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USING A FUZZY APPROACH IN MULTI-CRITERIA DECISION MAKING WITH MULTIPLE ALTERNATIVES IN HEALTH CARE

Abstract

One of the responsibilities of the health care sector regulator is to decide which health technologies (drugs, procedures, diagnostic tests, etc.) should be financed using public resources. That requires taking into account multiple criteria, of which two important ones are: cost and effectiveness of a technology (others being, e.g., prevalence, safety, ethical and social implications). Hence, health and wealth need to be traded off against each other, and hence the willingness-to-pay (WTP) has to be determined. Various approaches to setting WTP have been taken, yet the results differ substantially. In the present paper I claim that the proper approach is to treat WTP as a fuzzy concept (the decision maker may not be able to decidedly state that a given health-wealth trade-off coefficient is acceptable/unacceptable – an idea backed up by the survey presented in the paper). Previous research shows how this fuzzy approach can be embedded in defining the preference relation and pairwise comparisons. In the present paper I account for the fact that there are often more than two alternatives available. To avoid difficulties that might arise (e.g., incompleteness or intransitivity of preferences) I show how the fuzzy approach can be used to define a fuzzy choice function based on the axiomatic approach. Some properties are discussed (e.g., how the approach handles the dominance and extended dominance), and the directions of further research are hinted at.

Keywords: health technology assessment, cost-effectiveness analysis, fuzziness, choice functions, willingness to pay, net benefit.

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1 Introduction

The health care market is often regulated due to its specificity as pointed out by Arrow (1963). The regulation encompasses, e.g., the decisions on which technologies should be financed using public resources. To make such decisions, the public regulator should analyse the clinical and financial consequences of using the technology. Health technology assessment (HTA) is an interdisciplinary approach (linking medicine, economics, statistics) developing methods allowing to define and measure these consequences. HTA is used more and more often, e.g., in Poland (Ustawa z dnia 12 maja 2011 r…).

Using financial and clinical criteria requires, explicitly or implicitly, a trade-off between money and health: the willingness to pay (WTP) of the decision maker needs to be determined. There have been various approaches to setting WTP (cf. Section 3), yet the results differ substantially. I argue here that determining the WTP is difficult due to the peculiarity of health as economic good and results from an inherent reluctance to report a precise price for health. The problem with determining WTP is not of statistical nature and requires a particular approach. Fuzzy-set modelling is suggested below.

The goal of the present paper is to show, from the theoretical point of view, how the fuzzy approach can be embedded in the decision making process. Jakubczyk and Kamiński (2015) showed how fuzzy preference relations can be used to model comparisons between two alternatives under uncertainty. Here I extend these ideas in one direction, modifying them to support choice from among multiple alternatives (I neglect the uncertainty, however). To avoid technical difficulties (e.g., lack of completeness or transitivity) I approach this problem by defining a fuzzy choice function. After all, ultimately the decision maker needs to make a choice, rather than simply express her preferences.

In Section 2 I present the typical approach to decision making in HTA and the concepts defined therein. In Section 3, I briefly discuss the attempts to determine the value of WTP presented in the literature and the results of the survey conducted by Jakubczyk and Kamiński (2015). In Section 4 I present the proposed model for decision making – axioms, properties, and the decision making approach. The last section is a summary. The proofs are in the appendix.

2 Decision making in health technology assessment

2.1 Nature of decision problems in HTA

Arrow (1963) pointed out that the health care services sector has many peculiarities: e.g., the demand for health care services is stochastic; there is a strong asymmetry of information between the recipients and the providers; the product quality is uncertain and difficult to verify; there are externalities, related, for instance, to ethical issues. Partly for these reasons the health care markets are often regulated with the goal to improve the efficiency in their functioning. Numerous decisions have to be made centrally, and as public money is spent, there must be a clear rationale behind the decisions. One of them is the choice of health technologies to be financed using public resources (e.g., which drugs should be reimbursed). The public regulator needs to weigh benefits and costs in a process called health technology assessment, and there are multiple criteria to be used. Reimbursing drugs uses the limited public resources, and hence the total cost needs to be assessed. Obviously, the public regulator wants to maximize the positive impact on health, and hence the clinical effectiveness of technologies is measured. As reimbursement decisions are performed across various illnesses, and the drugs compete for a single budget, the varying clinical effects have to be measured along one scale to allow comparisons. Usually a so called quality-adjusted life years (QALYs) are used, the concept combining the duration of life with its quality (cf. Pliskin et al., 1980; Bleichrodt et al., 1997). Other criteria may also be used, e.g., ethical aspects (e.g., providing extra care for patients with rare or ultra-rare diseases). Still, in the present paper I restrict my attention to two criteria only: cost and effect per single treated patient.

