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IMPROVING THE GAME APPROACH TO FUZZY MADM

Abstract

In the FSS paper 157 (2005, p. 34-51) we presented a game approach for solving MADM problems with fuzzy decision matrix. The results of the paper essentially depend on the assumption that the entries of the fuzzy decision matrix are triangular fuzzy numbers and dependent via a real parameter λ. In this paper we present a more general game approach for solving fuzzy MADM problems free of these restrictions. The entries of the decision matrix are assumed to be not necessarily dependent fuzzy intervals with bounded support as defined by Dubois and Prade.

Keywords: Fuzzy MADM, Fuzzy interval, game against Nature, Nash equilibrium.

1. Introduction

In traditional Multiple Attribute Decision Making (MADM) problems it is assumed that the evaluations of alternatives with respect to attributes are known exactly by the decision maker (DM) (Hwang, Yun, 1981). This restriction limits the scope of real-world application of the traditional approaches. Indeed, it often happens that the DM doesn't know exactly the evaluations of the alternatives with respect to attributes. This situation occurs when the DM is uncertain about the behavior of the environment. The uncertainty in evaluations may be of dif-

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ferent types: probabilistic, fuzzy, fuzzy-probabilistic, etc. In this paper we deal with uncertainty of fuzzy type. When fuzzy uncertainty is involved, we say that the DM faces a fuzzy MADM problem. The most adequate tool to handle such type of problems is the fuzzy set theory introduced by Zadeh (1965). Several approaches have been developed for solving fuzzy MADM problems. We can classify them into two classes. The first class consists of methods that use different ways of ranking fuzzy numbers; for each alternative a fuzzy score is calculated, then the best alternative is selected based on the ranking method used. The second one is based on different ordering of fuzzy numbers. In Chen, Hwang (1992), the most important methods for solving fuzzy MADM problems are described. In our paper (Chen, Larbani, 2005), we have introduced a new approach for solving a fuzzy MADM problem by transforming it into a game against Nature, via α-cuts and maxmin criterion of decision making under uncertainty (Chen, Larbani, 2005; Larbani, 2009a; Larbani, 2009b). And our work inspired several papers dealing with the fuzzy game approach for MADM later; for example, see the papers by Kahraman (2008), Larbani (2009a; 2009b), Clemente et al. (2011), Yang and Wang (2012), etc. The results of the paper essentially depend on the assumption that the entries of the fuzzy decision matrix are triangular fuzzy numbers and are dependent via a real parameter λ. In this paper we present a more general game approach for solving fuzzy MADM problems free of these two restrictions. Indeed, in this approach, unlike in Chen, Larbani (2005), the entries of the decision matrix are assumed to be fuzzy intervals with bounded support as defined by Dubois and Prade (2000) and not necessarily dependent. Thus, the scope of application to real-world problems will be much larger than the one of the approach developed in Chen, Larbani (2005). As in Chen, Larbani (2005), in this paper, we also formulate the fuzzy MADM problem as a two-person zero-sum game against Nature with an uncertain payoff matrix via α-cuts and maxmin principle. However, the game we obtain and the solution we propose and its computation method are totally different from those developed in Chen, Larbani (2005).

The paper is organized as follows. In section 2, we present the fuzzy MADM problem. In Section 3, we present our method step by step. Then we provide a procedure for computation of the solution we propose. In section 4, we illustrate the method by an application. Section 5 concludes the paper.

2. Problem Statement

Let us consider an MADM problem with the following fuzzy decision matrix:

$$\widetilde{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} \widetilde{a}_{11} & \widetilde{a}_{12} & \dots & \widetilde{a}_{1n} \\ \widetilde{a}_{21} & \widetilde{a}_{22} & \dots & \widetilde{a}_{2n} \\ \dots & & & \\ \widetilde{a}_{m1} & \widetilde{a}_{m2} & \dots & \widetilde{a}_{mn} \end{bmatrix} \end{array} \quad (1)$$

where m alternatives A_i, $i = 1,2,\dots,m$ are evaluated with respect to n attributes C_j, $j = 1,2,\dots,n$; $\widetilde{a}_{ij}$ represents the evaluation of alternative i with respect to attribute j. The objective of the decision maker (DM) is to select the *best* alternative according to the available information in the fuzzy matrix (1). Let us recall the definition of a fuzzy interval with bounded support as defined by Dubois and Prade (2000).

