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ESTIMATING PRIORITIES IN GROUP AHP USING INTERVAL COMPARISON MATRICES

Abstract

In this paper analytic hierarchy process (AHP), a well-known approach for handling multi-criteria decision making problems, is discussed. It is based on pairwise comparisons. The methods for deriving the priority vectors from comparison matrices are examined. The existing methods for aggregating the individual comparison matrices into a group comparison matrix are revised. A method for aggregation, called WGMDEA, is proposed for application in the case study.

Because exact (crisp) values cannot always express the subjectivity and the lack of information on the part of a decision maker, the interval judgments are more suitable in such cases. Two main methodological problems emerge when dealing with interval comparison matrices in group AHP:

- a) to aggregate individual crisp preferences into the joint interval matrix,
- b) to calculate the weights from the joint interval comparison matrix.

In the paper we first discuss the already proposed approaches to the aggregation of individual matrices, and the derivation of weights from interval comparison matrices, pertained to AHP group decision making methodology. Then, a new method, ADEXTREME, for generating the interval group judgments from individual judgments is proposed. A numerical example based on Rural Development Program of the Republic of Slovenia in 2007-2013 is presented to illustrate the new methodology for deriving the weights from interval comparison matrices. The results obtained by WGMDEA, MEDINT and ADEXTREME methods are compared.

Keywords: multiple criteria decision making, group decision making, analytic hierarchy process, aggregating individual comparison matrices, interval judgments, deriving the weights from interval comparison matrices, management of natural resources.

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1 Introduction

The analytic hierarchy process (AHP) (Saaty, 1980) is a well-known method for solving multiple criteria problems; it has already been applied to several problems from different domains (Vaidya and Kumar, 2006). AHP is very flexible since it allows for combining empirical data and subjective judgments, and also intangible and non-measurable criteria. Thus, AHP is suitable for evaluating and ranking alternatives, and supporting the selection of the best alternative. It is based on a hierarchical structure of criteria, subcriteria, and alternatives. The pairwise comparisons of objects (criteria, subcriteria, alternatives) on the same level with respect to the object on the next higher level are gathered in a comparison matrix. The 1-9 ratio scale is used to express the strength of preference between the compared objects. The priority vectors are derived from a pairwise comparison matrix by one of the known methods and synthesized in the final priority vector (Saaty, 2006).

As a number of stakeholders (decision makers) who have different goals, or common interests are getting gradually more important when solving the multi-criteria problems, group (participatory) decision making methods replace the single decision maker multi-criteria methods. Among multi-criteria group methods, group AHP gained wide acceptance (Peniwati, 2007). The main problem in group AHP is to aggregate the individual judgments, i.e. the individual comparison matrices, into a group matrix, or individual priority vectors into a group priority vector. Although the group AHP has already been extensively used in practice, the problem of choosing an appropriate aggregation method to aggregate individual judgments (priorities) is still not fully solved. In the literature there are many approaches to aggregation (Bryson and Joseph, 1999; Cho and Cho, 2008; Escobar and Moreno-Jimenez, 2007; Forman and Peniwati, 1998; Grošelj et al., 2011; Hosseinian et al., 2009; Huang et al., 2009; Mikhailov, 2004; Moreno-Jimenez et al., 2008; Ramanathan and Ganesh, 1994; Regan et al., 2006; Sun and Greenberg, 2006), but it is not clear how good they are. In the literature, we seldom find studies comparing several group aggregation methods.

The complexity and uncertainty of the decision problems, the subjectivity, and the lack of information on the part of decision makers can sometimes be hard to express with exact values. Interval judgments can be more suitable in such cases. Another necessity for interval judgments occurs when the group is not satisfied with the aggregated judgment and expresses it with an interval (Arbel and Vargas, 2007; Chandran et al., 2005). If every decision maker in the group provides an interval comparison matrix, their aggregation is even more complicated than in the case of crisp matrices (Entani and Inuiguchi, 2010; Yang et al., 2010). Thus, two main methodological issues emerge when dealing with interval comparison matrices in group AHP:

- a) aggregation of individual crisp judgments into the group interval matrix,

b) determination the weights from the interval comparison matrix, what was already studied in many ways (Arbel and Vargas, 2007; Arbel and Vargas, 1993; Conde and de la Paz Rivera Pérez, 2010; Cox, 2007; Lan et al., 2009; Wang et al., 2005a; Wang et al., 2005b).