Let us assume we are interested in the average values of these two, i.e, the decision maker is risk neutral. Risk neutrality for cost results from averaging out the actual cost for many patients treated. Risk neutrality with respect to the effect stems from QALY being defined *à la* von Neumann-Morgenstern utility, for which the expectation is maximized. Thus, we can neglect the cost & effect variability among individual patients (first-order uncertainty). In the current study, due to space limitation, I neglect also the second-order uncertainty (average values of cost and effect being given only as estimates).

2.2 Decision analysis in HTA

To make the paper more self-contained, I present here the standard approach used in HTA and introduce the most important definitions and notation. These concepts are then redefined when fuzziness has been introduced. The interested reader might consult Gold et al. (1996), Karlsson and Johannesson (1996), or Garber (2000) for more details.

Under certainty, the decision maker knows the expected costs and effects of decision alternatives: c_i, e_i, respectively, where $i = 1, \dots, n$ enumerates the alternatives. When referring to technologies being compared, we will use $T_1, T_2, \dots$ (or capital letters A, B, …). I assume that the cost and effect are measured relative to some null option, denoting, depending on the context, no treatment, basic supportive care only, or a standard treatment. Importantly, I assume that all $e_i \geq 0$, and hence the considered technologies can only increase the effectiveness as compared to the null option. I do not impose, however, $c_i \geq 0$, as it may be the case that active treatment allows to avoid, e.g., the cost of treating complications.

If $n = 2$ then we can simply calculate the *incremental cost-effectiveness ratio*:

$$ICER = \frac{c_2 - c_1}{e_2 - e_1},$$

assuming that T_2 is more effective and more costly (otherwise the choice is trivial, or we reverse the notation). ICER measures the additional cost of obtaining an additional unit of effect. It is natural then to treat ICER as a price of health that we can pay when switching from T_1 to T_2. Hence, we should compare it to the decision maker's WTP and switch if $ICER < WTP$, the interpretation being that the market price is smaller than our reservation price. This is, in turn, algebraically equivalent to calculating *net benefit* (NB):

$$NB_i = WTP \times e_i - c_i,$$

and selecting T_i maximizing this expression. The net benefit approach is mathematically more convenient as we don't have to worry about possible dominance (when ICER is meaningless).

In the case of $n > 2$ alternatives we need to decide which pairwise comparisons to make to calculate ICERs. It has been shown that this should be done in the form of a league table, i.e., first removing some technologies, then sorting the remaining ones according to effectiveness, and finally calculating ICERs between consecutive technologies in the table (e.g., Table 1). We remove dominated technologies; we should, e.g., remove T_1 from Table 1 (dominated by T_2, i.e., is more costly and less effective). We also disregard technologies subject to extended dominance, i.e., dominated by convex combinations of two other alternatives. We should remove T_3 from Table 1 (dominated by a simple average of T_2 and T_4). Another rationale is that the ICER between T_3 and T_2 amounts to 2, and the ICER between T_4 and T_3 amounts to 1, and hence if it makes sense to upgrade from T_2 to T_3, it makes even more sense to upgrade further to T_4. We then sort the technologies by effectiveness (sorting by cost yields the same results after removing the dominated alternatives), and calculate the ICERs between consecutive technologies (the ICER for the first technology is calculated with respect to the null option).

The decision making rule for a known WTP is to proceed in this table as long as $ICER < WTP$. E.g., if $WTP = 1.8$ in our example, then we should adopt technology T_4.

Table 1: Health technologies comparison in the form of a league table

Alternative	Effect	Cost	Comment	ICER
T_1	1	3	dominated	n.a.
T_2	2	2	compared with null	1
T_3	3	4	ext. dominated	n.a.
T_4	4	5	compared with T_2	1.5
T_5	7	11	compared with T_4	2

In the actual decision making (e_i, c_i) are almost never known precisely. They are based on estimates from randomized controlled trials (RCTs), observational trials, patients' registries, etc., and hence are based on parameters given with statistical error. The values of (e_i, c_i) are often calculated using modelling, combining different parameters, extending the time horizon of the RCTs, etc. (Buxton et al., 1997). Often a Bayesian interpretation is used, in which the *a posteriori* distribution of (e_i, c_i) is available to the decision maker (Hoch and Blume, 2008). Various tools for sensitivity analysis have been proposed in the HTA literature, e.g., confidence intervals for ICER, cost-effectiveness acceptability curves (CEACs), expected value of perfect information (EVPI), cost-disutility plane, and others (cf. e.g., Eckermann and Willan, 2011). It was also pointed out that the situation becomes more complicated when more than two alternatives are considered (Barton et al., 2008; Sadatsafavi et al., 2008; Jakubczyk and Kamiński, 2010). Introducing fuzziness may complicate this further, and hence in the present paper I develop the model not accounting for uncertainty, leaving it for further research.

As can be seen in the above presentation, it is crucial to know the value of WTP to proceed with the decision making. Should the WTP be subject to (statistical type) estimation, the resulting uncertainty would be no different than parameters uncertainty and could be merged therewith and accounted for using standard techniques. In the next section, however, I argue that WTP should rather be defined using fuzzy sets concepts and hence requires a new toolbox.

