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# **SENSITIVITY AND ROBUSTNESS ANALYSIS OF SOLUTIONS OBTAINED IN THE EUROPEAN PROJECTS' RANKING PROCESS**

## **Abstract**

After entering the European Union on 1 May 2004 Poland has become eligible to benefit from the EU Structural Funds and the Cohesion Fund and projects co-financed by these means have become a crucial instrument supporting restructuring and modernization of Polish economy. Total financial assistance granted to Poland for the previous (2004-2006) and present (2007-2013) programming periods amounts to over 80 billion euro. An efficient allocation of these subsidies depends, among other things, on proper choice of projects that are going to be co-financed, which can be made with the aid of such multi-criteria techniques as ORESTE, EVAMIX, PROMETHEE II, EXPROM II or modified BIPOLAR method.

In the paper sensitivity and robustness analysis of solutions obtained with the help of the above-mentioned methods will be carried out. It will enable to show the influence of the information delivered by the decision-makers and choices made by them during the decision aiding process on the final European projects' ranking. In a real-life example concerning this issue 16 applications for project co-financing by the European Regional Development Fund submitted to Measure 1.2 Environmental protection infrastructure in one of Polish voivodships in the programming period 2004-2006 will be used.

## **Keywords**

Multi-criteria decision aiding methods: ORESTE, EVAMIX, PROMETHEE II, EXPROM II, modified BIPOLAR method, stochastic dominance rules, sensitivity and robustness analysis.

## **Introduction**

European regional policy is currently one of the crucial factors in strengthening the socio-economic development of Poland and other European Union countries, especially those that entered the EU in 2004 and 2007, whose

economies have lagged far behind the economies of the old Member States of EU-15 and whose needs in the areas of environment, infrastructure, research and innovation, industry, services and SMEs are truly significant [Górecka 2011b].

Regional policy helps to reduce disparities between countries, increase the regions' competitiveness and attractiveness, improve the employment prospects and support innovation and development of the knowledge society as well as environmental protection. Moreover, it strengthens cross-border co-operation through financing concrete projects for regions, towns and their inhabitants.

In the previous programming period 2000-2006 over 233 billion EUR was earmarked for all regional instruments for the 15 old Member States. Moreover, around 24 billion EUR was allocated for the 10 new Member States for years 2004-2006, not to mention 22 billion EUR granted for pre-accession aid. In the present programming period 2007-2013 cohesion policy benefits from total allocation of about 347 billion EUR, which represents nearly 36% of the entire Union's budget.

Because of the enormous amount of money devoted to the structural aid it is crucially important to allocate the means in the most effective way possible. And that depends, among other things, on the proper choice of projects to be co-financed. In order to help the decision-makers in this challenging and difficult task multi-criteria decision aiding techniques, which refers to making decision in the presence of multiple, usually conflicting criteria, should be applied as evaluation of the European projects requires taking into account many diverse aspects: economic, financial, environmental, ecological, technical, technological, social and legal [Górecka 2011b].

Sensitivity and robustness analysis of the obtained solutions to the changes of the parameters of the preference model is in the case of projects applying for co-financing from the European Union funds quite a risky undertaking – in the extreme case it may lead even to undermining the decisions taken as a result of the proceedings conducted, i.e. to contesting the list of projects selected for funding. However, such analysis will be carried out for the purposes of this paper, primarily to demonstrate the importance of the quality of both the information acquired from the participants of the decision-making process and choices made by the decision-makers during the decision-aiding process, and the extent of their influence on the final ranking of projects [Górecka 2011a<sup>1</sup>].

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<sup>1</sup> This publication is devoted to the sensitivity and robustness analysis of solutions obtained with the help of the following MCDA techniques: PROMETHEE II with stochastic dominance rules, EXPROM II with stochastic dominance rules and modified BIPOLAR method with stochastic dominance rules.

## 1. Sensitivity and robustness analysis

When solving real decision-making problems decision-makers and analysts encounter problems related to the imperfection of knowledge. This deficiency has several different causes but invariably leads to assigning arbitrary values to the certain parameters of models and algorithms used in the decision-making process. In this case parameters are very broadly defined and include both the data in the classical sense of the word and information about values and beliefs of the participants of the decision-making process as well as information regarding technical issues related to the algorithms operation. In the multi-criteria methods based on the outranking relation, among which the ELECTRE and PROMETHEE methods stand out, doubts resulting from the imperfection of available data may concern both parameters related to the modelling of preferences (weights, thresholds or categories profiles) and technical parameters such as, for example, the cutting level  $\lambda$  [Figueira et al. 2005, p. 149]. Since it is difficult to expect that the participants of the decision-making process will easily define the values of parameters, therefore each of their permissible combinations should be treated as a “working hypothesis”. The problem is that different “working hypothesis” may lead to different results [Dias and Climaco 1999, p. 74].

In practice, a reference system composed of central values of the parameters is often defined and on this basis the calculations are carried out, whose results are used to prepare recommendations for the decision-makers. Subsequently the sensitivity of the solution to changes in the values of the parameters is examined. This analysis is usually performed for each parameter separately (ignoring possible interdependencies among them). It allows you to define the scope of the changes in the values of the parameters which make no impact on the solution designated earlier and also specify these parameters, whose values, when varying from the central positions, particularly strongly affect the outcome [Figueira et al. 2005, p. 149-150].

