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OPTIMAL SELECTION FROM A SET OF OFFERS USING A SHORT LIST

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Abstract

The rise of the Internet has led to a huge amount of information being available at the touch of a button. This article presents a model of searching for a valuable good, e.g. a new flat, using the Internet to gain initial information about the offers available. This information is used to create a short list of offers to be observed more closely before making a final decision. Although there has been a lot of recent work on the use of short lists in decision making procedures, there has been very little work on how the length of a short list should depend on the parameters of the search problem. This article addresses this problem and gives results on the optimal length of a short list when a searcher is to choose one of *n* offers and the search costs are convex in the length of the short list. Several examples are considered.

Keywords: secretary problem, incomplete information, payment for additional information, agglomeration of multiple traits.

1 Introduction

This paper considers procedures of searching for a valuable good using short lists of promising offers to be inspected more closely. For example, suppose an individual living in a relatively large city is looking for a new flat. Nowadays, it is easy to obtain basic information on a large number of flats, e.g. price, floor space and location. Purchasing a flat simply on the basis of information from the Internet would be highly risky (without closer, physical inspection it would be difficult to assess how appropriate an offer is). However, viewing all the flats that might be suitable based on size, price and location might well be prohibitive in terms of the search costs involved. Hence, a strategy based on the concept of

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a short list, i.e a relatively small set of seemingly attractive offers that are inspected more closely before a purchasing decision is made, is a natural heuristic to use in such scenarios.

Heuristics are useful tools due to the cognitive limitations of decision makers (DMs). Successful heuristics should be adapted to the abilities of DMs and the structure of the information gained during search (see Simon, 1955, 1956; Todd and Gigerenzer, 2000, as well Bobadilla-Suarez and Love, 2018). There has been a lot of recent research on the concept of short lists and the situations in which they are useful in decision making. Short lists are useful when the amount of information available exceeds the cognitive abilities of DMs or the costs of exhaustive search are too high (see Masatlioglu et al., 2012; Lleras et al., 2017). For example, short lists of possible holiday destinations can be constructed using information from friends and colleagues (see Bora and Kops, 2019). The use of short lists may also be useful when offers can be categorized (Armouti-Hansen and Kops, 2018). When using the Internet to search for offers, filters may be used to search a list of offers with multiple attributes by arranging offers according to traits that are seen to be the most important in the decision process (see Rubinstein and Salant, 2006; Mandler et al., 2012; Kimya, 2018). Mandler et al. (2012) argue that by considering the traits of an offer in order of decreasing importance, a DM acts in a similar manner to a utility maximizer. Kimya (2018) considers a similar problem where offers that are assessed the least positively according to a given trait are eliminated, starting with the most important trait. Such a procedure can be thought of as a strategy based on constructing short lists of decreasing size until a final decision is made.

This paper considers a model in which information from the Internet (or other information-rich source) plays an instrumental role in searching for a valuable resource, such as a second-hand car or a flat, but is supplemented by closer inspection of offers, which is assumed to occur offline. The cost of physically visiting and inspecting an offer is assumed to be much more costly than finding an offer via the Internet and inspecting the information contained there. For example, Internet search for a flat could begin by eliminating offers that do not satisfy the hard constraints defined by the DM based on price, floor space, number of rooms and location. If a large number of offers still remain (which is likely in large towns or cities), then the DM can make a short list of flats to view based on his/her preferences. The model assumes that the market is large enough to ensure that the best of the offers observed from this short list is acceptable to the DM.

Although much research has been carried out recently on theoretical aspects of using short lists, little work has been carried out on the question of how long short lists should be depending on the parameters of the search problem and the structure of the information. For example, when searching for a specialist employee, an employer often interviews a short list of four or five promising candidates based on a set of written applications. One obvious question regards the scenarios in which using short lists of such moderate length are optimal or near-optimal. The model presented in this paper is a step towards answering such questions.

The approach used in this paper is broadly based on theory regarding choice from a set of offers observed in parallel. Even when offers can be, in theory, considered in parallel, Saad and Russo (1996) and Bobadilla-Suarez and Love (2018) note that the order in which offers are observed may be important. For example, since the physical appearance of two flats cannot be compared in parallel, it is difficult to assess the first flat to be visited, since it cannot even be compared to a mental picture of other offers. Similarly, an average offer might be regarded as attractive if it appears after one or two unattractive offers. Hence, the theory of sequential search is also applicable to such decision problems.

