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DEMATEL, ANP AND VICOR BASED HYBRID METHOD APPLICATION TO RESTORATION OF HISTORICAL ORGANS

Abstract

The aim of the paper is to perform ex-post analysis for of a portable organ restoration applying a hybrid method, which combines DEMATEL, ANP and VICOR multicriteria approaches. The analysis of results and comparison with the earlier research based in the scheme of ELECTRE I method is included.

Keywords

Portable organ reconstruction, hybrid multicriteria method, DEMATEL, ANP, VICOR.

Introduction

The goal of the paper [Trzaskalik-Wyrwa et al. 2006] was to determine the best way of renovation of a historic positive organ, found several years ago in the Podlasie region (part of Poland). Portable organs were very popular musical instrument in Poland in the seventeenth and eighteenth centuries. Its popularity was due above all to the ease of handling and possibility of easy transportation. Unfortunately, only several copies of this once so common instrument are nowadays extant in Poland.

The following decision problem arose: what is the best way to reconstruct the found instrument, taking into account a variety of criteria. In [Trzaskalik-Wyrwa et al. 2006], this issue has been presented as a multi-criteria decision-making problem and solved by means of ELECTRE I method. Reconstruction of the instrument was performed using the received. recommendation.

ELECTRE I method requires the decision maker to specify criteria weights and subsequent variant ratings. However, interactions among criteria are not analyzed. This possibility can be found in ANP method. Anyway, applying ANP in the source version is numerically troublesome. This is why a hybrid method which combines elements of three multicriteria approaches: DEMATEL [Fontela and Gabus 1974], ANP [Saaty 1999] and VIKOR [Opricovic and Tzeng 2007], allows to overcome numerical difficulties, emerging when performing the calculations using only the ANP method. A description of that hybrid method can be found in [Liu et al. 2012; Tzeng et al. 2007; Tzeng and Huang 2011]. This method is also presented in our paper.

The aim of the research is to perform ex-post analysis for the rediscovered instrument reconstruction by applying hybrid method mentioned above. We want to find out, how to take into account the mutual influence of criteria and whether these mutual influences will affect the selection of the final solution.

The paper is divided into four parts. In chapter 1, we present brief considerations of the decision-making problem, fully described in [Trzaskalik-Wyrwa et al. 2006]. The criteria considered and the decision variants are described. The second chapter includes a description of the hybrid method, which combines elements of DEMATEL, ANP, and VICOR. The third chapter presents the data provided by the expert (co-author of this paper – Małgorzata Trzaskalik-Wyrwa). Some of them (the evaluation of alternatives due to subsequent criteria) were used previously [Trzaskalik-Wyrwa et al. 2006], others (the specification of the mutual influence of criteria) have been prepared by the expert for the purposes of this study. An application of the hybrid method and details of numerical calculations are presented. The fourth chapter contains an analysis of the results and compare them with the earlier results, obtained in [Trzaskalik-Wyrwa et al. 2006] by means of ELECTRE I method.

1. Restoration of historical portable organ as a multicriteria decision process

1.1. Decision criteria

We consider a division of the values of historical organs into four groups: historic, artistic, musical and utilitarian values. We will describe the values constituting each of the four groups [Trzaskalik-Wyrwa et al. 2006].

Historic values determine the character of the object as a document and its influence on the development of historical knowledge. Among the values of this group are **scientific values**, due to the fact that an organ

is an historic object, requiring a scholarly description. Also in this group are **technical values**, determining the ingenuity of the construction, the quality of the workmanship and the scientific value of its current condition. Also **historic emotional values**, perceived not only by scientists and scholars, but also by the public at large, belong here.

The **ownership values**, i. e., values stemming from the ownership of the original item (without hypothetical additions) are connected with honest approach of the conservators to the historic object, in which that what is preserved should be emphasised above all, as opposed to that what we think might have been there.

The group of **artistic values** is related to the perception of historic organs as works of art, and this is connected with the instrument's case. To this group belong **historic-artistic values**, determining whether the solutions chosen by the builders are typical or atypical as well as the importance of the original, its copy or its hypothetical reconstruction. **Artistic qualities** affect the public independently of the current fashion or style. The **artistic effect** of the case of historic organ should match musical impressions received by the audience from the musical compositions heard by it.

