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MULTICRITERIA EVALUATION OF FUZZY NET PRESENT VALUE[1]

Abstract

In this paper it is shown how to assess the degree of influence of various factors on the value of the project (NPV). The assessment is based on grouping and ranking of cash flows linked to various factors. The formulas are generated both for crisp and fuzzy net present value analysis. The projects are then evaluated on the basis of at least two criteria: the NPV and the risk (positive or negative) linked to the factors which have most influence on the project's NPV whose change may change the NPV considerably. In applications, fuzzy present values of different factors are calculated and compared for two different cases.

Keywords

Fuzzy Logic, Fuzzy Net Present Value, Ranking Fuzzy Numbers.

Introduction

Investment decisions are strategic decisions, which directly affect the position of a firm in the market. One of the most important criteria used to select an investment project is its worth for the decision makers. The discounted cash flows analyses (mostly net present value (NPV) analysis) are preferred to evaluate the investment projects. The NPV of a project, calculated before the beginning of the project, is the most widely used criterion to evaluate a project – it is generally accepted that the higher the NPV, the better the project.

However, things are not as simple that. It is widely known that investment decisions are always exposed to a high degree of uncertainty

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and risk. Thus, the NPV cannot be the only project selection criterion, because it may change and be in reality, once the project is implemented, substantially different from its planned value. That is why each investment project has to be evaluated taking into account more than just this one criterion. It is widely agreed that the risk of the project has to be taken into account too.

If risk is understood both in the negative (as a possible threat) and positive (as a possible chance), it is good to know where risk lies in a given investment project and which factor can change or be changed in such a way that its influence on the NPV of the project would be high. Each investment project is influenced by several factors, which may be dependent or independent of the company in question (behavior and situation of the customers and suppliers, payment conditions, prices, "make or buy" decisions etc.). The aim of the paper is to show how these factors and their possible influence on the project value can be identified. Each project should then be evaluated on the basis of its NPV and of the risk of a change in the NPV, linked to various factors influencing the value of the project.

What is more, as each investment project is a long-term project and its parameters are always connected with risk and uncertainty, we consider here the fuzzy approach to the estimation of the project's parameters. Fuzzy numbers allow us to model the incomplete knowledge about cash flows in the future.

The outline of the paper is as follows: first we present the classical approach to the evaluation of investment projects, but extending the classical definition of the NPV so that different factors influencing it, as well as their degree of influence, may be identified. Then we briefly describe the fundamentals of the fuzzy logic [Zadeh 1965; Ross 1995]. Finally the fuzzy approach to investment project evaluation is presented [Chio and Park 1994; Kuchta 2000; Zhang et al. 2011; Sorenson and Lavelle 2008], into which we incorporate the proposed approach taking into account those factors which influence the NPV. Subsequently, we present one of several possible approaches to fuzzy numbers ranking, which will be needed while estimating the "force" of individual factors influencing the NPV. The approach chosen permits to adjust the method to the attitude of the decision maker (pessimistic or optimistic). The paper concludes with two numerical examples.

1. Net Present Value Analysis

One of the most used discounted cash flow analysis method is net present value analysis which is calculated by adding up the present values of all cash flows into or out of the project.

The basic formula of present value of a single future payment ($PV(F)$) at the end of n^{th} year from now is given in Eq. 1.1 where F stands for the amount of the payment and i stands for the compound interest rate.

$$PV(F) = F\frac{1}{(1+i)^n} \tag{1.1}$$

The formula of net present value of a cash flow series ($NPV(F_1, \dots F_m)$) which has m different payments is calculated by Eq. 1.2 where F_j stands for the amount of the payment and n_j stands for the time period of the payment.

$$NPV(F_1, \dots F_m) = -I + \left(F_1\frac{1}{(1+i)^{n_1}}\right) + \left(F_2\frac{1}{(1+i)^{n_2}}\right) + \\ + \dots + \left(F_m\frac{1}{(1+i)^{n_m}}\right) \tag{1.2}$$

Sometimes the initial investment of the project can be distributed over several years. Eq. 1.3 gives NPV when the initial investment is distributed over z years:

$$NPV = -I_0 - \left(I_1\frac{1}{(1+i)^{n_1}}\right) - \left(I_2\frac{1}{(1+i)^{n_2}}\right) - \dots - \left(I_z\frac{1}{(1+i)^{n_z}}\right) + \\ + \left(F_1\frac{1}{(1+i)^{n_1}}\right) + \left(F_2\frac{1}{(1+i)^{n_2}}\right) + \dots + \left(F_m\frac{1}{(1+i)^{n_m}}\right) \tag{1.3}$$

The NPV is influenced by several factors. Sometimes small changes in some cash flow factors could be introduced – or occur independently of us – which result in a substantially better or worse NPV. For example, if the NPV is strongly influenced by labor costs, the decision maker could prefer to hire lower qualified workers or to outsource the work. If the factors influencing strongly the NPV are the payment conditions offered to the customers, the deadlines of the payments could be changed. Sometimes the NPV may be very sensitive to the situation of one of the customers or the suppliers. At that point, the decision maker may want to know the degree of influence of the situation of a given customer or supplier, of the chosen resources, suppliers, payment conditions etc. on the net present value of the project.

