MULTIPLE CRITERIA DECISION MAKING
Vol. 12 2017

Paweł Tadeusz Kazibudzki*

NEW RESULTS ON THE QUALITY OF RECENTLY INTRODUCED INDEX FOR A CONSISTENCY CONTROL OF PAIRWISE JUDGMENTS

DOI: 10.22367/mcdm.2017.12.07

Abstract

A system exists which meets a prescription of the efficacious multiple criteria decision making support methodology. It is called the Analytic Hierarchy Process (AHP). The consistency control of human pairwise judgments about their preferences towards alternative choices appears to be the crucial issue in this concept. This research examines the efficiency of a recently proposed consistency index grounded on the redefined idea of triads inconsistency within Pairwise Comparison Matrices. The quality of the recently introduced proposal is studied and compared to other ideas with application of Monte Carlo simulations coded and run in Wolfram Mathematica 8.0.

Keywords: pairwise comparisons, consistency control, AHP, Monte Carlo simulations.

1 Introduction

It can be noticed that a world is a complex system of interacting elements. For instance, the contemporary economy depends mostly on energy. The availability of energy, on the other hand, depends on geography and politics but politics depends on military strength which depends on technology and access to energy. A technology depends on ideas, innovations and resources but ideas and innovations also depend on politics for their acceptance and support…, and so on. It is obvious that human minds have not yet evolved to the point where they can clearly perceive these ultimate relationships and solve crucial issues associated with them like for example nuclear energy, environmental regulations or global

* Jan Długosz University in Częstochowa, Department of Law, Administration and Management, Częstochowa, Poland, e-mail: p.kazibudzki@ajd.czest.pl.

crisis concerning third world poverty, population migration issues, society aging problems, etc. In order to deal with complex and fuzzy social, economic, and political issues, people must be supported and directed on their way to order priorities, to agree that one goal out-weighs another from a perspective of certain criterion, to make tradeoffs in order to be able to serve the greatest common interest.

Obviously, we cannot trust our intuition, although many of us commonly do it, devising solutions for complex problems which demand reliable answers. There are many examples showing that our intuition fails in such situations. Moreover, there are also many examples that our intuition fails anyway, even then when problems are relatively simple but their solution requires of involvement, not one, but two human's hemispheres.

Many examples exist indicating the fact that human's intuition misleads. There is a common riddle: a brick weighs a kilogram and a half of the brick. The question asks: what is a weight of the brick? For some reasons, a majority of people asked about it, although mathematical calculations are very trivial, provides the following incorrect answer: a brick weighs a kilogram and a half. It is presumably the principal reason why scientists continuously deal with explanation and modeling of decisional problems in the way they could be widely comprehended. That is why many supportive methodologies have been elaborated in order to make decision-making process easier, more credible and sometimes even possible.

An overwhelming scientific evidence indicates that the unaided human mind is simply not capable to analyze simultaneously many different competing factors and then synthesize them for the purpose of rational decision. Miller's well known experiment of 1956, titled, 'The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Information Processing' (Miller, 1956) made clear – almost a century ago – that the human mind is limited when considering short-term memory and discriminating skills of more than seven items. This indicates that when confronted with multiple variables, the choice made is less rational; and conversely, the less rational, the more alternatives available. This condition becomes more apparent when a choice is required from among several alternatives considered through a matrix of various criteria.

2 A methodology for decision making

An exceptionally popular tool designed especially to aid people in complex decision making, i.e. making a choice from various alternative based on a criteria matrix, is the 'Analytic Hierarchy Process' (AHP). The AHP seems to be the most widely used multicriteria decision making approach in the world today. The most recent list of application oriented papers one may want to find for instance in Grzybowski (2016). Actual applications in which the AHP results were accepted and used by the competent decision makers for instance can be found in: Saaty (2008), Ishizaka and Labib (2011), Ho (2008), Vaidya and Kumar (2006).

Currently the most popular method of assessing preferences regarding various decisional variations in an AHP is the 'Right Eigenvector Method' (REV). This approach takes advantage of information contained in the 'Pairwise Comparison Matrix' (PCM) which reflects the decision-maker's preferences expressed as linguistic variables – more or less fuzzy. Thus, it is possible to use words to compare qualitative factors and derive ratio scale priorities that can be combined with quantitative factors.