3 Willingness to pay for health

3.1 Elicitation methods and results – a review

When estimating WTP we should differentiate between the willingness-to-pay to avoid certain death, the willingness-to-pay to reduce the risk of own death, and the willingness-(of the society we are part of)-to-pay to reduce the risk of somebody's death. In the first case, almost by definition, we should be willing to sacrifice all our resources (as not having sacrificed them we are certain not to profit from them). We may be willing to take a loan to pay more, or not to pay and let our children come into our wealth. One way or another, the answer to this question is both very subjective (depends on the wealth, family situation) and very emotions-driven (facing immediate death).

In the second case we are considering only marginal impact on the risk of death, and that is referred to as measuring the value of statistical life (VSL). We may try to estimate this value using revealed preferences approach, i.e., assuming that people's choices affecting their wealth and risk of death are rational and based on optimisation, hence they reveal the trade-off between life and money. An example might be the analysis of the tendency to accept risky employments (or an employment in a city that generates additional risk, e.g., due to the pollution, etc.) accounting for the wage differences. Another approach would be to see the revealed preference of the public for safety precautions, e.g., smoke detectors, burglar alarms, or airbags. Viscusi and Aldy (2003) present the results of a systematic review of the values reported in the literature. They report, for the US labour market data, VSL in the range as wide as 0.5-20.8 million USD (year 2000 value). For the US housing and product markets, they report the values in the range of 0.77-9.9 million USD. Obviously, using non-USA data further widens the range. In a newer meta-analysis Bellavance et al. (2009) present average values of VSL (along with standard deviations) calculated based on studies identified for several countries – e.g. (in million USD), for USA: 6.27 (5.04); for Canada: 9.16 (10.39); for the UK: 17 (12.59); for Australia: 11.17 (9.62). Notably the standard deviations are in the same range as the averages, proving it is difficult to come up with a reliable estimate.

Yet another question is: *'how much do you think the society you are part of should be willing to pay to save somebody's life'*. In the early 2000s in Poland the answer used to be approximated by the revealed preferences of the public payer, taking the kidney dialysis as the procedure that, as is widely accepted, ought to be provided and financed from public resources, clearly prolongs life, and has a determined cost for the public payer. Lee et al. (2009) present a quantitative analysis of this approach, showing that this translates to the implicit willingness to pay ca. $130,000 for a QALY or $61,000 for a year of life in the USA.

In the UK, where HTA is a well-established method of making a choice regarding the availability of health technologies, no official threshold is given. There were attempts to deduce this threshold via econometric analysis based on the past choices, that located WTP to be around 35,000 GBP (Devlin and Parkin, 2004; Dakin et al., 2006). A similar analysis in Poland, conducted for HTA decisions made until the end of 2011, yielded no clear conclusions on WTP (Niewada et al., 2013).

Currently in Poland the value of one QALY was set to the triple annual gross domestic product per capita, as of now ca. 120,000 PLN/QALY (based on the idea presented by Tan-Torres Edejer et al., 2003; WHO, 2001). Even though the limit is officially stated, proving a technology to offer one additional QALY at a lower cost does not guarantee reimbursement, which, in practice, makes the official threshold more of an upper acceptable bound.

Claxton et al. (2015) present another approach to estimate the WTP and combine data on health care spending and changes in mortality in the UK. They end up with lower values, of around 13,000 GBP.

As can be seen from this brief review, various methodologies can be applied, and even a single methodology can lead to varying results. The interpretation motivating the present paper is that this is exactly what should be expected based on the nature of the question. First, health cannot actually be purchased in the market so that the society can learn its monetary value. It is the health services that are bought, but the actual impact of these services on health is uncertain. The question about WTP, therefore, does not refer to any direct past experience.

Second, there is most likely a great ethically-based reluctance to define a precise threshold, if that would mean that health would not be purchased for someone, if the price exceeded the threshold by some negligible amount. That is why giving a precise answer (or behaving consistently in life-decisions, so that a revealed preferences method yields consistent results) is not possible. At the same time, as members of the society, we may feel that some values are definitely too high (we shouldn't be spending that much, and should rather direct the financial resources somewhere else) and some other values are definitely acceptable. That is what motivates the use of the fuzzy set theory to model the attitudes towards WTP.

3.2 Fuzzy description of preferences – a survey

To better justify the use of fuzzy set theory, I present the results of a survey on the perception of WTP among Polish HTA experts. Jakubczyk and Kamiński (2015) conducted a survey to verify how difficult it is for the public to decide about the WTP that should be used to ration health care services. The aim was not to come up with the ultimate estimate of the WTP, but rather to see how crisp the opinions of individuals regarding the concrete value of WTP are. In order to make it easier to understand the question the HTA experts were surveyed (27 experts participated; three answered '*no*' to the Q1 and were removed according to the survey protocol; two showed pre-defined logical inconsistencies – increasing enthusiasm in Q4 – and were removed), working in pharmaceutical companies, HTA consulting companies, and public agencies. To reduce the impact of unmentioned factors, the respondents were asked to think in terms of diabetes-related treatment. Table 2 presents the questions asked and a summary of answers (the actual questions were asked in Polish).

