M U L T I P L E   C R I T E R I A   D E C I S I O N   M A K I N G
Vol. 14 2019

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SPARE PARTS QUANTITY PROBLEM UNDER UNCERTAINTY – THE CASE OF ENTIRELY NEW DEVICES WITH SHORT LIFE CYCLE

DOI: 10.22367/mcdm.2019.14.01

Abstract

The paper presents a new scenario-based decision rule for the spare parts quantity problem (SPQP) under uncertainty with unknown objective probabilities. The goal of SPQP is to ensure the right number of extra parts at the right place at the right time. In the literature, SPQP is usually regarded as a stochastic problem since the demand for extra parts is treated as a random variable with a known distribution. The optimal stock quantity minimizes the expected loss resulting from buying a given number of parts before potential failures.

The novel approach is designed for the purchase of non-repairable spare parts for entirely new seasonal devices, where the estimation of frequencies is complicated because there are no historical data about previous failures. Additionally, the decision maker's knowledge is limited due to the nature of the problem.

The new procedure is a three-criteria method. It is based on the Hurwicz and Bayes decision rules and supported with a forecasting stage enabling one to set the scenario with the greatest subjective chance of occurrence. The method takes into account the decision maker's attitude towards risk and the asymmetry of losses connected with particular stock quantities. We assume that the future unit purchase cost of a service part bought after the breakdown is also uncertain and given as an interval parameter. The approach is designed for short life cycle machines.

Keywords: spare parts quantity problem, new seasonal devices, uncertainty, interval payoffs, unknown objective probabilities, decision maker's preferences, short life cycle.

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1 Introduction

One of the main goals of the spare parts quantity problem (SPQP) is to ensure that right spare parts and resources are at the right place (where the broken part is) at the right time. Spare parts are kept in an inventory and should be in proximity to a functional item (engine, device, automobile, boat, machine) since they might be used to repair it or to replace failed units. They constitute an important element of logistic engineering and supply chain management. There are many synonyms for "spare parts" such as service parts, extra parts, repair parts, replacement parts and interchangeable parts.

SPQP may be analyzed in the context of repairable and non-repairable spare parts (Guide and Srivastava, 1997; Louit, Banjevic and Jardine, 2005). In this paper we focus on the latter.

In the literature spare parts optimization is usually regarded as a stochastic problem (Aronis et al., 2004; Gu and Li, 2015; Inderfurth and Mukherjee, 2008; Ravindran, 2007; Rodriguez et al., 2013; Sikora, 2008; Wong et al., 1997) since the demand for service parts is treated as a random variable with a known distribution. Here, however, we would like to consider SPQP as a strategic problem, i.e. an uncertain problem with unknown objective probabilities (frequencies). Such a situation may take place when a totally new device is bought and there are no historical data concerning previous breakdowns that could be used to estimate probabilities.

SPQP may be investigated as a single-period (SPP) or a multi-period problem (MPP) (Petrovic et al., 1988). In this contribution we analyze the first situation only since the multi-period horizon cannot be discussed in the context of SPQP under uncertainty with permanently unknown probabilities: in subsequent periods historical data (frequencies) become available and the objective likelihood can be estimated. Thus, in the case of totally new devices MPP would be a mixed problem. The first stage could be analyzed without known probabilities, but further stages could be based on probabilities. The second reason why we do not deal with MPP is that we assume that the purchase of additional spare parts for a given device is made only once for the whole period of usage since the life cycle of the considered device is relatively short due to its seasonal character and a constantly changing environment. It is worth emphasizing that when the chosen decision is supposed to be executed only once (one-shot decision), researchers advise the decision makers (DMs) against the use of probabilities, because only one event has the chance to occur (von Mises, 1949).

The research is related to totally new devices. Therefore, in contrast to the traditionally understood SPQP, we take into account not one but two types of uncertainty. The first one results from the unknown demand for extra parts given as a discrete random variable with an unknown probability distribution (discrete parameter). The second one is caused by the unknown future unit purchase cost of service parts (interval parameter).

The paper is organized as follows. Section 2 briefly presents the main features of the classical SPQP. Section 3 defines a new problem: SPQP for totally new devices, i.e. SPQP under uncertainty with unknown probabilities. Section 4 discusses the characteristics of the loss matrix connected with SPQP for different cases and analyzes the possible usefulness of classical decision rules in that field. Section 5 presents the assumptions of the scenario-based model and a 3-criteria decision rule that may be used for the aforementioned problem. The procedure takes into account DMs' preferences. Section 6 provides an illustrative example. Conclusions are gathered in the last section.

2 The classical spare parts quantity problem: description

In the original version of SPQP the goal is to find the optimal number of extra parts bought with the purchase of the whole device. "Optimal" means "minimizing the expected loss resulting from buying a given number of service parts before potential failures (breakdowns)". If we buy too many parts with the whole machine, we lose the money spent for the purchase of those parts. On the other hand, if we buy not enough spare parts with the whole item, we lose the difference between the current price of a spare part and the previous price of that part. SPQP is mainly related to DMSU − decision making under stochastic uncertainty −, and based upon the assumption of risk neutrality due to the fact that the demand (D) for extra parts is a random variable with a known probability distribution (Sikora, 2008).

The cumulative distribution function (F) of the demand may be continuous or discrete. In this paper we concentrate on the second variant. Within SPQP we can distinguish c_1, denoting the unit purchase cost of the subassembly together with the purchase of the whole device, and c_2, denoting the unit purchase cost of the subassembly just after the failure, where $c_1 < c_2$. Both costs allow us to compute two types of losses: $s_1 = c_1$ and $s_2 = c_2 - c_1$, where s_1 denotes the unit loss from buying a service part with the whole device (loss due to the excess of spare parts) and s_2 is the unit loss from buying an extra part just after the failure (loss due to the shortage of spare parts). The only decision variable in SPQP is q – the order quantity (number of spare parts bought with the device). Usually, the

DM considers possible discrete values of q from the interval $[D_{min}, D_{max}]$, where D_{min}, D_{max} are the lowest and the highest observed demand for spare parts, respectively.

