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WHAT KINDS OF HYBRID MODELS ARE USED IN MULTIPLE CRITERIA DECISION ANALYSIS AND WHY?*

Abstract

Currently, a growing number of papers presenting the hybrid models in the field of MCDA is observed. In this paper we try to explain the reasons for the growing popularity of hybrid models. The most important question considers the reasons for the use of these, rather complicated, structures. We also try to find out the differences between various models which are called hybrid.

Keywords

Discrete models, hybrid models, hybrid solvers, integrated models, multiple criteria decision analysis (MCDA), mixed models.

Introduction

If one reviews MCDA literature, it seems that the term “hybrid” became popular in the beginning of the 21st century. However, it is clear that mixed methods had been used earlier for many years without being named as such. The authors used other terms such as integrated or mixed models. In many cases, there was no particular emphasis put on the use of two or more different MCDA methods in a single model. Hence, it is very likely that many more papers that use a mix of MCDA methods have been published. Nevertheless, they are not easy to detect because the term “hybrid” does not appear in the title, key words or anywhere else in the text. This is why this review is limited only to papers that can be found with the key “hybrid”.

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Perhaps the term “hybrid” was borrowed by the MCDA/MCDM literature from the field of computer programming, mainly from the domain of programs that solve optimization problems. In a natural way, these programs are strongly related to MCDM (and MCDA) methods and sometimes are an integral part of the multiple criteria methodology.

Most search results for the keyword “hybrid” in the literature databases refer to programs or algorithms. Nowadays, many different heuristics (or meta-heuristics) became popular tools for solving hard problems. There is also a large group of papers in which only one MCDA method is applied but the authors use the term hybrid to point out that they include some other auxiliary or complementary techniques for special purposes. In our terminology such models are not really hybrid, so we will call them “pseudo-hybrids”.

In the next section we discuss the various examples of the use of the term “hybrid” in the literature related to MCDA. It appears that this term is used in different context in MCDA. As our main interest is in situations in which two or more MCDA methods are combined together into a single model, we distinguish that kind of models from all other hybrids. In this paper we assume that an MCDA method is one which can independently solve an MCDA problem.

Consequently, we classify the models into three categories:

- the models outside the strict MCDM/MCDA methodology,
- the models with unjustified use of the term hybrid,
- the models that present really hybridized MCDA methodology.

We will call the first category “hybrid solvers”, the second, “pseudo-hybrids”, and the third, “real MCDA hybrids”.

In view of the growing popularity of hybrid models in MCDA literature, this paper presents an attempt to answer the following questions:

- Why are hybrid models developed?
- What kinds of hybrid models occur in the MCDA field?
- What are the theoretical and practical reasons for hybrid models?
- What kinds of MCDA problems can be solved with hybrid models?

1. Hybrid models in the scientific literature

1.1. Hybrid solvers

As was said in Introduction, we divided the existing variety of papers in which the term hybrid is used into three categories. We start our brief review with the examples of “hybrid solvers”, hybridization that is closely related to MCDM but is essentially outside the strict MCDM. This category comprises

mixtures of various computation techniques which serve as solving tools for complicated MCDM/MCDA problems. It must be said that this category is very rich but we limit ourselves to a few examples only, as such models lie outside the field of our main interest.

The computationally difficult problem of vehicle routing with time windows became the starting point to develop a hybrid algorithm that used specialized genetic operators and variable-length chromosome representation to accommodate the sequence-oriented optimization [Tan et al. 2005]. It has been proved that this algorithm leads to better or competitive solutions when compared with the best known ones from literature.

In the paper entitled “A hybrid method for solving multi-objective global optimization problems” the authors presented their new hybrid method msPESA (mixed spreading PESA) which combines some aspects of PESA (Pareto Envelope-based Selection Algorithm), NSGA-II (non-dominated sorting genetic algorithm II), and LS (local search) – [Gil et al. 2006]. They also compared their method with other methods testing them with five two-objective problems and three three-objective problems.

Improving technical ability of meta-heuristics applied to solve multiple objective problems was the aim of a novel approach based on hybridizing simulated annealing and tabu search [Baños et al. 2007].

The combination of elements from simulated annealing and a variable neighborhood search was a basis for other hybrid meta-heuristics [Behnamian et al. 2009]. The authors use their method to solve the bi-objective scheduling problem. They say: “(...) using a hybrid meta-heuristic is to raise the level of generality so as to be able to apply the same solution method to several problems”.

In order to get a fast method for calculation of the Pareto optimal set for multiple objectives the genetic algorithm has been combined with an activity analysis as a local search method [Whittaker et al. 2009]. The authors incorporated a biophysical model and an economic model in the integrated optimization, and presented its application in the evaluation of agricultural and environmental policy.

1.2. Pseudo-hybrids

The next category of models we will call “pseudo-hybrid”. The reason for such a term comes from the observation that the models consist of only one authentic MCDA method which is usually assisted by one or more complementary tools which are not essentially multiple criteria methods. Here we present a selection of models which have been classified to this category.

The first example is a continuous method and a combination of goal programming and fuzzy set theory (El-Wahed and Abo-Sinna 2001). The fuzzy membership function has been used to determine the weights of objectives and the degree of conflict among the objectives. The developed method has been used to solve a real-life problem.

The next example is the paper entitled “A hybrid model of fuzzy and AHP for handling public assessments on transportation projects” (Arslan 2008). It presents the application of MCDA method to the decision concerning the choice of the type of public transport in a metropolitan city in Turkey. In this paper the decision model is based on only one MCDA method: AHP. This method is supported by the fuzzy logic for processing of incomplete and imprecise data.

Three different techniques – AHP, Monte Carlo simulation and fuzzy expert system – have been used together in (S. Li and J.Z. Li 2009). Among them only AHP belongs to the MCDA field. In fact, the authors did not call their method “MCDA hybrid”, they used the term “a hybrid intelligent decision support system or approach” instead.

The next paper differs from the previous examples as it employs one MCDA technique, DEMATEL (Decision Making Trial and Evaluation Laboratory), accompanied by structural equation modeling (SEM), the method taken from multivariate analysis [Tzeng et al. 2007]. It uses the DEMATEL technique to improve the causal model built with the aid of SEM. Then, this hybrid model has been used for selecting the most important factor affecting web-advertising.

1.3. Real MCDA hybrids

The third category, which we call “real MCDA hybrid” is the focus of our consideration. To this category we assign all models in which two or more MCDA methods are combined together.

We start our review with the hybrid consisting of ELECTRE and AHP [Rudolphi and Haider 2003]. According to the authors, such an approach enables to balance the frequently conflicting goals of visitor management and ecological integrity. A case study from the West Coast Trail in Pacific Rim National Park Reserve, BC, Canada, has been presented. From ELECTRE the model took the concordance and discordance analysis and also the definition of the different types of thresholds for criteria. From the AHP method

the hierarchical structure and pair-wise comparisons have been adopted. The reason for such a hybrid model was that the two methods complement each other.

The next paper has also aimed at solving the real-life problem of the Armenian energy sector [Goletsis et al. 2003]. The authors have developed a hybrid of ELECTRE III and PROMETHEE to get an integrated methodology for group decision making in project ranking. The procedure has been divided into two steps. In the first step the weights and thresholds for pseudo-criteria have been assigned according to the ELECTRE III technique. In the second stage the flows have been calculated in a way similar to the PROMETHEE method.

In the paper “The assessment of the information quality with the aid of multiple criteria analysis” [Michnik and Lo 2009] the authors have not used the term hybrid explicitly, however they combined two MCDA methods together to assess and improve the information quality in a firm. The AHP method has been utilized to get the hierarchy of criteria and the corresponding weights. To compare the decision alternatives, the modified version of ELECTRE method has been used. The two main techniques have been accompanied by two auxiliary tools: the stochastic dominances (for group decision making) and fuzzy measures (uncertainty of human judgment).

Both DEMATEL and ANP are methods that have been designed to solve the problem of interrelations between elements of a complex system. However, the authors of two papers [Ou Yang et al. 2008; Y.-C. Chen et al. 2010] have presented the opinion that ANP does not cope well enough with the weighting and interrelations between clusters. Hence, they have proposed to supplement the ANP method with DEMATEL to solve the dependence and feedback in real-world problems.

A quite different methodology has been developed in a recently published paper [Y. Chen et al. 2011]. In this case a hybrid approach means an incorporation of OWA's (Ordered Weighted Averaging) way of aggregation into TOPSIS (technique for order performance by similarity to ideal solution). The aim of such a procedure was to facilitate the methodology comprising many ideal and anti-ideal points (e.g. given by a group of decision makers).

An interesting case of a real hybrid is a paper in which two MCDA methods: DEMATEL and AHP have been supplied by fuzzy integral and factor analysis [Tzeng et al. 2007]. The experiments done by the authors have shown that this hybrid model is capable of producing effective evaluation of e-learning programs.

Conclusions

Our search in the literature databases shows the existence of various kinds of hybrid models that appear in the wide range of MCDA discipline. A close look at the papers that explicitly refer to the term “hybrid” led us to the idea of dividing all models into a few categories that contain more uniform models.

Many examples of hybrid models refer to the hybrid solving methods, mostly from artificial intelligence. They have been designed to assist the MCDM/MCDA models with efficient and fast computing algorithms. We called this category “hybrid solvers”.

This category, which perhaps is the richest one, contains models that use one MCDA method complemented by one or more auxiliary techniques. Usually, the main task of such supplementary techniques is to support the core MCDA method in solving the more complicated problems that arise from real-world problems. It comes from the observation that the classical MCDA models usually restricted by many assumptions appeared to be too idealized for the practical applications. To cope with a variety of qualitative and quantitative criteria, uncertainty conditions and other specific features of elements of the MCDA model, other techniques are incorporated into the model. It is often called hybrid, however as the auxiliary methods are not able to solve autonomously the MCDA problem, for this kind of models we use the term “pseudo-hybrid”.

In our opinion, a model which deserves the name “real MCDA hybrid” is one in which at least two independent MCDA methods have been used. It can be expected that there are plenty of examples of real hybrid MCDA models because in many cases, especially in the past, the authors did not use explicitly the term “hybrid”. In all cases, the authors advocate such a construct in order to deal with more complicated problems and real-world applications.

It is not easy to find any fundamental theoretical reasons for the use of hybrid models. The methods chosen to form a hybrid are driven mostly by a practical goal. However, their sort, range and number are – to some extent – a subjective choice depending on experience, intuition, and inclination of the authors.

The question “why do we need hybrid models?” is important from the point of view of future directions of MCDA development. The overview of current literature shows that the reason for using hybrid models is better approximation of the reality. In such models two or more MCDA methods complement each other and perform different tasks.

Another important issue is “how should a reasonable hybrid model be constructed?”. Some general suggestions emerge from the analysis of various existing hybrids:

1. The second, third, ... MCDA method may be added to the model only if they really are complementary to the core MCDA method and they really extend the ability of the hybrid in comparison to the single MCDA method.
2. The methods compounded together to make a hybrid should not be very similar to each other and should not be designated to do the same task.
3. In any case the subtle balance between the complexity of the hybrid model and its ability to solve the problem should be carefully kept. In other words, the gain acquired from the employment of another method should outweigh the disadvantages of the more complicated structure.

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MULTICRITERIA METHODS FOR EVALUATING COMPETITIVENESS OF REGIONS IN V4 COUNTRIES

Abstract

Regional competitiveness is the source of national competitiveness. This paper presents multi-criteria decision making methods for evaluation of the regional competitiveness and regional differences and disparities. Specific indicators reflect the economic productivity of the region in form of factors of production inside of the region. The technology for the evaluation of regional competitiveness is based on the application of two methods of multi-criteria decision making. The first one is the method of Ivanovic deviation, the second one is the well known DEA. The results of the applications of the methods are compared on the basis of the competitiveness of the NUTS2 regions (V4 – Visegrad Four countries) in the EU within the period of 7 years (2000-2006). In particular, the disparities between the Czech and Polish NUTS2 regions are discussed.

Keywords

Regional competitiveness, regional disparities, multi-criteria methods, Ivanovic deviation, DEA.

Introduction

This paper deals with multi-criteria decision making methods for evaluating the regional competitiveness and regional differences and disparities. Specific indicators reflect economic productivity of the region in form of factors of production and/or efficiency inside the region (effect of one-regional unit) and are revitalized by the capacity of actual employment in the region. In particular, we deal with the following indicators: Gross domestic product and Labour productivity per person employed, Gross fixed capital formation, Total intramural R&D expenditure, Income of households, Employment rates.

The technology of evaluation of regional competitiveness is based on the application of two methods of multi-criteria decision making. The first one is the method of Ivanovic deviation, the second one is the well known Data Envelopment Analysis – DEA. The results of both methods will be compared.

1. Method of Ivanovic deviation

There does not exist a “universal” methodology for assessing the degree of regional non-competitiveness. An “alternative way” for evaluating regional competitiveness is to define a group of specific economic indicators of efficiency [see: Melecký and Nevima 2010]. The basic idea is to assess the internal sources of regional competitiveness in detail [see: Krugman 1994]. The evaluation of the competitiveness through five specific indicators have been proposed and discussed in [Nevima and Ramik 2009].

The classical weighted average methods (WA) proved to be irrelevant to the problem of regional competitiveness as the usual assumption of independent criteria is not satisfied. That is why we were looking for other suitable methods. Here, we present an application of two methods of this kind: Ivanovic deviation and DEA.

To overcome the problem of dependent criteria, we propose the technique of evaluation of regional competitiveness called Ivanovic deviation (ID) [see: Nevima and Ramik 2009]. This method is a technique of multi-criteria decision-making and its purpose here is to assess the ranks of the regions, too. In comparison with the simple averaging [see e.g.: Ramík and Perzina 2008], it takes into account the importance and mutual dependence of the decision-making criteria, i.e. six specific indicators ranked by their relative importance, that is: Gross domestic product (GDP), Labour productivity per person employed (LP), Gross fixed capital formation (THFK_EUR), Total intramural R&D expenditure (GERD), Income of households (INDIC_NA) and Employment rates (Y15_MAX). This ranking is done by an expert evaluation; here, GDP is the most important indicator as it reflects the total economic efficiency of the region and it also includes the level of production. The second most important criterion is LP, the labour productivity per person employed. THFK_EUR is the gross fixed capital – an indicator of connections of expenditures for the creation of the fixed assets. These assets are also included in the regional production. GERD could be interpreted as the total R&D expenditures. INDIC_NA is the income of households and Y15_MAX is the criterion of employment rate. In this method, the weight of each criterion

is based on its relative importance – the ranking takes into account the correlation coefficients with the previous (i.e. more important) criteria. Then the weighted distance of the current variant to the ideal (fictitious) one is calculated as follows [see: Nevima and Ramik 2009]:

$$I_j = \frac{|x_1^f - x_{1j}|}{s_1} + \sum_{i=2}^n \frac{|x_i^f - x_{ij}|}{s_i} \prod_{k=1}^{i-1} (1 - |r_{ki}|), \quad (1.1)$$

where:

x_i^f – value of i -th criterion of ideal (fictitious) variant (i.e. region),

x_{ij} – value of i -th criterion j -th variant,

r_{ki} – correlation coefficient i -th a k -th criterion (i.e. specific coefficient),

s_i – standard deviation i -th criterion calculates:

$$s_i = \sqrt{\frac{1}{m} \sum_{j=1}^m (x_i^j)^2 - (\bar{x}_i)^2}, \quad (1.2)$$

where m – total value of variants, n – total number of criteria.

