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A SOLVING PROCEDURE FOR THE MULTIOBJECTIVE DYNAMIC PROBLEM WITH CHANGEABLE GROUP HIERARCHY OF STAGE CRITERIA DEPENDENT ON THE STAGE OF THE PROCESS

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Abstract

We consider multiobjective, multistage discrete dynamic decision processes. In this paper we propose an interactive procedure which allows to solve the problem of optimal control of such a process in the case when the decision maker has determined a group hierarchy of stage criteria. This hierarchy is changeable and depends on the stage of the process. The proposed algorithm is illustrated by a numerical example.

Keywords: multiobjective multistage decision process, multiobjective dynamic programming, hierarchical problem, group hierarchy.

1 Introduction

The present paper is a continuation of the discussion conducted in Trzaskalik (in press). We consider decision processes consisting of a finite number of stages, determined by the decision maker. The decisions are made at the beginning of the consecutive stages and evaluated using many evaluation criteria. In the evaluation of the feasible process realizations we will use both stage criteria, which are related to the specific stages of the process, and multistage criteria, used to evaluate the overall realization of the process. Problems of this type are classified as problems of multiobjective dynamic programming. We consider the most frequently occurring situation, in which multistage criteria are sums of stage criteria.

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When formulating the issue of process realization evaluation we refer to the general notion of optimality in multiobjective problems (Steuer, 1986). We assume that the components of the vector criteria function are the consecutive multistage criteria. As vector-optimal realizations we admit those which are non-dominated (in the criteria space) or efficient (in the decision space) (Trzaskalik, 1990).

Among the varied topics dealt with currently there are many problems in which the hierarchization of the evaluation criteria is an essential element. An overview of the problems discussed has been presented in Trzaskalik (in press).

A change in importance of the criteria often influences decision making. Not infrequently, to achieve a better stage evaluation of a criterion which is important at the given stage, the decision maker is inclined to give up on the optimization of the realization of the multistage objectives. Obtaining such immediate profits can, however, have a very negative impact on the evaluation of the entire process. For that reason, in the case of criteria hierarchization, it seems justified to focus the analyses on the values of both the stage and multistage criteria.

The present paper attempts to answer the question about the method of controlling a multistage process so as to take into account at the same time both the tendency to multiobjective optimization of the entire process and the time-varying group hierarchy of stage criteria. We will discuss in detail one of many possible situations, in which the stage hierarchy varies in the consecutive stages and depends on the stage. We will present an interactive proposal of the solution of this problem, in which the decision maker actively participates in the process of finding the final realization of the process.

The present paper consists of six sections. In Section 2, we define the notation used and present the notion of vector optimization for a multiobjective decision process. Section 3 describes the idea of the group hierarchy of criteria. In Section 4 we formulate the hierarchical problem discussed in the paper and propose a solution procedure. A detailed solution of an illustrative numerical example is in Section 5. A summary completes the paper.

2 Multistage, multiobjective discrete decision process (Trzaskalik, in press)

We define $\overline{1,T}$ to be the set of all integer numbers from 1 to T and denote it as follows:

$$\overline{1,T} = \{1,\ldots,T\} \tag{1}$$

We consider a discrete decision process consisting of T stages. Let y_t be the state variable at the beginning of the stage number t, Y_t – the set of all feasible state variables for stage t, x_t – the decision variable for stage t and $X(y_t)$ – the set of all feasible decision variables for stage t and state y_t. We assume that all sets of states and decisions are finite. A stage realization is defined as follows:

$$d_t \cong (y_t, x_t) \tag{2}$$

Let D_t be the set of all stage realizations in stage t. We define $d_t(y_t)$ as the stage realization which begins in state y_t. The set of all stage realizations which begin in a given state y_t is defined as follows:

$$D_t(y_t) = \{d_t(y_t) \in D_t\colon d_t = (y_t, x_t) \wedge x_t \in X_t(y_t)\} \tag{3}$$

We assume that for $t \in \overline{1,T}$ the transformations:

$$\Omega_t\colon D_t \rightarrow Y_{t+1} \tag{4}$$

are given. A sequence of stage realizations:

$$(d_1,\ldots,d_T) = (y_1, x_1, y_2, x_2,\ldots,y_T, x_T) \tag{5}$$

is called a process realization and denoted as d, if:

$$\forall_{t\in\overline{1,T}}\, y_{t+1} = \Omega_t(y_t, x_t) \tag{6}$$

Let D be the set of all process realizations.

We assume that we consider K criteria and that for each stage t and $k \in \overline{1,K}$, stage criteria functions $F_t^k : D_t \rightarrow R$ are defined. For the given realization d we obtain the values:

$$\begin{array}{cccc}
F_1^1(d_1) & F_1^2(d_1) & \ldots & F_1^K(d_1) \\
\ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\
F_T^1(d_T) & F_T^2(d_T) & \ldots & F_T^K(d_T)
\end{array}$$

F is a vector-valued criterion function for the evaluation of the entire process and its components F^k, $k \in \overline{1,K}$, are defined as follows:

$$F^k(d) = \sum_{t=1}^{T} F_k^T(d_t) \tag{7}$$

We postulate maximization of all the components of F.