Definition 2.1 (Dubois, Prade, 2000). A fuzzy interval $\widetilde{F}$ with bounded support is defined by $\widetilde{F} = (R, \mu(.))$ with $\mu_{\widetilde{F}}(.): R \to [0,1]$ satisfying the following conditions:

(i) $\mu_{\widetilde{F}}(x) = 0$ for all $x \in (-\infty, c]$,

(ii) $\mu_{\widetilde{F}}(.)$ is right-continuous non-decreasing on $[c,a]$,

(iii) $\mu_{\widetilde{F}}(x) = 1$ for all $x \in [a,b]$,

(iv) $\mu_{\widetilde{F}}(.)$ is left-continuous non-increasing on $[b,d]$,

(v) $\mu_{\widetilde{F}}(x) = 0$ for all $x \in [d, +\infty)$,

where $-\infty < c \le a \le b \le d < +\infty$, and R is the real line.

We say that a fuzzy interval with bounded support $\widetilde{F} = (R, \mu_{\widetilde{\mathrm{F}}}(x))$ is positive if its support satisfies:

$$Sup(\widetilde{F}) = \{ z / \mathrm{z} \in R,\ \mu_{\widetilde{F}}(z) > 0 \} \subset [0, +\infty).$$

We make the following assumption.

Assumption 2.1. The DM assumes that the entries of $\widetilde{D}$ are positive fuzzy intervals as defined by Dubois and Prade.

Thus, we obtain an MADM problem with a fuzzy decision matrix under Assumption 2.1.

Remark 2.1. It is important to note that the fuzzy MADM problem (1) under Assumption 2.1 is more general than the fuzzy MADM treated in Chen, Larbani (2005). Indeed, in Chen, Larbani (2005) the entries of the fuzzy decision matrix are assumed to be triangular fuzzy numbers and dependent via a real parameter λ. In the fuzzy MADM problem (1) the entries of the fuzzy matrix are not assumed to be dependent and belong to the class of fuzzy intervals with bounded support as defined by Dubois and Prade (2000), which is more general than the class of triangular fuzzy numbers. Thus, the class of MADM problems that can be solved using the model (1) is much larger than the class of MADM problems that can be solved using the model in Chen, Larbani (2005).

3. The Method

In this section we present our approach and the resolution procedure. We transform the initial fuzzy MADM problem into a two-person zero-sum game between the DM and Nature. Then based on the solution of this game, we provide a procedure for selecting the best alternative. As in Chen, Larbani (2005), this game is obtained via α-cuts and maxmin principle of decision making under uncertainty. The use of α-cuts is based on the approach of Sakawa and Yano (1989) for solving multiobjective non linear problems with fuzzy parameters. In addition to the differences we have mentioned in Remark 2.1, the game we obtain in this paper and the resolution procedure are totally different compared to those of Chen, Larbani (2005). We present the method in four steps. We start by constructing the α-cuts of the entries of the fuzzy decision matrix $\widetilde{D}$ of the problem (1). In the second step, we introduce the game against Nature. In the third step we solve the game obtained in the second step. Finally, we propose a procedure for the selection of the best alternative.

3.1. Defuzzification

Suppose that the DM has chosen an α-cut level α. Then, following the approach of Sakawa and Yano (1989), for each entry $\widetilde{a}_{ij}$ of the fuzzy decision matrix $\widetilde{D}$, we obtain the α-cut:

$$[\widetilde{a}_{ij}]^{\alpha} = \{ a_{ij} \mid \mu_{\widetilde{a}_{ij}}(a_{ij}) \geq \alpha \},\ i = \overline{1,m} \text{ and } j = \overline{1,n} \tag{2}$$