As an alternative for the sensitivity analysis the robustness analysis of the obtained solutions to changes in the values of the parameters may be considered. In this case the problem is defined as follows: assuming that the role of the analyst is to build such recommendations that will prove correct for the possibly wide range of the parameter values, we want to obtain

information on the solutions proposed, depending on the values of the parameters. Thus, we are interested in whether and how the solution of the problem will be changing with modification of the parameters within the sets of their admissible values.

The concept of robustness was introduced by Roy [Roy and Hugonnard 1982, p. 301-312; Roy 1998, p. 141-160] who has formulated a definition of the robust conclusion describing it as a formalized premise that is true for all plausible combinations of parameter values. Dias and Climaco, starting with the definition given by Roy, have distinguished the following types of the robust conclusions [Dias and Climaco 1999, p. 75]:

- an absolute robust conclusion – a premise intrinsic to one of the examined variants, which is valid for all acceptable combinations of parameter values; in the case of additive aggregation model the absolute conclusion may be “for example as follows: “the assessment of the variant  $a_i$  is less than 0,5”;
- a relative binary robust conclusion – a premise concerning a pair of variants, which is true for all possible values of parameters; for example: “ $a_i$  outranks  $a_j$  with credibility greater than 0,8”;
- a relative unary robust conclusion – a premise concerning one variant but referring to others, binding for each admissible combination of parameter values; for example: “ $a_i$  is placed on one of the top three positions in the ranking”.

## 2. The proposed procedure of appraising and selecting European projects

Meeting the need to improve the system of evaluation and selection of applications for project co-financing by the European Union funds and taking into account advantages and disadvantages of different multi-criteria decision aiding methods<sup>2</sup>, the procedure composed of the following elements has been proposed to aid the choice of European projects:

- identification of the participants of the decision-making process;
- selection of the criteria and determination of their weights with the help of:

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<sup>2</sup> [See Górecka 2010, p. 105-108].

- Analytic Hierarchy Process [Saaty 2006; Saaty and Vargas 1991],
- REMBRANDT system [Lootsma et al. 1990, p. 293-305; Olson et al. 1995, p. 522-531],
- revised Simos' method [Figueira and Roy 2002, p. 317-326],  
(depending on the preferences of the decision-makers);
- establishing indifference, preference and veto thresholds for each of the criteria;
- building a table of assessments (evaluation matrix) of the projects participating in the contest;
- application of:
  - ORESTE method [Roubens 1982, p. 51-55],
  - EVAMIX method [Voogd 1982, p. 221-236],
  - PROMETHEE II method with stochastic dominance rules<sup>3</sup>,
  - EXPROM II method with stochastic dominance rules<sup>4</sup>,
  - modified BIPOLAR method<sup>5</sup> with stochastic dominance rules [Górecka 2009, p. 223-230],
  - EVAMIX method with stochastic dominance rules<sup>6</sup> [Górecka 2010, p. 120-122](depending on the available data and the expectations and preferences of the decision-makers);
- taking final decision.

### 3. Case study

The proposed procedure was employed in the simulation of the process of appraising and ranking European projects carried out with the use of applications for project co-financing by the European Regional Development Fund submitted to Measure 1.2 *Environmental protection infrastructure* in one of Polish voivodships in the programming period 2004-2006. Measure 1.2 was implemented within the framework of the Priority 1 *Development and modernisation of the infrastructure to enhance the competitiveness of regions* of the Integrated Regional Operational Programme.

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<sup>3</sup> The indifference threshold has been introduced to the technique. [See Górecka 2009, p. 218-223, 263-277]. Compare with the original approach presented in Nowak [2005, p. 193-202].

<sup>4</sup> See Appendix A.

<sup>5</sup> Original version of BIPOLAR method was proposed in Konarzewska-Gubała [1991].

<sup>6</sup> See Appendix B.

Sixteen infrastructure projects were considered<sup>7</sup>. They concern the protection of surface waters, waste management and flood control and include:

- construction and modernisation of wastewater and rainwater collection networks and wastewater treatment plants,
- implementation of a system of communal waste management, i.a. construction of a sorting and composting plants and recultivation of landfills,
- modernisation of dikes.

Five experts – specialists in the field of environmental protection infrastructure – scored them<sup>8</sup> from 0 (the lowest evaluation) to 10 (the highest evaluation) taking into account 11 criteria<sup>9</sup> presented in Table 1.

Table 1

Preference model  
(with weighting coefficients established by means of REMBRANDT system)

No.	Criteria	Weights	Indifference thresholds	Preference thresholds	Veto thresholds	
					ELECTRE	BIPOLAR
1	2	3	4	5	6	7
1	Total cost	0,12	2	3	7	3
2	Efficiency	0,19	1	3	6	3
3	Influence on the environment	0,15	2	4	7	3
4	Influence on the employment	0,05	3	4	9	2
5	Influence on the inhabitants' health	0,14	3	5	8	2
6	Influence on the investment attractiveness	0,07	2	4	8	2

<sup>7</sup> They are denoted by letters from A to T.

<sup>8</sup> In order to keep the classified data confidential while enabling an objective evaluation, the descriptions of the projects were truncated and standardised.

<sup>9</sup> All criteria are treated as quality criteria, even if it is possible to use them as quantitative criteria as in the case of total cost or efficiency. This is due to the specificity of the applications, in which the influence of the projects was often described in very complex and diverse manner and by means of incomparable data. The reason for treating efficiency as a quality criterion is that in the programming period 2004-2006 the guidelines for the preparation of the documents by applicants were not very precise and allowed them to provide a free and sometimes even creative financial analysis and benefit cost analysis (BCA). In many cases an inappropriate financial analysis methodology was applied and in economic analyses not all transfers, corrections and benefits were taken into consideration. Therefore, the appraisal of the projects was very often intuitive and based on the expertise and experience of specialists. In the case of total cost, in turn, their reliability and validity was to be assessed. An exception is made for EVAMIX method (with and without stochastic dominance rules), in which case the total cost is treated as a quantitative criterion.