In problems involving the choice of one offer from a set presented in parallel, one might suppose that a search procedure should satisfy the Weak Axiom of Revealed Preferences (WARP, see Lleras et al., 2017). In words, this criterion states that when offers x and y belong to both of the sets S and T and x is chosen from the set S, then y is never chosen from the set *T*. In mathematical terminology,

$$[c(S) = x, y \in S, x \in T] \Rightarrow [c(T) \neq y]. \quad (1)$$

The following example shows that a search procedure based on forming a short list of fixed length does not satisfy the WARP criterion. Suppose x and y are two offers such that initial inspection indicates that offer y is more attractive than offer x, but closer inspection reveals that offer x is better than offer y. One may define sets of offers S and T, such that both x and y would be on the short list of offers from set S and x is chosen, but x is not on the short list of offers from set T and the offer y is chosen.

Stigler (1961) presents a model of costly search for a good. The strategy of the DM is the number of offers to investigate, k. After observing these offers, the DM accepts the best offer (i.e. offers are essentially observed in parallel). The optimal number of offers to observe, k^* is the smallest value k such that the expected gain from observing an additional offer is less than the cost of observing an offer. In order to derive this optimal strategy, the DM needs to know the distribution of the values of offers and the cost of observing an offer.

MacQueen (1964) derives the form of the optimal strategy for a sequential search problem of a similar form to the one presented here. The search costs are assumed to be linear in both the number of offers seen and the number of offers inspected closely. A DM first decides whether to inspect an offer more closely based on an initial signal. After closer investigation, the DM either accepts the offer or continues searching. No recall of previous offers is possible. Under the optimal strategy, the DM closely investigates an offer only when the initial indicator of the value of an offer exceeds a given threshold. An offer is only accepted when its value according to the two signals exceeds a given threshold. Ramsey (2015) gives theoretical and numerical results for such problems when the two signals come from a joint normal distribution. To realise a strategy of this form, the precise values of the signals must be observed and to derive the optimal strategy, the DM should know the joint distribution of the signals.

Simon (1955) introduces the concept of satisficing, which can be adapted to both parallel and sequential search. Suppose two signals indicate the value of an offer. In sequential search, the DM observes the second signal only if the initial signal exceeds a given threshold. Given the second signal is observed, an offer is accepted if the second signal (considered on its own) exceeds an appropriate threshold). Assuming that the traits are observed in decreasing order of importance, such a strategy is normally near optimal (see Bearden and Connolly, 2007, 2008; Chun 2015).

Hogarth and Karelaia (2005) consider a similar model of parallel search based on deterministic elimination by aspects. Traits are assessed in decreasing order of importance. Offers that do not satisfy constraints based on successive traits are eliminated until either only one offer is left (which is then chosen), none of the offers satisfy the current constraint or all traits have been considered. In the final two cases, choice is made at random from the final non-empty set of offers.

Analytis et al. (2014) presents a similar model to the one presented here. There are two rounds of inspection. In the first round (parallel search), offers are ranked on the basis of an initial signal. In the second round (sequential search), the DM closely observes offers starting with the highest ranked and stops when the value of an offer exceeds the expected reward from future search. To realise such a strategy, the values of the offers must be observed and deriving the optimal strategy requires knowledge of the distribution of the value of an offer given the signal observed in the first round.

The model considered here assumes that search is parallel in both rounds. In the first round, a fixed number, n, of offers are ranked according to an initial signal. The k most highly ranked offers are then investigated more closely, after which the currently highest ranked offer is accepted. The payoff obtained is assumed to be a function of the values of the signals (or the rank according to this function) minus the search costs incurred. To realise a strategy of this form, the DM must be able to rank the observations according to the signals observed. Derivation of the optimal strategy requires knowledge of the joint distribution of the signals. Future research will investigate how robust a given strategy is to changes in the parameters (distribution of the signals, search costs). The search costs are assumed to be convex in the length of the short list. This reflects the fact that both the time spent searching and the cognitive effort involved are increasing in the length of the short list.

Section 2 considers the model. Section 3 presents a general result regarding the optimal length of a short list. Numerical results for three examples are presented in section 4. Section 5 gives a discussion of these results and section 6 presents some conclusions and directions for future research.

2 A model of search using short lists

A decision maker (DM) must choose one of n offers. The DM first observes in parallel a signal of the value of each offer. Assume that a linear ranking (from 1 to n) can be assigned to these initial signals (the rank i corresponds to the i-th best offer). This ranking will be called the initial ranking. The strategy of the DM is defined by the length of the short list, k, where $1 \leq k \leq n$. When $1 < k < n$, then in the second round the DM observes another signal of the value of the k best offers from the initial ranking. If $k = 1$, then the DM automatically chooses the best offer according to the initial ranking without observing the additional signal. If $k = n$, then the DM observes all of the offers closely. It is assumed that given the DM closely observes all the offers, then he/she can assign a linear ranking to these offers based on these signals combined. This ranking will be called the overall ranking. Naturally, after closer inspection, the DM can only rank the k offers on the short list with respect to each other (the DM's partial ranking). Assume that this partial ranking is in accordance with the overall ranking, i.e. offer i is ranked more highly than offer j in a partial ranking if and only if i is ranked more highly than offer j in the overall ranking. For convenience, it is assumed that if a DM is using a short list of length k, then the partial ranking of any offer that is not included in the second round of inspection is $k + 1$.