Musical values become apparent during a musical performance. We deal here with the issue of style (**historical musical value**) and of sound (**musical quality**). All of them taken together may reinforce the **musical influence** on the amateur listener. It can happen that the regaining of musical value and the preservation of the original technical solutions are conflicting goals. In such case we face the problem of **utilitarian values** of the historic instrument. The notions of live organ and dead organ are related to this group of values. A **musically dead organ** is an instrument that nowadays cannot fulfil its function of a musical instrument. A **live instrument** is an instrument capable of being used in musical performance, affecting the audience in various ways. Like any historic object, an organ as a piece of furniture can be also visually dead – not suitable for being exhibited, or else visually alive (independently of its musical “vitality”) – beautiful, but unplayable.

1.2. Decision alternatives

On the basis of research and evaluation of the condition of the individual parts of the instrument (or their lack) 12 renovation treatments of the rediscovered instrument have been suggested. They are decision alternatives, described below [Trzaskalik-Wyrwa et al. 2006].

Alternative A1

Preservation of the instrument as a non-functional, visually unattractive object (“destrukt”) and its exhibition in the form of a group of museum exhibits.

Alternative A2

Integration of the elements of the instrument using racks necessary to place the individual elements in proper places.

Alternative A3

Integration of the parts of the instrument with full completion of the construction elements of the case (without covering the “windows” with reconstructed wood carved ornaments) according to their former shape as concluded from the elements preserved; completion of the missing parts of the mechanism. The pipes remain secured, but do not play.

Alternative A4

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome and covering of the “windows” by a neutral filling (canvas, wooden grill). The pipes remain secured, but do not play.

Alternative A5

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome. Hypothetical reconstruction of the wood carved ornaments filling out the “windows” (on the basis of comparative analysis – it is impossible to achieve the historical truth). The pipes remain secured, but do not play.

Alternative A6

Integration of the parts of the instrument with full completion of the construction elements of the case (without covering the “windows” by reconstructed wood carved ornaments) according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Bringing the extant pipes to working condition and reconstruction of the missing pipes, so as to match the sound capabilities of the extant pipes.

Alternative A7

Integration of the parts of the instrument with full completion of the construction elements of the case (without covering the “windows” by reconstructed wood carved ornaments) according to their former shape,

as concluded from the elements preserved; completion of the missing parts of the mechanism. Exhibition of the extant historic pipes in a display case without giving them their former technical functionality. Reconstruction of the entire sound system according to preserved models.

Alternative A8

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome and covering the “windows” by a neutral filling (canvas, wooden grill). Bringing the pipes to a working condition and reconstruction of the missing pipes, so as to match the sound capabilities of the extant pipes.

Alternative A9

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved; completion of the missing parts of the mechanism. Reconstruction of the polychrome and covering the “windows” by a neutral filling (canvas, wooden grill). Exposition of the extant historical pipes in a display case without bringing them to a working condition. Reconstruction of the whole sound system according to preserved models.

Alternative A10

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved and completion of the missing parts of the mechanism. Reconstruction of the polychrome. Hypothetical reconstruction of the wood carved ornaments filling out the “windows” (on the basis of comparative analysis – it is impossible to achieve historical truth). Bringing the pipes to a working condition and reconstruction of the missing pipes so as to match the sound of the sound capabilities of the preserved pipes.

Alternative A11

Integration of the parts of the instrument with full completion of the construction elements of the case according to their former shape, as concluded from the elements preserved and completion of the missing parts of the mechanism. Reconstruction of the polychrome. Hypothetical reconstruction of the wood carved ornaments filling out the “windows” (on the basis of comparative analysis – it is impossible to achieve historical truth). Exhibition of the preserved historic pipes in a display case without bringing them to a working condition. Reconstruction of the whole sound system according to preserved models.

Alternative A12

Preservation of the instrument in its non-functional, visually unattractive condition (as a “destrukt”). Making of an accurate copy. The evaluation focuses on the values of the copy, which is presented to the public.

2. The hybrid method

The hybrid method [Liu et al. 2012; Tzeng et al. 2007; Tzeng and Huang 2011] is a combination of:

- DEMATEL – applied to clarify relation between components,
- ANP – applied to determine the relationship between the criteria (in limited supermatrix),
- VIKOR – applied to obtain the index values in gaps.

