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R&D RIVALRY AND COOPERATION IN DUOPOLY: FIRM ORGANIZATION, WELFARE AND POLICY IMPLICATIONS

Abstract

The objective of the paper is to reveal the optimal organization of industry when firms, facing externalities, compete or cooperate in R&D as well as in the final output market. The model hinges on a two-stage game setting. A ranking of solutions is established for alternative organizations. We focus on welfare issues and allow for public intervention. Subsidizing R&D is used to draw the industry to match the social welfare solution. The paper shows that targeting the optimal level of R&D leaves final output fall short of the welfare solution. Whereas targeting the final output leads to overinvestment in R&D. The ranking of policies reveals that the most efficient industry organization occurs when firms cooperate and fully share R&D results, but remain competitive in the final good market.

Keywords

R&D, subsidies, welfare, spillover, two stage-game, Cournot equilibrium.

Introduction

The literature dealing with R&D cooperation and policy regulations has focussed on the main private advantages and disadvantages of such agreements as well as the main public costs and benefits*. There are more difficulties encountered in setting up R&D cooperation, compared to other fields in economy, even though social welfare benefits are more likely to occur from such agreements. Cooperation in R&D appears as an alternative to pure market transactions on one hand and to full integration within a firm on the other hand. A cooperative research arrangement for instance, can reduce problems of asymmetric information as market transactions are liable to be affected

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* The main papers related to the subject are those of A. Jacquemin [1988], M. Katz [1986] and M. Spence [1984].

by moral hazard and adverse selection. The opposite case of mergers can tend to create quite rigid structures curtailing attempts to switch research capacity and strategy, or more generally to respond quickly to innovation over time.

Despite their many private advantages, cooperative agreements in R&D are not very frequent to observe. When they occur, they are usually fragile constructions with various difficulties to overcome and are either dismantled or absorbed through merger operations. The main argument in favour of co-operation stems from market failure. Such a situation prevents the firm from appropriating completely the benefits of R&D activity. The amount of research produced and diffused by private firms may be socially inefficient, whatever the market structure is. We need to distinguish between two situations:

1. The one without externalities: that is when each firm's R&D affects only its own cost. Competition among firms will usually lead to wasteful duplication of research. Investment in R&D is greater than what is socially needed.
2. In the case of substantial R&D externalities or spillovers, the benefits of each firm's R&D effort flow freely to other firms. In such a situation there is underinvestment in R&D compared to what the social optimal level would require. Incentive to innovate will be reduced as the innovator is aware that competitors will strengthen, in a costless way, their competitive position through his R&D investment.

It can then be argued that cooperative R&D can improve both situations. According to M. Spence [1984], the incentive of a firm to invest in R&D needs a sufficient amount of appropriation of the benefits, therefore a limited diffusion of knowledge. At the same time tightening conditions, to create a nearly perfect appropriation, impedes spillovers of R&D to other firms and will thus prevent cost reduction to spread across the sector. Cooperative R&D is then viewed as the means through which these two objectives can be achieved simultaneously, that is:

– internalizing, into an appropriate organization, the externalities generated by a high level of R&D spillover, and
– providing a better sharing of information among the participant firms.

The incentive to invest in R&D is improved, at the same cooperation will avoid to devote resources to wasteful duplication.

Our objective is to address the question of the socially optimal organization of the industry when firms compete or cooperate in R&D as well as in the final good market. We also discuss the policy of subsidizing R&D activities (not final output) as an incentive to reach welfare objectives.

Our analysis shares a good deal of similarity with the pioneering approach introduced by C. d'Aspermont and A. Jacquemin [1988]. They consider a two-stage game in a duopolistic setting: in the first stage firms conduct research in order to reduce unit costs, and are Cournot competitors in the second stage; that is in the final output market. The focus of their analysis is on comparison of cost reducing achieved when firms conduct R&D cooperatively or as competitors, in the presence of spillover effects.

There are many extensions and related papers to the d'Aspermont--Jacquemin approach, but they do not explicitly address policies, such as subsidies, and their welfare issues related to the model*.

Having sketched out the economic background, some technical aspects of the model are now underlined:

1. Any one firm, while maximizing its objective function, or profit, decides on the level of R&D output (x) as well as on the final good production level (q). These two choice variables are technically determined by a sequence of operations: R&D in the first step and final output in the subsequent stage. Moreover the firm's own decision on output depends on the other firm's behaviour, thus reflecting the market structure. The underlying industry organization can range from full rivalry at both stages to a completely integrated monopoly.
2. The other feature is the spillover effect (β), as all output solutions depend on this parameter. It can also be interpreted as the proportion of R&D results, the firms are willing to share, either within a coalition or if they remain rivals. Alternative information sharing hypothesis and objective functions consistent with industry organization are summarized in the Appendix.
3. The main focus of our model is optimizing a social welfare objective. Together with profit maximization by firms, we allow for public intervention. Taking into account anti-trust regulations, subsidies are used to fund R&D activities in order to drive the industry organization to meet the welfare solution. The social planner controls one instrument, the subsidy, whereas there are two target variables: R&D as well as final good outputs. Some compromise has to be established, and welfare solutions will hinge on the spillover parameter β.

The paper is organized in the following way. In section 1, we introduce the two stage-game model. Two alternative cases are discussed, one when rivalry between firms occurs at both stages, the other when firms coordinate R&D activities aiming to maximize joint profits but remain competitive in the final product market. The analysis focuses on stability issues as well as the switch of the slope of the reaction functions as the spillover parameter increases. We use numerical simulations to plot the behaviour of some

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* Contributions like those of M. Kamien et al. [1992], K. Suzumura [1992], De Bondt et al. [1992] are direct extensions of the effect of spillovers in R&D with many firms and many stages (mainly two) in R&D operations. Welfare issues are limited to R&D levels in Suzumura and are imbedded in a broader comparison of cooperative issues, in M. Kamien et al., namely joint ventures. This last point will be discussed further in this paper.

significant variables. We show that investment in R&D is a decreasing function of spillovers in the competitive case, whereas it is increasing in the cooperative case. For this latter case, R&D spending grows faster than the final good output beyond the switch point, magnifying the increase in profits. These results are indications of the performances of cooperation relations among firms; from simply coordinating R&D to the full sharing of information in order to eliminate duplication and free riding (discussed in section 3). Section 2 examines two alternative cases when industry is fully integrated. The first case is a private monopoly, while the other can be considered as a public monopoly that seeks to maximize total surplus. This last case unambiguously yields the highest levels of R&D spending as well as final product output. This is the standard welfare case in the d'Aspermont-Jacquemin model against which all other equilibriums are compared. We can then establish a ranking of solutions as to guide the implementation of economic policy. This point is taken up in section 3 where we introduce subsidies in order to highlight the cost of drawing the industrial organization to match the social welfare solution. We show that targeting the optimal R&D investment level would still leave final output fall short from the welfare solution. Whereas targeting the final output objective will overshoot the optimal level of spending on R&D. Subsidies are by no means substitutes to cooperation among firms. The analysis reveals the most adequate industrial organization liable to fulfil the welfare objective: this is cooperation and full sharing of information in R&D, while remaining competitive in the final good market. Subsidies are viewed as efficient incentives to stabilize such cooperative agreements in R&D activities. The final section gathers conclusions and considers some extensions of the analysis.