Let us assume that two process realizations: $\bar{d}$, $\tilde{d}$ and vectors:

$$F(\bar{d}) \cong [F^1(\bar{d}), \ldots F^K(\bar{d})]$$
$$F(\tilde{d}) \cong [F^1(\tilde{d}), \ldots F^K(\tilde{d})]$$

are given. The relation of domination ≥ is defined as follows:

$$F(\bar{d}) \geq F(\tilde{d}) \Leftrightarrow \forall_{k\in\overline{1,K}} [F^k(\bar{d}) \geq F^k(\tilde{d})] \wedge \exists_{i\in\overline{1,K}} [F^i(\bar{d}) > F^i(\tilde{d})] \tag{8}$$

If $F(\bar{d}) \geq F(\tilde{d})$ we say that vector $F(\bar{d})$ dominates vector $F(\tilde{d})$ and realization $\bar{d}$ is better than realization $\tilde{d}$. Realization $\overset{*}{d}$ is said to be efficient if:

$$\sim \underset{\bar{d} \in D}{\exists} F(\bar{d}) \geq F(\overset{*}{d}) \tag{9}$$

Let $\overset{*}{D}$ be the set of all efficient realizations for the given criterion function F. The problem of finding $\overset{*}{D}$ is called the dynamic vector optimization problem. The set:

$$\overset{*}{D}(\bar{d}) \cong \{\overset{*}{d} \in \overset{*}{D} : F(\overset{*}{d}) \geq F(\bar{d})\} \tag{10}$$

consists of all efficient realizations which are better than $\bar{d}$. The algorithm of finding the set of all efficient realizations of the process and the algorithm of finding the set of efficient realizations better than the chosen one is described in Trzaskalik (1990).

3 Group hierarchy of criteria

The issue of hierarchization of criteria has been presented many times in the literature dealing with multiobjective decision making, in particular in papers on goal programming. This hierarchization is understood in two ways. In the first approach, the criteria are assigned weight coefficients and the importance of a criterion is reflected by the appropriate value of this coefficient: the more important the criterion, the larger the value of the weight coefficient. In the second approach, hierarchy levels are introduced. Criteria on higher levels are regarded as more important than those on lower levels; criteria on the same level are equally important for the decision maker. For criteria situated at the same hierarchy level weight coefficients can also be used (Jones, Tamiz, 2010).

When hierarchy levels are used, we can introduce a single hierarchy or a group hierarchy. In the former case, a hierarchy level contains only a single criterion. In the latter case, a hierarchy level can contain more than one criterion (Galas, Nykowski, Żółkiewski, 1987).

In a discussion of hierarchical problems with a single criteria hierarchy it is important to create an appropriate numbering of criteria. The criteria can be numbered so as to assign the number 1 to the most important criterion, the number 2 to the second-most important criterion – one that is less important than criterion number 1 but more important than all the remaining criteria, and so on. A similar method of numbering can be applied in the case of group hierarchy. Criteria from a more important group will have numbers lower than all the less important criteria; criteria from the same group are equally important. Therefore, the numbering of criteria within one group is ambiguous.

The issue of criteria hierarchization discussed above appears also when multistage decision processes are considered. In such cases, both stage criteria and multistage criteria can occur. When a hierarchy of stage criteria is established, we can hierarchize multistage criteria in the same way as described above.

A different situation occurs when the importance of stage criteria for the decision maker vary from stage to stage. This is the case of a changeable stage hierarchy. We assume that at the given stage, stage criteria have been divided into a certain number of groups, depending on their importance. Each group contains criteria which are equally important for the decision maker. Moreover, a hierarchy of stage criteria can undergo changes in the consecutive stages.

The issue of hierarchization of multistage and stage criteria was discussed before by the present author. In Trzaskalik (1997) the issue of searching for the best process realization was discussed, in the situation when a hierarchy of multistage criteria was given. Each time when the consecutive (with respect to importance) criteria were analyzed, the stage structure of the consecutive process realizations was analyzed. The changeability of hierarchies of stage criteria was discussed in other papers, too. In Trzaskalik (1995) a hierarchy dependent on the joint value of the stage criteria obtained in previous stages was discussed, while in Trzaskalik (1992), a hierarchy dependent on the current state of the process. These discussions were continued in Trzaskalik (1998a, 1998b), which dealt also with the case of group hierarchy. Changeable, weighted relevance of stage objectives was discussed in Trzaskalik (2009). In each of those cases, the process realization, which satisfies best the assumptions regarding the hierarchization of stage and multistage criteria, was compared with the set of efficient realizations.

The changeable group hierarchy of stage criteria discussed further will be illustrated by an example. We consider a 3-stage process. In stage 1, the stage criteria are F_1^1, F_1^2, F_1^3 i F_1^4, in stage 2 they are F_2^1, F_2^2, F_2^3 i F_2^4, in stage 3 they are F_3^1, F_3^2, F_3^3 i F_3^4. In the proposed notation the lower index is the stage number, while the upper index is the criterion number. We call the criteria with the same value of the upper index single-name criteria; they refer to the same aspect of the process under consideration.

An possible method of dividing the criteria could be the following. In stage 1 the decision maker divided the criteria into two groups: more important: F_1^2 and F_1^3 and less important: F_1^1 and F_1^4. In stage 2 all the criteria are equally important, therefore we have a single group of stage criteria. In stage 3 the criteria were divided into three groups. The first group contains only one, the most important, criterion $F_3^{2.}$ The second group contains the second-most important criterion F_3^1. The third group contains the two least important criteria F_3^3 and F_3^4.