Table 2: The results of a survey on the willingness-to-pay in Poland

ID	Question	Answer type	Results
Q1	Cost should also be used as a criterion		88% agree/strongly agree
Q2	Exact WTP should be used in decision making		90% agree/strongly agree
Q3	This threshold should be publicly known	5-point Likert	100% agree/strongly agree
Q4	If $e_2 - e_1 = 1$, is i=2 better for various $c_2 - c_1$		(see Figure 1)
Q5	(similar to Q4, willingness to accept)		(irrelevant to this paper)
Q6	What range contains your WTP (PLN/QALY)	a range	ca. 89,000-125,000
Q7	What value equals to your WTP (PLN/QALY)	a number	ca. 105,000
Q8	How convinced are you by the answer to Q7's	4-point scale	45% level 1&2

A 5-point Likert scale used in Q1-Q5 contains the categories: *completely disagree*, *rather disagree*, *no opinion*, *rather agree*, *completely agree.* As can be seen, the respondents strongly supported the use of some kind of WTP parameter, that should be defined and publicly known in the decision making process (Q1-Q3). Hence, our respondents may be regarded as motivated to try to pinpoint the exact value of WTP.

In Q4 the respondents were asked to decide whether or not the technology that yields an additional unit of effect should be adopted if it also involves additional cost, depending on the exact value of this cost. The results are depicted in Figure 1: the fraction of respondents selecting a given answer is proportional to the area of the circle; the median answers are marked in black. We can see that there are differences between the respondents, as shown by the vertical span of responses for various suggested levels of WTP. This is especially the case for values between 100,000 and 150,000 PLN/QALY, but to a lesser degree for as wide a range as 75,000-300,000 PLN/QALY. Second, the individual respondents quite often have absolutely no opinion whether a given value should be regarded as a WTP (e.g., for WTPs = 125,000 27.3% neither agreed nor disagreed). For all the values in the range 125,000-175,000 less than a third had a definite opinion (either completely disagreed, or completely agreed).

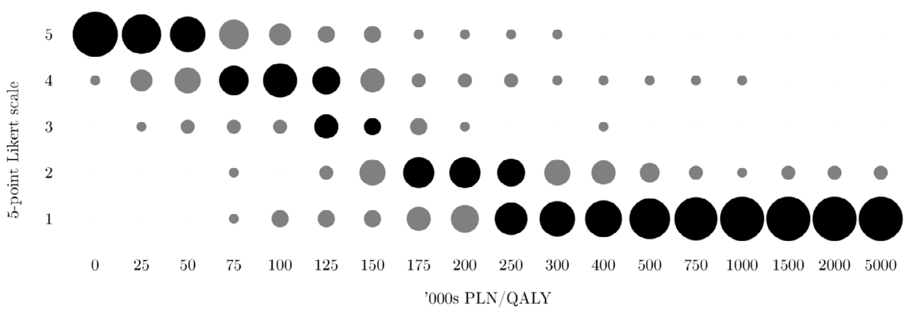

Figure 1. Respondents opinions about various levels of WTP (responses on a 5-point Likert scale: 1 = completely disagree, 5 = completely agree). The area of the circle is proportional to the percentage of responses. Median responses are in black

Third, when we combine the two above phenomena, and try to base the WTP assessment on a median voter approach, for some values of WTP we have no opinion as a society, i.e., for 125,000 and 150,000 PLN/QALY the median answer lies in the middle of the Likert scale. That means that we, as a society, are undecided whether or not the currently valid threshold (ca. 120,000 PLN/QALY) is correct.

In the survey an analogous question (Q5) was asked for the willingness-to-accept (WTA), when effectiveness was reduced, but that is of no relevance to the present study. In Q6 & Q7 the respondents were asked to give a value and a range that present their WTP. In Q8 they were asked to evaluate their satisfaction with their own answer, and almost a half was less than half-satisfied.

The results of the survey confirm that it is rather difficult, even for people with a large expertise in the area, to present a single estimate of WTP, and hence a fuzzy approach is appropriate.

4 Fuzzy decision making with multiple criteria and many alternatives – a formal model

4.1 Axioms for preferences

The axiomatic approach presented below follows the one of Jakubczyk and Kamiński (2015), but here I consider the case of more than two alternatives. To avoid difficulties with directly modelling preferences between any two alternatives (e.g., lack of transitivity), I assume that each alternative is compared to the null option only. The results of these individual comparisons are then used to select the best alternative, using a choice function approach. As all the alternatives are assumed more effective than the null option, we do not consider the relation between the WTP and willingness-to-accept (cf. Jakubczyk and Kamiński, 2015).