The Appendix contains tables summarizing the notations for alternative models discussed in the paper.

1. Competition and cooperation in R&D with spillovers

1.1. General assumptions

We consider an industry with two firms. They face the linear inverse demand function given by:

$$p = a - bQ \tag{1}$$

where p is the price and $Q = q_i + q_j$ is the total amount of a homogenous good produced by firms i and j, with $a, b > 0$ and $Q \leq \frac{a}{b}$.

Production cost is such as $C_i(q_i, x_i, x_j)$ is a function of its own final good output q_i, the amount of research x_i that it undertakes and the amount of the rival's firm research x_j:

$$C_i = (q_i, x_i, x_j) = (A - x_i - \beta x_j) q_i, \quad i = 1,2 \quad i \neq j. \tag{2}$$

With unit cost $c_i = (A - x_i - \beta x_j) \geq 0$, $0 < A < a$ and $0 \leq \beta \leq 1$ where β is the spillover parameter*. Moreover the cost of R&D is chosen to be quadratic as we may assume the production process to exhibit diminishing returns to scale; that is: $\frac{\gamma}{2} x_i^2$, $(i = 1,2)$.

The model and its variants feature a two-stage game with two firms. In this section, during the second stage, firms are assumed to engage in Cournot competition, while in the first stage they invest in R&D. For the first variant, there is R&D competition in which firms maximize their individual profits by deciding unilaterally on their R&D investments. In the second case firms coordinate R&D activities such as to maximize joint profits while maintaining competition in the final output production stage.

There are alternative organizations and different levels of cooperation in which firms can be involved while coordinating R&D activities. In one sub--case, we may consider that coordination does not necessarily mean total sharing of results between partners. One participant firm may be allowed (by an agreement) to carry out some propriety research; and hence duplication is not completely eliminated ($\beta < 1$). When results of R&D activities are fully shared, the spillover rate is at its maximal level, that is $\beta = 1$**.

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* When $\beta = 0$, we have the Brander/Spencer [1983] two- stage duopoly in R&D game. When $\beta = 1$, in a cooperative duopoly in R&D, then means full sharing of information as in M. Kamien et al. [1992].

** These cases are used by M. Kamien et al. [1992] to distinguish between *research joint venture* ($\beta = 1$) and R&D *cartelization* $\beta < 1$. Further cases are discussed, particularly the one with competition in both stages, but with fully sharing of R&D results. These "other" cases are imbedded in our analysis in this section and also while discussing policy implications in section 3. The authors do not introduce the two full cooperation cases of our section 2.

A firm's payoff consists of the second stage production profit less the first stage R&D cost. The cooperative or non-cooperative solutions to this first stage are then obtained by maximizing profits with respect to levels of R&D(x_i, x_j). We can then compare the corresponding sub--game perfect equilibrium.

1.2. The rivalry solution

Both firms act non-cooperatively at both stages of the game. Firm i maximizes its second stage profit, conditional on x_i and x_j, by choosing its output and assuming the output of the rival firm j is fixed: $\underset{q_i / q_j fixed}{Max} \pi_i = pq_i - C_i - \frac{\gamma}{2} x_i^2$.

The first order condition yields:

$$\frac{\partial \pi_i}{\partial q_i} = -bq_i + \left[a - b\left(q_i + q_j\right)\right] - \left[A - x_i - \beta x_j\right] = 0. \tag{3}$$

By collecting terms and using relation (2), the profit maximization condition (3) gives:

$$\left(p - c_i\right) = bq_i \text{, and the maximized profit is: } \pi_i^* = bq_i^2 - \frac{\gamma}{2} x_i^2 \tag{4}$$

where q_i and x_i are to be replaced by the optimal levels of output and R&D expenditure.

Solving for condition (3) also yields the reaction function for output: $q_i = -\frac{1}{2} q_j + \frac{\left(a - c_i\right)}{2b}$. Using relation (2) and arranging terms gives:

$$q_i = -\frac{1}{2} q_j + \frac{\left(a - A\right)}{2b} + \frac{x_i + \beta x_j}{2b}. \tag{5}$$

By the symmetry assumption there is a similar function for firm j. Solving the two reaction functions yields the second stage output:

$$q_i = \frac{\left(a - A\right) + \left(2 - \beta\right) x_i + \left(2\beta - 1\right) x_j}{3b}. \tag{6}$$

The maximized profit expression for firm i in (4) can be written as:

$$\pi_i^* = \frac{1}{9b}\left[\left(a - A\right) + \left(2 - \beta\right) x_i + \left(2\beta - 1\right) x_j\right]^2 - \frac{\gamma}{2} x_i^2 \quad i = 1,2 \quad i \neq j. \tag{7}$$

Expression (7) shows the influence of R&D levels on the profit through the output of the final good, the unit cost of production and the expenses devoted to R&D levels themselves. At the initial stage of the game, the non--cooperative level of R&D, x_i, is chosen to maximize the profit given in (7), assuming that the rival's investment in R&D x_j is fixed: $\underset{x_i / x_j fixed}{Max}\ \pi_i^*$.

The first order condition for profit maximization is given by:

$$\frac{\partial \pi_i}{\partial x_i} = \frac{1}{9b} 2(2-\beta)\left[(a-A)+(2-\beta)x_i+(2\beta-1)x_j\right]-\gamma x_i = 0. \tag{8}$$

The second order condition for a maximum: $\frac{\partial^2 \pi_i}{\partial x_i} < 0$, requires: $\frac{9}{2} b\gamma > (2-\beta)^2$ *.

The reaction function for R&D levels associated with this initial stage of the game is found by solving (8):

$$x_i = \frac{2(2-\beta)(2\beta-1)x_j + 2(2-\beta)(a-A)}{9b\gamma - 2(2-\beta)^2} \qquad i=1,2 \quad i \neq j. \tag{9}$$

Solving the reaction functions for R&D levels yields:

$$x_i^* = x_j^* = \frac{(a-A)(2-\beta)}{\frac{9}{2} b\gamma - (2-\beta)(1+\beta)}. \tag{10}$$

The optimal output for final goods is obtained using relation (6):

$$q_i^* = q_j^* = \frac{(a-A)}{3b}\left[\frac{\frac{9}{2} b\gamma}{\frac{9}{2} b\gamma - (2-\beta)(1+\beta)}\right]. \tag{11}$$

Total industry output is: $Q^* = q_i^* + q_j^*$. We can revert to relation (4) and compute the maximized profit for any one firm:

$$\pi_i^* = \frac{(a-A)^2 \gamma \left[9b\gamma - 2(2-\beta)^2\right]}{4\left[\frac{9}{2} b\gamma - (2-\beta)(1+\beta)\right]^2}. \tag{12}$$

* For $\gamma = b = 1$, even if β is at its maximum value, $\beta = 1$, the condition is always met. More constraining conditions appear when we discuss elaborate organization and policy issues as in section 3.