In our model we interpret confidence as "degree of certainty of truth", then an α-cut level can be interpreted as a degree of necessity (Dubois, Prade, 2000). We assume that once the DM has chosen the level α, then he is certain (with degree of necessity 1) that for each alternative *i* and attribute *j*, the evaluation of *i* with respect to *j* is in the α-cut $[\widetilde{a}_{ij}]^{\alpha}$, but he doesn't know which particular $a_{ij} \in [\widetilde{a}_{ij}]^{\alpha}$ is the actual evaluation of the alternative i with respect to attribute j. Hence, the decision maker faces a MADM problem with crisp uncertain evaluations that vary in the α-cuts (2). This problem can be represented as follows:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \ldots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & & & \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix} \end{array} \tag{3}$$

where each entry a_{ij} is a crisp parameter that can take any value in the α-cut $[\widetilde{a}_{ij}]^{\alpha}$, for $i = \overline{1,m}$ and $j = \overline{1,n}$. Such a problem is known in literature as the decision making problem under uncertainty in the case of complete ignorance (Luce, Raiffa, 1957; Nash, 1951; Rosen, 1965). In the next section we introduce a game approach to solve it.

3.2. The Game and the Selection of the Best Alternative

Since the problem (3) is a special decision making problem under uncertainty in the case of complete ignorance, we can use one of the criteria of decision making under uncertainty to solve it Luce, Raiffa (1957). In this paper we assume that the decision maker is conservative with respect to the possible realizations of the unknown parameters (evaluations) a_{ij} in $[\widetilde{a}_{ij}]^{\alpha}$, for $i = \overline{1,m}$ and $j = \overline{1,n}$. Then the most adequate criterion to use is the maxmin (Wald) criterion. Consequently, the problem can be treated as a game against Nature. In this game the DM wants to maximize his payoff and Nature wants to minimize the same payoff. The DM chooses the alternatives A_i, $i = 1,2,\ldots,m$; Nature chooses the evaluations a_{ij}, $i = \overline{1,m}$ and $j = \overline{1,n}$, i.e. the entries of the matrix D. Here the DM considers Nature as an "intelligent player" who wants to minimize his payoff. Formally, this crisp zero-sum two person game can be represented as follows:

$$G_2 = (S^m, \prod_{\substack{1 \le i \le m \\ 1 \le j \le n}} [\widetilde{a}_{ij}]^{\alpha}, N(x,a)) \tag{4}$$

where $S^m = \{x = (x_1,x_2,\ldots,x_m),\ x_i \ge 0,\ i = \overline{1,m},\ \sum_{1}^{m} x_i = 1\}$, is the set of mixed strategies of the DM; Nature chooses the entries a_{ij} for $i = \overline{1,m}$ and $j = \overline{1,n}$ of the matrix D in the set $\prod_{\substack{1 \le i \le m \\ 1 \le j \le n}} [\widetilde{a}_{ij}]^{\alpha}$.

The payoff function of the decision maker is $\mathrm{N}(x,a) = \sum_{\substack{i=1..m \\ j=1..n}} x_i a_{ij}$, where $a = (a_{ij})_{\substack{1\le i\le m \\ 1\le j\le n}} \in \prod_{\substack{1\le i\le m \\ 1\le j\le n}} [\widetilde{\mathrm{a}}_{\mathrm{ij}}]^{\alpha}$ and $x \in S^m$, the payoff of Nature is just the negative of the DM's payoff i.e. –N(x,a). We justify the definition of the payoff function of the DM as follows. Once the DM and Nature have chosen their strategies $x = (x_1,x_2,\dots,x_m) \in S^m$ and $a = (a_{ij})_{\substack{1\le i\le m \\ 1\le j\le n}} \in \prod_{\substack{1\le i\le m \\ 1\le j\le n}} [\widetilde{\mathrm{a}}_{\mathrm{ij}}]^{\alpha}$. The payoff of the DM with respect to any alternative i can be naturally defined as:

$$x_i \sum_{j=1}^{n} a_{ij} \tag{5}$$

Indeed, x_i is the probability (or weight) that he assigns to the alternative i and the sum $\sum_{j=1}^{n} a_{ij}$ is just the aggregated score of the alternative i with respect to all the n attributes if it was chosen with probability $x_i = 1$. Then the overall payoff of the DM can be rationally defined as the sum of the payoffs with respect to all the alternatives i.e.:

$$\sum_{i=1}^{m} x_i \sum_{j=1}^{n} a_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} x_i = \mathrm{N}(x,a)$$

On the other hand, x_i can also be interpreted as the proportion of times the DM should selects the alternative i as best alternative if the decision making problem is repeated a certain number of times.

