	**D**	**E**	**F**
A		3	−2	4	−3	−1
B	−3		−5	1	−6	−4
C	2	5		6	−1	1
D	−4	−1	−6		−7	−5
E	3	6	1	7		2
F	1	4	−1	5	−2	

P^2_{ij}	**A**	**B**	**C**	**D**	**E**	**F**
A		P^2_{12}	0	P^2_{14}	0	0
B	0		0	P^2_{24}	0	0
C	P^2_{31}	P^2_{32}		P^2_{34}	0	P^2_{36}
D	0	0	0		0	0
E	P^2_{51}	P^2_{52}	P^2_{53}	P^2_{54}		P^2_{56}
F	P^2_{61}	P^2_{62}	0	P^2_{64}	0	

For $k = 3$

d^3_{ij}	A	B	C	D	E	F
A		–3	7	–1	2	–2
B	3		10	2	5	1
C	–7	–10		–8	–5	–9
D	1	–2	8		3	–1
E	–2	–5	5	–3		–4
F	2	–1	9	1	4	

P^3_{ij}	A	B	C	D	E	F
A		0	P^3_{13}	0	P^3_{15}	0
B	P^3_{21}		P^3_{23}	P^3_{24}	P^3_{25}	P^3_{26}
C	0	0		0	0	0
D	P^3_{41}	0	P^3_{43}		P^3_{45}	0
E	0	0	P^3_{53}	0		0
F	P^3_{61}	0	P^3_{63}	P^3_{64}	P^3_{65}	

We deduce the preference index matrix C_{ij}, which is given in Table 3. From this matrix, we calculate the incoming flows, the outgoing flows as well as the net flows.

In order to determine the indifference threshold values q_k, we model the information provided by the decision-maker in a mathematical program 2.

Table 3

Preference index matrix

C_{ij}	A	B	C	D	E	F
A		$0.3P^1_{12}+0.5P^2_{12}$	$0.3P^1_{13}+0.2P^3_{13}$	$0.5P^2_{14}$	$0.2P^3_{15}$	$0.3P^1_{16}$
B	$0.2P^3_{21}$		$0.2P^3_{23}$	$0.5P^2_{24}+0.2P^3_{24}$	$0.2P^3_{25}$	$0.2P^3_{26}$
C	$0.5P^2_{31}$	$0.3P^1_{32}+0.5P^2_{32}$		$0.5P^2_{34}$	0	$0.5P^2_{36}$
D	$0.2P^3_{41}$	$0.3P^1_{42}$	$0.3P^1_{43}+0.2P^3_{43}$		$0.2P^3_{45}$	$0.3P^1_{46}$
E	$0.5P^2_{51}$	$0.3P^1_{52}+0.5P^2_{52}$	$0.3P^1_{53}+0.5P^2_{53}+0.2P^3_{53}$	$0.5P^2_{54}$		$0.3P^1_{56}+0.5P^2_{56}$
F	$0.5P^2_{61}+0.2P^3_{61}$	$0.3P^1_{62}+0.5P^2_{62}$	$0.2P^3_{63}$	$0.5P^2_{64}+0.2P^3_{64}$	$0.2P^3_{65}$	

Program 2:

$Min\ S_1^- + S_2^- + S_3^- + S_4^- + S_5^-$

Subject to

$E \succ F:\ 0.3P^1_{52} + 0.3P^1_{53} + 0.3P^1_{56} - 0.3P^1_{62} + 0.3P^1_{16} + 0.3P^1_{46} + 0.3P^1_{56} + 0.5P^2_{51} + 0.5P^2_{52} + 0.5P^2_{53} + 0.5P^2_{54} + 0.5P^2_{56} - 0.5P^2_{61} - 0.5P^2_{62}$
$-0.5P^2_{64} + 0.5P^2_{36} + 0.5P^2_{56} + 0.2P^3_{53} - 0.2P^3_{15} - 0.2P^3_{25} - 0.2P^3_{45} - 0.2P^3_{65} - 0.2P^3_{61} - 0.2P^3_{63} - 0.2P^3_{64} - 0.2P^3_{65} + 0.2P^3_{26} + S^-_1 - S^+_1 = 0;$