We propose, thus, to reorder the cash flow from Eq. (1.3) into groups of influence: each group consists of cash flows which depend on one specific factor (e.g. the situation of one customer, the decision to hire a certain workforce etc.). Of course, we assume that such reordering is possible – that it is possible to classify the cash flows into such groups which are influenced mainly by one factor. It is a limiting assumption, but less limiting than that – generally assumed – about the independence of the cash flows in subsequent years – the flows in various years are usually not independent, as the choice of one supplier, of the payment conditions or of resources used has its consequences for several cash flows in different years.

Thus let F_{jl} ($j = 1,\ldots, n$, $l = 1,\ldots, k$) denote the groups of cash flows occurring in the j-th year influenced by the l-th factor. The cash flows dependent on the l-th factor in the whole project are represented in Eq. 1.4, and the present value of these flows are calculated by Eq. 1.5 (we assume here that the investment is excluded from the influence of the factors, which is not a limiting assumption – the approach will be identical to the one that includes those factors).

$$F_l = F_{1l} + F_{2l} + F_{3l} + \cdots + F_{nl} \qquad l = 1, \ldots k \tag{1.4}$$

$$PV^l = \sum_{j=1}^{n} \frac{F_{jl}}{(1+i)^{n_j}} \qquad l = 1, \ldots k \tag{1.5}$$

As shown in Eq. 1.7, the NPV of the project is equal to the sum of the NPVs of the cash flows linked to the individual factors and to the PV^I, which is the present value of the initial investment given by Eq. 1.6.

$$PV^I = PV(I_0) + PV(I_1) + \cdots + PV(I_z) \tag{1.6}$$

$$NPV = -PV^I + PV^1 + PV^2 + PV^3 + \cdots + PV^k \tag{1.7}$$

Let us assume that we have the following inequality:

$$PV^1 > PV^2 > PV^3 > \cdots > PV^k \tag{1.8}$$

If the “greater-than-relations” stand for substantial differences between values $PV^1, PV^2, PV^3, \ldots, PV^k$, then a unitary change (intended by us or independent of us) in the values $PV^1, PV^2, PV^3, \ldots, PV^k$ leads to a different change in the NPV of the project. Thus, if Eq. 1.8 holds, the cash flows F_1 constitutes the main risk source (positive or negative) for the project, F_2 the next one etc. Now we have to evaluate, how easy it is for F_1 to change. If we can easily increase it, then F_1 constitutes a chance. If our environment can easily decrease it, then it constitutes a negative risk. The same analysis may be applied to F_2, F_3 etc.,

For another project we may have another ranking:

$$PV^k > PV^{k-1} > PV^{k-2} > \cdots > PV^1 \tag{1.9}$$

In such a case we would have to start our analysis of the project from F_k, F_{k-1} etc. Thus for one project the main positive or negative risk may be due to such factors as the inventory level or the payment conditions, while for another project the main risk factor may be the choice of suppliers, of resources etc. These factors may be more or less susceptible to change – and this information has to influence our project evaluation apart from the basic criterion, the NPV.

2. Fuzzy Logic

Fuzzy set theory was introduced by Zadeh in 1965. A fuzzy set is defined as a class of objects with a continuum of grades of membership, which is characterized by a membership function that assigns to each object a grade of membership ranging between zero and one. A fuzzy set A in U is characterized by a membership function $\mu_A(x)$ which associates with each point in U a real number in interval $[0, 1]$, with the value of $\mu_A(x)$ at x representing "the grade of membership" of x in A [Zadeh 1965]. We can also interpret $\mu_A(x)$ as the possibility degree of x being the actual value of a magnitude which is not known to us for the moment

The fuzzy number type most often used is a so-called triangular fuzzy number, in short TFN. The membership function for a triangular fuzzy number $\widetilde{M} = (m_l, m_m, m_r)$, characterized by three crisp parameters $m_l < m_m < m_r$ is given in Eq. 2.1.

$$\mu_M(x) = \begin{cases} 0 & \\ 1 + \dfrac{x - m_m}{m_m - m_l} & x \le m_l \\ & m_l < x < m_m \\ 1 - \dfrac{x - m_m}{m_r - m_m} \quad \text{if} & m_m \le x < m_r \\ & x \ge m_r \\ 0 & \end{cases} \tag{2.1}$$

