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APPLICATION OF THE GENERALISED DISTANCE MEASURE TO LOCATION SELECTION DURING ORDER-PICKING

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Abstract

When a shared storage system is used, the selection of locations from which products should be picked becomes a significant decision problem. Every storage location can be described using several criteria, such as: storage time, distance from the I/O point, degree of demand satisfaction, the number of other products to be picked near the analysed location, or others. Based on such criteria, a synthetic variable can be created to rank all these locations; the highest-ranking one is selected. Such a ranking is created using the Generalised Distance Measure (GDM); the selected locations and the picker's route based on them are compared to the results obtained using the Taxonomic Measure of Location's Attractiveness (TMAL). Both route length and picking time are compared. Also, the influence of the system of criteria weights within each method on the route length and the picking time is analysed using simulation methods.

Keywords: order-picking, Generalised Distance Measure, Taxonomic Measure of Location's Attractiveness, multiple-criteria decision making, simulation analysis.

1 Introduction

Order-picking is the most time- and cost-consuming activity in warehouse management, for both manual and automated systems (De Koster et al., 2007). Therefore, there is still room for improvement in this area, which can be done in three ways, by optimising storage assignment, orders batching, or routing methods. Every area uses different methods of improvement. Storage assignment can be improved, for example, by implementing class-based storage; orders batching, by reducing order picking time; and routing methods, by adjusting the method of travelling to the warehouse type.

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There are two main methods of storing goods in a warehouse: dedicated storage and shared storage (Bartholdi and Heckman, 2016). Dedicated storage consists in storing each product in one location containing one product only. It enables the picker (to a certain degree, depending on the number of SKUs¹ stored in the warehouse) to easily remember storage locations and makes order picking relatively fast and efficient. Its main drawback is its inefficiency (the storing space is utilised, on the average, in about 50% (Bartholdi and Heckman, 2016). Shared storage, on the other hand, consists in storing each product in any one of many locations, with many products stored in each location (Bartholdi and Heckman, 2016). This storage method greatly improves the utilisation of the storing space, but results in the products being scattered among various locations, often very distant from each other. Also, locations of all products change continually, which makes it impossible for the picker to remember them. It is then necessary to use a warehouse management system.

When a shared storage system is used, the products ordered can be picked from many locations. The question arises: Which location should be selected to pick the given product? The problem remains pretty much unsolved in the literature. Bartholdi and Heckman (2016) mention that during order-picking, the picker can select the most convenient location (to reduce labour) or the least-filled locations (which is more labour-intensive, but frees storage space for future replenishment orders). Gudehus and Kotzab (2012) specified several take-out strategies for a product which can be accessed from more than one location:

- FIFO – units are picked according to their arrival time to the warehouse.
- Priority of partial units – locations with the lowest content of the product are accessed first, even if it increases labour.
- Quantity adjustment (the opposite to the previous one) – the picker retrieves the product from the locations containing the entire requested quantity, even if it generates low amounts of products at these locations.
- Taking the access unit – if the amount of the product at the given location exceeds or is equal to the quantity requested, the entire unit is taken after the excess quantity is put aside.

There are thus several criteria relevant to the strategy of location selection during order picking. From the above-listed take-out strategies, we can think of at least two of them: storage time and the amount of product at a given location. However, other criteria can be also taken into account. To improve the picker's travel time, we can select locations close to the I/O point². Also, if there are

¹ SKU (*Stock Keeping Unit*) – the smallest physical unit of a product.

² The I/O (input/output) point is the location from where the picker starts picking the products ordered and collects picked products.

many products in the order, we can select those storage locations which are close to each other, so that the picker can complete the order without too much travelling around the warehouse. Moreover, in a high-storage warehouse the storage level is also an important criterion. The products should be picked from low levels first, since the picker can reach them directly from the floor. To reach locations on higher levels, he/she must use a ladder or forklift; therefore, products from those levels should be picked next.