The optimization model enabling one to find the optimal order quantity can be presented in the following way:

$$q^* = \arg\min_q l(q) \quad (1)$$

$$l(q) = \sum_{D=D_{min}}^{D_{max}} l(q,D) \cdot P(D) \quad (2)$$

$$l(q,D) = \begin{cases} s_1(q-D), & if \quad q > D \\ 0, & if \quad q = D \\ s_2(D-q), & if \quad q < D \end{cases}, \quad (3)$$

where q^* is the optimal order quantity, $l(q)$ denotes the expected loss, $l(q,D)$ is the loss incurred when the number of spare parts bought with the device equals q and the demand for spare parts is equal to D. $P(D)$ denotes the probability that the demand will be equal to D. Zero loss occurs if the order quantity is exactly the same as the demand.

There are many possible optimization methods to solve the aforementioned problem, such as the use of optimization software (SAS/OR, Solver in Excel, minizinc, R, cplex, etc.) and formulas (the recurrence equation, the critical ratio or the loss matrix; Sikora, 2008). In this paper we concentrate on loss matrices. Table 1 presents losses $l(q,D)$ for all possible combinations of pairs (q,D), see equation (3). Expected losses are generated in the last column (equation 2). The optimal solution is indicated by the lowest expected loss.

Table 1: Loss matrix for the classical version of SPQP (general case)

$q \setminus D$	$D_{min}=q_{min}$	$D_{min}+1$	…	$D_{max}-1$	$D_{max}=q_{max}$	$l(q)$
q_{min}	0	$s_2 \cdot (D-q)$	$s_2 \cdot (D-q)$	$s_2 \cdot (D-q)$	$s_2 \cdot (D-q)$	$l(q_{min})$
$q_{min}+1$	$s_1 \cdot (q-D)$	0	$s_2 \cdot (D-q)$	$s_2 \cdot (D-q)$	$s_2 \cdot (D-q)$	$l(q_{min}+1)$
…	$s_1 \cdot (q-D)$	$s_1 \cdot (q-D)$	0	$s_2 \cdot (D-q)$	$s_2 \cdot (D-q)$	…
$q_{max}-1$	$s_1 \cdot (q-D)$	$s_1 \cdot (q-D)$	$s_1 \cdot (q-D)$	0	$s_2 \cdot (D-q)$	$l(q_{max}-1)$
q_{max}	$s_1 \cdot (q-D)$	$s_1 \cdot (q-D)$	$s_1 \cdot (q-D)$	$s_1 \cdot (q-D)$	0	$l(q_{max})$

Source: Prepared by the author.

Interesting overviews of SPQP can be found, for instance, in Kennedy et al., 2002; Qu and Zhang, 2006; Rego and Mesquita, 2011. Extended SPQ models are variations of the classical SPQ model, involving additional losses due to the broken device, the purchase of parts at different moments, etc. (Bartakke, 1981;

Bian et al., 2013; Fera et al., 2010; Fortuin, 1981; Gu and Li, 2015; Papathanassiu and Tsouros, 1986; Pastore et al., 2015; Petrovic et al., 1986; Rodriguez et al., 2013; Rustenburg et al., 2000; Schuh et al., 2015; Sheikh et al., 2000; Verrijdt et al., 1998).

3 Spare parts quantity problem and entirely new seasonal devices

As it has been already mentioned, SPQP is usually regarded in the literature as a stochastic problem. However, in some circumstances it is extremely difficult to estimate the probability distribution (Gaspars-Wieloch, 2016a, 2017a, 2017b, 2018b, 2019a, 2019b).

Here we would like to investigate the case when totally new seasonal devices are bought. This entails: (1) the lack of historical data about previous failures, (2) the lack of sufficient knowledge about the mechanism of particular machines, (3) the inability to precisely define the whole sample space (Kolmogorov, 1993) and (4) perhaps a feeling anticipating new future factors which can radically change the trend up to now. Under such conditions objective probability quantification is impossible.

We focus on machines with very short life cycle. In such a case, the purchase of additional spare parts for these devices is made only once for the whole period of use (until the machine is withdrawn from service). Under such assumptions SPQP can be reduced to a one-shot decision problem (Guo, 2011; Zhu and Guo, 2016), since for each device only one scenario can occur. Czerwiński (1960) and von Mises (1949) state that the mathematical probability (understood as frequency) and expected value cannot be used for a single event, but only for repetitive events. Hence, this is the second reason why the use of probability in SPQP is not always justified.

There is also a general drawback related to the application of likelihood (not necessarily connected with SPQP and new devices). The term "probability" has many discrepant definitions, e.g. objective, subjective, classical, geometrical, frequency, logic, Bayes, Kolmogorov, Springer, Piegat, propensity (Carnap, 1950; de Finetti, 1975; Frechet, 1938; Hau et al., 2009; Knight, 1921; Kolmogorov, 1933; Piegat, 2010; Popper, 1959; Ramsey, 1931; Van Lambalgen, 1996; von Mises, 1949, 1957). The lack of unanimity leads to numerous doubts: what approach should be used? How to estimate the type of probability selected? Caplan (2001) adds that people are even unable to declare subjective probabilities: "they implicitly set them in acting".

In connection with all those facts, we would like to investigate SPQP for totally new seasonal devices as an SPQP under complete uncertainty, i.e. uncertainty with unknown probabilities (UUP).

It is worth mentioning that the term "uncertainty" also has diverse interpretations and types (Gaspars-Wieloch, 2016a, 2018b). According to decision theory, uncertainty may only be associated with situations where probabilities are unknown (in other situations this theory refers to risk or partial uncertainty). According to the theory of economics, there are diverse degrees of uncertainty but all of them involve situations with non-deterministic parameters (with risk understood as the possibility that some unfavorable or unpredicted event will happen). This paper is based on the second aforementioned theory. We consider both epistemic and aleatory uncertainty (Stirling, 2003; Zio and Pedroni, 2013).

Note that SPQP can be easily combined with scenario planning (Pomerol, 2001) thanks to (1) well-defined discrete sets of decisions (order quantities) and states of nature (demand quantities) and (2) the possibility to compute the loss matrix precisely (see Table 2).