Note that any method for ranking based on multiple criteria (e.g. TOPSIS, see Yoon and Hwang, 1995) may be used to rank offers according to the initial signal and then rank the offers on the short list according to both signals. Such an approach will be adopted in the future.

Denote the set of permutations of (1, 2,…, n) by S. Let $\pi = (a_1, a_2, \ldots, a_n) \in S$. Such a permutation can be used to denote the initial ranking of the offers according to their overall rankings such that the offer ranked i overall has ranking a_i in the initial ranking. Let $p(\pi)$ be the probability that the initial ranking is given by π and define $p_i(j)$ to be the probability that the overall rank of an offer is i given that its initial rank is j. It follows that

$$p_i(j) = \sum_{\pi:\, a_i=j} p(\pi). \tag{2}$$

Let R_j denote the random variable describing the overall rank of the offer with initial rank j. Assume that when $i < j$, then R_i is stochastically dominated by R_j, i.e. if object i has a better rank than object j based on the initial signal, then offer i is expected to have a better rank than offer j overall.

For the purposes of this article, it will be assumed that each offer can be described by a pair of random variables, X_1 and X_2, which come from a continuous joint distribution. The random variable X_1 describes the signal observed on initial inspection (the initial ranking is based on this signal) and the random variable X_2 describes the signal received after closer inspection. Note that X_1 and X_2 may be correlated with each other, but the pairs of numbers describing each offer are independent realizations from this joint distribution. The overall ranking of the object is based on U, where $U = X_1 + X_2$. It is assumed that U is stochastically increasing in the value of X_1 (this is automatically satisfied when X_1 and X_2 are independent).

We consider two models. According to Model A, the value of an offer to the DM is a non-increasing function of its overall rank. Under Model B, the value of an offer to the DM is given by U. It should be noted that the DM does not observe the numerical values of these signals, and hence does not observe the value of an offer, but can rank all the offers according to the initial signal and rank the offers that are closely inspected according to their value (the partial ranking). In other words, it is assumed that the DM can make perfect pairwise comparisons between objects, e.g. if $i < j$, then an offer of partial rank i has a greater value to the DM than an offer of partial rank j. Define W_i to be the value of the offer with initial rank i. Note that when $i < j$, then under both models R_i and W_j are stochastically dominated by R_j and W_i, respectively.

The DM's goal is to maximize the expected reward from search, which is defined to be the value of the offer accepted minus the search costs. Under Model A, the expected value of the offer accepted should be calculated with respect to the distribution of the overall rank of an offer given that its partial rank is equal to one. Under Model B, the expected value of the offer accepted should be calculated with respect to the distribution of U given that the partial rank of the offer is equal to one. The following two problems appear with regard to these assumptions. Firstly, calculation of the expected value of an offer given that its partial ranking is equal to one can be highly complex. Although in the particular cases of $k = 1$ and $k = n$, such calculations may be tractable, the results presented here are derived from simulations. Secondly, although the DM aims to maximize the expected reward from search, the DM cannot actually measure this reward. In order to solve this problem, it is assumed that the reward from search is a measure of the utility of an individual from the search procedure. Additionally, such rules are used by a population in which individuals copy/learn search procedures that are seen to be successful and/or the reproductive success of individuals depends on their success in such search procedures. Under such assumptions, the search rules that evolve should be near optimal in terms of the expected reward from search given the limits on the perceiptive abilities of the searchers.

It is intuitively clear that a short list of length k should consist of the k highest ranked offers from the initial observations. This results from the fact that the distribution of the reward obtained by selecting from these offers stochastically dominates the reward obtained by selecting from any other set of k offers given the initial ranking. The search costs are split into the costs of initial inspection and costs of closer inspection. The costs of initial inspection, given by $f_1(k, n)$, are strictly increasing in both the number of offers and the length of the short list, n and k respectively. These costs cover the effort needed for initial inspection of the offers and maintainence of the short list. Also, assume that f_1 is convex in k, i.e. $f_1(k, n) - f_1(k-1, n)$ is non-decreasing in k. This reflects the cognitive effort required to control short lists of long length. This is a simplification, since when $k = n$ the DM automatically inspects all the offers closely and hence, in this case, the search costs should not include the costs of controlling a short list.