Denoting as $\boldsymbol{K}_t^i$ the ith most important group of criteria at stage t, and by I_t^i , the set of indices corresponding to the numbers of stage criteria in set $\boldsymbol{K}_t^i$, we can write:

$\boldsymbol{K}_1^1 = \{F_1^2, F_1^3\}$ $\boldsymbol{K}_1^2 = \{F_1^1, F_1^4\}$

$\boldsymbol{K}_2^1 = \{F_2^1, F_2^2, F_2^3, F_2^4\}$

$\boldsymbol{K}_3^1 = \{F_3^2\}$ $\boldsymbol{K}_3^2 = \{F_3^1\}$ $\boldsymbol{K}_3^3 = \{F_3^3, F_3^4\}$

$I_1^1 = \{2, 3\}$, $I_1^2 = \{1, 4\}$

$I_2^1 = \{1, 2, 3, 4\}$

$I_3^1 = \{2\}$ $I_3^2 = \{1\}$ $I_3^3 = \{3, 4\}$

At stage 1 the criteria from group $\boldsymbol{K}_1^1$, that is, F_1^2 and F_1^3, are equally important; at the same time, each of them is more important than the remaining criteria for this stage, that is, F_1^1 and F_1^4. On the other hand, criteria F_1^2 and F_1^3 are equally important and less important than both F_1^1 and F_1^4.

At stage 2 all the criteria are equally important: none is less or more important, hence all of them belong to the same group $\boldsymbol{K}_2^2$.

At stage 3 criterion F_3^2 belonging to the (1-element) group $\boldsymbol{K}_t^i$ is more important than all the remaining criteria, that is, F_3^2, F_3^3 and F_3^4. Criterion F_3^1, belonging to the second-most important (1-element) group $\boldsymbol{K}_t^i$, is less important than F_3^2 but more important than the criteria from group $\boldsymbol{K}_3^3$, that is, F_3^3 and F_3^4. Finally, criteria F_3^3 and F_3^4 are equally important.

This example shows that the number of groups into which the criteria are divided can vary from stage to stage. In particular, at some stages, all criteria can be equally important. One-element criteria groups can also occur. Also, the composition of the groups can vary from stage to stage.

Further in the paper we assume that in each process stage t (t = 1, …, T) the decision maker divided the stage criteria into i_t groups denoted $\boldsymbol{K}_t^i$, with the corresponding sets of stage criteria numbers denoted $I_t^i,\ i \in \overline{1, i_t}$. This way we obtain a division into the following groups of criteria:

for stage 1: $\boldsymbol{K}_1^1, \boldsymbol{K}_1^2, \ldots, \boldsymbol{K}_1^{i_1}$

for stage 2: $\boldsymbol{K}_2^1, \boldsymbol{K}_2^2, \ldots, \boldsymbol{K}_2^{i_2}$

……………………………..

for stage T: $\boldsymbol{K}_T^1, \boldsymbol{K}_T^2, \ldots, \boldsymbol{K}_T^{i_T}$

and sets of indices:

for stage 1: $I_1^1, I_1^2, \ldots, I_1^{i_1}$

for stage 2: $I_2^1, I_2^2, \ldots, I_2^{i_2}$

……………………………..

for stage T: $I_T^1, I_T^2, \ldots, I_T^{i_T}$

We assume that:

$$\forall_{t\in\overline{1,T}}\forall_{k,l\in\overline{1,i_t}} I_t^k \cap I_t^l = \varnothing$$

4 Description of the procedure

The purpose of the procedure described in this section is the selection of a process realization which, on the one hand, fulfills the decision maker's expectations as to the achievement of stage goals according to the group hierarchy described in the previous section and, on the other hand, realizes the multistage objectives in the best possible way. This procedure makes the decision maker aware of the consequences of the stage decisions which he/she makes to realize the multistage objectives. It also points out new possibilities which result from the analysis of both the stage and multistage objectives. Below we describe the consecutive stages of the procedure, comparing it with the procedure proposed in Trzaskalik (in press) for a single hierarchy of stage criteria.

Selection of the initial stage
We find the maximum value for each stage criterion F_1^j from group $\boldsymbol{K}_1^1$. We normalize the stage values for each criterion from this group. This allows to sum up the normalized values for each process state under consideration. As the initial state we propose to select the one for which the sum of normalized values is largest. If there are more than one such states, we can select any of them; the consecutive states will be considered when the procedure is repeated (if at all).

Satisfactory stage realizations
Stage realizations satisfactory with respect to the given group of multistage criteria are such process realizations for which the values of stage criteria are optimal or almost optimal in the given state. That is, their stage values are within the tolerance intervals given by the decision maker.

We solve the problem for the consecutive stages, starting with the first stage. At any given stage, we consider all the criteria groups consecutively, according to the hierarchy determined by the decision maker, starting with the group of the most important criteria.

When considering a given group of stage criteria, we take into account all stage decisions admissible for the given process state. We ask the decision maker to give a preliminary tolerance interval for the maximum values for all the stage criteria from the criteria group under consideration. As the initial set of satisfactory realizations we take those realizations for which all the stage values are within the given intervals. The cardinality of this set depends on the extent to which the decision maker is willing to give up the optimal values for the stage criteria from the given group. For that reason, if the tolerance intervals determined by the decision maker turn out to be too narrow, we suggest than he/she extends them. As a result, the decision maker agrees to lower even more the requirements as regards the criterion under consideration. On the other hand, if the cardinality of the realization set is too large, the decision maker can narrow the suggested tolerance interval, which guarantees better values for the criteria from this group in the final solution. When the decision maker accepts the tolerance interval, we obtain a set of realizations satisfactory with respect to the given group of stage criteria. This allows to consider the next most important group of stage criteria (if it exists).