We can check that these results, contained in relations (10), (11) and (12) for the rivalry case in both stages are consistent when the following condition holds:

$$b\gamma > \frac{2}{9}(2-\beta)(1+\beta)^{*}.$$

1.3. The cooperative R&D solution

Let us consider that firms, while still being competitors in the product market, coordinate their R&D effort in order to maximize the joint profit. The second stage of the game is therefore unchanged and equation (6) and (7) still hold. The joint profit is given by $\Pi = \pi_i + \pi_j$. Considering the symmetric solution $x_i = x_j = x$, and using equation (7) yields the joint profit as a function of the R&D investment of any one firm:

$$\Pi = \frac{2}{9}\left[(a-A)+(2-\beta)x+(2\beta-1)x\right]^2 - \gamma x^2.$$

When arranging the terms in the brackets the joint profit can be written as:

$$\Pi = \frac{2}{9}\left[(a-A)+(1+\beta)x\right]^2 - \gamma x^2. \tag{13}$$

The first order condition $\frac{\partial \Pi}{\partial x} = 0,$ yields the solution for the R&D level of expenditure**:

$$\bar{x} = \frac{(a-A)(1+\beta)}{\frac{9}{2}b\gamma - (1+\beta)^2}. \tag{14}$$

The corresponding output of the final good for any firm is:

$$\bar{q} = \frac{(a-A)}{3b}\left[\frac{\frac{9}{2}b\gamma}{\frac{9}{2}b\gamma - (1+\beta)^2}\right]. \tag{15}$$

* We can compare this result with the second order condition on the profit and the indication given in footnote on p. 110.

** The second order condition for profit maximization is given by: $b\gamma > \frac{2}{9}(1+\beta)^2.$

We can then easily compute total output as $\overline{Q} = 2\overline{q}$. The profit of any one firm is given by:

$$\overline{\pi} = \frac{(a-A)^2 \gamma \left[9b\gamma - 2(1+\beta)^2\right]}{4\left[\frac{9}{2}b\gamma - (1+\beta)^2\right]^2} . \tag{16}$$

Results given by (14), (15) and (16) depend on the degree of spillovers which does not necessarily reach its maximum value ($\beta = 1$) because of cooperation. The agreement set between firms does not totally eliminate duplication because the sharing of information is not complete among the participants. The spillover parameter β plays a crucial role in the analysis as the following comparisons of output and R&D expenditure show. We can easily check that $\overline{x} > x^*$ if $\beta > \frac{1}{2}$, that is R&D effort is greater when firms cooperate compared to the case when they are rivals, only if externalities are high. We can also establish that $\overline{Q} > Q^*$, total output level is also higher when firms cooperate under the same condition on the spillover parameter β.

1.4. The potential gains from cooperation

In the traditional quantity competition Cournot model, reaction functions are downward sloping, and stability of the solution can be examined by comparing these slopes in the quantity space. When plotted in the (x_i, x_j) space, the slope of the reaction functions of firm i and j, using (9), are given by the following expressions:

Slope for firm $j = \frac{2(2-\beta)(2\beta-1)}{9b\gamma - 2(2-\beta)^2}$, and by the symmetry assumption,

Slope for firm $i = \frac{9b\gamma - 2(2-\beta)^2}{2(2-\beta)(2\beta-1)}$. By the second order condition on profit maximization, $9b\gamma - 2(2-\beta)^2 > 0$, therefore expressions of the slopes are negative if: $2(2-\beta)(2\beta-1) < 0$, but as $\beta \le 1$, the condition reduces to $\beta < \frac{1}{2}$. Reaction functions are downward sloping for weak values of the spillover parameter*.

* The useful reference for discussing stability issues is I. Henriques [1990].

As Figure 1 shows, there is a stable equilibrium when the slope of the reaction function of firm i is greater, in absolute value, than the slope of firm j ‘s reaction function. Such a condition holds when:

$$\frac{2(2-\beta)^2-9b\gamma}{2(2-\beta)(2\beta-1)}>1. \tag{17}$$

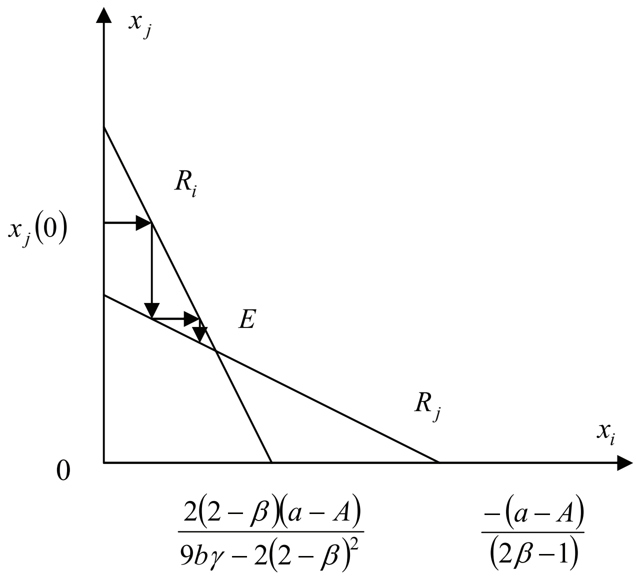

Figure 1. Adjustment path with downward sloping reaction functions

R_i and R_j are the reaction functions of firm i and firm j respectively. They are shown for $\beta<\frac{1}{2}$.

In order to maintain comparisons with other key variables, we set $b=\gamma=1$. From equation (17), we get the equivalent condition*: $2\beta^2-6\beta+1>0$. It can easily be checked that this inequality holds for $\beta>0.177$. Therefore for low spillovers there is a stable solution only if: $0;177<\beta<\frac{1}{2}$.

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* The condition is equivalent to show that the intercept with the x_i axis of reaction function R_j is greater than the intercept of the R_i reaction function.

Beyond the critical value of $\beta = \frac{1}{2}$, the slope of the reaction functions is reversed and the level of R&D of any one firm is an increasing function of the rival's expenditure on R&D as depicted in Figure 2.

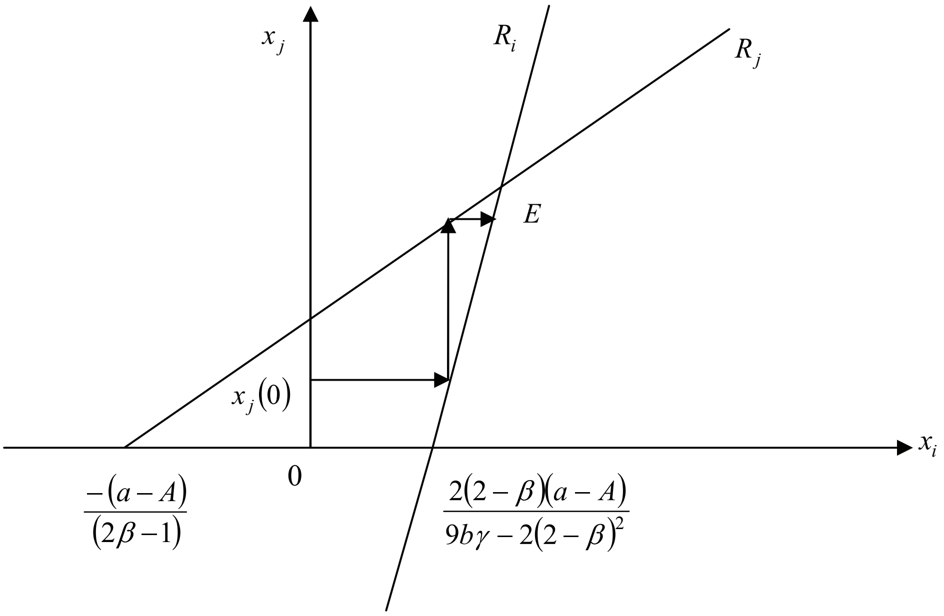

Figure 2. Adjustment path with upward sloping reaction functions

It can easily be shown that stability then holds for all values of $\beta > \frac{1}{2}$*