The costs of closer inspection of the offers on the short list, given by $f_2(k)$ are assumed to be increasing and convex in length of the short list. Note that it may be natural to assume that the costs of closer inspection are linear in k (when $k \geq 2$, then each successive offer on the short list is inspected and compared to the best ranked of previously inspected offers). Let $f(k, n) = f_1(k, n) + f_2(k)$ denote the total search costs and $C_k = f(k, n) - f(k-1, n)$ denote the marginal costs of increasing the length of the short list from $k - 1$ to k.

3 Form of the optimal strategy

Let V_k be the value of the offer accepted when the length of the short list is k, i.e. $V_k = \max_{1 \leq i \leq k} W_k$. It follows that when $i < j$, then V_i is stochastically dominated by V_j. Denote by M_k the marginal increase in the expected value of the offer accepted when the length of the short list is increased from $k - 1$ to k, i.e. $M_k = E[V_k - V_{k-1}]$.

The criterion determining the optimal length of the short list is based on the following theorem:

Theorem 1
The marginal increase in the expected value of the offer accepted, M_k, is non--increasing in k.

Proof
By definition
$M_k = E[\max\ \{0, W_k - V_{k-1}\}]; M_{k+1} = E[\max\ \{0, W_{k+1} - V_k\}]$.

The fact that $M_k \geq M_{k+1}$ follows directly from the facts that W_k stochastically dominates W_{k+1} and V_k stochastically dominates V_{k-1}. ■

Corollary
The optimal length of the short list, k^*, satisfies the following condition:

i) when $M_2 \leq C_2$, then the optimal strategy is to automatically accept the object with initial ranking 1, i.e. $k^* = 1$ and none of the objects are inspected closely.

ii) when the above condition does not hold, then k^* is the largest integer k, such that $k \leq n$ and $M_k > C_k$.

This corollary follows from the fact that M_k is non-increasing in k and C_k is non-decreasing in k. Hence, when the first condition holds, then the marginal costs of increasing the length of the short list always exceed the marginal gain. When the first condition does not hold, then for all $k \leq k^*$, it follows that $M_k > C_k$ and for $k > k^*$, then $M_k \leq C_k$. Hence, the DM always gains by increasing the length of the short list when $k < k^*$, but when $k \geq k^*$the costs of increasing the length of the short list always outweigh the gains. Thus the optimal length of the short list is given by k^*. Note that when $M_k = C_k$, then the DM is indifferent between using a short list of length $k - 1$ and using a short list of length k. The condition given above assumes that when the optimal length of the short list is not unique, then the smallest length from the set of optimal lengths is used.

4 Examples

In this section, we consider three examples.

4.1 Example A: Independent signals

Suppose X_1 and X_2 are independent random variables from the N(0, 1) and the N(0, σ^2) normal distributions, respectively, where the first parameter denotes the mean and the second parameter the variance. The value of an offer is given by

$U = X_1 + X_2$ and thus has a normal distribution with mean 0 and variance $1 + \sigma^2$. The DM makes an initial ranking of the offers based on X_1. The best k offers according to this initial ranking are then observed more closely and the DM then accepts the offer with the highest value of U from these offers. The search costs associated with initial inspection and maintanence of the short list are $f_1(k, n) = 0.0001(n + k^2)$ and the costs of closer inspection are $f_2(k) = c\sigma$, where c is a constant. The choice of the cost parameters should reflect the logic that strategies based on short lists should be successful when the costs of initial observation are low relative to the costs of closer inspection. The costs of close inspection are assumed to be proportional to the standard deviation of the second signal, since under sequential search based purely on the second signal the expected number of offers that are seen when c is fixed is independent of σ (see Ramsey, 2015). Hence, any changes in the optimal length of the short list when σ increases and c is fixed result from the amount of information contained in the second signal relative to the information contained in the first signal (as σ increases, the importance of the second signal compared to the first signal increases).

4.2 Example B: A best choice problem

The goal of the DM is to choose one of the best two offers overall. The initial rankings are based on X_1 (as defined in Example A) and the overall rankings of the offers are based on $U = X_1 + X_2$. The best offer overall is accepted if and only if it appears on the short list. The second best offer overall is accepted if and only if this offer appears on the short list and the best offer overall does not appear. The value of the second best object overall relative to the best object is r, where $0 \leq r \leq 1$. No reward is obtained by accepting any other object. Note that when $r = 0$, this problem is similar to the classical secretary problem (see Gilbert and Mosteller, 1966).