Selection of the stage decision
When all the hierarchized criteria from each group of the consecutive hierarchy levels are considered in this manner, the decision maker selects the final stage decision from the last set of satisfactory stage realizations. To select this decision one can use the value of the index which characterizes the joint relative change of the value of the given realization with respect to the possible maximal changes of the individual stage criteria. A method of the construction of this index, analogous to that proposed in Trzaskalik (in press), will be presented in the detailed description of the algorithm. Once we know the stage decision we use the transfer function and determine the initial state in the next stage.

Generating a satisfactory process realization
This procedure is repeated for the consecutive stages, including the last one. The result is a satisfactory process realization which fulfills the decision maker's expectations as regards the levels of the stage criteria (according to the group hierarchy assumed). As in Trzaskalik (in press), we call this realization a satisfactory realization for short. It is added to the set of potential realizations, from among which the decision maker will make the final selection.

Testing of the efficiency of the satisfactory realization
This part of the procedure is analogous to the procedure for a single hierarchy described in Trzaskalik (in press). Using the procedure for efficiency testing we check if the generated satisfactory realization is an efficient realization. If it is not, we generate better efficient realizations and add them to the set of potential realizations, i.e., the realizations from among which the decision maker will select the final realization. Therefore, in the set of potential realizations we have a satisfactory realization and efficient realizations better than this one (if they exist).

Generating the consecutive satisfactory realizations

This procedure fragment is analogous to the one described in Trzaskalik (in press). The decision maker performs a preliminary analysis of the set of potential realizations; he/she can decide that this set suffices to make the final decision (which is to indicate one of the potential realizations as the final realization) or else may conclude that it is necessary to extend the set of potential realizations by repeating the entire procedure, taking as the initial state of the process one of the states not yet considered.

Selection of the final realization

If the decision maker does not see the need to expand the set of potential realizations, then he/she uses expert knowledge to analyze in detail (jointly with an analyst) the values of the stage and multistage criteria of all the potential realizations generated. As a result, the decision maker can select as the final decision that satisfactory realization which is at the same time an efficient one (if it exists, of course). As the final realization, the decision maker can also select a satisfactory realization which is not efficient or else an efficient realization which is not satisfactory.

Below is a detailed description of the algorithm proposed.

Algorithm

Step 1. The decision maker determines a group hierarchy of stage criteria for each stage; this hierarchy is described in detail in the previous subsection.

Step 2. Denote by D^P the set of potential realizations and set $D^P = \varnothing$.

Step 3. Consider *stage* criteria from group $\boldsymbol{K}_1{}^1$. These are criteria $F_1{}^j$ with $j \in I_1$. The set Y_1 of states for the first stage is finite. Assume that it consists of N elements which can be written as the following sequence:

$$Y_1 = \{y_1{}^{(1)}, y_1{}^{(2)}, \ldots, y_1{}^{(N)}\} \tag{11}$$

For each stage criterion $F_1{}^j$ from set $\boldsymbol{K}_1{}^1$ calculate the maximum value:

$$F_1^{j^*} = \max_{d_1 \in \boldsymbol{D}_1} F_1^j(d_1) \tag{12}$$

For all stage realizations d_1 from set D_1 calculate the normalized values:

$$f_1^j(d_1) = \frac{F_1^j(d_1)}{F_1^{j^*}} \tag{13}$$

For the consecutive stage criteria from set $\boldsymbol{K}_1{}^1$ and for the consecutive initial states for Stage 1, that is for $y_1{}^{(n)} \in Y_1$ calculate:

$$S^j\left(y_1^{(n)}\right) = \sum_{x_1 \in \boldsymbol{X}_1\left(y_1^{(n)}\right)} f_1^j\left(y_1^{(n)}, x_1\right) \tag{14}$$

For the consecutive initial states for Stage 1, sum up the normalized values for all stage criteria from set $\boldsymbol{K}_1^{\ 1}$:

$$S\left(y^{(n)}\right)=\sum_{j\in \boldsymbol{I}_1^1} S^j\left(y_1^{(n)}\right) \tag{15}$$

As the initial state select state $y_1^{(p)}$, for which the sum $S(y_1^{(p)})$ is largest.
If the decision maker does not accept this proposal, ask him/her to indicate the preferred initial state.
Step 4. Set $t = 1$.
Step 5. Set $y_t = y^{(p)}$.
Step 6. Set $D_t = D_t(y^p)$.
Step 7. Set $i = 1$.
Step 8. Set $I = I_t^i$.
Step 9. If $I = \varnothing$, go to Step 13.
Step 10. Select $j\in I$, set $I = I \setminus \{j\}$.
Step 11. Find stage process realization $d_t^{j*}(y_t) \in D_t$, for which stage criterion F_t^j has its maximum value F_t^{j*}.
Step 12. Inform the decision maker what the value of F_t^j* is and ask him/her to give the value ε_t^j which determines the tolerance interval $[F_t^j* - \varepsilon_t^j, F_t*]$ for the criterion under consideration.
Step 13. Select those stage realizations from set D_t for which criterion F_t^j attains a value from the interval $[F_t^j* - \varepsilon_t^k, F_t^j*]$. Denote the set of these stage realizations by $D_t^{(j)}$. Return to Step 9.
Step 14. Find the intersection of sets $D_t^{(j)}$:

$$D_t^i = \bigcap_{j\in I_t^i} D_t^{(j)} \tag{16}$$

Step 15. Inform the decision maker about the cardinality of the set obtained and ask for approval. If the decision maker accepts this cardinality, go to Step 17.
Step 16. If the decision maker finds this cardinality too large or too small, ask him/her to repeat the analysis of set $\boldsymbol{K}_t^{\mathrm{i}}$. Return to Step 8.
Step 17. Check if $i = i_t$. If so, go to Step 19.
Step 18. Set $D_t = D_t^{i_t}$ and $i = i + 1$. Return to Step 8.
Step 19. Select the preferred stage realization from the reduced set D_t of realizations as described below. Check if there are dominated stage realizations in set D_t. If so, delete them. Assume that D' has cardinality P_t'. For each stage realization $d_t^{(p)} \in D_t'$ calculate the coefficient f_{pk} for the consecutive criteria, by dividing $F_t^k(d_t^{(p)})$ by the largest obtainable value of stage criterion F_t^k in $\boldsymbol{D}_t^{i_t}$. We obtain:

$$f_{pk} = \frac{F_t^k\left(d_t^{(p)}\right)}{\max\limits_{d\in D'} F^k\left(d_t\right)} \tag{17}$$

Form matrix $\boldsymbol{F} = [f_{pk}]$ of size $P' \times K$ with these values. As the stage decision, suggest to take that decision numbered p^o for which the sum of the elements of the corresponding row in matrix F is largest.
If the decision maker does not accept this suggestion, he/she should perform the selection independently, by analyzing the values of matrix $\boldsymbol{F}$.
Step 20. Check if $t = T$. If so, go to Step 23.
Step 21. Using the transfer function, determine the process state at the end of the stage. This state is at the same the initial state for the next stage. We have:

$$y_{t+1} = \Omega_t(y_t, x_t) \tag{18}$$

Step 22. Set $t = t + 1$, $y_t = y_{t+1}$, $D_t = D_t(y_t)$ and go to Step 7.
Step 23. Let d^o be the process realization obtained. Add d^o to the set D^P of potential realizations.

$$D^P = D^P \cup \{d^o\} \tag{19}$$

Step 24. Using the algorithm for efficiency testing, check if the generated realization is efficient. If not, generate the set $D^*(y^*)$ of efficient realizations better than the realization obtained.
Step 25. Add the realizations from set $D^*(y^*)$ (if any) to set D^P of potential realizations.

$$D^{P:} = D^P \cup D^*(y^*) \tag{20}$$

Step 26. Ask the decision maker to perform a preliminary analysis of set D^p. Ask the decision maker if he/she want to extend this set by repeating the procedure to obtain another satisfactory realization. If not, go to Step 28.
Step 27. Ask the decision maker to indicate as the next initial state a state not previously considered. Go to Step 3.
Step 28. The decision maker, using expert knowledge, analyzes the set of potential decision, taking into account the stage hierarchy and the value of the stage and multistage criteria. As a result, the decision maker:

a) indicates one of the potential realizations as the final realization,
b) repeats the procedure starting with Step 2, obtaining a new potential realization,
c) eliminates certain realizations obtained previously from the set of potential realizations,
d) changes the stage hierarchy and repeats the entire procedure,
e) gives up making the decision using the procedure described above.

5 Numerical example

We consider a two-stage decision process in which the transfer function is of the form:

$$y_{t+1}^{(j)} = \Omega_t(\mathrm{y}_t^{(i)}, x_t^{(j)}) = x_t^{(j)}$$

that is, the decision consists in the selection of the initial state for the next stage.

We denote stage realization d_t^{ij}, which begins in state $y_t^{(i)}$, when decision $x_t^{(j)}$ is taken, as follows:

$$d_t^{ij} = (y_t^{(i)}, x_t^{(j)})$$

The values of the stage criteria, the same in both stages, are shown in Tables 1-3.

Table 1: Values of stage criteria F_t^1 ($t = 1, 2$)

	Decision	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
State	0	420	451	433	494	462	400	455	459	438	452
	1	443	417	499	429	486	498	438	494	424	436
	2	429	490	491	434	494	484	420	480	458	482
	3	430	489	413	492	488	434	487	423	482	496
	4	414	407	418	409	460	456	454	452	419	446
	5	454	489	409	454	416	413	439	441	434	492
	6	455	462	427	483	460	437	456	493	468	436
	7	438	439	494	449	446	422	491	437	425	455
	8	490	418	449	410	429	454	439	422	434	438
	9	437	424	447	497	433	480	488	464	406	492

Table 2: Values of stage criteria F_t^2 ($t = 1, 2$)

	Decision	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
State	0	69	66	69	59	54	64	55	63	58	60
	1	59	55	63	62	53	67	61	65	62	69
	2	57	63	65	54	55	52	56	59	69	61
	3	52	67	61	61	69	59	65	51	52	69
	4	68	52	64	56	56	62	67	66	67	61
	5	53	65	69	63	68	50	50	58	64	54
	6	58	58	65	52	69	61	57	54	56	57
	7	51	56	63	58	52	53	52	60	53	62
	8	52	58	69	51	50	50	56	51	55	54
	9	60	63	60	52	51	53	69	59	63	53

Table 3: Values of stage criteria F_t^3 ($t = 1, 2$)

	Decision	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
State	0	153	162	177	180	182	189	182	178	189	157
	1	152	175	151	156	176	179	161	153	170	166
	2	186	175	168	176	174	173	175	152	188	151
	3	189	152	167	159	162	189	157	159	150	190
	4	180	177	158	176	186	158	170	172	172	180
	5	172	172	155	167	153	174	178	160	179	158
	6	162	156	172	186	180	157	155	150	172	162
	7	151	170	167	169	173	168	174	159	150	154
	8	159	158	165	154	155	171	152	185	165	162
	9	176	153	190	164	150	180	161	155	166	159

We proceed to show an application of the procedure proposed.