In the paper we first discuss the already proposed approaches to the aggregation of individual crisp judgments or priorities with respect to AHP group decision making methodology. Aggregation into the group interval matrix is also discussed. Then, a new way of aggregation of individual crisp matrices into the group interval matrix is proposed. A new approach using a modified minimum and maximum method is suggested. Further, the method of generating and ranking weights from the interval comparison matrix is discussed. Finally, a numerical example based on Rural Development Program of the Republic of Slovenia in 2007-2013 is presented (PRP, 2007) to illustrate the new methodology for deriving the weights from interval comparison matrices.

2 AHP and group AHP

2.1 Pairwise comparison matrix and priority vector in AHP for one decision maker

In AHP the decision maker pairwise compares all elements on the same level of the hierarchy (alternatives, subcriteria, criteria) with respect to the element to which they are connected on the next higher level. For comparing two elements a 1-9 Saaty's scale is used (Saaty, 2006). All comparisons are gathered in a pairwise comparison matrix A . The matrix $A = (a_{ij})_{n \times n}$ is reciprocal:

$$a_{ij} = \frac{1}{a_{ji}} \quad \text{for } i, j = 1, \dots, n \tag{1}$$

The matrix A is acceptably consistent if the consistency ratio $CR < 0.1$ (Saaty, 2006):

$$CR = \frac{CI}{RI_n}, \quad CI = \frac{\lambda_{max} - n}{n - 1} \tag{2}$$

Here n is the dimension of the matrix A resulting from the comparison of n elements and λ_{max} is the maximal eigenvalue of the matrix A . RI_n is the average consistency index (Saaty, 2006) which depends on the size of the matrix A .

In order to calculate the priority (preference) vector ω from the matrix A , several methods can be used (Bryson, 1995; Chandran et al., 2005; Chu et al., 1979; Cook and Kress, 1988; Crawford and Williams, 1985; Gass and Rapcsák, 2004; Mikhailov, 2000; Saaty, 2006; Wang and Chin, 2009; Zahedi, 1986; Zu, 2000). In applications, the eigenvector method (EV) (Saaty, 2006):

$$A\omega = \lambda_{max} \omega \tag{3}$$

or the logarithmic least-squares method (LLSM) (Crawford and Williams, 1985):

$$\min \sum_{i=1}^n \sum_{j>i}^n \left(\ln a_{ij} - (\ln \omega_i - \ln \omega_j) \right)^2 \quad (4)$$

the solution of which is the geometric mean of the row elements of the matrix A :

$$\omega_i = \sqrt[n]{\prod_{j=1}^n a_{ij}}, \quad i = 1, \dots, n \quad (5)$$

are most often applied.

2.2 Aggregation of individual matrices in group AHP and WGMDEA method

The main problem in group AHP is the aggregation of a set of individual judgments or preferences into the group judgment or preference. This problem is formulated as follows: let m be the number of decision makers and let n be the number of elements (criteria or alternatives) compared with respect to the element on the next higher level; further let $A_k = (a_{ij}^{(k)})_{n \times n}$, $k=1, \dots, m$ be their pairwise comparison matrices, and let $w^{(k)} = (w_1^{(k)}, \dots, w_n^{(k)})$, $k=1, \dots, m$ be priority vectors derived from A_k ; let α_k , $k=1, \dots, m$, $\alpha_k > 0$, $\sum_{k=1}^m \alpha_k = 1$ be the

weights of decision makers' importance. A group matrix A^{group} is finally derived from A_k , $k=1, \dots, m$ with one of the group methods. A group priority vector $w = (w_1, \dots, w_n)$ is then derived from A^{group} or directly from A_k , $k=1, \dots, m$.