Let A be a finite set of decision alternatives:

$$A = \{ A_1, A_2, \dots, A_l \}$$

C – a set of criteria, divided into n categories (called here aspects, dimensions, clusters):

$$C = \{ C_1, C_2, \dots, C_n \}$$

where:

$C_i = \{ c_{i1}, c_{i2}, \dots, c_{im_i} \}$, $i = 1, \dots, n$ – is a subset of criteria in i -th aspect

and F – matrix of values of the j -th alternative in the k -th criterion:

$$F = [f_{kj}], \quad j = 1, 2, \dots, l, \quad k = 1, 2, \dots, M$$

where $M = \sum_{i=1}^n m_i$

We assume that the criteria are defined so that a higher the value of the criterion is preferred to a lower one. Each criterion is assigned a positive number which reflects the valid contribution of that criteria.

The considered method is divided into following steps:

Step 1: Develop the structure of the problem.

The problem is broken down to a level structure.

Step 2: Develop the total influence matrix.

Based on the DEMATEL method, interactions between the aspects are explained to construct the map of the direct impact. This step is divided into three sub-steps:

Step 2a: Identify the average influence matrix A

The initial matrix $A = [a_{ij}^h]_{n \times n}$ is calculated, using experts' evaluations, where a_{ij}^h denotes the influence of i -th factor on j -th factor in h -th expert's opinion. If i -th element affects j -th element directly, then $a_{ij}^h \neq 0$; otherwise, $a_{ij}^h = 0$. We obtain:

$$A = [a_{ij}]_{n \times n} \quad a_{ij} = \frac{1}{H} \sum_{h=1}^H a_{ij}^h \tag{1}$$

where H denotes the number of experts and $h = 1, 2, \dots, H$. In particular, we can use expertise of one expert, then influence matrix A is obtained directly:

$$A = [a_{ij}]_{n \times n} \tag{2}$$

Step 2b: Calculate the normalized influence matrix X

We normalize the matrix A , applying (3) and (4). The diagonal in normalized matrix is equal to 0, and the maximum sum of each row or column is equal to 1:

$$X = sA \tag{3}$$

where:

$$s = \min \left\{ \frac{1}{\max_{1 \leq i \leq n} \sum_{j=1}^n a_{ij}}, \frac{1}{\max_{1 \leq j \leq n} \sum_{i=1}^n a_{ij}} \right\} \tag{4}$$

Step 2c: Compute the total-influence matrix T

The total-influence matrix T can be obtained according to (5) (I denotes the identity matrix):

$$T = X + X^2 + \dots + X^k = X(I - X)^{-1}, \text{ when } \lim_{k \rightarrow \infty} X^k = [0]_{n \times n} \tag{5}$$

The proof of this relationship can be found in [Tzeng and Huang 2011].

Step 2d: Set a threshold value α and obtain the normalized α -cut total-influence matrix T^α

We have total-influence matrix T in the form:

$$T = \begin{bmatrix} t_{11} & \dots & t_{1j} & \dots & t_{1n} \\ \vdots & & \vdots & & \vdots \\ t_{i1} & \dots & t_{ij} & \dots & t_{in} \\ \vdots & & \vdots & & \vdots \\ t_{n1} & \dots & t_{nj} & \dots & t_{nn} \end{bmatrix} \quad (6)$$

The α -cut total-influence matrix T^α will be given by Eq. (7)

$$T^\alpha = \begin{bmatrix} t_{11}^\alpha & \dots & t_{1j}^\alpha & \dots & t_{1n}^\alpha \\ \vdots & & \vdots & & \vdots \\ t_{i1}^\alpha & \dots & t_{ij}^\alpha & \dots & t_{in}^\alpha \\ \vdots & & \vdots & & \vdots \\ t_{n1}^\alpha & \dots & t_{nj}^\alpha & \dots & t_{nn}^\alpha \end{bmatrix} \quad (7)$$

where if $t_{ij} < \alpha$ then $t_{ij}^\alpha = 0$ else $t_{ij}^\alpha = t_{ij}$