Table 1 contd.

1	2	3	4	5	6	7
7	Influence on the tourist attractiveness	0,06	2	5	8	2
8	Validity of the technical solutions	0,08	1	3	7	2
9	Sustainability and institutional feasibility of the project	0,06	1	3	8	2
10	Complementarity with other projects	0,04	2	4	8	2
11	Comprehensiveness	0,04	2	4	8	2

The above-mentioned set of 11 criteria was constructed as follows: a list of the criteria (based on the data available in the considered applications considered for project co-financing and information contained within official documents related to the EU funds as well as on the criteria applied in the programming period 2004-2006 and the aims of regional development strategy) was presented to five specialists in the field of environmental protection infrastructure and European Union funds who could accept or reject each of them. They had also a possibility to add their own criteria to the preliminary list.

To obtain the essential data to use AHP method and REMBRANDT system, each of the five aforementioned experts in the scope of environmental protection infrastructure and the EU funds was asked to compare criteria pair-wise using the 1-to-9 Saaty's scale [Saaty 2006, p. 73]. As a result two different vectors of weighting coefficients have been produced. The third one was formed as a result of the application of the modified Simos' procedure. In this case the role of decision-maker was assumed by the author of the paper.

The experts were also asked to determine values of indifference, preference and veto thresholds within the meaning of ELECTRE method. Two extreme opinions were disregarded and with the remaining three the arithmetic mean was calculated. It was subsequently rounded to the nearest integer. Veto thresholds within the meaning of BIPOLAR method were established by the author of this paper.

Table 2 provides a summary of the results yielded by means of six multi-criteria techniques enumerated in the previous section of this paper. For comparison, the table includes also ranking of the projects obtained with the help of the arithmetic mean of the weighted sums of points assigned by experts, i.e. the method functioning so far in the system of evaluation and selection of the applications for project co-financing by the European Union funds.

Table 2

Rankings of the projects obtained using different MCDA methods

No.	ORESTE	EVAMIX	PROMETHEE II	EXPROM II	modified BIPOLAR method	EVAMIX with stochastic dominance rules	Arithmetic mean of the weighted sums of points	No.
1	C	C	P	P	C	C	C	1
2	P	D	C	C	D	D	D	2
3	D	M	D	D	M	G	P	3
4	M	G	G	M	G	M	M	4
5	R	P	M	K	R	P	G	5
6	G	H	K	G	T	T	T	6
7	T	T	T	R	E	R	R	7
8	H	R	R	H	N	H	H	8
9	K	K	H	N	P	E	K	9
10	E	E	N	T	H	L	E	10
11	L	L	B	B	F	K	N	11
12	N	N	E	E	K	N	B	12
13	B	B	F	S	B	F	L	13
14	S	F	S	F	S	B	F	14
15	F	S	A	A	L	S	S	15
16	A	A	L	L	A	A	A	16

The rankings presented in Table 2 show the sensitivity of the solutions to choice of the decision-aiding technique: depending on the method used to support the decision-making process and on the amount of available financial resources, different projects would receive subsidies.

The orders of the projects in the rankings are not in agreement. However, in spite of that it is possible to determine the set of projects which are the best (C, D, M and G) and the other one containing projects which are the worst (L, A, S, B and F). Project P may be regarded as controversial since on the one hand it is classified at the forefront of rankings in the case of PROMETHEE II and EXPROM II methods combined with stochastic dominance rules, but on the other hand, it is characterised by a very low appraisal of one of the criteria (namely influence of the project on the employment), which was clearly caught by the modified BIPOLAR method thanks to the veto procedure applied in this technique.



In this context it is worth mentioning that the ranking obtained with the help of arithmetic mean of the weighted sums of points granted by experts coincides fairly well with the results obtained using different multi-criteria decision aiding techniques. This is not surprising as high-quality projects should be classified at the top of the rankings and weak projects should be ranked low regardless of the method used. However, the assumptions of multi-criteria decision aiding methods based on the outranking relation are more congruent with reality than those of the method consisting in calculating weighted mean. Hence, they can definitely improve the procedure of appraising and selecting projects applying for co-financing from the European Union taking into account uncertainty and imprecision accompanying all the decision-making problems. Moreover, they can exclude – at least partly – the possibility of compensation a bad evaluation on one criterion by a good one on the other and limit – thanks to the earlier determination of the preference model – the risk of the manipulation of the outcomes. They prove correct especially in the case of projects with high appraisals with respect to some criteria and very low appraisals with respect to the others<sup>10</sup>.

#### **4. Sensitivity and robustness analysis of the solutions obtained**

In this part of the paper we will present a sensitivity and robustness analysis of solutions obtained by applying ORESTE, EVAMIX, PROMETHEE II, EXPROM II and the modified BIPOLAR method (see Table 2).

In the first step of the analysis the ranges of variations of indifference and preference thresholds, which do not result in modification of the rankings, were determined using optimization tools integrated with Excel. The analysis was carried out separately for each of the thresholds provided that they satisfy the condition  $q_k \leq p_k \leq v_k$  in the case of PROMETHEE II and EXPROM II methods with stochastic dominance rules and the condition  $q_k \leq p_k$  in the case of the modified BIPOLAR method. The results are displayed in Tables 3 and 4. They indicate that the results obtained for each MCDA technique considered are least sensitive to variations of the values of the thresholds for the criterion No. 5.