What are the potential gains expected from cooperation when we allow for externalities? This issue is best addressed when we compare cooperation and rivalry key variables of the model for the whole range of values of the spillover parameter β. The significant variables chosen for both cases are output, R&D levels and also profits. The numerical values have been computed using equations (10), (11) and (12) for the rivalry case and equations (14), (15) and (16) for the cooperative case. The values for parameters b and γ are unchanged from the preceding discussion; that is $b = \gamma = 1$.

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* We can check that the reaction function of firm i is steeper than the one of firm j, and stability occurs as the condition $2\beta^2 - 2\beta + 5 > 0$ is always satisfied for $\beta > 0.5$.

In equations (10) and (14), we set $m = (a - A) > 0$, and plot the behaviour of R&D levels from the values obtained by Table 1.

Table 1

β	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x^*	$0.77m$	$0.74m$	$0.70m$	$0.66m$	$0.6m$	$0.56m$	$0.51m$	$0.45m$	$0.4m$
$\bar{x}$	$0.39m$	$0.46m$	$0.55m$	$0.66m$	$0.82m$	$1.06m$	$1.42m$	$2.11m$	$4m$

These values are plotted in Figure 3.

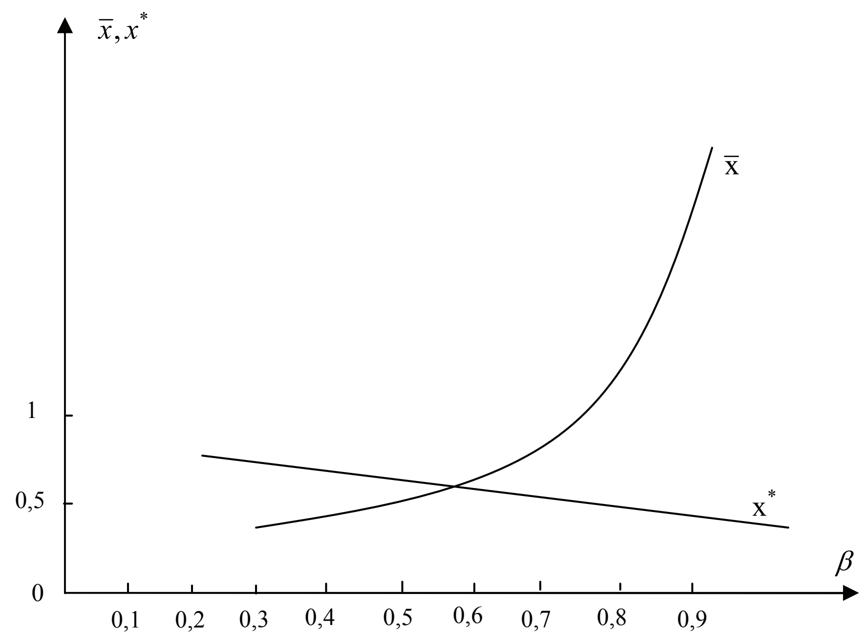

Figure 3. Comparative behaviour of R&D levels between competition and cooperation

The R&D level is decreasing in the non-cooperative situation for all ranges of β, whereas it is increasing in the cooperative case. R&D expenditures are greater, when rivalry prevails, compared to cooperation for weak values of the spillovers. The significant feature is the very rapid growth of R&D investment in the cooperation case when the curve crosses the switch point value $\beta = \frac{1}{2}$.

Final outputs behaviour, equations (11) and (15), reveals similar results as shown by Table 2.

Table 2

β	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q^*	0.64m	0.65m	0.66m	0.66m	0.66m	0.65m	0.64m	0.62m	0.60m
$\overline{q}$	0.49m	0.53m	0.59m	0.66m	0.77m	0.93m	1.12m	1.68m	3m

Final output rises, is stationary, and then declines very slowly in the non--cooperative case. It is an increasing function of β in the cooperative case, also showing a very quick growth after the switch point. Rather than plotting these results one against the other, it is significant for further interpretations to link the behaviour of R&D to output in each case as depicted in Figures 4 and 5.