$A \succ D:\ 0.3P^1_{12} + 0.3P^1_{13} + 0.3P^1_{16} - 0.3P^1_{42} - 0.3P^1_{43} - 0.3P^1_{46} + 0.5P^2_{12} + 0.5P^2_{14} - 0.5P^2_{31} - 0.5P^2_{51} - 0.5P^2_{61} + 0.5P^2_{14} + 0.5P^2_{24} + 0.5P^2_{34}$
$+0.5P^2_{54} + 0.5P^2_{64} + 0.2P^3_{13} + 0.2P^3_{15} - 0.2P^3_{21} - 0.2P^3_{41} - 0.2P^3_{61} - 0.2P^3_{41} - 0.2P^3_{43} - 0.2P^3_{45} + 0.2P^3_{24} + 0.2P^3_{64} + S^-_2 - S^+_2 = 0;$

$D \succ B:\ 0.3P^1_{42} + 0.3P^1_{43} + 0.3P^1_{46} + 0.3P^1_{12} + 0.3P^1_{32} + 0.3P^1_{42} + 0.3P^1_{52} + 0.3P^1_{62} - 0.5P^2_{14} - 0.5P^2_{24} - 0.5P^2_{34} - 0.5P^2_{54} - 0.5P^2_{64} - 0.5P^2_{24}$
$+0.5P^2_{12} + 0.5P^2_{32} + 0.5P^2_{52} + 0.5P^2_{62} + 0.2P^3_{41} + 0.2P^3_{43} + 0.2P^3_{45} - 0.2P^3_{24} - 0.2P^3_{64} - 0.2P^3_{21} - 0.2P^3_{23} - 0.2P^3_{24} - 0.2P^3_{25} - 0.2P^3_{26} + S^-_3 - S^+_3 = 0;$

$F \succ C:\ 0.3P^1_{62} - 0.3P^1_{16} - 0.3P^1_{46} - 0.3P^1_{56} - 0.3P^1_{32} + 0.3P^1_{13} + 0.3P^1_{43} + 0.3P^1_{53} + 0.5P^2_{61} + 0.5P^2_{62} + 0.5P^2_{64} - 0.5P^2_{36} - 0.5P^2_{56} - 0.5P^2_{31}$
$-0.5P^2_{32} - 0.5P^2_{34} - 0.2P^2_{36} + 0.5P^2_{53} + 0.2P^3_{61} + 0.2P^3_{63} + 0.2P^3_{64} + 0.2P^3_{65} - 0.2P^3_{26} + 0.2P^3_{13} + 0.2P^3_{23} + 0.2P^3_{43} + 0.2P^3_{53} + 0.2P^3_{63} + S^-_4 - S^+_4 = 0;$

$E \succ B:\ 0.3P^1_{52} + 0.3P^1_{53} + 0.3P^1_{56} + 0.3P^1_{12} + 0.3P^1_{32} + 0.3P^1_{42} + 0.3P^1_{52} + 0.3P^1_{62} + 0.5P^2_{51} + 0.5P^2_{52} + 0.5P^2_{53} + 0.5P^2_{54} + 0.5P^2_{56} - 0.5P^2_{24}$
$+0.5P^2_{12} + 0.5P^2_{32} + 0.5P^2_{52} + 0.5P^2_{62} + 0.2P^3_{53} - 0.2P^3_{15} - 0.2P^3_{25} - 0.2P^3_{45} - 0.2P^3_{65} - 0.2P^3_{21} - 0.2P^3_{23} - 0.2P^3_{24} - 0.2P^3_{25} - 0.2P^3_{26} + S^-_5 - S^+_5 = 0;$

If $d^k_{ij} \le d^k_{hl}$ then $P^k_{ij} \le P^k_{hl}$ $\forall\ i \ne j,\ i = 1,\dots,6$ and $j = 1,\dots,6$, $\forall\ k = 1,\dots,3$
$\forall\ h \ne l,\ h = 1,\dots,6$ and $l = 1,\dots,6$

$P^k_{ij} \in \{0,1\}$ $\forall\ i \ne j,\ i = 1,\dots,6$ and $j = 1,\dots,6$, $\forall\ k = 1,\dots,3$

$S^+_m \ge 0{,}01$ $\forall\ m = 1,\dots,5$

$S^-_m, S^+_m \ge 0$ $\forall\ m = 1,\dots,5$

By solving the second program, we notice that it has multiple solutions respecting all preference relations provided by the decision-maker.

From the P^k_{ij}'s matrices, the d^k_{ij}'s matrices and the relation (14), we determine the indifference threshold values q_k and we deduce the type associated with each criterion. The results are given in Table 4.