The approach based on the application of the Ivanovic deviation seems more relevant as compared to the results of the method of simple averaging. As we know the importance of the criteria and correlations (i.e. dependences) among the criteria, we are able to determine the “distance” to the ideal region in a more realistic way. Then the final rank of regions corresponds to the different economic importance of individual criteria (i.e. specific indicators of efficiency). Thanks to this fact we consider the final rank as another contribution of this alternative approach to the evaluation of regional competitiveness of the NUTS2 regions in the V4 countries, see Table 1 and Figures 3, 4 and 5.

2. DEA

Data Envelopment Analysis (DEA) is a relatively new data-oriented approach for evaluating the performance of a set of peer entities called Decision Making Units (DMUs) converting multiple inputs into multiple outputs. Here, we applied DEA to all 35 central European NUTS2 regions in Visegrad Four countries (V4). Recent years have seen a great variety of applications of DEA for use in evaluating the performances of many different kinds of entities engaged in many different activities in many different contexts in many

different countries [see: Cooper et al. 2000]. These DEA applications have used DMUs of various forms to evaluate the performance of entities, such as hospitals, US Air Force wings, universities, cities, courts, business firms, and others, including the performance of countries, regions, etc.

As pointed out in [Cooper et al. 2000], DEA has also been used to supply new insights into activities (and entities) that have previously been evaluated by other methods. Since DEA in its present form was first introduced in 1978 [see: Charnes et al. 1978], researchers in a number of fields have quickly recognized that it is an excellent and easily used methodology for modeling operational processes for performance evaluations. In their original study [Charnes et al. 1978], DEA is described as a “mathematical programming model applied to observational data that provides a new way of obtaining empirical estimates of relations – such as the production functions and/or efficient production possibility surfaces – that are cornerstones of modern economics”.

In most management or social science applications the theoretically possible levels of efficiency are not known. Our model is based on the inputs and outputs, which must be chosen carefully with regard to their definition in economic theory. This fact is vital for us to perceive the efficiency as a “mirror” of competitiveness. Moreover, here we present only one version of the DEA model, that is, the most popular input oriented CCR model and also the output-oriented CCR model [see: Charnes et al. 1978]. For more detailed analysis of efficient regions (coefficient of efficiency is equal to one) we applied DEA super-efficiency models [see e.g.: Cooper et al. 2000].

Now we introduce criteria for selecting inputs and outputs used in the DEA model as applied to efficiency of NUTS2 regions in V4 (i.e. Czech Republic, Slovakia, Poland, Hungary). It is evident that the overall performance of the regional economy affects the number of people employed in various sectors, their skills and working age (15-55 years). Therefore, we selected the criterion of employment rate and that of the creation of the THFK_EUR (Gross Fixed Capital Formation). This criterion includes, in general, investment activity of domestic companies and fixed assets of foreign companies, which are the “engine” of the innovation competitiveness. The total intramural R&D expenditure (GERD) is considered for the future development of the region. The third input included is the net disposable income of households (INDIC_NA). In terms of competitiveness the disposable income plays an important role, especially because it directly reflects the purchasing power of the region [see: Nevima and Ramik 2010]. The last input indicator is the employment rate (Y15_MAX).

There are two outputs in our DEA model [see: Zhu 2002]. The outputs are measured by GDP in purchasing parity standards and labor productivity per person employed. The GDP is the most important macroeconomic aggregate, and if it is measured per region, we can take into account a limited number of inputs for its calculation. Similarly, the labor productivity (LP) may be taken into account, as it shows what amount of production was created by economically active people or employed persons.

In Figure 1 and 2 we compare the Czech and Polish region super-efficiency and in Table 2 it is evident that the best results are traditionally achieved by economically powerful “capital” regions being efficient during the whole period 2000-2006. It is clear that whereas in the Czech regions the regional disparities between the capital region and the other NUTS2 regions diminish within the given period, in Poland the disparities in economic efficiency between the capital region and the other NUTS2 regions increase within the given period. Hence, the tendency in Poland is opposite to that in the Czech Republic.

Conclusions

The paper aims at presenting multi-criteria approaches to evaluating competitiveness (efficiency) and disparities of the European regions (NUTS2). This evaluation was based on the applications of two models (Ivanovic deviation and DEA) calculating an “efficiency index” of each region. Since no universal methodological approach to regional competitiveness exists, this paper should be understood as a contribution to the discussion of quantitative measurement of competitiveness at the regional level.

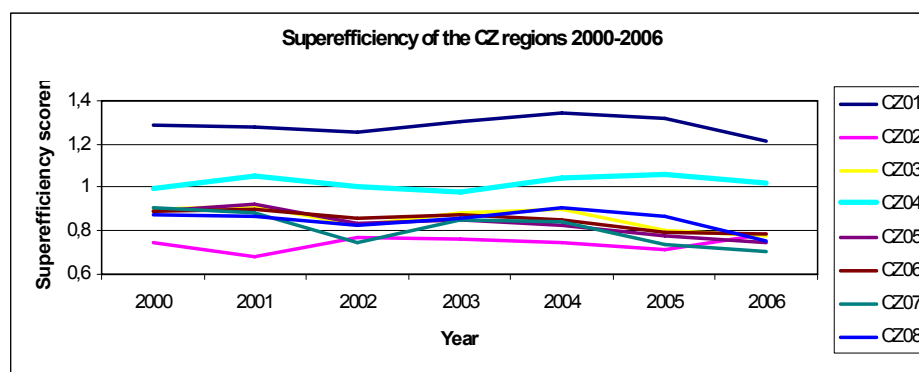


Figure 1. Superefficiency of the Czech NUTS2 regions

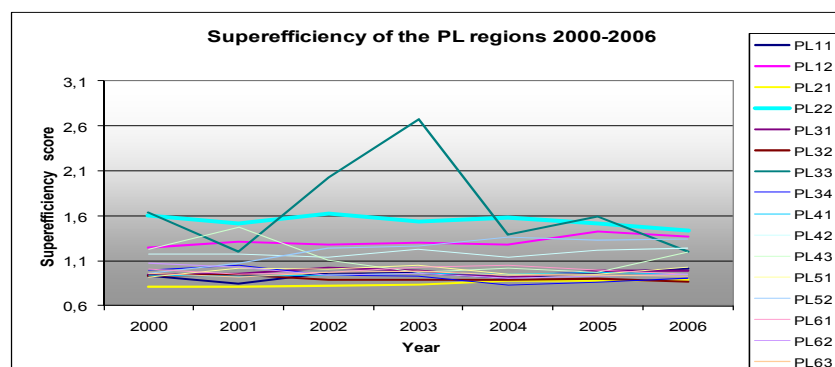


Figure 2. Superefficiency of the Polish NUTS2 regions

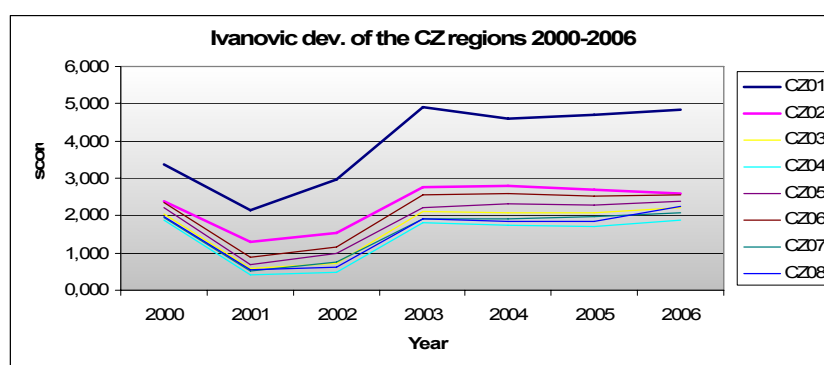


Figure 3. Ivanovic deviation of the Czech NUTS2 regions

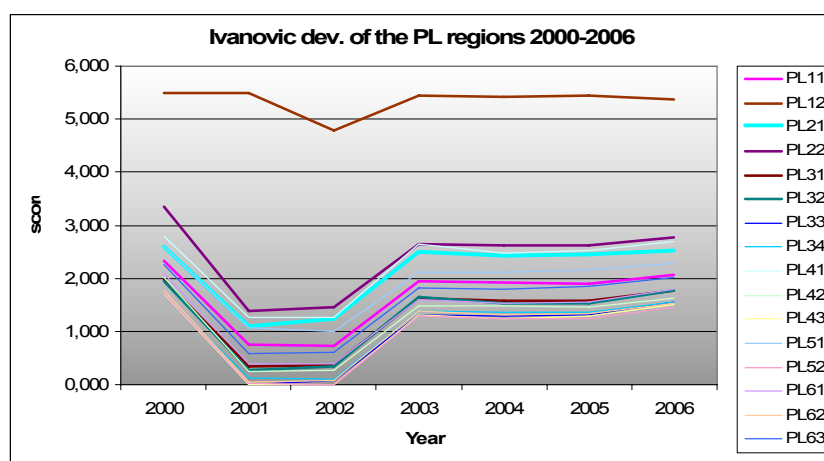


Figure 4. Ivanovic deviation of the Polish NUTS2 regions

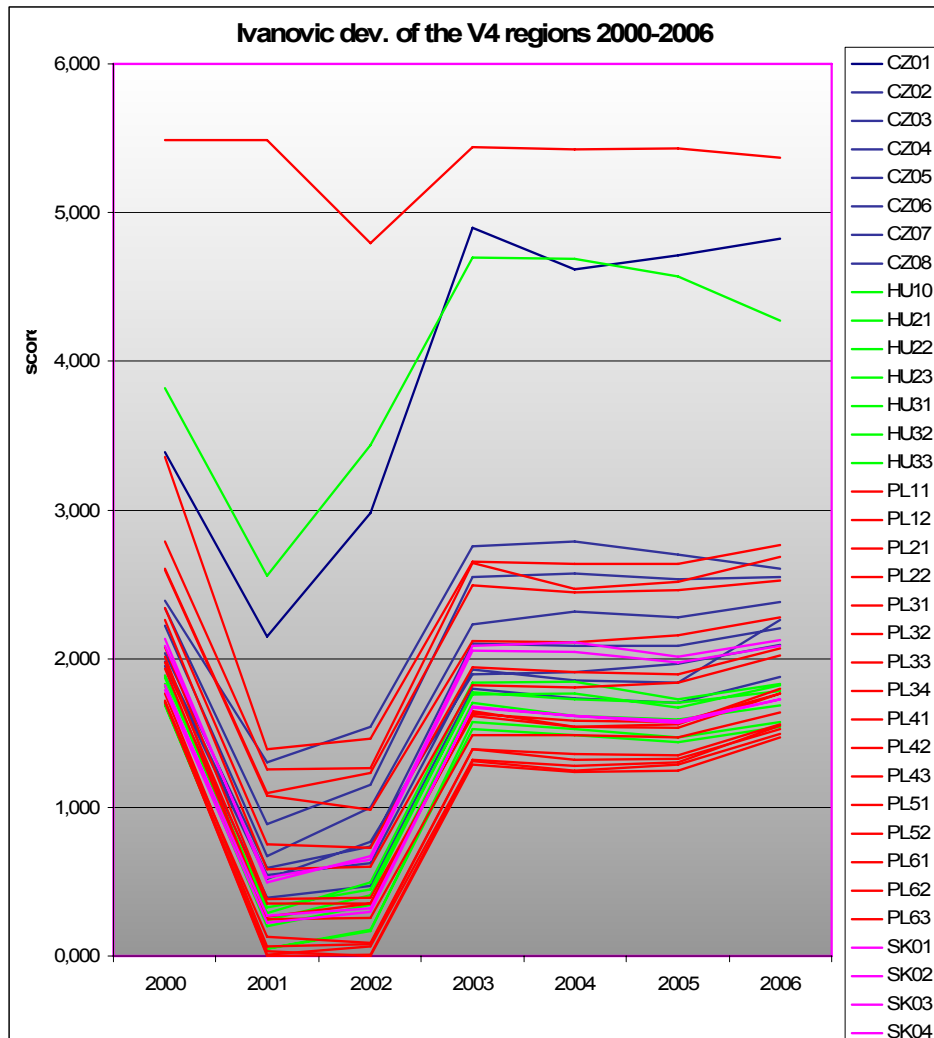


Figure 5. Ivanovic deviation of the V4 regions

Table 1

Application of Ivanovic deviation in NUTS 2 regions

Code	Region	2000	2001	2002	2003	2004	2005	2006
CZ01	Praha	3,388	2,150	2,981	4,898	4,617	4,714	4,829
CZ02	Střední Čechy	2,392	1,302	1,543	2,753	2,786	2,702	2,604
CZ03	Jihozápad	2,035	0,595	0,732	2,100	2,088	2,085	2,204
CZ04	Severozápad	1,883	0,393	0,469	1,801	1,735	1,706	1,877
CZ05	Severovýchod	2,220	0,675	1,001	2,226	2,321	2,276	2,380
CZ06	Jihovýchod	2,343	0,884	1,154	2,547	2,575	2,529	2,550
CZ07	Střední Morava	1,953	0,522	0,766	1,896	1,907	1,964	2,089
CZ08	Moravskoslezsko	1,984	0,540	0,625	1,925	1,855	1,838	2,261
HU10	Közép-Magyarország	3,816	2,555	3,437	4,699	4,691	4,567	4,275
HU21	Közép-Dunántúl	1,866	0,284	0,493	1,841	1,846	1,724	1,830
HU22	Nyugat-Dunántúl	1,888	0,325	0,443	1,761	1,765	1,673	1,824
HU23	Dél-Dunántúl	1,694	0,045	0,175	1,526	1,484	1,441	1,543
HU31	Észak-Magyarország	1,688	0,046	0,170	1,570	1,524	1,469	1,576
HU32	Észak-Alföld	1,812	0,236	0,408	1,777	1,722	1,701	1,779
HU33	Dél-Alföld	1,830	0,197	0,342	1,699	1,616	1,588	1,685
PL11	Lódzkie	2,341	0,752	0,731	1,939	1,912	1,897	2,070
PL12	Mazowieckie	5,486	5,489	4,796	5,442	5,427	5,435	5,370
PL21	Malopolskie	2,601	1,092	1,233	2,490	2,441	2,459	2,527
PL22	Slaskie	3,353	1,388	1,464	2,649	2,633	2,634	2,761
PL31	Lubelskie	1,971	0,352	0,353	1,628	1,580	1,577	1,767
PL32	Podkarpackie	1,934	0,265	0,352	1,646	1,541	1,535	1,767
PL33	Swietokrzyskie	1,712	0,007	0,066	1,322	1,280	1,301	1,548
PL34	Podlaskie	1,706	0,127	0,086	1,389	1,362	1,353	1,562
PL41	Wielkopolskie	2,788	1,254	1,266	2,643	2,471	2,521	2,685
PL42	Zachodniopomorskie	2,015	0,248	0,256	1,485	1,485	1,467	1,637
PL43	Lubuskie	1,707	0,000	0,010	1,314	1,250	1,289	1,498
PL51	Dolnoslaskie	2,599	1,080	0,985	2,113	2,111	2,154	2,279
PL52	Opolskie	1,714	0,034	0,000	1,288	1,235	1,248	1,467
PL61	Kujawsko-Pomorskie	2,079	0,381	0,388	1,613	1,540	1,561	1,796
PL62	Warminsko-Mazurskie	1,766	0,060	0,082	1,393	1,320	1,326	1,529
PL63	Pomorskie	2,260	0,587	0,602	1,824	1,806	1,838	2,019
SK01	Bratislavský kraj	2,131	0,492	0,668	2,086	2,106	2,010	2,125
SK02	Západné Slovensko	2,082	0,529	0,645	2,054	2,042	1,974	2,086
SK03	Stredné Slovensko	1,819	0,268	0,321	1,672	1,614	1,578	1,726
SK04	Východné Slovensko	1,792	0,227	0,298	1,674	1,617	1,566	1,724