For aggregation of individual priorities, weighted arithmetic mean method (WAMM) is normally used (Ramanathan and Ganesh, 1994) but also weighted geometric mean can be used (Forman and Peniwati, 1998).

In WAMM the individual priority vectors are synthesized into the group priority vector $w = (w_1, \dots, w_n)$ using the weighted arithmetic mean:

$$w_i = \sum_{k=1}^m \alpha_k w_i^{(k)}, \quad i=1, \dots, n \quad (6)$$

The Lehrer-Wagner model, adopted for AHP (Regan et al., 2006), is likewise the WAMM used for the aggregation of individual priorities.

In aggregating individual judgments the weighted geometric mean method (WGMM) is the only method that satisfies several required axiomatic conditions, such as separability, unanimity homogeneity and power conditions (Saaty and Peniwati, 2008).

Using the WGMM the group matrix $A^{group} = A^{WGMM}$ is calculated as:

$$a_{ij}^{(WGMM)} = \prod_{k=1}^m (a_{ij}^{(k)})^{\alpha_k} \tag{7}$$

To derive the priority vector from A^{WGMM} EV (3) is usually used.

Some other group AHP methods are:

1. Weighted group least-squares method for deriving group priorities which minimizes the weighted Minkowski distance (Sun and Greenberg, 2006).
2. Group method with aggregation on preferential differences and rankings which considers the differences of preference among criteria (or alternatives) and the ranks of the criteria (or alternatives) for each decision maker (Huang et al., 2009).
3. A method of aggregation of individual preference structures (Escobar and Moreno-Jimenez, 2007).
4. The weighted geometric mean DEA method (WGMDEA) (Grošelj et al., 2011) is a group method, based on data envelopment analysis, which uses weighted geometric mean for aggregation of individual judgments and linear programming for deriving the group priority vector. The solution of the linear program (8) for all $w_i, i = 1, \dots, n$ gives the group priority vector.

$$\begin{aligned} \max w_0 &= \sum_{j=1}^n \left(\prod_{k=1}^m (a_{0j}^{(k)})^{\alpha_k} \right) x_j \\ \text{subject to} & \sum_{j=1}^n \left(\sum_{i=1}^n \prod_{k=1}^m (a_{ij}^{(k)})^{\alpha_k} \right) x_j = 1 \\ & \sum_{j=1}^n \left(\prod_{k=1}^m (a_{ij}^{(k)})^{\alpha_k} \right) x_j \geq nx_i, i=1, \dots, n \\ & x_j \geq 0, j=1, \dots, n \end{aligned} \tag{8}$$

In the case study we applied WGMDEA method, since it is easily solved and provides good results as compared to some other group AHP methods (Grošelj et al., 2011).

3 Interval judgments in group AHP

3.1 Generating group interval matrix from individual crisp comparison matrices

Here we present the problem of combining individual judgments into a group interval judgment. Let $A^{group} = \left([l_{ij}, u_{ij}] \right)_{n \times n}$ be a group interval matrix, composed of intervals with lower bounds l_{ij} and upper bounds u_{ij} , derived

from the individual crisp comparison matrices. Intervals can be constructed using minimum and maximum judgments for the bounds of the intervals (Chandran et al., 2005; Wang et al., 2005b). If there are many intermediate judgments they do not influence the bounds of the intervals. One possibility to overcome this drawback is the MEDINT method (Grošelj and Zadnik Stirn, 2011) which uses values that are lower than the median for constructing the lower bound of the interval and values that are greater than the median for constructing the upper bound of the interval.

3.1.1 Method of aggregation of individual judgments into a group interval judgment using MEDINT method

Let m be the number of decision makers included in the process of evaluating n criteria (or alternatives) in respect to the element on the next higher level. Let $A^{(k)} = (a_{ij}^{(k)})_{n \times n}$, $k=1, \dots, n$ be their comparison matrices.