Let us assume that the decision maker can express her preference for each alternative $(e, c) \in \Re_+ \times \Re$, as compared to the null option. We assume that this preference is fuzzy, i.e., it is defined as $\mu(e, c) \to [0,1]$, where $\mu(e, c)$ measures the conviction that (e, c) is (weakly) preferred, i.e., is at least as good as the null option. Putting it differently, $\mu(e, c)$ is a fuzzy assessment that the sentence: *I'd like to use this technology* is true. I assume the following axioms.

Axiom 1 (reflexivity). We assume $\mu(0,0) = 1$, i.e., (something equivalent to) no treatment is as good as no treatment.

Axiom 1 serves only to clearly identify $\mu(\cdot,\cdot)$ as a fuzzy weak preference relation.

Axiom 2 (crisp preference for individual criteria). $\forall x > 0: \mu(x, 0) = 1$, $\mu(0, x) = 0$, i.e., even small gains in effect (cost) are liked (disliked) in a crisp fashion.

Axiom 3 (monotonicity). $\mu(\cdot,\cdot)$ is non-decreasing (non-increasing) in the first (second) argument.

Axioms 1-3 together imply that $\mu(e,c) = 1$ for $c \leq 0$.

Axiom 4 (limit behaviour). $\forall e \in R_{+} \exists c \in R: \mu(e,c) = 0$; $\forall c \in R \exists e \in R_{+}: \mu(e,c) = 1$

Axiom 4, being quite natural, is at the same time not vital, and is introduced mainly to make the proofs easier in borderline cases.

Axiom 5 (radiality). $\forall \alpha > 0$ $\mu(\alpha e, \alpha c)$ is constant.

Axiom 5 states that the decision maker is insensitive to scale, i.e., if she finds some technology (e,c) somewhat attractive, then a proportional scaling of effects and costs does not change her opinion. It might be interpreted that knowing the number of patients in which the technology might be used does not impact the evaluation. This is probably the least intuitive axiom and the first one to be dropped in further research.

4.2 Fuzzy willingness-to-pay and fuzzy net benefit

Based on the axioms presented in the previous subsection we can define the fuzzy WTP and the fuzzy net benefit. The former can be used to elicit the complete preference structure more easily (e.g., via surveys as presented in Section 3.2); the latter allows to compare alternatives with each other (even though originally the preferences are defined only between each alternative and the null option) and to define a choice function.

Note that $\mu(e,c)$ is defined trivially for $e = 0$ and for $c \leq 0$. Then, for all $(e,c), e > 0, c > 0$, $\mu(e,c) = \mu(1, \frac{c}{e})$. The value of $\mu(1,x)$ can be interpreted as the conviction that it is worth to pay x to get an additional unit of effect. Let us interpret the values of $\mu(1,x)$ as the membership function of a fuzzy set whose elements are values that are considered to be an acceptable cost to incur so as to gain one unit of effect. Hence, $\mu(1,x)$ defines the fuzzy willingness-to-pay.

Definition 1 (fuzzy willingness-to-pay, fWTP). Consider a preference structure as defined by axioms 1-5. Define the fuzzy set fWTP over the whole real axis by defining its membership function $\mu(1,x): \Re \to [0,1]$.

It is immediate to show that fWTP is a normal and convex fuzzy set, and that $\mu(1,x) = 1$ for $x \leq 0$. For brevity take $\mu(1,x) = fWTP(x)$. Note that under our axioms the whole preference structure can be rebuilt using fWTP as a starting point. That implies that questions like Q4 (section 0) could help to elicit fWTP, and hence the complete preference structure.

It is important that $\mu(\cdot,\cdot)$ allows to compare alternatives with the null option, but not with each other, and hence it cannot directly help to make a choice. I suggest an approach in which we measure the attractiveness of each alternative resulting from the comparison with the null option, and then make a choice using these measures of attractiveness for the individual alternatives. I suggest using the fuzzy net benefit measure, defined as in Jakubczyk and Kamiński (2015).

Definition 2 (fuzzy net benefit, fNB, of an alternative (e, c)). Consider a preference structure as defined by axioms 1-5 and a given alternative (e, c). Define a fuzzy set fNB over the whole real axis by defining its membership function $fNB_{(e,c)}(x)\colon \Re \to [0,1]$ as $fNB_{(e,c)}(x) = \mu(e, c + x)$ (the subscript will be omitted or replaced by another symbol denoting a technology when convenient)

The fNB measures the conviction that by adopting (e, c), instead of the null option, the decision maker effectively gains x (in monetary terms), i.e., would be indifferent to adopt (e, c) for an additional cost of x. We could alternatively define $fNB(x) = fWTP(\frac{c+b}{e})$. I will denote by $fNB^{\alpha}_{(e,c)}$ the α-cuts of fNB.