In the next section we deal with the problem of resolution of the game (4). Now based on Nash equilibrium of the game (4), we propose the following definition of the best alternative for the DM.

Definition 3.1. Assume that (x^0, a^0) is a Nash equilibrium (Luce, Raiffa, 1957; Nash, 1951; Rosen, 1965) of the game (4), then the best alternative for the DM is defined as the alternative that has the maximum score, that is, it is the alternative i_0 that satisfies:

$$\max_{1\le i\le m} \{ x_i^0 \sum_{j=1}^{n} a_{ij}^0 \} = x_{i_0}^0 \sum_{j=1}^{n} a_{i_0 j}^0 \tag{6}$$

We call it $\alpha -$ *maxmin best alternative*.

Remark 3.1. Note that in the definition (5) of the score of an alternative i, we assume that the DM considers the attributes equally important. If the DM wants to assign different positive weights w_j, j = 1,..n to attributes C_j, j = 1,..n respectively, then the score of any alternative i can be defined as follows:

$$x_i \sum_{j=1}^{n} w_j a_{ij} \qquad (7)$$

The results of this paper are also valid if the score (7) is used instead of the score (5). In the sequel of the paper, unless specified, for simplicity of the presentation, we will assume that the weights assigned by the DM are $w_j = 1, j = 1,..n$, that is, we will use the score (5) for alternatives.

3.3. Resolution of the Game

In this section we study the problem of existence of a solution to the game (4) and its computation. Note that the game (4) is not a traditional matrix two-person zero-sum game, because Nature chooses the entries a_{ij} for $i = \overline{1,m}$ and $j = \overline{1,n}$ of the matrix D in the set $\prod_{\substack{1\le i\le m\\ 1\le j\le n}} [\tilde{\mathrm{a}}_{\mathrm{ij}}]^{\alpha}$. The game (4) is an infinite two-person zero-sum game with variable payoff matrix. Consequently, the existence of Nash equilibrium is not guaranteed. Thus, we first deal with the problem of the existence of Nash equilibrium of the game (4), then address the problem of its computation.

Proposition 3.1. The game (4) has a Nash equilibrium.

Proof. By definition, the set S^m is convex and compact. Since the entries $\tilde{a}_{ij}$ for $i = \overline{1,m}$ and $j = \overline{1,n}$ of the fuzzy decision matrix in the problem (1) are fuzzy intervals with bounded support as defined by Dubois and Prade (2000), the α-cuts $[\tilde{a}_{ij}]^{\alpha}$, for $i = \overline{1,m}$ and $j = \overline{1,n}$ are closed intervals in the real line, hence they are convex and compact sets. The function $x \to \mathrm{N}(x,a) = \sum_{\substack{i=1..m\\ j=1..n}} x_i a_{ij}$ is linear for all $a \in \prod_{\substack{1\le i\le m\\ 1\le j\le n}} [\tilde{\mathrm{a}}_{\mathrm{ij}}]^{\alpha}$, hence it is concave on S^m, for all $a \in \prod_{\substack{1\le i\le m\\ 1\le j\le n}} [\tilde{\mathrm{a}}_{\mathrm{ij}}]^{\alpha}$. The function $a \to -\mathrm{N}(x,a) = -\sum_{\substack{i=1..m\\ j=1..n}} x_i a_{ij}$, is also linear for all $x \in S^m$, hence it is concave on $\prod_{\substack{1\le i\le m\\ 1\le j\le n}} [\tilde{\mathrm{a}}_{\mathrm{ij}}]^{\alpha}$, for all $x \in S^m$. From the foregoing we deduce that all the conditions of the theorem of the existence of Nash equilibrium (Luce, Raiffa, 1957; Nash, 1951; Rosen, 1965) are satisfied by the game (4). Thus, it has a Nash equilibrium.