To make it possible a scale is utilized in order to evaluate the preferences for each pair of items. Supposedly, the most popular is Saaty's numerical scale which comprises the integers from one (equivalent to the verbal judgment: "equally preferred") to nine (equivalent to the verbal judgment: "extremely preferred") and their reciprocals. However, in conventional AHP applications we may want to utilize also other scales, i.e.: geometric scale and numerical scale. The first one usually consists of the numbers computed in accordance with the formula $2^{n/2}$ where n comprises the integers from minus eight to eight. The latter involves arbitrary integers from one to n and their reciprocals.

The first step in using AHP is to develop a hierarchy by breaking the problem down into its components. The basic AHP model includes goal (a statement of the overall objective), criteria (the factors one should consider in reaching the ultimate decision) and alternatives (the feasible alternatives that are available to reach the ultimate goal). Although the most common and basic AHP structure consists of a goal-criteria-alternatives sequence (Figure 1), AHP can easily support more complex hierarchies.

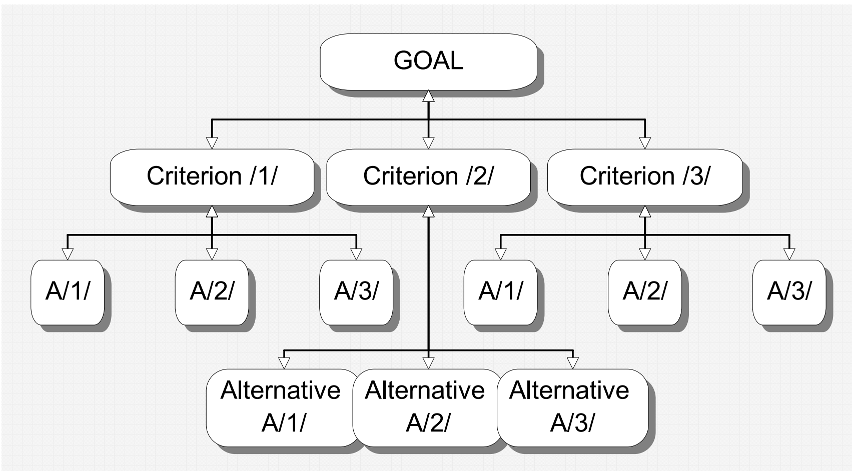


Figure 1. The most common exemplary hierarchy that consists of three levels: goal, three criteria, and three alternatives under each criterion

3 Introduction to the problem

One of the fundamental problems in AHP analysis is the priority weight assignment for the available decision alternatives. As it was stated earlier, the most popular method for estimating priority weights on the basis of the 'Pairwise Comparison Matrix' is the 'Right Eigenvector Method', proposed by Saaty and applied in the 'classic' AHP (Saaty, 1977). The conventional problem of AHP can be presented as:

$$\begin{bmatrix} x_1/x_1 & x_1/x_2 & x_1/x_3 & \ldots & x_1/x_n \\ x_2/x_1 & x_2/x_2 & x_2/x_3 & \ldots & x_2/x_n \\ x_3/x_1 & x_3/x_2 & x_3/x_3 & \ldots & x_3/x_n \\ \vdots & \vdots & \vdots & & \vdots \\ x_n/x_1 & x_n/x_2 & x_n/x_3 & \ldots & x_n/x_n \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} = \lambda_{\max} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} \quad (1)$$

and its outcome, i.e. the principal eigenvector $\boldsymbol{w} = [w_1,\ldots, w_n]^T$ is provided by a solution of $\boldsymbol{X}\,\boldsymbol{w} = \lambda_{\max}\,\boldsymbol{w}$ where: $w_i > 0$, and $i = 1,\ldots, n$.

Together with Saaty's method of priorities estimation, it was simultaneously proposed Saaty's 'Consistency Index'. What is important from the scientific point of view is that while the method contains several advantages, it also contains a series of very significant flaws which cannot be dismissed (Farkas, 2007).

It behooves mentioning those listed in literature on the subject, i.e. rank reversal, or the lack of any kind of quality criteria for the decision-maker to recognize why one decision vector weight is better than other evaluations. A significant drawback in the 'classic' approach of AHP is also the forced, reversed symmetry of the PCM which causes a loss of preference weight information contained in the elements of the 'ignored part' of a matrix (Grzybowski, 2012).

However, the most serious flaw of the AHP that was observed and stressed in current literature is the proposed, completely arbitrary method of recognizing (or not) the PCM as consistent enough for generating priority estimations (Grzybowski, 2012), and the very low correlation value between Saaty's sufficient consistency index values and the error value (absolute or relative) for the priority estimation weights (Grzybowski, 2016; Kazibudzki, 2016a). The examination of the latter issue is in order of this paper.