The result of a choice made under uncertainty with scenario planning depends on two factors: which decision will be selected and which scenario will occur. Thus, SPQP under complete uncertainty may be defined by means of a scenario-based decision model with m states of nature (scenarios, events, demand quantities): $S = \{S_1, \ldots, S_i, \ldots, S_m\}$, n possible alternatives (decisions, strategies, order quantities): $A = \{A_1, \ldots, A_j, \ldots, A_n\}$, and $n \times m$ losses ($a_{i,j}$ – loss incurred by the buyer if state S_i occurs and alternative A_j is selected) calculated according to formula (4). The distributions of losses are discrete. The interpretation of S_i is that until the end of the use of a given machine D_i spare parts will be needed.

$$a_{i,j} = \begin{cases} s_1(q-D), & if \quad j > i \\ 0, & if \quad j = i \\ s_2(D-q), & if \quad j < i \end{cases} \tag{4}$$

Table 2: Loss matrix for the SPQP presented as a scenario-based model (general case)

Scen. \ Altern.	A_1 **(q=q_{min})**	A_2 **(q=q_{min}+1)**	…	A_j	…	A_{n-1} **(q=q_{max}-1)**	A_n **(q=q_{max})**
S_1 **(D=D_{min})**	$a_{1,1}$	$a_{1,2}$	…	$a_{1,j}$	…	$a_{1,n-1}$	$a_{1,n}$
S_2 **(D=D_{min}+1)**	$a_{2,1}$	$a_{2,2}$	…	$a_{2,j}$	…	$a_{2,n-1}$	$a_{2,n}$
⋮	⋮	⋮	⋱	⋮	⋱	⋮	
S_i	$a_{i,1}$	$a_{i,2}$	…	$a_{i,j}$	…	$a_{i,n-1}$	$a_{i,n}$
⋮	⋮	⋮	⋱	⋮	⋱	⋮	
S_{m-1} **(D=D_{max}-1)**	$a_{m-1,1}$	$a_{m-1,2}$	…	$a_{m-1,j}$	…	$a_{m-1,n-1}$	$a_{m-1,n}$
S_m **(D=D_{max}1)**	$a_{m,1}$	$a_{m,2}$	…	$a_{m,j}$	…	$a_{m,n-1}$	$a_{m,n}$

Source: Prepared by the author.

Researchers discuss the pros and cons of using probability data in scenario planning in various papers (Gaspars-Wieloch, 2019b) and SPQP usually refers to the probability calculus. However, in this contribution, we do not assign a likelihood to the demand, since the novelty degree of the decisions considered is very high.

The original version of SPQP is based upon the assumption of risk neutrality. In this work, however, we would like to take into account various preferences of the DMs (predictions, attitudes towards future results) which can be measured by the coefficients of optimism (β) and pessimism (α): $\alpha, \beta \in [0,1]$ and $\alpha + \beta = 1$ (α is close to 1 for extreme pessimists – risk averse behaviour, β is close to 1 for radical optimists – risk prone behaviour). Thanks to these parameters, we can adjust the final decision to the DM's nature. Additionally, the estimation of the coefficients is relatively little time-consuming (less time-consuming than the estimation of scenario probability).

4 Classical decision rules and specificity of SPQP loss matrices

In this section we analyze the specificity of SPQP loss matrices and investigate the usefulness of classical decision rules applied to scenario planning and decision making under complete uncertainty (i.e. max-max rule, Wald rule, Hurwicz rule, Bayes rule, Savage rule and max-min joy criterion). This research may be helpful in constructing a suitable procedure for SPQP with totally new devices.

Table 3 presents losses for three possible situations: 1. s_1–> s_2, 2. $s_1 = s_2$, 3. $s_1 < s_2$) which can occur in real-life situations. Values for prices c_1 and c_2 are ficticious, but in each case the first one is lower than the second one. The first situation is the least dangerous for the DM since the difference between prices is the lowest. The highest risk is connected with the last situation where a spare part bought in the future is much more expensive than a current extra part.

Table 3: Loss matrices for SPQP ($q_{min} = D_{min} = 0$, $q_{max} = D_{max} = 4$) and rankings generated on the basis of classical rules – examples 1-3

Ex.	***1. c_1=50, c_2=51, s_1=50, s_2=1***					***2. c_1=5, c_2=10, s_1=5, s_2=5***					***3. c_1=1, c_2=51, s_1=1, s_2=50***				
S \ A	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
S_1	0	50	100	150	200	0	5	10	15	20	0	1	2	3	4
S_2	1	0	50	100	150	5	0	5	10	15	50	0	1	2	3
S_3	2	1	0	50	100	10	5	0	5	10	100	50	0	1	2
S_4	3	2	1	0	50	15	10	5	0	5	150	100	50	0	1
S_5	4	3	2	1	0	20	15	10	5	0	200	150	100	50	0

Table 3 cont.

Ex.	*1. c_1=50, c_2=51, s_1=50, s_2=1*					*2. c_1=5, c_2=10, s_1=5, s_2=5*					*3. c_1=1, c_2=51, s_1=1, s_2=50*				
S \ A	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
M	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
W	**4**	50	100	150	200	20	15	**10**	15	20	200	150	100	50	**4**
H ***α=0.2***	**0.8**	10	20	30	40	4	3	**2**	3	4	40	30	20	10	**0.8**
H ***α=0.8***	**3.2**	40	80	120	160	16	12	**8**	12	16	160	120	80	40	**3.2**
B	**2**	11.2	30.6	60.2	100	10	7	**6**	7	10	100	60.2	30.6	11.2	**2**
S	**4**	50	100	150	200	20	15	**10**	15	20	200	150	100	50	**4**

Source: Prepared by the author.

Conclusions regarding the specific structure of SPQP loss matrices are as follows:

a) for s_1 sufficiently larger than s_2 the average of losses is the smallest for $q = q_{min}$ and the range between $a_{j,min}$ (the smallest loss related to alternative A_j) and $a_{j,max}$ (the largest loss related to A_j) is an increasing function $f(q)$;
b) for s_1 close to s_2 the average of losses is the smallest for the middle q;
c) for s_1 sufficiently lower than s_2 the average of losses is the smallest for $q = q_{max}$ and the range between $a_{j,min}$ and $a_{j,max}$ is a decreasing function $f(q)$;
d) loss distributions connected with particular orders are usually asymmetric;
e) loss distributions are always symmetric for extreme alternatives (i.e. for the smallest and the largest numbers of spare parts);
f) for each decision particular losses $a_{1,j}$, …, $a_{i,j}$, …, $a_{m,j}$ are always ordered in the form of a convex function and the minimum loss is equal to zero.
g) ranges vary significantly for cases where s_1 significantly differs from s_2;
h) each decision is Pareto-optimal!

Hence, we see that in SPQP distributions of losses are usually asymmetric and loss ranges for particular order quantities can be extremely diverse.