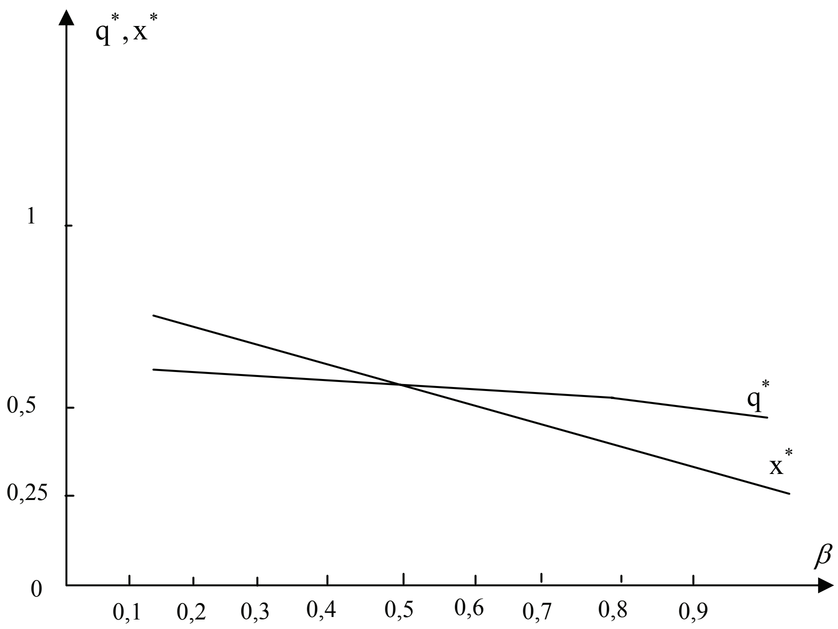

Figure 4. R&D and final output levels in the rivalry case

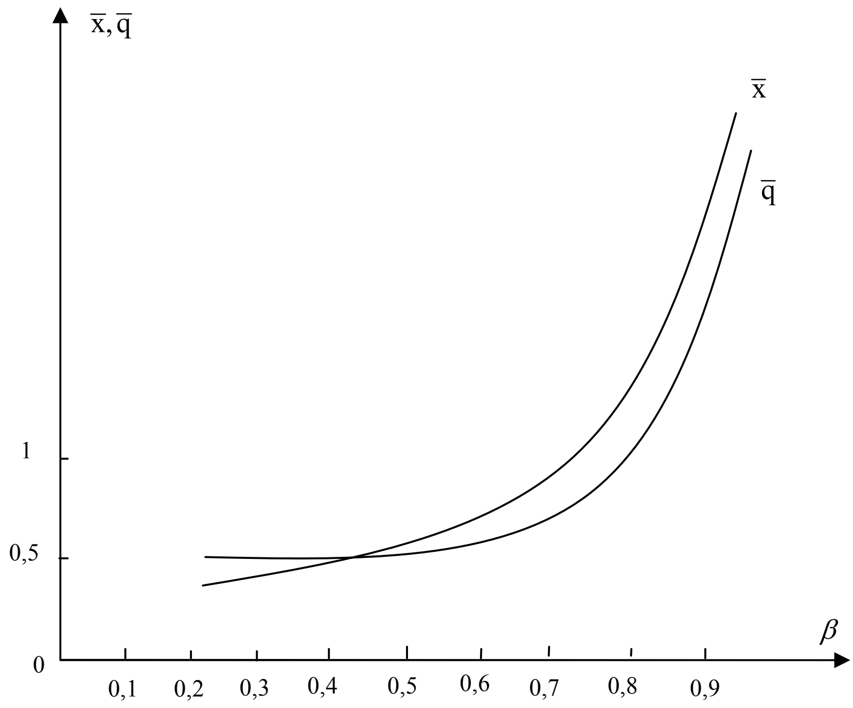

Figure 5. R&D and final output levels in the cooperative case

For high spillover rates, the reduction in cost is greater when firms coordinate R&D activities than when they remain rivals. There are two types of externalities generated by R&D activities when spillovers are meaningfully high:

– The first type is linked to a firm's competitiveness relative to its rivals. Any firm investing in R&D to reduce its unit cost, takes into account the fact that the spillover reduces to some extent the cost of the rival firm making it a tougher competitor.
– The other type, affects the performance of the industry as a whole. This second aspect is ignored under R&D competition. It is internalized in the process of choosing the level of R&D spending to maximize joint profits when firms cooperate within an adequate structure such as a cartel.

Another interesting way to look at the problem is to point out that cooperation acts to eliminate duplication. Moreover in the non-cooperative model, as the level of spillover rises firms tend to "free-ride" on the other firm's knowledge as we observe that R&D levels fall when β rises.

These results are also reflected by comparing the behaviour of profits in Table 3.

Table 3

β	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
π^*	$0.11\,m^2$	$0.15\,m^2$	$0.19\,m^2$	$0.22\,m^2$	$0.24\,m^2$	$0.26\,m^2$	$0.28\,m^2$	$0.28\,m^2$	$0.28\,m^2$
$\overline{\pi}$	$0.16\,m^2$	$0.18\,m^2$	$0.20\,m^2$	$0.22\,m^2$	$0.26\,m^2$	$0.30\,m^2$	$0.39\,m^2$	$0.56\,m^2$	m^2

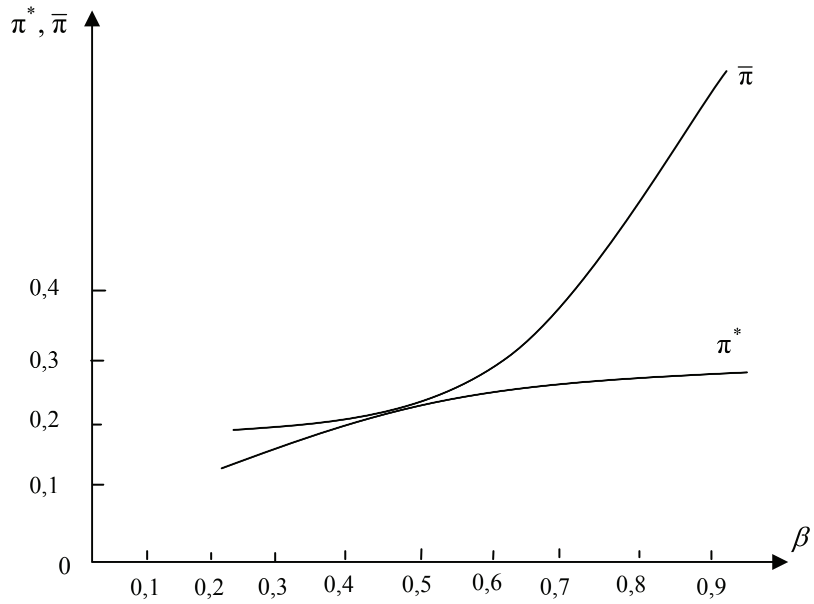

Figure 6. Comparative profit behaviour

In the rivalry case, the decline in both the R&D level and final output (the latter inhibits a sharp fall in price) drives the profit to a stationary value for high spillovers. Even if the firms form a *joint venture*, as discussed in M. Kamien et al. [1992], to share R&D results, it is not clear why there should be substantial gains from such an agreement. On the contrary when firms form a cartel and coordinate activities in order to maximize the joint profit, then the gains are magnified as R&D results are shared between participants (spillover reaches its maximal value).

2. The integrated industry

We assume in this section that firms cooperate in both stages of the game. In the first sub-case the fully integrated industry behaves like a private monopoly. In the other, which is the interesting alternative introduced by d'Aspermont and Jacquemin [1988], there is full cooperation in order to maximize total surplus. We look at this second situation as a public monopoly seeking to achieve a social welfare objective.

2.1. The private monopoly

As the industry is now fully integrated, the joint profit is given by:

$$\Pi = [a - bQ]Q - AQ + (x_i + \beta x_j)q_i + (x_j + \beta x_i)q_j - \frac{\gamma}{2}x_i^2 - \frac{\gamma}{2}x_j^2.$$

From the symmetry assumption, we can set the following equalities:

$$x_i = x_j = x^M \text{ and } q_i = q_j = q^M.$$

The expression for the joint profit is then given by:

$$\Pi = [a - bQ]Q - AQ + (1+\beta)xQ - \gamma x^2 \tag{18}$$

or by: $\Pi = pQ - [A - (1+\beta)x]Q - \gamma x^2$, which is also equivalent to $\Pi = (p-c)Q - \gamma x^2$
where $c = [A - (1+\beta)x]$ is the unit cost of producing the final output.

The first order condition for a maximum is given by:

$$\frac{d\Pi}{dQ} = -bQ + [A - bQ] - A + (1+\beta)x = 0.$$

Solving for Q leads to the monopoly output as a function of R&D expenditure:

$$Q^M = \frac{1}{2b}[(a - A) + (1+\beta)x]. \tag{19}$$

Substituting the value of monopoly output in (18) leads to:

$$\Pi \equiv \pi^M = \frac{1}{b}\left[\frac{(a-A) + (1+\beta)x}{2}\right]^2 - \gamma x^2.$$

The integrated firm now chooses the level of R&D to maximize the profit*, which yields:

$$x^M = \frac{(a-A)(1+\beta)}{\left[4b\gamma - (1+\beta)^2\right]}. \tag{20}$$

The second order condition on profit maximization is given by: $b\gamma > \frac{1}{4}(1+\beta)^2$. It is sufficient for a positive output level in R&D as well as for final output:

$$Q^M = \frac{1}{2b}\left[\frac{4b\gamma(a-A)}{4b\gamma - (1+\beta)^2}\right]. \tag{21}$$

2.2. The public monopoly

Let us look at the welfare objective of the monopoly. Total surplus is made up of consumer's surplus S_c and profits Π. We call this the welfare objective noted as:

$$W(Q) = S_c + \Pi \tag{22}$$

From the inverse demand function given in relation (1), we see that the maximum price, which drives output to zero is $p_{\max} = a$. The expression of consumer's surplus is given by:

$S_c = \frac{(a-p)Q}{2}$, using the inverse demand function given in relation (1) leads to:

$$S_c = \frac{bQ^2}{2}. \tag{23}$$

By using the expression of the joint profit in (18) and consumer's surplus given by (23), the welfare function can be expressed as:

$$W(Q) = -\frac{1}{2}bQ^2 + \left[(a-A) + (1+\beta)x\right]Q - \gamma x^2. \tag{24}$$

* It can be shown that $x^M > x^*$ if $\beta > 0.41$, and $x^M > \bar{x}$ always holds.