Algebraic operations for *TFN*s $\widetilde{M} = (m_l, m_m, m_r)$ and $\widetilde{N} = (n_l, n_m, n_r)$ are given by the following formulas with the order of addition, subtraction, multiplication, division and multiplication by a scalar [Chen et al. 1992]:

$$\widetilde{M} \oplus \widetilde{N} \cong (m_l + n_l, m_m + n_m, m_r + n_r) \tag{2.2}$$

$$\widetilde{M} \ominus \widetilde{N} \cong (m_l - n_r, m_m - n_m, m_r - n_l) \tag{2.3}$$

$$\widetilde{M}\otimes\widetilde{N} \cong \begin{cases} (m_l n_l, m_m n_m, m_r n_r) & \widetilde{M} \geq 0, \widetilde{N} \geq 0 \\ (m_l n_r, m_m n_m, m_r n_l) \quad \text{if} & \widetilde{M} \leq 0, \widetilde{N} \geq 0 \\ (m_r n_r, m_m n_m, m_l n_l) & \widetilde{M} \leq 0, \widetilde{N} \leq 0 \end{cases} \tag{2.4}$$

$$\widetilde{M} \oslash \widetilde{N} \cong \begin{cases} \left(\frac{m_l}{n_r}, \frac{m_m}{n_m}, \frac{m_r}{n_l}\right) & \widetilde{M} \geq 0, \widetilde{N} \geq 0 \\ \left(\frac{m_r}{n_r}, \frac{m_m}{n_m}, \frac{m_l}{n_l}\right) \quad \text{if} & \widetilde{M} \leq 0, \widetilde{N} \geq 0 \\ \left(\frac{m_r}{n_l}, \frac{m_m}{n_m}, \frac{m_l}{n_r}\right) & \widetilde{M} \leq 0, \widetilde{N} \leq 0 \end{cases} \tag{2.5}$$

$$\lambda\otimes\widetilde{M} \cong \begin{cases} (\lambda m_l, \lambda m_m, \lambda m_r) \\ (\lambda m_r, \lambda m_m, \lambda m_l) \end{cases} \text{if} \quad \begin{matrix} \lambda \geq 0 \\ \lambda \leq 0 \end{matrix} \quad \forall \lambda \in \mathcal{R} \tag{2.6}$$

The support of a fuzzy number $\widetilde{M} = (m_l, m_m, m_r)$ is the interval $[m_l, m_r]$ – thus the domain on which the membership function takes on positive values, together with its boundary. The support of $\widetilde{M}$ will be denoted as $\overline{M}$.

3. Fuzzy Net Present Value

Fuzzy present value of a single future payment ($\widetilde{PV}(F)$) occurred at the end of n^{th} year from now is given in Eq. 3.1 where $\tilde{F}$ stands for fuzzy amount of the payment and i stands for the compound interest rate.

$$\widetilde{PV}(\tilde{F}) = \frac{\tilde{F}}{(1+i)^n} \tag{3.1}$$

Kuchta [2000] defined the general formula of fuzzy net present value as given in Eq. 3.2, where $\widetilde{F}_i$ denotes net cash flows in the time period i and ĩ denotes the fuzzy interest rate.

$$\widetilde{NPV} = -\tilde{I} + \sum_{i=0}^{n} \frac{\widetilde{F}_i}{(1+\tilde{\imath})^i} \tag{3.2}$$

The formula of fuzzy net present value of a project ($\widetilde{NPV}$) which has m different payments and has an initial investment at the beginning of the project is calculated by Eq. 3.3.

$$\widetilde{NPV} = -\tilde{I} + \left(\tilde{F}_1 \frac{1}{(1+\tilde{\imath})^{n_1}}\right) + \left(\tilde{F}_2 \frac{1}{(1+\tilde{\imath})^{n_2}}\right) + \cdots + \left(\tilde{F}_m \frac{1}{(1+\tilde{\imath})^{n_m}}\right) \quad (3.3)$$

Eq. 3.4 gives $\widetilde{NPV}$ of a project which has m different payments and the investment distributed over z years:

$$\widetilde{NPV} = -\tilde{I}_0 - \left(\tilde{I}_1 \frac{1}{(1+\tilde{\imath})^{n_1}}\right) - \left(\tilde{I}_2 \frac{1}{(1+\tilde{\imath})^{n_2}}\right) - \cdots - \left(\tilde{I}_z \frac{1}{(1+\tilde{\imath})^{n_z}}\right) + $$
$$+ \left(\tilde{F}_1 \frac{1}{(1+\tilde{\imath})^{n_1}}\right) + \left(\tilde{F}_2 \frac{1}{(1+\tilde{\imath})^{n_2}}\right) + \cdots + \left(\tilde{F}_m \frac{1}{(1+\tilde{\imath})^{n_m}}\right) \quad (3.4)$$

Eq. 3.5 and Eq. 3.6 represent fuzzy equivalents of Eqs. 1.4 and 1.5.

$$\tilde{F}_l = \tilde{F}_{1l} + \tilde{F}_{2l} + \tilde{F}_{3l} + \cdots + \tilde{F}_{nl} \quad (3.5)$$

$$\widetilde{PV}^l = \sum_{j=1}^{n} \frac{\tilde{F}_{jl}}{(1+\tilde{\imath})^{n_j}} \quad (3.6)$$