All these criteria can be considered separately, partially or in their entirety. If the decision maker intends to consider some, or all of them, then his/her decision will be based on a multiple-criteria approach. There are many methods that support multiple-criteria decision making. In general, we deal with multiple-objective mathematical programming problems, or multiple-criteria evaluation problems (Trzaskalik, 2015). In the former, alternatives are not explicitly known: there may be even an infinite number of them. Such problems are solved by means of a mathematical decision model. In the latter, alternatives are known; they are described by multiple criteria and the best one is selected by ordering them. For the location selection problem, multiple-criteria evaluation methods can be applied. The decision maker knows all the alternatives in the problem: in this case they are the storage locations of products ordered. Some of the many multiple-criteria decision analysis methods are: AHP, ANP, ELECTRE, SAW, COPRAS, TOPSIS. The best ones are those that allow the decision maker to make many decisions in a short time. They should be easily implemented in software and should require minimum attention from the decision maker, therefore such methods as SAW (Podvezko, 2011) or TOPSIS (Hwang and Yoon, 1981) are the most obvious choice. They both can rank the decision alternatives (SAW does it without creating the so-called “pattern” object, while TOPSIS is based on the distances from both “pattern” and “anti-pattern”).

The present author, in his previous papers, designed a simple multiple-criteria decision-making technique, based on the Synthetic Measure of Development (Hellwig, 1968) and called the Taxonomic Measure of Location’s Attractiveness (Polish abbreviation TMAL) (Dmytrów, 2015). All the above-mentioned methods are based on Euclidean distances, which can be used only for criteria measured on an interval or a ratio scale. However, some criteria can be measured on a weaker, ordinal scale, for which Euclidean distances cannot be used. In this case, we can use the Generalised Distance Measure (GDM), proposed by Walesiak (2000). Although GDM was not meant as a multiple-criteria decision-making technique, but rather as a measure for the calculation of the distance matrix in object classification, or as a synthetic measure of development in methods of linear ordering. In the latter, its application is similar to the Synthetic

Measure of Development, used in the TMAL method. The goal of the present paper is to compare GDM with TMAL as multiple-criteria decision-making techniques for the selection of locations in order-picking.

2 Analytical methods applied

2.1 Specification of decision criteria and applied systems of weights

As mentioned before, to select the pick location of a product, various strategies can be used. In this paper, three criteria are applied:

x_1 – distance from the I/O point,

x_2 – degree of demand satisfaction,

x_3 – number of other products picked in the neighbourhood of the location analysed.

The first criterion is measured in contractual units, that is, shelf width. It is measured on a ratio scale and has negative impact.

The degree of demand satisfaction has positive impact. It is measured on a ratio scale and is calculated from the following formula:

$$x_2 = \begin{cases} \frac{l}{z}, & \text{if } z > l, \\ 1 & \text{if } l \geq z \end{cases} \quad (1)$$

where l – number of units of the product picked from the location analysed and z – demand for the picked product.

The third criterion – the number of other products picked in the neighbourhood of the location analysed – has positive impact. It is measured on a ratio scale and is a numerical and discrete variable. It should be mentioned here that the notion of a neighbourhood depends on the warehouse type. In a high-storage warehouse, this can be the rack. In a typical low-storage warehouse, this can be the racks within an aisle (which will be assumed here).

The criteria used to create the synthetic variable to classify the alternatives should be weighed. There are many methods to weigh the decision criteria, which can be classified as statistical and formal, and expert. Statistical methods can be based on the variability of criteria: The higher the share of variability of the given criterion in the total variability, the higher weight should the criterion have (Kukuła, 2000). Another statistical and formal method is based on the Shannon entropy (Lotfi and Fallahnejad, 2010). Among expert methods is AHP, in which experts specify their preferences by comparing the criteria pairwise (Trzaskalik, 2015). The weights can also be specified purely subjectively, with the decision-maker deciding the importance of each criterion. In our case, seven combinations of weights have been analysed (see table 1).

Table 1: Analysed combinations of weights

Combinations of weights	x_1	x_2	x_3
C1	0.333	0.333	0.333
C2	0.5	0.25	0.25
C3	0.25	0.5	0.25
C4	0.25	0.25	0.5
C5	0.4	0.4	0.2
C6	0.4	0.2	0.4
C7	0.2	0.4	0.4

Source: Author's own elaboration.

Combination C1, in which every criterion has the same weight, is the reference. In combinations C2, C3 and C4 one criterion is twice as important as the other two and therefore its impact on the final decision is the same as the total impact of the remaining two criteria. In combinations C5, C6 and C7 two criteria are twice as important as the remaining one. The combinations of weights have been selected so as to analyse how well the algorithm performs in each situation and whether making one or two criteria more important than the other(s) will improve the system's performance.