Now, let us check whether classical decision rules may be applied to SPQP. The **max-max rule** is designed for radical optimists only: it does not satisfy the assumption from the previous section since it is unable to adjust the decision to the DM's nature. Note that in the case of SPQP, the max-max rule has to be transformed prior to its use to a min-min rule because the matrix contains losses expressed as positive numbers. And then we can notice that, due to the very specific structure of the loss matrix it is impossible to generate a ranking on the basis of that procedure since all decisions are always treated as the best ones, regardless of the problem analyzed (the best value for each decision is equal to 0), see Table 3 (row *M*)!

The **Wald** (Wald, 1950) **decision rule** (max-min rule for profits and min-max rule for losses expressed as positive numbers) is designed for radical pessimists only, so again, this approach does not allow to consider diverse types of decision makers, either. In the case of SPQP, this method always suggests the decision with the smallest range of losses and focuses on extreme states, i.e. scenarios for which the demand is equal to D_{min} or D_{max} (other events are not significant), see Table 3 (row *W*). Those states are connected with the largest loss.

The next well-known decision rule is the **Hurwicz criterion** (Hurwicz, 1952). Here, the DM declares his/her coefficient of optimism/pessimism and two extreme scenarios are always taken into account: one with the highest loss and one with the lowest loss. In the case of SPQP the event with the highest loss is related to D_{min} or D_{max}. The event with the lowest loss is different for each decision and occurs when the order quantity is equal to the demand. The idea of the Hurwicz rule consists in (1) calculating for each decision the sum of two products: coefficient of optimism (β) multiplied by the highest profit (the lowest loss) and coefficient of pessimism (α) multiplied by the lowest profit (the highest loss), and (2) selecting the decision with the highest profit weighted average or the lowest loss weighted average. Theoretically, the Hurwicz rule may be applied by different decision makers (optimists, pessimists, moderate DMs). Nevertheless, the structure of the SPQP loss matrix is so unusual that the maximal profit (i.e. the minimal loss) is always equal to zero. Therefore, regardless of the level of α and β (with one exception: $\alpha = 0$), decisions recommended by the Hurwicz rule are exactly the same as alternatives suggested by… the Wald rule, see Table 3 (rows $H\ \alpha = 0.2$ and $H\ \alpha = 0.8$). Hence, as a matter of fact, there is no possibility to take into consideration different types of decision makers, although each strategy is Pareto-optimal! For instance, according to the Hurwicz rule, alternative A_1 is better than A_2 in Example 1 even for $\alpha = 0.2$, which is quite astonishing as A_2 dominates A_1 in the case of four out of five states! Even when the coefficient of pessimism decreases, the Hurwicz rule applied to SPQP indicates variants suitable for pessimists.

Additionally, we can easily notice that when computing weighted indices for each decision, the status of particular scenarios varies depending on the alternative (see, for instance, example 1, Table 3: S_1 is the best scenario for A_1, but it is the worst state for A_5), which may be quite surprising in SPQP, where we rather tend towards the view that the most optimistic (pessimistic) scenario is that with the lowest (highest) demand for extra parts, regardless of losses connected with particular decisions. Perhaps, a global status for each state would be more appropriate than a local one.

A general remark concerning the Hurwicz rule: the procedure does not take into account the nature of the outcome distribution connected with particular alternatives, which leads to illogical recommendations for decision problems with asymmetric profits (or losses) (Gaspars-Wieloch 2014a, 2014b, 2016a, 2017b). This drawback is worth considering since in SPQP losses are usually asymmetric.

As opposed to previous approaches, the **Bayes (Laplace) criterion**, thanks to the use of the arithmetical average, analyzes both extreme and intermediate losses (not only extreme ones), which is significant in the case of asymmetric outcomes (Table 3, row *B*). However, the Bayes rule is not suitable for SPQP under complete uncertainty, since it does not allow to declare our coefficients of pessimism and it is designed for multi-shot decisions (hence for a multi-period horizon) only, while in this paper we assume that the purchase of additional spare parts at cost c_1 for a given device is made once for the whole period of use (until the machine is withdrawn from service).

There are also two other classical decision rules for which the position of a given outcome in the profit (loss) matrix is extremely important. This is a feature characteristic of the Savage rule (Savage, 1961) and the max-min joy criterion (MJC) (Hayashi, 2008). The goal of MJC is to show the superiority of particular outcomes connected with a given scenario to its worst result, while in the Savage rule the aim is to demonstrate the inferiority of particular payoffs related to a state of nature to its best result (Gaspars-Wieloch, 2014c, 2018a).

The **Savage rule** (min-max rule) requires the DM to generate a relative loss matrix (regret matrix), but due to the occurrence of zero losses for each scenario in SPQP, the original loss matrix may be treated as a relative loss matrix (without any transformation). Hence, in the case of SPQP, rankings obtained by means of the Savage approach correspond to rankings offered by the Wald rule (Table 3, row *S*) and, again, that method is appropriate for pessimists only: there is no possibility to adjust recommendations to the DM's nature.

The idea of the max-min joy criterion (MJC) is very similar to the reasoning characteristic of the Savage procedure, but instead of a regret table a relative profits matrix is applied and the solution is set on the basis of the worst relative profits connected with particular alternatives. MJC is designed only for people exhibiting a risk-averse behavior. Note that in the case of SPQP the worst relative profits are always related to the first (q_{min}) or the last decision (q_{max}), which means that even for significant differences between s_1 and s_2 those extreme alternatives (q_{min} and q_{max}) will never be optimal in accordance with MJC (they have at least one zero value in their column): Table 4, MJC. This conclusion seems illogical since in some real-life situations the choice of extreme order quantities is desirable.

Table 4: Relative profit matrices and rankings generated by MJC − examples 1-3

Ex.	***1. c_1=50, c_2=51, s_1=50, s_2=1***					***2. c_1=5, c_2=10, s_1=5, s_2=5***					***3. c_1=1, c_2=51, s_1=1, s_2=50***				
S \ A	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
S_1	200	150	100	50	0	20	15	10	5	0	4	3	2	1	0
S_2	149	150	100	50	0	10	15	10	5	0	0	50	49	48	47
S_3	98	99	100	50	0	0	5	10	5	0	0	50	100	99	98
S_4	47	48	49	50	0	0	5	10	15	10	0	50	100	150	149
S_5	0	1	2	3	4	0	5	10	15	20	0	50	100	150	200
MJC	**0**	**1**	**2**	**<u>3</u>**	**0**	0	5	**<u>10</u>**	5	0	0	**<u>3</u>**	2	1	0

Source: Prepared by the author.