As shown in Eq. 3.8, $\widetilde{NPV}$ of the project is equal to the sum of the $\widetilde{PV}$s due to individual factors because of the linearity of $\widetilde{NPV}$ and the definition of the addition of fuzzy numbers, where $\widetilde{PV}^I$ denotes the present value of the initial investment which is given in Eq. 3.7:

$$\widetilde{PV}^I = \widetilde{PV}(I_0) + \widetilde{PV}(I_1) + \cdots + \widetilde{PV}(I_z) \quad (3.7)$$

$$\widetilde{NPV} = -\widetilde{PV}^I + \widetilde{PV}^1 + \widetilde{PV}^2 + \widetilde{PV}^3 + \cdots + \widetilde{PV}^k \quad (3.8)$$

To be able to perform in the fuzzy case the type of analysis illustrated in the crisp case by Eqs. 1.8 and 1.9, we have to be able to rank fuzzy numbers. The ranking of fuzzy numbers is not unambiguous and there are several methods which can be used to obtain it. The choice depends on the decision maker, on his preferences and attitude (he may be a pessimist or an optimist or someone “in between”). In the following section we present one method only – which allows us to differentiate between the pessimistic and optimistic attitudes of the decision maker, but other ranking methods may be also used, without modifying the proposed approach.

4. Ranking Method for Fuzzy Numbers

In decision-making problems, having the fuzzy data leads to fuzzy numbers as final solutions. A fuzzy number represents many possible real numbers that have different membership values. It is not easy to compare the fuzzy numbers to determine which alternatives are preferred. Many authors have proposed fuzzy ranking methods that can be used to compare fuzzy numbers [Chen et al. 1992].

According to Kahraman and Tolga [2009] the fuzzy ranking method of Dubois and Prade (1978) which will be used in our paper is one of the most cited ranking methods. Dubois and Prade (1978) proposed four indices to assess the position of a fuzzy number $\widetilde{N}$ relative to the position of a fuzzy number $\widetilde{M}$ to find out if $\widetilde{N}$ is smaller than $\widetilde{M}$ or not, out of which we chose two.

$$\Pi_M(]N,+\infty)) = Poss(x > N|x \text{ } is \text{ } M) = \sup_u \min\left(\mu_M(u), \inf_{\substack{v \\ v\geq u}} 1-\mu_N(v)\right) \tag{4.1}$$

$$= \sup_u \inf_{\substack{v \\ v\geq u}} min \left(\mu_M(u), 1-\mu_N(v)\right)$$

$$\mathcal{N}_M([N,+\infty)) = Ness(x \geq N|x \text{ } is \text{ } M) = \inf_u \max\left(1-\mu_M(u), \sup_{v\leq u} \mu_N(v)\right) = \tag{4.2}$$

$$= \inf_u \sup_{\substack{v \\ v\leq u}} \max(1-\mu_M(u), \mu_N(v))$$

The interpretation of the above indices, introduced by Dubois and Prade [1978], is as follows:

– $\Pi_M(]N,+\infty))$ is large if the values included in the support $\overline{N}$ and close to its upper bound are smaller or equal to the values in the support $\overline{M}$ and close to its upper bound
– $\mathcal{N}_M([N,+\infty))$ is large if the values included in the support $\overline{N}$ and close to its lower bound are smaller or equal to the values in the support $\overline{M}$ and close to its lower bound.

Thus, the two indices give the possibility to compare two fuzzy numbers according to the attitude of the decision maker (he may be a pessimist or an optimist – the first index corresponds to the positive attitude, we believe the flows will be rather rather be high, the second index corresponds to the pessimistic attitude).

5. Applications

In this section we will illustrate by means of examples how investment projects can be evaluated on the basis of different criteria.

Example 1: A manufacturer wants to decide to invest in a project which has 5 years of useful life. The company has five customers $C_i, i = 1,...5$. and three suppliers $S_i, i = 1,...3$. Supplier S_1 delivers materials for the production for customers C_1 and C_2, supplier S_2 delivers materials for the production for Customer C_3, and supplier S_3 delivers materials for the production for Customers C_4 and C_5. The sales (revenues) of the project for each customer are given as TFNs in Table 5.2. The profits before depreciation for each customer are given as TFNs in Table 5.3. The accounts payable for each customer are planned to be equal to the value of sales in 2 months for customer C_1, 1 month for customer C_2, 3 months for customer C_3, 1 month for customer C_4 and 3 months for customer C_5. The inventory of the project is planned to equal two months' sales, and liabilities of the suppliers are planned to be equal to the value of sales in 2 months, 1 month and 3 months, respectively for each supplier $S_i, i = 1,...3$. There will be no salvage value of the assets after 5 years. The interest rate is taken as $\tilde{\imath} = (8,10,12)\%$.

This is the basic information about the project; because of the uncertainty it is given in the form of fuzzy numbers.