Table 2

Superefficiency of NUTS2 regions in V4 countries

Code	Region	2000	2001	2002	2003	2004	2005	2006
CZ01	Praha	1,285	1,275	1,256	1,304	1,341	1,319	1,217
CZ02	Střední Čechy	0,749	0,681	0,768	0,761	0,747	0,716	0,784
CZ03	Jihozápad	0,901	0,918	0,824	0,883	0,903	0,803	0,776
CZ04	Severozápad	0,999	1,051	1,005	0,979	1,047	1,059	1,022
CZ05	Severovýchod	0,893	0,927	0,835	0,847	0,828	0,778	0,744
CZ06	Jihovýchod	0,892	0,902	0,855	0,875	0,851	0,796	0,790
CZ07	Střední Morava	0,910	0,884	0,743	0,848	0,842	0,734	0,705
CZ08	Moravskoslezsko	0,874	0,865	0,824	0,861	0,906	0,866	0,750
HU10	Közép-Magyarország	1,232	1,292	1,234	1,166	1,132	1,128	1,184
HU21	Közép-Dunántúl	0,954	0,955	0,813	0,850	0,898	0,898	0,863
HU22	Nyugat-Dunántúl	1,283	1,012	1,039	1,102	0,997	0,989	0,992
HU23	Dél-Dunántúl	1,090	1,038	0,948	0,936	0,970	0,909	0,886
HU31	Észak-Magyarország	0,926	1,065	0,946	0,926	1,016	0,996	0,904
HU32	Észak-Alföld	0,939	0,929	0,848	0,819	0,813	0,736	0,716
HU33	Dél-Alföld	1,102	1,009	0,890	0,831	0,835	0,769	0,756
PL11	Lódzkie	0,931	0,842	0,974	0,968	0,908	0,969	1,016
PL12	Mazowieckie	1,243	1,315	1,280	1,306	1,281	1,426	1,367
PL21	Małopolskie	0,809	0,816	0,825	0,837	0,878	0,879	0,887
PL22	Śląskie	1,610	1,516	1,622	1,541	1,579	1,514	1,441
PL31	Lubelskie	0,941	0,956	1,029	0,995	0,917	0,989	0,992
PL32	Podkarpackie	0,939	0,945	0,887	0,892	0,887	0,902	0,867
PL33	Świętokrzyskie	1,634	1,200	2,026	2,679	1,395	1,589	1,200
PL34	Podlaskie	0,993	1,049	0,939	0,924	0,830	0,859	0,902
PL41	Wielkopolskie	0,936	0,927	0,935	0,966	0,950	0,969	0,956
PL42	Zachodniopomorskie	1,169	1,180	1,138	1,231	1,140	1,223	1,246
PL43	Lubuskie	1,235	1,477	1,103	0,975	1,022	0,978	1,202
PL51	Dołnoslaskie	0,917	1,015	1,003	1,049	0,954	0,957	1,028
PL52	Opolskie	0,961	1,079	1,242	1,266	1,362	1,334	1,341
PL61	Kujawsko-Pomorskie	1,009	0,954	0,969	1,031	1,050	1,001	0,952
PL62	Warmińsko-Mazurskie	1,078	1,030	0,943	0,962	0,906	0,926	0,948
PL63	Pomorskie	0,958	0,919	0,974	0,986	0,944	0,944	0,933
SK01	Bratislavský kraj	1,681	1,742	1,831	1,800	1,758	1,839	1,876
SK02	Západné Slovensko	1,030	1,085	1,036	1,006	0,958	1,024	1,076
SK03	Stredné Slovensko	0,884	0,941	0,979	0,965	0,910	0,920	0,965
SK04	Východné Slovensko	0,939	1,033	1,080	0,981	0,901	0,959	0,910

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MULTIPLE CRITERIA EVALUATION OF PROJECT GOALS

Abstract

This text is focussed on the quantitative evaluation of project SMART goals using the ANP method. This approach should be used in the project initiation phase. The very first step in all projects: business, home, or education, is to define goals and objectives. It is important to develop several goals that will enable us to be successful. Goals should be SMART – S – specific, significant, stretching, M – measurable, meaningful, motivational, manageable, A – agreed upon, attainable, achievable, acceptable, action-oriented, R – realistic, relevant, reasonable, rewarding, results-oriented, resourced, T – time-based, timely, tangible, trackable.

In our paper we make complex decisions about satisfying project SMART goals based on the ANP method using Super Decisions Software. As criteria we used a general SMART (SMARTER) model, as sub-criteria we use S, M, A, R, T sub-goals and as alternatives different project schedules are applied. We experiment with their mutual dependencies and we try to propose the best methodology for evaluating projects using the Analytic Network Process.

Keywords

Project management, project proposal, evaluation of project goals achievement, Analytic Network Process, Super Decisions Software.

Introduction

Modern project management uses many methods, techniques and tools for evaluating the quality of a project, both in the phase of proposal and in the phase of realization. Any project proposal should look very nice but a deeper study of its aim, time schedule, and resource allocation can detect whether it is likely to fail/to succeed. The majority of methods used for project evaluation are not based on quantitative approaches; sophisticated mathematical methods of multiple criteria evaluation of alternatives are used only very rarely.

The BOSCARD (Background, Objectives, Scope, Constraints, Assumptions, Risks and Deliverables) is a tool used to provide the terms-of-reference for the newly proposed project [Haughey 2011]. It is used in the phase of project initiation. What future events may impact the project? For forecasting the future and customizing the project schedule the Delphi Step by Step technique can help. The MoSCoW method (Must have this, Should have this if at all possible, Could have this if it does not affect anything else, Won't have this time but Would like in the future) is applied when establishing a clear understanding of the customers' requirements and their priorities [Clegg and Barker 2004]. The PEST is a strategic planning tool for evaluating the possible impact of Political, Economic, Social, and Technological factors on a project. The RACI model (Responsibility, Accountability, Consultation, and Information) is a straightforward tool used for identifying roles and responsibilities and avoiding confusion over those roles and responsibilities during a project [Smith 2005]. SWOT analysis (Strengths, Weaknesses, Opportunities, Threats) is a well known strategic planning tool used to evaluate the strengths, weaknesses, opportunities, and threats to a project [Armstrong 2006]. It involves specifying the objective of the project and identifying the internal and external factors that are favourable and unfavourable to achieving that objective.

The tool we consider in this paper is called SMART Goals evaluation. Project goals should be SMART [Doran 1981], which very briefly means: S – specific, significant, M – measurable, manageable, A – agreed, action-oriented, R – realistic, relevant, resourced, T – time-based, trackable. SMARTI project adds I – Integrated criteria to SMART goals, SMARTER project is moreover E – Ethical, Excitable, Enjoyable, Engaging, Ecological and R – Rewarded, Reassess, Revisit, Recordable.

For our paper it is more important to evaluate a completed project, final proposals, or project baselines (schedules) whether the SMART goals have been achieved or not. These goals are hard to measure; they have no final quantitative features. That is why we first tried to apply the Analytic Hierarchy Process [Saaty 1980, 1999] for comparing a finite set of projects with respect to general SMART goals (criteria) and individual SMART specifications (sub-criteria). Upon receiving the AHP results we decided to abandon this approach and apply the Analytic Network Process [Saaty 2001, 2003] for this evaluation. In the AHP each element in the hierarchy is considered to be independent of all the others, the ANP does not require independence among elements. It is very

hard to make complex decision on satisfying project SMART goals without applying the ANP method because SMART specifications (sub-criteria) are not independent of each other. The majority of them are judged from sometimes very various points of views and one judgment strongly influences the others. R – goals (sub-criteria) “Realistic” and “Relevant” are typical examples of this dependency.

According to the survey [White and Fortune 2001] three crucial success factors mentioned most frequently by respondents were:

- Clear goals.
- Support from senior management.
- Adequate funds/resources.

That’s why we focus mostly on project goals.

1. The ANP process as a tool for SMART goals evaluation

Multiple criteria decision models are used by many industries to quantify, compare, and manage their performance. The Analytic Network Process is one of the most effective tools in cases where the interactions among qualitative and quantitative factors generate a hierarchical or a network structure. Isik at all [2007] presented a conceptual performance measurement framework that takes into account company-level factors (objectives, strategies, resources) as well as project-level (risks, opportunities) and market-level factors (competition, demand).

As a tool for SMART goals achievement evaluation the hierarchy evaluation by the Analytic Network Process (Saaty 2001, 2003) should be used. Two types of the ANP model are theoretically defined: the Feedback System model and the Series System model. The Series System model usually consists of a tree, where the root is a model goal; branches of various levels have the meaning of criteria or sub-criteria of various levels respectively and finally the leaves represent a set of alternatives. Branches and leaves together determine the so-called model clusters (criteria, sub-criteria, project proposals). A crucial role for the project proposal evaluation plays the Feedback System model, where the clusters are linked one by one into a complex network system. We assume that all sub-criteria (within S, M, A, R and T criteria) influence and interact with each other and in the same way all the criteria are interconnected, too. It means that the hierarchy structure can be transformed into

a network structure and the ANP feedback model seems to be a very suitable tool for solving this problem. The ANP super-matrices (non-weighted, weighted, limits) with possible cluster interactions and influences have to be defined and calculated and the most suitable project proposal will be selected according to the synthesis through addition of all the control criteria. The computation itself should be made using, for instance, the SuperDecisions software.

$$\mathbf{W} = \begin{matrix} & \begin{matrix} C & S & P \end{matrix} \\ \begin{matrix} C \\ S \\ P \end{matrix} & \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{W}_C \\ \mathbf{W}_S & \mathbf{W}_{S*} & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_A & \mathbf{0} \end{pmatrix} \end{matrix}$$

where:

- \mathbf{W}_C is the matrix of criteria weights with respect to projects,
- \mathbf{W}_S is the matrix of sub-criteria weights with respect to criteria,
- \mathbf{W}_{S*} is the matrix of sub-criteria weights with respect to each other,
- \mathbf{W}_A is the matrix of project weights with respect to sub-criteria.

1.1. The ANP Criteria Level

The criteria level in the ANP process includes the general SMART goals. The goal is a general statement about a desired outcome with one or more specific objectives that define in precise terms what is to be accomplished within a designated time frame. The goal may be performance-related, developmental, a special project, or some combination [Sheid 2011].

- **S-criteria** evaluate who, what, when, where, why and how provides a project.
- **M-criteria** include a numeric or descriptive measurement of a project.
- **A-criteria** consider the resources needed and set a realistic goal.
- **R-criteria** ensure the goal is consistent with the mission of a project.
- **T-criteria** set a realistic deadline.

The project's scope, goals and sub-goals should be clearly outlined, taking into consideration cost, time and quality factors. The project should also be within the capacity of the project team and with incentive and encouragement to push the project forward to reach a more general goal.

1.2. The ANP Sub-criteria Level

Many meanings of the S, M, A, R, T letters are known from the literature. For example, A-criteria should be divided into Actionable, Attainable, Ambitious, Aspirational, Accepted/Acceptable, Aligned, Accountable, Agreed, Adapted, As-if-now, Adjustable, Adaptable etc. [RapidBI 2011].

For the ANP process analysis we have decided to use the following sub-criteria. We don't aspire to actual project evaluation, our aim is to propose a methodology of using the ANP process in this type of problems. Any other sub-criteria can be set or applied.

- **Specific** – What exactly are we going to do, with or for whom? “Specific” in the context of developing objectives refers to an observable action, behaviour or achievement.
- **Significant** – Significant goals are the ones that will make a positive difference in reality.
- **Measurable** – A method or procedure allowing the tracking and recording the project behaviour or progress must exist.
- **Meaningful** – Realization of a project must have a meaning. The goal must be very important.
- **Manageable** – The project must be easy to manage!
- **Achievable** – It must be possible for the project to be done in the timeframe/in this political climate/with this amount of money.
- **Action Oriented** – The plan of “attack” to make each goal real.
- **Relevant** – The project goal being set with an individual is something that can impact, change or be important to the organization
- **Realistic** – It must be an objective toward which you are both willing and able to work.
- **Resourced** – The goal or target being set is something that must have relevant resources allocated to be satisfied.
- **Time Based** – Every project task must have clearly stated a finish and/or a start date.
- **Trackable** – All goals should be trackable so you can see what your progress is. In terms of Project Management, you are tracking progress of project tasks in time, earned value, work etc.

1.3. The ANP Alternatives Level

As the alternatives level the actual projects or project proposals are set. Criteria and sub-criteria weights differ from project to project, according to the different project types and scopes. But these differences are not very distinguished; every project must be built according to similar rules and principles. Until now this part of the ANP process has not been included in our approach.

2. Network Model for Criteria and Sub-criteria Weights

As a tool for setting a dependency network among criteria and sub-criteria, SuperDecisions® software has been used. One hierarchic level underneath the goal node, SMART criteria level as a unique cluster (there are no dependencies – relations among them) starts the Analytic Network Model. Weights of criteria were set identically to 0,2.

The next level, consisting of subcriteria divided into clusters, is a crucial element of the process evaluating importance of each of them within the ANP process. These relations have been set according to the authors' experiences with managing various types of projects. Very often the project managers correlate the time frame of a project and its specificity (originality). The more specific the project, the more time it needs, and the less trackable it is. The most crucial are relations within the clusters A and R. Sometimes, the achievement of certain project goals excludes the achievement of others, while the achievement of one goal accelerates the achievement of another one. Also, a relevant goal must be realistic to achieve. Similar relationships have been observed within and among other clusters (Figure 1). These current weights are based on expert evaluation and calculated using Saaty's pairwise comparisons matrix – as integral part of SuperDecision® Software.

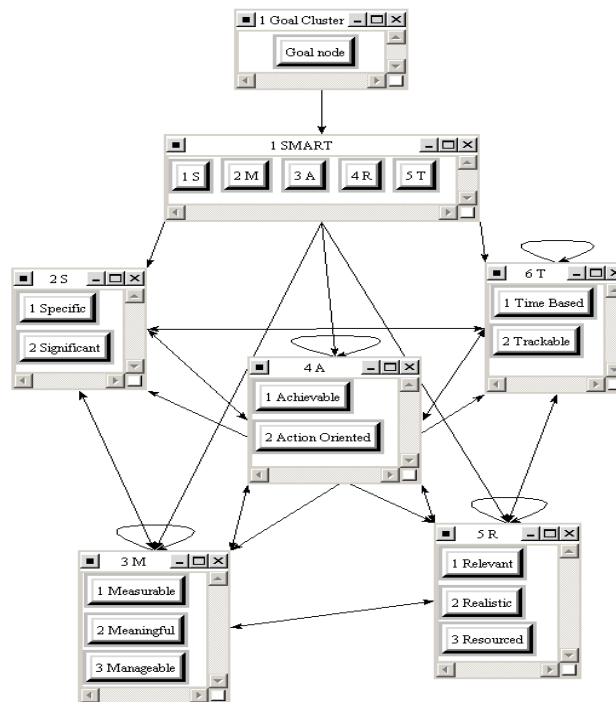


Figure 1. SMART Project Criteria Network (SuperDecisions Software)

2.1. The ANP Model Results

The first ANP result, un-weighted super-matrix for equal criteria weights, gives a good idea about clusters, established connections and their evaluation by weights.

