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FUZZY PARETO DOMINANCE IN MULTIPLE CRITERIA PROJECT SCHEDULING PROBLEM

Abstract

Planning is one of the most important aspects of project management. A project plan defines objectives, activities and timeframe for project realization. To be able to define the required timeframe for project realization it is important to prepare its schedule.

The purpose of this paper is to present the project scheduling problem as a multiple criteria decision making problem and to solve it using two evolutionary algorithms: SPEA2 and an evolutionary algorithm driven by the fuzzification of Pareto dominance. A comparison of these two approaches is conducted to investigate if it is reasonable to use the fuzzification of the Pareto dominance relation in evolutionary algorithms for the multiple criteria project scheduling problem.

Keywords: fuzzy Pareto dominance, project scheduling problem, multiple criteria optimization, evolutionary algorithms.

1 Introduction

A company's success depends on how it adapts to the changes in its current dynamic environment. Changes are conducted under pressure of time and cost and with limited access to the resources. Those changes should be managed as projects. In the current environment, when companies have to adapt to changes quickly, the number of projects conducted in companies is increasing. We can say that currently more than 25% of companies' activities should be managed as projects (Brilman, 2002). This is the case in such areas as engineering or IT. Projects managed properly lead to situations when companies' goals are met on time within the assumed budget and with limited resources.

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One of the most important phases of project management is project planning. Scheduling is one of the most important elements of a project plan. The most popular techniques used by companies for project scheduling are CPM and PERT which provide schedules optimal in terms of time. In real-life applications a project schedule should be optimized also in terms of other elements such as resources or cash flows generated in the project.

The multiple criteria project scheduling problem is not frequently discussed in the literature.

An example of describing and solving the multiple criteria project scheduling problem is presented in Viana, de Sousa (2000). The authors have considered a resource constrained problem whose objectives are: minimization of the project completion time, minimization of project delay, and minimization of the violation of resource constraints. They have presented two multiple objective techniques to solve this problem: Pareto Simulated Annealing and Multiple Objective Taboo Search.

Also Hapke et al. (1998) considered the multiple criteria project scheduling problem. They have described a problem using four components: the set of resources **R**, the set of activities **Z**, the set of precedence relationships on **Z,** and the set of objectives **C**. The project scheduling problem is a problem of allocation of resources from the set **R** to activities from the set **Z**, so that all activities can be completed, constraints can be met and the best compromise between the objectives from **C** is reached. The authors have considered a problem with three criteria: project cost minimization, project delay minimization, and resource usage optimization. To solve the problem, Pareto Simulated Annealing was used in the first stage and the interactive local search method was used to identify the final solution from the set of solutions obtained in the first stage.

The multiple criteria project scheduling problem is also considered in Krzeszowska (2013). The author has proposed a mathematical model with three objectives: minimization of penalty for project delay, minimization of the cost of additional resource usage, and maximization of NPV. The problem was solved in two stages. In the first stage the SPEA2 algorithm was used to find the set of non-dominated solutions. In the second stage an interactive method was used to identify the final solution from the set of solutions obtained in the first stage.

Another example is described in Leu et al. (1999). The authors have considered a resource constrained problem with three objectives: time, cost, and resource usage optimization. The problem is solved in two stages. In the first stage a compromise between time and cost is considered and resources are allocated to the project. In the second stage resource leveling is applied.

In the present paper the multiple criteria project scheduling problem is considered. Three objectives are taken into account: minimization of the penalty for project delay, minimization of the penalty for resource over-usage, and maximization of NP. The problem is solved with the fuzzy dominance-driven evolutionary algorithm. The results obtained are compared with the result obtained by the SPEA2 algorithm.

2 Multiple criteria project scheduling problem

We consider a project for which a schedule should be prepared. By scheduling we understand setting the start and finish times for each activity of the project. We are looking for a schedule which meets constraints and is the best compromise between the objectives.

For the problem described above the following assumption have been made:

- the project consists of J activities $j = 1,\dots,J$,
- the project has been described on an AON network (Activity On Node – using this type of network allows to use all precedence relationship types),
- each activity is described by three elements: duration, type and amount of required resources, cash flows generated by each activity,
- deterministic time is considered,
- if the project is finished with delay, a penalty is foreseen for each unit of delay,
- cash flows are generated at activity completion,
- only renewable resources are constrained (with the assumption that the amount of nonrenewable resources is sufficient to complete the project),
- we consider internal resources available for the project and external resources whose usage leads to penalty.

