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REMARKS ON DESIGNING ITERATIVE MULTICRITERIA PROCUREMENT AUCTIONS

Abstract

In this paper some mechanisms of the multicriteria procurement auctions are discussed, including the elements of the decision support to the auction organizer, as well as to the bidders. The auction mechanisms are considered in the context of attaining incentive compatible decisions. Using domination relations formulated in the criteria space, different rules for the improvement of offers in successive rounds of the auction process are analyzed. The general discussion is illustrated by an example of an iterative multicriteria closed-bidding auction conducted with the use of a multi-agent computer-based system. The system supports submission of offers, multicriteria analysis performed by the auction organizer, simulation, and analysis of the competing bidders' behavior. Experimental results of sessions conducted with the use of the system are analyzed.

Keywords: Multicriteria auctions, incentive compatible decision mechanisms, multi-agent systems, multicriteria optimization.

1. Introduction

For auctions with scalar valuation of offers (valuated by price only), there exists a rich bibliography dealing with auction theory, including the papers by Klemperer (2004); Milgrom, Weber (1982); Vickrey (1961). In such auctions we have to select a single offer with the best price. In the case of the iterative multicriteria

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auctions, in each round we have to deal with a set of promising offers valuated by the auction organizer with the help of a vector of criteria. It is reasonable to support multicriteria analysis performed by the auction organizer and to construct an auction mechanism that could lead to the best final offer, according to his true preferences. In most papers dealing with multicriteria auctions, aggregation models are applied, by aggregating multiple criteria to a scalar value using a vector of weights (see De Smet, 2007; Teich et al., 2006; Bichler Kalagnanam, 2005). In this case, the auction organizer has to reveal his model of preferences. Interesting are papers using the reference point approach of multicriteria optimization (Ogryczak, Kozłowski, 2011; Bellosta et al., 2004). This paper belongs to the last class.

The research presented here is a part of a wider research trend dealing with analysis of incentive compatible multicriteria decision mechanisms. Within this research, decision situations are analyzed where there is a number of independent agents that have private information and act according to their own interests. Each agent tries to achieve his own multiple egoistic goals, but the results depend on the actions of other agents. Our research subject includes investigation of the multicriteria decision mechanisms that could lead to incentive compatibility, by revealing true multiobjective preferences and by appropriate coordination of agents' activities, so that the efficiency of the whole process can assured. The incentive compatibility of the multi-commodity market mechanisms was analyzed previously by Toczyłowski (2003; 2009). The ideas developed in these papers have inspired the present study. An analysis of the incentive compatible multicriteria decisions has been presented in Kruś, Skorupiński, Toczyłowski (2012) for a particular case of the producer and buyers problem.

This paper deals with a multicriteria closed bidding-auction for procuring an object, realized in one or in many rounds. This is a case of the multicriteria reverse-type auction. Different forms and rules of the auction are analyzed that are not limited to the current rules of the public auctions defined by law. The auction organizer (buyer) and bidders make multicriteria decisions. The organizer and bidders have private knowledge about their own preferences and possibilities. This information is confidential. The organizer minimizes criteria (such as cost, realization time, etc.). Bidders know these criteria, but the organizer does not inform them about his preferences.

In the classic reverse type auction, we have a sequence of offers proposed by the bidders, with gradually decreasing prices. Each bidder has his reservation price (see Figure 1a). It is obvious that any possible contract below his reservation price is not profitable for him. The organizer has also his reservation price that defines an upper limit for the price he can accept. Information about the reservation prices is private and confidential.

In the multicriteria auction, possibilities of each bidder are defined by his profitability limits that can be presented in the criteria space formulated by the organizer. These limits restrict possible offers of the bidder (from below). The organizer has also his profitability limit of acceptable offers. An illustration of the profitability limits for two criteria – time and cost – are presented in Figure 1b. Information about profitability limits is private and confidential.

