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THE DISCRETE INTERACTIVE MULTIPLE GOAL PROGRAMMING UNDER RISK

Abstract

An Interactive Multiple Goal Programming (IMGP) is a popular method of multicriteria decision aiding. The discrete version of this method was proposed by Habenicht in 1984. In this paper we propose the modification of discrete version IMGP which enables us to take into consideration risk factors and it's also adopted for group decision making.. Risky criteria are described by probability distributions. The aggregation of local judgments making by individual decision makers to the group decision is carried out by voting system. In this paper proposed method is presented. In the last section the proposed method is illustrated by simple numerical example.

Keywords

Multicriteria decision making, decisions under risk, group decision making.

Introduction

In the beginning of the 21st century we observe a significant volatility of the macroeconomic environment, which has a considerable impact on the business world. First of all it is a consequence of rapid technological progress, particularly in the field of information and telecommunication technologies (ICT) and the increasing economic globalization.

In consequence, the influence that these factors exert on economic and business decisions has to be taken into account in the decision-making. The issues related to decision analysis and aiding under incomplete information remain an important part of operational research, in particular of multicriteria decision aiding. Uncertainty implies that in certain situations a person does not possess the information which is quantitatively and qualitatively appropriate to describe, prescribe or predict deterministically and numerically a system, its behavior or other characteristics [Zimmerman 2000].

In the MCDA approach we can find a wide range of methods and techniques to deal with uncertainty: sensitivity analysis [e.g. Rios Insua 1990], fuzzy set approach [e.g. Klir and Fogler 1988], rough set approach [e.g. Greco, Matarazzo and Slowinski 1999], probabilistic models and expected utility [Bazerman 2002; Rosquist 2001], pairwise comparisons based on stochastic dominance [e.g. Martel and Zaras 1995; Nowak 2008]. Risk measures as surrogate criteria are also applied [e.g. Millet and Wedley 2002; Jia and Dyer 1996].

In business organizations we observe that very often a Decision Maker (DM) is not a single person but a group of people responsible for making a decision. Thus the modeling of group preferences becomes an important part of the decision making process. The problems of group decision making is discussed, for example, by Ramanathan and Ganesh [1994], Van Den Honert [2001], Herrera, Martinez and Sanchez [2005].

In this paper we propose an interactive procedure which is a modification of the discrete version of Interactive Multiple Goal Programming (IMGP) which enables decision aiding under risk by a group of decision makers. The first section of this paper shortly describes the IMGP algorithm and its main advantages. The second section presents the proposed decision aiding method. A simple numerical example is presented in the last part of this paper.

1. The Interactive Multiple Goal Programming

The IMGP was proposed by Nijkamp and Spronk [1980]. According to this approach, the single criterion problems are solved first. Then, on the basis of the optimal solutions obtained the potency matrix is calculated. The potency matrix consists of the ideal solution and the current one. The DM chooses the criterion on which the current solution should be improved and describes the aspiration level for this criterion. The proper constraint is added to each single criterion problem considered and then those problems are solved again. The DM compares the obtained values in the potency matrixes and decides whether he/she accepts the new solution or not. The procedure is continued till the ideal and the current solutions become equal to each other. Some important advantages are connected with the IMGP. First of all, the DM does not have to give his preference information on an a priori basis but has to consider all kinds of choices and trade-off questions which may be relevant [see Nijkamp

1980, p. 104]. Another important advantage of IMGP is its relatively simply and easy to understand idea. During an interactive procedure the DM has to answer the simple questions:

1. Is the given solution acceptable or not?
2. Which goal value needs to be improved?
3. How much should this goal value be improved at least?
4. Does he accept the consequences of the proposed improvement of the value of the indicated goal variable? [see Nijkamp 1980, p. 250].

The discrete version of IMGP was proposed by Habenicht [1984]. In this case instead of a multicriteria linear programming problem we consider a finite set of alternatives. Each alternative is described by a finite set of attributes. The potency matrixes are calculated on the basis of criteria values within the set of alternatives and in each iteration according to the DM's decisions the set of alternatives is reduced.