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<sup>10</sup> [See Górecka and Pietrzak 2012].

Table 3

Ranges of variations of the indifference thresholds values

No.	Criteria	q <sub>min</sub>			q original	q <sub>max</sub>		
		PROMETHEE II	EXPROM II	BIPOLAR		BIPOLAR	EXPROM II	PROMETHEE II
1	Total cost	1,974	1,596	1,788	2	2,225	2,143	2,132
2	Efficiency	0,946	0,945	0,077	1	1,272	1,002	1,067
3	Influence on the environment	1,501	1,641	1,715	2	2,214	2,201	2,106
4	Influence on the employment	2,843	2,998	3,000	3	3,655	3,306	3,079
5	Influence on the inhabitants' health	2,122	2,196	2,412	3	5,000	5,000	5,000
6	Influence on the investment attractiveness	1,432	1,998	1,539	2	2,180	2,246	2,081
7	Influence on the tourist attractiveness	0,702	1,998	1,420	2	2,159	2,907	2,080
8	Validity of the technical solutions	0,930	0,995	0,000	1	1,379	1,115	1,330
9	Sustainability and institutional feasibility of the project	0,954	0,979	0,372	1	2,529	1,300	3,000
10	Complementarity with other projects	1,702	0,877	1,415	2	2,836	2,002	2,248
11	Comprehensiveness	1,812	1,996	1,509	2	3,733	3,383	2,735

Table 4

Ranges of variations of the preference thresholds values

No.	Criteria	P min			p original	P max		
		PROMETHEE II	EXPROM II	BIPOLAR		BIPOLAR	EXPROM II	PROMETHEE II
1	Total cost	2,954	2,998	2,606	3	3,411	3,159	3,154
2	Efficiency	2,796	2,774	2,527	3	3,192	3,003	3,040
3	Influence on the environment	2,200	3,002	3,334	4	4,647	5,612	6,036
4	Influence on the employment	3,907	4,000	3,345	4	4,428	4,200	4,064
5	Influence on the inhabitants' health	3,000	3,000	3,000	5	10,000	8,000	8,000
6	Influence on the investment attractiveness	3,163	3,995	2,500	4	4,450	5,211	4,169
7	Influence on the tourist attractiveness	3,418	4,995	3,464	5	5,380	8,000	5,116
8	Validity of the technical solutions	2,358	2,986	2,298	3	4,720	3,257	3,926
9	Sustainability and institutional feasibility of the project	2,806	2,625	2,177	3	5,732	4,403	8,000
10	Complementarity with other projects	3,941	3,416	3,312	4	4,901	4,001	4,246
11	Comprehensiveness	3,618	3,990	3,492	4	7,647	6,423	8,000

The analysis of robustness of the solutions to the changes of the weighting coefficients of evaluation criteria has been performed using the approach proposed by Hyde, Maier and Colby [Hyde et al. 2005, p. 278-290]. Its essence consists in determining for each pair of variants  $(a_i, a_j)$  the minimum admissible modification of criteria weights that is required to alter the total values of two selected variants such that rank equivalence occurs. This smallest change in the values of the criteria weights is obtained by solving an optimisation problem, in which the objective function is formulated as follows:

$$\min d_E = \sqrt{\sum_{k=1}^n (w_k - w'_k)^2}, \quad k = 1, \dots, n.$$

The aim is therefore to minimise a distance metric that provides the numerical measure to the amount of dissimilarity between the initial weights of the criteria  $w_k$  and the optimised criteria weights  $w'_k$ . The Euclidean distance has been selected as the most commonly used.

A set of constraints takes the following form:

$$\sum_{k=1}^n w_k = \sum_{k=1}^n w'_k = 1, \quad k = 1, \dots, n,$$

$$w_k, w'_k > 0,$$

$$w_k^d \leq w'_k \leq w_k^g, \quad k = 1, \dots, n,$$

where  $w_k^d$  and  $w_k^g$  are the lower and upper limits, respectively, of the values of the weighting coefficients assigned to each of the evaluation criteria  $f_k$ .

Applying the optimised criteria weights should cause the total values of two variants being assessed to be equal, thus we have in addition:

$$\phi'(a_i) = \phi'(a_j).$$

As a result of solving the non-linear programming task presented above the values of the minimum Euclidean distance for all pairs of variants are obtained. They can be presented in the form of a matrix.

In some situations one of the variants is always classified higher in ranking than the other, regardless of the values of the modified parameters. In this case, the ordering of these two variants – because of the insensitivity to variation of parameters – is called robust. Much more often, however, we have to deal with the situation, in which there are at least a few different combinations of the weighting coefficients for which  $\phi'(a_i) = \phi'(a_j)$ . Setting the smallest overall modification of the criteria weights allowing two variants to achieve the same position in ranking, enables determining whether their ordering is robust or not. Large values of the minimum Euclidean distance mean that one of the variants is generally better than the other, regardless of the values of the parameters changing within the range given by the decision-maker. If, on the other hand, the minimum Euclidean distances are small, minor changes in the values of the parameters will cause rank equivalence of variants being considered, thus their ordering may be concluded to be sensitive to the criteria weights [Hyde et al. 2005, p. 281-282].