An Ordered Weighted Geometric (OWG) operator (Chiclana et al., 2000) F of dimension m generates a weighting vector $W = (w_1, \dots, w_m)$ with the properties: $w_i \in [0,1]$, $\sum_{i=1}^m w_i = 1$, and such that: $F(a_1, \dots, a_m) = \prod_{i=1}^m c_i^{w_i}$,

where c_i is the i -th largest value from the set $\{a_1, \dots, a_m\}$.

The OWG operator preserves reciprocity. Different vectors W assign different weights to the values a_1, \dots, a_m .

Let all decision makers be equally important. Two vectors $W_L = (w_1^L, \dots, w_m^L)$ and $W_U = (w_1^U, \dots, w_m^U)$ for the lower and upper bounds of the intervals, respectively, are generated, depending on the odd or even number of individual judgments. If m is an odd number, then $\frac{m+1}{2}$ is the median of the numbers $1, 2, \dots, m$, $s_{\frac{m+1}{2}} = \frac{(m+1)(m+3)}{8}$ is the sum of the numbers from 1 to $\frac{m+1}{2}$ and:

$$W_L^{odd} = \left(0, \dots, 0, \frac{1}{s_{\frac{m-1}{2}}}, \frac{2}{s_{\frac{m+1}{2}}}, \dots, \frac{m-1}{s_{\frac{m-1}{2}}}, \frac{m+1}{s_{\frac{m+1}{2}}} \right); W_U^{odd} = \left(\frac{m+1}{s_{\frac{m+1}{2}}}, \frac{m-1}{s_{\frac{m-1}{2}}}, \dots, \frac{2}{s_{\frac{m+1}{2}}}, \frac{1}{s_{\frac{m-1}{2}}}, 0, \dots, 0 \right) \tag{9}$$

If m is an even number, then the median of the numbers $1, 2, \dots, m$ is not an integer and $s_{\frac{m}{2}} = \frac{m(m+2)}{8}$ is the sum of the numbers from 1 to $\frac{m}{2}$, which are smaller than the median. Then:

$$W_L^{even} = \left(0, \dots, 0, \frac{1}{s_{\frac{m}{2}}}, \frac{2}{s_{\frac{m}{2}}}, \dots, \frac{m-2}{s_{\frac{m}{2}}}, \frac{m}{s_{\frac{m}{2}}} \right); W_U^{even} = \left(\frac{m}{s_{\frac{m}{2}}}, \frac{m-2}{s_{\frac{m}{2}}}, \dots, \frac{2}{s_{\frac{m}{2}}}, \frac{1}{s_{\frac{m}{2}}}, 0, \dots, 0 \right) \tag{10}$$

The aggregated interval group matrix A^{group} is defined as:

$$A^{group} = \begin{bmatrix} 1 & \left[\prod_{k=1}^m (c_{12}^{(k)})^{w_k^L}, \prod_{k=1}^m (c_{12}^{(k)})^{w_k^U} \right] & \dots & \left[\prod_{k=1}^m (c_{1n}^{(k)})^{w_k^L}, \prod_{k=1}^m (c_{1n}^{(k)})^{w_k^U} \right] \\ \left[\prod_{k=1}^m (c_{21}^{(k)})^{w_k^L}, \prod_{k=1}^m (c_{21}^{(k)})^{w_k^U} \right] & 1 & \dots & \left[\prod_{k=1}^m (c_{2n}^{(k)})^{w_k^L}, \prod_{k=1}^m (c_{2n}^{(k)})^{w_k^U} \right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\prod_{k=1}^m (c_{n1}^{(k)})^{w_k^L}, \prod_{k=1}^m (c_{n1}^{(k)})^{w_k^U} \right] & \left[\prod_{k=1}^m (c_{n2}^{(k)})^{w_k^L}, \prod_{k=1}^m (c_{n2}^{(k)})^{w_k^U} \right] & \dots & 1 \end{bmatrix} \quad (11)$$

where $c_{ij}^{(k)}$ is the k -th largest value from the set $\{a_{ij}^1, \dots, a_{ij}^m\}$.