	1 Goal Cluster	1 SMART				
	Goal node	1 S	2 M	3 A	4 R	5 T
Goal node	0	0	0	0	0	0
1 S	0,2	0	0	0	0	0
2 M	0,2	0	0	0	0	0
3, A	0,2	0	0	0	0	0
4 R	0,2	0	0	0	0	0
5 T	0,2	0	0	0	0	0
1 Specific	0	0,6	0	0	0	0
2 Significant	0	0,4	0	0	0	0
1 Measurable	0	0	0,4	0	0	0
2 Meaningful	0	0	0,3	0	0	0
3 Manageable	0	0	0,3	0	0	0
1 Achievable	0	0	0	0,6	0	0
2 Action Oriented	0	0	0	0,4	0	0
1 Relevant	0	0	0	0	0,3	0
2 Realistic	0	0	0	0	0,4	0
3 Resourced	0	0	0	0	0,3	0
1 Time Based	0	0	0	0	0	0,6
2 Trackable	0	0	0	0	0	0,4

Figure 2. First part of un-weighted matrix – Goal node and criteria

	2S		3M			4A		5R			6T	
	1 Specific	2 Significant	1 Measurable	2 Meaningful	3 Manageable	1 Achievable	2 Action Oriented	1 Relevant	2 Realistic	3 Resourced	1 Time Based	2 Trackable
Goal node	0	0	0	0	0	0	0	0	0	0	0	0
1S	0	0	0	0	0	0	0	0	0	0	0	0
2M	0	0	0	0	0	0	0	0	0	0	0	0
3A	0	0	0	0	0	0	0	0	0	0	0	0
4R	0	0	0	0	0	0	0	0	0	0	0	0
5T	0	0	0	0	0	0	0	0	0	0	0	0
1 Specific	0	0	0	0,7	0	1	1	0,5	0,7	1	1	1
2 Significant	0	0	0	0,3	0	0	0	0,5	0,3	0	0	0
1 Measurable	0,7	0	0	0	1	0,6	0	0,4	0	0	0,8	0,7
2 Meaningful	0	1	0	0	0	0	0	0,6	0,8	0	0	0
3 Manageable	0,3	0	0	0	0	0,4	0	0	0,2	1	0,2	0,3
1 Achievable	0,6	0,7	0	1	0,8	0	1	1	0,8	0	0	0
2 Action Oriented	0,4	0,3	1	0	0,2	0	0	0	0,2	1	1	1
1 Relevant	0,3	0,4	0	0,4	0	0,25	0	0	0	0	0	0
2 Realistic	0,5	0,5	0	0,4	0	0,5	0	0,7	0	0	1	0
3 Resourced	0,2	0,1	1	0,2	1	0,25	0	0,3	1	0	0	1
1 Time Based	0,6	1	0	1	0,5	0,4	0,4	0	0	0,3	0	1
2 Trackable	0,4	0	1	0	0,5	0,6	0,6	0	0	0,7	1	0

Figure 3. Second part of un-weighted matrix – Sub-criteria

Next, the ANP results and the limit matrix are used to calculate the final weights. The limit calculation gives the following weights for SMART sub-criteria (Figure 4). As supposed, the most important sub-criteria are those, usually mentioned first within S, M, A, R, T – Specific, Measurable, Action Oriented, Resourced and Trackable.

Subcriteria	Weight
1 Specific	0,159
2 Significant	0,008
1 Measurable	0,096
2 Meaningful	0,017
3 Manageable	0,064
1 Achievable	0,102
2 Action Oriented	0,129
1 Relevant	0,021
2 Realistic	0,059
3 Resourced	0,107
1 Time Based	0,101
2 Trackable	0,136

Figure 4. Sub-criteria limit weights

Figure 5 shows the sub-criteria weights in graphical form. We tried to calculate some typical cases based on different input assumptions but these criteria weight values remained very similar or the same. It is surprising that the “Significant” sub-criteria have the lowest limit weight. Analyzing this situation we have discovered that sometimes project managers do not

understand what the expressions “Significant project, significant goal” mean. Often, they assume that every project is significant and therefore they have unrealistic expectations with regard to the values of this sub-criteria weight.

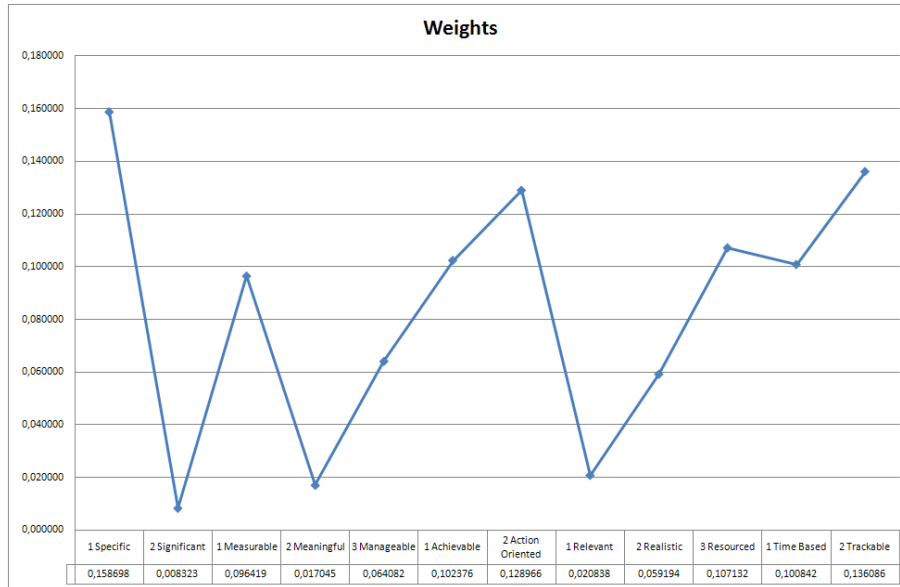


Figure 5. Sub-criteria limit weights – chart view

Conclusions

The aim of the paper was to analyse quality and quantity of various criteria frequently used while evaluating a project within a project management process. We have chosen the SMART approach to evaluate the fulfilment of project goals.

- The methodology used seems to be useful for the analysis of various projects according to more or less differing criteria.
- The ANP method allows description and research of complex dependencies among the important project criteria from various points of view. Network dependencies are typical for this problem.
- Our future research will be focused on criteria weights and on actual project proposal assessment. These weights have to be estimated by experts' judgement, because the set of SMART criteria requires the soft system approach.

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DEMATEL, ANP AND VICOR BASED HYBRID METHOD APPLICATION TO RESTORATION OF HISTORICAL ORGANS

Abstract

The aim of the paper is to perform ex-post analysis for of a portable organ restoration applying a hybrid method, which combines DEMATEL, ANP and VICOR multicriteria approaches. The analysis of results and comparison with the earlier research based in the scheme of ELECTRE I method is included.

Keywords

Portable organ reconstruction, hybrid multicriteria method, DEMATEL, ANP, VICOR.

Introduction

The goal of the paper [Trzaskalik-Wyrwa et al. 2006] was to determine the best way of renovation of a historic positive organ, found several years ago in the Podlasie region (part of Poland). Portable organs were very popular musical instrument in Poland in the seventeenth and eighteenth centuries. Its popularity was due above all to the ease of handling and possibility of easy transportation. Unfortunately, only several copies of this once so common instrument are nowadays extant in Poland.

The following decision problem arose: what is the best way to reconstruct the found instrument, taking into account a variety of criteria. In [Trzaskalik-Wyrwa et al. 2006], this issue has been presented as a multi-criteria decision-making problem and solved by means of ELECTRE I method. Reconstruction of the instrument was performed using the received. recommendation.

ELECTRE I method requires the decision maker to specify criteria weights and subsequent variant ratings. However, interactions among criteria are not analyzed. This possibility can be found in ANP method. Anyway, applying ANP in the source version is numerically troublesome. This is why a hybrid method which combines elements of three multicriteria approaches: DEMATEL [Fontela and Gabus 1974], ANP [Saaty 1999] and VIKOR [Opricovic and Tzeng 2007], allows to overcome numerical difficulties, emerging when performing the calculations using only the ANP method. A description of that hybrid method can be found in [Liu et al. 2012; Tzeng et al. 2007; Tzeng and Huang 2011]. This method is also presented in our paper.

The aim of the research is to perform ex-post analysis for the rediscovered instrument reconstruction by applying hybrid method mentioned above. We want to find out, how to take into account the mutual influence of criteria and whether these mutual influences will affect the selection of the final solution.

The paper is divided into four parts. In chapter 1, we present brief considerations of the decision-making problem, fully described in [Trzaskalik-Wyrwa et al. 2006]. The criteria considered and the decision variants are described. The second chapter includes a description of the hybrid method, which combines elements of DEMATEL, ANP, and VICOR. The third chapter presents the data provided by the expert (co-author of this paper – Małgorzata Trzaskalik-Wyrwa). Some of them (the evaluation of alternatives due to subsequent criteria) were used previously [Trzaskalik-Wyrwa et al. 2006], others (the specification of the mutual influence of criteria) have been prepared by the expert for the purposes of this study. An application of the hybrid method and details of numerical calculations are presented. The fourth chapter contains an analysis of the results and compare them with the earlier results, obtained in [Trzaskalik-Wyrwa et al. 2006] by means of ELECTRE I method.

1. Restoration of historical portable organ as a multicriteria decision process

1.1. Decision criteria

We consider a division of the values of historical organs into four groups: historic, artistic, musical and utilitarian values. We will describe the values constituting each of the four groups [Trzaskalik-Wyrwa et al. 2006].

Historic values determine the character of the object as a document and its influence on the development of historical knowledge. Among the values of this group are **scientific values**, due to the fact that an organ

is an historic object, requiring a scholarly description. Also in this group are **technical values**, determining the ingenuity of the construction, the quality of the workmanship and the scientific value of its current condition. Also **historic emotional values**, perceived not only by scientists and scholars, but also by the public at large, belong here.

The **ownership values**, i. e., values stemming from the ownership of the original item (without hypothetical additions) are connected with honest approach of the conservators to the historic object, in which that what is preserved should be emphasised above all, as opposed to that what we think might have been there.

The group of **artistic values** is related to the perception of historic organs as works of art, and this is connected with the instrument's case. To this group belong **historic-artistic values**, determining whether the solutions chosen by the builders are typical or atypical as well as the importance of the original, its copy or its hypothetical reconstruction. **Artistic qualities** affect the public independently of the current fashion or style. The **artistic effect** of the case of historic organ should match musical impressions received by the audience from the musical compositions heard by it.

Musical values become apparent during a musical performance. We deal here with the issue of style (**historical musical value**) and of sound (**musical quality**). All of them taken together may reinforce the **musical influence** on the amateur listener. It can happen that the regaining of musical value and the preservation of the original technical solutions are conflicting goals. In such case we face the problem of **utilitarian values** of the historic instrument. The notions of live organ and dead organ are related to this group of values. A **musically dead organ** is an instrument that nowadays cannot fulfil its function of a musical instrument. A **live instrument** is an instrument capable of being used in musical performance, affecting the audience in various ways. Like any historic object, an organ as a piece of furniture can be also visually dead – not suitable for being exhibited, or else visually alive (independently of its musical “vitality”) – beautiful, but unplayable.

1.2. Decision alternatives

On the basis of research and evaluation of the condition of the individual parts of the instrument (or their lack) 12 renovation treatments of the rediscovered instrument have been suggested. They are decision alternatives, described below [Trzaskalik-Wyrwa et al. 2006].

Alternative A1

Preservation of the instrument as a non-functional, visually unattractive object (“destrukt”) and its exhibition in the form of a group of museum exhibits.

Alternative A2

Integration of the elements of the instrument using racks necessary to place the individual elements in proper places.

Alternative A3

Integration of the parts of the instrument with full completion of the construction elements of the case (without covering the “windows” with reconstructed wood carved ornaments) according to their former shape as concluded from the elements preserved; completion of the missing parts of the mechanism. The pipes remain secured, but do not play.

Alternative A4

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome and covering of the “windows” by a neutral filling (canvas, wooden grill). The pipes remain secured, but do not play.

Alternative A5

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome. Hypothetical reconstruction of the wood carved ornaments filling out the “windows” (on the basis of comparative analysis – it is impossible to achieve the historical truth). The pipes remain secured, but do not play.

Alternative A6

Integration of the parts of the instrument with full completion of the construction elements of the case (without covering the “windows” by reconstructed wood carved ornaments) according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Bringing the extant pipes to working condition and reconstruction of the missing pipes, so as to match the sound capabilities of the extant pipes.

Alternative A7

Integration of the parts of the instrument with full completion of the construction elements of the case (without covering the “windows” by reconstructed wood carved ornaments) according to their former shape,

as concluded from the elements preserved; completion of the missing parts of the mechanism. Exhibition of the extant historic pipes in a display case without giving them their former technical functionality. Reconstruction of the entire sound system according to preserved models.

Alternative A8

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome and covering the “windows” by a neutral filling (canvas, wooden grill). Bringing the pipes to a working condition and reconstruction of the missing pipes, so as to match the sound capabilities of the extant pipes.

Alternative A9

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome and covering the “windows” by a neutral filling (canvas, wooden grill). Exposition of the extant historical pipes in a display case without bringing them to a working condition. Reconstruction of the whole sound system according to preserved models.

Alternative A10

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved and completion of the missing parts of the mechanism. Reconstruction of the polychrome. Hypothetical reconstruction of the wood carved ornaments filling out the “windows” (on the basis of comparative analysis – it is impossible to achieve historical truth). Bringing the pipes to a working condition and reconstruction of the missing pipes so as to match the sound of the sound capabilities of the preserved pipes.

Alternative A11

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved and completion of the missing parts of the mechanism. Reconstruction of the polychrome. Hypothetical reconstruction of the wood carved ornaments filling out the “windows” (on the basis of comparative analysis – it is impossible to achieve historical truth). Exhibition of the preserved historic pipes in a display case without bringing them to a working condition. Reconstruction of the whole sound system according to preserved models.

Alternative A12

Preservation of the instrument in its non-functional, visually unattractive condition (as a “destrukt”). Making of an accurate copy. The evaluation focuses on the values of the copy, which is presented to the public.

2. The hybrid method

The hybrid method [Liu et al. 2012; Tzeng et al. 2007; Tzeng and Huang 2011] is a combination of:

- DEMATEL – applied to clarify relation between components,
- ANP – applied to determine the relationship between the criteria (in limited supermatrix),
- VIKOR – applied to obtain the index values in gaps.

Let A be a finite set of decision alternatives:

$$A = \{ A_1, A_2, \dots, A_l \}$$

C – a set of criteria, divided into n categories (called here aspects, dimensions, clusters):

$$C = \{ C_1, C_2, \dots, C_n \}$$

where:

$C_i = \{ c_{i1}, c_{i2}, \dots, c_{im_i} \}$, $i = 1, \dots, n$ – is a subset of criteria in i -th aspect

and F – matrix of values of the j -th alternative in the k -th criterion:

$$F = [f_{kj}], \quad j = 1, 2, \dots, l, \quad k = 1, 2, \dots, M$$

where $M = \sum_{i=1}^n m_i$

We assume that the criteria are defined so that a higher the value of the criterion is preferred to a lower one. Each criterion is assigned a positive number which reflects the valid contribution of that criteria.

The considered method is divided into following steps:

Step 1: Develop the structure of the problem.