In the analysis performed the eight highest-ranked European projects in the orderings obtained with the help of EVAMIX method without stochastic dominance, PROMETHEE II technique with stochastic dominance rules and the modified BIPOLAR method have been taken into account. It has been assumed that the values of the weighting coefficients for evaluation criteria are within the following limits:

Table 5

The permissible range of variability in the values  
of the weighting coefficients for project evaluation criteria

No.	Criteria	Coefficients of importance		
		$W_{\min}$	$W_{\text{original}}$	$W_{\max}$
1	Total cost	0,05	0,12	0,20
2	Efficiency	0,10	0,19	0,20
3	Influence on the environment	0,10	0,15	0,20
4	Influence on the employment	0,03	0,05	0,10
5	Influence on the inhabitants' health	0,10	0,14	0,20
6	Influence on the investment attractiveness	0,03	0,07	0,10
7	Influence on the tourist attractiveness	0,03	0,06	0,10
8	Validity of the technical solutions	0,03	0,08	0,10
9	Sustainability and institutional feasibility of the project	0,03	0,06	0,10
10	Complementarity with other projects	0,03	0,04	0,10
11	Comprehensiveness	0,03	0,04	0,10

The values of the minimum Euclidean distance  $d_E$  for pairs of considered projects contained in Tables 6, 7 and 8 signify that the final rankings are not robust to changes in the criteria weights – in some cases only small modifications of the starting values are required for rank equivalence between the two examined variants.

The results of using the distance-based analysis approach for 28 pairs of projects also indicate that although the obtained solutions are sensitive to variations of input parameter values, the orderings of some projects are robust. For the acceptable ranges of weighting coefficients given in Table 5 in the case of:

- EVAMIX method without stochastic dominance projects C and D (not necessarily in that order) will always be superior to projects H, T and R (no feasible changes in criteria weights could be found); furthermore project C will be also superior to projects M, G and P;



Table 8

A minimum Euclidean distance matrix for pairs of European projects consisting of the 8 highest-ranked variants using the modified BIPOLAR method

Projects	C	D	M	G	R	T	E	N
C		0,0201	0,0437	–	0,1381	–	–	–
D			0,1353	0,0506	0,1242	–	–	–
M				0,0185	0,1060	–	–	–
G					0,0484	0,0901	0,0704	0,0721
R						0,0088	0,0104	0,0460
T							0,0048	0,0135
E								0,0112
N								

In order to show the impact of changes in the weights of evaluation criteria on the final rankings of projects obtained using ORESTE method, EVAMIX method with stochastic dominance rules and EXPROM II method with stochastic dominance rules, calculations with the aid of these techniques have been made again but for preference models in which the criteria weights obtained by means of REMBRANDT system (model I) have been replaced by the weights obtained by applying the Analytic Hierarchy Process (model II) and the weights obtained with the help of the revised Simos' procedure (model III). In both cases the modification of the vector of weights led to alterations in rankings, which indicates that the original solutions are not robust with respect to the variations of parameters. The values of parameters are determined on the basis of data provided by the participants of the decision-making process, thus the information and its skilful use is extremely important in the process of evaluation and selection of the European projects.

Table 9

Positions obtained by projects as a result of the application of ORESTE, EVAMIX and EXPROM II methods with different preference models

Projects	Variants of the preference model								
	I (with the weights obtained by means of the REMBRANDT system)			II (with the weights obtained by means of the AHP method)			III (with the weights obtained by means of the revised Simos' method)		
	ORESTE	EVAMIX	EXPROM	ORESTE	EVAMIX	EXPROM	ORESTE	EVAMIX	EXPROM
<b>A</b>	16	16	15	16	16	15	16	16	15
<b>B</b>	13	14	11	13	14	12	12	14	9
<b>C</b>	1	1	2	1	1	2	1	1	1
<b>D</b>	3	2	3	3	2	3	2	2	4
<b>E</b>	10	9	12	10,5	9	11	11	11	12
<b>F</b>	15	13	14	14	13	13	15	13	16
<b>G</b>	6	3	6	6	3	5	6	3	6
<b>H</b>	8	8	8	8	8	9	8	7	7
<b>K</b>	9	11	5	9	11	6	9	10	3
<b>L</b>	11	10	16	10,5	10	16	10	9	14
<b>M</b>	4	4	4	4	4	4	4	4	5
<b>N</b>	12	12	9	12	12	10	13	12	11
<b>P</b>	2	5	1	2	6	1	3	8	2
<b>R</b>	5	7	7	5	7	8	5	6	8
<b>S</b>	14	15	13	15	15	14	14	15	13
<b>T</b>	7	6	10	7	5	7	7	5	10

Table 9 contains ranks attributed to the 16 analysed European projects as a result of the utilisation of three different MCDA methods with three different preference models. It should be noted that rankings obtained during the analysis are similar. This observation can be confirmed by the Spearman rank correlation coefficients presented in Table 10. These coefficients, calculated separately for each of three considered MCDA techniques, indicate the existence of strong correlation dependencies between the obtained orderings of projects. However, the order of the projects in the rankings is not the same, and – depending on the method of determining the criteria weights and the available allocation of financial resources – different projects would be co-financed.