3.1.2 Method of aggregation of individual judgments into a group interval judgment using ADEXTREME method

We suggest a new approach, called adopted extreme values (ADEXTREME) method, for aggregating individual judgments into a group interval judgment, where all individual judgments have an impact on the bound of the group interval but not all have equal power. The highest power belongs to the minimum and the maximal values, respectively.

We assume that all decision makers are equally important. The smallest value has influence equal to one half and all the other values have influence equal to one half on the lower bound l_{ij} of the group interval. The highest value has influence equal to one half and all the other values have influence equal to one half on the upper bound u_{ij} of the group interval. Let t and T be indexes:

$$t, T \in \{1, \dots, m\} \text{ and } a_{ij}^{(t)} = \min_{k \in \{1, \dots, m\}} a_{ij}^{(k)} \text{ and } a_{ij}^{(T)} = \max_{k \in \{1, \dots, m\}} a_{ij}^{(k)}$$

Then:

$$l_{ij} = \left(a_{ij}^{(t)}\right)^{(1/2)} \prod_{\substack{k=1 \\ k \neq t}}^m \left(a_{ij}^{(k)}\right)^{1/(2m-2)} \text{ and } u_{ij} = \left(a_{ij}^{(T)}\right)^{(1/2)} \prod_{\substack{k=1 \\ k \neq T}}^m \left(a_{ij}^{(k)}\right)^{1/(2m-2)} \quad (12)$$

The following holds: $l_{ij} \leq u_{ij}$, $l_{ij} = 1/u_{ji}$ and $u_{ij} = 1/l_{ji}$ for all $i, j = 1, \dots, n$.

The new ADEXTREME method is easier for calculations than the MEDINT method and it enables all decision makers to influence the interval group judgment.

3.2 Calculating interval weights from interval comparison matrices

For deriving interval weights from the interval comparison matrix A^{group} (11) or (12) we propose the approach of splitting A^{group} into two crisp comparison matrices $A_L^{group} = (a_{ij}^L)$ and $A_U^{group} = (a_{ij}^U)$ (Liu, 2009), where:

$$a_{ij}^L = \begin{cases} l_{ij}, & i < j \\ 1, & i = j, \\ u_{ij}, & i > j \end{cases} \quad a_{ij}^U = \begin{cases} u_{ij}, & i < j \\ 1, & i = j, \\ l_{ij}, & i > j \end{cases} \quad (13)$$

Matrices A_L^{group} and A_U^{group} are reciprocal comparison matrices.

The weights can be obtained from A_L^{group} and A_U^{group} by the LLSM (4), (5).

The interval weights belonging to A^{group} are defined as:

$$\omega_i = [\omega_i^L, \omega_i^U] = [\min\{\omega_i^{A_L}, \omega_i^{A_U}\}, \max\{\omega_i^{A_L}, \omega_i^{A_U}\}] \quad (14)$$

For ranking interval weights the matrix of degrees of preference is used:

$$P = \begin{bmatrix} - & p_{12} & \cdots & p_{1n} \\ p_{21} & - & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & - \end{bmatrix} \quad (15)$$

In recent years the possibility-degree formula for p_{ij} has been used several times (Facchinetti et al., 1998; Wang et al., 2005b; Xu and Chen, 2008; Xu and Da, 2002):

$$p_{ij} = P(\omega_i > \omega_j) = \frac{\max\{0, \omega_i^U - \omega_j^L\} - \max\{0, \omega_i^L - \omega_j^U\}}{(\omega_i^U - \omega_i^L) + (\omega_j^U - \omega_j^L)} \quad , i, j = 1, \dots, n, i \neq j \quad (16)$$

The preference ranking order is obtained using row-column elimination method (Wang et al., 2005b).