4.3 Choosing with fNB

In the previous subsection I defined the fNB that can be calculated for each alternative. Comparing two technologies could then be reduced to comparing two fuzzy sets, fNBs. Choosing a technology from a larger set can, in turns, be defined as maximizing fNB, treated as a fuzzy number. It is important that the choice method should not violate intuition, and the following proposition says that fNB meets the basic properties.

Proposition 1 (fNB respects dominance). Assume axioms 1-5. Consider any two alternatives: $(e_1, c_1), (e_2, c_2)$, such that $e_2 \le e_1 \wedge c_2 \ge c_1$ and at least one inequality is strict. Then fNB_{e_2,c_2} is strictly smaller than fNB_{e_1,c_1} in the sense that: $\forall \alpha > 0\ fNB^{\alpha}_{e_2,c_2} \subset fNB^{\alpha}_{e_1,c_1}$ and $\exists \alpha > 0\ fNB^{\alpha}_{e_2,c_2} \ne fNB^{\alpha}_{e_1,c_1}$.

The above proposition guarantees that fuzzy approach to net benefit allows to maintain the information that a dominance holds, and hence the dominated alternative is not worth considering. The next proposition extends it to the extended dominance case.

Proposition 2 (fNB respects extended dominance). Assume axioms 1-5. Consider any three alternatives: $(e_1, c_1), (e_2, c_2), (e_3, c_3)$, such that $\exists \lambda \in (0,1)$ that $e_3 < \lambda e_1 + (1 - \lambda)e_2 \wedge c_3 > \lambda c_1 + (1 - \lambda)c_2$. Then for all $\forall \alpha > 0\ fNB^{\alpha}_{(e_3,c_3)} \subset (fNB^{\alpha}_{(e_1,c_1)} \cup fNB^{\alpha}_{(e_2,c_2)})$ and for some α it is a proper subset.

Propositions 1-2 justify the omission of the dominated or extended dominated alternatives in the comparisons. And vice versa: they suggests that comparing alternatives can be attempted by comparing the α-cuts of fNB sets, and, in particular, the suprema of the α-cuts. I propose the following choice function.

Definition 3 (fuzzy choice function, fC). Consider a finite set of alternatives $T_1, \dots, T_n$ described by (e_i, c_i), and the preference structure as defined by axioms 1-5. For each alternative T_i calculate the set A_i containing such an α that $fNB^{\alpha}_{e_i,c_i}$ is the largest of (or equal to) all α-cuts:

$$A_i = \left\{\alpha \in [0,1] : \forall_{j\in\{1,\dots,n\}} fNB^{\alpha}_{(e_j,c_j)} \subset fNB^{\alpha}_{(e_i,c_i)}\right\}.$$

A fuzzy choice function is then defined as:

$$fC(T_1, \dots, T_n) = (|A_1|, |A_2|, \dots, |A_n|),$$

where $|A_i|$ denotes the Lebesgue measure of A_i.

The fuzzy choice function returns then an ordered n-tuple of numbers between 0 and 1 that we will interpret as the conviction that a given alternative is the best choice. The next proposition claims that Definition 3 can be actually used, i.e., the resulting $|A_i|$ are intervals and hence have a well-defined measure.

Proposition 3 (definition of fC is formally correct). The sets A_i defined in Definition 3 are (perhaps empty) intervals, and hence the Lebesgue measure is well defined (and is, trivially, their length).

We can justify the use of fC appealing to intuition in several ways. First, it is in agreement with dominance and extended dominance as stated in Propositions 1-2. Second, consider crisp preferences, i.e., such that $\mu(\cdot,\cdot) \in \{0,1\}$ and take $WTP^* = \sup\{x \in \Re : fWTP(x) = 1\}$. Consider two technologies only: (e_1, c_1) and (e_2, c_2), and $ICER = \frac{e_2 - e_1}{c_2 - c_1}$. Then, if $ICER < WTP^*$ we get $fC(T_1, T_2) = (0,1)$, and hence T_2 is recommended. If $ICER > WTP^*$, $fC(T_1, T_2) = (1,0)$. In the limit case of $ICER = WTP^*$ we have $fC(T_1, T_2) = (1,1)$, and hence the decision maker can safely choose any alternative.

Third, let us return to fuzzy preferences, and compare two technologies: (e_1, c_1) and $(e_1 + e_2, c_1 + c_2)$. Using the additivity of fNB (cf. the proof of Proposition 3) it is interesting to measure the conviction that (e_2, c_2) offers a positive NB, and hence let $\alpha^* = \mu(e_2, c_2)$. It is easy to verify that fC yields exactly α^* as the conviction that $(e_1 + e_2, c_1 + c_2)$ should be chosen.

Thus, using fNB allows to define a fuzzy choice function that returns a (possibly non-normal) fuzzy set over the universe of all *a priori* alternatives. The membership function of fC combines the complete available information on the decision maker's (fuzzy) preferences and relative attractiveness of alternatives accounting for both criteria: effect and cost.

