

Maciej Nowak\*  
Tadeusz Trzaskalik\*\*

**INTERACTIVE APPROACH APPLICATION  
TO STOCHASTIC MULTIOBJECTIVE ALLOCATION  
PROBLEM – A TWO-PHASE APPROACH**

**Abstract**

In this paper a stochastic multiobjective allocation problem is considered. We assume that a particular resource should be allocated to  $T$  projects. Depending on the amount of allocated resource it is possible (with known probabilities) to obtain a specified level of each goal. The considered criteria are divided into two groups. The first group consists of financial criteria, the second one, of qualitative criteria, representing the degree to which the projects contribute to reaching strategic goals. We propose a two-phase procedure for identifying the strategy that should be implemented by a decision maker. Our technique combines multiobjective dynamic programming and interactive approach. First, efficient strategies are identified using Bellman's principle of optimality adapted to the multiobjective problem. Next, a dialog procedure is applied to identify the solution that satisfies the decision maker. A numerical example is presented to show the applicability of the procedure.

**Keywords:** stochastic allocation problem, multiobjective dynamic programming, interactive approach, stochastic dominance.

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\* Department of Operations Research, University of Economics in Katowice ul. 1 Maja 50, 40-287 Katowice, Poland, e-mail: maciej.nowak@ue.katowice.pl.

\*\* Department of Operations Research, University of Economics in Katowice ul. 1 Maja 50, 40-287 Katowice, Poland, e-mail: tadeusz.trzaskalik@ue.katowice.pl.

## 1. Introduction

Various types of stochastic allocation problems are considered in the literature. For example, sensor allocation management as a stochastic dynamic programming resource allocation problem is considered in Johansson et al. (2005). Techniques for approximately solving a certain class of resource allocation problems that arise in the context of stochastic sequential decision making problems that are computationally efficient are considered in Hariharan (2009). Another problem can be formulated as follows: A given quantity of a resource is to be allocated to several activities. The amount of the resource allocated to each activity is used to satisfy randomly occurring stochastic demands. The system operates as long as all the demands can be met (Mendelson, Pliskin, Yechiali, 1980). An application of dynamic resource allocation in wireless communication using stochastic optimization is described in Li (2012).

The single objective deterministic allocation problem, formulated as above, has been described and solved in Bellman (1957) and Bellman, Dreyfus (1962). A dynamic programming approach has been applied. A bi-objective allocation problem was analyzed in Trzaskalik (2008). The way in which the vector version of Bellman's optimality principle can be used for identifying the whole set of non-dominated solutions has been shown.

A stochastic multiobjective allocation problem (considered in our previous paper Nowak, Trzaskalik, 2014) can be described as follows. A company has a limited amount of a particular resource that can be allocated to various projects. We assume that the decision maker has defined  $K$  goals that he/she would like to achieve. For each goal several levels of achievement have been specified. The problem is a stochastic one: if a given amount of the resource is allocated to a given project, a given level of goal is achieved with a given probability. Hence, we are to decide which projects should be implemented and what should be the intensity of their realization.

The allocation problem considered in the present paper differs from the previous one. We assume that all projects are similar and provide an identical financial return. However, the return depends on the way in which the resource is split among projects. Let  $(x_1, x_2, \dots, x_T)$  mean that  $x_1$  units of the resource are allocated to project 1,  $x_2$  units are allocated to project 2 and so on. According to our assumption, the financial return of the allocation  $(x, 0, \dots, 0)$  is the same as for  $(0, x, \dots, 0)$  and  $(0, 0, \dots, x)$ . Since the financial result, while important, does not always reflect strategic importance, we also take into account non-financial criteria. We assume that the degree to which the projects contribute in reaching strategic goals is not the same, even when the financial return is identical.

Our goal is to propose a two-phase procedure for identifying the best allocation of the resource. Our method combines multiobjective dynamic programming and interactive approach. Since for most companies, financial result is of crucial importance, we propose to use a two-phase procedure. First, allocations are evaluated with respect to financial criteria using Monte-Carlo simulation. Then, non-dominated allocations with respect to financial criteria are identified using Bellman's principle of optimality adapted to the multiobjective problem. In the second phase experts are asked to evaluate non-dominated allocations with respect to non-financial criteria.

The evaluations of allocations with respect to financial criteria obtained by Monte-Carlo simulations are random variables. On the other hand, the evaluations with respect to non-financial criteria are measured on an ordinal scale and are deterministic. As a result, we have here a multicriteria mixed stochastic-deterministic problem. In our paper we propose an interactive procedure for identifying the final solution of the problem. A candidate solution is presented to the decision maker in each iteration. If he/she is satisfied with the proposal, the procedure ends, otherwise he/she is asked to express his/her preferences specifying the values that the criteria should achieve, or at least indicating the criterion that should be improved.

The rest of the paper is organized as follows. In section 2 dynamic programming with a partially ordered criteria space is considered. A stochastic allocation problem as an example of dynamic programming with partially ordered criteria space is presented in section 3. An interactive procedure for identifying the final solution is described in section 4. In section 5 a numerical example is presented. The last section consists of final remarks.

## 2. Dynamic programming with partially ordered criteria space

The presented description of the discrete dynamic decision process comes from Trzaskalik (1990; 1998). We consider multiobjective, multiperiod decision process, which consists of  $T$  periods.

We assume that for  $t \in \overline{1, T}$ :

$\mathbf{Y}_t$  is the set of all feasible states at the beginning of the period  $t$ ,

$\mathbf{Y}_{T+1}$  is the set of all feasible states at the end of the process,

$\mathbf{X}_t(y_t)$  is the set of all feasible decisions for the period  $t$  and the state  $y_t$ ,

$\mathbf{D}_t(y_t)$  is the set of all period realizations in the period  $t$ , defined as follows:

$$\mathbf{D}_t(y_t) = \{d_t(y_t, x_t) : y_t \in \mathbf{Y}_t, x_t \in \mathbf{X}_t(y_t)\} \quad (1)$$

$\Omega_t : D_t \rightarrow \mathbf{Y}_{t+1}$  is a given transformation.