Let us recall that a strategy profile (x^0, a^0) is a Nash equilibrium of the game (4) if x^0 is a best response of DM to the strategy a^0 of Nature, and a^0 is a best response of Nature to the strategy x^0 of DM, that is:

$$\underset{x\in S^m}{Max}\,\mathrm{N}(x,a^0) = N(x^0,a^0) \text{ and } \underset{a\in \prod_{\substack{1\le i\le m\\ \le 1 j\le n}}}{Max} -\mathrm{N}(x,a^0) = -N(x^0,a^0)$$

In the following proposition we show how a Nash equilibrium of the game (4) can be computed.

Proposition 3.2. Let $[\widetilde{a}_{ij}]^\alpha = [(a_{ij}^\alpha)^L, (a_{ij}^\alpha)^U]$, for all $i = \overline{1,m}$ and $j = \overline{1,n}$. Then the pair (x^α, a^α) where $a^\alpha = (a_{ij}^\alpha)^L_{\substack{1\le i\le m\\ 1\le j\le n}}$ and x^α is an optimal solution to the linear programming problem:

$$Max\,\mathrm{N}(x,a^\alpha) = \sum_{\substack{i=1..m\\ j=1..n}} x_i (a_{ij}^\alpha)^L , \tag{8}$$

$$x \in S^m$$

is a Nash equilibrium of the game (4).

Proof. Let us prove that the strategy of Nature $a^\alpha = (a_{ij}^\alpha)^L_{\substack{1\le i\le m\\ 1\le j\le n}}$ is the best response to any strategy $\overline{x} \in S^m$ of the DM. Indeed, for any $x \in S^m$, $x_i \ge 0$, $i = \overline{1,m}$, then $-\mathrm{N}(\overline{x},a) = -\sum_{\substack{i=1..m\\ j=1..n}} \overline{x}_i a_{ij} \le -\sum_{\substack{i=1..m\\ j=1..n}} \overline{x}_i (a_{ij}^\alpha)^L = -\mathrm{N}(\overline{x},a^\alpha)$, for all $a \in \prod_{\substack{1\le i\le m\\ 1\le j\le n}} [\widetilde{a}_{ij}]^\alpha$. In particular, for $\overline{x} = x^\alpha$, we get -N(x^α,a) $\le$ -N(x^α,a^α), for all $a \in \prod_{\substack{1\le i\le m\\ 1\le j\le n}} [\widetilde{a}_{ij}]^\alpha$. On the other hand, since x^α is an optimal solution to the problem (8), it is a best response to the strategy $a^\alpha = (a_{ij}^\alpha)^L_{\substack{1\le i\le m\\ 1\le j\le n}}$ of Nature. Thus, (x^α, a^α) is a Nash equilibrium of the game (4) (Luce, Raiffa, 1957).

3.4. Procedure for Selecting the Best Alternative

In this section we provide a procedure for selecting the best alternative. Moreover, the alternatives can also be ranked from the best to the worst.

Procedure 3.1

Step 1. Ask the DM to provide the α-cut level, then compute the α-cuts $[\widetilde{a}_{ij}]^\alpha = [(a_{ij}^\alpha)^L, (a_{ij}^\alpha)^U]$, $i = \overline{1,m}$ and $j = \overline{1,n}$.

Step 2. Solve the linear programming problem (8) with $a^{\alpha} = (a_{ij}^{\alpha})^{L}_{\substack{1\le i\le m\\ 1\le j\le n}}$. Let x^{α} be an optimal solution of (8), then (x^{α}, a^{α}) is a Nash equilibrium of the game (4).

Step 3. For each alternative i calculate its individual score $x_i^{\alpha}\sum_{j=1}^{n} a_{ij}^{\alpha}$. Then rank the alternatives based on their score, the best being the one with the largest score.