4 The problem description

It is obvious that even the best method of PVs estimation is useless until information about a scale of PCM inconsistency is provided. It is claimed and it is quite intuitive that serious errors in judgments about 'true' preferences of decision makers cause the data contained in PCM useless and result in poor estimates of decision makers' priorities (Saaty, 1980; Saaty, 2004; Saaty and Vargas, 1984). Therefore, we are presented with a number of papers dealing solely with the analysis of the inconsistency of the PCM. Undeniably, the consistency control and the evaluation of decision makers' inconsistency during the judgmental process is and should be a crucial part of every AHP study (Bulut et al., 2012; Aguaron et al., 2014; Altuzarra et al., 2010). The importance of the inconsistency control in the AHP practice was also emphasized in a number of application-oriented articles (Bulut et al., 2012; Pelaez and Lamata, 2003), group decision making oriented papers (Aguaron et al., 2014; Zhang et al., 2012), and research papers dedicated to elaboration of algorithms that lead to the consistency amelioration (Jarek, 2016; Xia et al., 2013; Benitez, 2012; Bozóki et al., 2011; Koczkodaj and Szarek, 2010).

In order to control the PCM consistency, different formulas (called indices) are proposed. These indices reflect in their way the degree of the PCM deviation from the one obtained in a perfect judgment case.

The first and the most popular inconsistency index (CI) was introduced by Saaty (1977) in his fundamental paper devoted to the AHP. His CI (denoted here as SI – formula 2) is closely related to the REV.

$$SI = \frac{\lambda_{\max} - n}{n - 1} \tag{2}$$

The other popular CI is connected with a prioritization procedure (PP) that is known as the Row Geometric Mean method (GM) that was introduced by Crawford and Williams (1985) together with the Geometric Consistency Index (denoted here as GI – formula 3).

$$GI = \frac{2}{(n-1)(n-2)} \sum_{i<j} \log^2\left(\frac{x_{ij} w_j}{w_i}\right) \tag{3}$$

Another interesting concept of CI devised Koczkodaj (1993) who proposed his CI (denoted here as KI – formulae 4 and 5) that is based on the notions of a triad and its inconsistency.

$$KI(TI) = \max[TI(\alpha, \beta, \chi)] \tag{4}$$

$$TI(\alpha, \beta, \chi) = \min\left[\left|1 - \frac{\beta}{\alpha\chi}\right|, \left|1 - \frac{\alpha\chi}{\beta}\right|\right] \tag{5}$$

for: α, β, χ that are called a *triad*, where: $\alpha = a_{ik}$, $\chi = a_{kj}$, $\beta = a_{ij}$ for some different $i \leq n, j \leq n$, and $k \leq n$, in a particular PCM denoted as: $\boldsymbol{A}(x) = [x_{ij}]_{nxn}$. It behooves mentioning that KI is not associated with any specific PP.

Apart from the indices SI, GI and KI, there exist and are promoted different other CI for PCMs, see for example: Kazibudzki (2016b), Dijkasra (2013), Grzybowski (2012). There are also some proposals for consistency control in the fuzzy pairwise comparison framework, such as the centric consistency index (which is based on GI) proposed by Bulut et al. (2012). However it seems undoubted, that these three above-mentioned indices (SI, GI and KI) are the most widely used ones in the pairwise comparisons methodology, see for instance Choo and Wedley (2004), Lin (2007), Grzybowski (2012), and Dong et al. (2008). All known from literature CI have one common characteristics, i.e. they are positive values and in the case of PCM perfect consistency they equal zero – what constitute a prerequisite of this theory. It is also believed that high CI values indicate poor consistency of decision makers' judgments what is supposed to indicate low quality of their preferences estimates. Obviously, such a belief is supported exclusively by some heuristic arguments which are mostly based on different intuitive psychological requirements, which according to the authors' opinions, should be reflected by CI properties.

It is important to underline that the most crucial and in the same time purely heuristic claim for common CI is the following assumptions: 'the more inconsistent judgments of decision makers are, the poorer are the estimates of priority weights'. Although it seems intuitive it turns out that it cannot be taken as granted (Grzybowski, 2016). Thus it is important to distinguish the following issues:

– the relation between the PCM consistency (reflected by CI) and the trustworthiness of decision makers judgments, and
– a dependence of the priority weights estimation errors from the level of PCM consistency designated by a given CI.