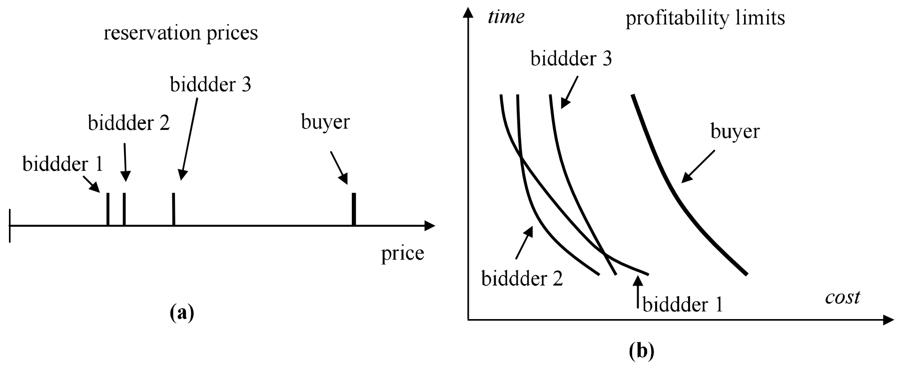

Figure 1. Examples of private information in auctions, (a) reservation prices in the classic reverse auction, (b) profitability limits in multicriteria auctions

In the multicriteria auction design, there are still some open questions regarding the rules for improvement of offers in consecutive rounds, range of information accessible to bidders, form of multicriteria decision support, and others. Regarding incentive compatibility of the multicriteria decisions, an interesting question arises: to what extent the auction mechanism can reveal private information of the bidders to the organizer, to attain efficiency of the allocation.

In this paper a general scheme of the multicriteria auction mechanism is discussed, including elements of the decision support to the auction organizer as well as to the bidders. Using domination relations formulated in the space of multiple criteria, different rules describing improvement of offers in the successive rounds of the auction process are analyzed. The general discussion is illustrated by an example of an iterative multicriteria closed-bidding auction conducted with the use of a multi-agent computer-based system. The system (Skorupiński, 2010; Kruś, Skorupiński, Toczyłowski, 2013) supports submission of offers, multicriteria analysis performed by the organizer of the bidding auction, simulation, and analysis of competing bidders' behavior. Experimental results of sessions conducted with the use of the system are presented and analyzed.

2. Problem formulation

Let a decision making authority organizes a procurement auction for the construction of a facility. We assume that there is a set of n bidders competing to obtain the order for the construction. Denote by $O = \{o^1, o^2, \dots, o^n\}$ the set of bidders participating in the auction. The offers $x \in X$, where X is a set of admissible offers, are valuated by a vector of m criteria $y = \{y_1, y_2, \dots, y_m\} \in \boldsymbol{R}^m$ defined by the auction organizer, further called also the buyer. Let $W: X \rightarrow \boldsymbol{R}^m$ be a mapping assigning a vector of the criteria to each offer. The buyer would like to obtain the best offer with the minimal values of the criteria.

We define two relations in $\boldsymbol{R}^m$: weak domination: $y^1 \succeq y^2 \Leftrightarrow y_i^1 \leq y_i^2$, for each $i = 1, 2, \dots, m$, and domination: $y^1 \succ y^2 \Leftrightarrow y_i^1 \leq y_i^2$, $y^1 \neq y^2$, for each $i = 1, 2, \dots, m$, where $y^1, y^2 \in \boldsymbol{R}^m$.

The buyer knows the profitability limits, defined as the set of acceptable offers X^0 and, related to them, the set of acceptable multicriteria valuations $Y^0 = W(X^0)$. The offers not belonging to the set X^0 are not accepted by the buyer.

The auction is conducted iteratively, in rounds $t = 1, 2, \dots$ In round t, the bidders present their offers $x^i(t)$, where $i = 1, 2, \dots n$ is the index of a bidder. Each bidder i has also his own profitability limits, defined by the set of admissible offers X^i and the related set of multicriteria valuations $Y^i = W(X^i)$. If the bidder cannot find a new offer in the set, which would beat the offers submitted so far, he gives up and cannot continue the bidding.

A general scheme of the auction carried out with use of a computer-based system is presented in Figure 2. The actions of the system operator and the decision-making processes of the auction organizer (buyer) and bidders are taken into account. The system operator starts the session and activates the computer agents that support the organizer and the bidders. The organizer specifies the requirements of the order for the construction of a facility. The specification is presented to the bidders. Before the bidding starts, the organizer and the bidders should define their profitability limits and the corresponding sets: the set of offers acceptable by the organizer and the sets of offers admissible for the bidders. The information about the profitability limits and about the sets is private and strictly confidential.