The value of the best object overall is defined such that the variance of the values of the offers in the second example is equal to the variance of U in the corresponding version of the first example. Hence, if the reward obtained by accepting the best offer overall is s, then

$$s^2(1 + r^2)\left(\frac{1}{n} - \frac{1}{n^2}\right) = 1 + \sigma^2. \tag{3}$$

This scaling is done in order to make the values of the offers and the search costs comparable for corresponding realizations of Example A and Example B. This enables comparison of the length of the optimal short list when the distribution of the values of the offers are non-skewed (Example A) and very highly skewed (Example B).

Obtaining analytical results for either of these problems is very difficult. In the first example, calculation of the marginal gains from increasing the length of the short list requires knowledge of the distribution of $W_k - V_{k-1}$. In the second example, this calculation requires knowledge of the distributions of the overall rank given the initial rank. Hence, the optimal length of the short list was obtained empirically on the basis of 100 000 simulations written in the R language of the search procedure when the number of offers $n \in \{20, 50, 100, 200\}$. These empirically obtained optimal lengths are described in Tables 1 and 2 (for relatively low and relatively high costs of close inspection, $c = 0.02$ and $c = 0.1$, respectively).

Several phenomena are visible from these results. Firstly, as the proportion of the information about the value of an offer contained in the second signal increases, the optimal length of the short list increases. Secondly, given the variance of the value of offers, the more skewed this distribution, then the greater the optimal length of the short list (it is shortest when the value comes from a normal distribution and greatest when a reward is obtained only when the best offer overall is accepted). Thirdly, as the costs of closer inspection relative to the amount of information contained by the second signal increase, then the optimal length of the short list decreases (as would be expected). Fourthly, when the values of offers come from a normal distribution, then the optimal length of the short list is almost independent of the total number of offers, particularly when the costs of closer inspection are relatively large.

In the best choice problems (Example B), the optimal length of the short list increases as the total number of offers increases. Due to the scaling of the value of the best offer according to the number of offers available, as n increases then the value of the best offer increases and it is often worthwile to carry out exhaustive search of all the offers in order to ensure obtaining the best of all the offers, especially when the costs of closer inspection are relatively low and the second signal contains a large proportion of the information regarding the ranking of an object. One interesting aspect is that when the costs of closer inspection are high and the total number of offers is large, then the optimal length of the short list can actually fall as the amount of information contained in the second signal increases. In such cases, it seems likely that the search costs required in order to ensure obtaining the best offer with a large probability become prohibitive and thus the intensity of search falls (the optimal length of the search list falls). This conclusion seems to be confirmed by the behaviour of the optimal length of the short list when the value of the best offer is not scaled according to the number of offers, i.e. is always equal to 1. In this case, as the amount of information in the second signal increases, the optimal length of the short list initially increases before falling to 1 and the optimal expected reward from search becomes negative. This indicates that when the number of offers becomes large and the amount of information contained in the first signal is small, then it does not pay to search for the best offer, since the search costs become too large.

Table 1: Optimal lengths of short lists for relatively low costs of close inspection ($c = 0.02$). The numbers in each cell give the optimal lengths of the short list for Example A, Example B with $r = 1$ and Example B with $r = 0$, respectively

σ	n=20	n=50	n=100	n=200
1/5	(2, 3, 5)	(3, 4, 7)	(3, 5, 9)	(3, 6, 10)
1/4	(3, 4, 6)	(3, 5, 8)	(3, 6, 10)	(3, 7, 12)
1/3	(3, 4, 7)	(4, 6, 10)	(4, 7, 14)	(5, 9, 16)
1/2	(5, 6, 9)	(5, 8, 15)	(6, 10, 20)	(5, 14, 25)
1	(6, 9, 15)	(7, 15, 27)	(9, 22, 39)	(9, 30, 56)
2	(10, 13, 20)	(11, 25, 42)	(14, 41, 67)	(14, 60, 106)
3	(12, 15, 20)	(15, 30, 49)	(14, 52, 89)	(15, 85, 148)
4	(14, 16, 20)	(15, 36, 50)	(16, 58, 96)	(18, 92, 168)
5	(14, 17, 20)	(16, 38, 50)	(17, 64, 99)	(19, 103, 187)

Source: Based on simulations written by the author in R.