The pronounced majority of research devoted to inconsistency analysis, to our best knowledge except two papers, i.e. Grzybowski (2016) and Kazibudzki (2016a), as far combined the above mentioned issues and the existence of the distinguished earlier relations, i.e. among CI values, judgment consistency, and magnitudes of priority weights estimation errors, altogether treated as granted.

However, we should distinguish these two areas of study. The first, which can be perceived from the perspective of decision makers expertise (Brunelli and Fedrizzi, 2013) and the second, which defines the estimation quality of priority weights.

In this study we focus on the second problem, which constitute the primary research area of multicriteria decision making theory. We intend to study the relation between the values of CI and the magnitude of priority weights estimation errors. Thus, we are primarily interested in examination of the usefulness of the PCM as a source of information for estimation of priority weights. Hopefully, the results of our examination will allow decision makers to select such CI that is the most suitable from the perspective of their designated objectives.

5 The problem analysis

In order to examine a performance of selected CI from the assumed perspective, the following simulation scenario was considered. Its assumptions were introduced by Grzybowski (2016) then discussed and implemented in the paper of Kazibudzki (2016a). The simulation scenario comprises the following steps:

Step /1/ Randomly generate a priority vector $\boldsymbol{k} = [k_1,\ldots, k_n]^T$ of assigned size $[n \text{ x } 1]$ and related perfect PCM(k) = $\boldsymbol{K}(k)$.

Step /2/ Randomly choose an element k_{xy} for $x < y$ of $\boldsymbol{K}(k)$ and replace it with $k_{xy}e_B$ where e_B is relatively a significant error which is randomly drawn from the interval D_B with assigned probability distribution π.

Step /3/ For each other element k_{ij}, $i < j \leq n$ randomly choose a value e_{ij} for the small error in accordance with the given probability distribution π and replace the element k_{ij} with the element $k_{ij}\, e_{ij}$.

Step /4/ Round all values of $k_{ij}\, e_{ij}$ for $i < j$ of $\boldsymbol{K}(k)$ to the closest value from a considered scale.

Step /5/ Replace all elements k_{ij} for $i > j$ of $\boldsymbol{K}(k)$ with $1/k_{ij}$.

Step /6/ After all replacements are done, calculate the value of the examined index as well as the estimates of the vector $\boldsymbol{k}$, denoted as $\boldsymbol{k}$*(EP), with application of assigned estimation procedure (EP). Then compute estimate errors AE($\boldsymbol{k}$*(EP), $\boldsymbol{k}$) and RE($\boldsymbol{k}$*(EP), $\boldsymbol{k}$) denoting the absolute and relative error respectively, where:

$$AE\left(k^*(EP),k\right)=\frac{1}{n}\sum_{i=1}^{n}\left|k_i-k_i^*(EP)\right|$$

$$RE\left(k^*(EP),k\right)=\frac{1}{n}\sum_{i=1}^{n}\left|\frac{k_i-k_i^*(EP)}{k_i}\right|$$

Remember values computed in this step as one record.

Step /7/ Repeat Steps 2 to 6 N_M times.

Step /8/ Repeat Steps 2 to 7 N_R times.

Step /9/ Return *all* records organized as one database.

Source: Kazibudzki (2016a, p. 75).

The probability distribution π attributed in Step /3/ for e_{ij} is applied in equal proportions as: *gamma*, *log-normal*, *truncated normal*, and *uniform* distribution. The simulation scenario assumes that the factor e_{ij} is drawn from the interval $e \in [0,5;1,5]$ with the expected value of e_{ij} EV(e_{ij}) = 1. The 'big error' applied in Step /2/ has the uniform distribution on the interval $e_B \in [2;4]$.

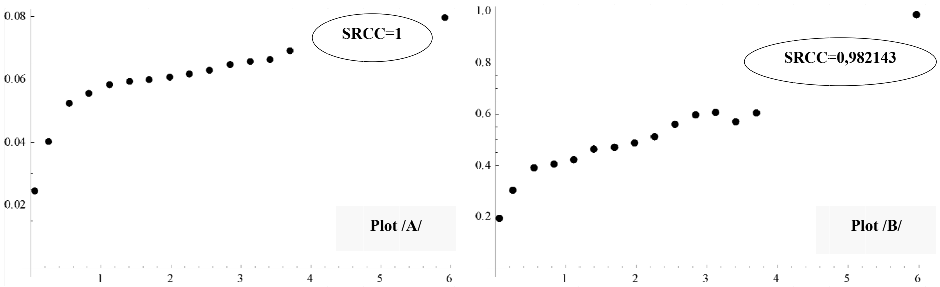
In this paper we examine simulation results for recently introduced by Kazibudzki (2016a) PCM consistency indicator – formula (6) with its yet not examined variation denoted by formula (7).