The α -cut total-influence matrix T_α needs to be normalized by dividing by the value:

$$d_i = \sum_{j=1}^n t_{ij}^\alpha, \quad i = 1, 2, \dots, n \quad (8)$$

Finally we obtain T_D as follow:

$$T_D = \begin{bmatrix} t_{11}^\alpha / d_1 & \dots & t_{1j}^\alpha / d_1 & \dots & t_{1n}^\alpha / d_1 \\ \vdots & & \vdots & & \vdots \\ t_{i1}^\alpha / d_i & \dots & t_{ij}^\alpha / d_i & \dots & t_{in}^\alpha / d_i \\ \vdots & & \vdots & & \vdots \\ t_{n1}^\alpha / d_n & \dots & t_{nj}^\alpha / d_n & \dots & t_{nn}^\alpha / d_n \end{bmatrix} = \begin{bmatrix} t_{11}^D & \dots & t_{1j}^D & \dots & t_{1n}^D \\ \vdots & & \vdots & & \vdots \\ t_{i1}^D & \dots & t_{ij}^D & \dots & t_{in}^D \\ \vdots & & \vdots & & \vdots \\ t_{n1}^D & \dots & t_{nj}^D & \dots & t_{nn}^D \end{bmatrix} \quad (9)$$

Step 3: Compare all criteria to form the initial supermatrix.

The initial supermatrix can be obtained in two ways:

A. Initial (unweighted) supermatrix can be obtained by pairwise comparison of all criteria as in AHP method [Tzeng and Huang 2011, p. 161]:

$$\begin{array}{l}
 \begin{array}{c}
 c_{11} \\
 C_1 \begin{array}{c} c_{12} \\ \dots \\ c_{1m_1} \end{array} \\
 c_{21} \\
 C_2 \begin{array}{c} c_{22} \\ \dots \\ c_{2m_2} \end{array} \\
 \dots \\
 \dots \\
 C_n \begin{array}{c} c_{n1} \\ c_{n2} \\ \dots \\ c_{nm_n} \end{array}
 \end{array}
 \begin{array}{c}
 C_1 \\
 C_2 \\
 \dots \\
 C_n
 \end{array}
 \begin{array}{c}
 c_{11} \quad c_{12} \quad \dots \quad c_{1m_1} \quad c_{21} \quad c_{22} \quad \dots \quad c_{2m_2} \quad \dots \quad \dots \quad \dots \quad c_{n1} \quad c_{n2} \quad \dots \quad c_{nm_n} \\
 \\
 W_{11} \quad \quad \quad W_{12} \quad \quad \quad \dots \quad \quad \quad W_{1n} \\
 \\
 W_{21} \quad \quad \quad W_{22} \quad \quad \quad \dots \quad \quad \quad W_{2n} \\
 \\
 \dots \quad \quad \quad \dots \quad \quad \quad \dots \quad \quad \quad \dots \\
 \\
 W_{n1} \quad \quad \quad W_{n2} \quad \quad \quad \dots \quad \quad \quad W_{nn}
 \end{array}
 \end{array}
 \quad (10)$$

B. We can repeat the steps 2a-2d, as in [Liu et al. 2012], on initial influence matrix for all criteria

$$A = [a_{ij}]_{\sum_{i=1}^n m_i \times \sum_{i=1}^n m_i}$$

Then we obtain the matrix T_C and $W = (T_C)^T$

Finally we receive unweighted supermatrix W in the form:

$$W = \begin{bmatrix}
 W_{11} & \dots & W_{1j} & \dots & W_{1n} \\
 \vdots & & \vdots & & \vdots \\
 W_{i1} & \dots & W_{ij} & \dots & W_{in} \\
 \vdots & & \vdots & & \vdots \\
 W_{n1} & \dots & W_{nj} & \dots & W_{nn}
 \end{bmatrix}
 \quad (11)$$

Step 4: Obtain the weighted supermatrix.

The normalized T_D is multiplied by unweighted supermatrix W to obtain weighted supermatrix W^α . The results are shown in Eq. (12).

$$W^\alpha = T_D \times W = \begin{bmatrix} t_{11}^D \times W_{11} & \dots & t_{1l}^D \times W_{1j} & \dots & t_{1n}^D \times W_{1n} \\ \vdots & & \vdots & & \vdots \\ t_{il}^D \times W_{il} & \dots & t_{ij}^D \times W_{ij} & \dots & t_{in}^D \times W_{in} \\ \vdots & & \vdots & & \vdots \\ t_{nl}^D \times W_{nl} & \dots & t_{nj}^D \times W_{nj} & \dots & t_{nn}^D \times W_{nn} \end{bmatrix} \quad (12)$$

Step 5: Obtain the limit supermatrix.