We illustrate Procedure 3.1 by Example 3.1 below.

Remark 3.2. If the DM provides a specific level α_{ij} for each alternative i and attribute j, $i = \overline{1,m}$ and $j = \overline{1,n}$, in Step 1 of Procedure 3.1, the value $\alpha = \max(\alpha_{ij})$ can be chosen as a common level. This choice can be justified by the fact that $[\tilde{a}_{ij}]^{\alpha} \subset [\tilde{a}_{ij}]^{\alpha_{ij}}$, for all $i = \overline{1,m}$ and $j = \overline{1,n}$.

3.5. A more General Model

In this section we assume that in order to face the uncertainty in evaluations, the DM chooses not only the mixed strategy $x \in S^m$ but the attribute weights w_j, $j = 1,..n$ as well (7). Using the same approach, we obtain the following extension of the game (4):

$$G_2 = (S^m \times S^n,\ \prod_{\substack{1\le i\le m\\ 1\le j\le n}} [\tilde{\mathrm{a}}_{\mathrm{ij}}]^{\alpha},\ N((x,w),a)) \tag{9}$$

where $S^n = \{w = (w_1,w_2,\ldots,w_n),\ w_j \ge 0,\ j = \overline{1,n},\ \sum_{1}^{n} w_j = 1\}$, the strategies of the DM are pairs $(x,w) \in S^m \times S^n$; the payoff of the DM is $\mathrm{N}((x,w),a) = \sum_{\substack{i=1..m\\ j=1..n}} x_i w_j a_{ij}$ and the payoff of Nature is $-\mathrm{N}((x,w),a)$. We have the following definition.

Definition 3.2. Assume that $((x^0, w^0), a^0)$ is a Nash equilibrium of the game (9), then the best alternative for the DM is defined as the alternative that has the maximum score, that is, it is the alternative i_0 that satisfies:

$$\max_{1\le i\le m} \{ x_i^0 \sum_{j=1}^{n} w_j^0 a_{ij}^0 \} = x_{i_0}^0 \sum_{j=1}^{n} w_j^0 a_{i_0 j}^0 \tag{10}$$

We call it *α w − maxmin best alternative*. We have the following proposition which is similar to Proposition 3.2.

Proposition 3.3. Let $[\widetilde{a}_{ij}]^{\alpha} = [(a_{ij}^{\alpha})^{L}, (a_{ij}^{\alpha})^{U}]$, for all $i = \overline{1, m}$ and $j = \overline{1, n}$. Then the pair (x^{α}, a^{α}), where $a^{\alpha} = \prod (a_{ij}^{\alpha})^{L}$ and x^{α} is an optimal solution to the linear programming problem:

$$Max\, \mathrm{N}((x, w), a^{\alpha}) = \sum_{\substack{i=1..m \\ j=1..n}} x_i w_j a_{ij} \tag{11}$$

$$x \in S^m,\ w \in S^n$$

is a Nash equilibrium of the game (9). The proof is similar to that of Proposition 3.2.

Procedure 3.2

Step 1. Ask the DM to provide the α-cuts level α, then determine $[\widetilde{a}_{ij}]^{\alpha} = [(a_{ij}^{\alpha})^{L}, (a_{ij}^{\alpha})^{U}]$, for all $i = \overline{1, m}$ and $j = \overline{1, n}$.

Step 2. Find a Nash equilibrium $((x^0, w^0), a^0)$ of the game (9) using Proposition 3.3.

Step 3. For each alternative *i* calculate its individual score $x_i^0 \sum_{j=1}^{n} w_j^0 a_{ij}^0$. Then rank the alternatives based on their score, the best being the one with the largest score.