2. The discrete IMGP under Risk

In this section we propose an interactive multicriteria decision aiding method which supports multicriteria group decision making under uncertainty for discrete decision problems. Let us assume that:

m – is the number of alternatives,

k – is the number of criteria, all criteria are maximized,

X_{ij} – is the probability distribution of the j -th criterion of the i -th alternative,

p_j – is the probability at which the j -th criterion is evaluated.

Moreover, let us assume that the matrix

$$X(p_j) = [x_{ij}]_{mk}$$

includes the values of the j -th evaluation criterion for the assumed probability value p_j , and these values guarantee the probability that a particular variable will have a lower value of at least p_j , which is defined as follows:

$$P(X_{i,j} \geq x_{i,j}) = p_j$$

Let x_{ideal} denote the **ideal** solution, defined below:

$$x_{ideal}(p_j) = [x_{i,j} : x_{i,j} = \max_{i=1,\dots,m} x_{ij}; j = 1, \dots, k]$$

Whereas x_{current} is a **current** solution:

$$x_{\text{current}}(p_j) = [x_{i,j} : x_{i,j} = \min_{i=1,\dots,m} x_{ij} ; j = 1, \dots, k]$$

Let P^0 be the initial matrix which consists (for all criteria) of all possible ideal values for different probabilities defined as follows:

$$P^0 = \begin{bmatrix} x_{\text{ideal}}(0,95) \\ x_{\text{ideal}}(0,75) \\ x_{\text{ideal}}(0,50) \\ x_{\text{ideal}}(0,25) \end{bmatrix}$$

The potency matrix P^r is written as follows:

$$P^r = \begin{bmatrix} x_{\text{ideal}}(p_j) \\ x_{\text{current}}(p_j) \end{bmatrix}$$

Where the index $r = 1, 2, 3, \dots$ denotes the number of the algorithm iteration which generated the matrix P .

STEP 1

The DM is presented with the potency matrix P^0 . Then, for each criterion, the DM defines the probability value at which he will analyse the values of a given evaluation criterion. The first potency matrix P^1 is calculated and presented to the DM. The DM chooses either to accept the values and move to Step 2 or to correct the adopted probabilities values p_j .

STEP 2

Following the analysis of the potency matrix, the DM chooses the criterion “j” for which the value of the current solution should be improved. He specifies the accepted value of the pessimistic solution of the criterion d_j , which should be greater than the current solution and should be lower than or equal to the ideal solution.

STEP 3

The alternatives that do not fulfil the condition specified by the DM in Step 2 are deleted from the set of the decision alternatives and a new potency matrix P^r is calculated. The DM compares the values in the potency matrixes P^r and P^{r-1} and decides whether he accepts the consequences of his requirements.

3a) If the DM accepts the new solution, we go back to Step 2. The DM can change the required probability values p_j for the particular evaluation criteria and is then presented with the accordingly improved potency matrix.

3b) If the DM rejects the new solution, we restore the deleted alternatives and then go back to Step 2.

STOP CONDITION

The procedure stops when there is only one alternative left in the set of decision alternatives and the DM accepts the solution.

3. The Group decision making

In the case when the decision maker is not a single person but a group of people we need to develop rules which enable us to aggregate individual preferences into a decision accepted by a group of decision makers. We consider the following types of decisions: choosing the criterion on which the current value should be improved, choosing the aspiration level for the selected criterion, changing the probability value at which the criterion is considered and deciding the current solution is accepted or not. We propose the following scheme of group decision making:

1. The group chooses the criterion to be improved by a series of voting,
2. Changing the probability (if desired) by vote,
3. Decision makers describe the aspiration levels individually,
4. Sort aspiration levels from weakest to the strongest,
5. Calculate the potency matrix for the aspiration level considered,
6. Voting on whether the current solution is acceptable or not.