The organizer starts the first round of the auction. The bidders prepare and present their offers. The organizer collects the offers and analyzes them by looking for the preferred ones. He can either finish the bidding process or start the next round of auction. At the start of the new round the bidders obtain information about the previous non-dominated offers. Then they can prepare and submit new improved offers, which are again analyzed by the organizer.

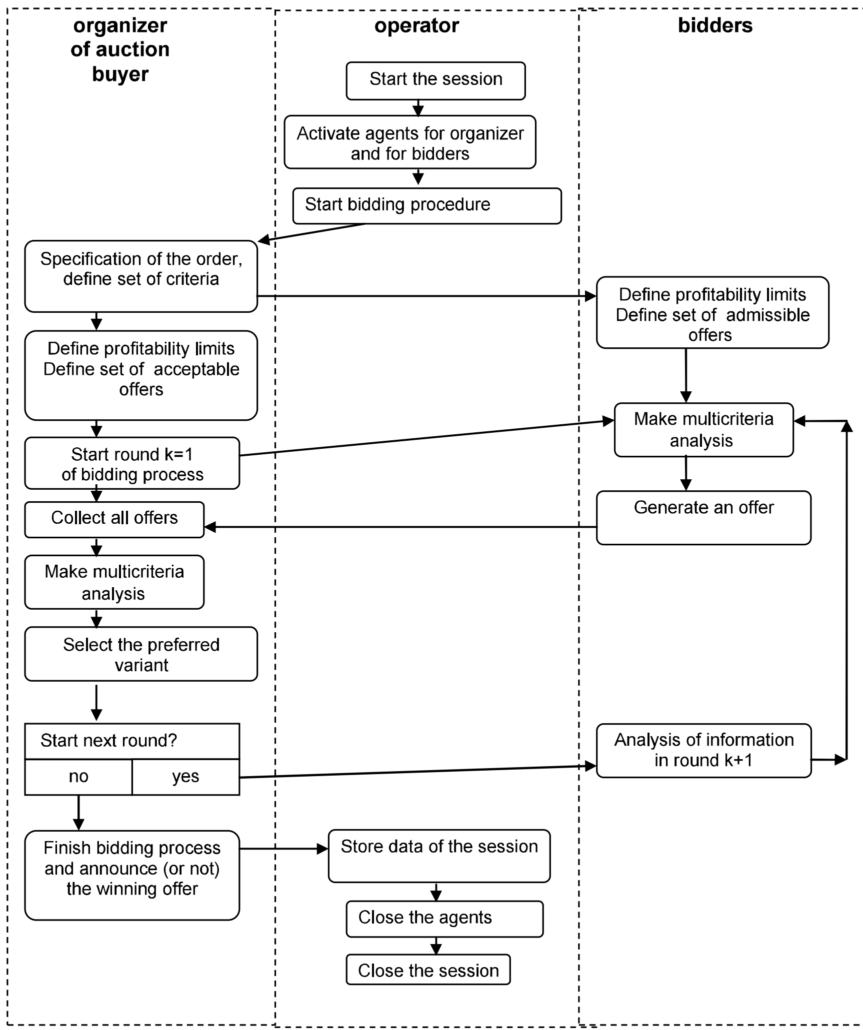

Figure 2. General scheme of the decision making processes in a multicriteria auction

The auction organizer – buyer – would like to obtain an offer that is the best with respect to his preferences. On the other hand, each bidder would also like to obtain a contract which satisfies his profitability limits and is the best with respect to his preferences.

In the case of the classic reverse auction, in successive rounds bidders propose offers with gradually decreasing prices. In the case of the multicriteria auction, in each round there can be a set of offers proposed by bidders which can be non-comparable in the sense of the domination relations mentioned above. Since the buyer performs multicriteria analysis in each round, it is necessary to support the analysis.

An example of a set of offers analyzed by the buyer is presented in Figure 3, as a set of black dots in the space of two criteria y_1, y_2. In the set there are non-dominated (Pareto-optimal) points, from the point of view of the buyer, denoted by $y^1, y^2, y^3, y^4, y^5, y^6$ (see Figure 3c).