Table 4

The multiple solutions of the mathematical program

Solutions	***1st criterion***		***2nd criterion***		***3rd criterion***	
	q_1	***Criterion type***	q_2	***Criterion type***	q_3	***Criterion type***
1st solution	0	Usual-criterion	6	Quasi-criterion	8	Quasi-criterion
2nd solution	0	Usual-criterion	2	Quasi-criterion	4	Quasi-criterion
3rd solution	0	Usual-criterion	0	Usual-criterion	4	Quasi-criterion
4th solution	0	Usual-criterion	1	Quasi-criterion	3	Quasi-criterion
5th solution	0	Usual-criterion	2	Quasi-criterion	2	Quasi-criterion
6th solution	0	Usual-criterion	1	Quasi-criterion	1	Quasi-criterion
7th solution	0	Usual-criterion	2	Quasi-criterion	1	Quasi-criterion
8th solution	0	Usual-criterion	2	Quasi-criterion	0	Usual-criterion
9th solution	0	Usual-criterion	0	Usual-criterion	2	Quasi-criterion
10th solution	0	Usual-criterion	2	Quasi-criterion	3	Quasi-criterion
11th solution	0	Usual-criterion	0	Usual-criterion	0	Usual-criterion
12th solution	0	Usual-criterion	0	Usual-criterion	1	Quasi-criterion
13th solution	0	Usual-criterion	1	Quasi-criterion	0	Usual-criterion
14th solution	0	Usual-criterion	1	Quasi-criterion	2	Quasi-criterion
15th solution	0	Usual-criterion	6	Quasi-criterion	4	Quasi-criterion
16th solution	0	Usual-criterion	5	Quasi-criterion	1	Quasi-criterion
17th solution	0	Usual-criterion	4	Quasi-criterion	1	Quasi-criterion
18th solution	0	Usual-criterion	4	Quasi-criterion	0	Usual-criterion
19th solution	0	Usual-criterion	6	Quasi-criterion	2	Quasi-criterion
20th solution	0	Usual-criterion	5	Quasi-criterion	0	Usual-criterion
21th solution	0	Usual-criterion	6	Quasi-criterion	3	Quasi-criterion

All the indifference threshold values q_k of each solution permit to respect preference relations provided by the decision-maker.

The decision-maker communicates to us the following information concerning intervals of indifference threshold values: $q_1 \in [0, 2]$, $q_2 \in [0, 4]$, $q_3 \in [2, 6]$. Among the solutions found, and taking into account the assumption that the indifference thresholds must have integer values, seven solutions belong to the fixed intervals (the 2nd, the 3rd, the 4th, the 5th, the 9th, the 10th and the 14th). In this case, we ask the decision-maker to reduce the intervals already fixed. Then, he presents to us the following new intervals: $q_1 \in [0, 2]$, $q_2 \in [2, 4]$,

$q_3 \in [4, 6]$. The 2nd solution belongs to the given intervals, therefore $\boldsymbol{q_1 = 0}$, $\boldsymbol{q_2 = 2}$, $\boldsymbol{q_3 = 4}$. The first criterion is then a true-criterion, whereas the second and the third are quasi-criteria.

While applying the **PROMETHEE** method with the indifference thresholds found, we get the following alternative ranking: *E*, *A*, *F*, *C*, *D*, *B* which satisfy the decision-maker.

Conclusions

In this paper, we clarified and illustrated an approach which permits to determine the indifference threshold values associated with each criterion in the framework of the PROMETHEE II method. This approach of indifference threshold determination presents the advantage of modelling with the unavoidable subjectivity and uncertainty at the level of the alternative assessment, as well as the direct intervention of the decision-maker in the decision process. In addition, it offers us the possibility to start from partial information concerning the preference relations on some pairs of alternatives in order to reach a total ranking, and this is in the context of PROMETHEE II method.

The extension of the methodology for the simultaneous determination of indifference and preference threshold values associated with the criteria function of type criterion with linear preference, level criterion, criterion with linear preferences and indifference area is a direction of research that we pursue presently, the preference threshold (p) corresponds to the minimum value of d^k_{ij} above which we consider that the alternative x_i is strictly preferred to x_j.

References

Bous G., Fortemps P., Glineur F. and Pirlot M. (2010): *ACUTA: A Novel Method for Eliciting Additive Value Functions on the Basis of Holistic Preference Statements.* “European Journal of Operational Research”, 206, pp. 435-444.

Brans J.P. and Vincke Ph. (1985): *A Preference Ranking Organization Method: The PROMETHEE Method*. “Management Science”, 31, pp. 647-656.