The problem is broken down to a level structure.

Step 2: Develop the total influence matrix.

Based on the DEMATEL method, interactions between the aspects are explained to construct the map of the direct impact. This step is divided into three sub-steps:

Step 2a: Identify the average influence matrix A

The initial matrix $A = [a_{ij}^h]_{n \times n}$ is calculated, using experts' evaluations, where a_{ij}^h denotes the influence of i -th factor on j -th factor in h -th expert's opinion. If i -th element affects j -th element directly, then $a_{ij}^h \neq 0$; otherwise, $a_{ij}^h = 0$. We obtain:

$$A = [a_{ij}]_{n \times n} \quad a_{ij} = \frac{1}{H} \sum_{h=1}^H a_{ij}^h \quad (1)$$

where H denotes the number of experts and $h = 1, 2, \dots, H$. In particular, we can use expertise of one expert, then influence matrix A is obtained directly:

$$A = [a_{ij}]_{n \times n} \quad (2)$$

Step 2b: Calculate the normalized influence matrix X

We normalize the matrix A , applying (3) and (4). The diagonal in normalized matrix is equal to 0, and the maximum sum of each row or column is equal to 1:

$$X = sA \quad (3)$$

where:

$$s = \min \left\{ \frac{1}{\max_{1 \leq i \leq n} \sum_{j=1}^n a_{ij}}, \frac{1}{\max_{1 \leq j \leq n} \sum_{i=1}^n a_{ij}} \right\} \quad (4)$$

Step 2c: Compute the total-influence matrix T

The total-influence matrix T can be obtained according to (5) (I denotes the identity matrix):

$$T = X + X^2 + \dots + X^k = X(I - X)^{-1}, \quad \text{when } \lim_{k \rightarrow \infty} X^k = [0]_{n \times n} \quad (5)$$

The proof of this relationship can be found in [Tzeng and Huang 2011].

Step 2d: Set a threshold value α and obtain the normalized α -cut total-influence matrix T^α

We have total-influence matrix T in the form:

$$T = \begin{bmatrix} t_{11} & \dots & t_{1j} & \dots & t_{1n} \\ \vdots & & \vdots & & \vdots \\ t_{i1} & \dots & t_{ij} & \dots & t_{in} \\ \vdots & & \vdots & & \vdots \\ t_{n1} & \dots & t_{nj} & \dots & t_{nn} \end{bmatrix} \quad (6)$$

The α -cut total-influence matrix T^α will be given by Eq. (7)

$$T^\alpha = \begin{bmatrix} t_{11}^\alpha & \dots & t_{1j}^\alpha & \dots & t_{1n}^\alpha \\ \vdots & & \vdots & & \vdots \\ t_{i1}^\alpha & \dots & t_{ij}^\alpha & \dots & t_{in}^\alpha \\ \vdots & & \vdots & & \vdots \\ t_{n1}^\alpha & \dots & t_{nj}^\alpha & \dots & t_{nn}^\alpha \end{bmatrix} \quad (7)$$

where if $t_{ij} < \alpha$ then $t_{ij}^\alpha = 0$ else $t_{ij}^\alpha = t_{ij}$

The α -cut total-influence matrix T_α needs to be normalized by dividing by the value:

$$d_i = \sum_{j=1}^n t_{ij}^\alpha, \quad i = 1, 2, \dots, n \quad (8)$$

Finally we obtain T_D as follow:

$$T_D = \begin{bmatrix} t_{11}^\alpha / d_1 & \dots & t_{1j}^\alpha / d_1 & \dots & t_{1n}^\alpha / d_1 \\ \vdots & & \vdots & & \vdots \\ t_{i1}^\alpha / d_i & \dots & t_{ij}^\alpha / d_i & \dots & t_{in}^\alpha / d_i \\ \vdots & & \vdots & & \vdots \\ t_{n1}^\alpha / d_n & \dots & t_{nj}^\alpha / d_n & \dots & t_{nn}^\alpha / d_n \end{bmatrix} = \begin{bmatrix} t_{11}^D & \dots & t_{1j}^D & \dots & t_{1n}^D \\ \vdots & & \vdots & & \vdots \\ t_{i1}^D & \dots & t_{ij}^D & \dots & t_{in}^D \\ \vdots & & \vdots & & \vdots \\ t_{n1}^D & \dots & t_{nj}^D & \dots & t_{nn}^D \end{bmatrix} \quad (9)$$

Step 3: Compare all criteria to form the initial supermatrix.

The initial supermatrix can be obtained in two ways:

A. Initial (unweighted) supermatrix can be obtained by pairwise comparison of all criteria as in AHP method [Tzeng and Huang 2011, p. 161]:

$$W = \begin{matrix} & \begin{matrix} C_1 & C_2 & \dots & C_n \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{matrix} & \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1m_1} & c_{21} & c_{22} & \dots & c_{2m_2} & \dots & c_{n1} & c_{n2} & \dots & c_{nm_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ W_{11} & & & W_{12} & & & & & & W_{1n} & & & \\ \vdots & & & \vdots & & & & & & \vdots & & & \\ W_{21} & & & W_{22} & & & & & & W_{2n} & & & \\ \vdots & & & \vdots & & & & & & \vdots & & & \\ W_{n1} & & & W_{n2} & & & & & & W_{nn} & & & \end{bmatrix} \end{matrix} \quad (10)$$

B. We can repeat the steps 2a-2d, as in [Liu et al. 2012], on initial influence matrix for all criteria

$$A = [a_{ij}] \sum_{i=1}^n m_i \times \sum_{i=1}^n m_i$$

Then we obtain the matrix T_C and $W = (T_C)^T$

Finally we receive unweighted supermatrix W in the form:

$$W = \begin{bmatrix} W_{11} & \dots & W_{1j} & \dots & W_{1n} \\ \vdots & & \vdots & & \vdots \\ W_{il} & \dots & W_{ij} & \dots & W_{in} \\ \vdots & & \vdots & & \vdots \\ W_{nl} & \dots & W_{nj} & \dots & W_{nn} \end{bmatrix} \quad (11)$$

Step 4: Obtain the weighted supermatrix.

The normalized T_D is multiplied by unweighted supermatrix W to obtain weighted supermatrix W^α . The results are shown in Eq. (12).

$$W^\alpha = T_D \times W = \begin{bmatrix} t_{11}^D \times W_{11} & \dots & t_{1l}^D \times W_{1j} & \dots & t_{1n}^D \times W_{1n} \\ \vdots & & \vdots & & \vdots \\ t_{il}^D \times W_{il} & \dots & t_{ij}^D \times W_{ij} & \dots & t_{in}^D \times W_{in} \\ \vdots & & \vdots & & \vdots \\ t_{nl}^D \times W_{nl} & \dots & t_{nj}^D \times W_{nj} & \dots & t_{nn}^D \times W_{nn} \end{bmatrix} \quad (12)$$

Step 5: Obtain the limit supermatrix.

The ANP weights of each criterion can be obtained from limit supermatrix:

$$W^{lim} = \lim_{k \rightarrow \infty} (W^\alpha)^k \quad (13)$$

The evaluation of the total preference is performed by means of VIKOR method, which can be divided into following steps:

Step 6: Check the best value f_k^* and the worse value f_k^- .

f_k^* represents the positive-ideal point, that means the expert gives the scores of the best value (aspired levels) for each criterion and f_k^- represents the negative-ideal point, that means the expert gives the scores of the worst values for each criterion. Those values can be computed by the traditional approach, using Eqs. (14) and (15) to obtain the results:

$$f_k^* = \max_i f_{ki}, \quad i = 1, 2, \dots, l \quad (14)$$

$$f_k^- = \min_i f_{ki}, \quad i = 1, 2, \dots, l \quad (15)$$

or by setting aspire levels vector as in Eq. (16)

$$f^* = (f_1^*, f_2^*, \dots, f_M^*) \quad (16)$$

and setting worst value vector as in Eq. (17)

$$f^- = (f_1^-, f_2^-, \dots, f_M^-) \quad (17)$$

Step 7: Calculate the mean of group utility S_i and the maximal regret Q_i .

The S_i represents the ratios of distance to the positive-ideal, it means the synthesized gap for all criteria. The Q_i represents the maximal gap-ratios (regret) of normalized distance to the aspired level in all criteria, that is, the maximal gap for prior improvement. Those values can be computed respectively by Eqs. (18) and (19):

$$S_i = \sum_{k=1}^M w_k r_{ki} \quad (18)$$

$$Q_i = \max_k \{w_k r_{ki} \mid k = 1, 2, \dots, M\} \quad (19)$$

where:

w_k – represents the influential weights of the k-th criterion from previous step,

$r_{ki} = \frac{|f_k^* - f_{ki}|}{|f_k^* - f_k^-|}$ – represents the gap-ratios (regret) of normalized distance to the aspired level point

Step 8: Obtain the comprehensive indicator R_i .

The values can be computed using Eq. (20).

$$R_i = v(S_i - S^*) / (S^- - S^*) + (1 - v)(Q_i - Q^*) / (Q^- - Q^*) \quad (20)$$

where:

$$S^* = \min_k S_k \text{ or we can set } S^* = 0 \text{ (the aspired level)}$$

$$S^- = \max_k S_k \text{ or we can set } S^- = 1 \text{ (the worst situation)}$$

$$Q^* = \min_k Q_k \text{ or we can set } Q^* = 0 \text{ (the aspired level)}$$

$$Q^- = \max_k Q_k \text{ or we can set } Q^- = 1 \text{ (the worst situation)}$$

Therefore, when $S^* = 0$ and $S^- = 1$, and $Q^* = 0$ and $Q^- = 1$, we can re-write the Eq. (21) as:

$$R_i = vS_i + (1 - v)Q_i \quad (21)$$

The coefficient $v = 1$ represents situation where only the average gap (average regret) is considered. Coefficient $v = 0$ represents situation where only the maximum gap to the prior improvement is considered. Generally the coefficient is adjusted according to the situation. In the most situations we can use $v = 0.5$.

Step 9: Rank the alternatives, sorting by the value of $\min \{ R_i \mid i = 1, 2, \dots, l \}$.

3. Application of the hybrid method to the problem of restoration of historical organs

According to the expert's evaluations, we will consider the set of values, gathered in Table 1.

Table 1

Value Evaluation by Criteria												
Criteria	<i>A1</i>	<i>A2</i>	<i>A3</i>	<i>A4</i>	<i>A5</i>	<i>A6</i>	<i>A7</i>	<i>A8</i>	<i>A9</i>	<i>A10</i>	<i>A11</i>	<i>A12</i>
Historical-scientific value	10	8	6	6	6	6	6	6	6	6	6	0
Historical-technical value	4	6	10	8	8	6	10	6	10	6	10	0
Emotional value	10	10	10	8	6	10	6	6	4	4	2	0
Ownership value	10	10	10	9	5	8	5	9	4	5	0	0
Historical-artistic value	0	2	4	6	6	4	4	6	6	8	8	0
Artistic quality	0	0	2	4	8	2	2	4	4	8	8	8
Artistic influence	2	2	6	8	10	6	6	8	8	10	10	10
Historical-musical value	0	0	0	0	0	10	4	10	4	10	4	4
Musical quality	0	0	0	0	0	8	10	8	10	8	10	10
Musical influence	0	0	0	0	0	8	10	8	10	8	10	10
Visual-utilitarian value	2	4	6	8	10	6	6	8	8	10	10	10
Musical-utilitarian value	0	0	0	0	0	8	10	8	10	8	10	10

Step 1: According to the literature review and expert experiences, an value evaluation system including four dimensions and 12 criteria is established, as given in Table 2.

Table 2

The structure of evaluation criteria		
Aspects/Dimensions	Criteria	
C_1 Historical Values	c_{11}	Historical-scientific value
	c_{12}	Historical-technical value
	c_{13}	Emotional value
	c_{14}	Ownership value
C_2 Artistic values	c_{21}	Historical-artistic value
	c_{22}	Artistic quality
	c_{23}	Artistic influence
C_3 Music values	c_{31}	Historical-musical value
	c_{32}	Musical quality
	c_{33}	Musical influence
C_4 Utilitarian values	c_{41}	Visual-utilitarian value
	c_{42}	Musical-utilitarian value

Step 2: The ratings for each criterion's relationship to sustainable development using a five-point scale ranging from 0 (no effect) to 4 (extremely influential) were collected.

Table 3

Influence between aspects					
		C_1	C_2	C_3	C_4
Historical Values	C_1	x	2	2	3
Artistic values	C_2	3	x	0	3
Music values	C_3	3	0	x	2
Utilitarian values	C_4	0	4	4	x

Step 2a: Identify the average influence matrix A

As it was difficult to the decision maker to determine the influence between aspects themselves, we calculated the influence as rounded average of all influences between the criteria in the aspects. The result is presented in Table 4.

Table 4

Influence matrix					
		C_1	C_2	C_3	C_4
Historical Values	C_1	2	2	2	3
Artistic values	C_2	3	2	0	3
Music values	C_3	3	0	3	2
Utilitarian values	C_4	0	4	4	0

Step 2b-2d: Total-influential dimensions matrix T_D .

We used $\alpha = 0,1$ so it was necessary to normalize the resulting matrix. Result is presented in Table 5.

Table 5

Total-influential aspects (dimensions) matrix T_D .

	C_1	C_2	C_3	C_4
C_1	0,00	0,41	0,28	0,31
C_2	0,32	0,00	0,32	0,36
C_3	0,28	0,41	0,00	0,31
C_4	0,29	0,43	0,29	0,00

Step 3: Compare all the criteria to form the initial supermatrix. Because only direct impacts of the criteria were available, we use method **B**, repeating steps 2a-2d to the matrix presented in Table 6.

Table 6

Influential matrix A on criteria												
	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}
c_{11}	0	3	1	4	4	4	1	4	1	1	2	2
c_{12}	3	0	1	4	2	1	0	2	1	1	2	2
c_{13}	2	2	0	3	2	2	1	2	1	2	4	4
c_{14}	4	4	4	0	4	4	4	4	4	4	3	3
c_{21}	3	3	3	4	0	4	4	2	2	2	4	0
c_{22}	2	2	2	2	3	0	2	0	0	0	4	0
c_{23}	1	1	1	4	4	4	0	0	0	0	4	0
c_{31}	3	3	2	4	0	0	0	0	4	4	0	4
c_{32}	1	1	0	2	0	0	0	4	0	4	0	4
c_{33}	1	1	0	2	0	0	0	4	4	0	0	4
c_{41}	0	0	0	0	4	4	4	0	0	0	0	0
c_{42}	0	0	0	0	0	0	0	4	4	4	0	0

The result of repeating steps 2a-2b is shown in Table 7. Now we use $\alpha = 0$ but it is also necessary to normalize the resulting matrix.

Table 7

Total-influential criteria matrix T_C												
	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}
c_{11}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0
c_{12}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0
c_{13}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0
c_{14}	0	0	0	0	0,068	0,14	0,257	0,043	0,23	0,191	0,072	0
c_{21}	0	0	0	0,099	0	0,135	0,248	0,042	0,223	0,184	0,069	0
c_{22}	0	0	0	0,106	0,071	0	0,267	0,045	0,239	0,198	0,075	0
c_{23}	0	0	0	0,121	0,08	0,165	0	0,051	0,272	0,225	0,085	0
c_{31}	0	0	0	0,096	0,064	0,132	0,242	0	0,217	0,18	0,068	0
c_{32}	0	0	0	0,117	0,078	0,16	0,294	0,05	0	0,219	0,082	0
c_{33}	0	0	0	0,112	0,075	0,153	0,282	0,048	0,253	0	0,079	0
c_{41}	0	0	0	0,099	0,066	0,136	0,249	0,042	0,223	0,185	0	0
c_{42}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0

Step 4: We obtain the weighted supermatrix by multiplying matrixes $(T_C)^T$ and W presented in tables 5 and 7. The result is presented in Table 8.