2.2 Construction of TMAL and GDM

The construction of both TMAL and GDM consist of several steps, repeated for each product ordered. The steps for TMAL are as follows:

- The distance from the I/O point (x_1) is changed into a criterion with positive impact by calculating its inverse.
- The values of all criteria are normalised. We use quotient inversion:

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^n x_{ij}^2}}, \quad (2)$$

where x_{ij} – value of j -th criterion in i -th alternative (location). Many other normalisation formulas are possible (Walesiak, 2016). This formula was selected so as to preserve the differences in mean values and variability.

- The maximum normalised values form the so-called “perfect alternative” or the pattern.
- Euclidean distances between the pattern and each location are calculated.
- Mean weighed distances from the pattern for all combinations of weights are calculated.

- TMAL is calculated as the complement to unity from the ratio of the mean weighed distance of each location from the maximum value obtained in the previous step.
- TMAL values are sorted in the descending order.
- The highest-ranking locations are selected, until the demand for each product is satisfied.

The Generalised Distance Measure (GDM) is based on the generalised correlation coefficient, using the Pearson Product-Moment Correlation Coefficient and the Kendall τ correlation coefficient (Walesiak, 2011):

$$d_{ik} = \frac{1}{2} - \frac{\sum_{j=1}^m w_j a_{ikj} b_{kij} + \sum_{j=1}^m \sum_{l=1, l \neq i, k}^n w_j a_{ilj} b_{klj}}{2 \left[\sum_{j=1}^m \sum_{i=1}^n w_j a_{ilj}^2 \cdot \sum_{j=1}^m \sum_{i=1}^n w_j b_{klj}^2 \right]^{\frac{1}{2}}}, \quad (3)$$

where:

d_{ik} – distance (similarity) measure,

$i, k, l = 1, 2, \dots, n$ – index of the alternative (location),

$j = 1, 2, \dots, m$ – index of the criterion,

w_j – weight of j -th criterion.

For variables measured on an interval or a ratio scale, the values of a and b are calculated from the following formulas:

$$\begin{aligned} a_{ipj} &= x_{ij} - x_{pj} \text{ for } p = k, l, \\ b_{krj} &= x_{kj} - x_{rj} \text{ for } r = i, l, \end{aligned} \quad (4)$$

where x_{ij} (x_{kj} , x_{ij}) is i -th (k -th, l -th) value of j -th criterion.

Using GDM we can calculate the distance between objects (in multivariate statistical analysis) or decision alternatives (in multiple-criteria decision-making problems). The main advantage of GDM over the most commonly used distance measures, such as Euclidean, Mahalanobis or Manhattan, is that it allows to use criteria measured on an ordinal scale. It can be used for the determination of the distance matrix in classification procedures or in linear ordering of objects (in multivariate statistical analysis) or decision alternatives (in multiple-criteria decision-making problems).

The procedure of using GDM in linear ordering is as follows (Walesiak, 2003):

- There is no need to change the criteria with negative impact into ones with positive impact.
- The values of each criterion are normalised using formula (2).
- The so-called “perfect alternative”, or the pattern, is created. For the criteria with negative impact, the pattern values are the minimum values among all the alternatives. For the criteria with positive impact, the pattern consists of the maximum values among all the alternatives.

- The distance of each alternative (location) from the pattern is calculated using formula (3), applying the substitutions given by (4).
- The values of GDM for the alternatives (locations) are sorted in the ascending order.
- The highest-ranking locations are selected, until the demand for each product is satisfied.

2.3 Assumptions of the simulation experiment

A simulation experiment has been performed, with the following assumptions:

- A simple, rectangular warehouse was assumed.
- The warehouse contained 1000 locations with one main aisle and 20 aisles between racks. Every rack contained 25 locations.
- The warehouse used chaotic storage system.
- Every order consisted of ten products.
- Every product was stored in four locations.
- The available amounts of products in each location varied from a single unit to the amount that satisfied the demand twice.
- For both TMAL and GDM and all combinations of weights, 100 orders were generated.
- For every product picked, every method and every combination of weights, a ranking of locations was created.
- The highest-ranking locations were selected until the demand was satisfied.
- Once the locations had been selected, the picker's route was determined using *s-shape* heuristics (Le-Duc, 2005).
- For each route, its length was measured, and the order-picking time was calculated.
- The order-picking time was the sum of the picker's travel and collection times. It was assumed that the time of traversing a distance unit (shelf width) was 2 seconds and the time of collecting the product from the location, 10 seconds.
- For TMAL and GDM, it was analysed, using the one-way ANOVA, whether both route lengths and picking times were significantly different.
- If the null hypothesis was to be rejected, using *post-hoc* Tukey's HSD test, pairwise comparisons were performed.
- For every combination of weights, mean route length and order-picking time obtained using TMAL and GDM were compared using the paired *z*-test for independent samples.