Due to all these factors, we can conclude that the aforementioned decision rules should not be applied to SPQP (lack of possibility to consider the DM's nature; lack of application to one-shot decisions or asymmetric distribution of losses; generation of irrational rankings). Besides classical decision rules, there are of course many extended decision rules designed for uncertain decision making, but they refer to the probability calculus (e.g. Basili and Chateauneuf, 2011; Ellsberg, 2001; Etner et al., 2012; Garcia et al., 2012; Ghirardato et al., 2004; Gilboa, 2009; Gilboa and Schmeidler, 1989; Hildebrandt and Knoke, 2011; Marinacci, 2002; Pereira et al., 2015; Perez et al., 2015; Tversky and Kahneman, 1992).

In the next section we are going to describe in detail the problem to be solved, and suggest a new decision rule for that purpose.

5 Three-criteria decision rule for SPQP and entirely new devices

In previous sections we have demonstrated that (1) SPQP under complete uncertainty was worth investigating and (2) classical and extended decision rules were not appropriate to solve that problem. In this section all the assumptions connected with the chosen problem are gathered and a novel procedure is proposed.

The scenario-based SPQP model contains the following assumptions:

1) states and the loss matrix are known, but the probability (frequency) of particular scenarios is not known (entirely new devices, lack of historical data);
2) cost c_2 is not treated as a deterministic parameter since it concerns the future: it is given as an interval parameter, which means that parameter s_2 is also interval and the loss matrix is partially interval (Table 5 presents fictitious illustrative prices);

3) the problem concerns a one-period horizon (short life cycle devices) and the period ends when the machine is withdrawn from service (spare parts at cost c_1 are purchased only together with the purchase of the machine);
4) the final recommendation takes into account the DM's nature, i.e. his/her attitude towards a given problem (coefficients α and β);
5) the optimal decision is performed only once (one-shot decision): in the future, due to new experiences, the DM's nature and the loss matrix may change;
6) the optimality is checked using the weighted and arithmetical averages and the standard deviation of all losses (each loss connected with a given alternative has an impact on the final choice, not only extreme losses);
7) each order quantity may be optimal (depending on coefficients α and β) since each one is Pareto-optimal (which is not the case for classical decision rules);
8) the model is useful for both active and passive DMs (we assume that an active DM is a person who intends to analyze all the values very carefully and even influence the particular steps of the algorithm; while a passive DM only declares his/her coefficient of optimism and waits for the final recommendation);
9) the status of each scenario is defined globally, not locally.

Table 5: Partially interval loss matrices – examples 4-5

Ex.	***4. c_1=50, $c_2 \in$ [51,52], s_1=50, $s_2 \in$ [1,2]***					***5. c_1=1, $c_2 \in$ [41,51], s_1=1, $s_2 \in$ [40,50]***				
S \ A	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
S_1	0	50	100	150	200	0	1	2	3	4
S_2	[1,2]	0	50	100	150	[40,50]	0	1	2	3
S_3	[2,4]	[1,2]	0	50	100	[80,100]	[40,50]	0	1	2
S_4	[3,6]	[2,4]	[1,2]	0	50	[120,150]	[80,100]	[40,50]	0	1
S_5	[4,8]	[3,6]	[2,4]	[1,2]	0	[160,200]	[120,150]	[80,100]	[40,50]	0

Source: Prepared by the author.

The investigation of SPQP under complete uncertainty with interval unit purchase costs of the subassembly just after the failure (see assumption 2) is desirable because that price is related to the future and future purchase times and circumstances are not known exactly, especially in the case of totally new devices. The interval cost c_2 influences particular states of nature to a different extent (compare, for instance, scenarios S_1 and S_4, Table 5). Intervals in the matrix have different widths, i.e., differences between their endpoints (e.g., 50-40 = 10 and 150-120 = 30), and they occur only in the bottom left corner of the matrix. Losses connected with the first scenario and the last decision are given as point values.

The analysis of the standard deviation (assumption 6) is crucial because the ranges of losses related to particular order quantities vary rather significantly.

The procedure developed for the aforementioned problem refers to several other approaches described in the literature.

First, we are going to apply some elements of the (H+B) rule presented in Gaspars-Wieloch (2014a, 2015b, 2016b), which is a hybrid of the Hurwicz and Bayes decision rules. That method was originally worked out for profit matrices, but it can be easily modified for loss matrices. The hybrid, thanks to parameters $\alpha \in [0,1]$ and $\beta = 1 - \alpha \in [0,1]$, takes into account the DM's preferences (as does the Hurwicz rule). In (H+B) rule, in contrast to the Hurwicz, Wald, Hayashi, and Savage approaches, all the outcomes influence the value of the final measure, which is quite advantageous for cases where alternatives contain many payoffs almost equal to the extreme values. The general idea of H+B is to assign, for a pessimist, α to the last term of the non-increasing sequence of all the payoffs related to a given decision and β to the remaining terms of that sequence. For an optimist, weights are set in a different way: β is connected with the first term of the sequence and α with the remaining ones. The assignment of parameters α and β to particular payoffs, depending on the level of optimism, is justified in Gaspars-Wieloch (2014a, 2017b) where the author suggests a significant modification of the classical Hurwicz decision rule and adds to that procedure certain features characteristic for the Bayes rule. The idea of the hybrid presented in Gaspars-Wieloch (2014a) is to recommend, for a strong pessimist, an alternative with a relatively high payoff $a_{j,min}$ or with quite frequent payoffs (almost) equal to $a_{j,max}$ since the pessimist fears the worst, regardless of the decision selected, and that is why such a DM needs an alternative which is attractive even if the worst state occurs and which gives a feeling of security. On the other hand, that rule suggests, for a strong optimist, an alternative with the highest (or almost the highest) payoff $a_{j,max}$, but its highest payoffs do not have to be frequent since the optimist is almost or even completely sure that the best scenario will occur regardless of the decision selected.

Second, due to the fact that in SPQP the ranges of losses related to particular alternatives vary rather significantly, we will support the (H+B) rule with an additional auxiliary decision tool, which analyzes the deviations between outcomes (Gaspars-Wieloch, 2015a, 2017b; Ioan and Ioan, 2011).