Table 8

Weighted supermatrix W^{α}												
	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}
c_{11}	0	0	0	0	0	0	0	0	0	0	0	0
c_{12}	0	0	0	0	0	0	0	0	0	0	0	0
c_{13}	0	0	0	0	0	0	0	0	0	0	0	0
c_{14}	0	0	0	0	0,041	0,044	0,05	0,027	0,032	0,031	0,031	0,029
c_{21}	0,02	0,02	0,02	0,022	0	0	0	0,021	0,025	0,024	0,024	0,022
c_{22}	0,041	0,041	0,041	0,045	0	0	0	0,042	0,051	0,049	0,049	0,046
c_{23}	0,075	0,075	0,075	0,082	0	0	0	0,078	0,094	0,09	0,09	0,084
c_{31}	0,011	0,011	0,011	0,012	0,017	0,019	0,021	0	0	0	0,013	0,012
c_{32}	0,058	0,058	0,058	0,064	0,092	0,099	0,113	0	0	0	0,069	0,065
c_{33}	0,048	0,048	0,048	0,053	0,076	0,082	0,093	0	0	0	0,057	0,054
c_{41}	0,019	0,019	0,019	0,02	0,03	0,032	0,036	0,019	0,024	0,022	0	0
c_{42}	0	0	0	0	0	0	0	0	0	0	0	0

Step 5: By multiplying weighted supermatrix W^{α} we obtain the limit supermatrix W^{\lim} presented in Table 9.

Table 9

Limit supermatrix W^{\lim}												
	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}
c_{11}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
c_{12}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
c_{13}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
c_{14}	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126
c_{21}	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058
c_{22}	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119
c_{23}	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220
c_{31}	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036
c_{32}	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189
c_{33}	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157
c_{41}	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094
c_{42}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000

The weights obtained are shown in Table 10.

Table 10

The evaluation criteria			
Aspects/Dimensions	Criteria		Weight
C_1 Historical Values	c_{11}	Historical-scientific value	0,000
	c_{12}	Historical-technical value	0,000
	c_{13}	Emotional value	0,000
	c_{14}	Ownership value	0,126
C_2 Artistic values	c_{21}	Historical-artistic value	0,058
	c_{22}	Artistic quality	0,119
	c_{23}	Artistic influence	0,220
C_3 Music values	c_{31}	Historical-musical value	0,036
	c_{32}	Musical quality	0,189
	c_{33}	Musical influence	0,157
C_4 Utilitarian values	c_{41}	Visual-utilitarian value	0,094
	c_{42}	Musical-utilitarian value	0,000

Steps 7-9: The results are presented in Table 11.

Vikor Method Evaluation of Alternatives

Table 11

Dimensions/Criteria	Weight	f^*	f	$A1$	$A2$	$A3$	$A4$	$A5$	$A6$	$A7$	$A8$	$A9$	$A10$	$A11$	$A12$
C_1. Historical Values															
c_{11} Historical-scientific value	0	10	0	0	0	0	0	0	0	0	0	0	0	0	0
c_{12} Historical-technical value	0	10	0	0	0	0	0	0	0	0	0	0	0	0	0
c_{13} Emotional value	0	10	0	0	0	0	0	0	0	0	0	0	0	0	0
c_{14} Ownership value	0,126	10	0	0	0	0	0,013	0,063	0,025	0,063	0,013	0,076	0,063	0,126	0,126
C_2. Artistic values															
c_{21} Historical-artistic value	0,058	10	0	0,058	0,046	0,035	0,023	0,023	0,035	0,035	0,023	0,023	0,012	0,012	0,058
c_{22} Artistic quality	0,119	10	0	0,119	0,119	0,096	0,072	0,024	0,096	0,096	0,072	0,072	0,024	0,024	0,024
c_{23} Artistic influence	0,22	10	0	0,176	0,176	0,088	0,044	0	0,088	0,088	0,044	0,044	0	0	0
C_3. Music values															
c_{31} Historical-musical value	0,036	10	0	0,036	0,036	0,036	0,036	0,036	0	0,021	0	0,021	0	0,021	0,021
c_{32} Musical quality	0,189	10	0	0,189	0,189	0,189	0,189	0,189	0,038	0	0,038	0	0,038	0	0
c_{33} Musical influence	0,157	10	0	0,157	0,157	0,157	0,157	0,157	0,031	0	0,031	0	0,031	0	0
C_4. Utilitarian values															
c_{41} Visual-utilitarian value	0,094	10	0	0,076	0,057	0,038	0,019	0	0,038	0,038	0,019	0,019	0	0	0
c_{42} Musical-utilitarian value	0	10	0	0	0	0	0	0	0	0	0	0	0	0	0
<hr/>															
			S_i	0,811	0,78	0,638	0,552	0,492	0,351	0,341	0,24	0,255	0,168	0,183	0,23
			Q_i	0,189	0,189	0,189	0,189	0,189	0,096	0,096	0,072	0,076	0,063	0,126	0,126
$v = 0,5$			R_i	0,5	0,485	0,414	0,371	0,341	0,223	0,218	0,156	0,165	0,116	0,155	0,178
<hr/>															
Rank				12	11	10	9	8	7	6	3	4	1	2	5

Conclusions

According to the hybrid method considered in the paper alternative *A10* was classified as the best one.

We will compare results obtained in the present research by means of hybrid method with the results obtained previously applying ELECTRE I method.

Table 12

Ranks

No.	Metoda Electre	No.	Hybryd method	No.	Hybryd method
1.	<i>A8, A10</i>	1.	<i>A10</i>	7.	<i>A6</i>
2.	<i>A6, A9, A11</i>	2.	<i>A11</i>	8.	<i>A5</i>
3.	<i>A7</i>	3.	<i>A8</i>	9.	<i>A4</i>
4.	<i>A3, A12</i>	4.	<i>A9</i>	10.	<i>A3</i>
5.	<i>A1, A2, A4, A5</i>	5.	<i>A12</i>	11.	<i>A2</i>
		6.	<i>A7</i>	12.	<i>A1</i>

In both rankings alternative *A10* was classified as the best one. The alternative *A11* was classified as the second in the hybrid method and was better than the alternative *A8*, classified in ELECTRE I method into the first class and recommended for further realization. The rest of the alternatives were classified similarly in the both methods. It is less important, because the considered decision problem was formulated as the best alternative choice problem.

It is seen that taking into account the mutual influence of criteria causes a change of recommendation. When applying ELECTRE I method, the decision maker could choose between two alternatives: *A8* and *A10*. After the analysis of these alternatives the decision maker concluded that the alternative *A10* is better. When applying the hybrid method we obtained a ranking in which alternative *A10* was the best one. The alternative *A8*, recommended previously, was placed in the new ranking at the third position, so its chance to be recommended on the basis of the hybrid method is small.

The expert's ex post opinion (several years after reconstruction of the instrument) seems interesting. In perspective, it is seen that earlier choice of the alternative *A11* (which was second in the new ranking) would be better because of the possibility of the use of the instrument in musical performances. It is connected with the revision of criteria values for decision alternatives.

The alternative *A11* recommends reconstruction of the whole sound system according to preserved models. This solution gives the possibility of uncomprising use of the new pipes to obtain satisfactory level of sound. The conservation of several original pipes and the adjustment to them the rest of reconstructed pipes to their loudness caused an additional adverse result (among other problems) that the instrument plays too softly, and the “historical” timbre makes up for this insufficiently.

Also, the problem of hypothetical shape of wood-carver’s decorations in upper box windows (not hitherto reconstructed) could be positively solved now, as not causing damage to historical substance and, at the same time considerably increasing visual attraction of the monument.

The knowledge obtained during the decision process and later can be used in conservation works in the future.

The recommendation of the alternative *A8* (first compared with the alternative *A10* while performing ELECTRE I analysis) was prepared by the expert on the merits of the case. The merits and arguments from the field of historical objects restoration should be the most important – the ranking (of course significant) is of auxiliary importance.

A detailed analysis of the hybrid method assumptions and justification of joint applications of DEMATEL, ANP and VICOR methods is a separate problem. Such an analysis has not been performed yet and it should be presented in future research.

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MULTICRITERIA EVALUATION OF FUZZY NET PRESENT VALUE¹

Abstract

In this paper it is shown how to assess the degree of influence of various factors on the value of the project (NPV). The assessment is based on grouping and ranking of cash flows linked to various factors. The formulas are generated both for crisp and fuzzy net present value analysis. The projects are then evaluated on the basis of at least two criteria: the NPV and the risk (positive or negative) linked to the factors which have most influence on the project's NPV whose change may change the NPV considerably. In applications, fuzzy present values of different factors are calculated and compared for two different cases.

Keywords

Fuzzy Logic, Fuzzy Net Present Value, Ranking Fuzzy Numbers.

Introduction

Investment decisions are strategic decisions, which directly affect the position of a firm in the market. One of the most important criteria used to select an investment project is its worth for the decision makers. The discounted cash flows analyses (mostly net present value (NPV) analysis) are preferred to evaluate the investment projects. The NPV of a project, calculated before the beginning of the project, is the most widely used criterion to evaluate a project – it is generally accepted that the higher the NPV, the better the project.

However, things are not as simple that. It is widely known that investment decisions are always exposed to a high degree of uncertainty

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and risk. Thus, the NPV cannot be the only project selection criterion, because it may change and be in reality, once the project is implemented, substantially different from its planned value. That is why each investment project has to be evaluated taking into account more than just this one criterion. It is widely agreed that the risk of the project has to be taken into account too.

If risk is understood both in the negative (as a possible threat) and positive (as a possible chance), it is good to know where risk lies in a given investment project and which factor can change or be changed in such a way that its influence on the NPV of the project would be high. Each investment project is influenced by several factors, which may be dependent or independent of the company in question (behavior and situation of the customers and suppliers, payment conditions, prices, “make or buy” decisions etc.). The aim of the paper is to show how these factors and their possible influence on the project value can be identified. Each project should then be evaluated on the basis of its NPV and of the risk of a change in the NPV, linked to various factors influencing the value of the project.

What is more, as each investment project is a long-term project and its parameters are always connected with risk and uncertainty, we consider here the fuzzy approach to the estimation of the project’s parameters. Fuzzy numbers allow us to model the incomplete knowledge about cash flows in the future.

The outline of the paper is as follows: first we present the classical approach to the evaluation of investment projects, but extending the classical definition of the NPV so that different factors influencing it, as well as their degree of influence, may be identified. Then we briefly describe the fundamentals of the fuzzy logic [Zadeh 1965; Ross 1995]. Finally the fuzzy approach to investment project evaluation is presented [Chio and Park 1994; Kuchta 2000; Zhang et al. 2011; Sorenson and Lavelle 2008], into which we incorporate the proposed approach taking into account those factors which influence the NPV. Subsequently, we present one of several possible approaches to fuzzy numbers ranking, which will be needed while estimating the “force” of individual factors influencing the NPV. The approach chosen permits to adjust the method to the attitude of the decision maker (pessimistic or optimistic). The paper concludes with two numerical examples.

1. Net Present Value Analysis

One of the most used discounted cash flow analysis method is net present value analysis which is calculated by adding up the present values of all cash flows into or out of the project.

The basic formula of present value of a single future payment ($PV(F)$) at the end of n^{th} year from now is given in Eq. 1.1 where F stands for the amount of the payment and i stands for the compound interest rate.

$$PV(F) = F \frac{1}{(1+i)^n} \quad (1.1)$$

The formula of net present value of a cash flow series ($NPV(F_1, \dots, F_m)$) which has m different payments is calculated by Eq. 1.2 where F_j stands for the amount of the payment and n_j stands for the time period of the payment.

$$\begin{aligned} NPV(F_1, \dots, F_m) = & -I + \left(F_1 \frac{1}{(1+i)^{n_1}}\right) + \left(F_2 \frac{1}{(1+i)^{n_2}}\right) + \\ & + \dots + \left(F_m \frac{1}{(1+i)^{n_m}}\right) \end{aligned} \quad (1.2)$$

Sometimes the initial investment of the project can be distributed over several years. Eq. 1.3 gives NPV when the initial investment is distributed over z years:

$$\begin{aligned} NPV = & -I_0 - \left(I_1 \frac{1}{(1+i)^{n_1}}\right) - \left(I_2 \frac{1}{(1+i)^{n_2}}\right) - \dots - \left(I_z \frac{1}{(1+i)^{n_z}}\right) + \\ & + \left(F_1 \frac{1}{(1+i)^{n_1}}\right) + \left(F_2 \frac{1}{(1+i)^{n_2}}\right) + \dots + \left(F_m \frac{1}{(1+i)^{n_m}}\right) \end{aligned} \quad (1.3)$$

The NPV is influenced by several factors. Sometimes small changes in some cash flow factors could be introduced – or occur independently of us – which result in a substantially better or worse NPV. For example, if the NPV is strongly influenced by labor costs, the decision maker could prefer to hire lower qualified workers or to outsource the work. If the factors influencing strongly the NPV are the payment conditions offered to the customers, the deadlines of the payments could be changed. Sometimes the NPV may be very sensitive to the situation of one of the customers or the suppliers. At that point, the decision maker may want to know the degree of influence of the situation of a given customer or supplier, of the chosen resources, suppliers, payment conditions etc. on the net present value of the project.

We propose, thus, to reorder the cash flow from Eq. (1.3) into groups of influence: each group consists of cash flows which depend on one specific factor (e.g. the situation of one customer, the decision to hire a certain workforce etc.). Of course, we assume that such reordering is possible – that it is possible to classify the cash flows into such groups which are influenced mainly by one factor. It is a limiting assumption, but less limiting than that – generally assumed – about the independence of the cash flows in subsequent

years – the flows in various years are usually not independent, as the choice of one supplier, of the payment conditions or of resources used has its consequences for several cash flows in different years.

Thus let F_{jl} ($j = 1, \dots, n$, $l = 1, \dots, k$) denote the groups of cash flows occurring in the j -th year influenced by the l -th factor. The cash flows dependent on the l -th factor in the whole project are represented in Eq. 1.4, and the present value of these flows are calculated by Eq. 1.5 (we assume here that the investment is excluded from the influence of the factors, which is not a limiting assumption – the approach will be identical to the one that includes those factors).

$$F_l = F_{1l} + F_{2l} + F_{3l} + \dots + F_{nl} \quad l = 1, \dots, k \quad (1.4)$$

$$PV^l = \sum_{j=1}^n \frac{F_{jl}}{(1+i)^{n_j}} \quad l = 1, \dots, k \quad (1.5)$$

As shown in Eq. 1.7, the NPV of the project is equal to the sum of the NPVs of the cash flows linked to the individual factors and to the PV^I , which is the present value of the initial investment given by Eq. 1.6.

$$PV^I = PV(I_0) + PV(I_1) + \dots + PV(I_z) \quad (1.6)$$

$$NPV = -PV^I + PV^1 + PV^2 + PV^3 + \dots + PV^k \quad (1.7)$$

Let us assume that we have the following inequality:

$$PV^1 > PV^2 > PV^3 > \dots > PV^k \quad (1.8)$$

If the “greater-than-relations” stand for substantial differences between values $PV^1, PV^2, PV^3, \dots, PV^k$, then a unitary change (intended by us or independent of us) in the values $PV^1, PV^2, PV^3, \dots, PV^k$ leads to a different change in the NPV of the project. Thus, if Eq. 1.8 holds, the cash flows F_1 constitutes the main risk source (positive or negative) for the project, F_2 the next one etc. Now we have to evaluate, how easy it is for F_1 to change. If we can easily increase it, then F_1 constitutes a chance. If our environment can easily decrease it, then it constitutes a negative risk. The same analysis may be applied to F_2, F_3 etc.,

For another project we may have another ranking:

$$PV^k > PV^{k-1} > PV^{k-2} > \dots > PV^1 \quad (1.9)$$

In such a case we would have to start our analysis of the project from F_k, F_{k-1} etc. Thus for one project the main positive or negative risk may be due to such factors as the inventory level or the payment conditions, while for another project the main risk factor may be the choice of suppliers, of resources etc. These factors may be more or less susceptible to change – and this information has to influence our project evaluation apart from the basic criterion, the NPV.