$\mathbf{D}$  is a set of all process realizations, defined as follows:

$$\mathbf{D} = \{d = (d_1, \dots, d_T) : \forall_{t \in \overline{1, T}} y_{t+1} = \Omega_t(y_t, x_t)\} \quad (2)$$

The sets  $\mathbf{Y}_1, \dots, \mathbf{Y}_{T+1}, \mathbf{X}_1(y_1), \dots, \mathbf{X}_T(y_T)$  and functions  $\Omega_1, \dots, \Omega_T$  are identified. We assume that a partially ordered criteria space  $(\mathbf{W}, \leq, \circ)$  is given, which consists of a set  $\mathbf{W}$ , the preference relation  $\leq$  (named “not worse than”) and the binary operator  $\circ$ .

Let  $\mathbf{V} \subset \mathbf{W}$ . We denote by  $\max \mathbf{V}$  the set of all maximal elements of  $\mathbf{V}$ , defined as follows:

$$\max \mathbf{V} \{ \hat{v} \in \mathbf{V} : \sim \exists_{\bar{v} \in \mathbf{V}} \hat{v} < \bar{v} \} \quad (3)$$

If  $\hat{v} < \bar{v}$ , we say, that  $\hat{v}$  is worse than  $\bar{v}$ .

We consider period criteria functions  $f_t : \mathbf{D}_t \rightarrow \mathbf{W}$ . The multiperiod criteria function  $F$  is defined as follows:

$$F = f_1 \circ (f_2 \circ (\dots (f_{T-1} \circ f_T))) \quad (4)$$

A realization  $\hat{d} \in \mathbf{D}$  is said to be efficient, iff:

$$\sim \exists_{\bar{d} \in \mathbf{D}} F(\hat{d}) < F(\bar{d}) \quad (5)$$

The set of all efficient realizations is denoted by  $\hat{\mathbf{D}}$ .

The dynamic programming problem with a partially ordered criteria space is formulated as follows (Trzaskalik, Sitarz, 2002; Trzaskalik, Sitarz, 2007): find  $\hat{\mathbf{D}}$  in the decision space and  $F(\hat{\mathbf{D}})$  in the criteria space.

The optimality equations can be written as follows (Bellman, 1957; Trzaskalik, Sitarz, 2002):

$$G_T^*(y_T) = \max \{ f_T(y_T, x_T) : x_T \in \mathbf{X}_T(y_T) \} \quad (6)$$

$$G_t^*(y_t) = \max \{ F_T(y_t, x_t) \circ G_{t+1}^*(y_t, x_t) : x_t \in \mathbf{X}_T(y_t) \} \quad (7)$$

for  $t = T-1, \dots, 1$ .

**Theorem 1** (Trzaskalik, Sitarz, 2002). The condition:

$$F(\hat{\mathbf{D}}) = \max \bigcup_{y_1 \in \mathbf{Y}_1} G_1^*(y_1) \quad (8)$$

holds. By applying Theorem 1 we can find the sets  $F(\hat{\mathbf{D}})$  and  $\hat{\mathbf{D}}$ .

### 3. Stochastic allocation problem as an example of dynamic programming with partially ordered criteria space

Let us consider the set of probability sequences, defined in the following way:

$$\mathbf{W} = \left\{ (p_0, p_1, \dots, p_n) : n \in \mathbf{N}, p_n > 0, p_i \geq 0, \sum_{i=0}^n p_i = 1 \right\} \quad (9)$$

where  $p_i = P(X = i)$ ,  $n = \max\{k \in \mathbf{N}, p_k > 0\}$ . For simplicity we assume that random variables take only nonnegative integer values. This is an example of a partially ordered criteria space.

Let  $p, q \in \mathbf{W}$ ,  $p = (p_1, \dots, p_n)$ ,  $q = (q_1, \dots, q_m)$ . We define the operator  $\circ$  as follows:

$$p \circ q = (r_0, r_1, \dots, r_{n+m}) \quad (10)$$

where  $r_i = \sum_{\substack{k,l \\ k+l=i}} p_k q_l$  for  $i = \overline{1, n+m}$ .

To define the relation  $\leq$  we use the FSD (First Stochastic Dominance) and SSD (Second Stochastic Dominance) relations:

$$p \leq q \Leftrightarrow q \text{ FSD } p \vee q \text{ SSD } p \quad (11)$$

where:

$$q \text{ FSD } p \Leftrightarrow \forall_{i \in \overline{0, n}} \sum_{k=0}^i q_k \leq \sum_{k=0}^i p_k \quad (12)$$

$$q \text{ SSD } p \Leftrightarrow \forall_{i \in \overline{0, n}} \sum_{k=0}^i \sum_{l=0}^k q_l \leq \sum_{k=0}^i \sum_{l=0}^k p_l \quad (13)$$

The stochastic allocation problem, considered in this paper can be regarded as a  $T$  – stage decision problem in a partially ordered criteria space. The number of stages is determined by the number of analyzed projects. In the stage  $t$  ( $t \in \overline{1, T}$ ) the decision on the number of units allocated to a particular project is made.