The following notation is used:

J – number of all activities of the project ($j = 1,\dots,J$),
T – number of all time units ($t = 1,..,T$),
K – number of renewable resources ($k = 1,\dots,K$),
F^J – project completion time,
LF^J – project completion time defined by the decision maker,
Z^J – penalty for unit of project delay,
cf_j – net cash flows generated by activity j,
α – discount rate,
x_{jt} – decision variable,
d_j – duration of activity j,
F_j – completion time of activity j,
F_i – completion time of predecessor i,
S_j – start time of activity j,
S_i – start time of predecessor i,
r_{jk} – amount of kth resource required by activity j,

R_{kt} – amount of kth resource available at time unit t (both internal and external),
R^{w}_{kt} – amount of kth internal resource available at time unit t,
V_k – penalty for using external renewable resources,
A^{I}_{j}, A^{II}_{j}, A^{III}_{j}, A^{IV}_{j} – predecessors of activity j (precedence relationships are as follows: finish to start, start to start, start to finish, finish to finish).

A multiple objective model for the project scheduling problem can be formulated as follows:

Objectives

$$\max\{F^{J} - LF^{J}, 0\} \cdot Z^{J} \rightarrow \min \tag{2.1}$$

$$\sum_{t=1}^{T}[\sum_{k=1}^{K}[\max\{\sum_{j=1}^{J}(r_{jk} \cdot x_{jt}) - R^{w}_{kt}, 0\} \cdot V_k]] \rightarrow \min \tag{2.2}$$

$$\sum_{j=1}^{J} cf_j \cdot e^{-\alpha F_j} \rightarrow \max \tag{2.3}$$

Constraints

$$x_{jt} = \{0,1\} \tag{2.4}$$

$$\bigwedge_{j=1,\dots,J} \sum_{t=1}^{T} x_{jt} = d_j \tag{2.5}$$

$$F_j = \max_{t=1,..,T}(t \cdot x_{jt}) \tag{2.6}$$

$$\bigwedge_{x_{jt} \neq 0} S_j = \min_{t=1,..,T}(t \cdot x_{jt}) - 1 \tag{2.7}$$

$$F_j = S_j + d_j \tag{2.8}$$

$$S_j \geq F_i \qquad (i \in A^{I}_{j}) \tag{2.9}$$

$$S_j \geq S_i \qquad (i \in A^{II}_{j}) \tag{2.10}$$

$$F_j \geq S_i \qquad (i \in A^{III}_{j}) \tag{2.11}$$

$$F_j \geq F_i \qquad (i \in A^{IV}_{j}) \tag{2.12}$$

$$\bigwedge_{k=1,\dots,K} \bigwedge_{t=1,\dots,T} \sum_{j=1}^{J}(r_{jk} \cdot x_{jt}) \leq R_{kt} \tag{2.13}$$

The purpose of the criterion function (2.1) is to minimize the penalty for project delay. Delay is defined as a situation in which the project finishes later than it was assumed by the decision maker. If the decision maker did not provide a due date for project completion, then delay is calculated with respect to the latest finish time using the critical path method (CPM). The purpose of criterion (2.2) is to leverage resource usage. The project has its own resources available, but if needed, it can use other resources in the company; using those resources, however, leads to penalty. The criterion function (2.3) describes NPV maximization.

Constraint (2.4) defines a binary decision variable. This decision variable is equal to 1 when activity j lasts in time t, otherwise it is equal to 0. Each activity can be performed only once, and its duration is defined by (2.5). Equations (2.6) and (2.7) are used to calculate the activity completion and start time, respectively. An activity which has started cannot be stopped until it is completed (2.8). The lines (2.9)-(2.12) define precedence relationships of various types and (2.13) is the resource availability constraint.

3 Fuzzy Pareto dominance and fuzzy ranking

The subset of all vectors of a set A which are not dominated by any other vector of A is the Pareto set. The Pareto set for univariate data (single objective) contains solely the maximum of the data (Köppen et al., 2005).

Given two vectors a and b we say that a (Pareto-) dominates b when each component of a is less than or equal to the corresponding component of b, and at least one component is smaller:

$$a >_D b \leftrightarrow \forall i \ (a_i \le b_i) \ \wedge \ \exists k (a_k < b_k)\,. \tag{3.1}$$

The fuzzification of the Pareto dominance relation is given by the following definition:

We say that vector a dominates vector b with degree μ_a given by the formula:

$$\mu_a(a,b) = \frac{\Pi_i \min(a_i, b_i)}{\Pi_i a_i} \tag{3.2}$$

and that vector a is dominated by vector b with degree μ_p given by the formula:

$$\mu_p(a,b) = \frac{\Pi_i \min(a_i, b_i)}{\Pi_i b_i}\,. \tag{3.3}$$

The definition of Fuzzy Pareto Dominance is illustrated in Figure 1.