Step 1. The decision maker provides a group hierarchy of stage criteria for the consecutive stages. We have: $\boldsymbol{K}_1^1 = \{F_1^1, F_1^2\}$, $\boldsymbol{K}_1^2 = (F_1^3\}$, $\boldsymbol{K}_2^1 = \{F_2^3\}$, $\boldsymbol{K}_2^2 = \{F_2^1, F_2^2\}$, $I_1^1 = \{1, 2\}$, $I_1^2 = \{3\}$, $I_2^1 = \{3\}$, $I_2^2 = \{1, 2\}$.

Step 2. Set $D^P = \varnothing$.

<u>*Selection of the initial state*</u>

Step 3. The decision maker accepted the proposed selection of the initial state, presented in the description of the algorithm. The detailed calculations are shown in the appendix. As the initial state we take $y_1^{(2)}$.

<u>***Stage 1***</u>

Step 4. Set $t = 1$.

Step 5. Set $y_t = y_1^{(p)}$.

Step 6. Set $D_t = D_t(y_t^{(p)})$.

Step 7. Set $i = 1$.

<u>*First group of criteria*</u>

Step 8. Set $I = I_1^1 = \{1, 2\}$.

Step 9. We have $I \neq \varnothing$.

Step 10. Select $j = 1$, set $I = I \setminus \{1\} = 2$.

Step 11. Criterion F_1^1 has its maximum value for stage realization d_1^{24}. We have $F_1^{1*}(d^{24}) = 494$.

Step 12. The decision maker determined $\varepsilon_1^1 = 49$, hence the tolerance interval for criterion F_1^1 is [445, 494].

Step 13. The following realizations are in the interval determined by the decision maker:

$$D_1^{(1)} = \{d_1^{21}, d_1^{22}, d_1^{24}, d_1^{25}, d_1^{27}, d_1^{28}, d_1^{29}\}.$$

Step 9. We have $I \neq \varnothing$.

Step 10. Select $j = 2$, set $I = I \setminus \{2\} = \varnothing$.

Step 11. Criterion F_1^2 has its maximum value for stage realization d^{28}. We have $F_1^{2*}(d^{28}) = 69$.

Step 12. The decision maker determined $\varepsilon_1^2 = 9$, hence the tolerance interval for criterion F_1^1 is [60, 69].

Step 13. In the interval determined by the decision maker there are realizations from the set:

$$D_1^{(2)} = \{d_1^{21}, d_1^{22}, d_1^{28}, d_1^{29}\}$$

Step 9. Since $I = \varnothing$, go to Step 12.

Step 14. Find the set:

$$D_1^1 := D_1^{(1)} \cap D_1^{(2)} = \{\{d_1^{22}, d_1^{28}\ d_1^{29}\}$$

Step 15. The decision maker accepts the cardinality of set D_1^1.

Second group of criteria

Step 17. Set $i = i + 1 = 2$. We have $i = 2 \leq 2 = i_2$.

Step 18. Set $D_t = D_1^1$.

Step 8. Set $I = I_1^2 = \{3\}$.

Step 9. We have $I \neq \emptyset$.

Step 10. Select $j = 3$, set $I = \mathrm{I} \setminus \{3\} = \emptyset$.

Step 11. Criterion F_1^3 has its maximum value on set D^1 for stage realization d^{28}. We have $F_1^{3*}(d^{28}) = 188$.

Step 12. The decision maker determined $\varepsilon_1^3 = 18$, hence the tolerance interval for criterion F_1^3 is [170, 188].

Step 13. In the interval determined by the decision maker there are realizations from the set:

$$D_1^{(2)} = \{d_1^{22}, d^{28}\}$$

Step 9. We have $I = \emptyset$.

Step 14. Find:

$$D_1^2 := D_1^{(3)} = \{d_1^{22}, d_1^{28}\}$$

Step 15. The decision maker accepts the cardinality of set D_1^2.

Step 17. We have $i = 2 = i_2$.

Selection of the stage realization

Step 19. Compare the values of the stage criteria for the stage realizations from set D^2. We have:

$\mathrm{F}_1^1(\mathrm{d}^{22}) = 491$, $\mathrm{F}_1^1(\mathrm{d}^{22}) = 65$, $\mathrm{F}_1^1(\mathrm{d}^{22}) = 168$

$\mathrm{F}_1^1(\mathrm{d}^{28}) = 458$, $\mathrm{F}_1^1(\mathrm{d}^{28}) = 69$ $\mathrm{F}_1^1(\mathrm{d}^{28}) = 188$

Create the matrix:

$$\begin{bmatrix} 1 & 0{,}942 & 0{,}0897 \\ 0{,}923 & 1 & 1 \end{bmatrix}$$

The sums of the elements are: for $d^{22} = 2{,}839$, for $d^{28} = 2{,}923$. Select d^{28}.