The set  $\mathbf{Y}_t$  of feasible states  $y_t$  in the consecutive stages  $t \in \overline{1, T}$  is defined as follows:

$$\mathbf{Y}_t = \{y_t \in \mathbf{N}^0 : 0 \leq y_t \leq n_0\} \quad (14)$$

The set of feasible decisions for the consecutive states  $y_t \in \mathbf{Y}_t$  for  $t \in \overline{1, T}$  is defined in the following way:

$$\mathbf{X}_t(y_t) = \{x_t \in \mathbf{N}^0 : 0 \leq x_t \leq y_t\} \quad (15)$$

For  $t \in \overline{1, T}$  the transition functions are defined as follows:

$$y_{t+1} = y_t - x_t \quad (16)$$

Each of the  $K$  criteria functions is an element of the set  $\mathbf{W}$ , described by formula (1). The criteria space  $\mathbf{W}^K$  is the product of  $K$  structures  $\mathbf{W}$ . The operator  $\circ^K$  and relation  $\leq^K$  are defined as follows:

$$(p^1, p^2, \dots, p^K) \circ^K (q^1, q^2, \dots, q^K) \stackrel{\text{def}}{=} (p^1 \circ q^1, p^2 \circ q^2, \dots, p^K \circ q^K) \quad (17)$$

$$(p^1, p^2, \dots, p^K) \leq^K (q^1, q^2, \dots, q^K) \stackrel{\text{def}}{=} p^1 \leq q^1 \wedge p^2 \leq q^2 \wedge \dots \wedge p^K \leq q^K \quad (18)$$

The relation  $\leq^K$  holds, if the FSD or SSD relations hold for each criterion (Trzaskalik, Sitarz, 2004).

#### 4. Interactive procedure

Solving a multicriteria problem is possible when the information about the preferences of the decision maker is available. Interactive approach is often used for this purpose. The procedure presented here is a modification of the INSDECM method (Nowak, 2006; Nowak, Trzaskalik, 2013).

The general scheme of each interactive procedure is similar. In each iteration a single solution or a subset of feasible solutions is presented to the decision maker, who evaluates the proposals and specifies the way in which the solution should be improved. As in our problem the evaluation of each alternative with respect to each criterion is represented by a probability distribution, we must choose measures that will be used during the dialog phase of the procedure. These can be either measures of central tendency, or measures of dispersion. At least one measure must be used for each criterion. The measures should be chosen by the decision maker in the initial phase of the procedure.

The final solution is identified in a stepwise manner. In each iteration the potency matrix is generated and presented to the decision maker. It consists of two rows: the first grouping the worst (pessimistic), and the second – the best (optimistic) values of the considered measures attainable independently within the set of alternatives.

In addition to the potency matrix, a candidate solution is presented to the decision maker. As the criteria differ in nature, we use lexicographic approach to identify it. At the initial phase of the procedure the decision maker is asked to define a hierarchy of criteria. This information is used in each iteration to select the candidate solution.

Let  $\widehat{\mathbf{D}}$  be the set of efficient solutions,  $\mathbf{D}^{(l)}$  – the set of solutions analyzed in iteration  $l$ ,  $M$  – the number of measures that are analyzed, and  $\mathbf{G}^{(l)}$  – the potency matrix:

$$\mathbf{G}^{(l)} = \begin{bmatrix} \underline{g}_1^{(l)} & \cdots & \underline{g}_m^{(l)} & \cdots & \underline{g}_M^{(l)} \\ \overline{g}_1^{(l)} & \cdots & \overline{g}_m^{(l)} & \cdots & \overline{g}_M^{(l)} \end{bmatrix} \quad (19)$$

where:  $\underline{g}_m^{(l)}$  is the worst value of the  $m$ -th measure attainable in the set of solutions analyzed in iteration  $l$ , and  $\overline{g}_m^{(l)}$  is the best one.

The procedure for identifying the final solution operates as follows:

*Initial phase:*

1. Ask the decision maker to define the hierarchy of criteria.
2. Ask the decision maker to specify measures that will be used to evaluate solutions with respect to the financial criteria.
3.  $l = 1$ ,  $\mathbf{D}^{(1)} = \widehat{\mathbf{D}}$ .

*Iteration  $l$*

1. Identify potency matrix  $\mathbf{G}^{(l)}$ .
2. Identify the candidate solution taking into account the hierarchy of criteria defined by the decision maker in the initial phase.
3. Present the potency matrix and the candidate solution to the decision maker. Ask the decision maker if he/she is satisfied with the candidate solution. If the answer is “yes”, go to (8).
4. Ask the decision maker if he/she would like to define the aspiration levels for criteria. If the answer is “no” – go to (6).
5. Ask the decision maker to specify aspiration levels  $\tilde{g}_m^{(l)}$  for  $m = 1, \dots, M$ . Identify the set  $\mathbf{D}^{(l+1)}$  including the solutions satisfying the decision maker’s requirements. If  $\mathbf{D}^{(l+1)} = \emptyset$  – report it to the decision maker and go to (4), otherwise go to (7).
6. Ask the decision maker to indicate the index  $m$  of the measure for which the pessimistic value is unsatisfactory. Identify the set  $\mathbf{D}^{(l+1)}$  grouping the solutions for which the value of the  $m$ -th measure is better than the current pessimistic value  $\underline{g}_m^{(l)}$ .
7. Set  $l = l + 1$  and go to (2).
8. Ask the decision maker to indicate the index  $m$  of the measure that should achieve the optimistic value, select the solution for which the  $m$ -th measure is equal to the optimistic value  $\overline{g}_m^{(l)}$ .

## 5. Numerical example

The company is going to allocate 6 units of a resource among 3 projects. Three financial criteria are considered:

- Criterion 1: Net Present Value generated by the project.
- Criterion 2: Revenues made on new market.
- Criterion 3: Revenues made from a new product.

Additionally, the projects are evaluated with respect to the following qualitative criteria, representing the degree to which the projects contribute to reaching strategic goals:

- Criterion 4: Strategic fit.
- Criterion 5: The use of core competencies.
- Criterion 6: Improving customer loyalty.