The ANP weights of each criterion can be obtained from limit supermatrix:

$$W^{lim} = \lim_{k \rightarrow \infty} (W^\alpha)^k \quad (13)$$

The evaluation of the total preference is performed by means of VIKOR method, which can be divided into following steps:

Step 6: Check the best value f_k^* and the worse value f_k^- .

f_k^* represents the positive-ideal point, that means the expert gives the scores of the best value (aspired levels) for each criterion and f_k^- represents the negative-ideal point, that means the expert gives the scores of the worst values for each criterion. Those values can be computed by the traditional approach, using Eqs. (14) and (15) to obtain the results:

$$f_k^* = \max_i f_{ki}, \quad i = 1, 2, \dots, l \quad (14)$$

$$f_k^- = \min_i f_{ki}, \quad i = 1, 2, \dots, l \quad (15)$$

or by setting aspire levels vector as in Eq. (16)

$$f^* = (f_1^*, f_2^*, \dots, f_M^*) \tag{16}$$

and setting worst value vector as in Eq. (17)

$$f^- = (f_1^-, f_2^-, \dots, f_M^-) \tag{17}$$

Step 7: Calculate the mean of group utility S_i and the maximal regret Q_i .

The S_i represents the ratios of distance to the positive-ideal, it means the synthesized gap for all criteria. The Q_i represents the maximal gap-ratios (regret) of normalized distance to the aspired level in all criteria, that is, the maximal gap for prior improvement. Those values can be computed respectively by Eqs. (18) and (19):

$$S_i = \sum_{k=1}^M w_k r_{ki} \tag{18}$$

$$Q_i = \max_k \{w_k r_{ki} \mid k = 1, 2, \dots, M\} \tag{19}$$

where:

w_k – represents the influential weights of the k-th criterion from previous step,

$r_{ki} = \frac{|f_k^* - f_{ki}|}{|f_k^* - f_k^-|}$ – represents the gap-ratios (regret) of normalized distance to the aspired level point

Step 8: Obtain the comprehensive indicator R_i .

The values can be computed using Eq. (20).

$$R_i = v(S_i - S^*) / (S^- - S^*) + (1 - v)(Q_i - Q^*) / (Q^- - Q^*) \quad (20)$$

where:

$$S^* = \min_k S_k \quad \text{or we can set } S^* = 0 \quad (\text{the aspired level})$$

$$S^- = \max_k S_k \quad \text{or we can set } S^- = 1 \quad (\text{the worst situation})$$

$$Q^* = \min_k Q_k \quad \text{or we can set } Q^* = 0 \quad (\text{the aspired level})$$

$$Q^- = \max_k Q_k \quad \text{or we can set } Q^- = 1 \quad (\text{the worst situation})$$

Therefore, when $S^* = 0$ and $S^- = 1$, and $Q^* = 0$ and $Q^- = 1$, we can re-write the Eq. (21) as:

$$R_i = vS_i + (1 - v)Q_i \quad (21)$$

The coefficient $v = 1$ represents situation where only the average gap (average regret) is considered. Coefficient $v = 0$ represents situation where only the maximum gap to the prior improvement is considered. Generally the coefficient is adjusted according to the situation. In the most situations we can use $v = 0,5$.

Step 9: Rank the alternatives, sorting by the value of $\min\{R_i \mid i = 1, 2, \dots, l\}$.

3. Application of the hybrid method to the problem of restoration of historical organs

According to the expert’s evaluations, we will consider the set of values, gathered in Table 1.

Table 1

Value Evaluation by Criteria

Criteria	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12
Historical-scientific value	10	8	6	6	6	6	6	6	6	6	6	0
Historical-technical value	4	6	10	8	8	6	10	6	10	6	10	0
Emotional value	10	10	10	8	6	10	6	6	4	4	2	0
Ownership value	10	10	10	9	5	8	5	9	4	5	0	0
Historical-artistic value	0	2	4	6	6	4	4	6	6	8	8	0
Artistic quality	0	0	2	4	8	2	2	4	4	8	8	8
Artistic influence	2	2	6	8	10	6	6	8	8	10	10	10
Historical-musical value	0	0	0	0	0	10	4	10	4	10	4	4
Musical quality	0	0	0	0	0	8	10	8	10	8	10	10
Musical influence	0	0	0	0	0	8	10	8	10	8	10	10
Visual-utilitarian value	2	4	6	8	10	6	6	8	8	10	10	10
Musical-utilitarian value	0	0	0	0	0	8	10	8	10	8	10	10

Step 1: According to the literature review and expert experiences, an value evaluation system including four dimensions and 12 criteria is established, as given in Table 2.