2. Fuzzy Logic

Fuzzy set theory was introduced by Zadeh in 1965. A fuzzy set is defined as a class of objects with a continuum of grades of membership, which is characterized by a membership function that assigns to each object a grade of membership ranging between zero and one. A fuzzy set A in U is characterized by a membership function $\mu_A(x)$ which associates with each point in U a real number in interval $[0, 1]$, with the value of $\mu_A(x)$ at x representing “the grade of membership” of x in A [Zadeh 1965]. We can also interpret $\mu_A(x)$ as the possibility degree of x being the actual value of a magnitude which is not known to us for the moment

The fuzzy number type most often used is a so-called triangular fuzzy number, in short TFN. The membership function for a triangular fuzzy number $\tilde{M} = (m_l, m_m, m_r)$, characterized by three crisp parameters $m_l < m_m < m_r$ is given in Eq. 2.1.

$$\mu_M(x) = \begin{cases} 0 & x \leq m_l \\ 1 + \frac{x - m_m}{m_m - m_l} & m_l < x < m_m \\ 1 - \frac{x - m_m}{m_r - m_m} & m_m \leq x < m_r \\ 0 & x \geq m_r \end{cases} \quad \text{if} \quad \begin{matrix} x \leq m_l \\ m_l < x < m_m \\ m_m \leq x < m_r \\ x \geq m_r \end{matrix} \quad (2.1)$$

Algebraic operations for TFNs $\tilde{M} = (m_l, m_m, m_r)$ and $\tilde{N} = (n_l, n_m, n_r)$ are given by the following formulas with the order of addition, subtraction, multiplication, division and multiplication by a scalar [Chen et al. 1992]:

$$\tilde{M} \oplus \tilde{N} \cong (m_l + n_l, m_m + n_m, m_r + n_r) \quad (2.2)$$

$$\tilde{M} \ominus \tilde{N} \cong (m_l - n_r, m_m - n_m, m_r - n_l) \quad (2.3)$$

$$\tilde{M} \otimes \tilde{N} \cong \begin{cases} (m_l n_l, m_m n_m, m_r n_r) & \tilde{M} \geq 0, \tilde{N} \geq 0 \\ (m_l n_r, m_m n_m, m_r n_l) & \text{if } \tilde{M} \leq 0, \tilde{N} \geq 0 \\ (m_r n_r, m_m n_m, m_l n_l) & \tilde{M} \leq 0, \tilde{N} \leq 0 \end{cases} \quad (2.4)$$

$$\tilde{M} \oslash \tilde{N} \cong \begin{cases} \left(\frac{m_l}{n_r}, \frac{m_m}{n_m}, \frac{m_r}{n_l} \right) & \tilde{M} \geq 0, \tilde{N} \geq 0 \\ \left(\frac{m_r}{n_r}, \frac{m_m}{n_m}, \frac{m_l}{n_l} \right) & \text{if } \tilde{M} \leq 0, \tilde{N} \geq 0 \\ \left(\frac{m_r}{n_l}, \frac{m_m}{n_m}, \frac{m_l}{n_r} \right) & \tilde{M} \leq 0, \tilde{N} \leq 0 \end{cases} \quad (2.5)$$

$$\lambda \otimes \tilde{M} \cong \begin{cases} (\lambda m_l, \lambda m_m, \lambda m_r) & \text{if } \lambda \geq 0 \\ (\lambda m_r, \lambda m_m, \lambda m_l) & \lambda \leq 0 \end{cases} \quad \forall \lambda \in \mathcal{R} \quad (2.6)$$

The support of a fuzzy number $\tilde{M} = (m_l, m_m, m_r)$ is the interval $[m_l, m_r]$ – thus the domain on which the membership function takes on positive values, together with its boundary. The support of \tilde{M} will be denoted as \bar{M} .

3. Fuzzy Net Present Value

Fuzzy present value of a single future payment ($\tilde{P}\tilde{V}(F)$) occurred at the end of n^{th} year from now is given in Eq. 3.1 where \tilde{F} stands for fuzzy amount of the payment and i stands for the compound interest rate.

$$\tilde{P}\tilde{V}(\tilde{F}) = \frac{\tilde{F}}{(1+i)^n} \quad (3.1)$$

Kuchta [2000] defined the general formula of fuzzy net present value as given in Eq. 3.2, where \tilde{F}_i denotes net cash flows in the time period i and \tilde{i} denotes the fuzzy interest rate.

$$\tilde{NPV} = -\tilde{I} + \sum_{i=0}^n \frac{\tilde{F}_i}{(1+\tilde{i})^i} \quad (3.2)$$

The formula of fuzzy net present value of a project (\tilde{NPV}) which has m different payments and has an initial investment at the beginning of the project is calculated by Eq. 3.3.

$$\widetilde{NPV} = -\tilde{I} + \left(\tilde{F}_1 \frac{1}{(1+\tilde{i})^{n_1}} \right) + \left(\tilde{F}_2 \frac{1}{(1+\tilde{i})^{n_2}} \right) + \dots + \left(\tilde{F}_m \frac{1}{(1+\tilde{i})^{n_m}} \right) \quad (3.3)$$

Eq. 3.4 gives \widetilde{NPV} of a project which has m different payments and the investment distributed over z years:

$$\begin{aligned} \widetilde{NPV} = & -\tilde{I}_0 - \left(\tilde{I}_1 \frac{1}{(1+\tilde{i})^{n_1}} \right) - \left(\tilde{I}_2 \frac{1}{(1+\tilde{i})^{n_2}} \right) - \dots - \left(\tilde{I}_z \frac{1}{(1+\tilde{i})^{n_z}} \right) + \\ & + \left(\tilde{F}_1 \frac{1}{(1+\tilde{i})^{n_1}} \right) + \left(\tilde{F}_2 \frac{1}{(1+\tilde{i})^{n_2}} \right) + \dots + \left(\tilde{F}_m \frac{1}{(1+\tilde{i})^{n_m}} \right) \end{aligned} \quad (3.4)$$

Eq. 3.5 and Eq. 3.6 represent fuzzy equivalents of Eqs. 1.4 and 1.5.

$$\tilde{F}_l = \tilde{F}_{1l} + \tilde{F}_{2l} + \tilde{F}_{3l} + \dots + \tilde{F}_{nl} \quad (3.5)$$

$$\widetilde{PV}^l = \sum_{j=1}^n \frac{\tilde{F}_{jl}}{(1+\tilde{i})^{n_j}} \quad (3.6)$$

As shown in Eq. 3.8, \widetilde{NPV} of the project is equal to the sum of the \widetilde{PV} s due to individual factors because of the linearity of \widetilde{NPV} and the definition of the addition of fuzzy numbers, where \widetilde{PV}^l denotes the present value of the initial investment which is given in Eq. 3.7:

$$\widetilde{PV}^l = \widetilde{PV}(I_0) + \widetilde{PV}(I_1) + \dots + \widetilde{PV}(I_z) \quad (3.7)$$

$$\widetilde{NPV} = -\widetilde{PV}^l + \widetilde{PV}^1 + \widetilde{PV}^2 + \widetilde{PV}^3 + \dots + \widetilde{PV}^k \quad (3.8)$$

To be able to perform in the fuzzy case the type of analysis illustrated in the crisp case by Eqs. 1.8 and 1.9, we have to be able to rank fuzzy numbers. The ranking of fuzzy numbers is not unambiguous and there are several methods which can be used to obtain it. The choice depends on the decision maker, on his preferences and attitude (he may be a pessimist or an optimist or someone “in between”). In the following section we present one method only – which allows us to differentiate between the pessimistic and optimistic attitudes of the decision maker, but other ranking methods may be also used, without modifying the proposed approach.

4. Ranking Method for Fuzzy Numbers

In decision-making problems, having the fuzzy data leads to fuzzy numbers as final solutions. A fuzzy number represents many possible real numbers that have different membership values. It is not easy to compare the fuzzy numbers to determine which alternatives are preferred. Many authors have proposed fuzzy ranking methods that can be used to compare fuzzy numbers [Chen et al. 1992].

According to Kahraman and Tolga [2009] the fuzzy ranking method of Dubois and Prade (1978) which will be used in our paper is one of the most cited ranking methods. Dubois and Prade (1978) proposed four indices to assess the position of a fuzzy number \tilde{N} relative to the position of a fuzzy number \tilde{M} to find out if \tilde{N} is smaller than \tilde{M} or not, out of which we chose two.

$$\begin{aligned}\Pi_M([N, +\infty)) &= Poss(x > N | x \text{ is } M) = \sup_u \min \left(\mu_M(u), \inf_{\substack{v \\ v \geq u}} 1 - \mu_N(v) \right) \\ &= \sup_u \inf_{\substack{v \\ v \geq u}} \min(\mu_M(u), 1 - \mu_N(v))\end{aligned}\quad (4.1)$$

$$\begin{aligned}\mathcal{N}_M([N, +\infty)) &= Ness(x \geq N | x \text{ is } M) = \inf_u \max \left(1 - \mu_M(u), \sup_{v \leq u} \mu_N(v) \right) \\ &= \inf_u \sup_{\substack{v \\ v \leq u}} \max(1 - \mu_M(u), \mu_N(v))\end{aligned}\quad (4.2)$$

The interpretation of the above indices, introduced by Dubois and Prade [1978], is as follows:

- $\Pi_M([N, +\infty))$ is large if the values included in the support \tilde{N} and close to its upper bound are smaller or equal to the values in the support \tilde{M} and close to its upper bound
- $\mathcal{N}_M([N, +\infty))$ is large if the values included in the support \tilde{N} and close to its lower bound are smaller or equal to the values in the support \tilde{M} and close to its lower bound.

Thus, the two indices give the possibility to compare two fuzzy numbers according to the attitude of the decision maker (he may be a pessimist or an optimist – the first index corresponds to the positive attitude, we believe the flows will be rather high, the second index corresponds to the pessimistic attitude).

5. Applications

In this section we will illustrate by means of examples how investment projects can be evaluated on the basis of different criteria.

Example 1: A manufacturer wants to decide to invest in a project which has 5 years of useful life. The company has five customers $C_i, i = 1, \dots, 5$. and three suppliers $S_i, i = 1, \dots, 3$. Supplier S_1 delivers materials for the production for customers C_1 and C_2 , supplier S_2 delivers materials for the production for Customer C_3 , and supplier S_3 delivers materials for the production for Customers C_4 and C_5 . The sales (revenues) of the project for each customer are given as TFNs in Table 5.2. The profits before depreciation for each customer are given as TFNs in Table 5.3. The accounts payable for each customer are planned to be equal to the value of sales in 2 months for customer C_1 , 1 month for customer C_2 , 3 months for customer C_3 , 1 month for customer C_4 and 3 months for customer C_5 . The inventory of the project is planned to equal two months' sales, and liabilities of the suppliers are planned to be equal to the value of sales in 2 months, 1 month and 3 months, respectively for each supplier $S_i, i = 1, \dots, 3$. There will be no salvage value of the assets after 5 years. The interest rate is taken as $\tilde{r} = (8, 10, 12)\%$.

This is the basic information about the project; because of the uncertainty it is given in the form of fuzzy numbers.

Table 5.1

The planned payments for the assets

Year	Payment for the assets
0	(450000, 500000, 550000)
1	(270000, 300000, 330000)
2	(90000, 100000, 110000)
3	(90000, 100000, 110000)
4	None
5	None

As mentioned earlier, it is assumed that the assets are not dependent on any factors such as customers, resource choice, payment conditions etc. Thus they will be taken into account totally, as in Eq. 3.8. Then we have the data for different customers, including the sales (thus the revenue which is not necessarily cash in the same period, Table 5.2), the profit before depreciation (Table 5.3 – as depreciation is linked to the assets, it will not be distributed among factors, thus not among customers either).

Table 5.2

The sales of the project for each customer

<div>Sales Year</div>	Customer C_1 (x1000\$)	Customer C_2 (x1000\$)
0	–	–
1	(950, 1028, 1116)	(500, 520, 540)
2	(1023, 1087, 1251)	(622, 654, 686)
3	(1110, 1190, 1270)	(734, 772, 810)
4	(1230, 1310, 1390)	(760, 790, 820)
5	(1400, 1460, 1520)	(850, 910, 970)
<div>Sales Year</div>	Customer C_3 (x1000\$)	Customer C_4 (x1000\$)
0	–	–
1	(1104, 1200, 1296)	(780, 830, 880)
2	(1840, 2000, 2160)	(910, 960, 1050)
3	(2208, 2400, 2592)	(1100, 1230, 1350)
4	(2208, 2400, 2592)	(1150, 1250, 1350)
5	(2208, 2400, 2592)	(1200, 1330, 1460)
<div>Sales Year</div>	Customer C_5 (x1000\$)	
0	–	
1	(1090, 1130, 1170)	
2	(1220, 1270, 1320)	
3	(1760, 1850, 1940)	
4	(1935, 2046, 2157)	
5	(2100, 2330, 2560)	

Table 5.3

The profits before depreciation for each customer

Year	Profits before depreciation	
	Customer C_1 (x1000\$)	Customer C_2 (x1000\$)
0	–	–
1	(–30, 10, 40)	(–35, –10, 15)
2	(206, 219, 232)	(135, 150, 165)
3	(590, 622, 654)	(340, 367, 394)
4	(921, 992, 1063)	(560, 600, 640)
5	(935, 987, 1039)	(580, 610, 630)

Year	Profits before depreciation		
	Customer C_3 (x1000\$)	Customer C_4 (x1000\$)	Customer C_5 (x1000\$)
0	–	–	–
1	(–48, 5, 58)	(–35, 4, 43)	(–45, 12, 69)
2	(313, 330, 347)	(200, 210, 220)	(300, 327, 354)
3	(836, 880, 924)	(550, 590, 640)	(798, 875, 952)
4	(1235, 1300, 1365)	(885, 930, 975)	(1121, 1286, 1451)
5	(1130, 1190, 1250)	(900, 950, 1000)	(1240, 1275, 1310)

The payment and inventory conditions above given determine the change of accounts payable, liabilities and inventory level in each year. All these data together allow for calculating the cash flows. The cash flows (with the investment payment excluded) occurring in a time period can be calculated by the Eq. 5.1 below, which represents the indirect calculation method of cash flow in a given year:

$$\tilde{F}_t = \tilde{P}_t + \Delta\tilde{L}_t - \Delta\tilde{AP}_t - \Delta\tilde{I}_t \quad (5.1)$$

where \tilde{F}_t denotes total cash flows of the project in year t , \tilde{P}_t denotes profit before depreciation in year t , $\Delta\tilde{L}_t$ denotes change in year t in the liabilities, $\Delta\tilde{AP}_t$ denotes change in year t in the accounts payable, $\Delta\tilde{I}_t$ denotes change in year t in the inventory and \tilde{C}_t denotes payments for the newly purchased fixed assets in year t .