Multicriteria analysis of the set of offers and selection of the offer according to the preferences of the buyer can be done with the use of the reference point approach (Wierzbicki, 1986; Wierzbicki, Makowski, Wessels, 2000). The method has been used and implemented in the computer-based system constructed for experimental studies on a multicriteria bidding auction (Kruś, Skorupiński, Toczyłowski, 2013). The reference point method has been originally developed for the analysis of offers in multicriteria auction by Ogryczak & Kozłowski (2011).

3. Remarks on multicriteria auction mechanisms

Some questions arise regarding the rules of auction and the range of information accessible to bidders in each round. The rules of seeking improved offers can be formulated in different ways. We consider the following three variants:

a) the new offer is accepted if it cannot be dominated by offers given in previous rounds,

b) the new offer dominates at least one offer non-dominated in the previous round,

c) the offer proposed should dominate a non-dominated offer selected by the buyer in the previous round.

Figure 3 presents the sets of possible improved offers in variants **a**, **b**, **c**, as shadowed areas.

Variant **a** defines the weakest requirements regarding offers that can be submitted in each successive round. Each bidder can propose an offer which dominates an offer non-dominated in the previous round, but can also propose an offer noncomparable with the offers non-dominated in the previous round. The set to which the improved offers should belong is constructed as the sum of the shifted domination cones without their borders.

In variant **b** each proposed offer should dominate at least one offer non-dominated in the previous round. The set which defines the possible improved offers is constructed as the sum of the domination cones shifted to the points representing offers non-dominated in the previous round. Some offers which could be proposed in variant **a** cannot be proposed in this variant, though they could be of interest to the buyer. In variants **a** and **b**, the bidders should have information about all non-dominated offers proposed in the previous round. The buyer does not need to state which of the non-dominated offer he prefers, however, such information could speed up the auction process.

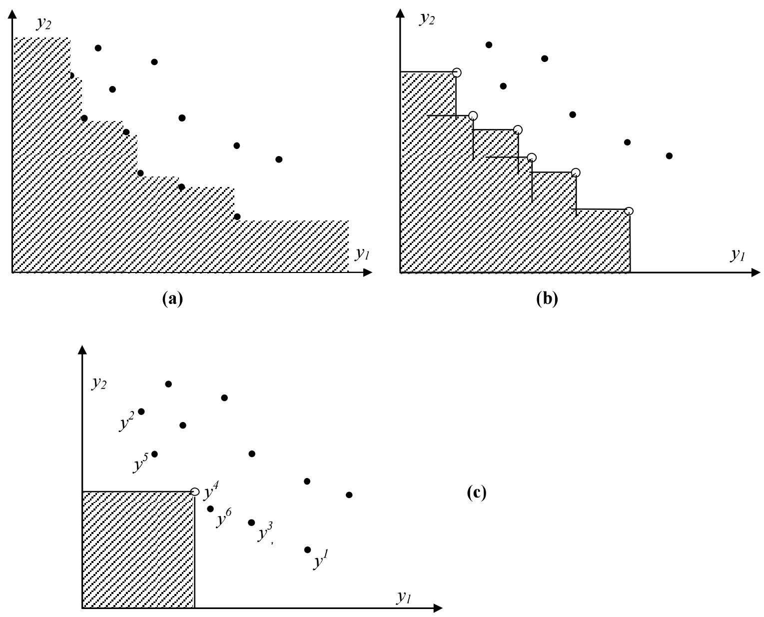

Figure 3. Sets of possible offers according to rules (a), (b), (c)

In variant **c** the buyer, after each round, informs the bidders about his preferred offer and expects that at least one of his criteria will be improved. This variant defines the strongest requirements regarding the offers proposed in the successive rounds. The auction process is speed up in comparison with variants **a** and **b**. On the other hand, some offers, which are non-dominated and of interest to the buyer, could be omitted. This is important especially at the end of the auction, when the bidders are close to their profitability limits.