In the first step the decision makers vote for the criterion on which the value should be improved.

If none of the criteria gets the desired number of votes the voting is repeated (without the criterion which gets lowest number of votes) till one criterion gets the necessary number of votes (e.g. 51%).

Next, any decision maker can propose to change the probability at which the chosen criterion is analyzed. He/she proposes the new probability value and the members of the group vote whether they agree with this proposal.

Then each decision maker describes the aspiration level for the chosen criterion. The values entered are sorted started with the weakest one. For the succeeding values of aspiration levels the following potency matrixes

are calculated and presented to the decision makers. The decision makers vote whether they accept the obtained solution (i.e. whether they accept the consequences of taking into account the aspiration level considered). If the considered solution is accepted by the group, we consider the next (better) value of the aspiration level till we check all values entered. Or, if the solution is not accepted, the remaining aspiration levels are omitted and we go to the next iteration of IMGP method.

4. The numerical example

Let us assume that the management board which consists of three persons (DM1, DM2, DM3) should choose one of the five investment alternatives A1, ..., A5. The following four criteria are taken into consideration:

K1 – NPV: net present value (max),

K2 – MS: market share (max),

K3 – DR: debt ratio (min),

K4 – CR: cover ratio i.e.: the proportion of operating profit to financial costs (max).

The following three tables show the percentile distributions obtained due to Monte Carlo simulation carried out for each alternative¹.

Table 1

Net Present Value in mln EUR

Centiles	A1	A2	A3	A4	A5
1,00%	30	25	16	23	25
5,00%	30	25	16	23	25
10,00%	30	25	17	23	25
25,00%	30	25	19	25	25
50,00%	33	29	25	32	31
75,00%	41	43	30	36	37
90,00%	46	49	34	40	42
95,00%	48	52	35	40	45
99,00%	49	54	35	41	45

¹ An examples of the use of Monte Carlo simulation in strategic decision making can be found in Dominiak [1998, 1999].

Table 2

Market share

Centiles	A1	A2	A3	A4	A5
1,00%	21%	14%	18%	21%	20%
5,00%	21%	15%	18%	22%	22%
10,00%	21%	15%	18%	22%	22%
25,00%	21%	15%	19%	22%	22%
50,00%	22%	17%	22%	24%	24%
75,00%	27%	18%	23%	26%	26%
90,00%	28%	20%	24%	27%	27%
95,00%	28%	22%	25%	27%	27%
99,00%	29%	23%	25%	28%	28%

Table 3

Debt ratio

Centiles	A1	A2	A3	A4	A5
1,00%	29%	39%	35%	44%	28%
5,00%	29%	39%	35%	44%	28%
10,00%	30%	40%	35%	45%	30%
25,00%	31%	41%	35%	46%	32%
50,00%	34%	45%	37%	47%	34%
75,00%	35%	46%	38%	48%	37%
90,00%	36%	47%	39%	49%	39%
95,00%	36%	47%	39%	49%	39%
99,00%	36%	47%	39%	49%	39%

Table 4

Cover ratio

Centiles	A1	A2	A3	A4	A5
1,00%	2,0	1,1	1,4	1,4	1,6
5,00%	2,1	1,1	1,5	1,4	1,7
10,00%	2,1	1,1	1,5	1,4	1,7
25,00%	2,1	1,1	1,5	1,4	1,8
50,00%	2,3	1,2	1,6	1,5	1,9
75,00%	2,4	1,3	1,7	1,6	2,1
90,00%	2,5	1,4	1,8	1,8	2,3
95,00%	2,5	1,4	1,8	1,9	2,3
99,00%	2,5	1,4	1,9	1,9	2,4

The initial potency matrix is presented in Table 5.