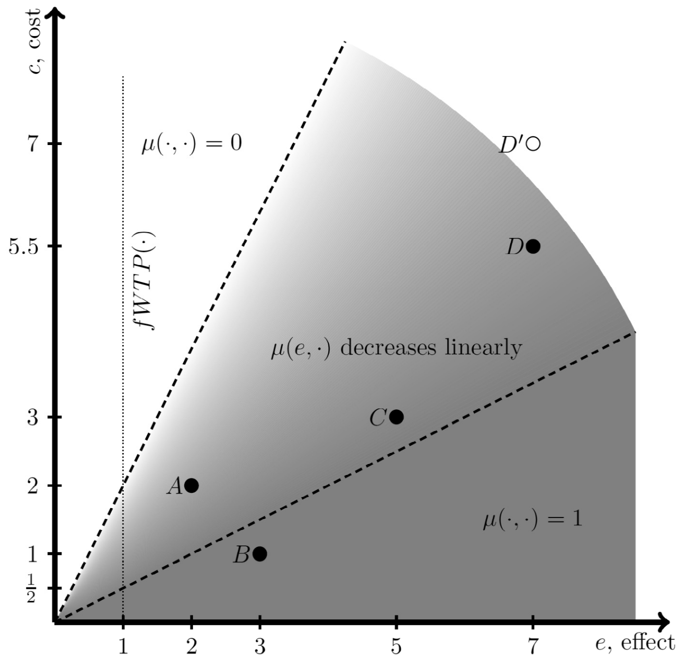

Figure 2. An example: four technologies shown in the cost-effect plane: A = (2,2), B = (3,1), C = (5,3), D = (7,5.5). We additionally consider D' = (7,7). Shades of grey represent the values of $\mu(e,c)$

Figure 2 and Figure 3 present an example. Figure 2 shows sample technologies A-D (and, additionally, D'). Note that A is dominated. I assume that $\mu(e,c) = 1$ below the line $2c = e$, and $\mu(e,c) = 0$ above the line $c = 2e$. Between these lines $\mu(e,c)$ decreases linearly with c, as shown by changing shades of grey. Specific values can also be projected radially from the membership function of $fWTP(\cdot)$, drawn as a horizontal line through (1,0). Figure 3 presents the membership functions of fNB for technologies A-D. fNB_A is moved to the left as compared to fNB_B due to the dominance. All other technologies offer the greatest net benefit with some conviction, while D maximizes the net benefit for the largest range of α's, which is reflected by the values of fC: $fC(A,B,C,D) = (0,\frac{1}{3},\frac{1}{6},\frac{1}{2})$. Note that if we considered D' instead of D, we would have to move fNB_D left by 1.5. Then $fC(A,B,C,D') = \left(0,\frac{1}{3},\frac{2}{3},0\right)$, and hence the technology D' is not recommended (not being dominated) as $ICER_{DvsC} = 2$, and $\mu(1,2) = 0$.

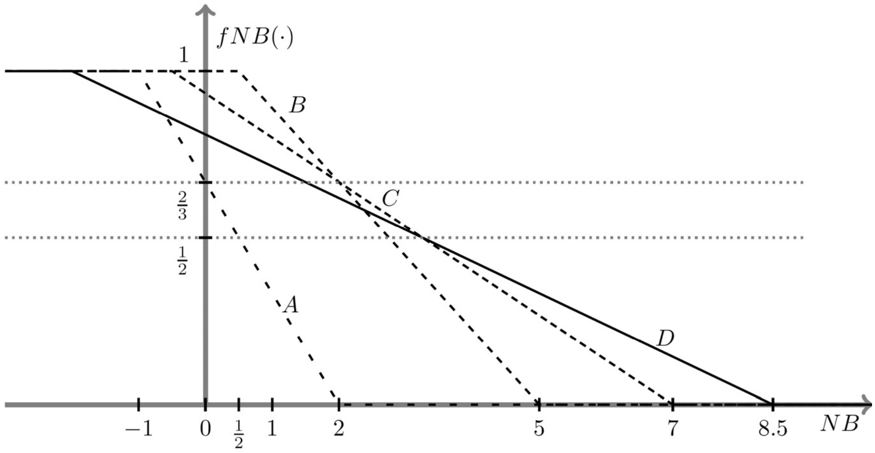

Figure 3. fNB for technologies presented in Figure 2. Horizontal dotted lines show crossings, and hence $fC(A,B,C,D) = (0,\frac{1}{3},\frac{1}{6},\frac{1}{2})$

5 Final remarks

The motivation for the present paper was the conviction that fuzzy approach is natural to WTP. Luckily, the fuzzy approach can be operationalized, i.e., axiomatically based, elicited using surveys, and used for decision making. The main outcome of the present paper is a conceptual framework allowing to use this fuzzy approach to compare several alternatives – health technologies. The paper is focused on the technical aspects of this framework, i.e., it is consistent with intuitive properties (e.g., respecting the dominance). Once the framework is developed (e.g., to encompass uncertainty) it can be used in the HTA process, i.e., in comparing health technologies, to inform the decision maker about the attractiveness of decision alternatives at hand.