Step 20. We have: $t = 1 < T$.

Step 21. We have: $y_2 = \Omega_1(d^{28}) = y_2^{(8)}$

Step 22. Set $t = 1 + 1 = 2$, $y_t = y_{t+1}$, $D_2 = D_2(y_2^{(8)})$ and go to Step 7.

Stage 2

First group of criteria

Step 7. Set $i = 1$.

Step 8. Set $I = I_2^1 = \{3\}$.

Step 9. We have $I \neq \emptyset$.

Step 10. Select $j = 3$, set $I = I / \{3\} = \emptyset$.

Step 11. Stage criterion F_2^3 has its maximum value for stage realization d_2^{87}. We have $F_2^{3*}(y^{(8)}) = 185$ for $x_2^7 \in X_1(y^{(2)})$.

Step 12. The decision maker determined $\varepsilon_1^1 = 15$, hence the tolerance interval for criterion F_2^3 is [170, 185].
Step 13. The following realizations are in the interval determined by the decision maker:

$$D_2^{(3)} = \{d_2^{85}, d_1^{87}\}.$$

Step 9. We have $I = \varnothing$.
Step 14. Find the set:

$$D_1^2 := D_1^{(3)} = \{\{d_1^{22}, \mathrm{d}_1^{28}\}$$

Step 15. The decision maker does not accept the cardinality of set D_1^2 and suggests that it be extended. Set $I = I_2^1 = \{3\}$ and return to Step 9.
<u>*Extension of set D_1^2*</u>
Step 9. We have $I \neq \varnothing$.
Step 10. Select $j = 3$, set $I = I/\{3\} = \varnothing$.
Step 11. Criterion F_1^1 has its maximum value for stage realization d_2^{87}. We have $F_2^{3*}(y^{(8)}) = 185$ for $x_2^7 \in X_1(y^{(2)})$.
Step 12. The decision maker determined $\varepsilon_1^1 = 25$, hence the tolerance interval for criterion F_2^3 is [160, 185].
Step 13. The following realizations are in the interval determined by the decision maker:

$$D_2^{(3)} = \{d_2^{82}, d_2^{85}, d_1^{87}, d_2^{88}, d_2^{89}\}$$

Step 9. We have $I = \varnothing$.
Step 14. Find the set:

$$D_2^2 := D_1^{(3)} = \{d_2^{82}, d_2^{85}, d_1^{87}, d_2^{88}, d_2^{89}\}$$

Step 15. The decision maker accept the cardinality of set D_1^2.
<u>*Second group of criteria*</u>
Step 17. We have $i = 1 < i_t$.
Step 18. Set $D_t = D_t^1$, $i = i + 1$.
Step 8. Set $I = I_2^2 = \{1, 2\}$.
Step 9. We have $I \neq \varnothing$.
Step 10. Select $j = 1$, set $I = I/\{1\} = \{2\}$.
Step 11. Criterion F_2^1 has its maximum value for stage realization d_2^{85}. We have $F_2^{1*}(y^{(8)}) = 454$ for $x_2^5 \in X_2(y_2^{(8)})$.
Step 12. The decision maker sets $\varepsilon_2^1 = 20$. The tolerance interval is [434, 454].
Step 13. Determine the stage realizations which fall within the given tolerance interval. We have:

$$D_2^{(1)} = \{d_2^{82}, d_2^{85}, d_2^{88}, d_2^{89}\}$$

Step 9. We have $I \neq \varnothing$.
Step 10. Select $j = 2$, set $I = I/\{2\} = \varnothing$.

Step 11. Criterion F_2^2 has its maximum value for stage realization d_2^{82}. We have $F_2^{1*}(y^{(8)}) = 69$ for $x_2^5 \in X_2(y_2^{(8)})$.
Step 12. The decision maker sets $\varepsilon_2^1 = 7$. The tolerance interval is [62, 69].
Step 13. Determine the stage realizations which fall within the given tolerance interval. We have:

$$D_2^{(2)} = \{d_2^{82}\}$$

Step 9. We have $I = \varnothing$.
Step 14. Find the intersection of sets $D_t^{(j)}$:

$$D_2^2 = D_2^{(1)} \cap D_2^{(2)} = \{d_2^{82}\}$$

Step 15. The decision maker accept the cardinality of this set.
Step 17. Set $i = 2 = i_2$.
<u>*Selection of the stage realization*</u>
Step 19. Since D_2^2 has one element only, the preferred stage realization is d_2^{82}.
<u>*Generating a satisfactory process realization*</u>
Step 20. We have $t = 2 = T$.
Step 23. Add the generated process realization $d^{282} = (d_1^{28}, d_2^{82})$ to the set of potential realizations. We have:

$$D^P = D^P \cup \{d^{282}\} = \{d^{282}\}$$

<u>*Testing the efficiency of the satisfactory realization*</u>
Step 24. Using the algorithm of efficiency testing, check that the generated realization is efficient.
<u>*Selection of the final realization*</u>
Step 26. The decision maker does not want to extend the set D^P of potential realizations.
Step 28. The decision maker indicates d^{282} as the final realization.