Our goal is to determine the amount of resource that should be allocated to each project. The first phase of the procedure starts with running simulation experiments. As we assume that the results generated by each project are identical, and the total value obtained for the project portfolio is the sum of values obtained for the individual projects, it is enough to run the simulations for a single project. The results of the simulations is presented in Table 1.

Table 1

The results of the simulation

Number of units allocated to project	Criterion 1		Criterion 2		Criterion 3	
	Value (10 <sup>5</sup> EUR)	Probability	Value (10 <sup>5</sup> EUR)	Probability	Value (10 <sup>5</sup> EUR)	Probability
0	0.0	1.0	0.0	1.0	0.0	1.0
1	0.0	0.5	0.0	0.3	0.0	0.8
	1.0	0.5	0.1	0.7	0.1	0.2
2	0.0	0.4	0.0	0.3	0.0	0.7
	1.0	0.5	0.1	0.5	0.1	0.2
	2.0	0.1	0.2	0.2	0.2	0.1
3	0.0	0.3	0.0	0.2	0.0	0.6
	1.0	0.5	0.2	0.4	0.2	0.2
	2.0	0.2	0.2	0.4	0.2	0.2
4	0.0	0.2	0.0	0.1	0.0	0.5
	1.0	0.5	0.1	0.3	0.1	0.2
	2.0	0.2	0.2	0.4	0.2	0.2
	3.0	0.1	0.2	0.2	0.2	0.1
5	0.0	0.2	0.0	0.1	0.0	0.4
	1.0	0.4	0.1	0.2	0.1	0.2
	2.0	0.2	0.2	0.4	0.2	0.2
	3.0	0.2	0.3	0.3	0.3	0.2
6	0.0	0.1	0.0	0.0	0.0	0.3
	1.0	0.3	0.1	0.1	0.1	0.3
	2.0	0.3	0.2	0.4	0.2	0.2
	3.0	0.3	0.3	0.5	0.3	0.2



By applying the dynamic programming procedure we obtain the following non-dominated allocations:

(6, 0, 0); (0, 6, 0); (0, 0, 6); (5, 1, 0); (5, 0, 1); (1, 5, 0);  
(1, 0, 5); (0, 5, 1); (0, 1, 5); (4, 1, 1); (1, 4, 1); (1, 1, 4)

All the computations are presented in the APPENDIX.

While the financial results for all allocations from the same profile are exactly the same, the evaluations with respect to qualitative criteria may be different. In the second phase of the procedure, experts are asked to rank the non-dominated allocations with respect to qualitative criteria. Each expert evaluates an allocation with respect to a single criterion. The results of this phase are presented in Table 2.

Table 2

Rankings of resource allocations based of experts' evaluations

Rank	Criterion 4	Criterion 5	Criterion 6
1	(6, 0, 0)	(0, 5, 1)	(0, 0, 6)
2	(5, 1, 0)	(1, 5, 0)	(0, 1, 5)
3	(1, 5, 0)	(0, 6, 0)	(1, 0, 5)
4	(0, 6, 0)	(5, 0, 1)	(5, 0, 1)
5	(0, 5, 1)	(1, 4, 1)	(5, 1, 0)
6	(4, 1, 1)	(5, 1, 0)	(6, 0, 0)
7	(5, 0, 1)	(6, 0, 0)	(0, 5, 1)
8	(1, 4, 1)	(4, 1, 1)	(0, 6, 0)
9	(0, 0, 6)	(1, 0, 5)	(1, 5, 0)
10	(0, 1, 5)	(0, 0, 6)	(1, 1, 4)
11	(1, 0, 5)	(1, 1, 4)	(1, 4, 1)
12	(1, 1, 4)	(0, 1, 5)	(4, 1, 1)

Finally, the interactive procedure proposed in the previous section is used to identify the final solution. The dialog with the decision maker is conducted as follows:

*Initial phase:*

The hierarchy of the criteria defined by the decision maker is as follows:

Criterion 1, Criterion 3, Criterion 2, Criterion 6, Criterion 5, Criterion 4

The decision maker defines also the data that should be presented:

- Criterion 1: probability that NPV is not less than  $2 \cdot 10^5$  EUR.
- Criterion 2: probability that revenues made on new market are not less than  $0,2 \cdot 10^5$  EUR.
- Criterion 3: probability that revenues made from a new product are not less than  $0,2 \cdot 10^5$  EUR.
- Project's ratings with respect to qualitative criteria.

Iteration 1:

1. The initial solution is identified taking into account the hierarchy defined by the DM. The allocations (4, 1, 1), (1, 4, 1) and (1, 1, 4) are the best with respect to criterion 1. Since they are equally evaluated with respect to criteria 2 and 3, in order to determine the initial solution, we consider criterion 6, which is next in the hierarchy. Among these three the allocation (1, 1, 4) is rated the best according to this criterion, and as a result it is the initial solution.
2. The potency matrix and the actual solution are presented to the decision maker (Table 3).

Table 3

Potency matrix no. 1

Solution	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
Optimistic	0.725	0.922	0.440	1	1	1
Pessimistic	0.600	0.840	0.392	12	12	12
(1, 1, 4)	0.725	0.922	0.392	12	11	10

3. The DM is satisfied with the results for criteria 1 – 3, but not with the evaluations with respect to the qualitative criteria. He asks to take into account only these solutions which rank not less than 8 with respect to criterion 6.
4. The set of allocations satisfying the requirement formulated by the DM is identified:  
(6, 0, 0); (0, 6, 0); (0, 0, 6); (5, 1, 0); (5, 0, 1); (1, 0, 5); (0, 5, 1); (0, 1, 5)
5. New potency matrix is constructed (Table 4). The DM is asked if he accepts the new results. The answer is YES.