One might be disappointed that the outcome is only a fuzzy choice function, i.e., a statement that, e.g., we are 0.4 convicted that T_1 should be selected, and 0.6 convicted that T_2 should be selected. It is important to stress that the goal was to show how far the fuzzy preferences, being the departure point, can be taken without forcibly changing fuzzy opinions into crisp ones. Obviously, the ultimate decision requires crispification, e.g., taking the argmax of the $fC(\cdot)$ choice function (and selecting D in the example in Figure 3).

Note that the current approach, i.e., comparing all the alternatives with the null option, allows to disregard the potential technical difficulties with the preference relation not being a total pre-order and also allows to focus on positive effects, and to disregard the potential difficulties with $WTP \neq WTA$, lack of transitivity, etc.

Further research should, in my opinion, focus on the following issues: i), discussing other possible approaches to making a crisp choice based on the fuzzy choice function outcomes (and to verifying their properties); ii), introducing uncertainty into the model; iii) trying to discuss and perhaps relax some axioms, e.g., radiality. Also, the present paper is a theoretical one, and further research should also present some sample applications of this methodology to actual decision problems.

Appendix

Proof of proposition 1

Let us start with a quick proof of the non-strict version. Take any $x \in \Re$. Then $fNB_{e_2,c_2}(x) = \mu(e_2, c_2 + x)$. Using the monotonicity axiom we immediately get that $\mu(e_1, c_1 + x)$ is not smaller. Now, let us proceed with the strict version, which we will prove for $\alpha = 1$. First, note that for $e_1 = 0$ we have also $e_2 = 0$, and hence $c_2 > c_1$ (for dominance to hold), which immediately gives the desired result, as α-cuts will be translated horizontally by the difference in cost. Assume henceforth that $e_1 > 0$. Denote $y = \sup\{x \in \Re: \mu(e_1, c_1 + x) \geq 1\}$, and hence y is the supremum of the considered α-cut (here $\alpha = 1$). Limit behaviour and radiality imply that $c_1 + y > 0$. Radiality further implies that $\sup\{x \in \Re: \mu(e_2, c_2 + x) \geq 1\} = (c_1 + y)\frac{e_2}{e_1} - c_2 = y\frac{e_2}{e_1} + \left(c_1\frac{e_2}{e_1} - c_2\right)$, where either $\frac{e_2}{e_1} < 1$ or the second term is negative, which finishes the proof for $\alpha = 1$. The proof for other $\alpha > 0$ follows analogously.

Proof of proposition 2

Let us consider, non-trivially, $e_2 > e_1 \wedge c_2 > c_1$ and $c_3 < c_2 \wedge e_3 > e_1$, as otherwise (e_3, c_3) is simply dominated by one of the other two alternatives. Note that $ICER_{3vs1} > ICER_{2vs3}$. Take any $\alpha \in (0,1]$. Denote $y = \sup\{x \in \Re: \mu(e_3, c_3 + x) \geq \alpha\}$. Limit behaviour, monotonicity, and radiality imply that $c_3 + y > 0$. Consider the slope of the line passing through the origin and the point $(e_3, c_3 + y)$, i.e., $\frac{c_3+y}{e_3}$. Assume that $\text{ICER}_{3vs1} > \frac{c_3+y}{e_3}$. Simple algebraic transformations yield that: $(c_3 + y)\frac{e_1}{e_3} - c_1 > y$, and hence the respective α-cut for technology 1 is larger than that for technology 3. If $\text{ICER}_{3vs1} \leq \frac{c_3+y}{e_3}$, then $\text{ICER}_{2vs3} < \frac{c_3+y}{e_3}$, and we get the required result for the α-cut for technology 2.

Proof of proposition 3

Consider any $(e_1, c_1), (e_2, c_2)$, $e_1 > 0, e_2 > 0$. Using radiality we can easily notice that fNB is additive, i.e., for any $\alpha > 0$, we have $\sup fNB^{\alpha}_{e_1,c_1} + + \sup fNB^{\alpha}_{e_2,c_2} = \sup fNB^{\alpha}_{(e_1+e_2),(c_1+c_2)}$. The monotonicity axiom implies that $\sup fNB^{\alpha}_{e_2,c_2}$ is non-increasing in . These two further imply that if for any α^* we have $\sup fNB^{\alpha^*}_{(e_1+e_2),(c_1+c_2)} \geq \sup fNB^{\alpha^*}_{e_1,c_1}$ then also for any $\alpha < \alpha^*$ we have $\sup fNB^{\alpha}_{(e_1+e_2),(c_1+c_2)} \geq \sup fNB^{\alpha}_{e_1,c_1}$. This yields the result.

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