Table 5.1

The planned payments for the assets

Year	Payment for the assets
0	(450000, 500000, 550000)
1	(270000, 300000, 330000)
2	(90000, 100000, 110000)
3	(90000, 100000, 110000)
4	None
5	None

As mentioned earlier, it is assumed that the assets are not dependent on any factors such as customers, resource choice, payment conditions etc. Thus they will be taken into account totally, as in Eq. 3.8. Then we have the data for different customers, including the sales (thus the revenue which is not necessarily cash in the same period, Table 5.2), the profit before depreciation (Table 5.3 – as depreciation is linked to the assets, it will not be distributed among factors, thus not among customers either).

Table 5.2

The sales of the project for each customer

Year \ Sales	Customer C_1 (x1000$)	Customer C_2 (x1000$)
0	–	–
1	(950, 1028, 1116)	(500, 520, 540)
2	(1023, 1087, 1251)	(622, 654, 686)
3	(1110, 1190, 1270)	(734, 772, 810)
4	(1230, 1310, 1390)	(760, 790, 820)
5	(1400, 1460, 1520)	(850, 910, 970)
Year \ Sales	Customer C_3 (x1000$)	Customer C_4 (x1000$)
0	–	–
1	(1104, 1200, 1296)	(780, 830, 880)
2	(1840, 2000, 2160)	(910, 960, 1050)
3	(2208, 2400, 2592)	(1100, 1230, 1350)
4	(2208, 2400, 2592)	(1150, 1250, 1350)
5	(2208, 2400, 2592)	(1200, 1330, 1460)
Year \ Sales	Customer C_5 (x1000$)	
0	–	
1	(1090, 1130, 1170)	
2	(1220, 1270, 1320)	
3	(1760, 1850, 1940)	
4	(1935, 2046, 2157)	
5	(2100, 2330, 2560)	

Table 5.3

The profits before depreciation for each customer

Year	Profits before depreciation	
	Customer C_1 (x1000$)	Customer C_2 (x1000$)
0	–	–
1	(–30, 10, 40)	(–35,–10, 15)
2	(206, 219, 232)	(135, 150, 165)
3	(590, 622, 654)	(340, 367, 394)
4	(921, 992, 1063)	(560, 600, 640)
5	(935, 987, 1039)	(580, 610, 630)

Year	Profits before depreciation		
	Customer C_3 (x1000$)	Customer C_4 (x1000$)	Customer C_5 (x1000$)
0	–	–	–
1	(–48, 5, 58)	(–35, 4, 43)	(–45, 12, 69)
2	(313, 330, 347)	(200, 210, 220)	(300, 327, 354)
3	(836, 880, 924)	(550, 590, 640)	(798, 875, 952)
4	(1235, 1300, 1365)	(885, 930, 975)	(1121, 1286, 1451)
5	(1130, 1190, 1250)	(900, 950, 1000)	(1240, 1275, 1310)

The payment and inventory conditions above given determine the change of accounts payable, liabilities and inventory level in each year. All these data together allow for calculating the cash flows. The cash flows (with the investment payment excluded) occurring in a time period can be calculated by the Eq. 5.1 below, which represents the indirect calculation method of cash flow in a given year:

$$\tilde{F}_t = \tilde{P}_t \quad + \Delta\tilde{L}_t - \Delta\widetilde{AP}_t - \Delta\tilde{I}_t \tag{5.1}$$

where $\tilde{F}_t$ denotes total cash flows of the project in year *t*, $\tilde{P}_t$ denotes profit before depreciation in year *t*, $\Delta\tilde{L}_t$ denotes change in year *t* in the liabilities, $\Delta\widetilde{AP}_t$ denotes change in year *t* in the accounts payable, $\Delta\tilde{I}_t$ denotes change in year *t* in the inventory and $\tilde{C}_t$ denotes payments for the newly purchased fixed assets in year *t*.

Then we have:

$$\tilde{P}_t = \widetilde{PC}_{1_t} + \widetilde{PC}_{2_t} + \widetilde{PC}_{3_t} + \widetilde{PC}_{4_t} + \widetilde{PC}_{5_t} \tag{5.2}$$

where $\widetilde{PC}_{i_t}$ denotes the profit before depreciation in year *t* from the sales to customer i.

$$\Delta\tilde{L}_t = \widetilde{LS}_{i_t} = \widetilde{LS}_{1_t} - \widetilde{LS}_{1_{t-1}} + \widetilde{LS}_{2_t} - \widetilde{LS}_{2_{t-1}} + \widetilde{LS}_{3_t} - \widetilde{LS}_{3_{t-1}} \quad (5.3)$$

Where $\widetilde{LS}_{i_t}$ denotes the liabilities in year *t* of seller i.