Third, we perceive a necessity to refer to the SF+AS (scenario forecasting and alternative selection) procedure recommended in Gaspars-Wieloch (2015a). Its general idea is to (1) forecast the set of scenarios with the largest subjective chance of occurrence (according to the DM's level of pessimism/optimism), see assumption 9, and (2) select a suitable alternative on the basis of a reduced payoff matrix. The use of certain SF+AS features is crucial in SPQP under complete uncertainty since, due to the existence of zero losses for each decision, the original (H+B) decision rule, just like the Hurwicz rule, unfortunately recommends the same optimal order quantities as the Wald rule does, regardless of the DM's nature.

Fourth, we intend to choose a tool enabling one to analyze interval values (see parameter c_2). One may apply, for instance, (1) fuzzy numbers and sets (which requires the estimation of additional parameters, such as degrees of membership), (2) the average cost c_2, (3) the level of c_2 which corresponds to the DM's nature, (4) a meta loss matrix (containing scenarios with the same demand and different values of c_2). Here, we decide to create two loss matrices for extreme cases (i.e. endpoints of interval $[c_{2,min}; c_{2,max}]$) and to compare the recommended solutions.

The suggested three-criteria rule for SPQP and totally new devices consist of the following steps:

1) Define $q_{min} = D_{min}$, $q_{max} = D_{max}$, m (number of scenarios), n (number of decisions), the set of alternatives (A) and the set of scenarios (S); this is performed mainly by experts;
2) Estimate cost c_1 as a point value and cost c_2 as an interval value: $[c_{2,min}; c_{2,max}]$. Compute s_1, s_2 and generate the partially interval loss matrix; this is performed mainly by experts;
3) Determine α and β (subjectively or on the basis of psychological tests). The coefficients should describe the DM's attitude towards a demand for spare parts. If $\alpha \in [0,0.5[$, then $\alpha = \alpha_o$, $\beta = \beta_o$ (α_o and β_o are optimist's coefficients). If $\alpha \in]0.5,1]$, then $\alpha = \alpha_p$, $\beta = \beta_p$ (α_p and β_p are pessimist's coefficients);
4) Assign an interval for the coefficient of optimism to each scenario. The width w of the range for each state of nature is defined as follows:

$$w = \frac{1}{m} \tag{5}$$

The extreme values (b_i and t_i) of interval $[b_i; t_i]$ set for scenario S_i, i.e. its endpoints, are computed according to Equations (6)–(7):

$$b_i = \frac{D_{\max} - D_i}{D_{\max} - D_{\min} + 1} \qquad i = 1,\ldots, m \tag{6}$$

$$t_i = \frac{D_{\max} - D_i + 1}{D_{\max} - D_{\min} + 1} = b_i + w \qquad i = 1,\ldots, m \tag{7}$$

Apart from the interval for the highest demand (i.e. the last scenario), the intervals are left-open, i.e. $]b_i; t_i]$ for $i = 1,2,\ldots, m$-1 and $[b_i; t_i]$ for $i = m$.

5) Find the scenario which corresponds to the coefficient of optimism declared by the DM (according to intervals computed in step 4). Let us denote this state of nature by S^*_i, and the losses connected with S^*_i by $a^*_{i,1}, \dots, a^*_{i,n-1}, a^*_{i,n}$;
6) Create two loss matrices: matrix I containing losses calculated on the basis of $c_{2,min}$ and matrix II for $c_{2,max}$. Perform steps 7-10 separately for each matrix;
7) Calculate, for each decision, index hb_j (hb^p_j, hb^o_j or $hb^{0.5}_j$ depending on the parameter α). If $\alpha\in\,]0.5,1]$, calculate hb^p_j (index for pessimists) from Equation (8). If $\alpha\in[0,0.5[$, compute hb^o_j (index for optimists) following formula (9). If $\alpha = 0.5$, calculate $hb^{0.5}_j$ using Equation (10), where b_j denotes the Bayes criterion, i.e. the average of all losses.

$$hb_j^p = \frac{\alpha_p \cdot a^*_{i,j} + \beta_p \cdot \left(\sum_{i=1}^{m} \left(a_{i,j}\right) - a^*_{i,j} \right)}{\alpha_p + (m-1)\cdot \beta_p} \tag{8}$$

$$hb_j^o = \frac{\alpha_o \cdot \left(\sum_{i=1}^{m} \left(a_{i,j}\right) - a^*_{i,j} \right) + \beta_o \cdot a^*_{i,j}}{(m-1)\cdot \alpha_o + \beta_o} \tag{9}$$

$$hb_j^{0.5} = hb_j^p = hb_j^o = b_j = \frac{1}{m}\cdot \sum_{i=1}^{m} a_{i,j} \tag{10}$$

The denominators in Equations (8)-(9) are introduced so that the final values of the particular indices belong to the interval $[a_{j,min};a_{j,max}]$. Denominators are not crucial and can be omitted when preparing the ranking;

8) Choose alternative A^*_j fulfilling condition (11). Options A^*_j chosen on the basis of matrices I and II belong to sets A^*_{I} and A^*_{II}, respectively. If, within a given matrix, there are alternatives with indices hb_j very close to the smallest one, they may also be selected by the DM as elements of sets A^*_{I} and A^*_{II};

$$A_j^* = \arg\min_j (hb_j) \tag{11}$$

9) If selected decisions A^*_j fulfill Equations (12)-(19), $A^*_j = A^{**}_j$. Go to step 11. Otherwise, go to step 10.