4 A case study – evaluation of Natura 2000 development scenarios

Natura 2000 is a European network of ecologically significant natural areas. Natura 2000 sites are managed through sectorial management plans. The agricultural priorities are outlined in the Rural Development Programme of the Republic of Slovenia 2007–2013 (PRP, 2007) and the objectives are divided into four development scenarios (alternatives):

1. Alternative 1 – improving the competitiveness of the agricultural and forestry sector. The activities included in the first alternative should support modernization and innovations and raise the qualification and competitive position. They should contribute to improved employment possibilities, increased productivity, and added value in agriculture and forestry.
2. Alternative 2 – improving the environment and rural areas. The activities included in the second alternative should contribute to environmental and water resource protection, conservation of natural resources, and

implementation of nature friendly technologies in agriculture and forestry. They should provide sustainable development of rural areas and ensure a favorable biodiversity status and the preservation of habitats in the Natura 2000 sites.

3. Alternative 3 – quality of life in the rural areas and diversification of the rural economy. The activities included in the third alternative promote entrepreneurship and raise the quality of life in rural areas through enhanced employment opportunities, rural economic development, and natural and cultural heritage conservation.
4. Alternative 4 – presents the scenario of providing support for rural development through implementing local development strategies. The activities included in the fourth alternative should stimulate the cooperation and connection of local action groups.

To assure the best results for the Natura 2000 sites we should rank these four aims (alternatives) and seek a balance between them. The weighting depends on the approach to the objectives which differs among stakeholders (decision makers). With an objective of incorporating different perspectives, we identified four main stakeholders for the Natura 2000 sites: representatives of environment protection (NGOs), representatives of farmers (owners), the government, and representatives of research/education institutions. Pairwise comparisons of the four objectives are represented by the matrices A, B, C and D for: environmentalist (NGOs representative), farmer (owner), representative of the government, and representative of the research and educational institution, respectively.

$$A = \begin{bmatrix} 1 & \frac{1}{4} & \frac{1}{3} & 2 \\ 4 & 1 & 2 & 3 \\ 3 & \frac{1}{2} & 1 & 2 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & 1 & 1 \\ \frac{1}{3} & 1 & \frac{1}{3} & \frac{1}{2} \\ 1 & 3 & 1 & 2 \\ 1 & 2 & \frac{1}{2} & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & \frac{1}{2} & 3 & 8 \\ 2 & 1 & 4 & 6 \\ \frac{1}{3} & \frac{1}{4} & 1 & 3 \\ \frac{1}{8} & \frac{1}{6} & \frac{1}{3} & 1 \end{bmatrix};$$

$$D = \begin{bmatrix} 1 & 2 & 1 & 2 \\ \frac{1}{2} & 1 & \frac{1}{4} & 1 \\ 1 & 4 & 1 & 1 \\ \frac{1}{2} & 1 & 1 & 1 \end{bmatrix}$$

All four matrices are acceptably consistent with CR = 0.058 (0.017; 0.039 and 0.070), respectively, calculated by (2). The priorities of the four decision makers, obtained by the EV method (3) are presented in Table 1.

Table 1

The priorities and the ranks of the four alternatives for four decision makers

	A		B		C		D	
	priorities	ranks	priorities	ranks	priorities	ranks	priorities	ranks
alternative 1	0.140	3	0.302	2	0.337	2	0.320	2
alternative 2	0.465	1	0.110	4	0.483	1	0.140	4
alternative 3	0.280	2	0.358	1	0.127	3	0.339	1
alternative 4	0.116	4	0.230	3	0.053	4	0.201	3

The ranking differs between the decision makers. The environmentalist prefers alternative 2, i.e., nature protection. Owners favor rural economic development, which is reflected by alternatives 3 and 1. The government’s weights indicate that the government is focused on sharing funds for particular objectives regarding protection and favors the alternative 2, while the institutional (research, education) representative favors the quality of rural life, i.e., alternative 3.

We calculate the group priorities using methods WGMDEA, MEDINT and ADEXTREME method.