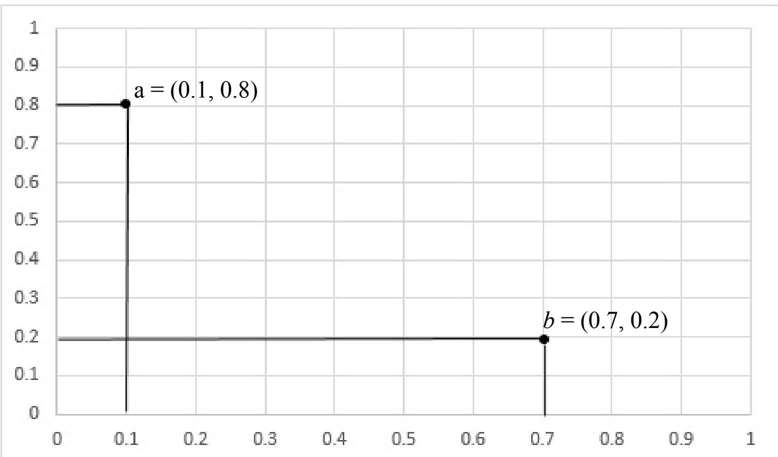

Figure 1. Definition of Fuzzy Pareto Dominance

Source: Based on: Köppen et al. (2005).

Of the two vectors $a = (0.1, 0.8)$ and $b = (0.7, 0.2)$, vector a dominates vector b with degree:

$$\mu_a(a,b) = \frac{0.1 \cdot 0.2}{0.1 \cdot 0.8} = 0.25,$$

and vector a is dominated by vector b with degree:

$$\mu_p(a,b) = \frac{0.1 \cdot 0.2}{0.7 \cdot 0.2} \approx 0.143.$$

We may use these dominance degrees to rank the elements of a set A of multivariate data (vectors) such as the fitness values of a multiple objective optimization problem.

Each element of A is assigned the maximum degree of being dominated by any other element of A:

$$r_A(a) = \max_{b \in A \setminus \{a\}} \mu_p(a,b). \tag{3.4}$$

Next, the elements of A are sorted in increasing order according to the ranking values.

4 Comparison of the Fuzzy Pareto Dominance-Driven Evolutionary Algorithm with the SPEA2 algorithm

The fuzzy Pareto dominance-driven algorithm has been developed on the basis of the SPEA2 (*Strength Pareto Evolutionary Approach 2*) algorithm, which is an elitist algorithm. As research shows (Zitzler, 1999), elitism in evolutionary algorithms can improve the results obtained.

4.1 Evolutionary algorithm scheme

The SPEA2 algorithm consists of the following steps (Zitzler et al., 2001):

Input:
N – population size,
$\overline{N}$ – size of external set,
G − maximum number of generations.

Output:
A − set of non-dominated solutions.

Step 1: Initialization

The initial population P_0 is generated and an empty external set $\overline{P_0}$ is created.

Step 2: Performance

Fitness assignment is performed for individuals from the sets P_0 and $\overline{P_0}$.

Step 3: Selection and external set updating

All non-dominated solutions are copied from the sets $\overline{P_g}$ and P_g to the set $\overline{P_{g+1}}$.

Step 4: Termination

If the stopping criterion is satisfied then set A is a set of decision vectors represented by the non-dominated individuals in $\overline{P_{g+1}}$.

Step 5: Mating selection

A tournament selection with replacement on $\overline{P_{g+1}}$ to fill the mating pool is conducted.

Step 6: Variation

Genetic operators are applied to individuals from the mating pool. The population P_{g+1} is the result of the variation.

4.2 Characteristics of the algorithm

The fuzzy Pareto dominance-driven evolutionary algorithm differs from the SPEA2 algorithm in two respects: performance and environmental selection.