$$MLTI\left(LTI\right)=\frac{1}{N}\sum_{i=1}^{N}\left[LTI_i\left(\alpha,\beta,\chi\right)\right] \tag{6}$$

$$LTI\left(\alpha,\beta,\chi\right)=\ln^2\left(\alpha\chi/\beta\right) \tag{7}$$

where $LTI(\alpha,\beta,\chi)$ defines a seminal formula intended for indication of triad's consistency.

Due to necessity of diminishing the volume of the paper we present only results for $n = 4$ (where n denotes the number of alternatives in the model). For the same reason we implement only one estimation procedure, i.e. Logarithmic Least Squares Method (LLSM). Our simulation scenario assumes application of the rounding procedure which in this research operates according to Saaty's scale. Finally, our scenario takes into account the obligatory assumption in conventional AHP applications, i.e.: the PCM reciprocity condition. The results are presented in Tables 1-2 and Figure 2.



Note: SRCC stands for Spearman rank correlation coefficient.

Figure 2. Performance of the index MLTI(*LTI*). Plots of correlation between average values of MLTI(*LTI*) within analyzed interval and – the average AE (Plot A) and the average RE (Plot B)

In order to compare simulation results for MLTI(*LTI*) with performance of other consistency indices commonly used or proposed as good inconsistency indicators we present relations between fluctuations of selected consistency indices and selected characteristics of absolute or relative estimation errors. In order to save a space of the paper the results are only pictured on Figures 3-4.

Table 1: Performance of the index MLTI(*LTI*) in relation to AE(*LLSM*) distribution

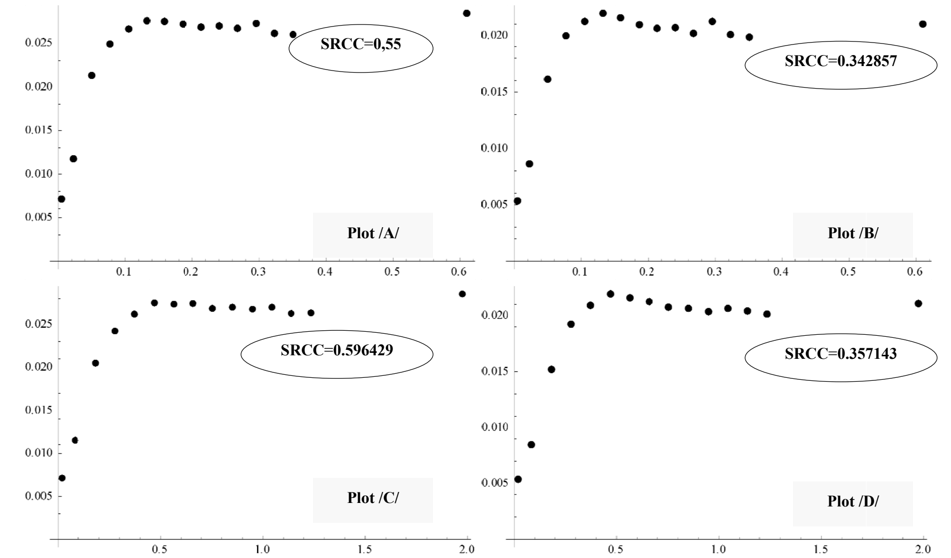
Average MLTI	***p*-quantiles of *AE*(LLSM)**			**Average *AE*(LLSM)**
	***p* = 0,1**	***p* = 0,5**	***p* = 0,9**	
0,05596	0,0071390	0,0176010	0,048430	0,0245701
0,25057	0,0114910	0,0309079	0,081851	0,0402469
0,54720	0,0204972	0,0467688	0,092151	0,0523990
0,83115	0,0241947	0,0490454	0,095879	0,0555646
1,12041	0,0262167	0,0531811	0,096393	0,0584429
1,40481	0,0275306	0,0552738	0,095133	0,0594058
1,68964	0,0273575	0,0553371	0,097423	0,0598936
1,97632	0,0274635	0,0555491	0,100479	0,0606786
2,26292	0,0268790	0,0559390	0,103806	0,0617115
2,55136	0,0270048	0,0565451	0,107156	0,0629664
2,84257	0,0267839	0,0570167	0,113131	0,0648082
3,12748	0,0270025	0,0579643	0,115005	0,0658326
3,41583	0,0262393	0,0594124	0,116590	0,0662670
3,70311	0,0263055	0,0614980	0,122258	0,0691538
5,92187	0,0285198	0,0721938	0,142200	0,0797634

Note: results based on 20 000 random reciprocal PCMs.