Similarly we have:

$$\Delta\widetilde{AP}_t = \widetilde{APC}_{1_t} - \widetilde{APC}_{1_{t-1}} + \widetilde{APC}_{2_t} - \widetilde{APC}_{2_{t-1}} + \widetilde{APC}_{3_t} - \widetilde{APC}_{3_{t-1}} + \\ + \widetilde{APC}_{4_t} - \widetilde{APC}_{4_{t-1}} + \widetilde{APC}_{5_t} - \widetilde{APC}_{5_{t-1}} \quad (5.4)$$

Where $\widetilde{APC}_{i_t}$ denotes the accounts payable in year *t* of customer i.

$$\tilde{F}_t = \tilde{P}_t + \Delta\tilde{L}_t - \Delta\widetilde{AP}_t - \Delta\tilde{I}_t - \tilde{C}_t = \\ = \sum_{i=1}^{5} \widetilde{PC}_{i_t} + \sum_{i=1}^{3} \left(\widetilde{LS}_{i_t} - \widetilde{LS}_{i_{t-1}}\right) - \sum_{i=1}^{5} \left(\widetilde{APC}_{i_t} - \widetilde{APC}_{i_{t-1}}\right) \quad (5.5)$$

The fuzzy net present value of the project is found by applying Eq. 3.4 to the values from Tables 5.1 and 5.4. Then we have $\widetilde{NPV} = (4950189, 7947617, 11289530)$\$.

Table 5.4

Total cash flows of the project

YEAR	CASH FLOWS TOTAL
0	(–550000, –500000, –450000)
1	(–1476333, –1082000, –699333)
2	(357333, 825000, 1277667)
3	(2262333, 2977000, 3718333)
4	(4121000, 5053833, 5986667)
5	(4060833, 4929667, 5788500)
$\widetilde{NPV}$	**(4950189, 7947617, 11289530)**

However, we want to see how the project is influenced by different factors. We start by choosing customers' situations as factors. We assume that it is not sensible to isolate flows year by year, because the yearly flows are not independent – if something happens in one year, it will have its consequences in the other years too. However, we think it is sensible to isolate the flows by customers – the customers may be assumed to be more independent from each other and we can imagine that we or the environment may influence, in a substantial way, only one customer at a time (but throughout all the years). Therefore, we group the flows not according to the years, but to the customers. The cash flows in the time period *t* for each customer C_i are calculated by Eq. 5.6.

$$\widetilde{FC}_{i_t} = \widetilde{PC}_{i_t} + \Delta\widetilde{LSC}_{i_t} - \Delta\widetilde{APC}_{i_t} \tag{5.6}$$

where $\widetilde{FC}_{i_t}$ denotes cash flows of the project for customer i in year t, $\widetilde{PC}_{i_t}$ denotes profit before depreciation for customer i in year t, $\Delta\widetilde{LSC}_{i_t}$ denotes change in year t in the liabilities of the supplier (suppliers) of the product for customer i, $\Delta\widetilde{APC}_{i_t}$ denotes change in year t in the accounts payable for customer i.

The present value of the total cash flows linked to each customer is found from Eq. 5.7.

$$PV(C_i) = \sum_{t=1}^{n} \frac{\sum_{i=1}^{m} \widetilde{FC}_{i_t}}{(1+i)^t} = \sum_{t=1}^{n} \frac{\sum_{i=1}^{m} \left(\widetilde{PC}_{i_t} + \Delta\widetilde{LSC}_{i_n} - \Delta\widetilde{APC}_{i_t}\right)}{(1+i)^t} \tag{5.7}$$

Here are the values $PV(C_i),\ i = 1, \dots, 5$

Table 5.5

The values $PV(C_i), i = 1, \dots, 5$

Year	Present value of cash flows	
	Customer C_1 (x1000$)	Customer C_2 (x1000$)
0	–	–
1	(–217560, –146667, –83951)	(–77381, –48485, –18519)
2	(71880, 172865, 268490)	(81447, 114738, 150463)
3	(344620, 454420, 589156)	(214246, 268345, 328117)
4	(521760, 663889, 825439)	(336931, 408784, 490142)
5	(476639, 597326, 737752)	(302155, 372553, 447483)

Year	Present value of cash flows		
	Customer C_3 (x1000$)	Customer C_4 (x1000$)	Customer C_5 (x1000$)
0	–	–	–
1	(–442857, –359091, –272222)	(–53571, 3636, 62963)	(–232143, –160303, –85802
2	(–65104, 52342, 178612)	(111607, 173554, 240055)	(172725, 250964, 334934)
3	(374871, 560982, 767371)	(322081, 443276, 585451)	(432762, 584773, 753082)
4	(662845, 887917, 1144441)	(490938, 635203, 799345)	(606496, 856044, 1141011)
5	(532246, 738896, 981401)	(445430, 589875, 758850)	(547756, 762284, 1014069)

Now we want to analyze the influence of the situation of each customer on the NPV of the project. We use the ranking method for fuzzy numbers described in the previous section.