Maximizing with respect to Q leads to: $Q = \dfrac{(a-A)+(1+\beta)x}{b}$, and the maximized welfare by the choice of Q is given by:

$$W(Q) = \frac{1}{2}bQ^2 - \gamma x^2 \quad \text{or as: } W(Q) = \frac{1}{2}b\left[\frac{(a-A)+(1+\beta)x}{b}\right]^2 - \gamma x^2.$$

The social planner now chooses the level of R&D to maximize total surplus. The second order condition is satisfied if: $b\gamma > \frac{1}{2}(1+\beta)^2$, and the solution is given by:

$$\overset{\#}{x} = \frac{(a-A)(1+\beta)}{2b\gamma - (1+\beta)^2}. \tag{25}$$

and the corresponding output is:

$$\overset{\#}{Q} = \frac{(a-A)}{b}\left[\frac{2b\gamma}{2b\gamma - (1+\beta)^2}\right]. \tag{26}$$

Solutions given by (25) and (26) are the social standard to classify the various results. By reverting to equations (10), (11), (14), (15), (20) and (21), we can establish the following rankings:

$$\overset{\#}{x} > x^M > \overline{x} > x^*$$

and

$$\overset{\#}{Q} > \overline{Q} > Q^* > Q^M. \tag{27}$$

Being fully integrated, the monopoly is more efficient in R&D activities. This effort is devoted to the sole objective of maximizing the profit when the organization is a private monopoly. Final output, and therefore consumer's surplus, then shows to be the least compared to all other situations. The case of the public monopoly maximizing social welfare is the main feature of the model as it shows that both, the R&D and final output, levels can be increased.

There are many reasons, such as anti-trust regulations, that prevent firms from cooperating in the final output market. The policy maker may revert to subsidies, in funding R&D activities to reach welfare objectives. This point is picked up in the following section.

3. Achieving welfare: subsidies and firm organization

Let "s" be the "marginal cost reducing subsidy" used to fund R&D activities. We shall henceforth refer to "s" as the "unit marginal subsidy". The quadratic cost function* for producing R&D is now changed to: $\frac{1}{2}(\gamma - s)x^2$. We also assume that $\gamma > s$, that is funding will only pay a share of total costs devoted to R&D expenditures.

3.1. Funding R&D of the private monopoly

From the ranking conditions given in (27), we notice that the private monopoly's R&D level is the nearest to the socially optimal level compared to the other cases discussed earlier. It is therefore tempting to ask what would be the consequence of subsidizing R&D activities to attain a welfare objective.

1. The optimal R&D objective.

The maximized profit by the choice of output is $\Pi = bQ^2 - (\gamma - s)x^2$, but monopoly output is still given by equation (19). Maximizing the profit when subsidies enter the R&D cost function yields:

$$x^{Ms} = \frac{(a-A)(1+\beta)}{4b(\gamma - s) - (1+\beta)^2}. \tag{28}$$

We can easily check that monopoly output given in equation (21), changes to:

$$Q^{Ms} = \frac{1}{2b}\left[\frac{4b(\gamma - s)(a-A)}{4b(\gamma - s) - (1+\beta)^2}\right]. \tag{29}$$

All solutions are fundamentally unaltered in their general structure; the term γ being replaced by $(\gamma - s)$. If the policy maker sets "s" to reach the socially optimal R&D investment level $\overset{\#}{x}$, we can then compute the adequate subsidy by setting: $x^{Ms} = \overset{\#}{x}$, that is:

* Introduced in section 1's general assumptions.

$$\frac{(a-A)}{\left[4b(\gamma-s)-(1+\beta)^2\right]}=\frac{(a-A)(1+\beta)}{\left[2b\gamma-(1+\beta)^2\right]}.$$

The unit marginal subsidy, when the optimal level of R&D is targeted, is given as:

$$s^{MR}=\frac{1}{2}\gamma. \tag{30}$$

What is the impact of such funding on the output of the final good? Monopoly output when R&D is chosen as a target, Q^{MsR}, is obtained by substituting the value of the unit subsidy given by (30) in expression (29), which yields:

$$Q^{MsR}=\frac{1}{2b}\left[\frac{2b\gamma(a-A)}{2b\gamma-(1+\beta)^2}\right]. \tag{31}$$

When we compare this result with the socially optimal output of final goods, $\overset{\#}{Q}$ given in (26), we get:

$$Q^{MsR}=\frac{1}{2}\overset{\#}{Q}. \tag{32}$$

Even if output is increased*, the cost reduction due to subsidies, will be used to improve profits by charging consumers a relatively high price compared to the public monopoly. Nevertheless, with higher output for final goods, the subsidy will shift some of the rent captured by the private monopoly to consumers.

2. The optimal output objective.

If we seek to reach the welfare output using subsidies that reduce costs of producing R&D, we have to solve for "*s*" such that $Q^{Ms}=\overset{\#}{Q}$:

$$\frac{1}{2b}\left[\frac{4b(\gamma-s)(a-A)}{4b(\gamma-s)-(1+\beta)^2}\right]=\frac{1}{b}\left[\frac{2b\gamma(a-A)}{2b\gamma-(1+\beta)^2}\right].$$

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* We can check that $Q^{MsR}>Q^{M}$.

Solving for "s" yields the subsidy needed to attain this objective:

$$s^{MQ} = \frac{2b\gamma^2}{\left[2b\gamma + (1+\beta)^2\right]}. \tag{33}$$

We can check that $s^{MQ} > s^{MR}$ if $2b\gamma > (1+\beta)^2$. This last inequality is the second order condition for maximizing the welfare objective. This result is predictable as it is more costly to "pull" monopoly output to the socially optimal value than to do the same with R&D levels when the ranking given in (27) holds. This higher value of unit marginal subsidy will mechanically increase the R&D level beyond its socially optimal value. Substituting (23) in equation (28), gives the subsidized R&D level, when final output is the target:

$$x^{MsQ} = \frac{(a-A)\left[2b\gamma + (1+\beta)^2\right]}{(1+\beta)\left[2b\gamma - (1+\beta)^2\right]}. \tag{34}$$

Comparing x^{MsQ} given by (34) with the value of $\overset{\#}{x}$ reveals that: $x^{MsQ} > \overset{\#}{x}$. There is excessive R&D compared to the socially necessary level when subsidies are used to attain the welfare maximizing output.