3 Results of the simulation analysis

3.1 Comparison of results for each combination of weights within each method

Mean route lengths, order-picking times and results of the ANOVA for TMAL are presented in Table 2.

Table 2: Mean route lengths, order-picking times (in minutes) and results of the ANOVA for TMAL

Specification	C1	C2	C3	C4	C5	C6	C7
Route length	367.88	362.92	350.28	383.64	346.98	376.10	378.42
	ANOVA $F = 9.612$, p -value $p < 0.0001$						
Order-picking time	14:38	14:33	13:55	15:15	13:49	15:05	14:56
	ANOVA $F = 8.516$, p -value $p < 0.0001$						

Source: Author's own elaboration.

The one-way ANOVA showed that both mean route lengths and order-picking times varied depending on the combination of weights. The results of *post-hoc* Tukey's test are presented in Table 3.

Table 3: Results of Tukey's test for TMAL (significant differences are marked in bold)

	C2	C3	C4	C5	C6	C7
Route length						
Tukey's criterion $T = 17.736$						
C1	4.96	17.60	15.76	20.90	8.22	10.54
C2		12.64	20.72	15.94	13.18	15.50
C3			33.36	3.30	25.82	28.14
C4				36.66	7.54	5.22
C5					29.12	31.44
C6						2.32
Order-picking time						
Tukey's criterion $T = 39.622$						
C1	5.12	43.20	37.52	48.50	27.04	18.18
C2		38.08	42.64	43.38	32.16	23.30
C3			80.72	5.30	70.24	61.38
C4				86.02	10.48	19.34
C5					75.54	66.68
C6						8.86

Source: Author's own elaboration.

For the route length, the best results (the shortest route lengths) were obtained for combination C5 (0.4; 0.4; 0.2). This means that in order to minimise the picker's route length, the decision-maker should weigh both the distance from the I/O point and the degree of demand satisfaction twice as much as the number of other products in the neighbourhood of the location analysed. The mean route length for this combination was significantly shorter than the route lengths obtained for combinations C1, C4, C6 and C7. The mean route length for combination C5 was shorter by 9.6% than that for the worst combination, that is, C4. The results obtained by the reference combination C1 were exactly in the middle.

For the order-picking time, the best results were also obtained for combination C5. The worst results (longest order-picking times) were obtained for combination C4. The mean order-picking time obtained for combination C5 was shorter by 9.4% than that for the worst combination C4. Also, the results for combination C5 were significantly better than those obtained for combinations C1, C2, C4, C6 and C7. The results obtained for the reference combination C1 were exactly in the middle, as in the case of route length.

The mean route lengths, order-picking times and the results of the ANOVA for GDM are presented in Table 4.

Table 4: Mean route lengths, order-picking times (in minutes) and results of the ANOVA for GDM

Specification	C1	C2	C3	C4	C5	C6	C7
Route length	366.66	361.36	343.72	383.08	340.72	369.26	372.94
	ANOVA $F = 10.306$, p -value $p < 0.0001$						
Order-picking time	14:34	14:29	13:40	15:14	13:33	14:49	14:43
	ANOVA $F = 11.647$, p -value $p < 0.0001$						

Source: Author's own elaboration.

Similarly as in the case of TMAL, the one-way ANOVA for GDM showed that both mean route lengths and order-picking times varied significantly depending on the combination of weights. The results of *post-hoc* Tukey's test for the results obtained by GDM are presented in Table 5.

As in the case of TMAL, the shortest route lengths for GDM were obtained for combination C5 (0.4; 0.4; 0.2). The mean route length for this combination was significantly shorter than the route lengths obtained for combinations C1, C2, C4, C6 and C7. The mean route length for combination C5 was shorter by over 11% than that for the worst combination, that is, C4. The results obtained by the reference combination C1 for GDM were exactly in the middle, as previously.