Then we have:

$$\tilde{P}_t = \tilde{P}C_{1t} + \tilde{P}C_{2t} + \tilde{P}C_{3t} + \tilde{P}C_{4t} + \tilde{P}C_{5t} \quad (5.2)$$

where $\tilde{P}C_{it}$ denotes the profit before depreciation in year t from the sales to customer i .

$$\Delta \tilde{L}_t = \tilde{L}_{S_t} = \tilde{L}_{S_{1t}} - \tilde{L}_{S_{1t-1}} + \tilde{L}_{S_{2t}} - \tilde{L}_{S_{2t-1}} + \tilde{L}_{S_{3t}} - \tilde{L}_{S_{3t-1}} \quad (5.3)$$

Where \tilde{L}_{S_t} denotes the liabilities in year t of seller i .

Similarly we have:

$$\begin{aligned} \Delta \tilde{A}P_t = & \tilde{A}P_{C_{1t}} - \tilde{A}P_{C_{1t-1}} + \tilde{A}P_{C_{2t}} - \tilde{A}P_{C_{2t-1}} + \tilde{A}P_{C_{3t}} - \tilde{A}P_{C_{3t-1}} + \\ & + \tilde{A}P_{C_{4t}} - \tilde{A}P_{C_{4t-1}} + \tilde{A}P_{C_{5t}} - \tilde{A}P_{C_{5t-1}} \end{aligned} \quad (5.4)$$

Where $\tilde{A}P_{C_t}$ denotes the accounts payable in year t of customer i .

$$\begin{aligned} \tilde{F}_t = & \tilde{P}_t + \Delta \tilde{L}_t - \Delta \tilde{A}P_t - \Delta \tilde{I}_t - \tilde{C}_t = \\ = & \sum_{i=1}^5 \tilde{P}_{C_{it}} + \sum_{i=1}^3 (\tilde{L}_{S_{it}} - \tilde{L}_{S_{it-1}}) - \sum_{i=1}^5 (\tilde{A}P_{C_{it}} - \tilde{A}P_{C_{it-1}}) \end{aligned} \quad (5.5)$$

The fuzzy net present value of the project is found by applying Eq. 3.4 to the values from Tables 5.1 and 5.4. Then we have $\tilde{NPV} = (4950189, 7947617, 11289530)\$$.

Table 5.4

Total cash flows of the project

YEAR	CASH FLOWS TOTAL
0	(-550000, -500000, -450000)
1	(-1476333, -1082000, -699333)
2	(357333, 825000, 1277667)
3	(2262333, 2977000, 3718333)
4	(4121000, 5053833, 5986667)
5	(4060833, 4929667, 5788500)
\tilde{NPV}	(4950189, 7947617, 11289530)

However, we want to see how the project is influenced by different factors. We start by choosing customers' situations as factors. We assume that it is not sensible to isolate flows year by year, because the yearly flows are not independent – if something happens in one year, it will have its consequences in the other years too. However, we think it is sensible to isolate the flows by customers – the customers may be assumed to be more independent from each other and we can imagine that we or the environment may influence, in a substantial way, only one customer at a time (but throughout all the years). Therefore, we group the flows not according to the years, but to the customers. The cash flows in the time period t for each customer C_i are calculated by Eq. 5.6.

$$\widetilde{FC}_{it} = \widetilde{PC}_{it} + \widetilde{\Delta LSC}_{it} - \widetilde{\Delta APC}_{it} \quad (5.6)$$

where \widetilde{FC}_{it} denotes cash flows of the project for customer i in year t , \widetilde{PC}_{it} denotes profit before depreciation for customer i in year t , $\widetilde{\Delta LSC}_{it}$ denotes change in year t in the liabilities of the supplier (suppliers) of the product for customer i , $\widetilde{\Delta APC}_{it}$ denotes change in year t in the accounts payable for customer i .

The present value of the total cash flows linked to each customer is found from Eq. 5.7.

$$PV(C_i) = \sum_{t=1}^n \frac{\sum_{i=1}^m \widetilde{FC}_{it}}{(1+i)^t} = \sum_{t=1}^n \frac{\sum_{i=1}^m (\widetilde{PC}_{it} + \widetilde{\Delta LSC}_{it} - \widetilde{\Delta APC}_{it})}{(1+i)^t} \quad (5.7)$$

Here are the values $PV(C_i)$, $i = 1, \dots, 5$

Table 5.5

The values $PV(C_i)$, $i = 1, \dots, 5$

Year	Present value of cash flows		
	Customer C_1 (x1000\$)	Customer C_2 (x1000\$)	
0	–	–	
1	(–217560, –146667, –83951)	(–77381, –48485, –18519)	
2	(71880, 172865, 268490)	(81447, 114738, 150463)	
3	(344620, 454420, 589156)	(214246, 268345, 328117)	
4	(521760, 663889, 825439)	(336931, 408784, 490142)	
5	(476639, 597326, 737752)	(302155, 372553, 447483)	
Year	Present value of cash flows		
	Customer C_3 (x1000\$)	Customer C_4 (x1000\$)	Customer C_5 (x1000\$)
0	–	–	–
1	(–442857, –359091, –272222)	(–53571, 3636, 62963)	(–232143, –160303, –85802)
2	(–65104, 52342, 178612)	(111607, 173554, 240055)	(172725, 250964, 334934)
3	(374871, 560982, 767371)	(322081, 443276, 585451)	(432762, 584773, 753082)
4	(662845, 887917, 1144441)	(490938, 635203, 799345)	(606496, 856044, 1141011)
5	(532246, 738896, 981401)	(445430, 589875, 758850)	(547756, 762284, 1014069)

Now we want to analyze the influence of the situation of each customer on the NPV of the project. We use the ranking method for fuzzy numbers described in the previous section.

Table 5.6

The possibility and necessity indices of the present values
of cash flows determined for customers

Cases \ Indices	$\Pi_M([N, +\infty))$	$\mathcal{N}_M([N, +\infty))$
$\bar{P}V_{C1} > \bar{P}V_{C2}$	1	1
$\bar{P}V_{C2} > \bar{P}V_{C1}$	0	0
$\bar{P}V_{C1} > \bar{P}V_{C3}$	0.3	0.5
$\bar{P}V_{C3} > \bar{P}V_{C1}$	0.7	0.5
$\bar{P}V_{C1} > \bar{P}V_{C4}$	0.42	0.4
$\bar{P}V_{C4} > \bar{P}V_{C1}$	0.58	0.6
$\bar{P}V_{C1} > \bar{P}V_{C5}$	0.05	0.16
$\bar{P}V_{C5} > \bar{P}V_{C1}$	0.95	0.84
$\bar{P}V_{C2} > \bar{P}V_{C3}$	0	0.05
$\bar{P}V_{C3} > \bar{P}V_{C2}$	1	0.95
$\bar{P}V_{C2} > \bar{P}V_{C4}$	0	0
$\bar{P}V_{C4} > \bar{P}V_{C2}$	1	1
$\bar{P}V_{C2} > \bar{P}V_{C5}$	0	0
$\bar{P}V_{C5} > \bar{P}V_{C2}$	1	1
$\bar{P}V_{C3} > \bar{P}V_{C4}$	0.62	0.42
$\bar{P}V_{C4} > \bar{P}V_{C3}$	0.38	0.58
$\bar{P}V_{C3} > \bar{P}V_{C5}$	0.28	0.23
$\bar{P}V_{C5} > \bar{P}V_{C3}$	0.72	0.77
$\bar{P}V_{C4} > \bar{P}V_{C5}$	0.1	0.24
$\bar{P}V_{C5} > \bar{P}V_{C4}$	0.9	0.76

We can now rank the discounted flows influenced by each customer in the pessimistic (when we assume that rather small flow values will occur) and in the optimistic cases. We assume that the corresponding relation is true if the respective possibility (necessity) index is greater than or equal to 0.7. Then in the optimistic case (the possibility measure) we get the following partial order: $\bar{P}V_{C5} > \bar{P}V_{C3} > \bar{P}V_{C1} > \bar{P}V_{C2}$, $\bar{P}V_{C5} > \bar{P}V_{C4}$, and in the pessimistic case the following one: $\bar{P}V_{C5} > \bar{P}V_{C1} > \bar{P}V_{C2}$, $\bar{P}V_{C5} > \bar{P}V_{C3} > \bar{P}V_{C2}$, $\bar{P}V_{C5} > \bar{P}V_{C4}$. Thus in both cases customer C_5 is responsible for the greatest flow contributing to the NPV of the project.

Let us now suppose that we have another project with the same NPV, but in this project customer C_2 has most influence on the NPV of the project. Now the selection of one of the two projects will be depending on the stability of the customer's situation, on the probability of their buying less, paying too late, etc. and also on the probability of the success of our endeavors influencing them and make them buying more, paying less, using cheaper suppliers, using less inventory etc.

We might also group the flows in different groups, e.g. flows due to accounts payable, liabilities and inventory level. In this way we might see which factor: the payment conditions offered by us to the customers, the payment conditions given to us by the suppliers, our decision about keeping a certain amount of inventory, has the most influence on the project's NPV and what the risk (positive or negative) connected to this factor is. This would constitute an additional criterion, apart from the NVP, to select an investment project.

Example 2: A company decides to manufacture a new product. One of two different machines X and Y, with different production volumes, can be used to manufacture the product. The new product needs two kinds of raw materials (R_A and R_B). The production requirements for the machines and cost and revenues for the product are given in Tables 5.7 and 5.8. The interest rate is taken as $i = (8,10,12)\%$.

Table 5.7

Parameters of the machines

	Machine X	Machine Y
Initial Investment	(1250000,1430000,1610000)\$	(1240000,1320000,1400000)\$
Production capacity per year	(1620000, 1800000, 1980000) units	(1140000, 1200000, 1260000) units
Work power per month	(3360, 3600, 3840) hours	(2016, 2160, 2304) hours
Raw material A per product unit	2	2
Raw material B per product unit	1	1
Energy cost per year	(11760,12000,12240)\$	(8340,8400,8460)\$
Useful life	10 years	10 years

Table 5.8

Costs and revenues generated by the new product

	Cost or Revenue
Labor cost per hour	(4.5, 5, 5.5)\$
Cost of Raw material A per product	(0,8,0,9,1)\$
Cost of Raw Material B per product	(0.9, 1, 1.1)\$
Price of the product per unit	(9, 10, 11)\$

Net annual cash flows and fuzzy present values of the cash flows for each factor are calculated and given in Table 5.9.

Table 5.9

Cash Flows and Present Values of the Factors

	TOTAL CASH FLOWS	
	Machine X	Machine Y
1	2	3
Net annual cash flows (total)	(8408640, 12930000, 17703120)	(6332868, 8620800, 10992588)
$\bar{P}\bar{V}$ of total cash flows	(18660829, 49014873, 114812908)	(14054183, 32679615, 71292009)
$\bar{NP}\bar{V}$ of the project	(17050829, 47584873, 113562908)	(12654183, 31359615, 70052009)
Annual Energy Cost	(12240, 12000, 11760)	(8460, 8400, 8340)
$\bar{P}\bar{V}$ of Energy Costs ($\bar{P}\bar{V}_E$)	(76269, 45489, 27164)	(54089, 31843, 18775)
Annual Labor Cost	(21120, 18000, 15120)	(12672, 10800, 9072)
$\bar{P}\bar{V}$ of Labor Costs ($\bar{P}\bar{V}_L$)	(98060, 68234, 46870)	(58836, 40940, 28122)
Annual raw material cost	(6138000, 5040000, 4050000)	(3906000, 3360000, 2850000)
$\bar{P}\bar{V}$ of total raw material costs ($\bar{P}\bar{V}_{RT}$)	(26266120, 19105565, 13621724)	(18483566, 12737044, 8668370)
Annual cost of raw material A	(3960000, 3240000, 2592000)	(2520000, 2160000, 1824000)
$\bar{P}\bar{V}$ of raw material A costs ($\bar{P}\bar{V}_{RA}$)	(16810317, 12282149, 8788209)	(11829482, 8188099, 5592496)
Annual cost of raw material B	(2178000, 1800000, 1458000)	(1386000, 1200000, 1026000)

Table 5.9 contd.

1	2	3
\widetilde{PV} of raw material B costs (\widetilde{PV}_{RB})	(9455803, 6823416, 4833515)	(6654084, 4548944, 3075873)
\widetilde{PV} of initial investment (\widetilde{PV}_I)	(1610000, 1430000, 1250000)	(1400000, 1320000, 1240000)

To determine which machine has higher \widetilde{NPV} , the possibility indices of ranking cases are calculated and given in Table 5.10.

Table 5.10

Possibility and necessity indices for Fuzzy Net present Values of Two Machines

Cases \ Indices	$\Pi_M([N, +\infty))$	$\mathcal{N}_M([N, +\infty))$
$\widetilde{NPV}(X) > \widetilde{NPV}(Y)$	0.78	0.71
$\widetilde{NPV}(Y) > \widetilde{NPV}(X)$	0.22	0.29

We can see that the decision maker, regardless of his pessimistic or optimistic attitude, should decide to invest in Machine X. However, we want to evaluate this decision also according to other criteria, because the opposite relation $(\widetilde{NPV}(Y) > \widetilde{NPV}(X))$ is also true to some extent (this is the feature of fuzzy values, which are based on uncertainty and incomplete knowledge), thus we may still consider the choice of machine Y – if there is a too high negative risk linked to Machine X or a high positive risk (chance) linked to Machine Y.

To determine the importance of the factors which affect fuzzy net present value of the project “buy Machine X”, the possibility and necessity indices are calculated. We get only the values 0 or 1 which means the numbers don’t have intersections and the ranking is exactly known. We get the following ranking: $\widetilde{PV}_{RT} > \widetilde{PV}_{RA} > \widetilde{PV}_{RB} > \widetilde{PV}_I > \widetilde{PV}_L > \widetilde{PV}_E$. Thus the factors linked to the raw materials have the greatest influence here. If we judge that the prices of raw materials may change considerably and in the unfavorable direction, we may want to see what is the influence of this factor on the NPV of the project “buy machine Y” and we may found out that this project is biased in the first place by other types of risk and/or offers new chances. In such a situation the fuzzy NPV and the fuzzy ranking shown in Table 5.10 would not constitute the only criterion to choose a machine.

Conclusions

The fuzzy net present value (NPV) method is one of the most preferred investment analysis methods which can deal with the uncertainty of forecasting cash flows of investment projects. In the fuzzy net present value method the worth of an investment is defined by a fuzzy number and the decision maker has to decide whether to invest in the project by considering this number. In reality all the factors influencing fuzzy net present value of an investment should also be taken into account (e.g. the credibility of customers or suppliers, the cash flows resulting from work power, raw material selection, payment conditions etc.), together with their variability/stability. The decision maker should analyze, apart from the NPV, what is the factor which influences it most and how probable (possible) are changes in the flows linked to this factor. The final decision in the evaluation of a project should be made on the basis of NPV and the structure of its components dependent on individual factors. In the present paper we propose a method to perform such an evaluation. Further research is needed to propose a methodology of identifying different factors and verify the independence between selected groups of cash flows.

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