Table 2: Performance of the index MLTI(*LTI*) in relation to RE(*LLSM*) distribution

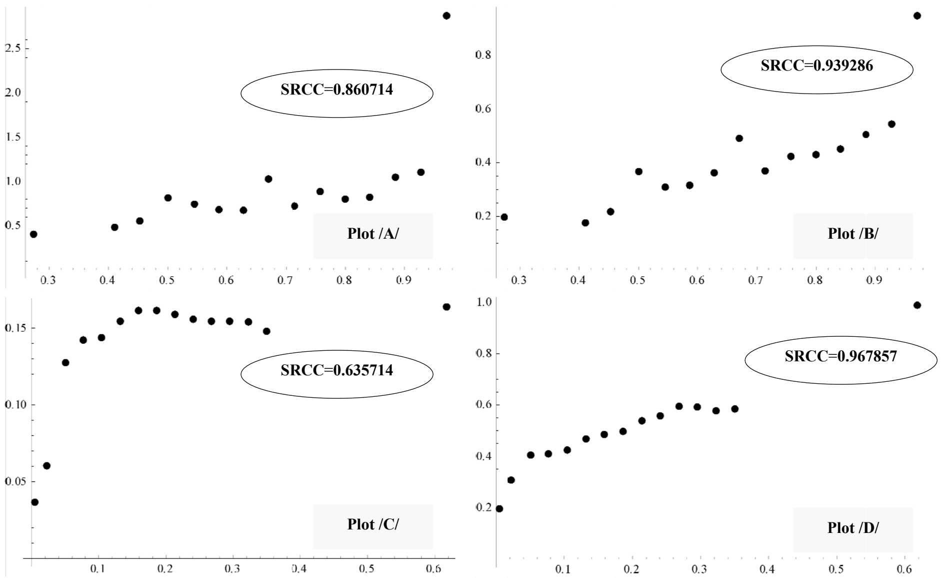
Average MLTI	***p*-quantiles of *RE*(LLSM)**			**Average *RE*(LLSM)**
	***p* = 0,1**	***p* = 0,5**	***p* = 0,9**	
0,05629	0,036686	0,083857	0,242085	0,195402
0,24889	0,058777	0,154707	0,463732	0,303042
0,54581	0,118697	0,233932	0,562547	0,391110
0,82979	0,141469	0,250504	0,571199	0,406388
1,11668	0,142768	0,274579	0,562745	0,423317
1,40159	0,151422	0,271206	0,551958	0,463763
1,68759	0,159168	0,266687	0,593866	0,471272
1,97225	0,162532	0,267185	0,624678	0,487771
2,26086	0,160420	0,274621	0,678569	0,511222
2,54802	0,157078	0,283623	0,716009	0,561110
2,83765	0,154172	0,289968	0,748193	0,598254
3,12254	0,154614	0,304405	0,764992	0,607791
3,41062	0,153110	0,308901	0,832981	0,570705
3,69985	0,150794	0,325909	0,835161	0,605517
5,96260	0,164129	0,393369	1,497960	0,987263

Note: results based on 10 000 random reciprocal PCMs.



Note: SRCC stands for Spearman rank correlation coefficient.

Figure 3. Performance of the indices SI and GI. Plots depict correlations between mean values of SI within analyzed interval and – AE quantiles of order 0,1 (Plot A) and AE quantiles of order 0,05 (Plot B), as well mean values of GI within analyzed interval and – AE quantiles of order 0,1 (Plot C) and AE quantiles of order 0,05 (Plot D)



Note: SRCC stands for Spearman rank correlation coefficient

Figure 4. Performance of the indices KI and SI. Plots depict correlations between mean values of KI within analyzed interval and – RE quantiles of order 0,95 (Plot A) and mean RE (Plot B), as well mean values of SI within analyzed interval and – RE quantiles of order 0,1 (Plot C) and mean RE (Plot D)

6 Discussion

It is believed that high CI values mean poor consistency of judgments what is supposed to entail low quality of decision makers' preferences estimates. This examination manifested that such a belief is supported exclusively by some heuristic arguments which according to some opinions, should be reflected by CI properties. The common assumption: 'the more inconsistent judgments of decision makers are, the poorer are the estimates of priority weights', cannot be taken as granted any more. Thus, we studied the relation between the values of selected CI and the magnitude of priority weights estimation errors.