Figure 4 presents sets Y^1, Y^2, Y^3 of admissible offers of three bidders in the space of criteria y_1, y_2 of the buyer. These sets correspond to the profitability limits of the bidders. Black dots represent offers given in round $t-1$. The offer $y(t-1)$ represented by a small circle has been selected by the buyer as the preferred one in round $t-1$. At this place starts the set of offers that can be proposed by the bidders in round t according to variant **c**. This set is the domination cone shifted to the point $y(t-1)$. Black diamonds represent offers given in round t. The offer $y(t)$ represented by a small circle has been selected by the buyer as his preferred one in round t. At this point starts the set of possible offers in the next round. The sets of offers that can be proposed by the bidders are limited by their profitability limits and are decreased in successive rounds. Finally, individual bidders have to interrupt the auction and some offers that could be of interest to the buyer can be omitted. This results from the rule which defines the improvement of offers assumed in variant **c**.

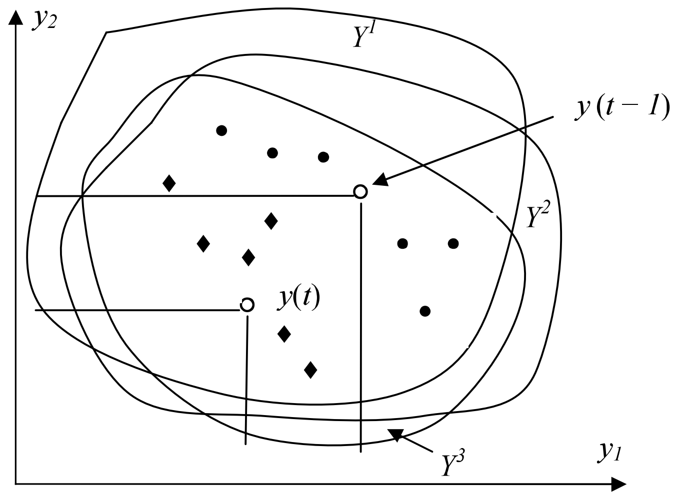

Figure 4. Sets of admissible offers. Examples

When the auction mechanism is constructed, different rules can be used at different stages of the auction process. For example, variant **c** can be assumed as the basic one. However, at the beginning and in the final rounds, variant **a** or **b** can be applied instead. At the beginning of the auction the buyer is not fully aware of his preferences, therefore the bidders should have an opportunity to present a wide portfolio of offers, and this is enabled by variants **a** and **b**. Similarly, in the final rounds it would be a pity to miss some offers, which are non-dominated and lie near the border of the domination cone, excluded from consideration by variant **c.**

The questions discussed above have been solved in a specific way in the case of a closed bidding-auction analyzed in our study. Let us assume that a decision making authority organizes an auction for the construction of a public facility, for example a bridge. The authority is interested in constructing the facility in the shortest time and with the lowest cost possible. The authority – the auction organizer and buyer, defines a discrete set T of several construction time variants, with realization times $tr \in T$. We assume that the organizer and each bidder have their own profitability limit for each time variant. For the organizer, it is the maximal accepted cost of realization of the object. For the bidders, it is assumed that each of them has conducted multicriteria analysis of the possible realization of the facility. On this basis, the organizer has defined values of minimal payments for the facility construction for the time variants. For lower values, the construction of the facility is not profitable for him. Confidentiality of information is approved. The bidders do not know which time variant will be finally accepted by the organizer. No bidder knows the profitability limit of the organizer or the profitability limits of the competitors. The organizer does not know the profitability limits of the bidders. The auction mechanism should lead to finding the contractor and the best variant of project realization according to the preferences of the organizer.

A special multiagent system has been constructed to simulate different variants of a bidding auction. The system has been written in the AIMMS (see Bisschop, Roelofs, 2009) environment. Users play the roles of the organizer of the auction and of the bidding competitors. The system is started by an operator who initiates actions of a computer agent acting for the organizer and setting a required number of agents for the competitors. The system supports confidentiality of information. The auction is carried out according to the general scheme presented in Figure 2. In each round, the bidders can present their offers with prices for each time variant. The organizer performs multicriteria analysis of the offers submitted. He does not inform the bidders about his preferences. They obtain information about the best offers for each time variant, but do not know who has proposed these offers.