In the SPEA2 algorithm the performance *F(i)* is calculated using the following equation:

$$F(i) = R(i) + D(i). \tag{4.1}$$

At first a strength value *S(i)* is assigned to each individual. It represents the number of individuals that the individual *i* dominates:

$$S(i) = |\{j \mid j \in P_g + \overline{P_g} \wedge i \succ j\}|. \tag{4.2}$$

Then a raw fitness of individual *i* is calculated:

$$R(i) = \sum_{j \in P_g + \overline{P_g}, j \succ i} S(j). \tag{4.3}$$

Individuals are discriminated from each other using density information. The density estimation technique is an adaptation of the *k-th* nearest neighbor method (Silverman,1986), where the density at any point is a (decreasing) function of the distance to the *k-th* nearest data point. For each individual the distances (in objective space) to all individuals in archive and population are calculated and stored in a list. Once the list is sorted in increasing order, the *k-th* element gives the distance sought, denoted by σ_i^k.

The density is defined by:

$$D(i) = \frac{1}{\sigma_i^k + 2}. \tag{4.4}$$

Individuals with the fitness value *F(i)* lower than 1 are non-dominated.

In the fuzzy Pareto dominance-driven evolutionary algorithm a ranking of all individuals is calculated (according to the scheme described in section 3). After assigning to each element of A the maximum degree of being dominated by any other elements of A, we sort the individuals in increasing order:

$$r_A(a) = \max_{b \in A \setminus \{a\}} \mu_p(a,b) . \tag{4.5}$$

The higher the position in the ranking, the better the individual performance.

The next aspect in which the SPEA2 algorithm and the fuzzy Pareto dominance-driven evolutionary algorithm differ is environmental selection.

In the SPEA2 algorithm the individuals are selected to the external set according to the following rule:

$$\overline{P}_{g+1} = \{i \mid i \in P_g + \overline{P_g} \wedge F(i) < 1\} . \tag{4.6}$$

If $\overline{P_{g+1}}$ is larger than the external set, it is reduced; if it is smaller, it is filled with dominated individuals from $\overline{P_g}$ and $P_{g.}$

In the fuzzy Pareto dominance-driven evolutionary algorithm the first N individuals from the ranking are copied to the external set. No additional set reduction or selection of individuals to the external set is required.

4.3 Other elements of algorithm

Individual

Binary variables are used in the scheduling problem described in section 2. In this paper an individual is a binary matrix with J rows and T columns. Activities are presented in rows and time units are presented in columns. The individual i can be presented as follows:

$$Ch_i = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,T} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,T} \\ \vdots & \vdots & \vdots & \vdots \\ x_{J,1} & x_{J,2} & \dots & x_{J,T} \end{bmatrix} .$$

For the initial population only feasible solutions are generated.

Crossover

A crossover process proposed in this paper is conducted in two phases. In the first phase the individuals for which crossover will be performed are randomly chosen from the population and the crossover point is chosen, also randomly. The crossover point is the row number. In the second stage the chosen row is exchanged between the two individuals.

Mutation

In the proposed solution, mutation is a process of delaying a randomly chosen activity. The activity is delayed by one time unit.

Constraints considering

The mathematical model presented in section 2 contains constraints which should be taken into account in the algorithm. In the approach proposed in this paper a penalty is foreseen for each not feasible solution. The penalty makes the individual performance dramatically worse, to reduce the probability of such individual reproduction.

5 Experimental results

In this section the fuzzy Pareto dominance-driven evolutionary algorithm will be used to solve an example of the multiple objective project scheduling problem.

A project consisting of 13 activities (Figure 2) is to be scheduled.

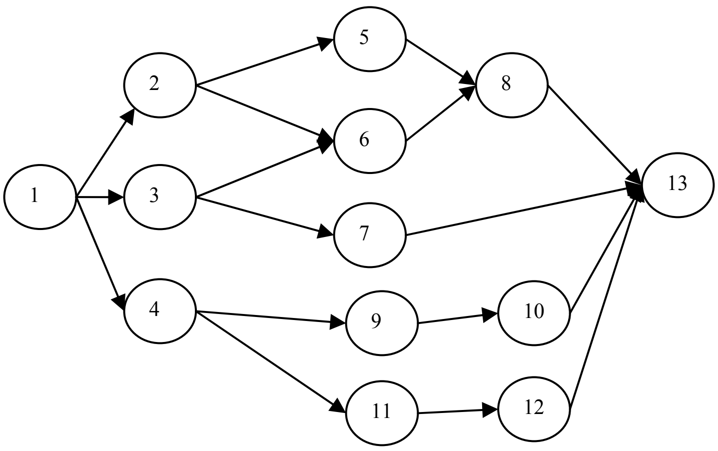

Figure 2. AON network for the example

Source: Prepared by the author.