Table 2: Optimal lengths of short lists for relatively high costs of close inspection ($c = 0.1$). The numbers in each cell give the optimal lengths of the short list for Example A, Example B with $r = 1$ and Example B with $r = 0$, respectively

σ	$n = 20$	$n = 50$	$n = 100$	$n = 200$
1/5	(2, 3, 4)	(2, 3, 5)	(2, 4, 7)	(2, 5, 8)
1/4	(2, 3, 5)	(2, 4, 6)	(2, 4, 7)	(2, 5, 9)
1/3	(2, 3, 5)	(2, 4, 7)	(2, 5, 9)	(2, 6, 11)
1/2	(2, 5, 7)	(3, 6, 10)	(3, 7, 12)	(3, 9, 17)
1	(3, 6, 10)	(3, 9, 16)	(4, 12, 23)	(4, 18, 32)
2	(4, 8, 14)	(4, 14, 22)	(4, 19, 29)	(5, 27, 45)
3	(4, 9, 17)	(5, 17, 29)	(5, 23, 38)	(5, 32, 43)
4	(5, 10, 19)	(5, 17, 31)	(5, 26, 38)	(5, 30, 41)
5	(5, 11, 20)	(5, 18, 36)	(5, 23, 42)	(5, 35, 35)

Source: Based on simulations written by the author in R.

Note that due to the nature of the simulations, the empirically derived optimal lengths of short lists show some fluctuation around what would be the theoretically optimal length. This is particularly true when the optimal length of the short list is large (in such cases, slightly changing the length of the short list from the optimal length has very little impact on the expected reward). The expected reward from search in Example A as a function of the length of the short list is presented in Figure 1 for the case $n = 200, c = 0.02, \sigma = 1$.

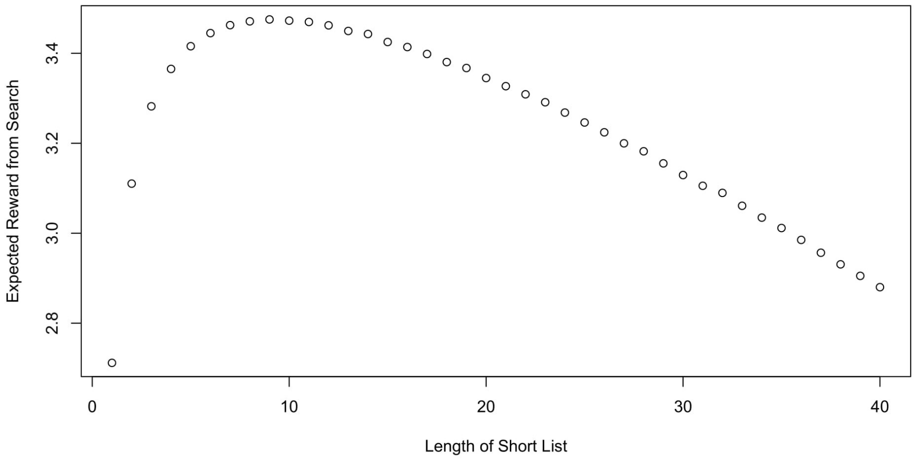

Figure 1: Expected reward as a function of the length of the short list for Example A when the number of offers is large, the costs of closer inspection relatively small and the initial and second signal contain the same amount of information ($n = 200, c = 0.02, \sigma = 1$)

4.3 Example C: Correlated signals

Since the signals indicating the value of an offer may be correlated, it is assumed that the value of an offer is based on two signals (X_1, X_2) from a bivariate normal distribution. X_1 comes from a standard normal distribution, the coefficient of correlation between X_1 and X_2 is ρ and the residual variance of X_2, i.e. the variance in X_2 that is not explained by X_1, is σ^2. Hence, given X_1, X_2 has a normal distribution with mean ρX_1 and variance σ^2. The overall variance of the signal X_2 is $\frac{\sigma^2}{1-\rho^2}$. As in Example A, the value of an offer is $U = X_1 + X_2$. From these assumptions, $E(U) = 0$ and

$$\begin{aligned} Var(U) &= Var(X_1) + Var(X_2) + 2\rho\sqrt{Var(X_1)Var(X_2)} \\ &= 1 + \frac{\sigma^2}{1-\rho^2} + \frac{2\rho\sigma}{\sqrt{1-\rho^2}}. \end{aligned} \quad (4)$$

It can be shown by differentiation that this variance is increasing in ρ for $\rho > 0$.

The search costs are defined as in Example A, i.e. the costs of closer inspection are proportional to the residual variance of X_2. On one hand, the increase in the overall variance of the offer favours more intense search (a longer short list). On the other hand, observing X_1 gives us information about the value of the second signal (hence, all other things being equal, as ρ increases X_1 contains relatively more information about the value of an offer). This effect favours short lists with fewer items. The optimal lengths of short lists were derived empirically for $\rho \in \{0.2, 0.4, 0.6, 0.8\}$ on the basis of 100 000 simulations, written in R, of the search process. Table 3 presents the empirically obtained optimal lengths of short lists when the relative costs of closer inspection are $c = 0.1$.