Remark 3.3. Let A=$\{ A_i \mid x_i^{\alpha} = 0 \}$ be the set of alternatives with zero weight, and $\overline{\mathrm{A}}$=$\{ A_i \mid x_i^{\alpha} > 0 \}$ be the set of alternatives with positive weights. It may happen in Procedure 3.1 or 3.2 that A ≠ ∅. In this case the implemented procedure divides the set of alternatives into two classes A and $\overline{\mathrm{A}}$. The DM is indifferent regarding the alternatives in the class A, moreover they are the least alternatives. On the other hand, he can rank the alternatives in $\overline{\mathrm{A}}$ according to their scores. As an extreme case it may happen that for an alternative A_{i_0}, $x_{i_0}^{\alpha} = 1$, then we have, $x_i^{\alpha} = 0$, for all $i \neq i_0$, i.e. A=$\{ A_i \mid i \neq i_0 \}$ and $\overline{\mathrm{A}}$=$\{i_0\}$. It clear that i_0 is, absolutely, the best decision for its score is better than the score of any other alternative. This case happens when the alternative i_0 dominates all the other alternatives for all $a \in \prod_{\substack{1 \le i \le m \\ 1 \le j \le n}} [\widetilde{\mathrm{a}}_{\mathrm{ij}}]^{\alpha}$ i.e $a_{i_0 j}^{\alpha} > a_{ij}^{\alpha}$, for all $i \neq i_0$.

Remark 3.4. From a computational point of view, the Procedures 3.1 and 3.2 are simpler than the procedure developed in Chen, Larbani (2005). In Procedures 3.1 and 3.2 one has to solve only one linear programming problem (8) and (11) respectively, while in the procedure of Chen, Larbani (2005) several linear programming problems have to be solved.

4. Example and Discussion

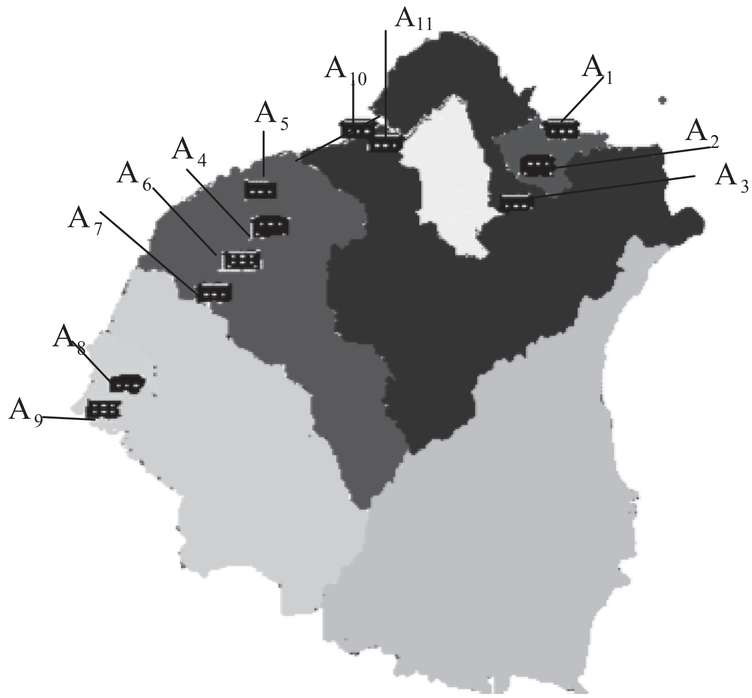

Figure 1. The Candidate Locations around the Taipei Metropolitan (Yellow District)

In this section, we will apply Procedure 3.1 to validate the presented game approach. The case study is describes a logistics company in Taiwan, which wants to select an appropriate location in the northern part of Taiwan for business expansion. Five experienced experts from various vendors and customers of this logistics company are invited to rank eleven candidate warehouse locations in Figure 1. The necessary attributes for appropriately ranking the location of warehouse are collected – these attributes are land cost (C_1), labor cost (C_2), degree of traffic congestion (C_3), accessibility to the rapid transit system (C_4), accessibility to the industrial park (C_5), accessibility to the international airport (C_6) and accessibility to the international harbor (C_7). These experienced logistics managers are asked to provide the evaluations of the locations with respect to the attributes. These fuzzy values are ranged within the quality interval from 1 to 10, where "1" means the lowest degree and "10" means the highest degree. When using the information in a decision matrix, the values of all attributes are normalized so that for all of them, beneficial or not, higher values are preferred. A matrix of type (1) with triangular fuzzy entries is obtained as a result (see appendix). The DM with the help of logistic managers fix the α-cut level α, then the α-cuts $[\widetilde{a}_{ij}]^{\alpha}$ are determined, and a decision matrix of type (3) is obtained. Next, a game of type (4) is solved. The computed scores of $x_i^{\alpha}\sum_{j=1}^{n} a_{ij}^{\alpha}$ with respect to various α-cut levels are shown in Table 1.