WGMDEA:

	WGMDEA	
	priorities	ranks
alternative 1, (ω_1)	0.294	1
alternative 2, (ω_2)	0.274	3
alternative 3, (ω_3)	0.287	2
alternative 4, (ω_4)	0.145	4

The geometric mean matrix is calculated by (7), while the priorities by (8):

$$A^{WGM} = \begin{bmatrix} 1 & 0.931 & 1 & 2.378 \\ 1.075 & 1 & 0.904 & 1.732 \\ 1 & 1.107 & 1 & 1.861 \\ 0.420 & 0.577 & 0.537 & 1 \end{bmatrix},$$

The ranks of the alternatives obtained by the WGMDEA method are: $\omega_1 > \omega_3 > \omega_2 > \omega_4$.

MEDINT:

The comparison matrices A , B , C and D are aggregated in the A^{group} according to (11). The associated lower and upper weighted vectors (10) are defined as $W_L = (0, 0, \frac{1}{3}, \frac{2}{3})$ and $W_U = (\frac{2}{3}, \frac{1}{3}, 0, 0)$. The intervals in the matrix A^{group} are:

$$A^{group} = \begin{bmatrix} 1 & [0.3150, 2.6207] & [0.4807, 2.0801] & [1.2599, 5.0397] \\ [0.3816, 3.1748] & 1 & [0.2752, 3.1748] & [0.6300, 4.7622] \\ [0.4807, 2.0801] & [0.3150, 3.6342] & 1 & [1.2599, 2.6207] \\ [0.1984, 0.7937] & [0.2100, 1.5874] & [0.3816, 0.7937] & 1 \end{bmatrix}$$

Then A^{group} is split into two crisp matrices A_L^{group} and A_U^{group} (13) and priorities are calculated by the logarithmic least-squares method (4), (5). The interval weights are then given by (14).

	MEDINT
	interval weights
alternative 1, (ω_1)	[0.154, 0.459]
alternative 2, (ω_2)	[0.201, 0.311]
alternative 3, (ω_3)	[0.159, 0.411]
alternative 4, (ω_4)	[0.071, 0.234]

The matrix of degrees of preference P (15), (16):

$$P = \begin{bmatrix} - & 0.623 & 0.539 & 0.830 \\ 0.377 & - & 0.420 & 0.881 \\ 0.461 & 0.580 & - & 0.820 \\ 0.170 & 0.119 & 0.180 & - \end{bmatrix}$$

and the final preference order of alternatives obtained by the MEDINT method: $w_1 \succ w_3 \succ w_2 \succ w_4$.
53.9% 58% 88.1%

ADEXTREME:

The comparison matrices A , B , C and D are aggregated into the A^{group} according to (12). The intervals in the matrix A^{group} are:

$$A^{group} = \begin{bmatrix} 1 & [0.6005, 1.3747] & [0.6934, 1.4422] & [1.7818, 3.5636] \\ [0.7274, 1.6654] & 1 & [0.5888, 1.4837] & [1.1447, 2.6207] \\ [0.6934, 1.4422] & [0.6740, 1.6984] & 1 & [1.5131, 2.1822] \\ [0.2806, 0.5612] & [0.3816, 0.8736] & [0.4582, 0.6609] & 1 \end{bmatrix}$$

Then, A^{group} is split into two crisp matrices A_L^{group} and A_U^{group} . Priorities are calculated by the logarithmic least-squares method (4), (5). The interval weights are then given by (14).

	ADEXTREME
	interval priorities
alternative 1, (ω_1)	[0.226, 0.370]
alternative 2, (ω_2),	[0.251, 0.295]
alternative 3, (ω_3)	[0.228, 0.338]
alternative 4, (ω_4)	[0.107, 0.184]

The matrix of degrees of preference P (15), (16):

$$P = \begin{bmatrix} - & 0.636 & 0.559 & 1 \\ 0.364 & - & 0.431 & 1 \\ 0.441 & 0.569 & - & 1 \\ 0.000 & 0.000 & 0.000 & - \end{bmatrix}$$

and the final preference order of alternatives obtained by the ADEXTREME method $w_1 \succ w_3 \succ w_2 \succ w_4$.