Table 10

Spearman rank correlation coefficients

ORESTE			
Method	REMBRANDT	AHP	Simos'
REMBRANDT	1,000	0,996	0,991
AHP	0,996	1,000	0,990
Simos'	0,991	0,990	1,000
EVAMIX			
Method	REMBRANDT	AHP	Simos'
REMBRANDT	1,000	0,997	0,974
AHP	0,997	1,000	0,982
Simos'	0,974	0,982	1,000
EXPROM			
Method	REMBRANDT	AHP	Simos'
REMBRANDT	1,000	0,974	0,962
AHP	0,974	1,000	0,924
Simos'	0,962	0,924	1,000

## Conclusions

The results of the case study as well as the sensitivity and robustness analysis undertaken in the framework of it have clearly illustrated that the output of MCDA methods depends significantly on the data input. Therefore, for the proper choice of projects that are going to be co-financed it is extremely important to determine the values of the parameters of the preference model consciously and precisely. It is an essential condition for the effective and efficient utilisation of the European Union funds.

A key decision parameter in the models used in the paper, on which the preference structure is based, is the vector of criteria weights. Conducted research has shown that the solutions obtained with the help of different multi-criteria decision aiding techniques are not robust to the modifications of this parameter – it turned out that changes in weighting coefficients affect the rankings of examined projects. Thus, on the one hand identification of the most critical (most sensitive to the variations of the values) criteria

weights is extremely beneficial, and on the other – the assignment of importance weightings to each criterion is a crucial step within the methods considered. As there are many different techniques of criterion weighting, the choice of one of them may be directed by the simplicity of its application, explanation and interpretation.

## Appendix A

### APPLICATION OF THE EXPROM METHOD WITH STOCHASTIC DOMINANCE RULES TO THE EUROPEAN PROJECTS' SELECTION

EXPROM is a modification and extension of PROMETHEE method<sup>11</sup> that was proposed in Diakoulaki and Koumoutsos [1991]. It is based on the notion of ideal and anti-ideal solutions and enables the decision-maker to rank variants on a cardinal scale. Assuming that all criteria are to be maximized, the ideal and anti-ideal solutions' values are defined as follows:

- ideal variant:  $f_k(a^*) = \max_{a_i \in A} f_k(a_i)$ ,
- anti-ideal variant:  $f_k(a_*) = \min_{a_i \in A} f_k(a_i)$ <sup>12</sup>,

where  $A = \{a_1, a_2, \dots, a_m\}$  is finite set of  $m$  variants and  $F = \{f_1, f_2, \dots, f_n\}$  is set of  $n$  criteria examined.

After introducing stochastic dominance rules to EXPROM method the procedure of ordering projects consists of the following steps<sup>13</sup>:

1. Identifying stochastic dominances for all pairs of projects with respect to all criteria<sup>14</sup>. Because all criteria are measured on ordinal scale the ordinal stochastic dominance approach proposed in Spector et al. [1996] is applied:

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<sup>11</sup> The idea of PROMETHEE methodology is presented in Brans and Vincke [1985] and a description of PROMETHEE techniques can be found in Brans et al. [1986].

<sup>12</sup> The values can be also defined independently from the examined variants, representing – in the case of an ideal solution – some realistic goals and in the case of an anti-ideal solution – a situation that should be avoided.

<sup>13</sup> The PROMETHEE method with stochastic dominance rules was proposed by Nowak. A detailed description of this method is presented in Nowak [2005].

<sup>14</sup> According to the results of experiments presented in Kahneman and Tversky [1979] it is assumed that the decision-maker(s) is (are) risk-averse.

**Definition 1:** Ordinal First-Degree Stochastic Dominance (OFSD):

$$X_k^i \text{ OFSD } X_k^j \text{ if and only if } \sum_{l=1}^s p_{kl}^i \leq \sum_{l=1}^s p_{kl}^j \text{ for all } s = 1, \dots, z,$$

where:

$X_k^i$  – distribution of the evaluations of project  $a_i$  with respect to criterion  $f_k$ ,  
 $p_{kl}$  – probability of obtaining given evaluation by the project in case of criterion  $f_k$ .

**Definition 2:** Ordinal Second-Degree Stochastic Dominance (OSSD):

$$X_k^i \text{ OSSD } X_k^j \text{ if and only if } \sum_{r=1}^s \sum_{l=1}^r p_{kl}^i \leq \sum_{r=1}^s \sum_{l=1}^r p_{kl}^j \text{ for all } s = 1, \dots, z.$$

For modelling preferences the ordinal almost stochastic dominances are also used<sup>15</sup>:

**Definition 3:** Ordinal Almost First-Degree Stochastic Dominance (OAFSD):

$$X_k^i \varepsilon_1^* \text{ - OAFSD } X_k^j, \text{ if for } 0 < \varepsilon_1^* < 0,5$$

$$\sum \left( \sum_{l=1}^{s_1} p_{kl}^i - \sum_{l=1}^{s_1} p_{kl}^j \right) \leq \varepsilon_1^* \|X_k^i - X_k^j\| \text{ for all } s_1 = 1, \dots, z,$$

$$\text{where: } s_1 = \left\{ s : \sum_{l=1}^s p_{kl}^j < \sum_{l=1}^s p_{kl}^i \right\}, \|X_k^i - X_k^j\| = \sum \left( \left| \sum_{l=1}^s p_{kl}^i - \sum_{l=1}^s p_{kl}^j \right| \right),$$

$\varepsilon_1^*$  – allowed degree of OFSD rule violation, which reflects the decision-maker's preferences;  $\varepsilon_1^* \geq \varepsilon_1$ , where  $\varepsilon_1$  – the actual degree of OFSD rule violation.

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<sup>15</sup> Almost stochastic dominances were proposed in Leshno and Levy [2002].