Table 5

The initial matrix P^0

Probability	K1	K2	K3	K4
25%	43	27%	0,49	2,1
50%	33	24%	0,47	2,1
75%	30	22%	0,46	2,3
90%	30	22%	0,45	2,4

Looking at the values from Table 5, the decision makers determine the probabilities at which they want to analyze the criteria. Let us assume that for the criterion K1 the members of the management board determined the probabilities: 0.95, 0.99, and 0.99 respectively. Thus the first criterion will be analyzed at the probability level equal to 0.99. In the same way the probabilities for other criteria were determined. Let us assume that they are equal to: 0.95, 0.90, and 0.99 respectively. Then the first potency matrix is presented to the DMs:

Table 6

Potency Matrix P^1

Criterion:	K1	K2	K3	K4
Probability:	0,99	0,95	0,9	0,99
Solution:				
Ideal	30	22%	0,36	2,0
Current	16	15%	0,49	1,1

The decision makers vote which criterion should be improved. Let us assume that DM1 voted for K1, DM2 voted for K2 and DM3 voted for K1. Thus K1 was chosen to be improved. In the next step the decision makers describe the desired value of the chosen criterion in the solution. They propose the following values for K1: 25, 16, 20. First we check the lowest value. Since the desired value of K1 is equal to 16, which is the current solution, the solution remains unchanged. Then we check what happens when $K1 \geq 20$. In this case we obtain:

Table 7

Potency Matrix P^{1-1}

Criterion:	K1	K2	K3	K4
Probability:	0,99	0,95	0,9	0,99
Solution:				
Ideal	30	22%	0,36	2,0
Current	23	15%	0,49	1,1

All decision makers accept this solution, therefore we check the last value $K1 \geq 25$. In this case we get the following potency matrix:

Table 8

Potency Matrix P^{1-2}

Criterion:	K1	K2	K3	K4
Probability:	0,99	0,95	0,9	0,99
Solution:				
Ideal	30	22%	0,36	2,0
Current	25	15%	0,47	1,1

Let us assume that the DMs again accept the solution and we can go to the second iteration. The P^2 matrix is equal to P^{1-2} matrix. The decision makers chose the criterion that will be improved. They voted as follows: DM1-K3, DM2-K4, DM-K4. The fourth criterion is chosen to be improved. The decision makers determined the following aspiration levels: 1.2, 1.4, 1.5. When the constraint $K3 \geq 1.2$ is added, we get the following potency matrix:

Table 9

Potency Matrix P^{2-1}

Criterion:	K1	K2	K3	K4
Probability:	0,99	0,95	0,9	0,99
Solution:				
Ideal	30	22%	0,36	2,0
Current	25	21%	0,39	1,6

The decision makers accepted the consequences and the new solution. The values 1.4 and 1.5 don't change the last solution so we start the next iteration. The P^3 matrix is equal to P^{2-1} . DM2 proposed to change the probability at which K2 is analyzed to the level of 0.99. DM1 did not agree but DM3 did, thus the revised P^3 matrix is calculated and presented to the DMs:

Table 10

Potency Matrix P^3

Criterion:	K1	K2	K3	K4
Probability:	0,99	0,99	0,9	0,99
Solution:				
Ideal	30	21%	0,36	2,0
Current	25	20%	0,39	1,6

First DM votes that K1 should be improved but DM2 and DM3 decided to improve the value of K2. They wanted to increase the accepted market share to at least 21%. The following potency matrix is obtained:

Table 11

Potency Matrix P^4

Criterion:	K1	K2	K3	K4
Probability:	0,99	0,99	0,9	0,99
Solution:				
Ideal	30	21%	0,36	2,0
Current	30	21%	0,36	2,0

Each decision maker accepted the above solution. We can see that the ideal solution is equal to the current one and the set of alternatives consists of one object. Thus the decision aiding procedure stops. As the final decision according to DM's preferences, A1 should be chosen.

Conclusions

In this paper we proposed an interactive method for decision support in discrete multi criteria problems under risk. The proposed procedure is based on the discrete version of Interactive Multiple Goal Programming. The important part of the proposed method is the scheme of aggregating local judgments, made by individual decision makers, into a group decision made by a voting system.

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