Multicriteria analysis is performed by the organizer interactively with use of the reference point method developed by A.P. Wierzbicki (Wierzbicki, 1986; Wierzbicki, Makowski, Wessels, 2000). According to this method, the organizer can find and analyze non-dominated offers in the space of his criteria, assuming the reservation points r and the aspiration points a in this space. The subscripts i of the components r_i, a_i of vectors r and a, refer to the cost and the time, respectively, of the project realization. A set of the indexes is denoted by I, in our case $I = \{\text{cost, time}\}$. The following optimization tasks are solved:

$$\max z + \varepsilon \sum_{i \in I} z_i$$

subject to the constraints of the reference point method:

$$z \le z_i, \forall i \in I,$$
$$z_i \le \gamma (x_i - r_i)/(a_i - r_i), \forall i \in I,$$
$$z_i \le (x_i - r_i)/(a_i - r_i), \forall i \in I,$$
$$z_i \le \beta (x_i - a_i)/(a_i - r_i) + 1, \forall i \in I,$$

to the limits for minimized values for the time and the cost:

$$x_{cost} \geq p_{o,tr} - (p_{\max} - \mathrm{p}_{\min})(1 - w_{o,tr}), \forall o \in O, tr \in T,$$
$$x_{time} \geq d_{tr} - (d_{\max} - d_{\min})(1 - q_{tr}), \forall tr \in T,$$

and to the constraints related to the discrete form of the set T:

$$\sum_{o \in O, t \in T} w_{o,tr} = 1,$$
$$\sum_{o \in O} w_{o,tr} = q_{tr} \forall tr \in T.$$

In this formulation there are additional variables z, z_{cost}, $z_{time} \in \mathbf{R}^1$, and the coefficients of the reference point method ε, β, γ, where ε is a small positive number; $0 < \beta < 1 < \gamma$; p_{max} and p_{min} are the most costly and the cheapest offer for the given time variants; d_{max} and d_{min} are the shortest and the longest realization time; $w_{o,tr}$ for $o \in O$ and $tr \in T$, q_{tr} for $tr \in T$ are additional binary variables.

This is a mixed integer-programming problem. The reference point method is implemented for the considered multicriteria optimization problem of the auction organizer. The problem is solved by the system for the points *r* and *a,* set by the organizer. The solution of the problem – the point *x* in the criteria space – is non-dominated in the set of variants proposed by the bidders, due to the properties of the reference point method. The organizer can obtain a representation of the set of the non-dominated offers by changing the reference points.

The organizer finishes multicriteria analysis after having valuated and compared all non-dominated points of interest for him. Then he either selects the best solution, according to his preferences and announces the selected offer, completing the bidding auction, or decides to continue the auction for the next round.

If he decides to continue the auction, the bidders obtain information about the cheapest offers for the indicated time variants. However, they do not know which bidder has presented a given offer, and they do not know the preferences of the organizer. Each bidder can update his offers by decreasing costs. He cannot, however, retract his previous offer unless he wants to correct it. Moreover, he does not know if the auction will be continued in the next round. The organizer opens new offers and repeats the multicriteria analysis with the new set of offers. He can continue the process in the next round; he can either stop the process at any round and interrupt the auction if he has found all the offers unsatisfactory, or can complete the auction announcing the selected offer.

A number of simulated interactive auction sessions have been made with the use of the computer-based system. Human users of the system played roles of an auction organizer and of bidders. We wanted to investigate possible behavior of the organizer and of the bidders. An important question can be posed, whether a multi-round and multicriteria auction mechanism encourages to reveal some confidential information of the bidders about their true cost of realization of the public facility.

4. Experimental results

Selected results of one of the sessions are presented and analyzed below. The session is related to a bidding auction for the construction of a public facility. Three bidders have participated in it. The auction organizer has defined six possible time variants for the realization of the contract: 30, 33, 36, 39, 42 or 45 months. He has also defined his profitability limit, i.e. the maximal cost he can pay for the project realization for each time variant. We assume that each bidder has also defined his profitability limit i.e. the lowest price for which he can construct the facility in each given time variant.