**Definition 4:** Ordinal Almost Second-Degree Stochastic Dominance (OASSD):

$X_k^i \varepsilon_2^* - OASSD X_k^j$ , if for  $0 < \varepsilon_2^* < 0,5$

$$\sum \left( \sum_{l=1}^{s_2} p_{kl}^i - \sum_{l=1}^{s_2} p_{kl}^j \right) \leq \varepsilon_2^* \|X_k^i - X_k^j\| \quad \text{for all } s_2 = 1, \dots, z \quad \text{and } \mu_k^i \geq \mu_k^j,$$

$$\text{where } s_2 = \left\{ s_1 : \sum_{r=1}^{s_1} \sum_{l=1}^r p_{kl}^j < \sum_{r=1}^{s_1} \sum_{l=1}^r p_{kl}^i \right\},$$

$$\|X_k^i - X_k^j\| = \sum \left( \sum_{l=1}^s p_{kl}^i - \sum_{l=1}^s p_{kl}^j \right),$$

$\mu_k^i$  and  $\mu_k^j$  – average performances (expected values of the evaluations' distributions) of the projects  $a_i$  and  $a_j$  on the criterion  $f_k$ ,

$\varepsilon_2^*$  – allowed degree of OSSD rule violation, which reflects the decision-maker's preferences;  $\varepsilon_2^* \geq \varepsilon_2$ , where  $\varepsilon_2$  – the actual degree of OSSD rule violation.

2. Calculation of concordance indices for each pair of projects  $(a_i, a_j)$ :

$$c(a_i, a_j) = \sum_{k=1}^n w_k \varphi_k(a_i, a_j)$$

where:

$$\sum_{k=1}^n w_k = 1$$

$$\varphi_k(a_i, a_j) =$$

$$= \begin{cases} 1 & \text{if } X_k^i SD X_k^j \quad \text{and} \quad \mu_k^i > \mu_k^j + p_k[\mu_k^j], \\ \frac{\mu_k^i - q_k[\mu_k^j] - \mu_k^j}{p_k[\mu_k^i] - q_k[\mu_k^j]} & \text{if } X_k^i SD X_k^j \quad \text{and} \quad \mu_k^j + q_k[\mu_k^i] < \mu_k^i \leq \mu_k^j + p_k[\mu_k^i], \\ 0 & \text{otherwise,} \end{cases}$$

$w_k$  – coefficient of importance for criterion  $f_k$ ,

$q_k[\mu_k^i], p_k[\mu_k^i]$  – indifference and preference threshold for criterion  $f_k$  respectively.

3. Calculation of discordance indices for each pair of projects and for each criterion:

$$d_k(a_i, a_j) = \begin{cases} 1 & \text{if } X_k^j SD X_k^i \text{ and } \mu_k^j > \mu_k^i + v_k[\mu_k^i], \\ \frac{\mu_k^j - p_k[\mu_k^i] - \mu_k^i}{v_k[\mu_k^i] - p_k[\mu_k^i]} & \text{if } X_k^j SD X_k^i \text{ and } \mu_k^i + p_k[\mu_k^i] < \mu_k^j \leq \mu_k^i + v_k[\mu_k^i], \\ 0 & \text{otherwise,} \end{cases}$$

where  $v_k[\mu_k^i]$  – veto threshold for criterion  $f_k$ .

4. Calculation of credibility indices for each pair of projects  $(a_i, a_j)$ :

$$\sigma(a_i, a_j) = c(a_i, a_j) \prod_{k \in D(a_i, a_j)} \frac{1 - d_k(a_i, a_j)}{1 - c(a_i, a_j)}$$

where:  $D(a_i, a_j) = \{k : d_k(a_i, a_j) > c(a_i, a_j)\}$ .

5. Determination of strict preference indices for each pair of projects  $(a_i, a_j)$ :

$$\pi(a_i, a_j) = v(a_i, a_j) \cdot \sum_{k=1}^n w_k \pi_k(a_i, a_j),$$

where:

$$v(a_i, a_j) = \begin{cases} 1, & \text{if } \forall k : d_k(a_i, a_j) \leq c(a_i, a_j), \\ 0, & \text{if } \exists k : d_k(a_i, a_j) > c(a_i, a_j), \end{cases}$$

$$\pi_k(a_i, a_j) = \begin{cases} \frac{(\mu_k^i - \mu_k^j) - p_k[\mu_k^i]}{(\mu_k^* - \mu_{k*}) - p_k[\mu_k^i]} & \text{if } \varphi_k(a_i, a_j) = 1, \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_k^* = \max_{a_i \in A} \mu_k^i \quad \text{and} \quad \mu_{k*} = \min_{a_i \in A} \mu_k^i.$$

The aim of the strict preference function  $\pi_k(a_i, a_j)$  is to differentiate the state of the strict preference found to be valid for more than one pair of projects at a given criterion  $f_k$ . Their values belong to the interval  $[0, 1]$  and  $\pi_k(a_i, a_j) = 0$  denotes weak preference or indifference between two projects.

6. Calculation of total preference index for each pair of projects  $(a_i, a_j)$  :

$$\omega(a_i, a_j) = \min\{1; \sigma(a_i, a_j) + \pi(a_i, a_j)\}.$$

The total preference index gives an accurate measure of the intensity of preference of project  $a_i$  over  $a_j$  for all the criteria. It combines two aspects: a subjective one, expressed by the credibility index and referring only to the relation between two examined projects, and an objective one, expressed by the strict preference index and representing the relation between two considered projects with regard to other projects examined.