Table 2

The structure of evaluation criteria

Aspects/Dimensions	Criteria
C_1 Historical Values	c_{11} Historical-scientific value
	c_{12} Historical-technical value
	c_{13} Emotional value
	c_{14} Ownership value
C_2 Artistic values	c_{21} Historical-artistic value
	c_{22} Artistic quality
	c_{23} Artistic influence
C_3 Music values	c_{31} Historical-musical value
	c_{32} Musical quality
	c_{33} Musical influence
C_4 Utilitarian values	c_{41} Visual-utilitarian value
	c_{42} Musical-utilitarian value

Step 2: The ratings for each criterion’s relationship to sustainable development using a five-point scale ranging from 0 (no effect) to 4 (extremely influential) were collected.

Table 3

Influence between aspects

		C_1	C_2	C_3	C_4
Historical Values	C_1	x	2	2	3
Artistic values	C_2	3	x	0	3
Music values	C_3	3	0	x	2
Utilitarian values	C_4	0	4	4	x

Step 2a: Identify the average influence matrix A

As it was difficult to the decision maker to determine the influence between aspects themselves, we calculated the influence as rounded average of all influences between the criteria in the aspects. The result is presented in Table 4.

Table 4

Influence matrix

		C_1	C_2	C_3	C_4
Historical Values	C_1	2	2	2	3
Artistic values	C_2	3	2	0	3
Music values	C_3	3	0	3	2
Utilitarian values	C_4	0	4	4	0

Step 2b-2d: Total-influential dimensions matrix T_D .

We used $\alpha = 0,1$ so it was necessary to normalize the resulting matrix. Result is presented in Table 5.

Table 5

Total-influential aspects (dimensions) matrix T_D .

	C_1	C_2	C_3	C_4
C_1	0,00	0,41	0,28	0,31
C_2	0,32	0,00	0,32	0,36
C_3	0,28	0,41	0,00	0,31
C_4	0,29	0,43	0,29	0,00

Step 3: Compare all the criteria to form the initial supermatrix. Because only direct impacts of the criteria were available, we use method **B**, repeating steps 2a-2d to the matrix presented in Table 6.

Table 6

Influential matrix *A* on criteria

	<i>c</i> ₁₁	<i>c</i> ₁₂	<i>c</i> ₁₃	<i>c</i> ₁₄	<i>c</i> ₂₁	<i>c</i> ₂₂	<i>c</i> ₂₃	<i>c</i> ₃₁	<i>c</i> ₃₂	<i>c</i> ₃₃	<i>c</i> ₄₁	<i>c</i> ₄₂
<i>c</i> ₁₁	0	3	1	4	4	4	1	4	1	1	2	2
<i>c</i> ₁₂	3	0	1	4	2	1	0	2	1	1	2	2
<i>c</i> ₁₃	2	2	0	3	2	2	1	2	1	2	4	4
<i>c</i> ₁₄	4	4	4	0	4	4	4	4	4	4	3	3
<i>c</i> ₂₁	3	3	3	4	0	4	4	2	2	2	4	0
<i>c</i> ₂₂	2	2	2	2	3	0	2	0	0	0	4	0
<i>c</i> ₂₃	1	1	1	4	4	4	0	0	0	0	4	0
<i>c</i> ₃₁	3	3	2	4	0	0	0	0	4	4	0	4
<i>c</i> ₃₂	1	1	0	2	0	0	0	4	0	4	0	4
<i>c</i> ₃₃	1	1	0	2	0	0	0	4	4	0	0	4
<i>c</i> ₄₁	0	0	0	0	4	4	4	0	0	0	0	0
<i>c</i> ₄₂	0	0	0	0	0	0	0	4	4	4	0	0

The result of repeating steps 2a-2b is shown in Table 7. Now we use $\alpha = 0$ but it is also necessary to normalize the resulting matrix.