We examined three commonly proposed inconsistency indicators for Pairwise Comparison Matrices, i.e. Saaty's consistency index (SI), geometric consistency index (GI), Koczkodaj's consistency index (KI), and the alternative proposition for consistency control, recently introduced by Kazibudzki (2016a), i.e. MLTI(*LTI*) index. We found out on the basis of analyzed cases that it is not true that a lower value of consistency index directly lead to a better estimation accuracy of decision makers' preferences. If that was true, we could observe a high and positive correlation between average values of selected consistency indices and relative or absolute estimation errors of simulated priority vectors. However, this research indicates that for GI, KI and SI, we can actually witness the situation when a decrease of consistency may lead to the improvement of a priority vector estimation quality, and inversely, when a growth of consistency may lead to the deterioration of a priority vector estimation quality (Figures 3-4). Our research indicates that in many analyzed cases we witness a non-monotonous relationship between values of a given consistency indicator and absolute or relative estimation errors of decision makers' preferences. However, it is not the case of proposed herein and examined new proposition for consistency control, i.e. MLTI(*LTI*) index – Figure 2, Tables 1 and 2. Its most serious advantages in comparison with other consistency indicators are: it is not connected with any prioritization procedure, it performs better than other analyzed consistency indicators and it can work also with AHP models that assume application of nonreciprocal PCM.

7 Conclusions

We have analyzed a performance of selected inconsistency indicators for simulated pairwise judgments from the perspective of their relations to absolute or relative estimation errors of decision makers' preferences.

We found out on the basis of analyzed cases that there exists a discrepancy between a common belief and a reality, i.e. it is not true that a lower values of consistency indicator directly lead to a better estimation accuracy of decision makers' preferences. It is a very important discovery because many authors still dedicate their research to the methods or procedures which strive to diminish some targeted consistency indicator.

Our research indicates that in many analyzed cases we witness a non-monotonous relationship between values of a given consistency indicator and absolute or relative estimation errors of decision makers' preferences. It means we should reform the concept of pairwise judgments consistency and search for such consistency indicators which reflect better the estimation quality of decision makers' priorities. It is so because the most commonly used consistency indicators may mislead about the estimation quality of decision makers' preferences.

The research indicates that in some cases we witness a situation when diminishing of a particular consistency indicator can lead to the deterioration of estimation quality. However it is certainly not a point of many researchers' effort. Thus, we should learn how to search and find new consistency indicators which possess features that are desired.

In this article we examined the consistency indicator that performs relatively well and it was recently introduced as a competitive solution for a consistency control of pairwise judgments.

References

Aguarón J., Escobar M.T., Moreno-Jiménez J.M. (2014), *The Precise Consistency Consensus Matrix in a Local AHP-group Decision Making Context*, Annals of Operational Research, DOI: 10.1007/s10479-014-1576-8.

Aguarón J., Moreno-Jiménez J.M. (2003), *The Geometric Consistency Index Approximate Thresholds*, European Journal of Operational Research, 147(1), 137-145.

Altuzarra A., Moreno-Jiménez J.M., Salvador M. (2010), *Consensus Building in AHP-Group Decision Making: A Bayesian Approach*, Operations Research, 58, 1755-1773.

Benítez J., Delgado-Galván X., Izquierdo J., Pérez-García R. (2012), *Improving Consistency in AHP Decision-making Processes*, Applied Mathematics and Computation, 219, 2432-2441.

Bozóki S., Fülöp J., Koczkodaj W.W. (2011), *An LP-based Inconsistency Monitoring of Pairwise Comparison Matrices*, Mathematical and Computer Modeling, 54(1-2), 789-793.

Brunelli M., Fedrizzi M. (2015), *Boundary Properties of the Inconsistency of Pairwise Comparisons in Group Decisions*, European Journal of Operational Research, 240(3), 765-773.

Bulut E., Duru O., Kececi T., Yoshida S. (2012), *Use of Consistency Index, Expert Prioritization and Direct Numerical Inputs for Generic Fuzzy-AHP Modeling: A Process Model for Shipping Asset Management*, Expert Systems with Applications, 39, 1911-1923.