For each activity a deterministic duration is given (Table 1). For the realization of the project two resource types are required: *k1* and *k2*. The amount of resources required by each activity is given in Table 1. The availability of resource *k1* is restricted and equal to 1 units in the project and to 2 in the company (in each time unit). The availability of resource *k2* is restricted and equal to 3 units in the project (in each time unit) and to 5 in the company. For each activity the cash flows generated by it are determined.

Table 1: Data for the example

Activity	Duration	Resource *k1* requirement	Resource *k2* requirement	Net cash flows
1	4	1	3	-2000
2	9	1	2	-1000
3	10	0	3	-2000
4	8	0	3	-1000
5	13	0	2	-2000
6	8	1	1	-2000
7	4	0	2	2000
8	5	0	3	4000
9	13	1	2	6000
10	12	1	2	8000
11	10	0	2	10000
12	12	1	0	12000
13	10	1	5	15000

The following parameters have been set for computations:
- Population size: 10 individuals,
- Crossover probability: 90%,
- Mutation probability: 10%,
- Number of generations: 100,
- Size of external set: 5.

After 100 generations the following set has been obtained (Table 2):

Table 2: Set of solutions after 100 iterations of the fuzzy Pareto dominance-driven evolutionary algorithm

Solution	Objective 1 (min)	Objective 2 (min)	Objective 3 (max)
1A	1 500	144	7 537
2A	1 300	152	19 891
3A	1 100	150	8 334
4A	1 100	150	8 633
5A	1 100	141	8 846

The solutions are ordered according to their ranking, so we can assume that the first solution is the best one. Its maximum value with which this solution is dominated by the other solutions in this set is the smallest. During the analysis of these solutions, we can conclude that:
- solution 1A is dominated by solution 5A,
- solution 3A is dominated by solutions 4A and 5A,
- solution 4A is dominated by solution 5A.

Solutions 2A and 5A are non-dominated.

In the next step we have performed 100 iterations of the SPEA2 algorithm and the following solutions have been obtained (Table 3).

Table 3: Set of solutions after 100 iterations of SPEA2

Solution	Objective 1 (min)	Objective 2 (min)	Objective 3 (max)
1B	1 300	152	19 891
2B	1 000	151	12 256
3B	700	169	14 955
4B	1 100	156	13 702
5B	1 100	141	8 846

Comparing Tables 2 and 3 we can see that solution 1B is identical with 2A and solution 5B, with 5A. Solutions 1A, 3A and 4A are dominated by 2A and 5A, but not by 2B, 3B and 4B.

Now we will mutually compare all solutions using fuzzy Pareto dominance (Table 4).

Table 4: Comparison of all solutions

Solution	max
1A	0.0017629
2A, 1B	0.0015279
3A	0.0012928
4A	0.0012928
5A, 5B	0.0012928
4B	0.0012928
2B	0.0011753
3B	0.0008227

From Table 4 we can see that solutions 2B, 3B and 4B identified by the SPEA2 algorithm but not by the fuzzy Pareto dominance-driven algorithm are on the last 3 positions in the fuzzy ranking. What is interesting, solutions 1A, 3A and 4A are dominated by solutions 2A (1B) and 5A (5B), but solutions 2B, 3B and 4B are not dominated by solutions 2A (1B) and 5A (5B). Comparing solutions 1A, 3A and 4A with solutions 2B, 3B and 4B we are unable to find any dominance relationship between them.

6 Summary

In this paper a project scheduling problem has been described as a multiple objective decision making problem. It has been solved using the fuzzy Pareto dominance-driven evolutionary algorithm.

Applying fuzzy Pareto dominance in an evolutionary algorithm seems to make the performance of the individuals and environment selection (also selection to the external set) better. Additionally, thanks to the fuzzy ranking scheme it is clear which solution should be chosen as the final one – we should always choose the highest-ranking solution. In other evolutionary algorithms for multiple objective problems we obtain a set of solutions, and then we should choose one of them. In the case of the SPEA2 algorithm we can choose any solution from the set of solutions with the objective function *F(i)* lower than 1, as those are non-dominated solutions.

Both algorithms ended with similar solutions, and even though some papers report that the evolutionary algorithm using fuzzy Pareto dominance is more effective (Köppen et al., 2005), it is difficult to conclude the same from the present paper. This may be caused by the small size of the example presented in this paper and that is why additional experiments should be conducted. Therefore, in a future study a larger example will be considered.

Acknowledgments

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