Table 7

Total-influential criteria matrix T_C

	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}
c_{11}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0
c_{12}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0
c_{13}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0
c_{14}	0	0	0	0	0,068	0,14	0,257	0,043	0,23	0,191	0,072	0
c_{21}	0	0	0	0,099	0	0,135	0,248	0,042	0,223	0,184	0,069	0
c_{22}	0	0	0	0,106	0,071	0	0,267	0,045	0,239	0,198	0,075	0
c_{23}	0	0	0	0,121	0,08	0,165	0	0,051	0,272	0,225	0,085	0
c_{31}	0	0	0	0,096	0,064	0,132	0,242	0	0,217	0,18	0,068	0
c_{32}	0	0	0	0,117	0,078	0,16	0,294	0,05	0	0,219	0,082	0
c_{33}	0	0	0	0,112	0,075	0,153	0,282	0,048	0,253	0	0,079	0
c_{41}	0	0	0	0,099	0,066	0,136	0,249	0,042	0,223	0,185	0	0
c_{42}	0	0	0	0,092	0,062	0,127	0,233	0,039	0,209	0,173	0,065	0

Step 4: We obtain the weighted supermatrix by multiplying matrixes $(T_C)^T$ and W presented in tables 5 and 7. The result is presented in Table 8.

Table 8

Weighted supermatrix W^α

	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}
c_{11}	0	0	0	0	0	0	0	0	0	0	0	0
c_{12}	0	0	0	0	0	0	0	0	0	0	0	0
c_{13}	0	0	0	0	0	0	0	0	0	0	0	0
c_{14}	0	0	0	0	0,041	0,044	0,05	0,027	0,032	0,031	0,031	0,029
c_{21}	0,02	0,02	0,02	0,022	0	0	0	0,021	0,025	0,024	0,024	0,022
c_{22}	0,041	0,041	0,041	0,045	0	0	0	0,042	0,051	0,049	0,049	0,046
c_{23}	0,075	0,075	0,075	0,082	0	0	0	0,078	0,094	0,09	0,09	0,084
c_{31}	0,011	0,011	0,011	0,012	0,017	0,019	0,021	0	0	0	0,013	0,012
c_{32}	0,058	0,058	0,058	0,064	0,092	0,099	0,113	0	0	0	0,069	0,065
c_{33}	0,048	0,048	0,048	0,053	0,076	0,082	0,093	0	0	0	0,057	0,054
c_{41}	0,019	0,019	0,019	0,02	0,03	0,032	0,036	0,019	0,024	0,022	0	0
c_{42}	0	0	0	0	0	0	0	0	0	0	0	0

Step 5: By multiplying weighted supermatrix W^α we obtain the limit supermatrix W^{lim} presented in Table 9.

Table 9

Limit supermatrix W^{lim}

	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}
c_{11}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
c_{12}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
c_{13}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
c_{14}	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126	0,126
c_{21}	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058	0,058
c_{22}	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119	0,119
c_{23}	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220	0,220
c_{31}	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036	0,036
c_{32}	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189	0,189
c_{33}	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157	0,157
c_{41}	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094	0,094
c_{42}	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000

The weights obtained are shown in Table 10.

Table 10

The evaluation criteria

Aspects/Dimensions	Criteria	Weight	
C_1 Historical Values	c_{11}	Historical-scientific value	0,000
	c_{12}	Historical-technical value	0,000
	c_{13}	Emotional value	0,000
	c_{14}	Ownership value	0,126
C_2 Artistic values	c_{21}	Historical-artistic value	0,058
	c_{22}	Artistic quality	0,119
	c_{23}	Artistic influence	0,220
C_3 Music values	c_{31}	Historical-musical value	0,036
	c_{32}	Musical quality	0,189
	c_{33}	Musical influence	0,157
C_4 Utilitarian values	c_{41}	Visual-utilitarian value	0,094
	c_{42}	Musical-utilitarian value	0,000

Steps 7-9: The results are presented in Table 11.