$$b_j^* \le \beta \cdot \left(b_{\max} - b_{\min}\right) + b_{\min} \tag{12}$$

$$b_j = \frac{1}{m}\cdot \sum_{i=1}^{m} a_{i,j} \quad j = 1, \dots, n \tag{13}$$

$$b_{\max} = \max_j \{b_j\} \tag{14}$$

$$b_{\min} = \min_j \{b_j\} \tag{15}$$

$$\sigma_j^* \le \beta \cdot (\sigma_{\max} - \sigma_{\min}) + \sigma_{\min} \tag{16}$$

$$\sigma_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(a_{i,j} - b_j\right)^2} \quad j = 1, \ldots, n \tag{17}$$

$$\sigma_{\max} = \max_j \{\sigma_j\} \tag{18}$$

$$\sigma_{\min} = \min_j \{\sigma_j\} \tag{19}$$

10) Find the nearest decision (by gradually increasing or decreasing the order quantity) satisfying Equations (12)-(19) and denote it by A^{**}_j;
11) Decisions A^{**}_j chosen on the basis of matrices I and II belong to sets A^{**}_{I} and A^{**}_{II}, respectively. If both sets are singleton sets and $A^{**}_{\mathrm{I}} = A^{**}_{\mathrm{II}}$, then decision A^{**}_j (equivalent for both sets) is the suitable one (let us denote it by A^{***}_j); Otherwise, go to step 12;
12) If at least one set (A^{**}_{I} or A^{**}_{II}) is a multi-element one and both sets contain exactly one common decision A^{**}_j, then that decision is the suitable one (i.e. A^{***}_j). Otherwise, go to step 13;
13) If both sets contain more than one common decision A^{**}_j, choose option A^{***}_j according to Equations (20)-(22). Otherwise, go to step 14;

$$A_j^{***} = A_j^{**} \mid \begin{cases} \left\lceil \dfrac{j^{**}_{\min} + j^{**}_{\max}}{2} \right\rceil, & if \quad \beta \in [0;0.5[\\ \left\lfloor \dfrac{j^{**}_{\min} + j^{**}_{\max}}{2} \right\rfloor, & if \quad \beta \in [0.5;1] \end{cases} \tag{20}$$

$$j^{**}_{\min} = \min_{A_j^{**} \in A_I^{**} \wedge A_j^{**} \in A_{II}^{**}} \{j\} \tag{21}$$

$$j^{**}_{\max} = \max_{A_j^{**} \in A_I^{**} \wedge A_j^{**} \in A_{II}^{**}} \{j\} \tag{22}$$

14) If the two sets are disjoint, choose option A^{***}_j according to formulas (23)-(25).

$$A_j^{***} = A_j^{**} \mid \begin{cases} \left\lceil \dfrac{j^{**}_{\min} + j^{**}_{\max}}{2} \right\rceil, & if \quad \beta \in [0;0.5[\\ \left\lfloor \dfrac{j^{**}_{\min} + j^{**}_{\max}}{2} \right\rfloor, & if \quad \beta \in [0.5;1] \end{cases} \tag{23}$$

$$j^{**}_{\min} = \max_{A_j^{**} \in A_I^{**}} \{j\} \tag{24}$$

$$j^{**}_{\max} = \min_{A_j^{**} \in A_{II}^{**}} \{j\} \tag{25}$$

In the last part of section 5 we explain in detail steps, terms and equations of the above algorithm.

Steps 4 and 5 refer to the SF+AS procedure (Gaspars-Wieloch, 2015a), which consists in predicting the scenario with the greatest subjective chance of occurrence on the basis of the coefficient of optimism, but this time, instead of dominance cases used in the original version, a new method is applied. The reasoning is as follows: the more optimist the DM is, the more probable is the minimal demand for extra parts, so the largest values of β are assigned to scenario D_{min}. The use of a different approach (as compared to the original SF+AS procedure) is justified below. In SPQP the situation is very specific: each successive state of nature is connected with a greater number of failures, hence with worse conditions. Therefore, the status of the particular scenarios can be assessed even without the knowledge of all the losses connected with a given event.

In steps 7 and 8 we refer to the hybrid of Hurwicz and Bayes rule. However, this time, we assign the highest coefficient (α or β) not to the extreme value (the lowest or the highest one), but to the value connected with the scenario with the highest subjective chance of occurrence. Such a modification results from the fact that in SPQP the status of particular scenarios can be evaluated in a global way, thus it does not change depending on the order quantity considered. The idea to treat the scenario status globally (not locally) has been already suggested by Milnor (1954) who stated that each decision rule theoretically designed for games against nature, which treats nature as a conscious opponent who is altering strategies depending on the outcomes, is wrong and unsatisfactory.

Steps 7 and 8 use the first criterion in the three-criteria decision rule, i.e. the weighted average of losses. The second and third criteria (arithmetical average and standard deviation, see Equations 12 and 16, step 9) are introduced in order to find a relatively safe strategy (i.e. an alternative with a relatively small range of losses and as few high losses as possible), which is particularly important in the case of cautious DMs. Of course, the arithmetical average and the standard deviation are just suggestions. One may use other measures, such as ranges of losses connected with particular order quantities. Note that the last two criteria are applied in the algorithm only to decisions satisfying the first criterion. However, if there are other decisions with indices hb_j very close to the lowest one, we recommend calculating and comparing the values of the second and third measures for the whole subset containing the best strategies according to the first criterion. We purposely do not define the acceptable distance between the lowest index hb_j and the other ones: we leave it to the DM.

Step 11 results from the fact that if for $c_{2,min}$ the only solution A^{**}_j is the same as for $c_{2,max}$, then for any value from interval $[c_{2,min}; c_{2,max}]$, solution A^{**}_j will be the same.

As a matter of fact, the ceiling and floor in Equations (20) and (23) (steps 13-14) are useful only if one device is bought. When more than one machine is bought, the ceiling and floor in that formula are not crucial, since the final result may be non-discrete (e.g. we might buy 5.5 spare parts on average, i.e. for some devices 5 and for others, 6). The non-discrete average is appropriate only if all devices are identical and purchased for the same project (company). In the case of the purchase of one machine, we assume that for optimists (pessimists) we search for the floor (ceiling) of that ratio (Equation 20 or 28) since optimists (pessimists) expect a low (high) demand and a low (high) cost c_2.