Table 5.6

The possibility and necessity indices of the present values of cash flows determined for customers

Cases \ Indices	$\Pi_M(]N,+\infty))$	$\mathcal{N}_M([N,+\infty))$
$\widetilde{PV}_{C1} > \widetilde{PV}_{C2}$	1	1
$\widetilde{PV}_{C2} > \widetilde{PV}_{C1}$	0	0
$\widetilde{PV}_{C1} > \widetilde{PV}_{C3}$	0.3	0.5
$\widetilde{PV}_{C3} > \widetilde{PV}_{C1}$	0.7	0.5
$\widetilde{PV}_{C1} > \widetilde{PV}_{C4}$	0.42	0.4
$\widetilde{PV}_{C4} > \widetilde{PV}_{C1}$	0.58	0.6
$\widetilde{PV}_{C1} > \widetilde{PV}_{C5}$	0.05	0.16
$\widetilde{PV}_{C5} > \widetilde{PV}_{C1}$	0,95	0.84
$\widetilde{PV}_{C2} > \widetilde{PV}_{C3}$	0	0.05
$\widetilde{PV}_{C3} > \widetilde{PV}_{C2}$	1	0.95
$\widetilde{PV}_{C2} > \widetilde{PV}_{C4}$	0	0
$\widetilde{PV}_{C4} > \widetilde{PV}_{C2}$	1	1
$\widetilde{PV}_{C2} > \widetilde{PV}_{C5}$	0	0
$\widetilde{PV}_{C5} > \widetilde{PV}_{C2}$	1	1
$\widetilde{PV}_{C3} > \widetilde{PV}_{C4}$	0.62	0.42
$\widetilde{PV}_{C4} > \widetilde{PV}_{C3}$	0.38	0.58
$\widetilde{PV}_{C3} > \widetilde{PV}_{C5}$	0.28	0.23
$\widetilde{PV}_{C5} > \widetilde{PV}_{C3}$	0.72	0.77
$\widetilde{PV}_{C4} > \widetilde{PV}_{C5}$	0.1	0.24
$\widetilde{PV}_{C5} > \widetilde{PV}_{C4}$	0.9	0.76

We can now rank the discounted flows influenced by each customer in the pessimistic (when we assume that rather small flow values will occur) and in the optimistic cases. We assume that the corresponding relation is true if the respective possibility (necessity) index is greater than or equal to 0.7. Then in the optimistic case (the possibility measure) we get the following partial order: $\widetilde{PV}_{C5} > \widetilde{PV}_{C3} > \widetilde{PV}_{C1} > \widetilde{PV}_{C2}$, $\widetilde{PV}_{C5} > \widetilde{PV}_{C4}$, and in the pessimistic case the following one: $\widetilde{PV}_{C5} > \widetilde{PV}_{C1} > \widetilde{PV}_{C2}, \widetilde{PV}_{C5} > \widetilde{PV}_{C3} > \widetilde{PV}_{C2}$, $\widetilde{PV}_{C5} > \widetilde{PV}_{C4}$. Thus in both cases customer C_5 is responsible for the greatest flow contributing to the NPV of the project.

Let us now suppose that we have another project with the same NPV, but in this project customer C_2 has most influence on the NPV of the project. Now the selection of one of the two projects will be depending on the stability of the customer's situation, on the probability of their buying less, paying too late, etc. and also on the probability of the success of our endeavors influencing them and make them buying more, paying less, using cheaper suppliers, using less inventory etc.

We might also group the flows in different groups, e.g. flows due to accounts payable, liabilities and inventory level. In this way we might see which factor: the payment conditions offered by us to the customers, the payment conditions given to us by the suppliers, our decision about keeping a certain amount of inventory, has the most influence on the project's NPV and what the risk (positive or negative) connected to this factor is. This would constitute an additional criterion, apart from the NVP, to select an investment project.

Example 2: A company decides to manufacture a new product. One of two different machines X and Y, with different production volumes, can be used to manufacture the product. The new product needs two kinds of raw materials (R_A and R_B). The production requirements for the machines and cost and revenues for the product are given in Tables 5.7 and 5.8. The interest rate is taken as $i = (8,10,12)\%$.

Table 5.7

Parameters of the machines

	Machine X	Machine Y
Initial Investment	$(1250000,1430000,1610000)$$	$(1240000,1320000,1400000)$$
Production capacity per year	$(1620000, 1800000, 1980000)$ units	$(1140000, 1200000, 1260000)$ units
Work power per month	$(3360, 3600, 3840)$ hours	$(2016, 2160, 2304)$ hours
Raw material A per product unit	2	2
Raw material B per product unit	1	1
Energy cost per year	$(11760,12000,12240)$$	$(8340,8400,8460)$$
Useful life	10 years	10 years

Table 5.8

Costs and revenues generated by the new product

	Cost or Revenue
Labor cost per hour	(4.5, 5, 5.5)$
Cost of Raw material A per product	(0,8,0,9,1)$
Cost of Raw Material B per product	(0.9, 1, 1.1)$
Price of the product per unit	(9, 10, 11)$

Net annual cash flows and fuzzy present values of the cash flows for each factor are calculated and given in Table 5.9.