Table 5: Results of Tukey's test for GDM (significant differences are marked in bold)

	C2	C3	C4	C5	C6	C7
Route length						
Tukey's criterion $T = 17.659$						
C1	5.30	22.94	16.42	25.94	2.60	6.28
C2		17.64	21.72	20.64	7.90	11.58
C3			39.36	3.00	25.54	29.22
C4				42.36	13.82	10.14
C5					28.54	32.22
C6						3.68
Order-picking time						
Tukey's criterion $T = 39.603$						
C1	4.90	54.18	39.64	60.88	14.8	9.36
C2		49.28	44.54	55.98	19.70	14.26
C3			93.82	6.70	68.98	63.54
C4				100.52	24.84	30.28
C5					75.68	70.24
C6						5.44

Source: Author's own elaboration.

For GDM the shortest order-picking times were obtained for combination C5. The worst results – longest order-picking times – were obtained for combination C4. The mean order-picking time obtained for combination C5 was shorter by 11% than that for the worst combination C4. Also, the results for combination C5 were significantly better than those obtained for all other combinations, except for C3. The route lengths obtained for the reference combination C1 were exactly in the middle, as previously.

3.2 Comparison of the results obtained using each method

The results of the paired z -test for independent samples for both methods are presented in Table 6.

Table 6: Mean route lengths, order-picking times (in minutes) for TMAL and GDM and the results of the paired z -test

Specification	C1	C2	C3	C4	C5	C6	C7
Route length							
TMAL	367.88	362.92	350.28	383.64	346.98	376.10	378.42
GDM	366.66	361.36	343.72	383.08	340.72	369.26	372.94
z	0.200	0.228	0.907	0.084	0.879	0.989	0.856
p -value	0.710	0.705	0.591	0.733	0.595	0.581	0.598

Table 6 cont.

Specification	C1	C2	C3	C4	C5	C6	C7
Order-picking time							
TMAL	14:38	14:33	13:55	15:15	13:49	15:05	14:56
GDM	14:34	14:29	13:40	15:14	13:33	14:49	14:43
<i>z</i>	0.271	0.234	0.932	0.105	1.033	0.990	0.891
<i>p-value</i>	0.697	0.704	0.588	0.729	0.575	0.581	0.593

Source: Author's own elaboration.

For both route length and order-picking time, GDM performed always better than TMAL, although the differences were not statistically significant. The difference between the mean route lengths varied from less than half of a unit for combination C4 to almost seven units for combination C6. For the order-picking time, the differences varied from one second for combination C4 to sixteen seconds for combination C6. Therefore, in this case, both methods are practically equally efficient.

4 Conclusions

In this paper, GDM has been applied as a multiple-criteria decision-making technique for the selection of locations in order-picking. Although GDM is usually not regarded as a decision-making support tool, its construction enables us to use it for this purpose. This measure has been previously used in decision making, as a distance measure in other techniques, such as TOPSIS (Wachowicz, 2011). Here, it has been used as a technique to create a ranking of the alternatives. The alternatives were the locations in a warehouse to be visited by the picker to complete the orders.

In the analysed simulation example, GDM generated similar results as TMAL, which is based on the classical Hellwig's Synthetic Measure of Development. Although the results obtained by GDM were slightly better than those obtained by TMAL, the differences were not statistically significant. Within each method, seven combinations of weights were analysed. Once the locations had been selected, route lengths and order-picking times were calculated. As regards both route length and order-picking time, the best results for both methods were obtained for combination C5 (0.4; 0.4; 0.2). For both methods this combination generated significantly better results than most of the other combinations. This means that the decision-maker should attach particular importance to the distance from the I/O point and the degree of demand satisfaction. The best locations are those closest to the I/O point and with the highest degree of demand satisfaction. The number of other products picked in the neighbourhood

of the location analysed is not as important as these two criteria. A comparison of the results obtained by the best method and combination of weights (GDM with C5) with the results obtained by the worst method and combination of weights (TMAL with C4) shows that both the picker's route length and the order-picking time can be shortened by about 11%. Of course, these results were obtained with the assumption that a chaotic storage system was used, hence they cannot be generalised for other storage systems, such as ABC or XYZ class-based storage systems.

Further research will include a comparison of multiple-criteria decision-making techniques for an ABC class-based storage system with within-isle and across-isle storing strategies. As the GDM method allows for using criteria measured on other than interval and ratio scales, other criteria, such as storage level in a high-storage warehouse or the presence (or absence) of complete packages at every location, will be added. Also, other methods of heuristics for the determination of the picker's route (*return*, *midpoint*, *largest gap*, or *combined*) will be analysed.

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