A comparison of WGMDEA, MEDINT and PEINT shows that all three methods, the crisp group method WGMDEA, and the interval methods MEDINT and ADEXTREME give the same ranking of the four alternatives discussed, i.e., $\omega_1 > \omega_3 > \omega_2 > \omega_4$. These results are given in Table 2 and in graphical form in Figure 1. We see that MEDMINT intervals are longer than PEINT, while weights are similar.

Table 2

WGMDEA, MEDINT and ADEXTREME method results for 4 alternatives and 4 decision makers

	WGMDEA		MEDINT		ADEXTREME	
	priorities	ranks	priorities (intervals)	ranks	priorities (intervals)	ranks
alternative 1, (ω_1)	0,294	1	[0.154, 0.459]	1	[0.226, 0.370]	1
alternative 2, (ω_2)	0,274	3	[0.201, 0.311]	3	[0.251, 0.295]	3
alternative 3, (ω_3)	0,287	2	[0.159, 0.411]	2	[0.228, 0.338]	2
alternative 4, (ω_4)	0,145	4	[0.071, 0.234]	4	[0.107, 0.184]	4

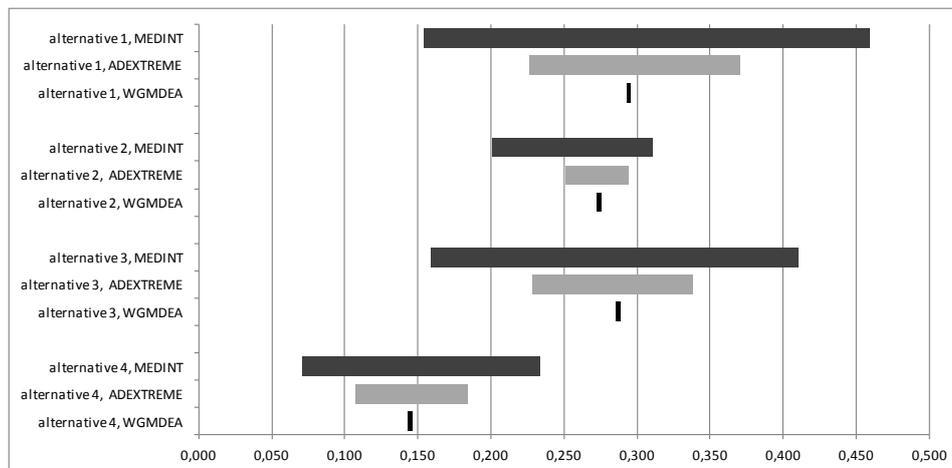


Figure 1. Graphical presentation of WGMDEA, MEDINT and ADEXTREME method results for 4 alternatives and 4 decision makers

5 Concluding remarks

Regarding the theoretical component of operations research, the paper addresses multiple criteria group methods employed in AHP. First, an overview of the methods for aggregating the individual comparison matrices into a group comparison matrix has been presented. A WGMDEA method, which preserves reciprocity, has been proposed for application in a case study. Its advantage is that it uses a linear program, while the most popular method, WGMM, uses an eigenvector procedure (3), which is not linear. Further, interval group matrices in group AHP aggregated from individual comparison matrices were introduced. The method MEDINT was presented and a new method ADEXTREME was suggested.

In the second part of the paper the an application of the proposed group AHP methodology was described. The main goal was the selection of the optimal strategy (alternative) for Natura 2000 site development with group AHP method and interval judgments. The results obtained by WGMDEA, MEDINT and ADEXTREME methods were compared. The results show that all stakeholders (decision makers) support modernization and innovations in agriculture and forestry which should contribute to improved employment opportunities, increased productivity, and added value in agriculture and forestry. The selected alternative can contribute to an enhanced management plan of the area. It can serve as a basis for the establishment of strategic and operational management goals of the area.

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