Table 5.9

Cash Flows and Present Values of the Factors

	TOTAL CASH FLOWS	
	Machine X	Machine Y
1	2	3
Net annual cash flows (total)	(8408640, 12930000, 17703120)	(6332868, 8620800, 10992588)
$\widetilde{PV}$ of total cash flows	(18660829, 49014873, 114812908)	(14054183, 32679615, 71292009)
$\widetilde{NPV}$ of the project	(17050829, 47584873, 113562908)	(12654183, 31359615, 70052009)
Annual Energy Cost	(12240, 12000, 11760)	(8460, 8400, 8340)
$\widetilde{PV}$ of Energy Costs ($\widetilde{PV}_E$)	(76269, 45489, 27164)	(54089, 31843, 18775)
Annual Labor Cost	(21120, 18000, 15120)	(12672, 10800, 9072)
$\widetilde{PV}$ of Labor Costs ($\widetilde{PV}_L$)	(98060, 68234, 46870)	(58836, 40940, 28122)
Annual raw material cost	(6138000, 5040000, 4050000)	(3906000, 3360000, 2850000)
$\widetilde{PV}$ of total raw material costs ($\widetilde{PV}_{RT}$)	(26266120, 19105565, 13621724)	(18483566, 12737044, 8668370)
Annual cost of raw material A	(3960000, 3240000, 2592000)	(2520000, 2160000, 1824000)
$\widetilde{PV}$ of raw material A costs ($\widetilde{PV}_{RA}$)	(16810317, 12282149, 8788209)	(11829482, 8188099, 5592496)
Annual cost of raw material B	(2178000, 1800000, 1458000)	(1386000, 1200000, 1026000)

Table 5.9 contd.

1	2	3
$\widetilde{PV}$ of raw material B costs ($\widetilde{PV}_{RB}$)	(9455803, 6823416, 4833515)	(6654084, 4548944, 3075873)
$\widetilde{PV}$ of initial investment ($\widetilde{PV}_I$)	(1610000, 1430000, 1250000)	(1400000, 1320000, 1240000)

To determine which machine has higher $\widetilde{NPV}$, the possibility indices of ranking cases are calculated and given in Table 5.10.

Table 5.10

Possibility and necessity indices for Fuzzy Net present Values of Two Machines

Cases \ Indices	$\Pi_M(]N, +\infty))$	$\mathcal{N}_M([N, +\infty))$
$\widetilde{NPV}(X) > \widetilde{NPV}(Y)$	0.78	0.71
$\widetilde{NPV}(Y) > \widetilde{NPV}(X)$	0.22	0.29

We can see that the decision maker, regardless of his pessimistic or optimistic attitude, should decide to invest in Machine X. However, we want to evaluate this decision also according to other criteria, because the opposite relation $(\widetilde{NPV}(Y) > \widetilde{NPV}(X))$ is also true to some extent (this is the feature of fuzzy values, which are based on uncertainty and incomplete knowledge), thus we may still consider the choice of machine Y − if there is a too high negative risk linked to Machine X or a high positive risk (chance) linked to Machine Y.

To determine the importance of the factors which affect fuzzy net present value of the project "buy Machine X", the possibility and necessity indices are calculated. We get only the values 0 or 1 which means the numbers don't have intersections and the ranking is exactly known. We get the following ranking: $\widetilde{PV}_{RT} > \widetilde{PV}_{RA} > \widetilde{PV}_{RB} > \widetilde{PV}_I > \widetilde{PV}_L > \widetilde{PV}_E$. Thus the factors linked to the raw materials have the greatest influence here. If we judge that the prices of raw materials may change considerably and in the unfavorable direction, we may want to see what is the influence of this factor on the NPV of the project "buy machine Y" and we may found out that this project is biased in the first place by other types of risk and/or offers new chances. In such a situation the fuzzy NPV and the fuzzy ranking shown in Table 5.10 would not constitute the only criterion to choose a machine.

Conclusions

The fuzzy net present value (NPV) method is one of the most preferred investment analysis methods which can deal with the uncertainty of forecasting cash flows of investment projects. In the fuzzy net present value method the worth of an investment is defined by a fuzzy number and the decision maker has to decide whether to invest in the project by considering this number. In reality all the factors influencing fuzzy net present value of an investment should also be taken into account (e.g. the credibility of customers or suppliers, the cash flows resulting from work power, raw material selection, payment conditions etc.), together with their variability/stability. The decision maker should analyze, apart from the NPV, what is the factor which influences it most and how probable (possible) are changes in the flows linked to this factor. The final decision in the evaluation of a project should be made on the basis of NPV and the structure of its components dependent on individual factors. In the present paper we propose a method to perform such an evaluation. Further research is needed to propose a methodology of identifying different factors and verify the independence between selected groups of cash flows.

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