Choo E.U., Wedley W.C. (2004), *A Common Framework for Deriving Preference Values from Pairwise Comparison Matrices*, Computers & Operations Research, 31, 893-908.

Crawford G., Williams C.A. (1985), *A Note on the Analysis of Subjective Judgment Matrices*, Journal of Mathematical Psychology, 29, 387-405.

Dijkstra T.K. (2013), *On the Extraction of Weights from Pairwise Comparison Matrices*, Central European Journal of Operations Research, 21(1), 103-123.

Dong Y., Xu Y., Li H., Dai M. (2008), *A Comparative Study of the Numerical Scales and the Prioritization Methods in AHP*, European Journal of Operational Research, 186, 229-242.

Farkas A. (2007), *The Analysis of the Principal Eigenvector of Pairwise Comparison Matrices*, Acta Polytech. Hung., 4(2), http://uni-obuda.hu/journal/ Farkas_10.pdf.

Grzybowski A.Z. (2016), *New Results on Inconsistency Indices and Their Relationship with the Quality of Priority Vector Estimation*, Expert Systems with Applications, 43, 197-212.

Grzybowski A.Z. (2012), *Note on a New Optimization Based Approach for Estimating Priority Weights and Related Consistency Index*, Expert Systems with Applications, 39, 11699-11708.

Ho W. (2008), *Integrated Analytic Hierarchy Process and Its Applications – A Literature Review*, European Journal of Operation Research, 186, 211-228.

Ishizaka A., Labib A. (2011), *Review of the Main Developments in the Analytic Hierarchy Process*, Expert Systems with Applications, 11(38), 14336-14345.

Jarek S. (2016), *Removing Inconsistency in Pairwise Comparison Matrix in the AHP*, Multiple Criteria Decision Making, 11, 63-76.

Kazibudzki P. (2016a), *Redefinition of Triad's Inconsistency and Its Impact on the Consistency Measurement of Pairwise Comparison Matrix*, Journal of Applied Mathematics and Computational Mechanics, 15(1), 71-78.

Kazibudzki P. (2016b), *An Examination of Performance Relations among Selected Consistency Measures for Simulated Pairwise Judgments*, Annals of Operations Research, 244(2), 525-544.

Koczkodaj W.W., Szarek S.J. (2010), *On Distance-based Inconsistency Reduction Algorithms for Pairwise Comparisons*, Logic Journal of the IGPL, 18(6), 859-869.

Koczkodaj W.W. (1993), *A New Definition of Consistency of Pairwise Comparisons*, Mathematical and Computer Modeling, 18(7), 79-84.

Lin CC. (2007), *A Revised Framework for Deriving Preference Values from Pairwise Comparison Matrices*, European Journal of Operational Research, 176, 1145-1150.

Miller G.A. (1956), *The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Information Processing*, Psychological Review, 63, 81-97.

Peláez P., Lamata M. (2003), *A New Measure of Consistency for Positive Reciprocal Matrices*, Computers & Mathematics with Applications, 46, 1839-1845.

Saaty T.L. (2008), *Decision Making with the Analytic Hierarchy Process*, International Journal of Services Sciences, 1(1), 83-98.

Saaty T.L. (1977), *Scaling Method for Priorities in Hierarchical Structures*, Journal of Mathematical Psychology, 15(3), 234-281.

Saaty T.L. (1980), *The Analytic Hierarchy Process*, McGraw-Hill, New York.

Saaty T.L. (2004), *Decision Making – The Analytic Hierarchy and Network Processes*, Journal of Systems Science and Systems Engineering, 13(1), 1-35.

Saaty T.L., Vargas L.G. (1984), *Comparison of Eigenvalue, Logarithmic Least Square and Least Square Methods in Estimating Ratio*, Mathematical Modeling, 5, 309-324.

Vaidya O.S., Kumar S. (2006), *Analytic Hierarchy Process: An Overview of Applications*, European Journal of Operation Research, 169, 1-29.

Xia M., Xu Z., Chen J. (2013), *Algorithms for Improving Consistency or Consensus of Reciprocal [0,1]-valued Preference Relations*, Fuzzy Sets and Systems, 216, 108-133.

Zhang G., Dong Y., Xu Y. (2012), *Linear Optimization Modeling of Consistency Issues in Group Decision Making Based on Fuzzy Preference Relations*, Expert Systems with Applications, 39, 2415-2420.