3.2. Subsidizing the R&D cartel

1. The optimal R&D objective.

We use the same device as previously to compute subsidies when firms remain competitive in the final good market but cooperate in the R&D stage. The subsidized R&D level is given using equation (14) as:

$$\overset{-s}{x} = \frac{(a-A)(1+\beta)}{\frac{9}{2}b(\gamma - s) - (1+\beta)^2}.$$

When the social planner targets the welfare solution for R&D, he (or she) solves "s" such that $\overset{-s}{x} = \overset{\#}{x}$, which yields: $\overset{-R}{s} = \frac{5}{9}\gamma$. We can immediately notice that:

$$\overset{-R}{s} > s^{MR} \tag{35}$$

The unit marginal subsidy needed to reach the welfare R&D level is slightly greater (0.55 compared to 0.50) than the one needed for the monopoly to hit the same objective. The impact of this first type of subsidy on final output can be deduced from equation (15)*:

$$\bar{Q}^{s} = \frac{2(a-A)}{3b}\left[\frac{\frac{9}{2}b(\gamma - s)}{\frac{9}{2}b(\gamma - s) - (1+\beta)^2}\right].$$

For $\bar{s}^{R} = \frac{5}{9}\gamma,$ the corresponding output is:

$$\bar{Q}^{R} = \frac{2(a-A)}{3b}\left[\frac{2b\gamma}{2b\gamma - (1+\beta)^2}\right]. \tag{36}$$

Comparing with the welfare output given in (26), we get:

$$\bar{Q}^{sR} = \frac{2}{3}\overset{\#}{Q}. \tag{37}$$

The performance of subsidies in the cartel is greater than what it would achieve in the monopoly organization, when the level of R&D is the objective.

2. The optimal output objective,

In the case the welfare output is targeted, we solve for "*s*" equating the subsidy driven output of the cartel $\bar{Q}^{s}$ to its welfare counterpart $\overset{\#}{Q}$:

$$\frac{2(a-A)}{3b}\left[\frac{\frac{9}{2}b(\gamma - s)}{\frac{9}{2}b(\gamma - s) - (1+\beta)^2}\right] = \frac{(a-A)}{b}\left[\frac{2b\gamma}{2b\gamma - (1+\beta)^2}\right].$$

Solving for "*s*" we find:

$$\bar{s}^{Q} = \frac{\gamma}{3}\left[\frac{3b\gamma + (1+\beta)^2}{b\gamma + (1+\beta)^2}\right]. \tag{38}$$

* Recall that $\overline{Q} = 2\overline{q}$.

For this situation also, we can show that: $\overset{-Q}{s} > \overset{-R}{s}$ A greater amount of subsidies is needed to reach the welfare output compared to the former case. Once again the level of R&D, when final output is targeted by subsidies exceeds the one obtained in the welfare solution:

$$\overset{-sQ}{x} = \frac{(a-A)\left[b\gamma + (1+\beta)^2\right]}{(1+\beta)\left[2b\gamma - (1+\beta)^2\right]} \tag{39}$$

and $\overset{-sQ}{x} > \overset{\#}{x}$.

It will be more efficient to subsidize the cartel in order to attain the welfare output solution compared to the private subsidized monopoly if:

$$\overset{-Q}{s} < s^{MQ}. \text{ This inequality holds if: } b\gamma > (1+\beta)^2. \tag{40}$$

The inequality may go either way as it is not supported by any second order condition on profit maximization. Therefore the ranking of subsidies may be reversed: $\overset{-Q}{s} > s^{MQ}$.

The reasons why monopoly may be more efficient in using subsidies are discussed below.

There are sets of values of b and γ for which the inequality in (40) should hold as the value of β increases. When there is strong cooperation in R&D, let it be in the monopoly organization or within the cartel with substantial result sharing, the spillover parameter is set to its maximal value: $\beta = 1$.

In such a case, $(1+\beta)^2 = 4$, we need to set $b\gamma > 2$ for the second order condition on profit maximization to hold in the welfare case*. Recall that satisfying the second order condition for monopoly was less restrictive. If we assume that the inequality $b\gamma > (1+\beta)^2$ holds for all values of β, then all other conditions are automatically satisfied and comparisons can be carried out for all cases.

Away from these technical considerations, it is important to ask under what circumstances and for what organization reasons this inequality may be reversed. Put differently, we may ask why is the monopoly more efficient, when R&D is subsidized, to achieve the welfare output compared to the situation where firms, in the R&D cartel, remain competitive in the final good market.

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* In section 2, the values $b = \gamma = 1$, while correct for comparisons with rivalry and cooperation in R&D, do not carry for the monopoly case as $4b\gamma > (1+\beta)^2$. $b\gamma$ will fall short of $(1+\beta)^2$ when β reaches its maximal value $\beta = 1$.

Expression $b\gamma$ puts together elements of cost of producing R&D, γ, and market power considerations for the final output through slope of the inverse demand function parameter b. A lower value for $b\gamma$ could simultaneously mean that the monopoly is more efficient if developing research can be undertaken by one firm and that the elasticity of demand is relatively high (as linked to the inverse of the slope given by b). The opposite case necessitates cooperation among a number of firms, to cut costs, combined with a high market power.

The economic reasons supporting a greater efficiency of the monopoly in using R&D cost reduction are more subtle. These were discussed in the introductory section. They are linked to the degree of appropriating benefits in the final good market. These benefits stem from cost improvements incurred by R&D realized in the first stage. The monopoly has a greater incentive to use efficiently the subsidies when it can capture profits in the second stage of the game.

We look at this question by setting $b = 2$ and $\gamma = 3$ so that the inequality in (40) holds. When targeting outputs, subsidies depend on the spillover parameter β. Either in monopoly or when cooperation in the cartel is based on full sharing of information, externalities reach their maximal value $\beta = 1$. Comparing subsidies needed to reach the welfare output reveals that:

$s^{MQ} = 1.5$, whereas $\bar{s}^{Q} = 1.46$. It will cost less to subsidize the cartel than the private monopoly when $\beta = 1$. On the contrary if cooperation among the participants is loose and the spillover parameter is set to equal $\beta = 0.6$ and $\bar{s}^{Q} = 1.6$. This value is greater than the amount of the subsidy devoted to the monopoly to reach the same objective.

Targeting output with many sellers dissipates the incentive to supply R&D at the first stage because the opportunity of capturing profits in the second stage decreases with competition. However comparing subsidies in the monopoly case and in the cooperative situation is meaningful only if $\beta = 1$. As long as the cost of developing and improving technology is high and the firm has a strong market power, which are the main features of modern industrial structures, cooperation in producing R&D, will prove to be an efficient organization only if results are fully shared.

The above discussion is intended to reveal the performance of industry organization that simultaneously internalizes externalities generated by a high level of R&D spillover and provides a better sharing of information among participant firms.

We may ask to what extent targeting welfare output with subsidies is a socially efficient policy. The output of R&D overshoots the optimal level given by the welfare solution. This outcome means that resources are wasted by excessive investment. A private firm may be willing to reach such high levels of R&D if the objective is to deter entry of potential competitors. The consequence of subsidizing excessively R&D in order to reach the socially optimal output is to shift potential producer's rent to increase consumer surplus. This situation inhibits the incentives of firms to use efficiently the subsidies to increase output. Economic policy may gain in efficiency if it was to be limited in funding R&D activities to reach the socially optimal level and allow competition in the final good market to increase output and consumer surplus.