6 Example

In this section we are going to solve Example 6 (Table 6) by means of the three--criteria decision rule. Let us assume that an engine with a totally new technology is bought. All steps are analyzed below:

1) $q_{min} = D_{min} = 0$, $q_{max} = D_{max} = 4$, $n = m = 5$, $S = \{S_1,S_2,S_3,S_4,S_5\}$, $A = \{A_1,A_2,A_3,A_4,A_5\}$;
2) $c_1 = 10$; $17 \leq c_2 \leq 25$; $s_1 = 10$; $7 \leq s_2 \leq 15$. The loss matrix is shown in Table 6.
3) $\alpha = 0.8$, $\beta = 0.2$ (the DM is a moderate pessimist) → $\alpha = \alpha_p$, $\beta = \beta_p$;
4) $w = 1/m = 0.2$. Intervals: [0;0.2] for S_5,]0.2;0.4] for S_4,]0.4;0.6] for S_3,]0.6;0.8] for S_2 and]0.8;1.0] for S_1;
5) The scenario with the greatest chance of occurrence is $S_i^* = S_5$ since $\beta_p = 0.2 \in [0;0.2]$. The most "probable" losses are: $a^*_{i,1} = [28,60]$, $a^*_{i,2} = [21,45]$, $a^*_{i,3} = [14,30]$, $a^*_{i,4} = [7,15]$, $a^*_{i,5} = 0$;
6) Matrices I and II contain losses equal to the left and right interval endpoints, respectively (Table 7);

Table 6: Partially interval loss matrix – example 6

Ex.	***6. c_1=10, $c_2 \in$ [17,25], s_1=10, $s_2 \in$ [7,15]***				
S \ A	A_1	A_2	A_3	A_4	A_5
S_1	0	10	20	30	40
S_2	[7,15]	0	10	20	30
S_3	[14,30]	[7,15]	0	10	20
S_4	[21,45]	[14,30]	[7,15]	0	10
S_5	[28,60]	[21,45]	[14,30]	[7,15]	0

Source: Prepared by the author.

Table 7: Matrices I and II (losses and computations) – example 6

Ex.	***Matrix I. $c_1=10, c_2=17, s_1=10, s_2=7$***					***Matrix II. $c_1=10, c_2=25, s_1=10, s_2=15$***				
S \ A	A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
S_1	0	10	20	30	40	0	10	20	30	40
S_2	7	0	10	20	30	15	0	10	20	30
S_3	14	7	0	10	20	30	15	0	10	20
S_4	21	14	7	0	10	45	30	15	0	10
S_5	28	21	14	7	0	60	45	30	15	0
HB_p	19.25	14.37	11.62	11.00	12.50	41.25	29.38	20.63	15.00	12.50
Constraints	Average <= 12.16; st. deviation <= 12.49					Average <= 18.00; st. deviation <= 13.69				
Average b_j	14.00	10.40	10.20	13.40	20.00	30.00	20.00	15.00	15.00	20.00
Standard deviation	11.07	7.83	7.50	11.74	15.81	23.72	17.68	11.18	11.18	15.81
HB_p (revised)	19.25	14.37	11.62	11.00	12.50	41.25	29.38	20.63	15.00	12.50

Source: Prepared by the author.

7)-10) Computations for both matrices are also presented in Table 7. As we can see, A_4 is selected in step 8 in matrix I (due to the lowest value hb_j): $A^*_{\mathrm{I}} = \{A_4\}$, but the average b_4 for that decision exceeds the allowed one (13.4 > 12.16). Thus, although its standard deviation satisfies Equation (16): 11.74 < 12.49, one should find another alternative. The nearest acceptable is $A_j^{**} = A_3$, since 10.20 < 12.16 and 7.5 < 12.49. Hence $A^{**}_{\mathrm{I}} =$ $=\{A_3\}$. A similar procedure is applied to matrix II. This time, $A^*_{\mathrm{II}} = \{A_5\}$, but the average and standard deviation are too high: 20.00 > 18.00 and 15.81 > 13.69. Therefore, we have to search for A_j^{**}: $A^{**}_{\mathrm{II}} = \{A_4\}$;

11)-14) Sets A^{**}_{I} and A^{**}_{II} contain one element each, but they are disjoint. That is why we move directly to step 14 and choose the final decision: $A^{***} = \{A_4\}$ since $j^{**}_{\min} = 3$, $j^{**}_{\max} = 4$ and $\beta = 0.2$. The optimal order quantity is 3.

At the end of this section we may check the results given by the Hurwicz rule and the original (H+B) rule which theoretically take into account the DM's nature. They also recommend A_3 (matrix I) and A_4 (matrix II), but note that their recommendations will not change after the modification of the coefficient values! If e.g. $\alpha = 0.2$, $\beta = 0.8$ (moderate optimist), the solution suggested by both procedures will be still the same and that is alarming (the reason has been given in previous sections: rankings do not change due to the occurrence of a zero loss for each decision). Fortunately, such a situation will not occur if we apply the three-criteria approach. For a moderate optimist that method recommends A_2.

7 Conclusions

The spare parts quantity problem (SPQP) under complete uncertainty has not been discussed yet in the literature, but we perceive the necessity to investigate this issue since in some cases the probability (frequency) estimation may be onerous (devices with a new technology). We have demonstrated that, due to a very specific structure of the loss matrix, classical decision rules designed for decision making under uncertainty with unknown probabilities cannot be applied to this problem, especially if one intends to take into account the decision maker's attitude towards risk. This paper contains a description of a three--criteria procedure that may be useful for the uncertain version of SPQP with totally new seasonal devices. The novel approach combines a hybrid of the Hurwicz and Bayes decision rules with the average and standard deviation criteria. It also refers to a two-stage procedure (SF+AS) consisting in forecasting the scenario with the largest subjective chance of occurrence before the final selection of the appropriate decision. Another method for spare parts demand forecasting has been already proposed by Romeijnders, Teunter and van Jaarsveld, 2012), for instance.

The three-criteria approach has several significant advantages. First, it takes into consideration the decision maker's attitude towards a given problem and leads to logical results for each kind of decision maker. Second, it may be applied even if the distribution of losses connected with particular alternatives is not symmetric since it examines each loss (not only extreme ones). Third, it has been worked out for the case where the future unit purchase cost of a spare part is given as an interval parameter. Fourth, it analyzes two kinds of uncertainties: uncertain demand for spare parts (discrete random variable with unknown probability distribution) and uncertain future cost of missing parts (interval value). Fifth, it does not require any information about the likelihood, which is useful in the case of new machines and one-shot decisions. It only applies certain secondary probability – like quantities which are not estimated by the DM, but are generated using the coefficient of optimism. Sixth, depending on the DM's commitment, the procedure may be applied by both active and passive decision makers. Seventh, the method is designed for one-shot decisions (i.e. single-period problems), but the obtained recommendation can be used simultaneously for each identical device belonging to a given company.

Note that the new procedure can support the SPQP decision making process, but it is reasonable to use it only in the case of expensive purchases. Otherwise, simple reasoning seems sufficient.

In the future, it would be desirable to analyze SPQP in the context of the length of the period of use (when is the machine going to be withdrawn from service?). This factor is also uncertain and may affect the final decision as well. A similar problem is discussed, e.g., in de Jonge et al. (2015).

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