The profitability limits of the organizer and of the bidders are presented in Figure 5. In this example, the profitability limits of bidders are below the profitability limit of the organizer. There exist cost intervals in which the possible solutions of the auction can be profitable for both the organizer and the winning bidder. A comparison of these profitability limits is presented here for the purpose of our analysis only. The organizer does not know the profitability limits of the bidders, and the bidders do not know the profitability limit of the organizer.

The organizer is interested in the construction of the facility in the shortest time possible and at a minimal cost. He understands that the construction of the facility in a shorter time requires a greater cost.

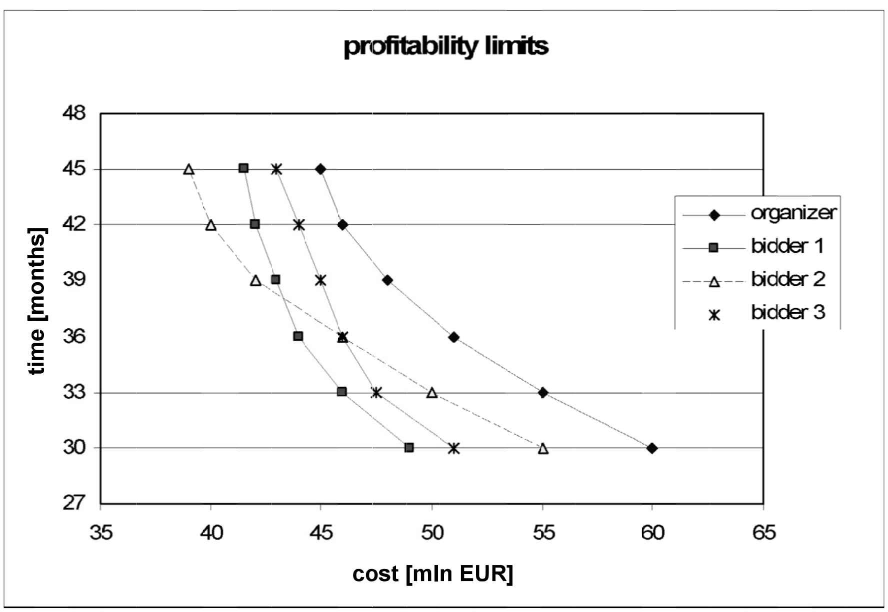

Figure 5. Profitability limits of the organizer and of the bidders

In each round, the organizer performs multicriteria analysis after all offers have been collected. The analysis consists of a number of iterations of the reference point method. In each iteration, the organizer assumes a reservation point and an aspiration point in his criteria space. The computer-based system solves the optimization task formulated in the previous section and derives the corresponding non-dominated point. The organizer obtains a representation of the set of non-dominated points by assuming different aspiration and reservation points, and can then select the best point, which is close to his preferences, but he informs the bidders about the decision after having decided to end the auction.

Figure 6 presents offers in the final (fourth) round. If the realization time is equal to 30, 33, or 36 months, the best offers are those of bidder 1, while in the case of 39, 42 or 45 months, the best offers are those of bidder 2. The organizer has obtained a significant improvement of offers in comparison with the best initial offers given in round one. Concurrent offers have been shown for each time variant.