Devaud J.M., Groussaud G. and Jacquet-Lagrèze E. (1980): *UTADIS: une méthode de construction de fonctions d'utilité additives rendant compte de jugements globaux*. European Working Group on Multicriteria Decision Aid, Bochum.

Dias L.C. and Mousseau V. (2006): *Inferring ELECTRE's Veto Related Parameters from Outranking Examples*. “European Journal of Operational Research”, 170, pp. 172-191.

Figueira J.R., Greco S. and Slowinski R. (2009): *Building a Set of Additive Value Functions Representing a Reference Pre-order and Intensities of Preference: GRIP Method*. "European Journal of Operational Research", 195, pp. 460-486.

Frikha H.M., Chabchoub H., and Martel J.M. (2010): *Inferring Criteria's Relative Importance Coefficients in PROMETHEE II*. "International Journal of Operational Research", 7, pp. 257-275.

Geoffrion A.M., Dyer J.S. and Feinberg A. (1972): *An Interactive Approach for Multi--Criterion Optimization with An Application to the Operation of An Academic Department*. Management Science, 19, pp. 357-368.

Greco S., Mousseau V. and Slowinski R. (2008): *Ordinal Regression Revisited: Multiple Criteria Ranking Using a Set of Additive Value Functions*. "European Journal of Operational Research", 191, pp. 416-436.

Greco S., Mousseau V. and Slowinski R. (2010): *Multiple Criteria Sorting With a Set of Additive Value Functions*. "European Journal of Operational Research".

Jacquet-Lagreze E. (1979): *De la logique d'agrégation des critères à une logique d'agrégation-désagrégation de préférences et de jugements*. Cahiers de l'ISMEA – série Sciences de Gestion, 13, pp. 839-859.

Jacquet-Lagrèze E., Meziani R. and Slowinski R. (1987): *MOLP with an Interactive Assessment of a Piecewise Linear Utility Function*. "European Journal of Operational Research", 31, pp. 350-357.

Jacquet-Lagrèze E. and Siskos Y. (1982): *Assessing a Set of Additive Utility Functions for Multicriteria Decision Making: The UTA Method*. "European Journal of Operational Research", 10, pp. 151-164.

Kiss L.N., Martel J.M. and Nadeau R. (1994): *An Interactive Software for Modeling the Decision Maker's Preferences*. Decision Support Systems, 12, pp. 311-326.

Köksalan M. and Özpeynirci S.B. (2009): *An Interactive Sorting Method for Additive Utility Functions*. "Computers and Operations Research", 36, pp. 2565-2572.

Mousseau V., Figueira J. and Naux J.P. (2001): *Using Assignment Examples to Infer Weights for ELECTRE TRI Method: Some Experimental Results*. "European Journal of Operational Research", 130, pp. 263-275.

Mousseau V. and Slowinski R. (1998): *Inferring an ELECTRE TRI Model from Assignment Examples*. "Journal of Global Optimization », 12, pp. 157-174.

Ngo The A. and Mousseau V. (2002): *Using Assignment Examples to Infer Category Limits for the ELECTRE TRI Method*. "Journal of Multi-Criteria Decision Analysis", 11, pp. 29-43.

Richard J.L. (1981): *Procédure multicritère d'aide à la décision en matière d'audit de stratégie: cas des moyennes et petites industries*. Thèse de 3ème cycle, Université de Paris Dauphine, Paris.

Siskos J. (1983): *Analyse de systèmes de décision multicritère en univers aléatoire*. „Foundations of Control Engineering", 8, pp. 193-212.

Siskos J. (1980): *Comment modéliser les préférences au moyen de fonctions d'utilité additives*. "RAIRO Recherche Opérationnelle", 14, pp. 53-82.

Siskos J. and Despotis D.K. (1989): *A DSS Oriented Method for Multiobjective Linear Programming Problems.* "Decision Support Systems", 5, pp. 47-55.

Stewart T.J. (1987): *Pruning of Decision Alternatives in Multiple Criteria Decision Making, Based on the UTA Method for Estimating Utilities*. "European Journal of Operational Research", 28, pp. 79-88.

Tangian A. (2001): *Constructing a Monotonic Quadratic Objective Function in n Variables from a Few 2-Dimensional Indifferences.* "European Journal of Operational Research", 130, pp. 276-304.

Zionts S. and Wallenius J. (1976): *An Interactive Programming Method for Solving the Multiple Criteria Problem*. "Management Science », 22, pp. 652-663.