Table 1

Ranking Scores of the Eleven Candidate Locations

Location \ α	0.1	0.6	0.8
A_1	0.00 (2)	0.00 (2)	0.00 (2)
A_2	0.00 (2)	0.00 (2)	0.00 (2)
A_3	0.00 (2)	0.00 (2)	0.00 (2)
A_4	47.9 (1)	52.4 (1)	54.2 (1)
A_5	0.00 (2)	0.00 (2)	0.00 (2)
A_6	0.00 (2)	0.00 (2)	0.00 (2)
A_7	0.00 (2)	0.00 (2)	0.00 (2)
A_8	0.00 (2)	0.00 (2)	0.00 (2)
A_9	0.00 (2)	0.00 (2)	0.00 (2)
A_{10}	0.00 (2)	0.00 (2)	0.00 (2)
A_{11}	0.00 (2)	0.00 (2)	0.00 (2)

Note: ( ) denotes the rank.

For the three different levels of α, we obtained the same optimal strategy for the DM, $x_4^{\alpha} = 1$ and x_j^{α}=0, for $j \neq 4$. It is clear that Alternative 4 is the best choice because $x_4^{\alpha} = 1$ and x_j^{α}=0, for $j \neq 4$. The computed priority of each alternative is quite stable: as the α-cut level changes, the fuzzy score of alternatives varies but the priority of each alternative is still the same. The logistics practitioners were very satisfied with the simplicity, effectiveness and outcome of the proposed method. Note that in our approach the DM can choose different α-cut levels in order to check the sensitivity of the best solution with respect to the level α.

5. Conclusions

In this paper we have considerably improved the game approach to fuzzy MADM proposed in Chen, Larbani (2005). Compared to the approach in Chen, Larbani (2005), our approach is more general in the sense that it doesn't require the dependence of the evaluations of alternatives with respect to attributes and the fuzziness of these evaluations is of a more general type: fuzzy intervals with bounded support. Thus, this approach is capable of handling a wider class of fuzzy MADM problems. We think that the game approach for solving fuzzy decision making problems is not well explored; more interesting results can be obtained in this direction of research.

Appendix

Fuzzy Decision Matrix for Location Decision

Alternatives/Attributes	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	5,6,7	7,8,9	5,6,7	2,4,5	3,4,5	6,6,7	3,3,4
A_2	6,7,8	7,9,10	6,8,9	3,4,5	4,5,5	6,7,7	3,4,4
A_3	8,9,10	7,9,10	6,8,9	4,5,6	5,5,6	6,6,7	4,5,6
A_4	7,9,10	4,5,6	7,8,9	8,9,10	8,9,10	7,8,9	6,8,9
A_5	8,8,9	3,4,5	5,6,7	6,7,8	7,8,8	7,7,8	6,7,8
A_6	8,8,9	5,6,8	7,8,8	6,7,8	7,7,8	5,6,7	6,7,8
A_7	5,6,8	6,7,7	7,8,8	7,7,8	7,8,9	5,5,6	6,7,8
A_8	8,8,10	4,5,5	7,8,9	5,6,7	4,5,5	3,4,5	8,8,9
A_9	7,8,9	8,9,10	4,5,6	5,6,7	4,5,6	4,5,6	7,8,9
A_{10}	3,4,5	7,8,8	8,9,9	4,5,6	6,7,8	8,9,10	4,4,5
A_{11}	3,4,5	7,8,8	8,9,9	6,7,8	6,7,8	7,7,8	4,5,6

Each block is a triangular fuzzy number.

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