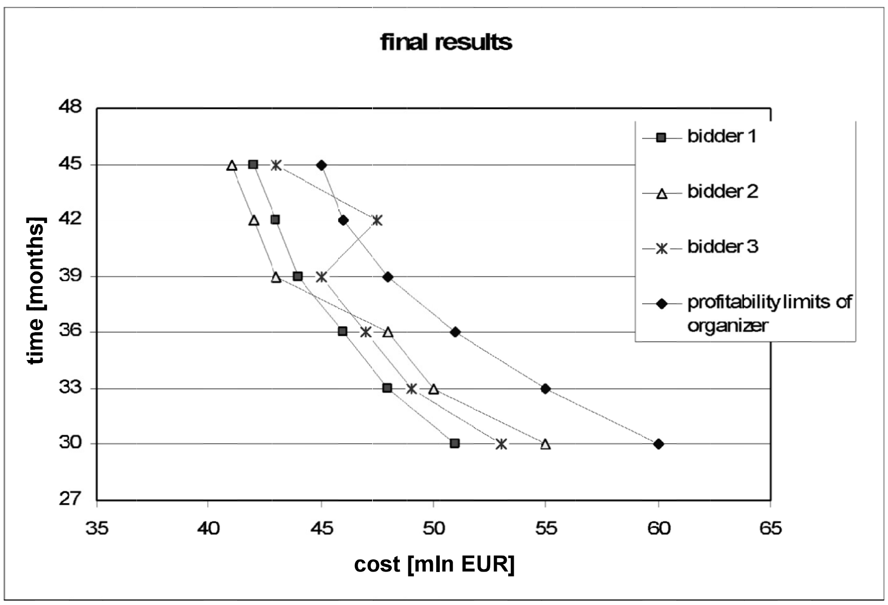

Figure 6. Offers in the final, fourth round

In the session presented here, we have observed that final offers tend to converge to the level of the second minimal profitability limit of the bidders. As we can see in Figure 5, the profitability limits of bidder 2 are lowest for the time variants 45, 42 and 39 months. Bidder 1 has the second minimal profitability limits for all the time variants. The profitability limits of bidder 3 are the second minimal ones for the time variants 36, 33 and 30 months. Let us compare the results of the final session presented in Figure 6. The winning offers of bidder 2 are on the level of the profitability limits of bidder 1 for 45, 42 and 39 months, and the winning offers of bidder 1 are on the level of the profitability limits of bidder 3 for 36, 33 and 30 months. It is understandable that the bidder with the lowest profitability limit for the given time variant has no incentive to decrease his offer and other bidders cannot beat it. In general, a large number of rounds can be required to obtain such a result, especially if the bidders are allowed to make only a small decrease in offers in each round.

5. Final remarks

The paper deals with mechanisms of multicriteria auctions in the context of incentive compatible decisions.

We have done an assessment of the rules for defining improvements of offers in successive rounds, based on the domination relation defined in the criteria space of the organizer. The rules differ with respect to the range of possible offers that can be proposed by bidders, and to the development of the auction process. It seems reasonable to apply different rules at different stages of the auction process. For example, at the beginning of the auction, the organizer may not be fully aware of his preferences. Therefore, a rule that enables the bidders to propose a wider range of offers can be applied, though the progress of such an auction can be rather slow. In further stages of the auction another, rather narrower, rule speeding up the progress could be applied, by limiting the range of possible offers.

We have constructed a mathematical model of the iterative multicriteria closed-bidding auction. It includes the formulation of the optimization task and implements the reference point approach of the multicriteria analysis performed by the organizer. The multi-agent computer-based system has been built to support the submission of offers, multicriteria analysis performed by the auction organizer, simulation and analysis of competing bidders' behavior.

The computer-based system used in the experimental studies ensures the confidentiality of private information about the profitability limits of the bidders and the organizer. We have done an assessment of the results of sessions conducted with the use of the system. We have observed that generally bidders are encouraged in the auction to gradually reveal their private information. Analogously to the Vickrey auction (see Vickrey, 1961), the proposed offers tend to converge in the consecutive rounds to the second minimal profitability limits of the bidders. This can be explained by the fact that the noncompetitive bidders, who must compete with the others to their limits, are motivated in the consecutive rounds to propose offers that tend to their profitability limits.

Further research may include development of the model and redesign of the multi agent computer-based system. Different rules of the multicriteria auction, and different strategies of bidders in the auction may be analyzed. Full confidentiality of individual information has been assumed in the model already proposed. The confidentiality relates to cost limits and preferences of the organizer and bidders. It is interesting to see how the access of bidders to some selected information, for example to the information on the organizer's preferences, may impact the behavior of the bidders and their strategies during the auction process. The bidders, in the model presented here, supply to the system data about their cost limits as well as the offers proposed. However, the corresponding multicriteria analysis leading to the calculation of that data has to be made outside the system. An additional module supporting such analysis would be useful. The cost limits of the organizer and the bidders impose obvious reservation points in multicriteria analysis performed by each of them. The cost limits can be calculated using the BATNA (Best Alternative to Negotiation Agreement) concept analogously to Kruś (2002; 2008; 2011). The BATNA concept (see Fisher, Ury, 1981) is commonly used in international negotiation processes.

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