Table 4

Potency matrix no. 2

Solution	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
Optimistic	0.600	0.900	0.440	1	1	1
Pessimistic	0.600	0.840	0.400	11	12	8

Iteration 2:

1. New proposal is identified taking into account the hierarchy of criteria: (0, 1, 5). The evaluations of the proposal are presented to the DM (Table 5).

Table 5

Proposal solution in iteration 2

Solution	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
(0, 1, 5)	0.600	0.840	0.440	10	12	2

2. The DM is satisfied with the results for criteria 1, 2, 3 and 6, but not with the evaluations with respect to criteria 4 and 5. He asks to take into account only those solutions which rank not less than 8 with respect to criterion 5.
3. The set of allocations satisfying the requirement formulated by the DM is identified:  
(6, 0, 0); (0, 6, 0); (5, 1, 0); (5, 0, 1); (0, 5, 1)
4. New potency matrix is constructed (Table 6). The DM is asked if he accepts the new results. The answer is YES.

Table 6

Potency matrix no. 3

Solution	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
Optimistic	0.600	0.900	0.440	1	1	4
Pessimistic	0.600	0.840	0.400	7	7	8

Iteration 3:

1. New proposal is identified taking into account the hierarchy of criteria: (5, 0, 1). The evaluations of the proposal are presented to the DM (Table 7).

Table 7

Proposal solution in iteration 3

Solution	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6
(0, 1, 5)	0.600	0.840	0.440	7	4	4

2. The DM is satisfied with the results and accepts the allocation (0, 1, 5) as a final solution.

## 6. Final remarks

The allocation problem considered in this paper can be used to describe a wide range of real-world problems. Dynamic programming is an efficient tool for solving it. However, in a multiobjective environment it must be used together with a procedure for the identification of the final solution. We propose to use an interactive method, in which preference information is obtained in a stepwise manner. This allows the decision maker to obtain more insight into trade-offs among different criteria. It is often pointed out that decision makers put much reliance in solutions generated via an interactive procedure, and as a result, such solutions have better chances of being implemented.

In our problem the decision maker defines his/her requirements in relation to the process realization. The nature of the problem is hierarchical. The advantage of the two-phase approach consists in reducing the number of alternatives evaluated by the experts. They evaluate only those which are likely to be accepted by the decision maker.

### Appendix

$$G_3^*(y_3) = \max \{ F_3(y_3, x_3) : x_3 \in X_3(y_3) \}$$

$$\begin{aligned} G_3^*(0) &= \{ F_3(0, 0) \} = \{ (1) \times (1) \times (1) \} & x_3^*(0) &= 0 \\ G_3^*(1) &= \{ F_3(1, 1) \} = \{ [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \} & x_3^*(1) &= 1 \\ G_3^*(2) &= \{ F_3(2, 2) \} = \{ [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \} & x_3^*(2) &= 2 \\ G_3^*(3) &= \{ F_3(3, 3) \} = \{ [(0.3; 0.5; 0.2) \times (0.2; 0.4; 0.4)] \times (0.6; 0.2; 0.2) \} & x_3^*(3) &= 3 \\ G_3^*(4) &= \{ F_3(4, 4) \} = \{ [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1)] \} & x_3^*(4) &= 4 \\ G_3^*(5) &= \{ F_3(5, 5) \} = \{ [(0.2; 0.4; 0.2; 0.2) \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2)] \} & x_3^*(5) &= 5 \\ G_3^*(6) &= \{ F_3(6, 6) \} = \{ [(0.1; 0.3; 0.3; 0.3) \times (0; 0.1; 0.4; 0.5) \times (0.3; 0.3; 0.2; 0.2)] \} & x_3^*(6) &= 6 \end{aligned}$$

$$G_2^*(y_2) = \max \{ F_2(y_2, x_2) \bullet G_3^*(y_2 - x_2) : x_2 \in X_2(y_2) \}$$

$$G_2^*(0) = \max \{ F_2(0, 0) \bullet G_3^*(0) \} = \max \{ [(1) \times (1) \times (1)] \bullet [(1) \times (1) \times (1)] \}$$

$$G_2^*(0) = \{ (1) \times (1) \times (1) \}$$

$$x_2^*(0) = 0$$

$$\begin{aligned} G_2^*(1) &= \max \{ F_2(1, 0) \bullet G_3^*(1) &= \max \{ (a) [(1) \times (1) \times (1)] \bullet [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \\ &F_2(0, 1) \bullet G_3^*(0) \} &= \max \{ (a) [(1) \times (1) \times (1)] \bullet [(1) \times (1) \times (1)] \} \\ & & & \text{(b) } [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \bullet [(1) \times (1) \times (1)] \} \end{aligned}$$

As a = b, we have:

$$G_2^*(1) = \{ [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \}$$

$$x_2^*(1) = 0 \text{ or } x_2^*(1) = 1$$

$$\begin{aligned} G_2^*(2) &= \max \{ F_2(2, 0) \bullet G_3^*(2) &= \max \{ (a) [(1) \times (1) \times (1)] \bullet [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \\ &F_2(1, 1) \bullet G_3^*(1) &= \max \{ (a) [(1) \times (1) \times (1)] \bullet [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \bullet [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \\ &F_2(0, 2) \bullet G_3^*(0) \} &= \max \{ (a) [(1) \times (1) \times (1)] \bullet [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \bullet [(1) \times (1) \times (1)] \} \end{aligned}$$

$$\begin{aligned} &= \max \{ (a) [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \\ &\text{(b) } [0.25; 0.5; 0.25] \times (0.09; 0.42; 0.49) \times (0.64; 0.32; 0.04) \\ &\text{(c) } [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \} \end{aligned}$$