3.3. Subsidizing R&D among competitors

It may still be interesting to look at the fully competitive case. We use the same device to compute the unit marginal subsidy that drives R&D expenditure to its welfare level, we find that:

$$s^{*R} = \gamma\left[1 - \frac{4}{9}\frac{(2-\beta)}{(1+\beta)}\right]. \tag{41}$$

The unit marginal subsidy now depends on the spillover parameter β, contrary to the other cases examined earlier. The subsidy will depend on the degree of information sharing within a coalition as discussed by M. Kamien et al. [1988]: firms are allowed to maintain competition in R&D but are willing to share all their results.

Once the socially optimal level of R&D is reached, the non-cooperative case will yield the same output of final goods as the cooperative solution, because in both cases firms are competitors in the second stage of the game. It can be readily shown that:

$$Q^{*sR} = \frac{2}{3}\overset{\#}{Q}. \tag{42}$$

This result is somewhat misleading if we are to carry out comparisons. The socially optimal output $\overset{\#}{Q}$ is computed as a function of β, as well as the subsidies for this case*. The spillover is actually fully internalized when the organization is a public monopoly that yields the value of $\overset{\#}{Q}$. The meaningful value of the externalities is to be set to $\beta = 1$, in order to allow comparisons.

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* In the former cases, the subsidies were given as a fixed proportion of γ, and were therefore independent of the spillover parameter. Comparisons with the socially efficient output could be conducted directly.

Firms in the non-cooperative case do not coordinate their action to maximize joint profit. The agreement is built on result sharing among participants. This organization is assumed to induce the spillover parameter to reach its maximal value: $\beta = 1$. As R&D output is a decreasing function of β (as discussed in section II), it will pay a high cost to incite participants to cooperate in order to reach the socially optimal level. The corresponding unit marginal subsidy is given by:

$$s^{*R} = \frac{7}{9}\gamma \quad \text{for} \quad \beta = 1. \qquad (43)$$

The policy maker may be only willing to subsidize coalitions that are significantly committed to cooperate*.

Conclusion

When we put together the results discussed in the precedent sections, economic policy will have to operate a compromise between conflicting goals while choosing the level and the destination of subsidies. In the case where the socially optimal level of R&D, is chosen as an objective, we can establish the following results:

Table 4

Firm organization, alternative policies and corresponding outcomes

	Unit marginal subsidy for β = 1	Corresponding output
Private monopoly	$s^{MR} = \frac{1}{2}\gamma$	$Q^{MR} = \frac{1}{2}Q^{\#}$
R&D cartel	$s^{-R} = \frac{5}{9}\gamma$	$Q^{-sR} = \frac{2}{3}Q^{\#}$
Duopoly competition	$s^{*R} = \frac{7}{9}\gamma$	$Q^{*sR} = \frac{2}{3}Q^{\#}$

The ranking of solutions is given by:

$$s^{MR} < s^{-R} < s^{*R}.$$

And the corresponding output by:

$$Q^{MR} < Q^{-sR} = Q^{*sR}.$$

* When firms are competitors the R&D output is higher, for weak spillovers, and there is no rationale for public policy to fund firms engaged in competition.

It will cost the policy planner a high subsidy, $\frac{7}{9}\gamma$, to fund a coalition in which firms are competitors in R&D but are willing to share completely their knowledge. The same result on final output will require a lesser amount of subsidy, $\bar{s}^{R} = \frac{5}{9}\gamma$, when firms coordinate the R&D activities in a cartel (and maximize joint profit) with full sharing of information.

In the monopoly case the organization is completely integrated and shows a higher degree of efficiency in using the subsidy to reach the R&D target. It will cost the policy maker a lesser amount, $s^{MR} = \frac{1}{2}\gamma$, compared to the cartel. There is however a social price to such a performance as output of final goods reaches a much weaker level than the one obtained by the cartel: $\frac{1}{2}Q^{\#}$ compared to $\frac{2}{3}Q^{\#}$. The cost/benefit ratio is unambiguously in favour of the cooperative cartel: $\dfrac{\frac{1}{2}\gamma}{\frac{1}{2}Q^{\#}} > \dfrac{\frac{5}{9}\gamma}{\frac{2}{3}Q^{\#}} \Leftrightarrow 1 > \frac{5}{6}$.

Cooperation in R&D activities among participants with full sharing of information (while keeping competitive in the final good market) proves to be the most efficient organization the policy maker in willing to fund.

We also explored the case of subsidizing R&D to reach the socially optimal level of final output. Such a policy may seem globally inefficient as improving consumer surplus by providing higher output is obtained at the expense of wasting resources, as subsidies induce excessive R&D levels. There may be an incentive to increase R&D output with the intention of using such a potential as a barrier to entry. However the discussion was intended to reveal industry performance when firms seek through their organization to simultaneously internalize externalities generated by high levels of R&D spillovers, and to provide a better sharing of information among participants. The fully integrated industry, as a monopoly, has an incentive to use efficiently the subsidies as it can capture a large share of profits in the final output stage of production. Competition among firms dissipates these potential gains. However for plausible assumptions such as high cost in developing research as well as for firms with large market power, the fully integrated industry is not necessarily more efficient. Comparing subsidies needed to reach the welfare output reveals that competition in the final good market and cooperation in producing R&D proves superior to full integration when these activities are coordinated and results completely shared.

Subsidizing integrated industries may be less acceptable by anti-trust regulations because the degree of capturing profits in the second stage is high. Policy makers may act indirectly to provide some protection to research consuming industries (so as to create barriers to entry) as well as improving consumer surplus. Encouraging coalitions with subsidies flowing to participants can be achieved through cooperation involving private or public organization should these be firms, research units or universities. One of the possible extensions of the analysis would be to define rules of cooperation within these structures. Distribution of subsidies between participants will have to provide mechanisms to cope with moral hazard and adverse selection.

Appendix

1. Monopoly.

	Decision structure	Objective	Spillover β	Outcome
Monopoly	Two stage cooperation: R&D and final output	Private monopoly: Max π	Full sharing of results: $\beta=1$	x^M, Q^M
	Two stage cooperation: R&D and final output	Public monopoly: Maximise social welfare: $S_c + \pi$	Full sharing of results: $\beta=1$	$x^\#, Q^\#$

2. R&D cartel.

	Decision rule	Objective	Spillover β	Outcome
R&D cartel	Coordination in R&D stage Competition in output stage	Max joint profit by coordinating R&D: Max: $\pi_1 + \pi_2$	Full sharing of results if $\beta = 1$ Or partial sharing of results if $\beta < 1$	$\bar{x}$,$\bar{Q}$ as functions of β

3. Competition.

	Decision structure	Objective	Spillover β	Outcome
Duopoly competition	Two stage competition: R&D and final output	Max: π_1 Max: π_2	Full sharing of results if $\beta=1$ Or partial sharing of results if $\beta < 1$	x^*, Q^* as functions of β

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