Table 3: Optimal lengths of short lists for relatively high costs of close inspection ($c = 0.1$) and correlated signals (Example C). The numbers in each cell give the optimal thresholds for $\rho = 0, 0.2, 0.4, 0.6$ and 0.8, sequentially

σ	$n = 20$	$n = 50$	$n = 100$	$n = 200$
1/5	(2, 2, 2, 2, 2)	(2, 2, 2, 2, 2)	(2, 2, 2, 2, 2)	(2, 2, 2, 2, 2)
1/4	(2, 2, 2, 2, 2)	(2, 2, 2, 2, 2)	(2, 2, 2, 2, 2)	(2, 2, 2, 2, 2)
1/3	(2, 2, 2, 2, 2)	(2, 2, 2, 2, 2)	(2, 2, 2, 2, 2)	(2, 2, 2, 2, 2)
1/2	(2, 2, 2, 2, 2)	(3, 3, 2, 2, 2)	(3, 3, 2, 2, 2)	(3, 3, 2, 2, 2)
1	(3, 3, 3, 2, 2)	(3, 3, 3, 3, 2)	(4, 3, 3, 3, 3)	(4, 3, 3, 3, 3)
2	(4, 4, 3, 3, 2)	(4, 4, 4, 3, 3)	(4, 4, 4, 3, 3)	(5, 4, 4, 3, 3)
3	(4, 4, 3, 3, 3)	(5, 4, 4, 3, 3)	(5, 4, 4, 3, 3)	(5, 5, 4, 4, 3)
4	(5, 4, 4, 3, 3)	(5, 4, 4, 3, 3)	(5, 5, 4, 4, 3)	(5, 4, 4, 4, 3)
5	(5, 4, 4, 3, 3)	(5, 5, 4, 3, 3)	(5, 4, 4, 4, 3)	(5, 5, 4, 4, 3)

Source: Based on simulations written by the author in R.

Table 3 indicates that when the residual variance of the second signal is fixed, there is a tendency for the optimal length of the short list to fall, especially when the residual variance of the second signal is relatively large. Hence, the increase in the overall variance in the value of an offer is outweighed by the fact that X_1 explains an increasing proportion of the variance in the value of an offer. Note that if the (overall) variances of the two signals were fixed, then this negative relationship of the coefficient of correlation with the optimal length of the short list would be stronger.

5 Discussion of the results from the simulations

Although a large number of articles have been recently published on the concept of short lists in decision making, there has been little work on models which indicate what the optimal length of a short list should be depending on the parameters of a search problem. Such an approach to decision making seems very fruitful in the Internet age, since when searching for a valuable resource in a large market, basic information about offers can be found at very little cost via the Internet. However, DMs should investigate promising offers more closely, before making a final decision. The model presented here illustrates the qualitative properties of optimal strategies based on short lists.

In practical terms, if there are a large number of candidates for a given position, it seems more reasonable to assume that the reward obtained by a real life DM will depend on the intrinsic value of an offer, rather than whether the offer accepted is the best offer or not. Also, the signals associated with an offer may be correlated. Hence, for practical purposes the results from Example C, where the two signals observed are correlated, are the most instructive. Of course, independent signals can be treated as a particular case within this framework.

The results indicate that the optimal length of the short list is only very weakly dependent on the total number of offers available. On the other hand, in such problems the optimal length of the short list is increasing in the importance of the second signal relative to the first signal. Although it may be difficult for a DM to estimate the total number of offers before the search procedure is realized, in general the DM will know the type of information gained in the two stages of the search procedure and be able to estimate the weight of the information gained at both stages. For example, procedures applied by employers to search for a specialist employee might be one interesting practical example of the use of short lists. The short list is made on the basis of written applications. The cost of collecting these applications and creating a short list are assumed to be small compared to the value of the employee. The members of this short list are invited for interview. Since the employer often covers the travel costs of the interviewees and interviews occupy a significant amount of time, the relative costs of closer inspection are high. In such problems, the rule of thumb "invite four or five candidates for interview and then offer the position to the best of these candidates" is often used. The results from this paper indicate that such rules are optimal or close to optimal when a) the values of employees come from a non-skewed distribution, b) the DM obtains a similar amount of novel information from the initial signal (written application) and closer inspection (interview), c) the costs of closer inspection are high compared to the costs of initial inspection. For example, when the two signals contain the same amount of information and the number of offers is large ($n = 200$) and $c = 0.1$ (high costs of closer inspection), the optimal length of the short list is 4. When $c = 0.02$ (low costs of closer inspection), the optimal length of the short list is 9. However, using a short list of length 4 ensures an expected payoff that is not much lower than the optimal payoff (see Figure 1). Hence, using short lists of moderate length ensures at least near-optimal rewards over a wide range of parameter sets.