As b (FSD, FSD, SSD) a and b (FSD, FSD, SSD) c, we have:

$$G_2^*(2) = \{ [0.25; 0.5; 0.25] \times (0.09; 0.42; 0.49) \times (0.64; 0.32; 0.04) \}$$

$$x_2^*(2) = 1$$

$$\begin{aligned}
 G_2^*(3) &= \max \{ F_2(3, 0) \bullet G_3^*(3) \\
 & F_2(3, 1) \bullet G_3^*(2) \\
 & F_2(3, 2) \bullet G_3^*(1) \\
 & F_2(3, 3) \bullet G_3^*(0) \} \\
 &= \max \{ (a) [(1) \times (1) \times (1)] \bullet [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \times (0.6; 0.2; 0.2) \\
 & (b) [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \bullet [0.4; 0.5; 0.1] \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1) \\
 & (c) [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \bullet [0.5; 0.5] \times (0.3; 0.7) \times (0.8; 0.2) \\
 & (d) [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \times (0.6; 0.2; 0.2)] \bullet [(1) \times (1) \times (1)] \} \\
 &= \max \{ (a) [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \times (0.6; 0.2; 0.2) \\
 & (b) [(0.15; 0.4; 0.35; 0.1) \times (0.06; 0.26; 0.4; 0.28) \times 0.48; 0.28; 0.2; 0.04] \\
 & (c) [(0.15; 0.4; 0.35; 0.1) \times (0.06; 0.26; 0.4; 0.28) \times 0.48; 0.28; 0.2; 0.04] \\
 & (d) [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \times (0.6; 0.2; 0.2)] \}
 \end{aligned}$$

As a = d and b = c, we have:

$$\begin{aligned}
 G_2^*(3) &= \{ [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \times (0.6; 0.2; 0.2) \\
 & [(0.15; 0.4; 0.35; 0.1) \times (0.06; 0.26; 0.4; 0.28) \times 0.48; 0.28; 0.2; 0.04] \} \\
 G_2^*(4) &= \max \{ F_2(4, 0) \bullet G_3^*(4) \\
 & F_2(4, 1) \bullet G_3^*(3) \\
 & F_2(4, 2) \bullet G_3^*(2) \\
 & F_2(4, 3) \bullet G_3^*(1) \\
 & F_2(4, 4) \bullet G_3^*(0) \} \\
 &= \max \{ (a) [(1) \times (1) \times (1)] \bullet [0.2; 0.5; 0.2; 0.1] \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1) \\
 & (b) [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \bullet [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \\
 & (c) [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \bullet [0.4; 0.5; 0.1] \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1) \\
 & (d) [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \bullet [0.5; 0.5] \times (0.3; 0.7) \times (0.8; 0.2) \\
 & (e) [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1)] \bullet [(1) \times (1) \times (1)] \} \\
 &= \max \{ (a) [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1)] \\
 & (b) [(0.15; 0.4; 0.35; 0.1) \times (0.06; 0.26; 0.4; 0.28) \times (0.48; 0.28; 0.2; 0.04)] \\
 & (c) [(0.16; 0.4; 0.33; 0.1; 0.01) \times (0.09; 0.3; 0.37; 0.2; 0.04) \times (0.49; 0.28; 0.18; 0.04; 0.01)] \\
 & (d) [(0.15; 0.4; 0.35; 0.1) \times (0.06; 0.26; 0.4; 0.28) \times (0.48; 0.28; 0.2; 0.04)] \\
 & (e) [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1)] \}
 \end{aligned}$$

As b (SSD, SSD, SSD) c, a = e, b = d, we have

$$\begin{aligned}
 G_2^*(4) &= \{ [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1)] \\
 & [(0.15; 0.4; 0.35; 0.1) \times (0.06; 0.26; 0.4; 0.28) \times (0.48; 0.28; 0.2; 0.04)] \} \\
 G_2^*(5) &= \max \{ F_2(5, 0) \bullet G_3^*(5) \\
 & F_2(5, 1) \bullet G_3^*(4) \\
 & F_2(5, 2) \bullet G_3^*(3) \\
 & F_2(5, 3) \bullet G_3^*(2) \\
 & F_2(5, 4) \bullet G_3^*(1) \\
 & F_2(5, 5) \bullet G_3^*(0) \} \\
 &= \max \{ (a) [(1) \times (1) \times (1)] \bullet [0.2; 0.4; 0.2; 0.2] \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2) \\
 & (b) [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \bullet [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1)] \\
 & (c) [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \bullet [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \\
 & (d) [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \bullet [0.4; 0.5; 0.1] \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1) \\
 & (e) [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1)] \bullet [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \\
 & (f) [(0.2; 0.4; 0.2; 0.2) \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2)] \bullet [(1) \times (1) \times (1)] \} \\
 &= \max \{ (a) [(0.2; 0.4; 0.2; 0.2) \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2)] \\
 & (b) [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \bullet [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1)] \\
 & (c) [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \bullet [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \\
 & (d) [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \bullet [0.4; 0.5; 0.1] \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1) \\
 & (e) [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1)] \bullet [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \\
 & (f) [(0.2; 0.4; 0.2; 0.2) \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2)] \bullet [(1) \times (1) \times (1)] \}
 \end{aligned}$$

$$= \max \{ \begin{array}{l} \text{(a)} [0.2; 0.4; 0.2; 0.2] \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2) \\ \text{(b)} [0.1; 0.3; 0.3; 0.2; 0.1] \times (0.03; 0.16; 0.33; 0.34; 0.14) \times (0.32; 0.24; 0.2; 0.2; 0.04) \\ \text{(c)} [0.12; 0.35; 0.36; 0.15; 0.02] \times (0.06; 0.22; 0.36; 0.28; 0.08) \times (0.42; 0.26; 0.24; 0.06; 0.02) \\ \text{(d)} [0.12; 0.35; 0.36; 0.15; 0.02] \times (0.06; 0.22; 0.36; 0.28; 0.08) \times (0.42; 0.26; 0.24; 0.06; 0.02) \\ \text{(e)} [0.1; 0.3; 0.3; 0.2; 0.1] \times (0.03; 0.16; 0.33; 0.34; 0.14) \times (0.32; 0.24; 0.2; 0.2; 0.04) \\ \text{(f)} [0.2; 0.4; 0.2; 0.2] \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2) \end{array} \}$$