7. Calculation of outgoing flow  $\phi^+(a_i)$  and incoming flow  $\phi^-(a_i)$  for each project:

$$\phi^+(a_i) = \frac{1}{m-1} \sum_{j=1}^m \omega(a_i, a_j)$$

$$\phi^-(a_i) = \frac{1}{m-1} \sum_{j=1}^m \omega(a_j, a_i)$$

In EXPROM I a final partial ranking is obtained as follows:

$$\left\{ \begin{array}{l} a_i Pa_j, \quad \text{if } \left\{ \begin{array}{l} \phi^+(a_i) > \phi^+(a_j) \quad \text{and } \phi^-(a_i) < \phi^-(a_j) \quad \text{or} \\ \phi^+(a_i) = \phi^+(a_j) \quad \text{and } \phi^-(a_i) < \phi^-(a_j) \quad \text{or} \\ \phi^+(a_i) > \phi^+(a_j) \quad \text{and } \phi^-(a_i) = \phi^-(a_j); \end{array} \right. \\ a_i Ia_j, \quad \text{if } \phi^+(a_i) = \phi^+(a_j) \quad \text{and } \phi^-(a_i) = \phi^-(a_j); \\ a_i Ra_j, \quad \text{if } \left\{ \begin{array}{l} \phi^+(a_i) > \phi^+(a_j) \quad \text{and } \phi^-(a_i) > \phi^-(a_j) \quad \text{or} \\ \phi^+(a_i) < \phi^+(a_j) \quad \text{and } \phi^-(a_i) < \phi^-(a_j); \end{array} \right. \end{array} \right.$$

where  $P$ ,  $I$  and  $R$  stand for preference, indifference and incomparability, respectively.

In EXPROM II a final complete ranking is constructed according to the descending order of the net flows  $\phi(a_i)$ , where  $\phi(a_i) = \phi^+(a_i) - \phi^-(a_i)$ .

## Appendix B

### APPLICATION OF THE EVAMIX METHOD WITH STOCHASTIC DOMINANCE RULES TO THE EUROPEAN PROJECTS' SELECTION

In EVAMIX method, proposed by H. Voogd, the qualitative and quantitative data are distinguished and the final appraisal score of a given variant is the result of a combination of the evaluations calculated separately for the qualitative and quantitative criteria.

After introducing stochastic dominance rules to EVAMIX method the procedure of ordering projects consists of the following steps:

1. Determination of the qualitative dominance measures for the ordinal criteria:

$$\alpha_{ij} = \left[ \sum_{k \in O} \{w_k \operatorname{sgn}(\mu_k(a_i) - \mu_k(a_j))\}^c \right]^{\frac{1}{c}}, \quad c = 1, 3, 5, \dots,$$

where:

- $c$  – an arbitrary scaling parameter, for which any positive odd value may be chosen; the higher the value of the parameter, the weaker the influence of the deviations between the evaluations for the less important criteria;
- $O$  – a set of qualitative (ordinal) criteria<sup>16</sup>;

$$\operatorname{sgn}(\mu_k(a_i) - \mu_k(a_j)) = \begin{cases} 1 & \text{if } X_k^i \text{ SD } X_k^j \text{ and } \mu_k(a_i) > \mu_k(a_j), \\ -1 & \text{if } X_k^j \text{ SD } X_k^i \text{ and } \mu_k(a_j) > \mu_k(a_i), \\ 0 & \text{otherwise,} \end{cases}$$

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<sup>16</sup> It is assumed that all the criteria are maximized.

$X_k^i$  – distribution of the evaluations of project  $a_i$  with respect to criterion  $f_k$ ,  
 $\mu_k(a_i)$  – average performance (expected value of the distribution of evaluations) of the project  $a_i$  on the criterion  $f_k$ ,  
 and  $SD$  denotes stochastic dominance relation (OFSD, OSSD, OAFSD, OASSD).

2. Calculation of the quantitative dominance measures for the cardinal criteria:

$$\gamma_{ij} = \left[ \sum_{k \in Q} \{w_k (\mu_k(a_i) - \mu_k(a_j))\}^c \right]^{\frac{1}{c}}, \quad c = 1, 3, 5, \dots,$$

if  $(F_{ik} \text{ SD } F_{jk} \text{ and } \mu_k(a_i) > \mu_k(a_j))$   
 or  $(F_{jk} \text{ SD } F_{ik} \text{ and } \mu_k(a_j) > \mu_k(a_i))$ ; otherwise  $\gamma_{ij} = 0$ ,

where:

$Q$  – a set of quantitative (cardinal) criteria<sup>17</sup>,  
 $F_{ik}$  – distribution function representing evaluations of project  $a_i$  with respect to criterion  $f_k$ .

3. Standardization of the dominance measures as follows:

$$\delta_{ij} = \alpha_{ij} \left( \sum_{i=1}^m \sum_{j=1}^m |\alpha_{ij}| \right)^{-1},$$

$$\sigma_{ij} = \gamma_{ij} \left( \sum_{i=1}^m \sum_{j=1}^m |\gamma_{ij}| \right)^{-1}.$$

4. Calculation of the overall dominance measure  $q_{ij}$  for each pair of projects:

$$q_{ij} = w_O \delta_{ij} + w_Q \sigma_{ij},$$

where:

$w_O$  – the sum of weights of quantitative criteria,  
 $w_Q$  – the sum of weights of qualitative criteria.

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<sup>17</sup> It is assumed that all the criteria are maximized.



5. Determination of the final appraisal score  $u_i$  for each project as follows:

$$u_i = \frac{1}{m} \sum_{j=1}^m q_{ij} .$$

6. Ranking projects according to the descending order of the final appraisal scores.

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