Fixing the marginal variances of the signal, the optimal length of the short list tends to decrease as the coefficient of correlation ρ, between the signals becomes larger, particularly when the overall variance of the second signal is large. The overall variance of the value of an offer is increasing in ρ, which might lead to an increase in the overall search effort (i.e. a longer short list). However, this effect is more than counteracted by the fact that the weight of the initial signal as an estimate of the overall value of an offer becomes greater. In terms of searching for a skilled employee, e.g. if the verbal skills of a candidate are highly correlated with the quality of the candidate as observed in a written application, then it is only necessary to interview a small number of candidates.

It should be noted that these results also indicate that the greater the skew in the distribution of the values of offers, then the greater is the optimal length of a short list. However, this should be investigated in more detail by considering signals with different distributions.

6 Conclusions and directions for future research

There are obvious weaknesses in this approach, as outlined below.

Firstly, it is assumed that a linear ranking of offers can be defined according to a) initial information, and also on b) initial information combined with information from closer inspection. This implicitly assumes that any pairwise comparison of offers always results in one offer being preferred to another and the results of such comparisons are always correct. In practice, comparison of offers based on the initial information may be difficult since it embraces multiple traits, e.g. when searching for a new flat, Internet sites will give quantitative information about price, floor space, number of rooms and location (e.g. distance from city centre). Hence, future work will concentrate on how such multivariate information can be used to create a short list. Approaches to multicriteria decision making, such as TOPSIS or AHP, could be used to create a short list (see Yoon and Hwang, 1995). Due to the ever increasing availability of information and the speed with which computers can analyze data, it seems that methods of automating at least the initial stage of such a search procedure would be beneficial. For example, the database of available flats in a city may be too large for efficient manual handling. Hence, it seems natural that an algorithm could be developed to suggest a short list of flats to view based on the stated preferences of the DM regarding price, floor space etc. This would decrease the search costs in the first round. However, there may exist a conflict of interest between the goals of the writer or implementer of the algorithm (a seller or intermediary) and the DM (e.g. Skulimowski, 2017). In this case, it would be natural to examine game theoretic models of such search procedures. Future research will model such algorithms and investigate their effectiveness in various types of problem (e.g. computer aided search procedures for selecting a short list of flats or second-hand cars to view).

Also, future models should investigate problems related to bounds on the ability of a DM to compare offers (the phenomena of inconsistent comparisons and the inability to state a preference between two offers). Similar phenomena also appear when comparing the offers on the short list after the round of closer inspection.

The assumption that the costs of maintaining the short list are convex in the length of the short list seem reasonable, due to the increased cognitive effort required to control longer short lists. However, the costs of initial observation and maintaining the short list should be modelled more formally. This might be done by defining the costs of initial observation according to the number of pairwise comparisons required. Such an approach will be used in future research. Also, when the length of the short list is large in comparison to the total number of offers, it is cognitively simpler to carry out an exhaustive search of all the offers as this only requires remembering the best offer seen so far on the basis of closer inspection. Future research should compare the expected reward under the optimal short list policy with the expected reward obtained using exhaustive search based on an appropriate model.

The optimal short list policy should also be compared with search based purely on the initial signal. Finally, comparison with the optimal reward obtained in the corresponding sequential decision problem where the search costs are linear in both the number of offers observed and the number of offers inspected closely (see Ramsey, 2015) will give a measure of the efficiency of short list rules.

The model does not take into account various factors that might be relevant in real-life decision problems. For example, a DM might feel, after closely inspecting the offers on the short list, that none of the offers are suitable and thus the search procedure should be repeated. Also, the price might be negotiable or the offer chosen after close inspection might become unavailable before the DM can make a decision. How should such factors be modelled? Also, the decision to purchase a valuable good is often made by households rather than individuals. Hence, models of group decision making or game theoretic models should be considered.

In practice, there is great uncertainty regarding the parameters of a problem. In order to determine the optimal length of the short list, the model presented here assumes that the DM should know a) the distribution of the value of the offers, b) the proportion of the value of an offer described by the first signal, c) the total number of offers, d) the search costs. There will be uncertainty regarding these parameters, which might be modelled using probability theory or fuzzy theory.

In conclusion, the frequency with which procedures based on short lists are applied when humans make decisions indicate that such heuristic policies are successful. Future research should investigate how the perceptive abilities of searchers can be supported using automatic systems designed, for example, to search a large database of offers in order to suggest a short list of offers appropriate for close inspection.

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