As b (FSD, FSD, FSD) a, b (FSD, SSD, FSD) c, a = f, b = e, c = d, we have

$$G_2^*(5) = \{ [0.1; 0.3; 0.3; 0.2; 0.1] \times (0.03; 0.16; 0.33; 0.34; 0.14) \times (0.32; 0.24; 0.2; 0.2; 0.04) \}$$

$$= \max \{ \begin{array}{l} \text{(a)} [(1) \times (1) \times (1)] \bullet [0.1; 0.3; 0.3; 0.3] \times (0; 0.1; 0.4; 0.5) \times (0.3; 0.3; 0.2; 0.2) \\ \text{(b)} [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \bullet [(0.2; 0.4; 0.2; 0.2) \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2)] \\ \text{(c)} [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \bullet [(0.2; 0.5; 0.2; 0.1) \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1)] \\ \text{(d)} [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \bullet [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \bullet [0.3; 0.5; 0.2] \times (0.2; 0.4; 0.4) \\ \text{(e)} [0.2; 0.5; 0.2; 0.1] \times (0.1; 0.3; 0.4; 0.2) \times (0.5; 0.2; 0.2; 0.1) \bullet [(0.4; 0.5; 0.1) \times (0.3; 0.5; 0.2) \times (0.7; 0.2; 0.1)] \\ \text{(f)} [0.2; 0.4; 0.2; 0.2] \times (0.1; 0.2; 0.3; 0.4) \times (0.4; 0.2; 0.2; 0.2) \bullet [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \\ \text{(g)} [0.1; 0.3; 0.3; 0.3] \times (0; 0.1; 0.4; 0.5) \times (0.3; 0.3; 0.2; 0.2) \bullet [(1) \times (1) \times (1)] \end{array} \}$$

$$= \max \{ \begin{array}{l} \text{(a)} [0.1; 0.3; 0.3; 0.3] \times (0; 0.1; 0.4; 0.5) \times (0.3; 0.3; 0.2; 0.2) \\ \text{(b)} [0.1; 0.3; 0.3; 0.2; 0.1] \times (0.03; 0.13; 0.26; 0.37; 0.21) \times (0.32; 0.24; 0.2; 0.2; 0.04) \\ \text{(c)} [0.08; 0.3; 0.35; 0.19; 0.07; 0.01] \times (0.35; 0.24; 0.23; 0.13; 0.04; 0.01) \\ \text{(d)} [0.04; 0.3; 0.37; 0.2; 0.04] \times (0.04; 0.16; 0.32; 0.16) \times (0.36; 0.24; 0.28; 0.08; 0.04) \\ \text{(e)} [0.08; 0.3; 0.35; 0.19; 0.07; 0.01] \times (0.35; 0.24; 0.23; 0.13; 0.04; 0.01) \\ \text{(f)} [0.1; 0.3; 0.3; 0.2; 0.1] \times (0.03; 0.13; 0.26; 0.37; 0.21) \times (0.32; 0.24; 0.2; 0.2; 0.04) \\ \text{(g)} [0.1; 0.3; 0.3; 0.3] \times (0; 0.1; 0.4; 0.5) \times (0.3; 0.3; 0.2; 0.2) \end{array} \}$$

As c (FSD, FSD, FSD) d, a = g, b = f, c = e, we have:

$$G_2^*(6) = \{ [0.1; 0.3; 0.3; 0.3] \times (0; 0.1; 0.4; 0.5) \times (0.3; 0.3; 0.2; 0.2) \}$$

$$= \{ [0.1; 0.3; 0.3; 0.2; 0.1] \times (0.03; 0.13; 0.26; 0.37; 0.21) \times (0.32; 0.24; 0.2; 0.2; 0.04) \}$$

$$= \{ [0.08; 0.3; 0.35; 0.19; 0.07; 0.01] \times (0.35; 0.24; 0.23; 0.13; 0.04; 0.01) \}$$

$$G_1^*(6) = \max \{ F_1(6, 0) \bullet G_2^*(6) = \max \{ \begin{array}{l} \text{(a)} [(1) \times (1) \times (1)] \bullet [0.1; 0.3; 0.3; 0.3] \times (0; 0.1; 0.4; 0.5) \times (0.3; 0.3; 0.2; 0.2) \\ \text{(b)} [(1) \times (1) \times (1)] \bullet [0.1; 0.3; 0.3; 0.2; 0.1] \times (0.03; 0.13; 0.26; 0.37; 0.21) \times (0.32; 0.24; 0.2; 0.2; 0.04) \\ \text{(c)} [(1) \times (1) \times (1)] \bullet [0.08; 0.3; 0.35; 0.19; 0.07; 0.01] \times (0.35; 0.24; 0.23; 0.13; 0.04; 0.01) \\ \text{(d)} [(0.5; 0.5) \times (0.3; 0.7) \times (0.8; 0.2)] \bullet \{ [0.1; 0.3; 0.3; 0.2; 0.1] \times (0.03; 0.16; 0.33; 0.34; 0.14) \times (0.32; 0.24; 0.2; 0.2; 0.04) \} \end{array} \}$$