Table 11
Vikor Method Evaluation of Alternatives

Dimensions/Criteria	Weight	f^*	f	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12
C₁. Historical Values															
<i>c₁₁</i>	0,10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>c₁₂</i>	0,10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>c₁₃</i>	0,10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>c₁₄</i>	0,126	10	0	0	0	0,013	0,063	0,025	0,063	0,013	0,076	0,063	0,126	0,126	0,126
C₂. Artistic values															
<i>c₂₁</i>	0,058	10	0	0,058	0,046	0,035	0,023	0,023	0,035	0,035	0,023	0,023	0,012	0,012	0,058
<i>c₂₂</i>	0,119	10	0	0,119	0,119	0,096	0,072	0,024	0,096	0,096	0,072	0,072	0,024	0,024	0,024
<i>c₂₃</i>	0,22	10	0	0,176	0,176	0,088	0,044	0	0,088	0,088	0,044	0,044	0	0	0
C₃. Music values															
<i>c₃₁</i>	0,036	10	0	0,036	0,036	0,036	0,036	0,036	0	0,021	0	0,021	0	0,021	0,021
<i>c₃₂</i>	0,189	10	0	0,189	0,189	0,189	0,189	0,189	0,038	0	0,038	0	0,038	0	0
<i>c₃₃</i>	0,157	10	0	0,157	0,157	0,157	0,157	0,157	0,031	0	0,031	0	0,031	0	0
C₄. Utilitarian values															
<i>c₄₁</i>	0,094	10	0	0,076	0,057	0,038	0,019	0	0,038	0,038	0,019	0,019	0	0	0
<i>c₄₂</i>	0	10	0	0	0	0	0	0	0	0	0	0	0	0	0
<hr/>															
			S_i	0,811	0,78	0,638	0,552	0,492	0,351	0,341	0,24	0,255	0,168	0,183	0,23
<hr/>															
			Q_i	0,189	0,189	0,189	0,189	0,189	0,096	0,096	0,072	0,076	0,063	0,126	0,126
<hr/>															
	$v = 0,5$		R_i	0,5	0,485	0,414	0,371	0,341	0,223	0,218	0,156	0,165	0,116	0,155	0,178
<hr/>															
	Rank			12	11	10	9	8	7	6	3	4	1	2	5

Conclusions

According to the hybrid method considered in the paper alternative *A10* was classified as the best one.

We will compare results obtained in the present research by means of hybrid method with the results obtained previously applying ELECTRE I method.

Table 12

Ranks

No.	Metoda Electre	No.	Hybryd method	No.	Hybryd method
1.	<i>A8, A10</i>	1.	<i>A10</i>	7.	<i>A6</i>
2.	<i>A6, A9, A11</i>	2.	<i>A11</i>	8.	<i>A5</i>
3.	<i>A7</i>	3.	<i>A8</i>	9.	<i>A4</i>
4.	<i>A3, A12</i>	4.	<i>A9</i>	10.	<i>A3</i>
5.	<i>A1, A2, A4, A5</i>	5.	<i>A12</i>	11.	<i>A2</i>
		6.	<i>A7</i>	12.	<i>A1</i>

In both rankings alternative *A10* was classified as the best one. The alternative *A11* was classified as the second in the hybrid method and was better than the alternative *A8*, classified in ELECTRE I method into the first class and recommended for further realization. The rest of the alternatives were classified similarly in the both methods. It is less important, because the considered decision problem was formulated as the best alternative choice problem.

It is seen that taking into account the mutual influence of criteria causes a change of recommendation. When applying ELECTRE I method, the decision maker could choose between two alternatives: *A8* and *A10*. After the analysis of these alternatives the decision maker concluded that the alternative *A10* is better. When applying the hybrid method we obtained a ranking in which alternative *A10* was the best one. The alternative *A8*, recommended previously, was placed in the new ranking at the third position, so its chance to be recommended on the basis of the hybrid method is small.

The expert's ex post opinion (several years after reconstruction of the instrument) seems interesting. In perspective, it is seen that earlier choice of the alternative *A11* (which was second in the new ranking) would be better because of the possibility of the use of the instrument in musical performances. It is connected with the revision of criteria values for decision alternatives.

The alternative *A11* recommends reconstruction of the whole sound system according to preserved models. This solution gives the possibility of uncomprising use of the new pipes to obtain satisfactory level of sound. The conservation of several original pipes and the adjustment to them the rest of reconstructed pipes to their loudness caused an additional adverse result (among other problems) that the instrument plays too softly, and the “historical” timbre makes up for this insufficiently.

Also, the problem of hypothetical shape of wood-carver’s decorations in upper box windows (not hitherto reconstructed) could be positively solved now, as not causing damage to historical substance and, at the same time considerably increasing visual attraction of the monument.

The knowledge obtained during the decision process and later can be used in conservation works in the future.

The recommendation of the alternative *A8* (first compared with the alternative *A10* while performing ELECTRE I analysis) was prepared by the expert on the merits of the case. The merits and arguments from the field of historical objects restoration should be the most important – the ranking (of course significant) is of auxiliary importance.

A detailed analysis of the hybrid method assumptions and justification of joint applications of DEMATEL, ANP and VICOR methods is a separate problem. Such an analysis has not been performed yet and it should be presented in future research.

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