6 Summary

The interactive procedure proposed in this paper allows to include the decision maker into the process of solving the problem. Of fundamental importance is here the decision maker's (or the advisory team's) expert knowledge. The key theoretical aspect of the proposed procedure is the use of the algorithm for testing the efficiency of the potential realizations generated at each stage and, related to this, the possibility of generating better efficient realizations (if they exist) and of performing appropriate comparisons. Such a situation did not occur in the presented example because the potential realization generated as a result of the algorithm turned out to be efficient, but it occurred in the numerical example in Trzaskalik (in press), which can be a model for such situations. The selection of the final realization is then performed using the decision maker's expert knowledge.

Further research should take into account numerical aspects of the proposed solutions, both for single hierarchy and for group hierarchy, discussed in the present paper. For this purpose one should perform simulations with randomly generated criteria values. Taking into account the significant number of the necessary courses of action, one should discuss the possibility of determining the proposed rules of behavior for the decision maker in the situations when he/she makes decisions and of automating these decisions.

Another direction of theoretical research should deal with extending the hierarchical approach to stochastic and fuzzy decision processes.

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Appendix. Determination of the initial state

We find the largest values for each state for the first-level criteria.

Criterion F_1^1

State	0	1	2	3	4	5	6	7	8	9	max
0	420	451	433	494	462	400	455	459	438	452	494
1	443	417	499	429	486	498	438	494	424	436	499
2	429	490	491	434	494	484	420	480	458	482	494
3	430	489	413	492	488	434	487	423	482	496	496
4	414	407	418	409	460	456	454	452	419	446	460
5	454	489	409	454	416	413	439	441	434	492	492
6	455	462	427	483	460	437	456	493	468	436	493
7	438	439	494	449	446	422	491	437	425	455	494
8	490	418	449	410	429	454	439	422	434	438	490
9	437	424	447	497	433	480	488	464	406	492	497

Criterion F_1^2

State	0	1	2	3	4	5	6	7	8	9	max
0	69	66	69	59	54	64	55	63	58	60	69
1	59	55	63	62	53	67	61	65	62	69	69
2	57	63	65	54	55	52	56	59	69	61	69
3	52	67	61	61	69	59	65	51	52	69	69
4	68	52	64	56	56	62	67	66	67	61	68
5	53	65	69	63	68	50	50	58	64	54	69
6	58	58	65	52	69	61	57	54	56	57	69
7	51	56	63	58	52	53	52	60	53	62	63
8	52	58	69	51	50	50	56	51	55	54	69
9	60	63	60	52	51	53	69	59	63	53	69

We normalize the values, sum them up for the consecutive states and select the largest value.

Criterion 1	State	0	1	2	3	4	5	6	7	8	9
	0	0,841683	0,903808	0,867735	0,98998	0,925852	0,801603	0,911824	0,91984	0,877756	0,905812
	1	0,887776	0,835671	1	0,859719	0,973948	0,997996	0,877756	0,98998	0,849699	0,873747
	2	0,859719	0,981964	0,983968	0,869739	0,98998	0,96994	0,841683	0,961924	0,917836	0,965932
	3	0,861723	0,97996	0,827655	0,985972	0,977956	0,869739	0,975952	0,847695	0,965932	0,993988
	4	0,829659	0,815631	0,837675	0,819639	0,921844	0,913828	0,90982	0,905812	0,839679	0,893788
	5	0,90982	0,97996	0,819639	0,90982	0,833667	0,827655	0,87976	0,883768	0,869739	0,985972
	6	0,911824	0,925852	0,855711	0,967936	0,921844	0,875752	0,913828	0,987976	0,937876	0,873747
	7	0,877756	0,87976	0,98998	0,8998	0,893788	0,845691	0,983968	0,875752	0,851703	0,911824
	8	0,981964	0,837675	0,8998	0,821643	0,859719	0,90982	0,87976	0,845691	0,869739	0,877756
	9	0,875752	0,849699	0,895792	0,995992	0,867735	0,961924	0,977956	0,92986	0,813627	0,985972
Criterion 2	0	1	0,956522	1	0,855072	0,782609	0,927536	0,797101	0,913043	0,84058	0,869565
	1	0,855072	0,797101	0,913043	0,898551	0,768116	0,971014	0,884058	0,942029	0,898551	1
	2	0,826087	0,913043	0,942029	0,782609	0,797101	0,753623	0,811594	0,855072	1	0,884058
	3	0,753623	0,971014	0,884058	0,884058	1	0,855072	0,942029	0,73913	0,753623	1
	4	0,985507	0,753623	0,927536	0,811594	0,811594	0,898551	0,971014	0,956522	0,971014	0,884058
	5	0,768116	0,942029	1	0,913043	0,985507	0,724638	0,724638	0,84058	0,927536	0,782609
	6	0,84058	0,84058	0,942029	0,753623	1	0,884058	0,826087	0,782609	0,811594	0,826087
	7	0,73913	0,811594	0,913043	0,84058	0,753623	0,768116	0,753623	0,869565	0,768116	0,898551
	8	0,753623	0,84058	1	0,73913	0,724638	0,724638	0,811594	0,73913	0,797101	0,782609
	9	0,869565	0,913043	0,869565	0,753623	0,73913	0,768116	1	0,855072	0,913043	0,768116
Σ		17,22898	17,72911	18,36926	17,35212	17,52865	17,24931	17,67404	17,64105	17,47475	17,96419

The suggested initial state is $y_1^{(2)}$.