$$\begin{aligned}
 &F_1(6, 2) \bullet G_2^*(4) \\
 &F_1(6, 3) \bullet G_2^*(3) \\
 &F_1(6, 4) \bullet G_2^*(2) \\
 &F_1(6, 5) \bullet G_2^*(1) \\
 &F_1(6, 6) \bullet G_2^*(0) \\
 = &\max \{ (a) [0,1; 0,3; 0,3] \times (0; 0,1; 0,4; 0,5) \times (0,3; 0,3; 0,3; 0,2; 0,2) \\
 &(b) [0,1; 0,3; 0,3; 0,2; 0,1] \times (0,03; 0,13; 0,26; 0,37; 0,21) \times (0,32; 0,24; 0,2; 0,2; 0,04) \\
 &(c) [0,08; 0,3; 0,35; 0,19; 0,07; 0,01] \times (0,03; 0,14; 0,29; 0,32; 0,18; 0,04) \times (0,35; 0,24; 0,23; 0,13; 0,04; 0,01) \\
 &(d) [0,05; 0,225; 0,35; 0,25; 0,1; 0,025] \times (0,009; 0,069; 0,211; 0,333; 0,28; 0,098) \times (0,32; 0,288; 0,212; 0,136; 0,004; 0,004) \\
 &(e) [0,08; 0,3; 0,35; 0,19; 0,07; 0,01] \times (0,03; 0,14; 0,29; 0,32; 0,18; 0,04) \times (0,35; 0,24; 0,23; 0,13; 0,04; 0,01) \\
 &(f) [0,06; 0,235; 0,355; 0,255; 0,085; 0,01] \times (0,18; 0,108; 0,262; 0,336; 0,22; 0,056) \times (0,336; 0,292; 0,244; 0,096; 0,028; 0,004) \\
 &(g) [0,09; 0,3; 0,37; 0,2; 0,04] \times (0,04; 0,16; 0,32; 0,32; 0,16) \times (0,36; 0,24; 0,28; 0,08; 0,04) \\
 &(h) [0,06; 0,235; 0,355; 0,255; 0,085; 0,01] \times (0,18; 0,108; 0,262; 0,336; 0,22; 0,056) \times (0,336; 0,292; 0,244; 0,096; 0,028; 0,004) \\
 &(i) [0,05; 0,225; 0,35; 0,25; 0,1; 0,025] \times (0,009; 0,069; 0,211; 0,333; 0,28; 0,098) \times (0,32; 0,288; 0,212; 0,136; 0,004; 0,004) \\
 &(j) [0,1; 0,3; 0,3; 0,2; 0,1] \times (0,03; 0,13; 0,26; 0,37; 0,21) \times (0,32; 0,24; 0,2; 0,2; 0,04) \\
 &(k) [0,1; 0,3; 0,3; 0,3] \times (0; 0,1; 0,4; 0,5) \times (0,3; 0,3; 0,3; 0,2; 0,2) \} \\
 \end{aligned}$$

As  $c(\text{FSD}, \text{FSD}, \text{FSD}), g, e(\text{FSD}, \text{FSD}, \text{FSD}), g, d(\text{FSD}, \text{FSD}, \text{SSD}), c, i(\text{FSD}, \text{FSD}, \text{SSD}), e, d(\text{FSD}, \text{FSD}, \text{SSD}), g,$   
 $I(\text{FSD}, \text{FSD}, \text{SSD}), g, d(\text{FSD}, \text{FSD}, \text{FSD}), f, a=k, b=j, c=e, d=i, f=h$  we have:

$$\begin{aligned}
 G_6^*(6) &= \{ [0,1; 0,3; 0,3; 0,3] \times (0; 0,1; 0,4; 0,5) \times (0,3; 0,3; 0,3; 0,2; 0,2) \\
 &[0,1; 0,3; 0,3; 0,2; 0,1] \times (0,03; 0,13; 0,26; 0,37; 0,21) \times (0,32; 0,24; 0,2; 0,2; 0,04) \\
 &[(0,05; 0,225; 0,35; 0,25; 0,1; 0,025) \times (0,009; 0,069; 0,211; 0,333; 0,28; 0,098) \times (0,32; 0,288; 0,212; 0,136; 0,004; 0,004)] \} \\
 \end{aligned}$$

The non-dominated realizations of the process are as follows:

- $(y_1 = 6, x_1 = 6; y_2 = 0, x_2 = 0; y_3 = 0, x_3 = 0)$
- $(y_1 = 6, x_1 = 0; y_2 = 6, x_2 = 6; y_3 = 0, x_3 = 0)$
- $(y_1 = 6, x_1 = 0; y_2 = 6, x_2 = 0; y_3 = 6, x_3 = 6)$
- $(y_1 = 6, x_1 = 5; y_2 = 1, x_2 = 1; y_3 = 0, x_3 = 0)$

The non-dominated allocations are as follows:

- $(6, 0, 0)$
- $(0, 6, 0)$
- $(0, 0, 6)$
- $(5, 1, 0)$

- $(y_1 = 6, x_1 = 5; y_2 = 1, x_2 = 0; y_3 = 1, x_3 = 1)$
- $(y_1 = 6, x_1 = 1; y_2 = 5, x_2 = 5; y_3 = 0, x_3 = 0)$
- $(y_1 = 6, x_1 = 1; y_2 = 5, x_2 = 0; y_3 = 5, x_3 = 5)$
- $(y_1 = 6, x_1 = 0; y_2 = 6, x_2 = 5; y_3 = 1, x_3 = 1)$

- $(0, 1, 5)$
- $(4, 1, 1)$
- $(1, 4, 1)$
- $(1, 1, 4)$

$$x_3^*(6) = 0 \text{ or } x_3^*(6) = 1 \text{ or } x_3^*(6) = 4 \text{ or } x_3^*(6) = 5 \text{ or } x_3^*(6) = 6.$$

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